UNIVERSITY OF EDUCATION, WINNEBA

JUNIOR HIGH SCHOOL MATHEMATICS TEACHERS' PERCEIVED KNOWLEDGE OF PROBLEM-SOLVING AND PRACTICES OF TEACHING MATHEMATICS THROUGH PROBLEM-SOLVING IN THE TAMALE METROPOLIS

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DECLARATION

Student's Declaration

I, Issah Alhassan declare that this dissertation, the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole for another degree elsewhere.

SIGNATURE :………………………………………………

DATE:………………………………………………………..

Supervisor's Declaration

I hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of thesis as laid down by the University of Education, Winneba

NAME OF SUPERVISOR: DR. FRIMPONG ALI SYLVESTER

SIGNATURE :………………………………………………

DATE:………………………………………………………..

DEDICATION

This thesis is dedicated to the Almighty God.

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ABSTRACT

This study focused on Public Junior High School (JHS) mathematics teachers' perceived knowledge about problem-solving, the extent to which mathematics teachers employed problem-solving strategies, the engagement of pupils through problem-solving and the challenges they encounter when teaching through problem-solving. The sequential explanatory research design was adopted for this research. The sample of the study consisted of eighty-six JHS mathematics teachers in the Tamale metropolis. Questionnaire and interview guide were used for the data collection. Means, standard deviations and percentages were utilised for the quantitative data. Thematic analysis was done on the qualitative data after it was transcribed. The results demonstrated that even though JHS mathematics teachers have good knowledge about problem-solving and they moderately used problem-solving instructional strategies in their lessons and highly used task-based instruction and cooperative learning strategies. Who highly engaged learners and moderately applied manipulative materials other than the standard procedures espoused in problem-solving techniques. Lack of TLMS, little content knowledge of problem-solving, large class size, teacher workload and little time allocation for mathematics and lack of motivation were identified as challenges encountered when teaching through problem-solving. Therefore, the researcher recommends a new paradigm of training teachers on problem-solving strategies for public JHS mathematics instruction. For mathematics teachers to be able to integrate problem-solving into their lessons, it is recommended that TLMS are made available to the teachers.

CHAPTER 1

INTRODUCTION

1.0 Overview

This chapter discusses the background of the study, the statement of the problem, the purpose of the study, the objectives of the study, the research questions, the significance of the study, limitations and delimitations, and the organizational plan of the study.

1.1 Background

The importance of mathematics education cannot be overstated. The importance of mathematical knowledge in this technological time is recognised globally, for it is a tool for developing a rational personality (Kavkler, Magajna, & Babuder, 2014). Problem-solving is widely acknowledged as a pivotal skill that nurtures critical thinking, logical reasoning, and the practical application of mathematical principles (Polya, 1945; Schoenfeld, 1985). Mereku (1992) believes that the subject occupies a privileged position in the school curriculum because the ability to cope with more of it improves one's chances of social development.

Undoubtedly, mathematics education is of essence and it is not surprising that in our modern society, there are growing need for mathematical skills and proficiency because students must master advanced skills to stay on track for promising careers (Njagi, 2015). The desire among educators to see pupils becoming mathematically competent is therefore of immense relevance though this quest remains far-fetched. From my years of experience as a mathematics teacher, it is evident that effective curriculum delivery yields numerous advantages for learners. Enhancing students' reasoning skills, fostering critical thinking abilities, and honing problem-solving competence are among the array of benefits that stem from a comprehensive study of mathematics. Legner (2013) supports the notion that a foundational grasp of arithmetic, which is essential for many professions, is generally acquired by the end of primary school. Hence, underscoring the necessity of prioritising the thorough teaching and learning of its core concepts. Building upon this perspective and the research findings of other scholars (e.g Kavkler, Magajna, & Babuder, 2014) who have systematically revealed the advantages of the subject, it is reasonable to assert that furnishing young students with a robust understanding of mathematics remains paramount. This responsibility should be embraced by all educators in the field. Advancing the quality of mathematics education in schools, especially at the pre-tertiary level, has evolved into a worldwide concern over the past three decades.

Consequently, multiple countries have undergone substantial revisions to their mathematics curricula. A major principle in many of these contemporary curricula involves the transition from a teacher-centred approach to a more student-centred approach. For instance, as exemplified by Mosvold (2005), Norway took significant steps to enhance mathematics education by introducing a new mathematics curriculum in 1997. This curriculum places substantial emphasis on student-centred teaching methods, aiming to establish tangible connections between school mathematics and real-life encounters for students. Correspondingly, Chambers (2008) discloses that England introduced a national mathematics curriculum in the 1980s due to subpar mathematics achievement among students and the pressing need for improved pedagogical methodologies. This initiative involved outlining diverse teaching and learning strategies within these mathematics syllabi to foster students' comprehensive grasp of mathematical concepts. Problem-solving is perceived as addressing everyday challenges that compel the solver to employ their forecasting and analytical abilities (Pertersen, 2016). In such scenarios, the problem solver must utilize prediction and analysis to arrive at a solution. Within the realm of mathematics education, problem-solving assumes a significant role. Problem-solving affords students opportunities to engage in meaningful mathematical discussions, involving the scrutiny of diverse representations and justifications for their solutions. Instructional practices centred around problem-solving empower students to become active participants in the learning process and encourage educators to be actively engaged in the classroom. Within this context, Bay (2000) expounds on teaching through problem-solving as an avenue through which mathematics instructors can have a substantial instruction. Elaborating on this perspective, the researcher explains that teaching through problem-solving entails applying mathematical content via problem-solving strategies and employing suitable tools. Students foster, extend, and enrich their comprehension through problem-solving endeavours (Hieber & Wearne, 2003).

Teaching through problem-solving equips learners for a life enriched with adeptness. Moreover, it contributes to enhancing students' self-assurance as adept problem solvers and cultivates their readiness to embrace mathematical challenges (Tratton & Midgett, 2001). Furthermore, Van de Walle (2007) observes that adopting a problem-solving approach to teaching mathematics engages students comprehensively in vital mathematical learning. This signifies that problem-solving underpins all mathematical tasks and functions as a versatile skill. Its pertinence extends to lifelong learning, demanding independent thinking and critical evaluation of matters. Hence, nurturing mathematical problem-solving abilities in students during their foundational education holds significance, indicating their readiness for both professional pursuits and life challenges. In Ghana, the issue of teachers lacking the requisite knowledge and competencies to employ effective problem-solving techniques in mathematics instruction persists (Mereku, 2015) and contends that teachers are falling short in

Aiding students' development of problem-solving capabilities within Ghanaian basic schools. To enhance these techniques, educators need to integrate the fundamental principles of teaching through problem-solving into Polya's model. This model signifies a strategy through which mathematics instructors deliver more profound mathematics instruction. In this instructional approach, students are encouraged to build, expand, and enhance their comprehension through problem-solving endeavours (Hiebert, 2003). This practice fosters students' confidence and proficiency in mathematics. Cai and Lester (2012) observed that instructing through problem-solving facilitates the comprehension of concepts, cultivates reasoning abilities, and enhances mathematical communication, while also fostering an interest and curiosity in mathematics. According to Polya's model, problem-solving can be regarded as an endeavour encompassing a range of skills. Problem-solving is envisioned as a practical art, involving both teaching and learning. It centres on a child's capability to employ prior knowledge and apply it in new situations (Polya, 1945). The child's aptitude to recall fundamental arithmetic skills, comprehend when and how to incorporate them into new scenarios, and execute such actions represents three distinct competencies. Although a child might possess all three skills that facilitate problem-solving, an absence of proficiency in one skill doesn't necessarily imply a lack of understanding of the problem.

Rather, it could indicate that the child's preferred learning style has not been effectively accommodated (Polya, 1954). Likewise, the mere ability of a child to execute isolated procedures doesn't necessarily indicate their capacity to apply or interpret the involved numbers (Bley & Thornton, 2001). Crafting effective problems involves tailoring them to cater to the varying skill levels, capabilities, and learning preferences of students.

Consequently, educators are encouraged to embrace diverse solutions and techniques within the classroom to foster problem-solving abilities.

Furthermore, in light of the global technological and scientific progress, Ghanaian learners must be guided beyond superficial comprehension and rote memorization of facts and formulas to become future problem solvers. Consequently, teachers need to be adequately equipped to nurture advanced mathematical thinking abilities in their students. The development and deepening of students' problem-solving skills significantly rest on the educator's shoulders. Specifically, teachers must make judicious choices, encompassing the selection of engaging learning materials, appropriate teaching methodologies, the creation of conducive learning environments for exploration, prudent decision-making to mitigate risks, and the sharing of both failures and successes from real-world applications. The traditional role of teachers, serving as mere sources of knowledge and authority, undergoes a substantial transformation in the context of developing problem-solving skills in mathematics. Teachers role evolves into that of guides and facilitators. Proficiency in problem-solving has diverse applications across numerous domains of human endeavour, including commerce, industry, and science. For this reason, problem-solving is advocated as a potent instructional approach in mathematics (Roberts, Sharma, Britton, & New, 2009). Teachers possess the capacity to equip students with problem-solving skills that empower them to address real-life problems, but this achievement hinges on aligning teaching practices with the problem-solving objectives outlined in the curriculum. While an analysis of the Junior High School Mathematics curriculum in Ghana indicates sufficient guidance for teachers to incorporate problem-solving into their instructional approach, it remains uncertain whether teachers universally interpret and implement these guidelines in a similar manner. Also, teachers limited knowledge of problem-solving influences how they apply it in the classroom when teaching mathematics.

1.2 Statement of the problem

All mathematical concepts and ideas should be imparted to students through hands-on activities and the guidance of teachers, enabling students to eventually grasp these concepts independently. The primary aim of teaching and learning mathematics is to equip students with the skills to tackle diverse and complex mathematical problems and to apply mathematical principles in real-world situations.

The educational curriculum in Ghana emphasises the importance of incorporating mathematics into daily life by fostering the identification and application of appropriate problem-solving strategies (Ministry of Education, 2007). Mereku (2004) observed that more emphasis on teaching is based on the theoretical aspects, while equal attention is not dedicated to the practical aspects of teaching and that problem-solving appears unpopular among Ghanaian students.

In Ghana where this study carries out, pupils' attainment in mathematics is generally low (Mereku, 2003) and the area of non-routine problems is not an exception. Evidence of such poor performances is enshrined in various reports focussing on outcomes from national assessments National Education Assessment (NEA), Early Grade Reading (EGRA), and Early Grade Mathematics Assessment (EGMA), national examinations (BECE & WASSCE) including international examinations (Mereku, 2012). Debilitating standards and records of worrying grades attained by some BECE candidates, for instance, indicate that most students underperform in the subject area; a challenge mainly caused by a lack of basic concepts (Mills & Mereku, 2016).

Over the years, several concerned scholars, well versed in the subject area, have carried out extensive studies to find some solutions to the current predicament. For example, Mereku and Cofie (2008) have looked at how overcoming language difficulties can help minimise mathematical difficulties among students. Anamuah-Mensah, and Mereku (2005) also analysed the report of 2003 TIMMS report, highlighting some struggles and challenges educators have to pay attention to if learners have to do better on non-routine problems. Atteh, Andam, Obeng-Dneteh, Okpoti, and Johnson (2014) also looked at how the constructivist approach could enhance students' competence in problem-solving. Armah (2015) delved into problem-solving from the perspective of teachers' beliefs, intentions and behaviour. Nyala, Assuah, Ayebo, and Tse (2016) have also looked at the prevalent rate of problem-solving approaches in teaching mathematics in Ghanaian basic schools. Finally, Atteh, Andam, and Obeng-Denteh (2017) in their contribution to the enhancement of problem-solving approach looked specifically at a four-step framework to foster students' progress.

As the mathematics syllabus recommended the use of mathematics in daily life by recognizing and applying appropriate mathematics Problem-solving strategies (MOE, 2007). However, studies have shown that students are not able to solve non routine mathematical problems and that Problem-solving is unpopular among Ghanaian students (Mereku; 1998). The solutions to the students' assignments and their end-of-term examination scripts were carefully examined. The researcher realised that most Public Junior High School students in the Tamale Metropolis could not solve non-routine mathematical problems. Most students remember the routine techniques without understanding why and how they work and therefore find difficulties in interpreting their answers and it is also observed that mathematical skills are often lacking because students are not given enough problem-solving strategies.

Students' ability to use appropriate Problem-solving strategies to deal with new mathematical situations and the unpopularity of Problem-solving in schools is a reflection of how teachers conceive and practice Problem-solving in the classroom. Hence this study was designed to examine JHS mathematics teachers' conceptions of Problem-solving and how they practice Problem-solving in the Tamale Metropolis of Northern Region of Ghana.

1.3 Purpose of the Study

The purpose of the study is to examine Public Junior High School mathematics teachers' perceived knowledge of problem-solving and practices of teaching mathematics through problem-solving in the Tamale Metropolis.

1.4 Objectives of the Study

The following objectives were considered:

1. To find out mathematics teachers' perceived knowledge for teaching through problem-solving among public junior high schools in the Tamale Metropolis.
2. To establish the extent to which mathematics teachers employ problem- solving strategies in teaching among public junior high schools in the Tamale Metropolis.
3. To find out how often mathematics teachers engage junior high school pupils through Problem-Solving in the Tamale Metropolis.
4. To identify the challenges junior high school mathematics teachers, encounter when using problem-solving as a teaching strategy in the Tamale Metropolis.

1.5 Research questions

The study presented the following research questions to guide the study.

1. What is mathematics teachers' perceived knowledge for teaching through problem-solving among public junior high schools in the Tamale Metropolis?
2. To what extent do mathematics teachers employ problem-solving strategies in

teaching mathematics among public junior high schools in the Tamale Metropolis?

3. How often do mathematics teachers engage junior high school pupils through Problem-Solving in the Tamale Metropolis?
4. What challenges do junior high school mathematics teachers encounter when using problem-solving as a teaching strategy?

1.6 Significance of the Study

The study would benefit Junior High School mathematics teachers and students in the Tamale Metropolis, the Northern region and the nation. The findings of this study would contribute to the knowledge of literature in research on mathematics education and greater comprehension of the utilization of the problem-solving method in teaching mathematics. The findings of this study are useful to stakeholders and policymakers in education about teachers' problem-solving practices in the mathematics classroom. That is, future education policy formulation and directions can base decisions on the findings of the research. This study's result can guide curriculum developers in planning and designing problem-solving and enriching the mathematics curriculum for Ghanaian pre-university institutions. It can also serve as a basis for organizing professional development courses and in-service training programs for teachers teaching mathematics through problem-solving. In this context, the study would provide vital information for designing functional mathematics programs for teacher education institutions.

1.7 Delimitations

Although the study intended to focus on the mathematics teachers' perceived knowledge about problem-solving in the teaching and learning of mathematics in Ghana as a whole, it was delimited to only the Public Junior High School mathematics teachers' perceived knowledge about problem-solving in the teaching of mathematics in the Tamale Metropolis in the Northern region of Ghana. The delimitations of a study are those characteristics that arise from limitations in the scope of the study (defining the boundaries) and by the conscious exclusionary and inclusionary decisions made during the development of the study plan (Simon & Goes, 2013).

For reliability and representation of the outcome of the study, it would have been prudent to use a larger sample from the population than the targeted sample, but due to constraints such as finance and the ability to reach the entire population in time the study looked at the Public Junior High Schools in the Tamale Metropolis.

1.8 Limitations

The limitations of a study are those situations which are out of the control of the researcher. The researcher can do little or sometimes nothing to avert those situations. Simon and Goes (2013) defined limitations as those shortcomings, conditions or influences that cannot be controlled by the researcher that place restrictions on your methodology and conclusion. Some of the limitations which this study encountered are: some teachers were not willing to partake in answering the questionnaire as well as subjecting themselves to be interviewed.

1.9 Organisation of the Study

This research consists of five chapters. Chapter One gives a brief knowledge about the main content of the study. Specifically, the chapter consists of the introduction, problem Statement, research questions and objectives, the significance of the study, the Delimitations and Limitations, and organisation of the study. Chapter two consists of the theoretical framework and review of related literature on perceptions of problem-solving, problem-solving strategies, engagements in problem-solving and challenges in problem-solving instructions. The chapter will end with a summary of the literature.

Chapter Three describes the methods used in the conduction of the research. It comprises the research design, population of the study, sample size, sampling procedure, data collection instruments, pilot testing, administration of instruments and data analysis plan. The analyses of the results and findings from the study are discussed in the fourth chapter and finally Chapter Five contains summary of the main findings, conclusions drawn from the findings, recommendations of the study and suggestions for further study.

1.10 Definition of key terms

The following are definitions applicable to the study in the way that the study denotes them to mean:

Perceived knowledge: the views and understanding of problem-solving.

Junior High School (JHS): This is the pre-secondary education for 7th, 8th, and 9th grades.

Problem-solving (PS): It is what a person performs in order to accomplish a specific objective without being aware of the solution approach beforehand.

Problem-solving strategy: A technique used to deliver lessons in a meaningful, contextualized, personalized, and real-world manner. By giving students sound educational guidance and insightful instruction, a problem-solving strategy seeks to assist students in developing their problem-solving skills.

Routine problem: A problem that can be solved by applying an already-known process, computational strategies, making use of formulas and only needing a single step.

Non-routine problem: A problem requiring the solution method to be ascertained as a component embedded within the problem's process. It exists when one does not have a clue, or has relatively little information, regarding the solution process and is unable to see the solution because it is not conspicuous (Mayer & Hegarity, 1996).

Learning engagement: active participation and involvement in learning activities.

CHAPTER 2

LITERATURE REVIEW

2.0 Overview

This chapter discusses the study's theoretical framework and related literature on problem-solving as an instructional method of teaching mathematics. This chapter focuses on the problem-solving framework model developed by Polya (1957) that includes four problem-solving phases: understanding the problem, devising a plan, carrying out the plan, and looking back. Also, the study describes Schoenfeld's (1985, 1992) framework for mathematical problem-solving and Verchaffed et al. (1999) problem-solving strategy, teachers' perceptions of problem-solving, problem-solving strategies and engagements, and challenges of teaching mathematics through problem-solving.

2.1 Theoretical framework

The constructivism theory was adopted as the underpinning theory of the framework. Constructivism, as outlined by Hausfather (2001), guides how teachers approach their work. It is not a specific teaching method but rather a theory concerning knowledge and learning. This theory places importance on the teaching context, students' prior knowledge, and the interaction between students and the subject matter. Tobin and Fraser (1998) suggest that constructivism should serve as the foundation for teachers' thoughts and actions. Consequently, Leach and Scott (2003) describe the teacher's role within constructivism as introducing and supporting the application of new knowledge in a social context, while the student's role is to internalize these ideas for personal use. According to Hausfather (2001), the teacher's depth of content knowledge is crucial in the constructivist theory, as it plays a vital role in students' understanding. Teachers support the use of new knowledge by creating situations in which students interact with information, apply it to solve problems, engage in discussions, and answer questions, allowing students to make the knowledge their own. This process necessitates continuous restructuring of subject matter knowledge by the teacher, which Cochran, DeRuiter, and King (1993) refer to as pedagogical content knowing. Constructivist theories come in various forms, as noted by Hausfather (2001), Mathews (1995), and Geelan (1997). Some of these forms include personal, radical, social, critical, and contextual constructivism. Geelan (1997) further categorizes them as either social or personal, and Mathews (1995) classifies them as objectivist or relativist, such as the social and critical constructivist theories. However, according to Leach and Scott (2003), these numerous forms of constructivism can be simplified into two broad strands: individual views and sociocultural learning. The individual view, based on Piaget (1970) work, focuses on the mental structure of the student. The sociocultural view, influenced by Vygotsky (1978), integrates both individual and social environments, emphasizing that learning occurs within a social context as well as within the individual's mental structure.

This perspective, where learning is both individual and socially influenced, is also supported by Kim (2001), Lemke (2001), Davydov (1995), and Vygotsky (1978). Carr, Barker, Beverley, Biddulph, Jones, Kirkwood, Pearson, and Symington (1994) consider science as human and social construct, suggesting that learning is a personal construct influenced by the social context. Social constructivism, according to Hausfather (2001), posits that knowledge emerges from human activity as people interact with one another and the physical world, utilizing their minds, bodies, and cultural tools. This perspective acknowledges the importance of the social context in learning.

To provide a framework for this research, social constructivism is used as the overarching theory, with additional adoption of Shulman (1986) knowledge domains in teaching. Shulman proposes three theoretical content domains that include; Subject Matter Content Knowledge (SMCK), Pedagogical Content Knowledge (PCK) and Curriculum Knowledge (CK). Teaching is more than simply conveying concepts and ideas to students, but involves incorporating the ideas and experiences that students experience in the classroom and working on those ideas and experiences with students in a way that refines, re-organizes, integrates and refines these ideas and experiences into a logical and understandable form of students' motivation (Shulman, 2000). This forms the basis upon which teaching mathematics through Problem solving depends. This means that in order for teachers to teach mathematics through Problem solving, they need to have instructional goals in Problem solving and computer-aided learning materials. Most importantly, they need integrated knowledge of these knowledge domains. Shulman (1986) knowledge domains provide a coherent framework for teaching mathematics through Problem solving Pedagogical Content Knowledge (PCK) models. PCK models, such as those proposed by Rollnick, Bennett, Dharsey, and Ndlovu (2008) and Magnusson, Krajcik, and Borko (1999), were chosen to enable a focus on specific classroom practices, teacher knowledge, and instructional strategies. PCK, as defined by Shulman (1986), encompasses an understanding of what makes the learning of a specific topic easy or difficult, including students' presumptions. Teachers require knowledge of strategies that can reorganize students' understanding to effectively teach and facilitate learning.

PCK is a skill and knowledge which is realised when the teacher accesses what he or she knows in terms of curriculum, student understanding, subject matter, pedagogic principles and all this embedded in his/her experiences. So it is not only about the knowledge of various domains which amalgamate to result in the PCK, but the teacher's craft is the ultimate variable in the classroom practice for students' learning. This is also a claim by Lee and Luft (2008), who indicate that PCK is the experiential knowledge and skills acquired through experience in the classroom. Within social constructivist theory, the interactions between the student and the teacher, the students themselves and the social milieu are the fundamental basis for knowledge construction by students (Leach & Scott, 2003)

2.2 Conceptual Framework

According to Maxwell (2005:33), the conceptual framework is “the systems of concepts, assumptions, expectations, beliefs and theories that support and inform your research”. Furthermore, it is something that one builds from pieces borrowed elsewhere and so it is not something that is readymade (Maxwell, 2005). In the pursuit of enhancing mathematics education in public Junior High Schools, it is imperative to consider the intricate interplay of various elements that shape the teaching and learning experience. This conceptual has been meticulously crafted to illuminate these essential components and their relationships.

The study canters on the premise that mathematics teachers' perceived knowledge for teaching through problem-solving serves as a cornerstone in the process of enriching mathematics instruction. The framework elucidates how this perceived knowledge influences the adoption of Problem-Solving Strategies in Teaching Mathematics.

2.3 Subject Matter Content Knowledge (SMCK)

According to Shulman (2000), subject matter content knowledge (SMCK) is the amount and organization of knowledge deeply in the teacher's mind. Shulman (2000) argues that knowledge of the subject matter of teachers should not be limited to the knowledge of facts and processes but also the understanding of both the substantive and syntactic structures of the subject matter. Substantive structures are various ways in which basic concepts and principles of discipline are organized to incorporate their truths.

Teachers will thus be able to use problem-solving teaching methods to teach mathematics well when they understand the network of basic concepts and principles of problem-solving (Shulman, 2000). The syntactic structure of a discipline is one of how truth or falsehood, authenticity or inefficiency are established (Shulman, 2000). A syntactic property is used to set up a valid claim where there is a competing claim for something. Teachers' knowledge must therefore go beyond mere definitions of accepted facts in the learning domain of the student to the extent of explaining why a particular definition is considered appropriate.

Understanding mathematical concepts should not be the main goal of a teacher, but the teacher must further understand why it is so. For junior high school mathematics teachers to effectively teach mathematics through problem-solving, they need to possess both the syntactic and substantive structures of problem-solving (Shulman, 2000). The syntactic and substantive structures allow teachers to critically evaluate, analyse, clarify, compare and contrast student solution processes to clarify their shortcomings and misconceptions in the teaching and learning process. Ball, Hill, and Bass (2005), in the information structure of Shulman (2000), noted that teachers' use of teaching materials, their methods of assessing student progress and how they make rational decisions about presentation, emphasis and sequence are based on their mathematical content teaching (MCT).

Hence, in teaching mathematics through problem-solving, it is necessary to have a thorough knowledge of selecting, designing and implementing appropriate teaching materials. Also, teachers' ability to choose practical approaches and set appropriate examples for students in problem-solving lessons is included in their mathematical content teaching (MCT). Knowing mathematics for teaching demands in-depth and detailed knowledge that goes well beyond what is needed to carry out the algorithm reliably to include consideration in choosing good examples for instructional purposes (Ball, Hill, & Bass, 2005). Problem-solving teaching depends heavily on teachers' knowledge of the topics because teachers need to explore the problem-solving techniques often used by students to find the right solutions, but their generalizability or mathematical validity is immediately unclear. When a teacher is deficient in the subject matter and knowledge of problem-solving, it becomes practically impossible for them to effectively engage in mathematics through problem-solving.

2.4 Pedagogical Content Knowledge (P.C.K.)

Shulman (1986) P.C.K. goes beyond the subject matter content knowledge to include the dimension of subject matter knowledge for teaching. According to Shulman (1986), this knowledge combines the topics most frequently taught, the most effective ways to represent those ideas, and the most powerful analogies, examples, illustrations, demonstrations and explanations in the art of teaching. P.C.K. also includes the methods for presenting and constructing the subject matter in a way that is understandable to students with various perspectives and understandings. To teach mathematics through problem-solving, teachers need to be able to design and present mathematics comprehensively for students since there is no single most powerful form of representation. (Shulman, 2000). Teachers must have at hand an armamentarium of alternative forms of problem-solving representations. Teachers' need knowledge of pedagogical strategies, which must be appropriate for recognizing the understanding of learners who might appear before them as blank slates (Shulman, 2000).

In Shulman's (1986), P.C.K. also entails understanding what makes learning specific topics difficult, the conception and preconception that learners of different ages and backgrounds frequently bring to the learning environment. Most of these perceptions are often misconceptions. P.C.K. helps teachers anticipate students' difficulty in learning and provide available alternative models or explanations to mediate those challenges. Shulman (2000) informs that a group-based strategy will improve understanding among students. Also, argues that if teachers could encourage learners to actively think about what they already know and create conditions where they can discuss what they know with other learners, this will minimize the problem of illusory understanding. Ball and Bass (2000) described P.C.K. for teaching mathematics as a specialized form of knowledge that combines mathematical knowledge with knowledge of learners, learning and pedagogy. This implies that teachers should have control of the subject matter, knowledge about the learners, their strengths and weaknesses, and resources with varied instructional strategies before they can teach mathematics through problem-solving. However, Ball and Bass (2000) cautioned that no amount of P.C.K. can prepare a teacher for all classroom practices because a great proportion of teaching is full of uncertainties. When teachers are prepared to harness all possible pedagogical teaching and learning strategies and use them in the classroom, it is likely to improve the teaching of mathematics in the curriculum.

2.5 Curriculum Knowledge

The mathematics curriculum is represented by a comprehensive list of programs designed to teach mathematical topics at a given level. It includes the variety of teaching aids available relating to the issue to be addressed and the set of factors that govern the use of curriculum resources in particular contexts (Shulman, 1986). Teachers must think carefully about students' mathematical ideas, analyse literature presentations, and assess the relative merits of two different presentations when dealing with a particular mathematical issue (Ball & Bass, 2000). Mathematics teachers must thoroughly understand the curricular resources available in problem-solving instruction to make them available to students in teaching math problem-solving. The concept of contextual information, knowledge of learning content, and knowledge of the curriculum form the foundation of the problem-solving teaching concept. Therefore, teachers must fully understand the curriculum knowledge to teach mathematics through effective problem-solving.

Tambychik and Meerah (2012) note that cultivating problem-solving abilities has become an essential component of national curricula across several countries. They assert that the significance of problem-solving skills in mathematics classrooms has escalated due to students' struggles in applying acquired mathematical skills to real-world problems. While these proficiencies are delineated in the curriculum, it lacks specific pedagogical guidance for educators to assist students in cultivating these pivotal skills necessary for mathematical success. Notably, many teachers completed their teacher training before the adoption of this new curriculum, potentially leaving them unfamiliar with these problem-solving skills and strategies. The attitudes of mathematics teachers towards the subject and mathematics education at large play a substantial role in shaping the quality of mathematics teaching and learning. Ernest (1989) suggests that alterations in teaching practices are contingent upon a shift in teachers' deep-seated beliefs about mathematics education, aligned with policy objectives. The execution of novel teaching methodologies hinges on teachers' foundational beliefs and their conceptualization of mathematics, as stated by Ernest (1994). Research demonstrates a correlation between teachers' beliefs and their instructional practices (e.g Handal & Herrington, 2003; Perkkila, 2003). For instance, Perkkila (2003) research involving Finnish primary school teachers illustrates that their past experiences significantly shape their teaching approaches. Mereku (2003) highlights that despite a uniform mathematics curriculum in Ghana, differences in teaching methods and learning strategies arise from varied teacher beliefs. These beliefs exert a direct influence on teaching practices, underscoring the potential to impact practices by comprehending educators' viewpoints.

2.6 Meaning and Processes of Problem-Solving

Problem-solving cannot be ignored when we touch the subject of non-routine mathematical problems. This means that the nature of problem-solving when working on non-routine issues cannot be overemphasized. Callejo and Vila (2009) it is not surprising that this area of mathematics education has received in-depth attention over the years, prompting more researchers to delve deeper into and identify the processes involved in teaching students how to solve problems more accurately. Simply put, problem-solving focuses on a mathematical question whose solution is unknown (Callejo & Vila, 2009). D'Ambrosio (2003) defines problem-solving as a process that often provides an environment for the student to work independently while discovering precise mathematical paths to answers.

Therefore, the path a student takes in solving a problem can be called a problem-solving process. Literally, this means that problem-solving begins when students are tasked with working on routine or non-routine problems. However, it should be noted that for the student to be able to carry out an effective problem-solving process, he needs to have a commendable level of mental agility, because if the student is less equipped with this skill, there will be a gross deficiency in the problem solver's ability to work accurately (D'Ambrosio, 2003). Also, further believes that problem-solving offers students opportunities to use their existing knowledge to solve problems. The process is essential in that students can create new knowledge and new understanding in the process. From this perspective, we can conclude that problem-solving as a process is a driving force for developing a deeper understanding of mathematical ideas and processes. For this reason, when students are presented with routine or non-routine tasks, it is important to read the problem carefully in order to understand the questions effectively; this may mean breaking down important information so that you have a better understanding of choosing the appropriate strategies to adopt. Against such background, Bruder and Collet (2011) believe that learning how to solve problems can be established as a long-term teaching and learning process that should involve four stages. These include intuitive familiarization with heuristic methods and techniques, introducing students to special heuristics using prominent examples (explicit strategy acquisition), a short conscious practice phase to use the newly acquired heuristics with different task difficulties, and broadening the context of the strategies used.

Similarly, Joseph (2011) outlined an eight-step problem-solving procedure and stressed several key facts including carefully reading out the questions, making note of the necessary information, looking out for the underlining goal of the problem, organizing one's plan for working out the problem and after arriving at the answer, going back to recheck if the solution is right. According to Joseph (2011), these are the necessary processes that students should follow in their thinking and problem-solving processes. Especially for young students, teachers need to take some of these models and present them in a child-friendly way, so students have a realistic idea of how to work in their jobs. Ang (2010) further argues that if we aim to follow modelling as a process for enabling students to get better at problem-solving, we should bear in mind that it must be seen as a process where one must be encouraged to work step by step on an array of exercises with the view of coming up with reasonable answers. On the other hand, Cirillo, Pelesko, Fellon-Koestler, and Rubel (2016) hold that mathematical modelling involves the construction of mathematical structures and concepts with mathematical representatives, such as counting blocks, counting scales, fraction cards, etc., and is used to visualize and reduce the complexity of mathematical structures. Kaur and Dindyal (2010) also hold that despite these different views, the process of mathematical modelling problems reveals a distinct and common characteristic of all different beliefs and that mathematical modelling is connected to real-life problems. Based on this judgment, Blum (2002) believes that through modelling, learners benefit better from mathematical concepts. With fewer modelling opportunities for students to have a better knowledge of problem-solving processes, it remains undeniable that learners continue to face challenges when confronted with problems. In their study, McGinn and Boote (2003) also stressed four basic factors that had to do with one's perception of the difficulty of problems: Categorisation- recognizing the category a problem or question appropriately fits. Interpretation of aims - Identifying the best solution to solve a problem from start to finish. The importance of resources - Figuring out the relevance of resources in the problem-solving process. Complicated nature of the problem – carrying out several problem-solving processes to derive an answer.

Mc Ginn and Boote (2003) further stated that the nature of the problems in terms of their difficulty always depended on the student's beliefs. For example, how well the problem was placed, as well as its level of difficulty. For example, Singaporean studies carried out by Yeo (2009) indicate that Singaporean learners' problem-solving difficulties were grouped into the following: lack of understanding of questions, inability to use appropriate strategies and difficulty translating problems into mathematical states. Francisco and Maher (2005) also examined students' mathematical problem-solving thinking skills and concluded that to successfully promote problem-solving reasoning, we must provide students with opportunities to work on complex tasks as opposed to simple tasks that are key to stimulating their mathematical reasoning. To this end, Wilburne (2006) stated that the best math problems to use in the classroom are non-routine math problems that encourage rich and meaningful mathematical discussions, those that show no obvious solutions, and those that require the student to employ a variety of strategies as they work. Cai (2003) emphasised that invented strategies can stimulate students' mathematical understanding during the problem-solving process, but they were quick to point out that to help a student develop effective problem-solving strategies, they must be properly guided. It is also appropriate to add that the problem-solving process has a key goal of supporting mathematics teaching and learning (NCTM, 2000). This is because the process of finding solutions to problems offers learners an opportunity to engage in more detailed problem-solving activities. This is why some scholars believe that the process of working out problems also develops pupils' problem-solving skills and strategies (Lesh & Zaworjewski, 2007). Using these skills, learners can recognize and solve problems using their innate critical thinking and creative abilities, engage more actively in group work and realise that problem-solving contexts do not exist in isolation. According to Brenner, Herman, Ho, and Zimmer (1999), one of the reasons why learners from Singapore do excel in international mathematics tests is a result of their brilliant problem-solving skills, which is a deficiency among most students in other countries. To enable more learners to engage in the process of problem-solving actively, Lesh and Zaworjewski (2007) believe that every student is capable of developing the skill since the skill at problem-solving is highly connected with the social environment. This follows that the more problem-solving is accomplished through interactive teaching, learners get conversant with the skill itself thereby reaching better tiers of achievement (Rigelman, 2007).

2.7 Benefits of Teaching Mathematics through Problem-Solving

To succeed in the 21st century, students must have the ability to think critically and solve problems. Educators suggest that problem-solving be used as a general approach to teaching and learning because it has a unifying purpose in the mathematics curriculum (Cai, 2003; Cockroft, 1982; National Council of Teachers of Mathematics [NCTM], 1989). This approach is believed to help students gain a much deeper and better understanding of mathematics. Because comprehension is an internal and unobservable phenomenon that occurs when students' minds incorporate new information with prior understanding, it cannot be taught directly. Therefore, using problem-solving as a teaching strategy is a powerful way of promoting this thinking (Lambdin, 2003). Yavuz, Karatas, Arslan, and Erbay (2015) argued that problem-solving is valuable in teaching and learning mathematics lessons. Because problem-solving is a scientific method, it requires reflective thinking, critical thinking, analysis, and creative and synthesizing abilities. Ersoy (2016) stated that focusing on problem-solving in class builds students' high-level thinking. In professional and everyday settings, problem-solving is generally considered a fundamental and significant cognitive articulation (Aksoy, Bayazit, and Donmez, (2015). For this reason, students can learn independently through the process of solving problems in mathematics classes. Chauraya and Mhlolo (2008) argued that the benefits of problem-solving include the fact that it is a learner-centred approach where students explore as well as explore mathematical ideas themselves. As students verify solutions, they develop evaluation and reflection skills throughout the problem-solving process.

According to Brehmer (2015), teaching problem-solving in mathematics to teachers develops general cognitive skills and encourages students to learn mathematics. Brehmer (2015) emphasized the importance of developing students' ability to solve mathematical problems agreed upon by educators. This is reflected in the national guidance documents of many countries (MOE, 2007; NCTM, 2000), which focus on solving mathematical problems. In addition, through solving math problems, students can learn and develop the practical and logical skills they need to be successful in everyday life (www.kevbotlearning.weekly.com). Chauraya and Mhlolo (2008) agreed that problem-solving in mathematics education has rich educational benefits. The benefits appear to students to be actively involved, provide opportunities to apply their mathematical knowledge and skills, provide rich experiences for students to enjoy discovery, learn new mathematical concepts with greater understanding, foster positive attitudes towards mathematics, develop thinking, problem-solving and cooperative skills and the development of flexibility and creativity.

2.8 The Rationale of the Problem-Solving Approach in Teaching Mathematics

A problem-solving approach is an essential method that today's students need. Guided by recent problem-solving research, changing professional standards, new workplace demands, and recent changes in learning theory, educators and trainers are revising curricula to include integrated learning environments that encourage students to use higher-order thinking skills, especially problem-solving skills (Cai, 2003). As education has come under criticism from many sectors, educators have sought ways to reform teaching, learning and the curriculum. Students often learn facts and rote procedures with little connection to context and application of knowledge. Problem-solving has become a means to reconnect content and application in the learning environment for basic skills as well as their applications in different contexts (Hiebert, 1996). Today, there is a strong movement in education to incorporate problem-solving as a key component of the curriculum. The need for learners to become successful problem solvers has become a dominant theme in many national standards (AAAS,

1993; NCTE, 1996; NCTM, 1991; NCTM, 1980). For example, the 1989 Curriculum Standards of the National Council of Teachers of Mathematics (NCTM) states: “Problem-solving should be the central focus of the mathematics curriculum’’. Several kinds of literature in the field offered many reasons as a rationale for the implementation of a problem-solving approach. For example, NCTM (1980) and NCTM (1989) have strongly approved the inclusion of problem-solving in school mathematics and their many reasons are:

First, problem-solving is a major part of mathematics. It is the sum and substance of our discipline and to reduce the discipline to a set of exercises and skills devoid of problem-solving is misrepresenting mathematics as a discipline. Second, mathematics has many applications, often representing important problems in mathematics. The subject is used in the work, understanding, and communication within other disciplines. Third, there is an intrinsic motivation embedded in solving mathematics problems. Problem-solving approach claims in school mathematics can stimulate the interest and enthusiasm of the students. Fourth, problem-solving can be fun. Many of us do mathematics problems for recreation. Finally, problem-solving must be in the school mathematics curriculum to allow students to develop the art of problem-solving. This art is essential to understanding and appreciating mathematics that should be an instructional goal.

2.9 Methods of Teaching Mathematics

Regarding the NCTM (2000) noted that several researchers and educators suggest that what we teach may be less important than how we teach. Also notes that although no single teaching method has been found to exclusively meet all needs, the literature emphasizes the importance and benefits of teaching mathematics through a problem-solving approach. Many other authors in this field have also argued the importance of using problem-solving approaches in teaching mathematics (Allevato & Onuchic, 2008; Cai, 2003; NCTM, 2010).

Resnick (1989) noted that problem-solving instructional methods tend to focus on developing children's mathematical thinking rather than simply mastering the facts and procedures of a given subject. Research verifies the superior effect of a contemporary problem-solving approach to teaching in which mathematics is seen as a dynamic subject to be explored by incorporating group work and the use of non-routine questions that encourage mathematical thinking and problem development, problem-solving skills, over the traditional problem-solving approach in which mathematics is seen as a fixed set of facts to be imparted by teachers and learned by students as they work with individual students, practicing routine questions and relying on textbooks or worksheets to achieve results in mathematics and attitudes and skills (Levin & Ammon, 1992). Many kinds of literature have argued that mathematics is not just about plugging numbers into an algorithm or calculator to find a solution, nor is it just a subject in school or a set of rules to memorize, rather mathematics is about thinking and reasoning, solving problems, making connections and being able to communicate ideas mathematically (NCTM, 2000). Therefore, at all stages, mathematics should be thought of to emphasise the development of student's ability to solve a wide range of complex mathematical problems. Many researchers (eg. Kamii & Housman, 1989; Maher & Martino, 1996) have examined students' mathematical thinking and suggested that young children can explore problem situations and "invent" ways to solve problems.

Based on this fact, most mathematics teachers have proposed and argued that problem-solving methods are the best way to teach and learn mathematics to help students make connections and apply mathematical knowledge in the real world (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Kamii & Housman, 1989; Polya, 1965). To summarize, the above literature show that the nature of mathematics courses recommends certain teaching methods other than problem-solving approaches, such as cooperative, inquiry, discussion, and discovery methods.

2.10 Choosing Teaching Methods to Facilitate a Lesson

Even (2005) as cited in Rapoo (2011) stated that it is a generally understood fact that the actual teaching of mathematics is much more perplexing than the straightforward implementation of the national curriculum. Thus, many teachers find it difficult to balance their focus on the learner, the curriculum, and the subject. By focusing on delivering textbook content regardless of students' understanding, some mathematics teachers may teach content at the expense of the other two aspects. As a result, a mathematics module for Unisa (2011) has urged teachers to put emphasis on helping students comprehend mathematics while selecting a teaching strategy for a lesson and the significant curriculum outcomes over covering the entirety of the content.

2.11 Using Problem-Solving as a Teaching Strategy

Mathematics educators have widely accepted that the principal objective of mathematics instruction should be developing learners' problem-solving abilities, and that problem-solving must play an integral role in the curriculum of mathematics programmes (Lester, 1994). Mills and Kim (2017) also argued that problem-solving skills do not necessarily develop naturally. The problem-solving skills must be taught explicitly so they can be moved across multiple contexts and settings.

According to Alsawaie (2003), the NCTM (200) calls for programmes of teaching that will make all learners to be capable of constructing new knowledge in mathematics through problem-solving, solving mathematical problems that may arise and those appearing in other settings, employing and adapting a lot of suitable techniques in finding solutions to problems and reflecting and monitoring the methods and strategies utilized to work out a mathematics solution to a given problem. The researcher believed that using problem-solving as a teaching strategy was educationally important. Similarly, Mayo, Donnelly, Nash, and Schwartz (1993) justified that problem-solving strategy as a "teaching strategy where important, contextualized, real-world situations are presented and guidance, resources, and instruction to learners are offered as they acquire problem-solving skills and content knowledge" (p.227). Mayo, Donnelly, Nash, and Schwartz‘s definition of the problem-solving strategy was applicable to the study.

2.11.1 Problem-Solving Classroom Ecology

This study primarily concerns the processes teachers can use to teach through problem-solving. This means using problem-solving as a technique to help students to construct new knowledge and take responsibility of learning. Mathematics teachers should create and maintain an appropriate learning climate for students to learn through problem-solving. Alsawaie (2003) argued that teachers need to select rich and appropriate problems, organize their use, assess students' understanding, and apply strategies to help their students become problem solvers. According to Donaldson (2010), "Problem- based learning is an instructional approach where teachers use problem-solving as a primary means of teaching mathematical concepts and helping students synthesize their mathematical knowledge" (p. 5). Problem-solving as a teaching strategy aims for students to develop, extend and improve their understanding by finding solutions to problems (Hiebert & Wearne, 2003). Taplin (2015) agreed that focusing on learning topics in mathematics through problem-solving contexts is observed when the teacher helps students to articulate their own interpretation of ideas in mathematics to a depth of understanding. This can be achieved by "having them do mathematics such as generating, testing, investigating, verifying and conjecturing" (Lester, Masingila, Mau, Lambdin, Santon, & Raymond, 1994).

According to McDougal and Takahashi (2014), a problem-based learning lesson begins with the teacher setting the context and presenting the problem. Students then spend about ten minutes trying to find a solution to the problem using problem-solving strategies while the teacher monitors their progress. The teacher also records the approaches students use. The teacher would then model problem-solving. "Modelling problem solving consists of the following: demonstrating skills and mathematical concepts, thinking aloud to offer students insight into the metacognitive aspects of problem-solving, and demonstrating a positive attitude and persistence when confronted with challenges" (Donaldson, 2011) Then the teacher begins a discussion with the whole class. As in lessons with a learning problem-solving approach, the teacher can ask students to share their ideas. However, instead of ending the lesson, the teacher goes on to ask the students to think and compare their different ideas; which ideas are wrong and why, which are right, which are similar, and which are more effective or subtle. Through such discussions during class, students acquire new ideas or practices in mathematics.

2.11.2 Problem-Solving Strategies

Students use problem-solving strategies as they engage in self-directed learning that consists of one-hand tasks and whole-class discussions. Pressley (1996) described strategies as conscious and manageable activities carrying out cognitive goals. An appropriate strategy will make the problem solver think about the meaning of both the mathematical equation and the problem sentence (Aydogdu, 2014). According to Posamentier and Krulik (1998) Problem-solving strategies may include working backward, taking different points of view, discovering a pattern, making a drawing, solving a simpler or analogous problem, considering extreme cases, making informed guesses and testing (approximation), explaining all possibilities, logical reasoning an data coordination. Students who solve problems during self-study or whole-class discussions use problem-solving strategies in the math classroom. By taking careful steps such as following productive leads and abandoning fruitless paths, the problem solver succeeds in solving the problem (Schoenfeld, 2014).

Avcu, and Avcu, (2010) emphasize that problem-solving strategies help students make progress in solving more challenging and difficult problems. Problem-solving strategies are methods that can be used to solve different types of problems (Ontario Ministry of Education, 2007). Common problem-solving techniques include modelling, picturing or drawing; finding patterns; guessing and searching for ideas; creating a formal list; creating a table or chart; solving a simple problem, working backward; and Application of Rational Thinking (Ontario Ministry of Education, 2007).

2.11.3 Polya's Problem-Solving Technique

Polya (1985) as cited in Hensberry and Jacobbe (2012) proposed a four-phase problem-solving model that includes: understanding and evaluating the problem, finding a strategy, using a problem-solving strategy, and then looking back and thinking about a solution. Although the four stages of problem-solving are listed in order of development, for complex problems it may not be possible to go through them to get a simple answer. Students move back and forth between steps in the problem-solving process. This framework is infused with the principles of teaching through problem-solving, as shown in Figure 2.1.

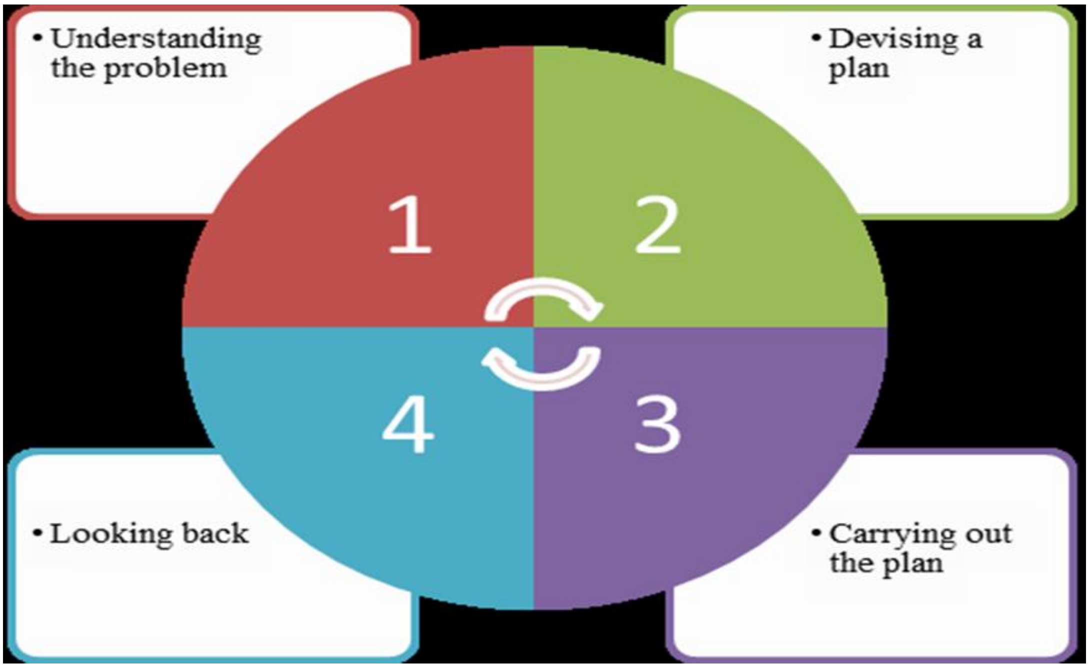

Figure 1: Polya's problem-solving model (Polya, 1945)

i. First step: Understand the problem

In this step, students should understand the nature of the problem and its related goals. Teachers find, show, grasp, or estimate conditions, assumptions, information and students are encouraged to frame the problem in their own words.

ii. Second step: Devise a plan

After the nature and parameters of a problem are understood, students need to select one or more appropriate strategies to help resolve the problem. Students need to know that they have many strategies available to them and no single strategy work for all problems. Here are some of the possible strategies:

Draw a picture or diagram: Many problem solvers find it useful to draw a picture or a diagram of a problem and its potential solutions prior to working on the problem. This allows students to think on the many dimensions of the problem.

Make an organized list: Recording information in list form is a process used quite frequently to map out a plan of attack for defining and solving problems. Encourage students to record their ideas in an organized list to determine regularities, patterns, or similarities between problem elements.

Look for a pattern: Looking for patterns is an important problem-solving strategy because many problems are similar and fall into predictable patterns. A pattern, by definition, is a regular, systematic repetition and may be numerical, visual, or behavioural.

Make a table: A table is an orderly arrangement of data. When students have opportunities to design and create tables of information, they begin to understand that they can group and organize the data of the problem.

Guess and check: Give students opportunities to engage in some trial-and-error approaches to problem-solving. "Guess and Check" is also referred to as "Trial and Error". However, Polya Prefers to call this "Trial and Success".

Use a variable, and write an equation: Students are highly advised to use this strategy where a problem contains two or more quantities. This can be done by looking at the relationship among the quantities (variables) and bring them to an equation.

Work backward: It's frequently helpful for students to take the data presented at the end of a problem and use a series of computations to arrive at the data presented at the beginning of the problem.

Act it out: Here students are expected to act on the situation of the problem by themselves or with friends on behalf of the subject(s) of the problem.

Change point of view: Sometimes students become frustrated with the first attempt by insisting one assumption of the solution. But, removing this unnecessary restriction opens the door of the solution.

Solve a simple problem: There are times where problems are easier to solve with compartmenting to simple problems than to solve as it is. In this case, solving these simple problems step by step arrives at the solution of the main problem.

Use deduction (making suppositions): This strategy is a process of reaching a conclusion or solution through logic or reasoning.

iii. Third step: Carry out the plan

When working with a strategy or combination of strategies, it is important that students keep accurate and up-to-date records of their thoughts and actions. Recording the data collected, predictions made, and strategies used is an important part of the problem-solving process.

Try to work through the chosen strategy or combination of strategies until it is clear that it is not working, needs to be modified, or is providing inappropriate data. As students become more adept at solving problems, they should feel comfortable rejecting potential strategies at any point during the search for a solution.

Follow the steps taken in the solution with great care. Although students may have a natural tendency to "rush" the strategy to arrive at a quick answer, encourage them to carefully evaluate and track their progress. Put the problem aside for some time and deal with it later. For example, scientists rarely come up with a solution when they first approach a problem. Students should also feel comfortable letting the problem rest for a while and returning to it later.

iv. Fourth step: Look back (reflect)

It's very important that students have multiple opportunities to assess their own problem-solving skills and the solutions they generate from using those skills. Frequently, students are overly dependent upon teachers to evaluate their performance in the classroom. The process of self-assessment is not easy, however. It involves risk-taking, self-assurance, and a certain level of independence. But it can be effectively promoted by asking students question such as “How do you feel about your progress so far?" "Are you satisfied with the results you obtained?" and "Why do you believe this is an appropriate response to the problem?".

In this section, an attempt has been made to show and discuss the procedures of the problem-solving approach. It is hoped that this helps in the forthcoming discussions of the teachers' and students' roles in the classroom. According to Polya (1985), there is no possibility of being able to solve the problem unless you first understand the problem. Understanding the problem involves more than just knowing what to do but also an important part of the information that needs to be integrated in some way to find the answer. As one is often unable to get all the important information about a problem simultaneously. Polya (1985) advised that reading the problem thoroughly from the beginning and during the work is always necessary.

During the troubleshooting process, one may need to periodically reflect on the first question to ensure they are on the right track. Also emphasized that understanding the problem can be confirmed by asking the following questions in these categories: what is unknown? What are the details? And what is the situation like? Polya's second stage, finding a strategy, often suggests that it is really easy to think of the right strategy.

However, there are certainly problems where children may feel the need to play with this knowledge before they can think of a plan to produce a solution. This assessment phase will also help students better understand the problem and see some of the information they have praised after the first reading. At this stage, one may ask: Do I know the related problem? Then try to think of a common problem of having the same or similar issue (Polya, 1985). Polya's third phase applies the selected strategy to complete the problem step-by-step; if a solution is unavailable, the strategy is changed. The final stage involves looking at the solution according to the original problem to see whether the answer is reasonable and whether there is a way out of the solution. Polya's phases provide a helpful framework for looking at problem-solving, but they constitute only part of his contribution. A distinctive feature of Polya's conception of problem-solving is the notion of heuristic.

More recently, some mathematics educators have considered heuristic strategies, or simply heuristics, to be synonymous with problem-solving strategies, whereas others describe heuristics as being contained in a larger set of problem-solving strategies. Schoenfeld (1987) described heuristic strategies this way: Heuristic strategies are rules of thumb for making progress on difficult problems. There are, for example, heuristic strategies for understanding a problem (focusing on the unknown, on the data, drawing a diagram, etc.), for devising a plan (exploiting related problems, analogous problems, working backwards, etc.), and for carrying out and checking a solution.

In this research, the terms heuristic and heuristic strategy refer to items such as those Polya suggested in How to Solve It (e.g., add auxiliary lines to a geometric figure, solve a related problem, examine a special case, or work backwards), and I use problem-solving strategy to denote elements of a larger set of both general and specific strategies for problem-solving. For example, there are specific strategies that a problem solver can use to perform tasks such as simplifying an algebraic expression, and there are general strategies that a problem solver can use with any problem. An example of a general strategy is using intuition to make a conjecture.

2.12 Schoenfeld Problem-solving Strategies

Several decades after Polya's work, Schoenfeld (1985, 1992) developed a framework for mathematical problem solving that built on Polya's framework and added to our understanding of what it means to be a good problem solver and suggested phases of problem-solving similar to Polya's: read, analyses, explore, plan, implement, and verify. In Schoenfeld's description, as in Polya's, a good problem solver moves among the phases in a non-sequential fashion. Schoenfeld (1985) employed a think-aloud protocol to gain insight into problem solvers' decision-making. Schoenfeld (1985) devised a method for charting the progress of problem solvers as they worked on a problem and noted that novice problem solvers moved in one direction, from reading the problem statement to implementing a strategy, without considering whether the strategy was leading to a solution.

On the other hand, expert problem solvers analysed the problem before implementing a strategy and then moved back and forth among the different problem-solving phases (see also Schoenfeld, 1992). Several elements must be in place for someone to be a successful problem solver. In Schoenfeld's (1985, 1992) framework, these elements are resources, problem-solving strategies (including heuristics), control, and beliefs and affect. A set of resources or the knowledge base refers to mathematical knowledge at a problem solver's disposal. Resources include facts, concepts, algorithms, and routine procedures. Schoenfeld (1992) made a distinction between algorithms and routine procedures, noting that algorithms are guaranteed to work, whereas "routine procedures are likely to work, but with no guarantees" (p. 350). For example, the long division algorithm for dividing polynomials is guaranteed to work if one follows the steps correctly. As an example of a "routine [procedure that is] likely to work, but with no guarantees," Schoenfeld (1992) described a common strategy for proving elements of a geometric figure are congruent: First, show that the elements are corresponding parts of congruent triangles.

Also, noted that this strategy is one of several "proof techniques [that are] not algorithmic, but they are somewhat routine" (p. 350). Mathematical knowledge alone is not enough to make someone a good problem solver; problem-solving strategies are necessary in order to help problem solvers use their resources effectively and efficiently. For example, suppose a problem requires one to calculate the area of an irregular shape. A problem solver needs resources such as the area formulas for rectangles, triangles, or other figures, but the solver also needs strategies to use these resources. One potential strategy is to divide the irregular shape into familiar shapes whose area formulas are known, then add the areas of the individual shapes to determine the area of the irregular shape. The third component of problem-solving Schoenfeld (1985) mentioned is control. Control falls under the category of metacognition, a broad term that includes knowledge of one's own cognition, monitoring or control of cognitive processes, and reflection. Schoenfeld (1985) described control as "resource allocation during problem- solving performance" (p. 143). More specifically, control involves deciding what resources may be useful, identifying what strategies will provide an efficient way to solve a problem, "recovering from inappropriate choices," and monitoring one's progress while solving a problem.

Kilpatrick (1985) also noted the importance of resources, strategies, and control: "Successful problem-solving in a given domain depends upon the possession of a large store of organized knowledge about that domain, techniques for representing and transforming the problem, and metacognitive processes to monitor and guid performance. Finally, beliefs and affected refer to "an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behaviour" (Schoenfeld, 1992). Confidence in one's ability to solve a problem, the belief that the problem is worth solving, and conviction that mathematics itself is a sensible and worthwhile endeavour all play a part in successful problem-solving. According to Silver (1982), one of the "components of a mathematical belief system which may have important implications for how one approaches mathematical problems is the belief that there is usually more than one way to solve a problem" (p. 21). Schoenfeld (1992) discussed typical student beliefs that can be a hindrance to the student's ability to be good problem solvers. For example, "Mathematics problems have one and only one right answer," "Students will be able to solve any assigned problem in 5 minutes or less," and "Mathematics is a solitary activity" are beliefs that have a negative impact on students' ability to solve problems (Schoenfeld, 1992, p. 359; see also Schoenfeld, 1985).

2.13 The Importance of Metacognition

Metacognition warrants special attention because of its significant role in problem-solving. As Polya implied (1957, 1962, 1965) and Schoenfeld (1985) stated explicitly, metacognitive behaviour can be the difference between success and failure for the problem solver. Simply stated, metacognition is thinking about thinking. Evidence of problem solvers' metacognitive behaviour includes awareness, not simply the use of some or all of the following: understanding what the problem is asking, choosing a particular strategy to solve the problem, evaluating whether the strategy is leading closer to a solution, and examining whether the answer makes sense. One could argue that Polya valued metacognition in mathematical problem-solving despite never using the term.

According to Silver (1982), "If we adopt a metacognitive perspective, we can view many of Polya's (1957) heuristic suggestions as metacognitive prompts" (p. 21). In Polya's (1962), looking back phase, the problem solver may ask himself many useful questions: "What was the decisive point? What was the main difficulty? What could I have done better? I failed to see this point: which item of knowledge, which attitude of mind should I have had to see it?" (p. xii) Metacognition includes not only knowledge of one's own cognition but also the regulation of one's behaviour in response to that knowledge (Garofalo, & Lester, 1985). This concept is known as self-regulation, which is closely related to control. Schoenfeld (1987a) summed up the notion of self-regulation: "It's not only what you know, but how you use it (if at all) that matters" (p. 192). Metacognitive behaviour during problem-solving was a hot topic during the 1980s (e.g., Campione, Brown, & Connell, 1988; Garofalo & Lester, 1985; Schoenfeld, 1983, 1987a; Silver, 1982), but research on metacognition is no longer prominent in mathematics education. Metacognition does, however, remain an implicit part of the problem-solving discussion. For example, the mathematics education literature is replete with terms such as monitoring and reflection and ideas such as self-assessment and knowledge of one's own cognition. The NCTM (2000) claimed that the development of students' metacognitive abilities is an important part of classroom instruction: "Students should be encouraged to monitor and assess themselves. Good problem solvers realize what they know and don't know and what they are good at and not so good at" (p. 260). This shows that the multiple dimensions of metacognition that the NCTM (2000) values in mathematics instruction. These dimensions include reflection, metacognitive questions, and monitoring. Reflective skills (called metacognition) are much more likely to develop in a classroom environment that supports them.

Teachers play an important role in helping to enable the development of these reflective habits of mind by asking questions such as "Before we go on, are we sure we understand this?" "What are our options?" "Do we have a plan?" "Are we making progress, or should we reconsider what we are doing?" "Why do we think this is true?" Such questions help students get in the habit of checking their understanding as they go along. As teachers maintain an environment in which the development of understanding is consistently monitored through reflection, students are more likely to learn to take responsibility for reflecting on their work and make the adjustments necessary when solving problems. Teachers are responsible for creating classroom environments that encourage metacognitive behaviour and allow students to reflect on their work. Teachers encourage metacognition by modelling metacognitive behaviour, such as thinking aloud and asking metacognitive questions.

Schoenfeld's (1985, 1992) framework for mathematical problem-solving summarised my assumptions about problem-solving and serves as the theoretical framework for the present study. As Schoenfeld's framework suggests, problem-solving is multifaceted. One consequence of this multi-faceted nature is that the teacher's role in helping students develop their problem-solving ability is complex. Because successful problem-solving requires mathematical knowledge, problem-solving strategies, metacognitive control, and positive beliefs, it is fitting to investigate actions teachers can take to facilitate their student's development of these aspects of problem-solving. Verchaffel, Corte, and Ratinckx (1999) designed, developed and explored a learning environment that emphasized students' acquisition of a five-phase mathematical Problem-solving strategy. These categories are: (1) construct mental representations of a problem; (2) decide how to solve the problem; (3) issue the necessary statistics; (4) interpret the result and formulate a response; and (5) evaluate the solution. Paying attention to these phases during instructional time is helpful and can improve students' Problem-solving abilities. The most common problem-solving techniques are; Listings, guesses and observations, drawing a diagram, writing an equation, searching for a pattern, making a table, consulting and simplifying a problem. Success in problem-solving is directly related to selecting the right strategy (Cai, 2003).

2.14 The Teacher's Role and Beliefs about Problem-Solving

Anderson (2000) believes that teachers' views and approaches to problem-solving vary widely. Anderson (2000) argues that teachers hold different views on the term problem and problem-solving, as well as on teaching practices in problem-solving and further claimed that, there is a congruence among teachers between the stated views and practices and between the stated practices and what happens in the classrooms.

However, for some teachers, the stated procedures are not easily followed, a situation that suggests either a lack of reflection or the possibility of limiting factors that could adversely affect teachers' plans. If this is the case, it means that most teachers have different perceptions when it comes to problem-solving. Funkhouser (1993), in an attempt to find out teachers' views on problem-solving, tried to interview several of them who gave different answers.

According to Funkhouser (1993), two-thirds of the answers were categorized as vague, for example: 'Problem-solving is finding a solution to a problem and problem solving is using thinking skills and also said that, only one-third of teachers were able to give a precise definition that included either reference to strategies or skills, such as "problem-solving is identifying a problem, determining steps, and then solving the problem." Grouws, Good, and Dougherty (1990) also sought to interview teachers to discover their problem-solving perceptions and, like Funkhouser (1993) different ideas were captured as Problem-solving is problem-solving, problem-solving is practical problem-solving, and problem-solving is thinking problem-solving. It was noted that when it came to definitions, teachers expressed their opinions based on what was happening in their classrooms. Phipps and Wagner (2017) argue that teachers' beliefs about teaching and learning strongly influence their pedagogical decisions. Haflu (2008) states that teachers should play an important role in solving mathematical problems, so they must choose problems that engage students for this purpose.

Teachers must also create a supportive environment that encourages students to explore, take risks, share failures and successes, and question each other (Haflu, 2008). It is believed that in such a classroom, students can build on their self-esteem and confidence while developing an interest in exploring problems and the ability to adjust their problem-solving strategies (NCTM, 2004). Again, if a teacher believes in developing problem-solving skills as part of their beliefs, then they will be more tempted to provide more open-ended questions.

In addition, teacher beliefs about the importance and skills of implementing problem-based instructional approaches significantly affect student achievement and problem-solving behaviour (Thompson, 1992). If teachers believe that problem-solving is learning, the classroom environment will be characterized by effective teaching of problem-solving as a process, particularly through heuristics, which are general rules and lines of procedural skills that help the problem-solver understand and find solutions to a given problem (Good & Brophy, 1990). However, it should be noted that teachers' beliefs and classroom practices are based on their beliefs in the process.

During the problem-solving process, it is also necessary for teachers to take on more responsible roles to facilitate students' understanding of methodologies and processes. One effective way for teachers to become more useful is to have more professional growth, development and training opportunities. Kaur (2001) believes that increasing teachers' support level through appropriate learning experiences is one strategy that can address the lack of problem-solving approaches in teaching.

Kaur (2001) revealed that with particular reference to Singapore, where educators focus heavily on problem-solving, the government ensures that teachers complete a hundred hours of in-service training each year to ground them firmly in delivering the curriculum. This puts teachers in a good position to be more confident in the classroom and more knowledgeable about choosing friendly approaches to solving seemingly complex problems. The teacher's role in helping students develop the needed problem-solving skills also focuses on another key area: the facilitators' ability to engage in what Jaworski (1994) described as investigative learning. Jaworski (1994) also called this the "teaching triad", describing it as teaching that offers students the chance to explore issues through active interaction with students.

2.15 Problem-solving approaches

Three approaches to problem-solving in mathematics education have been identified in the problem-solving literature. They are: teaching for problem-solving, teaching about problem-solving and teaching through problem-solving (Anderson, 2000; Fong, 2002; Schroeder & Lester, 1989; Siemon & Booker, 1990). Each of these approaches has implications for the types of activities and strategies that exist for students in mathematics lessons. All the three approaches involve the use of problem-solving strategies and heuristics. However, problem-solving learning sees problem-solving as a process of inquiry, while problem-solving teaching and problem-solving teaching see problem-solving as the object of inquiry. It has been argued that teaching has a place for all three approaches in mathematics, although teaching through problem-solving is considered the most appropriate. Schroeder and Lester (1989) also emphasized that all three approaches are important, but teachers should be aware of the shortcomings of problem-solving instruction when used in isolation. Schroeder and Lester (1989) argued that when teaching problem-solving, problems can be reduced to the application of recently learned concepts and do not require deep mathematical thinking from students. Schroeder and Lester (1989) also suggest that problem-solving can be treated as a second-course topic by teaching about problem-solving. Finally, Schroeder and Lester (1989) recommended that teaching through problem-solving most likely promotes understanding.

2.15.1 Teaching for Problem-Solving

Teaching for problem-solving involves students learning mathematical content so that they can use it to solve problems related to that content area (Anderson, 2000; Fong, 2002). In this approach, teachers provide students with the needed skills and knowledge to solve mathematical problems. Problems usually relate only to the mathematical content studied and students are provided with various applications in which mathematics can be used (Anderson, 2000). In teaching for problem-solving, the focus is on learning mathematics to use it to solve problems in a wide range of situations to learn a specific situation after learning a particular subject. This approach is often linked with closed-end problems in the context of an explicitly formulated task, where in the case of a problem, a definitive answer can always be determined in some way from the required data given. These closed problems will include content-specific routines, multi-routine problems, and non-routine-based problems (Fong 2002).

2.15.2 Teaching about Problem-Solving

Teaching about problem-solving includes guidance about problem-solving processes and instructions about various problem-solving strategies. It often includes Polya's (1985) recommendations for problem strategies. When teaching about problem-solving, students learn to use various problem-solving strategies or heuristics, such as drawing a list, drawing a diagram, drawing it out, solving a similar problem and estimating and checking (Anderson, 2000). In teaching about problem-solving, the emphasis is on using heuristic strategies approach and solving unfamiliar problems that are usually not domain-specific to any topics in the syllabus. It involves using non-routine problems to teach thinking skills and problem-solving heuristics (Fong, 2002)

2.15.3 Teaching through Problem-Solving

According to Aydogdu and Ayaz (2008), teaching through problem-solving will allow students to construct their mathematical concepts and take responsibility for their own learning. Teaching through problem-solving focuses more on student understanding. This approach uses problems to learn mathematics (Anderson, 2000; Fong, 2002). It makes problem-solving a means to an end rather than an end. Learning through problem-solving starts with a problem. Teachers pose problems to challenge students' knowledge, thus providing students with the need to organize their understanding in order to solve the problem (Anderson, 2000). Students gain knowledge and understand key ideas in concepts by examining a problem state (Cai, 2003).

In problem-solving teaching, problems come from organizational focus and excitement for student learning and serve as a vehicle for exploring mathematics (Cai, 2003). Students actively participate in their learning by exploring problem situations with leadership and "inventing" their solution strategies. Student exploration of a problem is essential to teaching and learning through problem-solving. In problem-solving learning, learning occurs during the problem-solving process. When students solve problems, they use whatever approach they can think of, rely on whatever piece of knowledge they have, and justify their ideas in ways they believe to be certain (Cai, 2003). The learning environment of problem-solving learning provides a natural environment for students to present multiple answers or solutions to their group or class and learn mathematics for social interaction, semantic interaction, and shared understanding (Cai, 2003). Such activities help students articulate their thoughts and gain different perspectives on the concept or idea they are learning. Corte (2000) stated that learning through problem-solving is process and strategy-oriented compared to the product. According to Baki (2004) and Chapman (2008), problem-solving learning involves creating an environment where students can discuss their ideas about a problem and explain ways of questioning and generalizing to classmates. Van de Walle (2004) stated that learning through problem-solving requires students to read the problems carefully, analyse them to find any information, and then look at their mathematical knowledge to develop a strategy that can help them find a solution. This process focuses on structuring already existing ideas and developing new ones. The student solves problems with the help of the teacher, who serves as a facilitator by asking questions to help students review their knowledge and make connections. The teacher's role in this approach is essentially non-decisive, not dictatorial. Rather than being the sole source of knowledge and solutions, it creates a classroom climate and culture that supports and facilitates student initiative and stimulates interactive and collaborative problem-solving (Corte, 2000).

Therefore, Norton, MacRoby and Copper (2002) see problem-solving mathematics teaching as an approach in which teachers see themselves as guides, listeners and observers rather than authorities and providers of knowledge and information.

However, teaching mathematics through problem-solving is a relatively new concept in the history of mathematics curriculum that emphasizes problem-solving (Cai, 2003).

2.16 Varying Perspectives on the Goal of Problem-Solving

For several years, both mathematicians and mathematics educators have presented various perspectives on problem-solving within the realm of mathematics education (Yuan, 2016). Teaching problem-solving can generally be categorized into two main viewpoints: direct teaching and constructivist teaching. Lester (2013) notes that the debate surrounding the merits of these two perspectives has persisted over time, influencing the design of mathematics programs and shaping the instructional approach employed in classrooms. Yuan (2016) contends that the primary distinction between direct and constructivist instruction lies in the ultimate objective of the problem-solving task. Lester (2013) stresses that if problem-solving is the intended outcome of learning, then students should engage in learning about problem-solving itself. Similarly, if problem-solving serves as the method through which mathematical concepts are grasped, then students should learn through problem-solving activities.

2.17 The Role of the Teacher in Mathematics Problem-Solving Instructions

A teacher attains excellence in their profession by demonstrating sincerity and enthusiasm, wholeheartedly fulfilling their responsibilities and commitments. The role of a teacher as a facilitator involves encouraging and aiding students with the necessary resources to become more independent and focused on overcoming challenges. This role is crucial in the context of constructivist learning, where knowledge is actively constructed within the student's mind and continuously evolving (Harden & Crosby, 2000). The facilitative teacher guides this process, creating suitable situations for students to acquire the desired knowledge, rather than merely supplying information. The teacher facilitators task is to optimize students' learning experiences through diverse strategies, methods, media, and resources.

Brody and Davidson (1998) underscore the facilitative role of the teacher, whether offering direct assistance or working collaboratively with students. The learning process comprises two centres: Teacher-Centred Learning (TCL) and Student-Centred Learning (SCL). Matheson (2012) highlights the shift in the teacher's role from being a provider of information to knowing when and how to deliver pertinent information, especially when mathematics poses challenges for learners. Alsawaie (2003) stresses the importance of teachers recognizing when learners require assistance versus when they can make productive progress independently. Learners should be given opportunities to explore problems and make discoveries in mathematics. Offering help prematurely can hinder their capacity for independent exploration. Moreover, learners should understand that complex problems demand time and persistence, crucial elements of mastering mathematics and problem-solving. Sweetland (2016) outlines the teacher's role during problem-solving, advocating for the provision of hints rather than direct answers, instruction in various problem-solving strategies, allowing learners ample time to grapple with challenges, presenting problems in diverse formats, and modelling effective problem-solving. The teacher should offer encouragement and recognition, promoting multiple solution approaches, resilience, error correction, and appreciation of diverse strategies. This facilitative role commences before problem-solving and extends throughout the teaching and learning journey (Alsawaie, 2003).

2.18 Teachers' Knowledge for Incorporating Problem-Solving in Instruction

Teachers have reported that they lack the knowledge and skills they need to meaningfully incorporate a problem-solving approach into their teaching of mathematics (Buschman, 2004; Cai & Lester, 2010). Little empirical research has been done in Jamaica and the wider Caribbean to help teachers gain greater insight into how they can incorporate problem-solving in their classrooms to advance better students' problem-solving competence (Lester & Cai, 2015). However, the research literature has been published elsewhere that largely focuses on the different ways teachers integrate problem-solving in their classrooms and the challenges faced in using a problem-solving approach (Lester, 2013; Lester & Cai, 2015; Polya, 1945).

There are varied ways in which mathematics teachers can guide students' engagement with mathematical problems. The first and perhaps most important relates to selecting a “good” or worthwhile problem (Cai & Lester, 2010; Lester, 2013). Good problems can "inspire the exploration of important mathematical ideas, nurture persistence, and reinforce the need to understand and use various strategies, mathematical properties, and relationships" (National Council of Teachers of Mathematics, 2000, p. 182). And also concretize and extend students' prior knowledge; promote understanding of concepts; foster mathematical reasoning, discourse and communication; and capture interests and curiosity.

Both Cai (2003) and Buschman (2004) agree that the use of good problems is vital but posit that the success or failure of teaching using a problem-solving approach is not solely dependent on this. Buschman (2004) explains that everyday classroom activities can provide the basis for excellent problems, but it is imperative that teachers "listen to what children say and do and then help them to become aware of the mathematics embedded in these events" (p. 304).

In addition, the teacher should help students to understand the problem, including: posing questions that allow learners to extract essential information from the problem, helping learners to determine the goal of the problem, reconstructing the problem if necessary, and identifying suitable mathematical notations for easy reference and manipulation (Fan & Zhu, 2007). And also argued that after students have understood the problem, the teacher can help them decide on a possible plan of action that may lead to a solution; encourage them to "carry out the plan of action", and to persist with the plan if it is leading to an appropriate solution; but discard the plan and try a new one if the initial plan is not leading to a solution. Also, a teacher might encourage students to reflect on the plausibility of their solution and make judgments where necessary.

2.19 Learning Engagement in Problem-Solving

There are many conceptual and operational definitions of learning engagement. However, it generally refers to active participation and involvement in learning activities and is a motivational variable that captures the quality of learning (Skinner, Kindermann, Connel, & Welborn, 2009). Learning engagement is often divided into three broad categories: behavioural, emotional and cognitive (Fredricks, Blumenfeld & Paris, 2004). Behavioural engagement is behavioural involvement in learning, such as initiating action, effort, exertion, and hard work. Emotional engagement is a positive emotional response to learning, such as enthusiasm, interest, and pleasure. Cognitive engagement is cognitive involvement in learning, such as striving for goals and seeking strategies. Behavioural, emotional, and cognitive engagement in learning mathematics has been shown to positively influence academic performance in mathematics and performance in solving mathematical problems. Student-engaged learning does require the teacher to play an important role. Actually, teachers are key players in fostering student engagement, (Akey, 2006; Garcia-Reid & Pert, 2005) however, not the role that the teacher-centred educator plays. Instead, teachers should take on the responsibility of creating a culture of achievement in their classrooms, developing interactive and relevant lessons and activities, and being encouraging and supportive to students, these are all ways in which teachers can foster student engagement in the classroom. The teachers create opportunities for students to work cooperatively, to solve problems, do authentic tasks, and construct their own meaning. The teachers learn along with the students". In order for engaged learning to take place there must be certain freedoms granted to students by teachers. In an engaged learning environment, the students are allowed to explore, discover, and interact. According to Talbot (1998), Students should have choices in learning activities whenever possible, and they should be allowed to formulate questions and explore topics that interest them. Another responsibility of educators which is extremely vital to student achievement and engagement yet does not necessarily take place inside a classroom is that of developing professional learning communities among staff to ensure that teachers develop the skills they need to provid these conditions (Klem & Connell, 2004).

An environment that promotes encouragement and achievement as opposed to humiliation and competition is critical in engaging students. A classroom culture where instruction is challenging, students feel comfortable asking questions, and students are expected to do their best is an integral piece of the puzzle (www.centerforcsri.org newsletter, April 2007). In many cases, teachers give students the impression, through negative body language and offering responses to student inquiries that are highly destructive, that asking questions when they do not understand the material is not alright. Teachers should aim to create a culture in the classroom where learning is cool, and asking questions is not only okay but expected (Akey, 2006). An environment where being wrong leads to an onslaught of laughter and ridicule is a detriment to engaged learning and students being willing to take chances when learning something new. This type of classroom environment may take some time to develop, but it can be done if the teacher sets clear expectations and models appropriate responses to questions, encouragement after student mistakes, and appropriate praise after student triumphs (Akey, 2006).

2.19.1 Active Engagement of Learners in Problem-Solving

Klem and Connell (2004) strongly advocate for active student involvement in the educational process. And emphasised the significance of universally engaging all students, asserting that numerous studies underscore the positive outcomes associated with increased school engagement. This connection between engagement and favourable results, such as enhanced academic performance, remains consistent regardless of a student's economic or social background. And also contend that student engagement serves as a robust predictor of both academic achievement and behaviour within the school context. Students who exhibit higher levels of engagement tend to attain superior grades and test scores, while also displaying lower rates of dropping out. Conversely, students with a low level of engagement face a multitude of potential negative consequences in the long term, including disruptive classroom behaviour, frequent absences, and ultimately leaving school early (Klem & Connell, 2004).

2.19.2 Cooperative Learning Strategies in Problem-Solving

"Engaged learning fosters collaborative opportunities for students to collectively pursue objectives. Learning groups are assembled based on instructional goals, shared requirements, and mutual interests. Through cooperative endeavours, students can cultivate social aptitudes and the ability to solve problems together" (Judy, 1999 as referenced in Janes, Koutsopanago, Mason, & Villaranda, 2000, p.28).

Judy Willis, a medical doctor, neurologist, and educator at Santa Barbara Middle School in California, offers her perspective on collaborative learning: "I realized that relinquishing traditional authoritative control and granting students the chance to collaboratively work with peers towards shared accomplishments led to a heightened sense of investment and engagement in our learning journey" (Willis, 2007, p.4).

Cooperative learning emerges when teachers step back from solely being the primary source of information and knowledge in the classroom. However, they still establish behavioural guidelines and interaction frameworks while maintaining high expectations for students. According to Goodwin (1999), within such a context, small groups of students engage in discussions and collaboratively assume responsibility for their own learning. The emphasis lies on cultivating team spirit over competition as students collaborate.

The objective of cooperative learning is to foster positive interdependence, making the group's success contingent on each member achieving both their individual learning goals and the group's collective objectives (Hall, 2006). This instructional approach empowers students to hold each other accountable and encourages them to take on roles aligned with their skills and interests, rather than feeling compelled to participate solely at the teacher's behest. These opportunities, encompassing mutual monitoring of participation and work quality, alongside the freedom to choose roles, capture students' attention and generate higher levels of investment in assignments and projects. Engaged learning experiences within well-structured cooperative groups effectively lower affective barriers, facilitating the flow of information. When groups are formed in a manner that assigns genuine significance to each member's strengths in achieving the collective task's success, a setting is created where individual learning styles, talents, and proficiencies are valued. In this dynamic, students excel in their areas of expertise while also learning from their peers in domains where they may be less adept (Willis, 2007). Cooperative learning groups offer numerous additional benefits that contribute to engaging students in the learning process. It cultivates a positive academic discourse among students and provides a receptive setting for sharing ideas and viewpoints. One significant drawback of a classroom where students passively listen to teacher lectures and independently practice skills is the lack of interaction among students, resulting in unexpressed thoughts and ideas. The misconception that silence equates to productive learning is erroneous. Conversely, genuine and valuable learning occurs when students collaboratively share their ideas, tackle problems as a team, and receive constructive feedback and encouragement from both peers and instructors.

In contrast to whole-class discussions or solitary work, cooperative learning group activities offer the most extensive platform for students to vocalize their thoughts, questions, conclusions, and associations. These activities prompt students to seek each other's input to solve relevant and intriguing challenges, fostering interpersonal skills through effective communication with partners (Willis, 2007). Cooperative learning groups offer multiple advantages to students, encompassing improved exam scores, enhanced self-esteem, heightened engagement, refined social skills, and a more comprehensive grasp of subject matter and competencies. Given the positive outcomes attributed to this pedagogical approach, many educators have endeavoured to integrate this methodology into their teaching practices.

2.19.3 Problem-Based, Project-Based, and Experiential Learning-Authentic Learning

Problem-Based Learning, Project-Based Learning, and Experiential Learning are instructional strategies that have much in common, primarily their emphasis on fostering critical thinking and involving students in genuine learning tasks. Often grouped under the umbrella of inquiry-based learning, these instructional approaches encourage active engagement. Carlson (2001) describes "authentic learning" as a concept encompassing real-world issues and projects that empower students to explore and discuss these matters in ways relevant to their lives. This essence underscores the significance of these methods, including similar ones, in promoting engaged learning. These strategies enable students to absorb content in manners that resonate with their actual experiences, both within and beyond the classroom (Carlson, 2001). As stated by Eggen (2001), problem-based learning centred around using a problem as a focal point for students to delve into investigation and inquiry (Hall, 2006). While comprehending the subject matter is important, it's equally crucial for students to actively participate in devising potential solutions to the presented problem. Project-based learning mirrors problem-based learning by emphasising personally meaningful learning experiences for students, positioning them as active participants in their learning process (Yarnzon, 1999).

The primary distinction between problem-based learning and project-based learning lies in the fact that project-based learning grants students the opportunity to select the topic or subject of interest they wish to explore, which may or may not involve addressing a specific problem. Katz and Chard (1989), asserts that when students have the freedom to choose their learning focus, their enthusiasm for that subject is generally higher compared to when topics are assigned to them (Yarnzon, 1999). An additional instructional approach rooted in authentic learning tasks is experiential learning. This method aims to actively involve students in the learning process, enabling them to comprehend and retain course content. It also offers a platform for community engagement or service, exposes students to potential career paths, and cultivates skills in decision-making, teamwork, communication, and problem-solving (Pierce & Adams, 2004, cited in Hall, 2006). Experiential learning involves students in immersive experiences through which they collect information, reflect on it, and construct understanding based on hands-on activities, diverging from conventional lecture-based learning. Within this experiential learning environment, students' ideas and preferences are respected, granting them the agency to navigate challenging situations and confront hurdles within a supportive and caring atmosphere (Hall, 2006).

2.19.4 Technology in the Classroom: Intriguing and Attracting Student Interest

The adoption of information technology in schools is swiftly expanding nationwide, encompassing not only communication and security but also an array of inventive tools and services that actively involve both students and teachers in the learning process (Carless, 2008). According to research complementing Carless (2008) article, students hold specific expectations regarding technology, to the extent that if schools are astute enough to meet these expectations, they stand a better chance of effectively engaging students. This engagement is pivotal for sustaining their interest and ultimately aiding them in achieving their educational goals (Carless, 2008). It's crucial to recognize that merely assigning tasks to students using computers for actions that could be accomplished without technology does not qualify as genuine technology integration within the curriculum, nor does it effectively engage students. However, when technology is creatively employed and integrated in meaningful ways, it has the potential to deliver captivating learning experiences for students (Carless, 2008).

2.19.5 Student Self-Assessment Learners Taking Ownership in Problem-Solving

Self-assessment has demonstrated many benefits within the classroom, including heightened motivation, enhanced performance, and increased engagement with the subject matter. Geoff Munns and Helen Woodward from the University of Western Sydney propose that cultivating more profound levels of student self-assessment is a vital component of pedagogical approaches aimed at encouraging students to be actively involved in their classroom learning (Munns & Woodward, 2006). Self-assessment provides students with chances to express their views and sentiments regarding their learning to both teachers and peers. This empowers them with a voice and allows them to assume a major role in shaping their learning experiences. Through self-assessment, students engage in metacognition, contemplating what they comprehend and what they find challenging, enabling them to effectively communicate their level of understanding to teachers throughout the learning process.

2.19.6 Curiosity, Motivation, Perseverance and Building on R.P.K in Problem-Solving

These enthusiastic learning tendencies are integral for skill development. Curiosity, akin to fundamental needs, can be defined as the aspiration to discover, understand, sense, or encounter, propelling the quest for new knowledge, much like essential requisites (Litman, 2005). Notably, individuals, particularly children, tend to learn more effortlessly and swiftly when they pursue what intrigues and captivates them. The role of problem-solving in mathematics holds a significant place in education and remains a focal point of attention in educational recommendations for school-level mathematics (NTCM, 2000). The multifaceted benefits of mathematics education at primary, secondary, and subsequent levels extend across social, cultural, and economic domains. Consequently, cultivating a profound understanding of mathematics from early childhood and integrating it into daily life holds immense significance. Within the realm of educational psychology, the concept of curiosity encompasses concentration, heightened cognitive functions, persistence, and emotional investment. This psychological state empowers children to channel their focus towards a specific activity, fostering motivation and transforming aimless curiosity into a conscious quest for knowledge, particularly within Montessori classrooms (Soydan, 2013).

In the realm of constructivist teaching, teachers should partake in research and discovery with a shared enthusiasm, kindling the same excitement within their students. Teachers should be adept at acknowledging their students' modest yet substantial inquiries and discoveries, nurturing their inherent curiosity. Mathematical curiosity plays a pivotal role in generating and analysing problems that arise after a solution being reached, facilitating the development of experiences that many students might not otherwise encounter. Learning motivation, a pivotal factor is often linked with problem-solving, exerting influence on cognitive processing's initiation, direction, and intensity (Baars, Wijnia & Paas, 2017). Motivation to engage in problem-solving tasks can originate from the learners themselves or be prompted by the nature of the task's design. In this context, Muenks, Miele, and Wigfield (2016) differentiated between self-initiated effort, stemming from learners' intrinsic motivation, and task-elicited effort, arising from the perceived difficulty of the task. Cultivating persistence among students is a gradual process that requires deliberate effort, yet there are strategies to facilitate their perseverance in the face of challenges. First, establish a classroom atmosphere that encourages students to undertake ambitious goals and recognizes mistakes as integral to the learning journey. Second, foster a growth mind set by conveying the idea that intelligence is not fixed and equipping students with specific strategies to employ when they encounter obstacles. By addressing both the learning environment and students' individual resources, educators can nurture perseverance, thus laying the groundwork for profound learning experiences. Prior knowledge, encompassing the information an individual gather before encountering a specific situation or context, holds vital significance.

In learning settings, individuals inherently possess prior knowledge, which educators can tap into to establish connections with learners and establish a sturdy foundation for comprehension. At the outset, instructors can employ diverse assessments to activate students' existing knowledge and ascertain their individual understanding. These assessment methods might include concept inventories, brainstorming exercises, self- or peer evaluations, and concept mapping. When students enter higher education, they bring with them years of educational background and personal narratives that influence their response to the learning process. Research in educational psychology stresses that students' prior knowledge directly influences their learning outcomes within the classroom. For instance, learners who possess a more extensive academic background tend to achieve greater academic success at the university level (Kurlaender & Howell, 2012).

2.19.7 Manipulatives Materials in Problem-solving

Sowell (1989) asserts that children comprehend mathematical concepts best when exposed to concrete, concrete-abstract, and pictorial-abstract learning encounters before encountering strictly abstract experiences. Therefore, learning experiences should be thoughtfully organized under the sequence of cognitive developmental stages. The application of cognitive development theory in the classroom context necessitates the inclusion of both concrete and symbolic models within the learning environment. This ensures that children at varying developmental levels can benefit from the educational approach (Fennema, 1972). Yıldız (2012) conducted a qualitative case study aimed at exploring the perspectives of middle school teachers and students on the utilization of manipulatives in teaching and learning mathematics. The study employed various manipulatives, including base-ten blocks, fraction bars, pattern blocks, geoboards, four-pan balance, and algebra tiles. The participants comprised four middle school math teachers and their 6th, 7th, and 8th-grade students within a private school.

The study's findings indicated that a significant number of middle school students expressed a strong preference for learning mathematics through manipulatives, which facilitated playful yet educational experiences. Students attested that using manipulatives contributed to positive attitudes toward mathematics and enhanced conceptual understanding. Manipulatives offer several advantages for students; however, specific considerations must be taken into account when incorporating them into mathematics classes. Firstly, there are potential challenges tied to using manipulatives in the classroom, including the possibility of students utilising them for play rather than for completing assignments.

Additionally, the distribution and collection of manipulatives can consume a considerable amount of instructional time. Consequently, before introducing manipulatives, teachers should evaluate the time allocation and acknowledge the risk of manipulatives being used as mere toys (Magruder, 2012). Secondly, the accurate utilization of manipulatives holds paramount importance. Effective learning through manipulatives is not guaranteed if they are not employed appropriately. To ensure their effectiveness, manipulatives should bridge the gap between informal and formal mathematics, align with students' developmental stages (Smith, 2009), and match their mathematical proficiency (Boggan, Harper, & Whitmire 2010 as cited in Caylan, 2018). Students must grasp the mathematical principles underlying manipulatives, moving beyond viewing them as mere toys. Therefore, teachers should allocate time for students to engage with manipulatives before delving into the associated concepts.

2.20 Challenges Encountered by Mathematics Teachers When Teaching through Problem-Solving

Problem-solving methods often contrast with teachers' teaching methods in the classroom or traditional lecture. Kim (2005) described conventional teaching methods as “methods that include the following steps: introduction, development, and review” (p. 13). Similarly, Akhter, Akhtar, and Abaidullah, (2015) argued that traditional teaching usually develops memorization and assessment of the student's content knowledge. Chauraya and Mhlolo (2008) surveyed other mathematics teachers regarding their problem conception and problem-solving in the subject. And found that the respondents' conception of a mathematical problem was one with an explicit solution procedure or one that required the application of learned and clear skills and procedures. With such a conception, mathematics problems in the classroom are likely to be limited to the type of exercises and applications encountered in most standard mathematics textbooks.

Zanzali (2003) stated that exams that define how and what should be taught in mathematics influence teachers' perceptions of problem-solving and its relevance. Chauraya and Mhlolo (2008) argued that teachers have little influence over the mathematics content of the curriculum and therefore believe that its reforms an adaptations are beyond their control.

Additionally, Zanzali (2003) argued that the perceptions of most teachers are still traditional despite various efforts by the Ministry of Education's Centre for Curriculum Development to transform them. Akhter, Akhtar and Abaidullah, (2015) argued that the lack of time and the vast amount of material for students to learn make it difficult always to use problem-solving in all elements of the teaching process. Akhter, Akhtar and Abaidullah (2015) argued that due to the large number of students in the class, which results in the need for more time than is usually expected, both in terms of preparation and implementation. In addition, "the problem-solving method is not properly integrated into a curriculum that relies heavily on textbooks and an assessment system overloaded with formal examinations that reinforce memorization skills" (Akhter, Akhtar & Abaidullah, 2015; p.4). The challenges of teaching mathematics through problem-solving discussed in the research literature can be divided into three broad categories. These are questions about teachers, students and the school curriculum.

2.20.1Challenges Related to Teachers

Elementary school mathematics teachers are trained to be generalists and often lack the mathematical knowledge to teach using problem-solving techniques (Swars et al.,2018). As educators, they won't have enough information to think about anything other than curriculum goals or instructional boundaries, so it can help them find a problem-solving orientation (Xenofontos, 2007). The situation in Ghana is no different from Xenophontos (2007) views on teacher reform. This phenomenon can lead to teachers not being equipped with the content and pedagogical knowledge to teach mathematics using problem-solving strategies. Xenofontos (2007) also adds that problem-solving methods require extensive preparation and development of methods to maintain at least a classroom management mode and, most importantly, the ability to anticipate the mathematical goals of instruction while attracting this attention. Mereku (1998), in a research study on the problem of language in solving problems in elementary school calculation, noted that Problem-solving is not common in primary schools in Ghana because most teachers do not know how to introduce it in the classroom; they cannot solve problems themselves; nor can they explain why students find it difficult to learn to solve problems. Mereku (1998) also pointed out that teaching through problem-solving is difficult for teachers. McIntosh, Jarrett, and Peixotto (2000) argued that teaching mathematics through problem-solving is difficult for teachers because they lack sufficient subject matter knowledge, and personal issues.

Additionally, teachers lack the mathematical background to understand the different methods students can use to solve a problem and identify promising ways to solve problems. However, teachers often provide strong reasons for not including problem-solving activities in their mathematics instruction, which include: teaching takes a lot of time; it is highly sought after and not measured and evaluated in public examinations. Jarrett, and Peixotto (2000) also note that teachers are often expected to teach large areas of mathematical content, but problem-solving takes more time; it is highly sought after and not measured and evaluated in public examinations. As a result, many teachers often feel unprepared to use the problem-solving method in teaching mathematics. In addition, teachers often find it difficult to see their students struggle with problem-solving frustration when they have to advise and intervene.

Another basis for teachers' struggles with implementing problem-solving in their classes may be a lack of the knowledge and skills to incorporate problem-solving meaningfully into the mathematics curriculum (Buschman, 2004; Cai & Lester, 2010). While some teachers acknowledged that enabling student interactions was a critical component of using the problem-solving approach, they were unsure how to foster (or promote) this mathematical knowledge about teaching is essential to effective teaching (Ball & Bass, 2000). Currently, some teachers lack the necessary knowledge, skills, and competencies to teach mathematics through problem-solving (Anderson, 2000). Matlala's (2015) revealed that mathematics teachers found it difficult to teach through problem-solving approaches and that the teachers still taught using the traditional approaches, including storytelling and stepping in to show learners how to solve mathematical problems. A lack of mathematical knowledge about teaching undermines teachers' confidence in teaching mathematics through problem-solving. These teachers rely on traditional ways for students to memorize rules detrimental to learning to gain meaningful problem-solving knowledge. And limited ownership of mathematics content in the curriculum among teachers does not encourage teachers to use problem-solving (Anderson, Sullivan, & White, 2004). Foong, Yap, and Koay (1996) noted that many teachers expressed their concern that they did not have the teaching skills to solve problems in mathematics. For example, teachers feel inadequate in their problem-solving methods, especially for unusual problems. Teachers were also interested in their ability to communicate new ideas to their students for understanding using the various methods proposed in the curriculum change. Teachers also pointed out that several teachers expressed concerns about their ability to think about the right question at the right time and engage students in discussions when using the problem-solving method to teach content set out in the mathematics curriculum. The teachers in this study (Foong, Yap, & Koay, 1996) noted that this problem-solving approach put the teacher in a good mood and noted that the teacher needed to understand the topic to be covered thoroughly. Teachers' beliefs influence their practices and decisions in the classroom. For example, the general belief that mathematics is missing from the knowledge base and should be systematically presented to students is a major obstacle for teachers who want to teach mathematics using problem-solving methods (Anderson, Sullivan, & White, 2004). Some educators believe more in classroom management than in learning to think (Zanzali, 2003). These categories of teachers believe that the best way to learn mathematics is to do general problems while students sit quietly and listen to what they say (Zanzali, 2003).

Anderson (2000) argued that parents' expectations of test scores pressure teachers to teach assessment instead of using problem-solving strategies to teach critical thinking, a symptom of problem-solving. Another comparison can be found in the variety of methods students can use to solve a mathematical problem (Burton, 2002) and the variety of solutions commonly found for a particular problem. If teachers are not pedagogically adept at meeting the challenge of distinguishing among the multiple strategies that students will use to solve mathematical problems, they prefer to avoid it. Teachers may find it challenging to teach using a problem-solving strategy because it goes against their core ideas about teaching mathematics and the instructor's role in the learning process (Lester & Cai, 2015). For example, suppose a mathematics teacher thinks that learning mathematics is all about memorizing and regurgitating facts and procedures. In that case, they will not be motivated to spend time on problems that do not require applying procedures. Aligned to this, Sakshaug and Wohlhuter (2010) reported that teachers tend to teach in like manner as they were taught (Battista, 1994; Oleson & Hora, 2014), and the way that they were taught often varies considerably from teaching through problem-solving. Lack of knowledge may be another reason why teachers fail to include problem-solving in teaching students and the skills necessary for effectively integrating problem-solving into the mathematics curriculum (Buschman, 2004; Cai & Lester, 2010). While some teachers agreed that encouraging student involvement was essential to use the problem-solving strategy; they were uncertain how to foster (or promote) this. To compound this challenge, there is little empirical research to guide teachers on the best way to incorporate problem-solving into the classroom in order to increase students' development of problem-solving skills (Lester & Cai, 2015).

2.20.2 Challenges Related to Students

A study by Henderson (2002) investigating faculty perceptions of teaching and learning Problem-Solving in Applied Mathematics described poor student knowledge/problem-related skills. Research conducted by Adesoji (2008) shows that high-performing students have a better understanding of problem-solving. Therefore, teaching mathematics through problem-solving is very easy. However, low-ability people can replicate their problem-solving skills when exposed to a problem-solving instructional strategy (Adesoji, 2008).

In a research study, Saleh (2009) confirmed students' knowledge base as a determinant in learning mathematics through problem-solving and concluded that problem-solving is not good for low-ability students. Anderson (2000) also found that students are sometimes confused with other approaches to solving mathematical problems. Such students are more resistant to teacher processes or programs that use problem-solving methods in teaching mathematics rather than being guided by the teacher to assess and build their understanding. It also finds that differences in classrooms, students understanding of languages and their context, and their beliefs about mathematics are major factors that can influence the implementation of the mathematical problem-solving process. Students' inability to read and understand teachers teaching mathematics using problem-solving strategies presents yet another challenge for teachers. Fletcher and Santoli (2003) stated that mathematics vocabulary is usually not taught in schools and unless students read good textbooks, they have no place to understand mathematical terms. Therefore, it is very important to emphasize vocabulary instruction as part of the mathematics program if students are to learn mathematics through complete problem-solving. Cai (2003) presents several challenges relating to students. And claims that students frequently strongly believe that there is only one "right" way to approach and solve a problem. This is not always the case because students may select various strategies to solve a particular problem. For example, to solve the problem, while one student may choose to write an equation, another may choose to use a table. Despite their different methods, both students can reach the same solution. Furthermore, various assessments show that a large number of students, "do not view mathematics as a creative and intellectually engaging activity, but rather as a set of rules that they must memorize in order to quickly pursue the single correct way to arrive at the single correct answer (Cai, 2003, p. 10). Whether students are open to learning will always depend on how they conceptualize mathematics or are resistant to engaging with mathematical problems. In the same way that teachers' beliefs influence their teaching. Likewise, students' views of mathematics strongly impact how they engage within the mathematics classroom. Problem-solving demands critical thinking and perseverance, and some students will fail at problem-solving since they perceive mathematics as a set of rules and procedures where they are not required to struggle and engage in deep reasoning. Notably, students' view of mathematics is regularly developed from their experience of learning mathematics (Cathcart, Pothier, Vance, & Bezuk, 2001). Therefore, based on how they have been taught mathematics over the years, students who come to see mathematics as a set of rules frequently reach this conclusion.

Tambychik and Meerah (2010) examined the main mathematics skills and learning cognitive abilities that a sample of 107 kids, aged 14, described as being difficult for them to employ in solving math problems. This study's key finding was that students struggled to solve problems successfully because of their poor content knowledge and related skills. This finding concurs with Lester (2013), who claims that having a solid grasp of mathematical concepts is essential for effective problem-solving. It may seem ideal in theory to let students approach problems in a way that makes sense to them since students have diverse backgrounds. Higher-order thinking and other specialized mathematical skills are needed for problem-solving. Students who lack the mathematical knowledge and abilities necessary for problem-solving frequently grow frustrated when require to participate.

2.20.3 Challenges Related to the School Curricular

The educational system faces several different challenges. Using a questionnaire and an in-depth interview to collect their data, Akhter, Akhtar, and Abaidullah, (2015) investigated the perception of problem-solving of 100 high school mathematics teachers from Pakistan. Akhter, Akhtar, and Abaidullah, (2015) discovered that the teachers cited challenges relating to preparation, a lack of training for teachers on implementing the strategies, and a lack of resources.

Additionally, their respondents claimed that a problem-solving strategy is incompatible with "a curriculum that over-relies on the textbook and an assessment system overloaded with formal examinations which reward recall skills" (p. 1). Another difficulty is that compared to when students work on routine problems, using a problem-solving technique demands more time to solve and discuss the problems (Cai, 2003; National Council of Teachers of Mathematics, 2014). Further, Cai and Lester (2010) note that students' problem-solving abilities frequently progress slowly, demanding long-term, sustained attention to making problem-solving an integral part of the mathematical programme. Since many teachers already believe they do not have sufficient time to cover the dense mathematics curriculum content using traditional approaches, it is not surprising that they are unsure whether teaching using a problem-solving approach is compatible with their present education system. Rickard (2005) found in his research that giving teachers a chance to dialogue with others who have more knowledge was the most important component needed to effectively implement a problem-solving teaching strategy and reflect on what happened in the classroom. This component is frequently absent in the educational environment. This could be one of the reasons contributing to the frequent ineffectiveness of problem-solving-based instruction.

McIntosh, Jarret, and Peixotto (2000) in a review of textbooks that teach mathematics through problem-solving found that most books did not provide a sufficient number of non- external problems for teachers to choose from. This affects how teachers use mathematical problem-solving strategies because they rely heavily on textbooks as a source of information. Ali, Hukamdad, Akhter, and Khanyi (2010) in a study commissioned to investigate the effect of problem-solving on student achievement in lower-grade mathematics education noted that traditional textbooks do not meet problem-solving methods. This happens in books that do not provide enough problem-solving questions with the potential to prevent teachers from teaching mathematics using problem-solving methods. The traditional teaching methods of other teachers in the school and parents' demands that their wards be prepared for competitive examinations were other factors identified as barriers to implementing problem-solving instruction in mathematics classrooms. As Anderson (2000) noted, school culture can sometimes hinder new educational practices. School curricula, mathematics textbooks, dissertations, evaluation methods and staff positions are some problems a school may face when implementing solutions to mathematical problems. School culture can interfere with teachers' planning and methods because of approved learning methods and the traditional beliefs of other staff. At school, many things compete for time. These include the compulsory school curriculum, the external assessment process and school curriculum obligations. Though, teachers believe that problem-solving teaching requires a lot of time; if they don't have enough time, they prefer to switch to teaching mathematics through storytelling. Teachers have expressed a lack of available problem-solving resources (Foong, Yap, & Koay, 1996). The research literature has stressed the importance of problem-based learning and problem-solving learning for students, putting pressure on teachers to increase their students' numbers, making them stick to textbooks instead of using problem-solving methods (Traiton & Midgett, 2001).

2.21 Traditional Method of Teaching Mathematics

Various instructional approaches, such as traditional and problem-solving methods, are utilized globally within the educational system. The Platonist, formalist, behaviourist, and structuralist models of teaching mathematics are classified under the traditional teaching paradigm (Unisa, 2011). While they vary slightly, these models share a common thread of employing transfer principles to convey mathematical concepts. As outlined by Bonato (2018), the instructor is typically regarded as the sole knowledgeable individual capable of imparting essential understanding to students who lack comprehensive knowledge. Consequently, in mathematics lessons following the traditional approach, the teacher's lectures, demonstrations, and illustrative examples serve as the means through which mathematical knowledge is transmitted to students (Unisa, 2011).

Bonato (2018) underscores that traditional methods typically involve the teaching of rules followed by students' memorization and application through problem-solving. For instance, in a conventional mathematics class, students might learn the rule that the order of multiplication for two numbers does not affect the result. The teacher could then illustrate this with examples on the board, after which students would engage in problem-solving related to the topic. Despite some educational institutions still adhering to the traditional approach in teaching mathematics, it is characterized by a teacher-centred nature, limited collaborative and group learning, a teacher acting predominantly as a source of information dissemination rather than a facilitator, and a greater emphasis on assessments over conceptual comprehension (Nazzal, 2014).

2.22 Mathematics Teachers' Perceptions about Problem-Solving

As stated by Ekici (2013), teachers' knowledge levels, perceptions, and viewpoints concerning the process of problem-solving hold significant rank in their ability to effectively teach students problem-solving skills and their applications in real-life situations. Chapman (2008) concurred with Ekici's (2013) viewpoint, emphasizing that teachers should possess a solid foundation in mathematical problem-solving both as solvers themselves and as guides to nurture students' problem-solving abilities. Teachers' proficiency in abstract thinking, comprehension of word problems, and adeptness in progressing through problem-solving stages are pivotal factors influencing their effectiveness in teaching mathematical problem-solving (Yee & Bostic,2014). Schukajlow,Leiss, Pekrum, Blum, Muller, and Messner (2012) observed that teachers often misinterpret problem-solving tasks as application problems. In another study by Lee and Kim (2005), the perception of "good problems" among basic school teacher candidates was investigated. The majority considered typical routine problems as favourable, while displaying resistance towards non-routine problems with unusual characteristics.

2.23 Summary

The chapter began with the theoretical framework of the study and the related literature on problem-solving as a teaching method for teaching mathematics. This was followed by a description of problem-solving in mathematics, importance of teaching through problem-solving, conceptual framework, the use of problem-solving as a teaching strategy, different views on the goal of problem-solving, the role of a mathematics teacher in problem-solving as a teaching strategy and engagement, and metacognition in problem-solving. The concluding remarks in this chapter also included a discussion of the potential challenges that teachers encounter when problem-solving is used as a teaching strategy.

CHAPTER 3

METHODOLOGY

3.0 Overview

This chapter describes the research paradigm for study, the research approach for the study, the research design for the study, the target population, sampling techniques and sample size, the research instruments, the source of data collection, the pilot study, reliability and the validity of the instruments, data collection procedure, Data analysis, ethical considerations and summary.

3.1 Research Paradigm

Paradigms serve as frameworks or maps for scientific and research communities to determine theories and methods for addressing problems or issues (Le Grange, 2004; Henning, Van Rensburg, & Smit, 2004). Positioning research within a particular paradigm has several benefits, including placing research in the field of study, helping researchers to make assumptions about the interconnection of things in the world, providing a framework for the field of study, leading to a conceptual framework, and anchoring research within a particular literature (Henning, Van Rensburg & Smith, 2004). A pragmatists paradigm was used to frame the investigation of the experiences of mathematics teachers and their beliefs regarding their engagement in problem-solving. Pragmatist scholars hold the view that there exists an objective reality beyond human experience, yet this reality is rooted in the environment and can only be accessed through human encounters (Goles & Hirschheim, 2000; Morgan, 2014a; Tashakkori & Teddlie, 2008). A fundamental tenet of pragmatist philosophy is that knowledge and reality are constructed socially based on beliefs and habits (Yefimov, 2004). Pragmatists generally agree that all knowledge in our world is socially constructed, though some of these social constructs align more closely with individual experiences than others (Morgan, 2014a).

The primary goal of research using a pragmatism lens is to understand the experiences or reality of others, such as the mathematical practices of teachers in real classroom situations (Cohen, Manion, & Morrison, 2000). To maintain the integrity of the phenomenon under investigation, it is important to comprehend the practices of teachers through their perspectives (Cohen, Manion, & Morrison, 2000).

3.2 Research Approach

This study adopted mixed method approaches because the findings from the qualitative data are to enrich the findings from the quantitative data (Mason, 2006) in order to generate new knowledge effectively (Creswell, 2009). Morrison (2012) outlines that adopting both quantitative and qualitative methods (two traditional approaches) give an in-depth understanding of a phenomenon. According to Creswell (2014), the mixed research approach when employed in a study provides quantitative and qualitative strengths which include:

1. Generation and testing of grounded theory;
2. Words, narratives, tables, and graphs can add meaning to numbers;
3. The numbers also add precision to words, narratives, tables, and graphs;
4. The strength of one method overcomes the weakness of another method;
5. For more insight and understanding that might be missed when only one method is used;
6. Two methods together produced more complete knowledge necessary to inform theory, practice, policymakers, mathematics educators, and the learners of mathematics.

Hence, the adoption of a mixed research approach type for this study is grounded on the above strengths.

3.3 Research Design

Creswell (2014) defined research design as the plan, structure, and strategy of inquiry conceived to obtain answers to research questions and control variance. Rennie (1997) also, posit that research design is a plan or blueprint that specifies how data in relation to a given problem should be collected and analysed. The study was carried out using the explanatory sequential design. According to Creswell (2017), an explanatory sequential design is one in which the researcher conducts quantitative research first, analyses the results and then uses qualitative research to expand the findings. One of the strengths of using the explanatory sequential design is that the two phases (quantitative and qualitative) make it straightforward to implement.

The explanatory sequential design procedure first involves the collection and analysis of quantitative data to identify specific quantitative results that need further explanation. This type of design is popular in fields with a strong quantitative orientation (hence the project begins with quantitative research), but it presents challenges of identifying the quantitative results to further explore and the unequal sample sizes for each phase of the study (Creswell, 2014). The sequential explanatory design (also referred to as a participant selection model) was employed in this study for the following reasons:

1. To help in explaining and interpreting quantitative results after collecting and analysing the data and then followed by qualitative data.
2. The phase one result can be used to develop and inform the purpose and design of the phase two component.
3. For stronger evidence of conclusion through convergence and corroboration o

findings (Triangulation)

From the structure of the design, the study data was gathered first by administering a questionnaire for the study. A follow-up interview was then conducted to understand or explain in detail teachers' perceived knowledge about problem-solving, instructional strategies used in teaching through problem-solving and how often JHS mathematics teachers engage pupils through problem-solving strategies in their instructions among public Junior High Schools in the Tamale Metropolis.

3.4 Population

The population of a study is a group of people, who have one or more common characteristics which the research study envisages (Ary, Jacobs, & Razavieh, 2006). The targeted population for the study consisted of all mathematics teachers working in all public Junior High Schools in the Tamale metropolis. The Tamale Metropolitan education office estimated a total of 74 public junior high schools in the metropolis, consisting of a total number of 110 mathematics teachers.

3.5 Sampling

Sampling is defined as the selection of a representative proportion of the target population for a study (James, McMillan, & Sally, 2014). The findings obtained from the sample can be used to generalise for the entire population under the study (Mweshi & Sakyi, 2020). The sample size used for the study was statistically determined using the Krejcie and Morgan (1970) standard sample size determination formulae below: At 95% confidence level ($\alpha = 0.05$) and a margin of error of 0.05 and a proportion of 0.05, a sample size of 86 was obtained.

Where, Z = z- scores

N = population size

e = margin of error

p = standard deviation

$$\frac{\frac{Z^2 \times p \times (1-p)}{e^2}}{N - 1 + \frac{Z^2 \times p \times (1-p)}{e^2}}$$

$$\frac{\frac{1.96^2 \text{ x } 0.5(1-0.5)}{0.05^2}}{110-1+\frac{1.96^2 \text{ x } 0.5(1-0.5)}{0.05^2}} = 86$$

A sample size of 86 mathematics teachers was obtained and used for the study and 5 mathematics teachers were selected from the 86 mathematics teachers for the interview.

3.6 Sampling Technique

Simple random sampling is a sampling technique in which every member of the population of the study is given an equal probability of being selected for a study (Cohen, Manion, & Morrison, 2007). This sampling technique can be used when the population is homogenous and there is the availability of a complete sampling frame. A complete list and names of all mathematics teachers in the 74 public Junior High Schools, making up the sampling frame, were obtained from the Tamale Metropolitan Education office. Based on that sampling frame, the lottery system was used to randomly select all the 86 mathematics teachers for the study. Also, purposive sampling was used select 5 of teachers for the interview.

3.7 Data Collection Instruments

Two instruments were used for data collection in this study. The instruments used were a structured questionnaire for quantitative data and an interview guide for qualitative data. A questionnaire is a list of questions used to collect information from a large number of people on a subject of interest. It serves as a medium through which the researcher and the respondent communicate indirectly. According to Strange, Forest, Oakley and Ripple (2003), respondents may feel more comfortable conveying their opinions through a questionnaire than in a face-to-face interview. An interview is a flexible tool for the collection of qualitative data which allows the use of a multi-sensory approach (verbal, non-verbal, spoken, and heard). Patton (1990) outlines four types; informal conversational interviews; interview guide approaches; standardized open-ended interviews; closed quantitative interviews (as cited in Cohen, Manion, & Morrison, 2007). Interviews are used to describe and understand the experiences of the interviewee. The interview was used as a follow-up to the questionnaire to validate the responses of the participants. A semi-structured interview was used. Semi-structured interviews are usually planned in advance for a specified time, at a specific place. The interview was structured around several pre-planned questions and the remaining questions emerge from the interviewer's discussion with the interviewee (Dicicco-Bloom & Crabtree, 2006). All participants are asked similar questions in semi-structured interviews formatted in such a way that the answers are open-ended (Gall, Gall, & Borg, 2003). In this type of interview, the interviewer can adjust the order of the questions based on respondents' answers and dig deeper into a given situation.

3.7.1 Questionnaire

This study employed the structured questionnaire format. The questionnaire for this study was adapted from Asoma, Ali, Adzifome, and Eric (2022). A 4-point Likert scale questionnaire was used in the study. There are four sections in the questionnaire A, B, C and D. Section A seeks to collect demographic information about the respondents. The background information about the teacher's age, gender, professional qualification and teaching experience in years.

Section B requests information on mathematics teachers' perceived knowledge about problem-solving (This sub-scale contained 6 closed-ended items). Responses using

Likert Scale item types follow the pattern of "strongly disagree, disagree, agree, strongly agree. The scale is based on statements classified as positive or negative. Each statement has four options among which respondents were to select one. Weights as 1, 2, 3, and 4 were put on respondents' responses of "Strongly disagree (SD)", "Disagree (D)", "Agree (A)", and "Strongly Agree (SA)". The study was designed in order to measure extreme opinions by not providing a neutral option. The researcher intended to encourage respondents to align themselves more definitively with one of the available response categories.

Section C requests information on the problem-solving instructional strategies mathematics teachers employ in their mathematics instructions (This sub-scale contained 9 closed-ended items). Responses using Likert Scale item types follow the pattern of "never used, low used, moderately used and highly used". Each statement has four options among which respondents were to select one. Weights as 1, 2, 3, and 4 were put on respondents' responses of "Never used", "Low used", "Moderately used", and "Highly used" respectively.

Section D requests information on how mathematics teachers engage students in teaching through problem strategy (This sub-scale contained 9 closed-ended items). Responses using Likert Scale item types follow the pattern of "never engage, rarely engage, occasionally engage and always engage". Each statement has four options among which respondents were to select one. Weights as 1, 2, 3, and 4 were put on respondents' responses of "Never engaged", "Rarely engage", "Occasionally engage", and "Always engage" respectively.

Section E was rated based on a four-point Likert scale, where SA = Strongly Agree, A = Agree, D= Disagree and SD= Strongly Disagree in a tabular form about the challenges teachers face when teaching through problem-solving their mathematics instructions (This sub-scale contained 6 closed-ended items).

Every section of the questionnaire began with specific instructions as to the intent of the items as well as how to respond to items in that section.

The researcher chose a 4 –point Likert scale in order to force participants to form an opinion on the construct (Brace, 2004). The advantage inherent in using the Likert Scale for this study was that it distinguished between the scale properly and it pointed out the difference between the two concepts. Also, preference for the Likert scale type of questionnaire was predicated upon the fact that it is very good to obtain a comprehensive assessment of a given topic.

3.71.2 Interviews

For this study, semi-structured interview was selected, based on the fact that it allows the researcher more freedom in conducting the interview and can collect detailed information on the topic under study by asking probing questions (Cohen, Manion, & Morrison, 2007). A structured interview guide consisting of five (5) questions was used to solicit responses from the participants on perceived knowledge about problem-solving, the extent to which mathematics adopt problem-solving strategies in mathematics lessons, engagement of pupils through problem-solving strategies and challenges faced when teaching through problem-solving (see Appendix B). In this study, efforts were taken to conduct the interviews in detail and on time. In order to reduce personal bias, efforts were taken to accurately reflect the overall view of all interview responses through the coding procedure.

3.8 Sources of Data Collection

The study employed primary data from 86 public JHS mathematics teachers who were randomly selected from 74 schools in the Tamale Metropolis. The data collected were made up of teachers who were teaching mathematics in JHS1, JHS2 and JHS3 in the first term of the Ghana Education Service (GES) 2022/2023 academic year.

3.9 Pilot Study

The questionnaire was pre-tested in 10 selected public JHSs in the Sagnarigu Municipal before actual data collection commenced. Those schools were isolated from the pool of schools selected for actual data collection. 20 Junior High School Mathematics teachers were used for the pilot study. The choice of schools for pre-testing of the research instruments was guided by the characteristics of the schools, the teachers, the location in the target district, and the availability of mathematics teachers with vast experience in teaching mathematics in the district. At the end of the pre-testing exercise, the researcher improved upon the data collection instruments in relation to clarity in the wording of interview questions, infusion of local terms to connect concepts with local content and knowledge, rephrasing of interview questions deemed sensitive, determination of appropriate time per interview, and providing additional instructions, among others.

3.10 Reliability and Validity of the Instruments

The credibility of a research study depends on the reliability and validity of the data collection instruments (Cohen, Manion, & Morrison, 2007).

3.10.1 Reliability

The extent to which a test can be repeated to produce similar results measures its reliability. For a test to the instruments to be valid, it has to be reliable. Neil (2004) stated that measurement procedures have the potential for error, so the aim is to minimize it. An observed test score is made up of the true score plus measurement error. The following ways can be used to measure the reliability of a test.

1. Test-retest Reliability

2. Internal Consistency

3. Interrater Reliability

The researcher to ascertain the reliability of the questionnaire for this work employed the internal consistency method, which is the use of the Cronbach's Alpha. Glen (2017) outlined that the Cronbach’s alpha tests are used to verify if multiple-questioned Likert scale surveys are reliable. These questions measure latent variables hidden or unobservable variables like a person’s perception, neurosis or openness by asking questions both open and close-ended to solicit a person’s views on issues. These are very difficult to measure in real life. Cronbach’s alpha will tell you if the questionnaire you have designed is accurately measuring the variable of interest. It is important to know how reliable the questionnaire for a research work is because the more reliable the test or questionnaire are, increases the ability to use the result of the study in similar situations. For example, if the findings of a study reveal that students benefit more from study groups as compared to individual learning in one school by using a particular questionnaire then a similar result is expected in a similar environment if the same questionnaire is used.

Each item was measured as to whether it is correlated significantly. The statistical analyses of the internal consistency, Cronbach’s alpha coefficient were obtained through SPSS software. The researcher found the Cronbach’s alpha for the questionnaire to be 0.71 which indicates that the questionnaire was reliable to be used for the study. According to Mugenda and Mugenda (2003), the coefficient is high when its absolute value is greater than or equal to 0.7: otherwise, it is low. A high coefficient implies a high correlation between variables indicating a high consistency among the variables. This study correlated items in the instruments to determine how best they relate using a Cronbach Coefficient Alpha of 0.71. According to GEBA (2013), a generally accepted rule is that Cronbach's alpha of 0.6-0.7 indicates an acceptable level of reliability and 0.8 or greater is a very good level.

3.10.2 Validity

The validity of the questionnaire is the measure of how the items on the questionnaire are able to measure the intent of the research. Bryman (2008) identified two forms of validity, content validity and face validity. The researcher Bryman (2008) further explained that validity involves stages where data is collected to ascertain the claim that the questionnaire measures the intent of the researcher as far as the subject being investigated is concerned. The validity of the questionnaire was pilot tested to be pronounced valid by experts in the field of mathematics education. According to Pasmore, Dobbie, Parchman, and Tysinger (2002), pilot testing helps to identify redundant or poor questions and provides an early indication of the reproducibility of the responses. This also enables the researcher not to pick up questions, or items that may affect any respondent or bias. According to Neil (2004), many researchers resort to simply selecting whatever they can get their hands on, or they resort to using instruments that have commonly been used in the past, resulting to availability bias which can affect the outcome of a study negatively since such an instrument might not measure what is intended to be measured by the study. The face validity of the questionnaire was established with the help of experts (research supervisor). This expert helped to correct any elements of ambiguity in the instruments before it was used in the pilot test and subsequently for the main study.

3.10 Data Collection Procedures

Before embarking on the collection of the data for the conduction of the research in the Tamale Metropolis, a letter of introduction from the Department of Mathematics Education of the University of Education, Winneba was obtained introducing me as a student researcher from the University to the Metro Education Directorate. The letter explained the reasons for the study and assured the confidentiality of responses. The Metro Directorate forwarded and authorized letters to the heads of schools for them to allow the researcher to undertake the study. Thereafter, the researcher discussed the time schedules and appointments with the participating teachers so that the processes did not interfere with teaching and learning. The head teachers instructed mathematics teachers to assist me in the data collection. The questionnaire was administered to the eighty-six (86) participants. The researcher personally distributed the questionnaire to the participants in the participating schools. Participants agreed to complete the questionnaire within 3 days. The researcher visited each school after administering the questionnaire to collect the completed questionnaires. A follow up interview was conducted to the selected participants after collecting and analysing the questionnaires.

3.11 Data Analysis

According to Kothari (2004), data analysis is a process of editing, coding, classification, and tabulation of collected data. The process involves operations that are performed to summarize and organize the collected data from the field. This section describes the statistical tools and thematic techniques used to analyse the data. The analysis is based on the purpose, the objectives, and the research questions of this study. The study used descriptive statistics such as mean and standard deviation, frequency and percentages to analyse the quantitative data, while the thematic analytical technique was used to analyse the qualitative data.

3.7 Ethical Considerations

The study followed all ethical procedures and guidelines for graduate student research. An introductory letter (see Appendix C) was obtained from the University of Education, Winneba, Mathematics Department. In addition, the researcher wrote to the Metro

Education Directorate (see Appendix D) to seek permission to use the junior high schools for the study. An approval letter (see Appendix E) was also developed and signed by the Metro Education Directorate to use the schools where the data was to be collected. The researcher explained the objectives of the research to the authorities of the schools. The researcher also informed the respondents of their right to withdraw when they feel like doing so. Before distributing the questionnaires, the researcher assured the participants that all data collected during the data collection will be kept securely and treated as confidential.

3.8 Summary

This chapter has addressed the research design used in the study. Also, explained the instruments used for data collection for the study. It also provided a detailed description of the methodology used for the study. This includes the description of data collection and analysis, information about instruments reliability and validity, and the ethical considerations for the study.

CHAPTER 4

PRESENTATION AND DISCUSSION OF RESULTS

4.0 Overview

The study was designed to find out public JHS mathematics teachers' perceived knowledge, practices and engagement levels about problem-solving strategies, as well as the challenges they encounter when using problem-solving as a teaching strategy. Questionnaire and interviews were used to collect quantitative and qualitative data for the study respectively. The quantitative data was analysed using descriptive statistics (mean scores, standard deviation, frequency and percentages), and thematic analysis was used to analysed the qualitative data. The study's results are presented and discussed in this chapter in accordance with the set research questions.

4.1 What is mathematics teachers' perceived knowledge for teaching through problem-solving among Junior High Schools in the Tamale Metropolis? (Research Question One)

The research question sought to determine JHS mathematics teachers perceived knowledge and view on problem-solving. To answer this research question, the respondents were asked to make explicit and thoughtful choices by indicating on a four-point Likert scale ranging from Strongly disagree, Disagree, Agree and Strongly agree, which makes the average/fair score to be 2.5 ($[1+2+3+4] \div 4$), the determination of the level of agreement of mathematics teachers' perceived knowledge on problem-solving was done using means and standard deviations. The statistical mean<2.50 is Low, a mean of $2.50 \leq$ mean<3.50 is Moderate, and mean≥ 3.50 is High. The results are shown in Table 4.1

Table 4.1: Mathematics Teachers' perceived knowledge on Problem-solving

S/N	Statements on perceived knowledge	Mean	SD	Level of agreement	4. SA	3. A	2. D	1.SD
1	Mathematics problems should task pupils to reason logically and critically	3.66	0.49	High	40 (46.51%)	38 (43.95%)	7 (8.14%)	1 (1.16%)
2	Mathematics problems should have a connection with pupils' real-life situation	3.62	0.53	High	41 (47.44%)	38 (43.95%)	6 (7.04%)	1 (1.16%)
3	Mathematics problems should challenge pupils to apply daily skills in solving it	3.64	0.55	High	42 (48.28%)	37 (42.86%)	6 (7.04%)	1 (1.16%)
4	Mathematics should guide pupils to self- develop strategies	3.53	0.50	High	39 (45.35%)	39 (45.35%)	6 (7.04%)	2 (2.3%)
5	Problem-solving involves tasks that challenge pupils' ability	3.37	0.62	Moderate	27 (31.36%)	34 (39.47%)	23 (26.74%)	2 (2.3%)
6	Mathematics problems should require pupils to conjecture their strategies for solving it	3.21	0.79	Moderate	21 (24.41%)	32 (36.84%)	26 (30.23%)	7 (8.14%)
	Overall perceived knowledge on problem-solving	3.51	0.58	High				

Table 4.1 provides a comprehensive insight into mathematics teachers' perceived knowledge regarding problem-solving should involve tasking learners to reason logically and critically with M=3.66 and SD=0.49 than problems connecting with learners' real-life situations with M=3.62 and SD=0.53. Mathematics problems should challenge pupils to apply their daily skills in solving them with M=3.64 and SD=0.55 High, mathematics lessons that should guide learners to develop strategies in solving problems with M = 3.53 and SD = 0.50, mathematical problems involving tasks that challenge the learners' ability with M = 3.37 and SD = 0.62, and mathematical problems requiring learners to conjecture their strategies in solving it with M=3.21 and SD = 0.79 components. Overall, Mathematics teachers' perceived knowledge generally yielded a mean of 3.51 and a standard deviation of 0.58, indicating the level of agreement to be high indicating good perceived knowledge about problem-solving. The participants demonstrated good perceived knowledge about problem-solving. This implies that the teachers have the requisite knowledge and awareness to effectively incorporate

problem-solving strategies into their teaching practices, which can potentially enhance students' learning experiences and outcomes.

From Table 4.1, majority of the participants ascribed positively to the problem-solving statement and so it can be argued that having such positive perceptions can be a great asset for the country in our quest for improving mathematics education using problem-solving strategies.

Results from five teachers who were interviewed were also used to corroborate some of these facts and were not different from the data gathered from the questionnaire. Respondents were asked to explain what they think problem-solving is all about. This question was meant to solicit the teachers' views of what they know about problem-solving. From the responses, it was clear that almost all the teachers had a good perceived knowledge about problem-solving. The following were some of the interview responses:

1. What do you think problem-solving is?

One of the respondents said,

…Ah, problem-solving is a way of helping pupils to devise their own strategies to find answers to problems that they encounter in their daily life... (Teacher1, Interview).

Another respondent revealed that,

…Mathematics problem-solving has to do with the efforts that students make to find a solution to a problem that probably you might have not met before… (Teacher 2, Interview).

Another respondent revealed that,

…Problem-solving is the process of finding a solution to a given problem without any prior knowledge of the solution… (Teacher 3, Interview).

Another respondent also said,

...From the experience I have, problem-solving has to do with word problems where children are required to think and sometimes work in a group to provide a solution to an unsolved problem with their skills… (Teacher 4, Interview).

One of the respondents said,

... Problem-solving is all about giving mathematics problems or tasks to children to think and reason very well before getting an answer and sometimes has to try solving it many times before getting the correct answer… (Teacher 5, Interview).

The next question the researcher asked was to find out when and how the teachers came to realise problem-solving as a necessary focus in teaching mathematics. The responses from the teachers indicated that most of them heard of the Problem-solving concept at their training colleges and universities they attended for their bachelor's degrees.

2. When and how do you come to realise problem solving strategy as a necessary focus in teaching mathematics?

The first respondent said,

... Through various books I've read, I've learned about problem-solving as an emphasis in teaching Mathematics. I got to know at college that problem-solving is one of the best approaches to evaluating students' application of Mathematics and the university of Education, Winneba. So as a teacher, I try to relate my lessons to students' daily life… (Teacher 1 Interview).

Another respondent also said,

...I learned that problem-solving is one of the techniques to assess a child's comprehension. I learned that way back at College and at the university, about 10 years ago and I sometimes use it in my lessons… (Teacher 2 Interview).

Also, one of the participants said,

…From the time I was in training college, I saw problem-solving as an emphasis in teaching Mathematics. We were taught and when I became a teacher, I sometimes try to also use problem-solving in my lessons… (Teacher 3 Interview).

Another participant responded,

… A workshop I attended a few years ago introduced me to problem-solving. I got to know that problem-solving challenges children's thinking so when I want children to think deeply, I use problem-solving… (Teacher 4 Interview).

One more participant also said,

… I heard about problem-solving from college and also from a workshop. It gives an in-depth understanding of both conceptual and procedural knowledge to pupils... (Teacher 5 Interview).

From these excerpts, the researcher deduced that most of the JHS Mathematics teachers have heard about problem-solving and have good perceived knowledge about it. For example, the definitions they gave were good. The participants who were interviewed gave responses demonstrating good perceived knowledge for teaching through problem-solving. Some of the teachers' views of what constitutes mathematical problem-solving were observed from the literature's definitions. Some of the teachers had in mind primarily the selection and presentation of "good" or effective problems to learners. All the five teachers who were interviewed indicated that they knew exactly what problem-solving was. That teacher's definition of problem-solving featured phrases like "giving learners a question and requiring them to think, without an obvious way to get to the solution", which is in line with the literature (The National Council of Teachers of Mathematics (NCTM) (2000). Problem-solving plays a crucial role in mathematics and should have a prominent role in the mathematics education of learners. The overall results from the questionnaire in Table 4.1 revealed that Tamale Metropolis

JHS mathematics teachers have good problem-solving knowledge with M =3.50 and SD =0.57. Earlier studies conducted by Xenofontos (2014), Mereku (2015) and McIntosh, Jarrett, and Peixotto (2000) attested that teachers lack expert knowledge in the area of problem-solving therefore, they do not teach through problem-solving in mathematics, this is against the findings of this study. Therefore, the findings of the study suggest that junior high school mathematics teachers in the Tamale Metropolis do have the required knowledge and skills to teach mathematics using a problem-solving approach.

4.2 To what extent do mathematics teachers employ problem-solving strategies in teaching at the Junior High School level in the Tamale Metropolis? (Research Question Two)

The second research question dwelled on the instructional strategies that were adopted by mathematics teachers in teaching through problem-solving to junior high school students in the Tamale metropolis. In ascertaining the extent of the practice of the problem-solving instructional strategies, mean scores and standard deviation of level of usage were used such that mean<2.50 indicated low used, 2.50≤ mean<3.50 indicated moderately usage, and mean≥3.50 indicated highly used. The results on the instructional strategies and the level of usage and practice are presented in Table 4.2. Item 7 to 15 of the Questionnaire was used to collect research question two data. The results of research question two are shown in Table 4.2.

Table 4.2: Teachers' level of adoption and practice of problem-solving strategies

S/N	Strategies	Mean	SD	Level ofusage	4.Always	3.Occasionally	2.Rarely	1.Never
1	Task-basedInstruction	3.50	0.55	High	40 (46.51%)	31 (35.96%)	12 (13.95%)	3 (3.57%)
2	CooperativeLearning	3.51	0.67	High	41 (47.44%)	31 (35.96%)	12 (13.95%)	2 (2.3%)
3	Brainstorming	3.41	0.81	Moderate	38 (43.95%)	29 (33.72%)	14 (16.28%)	5 (5.88%)
4	Guided Discovery	3.26	0.94	Moderate	31 (35.96%)	29 (33.72%)	22 (25.58%)	4 (4.65%)
5	Group Work	3.29	0.93	Moderate	32 (37.21%)	28 (32.56%)	22 (25.58%)	4 (4.65%)
6	Trial andError	3.31	0.72	Moderate	22 (25.58%)	26 (29.47%)	24 (27.91%)	4 (4.65%)
7	Look for apattern	3.15	0.75	Moderate	21 (24.41%)	25 (29.03%)	24 (27.91%)	6 (6.98%)
8	Inquiry Learning	3.05	0.91	Moderate	22 (25.58%)	25 (29.03%)	24 (27.91%)	5 (5.88%)
9	Working backwards	2.45	0.93	Low	8 (9.29%)	12 (13.95%)	48 (55.88%)	8 (9.29%)
	Overall level of usage	3.21	0.80	moderate				

Table 4.2 presents an analysis of the adoption and utilization of problem-solving strategies by teachers. The results in Table 4.2 shows that there were several instructional strategies that mathematics teachers employed in teaching through problem-solving in mathematics instruction. Particularly, the findings showed that the usage and adoption level of task-based instruction with M=3.50 and SD=0.55 and cooperative learning with M=3.51 and SD=0.67 are high compared to brainstorming with M=3.41and SD=0.81, guided discovery with M=3.26 and SD=0.94, group work with M=3.29 and SD=0.9.3, trial and error with M=3.31 and SD=0.72, look for a pattern with M=3.15 and SD=0.75 and inquiry learning with M=3.05 and SD=0.91. Working backwards with M=2.45 and SD=0.93 was low. The overall level of adoption and usage of the strategies outlined in the study yielded a mean of 3.21 and a standard deviation of 0.80, indicating moderate problem-solving strategies practices. Generally, the study's findings revealed that public JHS mathematics teachers in the Tamale Metropolis moderately used and adopted all the problem-solving instructional strategies outlined

in the study. However, the participants highly employed cooperative teaching and learning and task-based instruction. The results show an overreliance on only two major strategies, which indicates that the majority of teachers have limited knowledge of the role and benefits of other strategies such as trial and error, brainstorming, guided discovery, looking for a pattern, inquiry learning in solving problems. Though there is a gap between knowledge and practice, public JHS mathematics teachers in the Tamale Metropolis have good knowledge about problem- solving.

The implication of the research finding is that, the fact that teachers have good perceived knowledge of problem-solving does not necessarily mean that's what they would necessarily practice in their classrooms. It also means that someone knows something does not necessarily mean that's what the person would do in reality and this could be a lack of supervision, motivation and other factors.

The following questions were asked during the interview which was meant to solicit teachers' views on how they used problem-solving strategies in their teaching:

3. Which instructional strategies do you employ when teaching through problem-solving?

One of the teachers indicated that,

… I love giving students group work, assignments and sometimes encouraging students to guess their answers before solving it… (Teacher 1 Interview).

Another teacher indicated that

… I discuss the problem with them using drawings and diagrams to illustrate. Then I lead students to solve the problem by asking leading questions. Finally, I help them to work backward to be sure that the answer is correct… (Teacher 2 Interview).

Also, one of the teachers revealed that,

…I have various ways of adopting problem-solving by using brainstorming and trial and error strategies. For example, I use normal exercise in the form of puzzles so that students can reason more and write few… (Teacher 3 Interview).

It was further indicated by one of the participants that,

... In fact, I employ a range of techniques, including compiling a list, creating a chart or table, drawing a diagram, creating a model, reducing the problem and working backwards from a pattern. Sometimes, I even use a formula or equation or act out the problem situation using guesses and checks… (Teacher 4 Interview).

Apart from that, one teacher revealed that,

…through shared responsibilities between teacher and students [child-centred], teachers gained knowledge and practices on problem-solving and I also try to involve all the students in my lessons… (Teacher5 Interview).

These excerpts suggest that, teachers employ problem-solving strategies in their instructions but none of them could give a guiding principle of using the problem-solving strategies and describe properly how they incorporate it in their lesson. This may indicate that they are lacking the guiding principles of problem-solving.

The study findings are in harmony with those findings by Matlala's (2015) that mathematics teachers found it difficult to teach through problem-solving approaches and that the teachers still taught using the traditional approaches, including storytelling and stepping in to show learners how to solve mathematical problems. Anderson (2000), opines that some teachers lack the necessary knowledge, skills and competencies to teach mathematics through problem-solving. Lack of mathematical knowledge about teaching undermines teachers' confidence in teaching mathematics through problem-solving. Those teachers rely on traditional ways in which students memorize rules that harm the learning of students to build meaningful knowledge about problem-solving. Fong, Yap, and Koay (1996) noted that many teachers voiced their concern that they did not have the teaching skills to solve problems in mathematics. For example, teachers feel inadequate about problem-solving methods, especially with unusual problems.

Also, teachers' struggle with implementing problem-solving in their classes may be a lack of the knowledge and skills associated with incorporating problem-solving meaningfully into the mathematics curriculum (Buschman, 2004; Cai & Lester, 2010). Furthermore, Buschman (2004) argued that teaching through problem-solving is a challenge to many teachers. One of the challenges mentioned was teacher inability to prepare before teaching mathematics through problem-solving. The qualitative data from the interview also proved that Public Junior High School mathematics teachers in the Tamale Metropolis used problem-solving strategies in their teaching. The interviewees could not describe how they used problem-solving strategies in their teaching. Therefore, the findings show that most mathematics teachers reported using a problem-solving approach to teach mathematics daily. This suggests that the teachers appear to use this pedagogical approach regularly, which should have assisted them to become proficient in its use. Although the daily use of a problem-solving approach is encouraging, teachers must be careful and mindful to use a variety of teaching strategies as part of instruction (Strong, Thomas, Perini, & Silver, 2004; Van de Walle, Karp, Bay-Williams, & Wray, 2010), since other teaching methods are better at teaching mathematical skills. In other words, teachers need to select the most suitable approach that will enable students to gain understanding and competencies they seek to advance.

4.3 How often do mathematics teachers engage Junior High School pupils through problem-solving? (Research Question Three)

Research question three sought to establish how often junior high school mathematics teachers in the Tamale metropolis engage pupils through problem-solving. Item 16 to 24 of the questionnaire was used to answer research question three. To accomplish this objective, mean scores of the level of engagements were calculated under the following criteria: The mean<2.50 indicated Low, 2.50≤mean<3.50 indicated Moderate, and mean≥3.50 High. The results of research question three are presented in Table 4.3

Table 4.3: Mathematics teachers' engagement with pupils through problem-solving

Kind of Engagement	Mean	SD	Level of engagement	1.Never	2.Rarely	3.Occasional	4.Always
Use of manipulates	3.63	0.53	High	2 (2.33%)	11 (12.79%)	28 (32.56%)	45 (52.33%)
Active engagement of learners	3.51	0.76	High	5 (5.81%)	15 (17.44%)	32 (37.21%)	34 (39.53%)
Teacher as a facilitator	3.21	0.81	Moderate	10 (11.63%)	21 (24.42%)	31 (36.05%)	24 (27.91%)
Building lessons on learners RPK	3.15	0.76	Moderate	6 (6.98%)	19 (22.09%)	34 (39.53%)	27 (31.40%)
Motivation	3.12	0.85	Moderate	8 (9.30%)	21 (24.42%)	32 (37.21%)	25 (29.07%)
Application of knowledge	3.17	0.71	Moderate	7 (8.14%)	18 (20.93%)	36 (41.86%)	25 (29.07%)
assessment	3.41	0.61	Moderate	4 (4.65%)	14 (16.28%)	35 (40.70%)	33 (38.37%)
Curiosity	2.49	0.92	Low	28 (32.56%)	23 (26.74%)	23 (26.74%)	12 (13.95%)
Perseverance	2.59	0.81	Moderate	23 (26.74%)	28 (32.56%)	23 (26.74%)	12 (13.95%)
Overall engagement	3.14	0.75	Moderate				

Table 4.3 provides a comprehensive analysis of how mathematics teachers engage with pupils through various problem-solving strategies. The findings of the study revealed two items fall under the category of "Highly engage", as shown in Table 4.3, Items 16 and 17, representing the "use of manipulates" and "active engage of learners" with means and standard deviations of M=3.63, SD = 0.53 and M = 3.511, SD = 0.76 shows

high engagement of participants through problem-solving strategy. "Curiosity" was the only kind of low engagement by the participants, with M = 2.49 and SD = 0.92. Table 4.3 also revealed that six items were regarded as activities the mathematics teachers moderately engage in teaching through problem-solving to JHS pupils in the Tamale Metropolis. Among the moderate engaged activities, item 18, "Teacher as facilitator", has a M = 3.21 and SD = 0.81, item 20 ", Motivation", has M= 3.12 and SD=0.85, item 22 "Assessment" has M=3.41 and SD=0.61 and item 24 "Perseverance" has M=2.59n and SD=0.81. Item 19 "Building Lessons on Learners RPK", has M= 3.15 and SD = 0.96. Item 21, "Application of knowledge", has M= 3.17 and SD = 0.71.

Generally, Table 4.3 shows that mathematics teachers moderately engaged learners in all the kinds of engagement outlined in the study with a M=3.14 and SD=0.75.

However, the participants in the study show that they are highly and actively engage pupils and use manipulative materials in their instructions. According to Van de Walle (2007) and Florence (2012), mathematics manipulatives can help engage students for a longer period by helping them stay focused on particular tasks. Florence (2012) feels that while lecture-based teaching might be tedious, manipulatives allow students to participate actively in their learning. The following is how the selected teachers expressed their views through the interview on engagement of pupils through problem-solving as a teaching strategy.

4. How often do you engage pupils in your mathematics lessons when teaching through problem-solving?

One of the teachers said,

…I give them questions and give little instruction on how to solve them, also motivate them to solve them and I believe they can manage to get the answer correct and the perform… (Teacher1 interview).

Another respondent expressed that,

...I occasionally engage pupils in problem-solving by putting them in groups and providing them with the needed teaching and learning materials so that they can work together by sharing ideas and things to solve the problem… (Teacher 2, Interview).

Another respondent said,

…Children dislike Mathematics and at times difficult questions might scare them from even coming to school and as you know, our system too…., I try to motivate them So I give those questions I believe they can do with a guide… (Teacher 3, Interview).

One more teacher made it clear that,

...I try to teach students by building on their RPKs to develop their interest in my lessons, I also give them questions and I encourage and motivate them to follow the procedures I used to solve it. Even if it is difficult, I encourage them to reason well and at times some students will get the answer… (Teacher 4, Interview).

Another respondent also said,

….by allowing pupils to attempt problem-solving questions on their own before assisting them before putting them into groups to share ideas and put them together to solve it together, I also try to give them assignments… (Teacher 5, Interview).

The interview conducted indicated that the respondents engage pupils through the problem solving strategies but could not give a guiding principles of engagement through problem-solving. This finding contradicts that which was found in the literature. As outlined by Buschman (2004) and Cai and Lester (2010), teachers often report lacking the knowledge and skills to effectively incorporate a problem-solving approach into their mathematics teaching. The excerpts show that teachers need to the requisite knowledge and skills to enhance the level of engagement which can lead to increased student participation, motivation, and deeper understanding of mathematical concepts through problem-solving activities.

4.4 What challenges do Junior High School mathematics teachers encounter when using problem-solving as a teaching strategy? (Research Question Four)

The fourth research question was raised to assess JHS mathematics teachers' difficulties when teaching through problem-solving. To answer this research question, the respondents were asked to make explicit and thoughtful choices by indicating on a four-point Likert scale ranging from Strongly disagree, Disagree, Agree and Strongly agree, which makes the average/fair score to be 2.5 ([1+2+3+4] ÷4), the determination of the level of agreement of mathematics teachers' challenges concerning the integration of problem-solving in their instructions, the teachers were made to respond to a four-point Likert scale questionnaire was done using means and standard deviations. The statistical mean<2.50 is Low, a mean of 2.50≤ mean<3.50 is Moderate, and mean≥3.50 is High. The responses are shown in Table 4.4.

Table 4.4: Teachers' Challenges in teaching through problem-solving

Challenges in teaching through problem-solving	4.SA	3.A	2.D	1.SD	Mean	SD	Level of agreement
The time allocated for mathematics lessonsis not enough to employ problem-solving strategies in teaching mathematics	22 (25.6%)	18 (20.9%)	38 (44.2%)	8 (9.3%)	2.67	0.89	Moderate
Teachers are not given enough motivationto encourage them to integrate problem- solving in their teaching of mathematics	19 (22.1%)	46 (53.5%)	16 (18.6%)	5 (5.8%)	2.91	0.75	Moderate
Most of the mathematics teachers are askedto do many things in the cost of their job (workload)	31 (36.0%)	44 (51.2%)	6 (7.0%)	5 (5.8%)	2.83	0.59	Moderate
There are not enough TLMS for teachingthrough problem-Solving	57 (66.3%)	29 (33.7%)	0 (0.0%)	0 (0.0%)	3.92	0.54	High
The class size is large to teach mathematicsthrough problem-Solving	59 (68.6.%)	23 (26.7%)	4 (4.7%)	0 (0.0%)	3.86	0.47	High
Most mathematics teachers have limited requisite content knowledge on problem-solving strategies	19 (22.1%)	49 (56.9%)	17 (19.8%)	1 (1.2%)	2.74	0.73	Moderate

In Table 4.4, the researcher delved into the challenges that teachers grapple with when incorporating problem-solving methodologies into mathematics instruction. Table shows responses categorized as "Strongly Agree," "Agree," "Disagree," and "Strongly Disagree," alongside mean (M) and standard deviation (SD) values for each challenge. From Table 4.4, regarding the statement “teachers are not given enough motivation to encourage them to integrate problem-solving in the teaching of mathematics”, 46 (53.5 %) strongly agreed and 19 (22.1%) agreed that, whiles 16 (18.6%) of the teachers disagreed and 5 (5.8) strongly disagreed with M=2.9 and SD=0.75. Also, 31 (36.0%) strongly agreed to the fact that most of the mathematics teachers are asked to do many things in the cost of their job increasing workload which is a challenge in teaching mathematics through problem-solving and 44 (51.2%) agreed to this statement, whilst 6 (7.0%) of the teachers disagreed and 5 (5.8%) strongly disagreed with M = 2.83 and SD = 0.59. The statement “there are not enough TLMS to support when teaching through problem-solving, 57 (66.3. %) strongly agreed and 29 (33.7%) agreed to this, with none disagreed or strongly disagreed with M = 3.92 and SD = 0.54. Regarding large class as a challenge, 59 teachers representing 68.6% strongly agreed and 23 (26.7%) agreed to the statement whiles 4 (4.7%) disagreed to the statement and none of them strongly disagreed with a mean of 3.86 and standard deviation of 0.47. It is also observed that the statement “most mathematics teachers lack or have limited content knowledge on problem-solving strategies”, 19 (22.1%) strongly agreed and 49 (56.9%) agreed to that and 17 (19.8%) disagreed and 1(1.2%) strongly disagreed the statement with M = 2.74 and SD = 0.73. The statement regarding the time allocated for mathematics lessons is not enough to employ problem-solving strategies in teaching mathematics, 22 (25.6% strongly agreed and 18 (20.9%) agreed to the statement whiles 38 (44.2%) disagreed and 8(9.3%) strongly disagreed with M = 2.67 and SD = 0.89.

The data extracted from Table 4.4 unveils an array of formidable challenges confronting mathematics teachers as they endeavour to seamlessly integrate problem-solving strategies into their instructional practices. These factors must be looked at to ensure that mathematics teachers can incorporate problem-solving in their lessons and this could help to produce students who are problem-solvers and critical thinkers in Ghana. The participants reported that they are asked to do many things at the cost of their job which affects teaching through problem-solving. It is also observed that some of the mathematics teachers 68 (79.0%) noted that they have limited requisite knowledge and skills to enact a problem-solving approach to teach mathematics. This finding is consistent, to some extent, with Buschman (2004) and Cai and Lester (2010).

It is revealed that most challenges they faced when teaching through problem-solving were large class sizes, lack of TLMS and time-consuming. Except for the challenge of class size, the research done by Cai (2003) and Hiebert and Wearne (1993) provide similar findings. Large class sizes are one of Ghana's biggest struggles within the education system. This is likely to impact the quality of remediation and guidance teachers can provide to students who are engaged in problem-solving (Campbell Collaboration, 2018).

The results revealed that the participants have challenges as lack of enough time for mathematics lessons and lack of teaching and learning materials are in agreement with those obtained by Anderson (2005). For example, Anderson (2005) found that teachers agreed they needed considerable support in the form of time and resources so that they can implement a problem-solving approach in the classroom.

Five participants were interviewed to explain some challenges faced when using problem-solving as teaching strategy. The following excerpts are some of the views expressed by the respondents:

5. What are some of the challenges in engaging pupils through problem-solving?

One of the respondents revealed that,

…Some students prefer to be told Mathematics rather than guided by the teacher to explore and construct their understanding... (Teacher 1 Interview).

Another teacher expressed that,

…Teaching through problem-solving requires a lot of time, and if time is not sufficient, it is better to teach Mathematics by telling and our class size is large which makes it difficult the teach through problem-solving... (Teacher 2 Interview).

It was further revealed by a respondent that,

…to me some teachers have inadequate subject knowledge, pedagogical content knowledge and personal problems and consume a lot of time... (Teacher 3 Interview).

Another respondent also said,

...Some teachers lacked the requisite knowledge, skills and expertise to teach Mathematics through problem-solving and also lacked teaching and learning materials… (Teacher 4 Interview).

One of the teachers further added that

… identifying the correct procedure for solving a specific problem is a challenge of teaching Mathematics through problem-solving also curriculum constraints so the time allocated for mathematics is not enough for me... (Teacher5 Interview).

These excepts revealed that curriculum constraints, time consuming, inadequate subject knowledge, large class size, lack of teaching and learning materials are the challenges that make it difficult for junior high school mathematics teachers in the Tamale Metropolis to teach effectively through problem-solving.

CHAPTER 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Overview

In this chapter, the methods used and the findings are summarized. The result of the study was presented in the preceding chapter. A summary of findings, conclusion, recommendations and suggestions for further research into the problem based on the findings of this study are presented in this chapter.

5.1 Summary of the Study

This study was meant to examine the public JHS mathematics teachers' perceived knowledge about problem-solving, the extent to which mathematics teachers employ problem-solving strategies in teaching mathematics, how often JHS mathematics teachers engage pupils in mathematics instructions through problem-solving and the challenges JHS mathematics teachers encounter when using problem-solving as a teaching strategy in the Tamale Metropolis. Four research questions were formulated to achieve the objectives of the study.

What is mathematics teachers' perceived knowledge for teaching through problem-solving among public Junior High Schools in the Tamale Metropolis?; To what extent do mathematics teachers employ problem-solving strategies in teaching mathematics among Public Junior High Schools in the Tamale Metropolis?; How often do mathematics teachers engage pupils through a Problem-Solving strategy?; What challenges do JHS mathematics teachers encounter when using problem-solving as a teaching strategy in the Tamale Metropolis? The study was carried out with the mixed method approach and sequential explanatory research design was used with the view of being able to get in-depth information on the mathematics teachers' perceived knowledge about problem-solving, the extent to which mathematics teachers employed problem-solving strategies in teaching, engaging pupils in mathematics instructions through problem-solving strategies and the challenges JHS mathematics teachers encounter when using problem-solving as a teaching strategy in the Tamale Metropolis. The population for the study was 110 mathematics teachers working in all public Junior High Schools in the Tamale metropolis. The study used a sample size of eighty-six (86) mathematics teachers from the Tamale Metropolis of the Northern Region of Ghana. For the sample of the study, a random sampling technique was employed. A closed-ended questions were used for the questionnaire and an interview guide for the data collection. Some questions on the questionnaire were self-developed, whilst others were adapted from studies of Asoma, Ali, Adzifome and Eric (2022).

5.2 Summary of Key Findings

The results indicated that all the 86 public JHS mathematics teachers who participated in the study responded to all the statements given about problem-solving. The overall mathematics teachers' perception of the general knowledge yielded a mean response (rating) of 3.51 and a standard deviation of 0.58 which indicates a high level of agreement to be high. Meaning the participants have good perceived knowledge about problem-solving. The responses from the teachers indicated that most of them respectfully heard about problem-solving concepts at their teacher training colleges and university education for their diplomas and bachelor's degrees.

Therefore, the findings of the study suggest that Junior High School mathematics teachers in the Tamale Metropolis do have the required knowledge and skills to teach mathematics using a problem-solving strategy. The participants demonstrated good perceived knowledge about problem-solving. This implies that teachers' may have the necessary knowledge and awareness to effectively incorporate problem-solving strategies into their teaching practices, which can potentially enhance students' learning experiences and outcomes.

From Table 4.1 majority of the teachers ascribed positively to the problem-solving statement and so it can be argued that having such positive perceptions can be a great asset for the country in our quest for improving mathematics education using problem-solving strategies.

It was revealed that public JHS mathematics teachers in the Tamale Metropolis moderately or averagely adopted and used all the mathematical problem-solving strategies outlined in the study. The overall level of used strategies outlined in the study yielded a mean response or rating of 3.21 and a standard deviation of 0.80, indicating a moderately used and adoption of problem-solving strategies in mathematics.

However, only cooperative learning and learning and task-based instruction were highly adopted and used by the participants (3.51 ±0.67 and 3.50 ± 0.55 respectively) which they believed enhanced understanding of mathematics concepts and skills acquisition as well as improved performance in mathematics problem-solving. The participants had good knowledge about problem-solving but were not highly using most of the problem-solving strategies outlined in the study. This shows that there could be gab between perceived knowledge on problem-solving and its implementations in the classroom. This could be a lack of supervision, motivation and other factors. The excerpts from the interview suggest that most teachers did not employ problem-solving strategies in their instructions. They may probably lack the necessary knowledge, skills and competencies to teach mathematics through problem-solving. The findings show that most mathematics teachers reported using a problem-solving approach to teach mathematics daily. This suggests that the teachers appear to use this pedagogical approach regularly, which should have assisted them to become proficient in its use.

Although the daily use of a problem-solving approach is encouraging, teachers must be careful and mindful to use a variety of teaching strategies as part of instruction (Strong, Thomas, Perini & Silver, 2004; Van de Walle, Karp, Bay-Williams, & Wray, 2010), since other teaching methods are better at teaching mathematical skills. In other words, teachers need to select the most suitable strategies that will enable students to gain understanding and mastery of the competencies they seek to advance.

Again the finding of the study revealed that Six items were regarded as activities in which the JHS mathematics teachers moderately engage with pupils in teaching through problem-solving in the Tamale metropolis. Generally, Table 4.3 shows that Mathematics teachers moderately engaged learners in all the kinds of engagement outlined in the study as problem-solving instructional strategies. However, the participants in the study show that they are highly actively engage pupils and use manipulative materials in their instructions which is are geared toward developing students who can apply these mathematical skills in concepts in making decisions to solve real life problems.

The findings clearly show the challenges mathematics teachers at the Junior High School level face in adopting problem-solving in their delivery. Although the large class size, teacher workload, most mathematics teachers' have limited content knowledge on problem-solving, lack of motivation, and little time allocation for mathematics lessons were all agreed as challenges faced when adopting and using problem-solving, the most prevalent and the most mentioned of them all was large class size in the various Junior High Schools. Large class sizes are one of Ghana's biggest struggles within the education system. This is likely to impact the quality of remediation and guidance teachers can provide to students who are engaged in problem-solving (Campbell Collaboration, 2018).

From the interview conducted with five teachers, most teachers said it is time-consuming and lack of TLMS were the major hindrance when adopting problem-solving in the teaching and learning of mathematics at the junior high school level. Therefore, the finding indicates that more work needs to be done by the government and educational authorities to address these challenges to enable mathematics teachers teach through problem-solving as the educational curriculum in Ghana emphasises the importance of incorporating mathematics into daily life by fostering the identification and application of appropriate problem-solving strategies (Ministry of Education, 2007).

5.3 Conclusions

The purpose of this study was to explore the public JHS mathematics teachers' perceived knowledge about problem-solving, the extent to which mathematics teachers employed problem-solving strategies in teaching mathematics, how often JHS mathematics teachers engage pupils in mathematics instructions through problem-solving strategies and the challenges they encounter when using problem-solving as a teaching strategy in the Tamale Metropolis.

It can be concluded that Junior High School mathematics teachers in this study generally possess good perceived knowledge about problem-solving. Therefore, Junior High School mathematics teachers in the Tamale Metropolis do have the required knowledge and skills to teach mathematics using a problem-solving approach to develop and equip students who can apply mathematical skills for solving real life problems and decisions making.

The study's findings revealed that public JHS mathematics teachers in the Tamale Metropolis moderately used all the problem-solving strategies outlined in the study. However, cooperative learning and task-based instruction were highly used and adopted by the participants in the study, which they believed enhanced students' understanding of mathematics concepts and skills acquisition and improved their performance in mathematics problem-solving.

The finding shows that mathematics teachers moderately engaged learners in all the kinds of engagement outlined in the study. However, active engagement of learners and the use of manipulative materials were highly practiced among the participants.

Finally, the study revealed that little time allocation for mathematics lessons, teachers' limited content knowledge on problem-solving, Large class size, lack of TLMS, and lack of motivation were agreed as challenges faced when adopting and using problem-solving strategies in the teaching and learning of mathematics. Therefore, the finding indicates that more work needs to be done by the government and educational authorities to address these challenges to enable mathematics teachers teach through problem-solving as the educational curriculum in Ghana emphasises the importance of incorporating mathematics into daily life by fostering the identification and application of appropriate problem-solving strategies (Ministry of Education, 2007).

5.4 Recommendations

From the summary of key findings of this study, it is recommended that:

1. Educational Authorities, in collaboration with teacher training institutions and educational organisations, should organise regular professional development workshops or training sessions for enhancing mathematics teachers' practical implementation of problem-solving strategies in the classroom.
2. School administrators, educational policymakers, and government bodies responsible for educational resource allocation should prioritise the provision of adequate teaching and learning materials (TLMS) specifically designed to facilitate problem-solving activities in mathematics classrooms.

3. Educational policymakers, school administrators, and relevant stakeholders should work together to address the issue of large class sizes by constructing additional classrooms to enable teachers to effectively implement problem-solving instructional strategies.
4. Educational Authorities should supervise teachers to effectively use the problem-solving instructional strategies to enhance students' mathematics academic performance.

5.5 Suggestion for Further Research

The study suggested the following areas for further research:

1. The researcher recommends that further studies be conducted in this area, to provide a more detailed understanding of mathematics teachers' perceived knowledge on problem-solving and practices in the classroom at other regions of the country.

2. Further studies need to be investigated into the challenges encountered by the mathematics teachers when using problem-solving instructional strategies at the JHS and SHS levels.

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APPENDIX A

TEACHER'S QUESTIONNAIRE ON PROBLEM SOLVING UNIVERSITY OF EDUCATION, WINNEBA.

FACULTY OF SCIENCE EDUCATION. DEPARTMENT OF MATHEMATICS EDUCATION

This questionnaire is designed to collect information for the sake of research aiming at assessing JHS mathematics teachers' perceptions about problem-solving and practices of teaching mathematics through problem-solving in the Tamale metropolis. Realizing the purpose of this research is strongly dependent on the data you provided.

The information you give concerning your perceptions about teaching and learning of mathematics will be handled confidentially. Please respond to the items below as honestly as possible.

Put a [√] in the brackets corresponding to your answer. Section A: Demography of Participant

1. Sex: Male [ ] Female [ ]

2. Age:20-25yrs [ ] 26-30yrs[ ] 31-35yrs [ ] 36-40yrs [ ] Above 40yrs [ ]

3. Teaching experience (in years): 1-5 [ ] 6-10[ ] 11-15[ ] 16-20[ ] 20+ [ ]

4. Professional qualification:

a) Teacher Cert ''A'' [ ]

b) Diploma [ ]

c) Bachelor's degree [ ]

d) Master's degree [ ]

e) Other [ ]

Section B: Perceived Knowledge on Problem-Solving

Read the items and rate the extent to which you agree to each statement using the rating scale: 1= Strongly disagree, 2 = Disagree, 3 = Agree and 4 = Strongly agree.

S/N	Statements	1	2	3	4
1.	Mathematics problems should task pupils to reason logicallyand critically				
2.	Mathematics problems should have connection with pupils'real-life situation				
3.	Mathematics problem should challenge pupils to apply dailyskills in solving it				
4.	Mathematics should guide pupils to self-develop strategies				
5.	Problem-solving involves tasks that challenge pupils' ability				
6.	Mathematics problem should require pupils to conjecture their strategies in solving it				

Section C: Instructional strategies

Read the items and rate yourself depending on the instructional strategies employ and practice in problem-solving of these items in your mathematics classroom using the rating scale: 1= Never used, 2= Low used, 3= Moderately used and 4= Highly used

S/N	Statements	1	2	3	4
7	Task based Instruction				
8	Cooperative Learning				
9	Brainstorming				
10	Guided Discovery				
11	Group Work				
12	Trial and Error				
13	Look for a pattern				
14	Inquiry Learning				
15	Working backwards				

Section D: Level of Engagement with Students in Problem-Solving

Read the items and rate yourself depending on your level of engagement with students in problem-solving of these items in your mathematics classroom using the rating scale: 1= Never engage, 2= Rarely engage, 3= Occasional engage and 4= Always engage

S/N	**Items**	**1**	**2**	**3**	**4**
16	Use of Manipulates				
17	Active Engagement of Learners				
18	Teacher as a Facilitator				
19	Motivation				
20	Building Lessons on Learners RPK				
21	Application of Knowledge				
22	Curiosity				
23	Perseverance				
24	Assessment				

Section E: Teachers Challenges in Teaching Trough Problem-Solving

Tick appropriately how the statement is a challenge to you in teaching through problem-solving in your mathematics instruction. Please indicate the level of the challenge by ticking (√) the appropriate response using the symbol:

SA – Strongly Agree, A– Agree, D – Disagree, SD – Strongly Disagree.

	Statement	**SA**	**A**	**D**	**SD**
25	The time allocated for mathematics lessons is not enough to employ problem-solving strategies in teaching mathematics				
26	Teachers are not given enough motivation to encourage them to integrate problem-solving in their teaching of mathematics				
27	Mathematics' teachers are asked to do many things in the cost of their job (workload)				
28	There are not enough TLMS for teaching through problem-solving				
29	The class size is large to teach mathematics through problem-solving				
30	Most mathematics teachers have limited requisite content knowledge on problem-solving strategies				

APPENDIX B

TEACHERS INTERVIEW GUIDE

Thank you for showing an interest in this project. This interview is to get you views about problem solving in teaching mathematics. In the event that the line of questioning develops in such a way that you feel hesitant or uncomfortable you may decline to answer any particular question(s) and/or may withdraw from the project without any disadvantage of any kind. You are also reminded that any issue(s) you raised during this interview will only be used for the purpose of this study. Please try to be candid as much as possible in the responses to the following questions.

1. What do you think problem-solving is?
2. When and how do you come to realise problem solving strategy as a necessaryfocus in teaching mathematics?
3. Which instructional strategies do you employ when teaching through problem-solving?
4. How often do you engage pupils in your mathematics lessons when teachingthrough problem-solving?
5. What are some of the challenges in teaching through problem-solving?

APPENDIX C
INTRODUCTORY LETTER

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF SCIENCE EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION

January 12, 2023

Dear Sir/Madam,

LETTER OF INTRODUCTION: ISSAH ALHASSAN (220025947)

I write to introduce to you the bearer of this letter, Mr. Issah Alhassan, a postgraduate student in the University of Education, Winneba. He is reading for a Master of Philosophy degree in Mathematics Education and as part of the requirements of the programme, he is undertaking a research titled – ***Junior High Schools Mathematics Teachers Perceptions About Problem-Solving And The Practices of Teaching Mathematics Through Problem-Solving In The Tamale Metropolis.***

He needs to gather information to be analysed for the said research and he has chosen to do so in your institution. I would be grateful if he is given the needed assistance to carry out this exercise.

Thank you.

Yours faithfully,

DEPARTMENT OF MATHEMATICS EDUCATION
UNIVERSITY OF EDUCATION
WINNEBA

Dr. Ali Mohammed
Graduate Coordinator

APPENDIX D

PERMISSION LETTER

P O BOX 693 TL
Tamale, N/R
18th December, 2022

THE METRO DIRECTOR
GHANA EDUCATION SERVICE
P. O. BOX 6 E/R
TAMALE
NORTHER REGION

Dear Sir,

APPLICATION FOR PERMISSION FOR DATA COLLECTION

I am a second year M.Phil. student in the Department of mathematics Education, University of Education, Winneba. My thesis Supervisor is Dr. Ali Mohammed. Data collection is requirement for academic research, I am therefore conducting a research study on JHS mathematics teachers' perceptions about problem solving and practices of teaching mathematics through problem-solving in the Tamale Metropolis of Northern Region of Ghana. I wish to use your schools to collect data for the study.

The purpose of this study is to find out JHS mathematics teachers perceptions about problem-solving and practices of teaching mathematics through problem solving. I wish to administer questionnaire instruments to a maximum of 86 JHS mathematics teachers to seek their responses on problem-solving in a selected 30 Junior High Schools in the Tamale Metropolis.

I would like to request the participation of your teachers in this study by allowing me to administer the questionnaire. I will not use teachers' names or anything else that might identify them in written work, oral presentations or publications. The information remains confidential. I will destroy the questionnaires after the research has been presented and/or published which may take up to two years after the data has been collected. There are no known risks to you for assisting in this study.

This study has been approved by my supervisor and the mathematics department of the University of Education, Winneba. I hope you will grant me permission to carry out the exercise.

Thank you for your kind consideration.

Yours Faithfully,

Issah Alhassan

(0247794976)

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APPENDIX E

APPROVED LETTER

GHANA EDUCATION SERVICE

In case of reply the date and reference number of this letter should be quoted

Our Ref: GES/NR/MEO/ TT.12/VOL.3
Your Ref: ....

Email: *tmetroedu@gmail.com*

REPUBLIC OF GHANA

Metropolitan Education Office
P. O. Box 6, E/R
Tamale, Northern Region
Tel: 037-2022090

Date: February 20, 2023

APPLICATION FOR PERMISSION TO CONDUCT A STUDY

Reference your letter dated 18th December, 2022 on the above subject matter, and I wish to grant you permission to enter selected schools in the Metropolis for the purposes of collecting data for research thesis.

Master Issah Alhassan is an MPhil Mathematics Student of the University of Education, Winneba. He is seeking to collect data on the topic "**JHS Mathematics teachers perceptions about problem solving and practices of teaching mathematics through problem in the Tamale Metropolis**"

However, the permission is granted on the assurance that the Student (researcher) fully comply with requirements such as the safety,protection of the rights and confidentiality of selected students and teachers.

Find attached his personal letter, list of schools introductory letter from his head of department for your attention.

Counting on your usual cooperation.

(BRIGHT ARMSTON LAWOE)
METROPOLITAN DIRECTOR OF EDUCATION
TAMALE

ALL HEADMISTRESSES/HEADMASTERS OF
SCHOOLS CONCERN
TAMALE