UNIVERSITY OF EDUCATION, WINNEBA

INVESTIGATING TEACHERS' INTERPRETATIONS AND PERCEPTIONS OF SENIOR HIGH SCHOOL STUDENTS' ERRORS IN SOLVING ALGEBRAIC PROBLEMS.

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A thesis in the Department of Mathematics Education, Faculty of Science Education submitted to the school of Graduate Studies in partial fulfilment of the requirements for the award of the degree of Master of Philosophy (Mathematics Education) in the University of Education, Winneba

MARCH, 2023

DECLARATION

STUDENT'S DECLARATION

I, BABA LAARI, declare that this research project, with the exception of quotations and references cited in published works which have been identified and duly acknowledge, is entirely my original work and has not been presented for an award of degree in any University.

SIGNATURE…………………………………..

DATE…………………………………………..

SUPERVISOR'S DECLARATION

I hereby declare that the preparation of this research project was supervised in accordance with the guidelines for supervision of research work laid down by University of Education, Winneba.

NAME OF SUPERVISOR: DR. RICHARD ASUMADU OPPONG

SIGNATURE…………………………………

DATE……………………………....….….…

DEDICATION

I dedicate this project to my dear wife madam Nafisah, my beloved children for their co-operation, patient and understanding when I was unavoidably absent from home when my attention was needed most.

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I thank my God, my savior and creator of heaven and earth for the healthy mind and body throughout the time of this study. I register my heartfelt gratitude to all people who in their special ways have made this study a success.

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ABSTRACT

This study investigated senior high school teachers’ interpretations and perceptions of students’ errors in algebra. The study employed exploratory sequential mixed design method in which document (students’ exercises) and a set of questionnaires were used to collect data. Fifty mathematics teachers were purposefully selected from three schools. These teachers were asked to explain five common errors in algebra from students’ class exercise books (document analysis). Questionnaires also were administered to fifty mathematics teachers purposefully drawn from the three senior high schools for the study. The finding revealed that, teachers’ explanations of students’ errors in algebra were mainly procedural. Some of these explanations lacked clarity and incorrect. Teachers also perceived errors not solely due to the students’ related factors, but also due to other factors arising from teaching and the nature of the algebra. The study recommends the need for various senior high school management to incorporate teachers' professional development programs aimed at developing teachers’ understanding of the nature and role of errors in the teaching and learning of algebra.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This Chapter Presents the Background of the Study, Statement of the Problem, Objectives of the Problem, Research Questions, Purpose of the Study, Significant of the Study, Delimitation, Limitations, and Organization of the Study.

1.1 Background

Mathematical errors are worldwide phenomena. Students of any age, any country, and any era, irrespective of their performance in mathematics, have experienced getting mathematics wrong. It was, therefore, natural for educators and psychologists to demonstrate, from a very early an interest in algebraic errors. This interest resulted in the formation of many theories about the nature of mathematical errors, their interpretation, and the ways of overcoming them. (Omar et al, 2022)

In a psychological study by Kshetree (2018), the errors were initially conceptualized negatively: an error was considered a digression, the result of "confusion which should be avoided. A reversal of the traditional view on errors is found in the work of Piaget and the Geneva School: for the first time, errors were viewed positively since they allow the tracing of a reasoning mechanism adopted by the students. The existence of mechanisms that lead to errors were empirically supported by some studies, which revealed regularity in certain errors.

Davis (2022) proposed two kinds of regularity, the first regularity refers to certain errors made by different students that are extremely common. For example, if someone gives a wrong answer for $4 \times 4 =$, that wrong answer will probably be 8; or if someone gives a wrong answer for 23 =, that wrong answer will probably be 6. Davis (2022) defined this kind of regularity, that is, wrong answers given by many different people, as binary reversions. A well-established frame learned earlier, is ordinarily elicited by a visually similar cue. Since the eliciting cues are not sharply distinguished, the well-established frame is preferred to a tentative one. Thus, a question is asked about a recent (and seemingly insecure) piece of learning; this question is (erroneously) replaced by a question dealing with earlier material (and presumably more securely learned), as when 4×4 is answered as if it had been $4 + 4$. Davis (2022) explained that the term binary reversions were used because the student goes back to an earlier idea.

The second kind of regularity proposed by Davis (2022) refers to the wrong answers given by one person, in response to a sequence of questions. For example, when Erlwanger asked Benny, a 12-year-old in grade 6, to write 2/10 as a decimal number and Benny wrote 1.2. Then, Benny was asked to write 5/10 as a decimal number and he wrote 1.5. Finally, he was asked to write 27/15 as a decimal number and wrote 4.2. Although this method is wrong, Benny used it consistently. The terms "super procedure" and "sub-procedure" were used to explain this kind of regularity. As supported by Davis (2022) in nearly every case a "super-procedure" selects the wrong "sub-procedure". Likewise, there are mathematical errors based on the tendency of students to see the linear function everywhere. For example, students may consider $|a+b|$ equal to $|a|+|b|$ or $\sin(a+b)$as equal to $\sin a+\sin b$

To interpret error regularities, experts formulate hypotheses on the procedures through which students solve problems, attempt to identify mistakes, assess difficulties and suggest solutions to overcome them (Gagatsis & Christou, 2017). This approach has significant implications for teaching practice since it implies that the identification of mistakes helps teachers decide how to identify and meet students' learning needs and how to use their teaching time and their resources (Gagatsis & Kyriakides, 2000). Decisions about the next learning steps follow from the formative identification of students' errors (Michael-Chrysanthou & Gagatsis, 2014). A teaching plan, which is organized in such a way, might help teachers to plan class and individual programs of work according to the different performance levels of the students.

These views on errors suggest that teachers need to respond to students' errors in ways that involve understanding the students' thinking behind the error, which in turn can inform teaching. Such ways of dealing with errors require that teachers shift their understanding of students' errors; from viewing errors as obstacles to learning mathematics to an understanding of errors as integral to learning mathematics and as possible sources of learning mathematical concepts (Borasi, 2014).

Research on errors in mathematics highlights various pertinent issues relating to the nature of errors and teachers' conceptions of errors. In a research project conducted in South Africa called the Data Informed Practice Improvement Project (DIPIP) errors were defined as systematic, persistent, and pervasive mistakes performed by learners across a range of contexts (Brodie, 2014). In DIPIP, students' errors were regarded as evidence of learner thinking on which teachers could draw to help learners understand mathematical concepts. In that paper, the researchers share the same view of errors and argue that teachers need to view errors as integral to learning mathematics if teachers are to help students in teaching and learning situations. In the study, they sought to find out how a group of practicing mathematics teachers viewed and explained learner errors.

In mathematics, errors are different from slips. Slips are mistakes that are easily corrected (Brodie, 2014). In teaching and learning situations when students make slips, they are often easily identified and corrected either by the student or the teacher. Slips usually do not recur once they are corrected. Errors are mistakes that tend to recur. Errors arise independently of the teaching methods used (Brodie, 2014) and are often persistent even when corrected (Brodie, 2014). According to White (2015), teachers need to find out why students make errors in the first place. Errors have also been characterized as a world-wide phenomenon and are made by students of any age, country, or ability (Gagatsis & Kyriakides, 2000). These ideas highlight the pervasiveness and persistence of errors, which implies that irrespective of teaching methods, errors will always arise in the process of students' learning mathematics. It is the researcher's view that, teachers need to have this understanding of errors if they are to engage productively with errors in their teaching for the benefit of students' understanding of mathematical concepts.

A misconception is defined as "a student conception that produces a systematic pattern of errors" (Smith et al, 2017). This idea suggests that misconceptions are not easily discernible, but are manifested through error patterns that are observed in students' work.

As students faced with new situations, they draw on their prior knowledge or experiences to make sense of the new situations. The basic cognitive argument is that in making attempts to work with previously acquired knowledge in novel situations students' prior knowledge becomes inadequate for explaining phenomena and solving new problems, hence errors occur (Smith et al., 2017). Thus, errors are seen as reasonable and sensible for students in that they are a result of a student's reasoning within the context of existing mathematical knowledge and the student is normally convinced that the work is correct (Brodie, 2014; Lourens & Molefe, 2011). This view of errors suggests the need for teachers to engage with students' errors in ways that enable them to identify the students' thinking or conceptions behind any observed errors. Such knowledge will enable teachers to deal with students' errors in ways that support students in accessing the correct mathematical knowledge.

Regardless of the arguments for including algebra in the mathematics curriculum, algebraic problems are the most dreaded, feared, and disliked aspect of the subject (Bullock, 2015). Mathematics teachers are concerned about their students' dislike and anxiety about algebraic problems (Singh, 2022). Students' dislike for algebra severely undermines the utility of mathematics (Ihu & Kyeleve, 2021). Reinsburrow (2021), found that low- and average-achieving students disliked algebraic word problems in a study aimed at improving students' problem-solving skills in 37 middle school graders. Rembert et al (2019), discovered that approximately 70% of students disliked word problems due to their inability to solve them or because they did not find algebraic word problems relevant to their interests and identity. Without a doubt, algebra problems are difficult for most high school students to solve (Andam et al, 2015).

Adu et al (2015), investigated errors made by high school students in Ghana when solving word problems in linear equations. In their study, ten-word problem tasks were used. According to Adu et al (2015), a breakdown of students' difficulty in solving word problems in algebra revealed that 75% to 84% of the 130 students examined committed various errors, including comprehension and transformation errors. According to Adu et al (2015), while approximately 60% of the students attempted most of the questions, only 2% produced correct answers, highlighting students' inability to comprehend and transform worded problems into equations. As a result, they are unable to solve algebraic problems.

As a result, high school students in Ghana avoid answering algebraic tasks (Andam et al, 2015). The curriculum planners' recommendations led to the inundation of algebra in mathematics. The mathematics curriculum planners recommend that mathematics teachers, textbook authors, and curriculum implementers include algebra and its word problems in all topics (Ministry of Education, 2010). Because senior high school students in Ghana dislike word problem tasks, which may be attributed to the perceived difficulty in solving algebraic problems (Rembert et al, 2019), it is unclear whether the students are aware of the importance of algebra in high school mathematics study.

Errors are common in the subject's teaching and learning, as well as in students' written work. Mathematics teachers react differently to their students' errors during instruction. Some teachers respond by ignoring mistakes, while others make an effort to engage with them (Gardee & Brodie, 2022). For more than a decade, there has been an increase in interest in students' mathematical errors and misconceptions (Gardee & Brodie, 2022). There have been calls for teachers to embrace errors rather than avoid them because they are frequently the result of mathematical thinking on the part of students and thus are reasonable for the students (Gardee & Brodie, 2022).

These perspectives on errors suggest that teachers should respond to students' errors by understanding the students' thinking behind the error, which can then inform teaching. Such approaches to dealing with errors necessitate a shift in teachers' perceptions of students' errors, from viewing errors as impediments to learning mathematics to viewing errors as an integral part of learning mathematics and potential sources of learning mathematical concepts (Soncini, 2022).

1.2 Statement of the problem

During instruction, mathematics teachers respond differently to their students' errors in algebra. Some teachers ignore errors while others make efforts to analyze the errors (Hansen, 2020). For more than a decade there has been a growing interest in students' errors and misconceptions in mathematics (Chauraya, 2017). There have been calls for teachers to embrace errors rather than avoid them and such thinking is based on the justification that errors in mathematics are pervasive and systematic (Warshauer, 2015). During the researcher's interactions with colleagues mathematics teachers in Otumfuo

Osei Tutu II College and the other senior high schools in Afigya Kwabre North District and personal observation on how both teachers and students the researcher notice that, students do not pay much attention to the mathematical statements involved in answering the question and do not read the terms used in the algebraic problems very closely and therefore make errors in solving such problems. Students make more errors in questions that require higher analysis, such as problem-solving, evaluation, and application in mathematics examinations.

Despite, numerous interventions by governments, both past and present to improve the performance of students in mathematics, there had been a little decline in mathematics performance over previous years in senior high schools, Oduro (2015) using a 2007 report in the Trends in International Mathematics and Science Study (TIMSS), indicates that, in 2007, Ghana scored 309 which was lower than all the countries that participated in the assessment. This abysmal performance of students has implications for the country's advancement. The question then is what factors bring about excellent academic performance. One of these factors could be the teacher's ability to understand and interpret students' errors in algebraic problems.

For students to perform well in algebra, teachers need to respond to students' errors in a manner that involves understanding the students' thinking behind the error. Such ways of dealing with errors require that teachers shift their understanding of students' errors, from viewing errors as obstacles to learning mathematics, but rather understanding errors as possible sources of learning mathematical concepts (Hansen, 2020). However, there is no enough studies on the relationship between students' errors and senior high schools mathematics teachers' interpretations of students' errors in solving problems in algebra in Ghanaian, especially in Afigya Kwabre North District.

Most studies have concentrated on perceptions and identification of students' errors and they seem to agree that Ghanaian students make some common errors in solving problems in algebra (Adu et al, 2015; Zeina & Matthew, 2016; Capone, 2021). It is against this background of the literature gap that this study sought to analyze teachers' interpretations and perceptions of students' errors in solving algebra.

1.3 Purpose of the Study

The purpose of this study was to investigate senior high school teachers' interpretations and perceptions of students' errors in algebra in Afigya Kwabre North District.

1.4 Research Objectives

The objectives that guided this study were;

1. To examine how mathematics teachers interpret errors committed by students in solving problems in algebra in the Afigya Kwabre North district.
2. To investigate the perceptions of teachers on errors committed by their students toward solving problems in algebra in the Afigya Kwabre North district.
3. To determine whether there is uniformity of errors committed by the students among the senior high schools in solving problems algebra in the Afigya Kwabre North district.

1.5 Research Questions

These research questions were designed to help achieve the stated objectives;

1. How do mathematics teachers interpret students' errors in algebraic problems in Afigya Kwabre North district?
2. What are the perceptions of teachers on errors committed by their students toward solving algebraic problems in the Afigya Kwabre North district?

3. Is there any significant uniformity of errors committed by the students among various senior high schools in solving algebraic problems in the Afigya Kwabre North districts?

Hypotheses

The following hypothesis were formulated based on research question three.

H_0: There is no statistically significant uniformity of errors committed by students among various senior high schools in solving algebraic problems.

H_1 : There is a statistically significant uniformity of errors committed by students among various senior high schools in solving algebraic problems.

1.6 Significance of the Study

(i) Teachers

The findings of this study would benefit the teachers in senior high schools in undertaking remedial teaching in mathematics. This is in line with the ministry of education policy that schools should identify students with learning difficulties and design appropriate programs for them. By analyzing and establishing the errors in algebra solving, this study provides teachers with appropriate strategies and approaches in teaching of algebra problems consistently, persistently and full of patience that would improve achievement in mathematics in the senior schools. The study will also become a bench-mark for mathematics teachers to be better designers of the teaching and planning by looking at the strategies and the level of language that suit the ability of the senior high school students in solving algebraic problems in mathematics. Teachers have been shown that they should give the students more alternative approaches in solving mathematical algebraic problems so that they can arrive at correct solutions without errors.

(ii) Students

This study was instrumental in establishing the errors made by the students in senior high schools. The sources of these errors were highlighted. Other factors that contributed to students' errors were brought out and ways of reducing these errors were provided by this study.

(iii) Curriculum Development

Overall, this study has provided information to curriculum developers and officers who are responsible for the development and implementation of the senior high school mathematics curriculum. The study highlighted that there is need to review the resources used in learning of mathematics so as to reflect the everyday life. Objectives of mathematics should be achieved in the implementation stage if the most desired technological advancements are to be realized as envisaged in vision 2030.

1.7 Delimitation of the Study

The study was delimited to Senior High School teachers' perceptions of students' errors in algebra, and how they interpret those errors, though there are other aspects such as the students' perceptions towards the learning of algebra, factors that influence such beliefs, and others.

This study also confined itself to teachers of three Senior High Schools in Afigya Kwabre out of four (4) senior high schools in the Afigya Kwabre and about one hundred and twenty-two (122) senior high schools in the Ashanti Region. Only Forms three (3) students' class exercises, homework, tests, and assignments were used for the study since it is believed that they have been taught all aspects of algebra stipulated in the syllabus and their performance can be compared with the national standard. All students in form three (3) offering the various courses in the school were included.

1.8 Limitations of the Study

There were many limitations of the study and including: how perceptions would be scored or measured. Many external factors that could influence the perceptions and interpretations of teachers would not be accounted for. These factors include the influence of peers or other teachers and the influence of students and siblings.

Thirdly, the questionnaires which were used for collecting information for the study have their weakness. These weaknesses include; bias, incompleteness, variability in response, mechanical limitations or make-up, non-response errors, lack of clarity in definitions, ambiguities or inappropriate wording, limited responses, and briefness.

Besides, the study used only senior high school mathematics teachers and class exercises, homework, tests, and assignments from form three (3) students. On the other hand, samples of class exercises and assignments of students in form two (2) and form one (1) could have contributed to the study.

To add to that, the study would have been extended to cover all the Senior High schools in the entire region to arrive at a valid and more reliable outcome, but as a result of limited time, logistics, and financial constraints, the focus was based on only three Senior High Schools in Afigya Kwabre. These limited resources did not allow me to take a wide study.

1.9 Organization of the Study

The study would be organized into five chapters. The first chapter gives an overview of the background of the study and the key objectives of the research. The second chapter examines the relevant theories from both an appreciative inquiry and critical analysis point and also gives some perspectives on some empirical works. This provides relevant information on work done in the same field of study. The third chapter gives an account of the methodology. The collection and organization of the data have been spelled out here. The fourth chapter involves the analysis of the data as well as the interpretation of findings and the fifth chapter discusses the findings. The analysis and the conclusions and recommendations of the study.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

The literature review is discussed in the following; the first section justifies the theoretical framework, the second section discusses types of algebraic problems and difficulties in solving problems in algebra. The third section discusses the nature of errors in mathematical algebraic problems and the factors of errors in mathematical problems. The fourth section talks about difficulties in initial algebra learning and students' errors in algebraic problems. And the last section discusses teacher knowledge for teaching as well as the various strategies or teaching methods for developing students' critical thinking and mathematics problem-solving.

2.1 Theoretical Framework

The study was based on the Conditions of Learning Theory by Gagne (1985) which stipulates that there are several different types or levels of learning. The significance of these classifications is that each different type requires different types of instruction. Gagne (1985) identifies major categories of learning: verbal information, intellectual skills, cognitive strategies, motor skills, and attitudes. Different internal and external conditions are necessary for each type of learning. For example, for cognitive strategies to be learned, there must be a chance to practice developing new solutions to problems; to learn perceptions, the learner must be exposed to a credible role model or persuasive arguments.

Gagne (1985) suggested that learning tasks for intellectual skills can be organized in a hierarchy according to complexity: stimulus recognition, response generation, procedure following, use of terminology, discriminations, concept formation, rule application, and problem-solving. This is an essential aspect of learning mathematics where the use of mathematics terminologies, concept formation, and application problem-solving must be developed in the learners to enable them to perform mathematical tasks easily. The primary significance of the hierarchy is to identify prerequisites that should be completed to facilitate learning at each level.

Prerequisites are identified by doing a task analysis of learning or training tasks. Learning hierarchies provide a basis for the sequencing of instruction in a mathematics classroom. A lack of prerequisite skills in mathematics would lead the students in making errors when solving algebraic problems.

According to Prakitipong and Nakamura (2016), there are two kinds of obstacles that hinder students from arriving at correct answers in the process of solving algebraic problems:

1. Problems in linguistic fluency and conceptual understanding correspond with the level of simple reading and understanding the meaning of problems.
2. Problems in mathematical processing consist of transformation, process skills, and encoding answers.

This classification implies that the students have to interpret the meaning of the question before they proceed to mathematical processing to obtain an appropriate answer. Mathematics learning involves some variables such as the learner, teacher, mathematics content, methods of instruction, and resources. The learners are expected to interact with the content during instruction by the teacher. This content forms a mathematics curriculum that includes the four basic areas namely; concepts, computations, applications, and problem-solving. Inadequate understanding of concepts featuring in mathematics word expressions, wrong computations, and inability to choose the appropriate processes in solving algebraic problems usually lead students into making errors in mathematics (Sainah, 2018).

The errors made by the students in solving problems in mathematics will depend on reading, comprehension, transformation, process skills, and encoding is given questions in word problems. Sources of errors can therefore be attributed to methods of instruction, syllabus coverage, strategies used in solving word problems, and availability of learning resources in mathematics. Teaching experience is necessary to guide the students to have skills in problem interpretation in solving algebraic problems in mathematics.

2.2 Conceptual Framework

A conceptual framework, according to Ludviga (2023), is a written or visual presentation that explains either graphically, or in narrative form, the main things to be studied and the presumed relationship among them. A diagrammatic representation of the modeled variables' path is presented in figure 2.1.

According to Abdullah et al (2015). Students in senior high schools make errors when solving algebraic problems. He classified the errors according to student's inability to

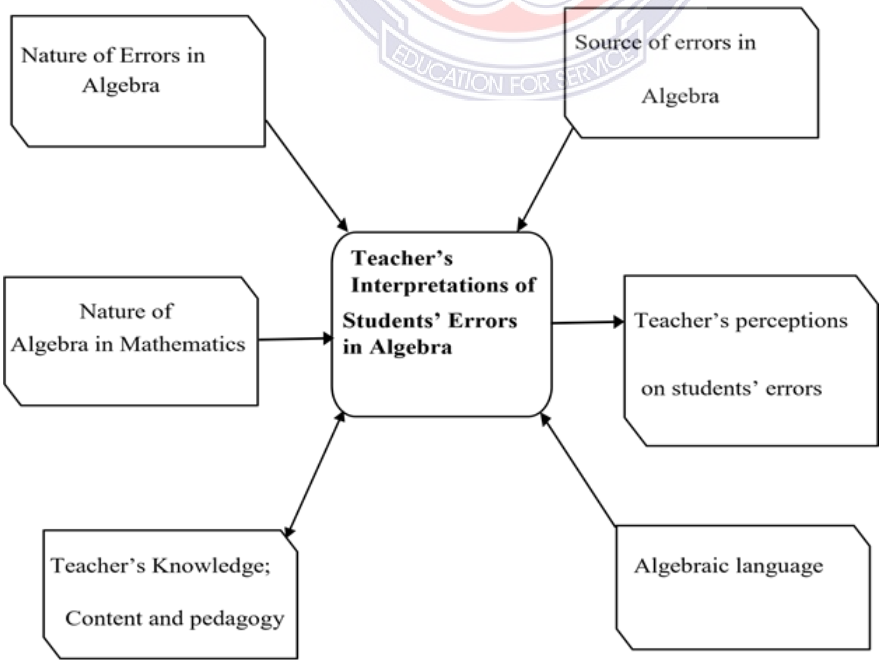

Figure 2.1 Diagrammatic Representation of the Modelled Variables' Path

They may also make errors due to the strategies used by the teachers during instruction. This study was concerned with the ability of teachers to interpret students' errors in algebra and their perceptions on those errors. If teachers are well trained in errors analysis in algebra, they are likely to conceptualize the ideas easily as they become more interested in analyzing algebraic problems. In situations where the schools are not providing in-service training to teachers and enough teaching resources such as mathematics models and teachers' handbooks, it impacts negatively on teachers' ability to make mathematical representations which may lead to teachers' inability to explain reason(s) of students thinking behind each error in solving problems in mathematics. Measures such as: elaborate teaching methods that would accord more time to the students in problem interpretation skills and familiarization with mathematics language in the classroom would be vital in their problem-solving ability. These would enhance skills in solving algebraic problems that would further impact positively in the overall students' achievement in solving algebraic problems in mathematics in senior high schools.

2.2.1 Types of Algebraic Problems

There are many types of algebraic problems including addition/subtraction problems, multiplication/division problems, and multi-step word problems. For example, consider the following question: There are 16 girls and 14 boys in a class. How many students are there in the class? The problem type here is combining or finding the total number of students. While if they were asked: How many more girls than boys are in the class? Then the problem type is finding the difference. Finding the total is different from finding the difference. Moleko and Mosimege, (2021), state understanding the structure of algebraic problems, helps students become better readers and problem solvers. Also, according to Arsenault and Powell (2022), once students determine the type of question, they can use a diagram and an equation to solve the problem. When students know the schema for each type, understand how to sort out the problem and write a solution method, they should be able to solve most word problems (Arsenault & Powell,.2022).

2.2.2 Difficulties in Solving Algebraic Problems

Algebraic problems are present in everyday life. For example, consider the problems: how will I get to my friend's house from my current location by 4:00 when it's already 2:30 PM and I don't have a car and not enough money to take a taxi? About how much is the total bill to pay in the grocery store? How much is the final price of a laptop if the original price was ghc2, 000 and the discount rate was 15%? Joe got 75 on his science exam, which is 5 points less than he achieved on the math exam. What was his score on the math exam? Five friends share the cost of 4 pizzas and a salad. Each pizza costs ghc20 and the salad costs ghc10. How much does each of the five friends pay? Problem-solving is a necessary skill not only in mathematics but also in everyday living (Swastika et al 2022).

Arsenault and Powell (2022), purport that regardless of the problem type, students need to learn a strategy for working through the problem. Some students cannot interpret word problems if they do not visualize the key elements in a diagram or a bar graph.

The ability to visualize the problem can lead to successful problem-solving (Dela & Lapinid, 2016). Another difficulty concerning the process of understanding the problem is students not understanding the assumption in the question inhibiting them from proceeding and translating the problem into a mathematical equation.

Some students have difficulties analyzing algebraic problems. They are either unable to translate or translate incorrectly. According to Salemeh (2016), it is important to teach students how to think in solving such problems and explain to them that they can develop a lot of skills through practice. Students will acquire a lot of reasoning when they observe how others solve problems. Some learners do not comprehend the problem and they tend to be confused. Others look for keywords when they read a problem instead of understanding it. This will lead to incorrect translation. When students fail to translate the problem, they end up with an erroneous solution (Dela & Lapinid, 2016).

In addition, some students tend to use the wrong operation. For example, my mother plans to buy 12 house decors worth ghc55 each. How much will she have to pay in all?

Some students tend to use addition instead of multiplication. It is important to make a plan for solving a word problem. Pearce et al (2022), assert reading skills played a significant role in solving word problems. The carelessness of students can also be a source of difficulty in solving word problems. Some might copy the number given incorrectly. Instead of writing 1500, they copy it as 500 or even add a digit to it as 11,500. Although they understand the given and what operation to use, copying the given numbers incorrectly will lead them to an incorrect answer (Dela & Lapinid, 2016). Some students tend to interchange the order of numbers in the question. Subtraction and division operations are not commutative. The minuend cannot be placed in the subtrahend's place and the same applies to divisor and dividend. For example, forty-eight taken away from a number gives ten. The number forty-eight here is in the subtrahend's place. The answer is obtained by adding the subtrahend to the difference and not by subtracting them as it may appear (Dela & Lapinid, 2016). Furthermore, the presence of unnecessary information in the problem can be distracting and considered a source of difficulty. For example, John has 30 dollars. Jane has 25 dollars more than John and Jane is 160 cm tall. Find the amount of money that Jane has. It is obvious

Jane's height is unnecessary information in the question (Gooding, 2009). Salemeh, and Etchells (2016) also suggests another source of the difficulty can be the child is not using jottings. When asked to perform a certain calculation, the child was doing it mentally instead of writing down the numbers and carrying out the calculation more effectively. Some students treated word problems too realistically. For example, Edward earns 5 dollars for every bundle of a newspaper he delivers. How many newspapers must he deliver to buy a toy car that costs him 28 dollars? If students were too realistic in their answer, they would say Edward must deliver 5.6 newspapers instead of 6. Another example can be Steven earned 33 dollars for delivering newspapers. How many did he deliver if each delivery was 5.5 dollars? (Salemeh & Etchells, 2016)

2.3.1 The nature of errors in mathematical problems

Research on errors in mathematics highlights various pertinent issues relating to the nature of errors and teachers' conceptions of errors. In a research project conducted in South Africa called the Data Informed Practice Improvement Project (DIPIP) errors were defined as "systematic, persistent, and pervasive mistakes performed by learners across a range of contexts" (Brodie, 2014). In DIPIP students' errors were regarded as evidence of learner thinking on which teachers could draw to help learners understand mathematical concepts. I share the same view of errors and argue that teachers need to view errors as integral to learning mathematics if they are to help students in teaching and learning situations. In the study, I sought to find out how a group of mathematics teachers viewed and explained learner errors.

In mathematics, errors are different from slips. Slips are mistakes that are easily corrected (Kaufmann et al, 2022). In teaching and learning situations when students make slips these are often easily identified and corrected either by the student or the teacher. Slips usually do not recur once they are corrected. Errors are mistakes that tend to recur. Errors arise independently of the teaching methods used (Peng et al, 2022). According to Bloome (2022), teachers need to find out why students make errors in the first place. Errors have also been characterized as a worldwide phenomenon and are made by students of any age, country, or ability (Chauraya & Mashingaidze, 2017). These ideas highlight the pervasiveness and persistence of errors, which implies that irrespective of teaching methods, errors will always arise in the process of students' learning mathematics. It was our view in this paper that teachers need to have this understanding of errors if they are to engage productively with errors in their teaching for the benefit of students' understanding of mathematical concepts.

2.3.2 Factors of error in mathematical algebraic problems

Abdullah (2015) "there are two factors that make the students unable to produce correct answers, namely: problems in the fluency of languages and understanding concepts, and problems process skill of mathematics (understanding, transformation errors, process skill, and writing answers)". According to Ismail (Abdullah, 2015), "student misconduct in completing mathematics deals with the following characteristics: (a) cognitive activity, (b) metacognitive ability, (c) attitudes (d) knowledge possessed by them. Various levels of characteristics have caused different errors in each student and different abilities for them to solve math problems. The problem-solving process skill errors are one of the cognitive and skill strategies that the individual must plan for achieving the goal. Therefore, low-ability students, do not have a strategy to solve the problem. Such a situation would be more difficult if students did not understand the given problem and could not identify mathematical operations".

Factors that cause errors when viewed from student learning difficulties and abilities are outlined as follows (Abdullah, 2015):

Students are not able to absorb information well

The information contained in the problem is not fully absorbed by the students. Students are confused in determining what is known in the matter, unable to abstract the matter into mathematical patterns, and find no solution formula. Some students confuse the meaning of words used in mathematical teaching by giving their meaning.

The lack of experienced students working on the problem

Students practice with various variations of the problem, especially the story in the form of a narrative without any illustrations and problems that are varied with a more complex form, so students are often confused about how to solve the problem. Justification, they are not used to thinking of alternative solutions to problems that are different from the examples that have been studied.

The weak ability of the concept of prerequisites

Students are not able to do the process because they do not master the prerequisite concepts related to the given material.

Negligence or carelessness of students

Students are not careful and not careful in the process of the problem, either at the time of writing the formula or when doing the count. In this study, students tend to rush through the process of working without first reviewing the right concepts to solve the problem and did not examine the answers that have been written. (Abdullah, 2015)

2.4.1 Difficulties in initial algebra learning

The term "difficulties" refers to obstacles that cause errors or mistakes by students when dealing with algebraic word problems.

From the existing research literature and an interview study, we earlier identified the following five categories of difficulties in initial algebra (Jupri et al, 2014):

The category of applying arithmetical operations in numerical and algebraic expressions (abbreviated as ARITH) includes difficulties in adding or subtracting similar algebraic terms, also difficulties in using associative, commutative, distributive, and inverses properties; and in applying priority rules of arithmetical operations (Jupri et al, 2014) The category of understanding the notion of variable concerns difficulties to distinguish a literal symbol as a variable that can play the role of a placeholder, a generalized number, an unknown, or a varying quantity (Jupri, & Drijvers, 2016).

The category of understanding algebraic expressions encompasses the parsing obstacle, the expected answer obstacle, the lack of closure obstacle, and the gestalt view of algebraic expressions. The parsing obstacle in this case refers to understanding the order in which the algebraic expressions must be processed, the expected answer obstacle concerns the expectation to get a numeric result rather than an algebraic expression, and the lack of closure obstacle refers to discomfort in handling algebraic expressions that cannot be simplified any further.

The category of understanding the different meanings of the equal sign concerns difficulties in dealing with the equal sign, as an equal sign in arithmetic usually invites a calculation, while it is a sign of equivalence in algebra (Jupri & Drijvers, 2016). Finally, the category of mathematization concerns the difficulty to translate back and forth between the world of the problem situation and the world of mathematics, and in the process of moving within the symbolic world (Yuhasriati et al, 2022).

2.4.2 Students' errors in algebra

For most students entering senior high school, learning algebra marks a departure from the world of numbers that they were used to in primary school to the use of letters in learning mathematics. Although the rules for manipulating numbers are the same as those for algebraic systems, students usually find it difficult to shift from working with numbers to working with letters and words (Chan et al, 2022). Research on students' errors in algebra highlights some common errors that are attributed to students' understanding of letters as used in algebra and algebraic processes (Chauraya, & Mashingaidze, 2017).

According to Chauraya and Mashingaidze (2017), students' errors in algebra can be attributed to a conception that answers have to be numerical; students' interpretation of operational symbols in algebra; perceptions of letters as objects rather than variables; and misunderstandings of arithmetic that are carried over to algebra. In their study, Chauraya and Mashingaidze (2017) found that students could not find the perimeter of an n-sided regular shape when given the length of each side. For the students, the perimeter could only be found if the numerical value of n was known. The study also found that when given expressions such as 2a+3b, students wrote 7ab as the 'answer', which indicated an interpretation of the '+' sign as a signal to add as normally used in arithmetic, and a belief that an answer has to be a 'single' term. Students could also not explain the meaning of letters in given expressions. For example, when asked what y represented in an expression such as 3+5y, students said that y stood for anything such as a 'yacht' or 'yam' (Booth, 20018). Such responses indicated students' conceptions of letters as standing for objects rather than variables, and such conceptions have been found to cause several common errors in learning algebra. Mathematics teachers need to understand and explain these errors to help students deal with some of the common errors in algebra.

2.5.1 Role of the Teacher of students solving algebraic problems

The role of any teacher in the classroom is to educate the students and prioritize the important things for them to learn. It can start by building self-confidence in every student. Teaching word problems is not an easy task. The most cited classroom practice was working on the problem independently (Kwok et al, 2022). Classroom practices and strategies teachers use are crucial to foster student problem-solving. Students must be able to recognize the types of word problems and the appropriate solution to solve the problem (Powell et al, 2022). The goal is to teach students a strategy to help them become more independent learners. For example, cognitive strategy instruction can be used to teach young students with mathematical difficulties to enhance learning and improve their performance (Powell et al, 2022).

Cognitive strategy instruction consists of teaching cognitive and metacognitive strategies that can guide students to understand and be self-aware of the requirements. A cognitive strategy helps students focus on the problem structure and increase their ability to understand the problem (Pfannenstiel et al, 2015). This strategy is a vital component for students in the younger grades to solve effectively word problems (Pfannenstiel et al, 2015).

The metacognitive strategy helps students plan, monitor, and modify their approach to solving a word problem (Pfannenstiel et al, 2015). It addresses six components of word problem-solving. First, to state the question, identify the important units and numbers, analyze the question, select the operation needed to solve, create a strategy to solve, and finally remove any unimportant information (Pfannenstiel et al, 2015). There are three main steps aligned with these six components. The first step is to inspect and find clues, a verbal strategy. Students must read the problem, circle important words, and numbers, and then cross out the unnecessary information. The second step is to plan and solve by drawing a diagram or a map and then writing an equation. This is intended to build algebraic readiness skills. The final step is to check the answer.

Both cognitive and metacognitive strategies have been shown to increase students' understanding of word problems. Teachers must focus on teaching these strategy steps to their students by engaging all learners in an interactive process. It is recommended teachers encourage their children to make a plan. Firstly, they read the word problem individually many times, then point out the important information in the question and the requirement, then they know what number sentence to use, solve independently, and finally check if the answer satisfies the question.

Students have to go one step at a time, reading and comprehending to be able to translate a word problem into a mathematical one (Salemeh, & Etchells, 2016). It might be a good suggestion to start by asking the student about the difficulties they encountered in solving a certain word problem. Most difficulties arise when the learner cannot imagine the word problem's context, so the child tends to find it hard and gets confused about what number of sentences to write. Therefore, the role of the teacher is to explore new methods of explaining the concept (Salemeh, & Etchells, 2016).

According to Barwell et al (2022), motivated learners are more excited to interact and learn. Teachers need to identify the students who are disengaged in their classes and find strategies to motivate them. Another suggestion is to give them a word problem based on a historical event or a sports game to maximize their engagement in class

(Kolta, 2022). Word problems can be challenging for some students. According to Swanson (2014), children need to be directed to consider relevant information within the context of increasingly irrelevant information. Thus, students must follow multiple steps in solving such problems. They should learn the skill to read the subtext and understand the given information, mathematize the problem by writing a detailed plan, make mathematical connections, analyze, and then check.

Dixon et al (2014), affirms learners will develop a deep conceptual understanding of word problems when their teachers provide them with rich, and meaningful learning activities. For example, if students are asked to write their word problem, it eludes to their interests and they will be more engaged in valuable and meaningful mathematical thinking. For instance, invite students to write a word problem to a relatable event in their lives, such as a trip, a football game, etc. Students that can create their math word problems will be positively influenced and this will reflect, not only on their understanding but also on their problem-solving skills and disposition toward mathematics (Dixon et al, 2014).

2.5.2 Teachers' perceptions of students' errors in algebra

Research on teachers' perceptions of students' errors can be classified into three categories. The first category relates to studies that sought to investigate teachers' interpretations of common students' errors in mathematics (Chauraya & Mashingaidze, 2017). The second category is that of studies that sought to explain the reasons for students' errors in mathematics (Chauraya, & Mashingaidze, 2017). The last category of the studies that investigated teachers' perceptions of students' errors (Chauraya, & Mashingaidze, 2017). The researcher's study fell in the first and the last categories of these studies.

The first category of studies analyzed how teachers at senior high school levels explained learner errors in solving word problems in an algebraic expression. In South Africa, Shalem et al (2016), developed a coding criterion for senior high school mathematics teachers' explanations of learner errors using data from the DIPIP project. In the project, 62 teachers drawn from 9 schools were asked by the researchers to explain learner errors on international standardized mathematics assessments. The study found that teachers drew mostly on their mathematical knowledge and knowledge of learners to explain learner errors, and less on other possible factors such as the nature of the test or the mathematics curriculum. The authors also found that there were more partially correct procedural explanations than correct and conceptual explanations of the errors. As well, the teachers described the errors without explaining the learners' reasoning behind the errors. A significant percentage of the explanations were found to be inaccurate. In another study, Chauraya, and Mashingaidze (2017), found that teachers relied on their knowledge of the mathematics content to explain learners' errors and that the explanations were mostly procedural rather than conceptual.

In their study, Hu et al (2022), asked teachers who participated in an in-service course in mathematics about their understanding of the causes of students' errors and their explanations of particular errors. The researchers found that the specialized course affected the teachers' understanding and explanations of errors. The teachers no longer attributed errors to student factors only such as students' attitudes, but saw and explained errors as a result of the nature of mathematical knowledge and the rules in mathematics, for example viewing errors as a result of previous correct knowledge which is not applicable in a new situation. The teachers also attributed students' errors to the 'didactic contract'. The didactic contract refers to "the widespread tendency by students to answer school math word problems with apparent disregard for the reality of the situations described by the text of the problems" (Chauraya, & Mashingaidze, 2017) Examples of adhering to the didactic contract occur when students accept a solution to a problem based on their computation, although the solution may not make sense in the context of the problem.

2.5.3 Teacher knowledge; content and pedagogy

Superior teacher knowledge both in content and pedagogy is a prerequisite to effective teaching and learning in all subject areas. Conflicting positions have been identified by mathematics teachers as to where the emphasis should be placed, and recent developments in teacher education have shifted focus primarily to pedagogy often at the expense of content knowledge (Ananin & Lovakov, 2022). According to them, the focus is only on the pedagogical practices in the classroom, isolated from any relevant subject matter.

However, teacher education programs should combine these knowledge bases to more effectively prepare teachers for classroom work. Mathematics Education Reforms in recent times have begun to bridge the gap between the pedagogical and content aspects of mathematics teachers' preparation by advocating the development of a cohesive knowledge base (Şen et al, 2022). In support of this view, Schiering et al (2022), proposed that it was not enough to teach content and pedagogy as two separate entities.

She indicated that good teaching required a complex integration and balance of the two. Such integration will greatly assist teachers to acquire the requisite knowledge and skills to effectively analyze the processes of working and marking or scoring students' work to effectively assist in reflective and remedial teaching.

In a more holistic dimension, Stronge (2018), describes teacher knowledge as having three components, notably teacher knowledge, the ability to identify typical students' errors, knowledge particularly associated with teaching strategies (pedagogy), and knowledge of content elaboration. All these components come to play when teaching mathematics, they stated. Furthermore, Metsäpelto et al (2022), emphasized the fact that understanding student errors by teachers and developing effective strategies to avoid them is an important aspect of Pedagogical Content Knowledge. Most curriculum documents emphasize both. However, it has been noted that one of the challenges in teaching is how to address both aspects (Chicks, 2003).

2.5.4 Teaching methods for developing mathematics problem-solving in algebra

Studies reveal that requiring students to explain their mathematical thinking provides teachers with the opportunity to determine the source of students' misconceptions and to adjust instruction accordingly (Klem, 2021). He studied the impact of a two-year professional development program in one basic school classroom using cooperative planning and teaching model. The study focused on using rich mathematical tasks, classroom discourse, and strategies to encourage student self-regulation. The researcher found that having students create written representations of their solutions and discuss their thinking regularly motivated students and led to improved mathematical understanding. In the end, he concluded that students in the class were now more able than previously to communicate mathematical understanding and justify their mathematical reasoning. They have been exposed to many strategic behaviors and are more able to articulate their strategies (Brendefur et al, 2022).

In a similar study conducted by Bywater et al (2022), techniques for having students explain their thinking in a teacher experiment were designed to determine the benefits of writing on the development of algebraic thinking. According to the study, students were given mathematical tasks without the teacher modelling the solutions or providing suggestions for strategies. Instead, students were asked to draw on prior knowledge to select and implement a strategy for each new problem situation. Appropriate problem-solving strategies for the tasks included creating diagrams, tables, and graphs, working backward, and writing equations. In an attempt to develop algebraic thinking, tasks were specifically sequenced to build on the previous task. Students were asked to provide written explanations of their problem-solving process to understand the ways students use writing to create knowledge. After writing preliminary solutions, students were able to discuss the problems with their peers and make revisions if they felt compelled to do so (Bywater et al, 2022). Data were collected from eight students which consisted of written work and student interviews.

From the study, they concluded that most students developed connections between their conceptual and procedural knowledge by analyzing and recognizing patterns, representing and generalizing quantitative relationships in patterns, and generalizing quantitative relationships toward a formal symbol system. Analysis of the data showed that students developed conceptual knowledge as well as an understanding of procedural knowledge as a result of their mathematical writing. (Davis & Witt, 2022).

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The role of communication in developing critical thinking skills is also evident in Pizziconi and Iwasaki (2022). This is the study of one teacher from within a research project that sought to identify teaching strategies that promote students' mathematical modeling abilities. Pizziconi and Iwasaki perceived students' ability to construct accurate models as an integral component of conceptual understanding. According to them, the actions of the teacher supported extensive student engagement with the task and led the students to revise and refine their mathematical thinking. This latter action reflects a significant shift in classroom practice from the role of the teacher as an evaluator of student ideas to the role of students as self-evaluators of their emerging ideas.

While most studies in communication focus on either oral discussion or written explanations, Yu (2022) investigated their combined impact in a quasi-experimental study of five teachers and more than 200 students. Four separate groups participated in the study. A treatment group that received traditional instruction, a group that participated in argumentation only, a group that participated in written explanations only, and finally a group that utilized both oral argumentation and written explanations. All students completed a pre-test and a post-test. Results showed that students who participated in both argumentation and writing demonstrated the highest achievement gains. Qualitative analysis of classroom discourse also revealed pupils in this group developed a deeper understanding of the content. These students were allowed to reflect and revise their thinking thereby maximizing their learning (Yu, 2022).

Sampson (2021) took the communication investigation one step further than Cross by examining the impact of instruction that focuses on speaking, listening, writing, and reading in a two-year quasi-experimental study. This study was conducted in science classrooms of four different schools; more than 200 middle-school students participated. While this study was not designed for mathematics students, the results mirrored those found by Cross. Sampson III noted the importance of building time into the curriculum for students to communicate their understandings as the process of learning unfolds. Communication is a learning opportunity that should, however, not be used solely for summative purposes. Sampson III cautions teachers not to assume that students comprehend simply because of the grand nature of a lesson. Teachers should use communication as a tool for verifying understanding throughout the lesson. The aforementioned studies relate communication to the development of critical thinking skills but they fall short in describing specific methods for implementing classroom discourse. In a synthesis of research, Wallace et al, (2014) suggested five explicit steps teachers can follow to maximize the benefits of classroom discussions.

Anticipating likely student responses to cognitively demanding mathematical tasks.

Monitoring students' responses to the tasks during the explore phase.

Selecting particular students to present their mathematical responses during the discuss-and-summarize phase.

Purposefully sequencing the student responses that will be displayed, and

Helping the class make mathematical connections between different pupils' responses and between students' responses and the key ideas.

The need for building connections in classroom discourse and specifically ordering students' presentations is echoed by (Trott, 2022). After analyzing videotapes of three teachers the authors deemed highly qualified in directing classroom discourse, Trott also determined it was important to carefully sequence students' presentations and to make connections from students' work to the larger mathematical picture. By previewing and avoiding the system of marking students' final answers either correct or wrong without recourse to their line of thinking, teachers can orchestrate an order of student presentations that will maximize student understanding and allow access to the important mathematical concepts of the lesson (Dixon et al, 2014)

Rodríguez -Nieto et al (2022), also suggested specific strategies to effectively develop mathematical discourse. These strategies were identified after reviewing videos of teacher interviews with students about their problem-solving methods. he suggests teachers: ensure students understand what the problem is asking, change the mathematics to match the students' level, when necessary, explore what was done in the solving process, remind students of relevant strategies, promote students' reflection, explore connections, make connections to symbols, and generate follow-up questions.

They, however, noted that this is only possible if teachers can analyze students' wrong answer solutions

2.5.5 Teachers' interpretation of students' errors.

Studies have shown that the analysis of the wrong answer solution method by teachers has the greatest impact on developing the critical thinking of students. This finding suggests that it allows the teacher to analyze and identify the source of error and the thinking process of students (Mueller & Yankelewitz, 2014). This finding was made when Mueller worked with 24 sixth-grade students participating in an urban afterschool program. The study observed that when students are given a supportive environment, open-ended tasks, and thoughtful teacher questioning, their reasoning can develop naturally. By analyzing their wrong answer solutions teachers can help students to explain and justify their solutions and strategies that promote reasoning. During their study, the researchers (Mueller & Yankelewitz, 2014) asked students to work in small groups on mathematical tasks, record their thinking through the use of pictorial representations or models, and present their solutions to the class. After solutions were shared, students were asked to revisit the problem and reflect on solution methods different from their own. Through this discourse and reflection, students developed an understanding of alternate problem-solving strategies.

In similar research, prospective elementary teachers were divided into two groups. One group of participants cooperatively studied worked solutions to proportion problems while the other group generated and collectively studied their solutions to the same problems. While discourse was the tool used for this investigation, the true focus of the study was to determine the impact of teachers' abilities to analyze wrong-answer solutions on students' abilities to solve proportion problems using multiple strategies (Türker et al, 2022).

According to the study, interviews of the prospective teachers revealed their ability to analyze students' wrong answer solutions positively impacted students' performance and ability to engage in critical thinking in mathematical problem-solving. According to the study, wrong answer solution analysis allows teachers to identify the source of the error, why the errors are made, and the thinking ability of their students (Türker et al 2022).

This would help the teacher to develop appropriate remedial and corrective measures to support the students. Error synthesis is important for building critical thinking and problem-solving style and concepts. This supports the idea that to develop critical thinking skills, students need to be exposed to multiple solution methods rather than the teachers' preferred method and the correct final answer. The study concluded that comparing solutions is an effective communication strategy that provides the opportunity for developing reasoning skills.

The research conducted by several researchers (Kong & Wang, 2021) emphasized the ability of the teacher to follow students' line of thinking, and his or her ability to analyze students' wrong answers and solutions is worth studying because they have far-reaching results. The researcher implemented a cooperative planning and teaching model to develop appropriate methods of classroom discourse and promote student self-regulation. They investigated the impact on student learning of oral and written communication through quasi-experimental studies. Pruitt (2022) provided students with high-quality mathematical tasks and investigated the effect of asking students to select an appropriate strategy and work through their solutions without teacher direction. Ledezma et al (2022) focused on mathematical modeling in their case study that identified strategies that promote students' critical thinking. Rupe and Borowski (2022) used video analysis used a synthesis of research to establish a list of critical components of oral discussions in mathematics classrooms. In these studies, the underlying strategy is to help the teacher analyze and evaluate the thinking process of students and develop teaching methods to impact positively on the thinking process and formulation of concepts and theories of the students.

Research suggests that in addition to oral and written communication, mathematical representations impact students' conceptual understanding (Bywater et al, 2022). One of the features of classroom instruction that emerged as critical to students' learning was the teacher's and classroom assessment.

Satsangi et al (2018) also emphasized the need for manipulative drawings and algebraic notation to visualize problems and effectively generate solutions. Lee and Hwang (2022), the study showed that students used the connections between various problems to establish patterns and construct representations which promoted algebraic thinking. In addition, collaborative discussion of various representations led to students' improved ability to create accurate diagrams. Tembo-Silungwe and Khatleli (2017), compared a diagram to use as a problem-solving tool in 300 Japanese students to a similar group of 300 students in New Zealand. The study revealed that students who used diagrams correctly solved more problems than students who did not use diagrams.

Student questionnaires showed a correlation between the number of time teachers spent teaching diagram use and students' abilities to arrive at correct solutions. TemboSilungwe and Khatleli conclude that diagram use is a critical component of developing conceptual understanding. Students need frequent exposure to diagrams, not simply as a tool for the teacher to deliver content, but also as a tool for students to use in problem-solving. Without ample opportunities to develop skills in creating and interpreting diagrams, students will not learn to value diagrams used as an effective problem-solving strategy.

Within the use of diagrams as a strategy to promote critical thinking skills, there exists a debate as to whether students should be provided with representations or are required to create their representations. To find an answer to this question, (Lee, & Hwang, 2022) conducted a study of more than 200 fifth-grade students from eight different primary schools in the Netherlands.

For this study, students were organized into an experimental group that received instruction in creating problem-solving models and a control group that received identical content instruction but was provided completed models for use in the assigned mathematical tasks. Students in the experimental group outscored students in the control group significantly. In constructing representations, students are focused on the structure of problems rather than simply following procedural steps involved in the solution of one specific problem. This focus on structure rather than on rule-following helps generate mathematical knowledge, especially when the learner's attention is directed to the underlying structure of classes of comparable problems.

Representations are an important part of building conceptual understanding and developing critical thinking skills in mathematics (Bywater et al, 2022). A debate exists, however, as to whether students should develop their representations or teachers should provide those representations (Dwijayani, 2019) suggesting allowing students the freedom to create pictorial representations of their own to ensure meaning. Rather than enter into that debate, this action research includes both student-created and teacher provided models as a means of developing critical thinking skills.

2.6 Summary

Many students at the senior high school level have the misconception that mathematics is a difficult subject and involves complex manipulations of concepts that can be understood and handled by exceptionally endowed students who are also mathematically oriented.

The problem deepens when mathematics moves beyond simple manipulations with resultant single-number answers and higher realms of conceptual and word problem solutions that require reading and comprehension as well as presentation of responses of final answers in written word form. Students' ability to read and comprehend word problems in mathematics is a function of their language proficiency, hence the need to develop good oral and written communication skill component of mathematics when the focus moves to algebraic thinking and problems.

Fear, misconception, and weak language base of students make them commit several errors in the process of solving mathematics problems. Similarly, with limited English language proficiency students usually have difficulty understanding their teachers and mathematics word problems in textbooks.

To facilitate understanding, many teachers combine both English and local languages to explain concepts and processes in mathematics. It is therefore evident that language and learning across the disciplines constitute the true problems of mathematics. Conceptual mathematics courses focus on proof and argument with an emphasis on the correct, clear, and concise expression of ideas.

Students' solutions to mathematics problems are replete with errors. Newman attributed the sources of errors to these factors namely reading, comprehension, transformation, application of processes skills, and encoding. However, reading, comprehension, and transformation collectively contribute over 50% of the sources of working errors that students commit in solving word problems. Apart from the five identified sources of errors, carelessness has also been noted to be another major source of error. Students may at times arrive at wrong solutions to problems not due to the lack of understanding of the problem or concept but due to carelessness. Teachers' inadequate knowledge in analyzing students' wrong answers makes them adopt the wrong approach to marking only the final answer arrived at by the students as right or wrong. Once they do not analyze the working process of the students, they are unable to detect the actual sources of errors.

Suggestions have been made to assist teachers to identify the sources of errors. In this direction, students must be made to provide a written explanation or justification for the solution to the problem. In the process of justification, they realize that their final answer is wrong or right. A reflective assessment or analysis of the answer may thus reveal the source of error leading to its correction or rectification.

Furthermore, students' explanations of their solutions to problems will also provide the teachers the opportunity and cues to determine the sources of students' misconceptions and adjust the mode and direction of instruction appropriately.

Ultimately, teachers' ability to analyze students' wrong answer solutions will impact positively students' performance, enabling them to engage in critical thinking in mathematical problem-solving.

It has been noted that, for students to develop critical thinking skills, students need to be exposed to multiple solution methods rather than the teacher's preferred method and correct final answer.

It has been suggested that developing students' mathematical beliefs system, providing students with frequent opportunities to address difficult tasks, and exposure to practice with multiple problem-solving strategies builds students' confidence and willingness to persist with difficult mathematical tasks.

Comparison of the solution is also another effective communication strategy that provides students the opportunity of the development of reasoning skills and confidence. To supplement students' efforts in providing teachers with clues to analyze wrong solutions; Newman has developed a model which presents a systematic and holistic approach to understanding a child's line of thinking in carrying out mathematics tasks. Newman maintained that a person wishing to obtain a correct solution to a one-step word problem must ultimately proceed according to some form of hierarchy. The hierarchy considers the issue of language as the first step in identifying the student's area of difficulty in mathematics word-problem solving.

The hierarchy progressively follows this sequence; read the problem, comprehend what is read; mentally transform the problem into a mathematical strategy; application of process skills and encode of answer into an acceptable written form.

Newman laid much emphasis on the hierarchy because failure at any level of the sequence prevents problem solvers from obtaining a satisfactory solution.

From the hierarchy, it could be derived that the factors of reading, comprehension, transformation, process skills, and encoding are mainly responsible for errors in solving word problems in mathematics. In addition to these, carelessness was also identified as a major source of error.

Studies have shown that more than 50% of the errors occur before the stage of application of process skills and encoding hence attention should be focused on the factor of language in mathematics learning. To this end, remedial mathematics programs need to pay attention to whether the students comprehend the mathematics word problems that they are tasked to solve.

Finally, it could be stated that teachers' inadequate knowledge about wrong answer analysis made them adopt the wrong approach to marking only the final answer arrived at by the students as either right or wrong.

More than 50% of the errors committed by the students in the course of solving mathematical word problems occur before the stage of process skills in Newman's hierarchy of errors.

To facilitate easy identification of sources of errors and their analysis, students must be made to explain how they arrived at the final answer of their solutions. This approach will assist teachers to identify areas and stages where the students went wrong thus assisting the teachers to plan correct and remedial processes of instruction.

Furthermore, teachers' adequate knowledge of wrong answer analysis will help correct students' misconceptions in mathematics and also improve the preparation and delivery of lessons to the benefit of the students.

Historically, teacher knowledge acquisition focused on content knowledge acquisition; however, recent developments in teacher education have shifted emphasis to pedagogy perfection. Fluctuations in emphasis on either content or pedagogy acquisition continued for some time.

To synthesize the two knowledge bases, Neumann et al (2019), suggested that teacher education programs should combine the question of both contents and pedagogical knowledge since teacher competence in these two areas of knowledge will prepare teachers effectively for instruction in mathematics and related subjects.

Shulman further indicated that it was not enough to teach content and pedagogy as separate entities, in that good teaching required a complex integration and balance of the two to achieve the required results at the end of an instructional activity. In a more holistic and comprehensive dimension, Bizuneh (2021) describes teacher knowledge to consist of three interrelated components each of which should be acquired comprehensively.

The identified components are teacher knowledge and ability in typical students' sources of error, knowledge associated with teaching strategies (pedagogy), and knowledge of content simplification and elaboration.

Lea (2020), buttressed the fact that understanding students' sources of error teachers and developing strategies of analyses (pedagogically) to help them avoid such errors is an important aspect of Pedagogical Content Knowledge and mastery of subject matter.

CHAPTER THREE

METHODOLOGY

3.0 Overview

This chapter describes the research paradigm, research approach, research design, population of the study, sampling and sampling technique, selection of instruments, data collection procedure as well as data analysis.

3.1 Research Paradigm

A research paradigm is a set of beliefs, values and assumptions that researchers have in common regarding the nature and conduct of research (Burke & Anthony, 2004). This study adopted the pragmatists' paradigm. Pragmatism involves research designs that incorporate operational decisions based on 'what will work best' in finding answers for the questions under investigation and this enables pragmatic researchers to conduct research in innovative and dynamic ways to find solutions to research problems. This study employed pragmatist paradigm because it offers an attractive philosophical partner for mixed method research as well as providing a framework for designing and conducting mixed method research (Burke & Anthony, 2004). This is one of the best philosophies when statistical tools are used to collect and analyze data on a phenomenon in a scientific way.

3.2 Research Approach

To reach the best answers for the researcher questions, the research approach should be carefully selected (Opoku & Akotia, 2016). The approach to the study is the mixed method approach. This is appropriate for the phenomena that regularly occurs and to examine any existence of relationships between the variables of the phenomena by gathering the data and applying a large number of cases to represent the target population.

Words, pictures and narratives can be used to add meaning to numbers and it is also useful in formulating conclusions for the population based on the data to be taken from the sample. In research related to assessing teachers' perceptions and interpretations of errors, as mentioned earlier in chapter two, this type of data collecting has been criticized for its time-consuming method, its small number of samples, and the need for well-trained to obtain good outcomes.

3.3 Research Design

Creswell (2014) defined research design as the plan, structure, and strategy of inquiry conceived to obtain answers to research questions and control variance. In this study, an exploratory sequential mixed method research design was selected. This was aimed at broadly exploring and understanding how teachers' interpret and perceive their students' errors in algebra. The design was considered most appropriate for this study since the selection of the participants (teachers) was randomly done. Consideration was given to significant factor such as the number of representations per school as well as sex per school in designing the study. A quota system was therefore employed to reflect equal representation of sexes.

The nature of the research problem determines which research strategy is best applied, so assessing teachers' interpretations and perceptions of students' errors, the needs and benefits of using mixed method in this type of investigation were clarified especially since it was found in the relevant literature to be the most widely used research methodology (Farrington, 2014; Kuhn et al, 2000; Hofer 2000). Therefore, there was the need to employ this design for this study.

3.4 Population

The population of a study is a group of people, who have one or more common characteristics which the research study envisages (Ary et al, 2010). The targeted population for the study were mathematics teachers and form three students from three Senior High Schools in the Afigya Kwabre North District of the Ashanti Region with a total population of 57 mathematics teachers of which 11 were females and 46 were males. The total enrolment for the schools was 3774 while that of the form three was 1159. The table below shows the summary of the targeted population.

Table 3.1: Target Population

School	**Total Enrolment**	**Form 3 Enrolment**	**Teachers**
School A	2055	602	27
School B	1219	450	19
School C	500	107	11
Total	3774	1159	57

Source: Field data, 2023

3.5 Sample and Sampling Techniques

A sample is defined as a subset of the population considered for a study (James, McMillan, & Sally, 2014). Information obtained from a good sample is representative of the total population under the study (Mweshi, G. K., & Sakyi, K. 2020). However, the three Senior High Schools have the total number of 57 teachers, of which 46 are males and 11 are females. Mathematics teachers were selected purposively for the study based on qualification and teaching experience. The purposive sampling technique is a non-probability sampling technique that is used to select participants based on the characteristics of the population and the objective of the study (Bridget, 2019). Teachers were sampled using Yamane's (1967) sampling method. Yamane

(1967) provides a simplified formula to calculate sample sizes. The formula shown below was used to calculate the sample sizes at a 95% confidence level and α = 0.05 are assumed.

$$n = \frac{N}{1+Ne^2}$$

Where n is the sample size, N is the population size, and e is the level of precision (Yamane, 1967). Three senior high school's mathematics teachers in Afigya Kwabre North established the sample frame for the research. For the sample size to be objectively represented, the sample size is calculated at a 95% confidence level (at a 0.05 significance level) and 57 as the total teacher population.

$$n = \frac{57}{1 + 57(0.05)^2}$$

$$n = 49.891$$

The sample size ($n = 49.891 \approx 50$) of the students was determined by employing the proportional method of sample size.

3.6. 0 Data Collection Instruments

The study used students' class exercises, assignments, homework and testes for teachers to explain errors and give reasons behind those errors in algebra, focus group discussions for data collection and teachers; perceptions questionnaires.

3.6.1 Content Analysis

To find out how the teachers explained some common errors in algebra, form three students' class exercises, assignments, homework, and class testes were used to analyze teachers' interpretations of students' errors in algebraic problems. These materials were given to teachers to identify, explain and give reason(s) behind each error found.

3.6.2 Questionnaire

A questionnaire is defined as a research instrument that consists of a set of questions or other types of prompts that aims to collect information from a respondent (Zuzana, 2017). The two most common types of questionnaires are close-ended questions and open-ended questions (Zuzana, 2017). The study chose questionnaires because the nature of this study was to get the opinion and views of the respondents. The questionnaire included two sections of information arranged as follows: 1) the demographic information; and 2) teachers' perceptions of students' errors in algebra.

The demographic background included information about the qualification of the respondent, gender (male and female) and teaching experience of the respondent. The purpose of this section was to collect simple personal and demographic data about the participants. The questionnaires used a three-point Likert-type scale where the participants' responses were: three for agreement, two for unsure and one for disagreement. The participants were asked to take all the time needed to answer all the questions.

Table 3.1 below, shows the three dimensions of the teachers' perceptions of students' errors in algebra, one item is given as an example of each dimension and the number of questions in each dimension.

Table 3.1: Teachers' perceptions of students in algebra Questionnaire

Dimensions	Item Example	No of Questions
Student-related (7 Questions)	Errors are associated to students' attitudes towards algebra.	7 Questions Q1, Q3, Q4, Q5, Q7, Q10 and Q13
Teacher-related (9 Questions)	Errors are associated the text of the problem.	9 Questions Q2, Q6, Q8, Q9, Q14, Q18, Q19, Q20, Q21 and Q22
Nature of algebra (5 Questions)	Errors in algebraic problems are unavoidable	5 Questions Q11, Q12, Q15, Q16, Q17

The specific-domain questionnaires of teachers' perceptions of students' errors were designed mainly to produce quantitative data following a close-ended structure. The participants were given three different options, from which they were asked to choose any one option for a single question in the survey questionnaire. The questionnaire was distributed and collected under the supervision of the researcher who clarified the instructions and answered any questions put by the participants. As there were no right or wrong answers the participants were encouraged to give their opinions and not to leave any questions unanswered.

3.7 Sources of data collection

The study employed primary data, the primary data was collected from students' class exercises, assignments, homework, testes from final year students who were preparing to write their West African Senior School Certificate Examination (WASSCE) and the administration of questionnaires.

3.8 Pilot Study

A pilot study is a trial run with a few subjects to assess the appropriateness and practicability of the procedures and data-collecting instruments (Ary, et al, 2010; Moon, 2014).

According to Cohen et al (2017), run a pilot study and make any necessary amendments to the procedure. Hence, the researcher conducted a pilot study before the final collection of data. The purpose of the pilot study was to determine the validity and reliability of the instruments. It was carried out in senior high school 'D' in the Afigya Kwabre North District of the Ashanti Region, the results were not included in the main study. The pilot study was carried out to check the appropriateness of the language used in the questionnaire as well as to determine the difficulty of the instrument items. This enabled the researcher to update the research instruments by making corrections and adjustments based on the observations made. This enhanced the reliability and validity of the instruments before the final administration.

3.9. 0 Reliability and Validity

The credibility of a research study depends on the reliability and validity of the data collection instruments (Cohen et al, 2017).

3.9.1 Reliability

Reliability is a critical issue if a study must meet the objective and the purpose for which it has been designed. For example, a suitable and appropriate question asked determines to a greater height how reliable an answer is arrived at. Reliability deals with the degree to which a measure has stability (Kyeremeh et al, 2019). In this study, reliability was measured by conducting a pilot study. Statistical tool, Cronbach Alpha Index was used to determine the reliability of the instrument. The reliability index of 0.83 was observed from piloting the instruments.

3.9.2 Validity

Validity is the extent to which any measuring instrument measures what it is intended to measure (Fereday & Muir-Cochrane, 2006; Carter, et al, 2014). The validity of an instrument concerns what an instrument measure and how well it does. The face validity of the questionnaire was done before piloting the instruments with the help of experts (research supervisors). These experts corrected elements of ambiguity in the instruments before it was used in the pilot test and subsequently for the main study.

3.10.0 Data Collection Procedure

3.10.1 Ethical Considerations

The study followed all ethical procedures and guidelines for graduate student research.

An introductory letter (SEE APPENDIX B) was obtained from the University of Education, Winneba, mathematics department. In addition, the researcher wrote to the headmasters to seek permission to use the schools for the study. A consent form was also developed and signed by the headmasters of the schools where the data was to be collected. The researcher explained the objectives of the research to the authorities of the schools. The researcher also informed the respondents of their right to withdraw when they feel like doing so. Before distributing the questionnaires, the researcher assured the participants that all data collected during the data collection will be kept securely and treated as confidential.

3.11. 0 Data Analysis

According to Singh and Singh (2014), data analysis is a process of editing, coding, classification, and tabulation of collected data. The process involves operations that are performed to summarize and organize the collected data from the field. This section describes the statistical tools and thematic techniques used to analyze the data. The analysis is based on the purpose, the objective, and the research questions of this study. Qualitative and quantitative data analysis techniques were employed.

3.11.1 Qualitative Data Analysis Procedure

Data on teachers' interpretations or explanations of some common student errors were analyzed using a coding framework developed through a thematic analysis of the teacher's

Table 3.2 Framework for analyzing teachers' explanations of the given errors

Nature of Explanation	Description
A mathematically correct or partially correct explanation	A correct explanation for the error, or a partially correct explanation that explains part of the error
Mathematically incorrect explanation	An explanation that does not match the error, or is mathematically incorrect
Mathematically imprecise explanations or blaming students	An explanation that lacks mathematical clarity in relation to the error, or blames students
Explaining or illustrating what should have been done	An explanation of what the student should have done or a presentation of a correct solution
Attributing errors to teaching	An explanation that describes the error as a result of teachers' teaching
Descriptions of what the student did	A description of what the students did without giving a possible reason

Framework for analyzing teachers' explanations of the given errors

In one category the researcher combined an explanation that was not mathematically precise in terms of what the error was with blaming students for the error. This was done because, in the majority of cases, the teachers expressed both in one explanation. An example of such an explanation was "Student has a problem in removing brackets on quadratic expressions". Such explanations tended to begin by blaming the student and then referring, in unclear terms, to what was the mathematical cause behind the error. In validating the framework, the researcher initially coded the teachers' reasons or explanations of each error separately and then discussed and agreed on the validity of each code.

It should be noted that although the teachers were asked to give a possible reason or explanation for each error, the framework shows that some of the teacher responses were not reasons for the errors. For example, explaining what the students should have done does not constitute a reason for the error. Similarly, simply describing what the student did is not a reason for why the student made the error. The researcher however classified these responses as significant codes due to their frequency in the teachers' responses and revisit this observation later in the study.

3.11.2 Quantitative Data Analysis Procedure

Quantitative data was gathered through the questionnaire administration. The data were analyzed and presented using, descriptive statistics (i.e., frequency distribution, percentages, mean, standard deviation, standard error, and mode), and inferential statistics (Chi-square) were employed to analyze, interpret and describe the participants' perceptions of their students' errors in algebra, and to ascertain if there is a statistically significant in the uniformity students' errors among the various senior high schools.

3.11.3 Descriptive Statistics

In presenting the characteristics of the samples, descriptive analysis of the raw data was employed. Statistical Package for Social Sciences (SPSS) version 26.0 was used to calculate the percentages of variables, a frequency distribution of variables; as well as the means and standard deviations for variables. In collecting samples, bias in the interpretation of analyses is unavoidable and could adulterate the inference drawn from the analysis. One of the possible sources of bias is the violation of the assumptions. Hence, some of the important assumptions before performing the analyses were carefully examined.

3.12 Summary

This chapter has addressed the research design used in the study. An explanation of the instruments used for data collection for the study. It also provided a detailed description of the methodology used for the study. This includes the description of data collection and analysis, information about instruments' reliability and validity, and the ethical considerations for the study.

CHAPTER FOUR

PRESENTATION AND DISCUSSION OF RESULTS

4.0 Overview

This chapter describes the demographic characteristics of respondents, the teachers' interpretation of some common students' errors in algebra, teachers' perceptions of students' errors in algebra and errors senior high school students make in solving algebraic problems.

4.1 Demographic characteristics of respondents

This section of the chapter surveyed teachers 'demographic characteristics including gender, qualification, and teaching experience. A detailed breakdown of the number of participants' gender is presented.

Table 4.1: Distribution of Respondents' Gender

Respondent	Frequency	Percentage
Male	40	80
Female	20	20
Total	**50**	**100**

Source; Field data, 2023

The sample selected for the study constitute 80% male and the rest 20% female as indicated in table 2.

Table 4.2: Distribution of Respondents' Qualification

Qualification	Frequency	Percentage
Masters	5	10
Bed/BSc	45	90
Total	**50**	**100**

Source; Field data, 2023

Table 2 shows the distribution of the professional qualification of respondents 10% and 90% of respondents have a second and first degrees in education respectively.

Table 4.3: Teaching Experience

Period (years)	Frequency	Percent	Cumulative percent
1 - 10	9	18	18
11 - 20	17	34	52
21 - 30	24	48	**100**
Total	**50**	**100**	

Source; Field data, 2023

The table above shown that 18% of the respondents had teaching experience of 1-10 years, 34% of the respondents had teaching experience of 11-20 years, 48% had teaching experience of 21-30 years. Most of the respondents had good experience in teaching mathematics in the Afigya Kwabre North. Effective mathematics teachers have a sound grasp of relevant content and can critically evaluate students' processes, solutions, and understanding in solving algebraic problems.

4.2.0 The teachers' interpretation of some common student errors in algebra

Researcher question 1: How do mathematics teachers interpret common errors students make in solving algebraic problems in the district?

In this section, the researcher presents the teachers' explanations of five common errors in algebra. The table and exhibits below show teachers' responses in each category for each error. The errors presented to the teachers varied according to the domains of algebra, and how the teachers explained each error varied considerably.

The researcher analyzed the teachers' explanations of each error separately. In the analysis, the researcher focused on responses that had a frequency of ten percent or higher, for expediency purposes.

Table 4. 4 The distribution of teachers' interpretation of some common student errors in algebra

Keys;

N; Number of respondents, CE: Correct Explanation, IE: Incorrect Explanation

BS: Blaming Students, TP: Teaching Problem, DWSD: Describing What Students Did

Questions	Students' answers	Teachers' Interpretations N CE%	N IE %	N BS %	N WSD, %	N TP%	N DWSD %
$(a+b)^2$	$a^2 + b^2$	4 (7.1)	13 (26.2)	25 (50)	2 (4.8)	1 (2.4)	5 (9.5)
$x+y$	xy	33 (66.7)	1 (2.4)	12 (23.8)	1 (2.4)	1 (2.4)	2 (4.8)
$2x^2 - 3x + 1 = 5$	$(2x-1)(x-1)=5$, $2x-1=5$, $2x-1=5$, x=3, *or* $x-1=5$, x= 4	7 (14.3)	8 (16.7)	21 (42.9)	5 (9.5)	4 (7.1)	2 (4.8)
$\frac{a}{b}+\frac{c}{d}$	$\frac{a+c}{b+d}$	24 (47.6)	4 (7.1)	13 (26.2)	1 (2.4)	6 (11.9)	2 (4.8)
$f(x) = \frac{(2x-9)+(x^2+1}{(3x+7)(x^2+6)}$	$\frac{(2x-9)(x^2+1)}{(x^2+6)}$	16 (31)	7 (14.3)	12 (23.8)	5 (9.5)	2 (4.8)	6 (11.9)

4.2.1 Teachers' explanations of the expanding brackets error

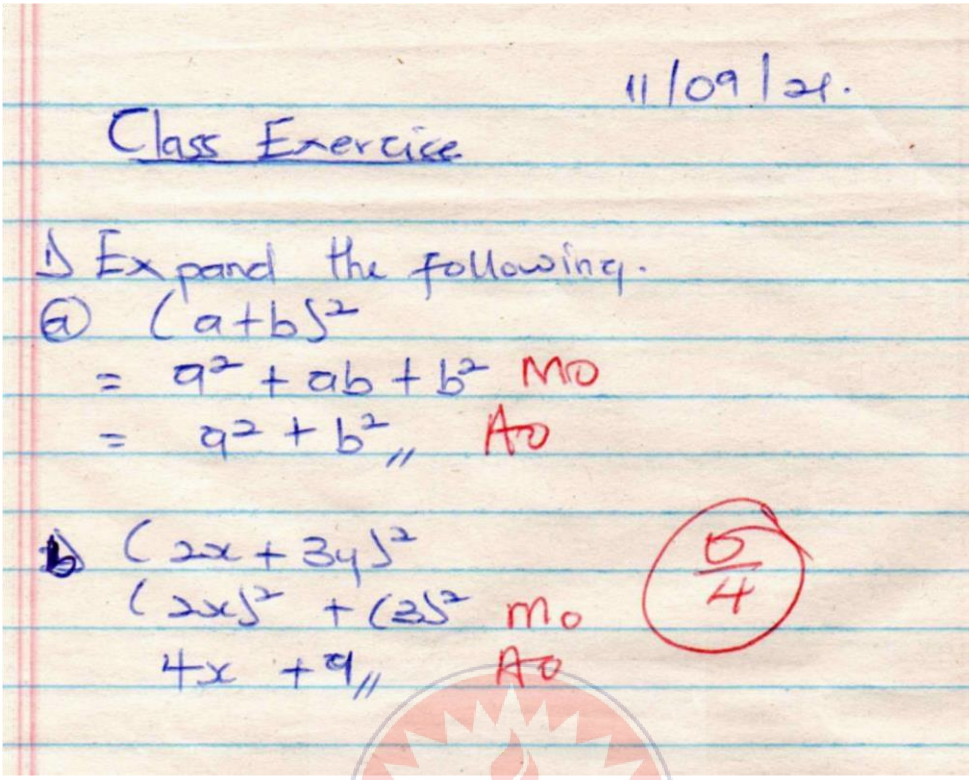

Figure 4. 1 Example 1

Errors in the example above, errors are common in algebraic problems and is normally associated with the over-generalization of statements such as $(ab)^2 = a^2b^2$ or $(a \times b)^2 = a^2 \times b^2$ Results in the table 4.4 generally show that the teachers found this error difficult to explain. Only 7% of the teachers gave correct explanations. 50% of the teachers gave mathematically imprecise explanations or simply blamed students for the error without giving reasons for the error. Examples of imprecise explanations were:

'Lack of knowledge of expansion of brackets or expressions';

'Misconception of expansion'

'Student has a problem in removing brackets on quadratic expressions'

'Violation of mathematical rules' and

'Students lazy to follow the expansion procedures.

These explanations were imprecise in that they did not indicate what was wrong; the tendency was to generalize the error as due to 'removing brackets'. The explanations also linked the error to the procedure for expanding $(a + b)^2$, rather than connecting the error to other mathematical structures and procedures. 26.2% of teachers gave mathematically incorrect explanations. Examples of such explanations were:

'Students confusing $a^2 - b^2$ with a^2+b^2';

'Lack of knowledge of the order of operations';

'Poor background of indices' and

'Incomplete knowledge on factorization'.

The researcher classified these explanations as incorrect because there was very little connection between the explanation given by teachers and the error committed by students. This number of teachers gave incorrect explanations for the error was a cause for concern to the researcher. Failure to correctly explain a student's error implies that the teacher would not be able to access the reasoning behind the error, and therefore may not be able to productively help the student in correcting the error.

4.2.2 Teachers' Explanations of the conjoining error

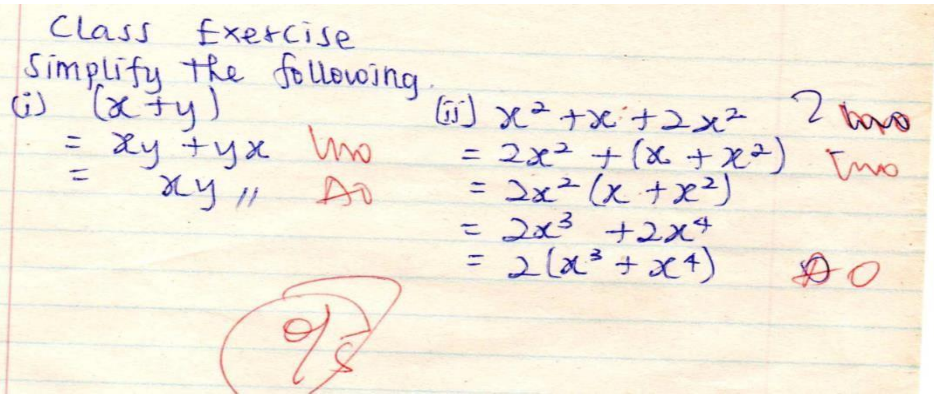

Figure 4.2 Example 2

In the example above, error x + y = xy was also common. Such an error is due to students' interpretation of the '+' and '=' signs. For twelve-year-old, up to fourteen-year-old students the two signs are signals that an action has to be performed those results in an answer (Ramdani et al, 2019). This is understandable given that in the domain of real numbers the result of addition is always another unique number. This understanding is normally carried forward into algebra, resulting in such errors. More than half of the teachers in the study (66.7%) provided mathematically correct or partially correct explanations of this error. Examples of the explanations were:

'Just as the addition of say 1+2=3 which is a single answer, the student seeks to get a single answer by eliminating the plus sign'

'Failure to note the difference between addition of numbers and that of letters'

'Student failed to understand terms that cannot be added together e.g. g+p, but they can be multiplied to give a term g x p = gp but g + p ≠ gp'.

These explanations show the teachers' understanding of how the addition of real numbers can contribute to the conjoining error. However, some of the explanations highlight the difference between the addition of real numbers and variables represented by letters in algebra without indicating what the difference is. There was no indication of an awareness of students' understanding of the '+' sign as a possible explanation for the error.

The next category of explanations for this error worth mentioning was that of imprecise explanations. 23.8% of teachers gave such explanations as

'Lack of mastery of the algebraic processes'

'Failure to apply knowledge on the addition of symbolic terms'

'Failure to interpret the meaning of basic operations and their applicability to algebra'

'Students cannot apply concepts taught on the addition of unlike terms' and

'Failure to understand the concept of addition and multiplication of algebraic terms'

A common theme in these explanations is blaming students without specifying the underlying cause or thinking behind the error. The explanations do not specify what it is that the students know or think that could have resulted in the error. Such explanations of errors are not helpful if teachers are to engage with errors in productive ways.

4.2.3 The teachers' explanations of the error in solving a quadratic equation through factorization

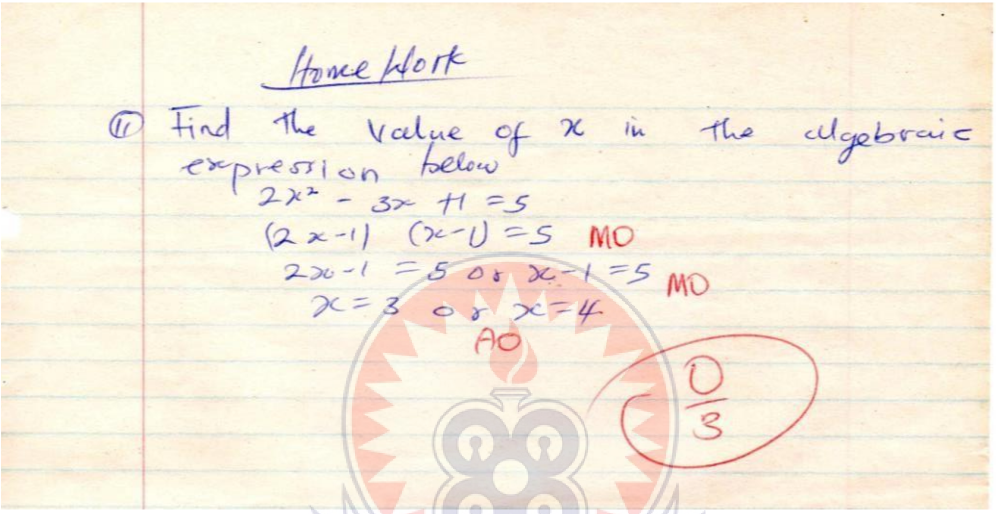

Figure 4.3 Example 3

In the example above, students committed common errors in solving quadratic equations using the method of factorization. The method is based on an application of one of the properties of the number '0', that is. if for any two numbers, a and b, a x b = 0 it implies that either a = 0 or b = 0. In teaching situations, this explanation of why the right-hand side has to be equal to zero is not usually provided to students. Teachers usually emphasize factorization, and once factors are easily identifiable students may rush into factorizing disregarding the other conditions necessary for the method to work. Only 14.3% of the teachers gave correct or partially correct explanations. Examples of these explanations were:

'Student lacks the knowledge that factorization can only be done when the RHS=0'

'Students are unaware of the standard form of the quadratic equation which is $ax^2+bx+c=0$'

'Generalization of solving quadratic equations in factorized form, i.e. (a)(b) = 0 where students are used to say either A=0 or B=0'

Although these explanations are correct, they are procedural in that they are based on descriptions of the procedures to follow in solving quadratic equations through factorization. There was no evidence to suggest that the teachers were aware of the reason why the right-hand side has to be zero or why the equation has to be in the standard form of a quadratic equation. 42.9% of the teachers gave mathematically imprecise explanations for this error. Examples of such explanations were:

'Failure to rearrange the equation'

'Students forgot to exclude 5, since 5 has not been moved to the other side when factorizing'

'Failure to understand the method of solving equations'

'Lack of techniques in solving equations, especially quadratic word problem equations'

'Misunderstanding of quadratic expressions' and

'Failure to grasp the concept of solving quadratic equations in word problems'

These explanations highlight blaming students for the error without specifying what it is that the students think, or know, that could have contributed to the error. Such imprecise explanations are not helpful for teachers who intend to engage with errors in ways that support students' learning.

There was 16.7% of teachers who gave incorrect explanations for this error. Examples of such explanations were:

'Student confused the use of the quadratic formula and factorization method of solving quadratic word problem equation'

'Failure to understand the meaning of a bracket and using the wrong concept'

'Student can factorize but the error is on simplification of equations on 2x-1' and

'Student failed to find the factors so that the result will mean two factors should replace 'the unlike term and when multiplied obtain the last term after **multiplying** ***by 2'***

For the researcher, these explanations were incorrect in the sense that they did not explain the error in any meaningful way. The explanations reflect an inadequate understanding of the error and the thinking behind the error.

4.2.4 Teachers' explanations of the error in adding algebraic fractions

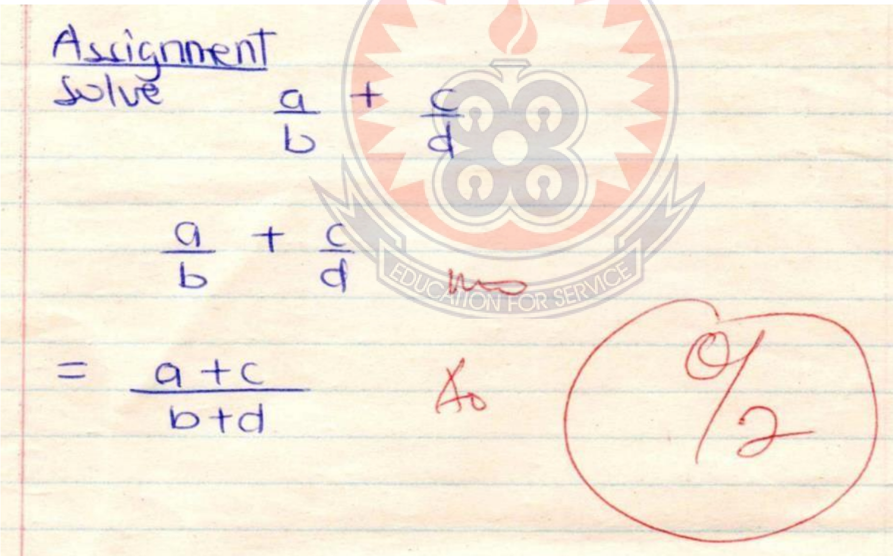

Figure 4.4 Example 4

In this example, error in adding two algebraic fractions is also common and is usually associated with the student's belief that when adding fractions, one has to add numerators and denominators separately (Brown & Quinn, 2016). Such an understanding can also be an over-generalization of the multiplication of common fractions to the addition of similar fractions. The error can also be linked to an inadequate understanding of what a common denominator is and why one has to find a common denominator when adding fractions. Almost an average of the teachers (46.7%) were able to give correct or partially correct explanations for this error. Examples of the explanations were:

'Student confuses addition of fractions with a multiplication of fractions'

'Failure to find the common denominator'

'No idea on the addition of fractions with different denominators'

'Lack of enough knowledge on expressing algebraic fractions as a single fraction where a common denominator should be calculated first and little knowledge on how to find a common denominator of algebraic fractions'

While these explanations were correct, they were all procedural explanations that described either what the student could not do or what steps the student did not know. This is consistent with how the addition of such fractions is taught. Teachers normally teach students the procedures of how to add such fractions, step by step, without engaging with the meaning of a denominator and a common denominator.

Twenty-six percent (26.2%) of the teachers gave explanations that were imprecise or blamed students for the error. Examples of such explanations were:

'Misunderstanding of the concepts of addition of fractions'

'Failing to apply the rule for addition of fractions'

'Misconceptions on addition and simplification of algebraic expressions'

'Violation of mathematical rules'

'Failure to find the LCM'

'Lacking knowledge of adding fractions by first dividing by the lowest common multiple and applied a wrong method for the addition of fractions'

These explanations were largely not explicit on what the error was. They all tended to blame students without specifying what the students' misunderstandings, misconceptions, or difficulties were. Such vague explanations of errors do not help provide information on how to deal with the error.

An interesting observation was that 11.9% of the teachers saw this error as a result of teaching algebra. Examples of these teachers' explanations were:

'Improper teaching of fractions, students believe that addition of fractions is when you add numerators on their own and denominators on their own'

'The basic rules for addition of fractions might have been missed from primary school level'

'Right from primary 6 the students did not grasp the method of simplifying fractions through addition'

'The concept of finding the common denominator was not well understood or it was not taught well'.

These explanations indicate that the teachers were able to connect the error to previously taught knowledge on the addition of numerical common fractions. For these teachers, the error was a result of some deficiency in teaching the addition of fractions involving real numbers. Linking the error to prior teaching and learning experiences situates teachers in positions in which they can make informed decisions on how to help students to deal with the error, or how to change their teaching.

4.2.5 Teachers' explanations of the error in simplifying algebraic fractions by identifying and cancelling common factors

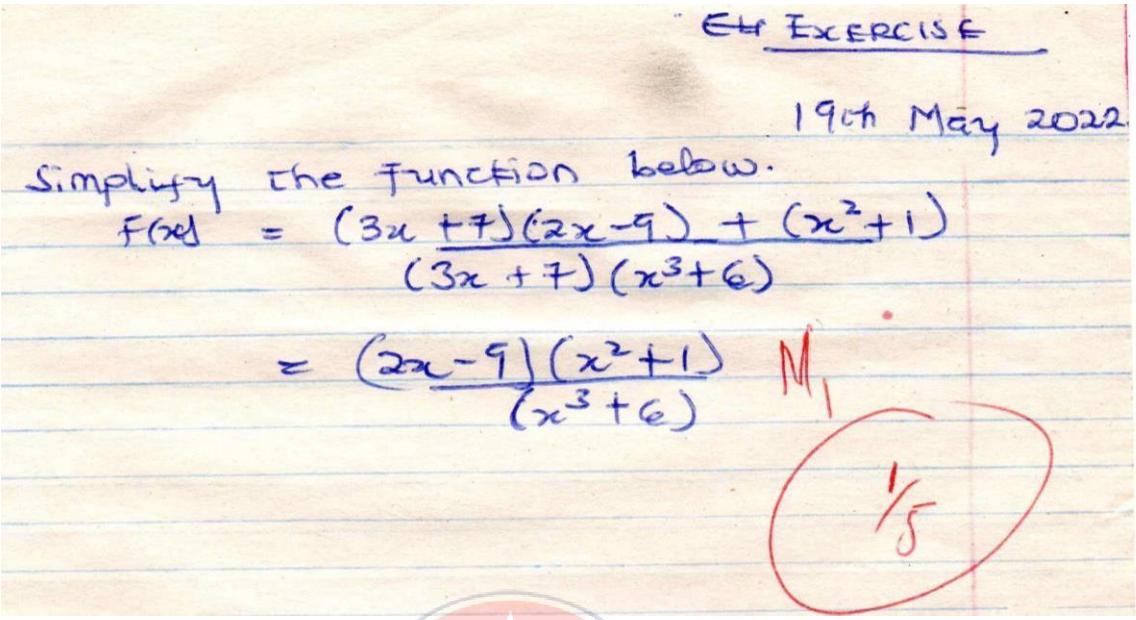

Figure 4.5 Example 5

In the example above, error arises in students' efforts to work with the idea of identifying common factors in the numerator and denominator of a given fraction and cancelling out the common factor(s). In teaching situations, teachers normally emphasize the need to identify and cancel factors that are common in both the numerator and the denominator. In this case, such knowledge leads the students to identify (3x+7) as a common factor in both the numerator and denominator and cancel it. This is a case in which mathematical knowledge which is valid and correct in some cases, is applied wrongly in a context where it leads to an error.

Thirty-one percent (31%) of the teachers gave correct or partially correct explanations for this error. Examples of these explanations were:

'Failed to recognize that there is the addition of expressions on the numerator and solved as if its multiplication throughout'

'Students have not grasped the concept that one can only divide when (3x+7) is a factor for both (2x - 9) and (x^2+1)'

'Lack of knowledge that (3x+7) has to be the factor of the whole numerator as well as of the denominator that one can cancel it out'

'Not aware that the numerator has two terms hence there is no common factor in the numerator and dividing by what is being thought to be a common factor'.

The explanations correctly attributed the error to failure to notice that the numerator consist of two terms and that (3x+7) is not a common factor of these two terms.

The next category of explanations in terms of popularity (23.8%) was that of mathematically imprecise explanations or blaming students: Examples of the explanations were:

'Students fail to identify when to cancel and when to simplify'

'Forgot that the denominator also affects (x^2+1) in the numerator'

'Violation of mathematical rules, student does not know how to compute the simplification of fractions'

'Lack of knowledge in solving functions with fractions'

'Factorization problem and expansion of algebraic terms and the expression is too long and the student may feel it's tiresome'

As with the other explanations in the same category for the other errors presented above, these explanations are not specific or detailed enough to show an understanding of the error. Each explanation is vague on what is involved in making the error.

Fourteen percent (14.3%) of the teachers gave mathematically incorrect explanations of the error. Examples of these explanations were:

'Failing to understand that multiplication is not distributive under addition, this may be caused by failure to open brackets'

'Students cannot factorize an expression completely and then identify the common factors of both numerator and denominator'

'The idea of a common denominator is lacking and lack of knowledge in addition of fractions'

These explanations are not linked to the error and do not explain the error in any way. The explanations indicate the teachers' inability to explain what is involved in the error, a situation that constrains them from engaging with errors and helping students deal with such errors.

Also eleven percent (11.9%) of the teachers gave explanations that were descriptions of what the students did or thought. These explanations were mostly correct but did not include why the students made the error. Examples of such explanations were:

'Students think that it's possible to cancel anyhow, (3x+7) is taken as a factor'

'Student applied the concept of dividing the fraction or reducing the fraction to its lowest term'

'Student saw (3x+7) as common, he/she thought it has already been factorized, has been betrayed by the brackets that follow'

Describing what students have done in making errors is an initial step towards engaging with the errors. Correct descriptions of what the students have done, or think, in making the error can lead to the next step which is interrogating why students think the way they do (Brodie, 2014).

Results on the teachers' explanations of the five errors show that the teachers were largely able to explain the conjoining error followed by the error on the addition of fractions. In both cases, the teachers gave correct procedural explanations that highlighted gaps in students' understanding or linked the error to previous knowledge involving numbers. Attributing errors to gaps in understanding and incorrect application of previously learned knowledge are some ways of explaining errors from a constructivist perspective (Ramdani et al 2019). The error in simplifying fractions by cancelling a common factor was correctly explained by 31% of the teachers, indicating that the majority found the error difficult to explain. Similarly, the teachers found the errors in the binomial expansion and solving quadratic equations by factorization difficult to explain. The three errors involved algebraic processes that are not easily linked to numerical processes in arithmetic, hence making it difficult for the teachers to explain. However, none of these correct explanations were conceptual, which highlights the tendency of teachers to focus more on the procedural aspects of a task than on the conceptual mathematics involved in the task (Shalem et al., 2014; Sheinuk, 2010).

Across all five errors, the researcher noticed that significant numbers of teachers provided imprecise explanations or simply blamed students for the errors. Providing vague explanations that were not specific enough to explain each error is a cause for concern. Mathematics teachers should be able to, at least, identify and describe students' errors correctly. This is an important initial step in engaging with students' errors in instructional situations. From being able to describe or explain, the next step is to interpret the error by finding out students' thinking that contributes to the error (Brodie, 2014). Blaming students for errors was also common in the teachers' imprecise explanations. While this is consistent with what research has shown (Gagatsis & Christou,2017), such explanations are counterproductive in that they limit access to other more meaningful explanations for errors. Teachers need to shift from blaming students for errors to viewing errors as integral to the process of learning mathematics.

The results also show that a significant number of teachers gave incorrect explanations, or did not explain the errors in any way. This was more pronounced in the errors in binomial expansion; solving a quadratic equation by factorization, and simplifying algebraic fractions by cancelling out a 'common' factor. That some teachers gave incorrect explanations is also a cause for concern. In instructional situations, if a teacher fails to correctly explain a student's error, he/she is likely to engage with the error in ways that do not help students to correct the error. It is therefore imperative that teachers take time to understand, and at least describe correctly students' errors, if they are to be able to assist students to deal with errors.

The researcher's findings also show that a low number of teachers gave explanations that explained what the students should have done. Explaining what should have been done, or illustrating the correct way of answering a task, are ways of avoiding an error. In instructional situations, the implication is that such teachers would just show students the correct solution without actually engaging with the observed errors. Similarly, there were low numbers of teachers who gave explanations that described what the students did or thought in making each error, which is also an unproductive way of engaging with errors because the erroneous thinking behind the error is not identified.

4.3 Teachers' Perceptions of Students' Errors in Algebra

Research question 2: What are teachers 'perceptions on students' errors in algebra at the Afigya Kwabre North senior high schools?

The second research question sought to find out the perceptions of teachers on students' errors in algebra in Afigya Kwabre senior high Schools. To investigate this, the researcher analyzed teachers' perceptions questionnaire intending to find out the perceptions' teachers hold on students' errors in solving algebraic problems. The descriptive statistics of the teachers' ratings of the statements are presented in Table 4.5

Table 4.5 Descriptive Statistics of Teachers' Perceptions of Student Errors in Algebra. (Students Related-Factors)

Key: D = Disagreement, U = Unsure, A = Agreement and SD = Standard Deviation

Teachers' Perception Source: field data, 2023	Number of Respondent	D	U	A	Mean	SD
1. Errors are associated with a lack of knowledge by a student in algebra	50	4	8	38	2.56	0.79
2.Errors are associated with the way the student studies and prepare himself/herself.	50	10	7	33	2.32	0.88
3. Errors are associated with student's attitudes toward algebra	50	14	7	29	2.13	1.01
4. Errors are associated with the psychological situation of the student.	50	13	6	31	2.26	0.86
5. Errors are due to the limited capabilities of students.	50	4	6	40	2.79	0.79
6. Errors are due to students' tendency to fulfill teacher's wishes without examining them	50	30	9	11	1.19	0.83
7. Errors are due to the violation of a rule.	50	1	2	47	2.87	0.95

Source: Field data, 2023

Most of the teachers in the study attributed errors to student-related factors. The mean scores of items 1, 2, 3, 4, and 5 were above the average mean score of 2.0, an indication that teachers strongly believed that errors in algebra are solely attributed students which is consistent with other research findings that show the tendency of teachers to perceive errors as results of student-related factors (Gagatsis & Christou, 2017; Gagatsis & Kyriakides, 2000). Accounting for students' errors through student-related deficiencies and/or in-capabilities can constrain teachers' efforts to understand more pertinent causes of students' errors such as the nature of the tasks and prior learning experiences. The finding was that the teachers agreed with the statement that students were to blame for errors. However, there were indications that the teachers perceived other factors besides students to be the cause of errors.

Whiles most teachers agreed to the fact that were limited capacities on the part of students with the mean score of 2.87. No other item on the students' related factors was seen to account for students' mistakes than this. The next two items that, according to some teachers are accounting for the students' errors are items 1 and 7. This have to do with students' knowledge in algebra. Also, a good number of teachers did not consider item 6, which concerned with the concept of didactic contract, as an explanation for the worst evident students' errors in algebra.

Table 4.6 Descriptive Statistics of Teachers' Perceptions of Student Errors in Algebra. (Teachers Related-Factors)

Key: D = Disagreement, U = Unsure, A = Agreement and SD = Standard Deviation

Teachers' Perceptions	Number of Respondent	D	U	A	Mean	SD
1. Errors are associated with the text of the problem.	50	14	0	36	2.72	0.92
2. Errors are associated with inappropriate ways of teaching algebra	50	25	0	25	1.50	1.08
3. Errors are due to wrong or incomplete knowledge about a concept taught previously.	50	10	4	36	2.72	0.92
4. Errors are due to previous correct knowledge which is not appropriate in a new situation.	50	39	4	7	1.71	0.95
5. Errors helps me in understanding students' lines of thinking	50	46	2	1	1.70	0.97
6. I think it is the teacher who can make algebraic problem learning easier	50	8	0	42	2.96	0.89
7. The best way to analysis students' errors is to understand the concept by oneself	50	7	3	40	2.79	0.79
8. I am well equipped with skills and provided with mathematics textbooks and other resources for interpreting students' errors in algebraic word problems and teaching the topic	50	7	8	35	2.70	0.97
9. I am responsible for the errors committed by students	50	47	0	3	1.80	0.89

Source: Field data, 2023

Nine statements in the Part B of the questionnaire were on teacher pedagogical practices as possible reasons for students' errors. The results of the teachers' responses to these statements. The mean scores of items 1, 3, 6, 7 and 8 shows that the teachers were generally in agreement with statements linking students' errors to teachers' pedagogical aspects such as the phrasing of the task; teachers' ways of teaching; incomplete knowledge of concepts taught previously; and confusing different methods that were used previously. While some of these statements could be associated with student-related factors such as forgetting previously taught knowledge, the research classified them in this category as they reflected connections to how students' understanding of the teachers' teaching could explain errors in algebra. The teachers' agreement with these statements could indicate their awareness of how their teaching could contribute to students' errors in algebra. However, there were mixed responses to the statement linking errors to the inappropriateness of mathematical tasks to students' capabilities with a mean score of 1.50 in item, which there was a split among teachers of the agreement and disagreement to this item. Agreement with the statement may indicate awareness that some mathematical tasks in algebra may be beyond students' capabilities and therefore could be the source of errors in algebra. Disagreement with the statement could be an indication that these teachers believed the mathematical tasks in algebra given to students were always within the students' capabilities, and therefore any errors should be a result of student-related factors. The teachers strongly disagreed with a mean score of 1.80 in item 9 statement that teachers were to blame for students' errors. Majority disagreed; an indication that these teachers regarded their teaching as not contributing to students' errors. Such a perception on the part of teachers reflects an incomplete understanding of errors as pervasive and often recurrent, irrespective of the way teachers teach (Brodie, 2014).

Table 4.7 Descriptive Statistics of Teachers' Perceptions of Student' Errors in Algebra. (Nature of Mathematics)

Key: D = Disagreement, U = Unsure, A = Agreement and SD = Standard Deviation

Teachers' Perceptions	Number of Respondent	D	U	A	Mean	SD.
1. Errors are normal part of learning algebraic problems in mathematics	50	1	4	40	2.79	0.79
2. Errors are useful resources for inquiry into mathematical concepts	50	2	3	47	2.87	0.95
3. Errors in algebraic problems are unavoidable.	50	27	0	23	1.61	0.79
4. Errors are due to confusion about the model needed for completing a task with an already-known model.	50	10	12	31	2.26	0.86

Source: Field data, 2023

The four statements in this category linked errors to the normal part of learning algebra in mathematics; the nature of errors as unavoidable in learning algebra in mathematics; confusion about the model needed for completing a task with an already known model and the potential of errors as useful resources for inquiry into algebraic mathematical concepts. The underlying theme in the statements is that errors are normal part of learning mathematics.

The mean scores of items 1, 2 and 4 shows that the respondents were mostly in agreement with three statements that linked errors to the nature of algebra in mathematics as a topic, whiles in item 3, teachers disagreed that students in algebra were unavoidable with a mean score of 1.61. Viewing errors as due to the generalization of correct mathematical knowledge to situations where such knowledge is inapplicable can be attributed to the teachers' understanding of the nature of algebra in mathematics where some forms of knowledge can be used to solve mathematical algebraic tasks in different areas of the subject. The teachers' agreement with the attribution of errors to violation of algebraic rules could be a result of their awareness that algebra in mathematics is a rule-dominated. And errors can be a result of confusing one rule with another rule. Students are usually taught to memorize rules or procedures without any understanding of the conceptual meanings of the procedures, hence forgetting or mixing up one procedure with another is expected in students' solutions to mathematical tasks. The teachers were also in agreement that errors are a normal part of learning mathematics and that errors can be useful resources for inquiry in mathematics. Agreement with these statements could be an indication of their appreciation that errors are part and parcel of algebra in mathematics and learning algebra. However, there was a split agreement with the statement that errors are unavoidable in mathematics. A slight majority of the teachers perceived errors as avoidable. Perceiving errors as avoidable is inconsistent with some theoretical explanations of the nature of errors in algebra, which highlight those errors show students' reasoning; are a necessary part of learning algebra in mathematics; and can provide teachers access to students' thinking (Brodie, 2014). Teachers who perceive errors as avoidable are likely to be frustrated when learners make errors.

The researcher's findings on the teachers' perceptions of errors raise some issues. The teachers were generally in agreement that errors were linked to student-related factors, something that was found in other studies (Gagatsis & Christou, 2017). While it is correct that these factors can contribute to errors, errors cannot be wholly attributed to students, and doing so reinforces the tendency to place the blame for errors on students.

The researcher believes that teachers need to view errors as due to other causes rather than student-related factors only. Hence the finding from the study that significant numbers of teachers were in agreement that teaching-related factors can contribute to the occurrence of errors was fortifying for the researcher. Such an understanding of errors can support shifts from blaming students for errors in algebraic problems.

Teachers were also in agreement that errors could be due to the nature of mathematics. The researcher thinks this understanding of the nature of errors is a useful and progressive step towards realizing that errors are pervasive, systematic, and persistent (Chauraya & Mashingaidze, 2017) and these are unavoidable in teaching situations. Teachers who have this understanding of the nature of mathematics are likely to engage with errors, rather than avoid them in their teaching.

Some disconcerting findings were that some teachers thought that teachers cannot carry the blame for students' errors in mathematics, and some teachers thought that errors were avoidable in teaching situations. Teachers who believe that they are not to blame for students' errors and that errors are avoidable are likely to view their teaching as 'perfect' and hence not a possible source of students' errors. This is contrary to the understanding that errors occur independently of methods of teaching (Brodie, 2014; Peng & Luo, 2019). In teaching situations, such teachers are likely to ignore errors when they come up. Such teachers are also likely to blame students for errors, and in the process constrain their capacity to productively engage with errors for the benefit of students' learning.

4.4.0 Research question 3:

Is there any significant uniformity of students' errors in algebraic problems among the three senior high schools in algebraic problems in Afigya Kwabre North District?

4.4.1 Nature of Errors Committed by in the three senior high Schools

The study sought to identify the errors made by students from school A, B and C in Afigya Kwebra North are summarized and presented in Table 4.8

Table 4.8 Nature of Errors across the of Schools

Nature of error	School			Total
	School A	School B	School C	
Comprehension	10	38	55	103
Transformation	6	11	54	71
Computation	5	2	22	29
Wrong facts	4	7	36	47
No errors	17	3	8	28
Total	**42**	**61**	**175**	**278**

Source: Field data, 2023

From Table 4.9, it can be shown that out of 103 students with comprehension errors, 10(9.7%) were in school A, 38(36.9%) in school B and 55(53.4%) in school C. Of those with transformational errors, 6(8.5%) were in school A, 11(15.5%) in school B and 54(76%) in school C. In carelessness error, 5(17.2%) were in school A, 2(6.9%) in school A, and 22(75.9%) in school C. In wrong facts error, 4(8.5%) were in school A, 7(14.9%) in school B, and 36(76.6%) in school C. Of those who had no errors in their work, 17(60.7%) were in school A, 3(10.7%) in school B, and 8(28.6%) in school C.

To test this hypothesis, a chi-square was used (of three senior high schools and error nature). The chi-square statistics were used because there are three schools and the researcher was looking for any significant uniformity of students' errors at $P \leq 0.05$. The results obtained are shown in table 6 below

Hypothesis

H_0: There is no any significant uniformity of students' errors among the three senior high schools in algebraic problems.

H_A: There is significant uniformity of students' errors among various the three senior high schools in algebraic problems.

Table 4.9: Chi-square test uniformity of students' errors among the three senior high schools in Afigya Kwabre North.

Statistic	Value	df	Significance
Pearson Chi-Square	72.409	8	.001
Likelihood Ratio	57.707	8	.001
Linear-by-Linear Association	5.627	1	.018
No Valid Cases	278		

χ^2 (8, N=278) = 72.409, p<0.001 Source: Field data, 2023

The value **χ^2 (8, N=278) = 72.409, p<0.001.** Since the p-value **0.001** was less than the alpha value of **0.05** significant level, the researcher failed to accept the null hypothesis **H_0**. Hence there is significant uniformity of students' errors among the three senior high schools in Afigya Kwabre North District of Ashanti Region.

The finding concerning the students' errors is generally not similar to the finding of some researchers such as Tsamir and Bazzini (2018).

4.5 Discussion of the findings

The evidence presented above can be discussed in terms of its implications for the development of a national policy on teaching and assessment of algebra in mathematics. However, it also raises more general issues regarding the development of educational theories on the reasons associated with mathematical errors.

The analysis of teachers' responses revealed that items concerned with reasons for errors can be classified into four broad categories. These are students' characteristics, the teachers' role, the mathematical knowledge, and the rules which students are supposed to follow in a typical mathematics classroom.

The first factor is related to items that imply that errors are a negative behavior. Errors are seen as the result of confusion (Gagatsis et al, 2019) and thereby students' lack of interest and/or preparation are the main reasons for errors in algebra as a topic. The second factor is concerned with the role that the teacher has to play to enable students to avoid mistakes and is very significant educationally. A significant contribution of this study to educational theory on reasons associated with mathematical errors in algebraic problems has to do with the other two factors which emerged. These factors are in line with the didactical and epistemological approach to the concept of errors in algebraic problems. More specifically, the items associated with the third factor partly derive from the epistemological approach to the concept of error and especially with the concept of obstacle. Finally, the fourth factor is highly correlated with items concerned with the concept of the didactic contract. It can therefore be claimed that even if we did not attempt to test a theory by using confirmatory factor analysis, the four factors which emerged from exploratory factor analysis can be linked with the approaches to the concept of error presented in this study and reveal the need of adopting a multidimensional approach to the concept of error.

The mean scores of the four factors revealed that teachers in the three senior high schools supported those errors in algebra are often associated with the characteristics of the students. This seems to be in line with the findings of some studies (Gagatsis et al 2019) which revealed that teachers attributed errors mainly to the student's lack of interest or lack of preparation. Although behaviorism has now declined in popularity in the field of psychology, many of its principles continue to have a great influence on Afigya Kwabre North senior high school teachers' perceptions concerning the teaching of algebra in mathematics and especially their attitudes concerning the reasons for their students' errors. However, Afigya Kwabre teachers considered the knowledge factor also as a significant source of mathematical errors in algebra. Moreover, a variation among teachers about the extent to which each factor was seen as a source of errors has been identified. Thus, the analysis revealed four homogeneous groups of teachers according to the extent to which they considered each factor as a source of error in mathematics. It is important to note that one of these groups considered all four factors as significant sources of errors. These findings are particularly encouraging since they imply that Afigya Kwabre teachers may accept a model for analyzing errors in algebraic problems based not only on the principles of behaviorism but also on the didactical and epistemological approach.

Analysis of teachers' responses on sampled students' class exercises, assignments, and tests on students' errors in Part C (concerned with the reasons why students make specific errors in algebra) revealed that teachers attributed most of these errors to the student and the knowledge factor. Moreover, they rarely pointed out that the way teachers taught mathematics might be a reason for students' errors. As for the ruling factor, this was seen as a source of error for only three of the five errors presented in the questionnaire. It may be claimed that the findings derived from teachers' responses to part C are in line with those derived from their responses to part B, and therefore suggest robust internal validity.

The fact that the five errors were not attributed only to the characteristics of the students seems to be in line with the fact that the mean score of each of the four factors revealed that teachers believe that often their students' errors can be attributed to each factor. Moreover, both content analysis of teachers' responses to part C and analysis of the five semi-structured interviews revealed that they considered error analysis as a significant way of improving their teaching practice. This finding should be taken into account by policymakers since error analysis is an area that has not been emphasized in official documents. By encouraging the analysis of students' errors, policymakers may enable teachers to seek specific information about individual students' thinking and understanding and then adjust the level of content to match individual students' performance levels.

By further analysis of the responses of teachers who attributed these five errors to the knowledge factor, it was found that this was sometimes done superficially and by not taking into account the epistemological approach to errors. For example, the majority of teachers (70%) who mentioned reasons associated with the knowledge factor as a source of the second error [$(a + b)^2 = a^2 + b^2$] did not point out that this error is due to previous knowledge about the expansion of brackets which is not appropriate in the case of the expanding variables in the square of brackets. This implies that the factors which may influence teachers' perceptions to develop a more global picture of the reasons associated with errors should be identified. Verschaffel et al. (2020) argue that: there is good reason to assume that these teacher cognitions and beliefs about the realworld knowledge in the interpretation and solution of school algebraic problems have, indeed, a strong impact on their actual teaching behavior and consequently on their student's learning processes and outcomes. The above argument remains to be systematically investigated in further research.

However, the fact that teachers' perceptions about teaching and learning algebra significantly affect the form and type of instruction they deliver (Mainali, 2022), reveals the significance of the above argument. It also implies that the findings of research on teachers' perceptions and interpretation of their students' errors may enable policymakers to identify how they can help teachers realize the value of the model of interpretation of errors, suggested in this study, and try to alter both their way of thinking and their teaching practice.

CHAPTER FIVE

SUMMARY, CONCLUSION, AND RECOMMENDATIONS

5.0 Overview

In this chapter, the methods employed and the findings are summarized. In addition, the conclusion, recommendations, and implications for future studies are presented.

5.1 Summary of the Study

The study was aimed at investigating senior high school teachers' interpretations and perceptions of students' errors in algebra in Afigya Kwabre North. The target population were all the senior high school mathematics teachers in the Afigya Kwabre North district. The sample population was taken from three senior high schools of which only form three students were considered. A purposive sampling technique was used to select the sample for the study, the whole population was 57 mathematics teachers and this population was considered as a sample for the study. Class exercises and assignments (document) were sampled from 115 students across the three senior high schools. The instrument employed for the study were document analysis and questionnaire.

5.2 Findings of the Study

The researcher's findings on the teachers' interpretations and perceptions of errors raise some pertinent issues. The teachers were generally in agreement that errors were linked to student-related factors, something that was found in other studies (Gagatsis & Christou, 2017). While it is correct that these factors can contribute to errors, errors cannot be wholly attributed to students, and doing so reinforces the tendency to place the blame for errors on students.

The researcher believes that teachers need to view errors as due to other causes rather than student-related factors only. The study found that, some teachers were in agreement that teaching-related factors also contributed to the occurrence of errors was fortifying for the researcher. Such an understanding of errors can support shifts from blaming students for errors. Teachers were also in agreement that errors could be due to the nature of mathematics. The researcher thinks this understanding of the nature of errors is a useful and progressive step towards realizing that errors are pervasive, systematic, persistent, and unavoidable in teaching situations (Smith et al., 2017). Teachers who have this understanding of the nature of mathematics are likely to engage with errors, rather than avoid them in their teaching.

Some disconcerting findings were that some teachers thought that, they cannot carry the blame for students' errors in mathematics, and some teachers thought that errors were avoidable in teaching situations. Teachers who believe that they are not to blame for students' errors and that errors are avoidable are likely to view their teaching as 'perfect' and hence not a possible source of students' errors. This is contrary to the understanding that errors occur independently of methods of teaching (Brodie, 2014; Peng & Luo, 2019). In teaching situations, such teachers are likely to ignore errors when they come up. Such teachers are also likely to blame students for errors, and in the process constrain their capacity to productively engage with errors for the benefit of students' learning.

Across all five errors, the researcher noticed that significant numbers of teachers provided imprecise explanations or simply blamed students for the errors. Providing vague explanations that were not specific enough to explain each error is a cause for concern. Mathematics teachers should be able to, at least, identify and describe students' errors correctly. This is an important initial step in engaging with students' errors in instructional situations. From being able to describe or explain, the next step is to interpret the error by finding out students' thinking that contributes to the error

(Brodie, 2014). Blaming students for errors was also common in the teachers' imprecise explanations. While this is consistent with what research has shown (Gagatsis & Christou, 2017), such explanations are counterproductive in that they limit access to other more meaningful explanations for errors. Teachers need to shift from blaming students for errors to viewing errors as integral to the process of learning mathematics. The results also show that a significant number of teachers gave incorrect explanations, or did not explain the errors in any way. This was more pronounced in the errors in binomial expansion; solving a quadratic equation by factorization, and simplifying algebraic fractions by cancelling out a 'common' factor and algebraic word problems. That some teachers gave incorrect explanations is also a cause for concern. In instructional situations, if a teacher fails to correctly explain a student's error, he/she is likely to engage with the error in ways that do not help students to correct the error. It is therefore imperative that teachers take time to understand, and at least describe correctly students' errors, if they are to be able to assist students to deal with errors.

The results also indicate that a low number of teachers gave explanations that explained what the students should have done. Explaining what should have been done, or illustrating the correct way of answering a task, are ways of avoiding an error. In instructional situations, the implication is that such teachers. would just show students the correct solution without actually engaging with the observed errors. Similarly, there were low numbers of teachers who gave explanations that described what the students did or thought in making each error, which is also an unproductive way of engaging with errors because the erroneous thinking behind the error is not identified.

5.3 Conclusion

Given that errors are very common in algebra, the need for teachers to engage with errors in teaching situations cannot continue to be overlooked. Research in this area can only help to illuminate some of the major issues in how teachers regard, and deal with, errors as they teach. The researcher's findings showed that teachers perceive errors differently. Some of the perceptions show an initial understanding of the steps in engaging with errors (Brodie, 2014), for example viewing errors as part of the mathematics and the process of learning the subject. However, some of the teachers' perceptions evidenced in the study show an inadequate understanding of the nature of errors, and how students come to make errors in algebra. These gaps in teachers' understanding of errors may need to be addressed if teachers are to engage with errors productively in instructional situations.

Algebra is a fundamental branch of mathematics that underpins most mathematics courses and mathematics-related careers, especially in post-primary school education (Chauraya & Mashingaidze, 2017). Students' early errors and misconceptions in some aspects of algebra can cause learning difficulties in their further learning of mathematics if left unaddressed. It is therefore the researcher's view that mathematics teachers need to be able to explain and account for some common errors in algebra.

Booth's work in this area has highlighted some of these errors and misconceptions (Booth et al., 2014; Booth & Koedinger, 2008). Evidence from this study shows that some of the teachers struggled to explain common errors in algebra. Where correct explanations were given, these were mostly procedural, without linking the errors to broader ideas and processes in algebra. For the researcher, this situation highlights the potential for mathematics teachers to continue ignoring errors in their teaching, or to engage with errors in superficial ways, if interventions are not put in place to capacitate teachers in this area.

5.5 Recommendation

The researcher recommends that senior high school teachers' development programs include developing mathematics teachers' understanding of the nature and role of errors in learning algebra in mathematics, especially Afigya Kwabre North District in the Ashanti Region. And also develop their capacity to be able to account for errors in algebra in ways that show a deep understanding of algebraic concepts and processes. Such knowledge can help teachers in improving their understanding and how to engage with errors productively in teaching and learning situations.

Recommendations for further research arising from the study include investigating the relationship between the variables over a wider geographical area within Ashanti Region. Future research can consider using other relevant demographic information (eg. gender, level of education, age, etc.) to test differences between perception and academic achievement.

Also, the use of fifty participants could have been increased. Large sample sizes are important. Shoaib et al (2020) used six hundred participants in their study. Small sample sizes prevent the results from having the necessary statistical power to support claims made by other researchers using bigger sample sizes.

Last, it is challenging to infer people's perceptions and interpretations from a single report using just one instrument. Perceptions of errors in mathematics theories continue to develop, while academics continue to develop methods for assessing perceptions and teachers' abilities to analyze students' errors in algebra. If a different instrument had been used, different results might have been obtained. Therefore, future researchers may evaluate individuals' perceptions of algebra in Mathematics with a mix of instruments (Colby, 2007; Schommer-Aikins & Easter, 2006).

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APPENDICES

APPENDIX A

UNIVERSITY OF EDUCATION, WINNEBA

MATHEMATICS TEACHERS QUESTIONARE

Students' errors in algebra are a major concern in Afigya Kwabre North. This study intends to establish teachers' interpretation and perception of errors committed by senior high school students in algebra. The results will help improve achievement in mathematics. The information provided will be treated with confidentiality. You are requested to answer the questions honestly. Tick as appropriate

SECTION A

Personal Data

a. Indicate your sex Male [ ] Female [ ]

b. Indicate your qualification

Med/MPhil [ ] BEd [ ] Diploma Ed [ ] Others [ ] For others specify-------------------

c. Your School

Osei Tutu II College [ ] St. Michael's Senior High [ ] Afigyamang Senior High [ ]

Teaching information

Teaching experience -------------------------------------

SECTION B

Instructions: This section has statements that you are to decide carefully whether you ***Disagreement (D) =1, Unsure (U) =2, Agreement (A) =3.*** Put a tick **[√]** against each statement depending on your feelings. If you make a mistake, cross by putting (X) through the tick [√] and then tick in the appropriate box in the table below.

Teachers' Perceptions	**D**	**U**	**A**
1. Errors are associated with lack of knowledge in algebra			
2. Errors are associated with the text of the problem.			
3. Errors are associated with the way the student studies and prepares himself/herself.			
4. Errors are associated with student's attitude towards algebraic problems			
5. Errors are associated with the psychological situation of the student.			
6. Errors are associated with inappropriate ways of teaching algebra			
7. Errors are due to the limited capabilities of students.			
8. Errors are due to wrong or incomplete knowledge about a concept taught previously.			
9. Errors are due to previous correct knowledge which is not appropriate in a new situation.			
10. Errors are due to the violation of a rule.			
11. Errors are due to a confusion of the model needed for completing a task with an already known model.			
12. Errors are due to the wrong processing of the models.			
13. Errors are due to students' tendency to fulfil teacher's wishes without examining them.			
14. Errors are due to the fact that an inappropriate question for the ability of the student is given.			
15. Errors are normal part of learning algebra in mathematics			
16. Errors are useful resources for inquiry into mathematical concepts			
17. Errors in algebra problems are unavoidable.			
18. Errors helps me in understanding students' lines of thinking			
I think it is the teacher who can make algebra learning easier			

19. The best way to analysis students' errors is to understand the concept by oneself			
20. I feel extremely anxious and fearful, when teaching algebra.			
21. I am well equipped with skills and provided with mathematics textbooks and other resources for interpreting students' errors in algebra.			

Source: (Gagatsis & Kyriakides, 2000)

PART C

The following are sampled errors from students' class exercises and assignments in algebra. Please, kindly explain/state the reason(s) associated with each error, and then indicate error type.

1. Twice the number decreased by 22 is 48. Find the number?

$2x - 22 = 48$

$2x = 48 - 22$

$2x = 26$

$\frac{2x}{2} = \frac{26}{2}$

$x = 13$

2. Seven times the number is 36 less than 10 times the number, find the number?

$3 \times x < 36 \times 10$

$\frac{3x}{3} < \frac{360}{3}$

$x < 51.4$

3. Find the values of x?

$2x^2 - 3x + 1 = 5,$

$(2x - 1)(x - 1) = 5$

$2x - 1 = 5 \; or \; x - 1 = 5,$

x = 3 or x = -6

4. Solve the following polynomial.

$$f(x) = \frac{(3x+7)(2x-9)+(x^2+1)}{(3x+7)(x^3+6)}$$

$$= \frac{(2x-9)(x^2+1)}{(x^3+6)}$$

5. Simplify $\frac{a}{b}+\frac{c}{d}$. $\frac{a}{b}+\frac{c}{d} = \frac{a+c}{b+d}$

6. Simplify (a + b) $^2 = a^2 + b^2$

7. Simplify x + y = xy

APPENDIX B
INTRODUCTORY LETTER

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF SCIENCE EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION
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+233 (020) 2041076

August 31, 2022

Dear Sir/Madam,

LETTER OF INTRODUCTION:

BABA LAARI (190011632)

I write to introduce to you the bearer of this letter, Mr. Baba Laari, a postgraduate student in the University of Education, Winneba. He is reading for a Master of Philosophy degree in Mathematics Education and as part of the requirements of the programme, he is undertaking a research titled — ***Investigating Teachers' Interpretations and Perceptions of Senior High School Students' Errors in Solving Algebra.*** He needs to gather information to be analysed for the said research and he has chosen to do so in your institution. I would be grateful if he is given the needed assistance to carry out this exercise. Thank you.

Yours faithfully,

Dr. Ali Mohammed
Graduate Coordinator

DEPARTMENT OF MATHEMATICS EDUCATION
UNIVERSITY OF EDUCATION
WINNEBA