UNIVERSITY OF EDUCATION, WINNEBA

THE VAN HIELE GEOMETRIC LEVELS OF STUDENTS' IN THE GA-MASHIE CIRCUIT: GENDER AND SCHOOL TYPE DIFFERENCES

BISMARK NII AYI ANKRAH

MASTER OF PHILOSOPHY

2023

UNIVERSITY OF EDUCATION, WINNEBA

THE VAN HIELE GEOMETRIC LEVELS OF STUDENTS' IN THE GA-MASHIE CIRCUIT: GENDER AND SCHOOL TYPE DIFFERENCES

BISMARK NII AYI ANKRAH
(202146544)

A thesis in the Department of Mathematics Education, Faculty of Science Education, submitted to the School of Graduate Studies, in partial fulfilment of the requirements for the award of the degree of Master of Philosophy (Mathematics Education) In the University of Education, Winneba

MARCH, 2023

DECLARATION

STUDENT'S DECLARATION

I, BISMARK NII AYI ANKRAH declared that this thesis, with the exception of quotations and references contained in published works which have all been identified and acknowledge is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

SIGNATURE: ……………………..……..…………..

DATE: ……………………..…………..….…………

SUPERVISOR'S DECLARATION

I hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on the supervision of thesis laid down by the University of Education, Winneba.

SUPERVISOR"S NAME: MR. MICHAEL AMPPIAH

SIGNATURE: ……………………..……..…………..

DATE: ……………………..…………..….…………

DEDICATION

This work is dedicated to my lovely beloved Kezia Prudence Nylander for her love, support and encouragement. It is also dedicated to my parents Mr and Mrs Ankrah and finally Edo Hatsu for his support and encouragement who was also a graduate student in the Department of Mathematics Education.

ACKNOWLEDGEMENT

First and foremost, the greater acknowledgement goes to the almighty God who has seen me through difficult time of my study. I would also like to express my heartfelt gratitude to Mr. Micheal Amppiah, my supervisor and a Senior Lecturer in the Department of Mathematics Education, University of Education Winneba for the time and pains he took for correcting the whole manuscript. Next to be thanked is my beloved Kezia Prudence Nylander who gave me support and encouragement during the period of this study.

I also thank my parents Sarah N.O. Ankrah and Osman Ankrah for the support they gave me throughout my education and also my brother Razak Ankrah and my sisters Linda Ankrah, Freda Ankrah and also to my colleagues and friends especially Mr. Edo Hatsu for the encouragement.

TABLE OF CONTENTS

Content **Page**

LIST OF TABLES

LIST OF FIGURES

Figure **Page**

ABSTRACT

This study sought to investigate students" van Hiele"s geometric thinking levels from public and private basic schools at the Ga-Mashie circuit of the Accra Metropolis. The population for the study was 562 J.H.S.3 students from private and public basic schools. Purposive and stratified sampling was used to select three hundred and thirty seven (337) students for the study, two hundred and seventy six (276) from public schools and one sixty one (61) from private schools. A quantitative descriptive survey approach was used for the study with van Hiele"s Geometric Test (VHGT) as the instrument. Four research questions were formulated for the study, research question one was on the van Hiele"s levels reached by private and public basic school students. Research questions two to four were meant to find out whether there was a difference in terms of gender and school type between the two schools (private and public). It was revealed that 190 (56.40%) of students attained No level, 126(37.40%) attained level 1, 20 (5.90%) attained level 2 and only 1 (0.30%) attained level 3 in both public and private school. The study also reveals that with regard to gender and school type there was no significant difference in students" van Hiele geometric levels. Recommendations are made for improving the geometric levels of students.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This chapter provides information to the study. It discusses the background to the study, statement of the problem, purpose of the study, significant of the study, objective of the study, research questions and hypotheses, limitations of the study, delimitations of the study and organization of the study.

1.1 Background to the Study

Algebra, geometry, probability and statistics, trigonometry, and calculus are all included in school mathematics. It emphasizes on numbers, geometry, probability, and statistics, as well as basic algebra (Riastuti et al., 2017). Mathematics is a field of study which is part of the curriculum and is highly significant for students in primary and secondary school. It is concerned with the manipulation of algorithms and axioms in mathematical research (Wachira, 2016). According to Ghana Education Service the purpose of teaching mathematics is to improve learners' acquisition of numerical and logical abilities and to help them think in a logical, accurate, and exact manner. The study of mathematics is very essential in our academics pursuit, especially at the basic level, which serves as foundation for higher academic work.

Geometry in the mathematics curriculum according to the Curriculum Research and Development Division (CRDD) of the Ghana Education Service (GES) (CRDD, 2010) covers the properties of solids, planes and shapes as well as the relationships between them.

According to Clements and Battista (1990), Geometry is a discipline of mathematics dealing with the relationship between a point, a straight line, planar figures, space, and spatial figures. Since it has various geometrical concepts, the geometry has a special place in the mathematics curriculum (Connolly, 2010). Goos et al., (2020) identified visualization and thinking processes as components of mathematical concepts.

Geometry, as a science, is described by van Hiele (1986, p. 60) as a type of study in which "we (as instructors) have no concern for space, nor for geometric objects in space, but (rather) solely for the links between qualities of those figures." van Hiele (1986, p.76) proposes an "intuitive introduction" to the study of geometry in which learners are given the opportunity for direct observation/manipulation of geometric figures such as triangles and quadrilaterals in order to abstract the relationships between the properties of those shapes.

Moreover, according to Duval (1998), geometric thinking comprises the cognitive processes of imagery and thinking. The key mental abilities necessary for mathematics are visualization and thinking, Visualization is an ability that assists pupils in perceiving forms, creating new shapes or objects, and revealing links between them (Battista et al., 1982). All these cognitive processes are linked, boosting pupils' geometry success (Duval, 1998). Battista (2007), stated that geometric reasoning refers to the act of "inventing and using formal conceptual systems to investigate shape and space" (p.843). Instruction approaches can increase visualization and reasoning skills (Arıcı & Aslan-Tutak, 2015; Goos et al., 2020; Jones, 2000). To use the appropriate instruction in delivery geometry content, student‟s geometric levels have to be taken into consideration.

Why study geometry? "Geometry organizes and clarifies our visual experiences and provides visual models of mathematical concepts" (Burger, 1988, p.2). People who understand geometry have a greater respect for the world in which they live. Geometric shape may be found in the structure of the solar system, geological formations, plants, animals, art, architecture, machines, and practically anything people make (Asemani et al 2017). Geometry is used on a daily basis by many professionals, including architects, engineers, and property developers. Geometry is intimately linked to the study of other mathematical concepts such as fractions, ratio and proportion, and measurement (Lee et al., 1995).

There has been much worry regarding students' comprehension of geometry in Ghanaian classrooms. Ghana participated in the Trends in International Mathematics and Science Study (TIMSS) in 2003, 2007, and 2011 to determine how her (JHS 2) performance in science and mathematics compared to that of other nations. Report shows that her performance declines with respect to the various years. The Chief Examiner‟s Report (2011) on the Basic Education Certificate Examination also stressed on the weaknesses of students‟ performance, adding that most basic schools lack the ability to apply the relevant rules of geometry. The evaluation of Ghanaian students' mathematics performance revealed that measurement, geometry, and algebra were the pupils' poor topic area (Anamuah-Mensah et al., 2004). Despite the relative significance of geometry, it is disheartening to notice that student performance in the subject in both internal and external examinations has remained continuously dismal. Mathematics educators have made every attempt to identify the key issues related with mathematics teaching and learning in the nation's schools. Despite all of these measures, the problem of low mathematics performance has persisted in the nation's public examinations (Adolphus, 2011).

According to Malpass, O'Neil and Hocevar (1999) and Saleh, Asyraf and Rahman (2016), there has also been a significant growth in the gender gap among talented or high-scoring pupils on mathematics tests. Prior accomplishment, valuation, categorizing mathematics as a male area, and curriculum appear to play important roles in the sex inequalities between boys and girls in mathematics (Council & Education, 1981, 1992; Grossman & Grossman, 1994; Halat, 2006; Haviger & Vojkůvková, 2014). According to future research, it would be extremely appropriate to split geometry instruction into distinct stages based on van Hiele theory. Secondary schools, depending on their specialty, should decide what levels they aim to accomplish and tailor their geometry instruction to that purpose (Haviger & Vojkůvková, 2014).

Van Hiele‟s levels are equally suitable for both genders (Halat, 2006). However, according to Yang and Chen (2010), spatial skills and gender differences are vital to geometric learning, and gender differences play a significant role in geometric learning since boys and girls perform differently in different learning settings while studying geometry (Yang & Chen, 2010, p.1221). It should be mentioned that the balance of spatial and logical ability can influence geometry performance in general (Battista, 1990). However, according to Ayten (2014), attitude is a key predictor of success in geometry, and gender is an essential element impacting achievement since cultural variables outweigh biological aspects.

Gender inequalities in geometry are diverse at the middle school level, according to research findings. For example Alex and Mammen (2014) study showed that there was a slight difference in the performance in favour of the female learners in terms of the mean score obtained in the van Hiele Geometric Test. Nonetheless, the independent two sample t-test showed no significant statistical difference between the genders in performance. Also, Halat (2006) stated that there was no statistically significant difference as in motivation between boys and girls, and that no significant difference was detected in the acquisition of the levels between boys and girls.

In other words, gender was not a factor in learning geometry. Again, according to Armstrong (1981) thirteen-year-old girls performed better at computation and spatial visualization than boys. This finding is consistent with the research of (Lloyd, Walsh & Yailagh, 2005) stating that girls" mathematics achievement met or exceeded that of boys. However, according to Humphrey (2008) performance in the van Hiele"s levels favour boys. In short, Gender is clearly an essential element influencing student performance in geometry, and study findings on this field vary.

In addition, school type also plays a significant role in students" performance in geometry, several studies have been conducted over the years to examine the differences in performance between students in public (government) and private schools, with mixed results (Akmal, 2016). Some researchers, such as Khan, Igbal and Tasneem (2015) hypothesized that there is no statistical difference in student performance.

In Ghana, there are essentially two sorts of schools. Individually owned (private) and publicly owned (public) schools (government). Students' academic achievement, on the other hand, is heavily determined by the sort of school they attend and school variables impacting performance. School structure, school composition, and school climate are examples of these characteristics. The institutional context in which one attends school determines the parameters of a student's learning experience and accomplishment (Ayodele & Muganyizi, 2014). However, the invasion of private schools into Ghana's educational system was noteworthy since the education reform began in 1987 failed to accomplish one of its key goals of giving universal access to all students at the basic education level (Akyeampong, 2009).

This global access remained a pipe dream, and educational quality was steadily deteriorating (Asiedu, 2002). The structures and facilities that might allow for universal access were severely inadequate, while teacher quality, morale, and dedication, which could assure basic education quality, were steadily dwindling, providing a significant opportunity for private schools. Private schools swooped in and have subsequently made significant gains into basic education provision (Ntim, 2014).

A private school, charges hefty tuition, which has detracted from the good contribution they might make to universal access. Nonetheless, they remain popular among those who can pay the costs (Ntim, 2014). The continued popularity of private schools can be attributed to the high level of education (at least academically) that they give. The majority of Ghanaians, particularly parents, viewed educational excellence solely in terms of the number of students who passed the final exams with high enough scores to get entrance to secondary school (Hatsu, 2019).

Nonetheless, the effort to get admission into Senior High institutions with low results following the Basic Education Certificate Examination can be a traumatic experience. Most private schools were able to guarantee their kids high results. The key, clearly, is tight monitoring and efficient time management. Private schools in Ghana have more effective and efficient supervision capability than state schools, resulting in higher success (Donkoh, 2014). The differences in performance between private and public schools in mathematics indicated that public schools have consistently performed poorly when compared to private schools. Several causes or variables influenced the discrepancies in mathematics performance between private and public schools (Ntim, 2014).

Parents and other individuals frequently believe that private schools, on average, do better academically, particularly in mathematics, than public schools. This empirical premise, however, is not supported by data (Gakure, Mukuria & Kithae, 2013). Parents' and policymakers' decisions on private school enrollment are frequently based on the belief that by enrolling their children in private schools, they would improve their children's academic performance (Ntim, 2014). This belief regarding private school performance is founded on a body of evidence indicating that private schools outperform public schools in mathematics.

However, the superior quality of private schools (in terms of teaching, teacher attendance, school performance, small class size, and discipline) as compared to public schools is a major factor in parents' preference for private schools (Asiedu, 2002). Recent global trends also reveal that many rich and developing nations are looking for public-private partnerships to share expenses and improve education supply. The Ghanaian government is exploring for alternatives to public-sector education delivery and finance. Budgetary constraints stimulate the adoption of government projects (Hatsu, 2019).

In Ghana, the boundary between public and private schools is rather simple. The public sector comprises both government and community schools, both of which get full government support for recurring expenditures (some of which are defrayed by the collection of school fees); the only distinction between them is how school building costs are funded. Such expenditures are borne by the government in the case of government schools, but by local communities in the case of community schools (Akyeampong, 2009). The private sector includes a wide range of schools, with the common trait that they all rely nearly entirely on school fees and private contributions to cover both recurring and capital expenditures. Although the bulk of private schools were founded by religious and other community organizations in the early 1990s, there are currently many new schools that are run for profit by individuals or groups of persons.

The fast expansion of private basic schools in Ghana over the last several decades teaches us some fascinating insights about the influence of government policy on private education. Excess demand was a primary motivator for the government's decision to eliminate impediments to private sector expansion in the early 1980s. The government's position regarding private education has been vague, particularly prior to the mid-1980s, and has hampered the growth of private schools in Ghana. When equitable aims dominated the policy agenda, the government would prevent the establishment of new private elementary schools in some years (Ntim, 2014).

In other years, the approach was reversed because the government was worried about the increasing excess demand for elementary schools. Ntim (2014) discovered that public pupils outperformed their private classmates in terms of national assessment success in a study of relatively new private schools in Ghana. Despite significant efforts to train teachers and provide in-service training and seminars for public school instructors, private schools in Ghana continue to outperform public schools in mathematics (Ntim, 2014). Despite the government's efforts to increase performance, public schools do badly in mathematics (Darfour, 2016).

The low performance of public school students aroused the attention of researchers to the issue of performance differences between private and public schools, particularly in mathematics. According to Ankomah et al., (2005), there are several variables that might explain for the disparities in mathematics performance between private and public schools in Ghana. However, further research into the topic of private and public schools geometry performance is required. Hence this study sought to explore the van Hiele‟s levels between private and public basic schools in the Ga-Mashie circuit of the Accra metropolis with emphasis on which school performed better on Van Hiele‟s Geometric Test (VHGT).

1.2 Statement of the Problem

School geometry has been taught from an axiomatic framework, much the way that Euclid‟s Element documented geometric learning in 300 BC. Yet many students struggle with geometry and do not take it serious in studying mathematics. Teachers becomes frustrated with the slow progress of most students (Connolly, 2010). Learning geometry may not be easy, and a larger number of students fail to develop an adequate understanding of geometric concepts, geometric reasoning and geometric problem-solving skills (Idris, 2009).

If the geometric levels remain under-developed, students will come to view geometry as fixed rules and an ensemble of meaningless executed calculation and drawing of shapes. Without a good foundation in geometry at the junior high school (JHS) level, the problem gets bad at senior high school (SHS) level where student have to advance in geometric topics such as mensuration and trigonometry. Furthermore, student taking Basic Education Certificate Examination (BECE) and the West Africa Senior Secondary Certificate Examination (WASSCE) have been performing poorly in questions involving geometry according to various examination report (BECE, 2017; WASSCE, 2020).

According to Asare (2021) a statement made during the School Performance Appraisal Meeting (SPAM) in Ashiedu-Keteke to improve standard indicated that out of 800 candidates who wrote the exams in Ashiedu Keteke of which Ga-Mashie is not excluded, 428 obtained aggregate 31 and above whilst 367 obtained aggregate 6 to 30. These schools in the district could perform well only when all stakeholders work together harmoniously. According to the Trends in International Mathematics and Science Study (TIMSS) 2011 study, algebra and geometry were the weakest content areas among Ghanaian pupils in mathematics (Mullis, Martin, Foy & Aaron, 2012).

When students have other options, they frequently avoid questions on geometry. On rare occasions, persons who attempt questions on geometry demonstrate little more than a lack of expertise in the subject area (Baffoe & Mereku, 2010). To understand why geometry learning is difficult, several elements have been proposed: geometry language, visualizing ability, and inefficient instruction (Idris, 2005). Another source of worry among secondary school students is a lack of thinking abilities. Many students struggle to extract relevant information from supplied data, and many more struggle to analyze responses and draw conclusions.

Techniques from the past to teaching geometry place more emphasis on how much students can recall and less emphasis on how effectively students can think and reason as a result, learning becomes forced and rarely provides pupils with happiness (Idris, 2009). Mathematics educators have put in effort aimed at identifying the major problems associated with junior high school geometry. Despite all these efforts, the problem of poor performance in geometry has continued to rear its head. Van Hiele

(1986) specifically state the inability of many teachers to match instruction with their leaners levels of geometric understanding is a contributing factor to their failure to promote meaningful understandings in geometry.

For this reason, it necessary to first identify students van Hiele Geometric levels (VHGL). The literature review reveals that the investigation of various issues related to students‟ geometrical reasoning (knowledge, abilities, strategies, difficulties) do not take the type of school into consideration. In other to fill in the gap of public and private schools VHGL the researcher believes it is necessary to gather empirical data to allow the comparison between the public and private schools and their level of geometric thinking.

1.3 Purpose of the Study

The purpose of this research was meant to investigate students‟ van Hiele‟s geometric thinking levels from public and private basic schools at Ga-Mashie in the Greater-Accra Region.

1.4 Objective of the Study

The objectives of the study were;

i. To assess the stages of van Hiele‟s levels JHS 3 students of private and public schools reach in the study of geometry.
ii. To investigate whether there is a significant difference in terms of van Hiele‟s geometric thinking levels between male and female in the Ga-Mashie Circuit.
iii. To determine if there is a significant difference in terms of van Hiele‟s geometric thinking levels between male and females in the school type in the Ga-Mashie Circuit.

iv. To investigate whether there is a significant difference in students van Hiele"s geometric levels between public and private schools in the Ga-Mashie Circuit.

1.5 Research Questions

In line with the objectives of the study, the following research questions were raised in this study:

i. Which stages of van Hiele levels of understanding do JHS 3 students of public and private schools in Ga-Mashie Circuit reach in the study of geometry?

ii. Is there a difference in terms of van Hiele geometric thinking levels between males and females in the Ga-Mashie Circuit?

iii. Is there a significant difference in terms of van Hiele"s geometric thinking levels between male and females in the school type in the Ga-Mashie Circuit.

iv. Are there any differences in students van Hiele"s geometric levels between public and private schools in the Ga-Mashie Circuit?

1.6 Research Hypothesis

To answer research questions 2 to 4, the researcher formulated the following null and alternative hypotheses:

H_0: There is no significant difference between students van Hiele"s thinking levels in terms of their gender.

H_1: There is significant difference between students van Hiele"s thinking levels in terms of their gender.

H_0 : There is a no significant difference in terms of van Hiele"s geometric thinking levels between male and females in the school type in the Ga-Mashie Circuit.

H_1: There is a significant difference in terms of van Hiele‟s geometric thinking levels between males and females in the school type in the Ga-Mashie Circuit.

H_0: There is no significant difference in van Hiele‟s thinking levels between Public and Private Basic School students.

H_1: There is a significant difference in van Hiele‟s thinking levels between Public and Private Basic School students.

1.7 Significant of the Study

Every human being needs the ability to solve problems throughout their lives, necessitating the acquisition of geometric skills. Geometric thinking is one of the skills needed to address non-routine challenges throughout one's life. As a result, someone who lacked either the basics or the skills and strategies is limited in their capacity to deal with everyday situations (Asemani et al., 2017). Therefore, this study will:

i. Provide basic for classifying Ga-Mashie students in public and private schools geometric thinking levels on the van hiele‟s theory. This will enable teachers to determine the best teaching and methodological strategies to enhance their student‟s geometric thinking levels.
ii. Reveal general strength and weakness of students in their geometric thinking levels that will assist teachers to plan an appropriate interventional strategy to help them at all levels of the model in their various schools.
iii. Provide information on students geometric thinking levels based on their gender.

iv. Provide information on whether students‟ geometric abilities are affected by their various schools.

1.8 Limitations of the Study

According to Kahn (2006), constraints are factors outside the researcher's control that limit the study's conclusions and applicability. Due to the following factors, the findings of this research study could not be generalized since the linked literature that supported the study was more foreign than local, cultural differences influenced the findings to some extent. Also, because society in Ghana is made up of numerous groups of students, the findings of the study from only Ga-Mashie in the Accra Metropolis cannot be generalized.

1.9 Delimitations of the Study

The study could have been more representative if all the schools in the metropolis were used. However, owing to limited time, materials, geographical locations of those schools and financial resources at the disposal of the researcher, the study was delimited to only schools in the Ga-Mashie Circuit of the Accra Metropolis.

1.10 Organization of the Study

This study basically was made up of five chapters. The first chapter which was the introduction was devoted solely to overview and the background to the study, statement of the problem, purpose of the study, objective of the study, research questions and hypotheses, significant of study, limitations of the study, delimitations of the study. Chapter two reviewed previous related studies on the topic as well as theories that are related to the study and this focused-on review of relevant literature and theoretical framework; examine what researchers have written and said about this particular topic. The third chapter considered the methodology; explain data collection, data processing, and methods used in analysing the field data among others. Chapter four presented the findings of the study and discussions relative to the literature review. The final chapter five summarized the findings of the study and conclusions that was reached and relevant recommendations based on the findings and for future research.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.0 Overview

This chapter reviewed literature and explores relevant and useful theory that best describes this study, including discussions of the study"s theoretical framework. A literature review includes a broader, continuing discussion of the literature, as well as completing and expanding on prior studies (Snyder, 2019). In other words, it facilitates theory development, closes areas where excess of research exists and also uncovers areas where research is needed. The unit of analysis was on assessing the van Hiele Geometric levels of students in the Ga-Mashie circuit: Gender and School type differences. The literature for this study, however, was essentially drawn from peer-reviewed articles published in journals, text books, research papers, newspaper publications as well as Google Search Engine.

2.1 Theoretical Framework

Van Hiele Geometry Theory is the theory chosen for this study. According to (Alex & Mammen, 2016; Vojkuvkova, 2012; Yegambaram & Naidoo, 2009), van Hiele theory is the most widely accepted explanation for pupils' geometric thinking levels. The theory was developed by two mathematics educators who completed their PhD studies in 1957 at the University of Utrecht in the Netherlands (Usiskin, 1982). Pierre Marie van Hiele and his wife Dina van Hiele Geldof developed the theory (Armah et al., 2017; Armah et al., 2018; Essays, 2013). Dina died shortly after completing her dissertation; therefore, Pierre has been the one to help us understand the theory.

The van Hiele geometry theory is the most important of the geometric thinking and development research that have been undertaken (Alex & Mammen, 2016; Armah et al., 2017; Abdullah & Zakaria, 2013; Essays, 2013). The theory gained popularity in the 1970s and has been widely utilised to enhance geometry instruction throughout the world since 1984 (Breyfogle & Lynch, 2010; Dindyal, 2007; Siew, Chong & Abdullah, 2013; Alex & Mammen. 2016).

Recognition, Analysis, Order (Informal Deduction), Deduction, and Rigor are the five sequential and hierarchical discrete Levels of geometric thought in the van Hiele theory (Usiskin, 1982). The van Hiele Levels are typically described using two alternative numbering schemes: Levels 0 through 4 and Levels 1 through 5. The van Hieles numbering method originally employed Levels 0 through 4, but more recent works [Mason (1998); Usiskin (1982) and van Hiele's (1986; 1999)] use the Level 1 through 5 numbering schemes instead. According to Mason (1998), a sixth Level, Pre-recognition Level (i.e., a level for learners who have not yet attained even the fundamental Level 1), might be designated as Level 0. The Level 1 to 5 numbering scheme was utilized in this study to allow for the usage of Level 0. The following is a description of the van Hiele Levels:

Level 1: Recognition (or visual level). Learners employ visual perception and nonverbal reasoning at this level. They recognize figures only on the basis of their appearance, comparing them to prototypes or daily objects ("that looks like a door"), and categorizing them ("it is / it is not a..."). They speak in plain terms (Vojkuvkova, 2012)". The properties of geometric figures are not identified by learners at this level (Van Hiele, 1999).

Level 2: Analysis (or descriptive level). "Figures are the bearers of their properties" at this level. A figure is no longer appraised just on the basis of its appearance, but rather on the basis of its qualities (Van Hiele, 1999)". Learners begin by studying and naming geometric qualities, but they do not comprehend the interrelationships between different types of figures, nor do they completely comprehend or appreciate the use of definitions at this level (Mason, 1998).

Level 3: Order (or informal deduction level). Learners at this level are able to see how different sorts of figures interact with one another. At this level, they may establish meaningful definitions and provide informal arguments to support their thinking. Squares are a form of rectangle, therefore logical consequences and class inclusions are understood (Halat, 2008; Mason, 1998).

Level 4: Deduction: Learners at this level can provide deductive geometric proofs. Definitions, theorems, axioms, and proofs are all understood. At this level, students may provide justifications for claims in formal proofs (Halat, 2008; Vojkuvkova, 2012).

Level 5: Rigor. This Level's students "understand the formal components of deduction, such as building and comparing mathematical systems" (Mason, 1998.p 7). Learners will understand how geometric systems are "constructed" and how they must be comprehended in the abstract. Learners at this level should be aware that various geometries exist and that the structure of axioms, postulates, and theorems is what matters (Crowley, 1987).

The van Hiele theory is relevant to this topic because it assisted the researcher in categorizing students' geometric learning abilities into five distinct and hierarchical Levels of geometric thinking, as well as providing a model of teaching that teachers could use to improve their students' geometric understanding levels. This will assist teachers in being aware of effective teaching strategies targeted at increasing student knowledge and allowing students to engage in the teaching process. Again, an examination of the literature on the van Hiele theory in geometry teaching has revealed a worldwide push to embrace it in order to enhance the way geometry is taught and learned (Alex & Mammen, 2016; Howse & Howse, 2014; Suwito, Yuwono, Parta & Irawati, 2017). Since the teacher is the driving force in any educational process, this advocacy for the adoption of the van Hiele theory has implications for mathematics teacher education.

Practically, Atebe (2008) investigated geometry teaching practices that may have contributed to the levels of geometric conceptualization displayed by a group of high school learners in Nigerian and South African schools, using the van Hiele model of geometry instruction as a lens. The conformance of videotaped courses to criteria on the van Hiele phase descriptors checklist prepared for the study was assessed. The majority of the classes recorded did not follow the van Hiele model of instruction in the geometry classroom, according to the findings of the observations.

Also, Atebe (2008) found that geometry teaching approaches in Nigerian and South African schools provide learners little chance to understand the subject by comparing the van Hiele model of geometry instruction with observable geometry instructional approaches. According to the researcher, this resulted in the cohorts of learners from these places having poor levels of geometric conceptualizing. However, recent studies by (Alex & Mammen, 2016; Armah, Cofie, & Okpoti, 2017; Anas, 2018; Armah, Cofie, & Okpoti, 2018) indicate that using the Van Hiele theoretical model to teach geometry has the potential to improve geometry performance at all levels of education. This and more informed the researcher‟s decision to adopt Van Hiele Geometry Theory for this study.

2.2 Conceptual Review

2.2.1 Geometry and mathematics

A strong mathematical culture in a country leads to the development of a strong industrial culture all over the world. For this and other reasons, practically every country in the world has made mathematics a compulsory subject from preschool to secondary school. In Ghana, the Ministry of Education (MoE) has developed a mathematics syllabus based on the premise that "all students can learn mathematics and should learn mathematics" through the National Council for Curriculum & Assessment (NaCCA) and the Curriculum Research and Development Division (CRDD) of the Ghana Education Service (GES) (CRDD, 2010). The decision was made as part of the government's attempts to build a strong human resource for the country's economic development. This approach, however, has not been without its challenges. There have been several theories put up to explain why mathematics is in such a bad state. Poor mathematics teaching in primary, junior, and senior high schools, a lack of motivation and incentives, and poor career prospects in mathematics in many sectors of the economy other than teaching are among the reasons, according to Adolphus (2011).

Ghana took part in the West African Secondary School Certificate Examination (WASSCE) April 2006, together with other English-speaking (Anglophonic) countries in West Africa. Chief Examiners Reports have consistently revealed that students continue to perform badly in mathematics across the country, according to WAEC, Chief Examiners Report (2014, 2015 & 2016). However, many of the reports point to geometry as a problem among Ghanaian students. Students' number one weakness, according to WAEC, is "solving problems combining geometry and trigonometry" (WAEC, Chief Examiners Report, 2014). Despite the fact that geometry is one of the most important topics for Ghanaian students to acquire in order to be prepared for life, it was discovered in WASSCE examinations that most students struggle with geometry-related questions.

Students' mathematical skill is linked to their geometrical conceptual ability, according to van Hiele (1999). One of the most important subjects in mathematics is geometry. Geometry is a field of mathematics and one of the most fundamental abilities to master (Hoffer & Hoffer, 1992). It is about their spatial relationships, the form of individual objects and their properties (Luneta, 2014; Bora & Ahmed, 2018). Geometry (Aktaş & Aktaş, 2012; Serin, 2018) is a discipline of mathematics that investigates forms and space in two-dimensional (2D) and three-dimensional (3D) dimensions, as well as its characteristics.

Geometry ideas are important and should be taught well in mathematics classes because they help students to analyze and interpret the world around them while also providing them with tools to utilize in other subjects (Özerem, 2012). Geometry learning needs pupils to visually understand objects and their attributes by comparing them to previous experiences with similar objects (Idris, 2005).

Geometry is a fundamental mathematical skill that is taught to learners and students at all levels of education (Robert et al., 2018). Geometry plays an important role in the early grades, as evidenced by the development of mathematics curriculum. Many students regard issues, concepts, and geometric manifestations as exciting and useful, but traditional geometry has become ambiguous and frustrating, and research in recent years has focused on establishing or highlighting the achievement of agreement between the mathematical and psychological points of view in the teaching of geometry (Awad, 2014).

Olkun and Aydlogdu (2003) asserted that, geometry is significant not just as a discipline but also as a conceptual building block of mathematics. Geometry, according to Asemani, Asiedu-Addo, and Oppong (2017), gives a more complete understanding of the world we live in (geometry, for example, emerges naturally in the structure of the solar system, in geological formations of plants and flowers, and even in animals). It is also a big element of our synthetic world, which includes items like art, architecture, automobiles, machines, and pretty much everything else humans make. Geometry's teaching and learning process differs from those of other mathematical disciplines like algebra, arithmetic and probability, according to Noraini (2005).

Visualization, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument, and evidence are all qualities that geometry helps students acquire. Geometric representations can also be used to help students understand other areas of mathematics, such as arithmetic fractions and multiplication, relationships between graphs of functions (of two and three variables), and graphical representations of data in statistics (National Council of Teachers of Mathematics) (NCTM, 2009). Geometry also aids in the development of spatial imagination and logical reasoning, as well as forming the foundation for a variety of mathematical and non-mathematical fields, in which it plays a crucial part (Reilly, Neumann & Andrews, 2017). The quality of geometric knowledge of students has been directly connected to their mathematical competencies (Van Hiele, 1986)

Recent studies by (Alex & Mammen, 2016; Armah, Cofie, & Okpoti, 2017; Anas, 2018; Armah, Cofie, & Okpoti, 2018) indicate that using the van Hiele theoretical model to teach geometry has the potential to improve geometry performance at all levels of education. The competence of mathematics teachers, the use of appropriate teaching methods and strategies, the characteristics of the teacher, the readiness of mathematics teachers and the connection of teaching to real life are all important factors influencing pupils' learning of mathematics, particularly geometry (Sunzuma, Masocha & Zezekwa, 2013).

Mann, Chamberlin, and Graefe (2017) remarked that, without mathematics the world would not move an inch as it is the birthplace of all creations. As a result, we may argue that mathematics is an essential discipline, particularly in the formal education system, since it improves students' cognitive abilities, which may be useful in their future fields of employment. Anderson (2013), on the other hand, noted that for all private and government institutions, low mathematic achievement in a variety of disciplines is now a worry. According to Alzhanova-Ericsson, Bergman, and Dinnétz (2017), pupils' mathematic achievement is very low, which might be due to a lack of practice, teaching techniques, or teaching facilities such as instructional media, games or computers.

However, in addressing the quality of education students receive, teacher quality and teaching methods have become a national concern (Mann, Chamberlin, & Graefe, 2017). Consequently, it is clear that mathematic is both important to study and valuable to national development. To Khun-Inkeeree, Omar-Fauzee, and Othman (2016), self-confidence and value in mathematic achievement have an insignificant positive relationship. Furthermore, Zan and Di Martino (2014) stated that if a student's self-confidence in mathematics is low, the aim of learning mathematics would be undermined.

Hence, self-confidence in mathematics was a key factor in achieving success in the subject. Furthermore, according to Belin (2016), students' attitude will improve their performance throughout their education and will be able to strengthen their self-confidence, enjoyment, values and motivation to learn. As a result, enhancing a student's self-confidence may help them gain better mathematics achievements.

Shaikh (2013), on the other hand, revealed that students' mathematic confidence was most likely learnt from their own parents or primary school teachers, who were similarly inept at working with numbers. Das, Das, and Kashyap (2016) agreed that, parents and schools are having difficulties with pupils' mathematics performance, which is regarded poor. If a student's self-confidence in mathematics is poor, it will be difficult for them to learn; they will find it boring and uninteresting, which will negatively affect their mathematic performances.

Thus, the relationship between attitude, accomplishments, and performance of students in mathematics has been studied by Akinbobola (2009); Brookstein, Hegedus, Dalton, Moniz, and Tapper (2014); Mata, Monteiro, and Peixoto (2012); Tran (2014). The outcomes of the correlation between these factors have been found to be significant.

2.2.2 Geometry content knowledge

Subject matter knowledge, also known as content knowledge, is a significant concern in teacher education, according to researchers (Shulman, 1986, 1987; Hill et al., 2008; Aslan-Tutak & Adams, 2015). Content knowledge is defined by Aksu and Kul (2016) as the "organization and amount of knowledge in a teacher's mind" (p. 35). A teacher's content understanding is a determining factor in all classroom activities. Pre-service teachers with limited content understanding confront several challenges in pedagogical training, according to Aslan-Tutak and Adams (2015). Several researchers have found that teachers' use of pedagogical tools is influenced by their lack of content knowledge (Aslan-Tutak & Adams, 2015).

To ensure their competency development in training, pre-service teachers must have a thorough understanding of the material or subject matter. Despite the various uses of geometry in our daily lives, it appears to be a topic that is usually disregarded in the mathematics classroom (Aslan-Tutak & Adams, 2015). To the authors, "beginning teachers are not equipped with an adequate content knowledge of geometry," according to a large body of research on the subject (p. 303). Concepts, facts, skills, theorems, theories, and understanding of relationships among themes in geometry make up content knowledge (Sunzuma & Maharaj, 2019). In geometry and other areas of study, knowledge of associations refers to how specific subject is linked to other concepts. This knowledge is required for teachers to be effective in their roles as educators.

2.2.3 Van Hiele phase-based learning

According to van Hiele (1959), the development of pupils' geometric thinking is dependent on their thinking abilities as well as their learning experiences, rather than their age and maturity. Therefore, van Hiele offered five learning phases to help students‟ progress from one step to the next in their geometric thinking (Crowley, 1987). The first is information, which consists of a dialogue between the teacher and the students. Teachers, according to Connolly (2010), must propose new information in each of the best-designed questions for students to express the relevance of the initial concepts to the content to be learned; secondly, guided orientation, in which students engage in guided activities to explore the content to be learned.

Teachers must instruct students to study the learned objects, as Clements and Battista (1992) and Crowley (1987) posit. Giving students instructions is a sequence of brief activities aimed at eliciting certain responses from them; third, explications, in which students are allowed to explain and share their thoughts about their work. To Clements and Battista (1992), in this phase, teachers use linguistic precision to bring a combination of objects (e.g., geometrical ideas, relationships, patterns, etc.) to the level of understanding; fourth, free orientation, in which students can solve increasingly complex questions.

Crowley (1987) proposed that, students be given the opportunity to learn themselves how to solve issues in order for them to see the relationships between solid properties more clearly. Fifth, students draw a conclusion from their learning through integration. Crowley (1987) emphasized that at the end of this phase, pupils have acquired a certain level of geometric thinking and are ready to repeat these five learning phases for the next level of geometric thinking.

However, in Indonesia, the learning approach still relies on teachers passing on their knowledge to pupils, causing students to memorize without understanding the mathematical concept. This is consistent with studies by Abu and Abidin (2013), which found that traditional geometry teaching and learning methods contributed to secondary school pupils' lack of geometric thinking. Nadjib (2014) found disappointing findings in his study in South Sulawesi. Only one student out of 25 reached the level of informal deduction (Level 2), whereas the majority was at the level of analysis (Level 1).

Internal and external variables had a role. The internal aspect, according to Djamarah (2011), was the students' psychology and mental or learning readiness, while the external factor was the environment and instrumental factors (example learning program, teacher, curriculum, activity, and facility). Furthermore, traditional geometry education based on lectures and school textbooks has yet to aid pupils in understanding geometry concepts (Nur & Nurvitasari, 2017).

2.2.4 Learning module

Modules are a collection of learning resources that are presented in a logical order, allowing users to study with or without an instructor or teacher (Mostafa, Mohammad Javad & Reza, 2016). The major goal of the writing module is for students to study autonomously, with or without teacher support. Mostafa, Mohammad Javad, and Reza (2016) added that, there are certain concerns about the developing module. These are; (1) The layout arrangement on the module should be easily structured, with a brief title, a list of content, a clear cognitive structure, a résumé, and a task for readers. (2) The language is simple to comprehend.

(3) There is content for the readers to examine. (4) Any stimulus that encourages the reader to think is recommended. (5) The module should be simple to understand, with a font size that is not too tiny or too large, a text structure that is easy to read, and a font size that is not too small or too large. (6) The module's content, including materials and worksheets, should be relevant to the learning goal. There are five characteristics, on the other hand, that make a module capable of motivating pupils to study. Self-instruction, self-contained, stand-alone, adaptable, and user-friendly are some of these characteristics (Depdiknas, 2008).

Furthermore, the phases of developing a module are (1) analysing the main and basic competition, (2) determining the title of the module, (3) assigning a module code to make organization easier, and (4) writing the module, which includes formulating the basic competition, determining the instrument of evaluation, material, learning sequences, and the completeness of the module structure (Depdiknas, 2008). Moreover, the properness of content, validity, presentation, and visual design should all be considered when evaluating the module's quality (Depdiknas, 2008).

2.3 Factors Influencing Learner Performance in Geometry in Secondary Schools

Some of the factors influencing learner performance in geometry in the secondary schools are discussed below;

2.3.1 Home environmental factors

The main factors relating learners' disadvantages to underachievement in geometry, according to Banergee (2016), are a lack of supportive home environment and support. Mahanta (2014), for example, indicated that the family environment, particularly socioeconomic status, has a major impact on students' educational attainment. To Mahanta (2014), the house is the first environment with which a child interacts during the learning process. The goal of this research is to add to this area of study by examining into the van Hiele Geometric levels of students in the Ga-Mashie.

2.3.2 Socioeconomic factors

Khaliq et al., (2016) argued that, notably there is a link between parental income/status/occupation and a learner's academic achievement in geometry. Similarly, Sonali (2017) found that students from low socioeconomic status experience more academic stress and, as a result, perform worse in geometry than those from higher socioeconomic status. Scholars tended to agree that parental education is a measure for socioeconomic status, and that it can lead to improved student performance (Mullis et al., 2016).

In contrast, Devenish and colleagues found in a review of the literature that the neighborhood level had a considerable impact on youth outcomes regardless of parental socioeconomic status (Devenish, Hooley & Mellor, 2017). The growing discrepancy in household income and wealth in the United States over the last half-century Autor (2014); Saez and Zucman (2016); Alvaredo et al., (2017) has heightened worries regarding the impact of a learner's socioeconomic status (SES) on academic attainment. Sean (2013) had shown in his extensive study how students from high-income homes performed better than those from low-income families.

His research was conducted in the United States of America. He claimed that, the influence of a parent's income may be seen in the early timing of a child's learning, and that higher-income parents send their children to school sooner than lower-income parents. Similarly, Brito, Piccolo, and Noble (2017) discovered that learners from higher socioeconomic backgrounds perform better in terms of cognitive performance than learners from lower socioeconomic backgrounds.

This is comparable to Pearce et al., (2016) findings, which showed that students from the most disadvantaged group were twice as likely to obtain the lowest grade in geometry. Ndebele (2015) found comparable results in research he did. He observed that the higher the family's income and socioeconomic position, the more probable it is that parents will assist their children with homework. Learners from lower socioeconomic backgrounds, on the other hand, are less likely to have parents that assist with homework.

Mucee et al., (2014) investigated the socioeconomic factors that impacted learners' academic achievement in geometry in secondary schools in Tharaka in a Kenyan research. They discovered that many Kenyan students were compelled to seek jobs and labor beyond school hours every day owing to their low socioeconomic condition (Ministry of National Education, 2015, p. 10). As a result, students' school performance suffered as a result of their inability to do homework on time. Letsoalo et al., (2017) did another study on factors influencing learners' geometry performance, and found that diverse socio-economic, psychological, and environmental factors all had a role.

He also stated that a wealthy family may provide a better education for their children by employing private tutors to help them with geometry. For example, wealthy families might pay for private organizations to provide extra tuition to their children to help them learn geometry. In addition, Soni and Kumari (2017) suggested that a parent's educational degree is an indication of SES because it is linked to children' school performance. Sikhwari (2016) ascribed socioeconomic factors to the predictor of student performance in geometry in South Africa, stating that nearly 75% of learners hailed from two-room families. As a result, all members of the family shared rooms, and students were unable to concentrate on their geometry homework due to a lack of space.

Parents from low-income households feel inferior and shun participating in school activities. According to Silvernail et al., (2014), educators frequently behave differently toward learners and parents depending on their socioeconomic status. Thus, the family's socioeconomic situation may have an impact on the learners' self-esteem. Learners with access to educational materials at home, on the other hand, tend to do better in geometry than those who do not (Visser, Juan & Feza, 2015). South Africa's future socioeconomic possibilities for learners, as well as the country's overall development, necessitate massive improvements in the teaching of geometry in public schools (McCarthy & Oliphant, 2013).

2.3.3 Parental educational background

The quality of parental participation in their children's learning is likely to be impacted by the parents' educational level, according to previous study (Kiadarbandsari et al., 2016; Kikas & Mägi, 2014). Research done in the United Arab Emirates, for example, found that students who considered their parents' attitudes about geometry to be positive had much higher performance rates (Areepattamannil, et al., 2015). Indeed, parental education has been demonstrated to be one of the most powerful factors of learners' educational performance; as a result, family educational level and attitudes about geometry may have a positive or negative impact on learners' geometry performance.

Skolverket (2012) observed an improvement in learner accomplishment between 2007 and 2011 with a more differentiated assessment of family educational level in research conducted in Sweden. Learners' academic achievement is linked to their parents' level of education and occupation. Parental occupational class, parental participation and parental income according to Leung, Chung, and Kim (2016), determine learners' goals and serve as an indirect indicator of the material resources available to aid the student in his education. Hence, the relationship between parental background and academic achievement includes factors such as home environment, parent income, ethnicity and profession.

In any educational setting, other studies revealed that parental characteristics such as parental education, family size, and parental income are connected to, and impact, children's academic performance (Ogbugo-Ololube, 2016). Kainuwa and Yusuf

(2013) looked at how parents" socioeconomic position and educational background influenced their children's education. It has been noticed that the level of education of parents often influences their support and goals for their children's education. In mathematics and science subjects, Chevalier (2013) agreed, stating that more-educated and richer parents can offer a better environment for their children's scholastic achievement.

Better-educated parents, according to Özcan and Erktin (2015), make investments such as sending their children to geometry lessons. Better educated parents are also aware of the benefits of education and may be ready to spend more money to ensure that their children receive a good education (Erola, Jalonen & Lehti, 2016). They are guiding their children towards better job opportunities in the workplace by doing so. There is a link between family background and students' intellectual achievement, according to research. In South African schools, Mutodi and Ngirande (2014) discovered a positive connection between parental education levels and pupils' geometry performance. Aliyu and Mohammed Isa (2016) found, parents are required to support and lead their children to become successful members of society. Similarly, Pangeni (2014) research highlighted the importance of parental education as a predictor of geometry achievement.

Pangeni (2014) indicated that, parents with a higher level of education have more access to a combination of economic and social resources that may be utilized to assist their children succeed in school. In encouraging their children to achieve high educational objectives and desires, parents serve as role models and guides. They accomplish this through providing educational resources in the home and adopting specific attitudes and ideals toward their children's education.

2.4 School Environmental Factors

Shamaki (2015) posits that, the school learning environment encompasses the entire climate in which staff and students operate. It is a dynamic and complete picture of all the forces that shape the members of the school's physical, emotional, psychological, and social lives. For Uharian (2016), the term "school learning environment" encompasses a number of strands, including the school's location, available resources, interpersonal relationships, structure, supervisory practices, organizations, communication patterns, administrative and, so on.

Research ascertained that, the attitude of educators‟ influence how they teach, which has a detrimental or good impact on students' academic performance (Ogembo, Otanga, & Yaki, 2015). Geometry, as well as the teaching and learning of geometry, is complicated, according to Ali, Bhagawati, and Sarmah (2014), since it necessitates the simultaneous presence of several cognitive processes. Gamlem and Smith (2013) added that, education is carried out through teaching, learning, assessment, and continuous monitoring in order to promote the improvement of learners' performance if it is properly done. In this sense, teaching and learning monitoring is viewed as an important leadership part of identifying gaps in the teaching and learning process in order to enhance instruction and learner performance (Du Plessis, 2013).

According to Tshabalala and Ncube (2013), in South Africa, learners' performance in geometry is primarily influenced by teaching methods, material, and the establishment of a strong foundation in the subject at the lower levels. One of the constraints in solving geometric high-order thinking skill test issues is the learners' lack of conceptual understanding (Alhassora, Abu & Abdullah, 2017). Accordingly, educators must adopt teaching methods that take into account the pupils' degree of geometry comprehension (Schoenfeld & Floden, 2014). Learners' inadequate knowledge of geometric concepts and lack of capability to make geometrical connections are the reasons why Clement et al., (2013) linked learners' poor performance in geometry with educators' teaching practices. Educators in South Africa, according to Clement, continue to use conventional teaching methodologies. Teaching quality suffered as a result of outdated teaching methods and a lack of basic content knowledge (Carey et al., 2015).

2.4.1 Cooperative learning in geometry classes

Cooperative learning is a form of learning approach that allows individuals to engage in learning in a variety of ways (Ching & Nunes, 2017). Learner-centered learning, which is arranged around the learners' interests, needs, abilities, and skills, is producing successful individuals (Good & Clarke, 2017). Educational research focuses on ways that encourage learners' active participation, learner-centered education that is relevant to everyday life, and learners' prior experiences. Furthermore, Mtitu (2014) highlighted that learner-centered methods, which require educators to actively include learners in the teaching and learning process, are required for successful and efficient teaching.

Eze, Ezenwafor, and Molokwu (2015) agreed with Mtitu and stated that educators should use appropriate teaching methods that best suit specific objectives and level exit outcomes to facilitate the process of information transmission. One of the likely causes of Indonesian students' poor geometry performance was an inappropriate learning model for the geometry learning process (Prahmana, Kusumah & Darhim, 2017). The cooperative learning model was proposed as one of the learning models that could be applied in Indonesia in the learning of geometry, according to

Prahmana, Kusumah, and Darhim (2017) studies. Cooperative learning is a method that allows for group work as well as self-exploration. Indeed, Chowa et al., (2015) found that conventional teaching methods utilized by Thailand's education system, such as memorizing textbook materials, had a detrimental impact on learner performance.

2.4.2 The quality of geometry educators

Alzhanova-Ericsson, Bergman, and Dinnétz (2017) pointed out that, the problem of learners' performance in geometry cannot be separated from how educators interact with students during the subject's learning. The influence of instructor quality on student success has been extensively documented in studies (Jimerson & Haddock, 2015). Other research found that the quality of geometry educators is the single most important factor in predicting student performance (Bear & Jones, 2017). Prahmana, Kusumah, and Darhim (2017) found that, an educator's influence in the learning process might affect learners' geometric problem-solving skills, independence, and curiosity in good or bad ways.

The quality of educators' instruction may, in fact, have a direct influence on students' performance (Venkat & Spaull, 2015). According to Alzhanova-Ericsson et al., (2017), learners' bad performance in geometry can be impacted by a lack of practice, teaching methods, and teaching facilities such as a game, computer, or instructional material that an educator uses. Despite the findings of Suleman, Aslam, and Hussain (2014), who asserted that well-equipped classrooms help educators to educate learners efficiently and that favorable outcomes in geometry may be on the way, these do not go together. While the quality of the educator's educational technique is critical, engaging in suitable geometrical activities is also essential (Jacobi-Vessels et al., 2016).

Professor Mohamed Ibrahim (Professor of Mathematics and President of the Mathematical Association of Nigeria) was quoted by Kukogho (2015) as arguing that inadequately qualified educators are to blame for Nigeria's low performance in geometry. According to Ibrahim, students have developed "geometrical phobia," which causes dread and failure. Ibrahim believes that educators' methods are to blame for students' disdain of geometry. He further stated that, most Nigerian educators are unable to handle contemporary technologies such as computers, which are now widely used in developed countries.

In addressing the quality of education learners receive, national priorities include the quality of geometry educators and teaching methods (Mann, Chamberlin & Graefe, 2017). Quality educators, according to Ramphele (2015), are the source of quality education. What an educator does not have, he cannot provide. Educator qualifications and experience, for example, are major drivers of learner performance (Kimani, Kara, & Njagi, 2013). Abe (2014), for example, looked at geometry classes and discovered that students taught by educators with higher credentials achieve much better results than students taught by educators with lesser qualifications.

Years of experience, degree level, and kind of certification impact learners' performance in geometry, according to Ellerhorst (2014). However, (Couto & Vale, 2014; Gegbe & Koroma, 2014) refuted the previous researchers' findings, claiming that educator credentials had no impact on student performance. According to Gegbe and Koroma (2014), in order to be a good professional capable of teaching geometry, one must have a thorough knowledge and grasp of the subject as well as the capacity to implement learning methodologies. Similarly, Kimani, Kara, and Njagi (2013) disagreed with the previous findings, stating that more experienced instructors had little effect on learners' performance.

Browning et al., (2014) suggested increasing spatial visualization and problem-solving skills through geometry curricular experiences that went beyond procedural and memorizing skills. Geometry is a major part of the school mathematics curriculum, according to Oladosu (2014), and it is important in learners' mathematics education because it allows them to acquire spatial awareness and geometric reasoning. When preparing lessons, educators must be able to pick appropriate examples and exercises, as well as sequence the material of the lesson and choose a technique for teaching the relevant procedures (Bansilal, Mkhwanazi & Brijlall 2014). By designing lessons with activities on more than one level in a class, quality educators must be able to recognize and reach all learners on different levels (Bleeker, Stols & Van Putten, 2013).

2.4.3 Educators' knowledge of geometry

Both content (subject matter) and procedural knowledge (pedagogical knowledge) of educators are crucial components in learners' understanding and accomplishment in education. Numerous studies have found a link between educators' intellectual abilities and students' success as evaluated by achievement tests (Campbell & Prew, 2017). Another key element impacting student performance in geometry, according to Fadzil and Saat (2014), is educators' lack of Pedagogical Content Knowledge (PCK), as described by Shulman (1986) and Hill et al., (2008). Learners' geometrical content knowledge is heavily influenced by the educator's geometrical content knowledge

(Couto & Vale, 2014). Learners' performance in geometry is influenced by the educator's expertise and mastery of the subject.

Due to a lack of confidence, ability, geometrical substance, and pedagogical understanding, many instructors avoid the subject (Beilock & Maloney, 2015). Some educators, particularly under-qualified and unskilled educators who teach in overcrowded and under-equipped classrooms, dislike teaching geometry and spend little time on it (Metzler, 2014). Participation in a geometry techniques course resulted in considerable gains in geometry teaching efficacy among educators (Zee & Koomen, 2016). In the geometry classroom, using non-traditional approaches such as small-group teaching techniques and addressing individual attitudes toward geometry, can help learners and instructors feel less anxious about geometry (Lake & Kelly, 2014). Confidence and understanding of the subject matter are also important in shaping one's attitude toward geometry, particularly educators' attitudes (Catlioglu et al., 2014).

2.5 Learners' Misconceptions in Geometry

According to Yang (2017), learners' academic performances are linked to their concepts. This is in line with the idea that educators need to understand the underlying causes of these misconceptions so they may take steps to improve learning environments (Ojose, 2015). Misconceptions and errors, according to Makhubele (2014), result in a set of emotions such as fear, anxiety, frustration, and fury, which can jeopardize both performance and participation in geometry. Gardee and Brodie (2015) advocate that, instructors should avoid re-teaching a geometry part by simply correcting errors and misconceptions while teaching, saving time.

Geometric ideas, according to Fyfe et al., (2015), are the key issues that cause the greatest challenges while teaching mathematics. According to Siyepu (2013), learners do not comprehend basic geometry ideas and do not acquire necessary problem-solving abilities. Geometry relies heavily on notation and symbols. As a result, it is true that grasping the notion necessitates knowledge of other components such as notation. However, studies have shown that learners frequently employ improper notation (Jojo, 2017; Siyepu, 2013). In reality, studying geometry necessitates not only the production of ideas, but also the knowledge of their standard names and notations, as well as the necessary verbal and geometrical syntax for referring to them in geometrical discourse (Jojo, 2017).

There are a variety of misconceptions and challenges that students have when learning about geometry at various levels of education (Browning et al., 2014). Additionally, (Herholdt & Sapire, 2014; Luneta, 2015) said that learner errors or misconceptions were other aspects that had been well investigated. Many of the learners' misconceptions occurred because educators and learners operated on separate geometric levels (here the author refers to van Hiele's (1986, p. 211) levels, according to Luneta.

The students' perceptions of geometry Learners' attitudes about geometry, according to Jennison and Beswick (2014), are a component that has been repeatedly investigated to see if there is a correlation between learner achievement and attitudes. Many research has been conducted across the world to study learner attitudes about geometry as well as the link between attitudes and student performance (Mahanta, 2014; Mensah, Okyere, & Kuranchie, 2013; Mutodi & Ngirande, 2014). These studies have found several aspects of geometry attitudes, as well as how these aspects affect student performance. Tall (2014) contends that learners' emotions play a crucial part in geometrical thinking and can have a massive effect on how they understand proofs.

Ngussa and Mbuti (2017) carried out a study with secondary school students in Arusha, Tanzania. When educators utilize humor as a teaching approach, they discovered a modest association between student attitudes and performance. They came to the conclusion that improving students' positive attitudes can improve their math‟s performance. Learners who lack self-confidence and an attitude toward geometry, according to Zan and Di Martino (2014), do not believe in themselves, and their performance drops as a result.

Learners who have great self-confidence and believe in their abilities, on the other hand, can succeed in studying geometry and overcome their fear of failure. These students are willing to take on geometrical a challenge, which improves their academic performance. Furthermore, Belin (2016) stated that, having a good attitude improved students' academic performance and increased their self-confidence, values, happiness, and willingness to learn throughout their education. Brookstein et al., (2014) performed study on the relationship between learners' attitudes, achievements, and performance in geometry and found that positive attitudes toward geometry increase learners' performance. Khun-Inkeeree, Omar-Fauzee, and Othman (2016) agreed, saying that increasing a learner's self-confidence can help them perform better in geometry.

Since they felt welcomed, Rimm-Kaufman and Sandilos (2015) reported that students who had positive relationships with their teachers performed better academically in geometry. The culture of respecting superiors without questioning their approaches may have resulted in their being uncontested, which has hampered learners' geometry performance (Strutchens et al., 2017). When learners develop close, good, and supportive relationships with their teachers, they will perform better in geometry than students who have more conflict with their teachers (Rimm-Kaufman & Sandilos, 2015).

2.6 Learner Motivation

The importance of learner motivation cannot be underestimated. According to Sumantri and Whardani (2017), motivation has a significant impact on learners' learning results. Learner motivation and engagement in learning geometry are closely linked, according to research. Tas (2016), for example, suggested that the learning environment and student willingness to learn are major indicators of their engagement in high school geometry learning.

Many learners dread geometry and feel helpless in their understanding of geometry concepts, according to Mutodi and Ngirande (2014). They see geometry as tough, cold, abstract, and a primarily male-dominated topic in most countries. Further research identified many aspects of learner motivation that contributed to their engagement in the geometry course. When students are interested in and like learning geometry, they are more likely to participate in geometry-related activities (Lin et al., 2013). In junior high schools, learners' mathematics preferences are also a good predictor of their interest in geometry study (Hsieh et al., 2016). Educators should be able to affect learner performance in geometry by exhibiting concern for their students, patience in their interactions, and the capacity to establish a pleasant environment, according to Cordes (2014).

Similarly, Prendergast et al., (2019) emphasized the need of educators focusing on developing positive academic relationships with students as well as positive pedagogical relationships that included a deep understanding of how students learn and a strong subject mastery of geometry. When educators offer praise to students, they assist to reduce anxiety and increase self-esteem (Canfield, 2013). Learners who are struggling or who have the same geometrical skill level as them frequently surround themselves with other learners who are also struggling or who have the same geometrical skill level as them (Mainali, 2014). Despite the fact that few students attribute their inability to laziness or a lack of effort, they nevertheless feel it is their responsibility to change their mindset and succeed. Learners feel motivated and have higher academic confidence when they can achieve at an adequate academic level in geometry (Cordes, 2014).

2.7 Second Language Complexity

One of the difficulties that geometry students have is a lack of proficiency in the geometry language. Geometry is sometimes regarded as a language with its own rules, conventions, symbols, and syntax, according to Schulte and Stevens (2015). Many learners regard geometry as arbitrary. In reality, language serves as a channel for understanding as well as communication (Planas et al., 2018). Geometry has a distinct vocabulary that conveys meaning, descriptions, and even properties. According to Müller and Ehmke (2012), learners must interpret or construct meaning in their language.

Since our educators encourage certain learners while disadvantaging others, language is one of the key drivers of marginalization. As school language differs from home language, some learners may feel excluded from classroom practice owing to a language barrier, resulting in a social class of learners with low participation and engagement in the classroom (Nero, 2014). Essien et al., (2016) suggest that most secondary school students have very low English proficiency. According to Essien et al. (2016), learners' geometry performance in secondary schools is influenced by their English proficiency.

Prediger et al., (2015), on the other hand, argued that if students are forced to learn in a language in which they are not fluent, the system will not function optimally, and if children are forced to learn in a poorly developed language, the quality and quantity of what they learn from the curriculum will be obviously inadequate. Improper language functioning can lead to poor cognitive functioning and performance. In general, communicating in the language of geometry necessitates a solid foundation in geometrical content and pedagogy, as well as a strong command of the English language, a strong number sense, and the ability to analyze critically (Riccomini et al., 2015).

2.8 Transition of Learners from Primary to Secondary School

Moss et al., (2015) conducted research in Japan and found that a learner's geometrical understanding in kindergarten is a powerful predictor of future academic performance. The research looked at how children develop core geometrical skills in their early years of education. For example, the format of a Japanese Lesson Study has been utilized in Japan for almost a century and is now well acknowledged as an effective tool for mathematics professional development (Groves et al., 2013). This method allowed educators to work with colleagues to rearrange their geometrical knowledge, understand underlying goals, build deep understandings of concepts, and relate learners' understandings to those goals. From kindergarten to eleventh grade, students' core geometry skills are a predictor of their success in mathematics and other subjects.

As higher levels of geometry are spatial in nature, spatial reasoning abilities, according to Bruce, Flynn, and Bennet (2016), are powerful predictors of future geometrical success. During the literature research, it became clear that educators needed to cooperate with others in order to make changes in their geometrical instruction. The educators in Bruce, Flynn, and Bennet (2016) and Moss et al., (2015) research actively participated in geometrical exercises and discussed them with others, allowing them to create personal relationships with the geometrical concepts before attempting to communicate them with their students. For constructing a learning environment with limitless geometrical possibilities, that seems to be the starting point.

To Fadzil and Saat (2014), primary schools have a substantial lack of geometry specialists, and there is an urgent need to discover an efficient strategy to train and hire primary educators with specialist understanding of geometry and the confidence to teach. Similarly, Fadzil and Saat (2014) found that learners' academic performance in geometry drops when transitioning from elementary to secondary school and that learners receive less social support throughout this transition. Prendergast et al., (2019) agreed that learners have significant problems when they transit from primary to secondary school, particularly in terms of geometry learning, because it takes time to adjust to the secondary school geometry curriculum. For example, Fadzil and Saat (2014) discovered that as students migrate from elementary to secondary school, they develop a gap in their ability to relate to the level of geometry content they learned in lower grades. During these transitions, interest in and enjoyment of geometry decreases.

2.9 Gender and School Type Differences in Mathematics

According to (Doris, O'neill, & Sweetman, 2013; Kim & Law, 2012), studies on gender variations in mathematics performance are gaining traction throughout the world. Gender differences in mathematics learning are not obvious throughout primary school years, but females begin to lag behind males at intermediate levels, according to some studies (Hyde, Geiringer, & Yen, 1975; Mann et al., 1990). During their high school years, females lag more behind (Fennema, 1980; Leder, 1985). Many of the research findings revealed that there are a variety of sex differences in mathematics. Because of its relevance, Forgasiz (2005) believes that gender should be a focus in mathematics education.

He also emphasized that gender should be considered as a variable in research studies, even if it is not the primary emphasis of the research on mathematics education. Furthermore, gender is an essential component in mathematics learning, according to Armstrong (1981), Lloyd, Walsh, and Yailagh (2005). The researcher was inspired by this reasoning to evaluate this variable. According to studies by Lloyd et al., (2005), there are disparities in accomplishment between male and female students in several topic areas of mathematics such as problem solving, computation, measurements, and spatial visualization, as reported by Halat (2008).

As some studies have shown gender differences in geometric thinking, the gender variable may have an effect on geometric thinking. For example, Hamzah's (2017) study found a difference in favor of females in engineering thinking among a sample of Jordanian class teacher students. Sudihartinih and Wahyudin's (2019) study also found that there are differences between males and females in favor of females. Armstrong (1981) found that female students do better in computation and spatial visualization than their male counterparts. Furthermore, other research suggests that female students do better than their male counterparts (Arnot, David & Weiner, 1999; Hydea & Mertzb, 2009).

When senior secondary school students were assessed in sequences and series, Ezeh (2005) found that male students fell behind their female counterparts in terms of delayed formalization approach. Ezeh further stated that the delayed formalization strategy improved student success, indicating that it is beneficial in the teaching and learning of mathematics. The goal of the study was to see how a delayed formalization approach affected students' achievement in sequences and series in senior secondary school. The research was conducted in the Obollo Education Zone in Nigeria's Enugu State.

A quasi-experimental approach was adopted, with 240 senior secondary school students (130 males and 110 females) serving as the participants. In addition, Ogbonna (2007) conducted another study in which female students outperformed their male counterparts. The research area was in Nigeria's Abia State. The goal of the study was to see how achievement and retention were affected when two constructivist teaching strategies were used in Number and Numeration. The sample size was 290 students, which was employed in a quasi-experimental approach. His findings revealed that students who were taught using the two constructivist instructional models (IEPT and TLC) performed better and retained more information than students who were taught using the traditional method.

According to Fox and Cohn (1980), when both male and female students took the Scholastic Aptitude exam in high school, male students performed higher. In addition, Smith and Walker (1988) discovered substantial sex-related disparities in favor of male students in tenth-grade geometry. Males outperformed girls in Mathematics during high school (Randhawa, 1994). Male pupils outperformed female students in mathematics, according to studies done by the following academics (Fennema, 2000; Kaiser-Messmer, 1994). Similarly, as noted by Asante (2010), research on standardized arithmetic examinations by (Fox, Brody, & Tobin, 1980; Hedges & Nowell, 1995; Fennema & Sherman, 1978; Randhawa, 1994) revealed that male pupils typically outperformed their female counterparts.

When VHGT was used to gather data, there was no statistical difference between male and female students in the acquisition of their geometric levels, according to Halat (2006). While using VHGT, Halat (2008) reported that there was no significant difference in geometric reasoning levels and mean between male and female students. In addition, when boys and girls in Senior High School form one was assessed on problem-solving ability, Arhim and Offoe (2015) found no gender differences. Armstrong (1981) shares his belief that there was no statistical difference in achievement between boys and girls in sixth grade when their skills were assessed in measuring, statistics, probability and geometry.

2.10 Geometric and Mathematics Performance in Public and Private Schools

Academic performance refers to the amount to which a person has met specified objectives that were the focus of activities in educational settings, such as elementary, college, and university (Steinmayr et al., 2014). There are very broad indicators of academic achievement, such as more curricular-based criteria, procedural and declarative knowledge acquired in an educational system such as grades or performance on an educational achievement test, and cumulative indicators of academic achievement, such as educational degrees and certificates, according to Steinmayr et al., (2014).

The quality of the learning process has been identified as an outcome of student academic success (World Bank, 2017). The degree of learning-based skills, experience, and knowledge is related to evaluating performance, just as the curriculum's learning objectives are linked to the level of performance (Levpuscek & Zupancic, 2009; Németh & Long, 2012). According to various academics, student academic achievement is influenced by their cognitive and noncognitive traits, as well as the socio-cultural environment in which they study (Lee & Stankov, 2016; Liem & Tan, 2018).

Therefore, student performance refers to the amount of competence demonstrated in academic work or conventionally acquired information in school topics, as measured by the proportion of marks received in examinations. As a result, the worth of academic achievement cannot be discounted everywhere in the globe since it is linked to social value and the means to a prosperous future (Indah et al., 2018). According to Borasi (1990), students' concepts, attitudes, and expectations about geometric and mathematical education are highly important variables underpinning their school experience and eventual success in the subject. Other research (e.g., Fraser & Kahle, 2007; Goh & Fraser, 1998) has indicated that geometric mathematics classroom learning settings have a significant impact on students' attitudes toward mathematics and studying mathematics.

Learners have difficulty learning Euclidean geometry, according to research, and numerous sources have been found. Geometry is one of the most difficult subjects in mathematics for students, according to Harris and Bourne (2017). Geometry, according to Adulyasa and Rahman (2014), is one of the most difficult disciplines in mathematics for both public and private institutions.

It is a discipline of human study concerned with the study of form and space and their interactions, particularly their generalizations and abstractions, as well as their application to real-world circumstances (Nicholson, 2014). Despite its relevance and importance, many students still struggle with geometry (Govender, 2017). Das, Das, and Kashyap (2016) found that instructors and parents are failing to motivate students to acquire self-confidence in geometry. For example, while parents shifted responsibility on motivating students to schools, there was no good communication between educators and students in geometry classrooms (Nenthien & Loima, 2016), resulting in low learner performance in geometry.

For example, various research and authority (Makeleni & Sethusha, 2014; Sa'ad, Adamu, & Sadiq, 2014) showed many elements impacting secondary school geometry learner performance. These factors include a lack of important school environmental elements, a paucity of well-trained educators, instructor quality, staff ratio, frequent educator transfers, home environmental factors and family backgrounds, and learners' automatic promotions and learner-related factors. In contrast, Mega, Ronconi, and De Beni (2014) asserted that, learners believe geometry is one of the most difficult areas of mathematics, and as a result, majority of them avoid pursuing mathematics courses because of the geometry sections.

2.11 Public and Private Schools

In developing countries, where the need for a great education system is growing every day, comparing public and private schools is always a hot issue. Comparing public and private schools is always a major topic in developing countries, where the need for a quality education system is growing every day (Caddell, 2007). Public schools are those that are sponsored or managed wholly or partially by the government (Thapa, 2011). Individuals and organizations such as trusts, private enterprises, charitable groups, and missionaries, on the other hand, run and fund private schools (Thapa, 2011). Others, such as missionaries, provide a social service and charge a nominal fee.

To Mathema (2013), public school is simply a place of refuge for low-income pupils who have been disbanded from public to private schools. As a result, within the same national education system, two separate education systems have formed, notably private for the wealthy and public for the poor, posing a threat to societal cohesiveness (Mathema, 2013). Public's confidence in public schools has eroded as a result of their poor quality, which has fueled expansion in private education, according to Carney and Bista (2009), Mathema (2007). According to the survey, private school pupils generally outperform public school students academically (Caddell, 2007; Mathema, 2007; Sharma, 2012; Thapa, 2012).

Teaching at a private school offers several advantages over teaching in a public school, according to Kennedy (2011), because the former's administrative systems have less bureaucracy. He explained that when you sign a contract to attend a private school, the regulations of the school are clearly spelled forth, and by signing the contract, you agree to comply by the provisions of the contract, which include repercussions for infractions of the disciplinary code. One has rights (Constitutional rights) that must be observed at public schools, which are characterized by a lot of bureaucratic tendencies.

For Otieno (2010), most public schools" children are disadvantaged since classes are overcrowded and learning facilities are inadequate. As a result, their teachers are unable to provide them with personalized attention. Students may lose hope in achieving well in academic work if they lack suitable textbooks and laboratory equipment. This is in sharp contrast to private schools, which have fewer pupils but better facilities and teachers who are prepared to go above and beyond to ensure that students perform well in both internal and external examinations.

2.12 Empirical Review

Anas (2018) conducted research in five Ghanaian mathematics/science colleges of education. The study's purpose was to look into van Hiele levels of geometric thinking among Mathematics Pre-service Teachers in Ghana's three northern regions. There were 412 Mathematics Pre-service Teachers in attendance. A total of 298 Mathematics Pre-service Teachers were employed as the study's sample. Ghana's Upper West, Upper East, and Northern areas provided the sample. The full sample distribution was fifty girls and 248 boys. Purposeful and random sampling procedures were used.

The Van Hiele Geometry Test was administered to Mathematics Pre-service Teachers during their second year second semester. Results from his study indicated that majority of 50.3% reached van Hiele Level 0. Another, 23.5% reached van Hiele Level 1. Also, 14.8% and 9.1% reached van Hiele Levels 2 and 3 respectively. Finally, 2.34% and 0 % attained VHL 4 and 5 respectively. Only 11.44% of those interviewed were qualified to teach in Ghana's primary schools, according to his findings. Colleges of Education should be encouraged to implement van Hiele phase-based Geometry instruction, according to Anas (2018).

Also, based on van Hiele geometry exam levels, Armah et al., (2017) found that 75.33% of pre-service instructors were lower than their projected future JHS 3 pupils. Their research focused on pre-service teachers' van Hiele geometric thinking in Ghana before they began their teaching careers. A total of 300 second-year students from four Ghanaian colleges from three (3) regions took part in the study. According to the findings, 16.33% of pre-service teachers were at level 0 according to van Hiele levels. In addition, 27%, 32%, and 17.67% of pre-service teachers, respectively, achieved levels 1, 2, and 3. Further investigation indicated that just 6% and 1% of pre-service teachers, respectively, attained level 4 and 5. Furthermore, Asemani et al., (2017) used a quantitative study technique on Ghana's Senior High School final year students to test Van Hiele degree of geometric thinking. For the study, 200 people were chosen at random from three (3) municipalities in the Central Region.

Males made up 44% of the sample, while females made up 56%. In the study, the lowest age was 14 years old and the highest age was 20 years old. The investigation revealed that 42.5% of the pupils failed to fulfill any of the van Hiele Geometric thinking levels. Furthermore, van Hiele level 1 was achieved by 33% of final year students, while level 2 was achieved by 22.5%. For stages 3 and 4, the rests are 1.5% and 0.5%, respectively. According to the van Hiele Geometric thinking publishing conclusion, roughly 43% of Ghana's final year Secondary School pupils did not achieve any van Hiele Geometric thinking level.

A total of 188 students from Winneba Senior High School and Zion Girls Schools were employed in Baffoe and Mereku's (2010) study. Both schools were from the city of Winneba. The goal of the study was to assess Ghanaian SHS 1 students' van Hiele levels of geometric reasoning. The study began as soon as the students arrived at SHS and continued for four weeks on campus. When the examination of van Hiele geometric thinking was done, the study found that Ghana's SHS 1 student performed worse than their counterparts from other countries. According to the findings, 59%, 11%, and 1% of Ghana's SHS 1 student achieved van Hiele levels 1, 2, and 3 correspondingly.

Luneta (2014) studied the content understanding of fundamental geometry that 128 South African student teachers (foundation phase) had. The framework of analysis was the van Hiele theory of geometric cognition. The results demonstrate that the majority of the research participants were working at level 1 (identification of forms by appearance), which is the same level as the class they were about to teach. This level is claimed to be lower than that required of Grade 12 students, who should be able to operate at levels 3 (awareness of the relevance of form attributes and connections) and 4 (students develop and evaluate definitions, axioms, and theorems) by the end of their studies.

Robichaux-Davis and Guarino (2016) evaluated primary pre-service teachers' knowledge in teaching geometry in their study. The van Hiele Levels of geometric thinking were used to evaluate three domains: (a) geometry content knowledge, (b) geometry pedagogical content knowledge, which includes knowledge of appropriate geometric materials and manipulatives and (c) spatial visualization skills.

The first three levels, visualization, analysis, and informal deduction, were addressed in the questions. The study's findings revealed that the teachers had a serious lack of understanding of geometry because they only taught at the first and second levels. It was alleged that these teachers lacked the necessary abilities to teach students in the grades they were assigned. Even though these teachers performed better than those in Luneta's research, they were still unable to achieve the curriculum's demands.

Students who were taught using van Hiele phase-based instruction (characterized by discussion, group work, hands-on investigations, and collaborative learning) managed to transit to higher levels of geometric thinking than their counterparts who were taught using conventional or traditional methods, according to Connolly (2010), Abdullah and Zakaria (2013), Abu and Abidin (2013), Alex and Mammen (2016), and Armah et al., (2018) (where students follow the instructions the teacher gives with no hands-on activities). Students were exposed to education that was aligned with all phases of learning before going on to the next level.

These results indicate that van Hiele phase-based instruction has a positive influence on students' geometric thinking and, as a result, has a greater potential for enhancing geometry teaching in schools than the traditional approach. Atebe and Schäfer (2011) examined geometry teaching in South African and Nigerian high schools, using a sample of three Nigerian mathematics instructors (T1, T2, and T3) and three South African mathematics teachers (T4, T5, and T6).

The six schools' instructional activities were filmed. The framework for analysis was the van Hiele levels of learning descriptors. The study's findings demonstrate that South African instructors offered students with more van Hiele phase-aligned learning opportunities than their Nigerian counterparts. Information (T1, T2 & T3), explication (T1, T2 & T3), and integration (T1, T2 & T3) were some of the missing stages detected in Nigerian high schools (T1 & T3). The missing phases in South African schools were information (T6), explication (T6), and integration (T6) (T4, T5 & T6).

It is possible to conclude from these data that South African students have a higher conceptual understanding of geometric concepts than Nigerian students.

Hassan (2015) conducted research to determine the degree of geometric thinking among students in the Mathematics Department at the University of Baghdad's Faculty of Education. To meet the research's goal, the Harby 2003 standard was applied for the Saudi context, which consists of 25 paragraphs, and the study sample consisted of 206 students from the mathematics department who were randomly selected. The study's first level (visual) was achieved by 84.5%, while the pupils did not go beyond the fourth level, and there were no statistically significant variations in the levels of geometric thinking by gender or school year.

Ibrahim (2014) conducted a study to investigate the change in van Hiele levels of Geometric thinking in students of teachers in the classroom (open education) in the Faculty of Education at the University of Damascus after their study of Geometric concepts and methods of teaching, as well as the relationship between their achievements in school. The study sample included 101 male and female fourth-year students. The findings of the study revealed that after studying the ideas and teaching methods, van Hiele levels of Geometric thinking in the pupils of the classroom teacher in (Open education) improved. The study's findings revealed that there was a high positive correlation between the students' degrees on van Hiele's experience of Geometric thinking and their degrees on the accomplishment exam in Geometry at the level of (0.01).

Abu Musa and Nimrawi (2014) did a study to determine the levels of Geometric reasoning among Mathematics students in the conical sections. For four years, a test was developed to examine the four levels of geometric reasoning proposed by van

Hiele. The study sample consisted of 203 students from the Mathematics Department at Zaytoonah University in Jordan. According to the findings, there were statistically significant disparities in student performance depending on the school year level. Ibrahim and Nansour (2015) sought to determine the distribution of van Hiele levels of Geometric thinking among eighth-grade pupils in their research. The sample included 400 eighth-grade students (male and female) from public schools in the governorate of Lattakia.

The van Hiele test for geometric thinking was employed by the researcher, and the results revealed that there was no statistically significant difference in mean scores between males and females in van Hiele's Geometric thinking. Al-Qurashi (2016) conducted research to assess the degree of geometric thinking among Umm Al-Qura University Mathematics students. The study sample consisted of 191 students who were subjected to Geometric thinking tests based on the van Hiele model. The findings revealed that students have a low level of Geometric thinking, with about 40% of students falling into the second level of Geometric thinking, which is the analytical level.

The study sample consisted of 148 teachers (49 males and 61 females) randomly selected from schools in Antalya, Turkey, to investigate the levels of Geometric thinking of middle and high school teachers during service. Teachers have all degrees of van Heile for Geometric thinking, according to the study's findings. There were no statistically significant differences in regard to stage or gender, according to the findings. Khasawneh performed research in 2007 to look at the levels of thinking in the field of space geometry among Jordanian tenth grade students.

The research included 310 students who answered questions on van Hiele's first four stages of thought. The study's findings revealed that (71, 94%) of the pupils fit into one of three categories. Non-formal reasoning, formal reasoning, analytical, or visual reasoning were the four levels in descending order, and 19.03% of students were categorized below the first level (cognitive), while 9.03% of students were not classified within any of the four levels.

Abssi (2016) conducted research to determine the influence of geometric thinking levels training for 7th grade mathematics instructors on their students' success in geometry, the growth of their geometry thinking levels, and their attitudes toward geometry. This study enlists the participation of 64 seventh-grade pupils, who are evenly divided into experimental and control groups. To meet the study's objective, the researchers formulated a training programmed on geometric thinking levels based on van Hiele's model, as well as an achievement exam, a test in geometric thinking, and a measure of students' attitudes toward geometry. The data analysis revealed statistically significant differences in attitudes of students toward geometry for the two experimental and control groups in favor of the experimental group, as well as statistically significant differences in attitudes of students toward geometry for the two experimental and control groups in favor of the experimental group.

Ding and Jones (2018) utilized class observations, interviews with teachers and students, and analysis of students' tests and homework to screen the geometric education in Shanghai schools in China, education tactics adopted by instructors, and thinking levels employed by the 8th grade students. The findings revealed that teachers follow a consistent instructional paradigm (introduction, review, new information, exercises, summary, and homework), and that students' geometric thinking skills ranged from 1 to 4 on the van Hiele scale. According to the van Hiele model, Al-Harbi (2015) performed a study to determine the geometric thinking levels of Faculty of Education students who were "pre-service mathematics teachers."

According to van Hiele standards, the researcher devised a test for geometric reasoning. The exam was conducted on a group of 120 male and female students from Sana'a University's Faculty of Education. According to the study's findings, 27.5% of the sample persons were grouped into one level, and 28.3% of students were in the 1st level. Salem (2014) investigated van Hiele geometric thinking requirements for pupils in Jerash Governorate's upper level of basic education. The study's sample consisted of 532 male and female students who took a geometric thinking level test. The findings of the study revealed the lack of thinking levels, with the sample persons falling into the second level of van Hiele standards, and no statistically significant variations between male and female geometric thinking levels.

Al-Harbi (2015) performed research in the governorate of Al-Qurayat to determine the impact of the van Hiele model on geometric thinking levels in middle-class pupils. The study was conducted on the geometry and spatial inference unit, and the findings revealed that there was a difference between the experimental and control groups on both the pictorial, analytical, and quasi-inference levels, with the experimental group outperforming the control group. Kilani (2013) performed research to see how the van Hiele model affected the development of geometric thinking and self-confidence among pupils in Damascus governorate's Fifth Scientific Governorate.

The study's findings revealed a difference in the arithmetic means of the two experimental and control groups in each geometry question, as well as a bias in favor of the experimental group in terms of self-confidence. The study's findings revealed that the experimental has an influence on developing geometry question competencies in order to get pupils to the level of abstract inference in stereoscopic geometry subjects.

Research by Yenilmez and Korkmaz (2013) looked at the link between geometric self-efficiency and geometric thinking. The study used the van Hiele scale in geometry thinking, in addition to analyzing differences in self-efficacy in geometry based on variables such as gender, grade, and educational achievement as well as a self-efficacy test, and the findings revealed a weak link between self-efficacy and geometric thinking, as the findings revealed variations in self-efficacy based on achievement, gender and class.

Skrbec and Cadez (2015) conducted a study in Slovenia to determine the levels of geometric thinking among primary school students; the study's findings revealed that 4% of students were at the zero level, 61% between the zero and first level, 32% at the first level, and only 1% at the second level.

The study's findings also revealed that pupils' preference for geometry language is minimal. Behçet and İlhan (2012) investigated the levels of geometric thinking among secondary and primary teachers before service, as well as their relationship to some variables. The study's findings revealed that the teachers did not reach the required level, that there are no differences in the level of geometric thinking between teachers at the secondary and primary levels, and that there are no differences in the level of geometric thinking related to gender or the type of secondary school certificate.

According to the findings of a study conducted by Cacmac and Kubra (2014) to investigate the levels of geometric thinking of pre-service teachers in Turkey, teachers are mostly in the third level, and there are no differences in the level of geometric thinking due to gender, age, or the overall average cumulative rate. Fidan and Turnuklu (2010) tested pupils in fifth grade. The study's findings revealed that nearly half of the pupils did not achieve the first level, while only 28% did, demonstrating a poor level of geometric thinking among primary school students. The findings also revealed disparities based on gender and parental education level. To crown it all, in the field of professional education, Yilmaz and Koparan (2016) conducted a study to determine the impact of designing geometry lessons on the development of geometric thinking among pre-service teachers in Turkey, and the findings revealed that geometry lessons have an impact on raising the level of geometric thinking among teachers.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

Merriam-Webster (2013) defines methodology as a set of methods in the fields of science or art with regard to rules, ideas or procedures. Merriam-Webster (2013) further stated that in research, methodology refers to the processes and procedures through which researchers discover knowledge systematically. This chapter deals with methodological steps such as research paradigm, research approach, research design, Sampling, population and sample size, data analysis and ethical considerations.

3.1 Research Paradigm

The study operated in the positivist‟s paradigm. Positivist paradigm relies on deductive logic, formulation of hypothesis, testing of those hypotheses, offering operational definitions and mathematical equations, calculations, extrapolations and expressions to arrive at conclusions (Kivunja & Kuyini, 2017). Positivists use quantitative research methods to gather data, analyze the data, interpret the data and establish relationships among the variables. The study employed this paradigm since quantitative data was collected, analyze, interpret and made the necessary deductions to establish the relationship between the independent variables (gender and school type) and the dependent variable (van Hiele‟s geometric levels).

3.2 Research Approach

This study employed the quantitative approach for data collection. Tavakol and Saunders (2015) noted that, quantitative studies are involved in exploring how and why phenomena vary. Mathematical models and statistics are used for quantitative studies to analyse, providing more objective numerical results. McMillan and

Schumacher (2016) have argued that in measuring and explaining a phenomenon, quantitative sample designs stress objectivity. As such, by using numbers, statistics, structure and control, the research design maximizes objectivity. The study explored and analyzed the issues with regards to assessing the van Hiele‟s Geometric levels of students in the Ga Mashie Circuit: Gender and School type differences.

3.3 Research Design

An appropriate framework for a study is research design. A very critical decision is the choice of research design in terms of how relevant information for a study can be obtained, however there are other relevant decision in the research design process (Jilcha, 2019). In this study, the descriptive research design was used. The descriptive research provides information on current phenomenon obtained from every specific valid general conclusion under discussion (Sekaran & Bougie, 2016). For Kothari (2017), the design provides adequate avenue for reliability and validity. Therefore, this research design enabled the researcher to gather data from a wide range of respondents on their van Hiele levels in public and private schools in the Ga-Mashie Circuit. And this helped in analyzing the response obtained on which school perform better and also whether there is a difference between males and females in their van Hiele levels.

3.4 Population of the Study

Generally, for the benefit of which the study is performed, populations are an extensive collection of individuals or objects that are the focus of the research (Bernard, 2017). Bernard (2017) indicates that a research population is a well-defined collection of persons or objects with similar characteristics and usually a common, binding trait or characteristic. For the purpose of this study, the targeted population was students in JHS 3 from both private and public basic schools at Ga-Mashie Circuit in the Accra metropolis. There were five hundred and sixty-two (562) JHS 3 students from both private and public schools in the Ga-Mashie Circuit of Accra metropolis.

3.5 Sample

Ga-Mashie Circuit is an area with four private schools and eight public schools; approximately there were 562 students from both private and public basic schools in JHS 3. A sample of three hundred and thirty-seven which represent sixty present of the population was sampled from both schools for the study. This was in line with the study of Taherdoost (2020), which state that the sample of every population should be large enough to represent the population of the study.

3.6 Sampling Procedures

The study used two types of sampling procedures which where purposive and stratified sampling**.** Purposive sampling is the deliberate selection of participants for their known attributes (Denscombe, 2009). Out of the eight public schools, six were purposively selected because two of the public schools where without Junior High School with form three classes (JHS3), therefore purposive sampling technique was necessary for the selection of the six public schools for the study. All the four private schools were selected since they all have Junior High Schools with form three classes (JHS3). Hence the study constitutes six public and four private schools in Ga-Mashie Circuit of the Accra Metropolis.

Stratified Sampling was used to classify all JHS 3 classes of the ten schools in the population as strata based on their location. According to Denscombe (2017), Stratified sampling subdivides the research population into different subgroups

(Strata) and then chooses the required number of people from within each subgroup using random sampling techniques.

The study also adopted a proportional representation for the selection of samples from each class (stratum) and this was done before choosing the sample randomly from each class. Proportional representation was very necessary in this study, since the schools in the Ga-Mashie Circuit of the Accra Metropolis do not have the same number of students in each JHS 3 class. There was the need to take a proportional representation of the JHS 3 classes in the Circuit depending on the number of students in each class.

With the concept of proportional representation, calculations were based on the total population of JHS3 students in public and private schools in the Ga-Mashie Circuit of the Accra Metropolis which was 562 and a sample of 337. That is the number of students selected from each JHS 3 class is proportional to the total number of JHS3 students in a particular school, hence the term proportional representation. Selecting students from each JHS 3 classes of the ten schools, the researcher took the total number of students in each class divided by the total population (562), multiplied by the sample (337) for the study.

$$\text{Sample from each Class} = \frac{\text{No. of students in Class}}{\text{Total Population}} \times \text{Sample (for the study)}$$

This calculation was done for all JHS 3 classes in the ten schools involved in the study (four private schools and six public schools). After which simple random sampling was then used to sample the required number from each of the JHS 3 classes. To avoid bias when choosing student as part of the sample, pieces of paper labelled Yes or No were put in a box. The number of „YES" written equals the sample size required from a class, the students were allowed to pick a piece of paper from the box. Those who picked papers written „Yes" were involved in the sample. This was done because in random sampling procedure, each member of the population in the group had an equal chance of being selected (Cohen, 2000).

Putting together the samples of all the JHS 3 classes from the ten schools constitute the sample of the study that is three hundred and thirty-seven (337). Out of the 337 students which form the sample of the study, sixty-one (61) are from private schools (32 males and 29 females) and Two Hundred and seventy-six (276) are from public schools (122 males and 154 females).

3.7 Research Instrument

Van Hiele Geometric Test used by Usiskin (1982) was adapted as the instrument for the study. This was done to match or reflect the geometry curricular used by students. The van Hiele Geometric Test was administered to JHS3 students of both private and public basic schools in Ga- Mashie. The van Hiele Geometric Test was made up of fifteen objective questions, five questions for each level and each of the questions had five options (A, B, C, D, E) of which one is correct under it. To achieve standard questions, each van Hiele geometric test was selected in reference to the topics in the teaching syllabus of the Ghana Education Service. The topics were also selected based on specific concepts in the teaching syllabus. The students were tested on the concepts of squares, triangles, rectangles, parallelograms among others. The marks from the van Hiele geometric test and the item analyses with regards to public and private basic schools were meant to answer the research questions appropriately.

3.8 Rubric for Scoring the Van Geometric Test

Two methods of grading were used to assign marks to the learners. The first grading method provided 1 point for each right response to the 15-item multiple-choice test. As a result, each student received a score ranging from 0 to 15. The percentage score was obtained for each student and an item analysis of each student's replies was performed using the Statistical Package for the Social Sciences (SPSS).

The second grading system approach was used to place students in Van Hiele Levels. This recommended, was based on Usiskin (1982) "3 of 5 accurate" success criterion (p. 33). According to this criterion, if a student successfully answers at least three out of five items in any of the three subsets of the van Hiele geometric test, the student was regarded to have mastered either level 1, 2, or 3. Using the scoring system created by Usiskin (1982), the learners were allocated weighted total points in the following manner: 1 point for completing van Hiele level 1 items 1-5; 2 points for completing van Hiele level 2 items 6-10, with 4 points for completing van Hiele level 3 items 11-15. The highest point total that each student was required to achieve was 1+2+4 =7 points. The used of the weighted sum made it easier for the researcher to match students van Hiele levels. That is, weighted score of 3 implies that the learner satisfied the level 1 and 2 requirements. A weighted score of 7 indicates that the leaner satisfied the criteria of levels 1, 2, and 3.

The second grading method was used to categorize learners into various van Hiele levels depending on their replies. Because the geometric thinking levels of junior high school students are projected to be optimum in the third level (Baffoe & Mereku, 2010), the top that the research group may achieve is the third level, and so, the first 15 questions were assessed.

3.9 Pilot Testing

Pilot-testing in research plays a key role in ensuring the research instrument assist in elimination of vague and incomprehensible items to ensure relevant data is collected for use in the research. Pilot testing of the research instruments was done to make the instrument reliable (Wachira, 2016). In this study, the van Hiele Geometric Test was piloted in a different school within the same geographical area with similar characteristics. The objectives of the piloted test were: to establish clarity, meaning and comprehensibility of each item in the tests, to validate the instruments by cross checking their validity and reliability and to gain basic administrative experience in conducting the research in preparation for the actual study.

3.10 Validity and Reliability

Reliability and validity are very important in research because the credibility of the research study depends on the reliability of data, methods of data collection and also on the validity of the findings (Cohen, 2000).

3.10.1 Validity of instrument

To establish validity of the instrument employed, the researcher conducted a pilot test prior to the actual data collection. The Van Hiele Geometric Test was also given to colleague teachers at the Ga Mashie Sub-Metro to ensure face validity. The instrument was also presented to the supervisor and other researchers in academia for further comments and improvement hence all necessary adjustments was made, this also help the researcher to ensure content validity of the achievement test. This was done to ensure that items which were found unsuitable were removed or replaced.

Finally, the test items were similar to the test items used by other researchers this was to ensure convergent validity. Convergent validity describes how well two measures represent a similar notion (Carlson & Herdman, 2010). According to Heale and Twycross (2015) it shows that an instrument is highly correlated with instruments measuring similar variables.

3.10.2 Reliability of Achievement Test

To ensure reliability of collected information, the test items, which were meant for the study, were administered to 25 students in a similar geographical area by the researcher. This was done twice to see if there are understanding and consistency in their responses (answers). This was in line with literature as Thatcher (2012) denoted that reliability is the extent to which an instrument will generate the same data after multiple applications. The tendency toward consistency found in repeated measurements is referred to as reliability (Dennick &Tavakol, 2011). The second test was administered 7 days after the first one was taken. The Peason r value of the two tests was 0.92 which denoted a very high relationship. Hence the test instrument was highly reliable.

3.11 Data Collection Procedure

The researcher wrote a letter to the Municipal Director of Education in charge of Accra Metropolis together with an introductory letter from the institution, for permission to carry out the research and the necessary information in the circuit. The approval letter from the Director was handed over to the heads of the schools for the pilot and actual study. The head teachers, and circuit supervisor in charge of Ga-Mashie were briefed about the purpose and the implications of the study. They were also assured that any information they gave were going to be kept confidential.

The students were informed through the head teachers of the various basic schools about the date, time and duration for the achievement test. Mathematics teachers in the various schools also assisted the researcher to invigilate the test. The Van Hiele Geometric Test was administered to students and the test lasted for twenty minutes. The test was meant to assess the van Hiele levels of both private and public basic school students in the circuit.

3.11 Data Analysis Procedures

Data analysis has been defined by Kothari (2017) as computational measures and the search for data group relationship patterns. The data collected was coded and analyzed using descriptive and inferential statistics. Descriptive statistics are a critical part of initial data analysis and provide the foundation for comparing variables with inferential statistical tests (Peace & Hsu, 2018). This involved presentation of statistical data in the form of frequency distribution tables and percentages. Percentages have a considerable advantage over more complex statistics because they are easy to interpret, this was used to analyze and discussed the research question one.

Inferential statistic was also used to compare the means of the data collected to answer the research question two, three and four. Each question (item) was also taken into consideration with regards to options chosen by students, with the weighted sum scoring method students were placed in their assigned van Hiele Levels. Statistical Package for Social Sciences (SPSS) was used to code the collected data.

The Statistical Package for Social Sciences (SPSS) is software for the rapid and effective application of statistical analysis used by students, teachers and researchers (Green & Salkind, 2014). The various tables constructed from the item analysis and the results obtained from the data analysis with the used of the (SPSS) were presented in chapter four of this study.

3.12 Ethical Procedures

Silverman (2013) is of the opinion that the researcher must recognize up-to-date standards required for research, and understand why it matters. Hence, the ethical issues addressed in this report also covered informed consent, secrecy and privacy. A letter was written to the Accra Metro Director for the necessary information and assistance, of which was used to carry out the pilot and actual study. All respondents were adequately educated on the purpose of the study. On the other hand, confidentiality was ensured in the sense that, the responses attained from the respondents was solely used for research purposes only.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.0 Overview

This study investigates students" van Hiele"s geometric thinking levels of public and private basic schools at Ga-Mashie in the Accra-Metropolis. The study was guided by four specific objectives and research questions. The first objective was to assess the stages of van Hiele"s levels JHS 3 students of private and public schools reach it the study of geometry. The second objective was to investigate whether there is a significant difference in terms of van Hiele"s geometric thinking levels between males and females. The third was to investigate whether there is a significant difference in terms of van Hiele"s geometric thinking levels between male and females in the school type. The fourth objective was meant to compare the difference in students" van Hiele"s levels between public and private schools. The study took place at Ga-Mashie, a town in Accra Metropolis. In line with the research objectives, the following research questions were formulated:

i. Which stages of van Hiele"s levels of understanding do JHS 3 students of public and private schools in Ga-Mashie Circuit reach in the study of geometry?

ii. Is there a difference in terms of van Hiele"s geometric thinking levels between males and females in the Ga-Mashie Circuit?

iii. Is there a significant difference in terms of van Hiele"s geometric thinking levels between male and females in the school type in the Ga-Mashie Circuit.

iv. Are there any differences in students van Hiele"s geometric levels between public and private schools in the Ga-Mashie Circuit?

To ascertain the degree of agreement or disagreement between the current study and the body of existing literature on the van Hiele‟s levels of private and public schools, the presentation of data and findings on the key themes was set against discussions of the existing literature.

4.1 Research Question 1

Which stages of van Hiele levels of understanding do JHS 3 students (of public and private schools) in Ga-Mashie Circuit reach in the study of geometry?

Table 4.1: Descriptive Statistics on the Total Score of the JHS 3 students

	N	Minimum	Maximum	Mean	Std. Deviation
Score of Pupils	337	0	9	4.46	1.566

Source: Field Data (2022)

According to Table 4.1, there were 337 students who took the Van Hiele's Geometric Test (VHGT). The least and maximum marks were 0 and 9, respectively. The student's score has a mean and a standard deviation of 4.46 and 1.566, respectively.

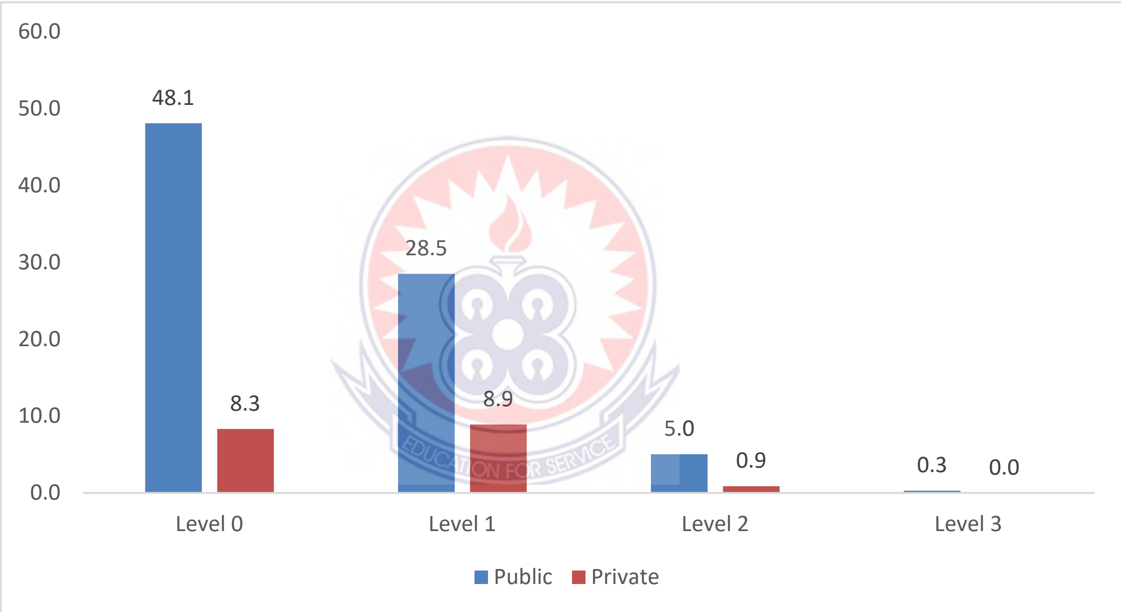
Table 4.2: JHS 3 students' Performance on VHGT level Attainment

	Level 0 N (%)	Visualization Level 1 N (%)	Analysis Level 2 N (%)	Order Level 3 N (%)
Public	162 (48.10)	96 (28.50)	17 (5.00)	1 (0.30)
Private	28 (8.30)	30 (8.90)	3 (0.90)	0 (0.00)
Total	**190 (56.40)**	**126 (37.40)**	**20 (5.90)**	**1 (0.30)**

Source: Field Data (2022)

Table 4.2 shows the overall attainment level reached by the JHS 3 students. The table shows that 190 (56.40%) of students attained No level, 126(37.40%) attained level 1, 20 (5.90%) attained level 2 and only 1 (0.30%) attained level 3 which should have been the level for all the students in JHS 3. Result from table 4.2 reveals that both private and public school students' overall performance fell far short of level 3 expectations for junior high school students. Therefore, according to Adulyasa and Rahman (2014), Geometry is one of the most difficult disciplines in mathematics for both public and private institutions.

In line with van Hiele Geometry Theory, Mason (1998) stated that, learners begin by studying and naming geometric qualities, but they do not comprehend the interrelationships between different types of figures, nor do they completely comprehend or appreciate the use of definitions at level 2.



Source: Field Data (2022)

Figure 4.1: Percentages of students from Private and Public Schools reaching the Three van Hiele's Levels

The figure shows that only 0.3% of public school students reached level 3 while none of the private school pupils reached this level. Comparatively the best level that both public and private schools could reach was level 1 (i.e., Visualization level) where they recorded 28.5% and 8.9% respectively. With regards to van Hiele Geometry

Theory, Vojkuvkova, (2012) was of the view that, learners employ visual perception and nonverbal reasoning at level 1 and they recognize figures only on the basis of their appearance, comparing them to prototypes or daily objects ("that looks like a door"), and categorizing them ("it is / it is not a...").

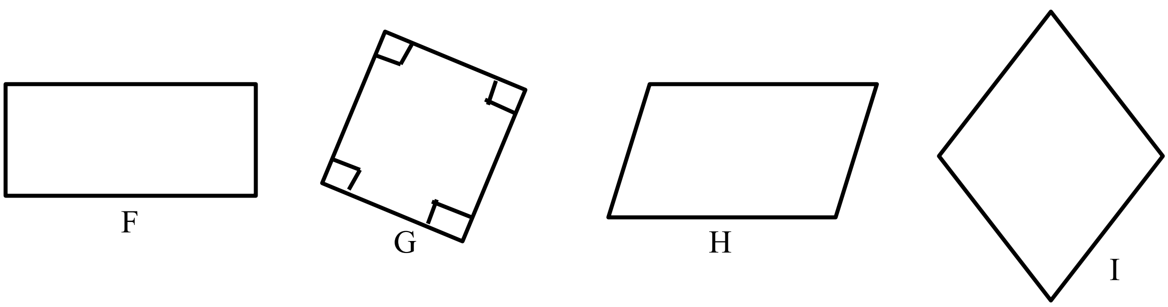
Table 4.3: Students' Subtest 1 Performance on each item in the VHGT Level1

Item	A N(%)	B N(%)	C N(%)	D N(%)	E N(%)	Blank N(%)
1	10(2.97)	7(2.08)	**308(91.39)**	11(3.26)	1(0.30)	0(0.00)
2	7(2.08)	2(0.59)	136(40.36)	**187(55.49)**	5(1.48)	0(0.00)
3	7(2.08)	**173(51.34)**	29(8.61)	123(36.50)	4(1.19)	1(0.30)
4	83(24.63)	**34(10.09)**	28(8.31)	183(54.30)	9(2.67)	0(0.00)
5	109(32.34)	52(15.43)	97(28.78)	13(3.86)	**66(19.58)**	0(0.00)

Source: Field Data (2022)

In level 1 of the VHGT, the performance of the JHS students was above average. In questions 1, 2, and 3 their performance was 91.39%, 55.49%, and 51.34% respectively. Whilst their performance in questions 4 and 5 was below average thus 10.09% and 19.58% respectively. Below is the question 4 from VHGT level 1.

Which of these is / are squares?



A. None of these are squares

B. G only

C. F and G only

D. G and I only

E. All are squares

The students were not able to answer the question correctly because they were not able to relate the interior angles of a square to the squares. Out of the total of 337 students, 183 representing 54.30% selected option D which says G and I only are squares. It means that the students lack the relation of the interior angles of a square. This was the reason why Crowley (1987) proposed that, students be given the opportunity to learn themselves how to solve issues in order for them to see the relationships between solid properties more clearly.

Below is the question 5 from VHGT level 1

Which of these is / are parallelograms?

A. J only

B. L only

C. J and M only

D. None of these are parallelograms

E. All are parallelograms

The students were unable to correctly answer this question because they were unaware that all quadrilaterals, except the trapezoid, are parallelograms. The majority of students chose J and L only, which could be due to the misconception that parallelograms are rectangles rather than squares and this could be due to teacher's influence. In this regard, Prahmana, Kusumah, and Darhim (2017) found that, an educator's influence in the learning process might affect learners' geometric problem-solving skills, independence, and curiosity in good or bad ways.

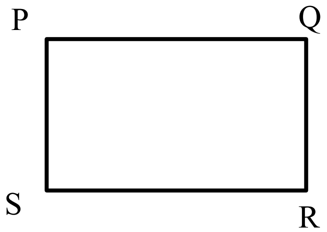
Table 4.4: Students' Subtest 2 Performances on each item in the VHGT Level 2

	A	B	C	D	E	Blank
Item	**N (%)**	**N (%)**	**N (%)**	**N (%)**	**N (%)**	**N (%)**
6	80(23.74)	**37(10.98)**	145(43.03)	71(21.07)	2(0.59)	2(0.59)
7	82(24.33)	28(8.31)	74(21.96)	49(14.54)	**100(29.67)**	4(1.19)
8	**62(18.40)**	69(20.47)	70(20.77)	80(23.74)	50(14.84)	6(1.78)
9	75(22.26)	39(11.57)	**172(51.04)**	36(10.68)	13(3.86)	2(0.59)
10	104(30.86)	55(16.32)	62(18.40)	71(21.07)	**44(13.06)**	1(0.30)

Source: Field Data (2022)

Aside the question 9 that 172 students representing 51.04% had it correct by choosing the correct option B, the students" performance was poor in the second level; that is 10.98%, 29.67%, 18.40% and 13.06% in questions 6, 7, 8 and 10 respectively. Also, for question in VHGT level 2 below;

Which relationship is true in all squares?



A. PR and PS have the same length.

B. QS and PR are perpendicular

C. PS and QR are perpendicular

D. PS and QS have the same length

E. Angle Q is larger than angle R

Students do not know the characteristics of the square. They did not also understand the meaning of perpendicular since 43.03% chose C which states that PS and QR are perpendicular. Here, according to Alzhanova-Ericsson et al., (2017), learners' bad performance in geometry can be impacted by a lack of practice, teaching methods, and teaching facilities such as a game, computer, or instructional material that an educator uses.

Table 4.5: Students' Subtest 3 Performances on each item in the VHGT Level 3

	A	**B**	**C**	**D**	**E**	**Blank**
Item	**N (%)**	**N (%)**	**N (%)**	**N (%)**	**N (%)**	**N (%)**
11	68(20.18)	**94(27.89)**	54(16.02)	86(25.52)	33(9.79)	2(0.59)
12	**82(24.33)**	106(31.45)	70(20.77)	47(13.95)	29(8.61)	3(0.89)
13	**34(10.09)**	51(15.13)	97(28.78)	83(24.63)	69(20.47)	3(0.89)
14	**6(1.78)**	65(19.29)	43(12.76)	25(7.42)	196(58.16)	2(0.59)
15	55(16.32)	49(14.54)	101(29.97)	**103(30.56)**	25(7.42)	4(1.19)

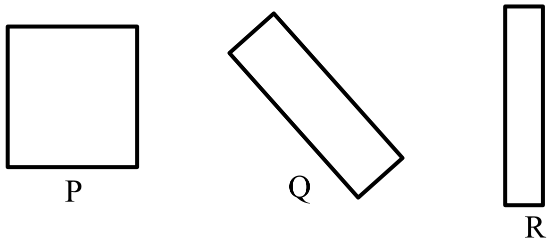
Source: Field Data (2022)

The third level of the VHGT was questions in relationship between different shapes. It determines whether students can recognize the relationships between forms. They identify students who respond correctly to questions in this group and have proven that they have knowledge of axioms however students‟ performance at this level was poor. The performance in the 3rd level was not encouraging at all. The number of students who chose the correct answer for each question in the Level 3 of the VHGT where all below average. With a record of 27.89%, 24.33%, 10.09%, 1.78% and 30.56% respectively in questions 11, 12, 13, 14 and 15.

It could be said in general that student‟s knowledge on the concepts of the properties of similar shapes triangles and various quadrilaterals was very low as reflected in their performances. This could clearly be seen in the responses as exhibited on table 4.5

question 14 with 58.16% of the students choosing Q and R only as the rectangles among the three shapes. In relation to the above findings, Bansilal, Brijlall and Mkhwanazi (2014) pointed out that, when preparing lessons, educators must be able to pick appropriate examples and exercises, as well as sequence the material of the lesson and choose a technique for teaching the relevant procedures.

Which of these can be called rectangles?



A. All the above are rectangles

B. Q only

C. R only

D. P and Q only

E. Q and R only

4.2 Research Question 2

Is there significant difference in terms of van Hiele's geometric thinking levels between males and females at Ga-Mashie Circuit?

Below are the analysis and findings done to investigate whether there is significant difference in terms of the van Hiele‟s geometric thinking levels between the males and females at Ga-Mashie Circuit.

Table 4.6: JHS 3 students Performance on VHGT Level Attainment

Gender Category	N	Mean	Std. Deviation	Sig. (2 tail)
Males	154	0.54	0.648	0.355
Females	183	0.48	0.610	0.358

Source: Field Data (2022)

From table 4.6 the mean score for male pupils was 0.54 with a standard deviation of 0.648 as against 0.48 and 0.610 as mean and standard deviation respectively score for female students.

Table 4.7 Shows Levene's Test for Equality of Variance

		Levene's Test for Equality of Variance				
		F	Sig	T	Df	Sig(2-tailed
Result of Test	Equal Variances Assumed	1.675	0.197	0.926	335	0.355
	Equal Variances not Assumed			0.921	317.629	0.358

The table 4.7 with p (0.197) > 0.05 shows no significant differences in their variance. It further indicates that, there is no statistically significant difference between males and females student in their geometric thinking levels with $t(335) = 0.926$, $p\ (0.355) > 0.05$. This finding is consistent with the work of Ibrahim and Nansour (2015) study which sought to determine the distribution of van Hiele levels of Geometric thinking among eighth-grade pupils in their research. The sample included 400 eighth-grade students (male and female) from public schools in the governorate of Lattakia. The Van Hiele test for geometric thinking was employed by the researcher, and the results revealed that there was no statistically significant difference in mean scores between males and females in van Hiele's Geometric thinking.

In this context, Salem (2014) investigated van Hiele geometric thinking requirements for pupils in Jerash Governorate's upper level of basic education. The study's sample consisted of 532 male and female students who took a geometric thinking level test. The findings of the study revealed the lack of thinking levels, with the sample persons falling into the second level of van Hiele standards, and no statistically significant variations between male and female geometric thinking levels. Halat (2008) also reported that there was no significant difference in geometric reasoning levels between male and female students. However, Hamzah's (2017) study found a difference in favour of females in engineering thinking among a sample of Jordanian class teacher students. Sudihartinih and Wahyudin's (2019) study also found that there are differences between males and females in favour of females.

4.3 Research Question 3

Is there is a significant difference in terms of van Hiele's geometric thinking levels between male and females in the school type in the Ga-Mashie Circuit?

Beneath are the analysis and findings done to investigate whether there is significant difference in terms of the van Hiele‟s geometric thinking levels between the males and females in the school type at Ga-Mashie Circuit.

Table 4.8: JHS 3 students Performance on VHGT Level Attainment

Gender Category	N	Mean	Std. Deviation	F	T	Df	Sig. (2 tail)
Males in Public School	122	0.49	0.646	0.527	-1.775	152	0.469
Males in Private School	32	0.72	0.634				

Source: Field Data (2022)

From table 4.8 the mean score of the students in public was 0.49 with a standard deviation of 0.646 as against 0.72 and 0.634 as mean and standard deviation respectively score by students in the private. The table 4.8 further indicates that, there is no statistically significant difference between the males in public and males in private schools in their geometric thinking levels $t(152) = -1.775,$

$p\ (0.469) > 0.05.$

Table 4.9: JHS 3 students Performance on VHGT Level 1 Attainment

Gender Category	N	Mean	Std. Deviation	F	T	Df	Sig. (2 tail)
Males in Public School	122	0.41	0.494	0.606	-2.196	152	0.437
Males in Private School	32	0.63	0.492				

Source: Field Data (2022)

Further analysis was also run on the performance of the pupils in the VHGT sub levels. From table 4.9 the mean score of the students in public was 0.41 with a standard deviation of 0.494 as against 0.63 and 0.492 as mean and standard deviation respectively score by students in the private in VHGT level 1. The table 4.9 further indicates that, there is no statistically significant difference between the males in public and males in private schools in their geometric thinking level 1 with

$t(152) = -2.196, p\ (0.437) > 0.05.$

Table 4.10: JHS 3 students Performance on VHGT Level 2 Attainment

Gender Category	N	Mean	Std. Deviation	F	T	Df	Sig. (2 tail)
Males in Public School	122	0.08	0.275	0.177	-0.212	152	0.675
Males in Private School	32	0.09	0.296				

Source: Field Data (2022)

Also, table 4.10 indicate that the mean score and standard deviation of the males‟ student in public school respectively was 0.08 and 0.275 whilst the males‟ from the private schools has a mean score of 0.09 with a standard deviation of 0.296 in VHGT level 2. The table 4.10 further specifies that, there is no statistically significant difference between males in public and males in private schools in their geometric thinking level 2 with $t(152) = -0.212, p\ (0.675) > 0.05$.

Table 4.11: JHS 3 students Performance on VHGT Level Attainment

Gender Category	N	Mean	Std. Deviation	F	T	Df	Sig. (2 tail)
Females in Public School	154	0.49	0.629	1.661	0.261	181	0.199
Females in Private School	29	0.45	0.506				

Source: Field Data (2022)

From table 4.11 the mean score of the students in public was 0.49 with a standard deviation of 0.629 as against 0.45 and 0.509 as mean and standard deviation respectively score by students in the private. The table 4.11 further points out that, there is no statistically significant difference between females in public and females in private schools in their geometric thinking levels with $t(181)= 0.261, p\ (0.199) > 0.05$.

Table 4.12: JHS 3 students Performance on VHGT Level 1 Attainment

Gender Category	N	Mean	Std. Deviation	F	T	df	Sig. (2 tail)
Females in Public School	154	0.42	0.494	0.306	-0.325	181	0.581
Females in Private School	29	0.45	0.506				

Source: Field Data (2022)

Further analysis was also run on the performance of the students in the VHGT sub levels. From table 4.12 the mean score of the students in public was 0.42 with a standard deviation of 0.494 as against 0.45 and 0.506 as mean and standard deviation respectively score by students in the private in VHGT level 1. The table 4.12 further illustrates that, there is no statistically significant difference between females in public and females in private schools in their geometric thinking level 1 with

$t(181)= -0.325, p\ (0.581) > 0.05$.

Table 4.13: JHS 3 students Performance on VHGT Level 2 Attainment

Gender Category	N	Mean	Std. Deviation	F	T	Df	Sig. (2 tail)
Females in Public School	154	0.05	0.223	7.037	1.254	181	0.009
Females in Private School	29	0.00	0.00				

Source: Field Data (2022)

Also, table 4.13 indicate that the mean score and standard deviation of the female students in public school respectively was 0.05 and 0.223 whilst the females from the private schools has a mean score of 0.00 with a standard deviation of 0.00 in VHGT level 2. The table 4.13 further displays that, there is statistically significant difference between females in public and females in private schools in their geometric thinking level 2 with $t(181)= 1.254, p\ (0.009) < 0.05$.

Table 4.14: JHS 3 students Performance on VHGT Level 3 Attainment

Gender Category	N	Mean	Std. Deviation	F	T	Df	Sig. (2 tail)
Females in Public School	154	0.01	0.081	0.760	0.433	181	0.385
Females in Private School	29	0.00	0.00				

Source: Field Data (2022)

Also, table 4.14 indicate that the mean score and standard deviation of the male students in public school respectively was 0.01 and 0.081 whilst the males from the private schools has a mean score of 0.00 with a standard deviation of 0.00 in VHGT level 3. The table 4.14 further demonstrates that, there is no statistically significant difference between females in public and females in private schools in their geometric thinking level 3 with $t(181) = 0.433, p\ (0.385) > 0.05$.

4.4 Research Question 4

Is there significant difference in students van Hiele's geometric levels between public and private schools in the Ga-Mashie Circuit?

The tables below shows the analysis and findings done to investigate whether there is significant difference in terms of the van Hiele"s geometric thinking levels between public and private schools in the Ga-Mashie Circuit in general and at the sub levels.

Table 4.15: JHS 3 students' Performance on VGHT Level

School Type	N	Mean	Std. Deviation	Sig. (2 tail)
Public School	276	0.49	0.635	0.239
Private School	61	0.59	0.588	0.218

Source: Field Data (2022)

Table 4.15 shows a mean score of 0.49 for public school students and 0.59 for private school students with a respective standard deviation of 0.635 and 0.588.

Table 4.16 Shows Levene's Test for Equality of Variance

		Levene"s Test for Equality of Variance				
		F	Sig	T	Df	Sig(2-tailed
Result of Test	Equal Variances Assumed	0.550	0.495	1.180	335	0.239
	Equal Variances not Assumed			1.238	93.583	0.218

The table 4.16 with p (0.495) > 0.05 shows no significant differences in their variance.It can also be deduced from table 14.16 that $t(335) = -1.180$,

$p(0.239) > 0.05$ .Which implies that there is no significant difference in students van Hiele"s geometric levels between public and private schools. On the contrary, according to a survey, private school students generally outperform public school students academically (Caddell, 2007; Mathema, 2007; Sharma, 2012; Thapa, 2012; Ntim, 2014; Hatsu, 2019).

Table 4.17: JHS 3 students' Performance on VGHT Level 1

School Type	**N**	**Mean**	**Std. Deviation**	**F**	**T**	**Df**	**Sig. (2 tail)**
Public School	276	0.41	0.493	1.090	-1.827	335	0.297
Private School	61	0.54	0.502				

Source: Field Data (2022)

Table 4.17 shows a mean score of 0.41 for public school students and 0.54 for private school students with a respective standard deviation of 0.493 and 0.502. It can also be deduced from the table 4.17 that $t(335) = -1.827, p(0.297) > 0.05$ which implies that there is no significant difference in students van Hiele‟s geometric thinking level 1 between public and private schools.

Table 4.18: JHS 3 students' Performance on VGHT Level 2

School Type	N	Mean	Std. Deviation	F	T	Df	Sig. (2 tail)
Public School	276	0.07	0.247	0.898	0.468	335	0.344
Private School	61	0.05	0.218				

Source: Field Data (2022)

Table 4.18 shows a mean score of 0.07 for public school students and 0.05 for private school students with a respective standard deviation of 0.247 and 0.218. It can also be deduced that there is no significant difference in students van Hiele‟s geometric thinking level 2 between public and private schools since from the table $\text{t}(335) = 0.468,\ p(0.344) > 0.05$.

Table 4.19: JHS 3 students' Performance on VGHT Level 3

School Type	N	Mean	Std. Deviation	F	T	Df	Sig. (2 tail)
Public School	276	0.004	0.060	0.888	0.470	335	0.347
Private School	61	0.000	0.000				

Source: Field Data (2022)

Table 4.19 shows a mean score of 0.004 for public school and 0.000 for private school students with a respective standard deviation of 0.060 and 0.000. It can also be deduced that there is no significant difference in students van Hiele‟s geometric thinking level 3 between public and private schools since from the table 4.19 $(335) = 0.470,\ p(0.347) > 0.05$.

Table 4.20: Comparison of student's performance between Males in Public Schools against Females Private Schools in the Van Hiele's Geometric Thinking Level

	Gender Category	N	Mean	Std. Deviation	F	T	Df	Sig (2 tail)
JHS 3 Pupil's performance on VHGT level attainment	Males in Public School	122	0.49	0.646	2.720	0.339	149	0.101
	Females in Private School	29	0.45	0.506				
Van Hiele"s Geometric level 1 attained	Males in Public School	122	0.41	0.494	0.401	-0.375	149	0.528
	Females in Private School	29	0.45	0.506				
Van Hiele"s Geometric level 2 attained	Males in Public School	122	0.08	0.275	12.32	1.598	149	0.001
	Females in Private School	29	0.00	0.000				

Source: Field Data (2022)

Table 4.20 gives the comparison of males in public school against the females in the private school. Over the mean scores for males and females in both public and private are 0.49 and 0.45 with respective standard deviation of 0.646 and 0.506. It can be asserted from the table 4.20 that there is no significant difference in van Hiele"s geometric thinking level between the males in the public school and the females in the private since from the table $t(149) = 0.339, p(0.101) > 0.05$.

However, in van Hiele"s geometric thinking level 1, the mean score of the males in the public schools 0.41 with a standard deviation of 0.494 and the females in the private school had a mean score of 0.45 with a standard deviation of 0.506. It can be deduced from the table that there is no significant difference in van Hiele"s Geometric thinking level 1 between the males in the public school and the females in the private since from the table $(149) = -0.375, p(0.528) > 0.05$ for level 3 both males in public and female in private did not attain. This was why Anderson (2013) noted that, for all private and government institutions, low mathematic achievement in a variety of disciplines is now a worry.

Also, in van Hiele‟s geometric thinking level 2, the mean score of the males in the public schools 0.08 with a standard deviation of 0.275 and the females in the private school had a mean score of 0.00 with a standard deviation of 0.000. However, it can be deduced from the table 4.20 that there is significant difference in van Hiele‟s Geometric thinking level 2 between the males in the public school and the females in the private since from the table 4.20 $t(149) = 1.598, p(0.001) < 0.05$. Concerning the van Hiele‟s geometric level 3, there was no t calculated for further conclusions since the standard deviations for both groups was 0.000.

Table 4.21: Comparison of student's performance between Females in Public Schools against Males Private Schools in the Van Hiele's Geometric Thinking Level

	Gender Category	N	Mean	Std. Deviation	F	t	Df	Sig (2 tail)
JHS 3 Pupil's performance on VHGT level attainment	Females in Public School	154	0.48	0.629	0.165	-1.948	184	0.685
	Males in Private School	32	0.72	0.634				
Van Hiele‟s Geometric level 1 attained	Females in Public School	154	0.42	0.494	0.918	-2.182	184	0.339
	Males in Private School	32	0.63	0.492				
Van Hiele‟s Geometric level 2 attained	Females in Public School	154	0.05	0.223	3.154	-0.909	184	0.77
	Males in Private School	32	0.09	0.296				
Van Hiele‟s Geometric level 3 attained	Females in Public School	154	0.01	0.081	0.839	0.455	184	0.361
	Males in Private School	32	0.00	0.000				

Source: Field Data (2022)

Table 4.21 gives the comparison of females in public school against the males in the private school. The mean scores for females and males in both public and private are 0.48 and 0.72 with respective standard deviation of 0.629 and 0.634. It could be concluded from the table 4.21 that there is no significant difference in van Hiele‟s geometric thinking level between the females in the public school and the males in the private since $t(184) = -1.948, p(0.685) > 0.05$.

However, in van Hiele‟s geometric thinking level 1, the mean score of the females in the public schools 0.42 with a standard deviation of 0.494 and the males in the private school had a mean score of 0.63 with a standard deviation of 0.492. It can be inferred from the table 4.21 that there is no significant difference in van Hiele‟s Geometric thinking level 1 between the females in the public school and the males in the private since from the table $t(184) = -2.182, p(0.339) > 0.05$.

Also, in van Hiele‟s geometric thinking level 2, the mean score of the females in the public schools 0.05 with a standard deviation of 0.223 and the males in the private school had a mean score of 0.09 with a standard deviation of 0.296. It can be deduced from the table 4.21 that there is no significant difference in van Hiele‟s geometric thinking level 2 between the females in the public school and the males in the private since $t(184) = -0.909,\ p(0.77) > 0.05$

Furthermore, the females in the public had a mean respective mean and standard deviation score of 0.01 and 0.081 as against males in private school 0.00 mean score and standard deviation 0.000 in the van Hiele‟s geometric thinking level 3. From the table, there is no significant difference between the females in public schools and males in private schools since $t(184) = 0.455, p(0.361) > 0.05$.

In general, some studies found that gender differences in mathematics learning are not obvious throughout primary school years, but females begin to lag behind males at intermediate levels, according to some studies (Hyde, Geiringer, & Yen, 1975; Mann et al., 1990).

4.5 Discussion

The purpose of this study was to investigate students‟ van Hiele‟s geometric thinking levels from public and private basic schools at Ga-Mashie in the Accra-Metropolis and whether there exists a difference in their levels based on their schools. The Van Hiele Geometric Test was administered to JHS3 students of both private and public basic schools in Ga- Mashie. The number of students from each school based on the concept of purposive and stratified sampling, put together represented the sample for the study (337).

4.5.1 Discussion of Research Objective 1

With regards to research question 1 that sort to assess the stages of van Hiele‟s levels of understanding JHS 3 students of public and private schools in Ga-Mashie Circuit reach in the study of geometry, analysis from the data of table 4.2 revealed that, both private and public-school students' overall performance fell far short of level 3 expectations for junior high school students. When the third level of the VHGT questions were asked in relationship between different shapes and they were to determine the relationships between forms, it was discovered that, the student performance in the 3rd level was not encouraging at all. The number of students recording the correct respond to each question in the Level 3 of the VHGT was all below average.

Moreover, the overall attainment levels (1, 2 and 3) reached by JHS 3 students as shown on table 4.2 reveals that, 190 (56.40%) of students couldn"t identify geometric shapes by their appearance, 126(37.40%) attained level 1, 20 (5.90%) attained level 2 and only 1 (0.30%) attained level 3 which should have been the level for all the JHS 3 students of public and private schools in Ga-Mashie Circuit. Also as illustrated on figure 4.1 that, only 0.3% of public school pupils reached level 3 while none of the private school students reached this level.

4.5.2 Discussion of Research Objective 2

To determine the existence of statistically significant difference in students' acquisition of van Hiele geometric thinking levels due to gender, the Arithmetic Mean, Standard Deviation and independence t -test was calculated for the Student's Marks due to their Gender, on all levels of the Geometric thinking. From table 4.6, the mean score for male students was 0.54 with a standard deviation of 0.648 as against 0.48 and 0.610 as mean and standard deviation respectively score for female students. The table 4.7 further indicates that, there is no statistically significant difference between the male and female students van Hiele"s geometric thinking levels with $t(335) = 0.926\,, p\;(0.355)\; > \;0.05$. This finding was consistent with previous literature that no statistically significant variations between male and female van Hiele geometric thinking levels (salem, 2014). The implication is that since geometry topics keeps featuring at the final basic school examinations, students both male and female will continue to record low grades in geometry. In line with the results of other researchers such as Asemani, Asiedu-Addo, and Oppong (2017) showed that, increased attention must be given to teaching geometry, in order to prepare future teachers for primary education.

4.5.3 Discussion of Research Objective 3

Research question 3 aimed at finding out whether there is a significant difference in terms of van Hiele‟s geometric thinking levels between males and females in the school type in the Ga-Mashie Circuit. The results from table 4.8 indicates that, there was no statistically significant difference between males in public school and males in private school in their van Hiele geometric thinking levels with $t(152) = -1.775,$

$p\ (0.469) > 0.05.$ The table 4.11 also pointed out that, there was no statistically significant difference between the females in public school and females in private school in their geometric thinking levels with $t(181) = 0.261, p\ (0.199) > 0.05.$

Additionally, van Hiele‟s geometric thinking levels between males and females in the public and private schools in the Ga-Mashie Circuit according to the van Hiele theory were also revealed. It could be inferred from Table 4.21 that, there was no significant difference in van Hiele‟s geometric thinking level 1 between the females in the public school and the males in the private schools with $t(184) = -2.182, p(0.339) > 0.05.$

In general, van Hiele's geometric thinking level between males and females in the public and private schools did not show significant difference with the exception of level 2 which shows significant difference in favour of females in public to that of females in private with $t(181) = 1.254, p\ (0.009) < 0.05$ from table 4.13. This implies that female students in public schools did better in identifying geometric shapes based on thier properties than female students in the private schools.

4.5.4 Discussion of Research Objective 4

The last research question 4, determined whether or not there is a significant difference in students van Hiele‟s geometric levels between public and private schools in the Ga-Mashie Circuit. Table 4.15 shows a mean score of 0.49 for public school students and 0.59 for private school students with a respective standard deviation of 0.635 and 0.588. It could also be deduced from the analysis as shown on table 4.16 that $t(335) = -1.180, p(0.239) > 0.05$ which inferred that, there was no significant difference in students van Hiele‟s geometric levels between public and private schools. This means that both schools perform at the same van Hiele‟s level in the Ga-mashie Circuit. It was again deduced that, there was no significant difference in the entire three sub van Hiele‟s levels (1, 2 and 3) of students between public and private schools as shown on the tables 4.17, 4.18 and 4.19 respectively with their p values greater than 0.05. This implies that, there are no differences in performance in identifying geometric shapes, their properties and the relationship between them in both public and private schools. This finding was contrary with reviewed literature that private school students generally outperform public school students academically (Ntim, 2014; Hatsu, 2019).

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.0 Introduction

This chapter presents the summary and conclusion of the entire study. In reference to the study, recommendation and suggestions were made from the major findings identified in the research work.

5.1 Summary of the Study

The primary purpose of the study was to investigate students‟ van Hiele‟s geometric thinking levels from public and private basic schools at the Ga-Mashie circuit of the Accra Metropolis. The research design was survey and the study used two types of sampling procedures which were purposive and stratified sampling**.** The population targeted in this study was students from JHS3 in both private and public basic schools at Ga-Mashie in the Accra metropolis. Ga-Mashie circuit is an area with four private schools and eight public schools. There were 562 students from both private and public basic schools in JHS 3. Four private schools and six public schools were selected from the circuit. A sample size of three hundred and thirty – seven (337) students from both private and public basic schools was selected for the study.

Van Hiele Geometric Test was the main instrument for the study. The van Hiele Geometric Tests were administered to JHS3 students of both private and public basic schools in the Circuit. In all the three hundred and thirty-seven (337) scripts that were collected from the students (from, both private and public schools) was analyzed one after the other. Each question (item) was taken into consideration. The individual marks of students from private and public schools in terms of achievement was coded and processed in to statistical package for social scientist (SPSS) for analysis.

5.2 Summary of Key Findings

These findings were derived from students‟ van Hiele levels in private and public schools with regard to their gender. The findings are:

1. Private and public schools in the Ga-mashie Circuit performed equally at the same level on the van Hiele‟s geometric test regarding all the required levels.
2. Geometry thinking viewed from a gender perspective, it was found that there was generally no significant difference between male and female respondents in van Hiele‟s geometric test.
3. 56.40% of students from public and private schools in the Ga-mashie Circuit were not able to identify geometric shapes in a different orientation.
4. 93.80% of students from public and private schools were not able to identify geometric shapes based on their properties.
5. 80.42% of students from public and private schools were not able to relate rhombus, rectangle and square as a parallelogram.
6. There are no differences in performance in identifying geometric shapes, their properties and the relationship between them in both public and private schools in the Ga-mashie Circuit.
7. Students, both public and private attained higher marks at level 1 (visualization).
8. Females from public school perform better than their fellow females in the private school in Van Hiele Level 2.

5.3 Conclusion

This study was aimed at investigating students‟ van Hiele‟s geometric thinking levels from public and private basic schools at the Ga-Mashie circuit of the Accra Metropolis. The study conducted at Ga-Mashie in the Accra Metropolis revealed that 190 (56.40%) of students attained No level, 126(37.40%) attained level 1, 20 (5.90%) attained level 2 and only 1 (0.30%) attained level 3 in both public and private schools. Also private schools performed equally the same as public schools in the Ga-Mashie Circuit of the Accra Metropolis. The mean performance and standard deviation of private schools was (0.59, 0.588) and that of public schools was (0.49, 0.635). The observed probability significance was $0.239 > 0.05$.

This implies that the null hypothesis was failed to be rejected, the study finally concludes that there was no significant difference in students‟ van Hiele‟s Geometric Thinking between Private and Public Basic School students in the Ga-mashie Circuit. This finding contradicts with previous literature review that private school students generally outperform public school students academically (Caddell, 2007; Mathema, 2007; Sharma, 2012; Thapa, 2012).

Van Hiele's geometric thinking level between males and females in the public and private schools did not show significant difference with the exception of level 2 which shows significant difference in favour of females in public to that of females in private with $t(181) = 1.254, p\ (0.009) < 0.05$

The study also revealed that the mean performance and standard deviation of males was (0.54, 0.648) and that of females was (0.48, 0.610). The observed probability significance was $0.355 > 0.05$. This implies that the null hypothesis was failed to be rejected, the study finally concludes that there was no significant difference in students‟ van Hiele‟s Geometric Thinking between males and females. The finding is consistent with previous literature review that there was no significant difference in geometric reasoning levels and mean between male and female students (Halat, 2008).

5.4 Recommendations

The following recommendations were made based on the findings of the study;

Students from public and private schools in the Ga-mashie attaining higher marks at level 1 (visualization) implies that they have good retention and when appropriate pedagogies are used by their teachers, is possible they will attain all the required levels. Teachers should design appropriate hands-on activities for their students to explore properties of geometric shapes. They should also guide their students in effectively exploring the interrelationship between shapes, this should be done in a thorough manner involving more challenging and thought-provoking exercises to enable student to operate at higher van Hiele level of geometric conceptualization.

Most students from public and private schools in the Ga-mashie were unable to identify sub set of geometric figures. The used of geometric shapes by teacher in teaching of geometry in both schools should be demonstrated in different orientation to enable them attain the van Hiele required levels.

More importantly, there is the need to review the geometric curricula, methods of teaching in both public and private schools and organize them in sequence according to the levels of van Hiele for geometric thinking.

To conclude, due to the poor performance of both public and private schools in the VHGT, school authorities and heads of both schools must emphasize on the use of learning materials and appropriate pedagogy in the classroom during teaching and learning of geometry. In-service training should be organized periodically to update teachers (public and private) on van Hiele phase-based instruction to improve their students van Hiele levels.

5.5 Suggestion for Future Study

This study focused on investigating students‟ van Hiele‟s geometric thinking levels from public and private basic schools and gender difference at Ga-Mashie in the Accra-Metropolis and whether there exists a difference in their levels based on their schools and gender. The study‟s variable was limited to gender and school difference hence future researchers should focus on other variables such as such as parents, learners‟ motivation, educators, and school managements. It is also suggested that the study should be replicated in many more circuit in the Accra Metropolis to get the general picture of students‟ geometric thinking levels.

REFERENCES

Abdullah, A. H., & Zakaria, E. (2013). The effects of Van Hiele‟ phases of learning geometry on students‟ degree of acquisition of Van Hiele levels. *Social and Behavioral Sciences, 102*(6), 251 – 266.

Abe, T. O. (2014). The effect of teachers' qualification on learners' performance in mathematics. *Sky Journal of Educational Research, 2*(1), 010-014

Abssi, I. (2016). *The impact of the training of Mathematics teachers on the levels of Geometric thinking in the achievement of their students and the development of levels of Geometric thinking and their orientation towards Geometry*. PhD unpublished thesis, University of Jordan.

Abu Musa, M & Nimrawi, Z. (2014). Levels of Geometric Thinking in the Conical Beams of Students of the Department of Mathematics at Al-Zaytoonah University of Jordan, *Educational Journal, Kuwait University, 28* (2), 111-138.

Abu, M. S. & Abidin, Z. Z. (2013). "Improving the Levels of Geometric Thinking of Secondary School Students Using Geometry Learning Video based on Van Hiele Theory,*" Int. J. Eval. Res. Educ., 2* (1), 16–22

Abu, M. S., & Abidin, Z. Z. (2013). Improving the levels of geometric thinking of secondary students using geometry learning video based on Van Hiele theory. *International Journal of Evaluation and Research in Education, 2*(1), 16-22.

Adolphus T, (2011). "Problems of teaching and learning of geometry in secondary schools,". *International Journal for Emerging Sciences, 1* (2), 143–152

Adulyasa, L., & Rahman, S. (2014). Lesson study incorporating phase – based instruction using Geometer's Sketchpad and its effects on Thai learners‟ geometric thinking. *International Journal for Lesson and Learning Studies, 3* (3), 252- 271.

Akmal, M. (2016). *The achievement gap between government and private schools in Pakistan*. Georgetown University.

Aksu, Z., & Kul, U. (2016). Exploring mathematics teachers‟ pedagogical content knowledge in the context of knowledge of students. *Journal of Education and Practice, 7*(30), 35-42.

Aktaş, M. C., & Aktaş, D. Y. (2012). Investigating high school students‟ attitudes towards geometry according to different variables: sample of Ordu City. *Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi, 18* (3), 156–167.

Akyeampong, K. (2009). Revisiting free compulsory universal basic education (FCUBE) in Ghana. *Comparative Education*, *45*(2), 175–195.

Alex, J. K., & Mammen, K. J. (2014). Gender differences amongst South African senior secondary school learners‟ geometric thinking levels. *Mediterranean Journal of Social Sciences*, *5*(20), 1908.

Alex, J. K., & Mammen, K. J. (2016). Lessons Learnt from Employing van Hiele Theory Based Instruction in Senior Secondary School Geometry Classrooms. *Eurasia Journal of Mathematics, Science & Technology Education, 12*(8), 2223-2236.

Al-Harbi, A. (2015). *The impact of Van Hiele model in the teaching of geometry and spatial inference unit in the development of geometric thinking levels among middle-class students in al-qurayat governorate*. (Unpublished master's thesis). Yarmouk University, Irbid, Jordan

Alhassora, N. S. A., Abu, M. S., & Abdullah, A. H. (2017). Inculcating higher-order thinking skills in mathematics: Why is it so hard? *Man in India, 97* (13), 51-62.

Ali, I., Bhagawati, S., & Sarmah, J. (2014). Performance of Geometry among the Secondary School Learners of Bhurbandha CD Block of Morigaon District, Assam, India. *International Journal of Innovative Research and Development.* ISSN 2278–0211, 3(11).

Aliyu, Y., & Mohammad Isa, H. (2016). Enhancing learners‟ academic performance in Islamic Studies Via parental involvements in North-Central Nigeria. *e-Bangi, 11*(2), 450–458

Al-Qurashi, A. (2016). *Levels of Geometric thinking for students of Mathematics at Umm Al-Qari University*. Unpublished Master's Thesis, Faculty of Education, Umm AlQura University, Makkah, Saudi Arabia.

Alvaredo, F., Chanel, L., Piketty, T., Saez, E., & Zucman, G. (2017). *World Inequality Report 2018*. Paris: World Inequality Lab.

Alzhanova-Ericsson, A. T., Bergman, C., & Dinnétz, P. (2017). Lecture attendance is a pivotal factor for improving prospective teachers‟ academic performance in Teaching and Learning Mathematics. *Journal of Further and Higher Education, 41*(1), 1-15.

Alzhanova-Ericsson, A. T., Bergman, C., & Dinnétz, P. (2017). Lecture attendance is a pivotal factor for improving prospective teachers‟ academic performance in Teaching and Learning Mathematics. *Journal of Further and Higher Education, 41*(1), 1-15.

Anamuah-Mensah, J., Mereku, D. K. & Asabere-Ameyaw, A. (2004). *Ghanaian Junior Secondary School Students' Achievement in Mathematics and Science: Results from Ghana's participation in the 2003 Trends in International Mathematics and Science Study*. Accra: Ministry of Education Youth and Sports.

Anas, S. S. (2018). The Geometric Thinking Levels of Mathematics Pre-Service Teachers‟ In Northern Ghana Colleges of Education. *Research Journal of Mathematics, 5*(3), 1-19

Anderson, L. B. (2013). *Gifted Learners and Mathematical Achievement: An Analysis of Gifted Instructional Models* (Doctoral dissertation, Liberty University).

Ankomah, Y., Koomson, J., Bosu, R., & Oduro, G. K. T. (2005). Implementing Quality Education in Low-Income Countries (Edqual). *University of Cape Coast, Ghana*.

Areepattamannil, S., Khine, M. S., Melkonian, M., Welch, A. G., Al Nuaimi, S. A., & Rashad, F. F. (2015). International note: Are Emirati parents' attitudes toward mathematics linked to their adolescent children's attitudes toward mathematics and mathematics achievement?. *Journal of Adolescence*, *44*(1), 17-20.

Arhin, A. K. & Offoe, A. K. (2015). Gender differences and mathematics achievements of senior high students: A case of Ghana national College. *Journal of Education and Practice, 6*(33), 67-74.

Arıcı, S., & Aslan-Tutak, F. (2015). The effect of origami-based instruction on spatial visualization, geometry achievement, and geometric reasoning. *International Journal of Science and Mathematics Education*, *13*(1), 179–200.

Armah, R.B., Cofie, P.O., & Okpoti , C. A. (2017). The Geometric Thinking Levels of Pre-service Teachers in Ghana. *Higher Education Research, 2*(3), 98-106.

Armah, R.B., Cofie, P.O., & Okpoti , C. A. (2018). Investigating the effect of Van Hiele phase based instruction on pre-service teachers‟ geometric thinking. *International Journal of Research in Education and Science, 4*(1), 314-330.

Armstrong, J. M. (1981). Achievement and participation of women in mathematics: Results of two national surveys. *Journal for Research in Mathematics Education, 12*(5), 356-372.

Arnot, M., David, M., & Weiner, G. (1999). *Closing the gender gap*. Cambridge: Polity Press.

Asante, K. O. (2010). *Sex differences in mathematics performance among senior high students in Ghana.* Retrievedfromhttp://www.faqs.org/periodicals/201012/2187713381 .html#ixzz1I5YvD0t3

Asare, B. (2021). AMA *holds School Performance Appraisal Meeting in Ashiedu Keteke to improve standards*. (n.d.). https://ama.gov.gh/news-

Asemani, E., Asiedu-Addo, S.K. & Oppong, R.A. (2017). The Geometric Thinking Level of Senior High School students in Ghana. *International Journal of Mathematics and Statistics Studies, 5*(3), 1-8.

Asiedu, S. (2002). *A comparative study of public and private schools in the provision of quality education at the basic level in urban centres in Ghana* (Doctoral dissertation).

Aslan-Tutak, F., & Adams, T. L. (2015). A study of geometry content knowledge of elementary pre-service teachers. *International Electronic Journal of Elementary Education, 7*(3), 301-318.

Atebe, H. U. (2008). *Student's van Hiele levels of geometric thought and conception in plane geometry: a collective case study of Nigeria and South Africa* (Doctoral dissertation, Rhodes University).

Atebe, H. U., & Schäfer, M. (2011). The nature of geometry instruction and observed learning-outcomes opportunities in Nigerian and South African high schools. *Education African Journal of Research in Mathematics, Science and Technology, 15(*2), 191-204.

Autor, H. (2014). "Skills, education, and the rise of earnings inequality among the "other 99 percent"." *Science, 344*(843), 843-851.

Awad, F. (2014). The impact of the use of a combination of real evaluation strategies in the development of geometry thinking among tenth graders in Jordan and their tendency towards mathematics. *Zarqa Journal of Research and Humanities Studies, 17*(1), 251-274.

Ayodele, A., & Muganyizi, L. (2014). School Type, Parental Influence and Mathematics Attitudes-Achievement Relationship. *Quantile Analysis 4*(6), 221–228.

Ayten, B. A. L. (2014). Predictor variables for primary school students related to van Hiele geometric thinking. *Eğitimde Kuram ve Uygulama*, *10*(1), 259–278.

Baffoe, E., & Mereku, D. K. (2010). The van hiele levels of understanding of students entering senior high school in Ghana. *African Journal of Educational Studies in Mathematics and Sciences*, *8*, 51–62.

Banergee, P. A. (2016). A systematic review of factors linked to poor academic performance of disadvantaged learners in science and Maths in schools. *Cogent Education, 3*(1), 1178441.

Bansilal, S. (2015). Using APOS theory to understand some of the demands of teaching and learning Mathematics. In *the proceedings of the 21st Annual National Congress of the Association for Mathematics Education of South Africa, 1*(2), 1-14

Bansilal, S., Mkhwanazi, T., & Brijlall, D. (2014). An exploration of the common content knowledge of high school mathematics teachers. *Perspectives in Education*, *32*(1), 34-50.

Battista, M T. (2007). *The development of geometric and spatial thinking. F. Lester (Ed.), Second Handbook of Research on Mathematics Teaching and Learning (s. 843-908). Charlotte, NC: NCTM*. Information Age Publishing.

Battista, M. T. (1990). Spatial visualization and gender differences in high school geometry. *Journal for Research in Mathematics Education*, *21*(1), 47–60.

Battista, Michael T, Wheatley, G. H., Talsma, G., & Wheatley, G. H. (1982). The Importance of Spatial Visualization and Cognitive Development for Geometry Learning in Preservice Elementary Teachers. *National Council of Teachers of Mathematics*, *13*(5), 332–340.

Bear, S., & Jones, G. (2017). Learners as protégés: Factors that lead to success. *Journal of Management Education, 41*(1), 146-168.

Behçet, O. R. A. L., & İLHAN, M. (2012). Analysis of Geometric Thinking Levels of Candidate Mathematics Teachers of Primary and Secondary Schools in Terms of Various Variables. *Necatibey Faculty of Education Electronic Journal of Science & Mathematics Education, 6*(1), 201-219.

Beilock, S., & Maloney, E. (2015). Math anxiety: A factor in math achievement not to be ignored. *Policy Insights from the Behavioral and Brain Sciences, 2*(1), 4-12

Belin, R. J. (2016). *The Impact of Learners' Attitudes after Implementing a Leadership Collaborative Grouping Method in a Collegiate Technical Mathematics Class*.

Bernard, H. R. (2017). *Research methods in anthropology: Qualitative and quantitative approaches.* California: Rowman & Littlefield.

Bleeker, C., Stols, G., & Van Putten, S. (2013). The Relationship between Teachers' Instructional 104 Practices and their Learners' Level of Geometrical Thinking. *Perspectives in Education, 31*(3), 66 – 78.

Bora, A., & Ahmed, S. (2018). Secondary school students" attitude towards their learning geometry: A survey of DIphu Town secondary schools. *International Journal of Research and Analytic Review, 5*(3), 265 – 267.

Borasi, R. (1990). The invisible hand operating on mathematics instruction; Students conceptions and expectations. In T.J. Cooney, (Ed.), *Teaching and learning mathematics in the 1990s,* (NCTM Yearbook), pp.174-182. Reston: NCTM.

Breyfogle, M. L. & Lynch, C. M. (2010). Van Hiele Revisited. *Mathematics Teaching in the Middle School, 16*(4), 232- 238

Brito, N.H., Piccolo,L.R., & Noble, K.G. (2017). Associations between cortical thickness and neurocognitive skills during childhood vary by family socioeconomic factor. *Brian and Cognition, 116*(3), 54 – 62.

Brookstein, A., Hegedus, S., Dalton, S., Moniz, R., & Tapper, J. (2014). *Measuring Learner Attitude in Mathematics Classrooms. Kaput Center for Research and Innovation in STEM education.* Retrieved from http://www. kaputcenter. umassd. Edu/downloads/products/technical reports/tr4_stude attitude. Pdf

Browning, C., Edson, A. J., Kimani, P., & Aslan-Tutak, F. (2014). Mathematical content knowledge for teaching elementary mathematics: A focus on geometry and measurement. *The Mathematics Enthusiast*, *11*(2), 333-383.

Bruce, C. D., Flynn, T. C., & Bennett, S. (2016). A focus on exploratory tasks in lesson study: The Canadian „Math for young children" project. *Zdm, 48*(4), 541-554.

Burger, W. F. (1988). One point of view: an active approach to geometry. *The Arithmetic Teacher*, *36*(3), 2.

Cacmac, D., & Kubra, G. (2014). Determining the geometric levels of pre-service elementary mathematics teachers. *Journal of Turkish Educational Sciences, 12*(1), 1-17.

Caddell, M. (2007). Private Schools and Political Conflict in Nepal. In: Srivastava, Prachi and Walford, Geoffrey eds. Private Schooling in Less Economically Developed Countries: Asian and African Perspectives. Oxford Studies in Comparative Education. Didcot, UK: Symposium.

Campbell, G. & Prew, M. (2017). *Behind the matric results: The story of Maths and science.* Mail & Guardian, pp. 15–17.

Canfield, B. (2013). *A phenomenological study of persistence of unsuccessful learners in developmental mathematics at a community college* (Doctoral dissertation). Available from ProQuest Dissertations and Theses database. (UMI No. 3597647).

Carey, E, Hill, F., Devine, A., & Szücs, D. (2015). The chicken or the egg? The direction of the relationship between mathematics anxiety and mathematics performance. *Front Psychology, 6*, 1987.

Carlson, K. D., & Herdman, A. O. (2010*). Understanding the Impact of Convergent Validity on Research Results,* 1–16. https://doi.org/10.1177/109442811039 2383

Carney, S., & Bista, M. B. (2009). Community schooling in Nepal: A genealogy of education reform since 1990. *Comparative Education Review, 53*(2), 189-211.

Catlioglu, H., Gurbuz, R., & Birgin, O. (2014). Do pre-service elementary school teachers still have mathematics anxiety? Some factors and correlates. *Bolema, Rio Claro, 28*(48).

Chevalier, S. (2013). *IZA Journal of Labor Economics 2013*. Retrieved on 22/10/2022, from **Error! Hyperlink reference not valid.**.

Ching, B. H. H., & Nunes, T. (2017). The importance of additive reasoning in children‟s mathematical achievement: A longitudinal study. *Journal of Educational Psychology*, *109*(4), 477.

Chowa, G. A., Masa, R. D., Ramos, Y., & Ansong, D. (2015). How do learner and school characteristics influence youth academic performance in Ghana? A hierarchical linear modeling of Ghana youth save baseline data. *International Journal of Educational Development, 45*(13), 129-140.

Clements D. H., & Battista M. T., (1992). "Geometry and Spatial Reasoning," in D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*, New York: Macmillan, pp. 420–464

Clements, D. H., & Battista, M. T. (1990). The effects of Logo on children‟s conceptualizations of angle and polygons. *Journal for Research in Mathematics Education*, *21*(5), 356–371.

Clements, D. H., & Sarama, J. (2013). Rethinking early mathematics: What is research-based curriculum for young children?. *Reconceptualizing Early Mathematics Learning*, 121-147.

Cohen, L.(2000). *Research Method in Method,* (5th ed.). London and New York: Routledge Falmer.

Connolly, S. (2010). *The impact of Van Hiele-based geometry instruction of students understanding.* (Mathematical and Computing Sciences Masters, paper 97. St John Fisher College.

Cordes, M. L. (2014). *A transcendental phenomenology study of developmental math learners' experiences and perceptions (Doctoral dissertation).* Retrieved from http://digitalcommons.liberty.edu/doctoral/947

Council, N., & Education, M. (1981). Differential Treatment of Females and Males in Mathematics Classes Author (s): Joanne Rossi Becker Source . *Journal for Research in Mathematics Education, 12*(1), 40–53.

Council, N., & Education, M. (1990). *T*he Effects of Logo on Children " s Conceptualizations of Angle and Polygons Author (s): Douglas H . Clements and Michael T. Battista Source. *Journal for Research in Mathematics Education , 21*(5), 356–371.

Council, N., & Education, M. (1992). Gender Differences in a Psychological Model of Mathematics Achievement Author (s): Corinna A . Ethington Source : Journal for Research in Mathematics Education , *JSTOR, 23*(2), 166–181.

Couto, A., & Vale, I. (2014). Pre-service teachers' knowledge on elementary geometry concepts. *Journal of the European Teacher Education Network, 9*(11), 57-73.

CRDD (2010). *Mathematics Syllabus for Senior High Schools,* Ghana Education Service, Accra, Ghana,

Crowley, M. L. (1987). "The van Hiele model of the development of geometric thought," Learn. Teach. *Geom., 12* (3), 1–16

Darfour, M. (2016). *Causes of poor students' academic performance in mathematics among junior high school students in Bantama sub–metro in Kumasi Metropolis*.(Doctoral dissertation University of Education Winneba).

Das, G. C., Das, R., & Kashyap, M. P. (2016). An Investigation on the Relationship between Performance in Mathematics and Learners" Attitude towards the Subject in Secondary Schools of Guwahati. *Indian Journal of Applied Research, 5*(6)

Dennick, R., & Tavakol, R.(2011). Making sense of Cronbach's alpha. *International Journal of Medical Education, 2*(3), 53 - 55.

Denscombe, M. (2009). *Ground rules for social research: Guidelines for good practice*. McGraw-Hill Education (UK).

Denscombe, M. (2017). *EBOOK: The good research guide: For small-scale social research projects*. McGraw-Hill Education (UK).

Depdiknas, D. D. (2008). *Teknik Penyusunan Modul*. Jakarta: Depdiknas.

Devenish, B., Hooley, M., & Mellor, D. (2017). The pathways between socioeconomic status and adolescent outcomes: A systematic review. *American Journal of Community Psychology, 59* (5), 219 –238

Dindyal, J. (2007). The Need for an Inclusive Framework for Students" Thinking in School Geometry. *The Montana Mathematics Enthusiast, 4*(1), 73-83.

Ding, L. & Jones, K. (2018). Teaching Geometry in Lower Secondary School in Shanghai, China. *Proceedings of the British Society for Research into Learning Mathematics, 26* (1), 41-46.

Djamarah, S. B., (2011). *Psikologi Belajar, (*2^{nd} Ed.). Jakarta: PT. Rineka Cipta.

Donkoh, K. (2014). Attaining school and educational goals: duties of headteachers of public basic schools in Ghana. *Journal of Education and Practice*, *5*(1), 64–72.

Doris, A., O" Neill, D., & Sweetman, O. (2013). Single sex schooling and Math"s achievement. *Economics of Education Review, 35* (1), 104-119

Du Plessis, P. (2013). The Principal as Instructional Leader: Guiding schools to improve instruction. *Education as Change*, S79 – S92.

Duval, R. (1998). Geometry from a cognitive point of view. *Perspectives on the Teaching of Geometry for the 21^< St> Century*.

Ellerhorst, A. M. (2014). A study of the relationship between teacher characteristics and student performance in high school geometry.

Erola, J., Jalonen, S., & Lehti, H. (2016). Parental education, class and income over early life course and children"s achievement. *Research on Social Stratification and Mobility, 44*(13), 33-43.

Essays, U. K. (2013). *The Van Hiele Theory of Geometric Thinking.* Retrieved from https://www.ukessays.com/essays/education/the-vanhiele-theory-of-geometric-thinking-educationessay.php?vref=1

Essien, A., Chitera, N., & Planas, N. (2016). *Language diversity in mathematics teacher education. Challenges across three countries.* In R. Barwell et al. (Eds.), Mathematics education and language diversity. The 21st ICMI Study (pp. 103–119). New York: Springer.

Eze, T. I., Ezenwafor, J. I., & Molokwu, L. I. (2015). Effect of meta-learning teaching method on the academic performance of building trades learners in technical colleges in South-east Nigeria. *International Journal of Vocational and Technical Education, 7*(10), 101-108.

Ezeh, S.I. (2005). *Effect of Delayed Formalization Approach on Senior Secondary School Students Achievement in Sequences and Series*. Unpublished M.E.D Thesis, University of Nigeria, Nsukka

Fadzil, H. M., & Saat, R. M. (2014). Enhancing STEM education during school transition: bridging the gap in science manipulative skills. *Eurasia journal of mathematics, science and technology education, 10*(3), 209-218.

Fennema, E. (2000). Gender and mathematics: What is known and what do I wish was known. In *Fifth annual forum of the national institute for science education* (pp. 22-23).

Fennema, E. (1980). Sex related differences in mathematics achievements: Where and why. In L.H. Fox, L. Brody, & D. Tobin (Eds.), *Women and the mathematical mystique* (pp76-93). Baltimore: The Johns Hopkins University Press.

Fennema, E. H., & Sherman, J. A. (1978). Sex-related differences in mathematics achievement and related factors: A further study. *Journal for Research in Mathematics education*, *9*(3), 189-203.

Fidan, Y., Turnuklu, E. (2010). Examination of 5th grade students' levels of geometric thinking in terms of some variables, *Pamukkale University Journal of Education, 27*(13), 185-197

Forgasız, H. (2005). Gender and mathematics: re-igniting the debate. *Mathematics Education Research Journal, 17*(1), 1-2

Fox, L., & Cohn, S. (1980). Sex differences in the development of precious mathematical talent. In L. Fox, L.A. Brody, & D. Tobin (Ed.), *women and the mathematical mystique*. Baltimore, GA: Johns Hopkins University Press

Fox, L.H. Brody, L. & Tobin, D. (Eds.) (1980). *Women and the mathematical mystique*. Baltimore, MD: Johns Hopkins University Press.

Fraser, B. J., & Kahle, J. B. (2007). Classroom, home and peer environment influences on student outcomes in science and mathematics: An analysis of systemic reform data. *International Journal of Science Education,* 29, pp 1891-1909.

Fyfe, E. R., McNeil, N. M., & Borjas, S. (2015). Benefits of "concreteness fading" for children's mathematics understanding. *Learning and Instruction*, *35*, 104-120.

Gakure, R. W., Mukuria, P., & Kithae, P. P. (2013). *An evaluation of factors that affect performance of primary schools in Kenya : A case study of Gatanga district*. *8*(13), 927–937.

Gamlem, S. M., & Smith, B. (2013). Learner Perceptions of Classroom Feedback Assessment in Education. *Principles, Policy and Practice, 20*(2), 150 – 169

Gardee, A., & Brodie, K. (2015). A teacher"s engagement with learner errors in her Grade 9 mathematics classroom. *Pythagoras, 36*(2), 1–9.

Gegbe, B., & Koroma, J. (2014). Learners and teachers" perception of the causes of poor academic performance in general and further mathematics in Sierra Leone: A case study of BO District Sourthern Province. *International Journal of Engineering Research and General Science, 2*(5), 240–253.

Goh, S. C., & Fraser, B. J. (1998). Teacher interpersonal behaviour, Classroom Environment and Student Outcomes in Primary Mathematics in Singapore. In B.J Fraser and K.G Tobin (Eds.), *Handbook of Science Education*, pp 199-229. Dordrecht, The Netherlands: Kluwer Academic Publishers

Good, S., & Clarke, V. B. (2017). An integral analysis of one urban school system"s efforts to support learner-centered teaching. In: Keengwe, J., Onchwari, G. (eds.) *Handbook of Research on Learner- Centered Pedagogy in Teacher Education and Professional Development*, 45–68. IGI Global, Hershey.

Goos, M., Vale, C., Stillman, G., Makar, K., Herbert, S., & Geiger, V. (2020). *Teaching secondary school mathematics: Research and practice for the 21st century*. Routledge.

Govender, V. (2017). *Factors contributing to the popularity of mathematics Olympiads and competitions in some schools: An interrogation of learners' and teachers' views*. Paper presented at the Proceedings of the 20th Annual National Congress of the Association of Mathematics of South Africa, Kimberly.

Green, S. B., & Salkind, N. J. (2014). *Using SPSS for Windows and Macintosh: Analyzing and understanding data* (7th ed.). Upper Saddle River, NJ: Pearson

Grossman, H., & Grossman, S. H. (1994). *Gender issues in education.* ERIC Press.

Groves, S., Doig, B., Widjaja, W., Garner, D., & Palmer, K. (2013). Implementing Japanese lesson study: An example of teacher - researcher collaboration. *Australian Mathematics Teacher, 69*(3), 10-17.

Halat (2006). Sex-related differences in the acquisition of the van Hiele levels and motivation in learning geometry. *Asia Pacific Education Review, 7*(2), 173-183

Halat, E. (2008). In-Service Middle and High School Mathematics Teachers: Geometric Reasoning Stages and Gender. *The Mathematics Educator, 18*(1), 8–14

Hamzah, M. (2017). Levels of Geometric thinking according to the Van Hill model among students of a university class teacher Israa in Jordan. *Hebron University Research Journal, (B), 12*(2), 173-191

Harris, J., & Bourne, P.A. (2017). Perception of teachers and pupils on factors influencing academic performance in mathematics among a group of fifth and sixth graders in Jamaica. *Int. J. Transf. Appl. Math. Stat., 2*(1), 1–23

Hassan, H. (2015). Levels of Geometric Thinking for Students of the Mathematics Department at the College of Education / Ibn Al-Haytham - University of Baghdad, *Journal of Al-Ostaz, 2* (214), 345-372.

Hatsu, E. (2019). *Investigating pupils' strategies on mathematics achievement from private and public basic schools in the Chorkor Circuit of the Accra Metropolis*. (Master‟s thesis University of Education, Winneba).

Haviger, J., & Vojkůvková, I. (2014). The Van Hiele Geometry Thinking Levels: Gender and School Type Differences. *Procedia - Social and Behavioral Sciences*, *112*(2013), 977–981. https://doi.org/10.1016/ j.sbspro. 2014 .01.1257.

Heale, R., & Twycross, A. (2015). *Validity and reliability in quantitative studies*. *18*(3), 66–67.

Hedges, L.V., & Nowell, A. (1995). Sex differences in mental test scores, variability and numbers of high scoring individuals. *Science, 269*(5), 41-45.

Herholdt, R., & Sapire, I. (2014). An error analysis in the early grades mathematics: A learning Opportunity? *South African Journal of Childhood Education, 4*(1), 43-60.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and instruction*, *26*(4), 430-511.

Hoffer, A.R. & Hoffer, S.A.K. (1992). Geometry and visual thinking. In T. R. Post (Eds.). *Teaching mathematics in grades K-8: Research based mathematics,* 2nd. ed. (pp. 249- 227). Boston: Allyn and Bacon.

Howse, T. D., & Howse, M. E. (2014). Linking the Van Hiele theory to instruction. *Teaching children mathematics*, *21*(5), 304-313.

Hsieh, C. H., Tuan, H. L., Chin, C. C., & Chen, S. C. (2016). The development and study of the relevant factors of the engagement in science learning questionnaire in junior high school. *Chinese Journal of Science Education, 24*(3), 249–273.

Humphrey, A. (2008). Students" Van Hiele Levels of Geometric Thought and Conception in Plane Geometry : a Collective Case Study of Nigeria and South Africa. *Journal of Science Education, 1*, 1–363.

Hyde, J. S., Geiringer, E. R., & Yen, W. M. (1975). On the empirical relation between spatial ability and sex differences in other aspects of cognitive performance. *Multivariate Behavioral Research, 10*(3), 289-309.

Hydea, J.S. & Mertzb, J.E. (2009). *Gender, culture, and mathematics performance*. Retrieved from http: //tctvideo. madison.com/uw/gender

Ibrahim, H (2014). Change in Van Hiele levels of Geometric thinking among students in the classroom in open education as a result of their study of basic concepts and methods of teaching and relation to their achievement, *University of Damascus Journal, 30* (1), 87-119.

Ibrahim, H, & Nasour, R. (2015). The Van Hiele Level of Geometric Thinking in Eighth Grade Students (A Field Study in Lattakia Governorate) Tishreen University. *Journal, 33* (3), 113-129

Idris, N. (2005). *Teaching and Learning of Mathematics.* Kuala Lumpur: Utusan Publications.

Idris, N. (2009). The impact of using Geometers" Sketchpad on Malaysian students" achievement and van Hiele geometric thinking. *Journal of mathematics Education*, *2*(2), 94-107.

Indah, Y. F. K., Lasan, B. B., & Ramli, M. (2018). The effectiveness of transtheoretical model to reduce compulsive internet use of senior high school students. *Jurnal Kajian Bimbingan dan Konseling*, *3*(1), 1-13.

Jacobi-Vessels, J. L., Todd Brown, E., Molfese, V. J., & Do, A. (2016). Teaching pre-schoolers to count: Effective strategies for achieving early mathematics milestones. *Early Childhood Education Journal, 44*(1), 1-9. doi: 10.1007/s10 643-014-0671-4.

Jennison, M., & Beswick, K. (2014). Learner attitude, learner understanding and mathematics anxiety. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 280–288). Fremantle, WA: MERGA.

Jilcha, K. (2019). Research design and methodology. *Cyberspace. IntechOpen. https://doi. org/10.5772/intechopen*, *85731*.

Jimerson, S. R., & Haddock, A. D. (2015). Understanding the importance of teachers in facilitating learner success: Contemporary science, practice, and policy. *School Psychology Quarterly, 30*(4), 488-493.

Jojo, Z. (2017). Disrupting a learning environment for promotion of geometry teaching. *Africa Education Review*, *14*(3-4), 245-262.

Jones, K. (2000). Critical issues in the design of the school geometry curriculum. In, *Readings in Mathematics Education.* Auckland, NZ: University of Auckland.

Kahn, J. H. (2006). Factor analysis in counseling psychology research, training, and practice: Principles, advances, and applications. *The counseling psychologist*, *34*(5), 684-718.

Kainuwa, A. & Yusuf, M. (2013). Influence of Socio-Economic and Educational Background of Parents on their Children"s Education in Nigeria. *International Journal of Scientific and Research Publications,* 3(10), 1-8

Kaiser-Messmer, G. (1994). Result of an empirical study into gender differences in attitudes towards mathematics. *Educational Studies in Mathematics Journal, 25*(5), 209-216.

Kennedy, O. O. (2011). Reappraising the work skill requirements for building technology education in senior secondary school for optimum performance in Nigeria. In *1st International Technology, Education and Environment Conference*.

Khaliq, A., Baig, I. F., Ameen, M., & Mirza, A. (2016). Socio-economic status and learners" achievement score at secondary level: A correlational study. *International Journal of Research in Education and Social Science, 1*(2), 1-7

Khan, R. M. A., Iqbal, N., & Tasneem, S. (2015). The Influence of Parents Educational Level on Secondary School Students Academic Achievements in District Rajanpur. *Journal of Education and Practice*, *6*(16), 76–79.

Khun-Inkeeree, H., Omar-Fauzee, M. S., & Othman, M. K. H. (2016). Learners‟ attitude towards achievement in mathematics: a cross sectional study of year six learners in Songklha province, Thailand. *European Journal of Education Studies, 2* (4), 89-99

Kiadarbandsari, A., Madon, Z., Hamsan, H. H., & Nouri, K. M. (2016). Role of parenting style and parents‟ education in positive youth development of adolescents. *Pertanika Journal of Social Sciences Humanities, 24*(4), 1465-1480.

Kikas, E., & Mägi, K. (2014). Transactional development of parental beliefs and academic skills in primary school. *Early Child Development and Care, 185*(11), 1148–1165

Kilani, R. (2013). *Van Hiele's model influenced the development of geometry thinking and selfconfidence among fifth-graders in mathematics.* Unpublished master's thesis. Mosul University, Iraq.

Kim, D.H., & Law, H. (2012). Gender gap in Math‟s test scores in South Korea and Hong Kong: role of family background and single-sex schooling. *International Journal of Educational Development, 32*(1), 92-103.

Kimani, G. N., Kara, A. M., & Njagi, L. W. (2013). Teacher factors influencing learners' academic achievement in secondary schools in Nyandarua County, Kenya. *International Journal of Education and Research, 1*(3)

Kivunja, C., & Kuyin, A.B. (2017). Understanding and applying research paradigms in educational context. *International Journal of Higher Education*, *6*(5),26-41.

Kothari, C. R. (2017). *Research Methodology methods and techniques* (2nd edn). New Delhi: New age International.

Kukogho, I. S. (2015). *Mathematical Phobia. Pulse*. Retrieved on 03/07/2022, from http://pulse.ng/learner/mathematical-phobialearners-fail-Math because many teachers are not well trained prof id3405539.html

Lake, V., & Kelly, L. (2014). Female preservice teachers and mathematics: Attitudes, beliefs, and stereotypes. *Journal of Early Childhood Teacher Education, 35*(1), 262-275.

Leder, G. C. (1985). Sex-related differences in mathematics: An overview. *Educational studies in Mathematics, 16* (7), 304- 309

Lee, J., & Stankov, L. (2016). Non-cognitive influences on academic achievement: Evidence from PISA and TIMSS. In *Non-cognitive Skills and Factors in Educational Attainment* (pp. 151-169): Brill Sense

Lee, V., Rogers, C., Lee, V., & Rogers, C. (1995). *Teaching geometry in the elementary classroom Education : Elementary Education*.

Letsoalo, M. E., Maoto, R. S., Masha, J. K., & Lesaoana, M. A. (2017). The effect of gender on learner achievement in Gauteng and the Western Cape Provinces of South Africa. *Gender and Behaviour, 15*(2), 9177 - 9184.

Leung, K.M., Chung, P.K., & Kim, S. (2016). Parental support of children‟s physical activity in Hong Kong. *European Physical Education Review, 23*(2), 141-156

Levpuscek, M. P., & Zupancic, M. (2009). Math Achievement in Early Adolescence: The Role of Parental Involvement, Teachers' Behavior, and Students' Motivational Beliefs about Math. *Journal of Early Adolescence, 29*(4), 541-570.

Liem, G. A. D., & Tan, S. H. (2018). *Asian education miracles: In search of sociocultural and psychological explanations:* Routledge.

Lin, H.S., Lawrenz, F., Lin, S.F., & Hong, Z.R. (2013). Relationships among affective factors and preferred engagement in science-related activities. *Public Understanding of Science, 22*(8), 941–954.

Lloyd, J. E. V., Walsh, J., & Yailagh, M. S. (2005). Sex differences in performance attributions, self-efficacy, and achievement in mathematics: If I‟m so smart, why don‟t I know it? *Canadian Journal of Education, 28*(3), 384-408.

Luneta, K. (2014). *Teaching Elementary Mathematics. Learning to teach elementary mathematics through mentorship and professional development.* Saarbrucken: LAP Lambert Academy Publishing GmbH & Co. KG

Luneta, K. (2015). Understanding students' misconceptions: an analysis of final Grade 12 examination questions in geometry. *Pythagoras*, *36*(1), 1-11.

Mahanta, D. (2014). Impact of attitude and self - concept of the learners towards mathematics upon their achievement in mathematics. *International Journal of Theoretical and Applied Sciences, 6*(1), 20– 35.

Mainali, B. (2014). Investigating the relationships between preferences, gender, and high school students' geometry performance.

Makeleni, N. T., & Sethusha, M. J. (2014). The experiences of foundation phase teachers in implementing the curriculum. *Mediterranian Journal of Social Sciences, 5*(2), 103-109.

Makhubele Y. E. (2014). *Misconceptions and Resulting Errors displayed by grade 11 learners in the learning of geometry.* Unpublished master‟s dissertation. University of Johannesburg, Johannesburg, South Africa

Malpass, J. R., O‟Neil, H. F., & Hocevar Jr, D. (1999). Self-regulation, goal orientation, self-efficacy, worry, and high-stakes math achievement for mathematically gifted high school students. *Roeper Review*, *21*(4), 281–288.

Mann, E. L., Chamberlin, S. A., & Graefe, A. K. (2017). The Prominence of Affect in Creativity: *Expanding the Conception of Creativity in Mathematical Problem Solving*. In Creativity and Giftedness (pp. 57-73). Springer International Publishing

Mann, V. A., Sasanuma, S., Sakuma, N., & Masaki, S. (1990). Sex differences in cognitive abilities: A cross-cultural perspective. *Neuropsychologia, 28* (4), 1063–1077.

Mason, M. (1998). *The van Hiele Levels of geometric understanding*. In L. McDougal (Ed.). The Professional handbook for teachers: Geometry (pp. 4–8). Boston: McDougalLittell/Houghton-Mifflin

Mata, M. D. L., Monteiro, V., & Peixoto, F. (2012). Attitudes towards mathematics: Effects of individual, motivational, and social support factors. *Child Development Research, 5*(6)

Mathema, K. B. (2007). Crisis in education and future challenges for Nepal. *European Bulletin of Himalayan Research, 31*(2), 46-66.

Mathema, K.B. (2013). *The great equalizer. My Republica.* Retrieved from http://e.myrepublica.com/2014/component/flippingbook/book/1221-republica-25-april-2013/1-republica.html

McCarthy, J. & Oliphant, R. (2013). *Mathematics Outcomes in South African Schools: What are the facts? What should be done?* Retrieved from https://www.cde.org.za/wp-content/ uploads/2013/10/ mathematics outcomes in South African schools.pdf.

McMillan, J. H. & Schumacher, S. (2016). *Research in education: Evidence-based inquiry* (8th ed.). New York, NY: Pearson

Mega, C., Ronconi, L. & De Beni, R. (2014). What makes a good learner? How emotions, self-regulated learning, and motivation contribute to academic achievement. *Journal of Educational Psychology, 106*(1), 121–131

Mensah, J. K., Okyere M. & Kuranchie, A. (2013). Learner attitude towards mathematics and performance: Does the teacher attitude matter. *Journal of Education and Practice, 4*(3), 132- 139

Merriam-Webster Dictionary (2013). *Capability.* Springfield, MA: Merriam-Webster Inc.

Metzler, M. W. (2014). Teacher effectiveness research in physical education: The future isn't what it used to be. *Research Quarterly for Exercise & Sport, 85*, 14e19.

Ministry of National Education. (2015). *Secondary school mathematics (9-12. grades).* Retrieved May 15, 2017, from Error! Hyperlink reference not valid..

Moss, J., Hawes, Z., Naqvi, S., & Caswell, B. (2015). Adapting japanese lesson study to enhance the teaching and learning of geometry and spatial reasoning in early years classrooms: A case study. *Zdm, 47*(3), 377-390.

Mostafa, M. Mohammad Javad L, & Reza O. H., (2016). "The Effect of Van Hiele Theory-Based Teaching Educational Package on Achievement Goal Orientation of Student Teachers." *Rev. Eur. Stud., 9*(1), 93–105.

Mtitu, E. A. (2014). *Learner-centred teaching in Tanzania: Geography teachers' perceptions and experiences.* Victoria University of Wellington

Mucee, J. N., Rechee, J., Bururia, D. & Gikunda, R. M. (2014). Socio-Cultural Factors that Influence Access to Secondary School Education in Tharaka South Sub-County, Kenya. *International Journal of Education and Research, 2*(10), 489-502.

Müller, K., & Ehmke, T. (2012). Social background as a condition of competence development. *Pisa*, 245-274.

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS (2015) International Results in Mathematics.* TIMSS and PIRLS International Study Centre.

Mullis, I. V., Martin, M. O., Foy, P., & Arora, A. (2012). *TIMSS 2011 international results in mathematics*. International Association for the Evaluation of Educational Achievement. Netherlands: Herengracht.

Mutodi, P., & Ngirande, H. (2014). Exploring mathematics anxiety: Mathematics learner‟s experiences. *Mediterranean Journal of Social Sciences, 5*(1), 282-294.

Nadjib A, (2014). "Analisis Kesalahan Pemahaman Dalam Materi Segiempat Menurut Tingkat Berpikir Van Hiele Pada Siswa SMP Negeri 1 Suppa Kabupaten Pinrang," *J. Pepatuzdu, 8*(1), 14–23.

National Council of Teachers of Mathematics (NCTM, 2009). *Guiding Principles for Mathematics Curriculum and Assessment.* Retrieved from **Error! Hyperlink reference not valid.**%206 209.pdf

Ndebele, M. (2015). Socio-economic factors affecting parents' involvement in homework: Practices and perceptions from eight Johannesburg public primary schools. *Perspectives in Education, 33*(3), 72-91.

Németh, J., & Long, J. G. (2012). Assessing learning outcomes in US planning studio courses. *Journal of planning education and research, 32*(4), 476-490.

Nenthien, S., & Loima, J. (2016). Teachers‟ Motivating Methods to Support Thai Ninth Grade Learners‟ Levels of Motivation and Learning in Mathematics Classrooms. *Journal of Education and Learning, 5*(2), 250.

Nero, S. J. (2014). De facto language education policy through teachers‟ attitudes and practices: A critical ethnographic study in three Jamaican schools. *Language Policy*, *13*, 221-242.

Ngussa, B. M., & Mbuti, E. E. (2017). The Influence of Humour on Learners‟ Attitude and Mathematics Achievement: A Case of Secondary Schools in Arusha City, Tanzania. *Journal of Educational Research, 2*(3), 170 -181.

Nicholson, J. (2014). *The concise Oxford Dictionary of Mathematics* (5 ed.). Oxford University Press.doi:10.1093/acref/9780199679591.001.0001.

Ntim, S. (2014). The gap in reading and mathematics achievement between basic public schools and private schools in two administrative regions of Ghana: Where to look for the causes. *American International Journal of Contemporary Research*, *4*(7), 109–119.

Nur A. S. & Nurvitasari E, (2017). "Geometry Skill Analysis In Problem Solving Reviewed

Ogbonna, C.C. (2007). *Effects of two constructivist Based Instructional Model on Students Achievement and Retention in Number and Numeration.* Unpublished Ph.D Thesis, University of Nigeria, Nsukka.

Ogbugo-Ololube, R. (2016). *Impact of learners' parental Background on academic achievement in secondary school in obio/ akpor LGA, River State, Nigeria, 9*(2), 115–126.

Ogembo, J. O., Otanga, H., & Yaki, R. N. (2015). Learners‟ and Teachers‟ Attitude and Performance in Chemistry in Secondary Schools in Kwale County, Kenya. *Global Journal of Interdisciplinary Social Science, 4* (3), 39-43.

Ojose, B. (2015). Learners" misconceptions in mathematics: Analysis of remedies and what research says. *Ohio Journal of School Mathematics, 72*(12), 30-34

Oladosu, L. O. (2014). *Secondary school learners' meaning and learning of circle geometry.* Doctoral dissertation, University of Calgary, Calgary, Alberta, CA.

Olkun, S., & Aydoğdu, T. (2003). Üçüncü uluslararasi matematik ve fen araştirmasi (TIMSS) nedir? neyi sorgular? örnek geometri sorulari ve etkinlikler. *İlköğretim Online, 2*(1), 28- 35.

Otieno, K. O. (2010). Teaching/learning resources and academic performance in mathematics in secondary schools in Bondo District of Kenya. *Asian social science*, *6*(12), 126.

Özcan, Z. Ç., & Erktin, E. (2015). Enhancing mathematics achievement of elementary school students through homework assignments enriched with metacognitive questions. *Eurasia Journal of Mathematics, Science and Technology Education*, *11*(6), 1415-1427.

Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *Procedia-Social and Behavioral Sciences, 55*(5), 720–729.

Pangeni, K. P. (2014). Factors determining educational quality: Learner mathematics achievement in Kenya. *International Journal of Educational Development, 34*(13), 30-41.

Peace K, & Hsu JP. (2018). *The Importance of Statistics in Medical Science*. Retrieved from: https://www.researchgate.net/publication/237518872_The_Importance_of_Statistics_in_Medical_Science.

Pearce, A., Sawyers, C. P., Chittleborough, C. R., Mittinty, M. N., Law, C., & Lynch, J.W. (2016). Do early life cognitive ability and self-regulation skills explain socioeconomic inequalities in achievement? An effect decomposition analysis in UK and Australian cohorts. *Social Science and Medicine, 165*(6), 108 – 118

Planas, N., Morgan, C., & Schütte, M. (2018). Mathematics education and language. Lessons from two decades of research. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger & K. Ruthven (Eds.), *Developing research in mathematics education. Twenty years of communication, cooperation and collaboration in Europe* (pp. 196–210). London: Routledge

Prahmana R C I, Kusumah Y S., & Darhim. (2017). Didactic trajectory of research in mathematics education using research-based learning *J. Phys.: Conf. Ser.* 893-1201.

Prediger, S., Wilhelm, N., Büchter, A., Gürsoy, E., & Benholz, C. (2015). Sprachkompetenz und Mathematik leistung Empirische Untersuchung sprachlich bedingter Hürden in den Zentralen Prüfungen 10. *Journal für Mathematik-Didaktik, 36*(1), 77–104.

Prendergast, M., O'Meara, N., O'Hara, C., Harbison, L., & Cantley, I. (2019). Bridging the primary to secondary school mathematics divide: Teachers' perspectives. *Issues in Educational Research*, *29*(1), 243-260.

Ramphele, M. M. (2015). What is learning for? *European Journal of Education, 50*(1), 10-13

Randhawa, B.S. (1994). Self-efficacy in mathematics, attitudes, and achievement of boys and girls from restricted samples in two countries. *Perpetual and Motor Skills, 79*(3), 1011-1018

Reilly, D., Neumann, D.L. & Andrews, G. (2017). Gender differences in spatial ability: Implications for STEM education and approaches to reducing the gender gap for parents and educators. In *Visual-Spatial Ability in STEM Education*; Kheine, M.S., Ed.; Springer: Cham, Switzerland, pp. 195–224.

Riastuti, N., Mardiyana, & Pramudya, I. (2017). Analysis of students geometry skills viewed from spatial intelligence. *AIP Conference Proceedings*, *1913* (December).

Riccomini, P. J., Smith, G. W., Hughes, E. M., & Fries, K. M. (2015). The language of mathematics: The importance of teaching and learning mathematical vocabulary. *Reading & Writing Quarterly, 31*(3), 235-252.

Rimm-Kaufman, S., & Sandilos, L. (2015). *Improving learners' relationships with teachers to provide essential supportsfor learning. Apa.org*.

Robert, B.A., Primrose, O.C. & Christopher, A.O (2018). Investigating the effect of van Hiele phase-based instruction on pre-service teachers‟ geometric thinking. *Int. J. Educ. Sci., 4* (2), 314–330.

Robichaux-Davis, R. R., & Guarino, A. J. (2016). Assessing Elementary Pre-service Teachers' Knowledge for Teaching Geometry. *International Journal of Mathematics and Statistics Invention, 4*(1), 12-20.

Sa‟ad, T. U., Adamu, A., & Sadiq, A. M. (2014). “The Causes of Poor Performance in Mathematics among Public Senior Secondary School Learners in Azari Metropolis of Bauchi State, Nigeria,”. *IOSR- Journal of Research and Method in Education, 4*(2), 32-40.

Saez, E. & Zucman, G. (2016). "Wealth inequality in the United States since 1913: Evidence from capitalized income tax data." *Quarterly Journal of Economics, 131* (2), 519-578.

Saleh, S., Asyraf, M., & Rahman, A. (2016). A study of students " achievement in algebra : Considering the effect of gender and types of schools. *Journal of Turkish Educational Sciences,1*, 19–26.

Salem, T (2014). *Levels of Geometric thinking among the students of the elementary stage in Jerash governorate and their relation to gender and achievement in Mathematics*. Unpublished Master's Thesis, Hashemite University, Jordan.

Schoenfeld, A. H., & Floden, R. E. (2014). *An introduction to the TRU Math Dimensions*. Berkeley, CA: Graduate School of Education, University of California, Berkeley & East Lansing, MI: College of Education, Michigan State University.

Schulte, A. C., & Stevens, J. J. (2015). Once, sometimes, or always in special education: Mathematics growth and achievement gaps. *Exceptional Children, 81*(3), 370- 387.

Sean, S.F. (2013).The widening income achievement gap. Brown University, Educational leadership, Secondary School Students. *Multilingual Academic Journal of Social Sciences, 3*(1).

Sekaran, U., & Bougie, R. (2016). R*esearch methods for business: A skill building approach.* New York: John Wiley & Sons.

Serin, H. (2018). Perspectives on the teaching of geometry: teaching and learning methods. *Journal of Education and Training, 5*(1), 131-137.

Shaikh, S. N. (2013). Mathematics anxiety factors and their influence on performance in mathematics in selected international schools in Bangkok. *Journal of Education and Vocational Research, 4*(3), 77-85.

Shamaki, S.K. (2015). The impact and role of the physical environment of the school in teaching and learning mathematics. *American School Board Journals, 157* (3), 179-187

Sharma, U. (2012). Public and private schools in Nepal: a comparative perspective in essays on the economics of education in developing countries. ProQuest LLC.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(11), 4-14.

Shulman, L. S. (1987). Knowledge for teaching: Foundations for the new reform. *Harvard Educational Review, 57*(1), 1-22

Siew, N.M., Chong, C.L., & Abdullah, M.R. (2013). Facilitating students" Geometric Thinking through Van Hiele"s Phase Based learning using Tangram. *Journal of Social Sciences, 9*(3), 101-111.

Sikhwari, T. D. (2016). Study habits, attitudes and academic achievements: Comparing Grade 12 learners between two secondary schools. *Journal of Educational Studies, 15*(2), 43 - 61.

Silverman, D. (2013). *Doing Qualitative Research: A Practical Handbook*. London: Sage Publications Limited.

Silvernail, D. L., Sloan, J. E., Paul, C. R., Johnson, A. F., Stump, E. K., & University of Southern Maine, C.E. (2014). *The relationships between school poverty and learner achievement in Maine.* Center for education policy applied research, and evaluation.

Siyepu, S. (2013). The zone of proximal development in the learning of mathematics. *South African Journal of Education, 33*(2), 1-13.

Skolverket D. (2012). *Likvärdig utbildning i svensk grundskola*? En kvantitativ analys av likvärdighet över tid [Equitable education in Swedish compulsory school? A quantitative analysis of equity over time] (Rapport 374). Stockholm: Skolverket

Skrbec, M., & Cadez, T.H (2015). Identifying and fostering higher levels of geometric thinking, *Eurasia Journal of Mathematics, Science & Technology Education, 11*(3), 601-617

Smith, S. E., & Walker, W. J. (1988). Sex differences on New York State Regents" Examinations: support for the differential course-taking hypothesis. *Journal for Research in Mathematics Education*, *8*(4), 81-85.

Snyder, H. (2019). Literature review as a research methodology: An overview and guidelines. *Journal of business research*, *104*, 333-339.

Sonali, S. (2017). Role of socio-economic status in academic stress of senior secondary learners. *International Journal of Advanced Education and Research, 1*(12), 44-50.

Soni, A., & Kumari, S. (2017). The role of parental math anxiety and math attitude in their children"s math achievement. *International Journal of Science and Mathematics Education, 15*(2), 331- 347.

Steinmayr, R., Meibner, A., Weidinger, A. F., & Wirthwein, L. (2014). Academic achievement. In L.H. Meyer (Ed.) *Oxford bibliographies online: Education*. University Press.

Strutchens, M. E., Huang, R., Losano, L., Potari, D., Cyrino, M. C. D. C. T., da Ponte, J. P., ... & da Ponte, J. P. (2017). Current research on prospective secondary mathematics teachers‟ knowledge. *The mathematics education of prospective secondary teachers around the world*, 3-15.

Sudihartinih, E., & Wahyudin, E. (2019). The van hiele levels of geometric of students in first semester reviewed from gender, *Journal of Physics, Conference Series*, *2*(8)

Suleman, Q., Aslam, H. D., & Hussain, D. I. (2014). Effects of classroom physical environment on the academic achievement scores of secondary school learners in Kohat Division, Pakistan. *International Journal of Learning and Development*, *4*(1)

Sumantri, M. S., & Whardani, P. A. (2017). Relationship between Motivation to Achieve and Professional Competence in the Performance of Elementary School Teachers. *International Education Studies, 10*(7), 118-125. https:// doi.org/10.5539/ies.v10n7p118.

Sunzuma, G., & Maharaj, A. (2019). In-service teachers‟ geometry content knowledge: Implications for how geometry is taught in teaching training institutions. *International Electronic Journal of Mathematics Education, 14*(3), 633-646.

Sunzuma, G.; Masocha, M.; & Zezekwa, N. (2013). Secondary school students‟ attitudes towards their learning of geometry: A survey of Bindura urban secondary schools. *Greener J. Educ. Res., 3*(4), 402–410

Suwito, A., Yuwono, I., Parta, I. N., & Irawati, S. (2017, June). Geometry High School Students Thinking Ability Based On level van Hiele. In *International Conference on Mathematics: Education, Theory and Application, Indonesia*.

Taherdoost, H. (2020). Sampling Methods in Research Methodology ; How to Choose a Sampling Technique for Research. *International Journal of Academic Research in Management (IJARM), 5*(2), 18-27.

Tall, D. (2014). Making Sense of Mathematical Reasoning and Proof. In M. Fried, & T. Dreyfus (Eds.), *Mathematics & Math0ematics Education: Searching for Common Ground* (pp. 223-235). Springer Netherlands.

Tas, Y. (2016). The contribution of perceived classroom learning environment and motivation to learner engagement in science. *European Journal of Psychology of Education, 31*(4), 557–577.

Tavakol, M., & Saunders, J. (2015). *Quantitative and quantitative methods in medical education research*: AMEE Guide No 90: Part 1. *Med Teach, 36*(9), 746-756.

Thapa, A. (2011). *Does Private School Competition Improve Public School Performance? The Case of Nepal.* Columbia University

Thapa, A. (2012). Public and Private school performance in Nepal: An analysis using the SLC examination. Education Economics, (ahead-of-print), 1-16.

Thatcher, R. (2012). Validity and Reliability of Quantitative Electroencephalograph. *Journal of Neurotherapy, 14*(2), 121 - 153.

Tran, V. D. (2014). The effects of cooperative learning on the academic achievement and knowledge retention. *International Journal of Higher Education, 3*(2), 131-140.

Tshabalala, T and Ncube, A. C. (2013). *"Causes of poor performance of ordinary level pupils in mathematics in rural secondary schools in Nkayi District. Learner's Attributions." Nova, 1*, 4-14

Uharian, L. (2016). School environmental variables affecting achievement mathematics. *West African Journal of Education, 2*(2), 79-87.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry: Cognitive development and achievement in secondary school geometry project.* Chicago: University of Chicago Press

Van Hiele, P. M. (1959). The child"s thought and geometry. *English translation of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele*, 243-252.

Van Hiele P. M., (1999). "Developing geometric thinking through activities that begin with play," *Teaching Children Mathematics, 5* (6), 310–316

Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. Orlando: Academic Press.

Venkat, H., & Spaull, N. (2015). What do we know about primary teachers" mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. *International Journal of Educational Development, 41*(12), 121–130.

Visser, M., Juan, A., & Feza, N. (2015). Home and school resources as predictors of mathematics performance in South Africa. *South African Journal of Education, 35*(1), 01-10.

Vojkuvkova, I. (2012). The van Hiele Model of Geometric Thinking. *WDS'12 Proceedings of Contributed Papers, 1*(3), 72– 75.

Wachira, C. (2016). *Causes of poor performance in Mathematics in Secondary Schools: A Case of Nyandarua North Sub County, Kenya.* (A Master‟s thesis). Karatina University, Karatina, Kenya.

WAEC, Chief Examiners Report, (2011). Retrieved from http://www.waecgh.org/ examiners- report, (The West African Examination Council, Lagos, Nigeria) http://www.waecgh.org/examinersreport.

WAEC, Chief Examiners Report, (2014). Retrieved from http://www.waecgh.org/ examiners- report, (The West African Examination Council, Lagos, Nigeria) http://www.waecgh.org/examinersreport.

WAEC, Chief Examiners" Report, (2015). Retrieved from http://www.waecgh.org/ examiners- report, (The West African Examination Council, Lagos, Nigeria) http://www.waecgh.org/examinersreport.

WAEC, Chief Examiners" Report, (2016). Retrieved from http://www.waecgh.org/ examiners- report, (The West African Examination Council, Lagos, Nigeria) http://www.waecgh.org/examinersreport.

WAEC, Chief Examiners" Report, (2017). Retrieved from http://www.waecgh.org/ examiners- report, (The West African Examination Council, Lagos, Nigeria) http://www.waecgh.org/examinersreport.

WAEC, Chief Examiners" Report, (2020). Retrieved from http://www.waecgh.org/ examiners- report, (The West African Examination Council, Lagos, Nigeria) http://www.waecgh.org/examinersreport.

World Bank. (2017). *World development report 2018: Learning to realize education's promise*. The World Bank.

Yang, J. C., & Chen, S. Y. (2010). Effects of gender differences and spatial abilities within a digital pentominoes game. *Computers & Education*, *55*(3), 1220–1233.

Yang, L. (2017). Understanding in School and Post-School Success of adolescents: An Integrative Perspective of Multidimensional Self-Concepts in Education and Career Development. *Psychology and Behavioral Science International Journal, 6*(3), 555-688.

Yegambaram, P. & Naidoo, R. (2009). *Better learning of geometry with computer*. Retrieved from http://atcm.mathandtech.org/EP2009/papers_full/281200917 080.

Yenilmez, K., & Korkmaz, D. (2013). Relationship between 6th, 7th and 8th grade students‟ self-efficacy towards geometry and their geometric thinking levels. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*, *7*(2), 268-283.

Yilmaz, G. K., & Koparan, T. (2016). The Effect of Designed Geometry Teaching Lesson to the Candidate Teachers' Van Hiele Geometric Thinking Level. *Journal of Education and Training Studies*, *4*(1), 129-141.

Zan, R., & Di Martino, P. (2014). *Learners' Attitude in Mathematics Education. In Encyclopedia of Mathematics Education* (pp. 572-577). Springer Netherlands

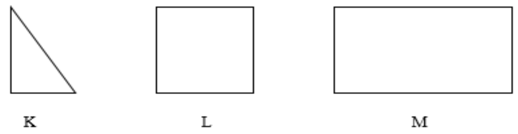
Zee, M., & Koomen, H. (2016). Teacher self-efficacy and its effects on classroom processes, learner academic adjustment, and teacher well-being: A synthesis of 40 years of research. *Review of Educational Research Monthly, 20*(10), 1-3.

APPENDIX A

VAN HIELE GEOMETRIC TEST

(To be answered by JHS 3 students from both public and private basic schools)

1. **Which of these is a rectangle?**

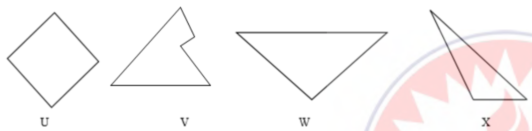


A. K only B. L only

C. M only D. L and M only

E. All are squares

2. **Which of these is /are triangles?**



A. None of these are triangles

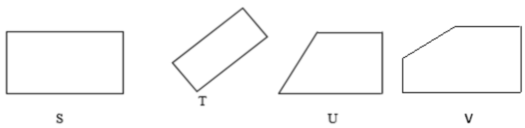
B. V only

C. W only

D. W and X only

E. V and W only

3. **Which of these is / are trapeziums?**

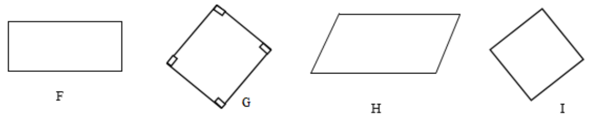


A. T only B. U only

C. V only D. U and V only

E. All are trapezium

4. **Which of these is / are squares?**



A. None of these are squares

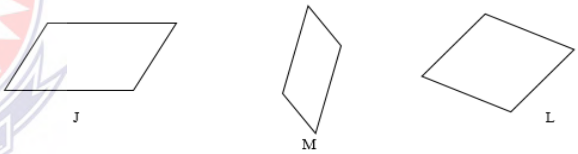
B. G only

C. F and G only

D. G and I only

E. All are squares.

5. **Which of these are parallelograms?**



A. J only

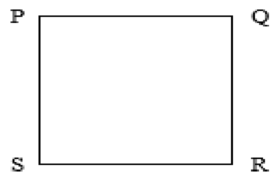
B. L only

C. J and M only

D. None of these are parallelograms.

E. All are parallelograms.

6. **Which relationship is true in all squares?**



PQRS is a square.

A. PR and PS have the same length.

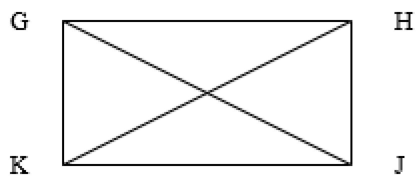
B. QS and PR are perpendicular

C. PS and QR are perpendicular.

D. PS and QS have the same length.

E. Angle Q is larger than angle R.

7. **Which of the following is <u>not</u> true in <u>every</u> rectangle?**



In rectangle GHJK, GJ and HK are <u>diagonals</u>.

A. There are four right angles

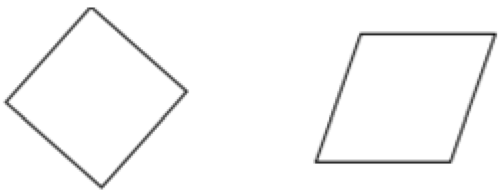
B. There are four sides

C. The diagonals have the same length

D. The opposite sides have the same length

E. All of the above are true in every rectangle.

8. A rhombus is a four- sided figure with all sides of the same length. Here are two examples, which of the following is <u>not</u> true in every rhombus?



A. the two diagonals have the same length.

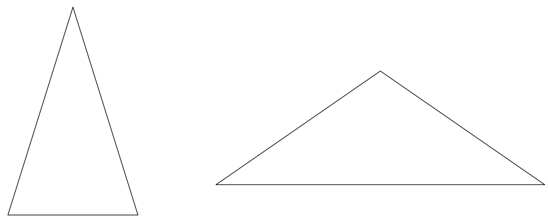
B. the two diagonals are perpendicular

C. All of the above are true in every rhombus

D. Each diagonal bisects two angles of the rhombus

E. The opposite angles have the same measure

9. An isosceles triangle is a triangle with two sides of equal length. Here are two examples, which of the following is true of isosceles triangle?



A. The three sides must have the same length

B. The three angles must have the same measure

C. There must be two angles with the same measure.

D. One side must have twice the length of another side

E. None of the above is true in isosceles triangle

10. A circle is a closed shape formed by tracing a point that moves in a plane such that its distance from a given point is fixed. Which of the following is true of every circle?

A. The circles are said to be same if they have equal radii

B. The diameter of a circles is the longest chord

C. Circles having different radius are similar

D. The radius drawn perpendicular to the chord bisects the chord

E. All the above are true in every circle.

11. Here are two statements. Which is correct?

Statement S: ΔABC has three sides of the same
Statement T: In ΔABC, $\angle B$ and $\angle C$ have the

A. Statements S and T cannot both be true

B. If S is true, then T is true

C. If T is true, then S is true

D. If S is false, then T is true

E. None of the above is correct.

12. Here are two statements. Which is correct?

Statement 1: Figure A is a square

Statement 2: Figure B is a triangle

A. Statements 1 and 2 cannot both be true

B. If Statement 1 is true, then Statement 2 is false

C. If Statement 1 is false, then Statement 2 is true

D. Statement 1 and 2 cannot both be false

E. None of the above is correct.

13. Which is true?

A. All properties of rectangles are properties of all squares

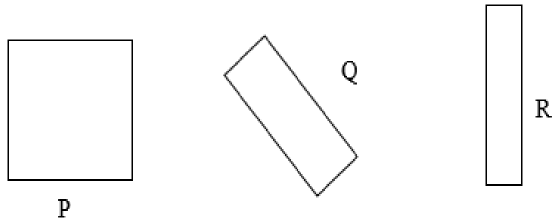
B. All properties of squares are properties of all rectangles

C. All properties of rectangles are properties of all parallelograms.

D. All properties of squares are properties of all parallelograms

E. None of the above is true.

14. Which of these can be called rectangles?



A. All of the above are rectangles

B. Q only

C. R only

D. P and Q only

E. Q and R only

15. What do all rhombus have that some squares do not have?

A. The opposite sides are equal

B. The diagonals are perpendicular to each other

C. The opposite sides are parallel

D. The diagonals are of different measures

E. None of the above is true

APPENDIX B

GHANA EDUCATION SERVICE

In case of reply the number and date of this letter should be quoted

My Ref. No: GES/PG 48 V13

Your Ref. No:

Republic of Ghana

METRO EDUCATION OFFICE
POST OFFICE BOX 337
ACCRA

Tel. No.: 030-2632370
030-2676199
054-0480229
Email: metroeducationoffice@gmail.com

1ST JUNE, 2022

THE SISO
GA-MASHIE
AMEO

**APPLICATION TO CONDUCT RESEARCH IN GA-MASHIE CIRCUIT
ACCRA METROPOLITAN EDUCATION DIRECTORATE**

I write to introduce **MR. BISMARK NII AYI ANKRAH**, a Post-Graduate student in the University of Education, Winneba offering Master of Philosophy degree in Mathematics Education.

Mr. Bismark Nii Ayi Ankrah as part of his requirements for the partial fulfillment of the award of degree, is expected to conduct and present a research to the University. His topic is: ***"Investigating Pupils' Van Hiele Geometry Thinking Levels from Private and Public Basic Schools in the Asiedu-Keteke Sub-Metro of the Accra Metropolis"***.

Permission has therefore, been granted for Mr. Bismark Nii Ayi Ankrah, to collect data on Students in the Ga-Mashie Circuit in the Accra Metropolis.

By this letter, the researcher is directed to contact you for further directives. Kindly ensure that students and teachers assist him to fulfill this academic requirement. Meanwhile you are entreated to ensure that contact hours are not compromised.

Thank you

**STEPHEN ABAMFO (MR.)
DIRECTOR OF EDUCATION
ACCRA METRO**

Cc: The Deputy Director, Supervision & Management, AMEO, Accra
The Basic School Coordinator, AMEO, Accra
Mr. Bismark Nii Ayi Ankrah, University of Education, Winneba

APPENDIX C

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF SCIENCE EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION
P. O. Box 25, Winneba, Ghana
+233 (020) 2041076
math@uew.edu.gh

LETTER OF INTRODUCTION

I write to introduce to you the bearer of this letter, Mr. Bismark Nii Ayi Ankrah, a postgraduate student in the University of Education, Winneba. He is reading for a Master of Philosophy degree in Mathematics Education and as part of the requirements of the programme, he is undertaking a research titled – ***Investigating Pupils' Van Hiele Geometry Thinking Levels From Private and Public Basic Schools in The Asiedu-Keteke Sub-Metro of The Accra Metropolis.***

He needs to gather information to be analysed for the said research and he has chosen to do so in your institution. I would be grateful if he is given the needed assistance to carry out this exercise. Thank you.

Yours faithfully,

DEPARTMENT OF MATHEMATICS EDUCATION
UNIVERSITY OF EDUCATION
WINNEBA

Dr. Jones Apawu
Head of Department