UNIVERSITY OF EDUCATION, WINNEBA

PRE-SERVICE TEACHERS' CONCEPTUAL AND PROCEDURAL UNDERSTANDING OF DERIVATIVES

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DECLARATION

STUDENT'S DECLARATION

I, ISAIAH BIGA, declare that this thesis, in an exception to quotations and references contained in already published works which have all been identified and acknowledged appropriately, is entirely my own original work, and it has been submitted, either in apart or whole, for another degree elsewhere.

SIGNATURE: ……………………………….……………

DATE: ………………………………….………………

SUPERVISOR'S DECLARATION

I, hereby declare that, the preparation and presentation of this research work was duly supervised by me and that the work was supervised in accordance with the guidelines on the supervision of research work by Faculty of Mathematics Education, Department of Mathematics Education of the University for Education, Winneba.

SUPERVISOR‟S NAME: DR. PETER AKAYUURE

SIGNATURE: ……………………………….………

DATE: …………………………………..…………

DEDICATION

I dedicate this work to my lovely wife, Ms Melody Bikala and my family. Also, a special dedication to the entire Lalam"s family. You have demonstrated time and again that, family is not only by blood. In fact, I am very much grateful to them and I owe all my success in life to them.

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ABSTRACT

This research explored pre-service teacher‟s conceptual and procedural understanding of derivatives. The research design that was used for the study was descriptive research design. Purposive sampling technique was used in the study to select sample of college students in the final year studying Mathematics and ICT in the Volta Region. The sample for the study was 61 pre-service teachers comprising 25 pre-service teachers from one college of education and 36 pre-service teachers from other college. Conceptual and Procedural Understanding Based Test (CPUBT) was the prime instrument used in collecting the data on pre-service teachers Conceptual and Procedural Understanding of derivatives. The results indicated that pre-service teachers have a lot of conceptual and procedural difficulties in derivatives. They were weak in both geometric and physical concepts used in finding derivatives, and lacked procedures used in working derivatives. Consequently, 60% of pre-service teachers who participated in the study were unable to make use of action, process, object and schema analysis of problems involving the application of derivatives. The implication is that, pre-service teachers would use rote memorization of the concept for the purpose of passing their exams without understanding the application of derivatives in real-life scenarios. To achieve pre-service teachers‟ conceptual and procedural understanding of derivatives, mathematics tutors in the study area should put more emphasis on geometric and physical concepts in teaching derivatives. They should use more activities and exercises in teaching the concept and should use the constructive approach for students to also use their construct knowledge and activities in understanding the concept of derivatives.

KEY WORDS: Conceptual Knowledge, Procedural Knowledge, Derivatives, Pre-service Teachers, Descriptive statistics.

CHAPTER ONE

INTRODUTION

1.0 Overview

This chapter discusses the following; background of the study, statement of the problem, rationale of the study, purpose of the study, objectives of the study, research questions, significance of the study, delimitation of the study, limitations of the study, definitions of terms and the organizational plan of the study.

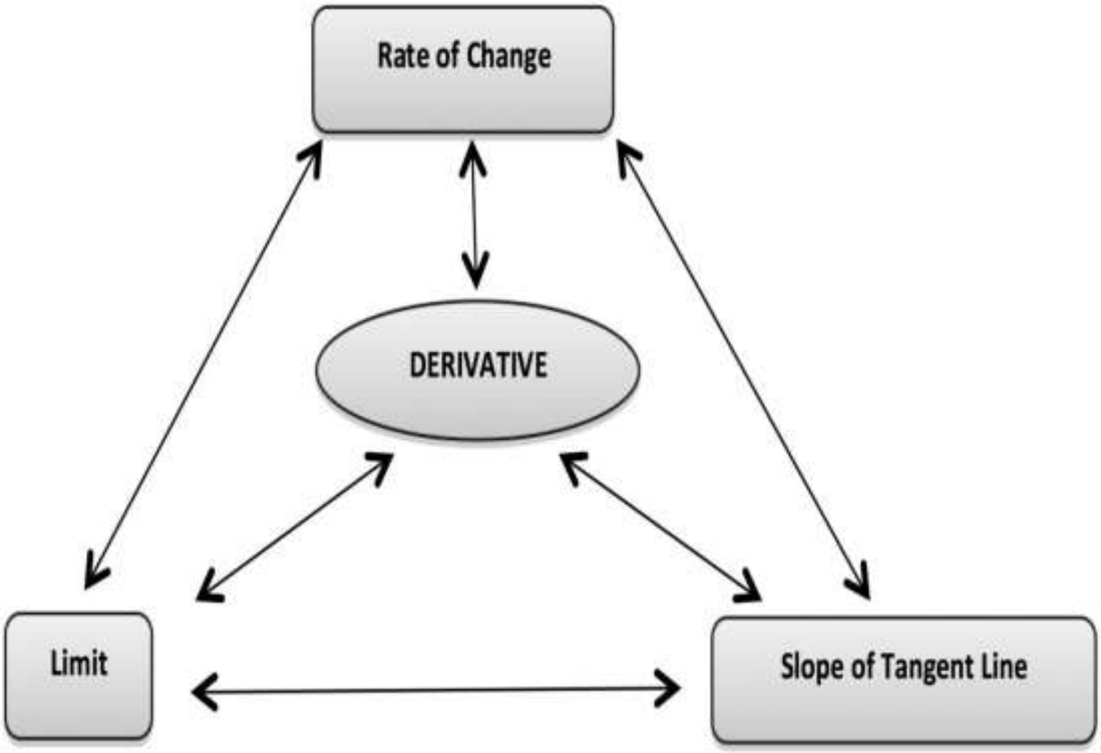
1.1 Background of the Study

The Ghanaian mathematics curriculum does not meet requirements that are currently valued globally in school mathematics (Mereku, 2010). The curriculum does not take accounts of different of mathematics in different local contexts in the planning of teaching and learning of mathematics. The teaching of derivatives falls into this category simply because learners are asked to find derivatives and integrals of functions without linking these to real-life situations. This suggests that the teaching and learning of derivatives in Ghanaian schools may not be relevant to the local context and may not prepare the students for the real-world applications of mathematics.

Carpenter et al., (2003); Obodo, (2004), indicated that, derivatives is a branch of mathematics that is studied both at the senior high schools, colleges of education and at the universities. It is the product whose value is derived from the value of one or more basic variables, called bases (underlying asset, index, or reference rate), in a contractual manner. Since mathematics education is seen as the backbone of every economic, scientific and technological advancement of any nation that is regarded as progressive. It is for this very reason that the educational systems of countries that are particular about their development lay much emphasis on the study of mathematics. In Ghana, mathematics is considered as a core subject from the basic level to the senior high level, elective subject at the senior high level and a tool for selecting students into tertiary institutions and other professions (Fletcher, 2005). Mathematics is a subject that anchors other subjects especially in the area of applied science (Gyasi-Agyei & Obeng-Denteh, 2014; Fletcher, 2005). It is in this vein that mathematics as a subject be made easier for students at both the pre-tertiary and tertiary levels through teaching and learning process to meet the demanding needs of students (Adu-Agyem & Osei-Poku, 2012; Gyasi-Agyei & Obeng-Denteh, 2014). Rhode, Jain, Poddar and Ghosh (2012) also identified derivatives (differential calculus) as an area of study which can be applied to something that moves or changes or has a shape. Tall (2009) postulates that "calculus begins with the desire to quantify how things change (the function concept), the rate at which they change (the derivative), the way in which they accumulate (the integral), and the relationship between the two (i.e., the fundamental theorem of calculus and the solutions of differential equation)". It is mostly used by mathematicians, physicists, economists, engineers and other experts to solve problems in real life (Jawurek et al., 2007). For instance, it can be applied to the study of machinery of all kinds, electric lighting and wireless, optics and thermodynamics (Rhode et al., 2012). In economics and commerce, derivatives help to solve problems in finding maximum profit or minimum cost and the like (Berresford & Rockett, 2015). Derivatives is an important topic; for this reason, if its fundamentals are not well understood, it becomes very difficult to solve issues relating to real-life situations (Rhode et al., 2012). Derivatives can also be used as a tool to model the behavior of changing quantities such as population dynamics, finding velocity and acceleration of moving object and many other more.

The teaching of derivatives at the colleges of education is designed not only to give students an in-dept knowledge of differential and integral calculus but also to provide pre-service teachers with the opportunity to apply these concepts both in other areas of mathematics learning and also in real-life situations. The syllabus at the colleges of education covered topics including, limits, continuity and derivatives of algebraic functions: derivatives of transcendental functions, implicit functions, inverse trigonometric functions and their derivatives: Hyperbolic functions and their inverse: applications of derivatives; curve sketching, maxima and minima, linear kinematics. It also covers concept of integration, techniques of integration-by substitution, by parts, the use of partial fractions, improper integrals, numerical integration (Trapezium and Simpson"s rule), reduction formulae, area between curves and volumes of solids of revolution. (Ministry of Education, 2010).

The teaching and learning of derivatives, an aspect of calculus, is not an easy task. The reason being that, it measures the level of steepness of a function, gradient of a tangent line to a curve at a given point, the rate of change output relative to input, and helps in finding critical points of a graph. Although students can solve differentiation problems correctly, they cannot explain derivative by relating it to the rate of change, the slope of tangent, and the limit (Bingölbali, 2008). Many studies demonstrate that students have difficulties in understanding related differential concepts such as functions, limits, tangents and derivatives (Mahir, 2009). The understanding of derivatives is related to rate of change, limits and slope of the tangents. The concept of derivatives is linked closely to these concepts. The relationship between these concepts is represented in the diagram below.



From the diagram above, it can be seen that the concept of derivatives spans from limits, through slope of Tangent line to the rate of change. Conceptual understanding of derivatives means one should have knowledge on these three major concepts before he/she understands the concept of derivatives better. Park (2012), noted that, derivative is regarded as a difficult concept to learn because of the poor manner in which it is taught to students. When these areas of derivatives are not well tackled, the concept of derivatives becomes difficult to be understood by students. This wasn't much different in the area research, either, as the majority of future instructors still had trouble understanding differential calculus. For instance, the majority of Colleges of Education final year pre-service teachers had negative opinions regarding derivatives since they are having trouble with the differential calculus course. The misconceptions pre-service teachers have about differential calculus fundamental theorems, which include serving as a foundation for the study of advanced mathematics such as vector analysis, complex analysis, and differential equations, are a result of mathematics tutors' inability to adequately explain their significance to them. The students were not ready to learn the idea of derivatives, as the math instructors had to add. Some mathematics tutors expressed their concerns about the poor performance of the student teachers in mathematics learning (Differential Calculus) by saying, "We have poor quality of student teachers, student teachers are lazy to learn mathematics courses, students are not ready and serious about their studies." While some of these criticisms may be partially accurate, we cannot entirely blame student teachers for their subpar results. Students' struggles to comprehend a subject can be attributed to a variety of circumstances on the part of the students, teachers, or curriculum. It is believed that a function is at the root of pupils' errors. (Brodie, 2014; Makonye, 2012; Shalem et al., 2014).

In the recent years, it is been realized that students are unable to understand and use appropriate concepts to answer questions on derivatives. The idea that students "chew and pour" instead of taking time to understand the concepts been is still rifted. The students fill unease in following the procedures in attempting a problem that needs procedures but rather concentrating on just answers to problem even if they do not understand what they are attempting. Tutors seem less concerned about knowing the origin of student teachers" errors, as they are unable to identify, interpret, evaluate and remediate. It is also reported by Khazanov (2008) that when mathematics tutors have in mind of the likely mistakes from a specific mathematics topic, their lesson preparations as well as their lesson evaluation methods will be sharper in addressing the students" likely mistakes more effectively. By doing so, the students acquire the needed and intended knowledge and skills efficiently. The issue of conceptual difficulties Ghanaian students face in achieving their goals in education needs to be addressed with importance. The conceptual difficulties can best be addressed when one is able to investigate the understanding that students have on the concept.

Research studies have attributed some of the students‟ difficulties with calculus problems to weak understanding of functions and other related graphs e.g Education, D.; (2014, 2015). Some students‟ difficulties are attributed simply to the procedures used when practicing the routine steps followed when solving calculus problems. As a result of the students‟ inability to understand and apply appropriately the conceptual understanding of derivatives, they have challenges working with the concept. Zachiarides et al. (2007) argue that procedural understanding should be focused more rather than conceptual understanding when teaching calculus which will contribute towards learners‟ difficulties in dealing with calculus problems. The argument made is that, the teaching of derivatives to students should lay more emphasis on the procedural knowledge in order to overcoming difficulties on the concept.

1.2 Problem Statement

The goal of this study is to investigate and analyze the level of conceptual and procedural comprehension of derivatives among pre-service teachers in mathematics education. With the ultimate goal of enhancing teaching methodologies and curriculum development to improve pre-service teachers' ability in teaching and learning derivatives effectively, the study aims to identify potential difficulties and gaps in their comprehension.

Derivative is regarded as a complex concept for students to understand because it contains many other several concepts such as; ratio, limit, and functions, and it can be represented in various ways - the slope of the tangent line, an instantaneous rate of change, and an expression using Leibniz‟s notation. Notwithstanding the complexities of derivatives, better understanding of derivatives is important to understanding other advanced topics such as integral, Mean Value Theorem, and Fundamental Theorem of Calculus, Ordinary Differential Equations (ODE), and Partial Differential Equations (PDE), (Park, 2012).

Research conducted by Sugan (2019), showed that the students have more difficulty in conceptual understanding of derivative which may discourage them to take the study of derivatives seriously. This lack of understanding can have far-reaching implications, as it may hinder pre-service teachers from effectively using the concept in their everyday applications. Furthermore, a weak foundation in derivatives may also affect pre-service teachers" confidence in learning calculus and its related mathematical concepts. Therefore, addressing the issue of pre-service teachers" conceptual and procedural understanding of derivatives becomes essential for enhancing the overall quality of mathematics education. In the contest of Ghanaian schools, derivatives are introduced from Senior High Schools, taught in colleges of education (especially those doing mathematics and science) and then to the universities but many students still have the notion that derivatives are a difficult concept and that they always try to memorize the formulae and the processes involved in finding derivatives. Students therefore tries to just only memorize the formulae and use it to solve differential problems (Tarmizi, 2010). Even though the students can solve the differentiation problems by route memorization. However, research indicates that many pre-service teachers struggle with both the conceptual and procedural. The conceptual understanding involves comprehending the geometric interpretation of derivatives as rates of change and understanding their significance in various real-world contexts. On the other hand, the procedural understanding requires the ability to efficiently calculate derivatives using differentiation rules and techniques.

Notwithstanding, the numerous changes to the old curriculum by the Transforming Teaching Education and Learning (T-TEL), teaching and learning of mathematics courses, including derivatives have been characterized by route memorization approaches at the college which has led to an increased in mathematical errors committed by student teachers. Sallah et al. (2021), argued out that student teachers perform poorly in finding derivatives. The study attributed the poor performance to possibly mathematics tutors‟ teaching approaches and student teachers‟ lack of procedural and conceptual knowledge. Developing targeted instructional approaches, providing opportunities for hands-on experiences, and incorporating technology-based learning tools may hold promise in addressing this challenge and better preparing pre-service teachers to excel in the teaching and learning of derivatives. It is therefore necessary to explore pre-service teachers‟ conceptual and procedural understanding of derivatives. Sallah et al. (2021), study was only limited to the Senior High School students. Though numerous works have been done on derivatives and misconception students have while learning derivatives, little is done on pre-service teachers‟ conceptual and procedural understanding of derivatives. Upon the review of Colleges of Education to Degree status, Derivatives is now a major area of study by Pre-service teachers studying Mathematics. Therefore, the purpose of this study is to bridge the knowledge gap among pre-service teachers about derivatives by investigating the Pre-service teachers‟ conceptual and procedural understanding of derivatives in two selected Colleges of Education in the Volta region of Ghana.

1.3 Purpose of the Study

The purpose of this study was to use descriptive research design to investigate the pre-service teachers conceptual and procedural understanding of derivatives in two selected Colleges of Education in the Volta region. The objectives of the study were to;

1) examine the difficulties pre-service teachers face in the learning of derivatives.
2) find out how students understand the concept of derivatives.
3) determine the procedures used by students in finding derivatives.

1.4 Research Questions

The research sort to answer the following questions;

1) What are the pre-service teachers" conceptual difficulties in finding derivatives?
2) What is pre-service teachers" knowledge on the concept of derivatives?
3) What procedures do pre-service teachers use in finding derivatives?

1.5 Significance of the Study

The study of pre-service teacher"s conceptual and procedural understanding of derivatives can be a learning paradigm in Colleges of Education to enhance the pre-service teachers" mathematical concepts and procedures as well. This is but a small contribution to Pre-service conceptual and procedural understanding of derivatives and other related concepts for Mathematics and ICT students. The study would be a fruitful and supportive document to help teach mathematics in a productive way. The research would also help College of Education mathematics tutors in the study area to also help them during their lesson"s delivery. From the study, College of Education mathematics tutors in the study area could be aware about the students" conceptual and procedural difficulties of learning derivatives and they could find alternative methods teaching and learning of the concept. This study would be of helpful to those who are interested to research in the field of derivative. In brief, the following were the significance of my study to stakeholders:

1) This research helps to do self- reflection for students so that they could improve upon their achievements in learning derivatives.
2) This research helps teachers for making their class effective and productive.
3) This research helps various stakeholders to guide in curriculum planning, textbook writing, making teaching strategy etc.
4) This research provides as a literature for those who wants to study on the topic derivatives.

1.6 Delimitations of the Study

The main delimitations of this study were as follows:

1) The study would focused only on college students enrolled particularly in two Colleges of education.
2) The study would be focused on final year students in two Colleges of Education studying Mathematics and ICT in the volta region of Ghana.
3) The study would analysed only the pre-service teachers῾῾ conceptual and procedural understanding of derivatives based on the scripts of pre-service teachers who wrote the test.
4) The study was descriptive in nature so the findings of the study cannot be overly generalized.

1.7 Limitations of the Study

The limitations of a pre-service teacher‟s conceptual and procedural understanding of derivatives can arise from various factors. Some of the limitations includes;

1. Insufficient mathematical background: Pre-service teachers may not have a strong foundation in calculus or may have gaps in their understanding of prerequisite mathematical concepts. This can hinder their grasp of the fundamental principles underlying derivatives.
2. Limited exposure to different representations. Pre-service teachers may have limited exposure to various representations of derivatives, such as graphs, equations, and real-life applications. Without experiencing diverse representations, their understanding of derivatives may be restricted to a narrow perspective.
3. Difficult in identifying and applying differentiation rules. Pre-service teachers may struggle with identifying and applying different differentiation rules accurately. They may have difficulty understanding the concept of the derivative as a limit and subsequently applying the quotient rule, chain rule and other rules correctly.
4. Difficult connecting concepts to real-life situations. Pre-service teachers may struggle to connect the abstract concept of derivatives to real-life situations, such as rates of change, optimization, and motion problems. This can hinder their ability to effectively teach the concept and provide meaningful examples to students.

1.8 Organization of the Study

The study was organized into five chapters. In chapter one, the study background, statement of the problem, rationale of the study, purpose of the study, objectives of the study, research questions, significance of the study, delimitation of the study, limitations of the study, and the organizational plan of the study were presented.

The relevant and related literature was presented in chapter two. The research described the research design and methodology in chapter three. Results and discussions were done in chapter four. Chapter five consisted of summary of key findings, implications for practice, conclusion, recommendations, and the areas for and further research.

1.9 Definitions of Terms

Difficulties in Procedural Understanding: It the difficulties in the procedural understanding and the hindrances that affect students in learning the proper use of algorithms, formulas, rules of solving problems etc. They are the factors that mostly gets students stacked when solving problems in mathematics. Most students start solving a mathematical problem correctly but perhaps because of procedural difficulties, they end up not getting the required solution to the problem. So, the weaknesses or difficulties in finding derivative from first principle, using power rule and chain rule are the difficulties in procedural understanding.

Difficulties in Conceptual Understanding**:** It is the difficulties in the conceptual understanding that hinders in learning the basic and fundamental elements in the larger structure of the content. The difficulties in conceptual understanding refers to the factors and elements that always hinder in learning the meaning of derivatives as a rate of change, derivative as a gradient of the tangent to a curve and the meaning of limit.

Conceptual Understanding**:** Conceptual understanding is when a learner is able to comprehend mathematical concepts, operations, and relations. It is the interrelationship between the basic elements within a larger structure that enables them to function together is the conceptual knowledge or understanding. Conceptual knowledge is a network of associations of mathematical procedures, integrated with the mathematical principles. It is the key of understanding the larger structure. The conceptual understanding is the meaning of derivative, as a rate of change and as a slope of a tangent line at given point in the curve and meaning of limit too.

Procedural Understanding**:** it is the knowledge of procedures that is how and when knowledge is used appropriately, and using the knowledge flexibly in performing skills. Also using the knowledge and skills accurately, and efficiently. The methods of inquiry and criteria for using skills, algorithms, techniques and methods is called the procedural knowledge. The understanding of the how the algorithms, formulae can be used in solving the problem can also be referred to as Procedural Understanding. Again, the understanding of process of finding derivatives using formula is the procedural understanding.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

This Chapter reviewed related literature relevant to the study. The literature review are the various contributions of other people"s works that have been done or similar works. This made it possible for the researcher to fine tune the contributions of other works related to the problem statement. Some of the works that were reviewed were based on both national and international works. The reason is because of the rare nature of national studies relating to the work. The review of the literature in this study was done under the following subheadings; Theoretical Framework, Conceptual framework, The concept of Derivatives, Difficulties in learning Derivatives, Derivatives misconception, Conceptual and Procedural Understanding of Derivatives, Multiple representation of derivatives concept, and Reflection on the various Literatures.

2.1 Theoretical Framework

The theoretical framework forms an important part of the research. The conceptual and procedural understanding of the students was analyzed according to the Action-Process-Object-Schema (APOS) theoretical framework according to Dubinsky (2001).

APOS Theory

The major mental mechanisms for building the mental structures of action, process, object and schema are referred to as interiorization and encapsulation (Maharaja, 2010). The mental structures of action, process, object and schema constitute the acronym APOS. APOS theory postulates that a mathematical concept develops as one tries to transform existing physical or mental objects. The descriptions of action, process, object and schema; given below; are based on those given by Weller (2009). APOS theory presumes hypothetically that mathematical understanding is made up of an individual capability, to be able to deal with perceived problems in mathematical situations by building mental action, mental processes and mental objects and its organization into schemas that makes sense of the situations and also solve the problem (Dubniskey & McDonald, 2001). APOS theory is the theory of how mathematical ideas can be studied. A brief summary of the component of the APOS theory is stated as follows; *Action*, *Process*, *Object* and *Schema*.

First, *action*. An action is a transformation of mathematical objects that is seen by an individual as externally important and as demanding, either explicitly or form memory, step-by-step teaching on how to perform the activity. Actional questions demands learners to execute the procedures to solve problems explicitly. If an action is repeated and the person meditates upon it, he or she can make an intrinsic mental development known as a process which the person can think of as performing the same kind of action, but no longer with the need of external stimuli. An individual can think of performing a process without necessarily doing it, and therefore can think about reversing it and composing it again with other different processes.

Second, the *process*. The process as part of this theory is for learners to be able reflect more on the action that have been performed. The steps of the process level are similar to the levels in action; however, the process level creates a very thoughtful process for the learner. The meaning that, learners would have to think deeply about the question before applying the technique in the action.

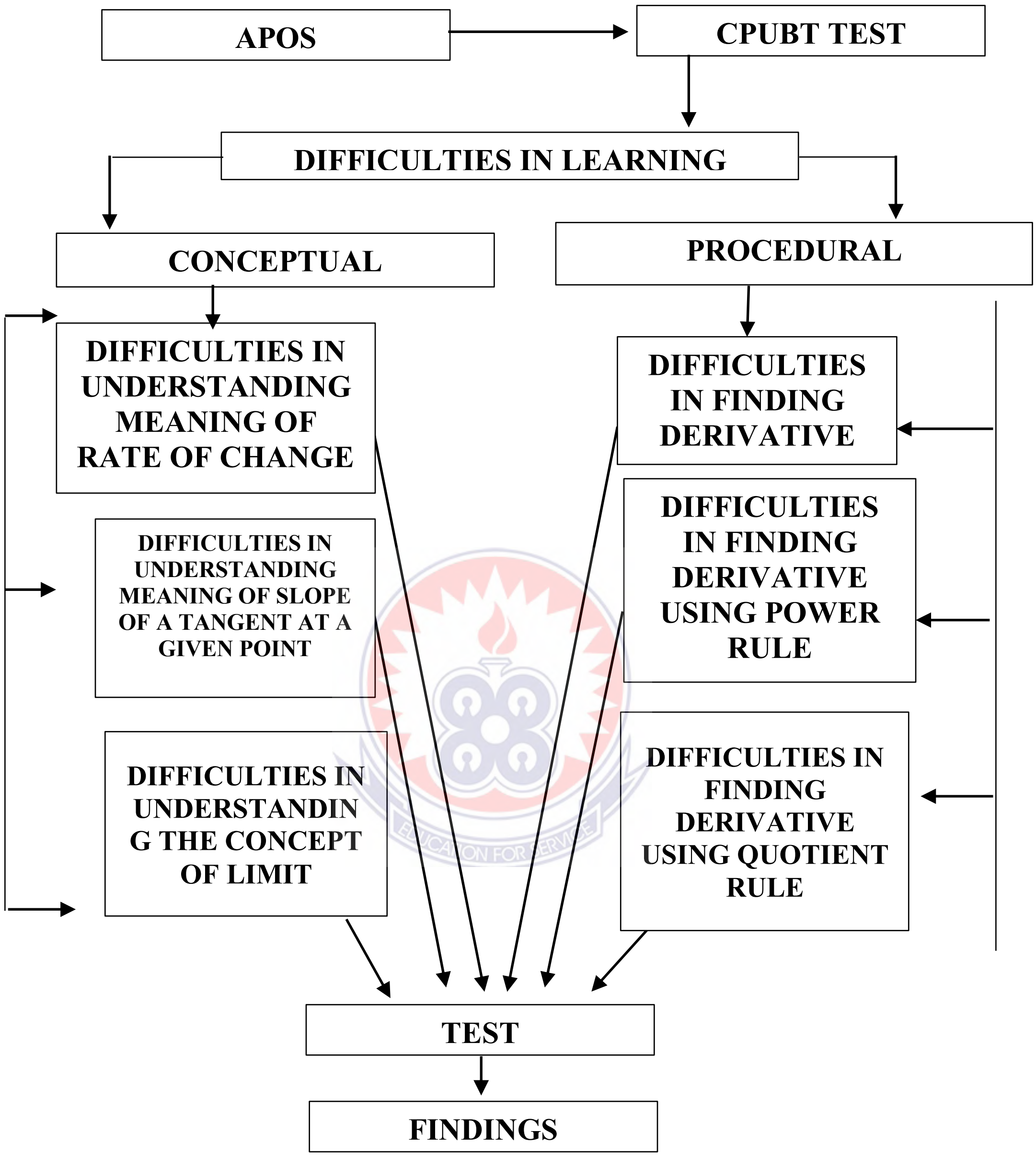
Third, Object. An object is developed from a process where the individual becomes aware of the procedures as a sum and realizes that changes can act on it. If a student is comfortable with an action and is familiar with the process level of this theory, then the learner can go up to the object level. The object level is where a student can think more deeply about the problem and critically about the problem.

Finally, the last level of the APOS theory is the schema. A schema for a certain mathematical concept is the individual putting together the actions, process, objects and other schemas which are connected with some general principles to form a framework in the individual's memory that will be brought to bear based on a problem involving those concepts. The schema is the highest level of understanding for learners. Schema connects the action, process and object procedures together. Schema questions indicate that students have a deeper knowledge of the topic. The processes, objects and previously developed schemas in the schema level that has been collected is coherent and that is invoked to deal with a mathematical problem situation. As with encapsulated processes, an object is created when a schema is put into themes to become another type of object which can also be removed from the themes to obtain the original contents of the schema. The four levels, action, process, object, and schema have been placed here orderly and hierarchically. This is an important way of talking about these constructions and, in some cases, each conception in the list must be constructed before the next step is possible. (Dubinsky, 2001). Hence, APOS theory was used as a theoretical framework of this study. According to This theory the test was constructed consisting the conceptual and procedural understanding questions. The whole questions are selected on the Conceptual and Procedural Understanding Based Test (CPUBT) and was on the basis of APOS framework which explores the conceptual and procedural difficulties in the learning of derivatives.

2.2 Conceptual Framework

Conceptual framework helps to see clear picture of the thesis. It is also called the operational road map. The problem of my study is the conceptual and procedural difficulties in learning derivative. Before designing this conceptual framework, I read several literatures on this topic. Thus, the conceptual framework for my study was constructed on the basis of review of the different literature. On the basis of the study of Tarmizi (2010), related to conceptual and procedural difficulties in learning derivative the following was the conceptual framework for the study.

The figure below illustrates how the conceptual framework was used examining the conceptual and procedural difficulties that College students experience in the study of derivative. The study found the result on the basis of conceptual framework above. How student feel difficulty in conceptual and procedural understanding and why they feel difficulty in learning derivative. That was found by using Conceptual and Procedural Understanding Based Test. The researcher found the result with help of student‟s performance in the written test. To find the difficulties in learning derivative, the researcher categorized the difficulties into two (2) types of difficulty. The first is conceptual difficulties and the second is procedural difficulties. On the levels of APOS theory the researcher constructed the Conceptual and Procedural Understanding Based Test and on the basis of test results, the researcher chose the participants and conducted the test to be able discover the two types of difficulties. The study found that students had difficulties in understanding the meaning of rate of change, gradient/slope of a tangent to a line at a given point of a curve and the concept of limit all under the conceptual difficulties. Similarly, under the procedural difficulties the study also found the students difficulties in, finding derivatives from first principle, finding derivatives using power rule and finding derivatives using chain rule.



The important insights into the relevant cognitive structures towards which teaching and learning should focus were revealed by the APOS genetic decomposition of the derivative concept and its applications. This was used to design learning activities for the concept of derivative, some of which were quite successful (Maharaja, 2013). The findings of his study suggested that students seem to face challenges when: (1)

differentiating a function that demands the application of the quotient rule, and (2) interpreting the derivative of function of limits. For the first findings, it appears that composition of functions positively influences the application of the chain rule, so this concept should preferably be focused on just before the quotient rule is introduced as a differentiation technique. In particular the detecting of which functions are involved in the composition of a given function could aid the application of the quotient rule. However, it seems that more emphasis should be placed on the detecting of embedded functions, to which the quotient rule should be applied.

For the second findings, it seems there is a need to help students set up an appropriate schema. This could include unpacking the information on the derivative represented in graphical form to a table of signs representation for $f'(x)$.

2.3 The Concept of Derivatives

The goal of mathematics as a discipline is to understand the complex relationships and patterns that underlie our environment. Derivatives are one such essential idea. Derivatives, which are derived from the latin verb derivare, or "to derive," comprise the core of mathematical concepts related to rates of change, slopes, and instantaneous velocities. This in depth, highlighting its significant significance in numerous mathematical applications and offering light on its formal nature. The derivative's fundamental representation is the rate of change or sensitivity of a function to tiny changes in its input. It reflects the instantaneous rate of change of a function f(x) at a certain point x and is formally represented as f'(x) or dy/dx. This idea, which was originally developed by Gottfried Leibniz and Sir Isaac Newton, transformed the study of calculus by giving mathematicians a method for solving issues that had previously looked intractable.

Let us explore the derivatives' mechanisms to obtain a greater grasp. Consider a straight forward real-valued function, f, with the formula y= f(x). The derivative of f, also known as dy/dx or f'(x), quantifies how the input value x affects the output value y. It relates to the specific point on the tangent line traced to the curve of f(x) geometrically. This illustration helps with intuitive understanding of the idea. Consider measuring the instantaneous velocity of a point travelling down a curve, which is identical to the slope of the tangent line at that specific location. Let's now examine the importance of derivatives in many disciplines of mathematics. Derivatives are fundamental to the study of limits, continuity, and differentiability in calculus. They offer a potent tool for analyzing how functions behave and making it easier to calculate important points. Mathematicians can objectively measure the behavior of functions thanks to derivatives, which reveal their intricate details.

Derivatives have several uses outside of calculus, including in physics, engineering, economics, and many other disciplines. Derivatives are essential for explaining motion, acceleration, and forces in physics, for example. We can accurately mod the physical world by finding the derivatives of an object's position with respect to time and its velocity and acceleration. Derivatives are used in economics to calculate the rate of change of variables like prices or interest rates, allowing economics to produce forecasts and guide decision-making.

Furthermore, derivatives go beyond functions of a single variable. Calculating the derivatives of functions with several variables is made possible by the concept's expansion, multivariable calculus. This area of mathematics reveals how variables interact in intricate systems. Derivatives, for instance, help us comprehend the geometric properties and connections between location, velocity, and acceleration while examining the motion of objects in three dimensions.

Although the idea of derivatives could seem complex and abstract, it actually forms the basis of mathematical problem-solving, modeling, and quantitative analysis. Its formal requirements call for accuracy and thorough reasoning since it combines basic mathematical concepts like limits, continuity, and differentiability. Mathematicians have used derivatives to solve the mysteries of the physical world, forecast future trends, and improve our comprehension of complicated systems.

The extended area of Mathematics Education goes from the fundamental mathematical notions to a more complicated mathematical notions that are taught in the Universities. From its epistemic, didactic, and cognitive aspects, its objective is to develop people who will be able to solve problems in their different expertise areas especially in engineering (Lopez et al. 2018). A lot of studies in the area have acknowledged the difficulties of improving teaching and learning activities with the carrying out of pedagogical methods aimed at improving student's challenges with the knowledge on concepts like limits, derivatives, and integrals (Quezada, 2020). Although different Calculus courses may have different objectives in higher education, all of them are mainly concerned with algorithms and traditional methods of mathematics teaching. With these methods learners tries to be able to apply integrals, derivatives, and basic limits, but they are unable to interpolate these concepts to a broader context. The mechanical methods by which students were taught dominate (Ruiz, and Gutiérrez, 2018).

There are investigations that have shown that in University Mathematics, when these courses are taught, they have different challenges that can be both pedagogical, epistemological and even psychological (Durand-Guerrier, and Arsac, 2005). Teachers complain that students are unable to understand and relate the main contents of these courses that they have continuity, derivatives and integrals (Sevimli, 2018). According to Jaafar, and Lin, (2017), mathematics teachers have made efforts to restructure differential calculus classroom learning and acknowledged its relevance in various countries because the notion is that this it is a challenging concept for learners. This seems to cause adverse consequences, even more when students need knowledge and abilities to solve practical and real-life problems in their careers (Vrancken, & Engler, 2014).

In conclusion, derivatives are the foundation of calculus and have a wide range of applications. Their formality necessitates careful consideration and conformity to mathematical rules. Mathematicians, physicists, and other scientists can decipher the basic patterns and relationships that govern our world by developing a thorough understanding of derivatives. Derivatives are ingrained in many fields of research, from motion analysis to economic forecasting, underscoring their crucial significance.

2.4 Difficulties in Learning Derivatives

The Derivatives represents the first time in which the student is confronted with the limit concept, involving calculations that are done no longer by simple arithmetic and algebra, and infinite processes that can only be carried out by indirect arguments. Teachers often attempt to rotate the problems by using an "informal" approach playing around the technical ideas involved (Tall, 1992). However, whatever procedural is used, a general dissatisfaction and misunderstanding with the calculus course has emerged in various countries round the world in the last decade. For instance, In France, the birthplace of the logical structures of Bourbaki, mathematics educators discovered that formal methods to learning had fundamental errors and the IREMs (Institute de Recherche sur l"Enseignement des Mathématiques) have not stopped pursued the need to make findings of the subject matter more meaningful and relevant to students learning (Artigue, 2020). In the UK a recent report of the London Mathematical Society acknowledges the difficulty of university mathematics and the need to reduce the workload and content and to rebuild the course (London Mathematical Society, 1992). The case study of Sello Makgakga (2012), found that the learners perform better in the finding derivative of a function by using rules of differentiation than using first principles. Learners have weak achievements in finding derivative from first principles but have a better knowledge finding by the use of rules of differentiation. The investigation by Orhun (2012), shows that the learners were unsuccessful in analyzing and interpreting derivative function.

In the USA, it is acknowledged that of the 600,000 students taking college calculus in 1987, only 46% obtained a pass at grade D or above (Anderson & Loftsgaarden, 1987). The atmosphere of general dissatisfaction has led to the creation of the "Calculus Reform Movement" in the United States of America, with a serious investment in the findings and technological tools is invested but with little initial investment in cognitive research. Difficulties are factors which affects the learning and deeper understanding of the concept of derivatives. This made it possible for several researchers to explore the difficulties of students in learning which helps to improve the learning abilities. This research will explore the main difficulties pre-service teachers experience in learning derivative which causes their poor performance in learning differential calculus. A similar research paper on the difficulties in derivatives by (Tall, 1992), showed that learners have difficulties in; limit and infinite process, various terminologies such as „"limit"", „"tends to"",

„"approaches"", „"as small as you please"" etc., handling quantifiers, symbolic numeric representation, subsequently, student preferred procedural methods rather than conceptual understanding. To help manage such difficulties good learning methods, basic skills and computer programming such as the Maple, GeoGebra, etc. are very effective in helping manage the difficulties in learning the concept. This paper also reveals the specific difficulties the students experience in learning derivative which at times causes their weak performance in learning derivatives. Araaya and Sideli (2012), recommended that mathematics teachers teaching concepts of calculus for beginners (secondary level) must take the necessary pedagogical-content care because students generally difficulties learning calculus.

2.5 Derivative Misconception

Misconception plays an important role in reducing the conceptual understanding. Misconception is defined by Zembat (2010), as the perception (conception) that is far from the consensus of the experts" perceptions for a specific subject. There is a thin fine line between mistakes and misconceptions. Mistakes are regarded as error in learning concept while misconception are considered as blockage to learning a concept (Keçeli, 2007; as cited in Kaplan, 2015).

According to Park (2012), the word "derivative" is colloquially used for both "derivative at a point" and "derivative of a function," which mostly a time confuses students about whether or not the word "derivative" refers to (a) a specific-point value or (b) a function. Students" misconception about derivatives is also mostly related to their thinking about the tangent to a line on graphical situation (Park, 2012). Another misconception is a tangent to line on the curve should meet the curve only once at the point tangent and students usually tries to find the equation of tangent to a line on the curve. They did not make the connection between the equation and the graph of tangent line because it intersects the curve at the other point also besides the tangency point when they are extended on the line (Kim, 2005; Park 2007; as cited in Park, 2012).

Kaplan et al. 2015, indicated that students are unable to use the operation with the meaning of the derivative. The intervals were not considered while finding the derivative. The misconception is grounded from the thinking that derivative of a function is the derivative at a specific point.44 The students have their misconceptions rooted from not knowing the symbolic representation, graphical representation and difficulty to construct a relation between the slopes, tangents and normal. Students equally face difficulties in finding derivative from the first principle. If students can actually relate the relationship between the average rate of change and the instantaneous rate of change and then connect it to the slope of a tangent to a curve then they would have a clear understanding of the concept of derivative.

Calculus is perceived as a challenging subject among most of the students due to the fact that they do not really understand the whole idea of function. For instance, the function $y = f(x)$ is a process: the input carried out in the procedural is x, and the output in the procedural is y. Students have some misconception in function so they need special treatment such as extra and more tutorial sessions in correcting their misconceptions and their confusing in Problem solving strategy according to George Polya (Tarmizi, 2010). Function is one of the major components of the derivative therefore the understanding of functions will go a very long way to help in understanding derivatives.

A study conducted by Tyne (2016), revealed that students have misperceptions about slope. Students‟ misperceptions about linear functions, and directly proportional relationships specifically (for which the slope is a ratio-of-totals), lead to a destitute (poor) understanding of slope. Aside the slope interpretation questions, students still performed badly, and those who perform badly especially on the slope interpretation questions were carried most likely by their misconception on derivative interpretation (Tyne, 2016). The study also revealed that students who used total-of-ratios approach in analyzing the slope often used the same idea in working derivatives. A ratio-of-totals approach to slope interpretation was the dominant incorrect reasoning. Students do not have the full glare of what gradient and derivative mean as a rate of change in the context of modeling situations, nor do they understand appropriately the uses of gradient and derivative to make predictions. Tyne (2016), also pointed out that students find it difficult with knowing what the gradient (slope) and derivative represent and how to use them correctly to make estimations. The prevailing incorrect reasoning by students was to see gradient (slope) as the ratio-of-totals ($\frac{y}{x}$) instead of ratio-difference ($\frac{\Delta y}{\Delta x}$). Considering gradient (slope) as a ratio-of-totals means that all linear functions are directly proportional ($y = mx, where\ y = 0$); students may be tempted to interpret the gradient (slope) as something that can be used to calculate the value of the dependent variable. This wrong thinking about slope influenced students‟ understanding of derivative. As a result, they often interpret derivative as something that could be used to find the value of the dependent variable (by multiplying the derivative by the value of the independent variable). Moreover, when students were quizzed to criticize the reasoning of a hypothetical person‟s predictions, they showed a very limited knowledge of how the derivative can be used to make valid predictions.

Instead of demonstrating understanding that the derivative can be used to estimate change only near the input value.

In a study conducted by Tarmizi (2010), Tarmizi discovered that students did not fully understand functional notations, memorized a procedure, and could not verbalize what they were doing in the problem. Even though this study was conducted with a simple function question, it still can be tied to derivatives. Students memorize the tricks and techniques to find derivatives without going through actual procedural; however, students often cannot make the meaning behind what they calculate verbal. The students think sometimes that x tends to 2 means exactly two which is a very wrong concept. The meaning of instantaneous is very important to understanding derivative. So, the concept of limit may be one of the causes which affect the understanding of derivative. Research conducted by Tarmizi explores the specific misconception which is not mention in other literature. A study conducted also by Zulal et al. 2015, postulates that the students were not aware of the linkage between the average rate of change and the instantaneous rate of change. On the other hand, they could not link the rate of change to the idea of limit. More to the point, it was realized that the participants did not make sense of the instantaneous rate of change as the slope of the tangent line. These results showed that none of the students in this study knew and could explain the meaning of the rate of change, why the rate of change is related to derivative, and how the rate of change is related to the limit and the slope of tangent line. Therefore, in the light of the theoretical framework used in this study, it can be concluded that the students" understanding of the rate of change in relation to the concept of derivative was rather instrumental. On the other hand, the students provided the meaning of derivative as the gradient (slope) of a tangent to a line drawn to the curve at a certain point. However, they interpreted the equation of the tangent line at a certain point as the derivative function of the function.

Another similar study done by Paparvripidon et al. 2014, indicated that Students do not understand the concept of a derivative fully and have trouble correctly answering derivative application questions. Teachers with a strong understanding of their students‟ knowledge can iron out their lessons to accommodate students who partially understand the concept of a derivative. Students use the principle of derivative in multiple classes through college depending on their majors. It is important for them to easily understand the concept of a derivative and be able to apply it. Teachers should review conceptual questions that are missed by students in order to help students understand derivatives at a higher reasoning level. The teacher plays an important role to reduce the difficulty in learning derivative.

2.6 Conceptual and Procedural Understanding of Derivatives

Notwithstanding the fact that conceptual and procedural understanding cannot always be separated, it is important to distinguish between the two types of understanding for better knowledge development. Conceptual and Procedural Understanding is one of the major components in learning derivatives. A concept is ‚an abstract or generic idea generalized from particular instances‟ (Merriam-Webster‟s Collegiate Dictionary, 2012). This knowledge is usually not linked to a specific problem type. It may be in the form of implicit or explicit, and does not necessarily need to be verbalized. Conceptual understanding can be defined as a functional hold and an integrated grasp of ideas of mathematics. On an important note, conceptual understanding is understanding what is more than an isolated fact thus, connecting between those facts and facts that are well organized. Understanding these concepts is mostly known as *conceptual understanding* (e.g., Canobi, 2009; Rittle-Johnson et al. 2001). Schneider and Stren (2005) also explained conceptual understanding as the understanding of the major concepts, facts and principles and their interconnections to a particular domain, knowledge that has already be consisted of those relationships constructed internally and connected to already existing understanding.

Engelbreeht, et al. 2016, had this to say about conceptual understanding, that conceptual understanding refers to the ability to show understanding of mathematical ideas by being able to interpret and apply them correctly to different situations as well as the ability to translate these ideas between verbal statements and their equal mathematical expressions. The condition of having conceptual understanding ability to show the connections between ideas or between ideas and procedures. Learners identify principles with the use of their knowledge on conceptual understanding, why, where, what and when to use definitions and facts in mathematical concepts, compare and contrast other related concepts. Conceptual understanding is knowing how or why to apply a concept that can be adapted, adjusted and applied on another situation. Conceptual understanding cannot be downplayed as far as the study of mathematics especially derivatives is concerned. In plays an integral part in the teaching and learning of derivatives on learners. Students demonstrate *conceptual understanding* in mathematics when they provide evidence that they can actually identify, sort out, and generate examples of ideas; use, link and interrelate models, labels, manipulations, and varied representations of ideas; identify, use and apply principles; know, use and apply facts and definitions that can be compared, contrasted, and integrated to related concepts and principles; identify, explain, and apply the labels, symbols, and terms used to represent ideas. *Conceptual understanding* reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either (The National Assessment of Educational Progress, 2003).

Let us consider procedural understanding on the other hand. A procedure is a series of steps, or actions, done to accomplish a goal or an aim. Knowledge of procedures is often termed *procedural knowledge* (e.g., Canobi, 2009; Rittle-Johnson et al., 2001). For instance, „Procedural understanding refers to „knowing how", or the understanding on the processes needed in attaining various goals or aims. Research has is that procedural understanding is the understanding involved in solving a problem while conceptual understanding is the deeper understanding of the content which helps in the procedural understanding.

As with procedural understanding, the meaning of procedural understanding has sometimes included additional constraints. Within the contest of mathematics education, Star (2005) acknowledged that sometimes: „the term procedural understanding shows not only what is known (knowledge of procedures/processes) but the one way those procedures/processes (algorithms) can be made known (e.g., superficially and without good connections)" (p. 408). Star (2005) noted that: „The term *conceptual understanding* has come to encompass not only what is known (understanding of concepts) but also one way that concepts can be known (e.g., deeply and with rich connections)" (p. 408).

Baroody et al. (2007), made a suggestion that conceptual knowledge should be defined as „knowledge about facts, (generalizations), and principles" (p. 107), without requiring that the knowledge be richly connected. Empirical support for this notion comes from research on conceptual change that shows that (1) novices" conceptual understanding is often disarrayed and need be merged over the course of learning and

(2) professionals" conceptual understanding continues to increase and grow better when organized (diSessa et al. 2004; Schneider & Stern, 2009). Thus, there is populace consensus that conceptual understanding should be referred to as understanding of concepts. A much more constrained meaning demanding that the knowledge be richly connected which has sometimes been used previously, but recent thinking argues that the richness of connections as a feature of conceptual understanding thus increases with expertise.

Mahir"s (2009), and Tatar and Zengin"s (2016) studies postulate that, one of the main reasons why most students experience challenges in learning calculus expanse from the inadequacy of conceptual understanding. Muzangwa and Chifamba (2012), also opined that the deficiency of conceptual knowledge of calculus is a problem that can limit learners when learning other related scientific courses.

Another study by Parker et al. 2008, indicates that learners" choice for procedural understanding, resulting from their previous achievements with those procedural skills, is inculcated in their conceptual understanding. They have differences between procedural understanding and conceptual understanding which is a key factor that resulting to their poor in understanding graphical functions from information about the derivatives. Making learners free from learning procedurally is an important part of increasing conceptual understanding in calculus. The study discovers procedural understanding alone is a cause of making poor understanding of the conceptual knowledge. So, in other to increase the conceptual understanding we have to reduce the procedural understanding in our teaching and learning. This is because when there are a lot of procedures involved in solving a problem, learners turn not to like the concept thereby affecting their conceptual understanding since procedural and conceptual understanding are very good bed fellows.

2.7 Multiple Representations of Derivative Concept

A lot of research reveals that the multiple representation approach is very necessary for teaching and learning. The representation of the concept from different ways enables the teacher and learners to make a deep sense of understanding of the concept. In this regard, the research by Hana, Samer, Iman, (28th ICTCM), captioned "Effects of Technology Aided Multiple Representation (numeric, symbolic, graphical) Approach on students understanding of derivatives" led to the methodology used in the uncontrolled (experimental) group (multiple representation approach) is useful in helping learners to develop a good understanding of concept of derivatives. Learners could explain the relationship between graphical, numerical and symbolic representation of derivative in the experimental group than that of control group. Therefore, the representation of the content with various methods makes the deeper conceptual understanding of derivative. It also helps to relate understanding of concept and blocks the knowledge of compartmentalization. Students increase their concept knowledge better when presented with concepts on images which contains both graphics and algebraic representations of similar or same concepts (Aspinwall & Miller, 2001; Aspinwall & Shaw, 2002a).

According to Aspinwall and Shaw (2002a), teachers enhance students learning by the presentation of graphics and algebraic models to solve the same mathematical problem (as cited in Abbey, 2008). Students may basically rely on skill such as geometric or analytic to solve derivatives problems; whereas either of them is incorrect, each provides a different understanding of the problem (Aspinwall & Shaw, 2002). (Ubuz, 2007, as cited in Abbey, 2008), shows that Learners struggle with linking graphical and algebraic representations and with changing information in symbols to information in graphics. Several researches indicate that, one way of presenting the content makes it difficult to understand the content conceptually. Therefore, multiple ways of presenting the topic plays an important role in reduce difficulties in teaching learner"s derivative.

2.8 Reflection on the Various Literatures

The literature reviewed above is mainly focused on the misconceptions of the derivative, the student's conceptual understanding & procedural understanding of the derivatives, the difficulties students face in learning derivatives and some methods that can be used to help reduce the difficulties. A lot of studies are mostly concerned with conceptual understanding and how to overcome some of the misconception. For instance, so in the study above the difficulties of the students and the causes of their poor performance in finding derivatives are not mentioned. The particular challenges that are faced by the learners while learning derivative are not mentioned in the literatures above. So, my study seeks to find out the difficulties that higher students (Colleges of Education) face and the various methods that College Tutors use in teaching the concept of derivatives.

CHAPTER THREE

METHODOLOGY

3.0 Overview

This chapter deals with the research paradigm and design of the study, population, samples and sampling procedures, the tools for data collection, reliability and validity of tools, procedures for data collection and the procedures for analyzing the data.

3.1 Research Paradigm

A paradigm is a shared world view that represents the beliefs and values in a discipline and that guides how problems are solved (Schwandt, 2001).

The research paradigm used for this study was the positivist research paradigm. The positivist research paradigm is a framework that emphasizes objectivity, knowability, and deductive logic. It is based on the belief that knowledge should be derived from empirical evidence and scientific methods. Positivism claims that knowledge is based directly on experience and focuses on facts and the causes of behaviour (Chilisa & awulich, 2012). Due to the nature of the data collected, positivist paradigm design was used to help the researcher align the study with the appropriate methods and provide a coherent and rigorous approach to explore the pre-service teachers‟ procedural and conceptual understanding of derivatives.

3.2 Research Design

Research design explains how the study was carried out. It describes the study‟s overall procedure: how the research is set up, what happens to the participants, and what data collection methods are used (McMillan and Schumacher 2006). A research design is a methodical inquiry that can be defined as a procedural strategy followed by the researcher to answer the research questions properly, objectively, and accurately (Kumar 1999).

Mouton (2001), also saw research design as a plan or blueprint of how one plans to perform the research. Its major goal is to lay forth a strategy for gathering empirical evidence to address the research questions (McMillan & Schumacher 2006). It is critical to select a research design that focuses on both the research challenge and the final output. The goal is to employ a design that allows the most valid and reliable conclusions to be drawn from the responses to the research questions. Research designs are useful because they assist and guide the decisions that researchers must make during their investigations and set the logic by which they develop conclusions at the end of their studies (Creswell & Piano-Clark, 2011). This study descriptive design.

In this particular study, a descriptive research design was used to explore pre-service teachers‟ conceptual and procedural understanding of derivatives. Descriptive design according siedlecki (2020) is a sort of design that tries to systematic gather data to characterise a phenomenon, circumstance, or population that is being examined.

3.3 Population

McMillan and Schumacher (2006) define a target population as a set of elements, which can be people or objects, who meet particular criteria and to whom a researcher aims to generalize the study‟s findings. The study was conducted in two Colleges of Education in the Volta region. The researcher purposely chose two Colleges for the study for reasons of "accessibility and convenience," a valid and helpful technique noted by McMillian and Schumacher (2006). Therefore, the population for the study comprised all final year pre-service teachers offering Mathematics and ICT of the selected Colleges. Final year pre-service teachers were considered because they had just completed the semester which derivatives was taught as at the time of the study.

3.4 Sample and Sampling Procedures

According to Mugoh (2002), sampling is the act, process, or technique of selecting a suitably representative part in order to determine the characteristics of the entire population. For the purposes of the study, a sample is a group of respondents (people) chosen from a wider population (Cohen et al, 2007). The sample size for this study was all the pre-service teachers in the two colleges offering Mathematics and ICT. But due to some circumstances beyond the control of the researcher, 61 of the pre-service teachers comprising of 25 from one of the colleges and 36 from the other college was what is used as the sample size for the study.

3.5 Instrumentation/Test

Considering the research questions and the type of research that was undertaken, data collected were both qualitative and quantitative. The method that was used in obtaining data was through test.

Conceptual and Procedural Understanding Based Test (CPUBT). The objective of the research was to investigate pre-service teachers conceptual and procedural understanding of derivative. To investigate the pre-service teachers conceptual and procedural understanding of derivatives, the researcher conducted a test having conceptual and procedural understanding questions. The instrument that was administered was put together by the *APOS (Action, Process, Object, Schema)* framework. The questions were set according to the order of difficulties based on the levels of *APOS.* The first problems were problems involving action, process problems, object and the finally on schema. The Pre-service teachers had one hour to use in the assessment. Problems on the assessment were logically arranged from the simple level of conceptual understanding to the highest level of conceptual understanding. Pre-service teachers were told to solve the problems to the best of their understanding and their abilities.

Test questions made for the respondents were made on the ideas of APOS model in order to observe the pre-service teachers‟ difficulties on derivatives and their conceptual and procedural understanding of derivative. The problems were to test pre-service teachers‟ difficulties conceptual understanding, procedural knowledge and the difficulties that students experience in dealing with problems on derivatives. The problems that were given were ten. Questions 1-5 were action level problem. Here, problems were straight forward and were simple for students to do. *Action* questions needed students to show the steps to solve problems clearly based on their conceptual understanding. It is worth to note that both *Action* and *Process* questions uses the *Object* and *Schema* in their approach. Critical attention was paid to how the pre-service approach the questions based on their understanding. The procedures used in working the questions is also being looked at to ascertain the procedural understanding the pre-service teachers have. The second level Problem were from 5-10 where pre-service teachers were to really demonstrate their conceptual understanding of derivatives to solve problems believed to involve *process* of the APOS model. Here too, both *Object* and *Schema* are both applied to the *Process* level to demonstrate their conceptual and procedural understanding of derivatives.

The feedback of the pre-service teachers in their solutions will indicate if they have conceptual and procedural difficulties in solving derivatives and it will elicit their conceptual and procedural understanding of derivatives.

To assess students‟ performance on the Conceptual and Procedural Based Test, a marking scheme (see Appendix C) was created. The scheme was created based on expects advice in the area of study and the supervisor. The marking scheme served as an observation guide or checklist to guide in analysis of the data. The answers of the students were evaluated holistically, and marks were assigned based on procedures as method mark (*M*) and correct answers as *A*. This was done to check the procedures used by pre-service teachers in other to have sufficient knowledge about the pre-service teachers procedural understanding of derivatives. Pre-service teachers were given marks for the sections of questions that they had corrected based on the marking scheme on their inputs.

3.6 Validity and Reliability of the Instrument

The researcher used two types of validity that are typically expected of test instruments. Content validity and face validity.

Content validity is one of the requirements a research instrument must meet. For instrument to be content valid, it must cover all the content that is required to measure all the variable understudy. Nikolopoulou (2022), said that test or assessment instrument has content validity if it covers all relevant parts of the construct it aims to measure.

The face validity is another important requirement of a research instrument should meet. A face validity is a type of validity concerned with whether a test appears to measure what it is supposed to measure (Gassett-Webb & Yolanda, 2022). Gassett-Webb & Yolanda (2022), added that face validity is determined by one looking over the test designed in research and deciding if the questions in the test are related to the topic.

Dikko (2016) stated that “with every research design, instruments chosen for the collection of data must pass the tests of validity and reliability before they can be considered good measures”. The issue of validity and reliability are major importance to every research and thus the credibility of the research study rely on the reliability of the data, methods of data collection and on the validity of the findings of the research (Cohen et al. 2007)

Validity does not connote the same meaning in qualitative research as it means in quantitative research, nor is it a companion of reliability (weighing the stability or how consistent the responses are) or generality (using the results to new settings, samples or participants in terms of external validity). Qualitative validity means the study investigates the accuracy of the results by adopting some particular procedures, whereas quantitative reliability shows that the researcher's procedure is consistent across different researchers and different projects (Green et al. 2007: as cited in Creswell, 2009,190). Reliability and validity of tools is among the major important parts in any study. In this study, the CPUBT was constructed on the ideas of the research by Constantinou (2014). However, the pilot test was conducted to ensure the reliability of the CPUBT. The researcher analysed the reliability of test by using the split half method after the pilot test and found out that reliability coefficient is 0.92 (See Appendix A). The reliability coefficient shows that the test questions were reliable. Also, the test items were guided by the supervisor in constructing the items. This affirms the reliability and validity of the Conceptual and Procedural Understanding Based Test.

3.7 Goodness and Trustworthiness of the Instrument

To ensure that there is goodness and trustworthiness; Lee, (2012), suggested the following criteria as cited in (McAninch, 2015). Therefore, these criteria were used to maintain the Goodness and Trustworthiness of the study. The criteria used are triangulation of data, member check, prolonged engagement in the field, and peer review, rich description, Researcher role, etc. These were used to ensure the trustworthiness of the instrument. Different text books were used, different course outlines were consulted from different colleges and the nature of the previous semester exams scripts from different colleges were what guided the preparation of the test items to ensure its goodness and trustworthiness.

3.8 Data Collection Procedure

The study was conducted in a sample selected from related population. The required data was collected from the Colleges. Through the help of the introductory letter from the university, the researcher got in touch with the respondents. The researcher collected the data through the Conceptual and Procedural Understanding Based Test (CPUBT). These test items were administered to any student in the study area. Pre-service teachers that were available at the time of the study took part in the study. A maximum amount of time was allowed for the pre-service teachers to use to answer the test items. The scripts of the pre-service teachers were collected from them when they were done. After conducting the test, the researcher analysed the test result and on the basis of test result.

3.9 Data Analysis and Interpretation Procedures

Result of the Conceptual Procedural Understanding Based Test was descriptive data. The data for this study was collected from the final year pre-service teachers in two colleges of education in the Volta region of Ghana. Participants were administered exam by using a Conceptual Procedural Understanding Based Test aforementioned. Every participant was given an assessment with various derivatives questions. Each derivatives question was categorized at a different level of the *APOS* model. Students" assessments were manually graded and scored out of over seventy (*70*) points by the researcher.

The data was thoroughly analyzed based on the pre-service teachers" difficulties in derivatives, based on the conceptual understanding according to the percentages of pre-service teachers who were able to solve the problems, and finally analyzed based on the procedural knowledge pre-service teachers have. According to Bhandari (2023), descriptive statistics is a statistical method used to summarise and describe the basic features of a data in a study. With descriptive data, you are simply describing what or what the data shows.

The data collected was analyzed using item-by-item analysis. This was done by analyzing the questions on the Conceptual and Procedural Understanding Based Test (CPUBT). Finally, the themes were generated from the *APOS* model such as understanding of derivative as a rate of change, understanding derivative as a slope of tangent, concept of limit, finding derivative using quotient rule, finding derivative using product rule and applications of derivatives.

3.9.1 Ethical considerations

Permission was sorted for the Pre-service teachers from the mathematics department of two of the Colleges of Education prior to the commencement of the research and the anonymity of the participants was secured. The researcher was to ensure that the confidentially of the respondents was maintained. The confidentially of all available information by the respondents were secured by considering only the scripts of the respondents without considering the names of students for those responses.

CHAPTER FOUR

RESULT AND DISCUSSION

4.0 Overview

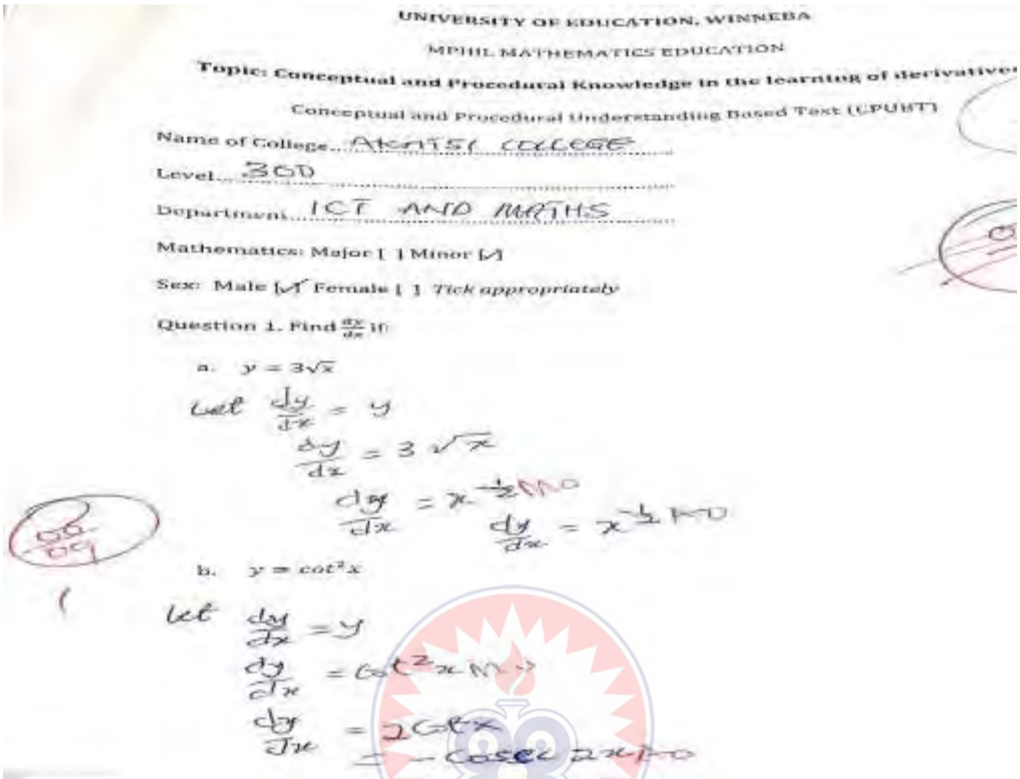
The results of the study were organized under the themes in the research questions.

4.1 What are the Pre-Service Teachers' difficulties in Learning Derivatives?

Students‟ conceptual difficulties in derivatives affect how they apply the knowledge in calculus and its related concepts (Chappell & Killpatrick, 2003). Students find it difficult to use the idea of derivatives in their daily lives since they have difficulties in the concept. They assume the concept is of no essence and the fact that they are not going to teach derivatives at the levels of teaching they will possibly find themselves in the teaching field. Some of the students think they only want to pass their end of semester exams. They do not make much efforts to understand the whole idea of derivatives.

The results obtained from the Conceptual and Procedural Knowledge-Based Test (CPUBT) shows that more than 50% of the students have difficulties in solving Derivatives and their application concepts. Students struggled to either solve the questions or use rules that are involved in the differentiation.

A lot of students had it challenging to find the $\frac{dy}{dx}$ of an equation involving a fraction. Analysis of the scripts of the students on Question One shows, students had conceptual difficulties. The difficulties are, most students could not use the knowledge of rewriting the equation before they differentiate. More than 50% of the students had difficulties in solving Question One (a) and (b). Both solutions needed students to rewrite the equation before finding the $\frac{dy}{dx}$ and rewriting the solution in the form of the question. The figure 1 below shows how some students tried to solve the question.


UNIVERSITY OF EDUCATION, WINNEBA

MPHIL MATHEMATICS EDUCATION

Topic: Conceptual and Procedural Knowledge in the learning of derivatives

Conceptual and Procedural Understanding Based Test (CPUBT)

Name of College: AKATSI COLLEGE

Level: 300

Department: ICT AND MATHS

Mathematics: Major [ ] Minor [✓]

Sex: Male [✓] Female [ ] *Tick appropriately*

Question 1. Find $\frac{dy}{dx}$ if:

a. $y = 3\sqrt{x}$

b. $y = \cot^2 x$

Figure 1: Sample Script 1

They have conceptual difficulties with the question. The inability of the students to use the conceptual and procedural knowledge in solving Question One shows that they had difficulties in the concept.

Question Two (b) was another difficulty that students faced when working on finding the gradient of a curve. The analysis shows, students were unable to use the product rule to differentiate the function. Most students were unable to state the formula used in the product rule. About 50% of the students could not either state or use the formula to find the derivative of the function. The script below is for one of the students who wanted to use the product rule to differentiate the curve but wrongly quoted the formula to be used in finding the derivative of the curve as the gradient of the curve.

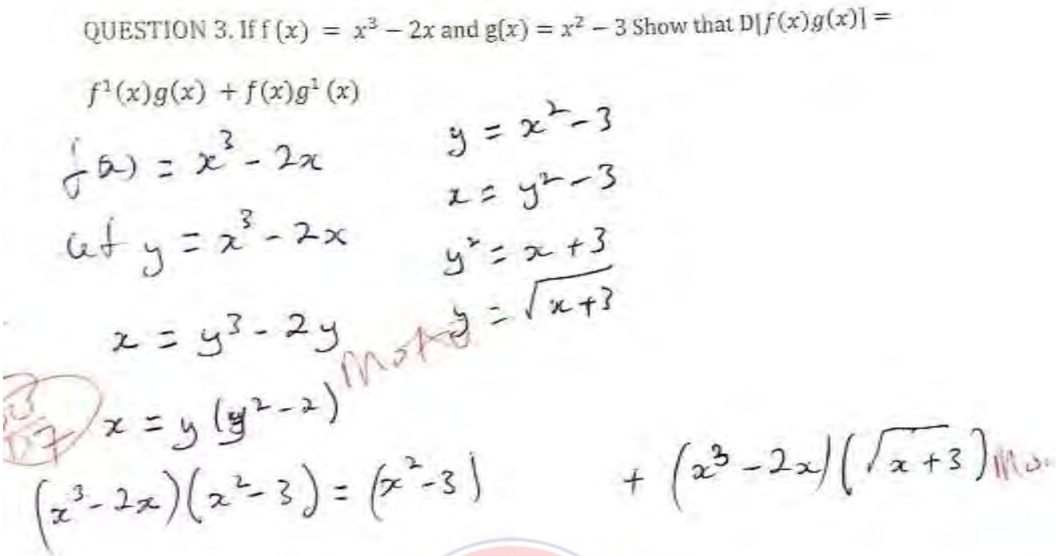


Figure 2: Sample Script 2

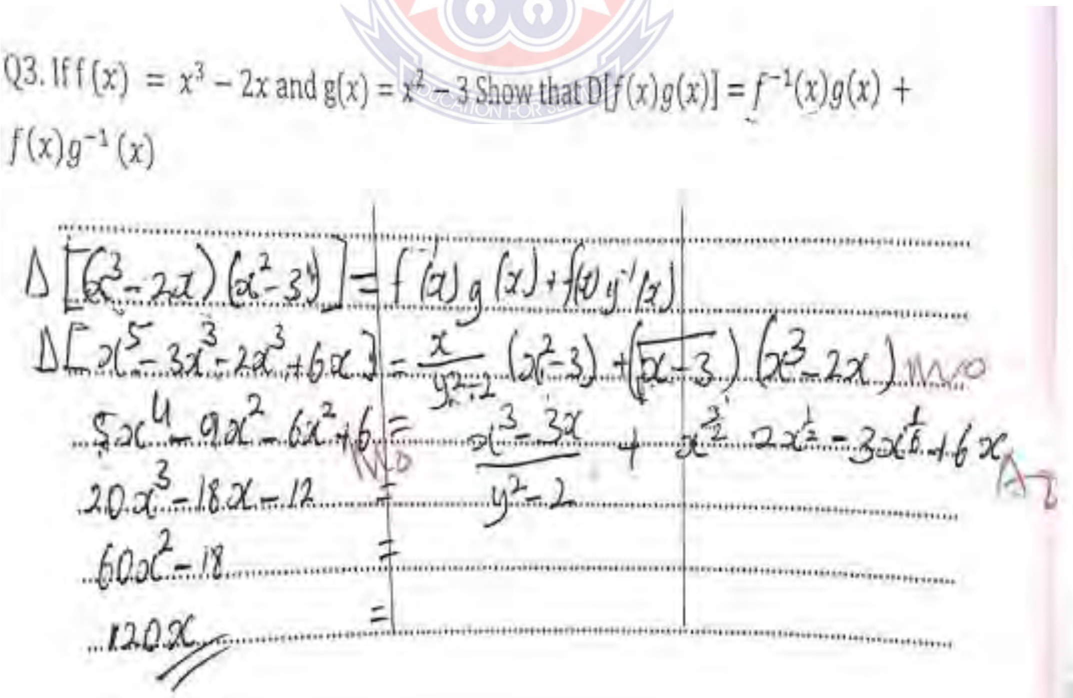


Figure 3: Sample Script 3

Students also had some conceptual difficulties with Question Three on how to write the composite function and use the product rule to differentiate. More than 50% were unable to differentiate the composite functions as they do not understand the demand of the question. The figure 4 and 5 below shows how some students could not find the derivative of the composite function.

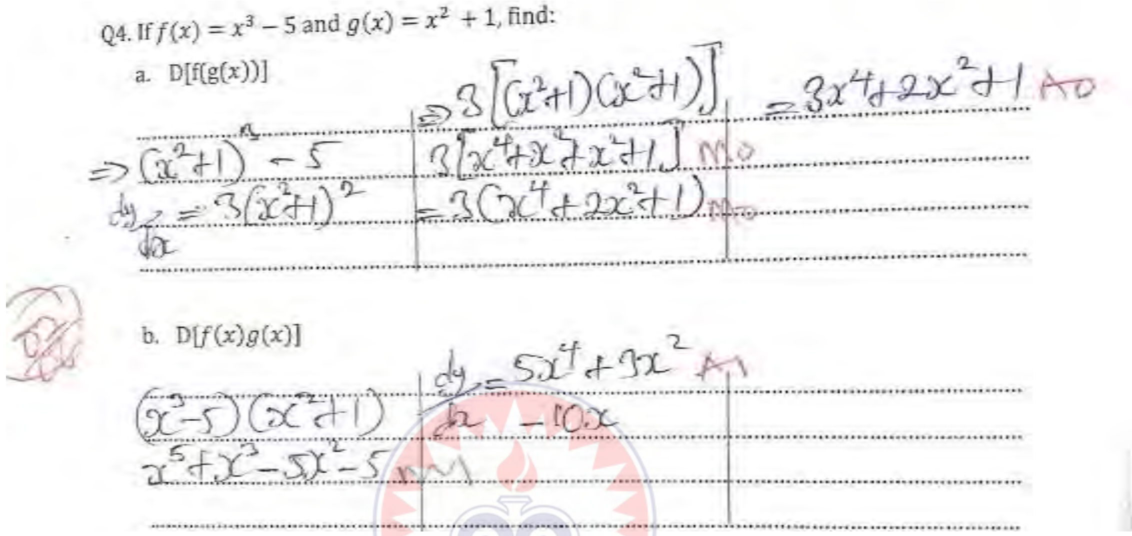


Figure 4: Sample Script 4

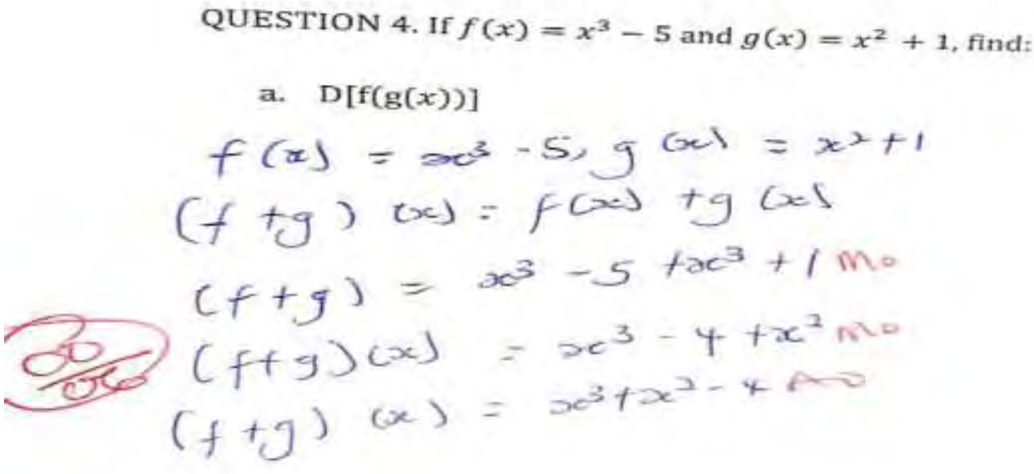


Figure 5: Sample Script 5

The concept in these questions was using D to mean differentiate of find $\frac{dy}{dx}$. Some students only understand $\frac{dy}{dx}$ to mean derivative. Some also understand $\frac{dy}{dx}$ and gradient to mean differentiate but do not know D in a composite function, it means differentiate. Some also had difficulties understanding that f^1 *and* g^1 also, mean to differentiate the function. This is because questions that involved finding $\frac{dy}{dx}$ were mostly attempted by the students trying to differentiate but other terms such as finding the gradient, f^1 *and* g^1, and D were not attempted by most students.

Another difficulty pre-service teachers faced was Question Six. Most Pre-service teachers had difficulties in evaluating with limits. Below are the scripts of pre-service teachers" difficulties in tackling Question Six.

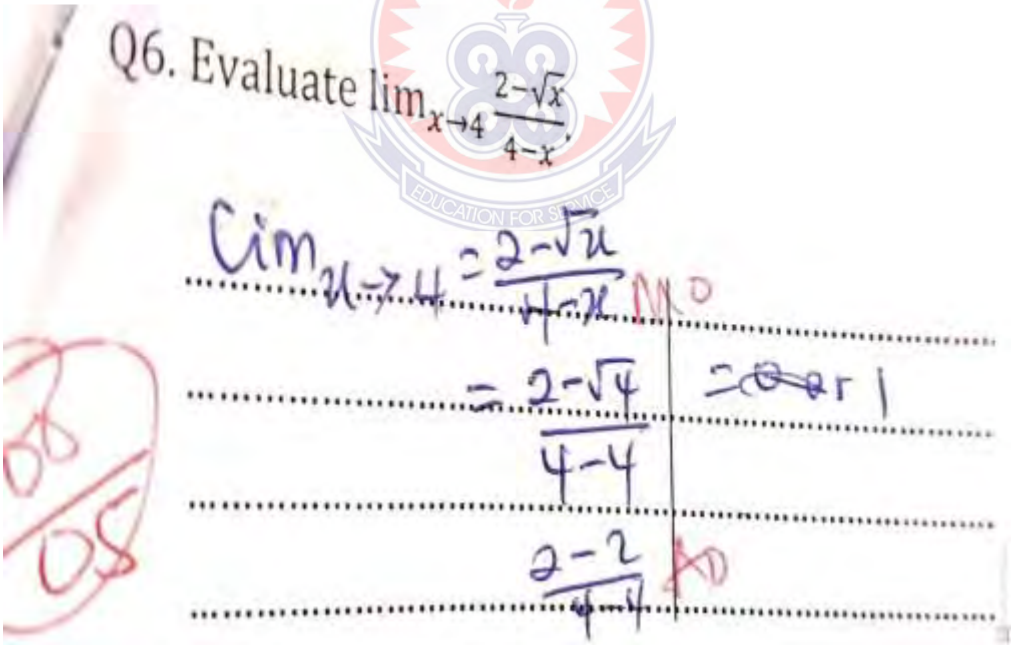


Figure 6: Sample Script 6

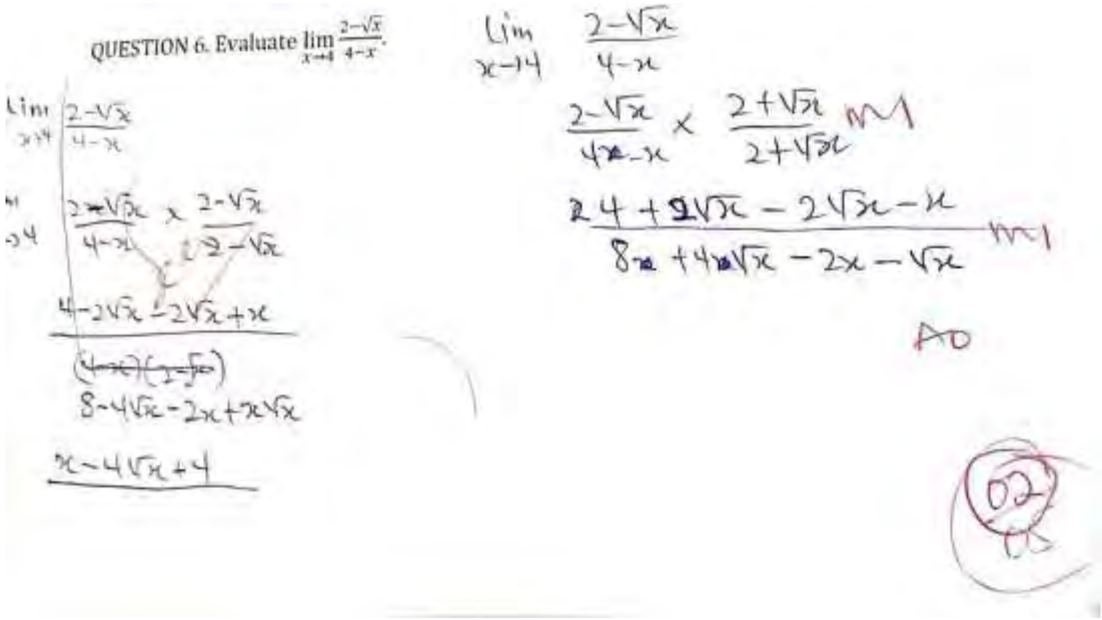


Figure 7: Sample Script 7

The above scripts show that pre-service teachers had both conceptual and procedural difficulties in applying the concept of derivatives. It is at the back of this that study seeks to find out the conceptual and procedural difficulties that pre-service teachers face in dealing with derivatives.

Moreover, most students were unable to find the equation of the tangent to the curve in Question Seven. More than 60% of the students could not state the equation of the tangent to the curve and be unable to state the formula used in finding the equation of a tangent to the curve. The gradient in the equation was for students to first differentiate to find the gradient and then substitute the values of the gradient and the coordinates of the points. Students had conceptual difficulties in finding the value of the constant used in a curve passing through points. Most students were unable to find the value because they had difficulties with the concept of the question. See figure 8 and 9 below the difficulties some pre-service teachers faced while attempting Question Seven.

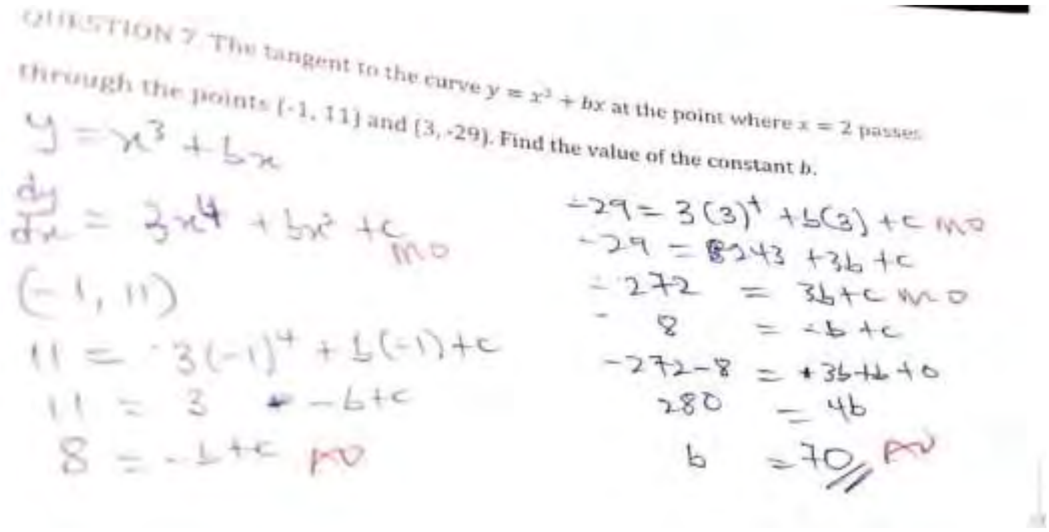


Figure 8: Sample Script 8

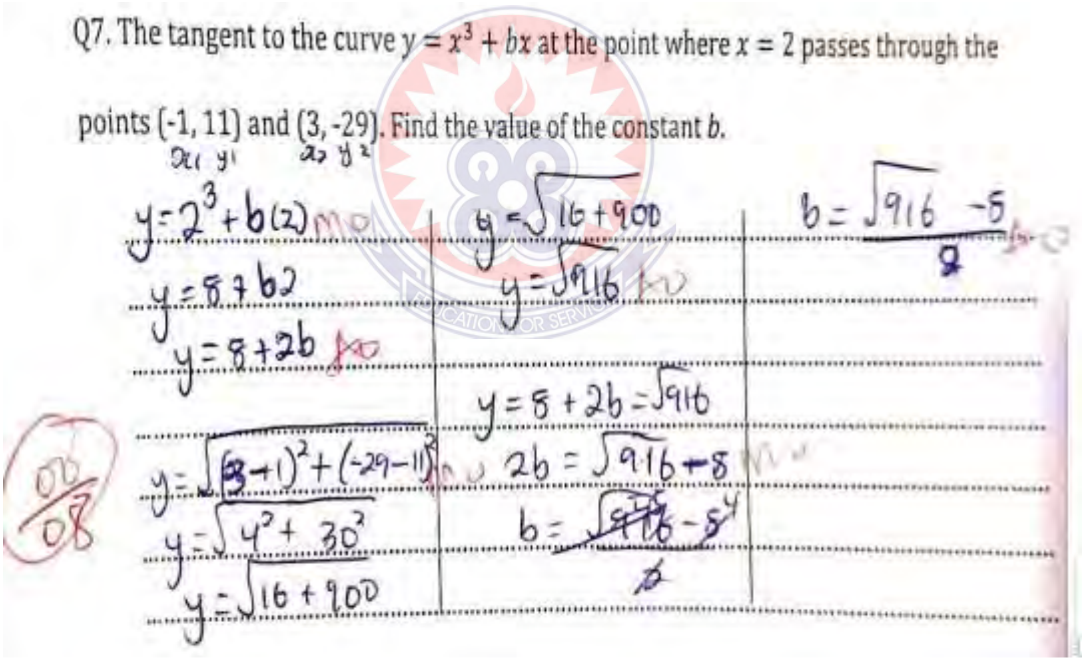


Figure 9: Sample Script 9

Question Eight was another difficulty students faced because they did not understand what the serpentine in the question means. See figure 10 below some of the difficulties students faced in finding the tangent of the line to the curve if the curve is serpentine.

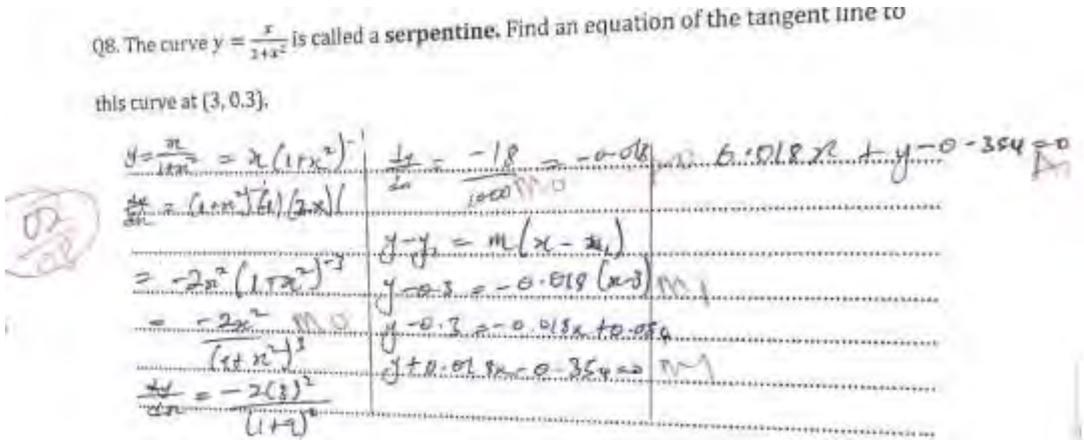


Figure 10: Sample Script 10

Despite the difficulties, some could use the knowledge of quotient to differentiate the curve and find the equation of the tangent line to the curve at the given point. See figure 12 below.

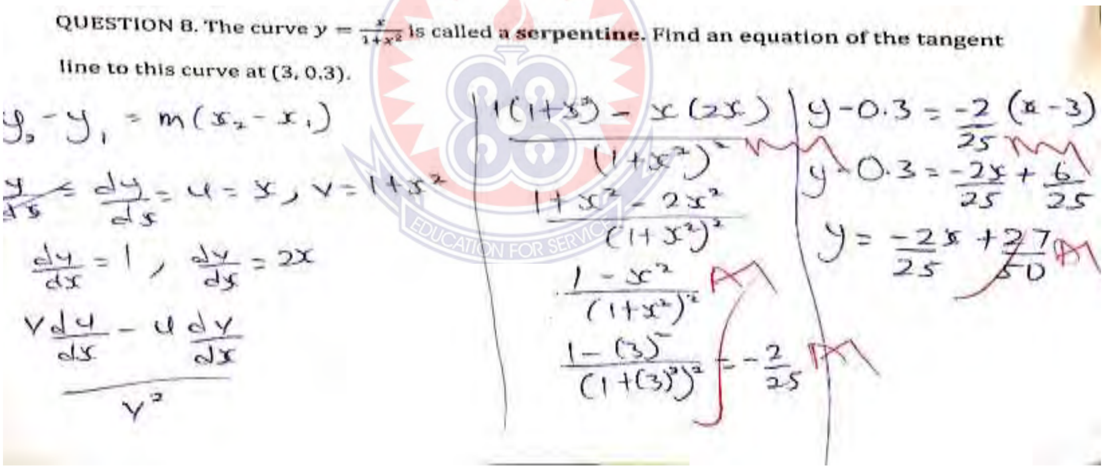


Figure 11: Sample Script 11

Furthermore, students had conceptual difficulties in verifying if the gradient of a function defined over the set of numbers exists at given points. Most pre-service teachers had difficulties in verifying if a function exists at a limit. Table 4.1 and Table 4.2 all pointed out that most pre-service teachers lack conceptual and procedural knowledge in dealing with trigonometric functions. See figure 12 below the difficulties faced by some pre-service teachers in verifying if a function exists at a limit in Question Nine.

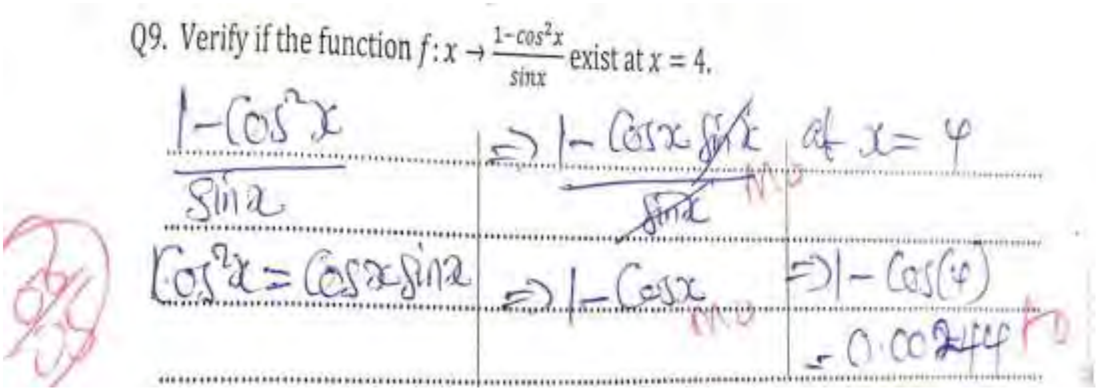


Figure 12: Sample Script 12

Finally, students had conceptual difficulties in the application of derivatives in question Ten. Some knew they were required to differentiate but were unable to state if the differential is the velocity and at which point will the derivative reach the highest point. Figure 13 below is a pre-service teacher script and the difficulties he has in solving the problem.

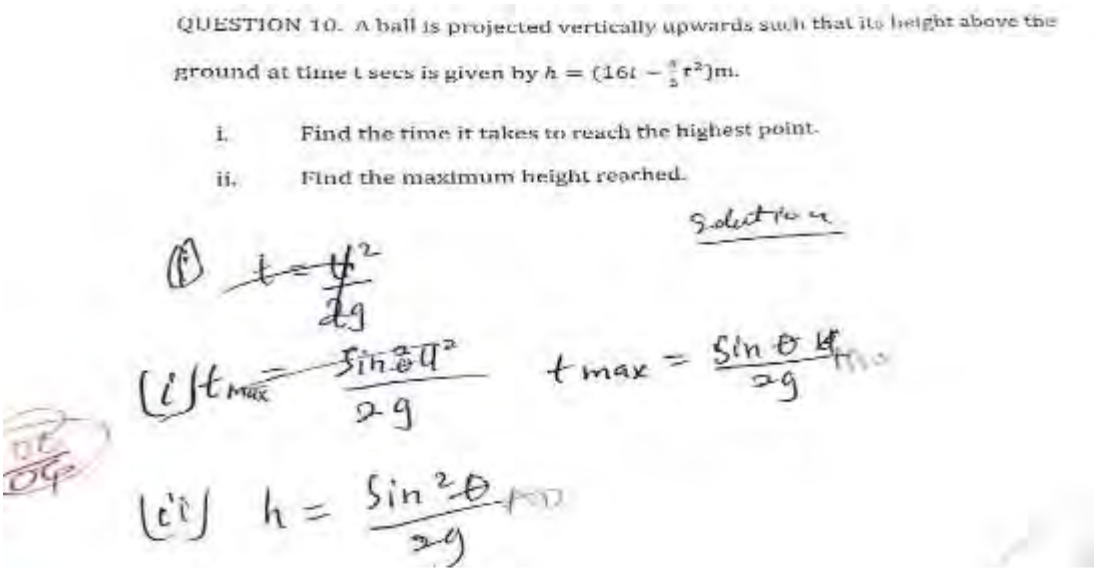
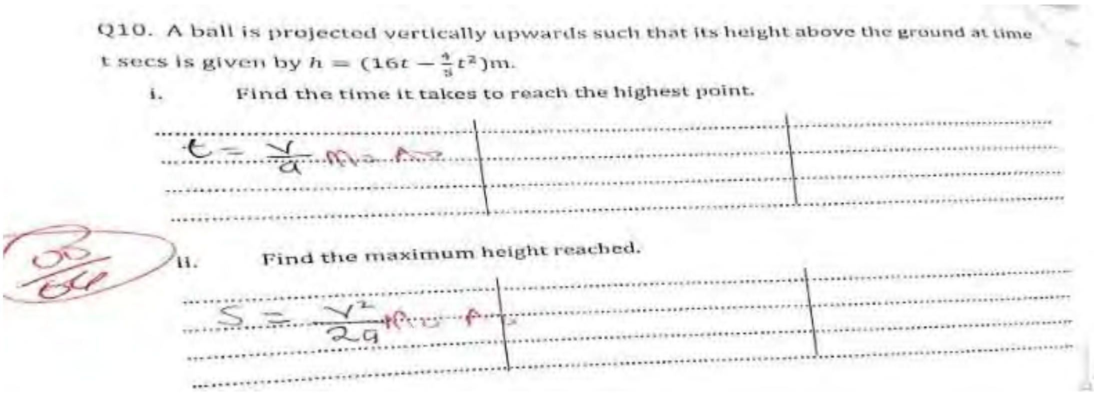


Figure 13: Sample Script 13

This is the script of one of the pre-service teachers who tried to answer Question Ten. It is very clear that this student had no idea about the concept under discussion. Another pre-service teacher had this to write about the same question. See below, figure 14 of the script.


Q10. A ball is projected vertically upwards such that its height above the ground at time t secs is given by $h = (16t - \frac{4}{5}t^2)$m.

i. Find the time it takes to reach the highest point.

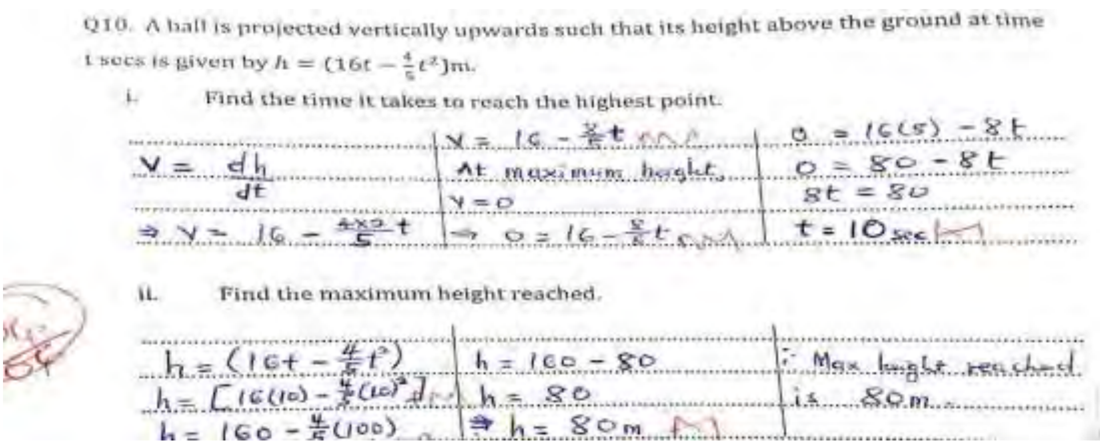
$t = \frac{v}{a}$

ii. Find the maximum height reached.

$S = \frac{v^2}{2g}$

Figure 14: Sample Script 14

It is crystal clear from the above scripts that, some pre-service teachers had conceptual and procedure difficulties on derivatives since they could not answer correctly the question given to them. Notwithstanding the difficulties some pre-service teachers faced, some of them had no difficulties dealing with Question Ten. Figures 15 and 16 below are some scripts of students who could apply conceptual and procedural knowledge in answering Question Ten.


Q10. A ball is projected vertically upwards such that its height above the ground at time t secs is given by $h = (16t - \frac{4}{5}t^2)$m.

i. Find the time it takes to reach the highest point.

$v = \frac{dh}{dt}$

$\Rightarrow v = 16 - \frac{4 \times 2}{5}t$

$v = 16 - \frac{8}{5}t$

At maximum height, $v = 0$

$\Rightarrow 0 = 16 - \frac{8}{5}t$

$0 = 16(5) - 8t$

$0 = 80 - 8t$

$8t = 80$

$t = 10$ sec

ii. Find the maximum height reached.

$h = (16t - \frac{4}{5}t^2)$

$h = [16(10) - \frac{4}{5}(10)^2]$

$h = 160 - \frac{4}{5}(100)$

$h = 160 - 80$

$h = 80$

$\Rightarrow h = 80$m

$\therefore$ Max height reached is 80m.

Figure 15: Sample Script 15

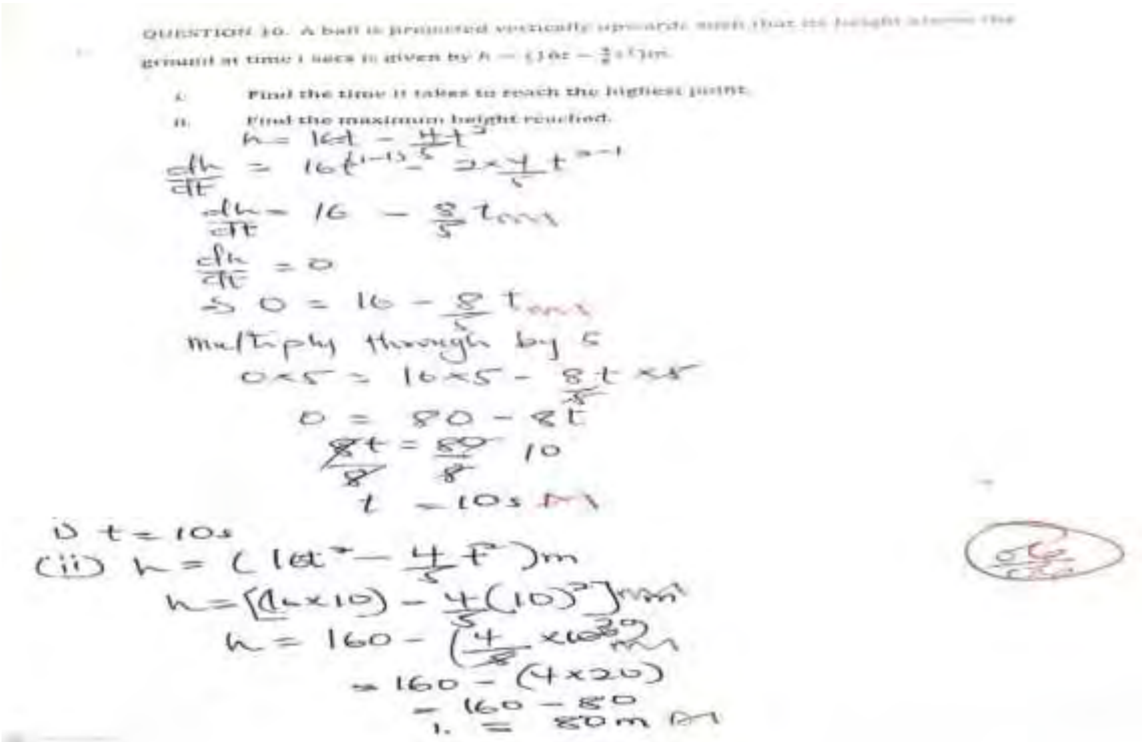


Figure 16: Sample Script 16

4.2 What is Pre-Service Teachers' Conceptual Understanding of Derivatives?

Conceptual understanding is the understanding of underlying principles and relationships of a mathematical topic (Delastri et al. 2020). In order to answer research question One, a 10-item Conceptual and Procedural Understanding Based Test (CPUBT) was given out to test students on their concept knowledge. Based on the APOS model in consideration of the research questions. Table 4.1 shows the result of analysis of participants" responses to questions on concept knowledge.

Table 4.1: Pre-Service Teachers' Responses to CPUBT ($n = 61$)

	Questions	Incorrect f (%)	Correct f (%)	N/A f (%)
1	Find $\frac{dy}{dx}$ if:			
	a. $y = 3\sqrt{x}$	25 (41.0%)	35 (57.4%)	1 (1.6%)
	b. $y = cot^2x$	43 (70.5%)	6 (9.8%)	12(19.7%)
2	a. Find the gradient of the curve $y = (x-3)(x^2+2)$ at the point where $x = 1$	20 (32.8%)	36 (59.0%)	5(8.2%)
	b. Find the slope of the function $2x + y^2 - 5 + 8y + 2xy = 0\ at\ (3, -4)$	26 (42.6%)	17 (27.9%)	18(29.5%)
3	If f $(x) = x^3 - 2x$ and g$(x) = x^2 - 3$ Show that D$[f(x)g(x) = f^1(x)g(x) + f(x)g^1(x)$	36 (59.0%)	5 (8.2%)	20(32.8%)
4	If $f(x) = x^3 - 5$ and $g(x) = x^2 + 1$, find:			
	a. D[*f*(g(x)]	33 (54.1%)	20 (32.8%)	8(13.1%)
	$b. D[f(x)g(x)]$	25 (41%)	15 (24.6%)	21(34.4%)
5	If a function, $f(x) = \frac{x^2}{4x+1}$, find an expression for the gradient function.	40 (65.6%)	9 (14.8%)	12(19.7%)
6	Evaluate $\lim_{x\to 4} \frac{2-\sqrt{x}}{4-x}$	35 (57.4%)	22 (36.1%)	4 (6.6%)
7	The tangent to the curve $y = x^3 + bx$ at the point where $x = 2$ passes through the points (-1, 11) and (3, -29). Find the value of the constant b.	18 (29.5%)	25 (41%)	18(29.5%)
8	The curve $y = \frac{x}{1+x^2}$ is called a **serpentine.** Find an equation of the tangent line to this curve at (3, 0.3).	20 (32.8%)	5 (8.2%)	36(59.0%)
9	Verify if the function $f: x \to \frac{1-cos^2x}{sinx}$ exist at $x = 4$	23 (37.7%)	4 (6.6%)	34(55.7%)
10	A ball is projected vertically upwards such that its height above the ground at time t secs is given by $h = (16t - \frac{4}{5}t^2)$m			
	i. find the time it takes to reach the highest point.	10 (16.4%)	31 (50.8%)	20(32.8%)
	ii. find the maximum height reached.	9 (14.8%)	31 (50.8%)	21(34.4%)

The Conceptual and Procedural Understanding Based Test (CPUBT) sought to find out the difficulties pre-service teachers face in learning derivatives, their conceptual and procedural knowledge and hence, the table above gives an overview of the pre-service teachers‟ knowledge on the concept of derivatives.

Question One, explored respondents‟ basic conceptual knowledge in finding $\frac{dy}{dx}$ of a function which is an *Action* type question. The Table 4.1 above indicates almost all the participants were unable to solve all the sub items of the questions correctly. Generally, the tasks seemed difficult for most pre-service teachers. Specifically, only 35 representing 57.4% of the pre-service teachers who took the test could solve question 1a correctly scoring all the marks assigned to the question. From Table 1, 25 out of the 61 pre-service teachers representing 41.0% could not score any mark or all the marks assigned to the question. It was realised they lacked some of the concepts applicable to solving the question. 1 out of 61 participants did not attempt the question representing 1.6% of the total participants. Question 1 b was another challenge faced by student teachers testing their concept knowledge on derivatives where they were required to do a differential equation involving trigonometry also an *Action* type question from *APOS*. Pre-service teachers were unable to differentiate trigonometric functions. Though evidence from their scheme of work and end-of-semester exams suggest that, it was in their syllabus. The response indicates that most students were aware of what to do but lack the manipulate skills to use in solving the problem. Generally, students were unable to solve the *Action* type problem given them in question 1.

Question One was marked over 9 marks out of 70 marks. Basically, 6 out of the total students representing 9.8% of the student teachers could solve correctly the problem. They were able to differentiate $y = cot^2x$ correctly. 43 out of the participants who took part in the test representing 70.5% could not solve the problem correctly. They had no concept knowledge on solving differentiation of trigonometric functions and hence could not solve the problem correctly. 12 out of 61 participants could not attempt the problem representing 19.7% of the total participants. Their inability to attempt the question cannot though be attributed to only a lack of concept understanding.

Question Two (a) tried sought the concept understanding of derivatives based on *Action* type question from *APOS* model. The students were to first find the derivatives of the curve and substitute the given values into the differentiated equation and come out with the coordinates. Question 2 also sort to find out if the students have the knowledge that the gradient of the function is the same as $\frac{dy}{dx}$. Question 2a was for students to use knowledge to find the coordinates of a curve. The response from the test reviewed that about only 10% of the student-teachers were able to use the concept knowledge on derivatives to find the coordinates on a given curve. Most students were able to find the derivative as the gradient but others were unable to realised that the gradient concept is the same as derivatives. 36 out of the total population representing 59.0% were able to solve the problem. Out of the total number who wrote the test, 20 of them representing 32.8% could not correctly solve the problem. They were unable to solve the problem because they did not have the required concept knowledge that is to be used in solving the problem. 5 out of the total number of participants could not solve the problem representing 8.2% of the total participants in the test.

Question Two (b) also an *Action* type question explored pre-service teachers‟ concept understanding on implicit differentiation. This concept explored pre-service teachers‟ understanding of differentiation with respect to a variable either than x but with respect to the variable y. The students were to use the understanding of differentiation „term by term‟ to solve the problem. The students were required to apply the knowledge of slope as the gradient and thus the differential of the function.

The students were to substitute the values of x *and* y into the gradient function to find the slope of the function. Here too, only 5 participants representing 8.2% of students were able to use the concept knowledge of product rule to find the gradient of the curve at a given point. 36 students representing 59.0% could not use the product rule to differentiate a function and put in the given values. 20 out of the total pre-service teachers who sat for the test representing 32.8% could not attempt the question. They lack the concept knowledge required to solve the problem. Some pre-service teachers also used other methods apart from the product rule to differentiate the function and were correct as indicated in the percentage above.

The question requires the use of the product rule in differentiation of a function. Generally, a lot of students struggled in finding the gradient of the function. The Conceptual and Procedural Based Test (CPUBT) seeks to still find out the *action* aspect of the model on the students" concept knowledge on derivatives. Question 2 was marked over 12 marks out of a total of 70 marks. The content was determined by the approaches pre-service teachers used is solving the problem. Much emphasis was paid to the various approaches that were used in solving the problem.

Majority of the students were unable to find the coordinates of the points on the curve. Table 4.1 gives a detailed description according to the percentage of students who could not appropriately find the coordinates of the points on the gradient function.

Question Three required pre-service teachers to show that two given functions when combined the compose form yields same results as when using the chain rule. This was also to explore to see how they understand which was an action type question from APOS model. Here, pre-service teachers were required to show their concept knowledge on derivatives using the chain rule. Student teachers were asked to show that D[$f(x)g(x) = f^1(x)g(x) + f(x)g^1(x)$. If f $(x) = x^3 - 2x$ and g$(x) = x^2 - 3$. The understanding of functions and derivatives were both required to answer this question. Table 1 below shows that only 8% of the respondents were able to show correctly using the chain rule. It is observed that a lot of students were unable to show because the question did not explicitly ask them to find $\frac{dy}{dx}$ and could not realize the question was simply asking them to differentiate. The action, process, object and schema of the APOS model were involved as students were required to think a bit deeper than just finding $\frac{dy}{dx}$ though the question was classified under *action* type question. It suggests that the students lack action understanding of the concept.

Question Four (a) asked pre-service teachers to find D[f(g(x))] if f$(x) = x^3 - 5$ and $g(x) = x^2 + 1$. This was based on the *action* type question. It examines student‟s on how to put a function into another function and differentiate. The conceptual understanding of the pre-service on derivatives is examined here too. Only 20 of the 61 students who took the test representing 32.8% of the students were able to solve the problem presented using the chain rule of differentiation. A lot of the students were able to initiate the process by putting the function $g(x)$ into the function $f(x)$ but could not continue from there as they were unable to use the chain rule to complete the differentiation. The understanding of the concept was not sufficient for the students to use in solving the problem contributing to lack of conceptual understanding. 20 out of the 61 pre-service teachers representing 32.8% who took the test were able to apply the concept correctly in solving the problem. Again, from Table 4.1, 33 out of the total number of respondents representing 54.1% were unable to solve the problem correctly. Eight (8) of the students representing 3.1% could not attempt the problem.

Question Four (b) also required students to use the knowledge concept in the 4a to find D[$f(x)g(x)$] given the same function. In this question, almost all students that attempted in the 4a equally tried to solve the 4b too but lack the process knowledge of the apos. They could not process the knowledge gained in differentiation in solving the problem as they were only able to multiply the two functions thus $f(x)$ *and* $g(x)$. Only the same students who were able to solve the 4a were able to solve this problem. A lot of the students are aware of the chain rule did were unable to apply it in solving two functions. In like manner, about 15 out of 61 students representing 24.6% of the pre-service teachers had knowledge on could solve the problem correctly. 25 out of the total number representing 41% of the students could not use the concept knowledge to solve the problem correctly but 21 of the participants representing 34.4% of the total student teachers could not attempt the problem suggesting they had no concept knowledge about the concept. Though the reason could not only be attributed to lack of conceptual understanding.

Question Five required the students to differentiate a function using the quotient rule. This was also an *action* type question. The question seeks to find out if the students really understood how to find the derivative of a function using the quotient rule. Only 9 out of the total number representing 14.8% of the entire students who sat for the test were able to attempt the question correctly. Most of the students could not state the quotient rule correctly. The performance in question 5 suggests that, 40 out of the total number representing 65.6% of the total participants were unable to us concept understanding of the question correctly. 12 out of the total number representing 19.7% of entire participants do not know how and when to use the quotient rule in finding the derivative of a given function and could not attempt the problem. Therefore, there is a lack of conceptual knowledge in finding the derivative of a function using the quotient rule.

Question Six incorporated the idea of derivatives and the equation of the tangent to a curve. This was a *process* type question from *APOS* model. It required students to differentiate a curve and use the value to find the equation of a curve which passes through a point. Table 4.1 above shows that 22 out of the 61 participants representing 36.1% of the participants could attempt the question correctly, 35 out of the total students representing 57.4% of the respondents attempted but were unsuccessful in solving the problem and 4 participants representing 6.6% were not able to attempt the problem at all.

Question Seven demanded students to find the values of a constant on a tangent to a curve which passes through two given points which was put imder the process from the APOS model. They were expected to differentiate the function to obtain the gradient and then use the two given points to get the gradient. They were to use the gradient from the two given points in the gradient function to get the constant value. Here, 25 out of the total students representing 41% could differentiate the given function correctly and also use the two given points to obtain the gradient in the question correctly. 18 out of 61 students were unable to solve the question correctly; they could not differentiate correctly to find the gradient or use the given points to evaluate to find the constant value. They did not have enough conceptual understanding of the concept and hence could not differentiate correctly the function. 18 pre-service teachers out of 61 of the sample who took the test could not attempt the question and hence left the question blank without attempt. They represent 29.5% of the student teachers who had no conceptual knowledge about the problem.

Question Eight tested the students" conceptual knowledge on serpentine functions. The question tested students" conceptual knowledge using the action, process, object and schema of the apos model. The question demanded students to use the quotient rule in differentiating the curve and put the value of x and get the gradient (m) and use the value of m in the equation $y - y_1 = m(x - x_1)$ since the values of x_1 and y_1 is what is given in the question. Only 3 students could use the quotient rule to differentiate the curve. From Table 4.1, only 5 pre-service teachers representing 8.2% could put the value of x into the differentiated function as the gradient and put the value of the gradient into the equation of the tangent with the coordinates of the line that the curve passed through to obtain the equation. From Table 4.1 above, 20 out of the participants who wrote the test representing 32.8% could not solve the problem correctly. They tried but were unsuccessful because they lack the conceptual knowledge about how to use the gradient in the equation to find the equation of the tangent to a curve. 36 out of the 61 representing 59.0% of the respondent could not attempt the problem. They lack conceptual knowledge about how the problem is to be solved. It can be drawn that a lot of pre-service teachers lacked conceptual knowledge on derivative (gradient) of a function in relation to the tangent.

Question Nine, required students to use the *process* which involves the *object* and *schema* stages of the APOS model to verify if the gradient of the function is defined on the set of real numbers that exist by a given function. The students needed to have the conceptual knowledge of the gradient of a function that exists at given points as limits. A lot of students had absolutely no knowledge about this question. Some also tried their understanding on the question. It"s obvious they lacked the conceptual understanding of the concept. Only 4 pre-service teachers representing 6.6% of the participants could verify a function exist. 23 out of the total respondents representing

37.7% could not verify if a function exists at a limit. Some of them only put the value of x into the function but did not know what to use the value for since they could not draw any conclusion from the value. They could not test the value of x into the function to obtain the y value and indicate that the values do exist and are defined on the set of real numbers. 34 out of the 61 pre-service teachers representing 55.7% could not attempt the problem. Clear they lacked the conceptual understanding on how to evaluate if a function exists at a particular limit. Majority of the pre-service teachers from the test clearly has little concept knowledge or has no conceptual understanding of the concept.

Question Ten tests the students" conceptual knowledge on the kinematics of differentiation. The students were tested on the action, process, object and schema of the APOS model on derivatives. They were expected to state that at the highest height, velocity is zero and differentiate the given equation with respect to time. They were to get the value of the Time t seconds after differentiation and equate the gradient to zero. Again, they were to put the Time t seconds into the given height equation to obtain the maximum height reach by the vertically upwards object. 31 out of those who responded to the test representing 50.8% could state that, at the highest point, velocity is zero and able to differentiate the given equation and equating it to zero and get the Time t seconds. 10 out of the 61 student teachers could not solve the problem correctly representing 16.4% of the total respondents. 20 of the student teachers were unable to solve the problem representing 32.8% of students having no conceptual understanding of the question. They lacked the conceptual understanding needed in solving the problem. 31 of the students representing 50.8% could put the value of the time to get the maximum height in the original height given correctly. The percentage indicates the students who could use the conceptual knowledge on derivatives to find the highest point and that of the maximum height reached by the vertically upwards object. 9 students attempted to find the maximum height representing 14.8% but were unsuccessful in the highest point reached by the vertical upwards. 21 out of the total participants representing 34.4% could not attempt the question on finding the maximum height reached when the object is projected vertically upwards.

4.3 What Procedures do Pre-Service use to Find Derivatives?

Another overarching question in the work was to find out the procedural understanding of pre-service teachers on derivatives. Here, the researcher wants to find out if students are able to solve problems on derivatives without any procedural difficulties. In order to answer the research question 2, a 10-item Conceptual and Procedural Understanding Based Test (CPUBT) was used to thoroughly examine students on how they are able to follow the procedures involved in solving problems on derivatives (calculus). Despite a lot of students having difficulties in the conceptual knowledge, other had some concept knowledge about the concept. The research, therefore, had the marking scheme of the CPUBT. The scheme covered almost every step that was involved in solving a problem in the test and students" the scripts were marked according to the test item. Where a procedure was wrong, the procedure was marked and the mark allocated was m_0 but where a procedure was correct, it was marked correct and awarded M_1. Final answers were not left out as some were correctly marked and awarded A_1 and those that were wrong wrongly were awarded an A_0. Content-by-content analysis was used to analyze the scripts. The action, process, object and schema of the APOS model were both engaged in solving the problems in the test. After the whole analysis, Table 4.1 was used to analyze students who could not correctly answer the questions based on the procedures they were to follow and those that were able to answer the questions correctly based on the procedures outlined in solving the problem.

The table below shows the item-by-item analysis based on the procedural understanding of pre-service teachers on derivatives.

Table 4.2: Procedural Understanding by Pre-Service Teachers on Derivatives

		Procedures	**Incorrect f (%)**	**Correct f (%)**
1a	i.	Rewrite $y = 3\sqrt{x}$ as $y = 3x^{\frac{1}{2}}$	12 (19.7%)	48 (78.7%)
	ii.	Find $\frac{dy}{dx}$ as $\frac{3}{2}x^{-\frac{1}{2}}$	12 (19.7%)	48 (78.7%)
	iii.	Rewrite $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}}$ as $\frac{dy}{dx} = \frac{3}{2\sqrt{x}}$	21 (34.4%)	39 (63.9%)
1b	i.	Let any be variable, $u = cotx$	24 (39.3%)	24 (39.3%)
	ii.	Indicate that $\frac{dy}{dx} = -cosec^2x$	31(50.8%)	18 (29.5%)
	iii.	Let $y = u^2$	39 (63.9%)	10 (16.4%)
	iv.	Indicate that $\frac{dy}{dx} = 2u$	37 (60.7%)	10 (16.4%)
	v.	State that $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	43 (70.5%)	6 (9.8%)
	vi.	State that $\frac{dy}{dx} = 2u \cdot (-cosec^2x)$	42 (68.9%)	7 (11.5%)
	vii.	Substitute to obtain, $\frac{dy}{dx} = -2cotxcosec^2x$	46 (75.4%)	4 (6.6%)
2a	i.	Let $u = x - 3$ and $v = x^2 + 2$	13 (21.3%)	43 (70.5%)
	ii.	Find that $\frac{du}{dx} = 1$	15 (24.6%)	41(67.2%)
	iii.	Find that $\frac{dv}{dx} = 2x$	17 (27.9%)	39 (63.9%)
	iv.	State that $\frac{dy}{dx} = (x^2 + 2)1 + (x - 3)2x$	17 (27.9%)	39 (63.9%)
	v.	Expand and simplify to obtain $\frac{dy}{dx} = 3x^2 - 6x + 2$	16 (26.2%)	40 (65.6%)
	vi.	Substitute $x = 1$ into $\frac{dy}{dx} = 3x^2 - 6x + 2$ to obtain $\frac{dy}{dx} = 1$ as the gradient at $x = -1$	18 (29.5%)	38 (62.3%)
2b	i.	Ability to differentiate implicitly to obtain $\frac{dy}{dx} = 2 + 2y\frac{dy}{dx} - 0 + 8\frac{dy}{dx} + 4xy\frac{dy}{dx} + 2y^2$.	20 (32.8%)	23 (37.7%)
	ii.	Ability to factorize appropriately to obtain $\frac{dy}{dx} = \frac{-(2+2y^2)}{2y+4xy+8}$.	21 (34.4%)	22 (36.1%)
	iii.	Substitute $x = 3$ and $y = -4$ into $\frac{dy}{dx} = \frac{-(2+2y^2)}{2y+4xy+8}$ to obtain $\frac{dy}{dx} = \frac{17}{24}$ as the gradient at (3,-4).	26 (42.6%)	17 (27.9%)
3	i.	Be able to expand and simplify to obtain $D[(x^2 - 2x)(x^2 -$	21 (34.4%)	20 (32.8%)

		$3)] = 5x^4 - 15x^2 + 6$		
	ii.	Be able to differentiate $x^5 - 5x^3 + 6x$ to obtain $D[f(x)g(x)] = 5x^4 - 15x^2 + 6$	27 (44.3%)	14 (23.0%)
	iii.	Be able to expand and simplify $(x^3 - 2x)(x^2 - 3)$ to obtain $x^4 - 5x^2 + 6x$	31 (50.8%)	11 (18.0%)
	iv.	Be able to carry out the differentiation to obtain $D[(x^2 - 2x)(x^2 - 3)] = (3x^2 - 2)(x^2 - 3) + (x^3 - 2x)(2x)$.	32 (32.8%)	9 (14.8%)
	v.	Be able to simplify to obtain $f^1(x)g(x) + f(x)g^1(x) = 5x^4 - 15x^2 + 6$	33 (54.1%)	7 (11.5%)
4a	i.	Ability to find that $f\left(g(x)\right) = (x^2 + 1)^3 - 5$	11 (18.0%)	42 (68.9%)
	ii.	Be able to find $D[(x^2 + 1)^3 - 5] = 3(x^2 + 1)^2 \cdot 2x - 0$	25 (41.0%)	28 (45.9%)
	iii.	Ability to obtain that $D[f\left(g(x)\right) = 6x(x^2 + 1)^2$	33 (54.1%)	20 (32.8%)
4b	i.	Be able to $f(x)g(x) = (x^3 - 5)(x^2 + 1)$	17 (27.9%)	23 (37.7%)
	ii.	Be able to carry out the differentiation of $D[f(x)g(x)]$	24 (39.3%)	16 (26.2%)
	iii.	Ability to write the answer as $D[f(x)g(x)] = 5x^4 + 3x^2 - 10x$	24 (39.3%)	16 (26.2%)
5	i.	Ability to identify that the quotient rule is required in this question by stating that $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ where $x = x^2$ *and* $v = 4x + 1$	20 (32.8%)	29 (47.5%)
	ii.	Be able to find $\frac{du}{dx} = 2x$	26 (42.6%)	23 (37.7%)
	iii.	Be able to find that $\frac{dv}{dx} = 4$	35 (57.4%)	14 (23.0%)
	iv.	Ability to substitute appropriately to obtain $\frac{dy}{dx} = \frac{4x^2+2x}{(4x+1)^2}$	31 (50.8%)	18 (29.5%)
6	i.	Be able to rewrite $\frac{2-\sqrt{x}}{4-x} = \frac{(2-\sqrt{x})}{4-x} \times \frac{(2+\sqrt{x})}{(2+\sqrt{x})}$	29 (47.5%)	27 (44.3%)
	ii.	Be able to expand and simplify to obtain that $\lim_{x\to 4} \frac{2-\sqrt{x}}{4-x} = \lim_{x\to 4} \frac{1}{(2+\sqrt{x})}$	30 (49.2%)	27 (44.3%)
	iii.	Be able to evaluate $\lim_{x\to 4} \frac{1}{2+\sqrt{x}} = \frac{1}{4}$	35 (57.4%)	22 (36.1%)
7	i.	Be able to differentiate the function to obtain $\frac{dy}{dx} = 3x^2$ +b as gradient.	13 (21.3%)	30 (49.2%)
	ii.	Ability to put the value of x into the gradient to obtain $\frac{dy}{dx} =$ 12+b	15 24.6%)	28 (45.9%)
	iii.	Be able to find the gradient through points as m = $\frac{y_2 - y_1}{x_2 - x_1}$ to obtain -10	12 (19.7%)	31 (50.8%)
	iv.	Ability to use the two gradients to find the value of *b* as 12+b = -10	18 (29.5%)	25 (41.0%)
8	i.	Ability to identify that the quotient rule is required in this question by stating that $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ where u= x *and* $v = 1 + x^2$	11 (18.0%)	14 (23.0%)
	ii.	Be able to find $\frac{du}{dx} = 1$ and $\frac{dv}{dx} = 2x$	13 (21.3%)	12 (19.7%)

	iii.	Ability to substitute appropriately to obtain $\frac{dy}{dx} = \frac{1-x^2}{(1+x^2)^2}$	14 (23.0%)	11 (18.0%)
	iv.	Be able to put the value of x into the gradient to obtain $-\frac{2}{25}$ or equivalent.	19 (31.1%)	6 (9.8%)
	v.	Ability to use the equation of the tangent as $y - y_1 = m(x - x_1)$ at (3, 0.3) to obtain $y - 0.3 = -\frac{2}{25}(x - 3)$.	19 (31.1%)	6 (9.8%)
	vi.	Ability to write the equation of the tangent to the curve as $2x + 25y - 13.5 = 0$ or equivalent.	20 (32.8%)	5 (8.2%)
9	i.	Ability to state that $1 - cos^2x = sin^2x$	19 (31.1%)	8 (13.1%)
	ii.	Be able to substitute sin^2x as $f(x) = \frac{sin^2x}{sinx}$	19 (31.1%)	8 (13.1%)
	iii.	Be able to simplify as $f(x) = sinx$	20 (32.8%)	7 (11.5%)
	iv.	Ability to substitute $x = 4$ in the function $f(x)$.	22 (36.1%)	5 (8.2%)
	v.	Ability to verify the function exist at $x = 4$ as 0.	23 (37.7%)	4 (6.6%)
10a	i.	Be able to differentiate the function as $\frac{dh}{dv} = 16 - \frac{8t}{5}$	10 (16.4%)	31 (50.8%)
	ii.	Be able to state that at the highest point $\frac{dh}{dv} = 0$ as $16 - \frac{8t}{5} = 0$	8 (13.1%)	33 (54.1%)
	iii.	Be able to find for time as 10s	9 (14.8%)	32 (32.8%)
10b	i.	Be able to put the time into the function for the maximum height as $h = 16(10) - \frac{4}{5}(10^2)$	9 (14.8%)	31 (50.8%)
	ii.	Be able to simplify for the maximum height as 80m.	9 (14.8%)	31 (50.8%)

From table 4.2 above, it could be seen that students had procedural difficulties in solving the problem on derivatives. The approach (procedures) to follow when solving problems involving derivatives is a challenge. A lot of pre-service teachers cannot use the appropriate methods/procedures to solve derivatives. The following procedures were analysed based on the scripts of pre-service teachers from the study.

Question One (a) from the table shows that the number of pre-service teachers who were about to follow the procedures used in solving the question with their percentage and the number of students who could not follow the procedures with their percentage and the number of student teachers who could not follow the procedures in solving the question. Below is the way the question was supposed to be answered by the participants. The figure 17 captured the marking scheme used in marking the methods expected from the Pre-service Teachers.

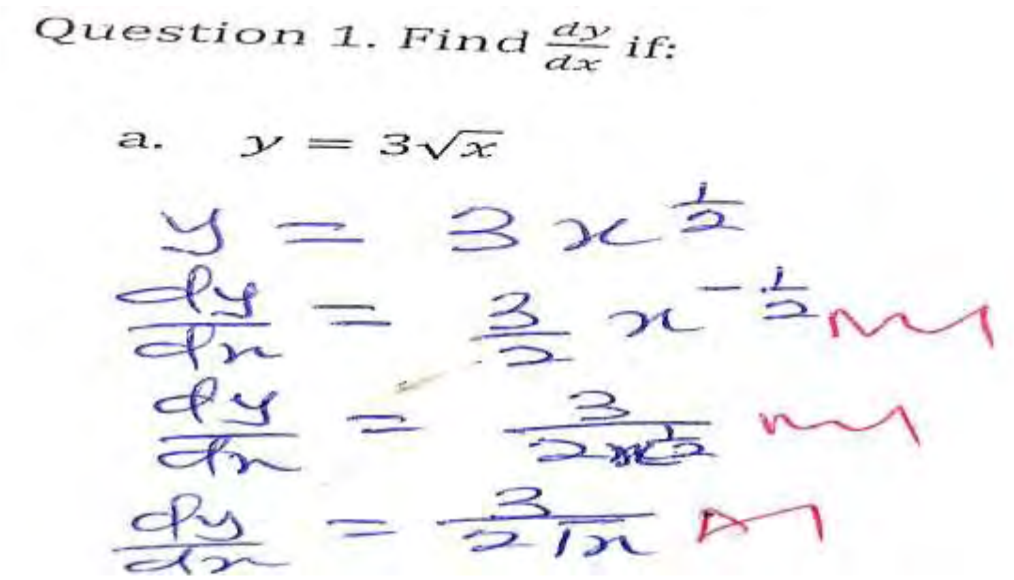


Figure 17: Suggested Procedures of Question 1(a)

Table 18 shows that 13 students representing 21.3% of the students could not rewrite the equation again before differentiating the function. 48 students out of 61 representing 78.7% were able to rewrite the equation before they differentiate. The percentage suggests that a lot of the students had enough procedures on the concept under discussion. 13 students out of the total participants representing 78.7% were not able to differentiate function while 48 students out of the 61 students representing 21.3% were not able to differentiate the function. 22 pre-service teachers representing 36.1 % could not write the final answer in the form of the question that was given whereas 39 out of the total participants were successful in writing the answer obtained in the form of the question given. It can be seen that, procedurally, some students had a procedural understanding of the concept derivatives on the question and very some had difficulties in differentiating a function in the form of the question.

Question One (b) also expected pre-service teachers to demonstrate how they will differentiate a trigonometric function. The required methods that were required from the pre-service teachers are shown in figure 18 two below;

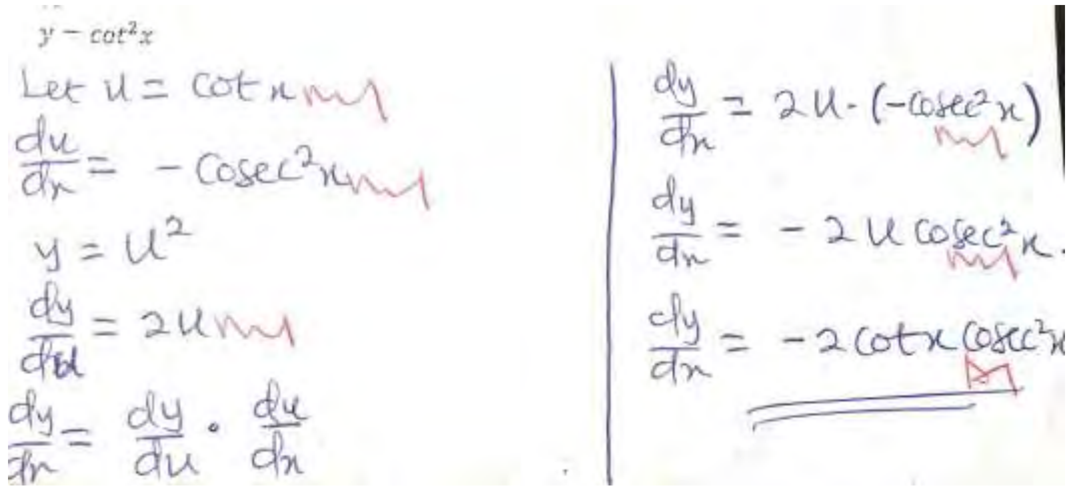


Figure 18: Suggested Procedures of Question 1(b)

The table 4.2 shows that 37 students out of the 61 total college students representing 60.7% were not able to represent the function by a variable before they differentiate as seen in the marking scheme. 24 out of 61 pre-service teachers representing 39.3% had it correctly in differentiating the given function by making a variable represent the given function. 31 of the students out of 61 respondents representing 50.8% could not state that the differential of $cotx \; is \; -cosec^2x.$ 18 of the pre-service teachers representing a percentage of 29.5% were able to follow the procedures in the marking scheme above in dealing with the problem. 43 out of a total of 61 pre-service teachers representing 70.5% were not able to represent the function as a variable and differentiate with respect to the variable as seen in the suggested scheme above.

18 participants representing 29.5% of the total participants were able to follow the procedure as seen in the scheme above in differentiating the given trigonometric function. 51 pre-service teachers representing 83.6% were not able to further

differentiate the variable that was to be represented as the function and differentiate whereas only 10 of the pre-service teachers were able to further differentiate the variable represented in dealing with the trigonometric function. 54 out of the total number who did the test representing 88.5% were not able to use the product rule to differentiate the function. Only 7 out of the total number who wrote the CPUT representing 11.5% were able to use the product rule in differentiating the trigonometric function. Finally, 57 pre-service teachers representing a percentage of 93.6% could not either solve, guess or attempt the question but only 4 pre-service teachers out of a sample of 61 pre-service teachers were able to follow the procedures in coming up with the solution to the trigonometric function. This suggests that a lot of pre-service teachers have a lot of procedural difficulties based on the data available in differentiating trigonometric functions.

Figure 19 below has the marking scheme used in marking the scripts

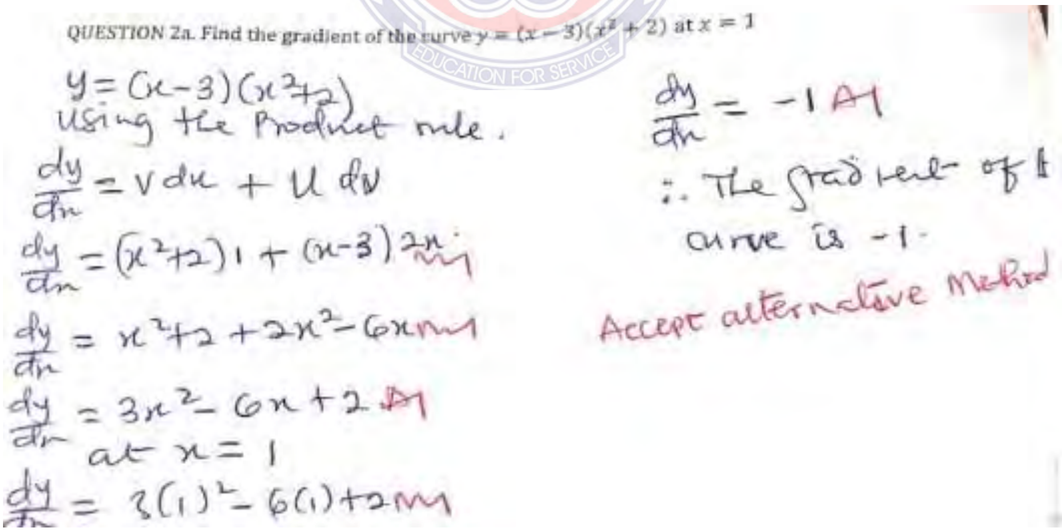


Figure 19: Suggested Procedures of Question 2a

Question Two (a) tested the pre-service procedural understanding of the product rule in finding the gradient of a curve. Above is what the pre-service teachers were expected to follow in finding the gradient of a curve at a given point in figure 3.

The Table 4.2 indicates that 18 of the students representing 29.5% were not able to follow the first step in solving the problem. 43 students were able to solve the problem by the first step involved in solving the problem. They represent 70.5% of the pre-service teachers who had the knowledge in solving the problem. The study seeks to find out whether pre-service teachers could procedurally follow the various methods involved in solving the question. 20 participants were not able to get the second procedure correctly in solving the question representing 32.8% of the total participants whereas 41 students representing 67.2% were able to follow the second step in solving the question.

The study was interested in all the procedures and hence statistics was taken on the next step involved in solving the question. 22 out of the total number of pre-service teachers representing 36.1% were not able to put correctly the various components into the product rule or expand the function completely before differentiating the function. 39 students representing 67.9% were able to follow the fourth step in differentiating since mathematics isn"t always all about the answers but the procedures in getting the answers. The next procedure was the fifth procedure where pre-service teachers were to expand and simplify the expression obtained. 21 pre-service teachers were not able to do that representing 34.4% while 40 students representing 65.6% were able to correctly do that. Finally, pre-service teachers were to evaluate the value of x into the expression as a gradient. 23 participants were not able to do that step and they represent 41.7% of the total participants whilst 38 of the respondents representing 62.3% were able to correctly follow the procedures in finding the gradient of the curve as -1 at $x = 1$.

Question Two (b) tested pre-service teachers‟ procedural understanding of how to differentiate a function implicitly. The procedures tested are stated in figure 20 below;

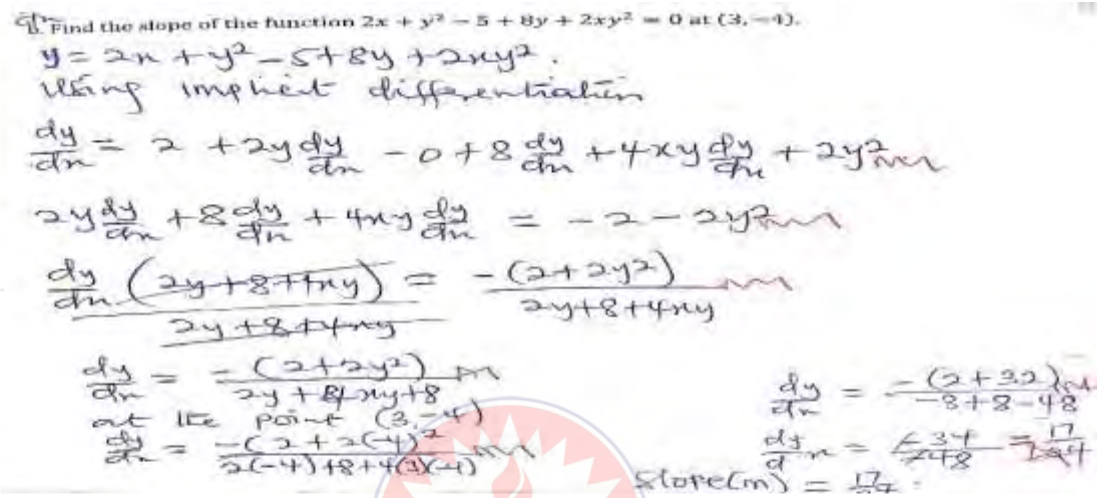


Figure 20: Suggested Procedures of Question 2b

Table 4.2 shows that 38 students out of the 61 participants that wrote the CPUT representing 62.3% could not follow the first procedural in solving the question. 23 of the pre-service teachers who took the CPUT representing 37.7% were able to start correctly the procedure in solving the problem. The second procedure involved in solving the problem, from the study shows that 39 of the pre-service teachers who took the CPUT representing 63.9% could not be able to follow the second required procedure involved in solving the problem. 22 pre-service teachers were able to follow correctly the procedure used in solving the problem. 44 pre-service teachers representing 72.1% were not able to follow the procedure used in solving the problem in the table above. 17 pre-service teachers representing 27.9% were able to solve the question according to the required procedure to be used in solving the problem correctly.

Figure 21 below Shows the Marking Scheme of Procedures used in solving the Problem

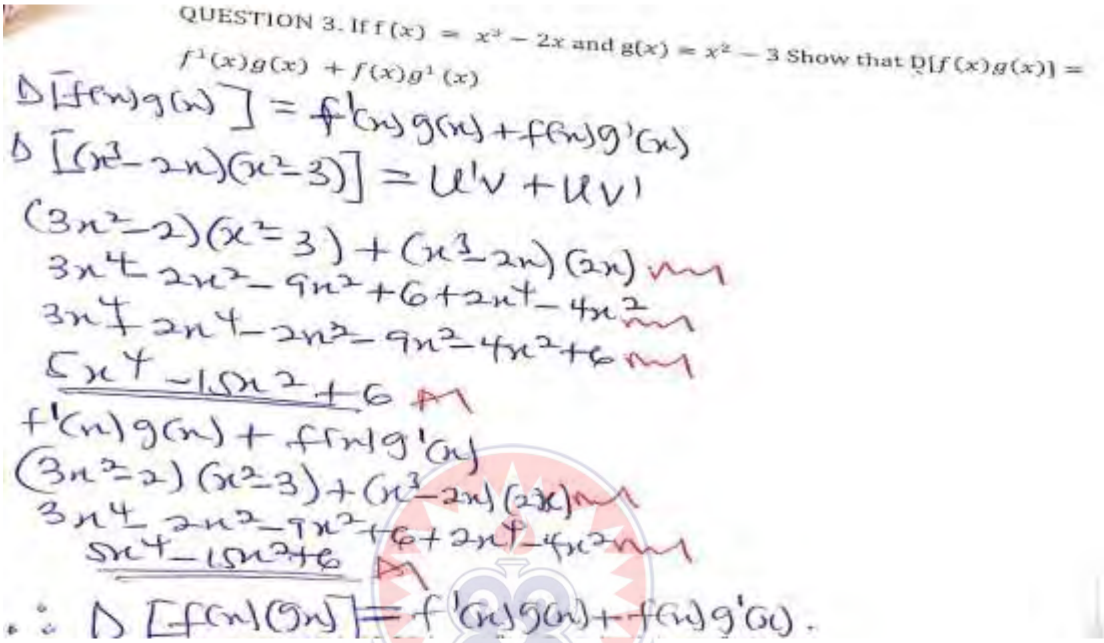


Figure 21: Suggested Procedures of Question 3

Table 4.2 above shows that 41 students representing 67.2% were not able to write the first step involved in solving the problem while 20 of the students representing 32.8% were able to get the first step involved in solving the problem. 47 pre-service teachers representing 77.0% were unable to differentiate the function at the left-hand side correctly. 40 students representing 82.0% were not able to correctly expand and simplify the functions at the right-hand side as the question demanded whereas 21 students representing 18% were able to carry out the step correctly as the procedure required in solving the problem. 52 of the participants representing 85.2% were not able to carry out the steps in differentiating the function at the right-hand side correctly as captured in the marking scheme either using the product rule or expanding and simplifying. Only 9 students representing 14.8% were able to go through the steps involved in the differentiation procedure involved at the right-hand side correctly. 54 pre-service students representing 88.5% were not able to differentiate the function at the right-hand side correctly. Only 7 of the pre-service teachers were able to differentiate the function at the right-hand side based on the required procedure and earned the A_1 mark.

Once again, the lack of conceptual knowledge has an impact on the students‟ procedural skills as most of the students were able to know that D$[f(x)g(x)]$ means to differentiate the product of the functions. Students could not work with the functions one by one to show that $D[f(x)g(x)] = f^1(x)g(x) + f(x)g^1(x)$. Again, it appeared students did not know that $f^1(x)$ means to find the differential of the function $f(x)$. Most of the students are only familiar with $\frac{dy}{dx}$. The inability of the students to decode the meaning of the sign as the first differential was also a bottleneck to the student‟s inability to relate it to differentiation. The test on the process and schema of the apos model both failed because students could not identify what they were to do before they will either do it right or wrong.

Question Four (a) explored the D concept of differentiation either than $\frac{dy}{dx}$ which students are not familiar with. The marking scheme for this problem is captured below from experts in the area of derivative. The figure 22 below is the procedures used in solving the problem.

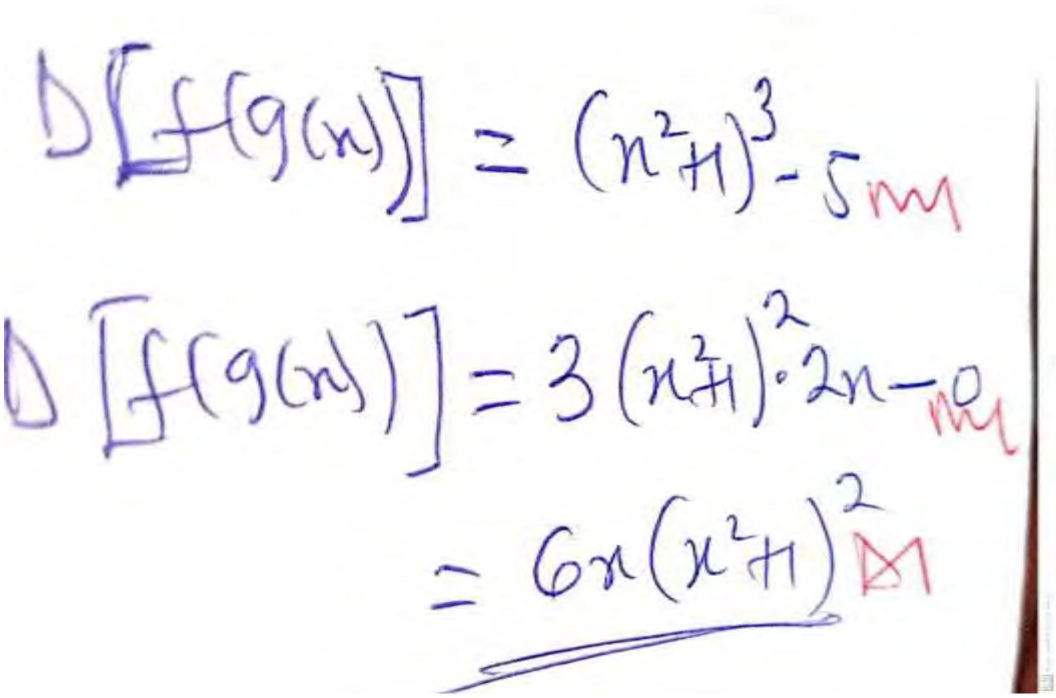


Figure 22: Suggested Procedures of Question 3a

The data in Table 4.2 indicates that, 19 students representing 31.1% were not able to use the chain rule putting the function $g(x)$ *into the* $f(x)$. Only 42 students representing 68.9% were able to use the chain rule to differentiate the composite function. 33 of the students representing 54.1% were not able to do the differentiation by either expanding the expression and simplifying or used the chain rule to differentiate. 28 pre-service teachers representing 45.9% were able to use either the chain rule or expand and simplify before doing the general differentiation. Here again, most of the students were unable to differentiate the products of the expansion since they did not know the D means to differentiate the composite function obtained.

Procedurally, a lot of the students were not able to find the derivative since they lack the concept knowledge about the D in the $D[f(g(x))]$ in the question.

Question Four (b) required students to multiply the two given functions and find its derivatives. The students were to first of all find $f(x)g(x)$ and find the derivative of the $f(x)g(x)$. The marking scheme required the procedures below in figure 23.

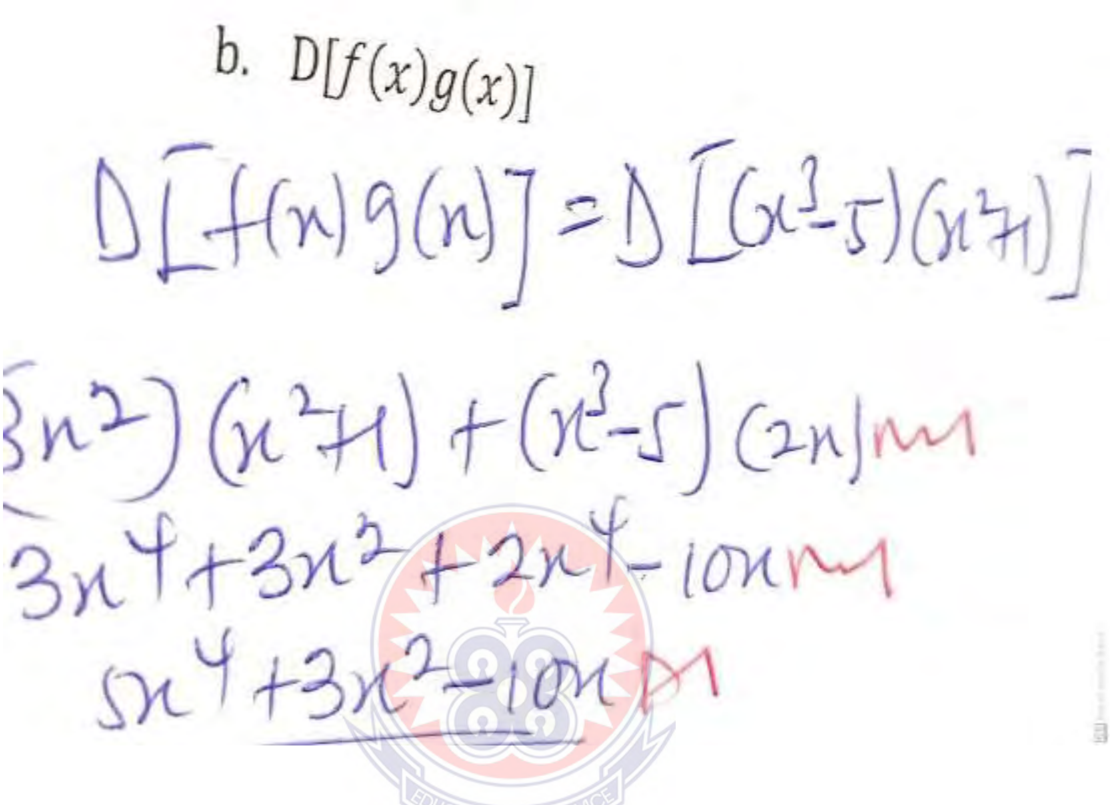


Figure 23: Suggested Procedures of Question 3b

From Table 4.2 above, 38 out of the total students representing 72.3% were unable to write the functions $f(x)g(x)$ and multiply the composite functions. 23 of the students representing 37.7% were able to write and multiply the functions $f(x) and\ g(x)$ correctly. 45 student teachers representing 73.8% as indicated in the Table above were not able to carry out the differentiation of the $f(x)g(x)$. 16 students representing 26.2% of the students were able to find the derivative of the $f(x)g(x)$ as the question demanded. 45 student teachers representing 73.8% were not able to write the final answer to earn the A_1 mark. 16 pre-service teachers representing 26.2% were able to write the final answer to earn the A_1.

Some of the pre-service teachers are not able to identify the fact that the concept was all about derivatives and just needed to differentiate the result of the expansion of the $f(x)g(x)$ or use the product rule of differentiation to find the derivatives. The procedural knowledge of the students was deficient in the demands of the question.

Question Five did not only test the conceptual knowledge of derivative as gradient but also sort to find out how students procedurally find derivatives as the gradient of a curve. The question expected students to use the quotient rule to differentiate the function, $f(x)$ and write the result as the expression of the gradient function. The procedures employed in solving the problem is captured in the marking scheme below in figure 24.

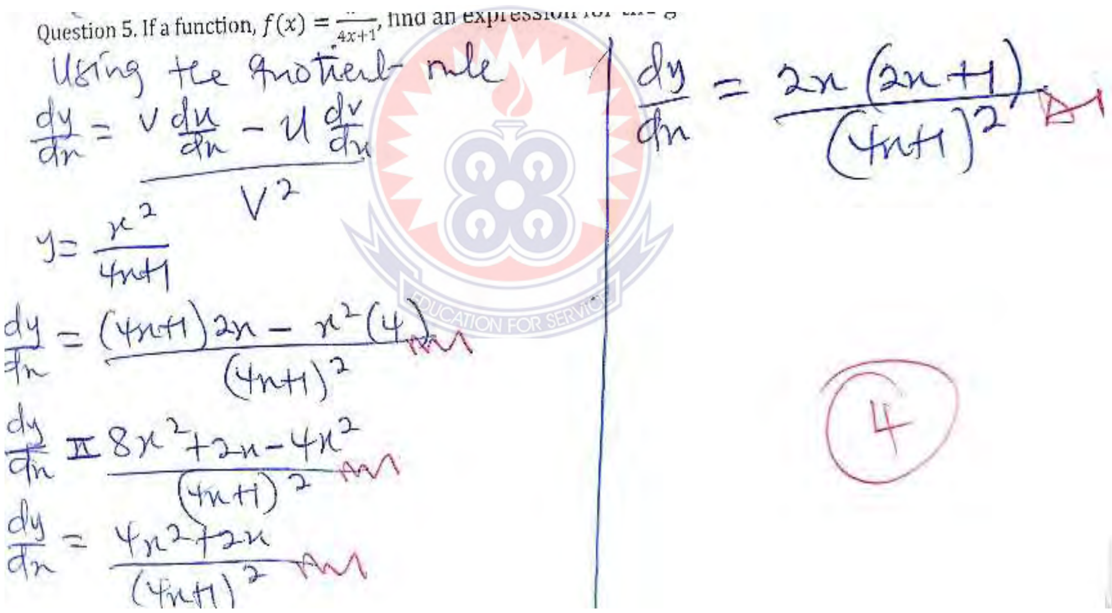


Figure 24: Suggested Procedures of Question 5

Table 4.2 above shows that, 32 student teachers representing 52.5% were not able to identify and state that the quotient rule with the variables involved in it is to be employed in solving the problem correctly. 29 students representing 47.5% were able to state the quotient rule or apply the quotient in attempting the question correctly. 38 students were not able to differentiate at the top called u with respect to the variable

x. 23 students representing 37.7% were able to differentiate the function at the top with respect to the variable x as seen in the marking scheme. Similarly, 47 students who took the test representing 77.0% were not able to deal with the function at the denominator called v with respect to the variable x but 14 of the students representing 23% were able to do that correctly and were marked m_1. Also, 43 students representing 70.5% who took the test were not able to either follow the required steps or use any other means to differentiate the function as the gradient of the function. 18 of the students were able to follow the procedures outlined above in the scheme or come out with their own procedures in determining the gradient of the function as the derivative of the function. All those students who were able to state the quotient rule correctly were all able to use the quotient rule correct in finding the derivatives. students could not expand and simplify the expression obtained in the differentiation. unfortunately, none of the students could state that the expression of $\frac{dy}{dx}$ is the gradient of the function as perhaps they did not know that the $\frac{dy}{dx}$ is the same as the gradient of the function.

The Conceptual and Procedural Understanding Based Test (CPUBT) Question 6 tested students‟ procedural understanding on how to find the equation of the tangent to the curve. The question required students to use the action, process, object and schema levels of the apos model to find the equation of the tangent to the curve.

The students were to, first of all differentiate the given curve and then put the value of x into the derivative to obtain the gradient value. They were secondly required to state the equation of the tangent to the curve as $y - y_1 = m(x - x_1)$. They were finally required to put the gradient(m) value and the coordinates of the points as x_1 and y_1

into the stated equation of the tangent to the curve and expand and simplify to find the equation of the tangent to the curve.

Figure 25 below shows what was expected from the Pre-service Teachers

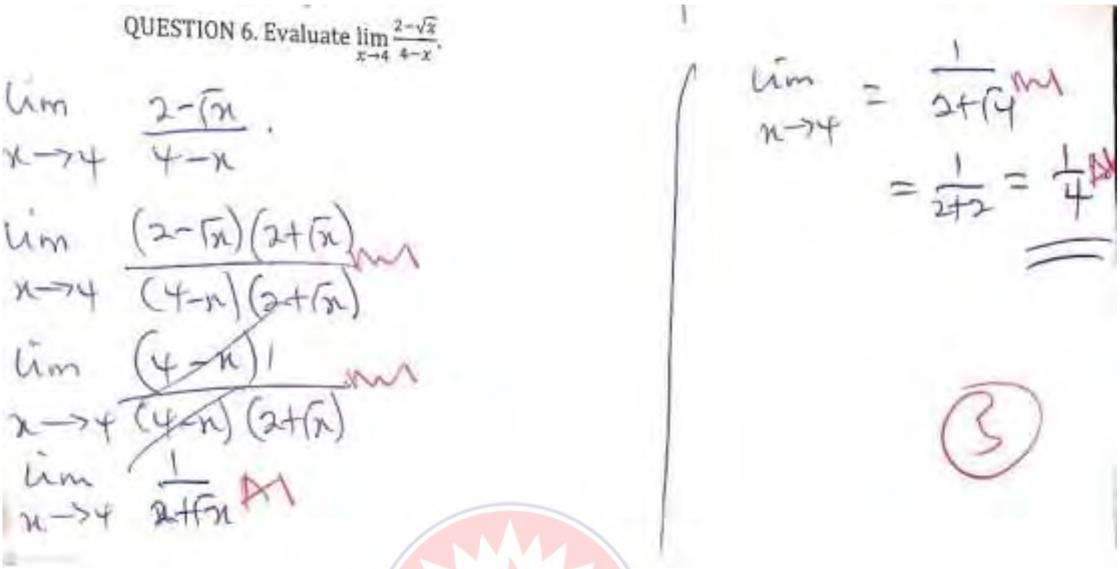


Figure 25: Suggested Procedures of Question 6

This was what the students were required to do in solving the problem in question six. Table 4.2 shows that 34 students representing 55.7% were not able to rationalize the function at the numerator. 27 of the students representing 44.3% were able to rationalize the function. 34 students who took the test representing 55.7% were not able to expand and simplify to obtain the solution in the simplest form. 27 of the students that represent a percentage of 44.3% were able to expand and simplify correctly. 39 students representing 63.9% were not successful in evaluating the value of x in the solution to determine the existence of the function at that limit. 22 of the students who took the test representing 36.1% were successful in evaluating that the function exists at the given limit of the derivative of the function while 86% were unable to differentiate the function.

It is observed also that most of the students only substituted the value of x as the limit in the question into the function without first simplifying the function. Most students learnt the concept but could not apply it in dealing with functions of this nature.

Question Seven was about finding the value of a constant used in the equation of the tangent to a curve that passes through a point. First, they were to differentiate the function to get the gradient. Secondly, they were to use the two other points through which the curve passes to obtain the value of the gradient. They were to equate the value of the gradient to the gradient of the curve. Finally, they were to solve the equation to obtain the value of the constant in the equation of the tangent to the curve. Figure 26 below shows the procedural knowledge that students were required to follow in solving the problem.

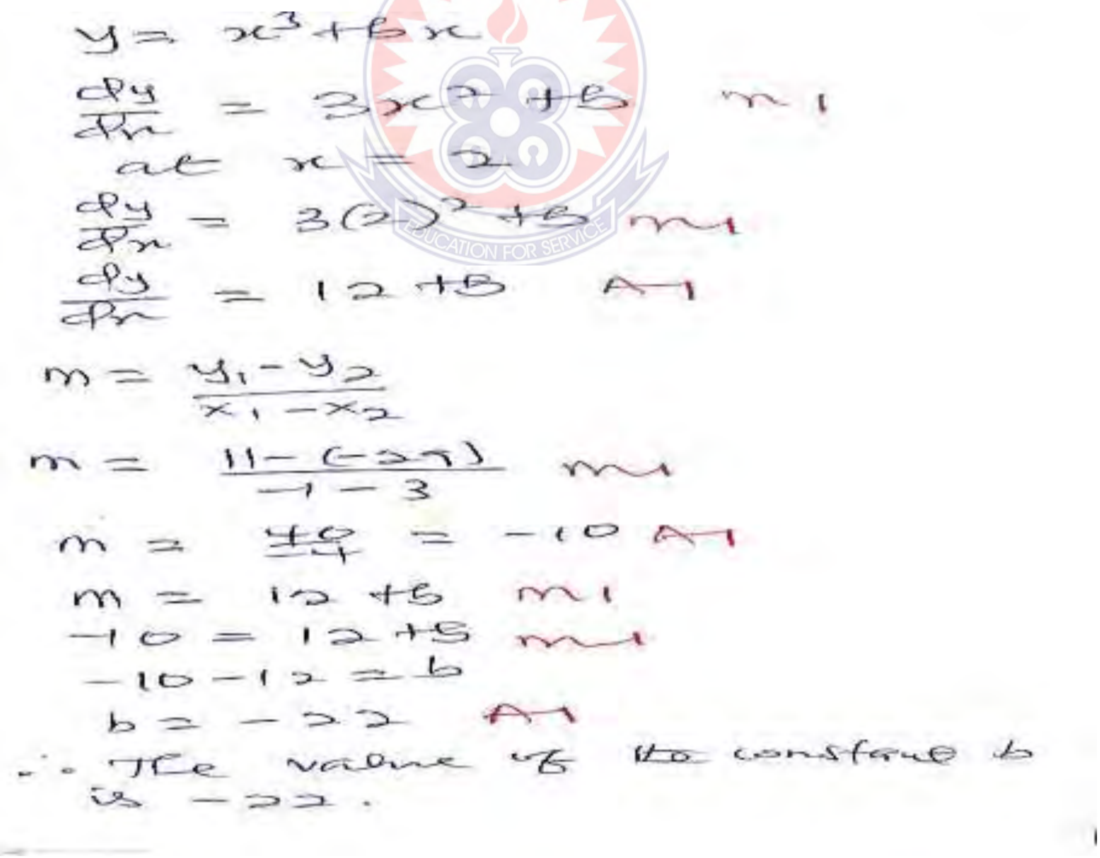


Figure 26: Suggested Procedures of Question 7

From the data in the Table 4.2, 31 students representing 50.8% of the students were not able to differentiate the function as the gradient while 30 0f the students representing 49.2% were able to differentiate the function as the gradient. 33 students representing a percentage of 54.1% from the table above indicates that those students could not correctly put the value through which the curve passes through to get the gradient whereas 28 of the students representing 45.9% were able to obtain the gradient of the curve as $12 + b$ after substituting the value of x into the gradient. Also, 30 students representing 49.2% could not correctly find the gradient through given points. 31 students representing 50.8% were able to find the gradient through points as -10. 36 students representing 59.0% of the data could not find the value b from the question using the two gradients whilst 25 of the students representing 41.0% were able to correctly solve for the value of b using the two gradients.

Question Eight explored students" procedural knowledge on how to use the product rule to differentiate an equation to get the equation of the tangent through a point if the curve is serpentine. Students were to differentiate the curve and put the value of x in the point into the differentiated function to get the gradient value as m. Again, they were to state the formula for finding the equation of the tangent to a curve as $y - y_1 = m(x - x_1)$ and put the value of the gradient into the equation as m and the point given as x_1 and y_1. They were expected to expand and solve to get the equation of the tangent to the curve. The scheme used in marking the question is captured in figure 27 below.

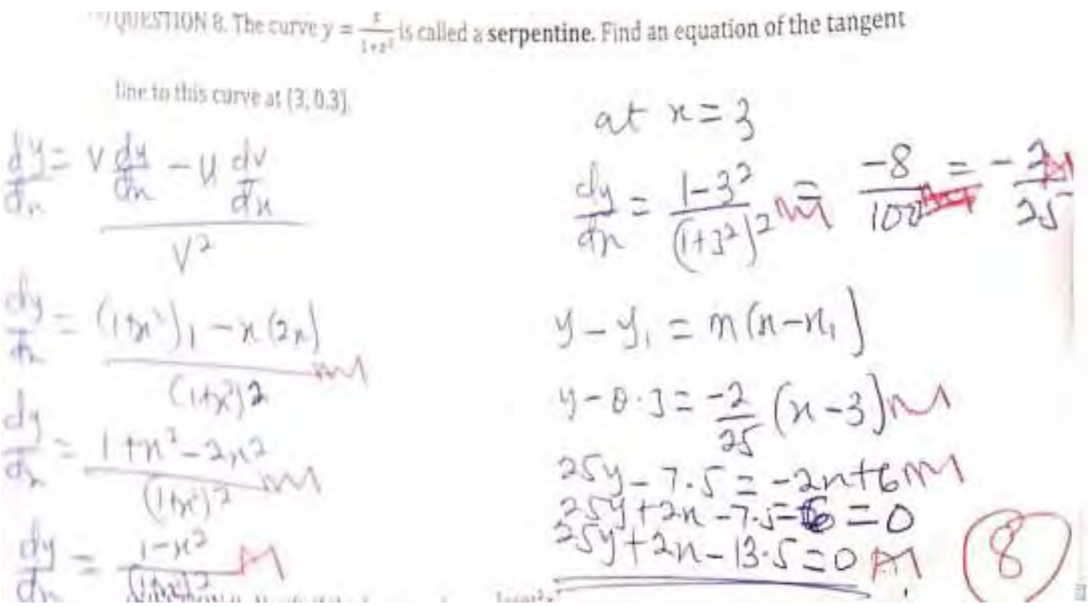


Figure 27: Suggested Procedures of Question 8

Table 4.2 above shows that 47 students representing 77.0% were not able to identify that they needed to use the quotient rule and to state it correctly. 14 of the pre-service teachers representing 23% were able to get the first procedure used in solving the problem by identifying the quotient rule to be used in solving the question. 49 out of the total students representing 80.3% could not differentiate the functions involved in the question. They had no either conceptual knowledge or procedural understanding of the question, only 12 students representing 19.7% were able to differentiate the functions involved in the questions.

Again, 50 students out of the total students representing 82.0% were not able to differentiate and write the answer of the gradient correctly while 11 students out of 61 of the student teachers were able to write the answer of the gradient after differentiating the functions both the numerator or denominator and deriving the answer as the gradient of the function. 55 out of the sample that took the test representing a percentage of 90.2% could not find the gradient by putting the value of

x into the gradient function. 6 students representing 9.8% were able to find the value of the gradient by putting the value of x into the gradient function. 55 students which represent 90.2% were not able to write out the equation of a line as demanded by the question. Only 6 students representing 9.8% were able to use the state of the equation of the tangent to the curve and also to put the value of x, y *and* m to obtain the equation of the tangent to the curve. Finally, 56 students which represents 91.8% were unable to expand and solve to obtain the equation of the tangent to the curve but only 5 out of the total pre-service teachers were able to correctly solve the question to get the final answer A_1. The table indicates that most learners had difficulties in finding the equation of the tangent to a curve through passing through a curve.

Question Nine explored the procedural knowledge of students on how to verify if a function exists at a given point. Students were to first expand the denominator of the function. They were to put the value of x into the function U to see if it will produce the y value to indicate it exists at the given point. If the result is the same as the value of y, then it can be concluded the function U exists at that given point or otherwise.

The procedures used in solving this problem are captured in the marking scheme in figure 28 below.

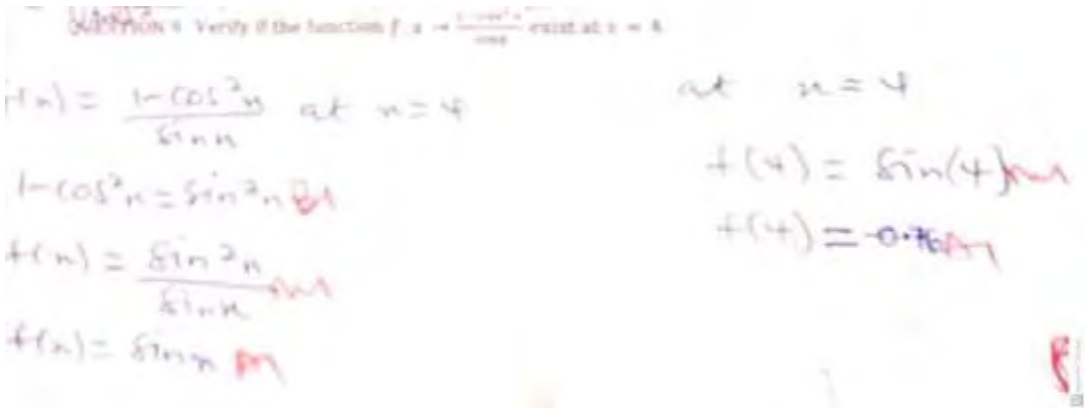


Figure 28: Suggested Procedures of Question 9

From Table 4.2, 53 students who wrote the test representing 86.9% were not able to state the identity that $1 - cos^2x = sin^2x$ will be used to solve the question with only 8 students representing 13.1% being able to identify the first procedure was to use the identity above to simplify the function. Though majority of the pre-service teachers were unable to state the identity, 8 of them representing 13.1% could simplify the function as $\frac{sin^2x}{sinx}$ whereas 53 of the students representing 86.9% could not either state the identity or simplify the given function as $\frac{sin^2x}{sinx}$. 54 student teachers representing 88.5% could not simplify the function to arrive at $sinx$ but only 7 student teachers representing a percentage of 11.5% were able to arrive at $sinx$ as their final answer to score the A_1score. They were to show if the function exists at the given limits. So, only 5 of the students representing 8.2% were able to put the limit into the final answer while 57 of the students representing 91.8% were unable to use the limit to show the function exists. Finally, only 4 out of the total number of participants who wrote the test representing 6.6% were the only pre-service teachers who could prove that the function exists at the given limit. Majority of the students could not go through the procedures involved in proving a function exists at a given limit.

It is observed that most of the college students studying mathematics and its related courses find it difficult in showing a function exists under a given limit despite the fact that, they are to have knowledge on this and go out there to also impact what they have learned.

Finally, Question Ten of the Conceptual and Procedural Based Test (CPUBT) seeks to find out the procedural knowledge students use in the application of derivatives. The 10 (i) requires students to state that, at the highest point, velocity=0 since the question wanted the Time t seconds at the highest point. They were to differentiate the height with respect to time $\frac{dh}{dt}$. They were to put the $\frac{dh}{dt}$ to zero since they were finding the Time t seconds at the highest point. They were to solve for t in the equation that represents the Time t seconds and at the highest point.

Below are the procedures used in solving the above problem in figure 29.

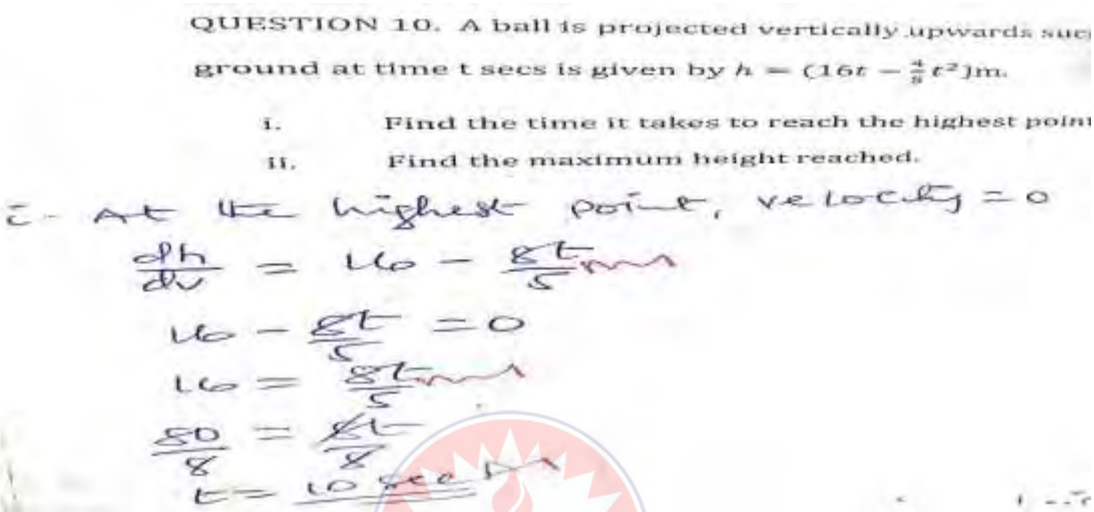


Figure 29: Suggested Procedures of Question 10(a)

From Table 4.2, 30 students representing 49.2% could not differentiate the function as the first procedure that will be used in finding the time at the highest or maximum point. 31 respondents representing 50.8% were able to differentiate the first as the first procedure that will be used in finding the time at the highest point. The second procedure was for student teachers to state that at the highest or maximum point, $\frac{dh}{dv}$ is zero. In doing this, 28 students representing 45.9% were not able to do this while 33 pre-service teachers representing 44.1% were able to get this procedure correct. The next procedure was for them to solve for time and arrives at 10s. 29 pre-service teachers representing 67.2% were not able to do this. 32 of the pre-service teachers were able to do this and hence were given the correct mark for the final answer. It is realized from the scripts that students had knowledge on what to do but the procedures to be used were the problem as the students were just solving it anyhow.

Question Ten (ii) tested students" knowledge on how to use time at the highest point to find the maximum height. Since the function was for height, students were expected to put the time in Question Ten (i) into the equation of the particle to find the height. The methods in doing this is stated in the figure 30 below.

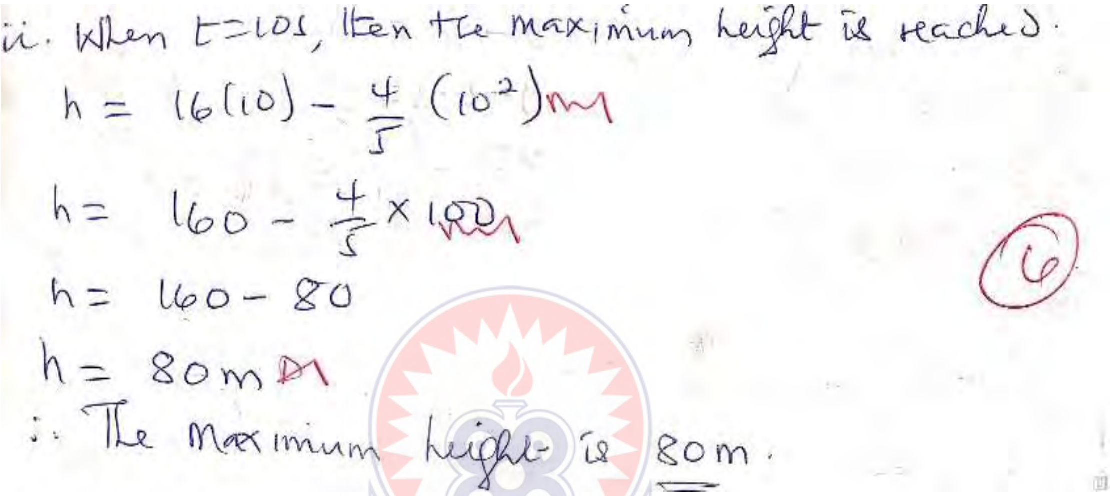


Figure 30: Suggested Procedures of Question 10(b)

Table 4.2 indicates that 30 students representing 49.2% were not able to find the maximum height reached by putting the time at the highest point in 10 (i) into the height of the particle to get the maximum height. 31 pre-service teachers representing 50.8% were able to put the time into the height of the particle to get the maximum height reached by the particle. Finally, 30 of the participants representing 49.2% were not able to simplify the results to get 80m as the maximum height. 31 students representing 50.8% were able to simplify to get the maximum height of the particle as 80m. Here, pre-service teachers who had conceptual knowledge about the concept had no or little difficulties with the procedures used in solving the problem.

Discussion and Major Findings

Students" lack of knowledge could be a major reason why they cannot solve certain problems consistently (Hudson & Miller, 2006). The findings of the study were drawn from the deep and systematic analysis and interpretation of the collected data. This study was a descriptive study about the conceptual and procedural knowledge in learning derivatives. The main purpose of this study was to explore conceptual understanding and procedural knowledge. For this purpose, the study included two types of derivatives (Geometric concept and physical concept). From this study, the results of the data analysis showed that students had little knowledge in the following areas: Weak Concept to Understand the Derivative as a Rate of Change. Though it was expected of students to perform well from the Questions 1-5 based on *APOS* model because the questions were *Action* questions that was required of students to do well, the case in this study was different as pre-service teachers could not solve some basic concepts of derivatives involving *Action*. Pre-service teachers could not understand the concept of derivatives in finding trigonometric functions. Students were not able to understand the meaning of rate of change as the gradient of a line. The symbolic form and the verbal form are very important in the derivative concept especially symbolic form. Students were not able to differentiate the symbolic form of derivatives as D or f^1. Students were not able to verify that a function exists at a particular limit. Also, students were not able to understand the concept of function and function notation. Also, they were not able to understand the derivative of a function is again a function not only the particular value. They were not able to explain on their answer sheets. They solved the problem which was asked in the Conceptual and Procedural Understanding Based Test (CPUBT) without understanding the meaning of rate of change.. All these things mentioned above justified the fact that the students had difficulties in understanding the meaning of derivative as a rate of change based from the APOS model used in the CBUBT. It is because of lack of conceptual understanding, lack of multiple representations of concept, teacher's weak performance, focusing on procedural understanding lack of understanding of language and phrases etc.

Students had difficulties in understanding the geometrical meaning of derivative as a slope of the tangent at a given point. Students were not able to understand the concept about slope. Students who have some concept about slope failed to differentiate the slope is not the ratio of totals but is the ratio of difference. Students were not able to find the slope of the function $y = (x - 3)(x^2 + 2)$ at $x = 1$. They were not able to understand the derivative of a given function is the slope function and if we put the value of x it gives the slope of the tangent at that point in the curve of a function. Another thing was they had a weak understanding of concept of limits, various terms and languages. Most of the students had not given the answer to question number Two (a) and Two (b). They were not able to find the slope when the curve of a derived function of a given function was given. This showed that they did not understand the curve of derivative function gives the slope. Also, a lot of students did not answer question number 2a and 2b. All the facts justified that the students had a very weak understanding of the geometrical meaning of derivatives. The main causes of these difficulties are the misconception about the slope as ratio totals instead the ratio of difference, misunderstanding of derivative as a particular value or derivative as a function, lack of pre-knowledge, lack of understanding of the multiple representation of the same mathematical concept, weak performance of teacher in using the various methods in teaching the concept, lack understanding of the meaning of limit.

Students were not able to understand the concept of limit of a function. They had a misconception that limit means directly putting the value of x in the function. Students did not understand the concept of limits. Students were not able to explain how the limit of function $x \to 4$ as x tends to 4. Students had a weak understanding of phrases like tends to, approaches to, limiting value, tending value, limit as an infinite process etc. These all things justified the fact that the students had the difficulty in conceptual understanding the limit. The main causes of difficulties are the lack of understanding of quantifiers like tends to, limit, approaches etc., focusing on procedural understanding, teacher"s weak performance, student"s irregularities, becoming exam oriented etc.

Students had difficulty using the power rule. Even some respondents use it correctly but one respondent was not able to use it but they were not able to explain in which condition we use the power rule. They could not understand the generalized power rule. They could not be able to connect the link between the generalized power rule and the chain rule. They did not explain that when differentiating the function cot^2x with respect to x, first we differentiate it with respect to $cotx$ because cot^2x is the function of $cotx$ and then we differentiate the $cotx$ with respect to x. Hence it can be seen that students have difficulty in the finding derivative using the power rule.

The researcher sought to find out if pre-service teachers have conceptual difficulties in derivatives. It is clear from the data gathered that; students face conceptual difficulties in the learning of derivatives. This is because pre-service teachers could not solve basic concepts involving *Action* questions modeled from *APOS*. The analysis of the scripts of the pre-service teachers" item-by-item indicated from the data obtained that, there had been a consistent difficulty faced by pre-service teachers to solve the problems conceptually and procedurally on derivatives. The percentages of the pre-service teachers who were able to solve the problems correctly fell below 40% as most incorrect percentages was always above 50%. The data also pointed to some of the errors that pre-service teachers made working derivatives. Some of the errors made by the students working derivatives were factual errors because the pre-service teachers made could not solve the problems because of lack of knowledge. Evidence from the scripts indicates that, the difficulties pre-service faced in solving the problems were as result of lack of knowledge resulting in factual error. Therefore, one of the major findings of the researcher is the fact that Pre-service teachers have difficulties in solving derivatives.

From the table 4.1 above, it can be observed that though most students were taught derivatives, they lacked conceptual understanding on the concept because most of the students could not solve problems that involve derivatives. The researcher also sought to find out how pre-service teachers conceptually understands derivatives. The findings suggest most of the pre-service teachers have troubles with conceptual understanding of derivatives because percentage of students who could apply conceptual knowledge were far less than those who had no conceptual knowledge based on the percentages from the data in table 4.1. Some of the pre-service teachers from the scripts also showed some conceptual errors because they had knowledge of the concept but made errors in working the problems resulting in conceptual errors. For instance, most pre-service teachers even tried to state the quotient rule that was supposed to be used in differentiating some of the functions but could not state it correctly. The inability to state some concepts that is to be used in solving a problem according to Hudson & Miller (2006), is classified as conceptual difficulties. The study concluded based on these conceptual difficulties and percentages of pre-service teachers incorrectly solving the problem as weak in both geometric and physical concepts of derivatives. It is therefore, important for pre-service teachers to have conceptual understanding of the concept instead of the traditional way of „chew, pour, pass and forget" since the concept is used in their daily activities if grasped well.

The research explored the pre-service procedural knowledge on derivatives. It is evidenced from the data that most pre-service teachers learn the concept of derivatives but find it difficult to procedurally use the understanding in solving derivatives. The data also shows that majority of pre-service teachers could not solve a problem related to trigonometric functions under derivatives as seen in Question One (b) and Question Nine. Pre-service performed poorly in the procedures used in solving trigonometric functions and showing that a function exists under a given limit. On the other hand, pre-service teachers performed fairly well in questions that required only differentiation *(Action*) as seen in Questions One (a) and Two (a). Questions that required the application of the concept knowledge were met with challenges by students. Students" procedure difficulties were reflected in their performance in Table 4.2. The scripts of the pre-service teachers show that, most of the pre-service teachers make procedural difficulties when working derivatives resulting in procedural difficulties by Hudson & Miller (2006). The procedural difficulties on some of the scripts is as a result of poor attention and carelessness. They do not give proper attention when solving the problem especially on the *Process* questions like limits of functions from modeled from *APOS* theory. Most pre-service teachers could not procedurally follow what was required of them in solving a lot of the problems in the Conceptual and Procedural Understanding Based Test (CPUBT). Some of the pre-service teachers are much interested in the answers without procedurally following the steps involved in getting the final answer. It is interesting to note that, much attention was not on the answers in answering the third research question but the methods and steps used in solving the problem. The findings suggest that, majority of the pre-service have difficulties in the procedures used in solving derivatives. Generally, there was not a clear-cut performance of pre-service teachers on the test based on the *APOS* model of questions.

CHAPTER FIVE

SUMMARY OF FINDINGS, DISCUSSION AND CONCLUSION

5.0 Overview

This chapter deals with the summary, findings, conclusion and recommendation concerning the student's conceptual and procedural difficulties in learning derivative. After rigorous analysis and interpretation of data, the findings of the study have been derived and conclusions have been made based on the findings, the implications have been forwarded to different levels. This chapter is divided into three sections findings, conclusion and recommendation for educational implication.

5.1 Summary

This study was titled "Pre-service teachers" conceptual and procedural understanding of derivatives". The purpose of this study was to explore the difficulties related to the conceptual and procedural understanding of derivatives. Though similar works were done on students" conceptual and procedural knowledge in derivatives (Rhode, Jain, Poddar and Ghosh (2012), Laridon, Jawurek, Kitto, Pike, Myburgh, Rhodes-Houghton, Sasman, Scheiber, Sigabi, & Van Rooyen (2007), Sugan Kafle (2019), Sallah et al., 2021). However, there are no or limited work on Pre-Service Teachers. The study sort to find out whether the Pre-Service Teachers who are learning Mathematics (Calculus) at the Colleges of Education level and also becoming mathematics teachers at the basic school level also face similar difficulties especially in concept and procedures in solving derivatives. To achieve this, the research was guided by the following questions.

1) What are the pre-service teachers" conceptual difficulties in learning derivatives?

2) What are pre-service teachers‟ conceptual understanding of derivatives?

3) What procedures do pre-service teachers use to find derivatives?

To achieve the objective of the study, data and information were collected through Conceptual and Procedural Understanding Based Test (CPUBT) in two colleges of education in Volta region of Ghana. The design of the research was a descriptive design. The study was conducted by purposively choosing 61 pre-service teachers in two colleges of education in the Volta region of Ghana. The findings of the study revealed that, Pre-Service Teachers are weak in both physical and geometric concept of derivatives, pre-service teachers have conceptual difficulties in finding derivatives, and pre-Service Teachers lack procedures use in solving derivatives.

5.2 Implication for Practice

This study found out that Pre-service teachers studying mathematics in the study area have relatively low conceptual and procedural understanding of derivatives. This trend of low understanding of college students studying mathematics is worrying because these are learners of higher cognitive order because learning mathematics and its related courses at the colleges of education level assumes that Pre-service teachers must be knowledgeable enough in mathematics. Teacher preparation should develop pre-service mathematics teachers' specialised content knowledge (Ndlovu at al. 2017). Derivative (Calculus) is one of the major areas in mathematics, thus not having enough understanding in the concept can go a very long way to affect the students‟ performance in mathematics in general. The study will put pressure on teachers (tutors) to adopt best and efficient methods in teaching student‟s concepts that will put indelible understanding of students to be able to solve problems related to derivatives. A cross section of the students were able to also follow concepts and procedures in solving of the problems in the test. This gain made by students should be maintained if possible improved upon especially on the approaches used in teaching and learning the concepts. For instance, 35 students representing 57.4% were able to solve the question one of the Conceptual and Procedural Based Test. This is quite a good performance and should only be improved upon.

The study also found out that pre-service teachers who participated in the study lacked conceptual understanding of both geometric and physical concepts of derivatives. They do not know how some of the concepts are applied in solving problems involving derivatives. For instance, Pre-service teachers could not apply the rules of differentiation on functions involving square roots. Also, students could not also evaluate limits, and differentiate trigonometric functions. This suggests that, College tutors of the course to use other activities and applications such as Maple to teach pre-service teachers derivatives. The teachers should therefore, take advantage in some of these challenges in other to address the problems in subsequent lessons.

The study identified that; Pre-Service Teachers could not solve problems procedurally involving derivatives. A lot of the Pre-service teachers only attempts the problem half-way and cannot work with the rightful or needed procedures in solving the problem. Other made procedural errors in solving problems in the test. To ensure that students stick to the appropriate procedures and reduce procedural errors in solving problems in derivatives, course tutors should make sure exercises, assignments, test, etc. involves a lot of procedures when working to inculcate in pre-service teachers how they should follow procedures in solving problems involving derivatives. It will also afford the teachers the opportunity to help eschew „chew and poor, pass and forget". The effective use of teaching and learning materials in teaching mathematical concepts should also be employed by teachers to make methods stick in the minds of students. Akkoyunlu (2002), also suggested that the use of teaching and learning materials is significant element in raising the quality of education.

5.3 Findings

The findings of the study revealed that;

- Majority of the pre-service teachers who part in the study have weak geometric and physical concepts of derivatives and hence Pre-service teachers have difficulties in solving problems relating to derivatives.
- 70% of Pre-service teachers who took the test lack the conceptual understanding of derivatives.
- 60% of Pre-service teachers who participated in the study lack procedural understanding of derivatives.

5.4 Conclusion

The findings of the study were drawn from the deep and systematic analysis and interpretation of the collected data. This study was case study about pre-service teachers conceptual and procedural understanding of derivatives in two selected colleges in the Volta region. The main purpose of this study was to explore the conceptual understanding and the procedural understanding. For this purpose, I included that the two concepts about derivatives thus geometric (Derivative as a rate of change, and derivative as a slope of a tangent) and physical (limits, functions and trigonometric functions). From this study, the following conclusions can be made.

1. Pre-service teachers who participated in the study had difficulties in solving problems in derivatives.
2. Pre-service teachers who took part of the study are weak in both geometric and physical concepts in derivatives.
3. Pre-service teachers in the study were unable to follow needed procedures in finding derivatives.

5.5 Recommendations

As a result of the findings that emerged from the study, the researcher wishes to recommend the following;

- College mathematics tutors in the area of study should put much more emphasis on both the geometric and physical concepts of derivatives to reduce Pre-service teachers difficulties in solving derivatives.
- College mathematics tutor in the area of the study should use more activities and exercises in teaching the concept in other to discourage pre-service teachers from rote memorization or learning to help improve upon their conceptual understanding.
- College mathematics tutors

5.1 Areas for Further Research

This is the case study of two selected Colleges of Education in the volta region. This study consisted of only the final year pre-service teachers of the two selected Colleges of Education in the Volta region. So, the findings and conclusions drawn from the study cannot be generalized. The researcher tried to make some suggestions for further studies in this field.

1) Further research can be done on all colleges of education reading mathematics and ICT in all the colleges in the volta region.
2) Further study can also be done on the factors that influences pre-service teachers conceptual and procedural understanding of derivatives.
3) Further study can also be done on the methods that can best be used in teaching pre-service teachers the concept of derivatives.

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APPENDICES

APPENDIX A

Inter -Rater Consistency

Reliability Statistics	
Cronbach's Alpha	N of Items
.923	2

Reliability

Reliability Statistics	
Cronbach's Alpha	N of Items
.997	2

APPENDIX B

TEST QUESTIONS

Conceptual and Procedural Understanding Based Test (CPUBT)

Name of College………………………………………………………….

Level……………………………………………………….…………….

Department………………………………………………………………

Mathematics: Major [ ] Minor [ ]

Sex: Male [ ] Female [ ] *Tick appropriately*

Question 1. Find $\frac{dy}{dx}$ if:

a. $y = 3\sqrt{x}$

b. $y = cot^2 x$

QUESTION 2a. Find the gradient of the curve $y = (x-3)(x^2+2)$ at $x = 1$

b. Find the slope of the function $2x + y^2 - 5 + 8y + 2xy^2 = 0$ at $(3, -4)$.

QUESTION 3. If f $(x) = x^3 - 2x$ and g$(x) = x^2 - 3$ Show that D$[f(x)g(x)] =$ $f^1(x)g(x) + f(x)g^1(x)$

QUESTION 4. If $f(x) = x^3 - 5$ and $g(x) = x^2 + 1$, find:

a. D[f(g(x))]

b. D$[f(x)g(x)]$

Question 5. If a function, $f(x) = \frac{x^2}{4x+1}$, find an expression for the gradient function.

QUESTION 6. Evaluate $\lim_{x \to 4} \frac{2-\sqrt{x}}{4-x}$.

QUESTION 7. The tangent to the curve $y = x^3 + bx$ at the point where $x = 2$ passes through the points (-1, 11) and (3, -29). Find the value of the constant b.

QUESTION 8. The curve $y = \frac{x}{1+x^2}$ is called a **serpentine.** Find an equation of the tangent line to this curve at (3, 0.3).

QUESTION 9. Verify if the function $f: x \to \frac{1-cos^2x}{sinx}$ exist at $x = 4$.

QUESTION 10. A ball is projected vertically upwards such that its height above the ground at time t secs is given by $h = (16t - \frac{4}{5}t^2)$m.

i. Find the time it takes to reach the highest point.

ii. Find the maximum height reached.

APPENDIX C

CONCEPTUAL AND PROCEDURAL UNDERSTANDING BASED TEST (CPUBT) MARKING SCHEME

METHOD/PROCEDURE	MARK(S)
Question 1. Find $\frac{dy}{dx}$ if:	
$y = 3\sqrt{x}$	
$y = 2x^{-3}$	
$\frac{dy}{dx} = -6x^{-4}$	M_1
$\frac{dy}{dx} = \frac{-6}{x^4}$	A_1
a. $y = cot^2x$	
$Let\ u = cotx$	M_1
$\frac{du}{dx} = -cosec^2x$	M_1
$y = u^2$	
$\frac{du}{dx} = 2u$	M_1
$\frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{du}{dx}$	
$\frac{dy}{dx} = 2u \cdot (-cosec^2x)$	M_1
$\frac{dy}{dx} = -2ucosec^2x$	M_1
$\frac{dy}{dx} = -2cotcosec^2x$	A_1
QUESTION 2a. Find the gradient of the curve $y = (x-3)(x^2+2)$ at $x = 1$	
$y = (x-3)(x^2+2)$ Using the product rule	
$\frac{dy}{dx} = Vdu + Udv$	
$\frac{dy}{dx} = (x^2+2)1 + (x-3)2x$	M_1

$\frac{dy}{dx} = x^2 + 2 + 2x^2 - 6x$	M_1
$\frac{dy}{dx} = 3x^2 - 6x + 2$	A_1
At $x = 1$ $\frac{dy}{dx} = 3(1)^2 - 6(1) + 2$	M_1
$\frac{dy}{dx} = -1$ $\therefore The\ gradient\ of\ the\ curve\ is -1$	
b. Find the slope of the function $2x + y^2 - 5 + 8y + 2xy^2 = 0$ at $(3, -4)$.	
$y = 2x + y^2 - 5 + 8y + 2xy^2$ Using implicit differentiation	
$\frac{dy}{dx} = 2 + 2y\frac{dy}{dx} - 0 + 8\frac{dy}{dx} + 4xy\frac{dy}{dx} + 2y^2$	M_1
$2y\frac{dy}{dx} - 0 + 8\frac{dy}{dx} + 4xy\frac{dy}{dx} = -2 - 2y^2$	M_1
$\frac{dy}{dx}\left(\frac{2y + 8 + 4xy}{2y + 8 + 4xy}\right) = -\frac{(2 + 2y^2)}{2y + 8 + 4xy}$	M_1
$\frac{dy}{dx} = -\frac{(2 + 2y^2)}{2y + 8 + 4xy}$	A_1
At the point $(3, -4)$ $\frac{dy}{dx} = -\frac{(2 + 2(-4)^2)}{2(-4) + 8 + 4(3)(-4)}$	M_1
$\frac{dy}{dx} = \frac{-(2 + 32)}{-8 + 8 - 48}$	M_1
$\frac{dy}{dx} = \frac{-34}{-48} = \frac{17}{24}$ $\therefore Slope\ (m) = \frac{17}{24}$	A_1
QUESTION 3. If $f(x) = x^3 - 2x$ and $g(x) = x^2 - 3$ Show that $D[f(x)g(x)] = f^1(x)g(x) + f(x)g^1(x)$	
$D[f(x)g(x) = f^1xg(x) + f(x)g^1x$	
$D[(x^3 - 2x)(x^2 - 3)] = u^1v + uv^1$	

$(x^3-2x)(x^2-3)+(x^3-2x)(2x)$	M_1
$3x^4-2x^2-9x^2+6+2x^4-4x^2$	M_1
$3x^4+2x^4-2x^2-9x^2-4x^2+6$	M_1
$\mathbf{5x^4-15x^2+6}$	A_1
$f^1(x)g(x)+f(x)g^1(x)$	
$(3x^2-2)(x^2-3)+(x^3-2x)(2x)$	M_1
$3x^4-2x^2-9x^2+6+2x^4-4x^2$	M_1
$\mathbf{5x^4-15x^2+6}$ $\therefore D[f(x)g(x)]=f^1g(x)+f(x)g^1x$	A_1
UESTION 4. If $f(x)=x^3-5$ and $g(x)=x^2+1$, find:	
b. D[f(g(x))]	
$D[f(g(x))]=(x^2+1)^3-5$	M_1
$D[f(g(x))]=3(x^2+1)^2\cdot 2x-0$	M_1
$\mathbf{D[f(g(x))]=6x(x^2+1)^2}$	A_1
c. $D[f(x)g(x)]$	
$D[f(g(x))]=D[(x^3-5)(x^2+1)]$	M_1
$(3x^2)(x^2+1)+(x^3-5)(2x)$	M_1
$\mathbf{5x^4+3x^2-10x}$	A_1
Question 5. If a function, $f(x)=\frac{x^2}{4x+1}$, find an expression for the gradient function.	

$$\frac{dy}{dx}=\frac{v\frac{du}{dv}-u\frac{dv}{du}}{v^2}$$ Using the Quotient rule.	
$$y=\frac{x^2}{4x+1}$$	
$$\frac{dy}{dx}=\frac{(4x+1)2x-x^2(4)}{(4x+1)^2}$$	M_1
$$\frac{dy}{dx}=\frac{8x^2+2x-4x^2}{(4x+1)^2}$$	M_1
$$\frac{dy}{dx}=\frac{4x^2+2x}{(4x+1)^2}$$	M_1
$$\frac{dy}{dx}=\frac{2x(2x+1)}{(4x+1)^2}$$	A_1
QUESTION 6. Evaluate $\lim_{x\to 4}\frac{2-\sqrt{x}}{4-x}$.	
$$\lim_{x\to 4}\frac{2-\sqrt{x}}{4-x}$$	
$$\lim_{x\to 4}\frac{(2-\sqrt{x})(2+\sqrt{x})}{(4-x)(2+\sqrt{x})}$$	M_1
$$\lim_{x\to 4}\frac{(4-x)}{(4-x)(2+\sqrt{x}}$$	M_1
$$\lim_{x\to 4}\frac{1}{(2+\sqrt{x}}$$	A_1
$$\lim_{x\to 4}\frac{1}{(2+\sqrt{4}}$$	M_1
$$\mathbf{\frac{1}{2+2}=\frac{1}{4}}$$	A_1
QUESTION 7. The tangent to the curve $y=x^3+bx$ at the point where $x=2$ passes through the points (-1, 11) and (3, -29). Find the value of the constant b.	
$$y=x^3+bx$$	
$$\frac{dy}{dx}=3x^2+b$$	M_1

$\frac{dy}{dx} = 3(2)^2 + b$	M_1
$\frac{dy}{dx} = 12 + b$	A_1
$m = \frac{y_1 - y_2}{x_1 - x_2}$	
$m = \frac{11 - (-29)}{-1 - 3}$	M_1
$m = \frac{40}{-4} = -10$	A_1
$m = 12 + b$	
$-10 = 12 + b$	M_1
$-10 - 12 = b$	M_1
$\boldsymbol{b = -22}$ $\therefore \textit{The value of the constant } b = -22$	A_1
QUESTION 8. The curve $y = \frac{x}{1+x^2}$ is called a **serpentine.** Find an equation of the tangent line to this curve at (3, 0.3).	
$\frac{dy}{dx} = \frac{v\frac{du}{dv} - u\frac{dv}{du}}{v^2}$ Using the Quotient rule.	
$\frac{dy}{dx} = \frac{(1 + x^2) - x(2x)}{(1 + x^2)^2}$	M_1
$\frac{dy}{dx} = \frac{1 + x^2 - 2x^2}{(1 + x^2)^2}$	M_1
$\frac{dy}{dx} = \frac{1 - x^2}{(1 + x^2)^2}$	A_1
At $x = 3$ $\frac{dy}{dx} = \frac{1-3^2}{(1+x^2)^2} = \frac{-8}{100} = \frac{-2}{25}$ Or -0.08	M_1A_1
$y - y_1 = m(x - x_1)$ Equation of a straight line	

$y - 0.3 = -\frac{2}{25}(x - 3)$	M_1
$25y - 7.5 = -2x + 6$	M_1
$25y + 2x - 7.5 - 6 = 0$ $25y + 2x - 13.5 = 0$	A_1
QUESTION 9. Verify if the function $f: x \to \frac{1-cos^2x}{sinx}$ exist at $x = 4$.	
$f(x) = \frac{1-cos^2x}{sinx}$ at $x = 4$	
Using $1 - cos^2x = sin^2x$	B_1
$f(x) = \frac{sin^2x}{sinx}$	M_1
$f(x) = sinx$	A_1
$f(4) = \sin(4)$ at $x = 4$ $\therefore f(x) = \frac{1-cos^2x}{sinx}$ at $x = 4$ is $\mathbf{0.070(3dp)}$	M_1A_1
QUESTION 10. A ball is projected vertically upwards such that its height above the ground at time t secs is given by $h = (16t - \frac{4}{5}t^2)$m. iii. Find the time it takes to reach the highest point. iv. Find the maximum height reached.	
i. At the highest point, velocity ($\frac{dh}{dv}$) =0	
$\frac{dh}{dv} = 16 - \frac{8}{5}t$	M_1
$16 - \frac{8}{5}t = 0$	
$16 = \frac{8}{5}t$	M_1
$80 = 8t$ $\therefore \boldsymbol{t = 10seconds}$	A_1
ii. At maximum height reached, $t = 0$	

$h = 16(10) - \frac{4}{5}(10)^2$	M_1
$h = 160 - \frac{4}{5}(100)$	M_1
$h = 160 - 80$	
$\boldsymbol{h = 80m}$ $\therefore$ *The maximum height is* $80m$	A_1

APPENDIX D

AKATSICO

HEAD HEART HANDS

AKATSI COLLEGE OF EDUCATION

MEMORANDUM

TO	:	HoD, Mathematics/ICT
FROM	:	College Secretary
DATE	:	11th October, 2022
REF	:	AKCE/19/VOL.2/0105
SUBJECT	:	Permission to Collect Data

Please be informed that Mr Isaiah Biga has been granted permission to collect data from the Mathematics/ICT Department.

Mr Biga is a student of the University of Education, Winneba, reading a Master of Philosophy Degree in Mathematics Education. He is conducting research on the topic **"Pre-Service Teacher's Conceptual and Procedural Understanding of Derivatives"** in fulfilment of the requirements of his programme.

Attached are copies of his introductory letter from the University of Education, Winneba, and questionnaire.

We would appreciate any courtesies that could be extended to him.

Thank you.

AKATSI COLLEGE OF EDUCATION

Our Ref: AKCE/50/VOL.3/0195

Your Ref: ....

HEAD HEART HANDS

Private Mail Bag
Akatsi, V/R
Tel: 0204371312
Fax: No.: 03626-44181
E-mail: info.akatsicoe@gmail.com

17th October, 2022

MR ISAIAH BIGA

UNIVERSITY OF EDUCATION, WINNEBA

P. O. BOX 25

WINNEBA.

Dear Mr Biga,

RE: PERMISSION TO GATHER DATA IN YOUR INSTITUTION

Your letter dated 17th October, 2022 on the subject above refers.

You are granted permission to collect data from our institution as requested for the purpose of research on the topic "Pre-Service Teacher's Conceptual and Procedural Understanding of Derivatives".

Please be reminded that the exercise is solely for academic purposes and must therefore be guided by all relevant ethical standards of academic research and under no circumstances should responses be used for any other purpose.

Thank you.

Yours faithfully,

BENEDICTA E. M. DANKU
COLLEGE SECRETARY
AKATSI COLLEGE OF EDUCATION
AKATSI, V/R.

ST. FRANCIS COLLEGE OF EDUCATION - HOHOE

CONTACTS: 0546253062/0500006690
BANK: GCB Bank PLC. Hohoe
EMAIL: info@franco.edu.gh

Post Office Box 100, Hohoe
Volta Region,
Ghana W/A
Digital Address – VC-0026-9502

Our Ref: COEG/VR/SFCE/220/37

Date: 27th September, 2022

MR. ISAIAH BIGA
UNIVERSITY OF EDUCATION
FACULTY OF SCIENCE EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION
P. O. BOX 25
WINNEBA

LETTER OF APPROVAL

This letter acknowledges that we have received and reviewed a request by Mr. Isaiah Biga, a Postgraduate student in the University of Education, Winneba, to conduct a research project entitled, ***"Pre-Service Teacher's Conceptual and Procedural Understanding of Derivatives"*** at St. Francis College of Education, Hohoe.

The College approves of this research to be conducted at its premises. He should be given access to collect the required data for the research project.

Thank you.

CLEMENT KANTAM KOLAMONG
(COLLEGE SECRETARY)

COLLEGE SECRETARY
ST. FRANCIS COLL. OF EDU.
P. O. BOX 100,
HOHOE

Cc: **HOD, MATHEMATICS DEPARTMENT**
ST. FRANCIS COLLEGE OF EDUCATION, HOHOE

STUDENTS' AFFAIRS OFFICER
ST. FRANCIS COLLEGE OF EDUCATION, HOHOE