UNIVERSITY OF EDUCATION, WINNEBA

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DEPARTMENT OF MATHEMATICS EDUCATION

# USING THE BALANCE MODEL TO IMPROVE THE UNDERSTANDING OF THE PRINCIPLES OF SOLVING LINEAR EQUATIONS IN ONE VARIABLE AT BUASI ROMAN CATHOLIC JUNIOR HIGH SCHOOL

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AUGUST, 2013

# DECLARATION

## STUDENT'S DECLARATION

I, Amoako Johnson, declare that this Dissertation, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

SIGNATURE:………………………………….

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## SUPERVISOR'S DECLARATION

I hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of Dissertation as laid down by the University of Education, Winneba.

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DATE:…………………………………………....................

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## ABSTRACT

This study investigated whether the use of the balance model in the teaching and learning of linear equations can improve students‟ conceptual understanding in the principles of solving linear equations in one variable. The data was gathered using tests. A pre-test conducted on thirty (30) Form two pupils of Buasi Roman Catholic Junior High School and an analysis of the interviews enabled the development and the use of the balance model in the teaching of linear equations in one variable. The study adopted a post-test after an intervention design on the 30 J.H.S 2B pupils. The descriptive analysis of the tests‟ results showed that the use of the balance model in teaching and learning of linear equations improves pupils‟ understanding of the principles of solving linear equations in one variable. This leads to improvement in pupils‟ performance in solving linear equations in one variable.

# CHAPTER 1

# INTRODUCTION

## 1.1 Background of the Study

Development in almost all areas of life is based on effective knowledge of science and mathematics. There simply cannot be any meaningful development in virtually any area of life without knowledge of science and mathematics (Sherrod, Dwyer and Narayan, 2009). It is for this reason that the education systems of countries that are concerned about their development put great deal of emphases on the study of mathematics (Ministry of Education, 2012). Mathematics is also widely regarded as one of the most important school subjects and a central aspect of the school curriculum in every society. Oyedeji (2000) also backs this idea by saying that, mathematics is an extremely advantageous tool in almost all spheres of human life, be it Science and Technology, Medicine, Engineering, the Economy, Industry, Arts and in Public Decision-Making. Again, from Umoren (2006) the practical value of mathematics is seen in the construction of roads and bridges, building of houses, communication, banking, insurance, politics and sociology, design of materials, music and many more.

In another context, the main purpose for teaching and learning mathematics according to National Mathematics Advisory Panel (2008) is to develop the ability of the learner to solve a wide variety of both simple and complex mathematics problems in their daily lives. Also, the study of mathematics is therefore seen as a means of sharpening the mind, shaping the reasoning abilities, and developing the personality of the individual to become a more scientifically and technologically minded person in the society (Asiedu-Addo & Yidana, 2001). That is why most countries put a great emphasis on the study of a well planned and effectively implemented mathematics education program especially those that are concerned about their scientific and technological development. Furthermore, the Curriculum Research Development Division (CRDD) in Ghana emphasized that mathematics together with other three subjects as a core and compulsory subject for all learners in the Junior High School. This makes mathematics as a critical filter for pupils or students seeking admissions to second circle and tertiary institutions as well as professional institutions such as College of Education and Polytechnics in Ghana (Adetunde, 2009). Adetunde (2009) then confirms that mathematics forms the foundation of any solid education.

Mathematics education has been highlighted above as a very important subject in these recent times since almost all domains of human knowledge apply conceptual and computational methods of mathematics (Eshun, 2000). This shows that the world has now become a global village and trends in all human endeavors have assumed new shapes and dimensions. Eshun (2000) therefore explained that is important that mathematics as a subject in Ghana should also be transformed in its teaching and learning to meet the challenges demanded of it by the ever-changing world. That is why a constructivist Sapkova (2011) beliefs that teachers in particular and educational planners in general are implored by society to design practical methods of teaching and learning that are applicable to the learner‟s environment and our everyday life situations. This is because, children possess a natural curiosity and interest in mathematics, and come to school with an understanding of mathematical concepts and problem solving strategies that they have discovered through exploration of the world around them (Scherer and Steinbring, 2006). Identifying these fundamental activities, mathematics educators are to provide experience that will continue to foster pupils‟

understanding and appreciation of mathematics to improve on their performance. According to Schifter (2001) and Rojano (2002) this can be done by providing mathematics activities in which pupils are encouraged to explore and make sense of mathematical pattern and relationship that will help them develop mathematical knowledge to solve problems and explore new ideas in the classroom and the technological world. Nabie (2001) supported this idea and argued that the traditional method of „talk and chalk" presentation of mathematical facts does not enable learners to apply what they learn to real life situations or solve problems. Nabie (2001) further explained that to enable learners apply what they learn to real life situations, teachers must teach mathematics within the pupils" experiential domain. This is however the opposite of what pertains in the classrooms.

Mereku (2001) continued to explain that at the basic level in Ghana most of the mathematics teachers taught largely through lecturing and teacher-centered approaches. Mereku (2001) again point out that these methods deny the pupil from experiencing the learning of mathematics using manipulative materials and this inhibited the development of the pupils" intuition, imagination and creative abilities thus leading to poor understanding of mathematics concepts. Also, some topics in mathematics are usually skipped by majority of those who teach it, especially the ones they find uncomfortable and those topics may however tend to be the bases upon which other important topics be built (Ingvarson, Beavis, Bishop, Peck and Elsworth, 2004). These contributed to the greater poor performance of pupils in mathematics (Anamuah-Mensah and Mereku, 2005). This is confirmed by Kraft (1994) and TIMSS (2007), who said that the pupils" performance in mathematics in Ghana remains among the lowest in Africa and the world as a whole. Also, a lot of concerns are frequently expressed by mathematics educators about the low achievement in mathematics at both Junior and Senior High levels of education in Ghana.

To overcome these challenges a variety of teaching and learning strategies have been advocated for use in mathematics classrooms moving away from the teacher-centered approaches to more pupils-centered approaches, and the use of concrete material is one of such potential strategies (Moyer, 2002). Also there is a substantial evidence of research work done by other researchers on the various effects on the use of concrete learning materials on pupils (Johnson, 1993). For example, Boggan, Harper and Whitmire, (2010) in recommending the use of concrete materials to teach mathematics to pupils suggested that practical work provides the most effective means by which understanding in mathematics can develop and also using concrete materials make classroom an interesting and exciting place for both the teacher and learner (McClung, 1998). Hence to overcome the challenges and difficulties of pupils, a variety of teaching and learning strategies have to be introduced for use in mathematics classroom moving away from the teachers-centered approach to more pupils-centered approach (Puchner, Taylor, O'Donnell and Fick, 2010). Therefore one paramount issue that comes to force concerning the improvement in mathematics education involves the use of concrete materials to teach the topics in the mathematics syllabus such as linear equation in one variable (Uttal, Scudder and Deloache, 1997).

A linear equation in one variable is one of the mathematical topics studied by second year pupils of the Junior High School in Ghana (Ministry of Education, 2012). The National Council of Teachers of Mathematics (NCTM, 2010) also places a heavy emphasis on linear equations as a central part of any mathematics course, especially at Lower Secondary level. They further point out that not only do they appear in their own right, but they are also an integral part of a wide variety of Algebraic, Geometric, and Trigonometric problems. These make Poon and Leung (2010) to confirm that linear equation is one of the most important topics in algebra and mathematics as a whole and also one of the first concepts a youngster encounters in pre-algebra course. This proves the teaching and learning of linear equations in one variable as an opener to the study of algebra. Historically, linear equations have also played a central role in the development of other aspects of mathematics, and in solving real-life problems (Dreyfus and Hock, 2004). Even though there has been a major shift in the landscape of school mathematics in recent years, learning to solve linear equations is still an essential element in the study of algebra (Chazan, 2008). For that reason, pupils must be able to solve linear equations while understanding the process, justifying, and explaining the steps. Unfortunately, many pupils still have a difficult time learning algebra, particularly learning the principles and skills related to the solving of linear equations in one variable (Richard, 2002). Cai & Moyer (2008) also supported this idea saying solving of linear equations causes confusion for pupils because of the improper way and manner it is taught by some teachers. The basic skills and pre-requisite knowledge needed to effect a fuller comprehension is normally lacking in the learners and sometimes those who teach them (Kieran, 2007).

This is confirmed by The WAEC chief examiner‟s reports of most of the Basic Education Certificate Examination in Ghana, stated that many of the pupils abstain from answering questions on linear equation in one variable and a couple of them who attempted showed little or no understanding of the principles of solving such equations. Solving linear equations in one variable has also been the problem of the second-year pupils of Buasi R/C JHS, Offinso in the Ashanti Region of Ghana. The problem was identified, when the pupils were given a test after teaching the said topic to see how best they understood the process of solving linear equation in one variable.

After the test, the pupils were given a different set of questions of relatively the same level of difficulty to solve in order to confirm the poor performance of the pupils in solving linear equations in one variable. After carefully analyzing the solutions of some of the pupils, the researcher realized that the pupils‟ performance in the test was abysmally poor. They applied wrong algebraic principles and used inappropriate methods, these made them arrive at wrong solutions. Also, pupils did not solve linear equations in one variable by using the principles of maintaining balance of the equation by performing the same operations on both sides. The above report and the results of the previous tests, together with how the pupils solved similar questions in class exercises and examination, called attention to the need to assist the pupils in Buasi R/C J.H.S to improve their performance in solving linear equation in one variable.

According to Fujii (2008), to overcome these challenges and difficulties of pupils, a variety of teaching and learning strategies has been introduced for use in mathematics classroom moving away from the teachers-centered approach to more pupils-centered approach. Also teaching strategies and curriculum design for pupils should therefore emphasize on the importance of giving meaning to algebraic symbols and using them in solving linear equations by applying real life situations (Kieran, 2007). Thus, from Vlassis (2002) a well planned use of balance model when implemented effectively in the mathematics lessons will help pupils in solving linear equations in one variable. Carraher, Schliemann and Brizuela (2001) further elaborated that an interactive nature of the balance model will enable pupils to apply what they learn to real life situations. Moreover, Mereku, Duedu and Atitsogbi (1995) supported this idea and stated that, the balance model provides a very powerful mental picture for solving linear equations. Also, the equation is viewed as a set of balance scales and the equal sign in the equation is represented by the scales being in balance (Carry, Lewis and Bernard, 1980; Pirie and Martin, 1997). They further elaborated that, it is standard to use this model diagrammatically as a thinking aid and also improve the process of solving the linear equation in one variable. Pirie and Martin (1997) further explained that the principles of beam balance is linked to the principles of equality, such that if each chip on the scale has the same weight, then the weight on each side is equal and hence there is a balance. Therefore in mathematics, whatever is done to one side of an equation must be done to the other side. Again, McClung (1998) added that a good way to picture a true equation is the use of a balance model. It is for these reasons that the study was designed to use the balance model in teaching linear equation in one variable. Hence, to improve the performance of the pupils in solving linear equation in one variable, the researcher propose using the balance model as an intervention.

## 1.2 Statement of the Problem

The main rationale for teaching mathematics in Junior High School ( J.H.S) as stated in the J.H.S Mathematics Teaching Syllabus in Ghana (Ministry of Education, 2012), is to enable all Ghanaian young person‟s acquire the mathematical skills, insights, attitudes and values that they will need to be successful in their chosen careers and daily lives. It also builds on their knowledge and competencies developed at the primary school level. In Ghana a pupil is expected at the Junior High School level to move beyond and use mathematical ideas in investigating real life situations. The strong mathematical competencies developed at the Junior High School level are necessary requirements for effective study in mathematics, science, commerce, industry and a variety of other professions and vocations for pupils terminating their education at the Junior High School level as well as for those continuing into tertiary education and beyond (Purdie, Hattie and Douglas, 1996).

Also, understanding and solving problems involving linear equation in one variable is one of the most important topics to be learned as a prerequisite to the study of algebra (Dugopolski, 2002). Moreover, the performance of pupils in application of sets (two set problems), powers of numbers, numeration systems, plane geometry, polygons and many more mathematical topics could be improve by a good knowledge of linear equation (Mathematical Sciences Education Board, 1998). Kieran (2007) further elaborated that, this can be achieved by the effective use of concrete materials. Gravemeijer (1991) also said that the use of concrete materials provides the most effective means by which understanding in mathematics especially linear equation can develop.

Unfortunately, pupils have challenges in learning linear equations in one variable (Richard, 2002). Their performance at external examination is also not impressive. This is confirmed in an external examination (WAEC, 2011), that most pupils could not answer questions on linear equation in one variable and those who attempted it showed little or no understanding of the principles of solving such equations. Also, studies have shown that pupils are not able to solve mathematical problems which linear equation in one variable is included (Brizuela and Schliemann, 2004). The researcher again observed that some of the pupils in Buasi R/C J.H.S., Offinso were also not able to solve linear equation in one variable. This was proved in some of the questions that they solved for exercises, tests and examinations. In it some of the pupils could not divide both sides by the co-efficient of the variable, group like terms, remove brackets and even not able to balance both sides of the equation in their simplification. This occurred because the pupils were probably not taught linear equations by using a manipulative materials and this could not develop their understanding of the principles of solving linear equations in one variable. Further interactions with my colleague teachers in basic schools revealed that many of them do not use teaching and learning materials that build on pupils‟ informal mathematical experiences in the teaching and learning of linear equation in one variable. Mereku (2001) and Johnson (1993) confirmed that the poor performance is as a result of the less used of the manipulative materials to teach linear equation in one variable. Johnson (1993) further concludes that effective use of manipulative materials would promote pupils confidence and effective pupils‟ mathematical contributions in the classroom.

The problem then is what must be done to the pupils to assist them improve upon their understanding, knowledge and skills as well as acquiring new principles to solve linear equation in one variable.

## 1.3 Purpose of the Study

The purpose of the study was to use the balance model joined with good pedagogical skills to teach linear equations in one variable and this will improve the understanding of the principles of solving linear equation in one variable and thereby brings about significant performances of pupils at Buasi R/C J.H.S in solving linear equations in one variable. This research is also to find out the effect the use of the balance model will have on pupils‟ knowledge and skills in solving linear equations in one variable at Buasi R/C J.H.S, Offinso. It is hoped that the use of the balance model possibly will to some extent motivate pupils to achieve better understanding of solving linear equation in one variable.

## 1.4 Research Questions

The following research questions were formulated to guide the study:

1. To what extent would the use of balanced model be effective tool in improving pupils understanding of the principles of solving linear equations in one variable?
2. How can the use of the balance model improve pupils‟ understanding of the principles involved in solving linear equations in one variable?

## 1.5 Research Hypothesis

The hypothesis designed to guide and direct the study is:

Null hypothesis: There is no significant difference in scores between the mean pre-test scores and the mean post- test scores of pupils.

Alternative hypothesis: There is significant difference in scores between the mean pre-test scores and mean post- test scores of pupils.

## 1.6 Objective of the Study

The Research focuses on the use of the balance model to improve pupils‟ performance in solving linear equations in one variable. The specific objectives therefore are to:

1. Improve pupils understanding in terms of dividing both sides, grouping like terms and removing of brackets in the principles of solving linear equations in one variable.
2. Determine the effects of using the balance model on pupils‟ performance in solving linear equations in one variable.
3. Enhance pupils‟ practical experience in solving linear equations in one variable.

## 1.7 Significance of the Study

The results from this study are intended to develop the understanding of the principles of solving linear equations in one variable through the use of the balance model at Buasi R/C J.H.S. The results of the study can also assist teachers to identify the mistakes pupils make when solving linear equations in one variable and guide them to vary their teaching methods to enable pupils to understand and relate the principle to real life situations. This would improve the performance of the pupils when solving linear equations in one variable.

For the part of the pupils the use of the balance model will encourage pupils to develop more confidence and interest in learning linear equations in one variable and this will develop in them the knowledge and skills in solving linear equations in one variable and other mathematics related topics. It would also enable the pupils to discover common mistakes they make when solving linear equations in one variable.

The results of the study would again help the educational policy makers, the curriculum research development division and teachers to accept that the balance model is effective and efficient when solving linear equations in one variable. This will make them ensure that the teachers use the balance model when teaching linear equations in one variable.

The balance model would also deepen existing mathematics curriculum by adding to the stock of teaching and learning materials (TLMs) and the methodology for teaching and learning of linear equations.

### 1.8 Delimitations of the Study

This research was confined to the second-year pupils of Buasi R/C J.H.S., Offinso in the Ashanti Region. The scope of the study was strictly at improving the understanding of the principles of solving linear equation in one variable of the pupils and thereby enhancing the performance of the pupils in solving linear equations in one variable through the use of the balance model. The study was also restricted to the use of the balance model to demonstrate the principles of linear equations in one variable without negative solutions or decimal coefficients.

### 1.9 Limitations of the Study

The main weakness of the study was that, the intervention was limited to only second-year pupils of the Buasi R/C J.H.S. in the Ashanti Region due to the financial difficulties, time constraints and then workload of the researcher. Hence the study could not be generalized.

Due to sickness and absenteeism, some of the pupils did not participate actively in the lessons. Others took part in the pre-test and post-test but failed to participate fully in the intervention as they absented themselves during some of the lessons hence did not benefit fully from the intervention. These resulted in a poor performance in the post-test.

### 1.10 Organization of the Study

The study has been organized into five chapters, which are introduction, literature review, methodology, results analyses and discussions and summary, conclusions and recommendations.

The first chapter entitled introduction, which consists of background to the study, statement of the problem, purpose of the study, research questions, objective of the study, research hypothesis, significance of the study, delimitation, limitation and organization of the study.

The second chapter presents literature related to the study. This literature examines what has been written by other researchers which are related to linear equations in one variable and strategies involved in solving them. The chapter also outlines the process that the researcher intends to use in improving the ability of the pupils to solve the linear equation in one variable.

The third chapter discusses the research methodology used for the study. It involves the research design population and sample selection, development of research instruments, pre-intervention, intervention, post-intervention, and data analysis plan.

The fourth chapter also reviews the results, findings and discussions indicates the data collected before and after the intervention, analysis of the data, discussion of the results and findings from the analysis of the pre-test and post-test.

The fifth chapter outlines the summary, conclusions and recommendations. It summarizes the report of the entire project, concludes, states suggested recommendations and presents issues for future research.

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 The Meaning of Equation

According to Howard, Asare-Ikoom and Nkani (2008), equation is the most significant tool in algebra and mathematics as a whole. They again point out that the more skillful the pupil becomes in working with equations, the greater will be his/her easiness in solving problems. Streeter, Hutchison and Hoelzle, (2001) therefore define equation as an algebraic statement in first degree that contains the equality sign „=" made up of three basic parts: the equal sign „=", the expression to the left of the equal sign and the expression to the right of the equal sign. Pirie and Martin (1997) and Dreyfus and Hock (2004) further explained that the components of an equation has a left hand side, the equality sign „=" and the right hand side. The components are further illustrated below:

| LEFT HAND SIDE | EQUALITY SIGN | RIGHT HAND SIDE |
|---|---|---|
| x + 1 | = | 2 |

Moreover, equation consists of two expressions, one on each side of an „equals" sign. They further explained that in an equation, the left side is always equal to the right side.

Supporting the previous definitions Bird (2001) also established that equations are mathematical statements that indicate equality between two expressions. That is a statement in which two expressions are equal in value. Confirming this definition Dugopolski (2006), define equation as two expressions set equal to each other.

Also from Blitzer (2002) an equation is a mathematical statement which contains an = symbol. He further stated that a simplest equation is an equation which has a single variable on one side of the equal sign and a single number on the other side. Falkner, Levi and Carpenter (1999) supported the Blitzer‟s properties of equality of an equation and stated that:

- if any expression is added to both sides of an equation the resulting equation is equivalent to the original equation.
- if the same number is subtracted from both sides of the equation the resulting equation is equivalent to the original equation.
- if both sides of an equation are multiplied by the same non-zero real number, the resulting equation is equivalent to the original equation.
- if both sides of an equation are divided by the same non-zero real number, the resulting equation is equivalent to the original equation.

Bowles and Metcalf (2000) further emphasized that, an equations are also like balances. They must have the same value on both sides of the equal (=) sign. A visual representation in the home is the beam balance and mobile. The beam balance and mobile will not balance unless the mass on each side is the same. That is, in an equation the left hand side must be equal to the right hand side (Bowles and Metcalf, 2000).

Figure 2.1

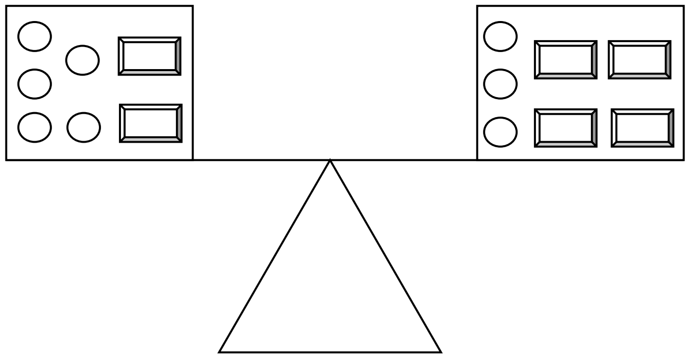


Figure 2.1: Equation compared to a balance model or scale

In using the balance scale, it is clear that any change made in one pan must be followed by an equal change in the other pan, or else the scale will not balance. Likewise, since the operations on equations are based on the same principle, the members must be kept balanced or the equality is lost.

## 2.2 The Meaning of Linear Equation in One Variable

According to Rittle-Johnson and Star (2007) graphs are used in algebra to give a visual picture containing a great deal of information about equations. Many numerical values, when substituted for the variables of an equation, will satisfy the conditions of the equation. On a particular type of graph, several of these values are plotted, and when enough are plotted, a line is drawn through these points where a certain type of curve results. For equations in the first degree in one or two variables, the resulting shape of the curve is a straight line. Thus the name Linear Equation is derived. The name Linear Equation now applies to equations of the first degree, regardless of the number of variables they contain (Hubbard and Robinson, 1995).

Also from Dugopolski (2006), the linear equation in one variable has been defined as an equation that can be written in the form "ax + b = c", where "a", "b" and "c" are real numbers and "a≠0". Moreover, Lial, Hornsby and McGinnis (2004) also stressed that a linear equation in one variable is a first degree equation that can be written in the form "ax = b", where "a" and "b" are real numbers and "a≠0".

Smith (1998) further explained that in linear equation in one variable, x, is a first-degree equation since the variable, x, has the highest exponent of one. Other researchers such as Caglayan and Olive (2010) proved this definition and explained that, a linear equation is an equation that can be written in the form ax + b = 0, where „a‟ and „b‟ are real numbers and a≠0. They further elaborated that when „b‟ is subtracted from both sides and divided by „a‟ as follows:

$$ax + b = 0,$$

$$ax = -b$$

$$x = -b/a.$$

Thus, the linear equation ax + b = 0 has exactly one solution, -b/a. The graph of the equation y = ax + b is a straight line that crosses the x-axis at (–b/a, 0).

In view of the above mentioned points, Ronald and Hubbard (1999) confirmed that the General Form of linear equation in one variable is ax + b = 0, where „a‟ and „b‟ are constants but a≠0, and „x‟ a variable. They further emphasized that the term General Form in mathematics means a form to which all expressions or equations of a certain type can be reduced. Ronald and Hubbard also explained that the first-degree term and constant term are the only possible terms in a linear equation in one variable. Once we transform a linear equation into the form ax + b = 0, it is quite easy to solve the equation. Moreover, Kaput (1999) suggested that by selecting various values for „a" and „b", this form can represent any linear equation in one variable after such an equation has been simplified to represents the numerical equation. For instance, when a = 2 and b = 3, then the numerical equation will be $2x + 3 = 0$.

Finally, the illustrations of linear equation in one variable as shown by some researchers above, confirmed the views of Streeter, et al (2001) that, a linear equation in one variable contains only one unknown quantity in the algebraic statement. The process of solving such an equation is therefore to find the value of the unknown quantity that makes the algebraic statement true. Bird (2001) back up their views and stated that linear equations in one variable are those in which an unknown quantity is raised only to the power of one and hence has exactly one solution.

### 2.3 Alternative Procedures for Solving Linear Equations in One Variable

According to Caglayan and Olive (2010), Witzel and Riccomini (2011) and other researches, there are numerous ways of solving linear equation in one variable; a few of such ways are illustrated below:

#### 2.3.1 Solving linear equation in one variable using the principle of equality (algebraic model)

The principle of equality uses the idea that the same mathematical operation must be done to each side of an equation to enhance learners solve linear equation in one variable (Falkner et al, 1999).

Asiedu (2010) and Cortes and Pfaff (2000) are some of the researchers who used the principle of equality in solving linear equation in one variable. They concluded that, the following principles should be followed when solving linear equations in one variable:

- For the addition property of equality, the same real number could be added to both sides of an equation. Thus, for all real numbers „a", „b", and „c", if a = b, then a + c = b + c.
- For the subtraction property of equality, the same real number could be subtracted from both sides of an equation without changing the solution set. Thus, for all real numbers „a", „b", and „c", if a = b then a – c = b – c.
- For the multiplication property of equality, both sides of an equation could be multiplied by the same non-zero number. Thus, if a = b and c≠0, then ac = bc, where „a", „b", and „c" are real numbers.
- For the division property of equality, both sides of an equation could be divided by the same non-zero number. Thus if a = b and c≠0, then a÷c = b÷c, where „a", „b", and „c" are real numbers.

In the same principle Bowles and Metcalf (2000) outline the following steps for solving linear equations in one variable.

a) Both sides must balance at all times.
b) We must do exactly the same to both sides in order to keep the equation balanced. To do this we can:
   - add the same amount to both sides,
   - subtract the same amount from both sides,

- multiply both sides by the same amount (this means each term on both sides),
- divide both sides by the same amount (this means each term on both sides),
- multiply out an equation with brackets in it,
- check the solution by substituting the answer back into the original equation.

They concluded that equations are easiest to solve if we collect all the unknown terms on one side and all the constant terms on the other. And also, it is best to put the unknown terms on the side that gives the positive value.

### 2.3.2 Solving linear equation in one variable using a general procedure

Pirie and Martin (1997), Van Amerom (2003), Brizuela and Schliemann (2004) and Carson, Gillespie and Jordan (2007), and other researchers also use arithmetical steps, which considers a set of rules to be followed by the learners in solving a linear equation in one variable.

These researchers outline the general procedures to follow in solving linear equation in one variable.

a) If fractions exist, then multiply all terms on both sides of the equation by the least common denominator of all the fractions to clear the fractions.
b) Use the distributive property of arithmetic to remove all parentheses if they occur.
c) Combine like terms on each side of the equation.

d) Use the addition and subtraction property of equality to collect the variable terms on one side of the equation.

e) Use the addition and subtraction property of equality to collect the constant terms on the other side of the equation.

f) Apply the multiplication and division property of equality to make the variable the subject of the equation and thus obtain the expected value of the equation.

g) Check the correctness of the solution by substituting the value of the variable into the original equation to obtain an identity.

Smith (1998) also uses the operations of adding, subtracting, multiplying and dividing (arithmetic operations) to solve linear equation in one variable.

He outlines these techniques with the following illustrations:

1) Solving linear equations with one operation.

5x = 30 5x means 5 times x.

$\frac{5x}{5} = \frac{30}{5}$ To get the x on its own you must divide both sides by 5

x = 6

2) Solving linear equations with two operations

3x = 15 – 2x 3x appears on one side of the equation, and -2x on the other side.

3x + 2x = 15 – 2x + 2x The first step is to add 2x from both sides of the equation.

$5x = 15$ — This leaves a simple one-operation problem to finish off.

$x = 15 \div 5$

$x = 3$

3) Solving harder linear equations

$7x + 6 = 41 + 2x$ — The first stage is to subtract 2x from both sides.

$7x - 2x + 6 = 41 + 2x - 2x$

$5x + 6 = 41$ — This makes the equation much simpler.

$5x + 6 - 6 = 41 - 6$ — The next stage is to subtract 6 from both sides.

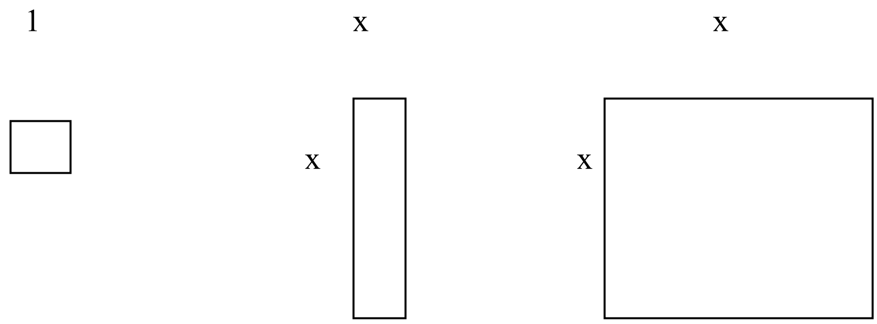
$5x = 35$ — This makes the equation even simpler.

$x = 35 \div 5$ — The final step is simply to divide by 5.

$x = 7$

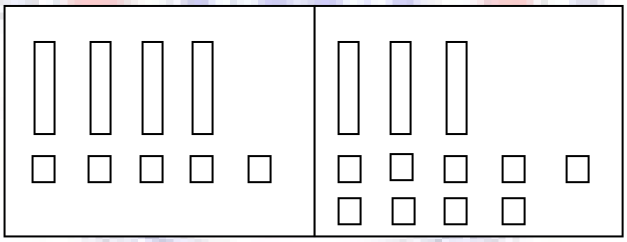
### 2.3.3 Solving linear equation in one variable by using the algebra tiles

According to Heddens and Speer (1997), one of the most powerful manipulative for exploring equations is a set of algebra tiles. Caglayan and Olive (2010) further explain algebra tiles as a manipulative that help pupils to visualize polynomial operations and solve equations. The tiles are based on an area model. A set of algebra tiles typically contains at least three different shapes with the following dimensions and values.

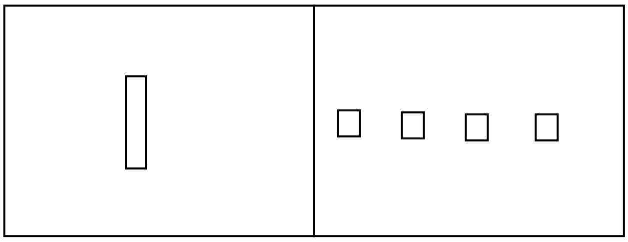



This concrete representation uses an area model. The area of the small square with dimensions 1 by 1 is 1; the rectangle has dimensions 1 by x and an area of x; and the large square has dimensions x by x and an area of $x^2$. With these tiles, a number of algebraic processes can be readily illustrated (Heddens and Speer, 1997).

Suppose that pupils were to solve this equation: $4x + 5 = 3x + 9$. They would begin by using a mat with a line drawn down the center representing the equals sign. With the algebra tiles, they would represent the quantities shown on each side of the line:



Because both sides of the paper are „in balance‟, pupils may remove any tiles they wish from each side, provided that they keep both sides in balance by removing the same number from each side. In a very concrete way, they can "remove" 5 squares and 3 rectangles from each side of the sheet, leaving a rectangle on the left side and 4 squares on the right.



Therefore, the „x" tile stood for 4 tiles and x = 4 is the solution to the equation.

## 2.4 Challenges in Solving Linear Equation in One Variable

According to Smith (1998) solving linear equation in one variable is just simply applying the operations of the arithmetic (that is addition, subtraction, multiplication and division). Falkner et al (1999) also see it as an application of the principles of equality. However, the solving of linear equation in one variable has its own challenges. These are:

- **Misinterpretation of techniques /lack of meaning for equation**

The first challenge is the belief of Sleeman (1986) and Payne and Squibb (1990) that, the poor performance of linear equation is simply a misinterpretation of techniques or lack of meaning for equations that prompt pupils to use wrong rules. Because of that the procedures are likely to become more complicated and end up causing even greater problems. This view is supported by the evidence that mathematical procedures have often been imposed on students in ways that do not necessarily develop mathematical thinking or understanding (Carpenter, Franke, and Levi, 2003). Carroll and Porter (1998) also elaborated that despite hours of instruction and practice, pupils often fail to master basic school algorithms or to apply them correctly in problem solving situations because of the misinterpretation of techniques. Carroll and Porter (1998) further suggests that imposing the standard algorithms on children gives students the idea that mathematics is a collection of mysterious and often magical rules and procedures need to be memorized and practiced. This could even result to algebra being frightening to pupils (Lima and Tall, 2008). These would not lead to better understanding of mathematical symbols when learning linear equation in one variable and most pupils will find the solving of linear equations as a more cumbersome task (Geary,1994).

- **Lack of meaning attributed to the mathematical symbols**

The next common difficulty for pupils solving linear equations according to Sherman and Bisanz (2009) is that pupils tend to take algebraic symbols as physical entities that can be moved around freely such that they can put them on the other side of the linear equation with the magic of changing signs in an effort to solve the equation (Lima and Tall, 2008). In supporting the previous point, Poon & Leung (2010) added that pupils find it difficult to move symbols from one side to the other of an equation with change of sign in order to maintain balance. Moreover, pupils‟ inability to solve linear equations at the Basic Level can be linked to pupils‟ inability to understand the meaning of "algebraic symbolization" (Borenson & Barber, 2008). This according to Kilpatrick & Izsak (2008) happens, because students struggle to develop symbolic understanding. Philipp (1999) further point out that, the previous knowledge of shifting symbol around in expressions fails when moving symbols over the equality sign. Sometimes, they even ignored the variables in algebraic expressions when solving linear equation in one variable. Furthermore, Vlassis (2008) described difficulties students experience with symbolic understanding, emphasizing that students have difficulty with symbolic understanding because of the multiple meanings that mathematical symbols hold. For example, the minus sign can be a unary sign (-*7*), which students cannot further simplify (*2x* ‑ *7y*), or a binary sign that students can further simplify (*7x* ‑ *3x*), or an operation sign (*7* - *3*). These will not assist pupils to understand and apply the equal sign when solving linear equation in one variable.

- **Misinterpretation of equal sign**

The third common difficulty for pupils solving equations involves interpreting the equal sign as a do something sign, rather than a symbol of equality (Behr, Erlwanger and Nichols, 1980). This implies that the pupils lack understanding of the meaning of the equal sign. Knuth, Stephens, McNeil and Alibali (2006) further point out that the equal sign is ubiquitous at all levels of mathematics, but little instructional time is spent describing its meaning. Erlwanger and Berlanger (1983) then concluded that without a proper understanding of equality, difficulties arise as students solve equations. Several authors also described a lack of understanding of the equal sign as a pervasive problem associated with algebra (Falkner et al., 1999).

Gonzalez, Ambrose, and Martinez (2004) as cited in Omaha (2011) agreed that many pupils perform poorly when solving linear equation in one variable because they interpret the equal sign as a command to produce an answer, rather than a relational symbol comparing two expressions, especially when the operation is on the left side and the answer located on the right side immediately after the equal sign. Knuth et al. (2006) further concluded that the unidirectional use which pupils commonly see during their arithmetic learning, leads to erroneous conceptions of the equal sign. These misconceptions can cause serious problems in learning linear equation in one variable and limits students‟ capacity to reflex on identities. This confusion will make pupils memorise the rules and procedures used when solving linear equation in one variable (Falkner et al., 1999).

- **Memorization of rules and procedurals**

Research by Loveless (2008) and Mooney, Briggs, Fletcher, Hansen and Muculloch (2011) suggest that conceptual proficiency is not developed through rote memorization when learning linear equation in one variable as the way some of the teachers teaches. Dahlin and Watkins (2000) also confirmed what Loveless said and stated that knowledge that results from simple memorizing rules and procedures without a true understanding often proves to be useless in novel situations. In supporting these points, the way much of mathematics topics and linear equation, in particular, are taught at the schools is so theoretical and abstract, it involves memorization, and students are taught a series of steps to follow, and they are told if they follow these steps, they will arrive at the right answer. But these set rules and the symbols upon which they operate, have as much meaning to the kids as Greek or Chinese symbols would have (Gu, Huang & Marton, 2004). And therefore, to the children, the process of getting the "right" answer, as traditionally taught, is often devoid of meaning (Anghileri, 1995). It is also clear that learning is not about accumulating random information, memorizing it, and then repeating it on some examination; learning is about understanding and applying principles, constructing meaning, and thinking about ideas" (Gordon, 2009, p. 743). Similarly, many educators hold the view that students should be encouraged to understand rather than to memorise what they are learning (Purdie, et al. 1996) as they believe that understanding is more likely to lead to high quality outcomes than memorizing (Dahlin & Watkins, 2000). Memorizing does not lead to a learning environment which is conducive to "good learning" and using a teaching method which is merely "passive transmission" and "rote drilling" (Gu, et al. 2004). One aspect of the criticism is that rote learning leads to poor learning outcomes. (Kadijevic, 1999).

- **The way teachers teach**

In view of the above points, Umoren (2006) noticed how teachers teach as the main causes of pupils‟ lack of understanding of mathematics especially equation. To support this point, Omaha (2011) stated in his work that due to the abstract nature of linear equation, it is difficult not only to teach, but also for students to grasp the principles of solving for an unknown quantity. However, he put the blame on teachers that, they often do not use concrete instructional materials which could improve pupils‟ understanding of mathematical concepts. Nabie (2001) also points out that the traditional method of "define and explain" technique in teaching mathematical concepts does not enable learners to apply what they learn to new situations or solve mathematical problems. He then encouraged teachers to adopt pragmatic teaching methods to facilitate mathematical understanding and to enable learners overcome learning difficulties. Also some of the mathematics classrooms used by the teachers often do not prepare students for algebraic thinking (Cai & Moyer, 2008). This does not allow students to think algebraically by using manipulative materials and these materials increases conceptual understanding. Having conceptual understanding enables pupils to meaningfully operate upon rules and procedures, and provides a strong basis for effective problems solving. Research indicates that pupils who use manipulative materials during mathematics instruction outperform pupils learning with more traditional methods (Moyer, 2002; Boggan et al., 2010). Again, Caglayan and Olive (2010) emphasized that, due to how some of the teachers teach, pupils experienced difficulty linking the physical activities of the manipulative materials and the mental operations necessary for solving equations. The National Research Council

(1989) summarized: Effective teachers as those who can stimulate students to learn mathematics. National Research Council (1989) then concluded that a greater proportion of students find mathematics difficult and they are unable to apply what they have learnt to their real life situations because of the way the subject is taught. One way that these ideas can be understood in the mathematics classroom is the shift from the teacher-centred approach of teaching with its accompanying rote learning, to the learner-centred approach which helps pupils to generate their own meaning and understanding of mathematical concepts (Anku, 2008).

## 2.5 Justification for Using Concrete Materials to Solve Linear Equations in One Variable

According to Ball (2003), children acquire knowledge about things consciously or unconsciously by exploring their environment through play and interacting with materials in their miniature world. Some of this knowledge is not even worth knowing. Learning by exploring the environment enables children to learn better even as building their own mathematical meanings. They again acquire the same knowledge through the use of concrete materials in a constructivist classroom whereby the children do not passively receive or copy input from teachers but instead actively mediates any new experience by trying to make sense of it and relating it to what they already know about the mathematical concept.

From Richardson (2001), this approach is based on the theoretical framework of constructivism. He further sees constructivism as a process of shifting learning and teaching from a teacher-centered toward a more learner-centered instruction. Moreover, Elbers (2003) also see constructivism as a view of learning suggesting that learners create and build their own understanding of the topics they study, rather than having it delivered to them by teachers and written materials. These sum up that learners be taught linear equation in one variable using their own experience from their environment as a necessary relevant knowledge to enable children to construct understandings that make sense to them, rather than having understanding delivered to them in already organized form. Thus, using constructivist approach to teach children linear equations through the use of their own playground experiences and interaction with concrete materials like seesaw can enable them to learn in a stimulating atmosphere (Gordon, 2009). Hence, using the seesaw (balance model) to teach pupils the understanding of the principles of solving linear equations in one variable seems a commendable approach. Also, there is evidence that earlier educational pioneers advocated the use of concrete equipment in the teaching of elementary numbers (Orton, 2002).

Burns and Hamm (2011) supported the view that understanding is best facilitated with the help of concrete materials. Also the NCTM (2010) in recommending the use of concrete materials to teach mathematics to pupils suggested that practical work provides the most effective means by which understanding in mathematics can develop. Furthermore, using concrete materials make classroom an interesting and exciting place for both the teacher and learner (McClung, 1998; Lee & Chen, 2010). In addition, Van de Walle and Lovin (2006) and Puchner et al (2010) also observed that learners generally find it difficult to understand abstract concepts if they are not given the opportunity to experience concrete representations. Uttal et al (1997) then concluded that, the most effective ways by which learners can be helped to understand abstractions is to look for manipulative materials for illustrations.

To support the above views, Moyer (2002) and McNeil and Uttal (2009) emphasized that the use of the concrete materials enhances pupils‟ ability and confidence to solve linear equations and thereby improve their performance. Boggan et al (2010) also confirmed that using the concrete materials in teaching linear equations takes away the abstractness seen in mathematical concepts. This according Anghileri (1995) provides good opportunities for children to handle and view models of concepts that form the basis of mathematical relationships. Li (2006) further stressed that, the use of concrete materials for practical activities is a good way of actively involving learners to gain relevant experiences in the learning process.

In other context, using the concrete materials in teaching linear equation in one variable could also present group members endless opportunities to investigate and reinforce understanding of key algebraic concepts and ask questions freely without fear (Abbasi and Iqbal, 2009). In addition, the use of the balance model to teach linear equations in one variable promotes cooperative and collaborative working group (Johnson, 1993; Caglayan and Olive, 2010). To substantiate that point, Moyer (2002) emphasized that, explaining a mathematical concept to pupils with logical reasoning and showing them concrete examples increase pupils‟ ability and help pupils to make good visualization thus making way for an interactive and a two-way learning environment. Sherman and Bisanz (2009) further elaborated that the use of concrete materials therefore provides a good opportunity to engage all learners, promote group learning and inquiry in the mathematics learning process. Bay-Williams and Livers (2009) also confirmed that frequent use of manipulative materials will enable the learner develop relevant vocabularies and other skills associated with concepts through interactions. They further established that interactions are essential components of making sense in learning linear equations in one variable.

Finally, performing a task with others provides an opportunity not only to imitate what others are doing, but also to discuss the task and make thinking visible

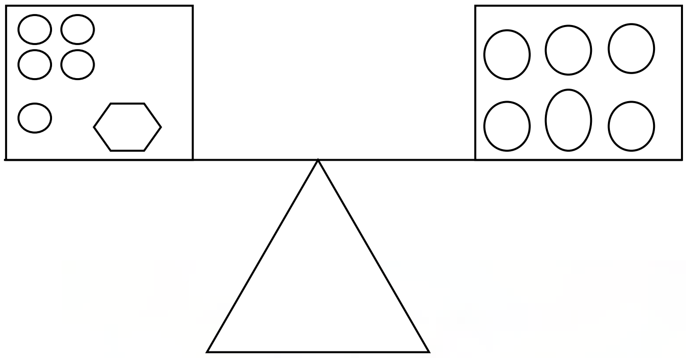
(Roschelle, Pea, Hoadley, Gordin and Means, 2000). Borenson and Barber (2008) also stated that the concrete model of the balance and the manipulative that are introduced during the lessons increase students‟ understanding of the principles as they move from a concrete model, to a pictorial representation, to finally, a symbolic (abstract) representation of a verbal problem in the form of an algebraic equation. Hence, the need to use a concrete material (balance model) to teach the understanding of the principles of solving linear equations in one variable becomes more imperative and beneficial.

## 2.6 The Balance Model

Balance model is a balance consisting of a lever with two equal arms and a pan suspended from each arm (Vlassis, 2002). A balance model depends on pull of gravity. The balance (also balance scale and beam balance) was the first mass measuring instrument invented. In its traditional form, it consists of a pivoted horizontal lever of equal length arms, called the beam, with a weighing pan, also called scale, suspended from each arm. The unknown mass is placed in one pan, and standard masses are added to the other pan until the beam is as close to equilibrium as possible. Although a balance technically compares weights, not masses, the weight of an object is proportional to its mass, and the standard weights used with balances are usually labeled in mass units.

The original form of a balance consisted of a beam with a fulcrum at its center. For highest accuracy, the fulcrum would consist of a sharp V-shaped pivot seated in a shallower V-shaped bearing. To determine the mass of the object, a combination of reference masses was hung on one end of the beam while the object of unknown mass was hung on the other end.

Fig 2.2



BALANCE MODEL

### 2.7 Justification for Using a Balance Model to Solve Linear Equation in One Variable

According to Vlassis (2002), a linear equation is therefore like a balance model in which the left-hand side is equal in value to the right-hand side. To maintain equality, what you do to one side, must be done on the other side. Warren and cooper (2005) further established that using the balance model to solve linear equations with the unknown on both sides shows that the balance model is a helpful metaphor for almost all the pupils in giving meaning to the equal sign as equality between the two sides of an equation. Vlassis (2002) continues to emphasize that, this model have been shown to be very effective in helping pupils understand the equality between the two sides of an equation. Moreover, Mereku et al. (1995) agrees that the balance model provides a very mental picture for solving linear equation in one variable and the equation is viewed, by analogy, as a set of balance scale and the equal sign in the equation is represented by the scale being „in balance". He further point out that, is usual to use this model diagrammatically as a thinking aid but there is scope to use a real set of scales to represent some equations in concrete form.

Orton (2002) also stated that the principles of the balance model and children‟s seesaw game activities they engage in at home are the same. He further declared that play activities from the pupils‟ environment and mathematics are interrelated and that they both operate on rules, employ, experiences, drills and practical applications. Therefore, the balance model as a game creates an exciting mathematical environment as they release boredom, tension and establish a friendly atmosphere, which allow for free growth of skills and knowledge (Umoren, 2006). The balance model is also viewed as a valuable educational tool for modeling relationships between the seesaw and linear equations in one variable, thus making connection between what is taught in school and children‟s everyday life activities (Warren and Cooper, 2005). Also, McClung (1998) is of the view that using balance model to solve linear equation makes classroom an interesting and exciting place for both the teacher and learner. With this idea, the balance model minimizes pupils‟ difficulties when solving linear equation in one variable (Vlassis, 2002). He then concludes that the balance model can even help pupils overcome their difficulties in solving linear equations in one variable.

Furthermore, using the balance model, Martin (1994) discussed the methods employed by the model to ease understanding and avoid mistakes when solving linear equations in one variable. Martin (1994) concluded that when these methods are employed correctly, it would improve pupils understanding of the principles involved in solving linear equations in one variable. Also, Roschelle et al. (2000) added that the balance model enables pupils to work both independently and in groups while the teacher provides guidance to the pupils. Therefore, working independently with the concrete materials enable every pupil derive maximum benefit, while working in groups enables them to socialize and communicate meaningfully at the children‟s own level of understanding.

Also, the process of using the balance model as a concrete learning material enables pupils to acquire the principles of problem solving in linear equations in one variable which can be transferred to paper and pencil problem solving format in mathematics studies (Brizuela and Schliemann, 2004). Caglayan and Olive (2010) further stated that the individuality, practicality, result-oriented and explorative nature of using a balance model in learning linear equations in one variable excite learners so much that they begin to emulate the work of their teachers. Children emulating and imitating their teachers result in frequent practice at home even without being given home exercises (Johnson, 1993). Therefore, using the balance model in teaching linear equations takes away the abstractness seen in mathematical concepts. Also from Kieran (2007), the steps performed by the use of the balance model enable the pupils to acquire the principle of maintaining balance in solving linear equations in one variable. Witzel and Riccomini (2011) also agreed to what Kieran said and they claimed that, the use of the balance model would enable pupils to perform the same operations on both side to maintain balance instead of shifting symbols around the equality sign. To add to the previous ideas, Lima and Tall (2008) emphasized that, the balance model seeks to reveal the symbolic nature of the principles involved in solving linear equations in one variable in a more practical approach. For this reason, the balance model problem solving ability leads to discovery which is aesthetic and favours both sexes thereby encouraging pupils‟ participation in the mathematics classroom (Austin & Vollrath, 1989). The mixed nature of sexes in the researcher‟s school of study is therefore suitable for use of the balance model in solving linear equations in one variable.

Finally, the use of the balance model in teaching pupils linear equations in one variable, is “something concrete and familiar to help them understand an unfamiliar, abstract idea”, thus linking a new principle to the learners‟ past experience and aiming to make that piece of mathematics more secure and memorable (Warren and cooper, 2005). Moreover, the use of the balance model enhances pupils‟ ability and confidence to solve linear equations in one variable and thereby improve their performance. In conclusion, Vlassis (2002) recommended the use of balance model to teach linear equations in one variable to pupils, because the practical work provides the most effective means by which understanding in mathematics can develop and Warren and cooper (2005) and Witzel and Riccomini (2011) and also supported the view that understanding of linear equation in one variable is best facilitated with the help of a balance model.

## 2.8 Challenges of using the Balance Model to Solve Linear Equations in One Variable.

Even though using the balance model in teaching and learning of linear equation is beneficial, yet it has its own challenges. Some of these challenges according to researchers are as follows;

- Mordant (1993) suggests that the balance model is a “very poor substitute” (p.22) for solving linear equations, because equations cannot be solved by physical operations with a scale balance and because the balance model "presents the pupil with an exceptionally poor notion of the algebraic expressions" (p.22).

- Kaput (1999) also confirms that, there is no way for linear equations involving negative signs or negative solutions to be rationalized using the balance model.
- Lins (1992) argues that using the balance model in linear equations contributes "to the constitution of obstacles to the development of an algebraic mode of thinking" (p.209).
- Vlassis (2002) further state that the balance models "does not lend itself readily to addition and subtraction of arbitrary rational numbers or to the more complex operations of multiplication and division" (p.57).
- Trelfall (1996) and Roberts (2007) has opined that there are benefits which accrue from the use of balance model. At the same time, he points out that the balance model will not show full value if it is used inappropriately.
- The balance model does not support situations in which negative and non-integer numbers are involved. Care should therefore be taken to caution pupils when using such models of their limitations and efforts should be made to introduce other models that overcome such limitations.

### 2.9 Principles of Solving Linear Equations using the Balance Model

Asiedu (2010) outlined the basic principles for solving linear equations based on the following procedures. For any given linear equation in one variable;

- If we add the same number (quantity) to both sides of the equation, we obtain a second equation which is equivalent to the first one.

- ❖ If we subtract the same number from both sides of an equation, we obtain a second equation which is equivalent to the first one.
- ❖ If we multiply both sides of an equation by the same non-zero number, we obtain a second equation which is equivalent to the first one.
- ❖ If we divide both sides of an equation by the same non-zero number, we obtain a second equation which is equivalent to the first one.

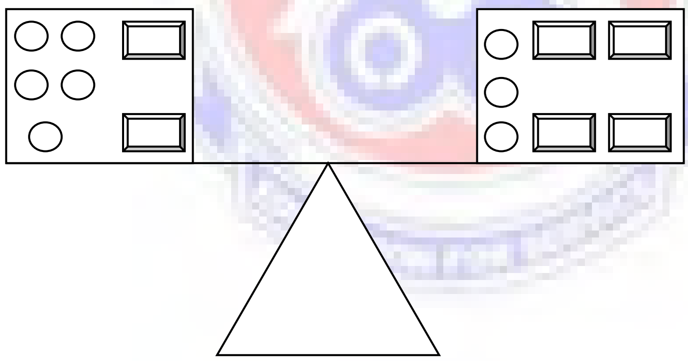
Bennett and Nelson (1998) further explained that the idea of balance scale or model is related to the concept of equality. When the balance scale or model is used, the same amount must be put on or removed from each side to maintain a balance. In other words, whatever is done to one side of the equation must be done to the other side. Hence, to solve linear equation in one variable, add the same number to or subtract the same number from both sides, multiply or divide both sides by the same non-zero number and replace an expression by an equivalent expression. To backup Bennett and Nelson views, Dugopolski (2002) also examined the principles of addition and multiplication for the solving of linear equation in one variable and stated that adding the same number to both sides of an equation does not change the solution set, for example if "a=b", then "a + c = b + c", where a, b, and c are real numbers. Similarly, multiplying both sides of an equation by the same non-zero number does not change the solution set; thus if "a = b" and c is not equal to zero, then "ca = cb". To support these views Dogbe, Mereku and Quarcoo (2004) also add that, "a – c = b – c" is equivalent to "a = b" for all real numbers. Thus, if the same number is added to or subtracted from both sides of an equation, the solution set is not altered. They added that, if "a = b" and c≠0, then "a † c = b † c". Thus, if both sides of an equation are divided by the same non-zero number, the result is an equivalent equation. However, dividing both sides by c = 0 gives an undefined solution.

## 2.10 Solving Linear Equation in One Variable using a Balance Model

In figure 2.3, the balance model uses the missing-addend form of subtraction to demonstrate the understanding of the principles of solving the linear equation in one variable, where each circle (chip) has an unknown value (that is a variable, x) and the rectangular (box) represents "1" in value. One approach to determining the number of boxes needed to balance the scale is to guess and check. Another approach is to notice that by removing a number of boxes and chips from both sides of the scale, the model will balance.

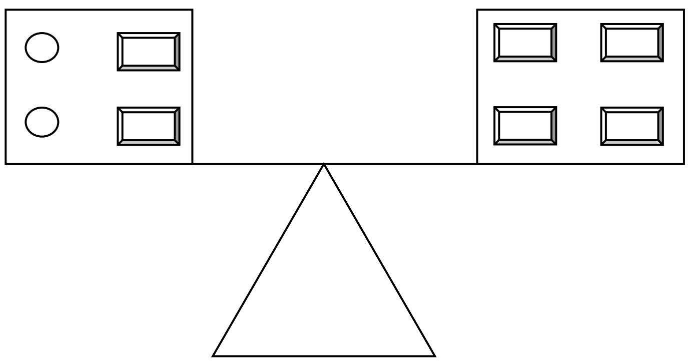
Using the balance model to solve linear equation in one variable practically demonstrate these principles. Each step in simplifying the balance scale corresponds to a step in solving the linear equation. For example, solve $5x + 2 = 3x + 4$

Visual Representation | Algebraic Representation



$$5x + 2 = 3x + 4$$

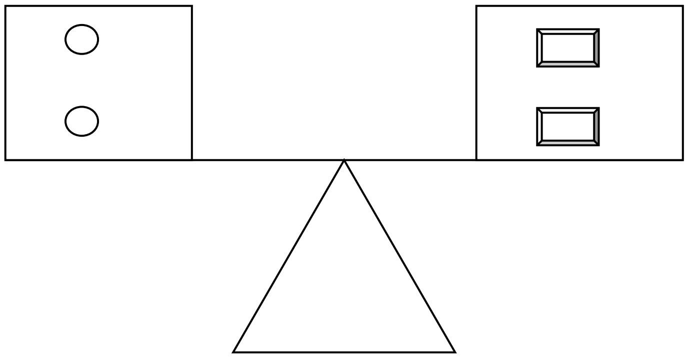
Remove 3 chips from each side | Subtract 3x from both sides



$$5x + 2 - 3x = 3x + 4 - 3x$$

$$2x + 2 = 4$$

Remove 2 boxes from each sides

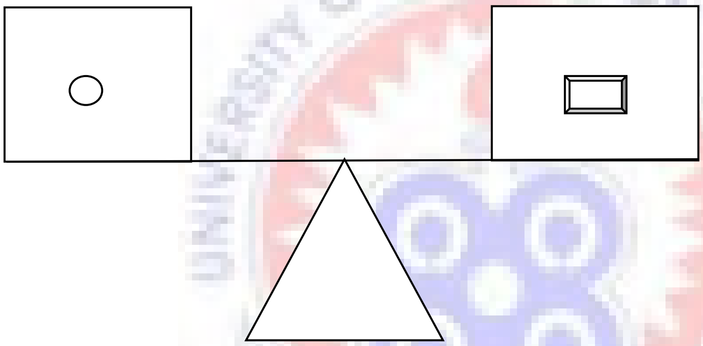


Subtract 2 from both sides.

$2x + 2 - 2 = 4 - 2$

$2x = 2$

Divide both the chips and boxes into 2 equal groups. One group for each box



Divide both sides by 2

$2x = 2$

$x = 1$

The balance model can therefore be used to demonstrate the principle of solving linear equation in one variable by trading and moving equal quantities (object) from both sides at the same time.

From the above example it is evident that the balance model has the following characteristics.

- Taking the same quantity off each side of the balance makes it easier to work out an unknown „weight".
- Taking the same number (known of unknown) off each side of an equation makes it easier to work out an unknown number.

- If one performs the same operations on each side of an equation, the answer is still the same.
- Performing the same operation on each side of an equation can sometimes make it easier to work out an unknown number (simplification).
- Finding an equation to represent a situation can sometimes make it easier to work out an unknown quantity (utilization).

## 2.11 Summary

In this final section, I summarize the key issues in the various sections of the chapter as a form of recapitulation. One key issue that arises with the study of linear equations is that pupils‟ ability to solve a linear equation in one variable depends on the ability to use the balance model to improve the understanding of the principles of solving linear equation in one variable. This model makes principles clearer, presentation easier, help maintain learner‟s interest and give learners the opportunity to extend their knowledge base at their own pace when learning linear equations in one variable. According to Nabie (2001), this makes learning easier and hence motivates children to learn further.

# CHAPTER 3

# RESEARCH METHODOLOGY

## 3.1 Research Design

The research design for this study is "Action Research" which is designed to deal with a classroom practice. The study is mainly concerned with how best certain techniques could be used to aids pupils overcome their difficulty in solving linear equations in one variable.

According to Kannae (2004), an action research is a process by which practitioner"s attempt to study their problems scientifically in order to guide, correct and evaluate their decision and actions. He further explained that, it involves the application of appropriate intervention strategies aimed at finding solutions to the problems identified in the teaching-learning situation in order to bring about change. Also, Cohen and Manion as cited in Bell (2004), described action research as an essential on-the-spot procedure designed to deal with a concrete problem located in an immediate situation. According to Bell (2004), action research is ideally the step-by-step process that is constantly monitored over varying periods of time and by a variety of mechanisms (observations, interviews, questionnaires and others) so that the ensuring feedback may be translated into modifications, adjustments, directional changes, redefinitions as necessary; so as to bring about lasting benefits to the ongoing process itself rather than to some future occasion.

## 3.2 Population and Sampling

The target population from which a sample was drawn for this purpose comprises of all the pupils of Buasi Roman Catholic Junior High School at Buasi in the Offinso Municipal, Ashanti Region of Ghana. This school was considered for the study because the researcher identified the pupils‟ difficulties when he was asked to teach mathematics in the school.

The researcher found it expedient to use the purposive sampling strategy in selecting the sample for the study. This is because the pupils that are drawn are based on available pupils and is judged to be representative of the total population. With purposive sampling, the sample is „hand-picked‟ for the research. The selected samples from the population were J.H.S 2B pupils of Buasi Roman Catholic Junior High School who had difficulties with the principles of solving linear equations in one variable. Also, according to the Junior High School Mathematics Syllabus (Ministry of Education, 2012), the teaching and learning of linear equations should begin at J.H.S 2.

The sample size for the study was thirty (30) pupils which comprises of 14 girls and 16 boys of the ages between 12 and 15 years.

## 3.3 Research Instruments

In conducting this research, pre-test and post-test were the methods used to gather information about the pupils‟ possible cause of poor performance in linear equation in one variable .The pre-test and the post-test were task given to pupils to carry out in order to know their level of performance. These tests also served as bases for evaluating the pupils.

### 3.3.1 Pre-test

A pre-test on the linear equation in one variable was given to the target group to find out their strength and weaknesses. It was also used to identify pupils‟ fundamental knowledge in solving linear equations in one variable and also the specific problems facing pupils in solving linear equations in one variable. This was to enable the researcher get to know how to handle the problem. The pre-test was made up of six (6) questions (see Appendix A). The pre-test was used to identify pupils‟ problem in linear equation in one variable.

The first two (2) questions were based on the application of dividing both sides by the coefficient of the variable. Questions 3 and 4 were based on the pupils‟ ability to group like terms and test the pupil‟s idea on equivalent equations. The last two questions tested pupil‟s idea on removing of brackets correctly taking the signs into consideration.

### 3.3.2 Administering pre-test

Each pupil was given a printed question paper and answer sheets. The duration for the pre-test was forty (40) minutes. Answers of pupils to the pre-test were marked using a marking scheme prepared by the researcher (see Appendix B).The researcher critically examined (evaluated) the wrong answers given by the pupils to find out the possible causes. Discussions were held with pupils to find out why they answered some questions the way they did. The pupils‟ score in the pre-test marked over 30 is recorded.

## 3.4 Data Collection Procedure

The research procedure was divided into three stages: pre-intervention stage, intervention stage, and post-intervention stage.

### 3.4.1 Pre-intervention stage

The pre-intervention was made up of a pre-test that was used as a diagnostic instrument to find out the level of pupils‟ difficulties in linear equations in one variable. The pre-test was also used to determine the most professional and appropriate way of intervening to address pupils‟ difficulties in linear equations in one variable. Also, it is used to gain adequate insight into the strengths and weaknesses of the pupils in the principles of solving linear equations in one variable. It was observed that the pupils did not understand linear equations in one variable even though they had been taught. The researcher was made to know that, the principles of solving linear equations were delivered to the class without the use of manipulative materials. The researcher therefore administered a six (6) question pre-test on linear equations in one variable to the sampled group to diagnose the extent of their difficulty. The pupils‟ responses were collected, marked, scored over thirty and the scores were then recorded. The scores were then used to decide on the seesaw/balance model as a type of intervention that could adequately address the pupils‟ difficulties.

### 3.4.2 Intervention stage

In the intervention stage, the researcher exploited the use of the balance model to address the lack of understanding of the principles of solving linear equations in one variable on the side of the pupils. This according to Lawrence (1999) means using the balance model is therefore a way of presenting the principles of solving linear equation in concrete terms to pupils. This would therefore help improve pupils understanding of the principles involved in solving linear equations in one variable. Again, the use of the balance model enhances pupils‟ ability and confidence to solve linear equations in one variable and thereby improve their performance.

In the intervention process, five miniature wooden seesaw models were designed and employed in the teaching of linear equations in one variable for five groups of pupils. Each group had a seesaw model and cut-out "variables" and "numerals" made from plywood.

The intervention period took 3 weeks of three mathematics lessons of three hours a week (60 minutes per lesson) which were taught and supervised by the researcher. These involved the use of five miniature wooden seesaw models with their „weights‟ („x‟ variables made of round pebbles of 1g, each representing an „x‟ and coloured red) and „ones‟ (rectangular plywood cut-outs, coloured blue and a size of 2cm by 1cm). Pupils were grouped into 5 with each group having six members. Each group was encouraged to work as a team with each group appointing their own leaders. Cooperative and independent learning strategies were used throughout the intervention stage. The topics discussed were divided into nine lessons for the 3 weeks of intervention.

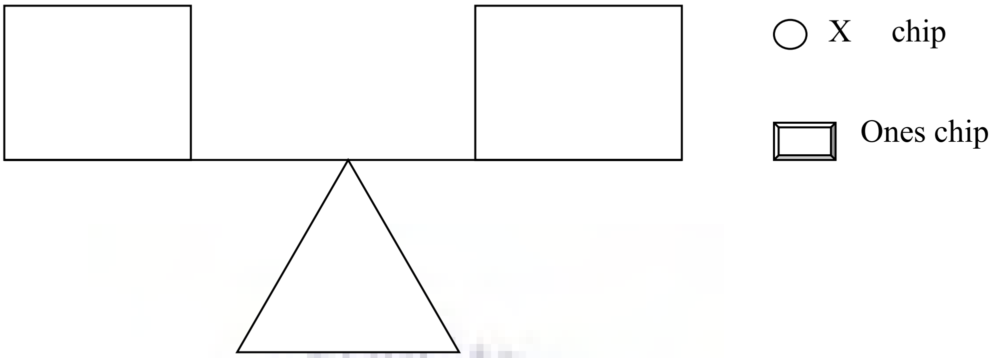
Discussions among the pupils in a group were vastly encouraged and teacher-pupil interactions created a friendly atmosphere for effective learning which also led to sustaining pupils‟ interest from the beginning to the end of each lesson. It was manifest from pupils‟ active interactions with the balance models and progression from one manipulative representation to the other that they learnt and understood the principles of solving linear equations in one variable and developed abstract logical thinking. Sherrod et al. (2009) agrees that, while performing these activities, pupils are nurtured in an environment that supports them in constructing a more comprehensive understanding of mathematics that empowers them to make use of their skills in the real word.

**Week One (Lesson 1, Duration: 60minutes)**

The first lesson began with pupils guided to define a linear equation in one variable as an equation that could be written in the form ax + b = 0, where „a" and „b" are real numbers and a ≠ 0. The pupils were able to pick out that when the graph of a linear equation is drawn, it takes the form of straight line, hence the name a linear equation.

In reviewing the principle of a solution to a mathematical equation, the pupils were taken through the meaning of solving linear equation in one variable as determining the value of the variable that results in a true mathematical statement, when substituted into the equation.

The lesson continued with pupils getting familiar with the balance models, the „variables" and the „ones" chips. The researcher set out the objectives of the lesson to the pupils and the type of teamwork needed. Learners" experience of the seesaw game formed the foundation of the teaching and learning process. The researcher served as a resource person, an instructor and a guide to the activities. Subsequent lessons began with the exploration of the learners" immediate environment and ended with the application of the lesson learnt to their immediate environment.

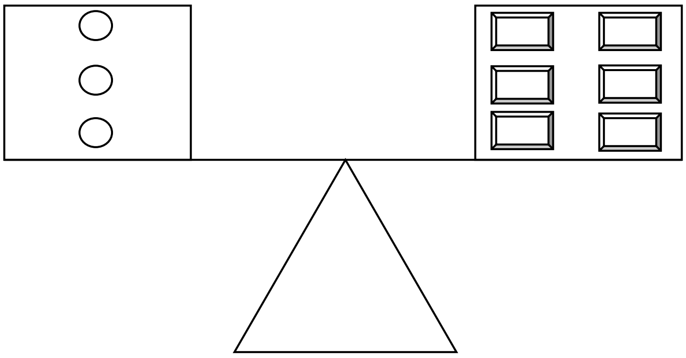



Seesaw (Balance Model)

**Week One (Lesson 2, Duration: 60minutes)**

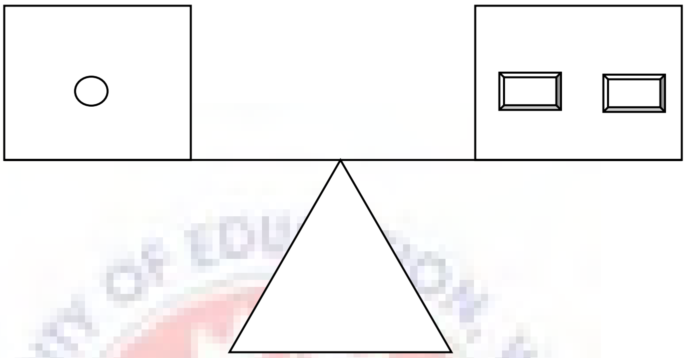
The learners were taken through a series of activities to solve linear equations of the form ax = b, where „a" and „b" are integers, on the balance model. The selection of the integers „a" and „b" under this form was such that „b" was always a multiple of „a" so as to avoid fractional solutions.

Pupils were guided to solve the equation 3x = 6, using the balance model as shown below;



The researcher guided the pupils to balance three „x" chips with six „ones" chips on the balance model. They were then guided to divide each side of the balance by 3 and record their observation. The pupils realized that one „x" was balanced by two „ones".

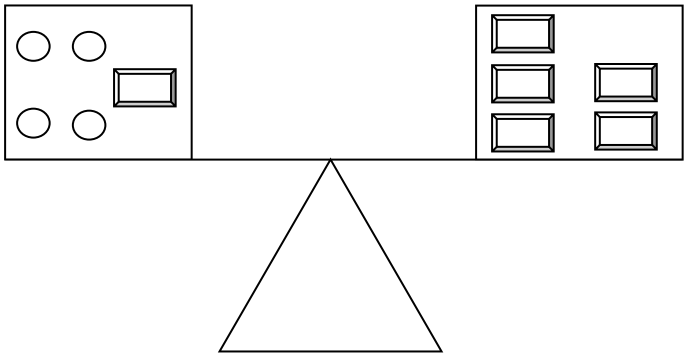
They then recorded x = 2.



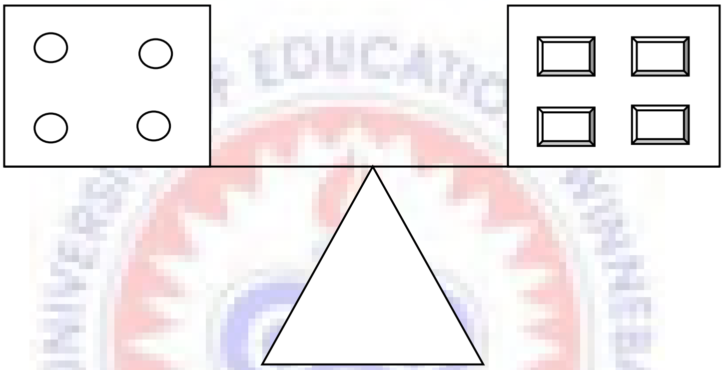
**Week One (Lesson 3, Duration: 60minutes)**

The pupils were assisted through explanation and demonstration to solve linear equations in one variable of the form $ax + b = c$, (where a, b, and c are Integers), on the balance model. The pupils were also guided systematically on how to solve each equation by the removal of equal quantities of „ones" chip(s) from both sides of the scale at the same time until the equation looked like what they learnt in lesson 2, (i.e. $ax = b$), and one „x" balances a specific quantity.

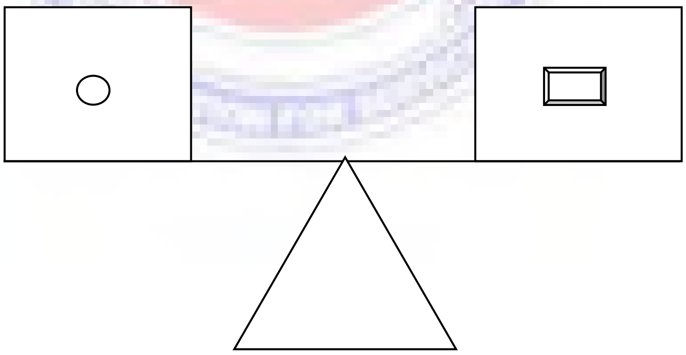
The pupils were then guided to solve $4x + 1 = 5$, using the balance model as shown below;



From the illustration, the pupils were led to remove one „ones‟ chips from each side of the balance model.



The pupils were then guided to share the four „ones‟ chips for the four „x‟ chips.



The pupils realized that one „x‟ was balanced by one „ones‟.

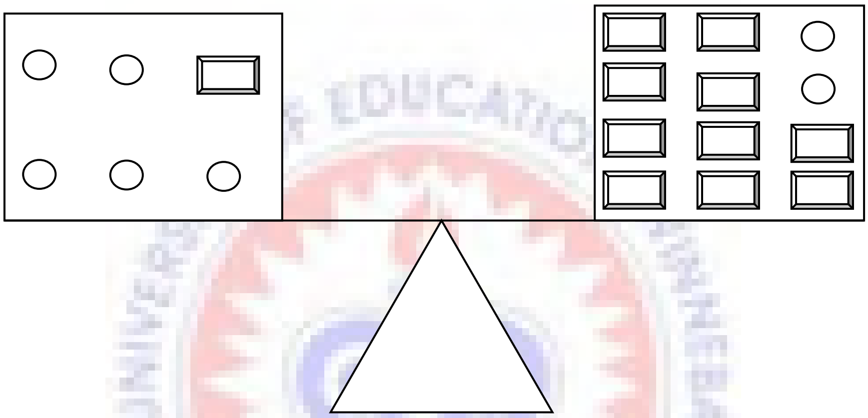
They then recorded x = 1.

**Week Two (Lesson 4, Duration: 60minutes)**

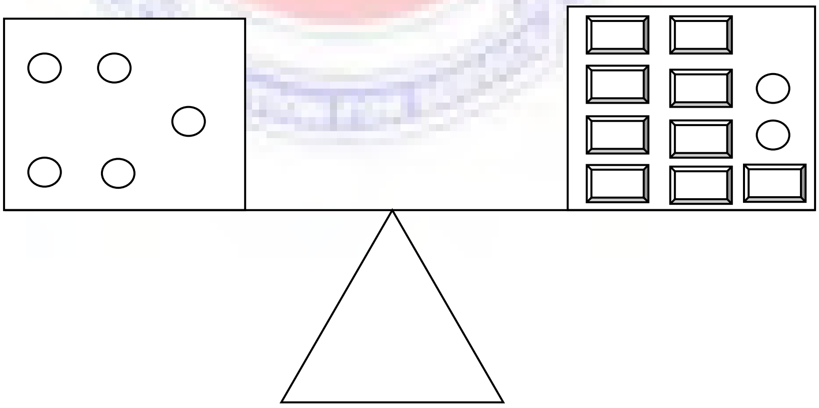
The pupils were again taken through a series of activities to solve linear equations of the form

ax + b = cx + d, (where a, b, c, and d are all integers), on the balance model.

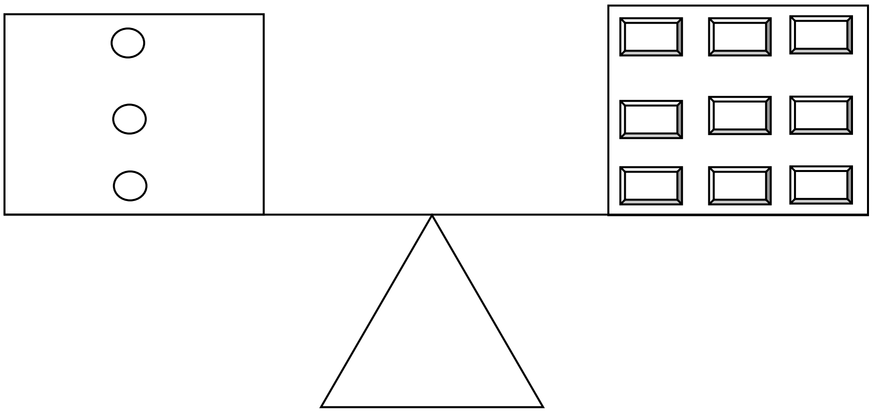
The pupils were tasked to represent the equation 5x + 1 = 2x + 10, on the balance model.



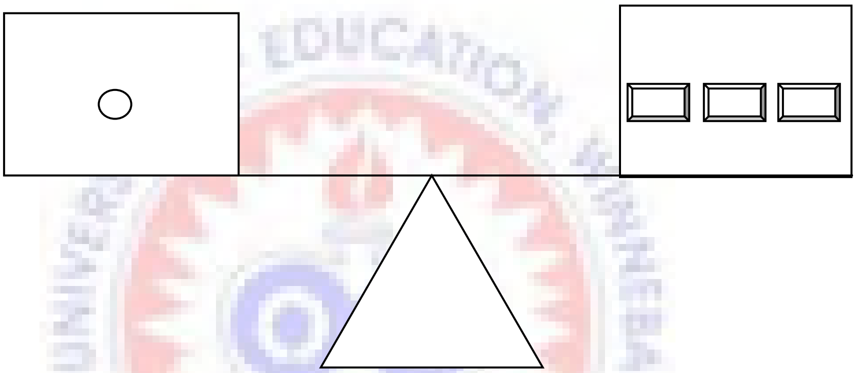
The pupils were assisted to remove one „ones‟ chip from both sides of the balance model.



The pupils were then guided to remove two „x‟ chips from both sides of the balance model.



The pupils were guided to share the nine „ones" chips by the three „x" chips.

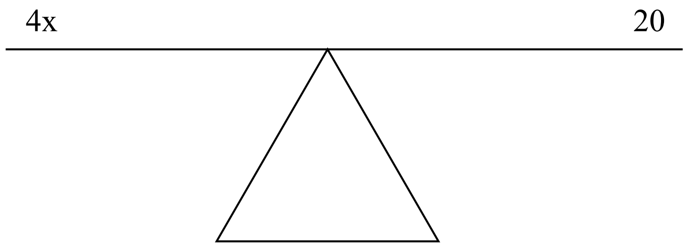


The pupils realized that one „x" was balanced by three „ones".

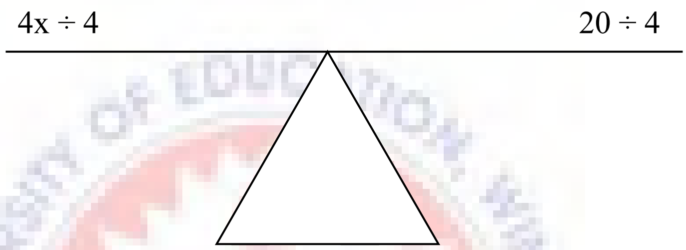
They then recorded x = 3.

**Week Two (Lesson 5, Duration: 60minutes)**

The pupils this time used balance diagrams to solve linear equations in one variable in the form ax = b. As Booth (1987) confirmed that, using both concrete and ideographic approaches to teach a mathematical principle may be unsuccessful unless pupils see the connection between the two. For example, pupils were tasked to solve 4x = 20, using the balance diagram as shown below;




To find the value of „x‟, pupils were guided to share the sixteen „ones‟ chips for the four „x‟ chips, was related to dividing both sides of the equation by the coefficient of x;

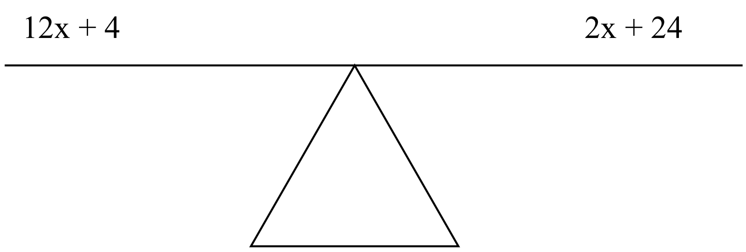



Thus, for $\frac{4x}{4} = \frac{20}{4}$

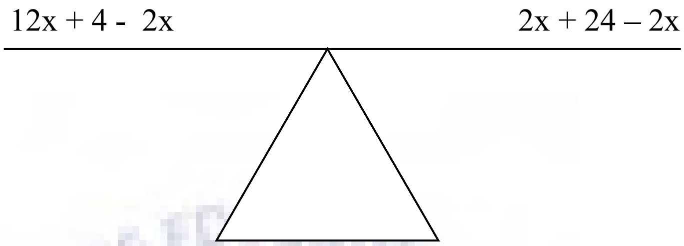
Therefore, we obtained x = 5.

**Week Two (lesson 6, Duration: 60 minutes)**

The pupils again used the balance diagrams to solve linear equations in one variable in the form ax + b = cx + d. The pupils were then guided to solve 12x + 4 = 2x + 24 on the balance diagram as shown below;

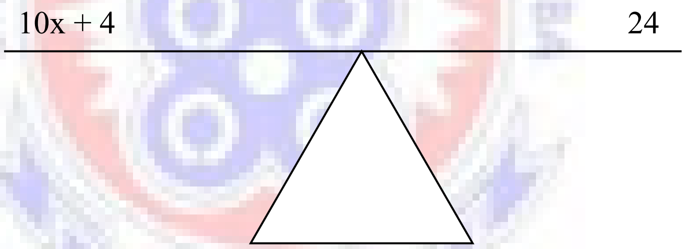


The researcher then encouraged the pupils to use the arithmetic procedure that represented the removal of chips for each step of the solution. That is, the pupils were guided to show when they added the additive inverse of „2x" to both sides of the equation to represent the removal of two "x" chips from both sides of the balance model.

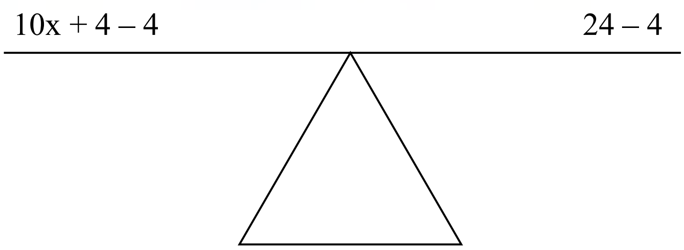



Thus, for $12x - 2x + 4 = 2x - 2x + 24$

We had $10x + 4 = 24$

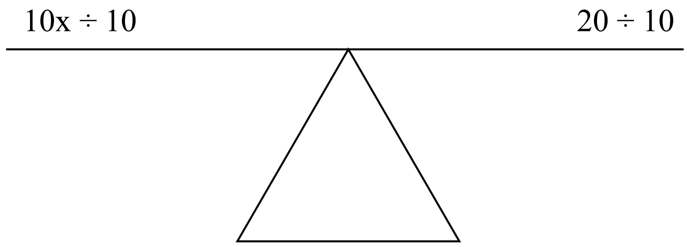



The pupils were again guided to add the additive inverse of 4 to both sides of the equation to remove the four „ones" chips from both sides of the balance model.




Thus, for $10x + 4 - 4 = 24 - 4$

We had $10x = 20$




To find the value of „x‟, pupils were guided to share the twenty „ones‟ chips for the ten „x‟ chips, was related to dividing both sides of the equation by the coefficient of x, which is 10;

Thus, for $\frac{10x}{10} = \frac{20}{10}$

Therefore, we obtained x = 2.

**Week Three (Lesson 7, Duration: 60minutes)**

The researcher at this time guided the pupils through explanations and exemplifications to solve given linear equations in one variable without the use of the balance model or the balance diagrams. They were taught to use the experiences, ideas and skills acquired from the previous lessons. These exercises covered the three forms of linear equations in one variable described in the previous units, that is $ax = b$, $ax + b = c$, and $ax + b = cx + d$.

**Form One : ax = b**

Example $7x = 21$

The pupils were guided to divide both sides of the equation by the coefficient of „x‟, which is 7.

$$\frac{7x}{7} = \frac{21}{7}$$

Thus, x = 3

Therefore, we concluded that the value of „x‟ which satisfies the given equation is 3.

**Week Three (Lesson 8, Duration: 60 minutes)**

**Form Two : ax + b = c**

Example 9x + 4 = 40

The pupils were guided to subtract „4‟ from either side of the equation as shown below,

$$9x + 4 – 4 = 40 – 4$$

We obtained, 9x = 36

The pupils were then guided to divide both sides of the equation by the coefficient of „x‟, which is 9.

$$\frac{9x}{9} = \frac{36}{9}$$

Thus, x = 4

Therefore, we concluded that the value of „x‟ which satisfies the given equation is 4.

**Week Three (Lesson 9, Duration: 60 minutes)**

**Form Three : ax + b = cx + d**

Example $12x + 9 = 7x + 39$

The pupils were guided to subtract „7x" from each side of the equation as shown below;

$$12x + 9 – 7x = 7x – 7x + 39$$

We obtained, $5x + 9 = 39$

In the next step, the pupils were tasked to subtract „9"from each side of the equation as shown below,

$$5x + 9 – 9 = 39 – 9$$

We obtained, $5x = 30$

The pupils were then guided to divide both sides of the equation by the coefficient of „x", which is 5.

$$\frac{5x}{5} = \frac{30}{5}$$

Thus, $x = 6$

Therefore, we concluded that the value of „x" which satisfies the given equation is 6.

### 3.4.3 Post-intervention stage

At the end of the intervention stage of three weeks, a post-test was immediately administered to the sampled pupils to assist in measuring the effectiveness of the intervention and also to evaluate the outcome of the intervention. This was to determine the extent to which their performance has improved in solving linear equations in one variable. That is, whether they had improved upon their knowledge and also understanding of the principles of solving linear equations in one variable. The post-test, consisting six (6) questions on linear equations in one variable, was administered to the sampled pupils (Appendix C). The conditions used in the pre-test were the same as those used in the post-test. The pre-tests and the post-tests were of the same difficulty level but different in form with the reason that if the intervention had been effective then pupils should be able to answer similar questions on the topic. The duration for the test was forty (40) minutes. Answers of pupils to the pre-test were marked using a marking scheme prepared by the researcher (see Appendix D).

The scripts were collected, marked and scored over thirty (30) and the pupils" scores recorded. As usual the researcher carefully examines wrong answers given in order to find out possible causes. Discussions were again held with pupils to find out why such blunders were committed. Finally the mean and the standard deviation of the pre-test and post-test were found to be used in analyses of the data.

## 3.5 Data Analysis Plan

The Data were analysed using quantitative approach based on the research questions formulated for this study. Quantitative research focuses on precise measurement of something and it determines facts and figures (Cooper and Schindler, 2006). In other words, quantitative analysis is used to figure out exactly what happened or how often things happened.

The results were inputted into SPSS software for descriptive statistical and inferential analyses and interpreted. The data collected from the pre-test and the post-test were analyzed by descriptive statistics. The results on how the given questions in both the pre-test and the post-test were analyzed. Frequency tables with percentages of pupils" performance in mathematics were constructed by the researcher based on the various question items covered in the pre-test and post-test (see Tables 4.2 and 4.4).

The means and the standard deviations, of the pre-test and the post- test were compared, discussed and analyzed

Bar and pie charts were also drawn to enhance the analysis of the results of the pre-test and post-test of the research.

# CHAPTER 4

# DATA ANALYSIS, RESULTS AND DISCUSSIONS

## 4.1 Descriptive Analysis

### 4.1.1 Analysis of pre – test results to test pupils understanding

After marking the pupils test items, the researcher took time to examine the questions one by one based on the three objectives of the test. That is dividing both sides, grouping like terms and removing brackets. The results obtained are recorded in the table below:

**Table 4.1: Pre-Test results of Pupils on Dividing Both Sides**

| Response | Frequency | Percentage |
|---|---|---|
| Correct | 12 | 40 |
| Wrong | 18 | 60 |
| **Total** | 30 | 100 |

From table 4.1, the number of pupils who answered questions 1 and 2 which was based on the principles of dividing both sides correctly was 12 representing 40% whereas 18 of the pupils representing 60% had the answer wrong. The researcher observed that the 12 pupils who were able to solve the questions correctly applied the rule of division property of equality (divide both sides of an equation by the same non-zero number) correctly. Most of the pupils who had the answers wrong found it difficult in dividing each term on both sides by the co-efficient of the variable. This indicated that the pupils lack basic principles in solving linear equation in one variable.

**Table 4.2.** **Pre-Test Results of Pupils on Grouping Like Terms.**

| Response | Frequency | Percentage |
|---|---|---|
| Correct | 8 | 26.67 |
| Wrong | 22 | 73.33 |
| **Total** | 30 | 100 |

The test items that required the pupils to use the principle of grouping like terms were found in question 3 and 4. Table 4.2 indicated that out of 30 pupils who wrote the test, 8 of the pupils representing 26.67% knew how to group like terms on both sides of the linear equation. However, 22 of the pupils representing 73.33% who attempted the questions did not know how to group like terms on both sides of the linear equation. The researcher observed that the 8 pupils who were able to solve the questions correctly applied the rules of addition and subtraction properties of equality correctly. Most of the pupils who had the answers wrong found it difficult in transposing positive (+) and negative (-) sign to where they are due. This indicated that the pupils lack basic principles in solving linear equation.

**Table 4.3** **Pre-Test Results of Pupils on Removal of Brackets**

| Response | Frequency | Percentage |
|---|---|---|
| Correct | 5 | 16.67 |
| Wrong | 25 | 83.33 |
| **Total** | 50 | 100 |

The last test items tested pupil"s knowledge on removing of brackets. Table 4.3 indicated that only 5 of the pupils representing 16.67% had the correct answer. For the wrong answers, 25 of the pupils representing 83.33% failed to remove the brackets of the linear equation. The researcher observed that 25 of the pupils representing 83.33% who attempted the questions did not know how to remove brackets from linear equation. The common mistake shown by most of the pupils who got the answer wrong was that they left the bracket unresolved. This made it difficult for them to get the final answer.

### 4.1.2 Analysis of post-test results to test pupils understanding

The post-test was administered after pupils were introduced to the intervention with regard to the use of the balance model. The post-test questions were similar to the pre-test questions and this assisted the researcher to assess its effectiveness. The researcher examined the questions one after the other in order to obtain sufficient information about the pupils" response to each question. The researcher examined the questions based on the three objectives (dividing both sides, grouping like terms and removing brackets) of the test.

The marks obtained out of 30 by the pupils after the post-test were as follows:

**Table 4.4 Post-Test Results of Pupils on Dividing Both Sides**

| Response | Frequency | Percentage |
|---|---|---|
| Correct | 27 | 90 |
| Wrong | 3 | 10 |
| **Total** | 30 | 100 |

From table 4.4, the number of pupils who answered questions 1 and 2 which was based on the principle of dividing both sides were 30. Only 3 pupils representing 10% failed to answer the questions correctly. However, the remaining 27 pupils representing 90% got the answer correct. Almost all the pupils were able to answer these questions correctly due to the fact that the balance model helped pupils to further understand the steps in answering the questions and mastering the principle of division property of equality involved. This assisted the pupils to apply the principle correctly. The only 3 pupils who were not able to solve the questions correctly found it difficult moving away from the use of the concrete balance model to the diagrammatic and algebraic representation of linear equation in one variable. There has therefore been an improvement.

**Table 4.5 Post-Test Results of Pupils on Grouping Like Terms**

| Response | Frequency | Percentage |
|---|---|---|
| Correct | 26 | 86.67 |
| Wrong | 4 | 13.33 |
| **Total** | 30 | 100 |

The test items that required the pupils to use the principle of grouping like terms were found in question 3 and 4. Table 4.5 indicated that out of 30 pupils who wrote the test, 26 of the students representing 86.67% knew how to group like terms on both sides of the linear equation whereas only 4 out of the total pupils representing 13.33% who attempted the questions did not know how to group like terms on both sides of the linear equation. The researcher observed that the 26 pupils who were able to solve the questions correctly applied the rules of addition and subtraction properties of equality correctly. This showed that students had clear understanding of the principles involved in the use of balance model in solving linear equation in one variable.

**Table 4.6 Post-Test Results of Pupils on Removal of Brackets**

| **Response** | **Frequency** | **Percentage** |
|---|---|---|
| Correct | 24 | 80 |
| Wrong | 6 | 20 |
| **Total** | 30 | 100 |

From the table 4.6 above, 6 pupils representing 20% were not able to answer questions 5 and 6 which were based on removal of brackets correctly. The rest of the pupils representing 80% answered the questions correctly. This was due to the fact that the balance model helped pupils to further understand the steps in answering the question and mastering the principles of expanding the bracket by multiplying out an equation with brackets in it. From the look of things there has been an improvement over what they did in the Pre-test.

### 4.1.3 Summary of tests results

**Table 4.7 Summary of Results of Thirty (30) Pupils in Pre-Test and Post-Test**

| QUE. | Wrong Answer | | Correct Answer | | % of Wrong Answer | | % of Correct Answer | |
|---|---|---|---|---|---|---|---|---|
| | Pre-T | Post-T | Pre-T | Post-T | Pre-T | Post-T | Pre-T | Post-T |
| 1 – 2 | 18 | 3 | 12 | 27 | 60.00 | 10.00 | 40.00 | 90.00 |
| 3 – 4 | 22 | 4 | 8 | 26 | 73.33 | 13.33 | 26.67 | 86.67 |
| 7 & 8 | 25 | 6 | 5 | 24 | 83.33 | 20.00 | 16.67 | 80.00 |

In question 1 and 2 of the pre-test, pupils were asked to apply the principle of dividing both sides by the coefficient of the variable to find the equivalent equation. Out of 30 pupils 60% had wrong answers and after the intervention, the post-test was 10% which was a decrease in the pre-test percentage. This means the use of the balance model as an interventional measure contributed to this decrease in the pupils" failure. The story was not different in the case of the right answers. As the pre-test score was 40%, the post-test percentage shot up to 90% indicating that there was great improvement in the pupils" performance.

Questions 3 and 4 also tested the pupils" knowledge in the grouping of like terms in linear equations in one variable. Out of the same total number of pupils who wrote the pre-test, 73.33% of the pupils had wrong answers where as in the post-test the percentage of the failures reduced to 13.33. For the right answers while the pre-test percentage was 26.67, the post-test also recorded an upward improvement of 86.67%.

Finally, question 5 and 6 required the pupils to remove brackets before simplifying the linear equation. Out of 30 pupils who took part in the pre-test 83.33% of the pupils got the answer wrong. The percentage reduced to 20 after the post-test. For the right answers while the pupils had 16.67% in the pre-test, the post-test increased to 80%. This confirmed the earlier assertion of the influence of the balance model on the pupil‟s performance.

## 4.2 Inferential Analysis of Pre-test and Post-test Score

The research was designed on assisting pupils in order to improve their performance in linear equation in one variable through the use of the balance model approach. The pre-test and the post-test scores were used in analyzing the data and the data collected was analyzed using inferential statistics employed on SPSS soft ware. The statistical test was set at $P < 0.05$. The table below shows the mean, standard deviation and standard error mean of the paired samples.

**Table 4.8 Paired Samples Statistics**

**Paired Samples Statistics**

| | | Mean | N | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|---|
| Pair 1 | Pretest | 7.00 | 30 | 3.939 | .719 |
| | Posttest | 27.37 | 30 | 3.079 | .562 |

### 4.2.1 Findings from pre-test

From Table 4.3, the mean of the pre-test score was 7.00 and that of post-test score was 27.37. Thus, comparing the mean score of the pupils in the pre-test (7.00) to the mean score of the pupils in the post-test (27.37) shows a significant improvement in pupils‟‟ performance in solving linear equations in one variable involving the use of the balance model.

Moreover, the standard deviation of the marks scored in the pre-test by the pupils was 3.939 and that of the post-test was 3.079. The comparison of the standard deviation of the pupils in the pre-test to the post-test shows that the marks scored in the post-test were closer to its‟‟ mean than that of the pre-test. This further indicated that the performance of the pupils in solving linear equation in one variable had improved, since the standard deviation of the post-test scores was less than that of the pre-test.

### 4.2.2 Testing the hypothesis

*Null hypothesis*: There is no significant difference in scores between the mean pre-test scores and the mean post-test scores of pupils at α = 0.05 level of significance (P< 0.05).

**Null Hypothesis** $H_O : \mu_1 = \mu_2$

*Alternative hypothesis*: There is significant difference in scores between the mean pre-test scores and mean post- test scores of pupils.

**Alternative Hypothesis** $H_a : \mu_1 \neq \mu_2$

**Table 4.9 Paired Samples Test**

**Paired Samples Test**

| | | Paired Differences | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | 95% Confidence Interval of the Difference | | | | |
| | | Mean | Std. Deviation | Std. Error Mean | Lower | Upper | T | df | Sig. (2-tailed) |
| Pair 1 | pretest – posttest | -20.367 | 5.353 | .977 | -22.365 | -18.368 | -20.840 | 29 | .000 |

*Decision*: From table 4.9 above, the paired sample test analysis of the data resulted in the value of P = 0.000. Since P is less than 0.05 (95% confidence interval), the level of significant; we reject the Null Hypothesis and rather accept the Alternative Hypothesis. Therefore, conclude that there is significant difference between the pre-test and the post-test which is in favour of the post-test.

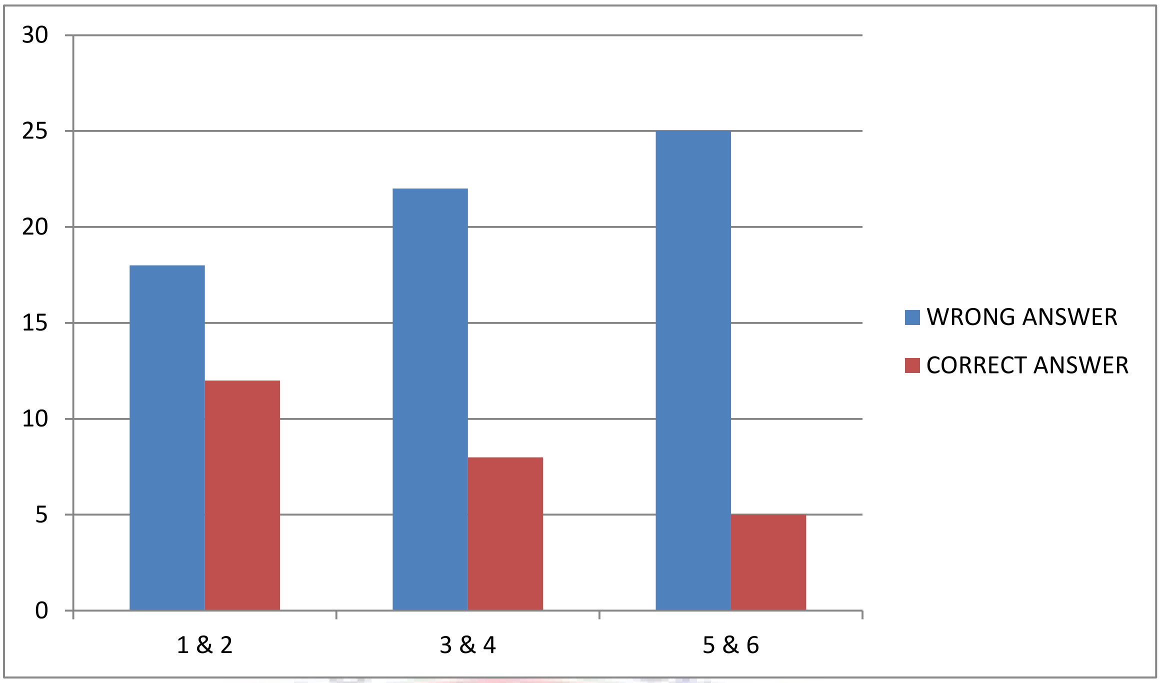



**Figure 4.1: Bar Chart showing Pupils' Wrong and Correct Answers to Questions in the Pre-test**

The diagram represents the performance of pupils in the pre-test conducted. In comparison, it was realized from figure 4.1 that pupils‟ performance was low in all the questions. All the questions indicate upward levels of wrong answers. This is an indication that pupils did not understand linear equations in one variable very well.

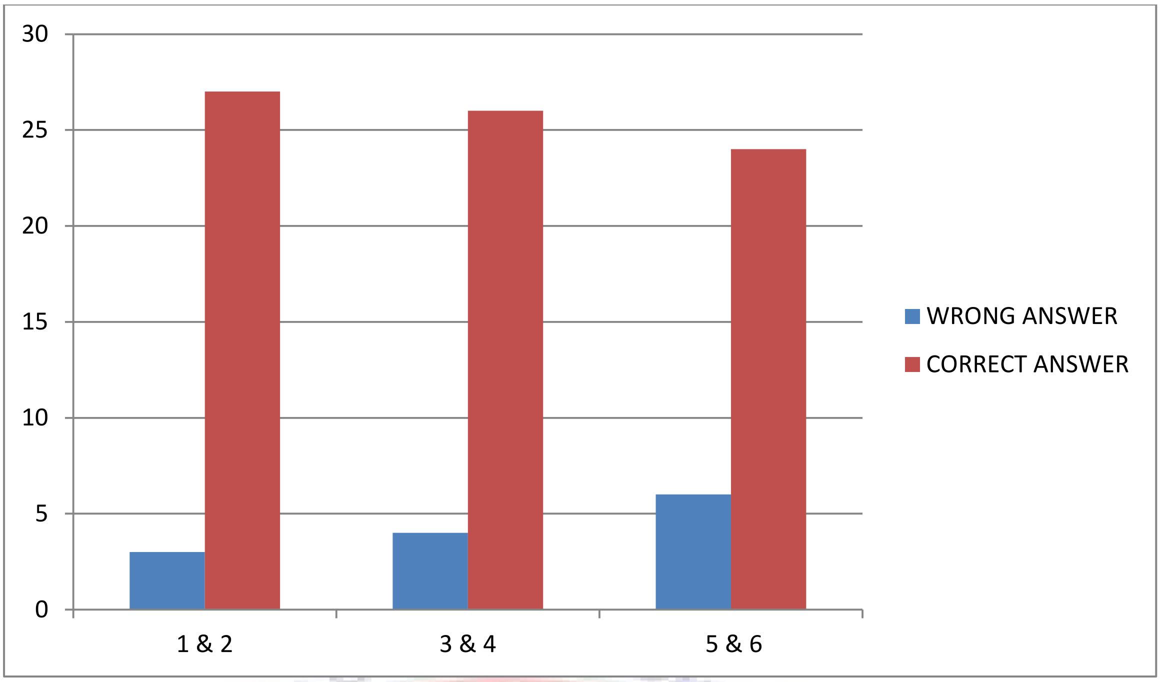



**Figure 4.2: Bar Chart showing Pupils' Wrong and Correct Answers to Questions in the Post-Test**

The diagram above represents the performance of pupils in the post-test conducted. In comparison, it was realized from figure 4.2 that pupils" performance was high in all the questions. All the questions indicate upward levels of correct answers. This is an indication that pupils did understand linear equations in one variable very well after the balance model had been introduced to pupils.

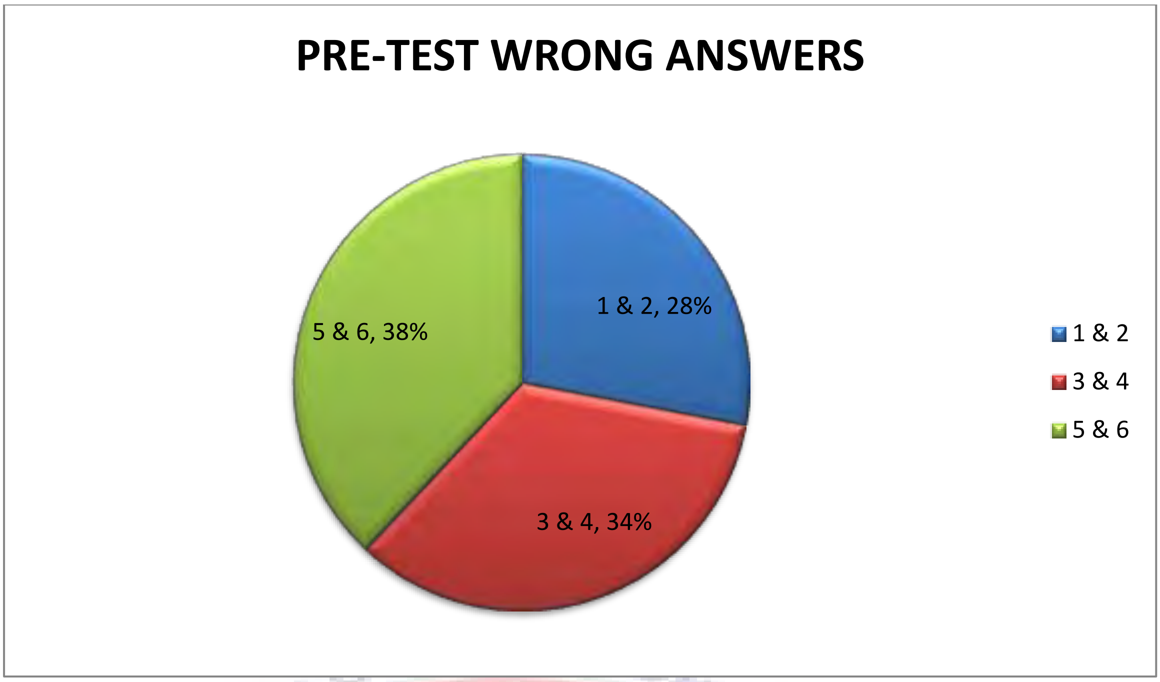



**Figure 4.3 Pie Chart showing Pupils' Wrong Answers in the Pre-Test**

In figure 4.3, it was observed that more than half of the pupils (72%) had higher percentage scores in wrong answers in the post-test. This indicates that a higher percentage of pupils scored wrongly in the pre-test.

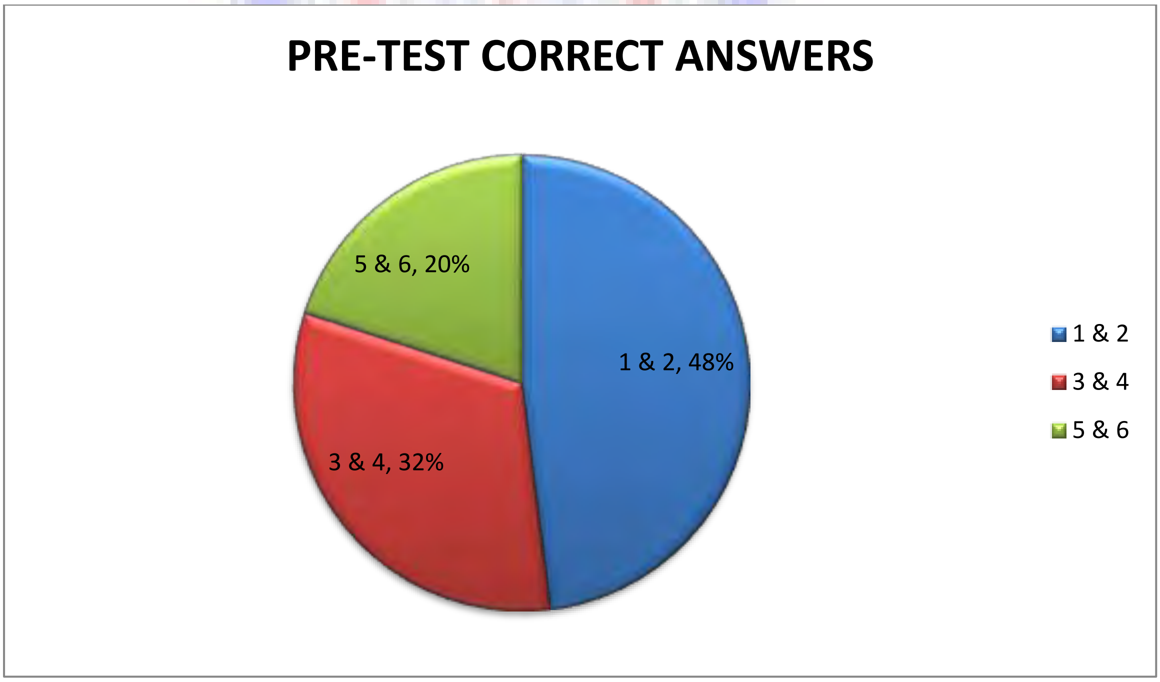



**Figure 4.4 Pie Chart showing Pupils' Correct Answers in the Pre-Test**

In figure 4.4, it was observed that more than half of the pupils (52%) had lower percentage scores in correct answers in the pre-test. This indicates that a lower percentage of pupils scored correctly in the post-test.

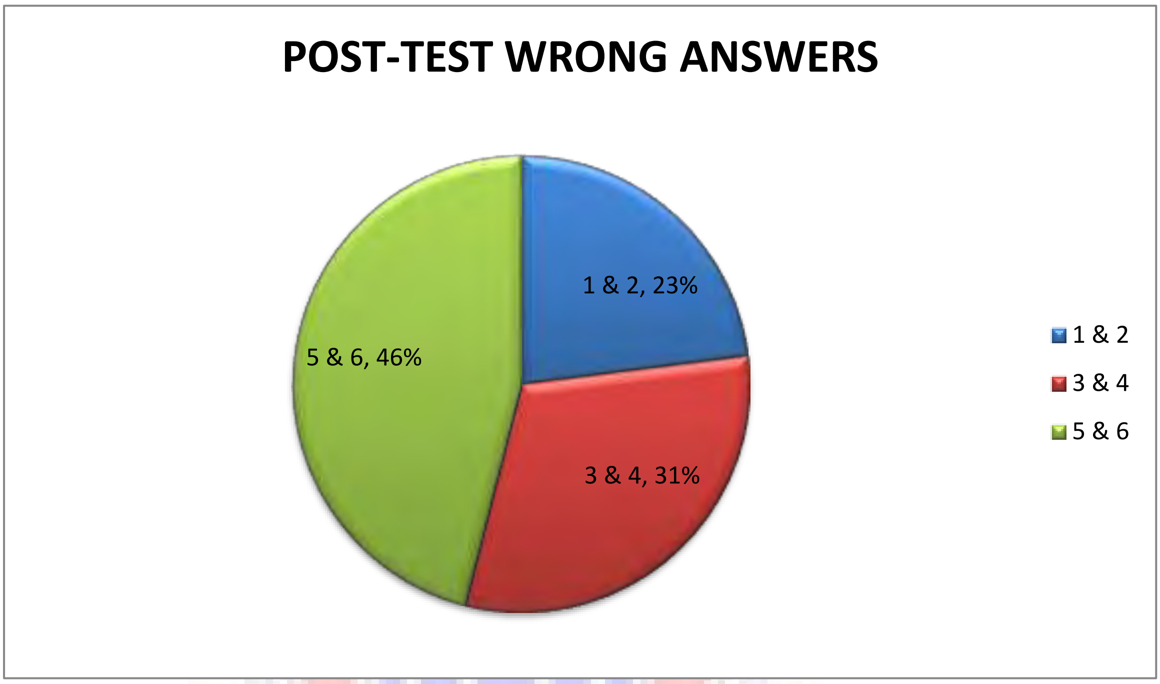



**Figure 4.5 Pie Chart showing Pupils' Wrong Answers in the Post-Test**

In figure 4.5, it was observed that more than half of the pupils (54%) had lower percentage scores in wrong answers in the post-test. This indicates that a higher percentage of pupils scored correctly in the post-test.

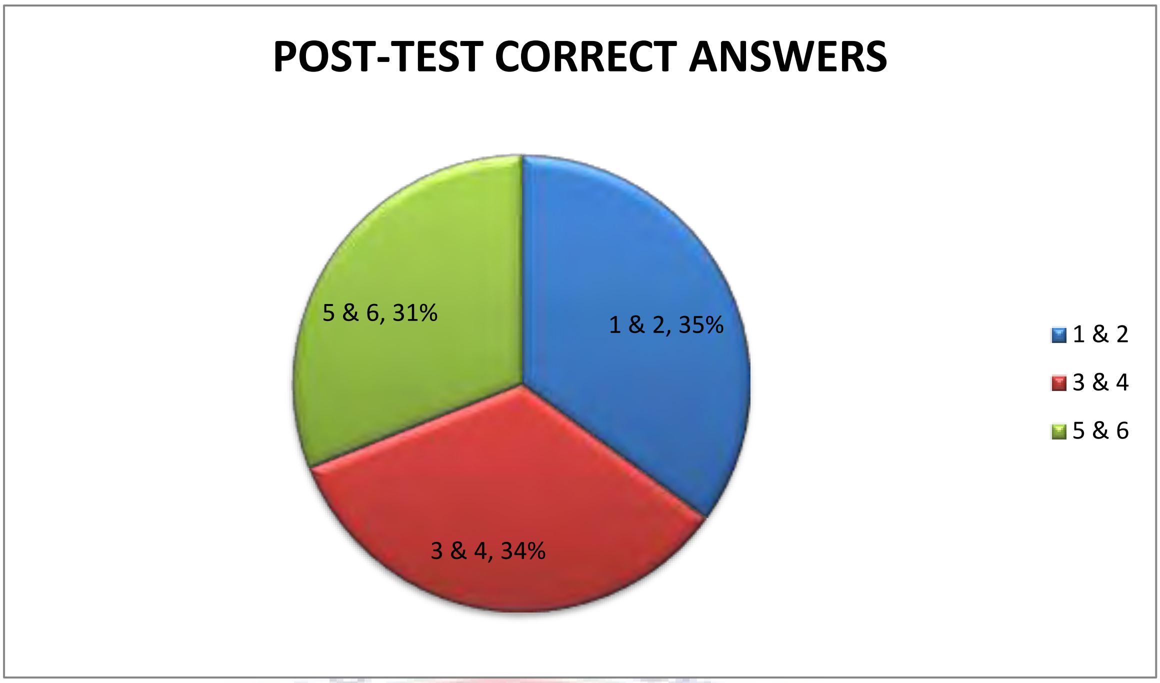



**Figure 4.6 Pie Chart showing Pupils' Correct Answers in the Post-Test**

In figure 4.6, it was observed that almost all the pupils (100%) had higher percentage scores in correct answers in the post-test. This indicates that almost all the pupils scored correctly in the post-test.

### 4.2.3 Discussion and analysis of findings

This part of the chapter discussed the results from the pre-test and the post-test based on the research questions. The analysis of the result enables the researcher to find out the level of improvement of pupils performance in relation to linear equation in one variable.

**Research Question 1:** To what extent would the use of balanced model be effective tool in improving pupils understanding of the principles of solving linear equation in one variable?

Table 4.7 revealed the result of the intervention by comparing the pre-test scores of the individual pupils‟ with their respective post test scores. It showed a significant improvement in pupils‟ performance after the use of the balance model to teach the topic. Also, the analysis of the mean, standard deviation and the t-test score supported the fact that, performance in equations can be improved if teachers incorporate the use of balance model in their teaching in the classrooms. That is, the post-test mean score of 27.37 (Standard deviation of 3.079) is significantly higher than the pre-test mean score of 7.00 (Standard deviation of 3.939). This shows that the use of balance model in teaching linear equation in one variable brought about a tremendous improvement in pupils‟ understanding in the principles (that is, addition property of equality, subtraction property of equality, multiplication property of equality and division property of equality) of solving linear equations in one variable.

Again, the Bar Chart on figure 4.1 & 4.2 shows the number of pupils who answered the questions rightly in the pre-test and the post-test. The results on the chart indicated that there was an upwards trend in the post-test result which showed that the intervention were effective in assisting the pupils in the learning of linear equations in one variable.

A further illustration was done using the Pie Chart as shown in figure 4.3 to 4.6. This implies that pupils‟ understanding of the principles such as addition property of equality, subtraction property of equality, multiplication property of equality and division property of equality in solving linear equation in one variable had improved when they were taught using a balance model.

From the intervention, pupils were able to grab the concepts of using the balance model to solve linear equations in one variable to the large extent that the balance model improves the teaching and learning of linear equations in one variable. The balance model as a concrete material is therefore useful and offers the pupils the necessary skills in algebra.

**Research Question 2:** How can the use of the balance model improve pupils" understanding of the principles involved in solving linear equations with one variable? With the help of the balance model, pupils were able to apply the principles (that is, addition property of equality, subtraction property of equality, multiplication property of equality and division property of equality) on both sides of the equal sign to maintain balance as in the activities of the intervention. The balance model was able to help pupils divide both sides by the coefficient of the variable in the given equations. Pupils were able to group like terms on both sides of the equation before simplifying and also expand the factors before performing same operations on both sides of the equation as stated by Corte & Pfaff (2000) that the use of the balance model enables pupils to perform the same operations on both sides to maintain balance instead of shifting symbols around the equality sign. Warren and Cooper (2005) also agree that the use of the balance model is a way of presenting the principle of an equation in concrete terms to pupils.

## 4.3 Summary of Major Findings based on the Research Questions

Following the findings from the pre-test and post-test after the intervention, it was established that:

a) The use of the balance model has improved the understanding of the principles of solving linear equations in one variable of the pupils.

b) The activity nature of the balance model has increased the knowledge of the pupils in solving linear equation in one variable.

c) The performance of the pupils in solving linear equation in one variable has improved through the use of the balance model.

# CHAPTER 5

# SUMMARY, CONCLUSION AND RECOMMENDATIONS

## 5.1 Summary

The purpose of the study was to use the balance model as a practical approach to help pupils of Buasi R/C JHS to improve their performance in solving linear equations in one variable. The application of the balance model was to improve the understanding of the principles (that is, addition property of equality, subtraction property of equality, multiplication property of equality and division property of equality) of solving linear equations in one variable of the second year pupils of the said school. The playground experience of the balance model through a seesaw game assisted the pupils to demonstrate the representation of an equation as the balancing of unknown weights on each side of the model. In this model, the balance scale must remain in balance, because of that the same operations must be performed at each side of the scale. By performing the same operation on each side of the equation means adding the same quantity to each side of the equation, or subtracting the same quantity from each side of the equation, multiplying each side of the equation by the same non-zero number and dividing each side of the equation by the same non-zero number. These four operations were applied to each side of an equation, one at a time until the value of the variable in the equation is known. Thus, the balance model was used diagrammatically, to provide mental picture to assist the pupils solve the linear equation in one variable.

Moreover, the study was conducted on thirty pupils in three stages; the pre-intervention, the intervention and the post-intervention stages.

At the pre-intervention stage, pupils were made to answer six questions on linear equations in one variable (See Appendix A). The questions were meant to test the pupils‟ understanding of the principles of solving linear equations in one variable. The pupils‟ responses were marked and scored over thirty to determine their individual performances. Based on the poor performance of the pupils at the pre-intervention stage, an intervention stage was designed and the balance model deployed to address the pupils‟ lack of understanding of the principles of solving linear equations in one variable.

The intervention stage lasted for three weeks during which period the pupils were taken through a series of practical activities in the use of the balance model in solving linear equations in one variable. The excellent implementation of the intervention stage, coupled with the interest shown by the pupils in the use of the balance model for solving linear equations in one variable, led to the post-intervention stage.

At the post-intervention stage, the pupils answered six questions on linear equations in one variable (See Appendix C). The structure of questions for the post-test in the post-intervention was basically the same as the pre-test questions for the pre-intervention. Results of both the pre-intervention and post-intervention tests were analyzed using descriptive statistics. Specifically, means and standard deviations were calculated and compared to determine the performance level of the pupils.

Furthermore, the study found that using the balance model in teaching linear equations in one variable brought about;

- A significant improvement in pupils‟ understanding of the principles (that is, addition property of equality, subtraction property of equality, multiplication property of equality and division property of equality) of solving linear equations in one variable.

- ❖ A gain in practical experiences. These practical experiences helped in the development of their intellectual and manipulative skills such that most of the pupils were able to solve the linear equations in one variable.
- ❖ A significant improvement in pupils" performance in solving linear equations in one variable.

## 5.2 Conclusion

The performance of the pupils for the study in the post-test was higher than that of the pre-test. Considering the performance of the pupils in solving the linear equation in one variable confirms the assertion that the balance model effectively represented a concrete example of the equality of the two sides of an equation. Thus, the balance model could therefore serve as a concrete experience for promoting the simplification and utilization of linear equations in one variable, and therefore a solid manipulative material for teaching algebraic equations. Moreover, the study could add to the existence knowledge in the use of the balance model in the teaching and learning of linear equations in one variable. The study could also promote the active participation and intellectual involvement of learners in the principles of solving linear equations in one variable using the balance model.

Another important aspect of the study was that it revealed the merits of cooperative-instructional strategies and the effectiveness of using appropriate concrete learning materials for teaching not only linear equations but also for teaching mathematics in general.

## 5.3 Recommendations

The balance model for the teaching and learning of linear equations in one variable was intended as an ideal learning resources, this study therefore recommend that what the pupils learned would be remembered and would play a significant part in their future learning of algebra.

The study examined linear equations with positive whole number coefficients. To make the balance model versatile, there is the need to probe in much detail just how the model could be used in the study of linear equations involving decimal or fractional coefficients, negative solutions, divisions and multiplications. Even so, I recommend that these deficiencies could be corrected to demonstrate that the balance model can naturally lead onto decimal coefficients, multiplication and division.

Using the balance model showed positive effect on pupils‟ performance and achievement. I therefore recommend that policy-makers organize in-service training for basic school teachers on the use of the balance model for classroom practice. The use of this model would boost their confidence in their instructional delivery. Moreover, educational planners should include the use of concrete classroom teaching materials (balance model) in the curriculum. Also, appropriate courses need to be introduced in the Universities and Colleges of Education for the training of teachers in the skills of designing, developing and applying the balance model in teaching linear equations in one variable at the basic level of education.

Although this study would contribute immensely towards the advancement of knowledge by inspiring other Mathematics educators to investigate aspects of using manipulative materials based on pupils‟ experience in teaching mathematical principles, it recommend however that a direct empirical comparison between the balance model and other models for solving linear equations would be desirable. The strengths and weaknesses, similarities and differences of the various models for solving linear equations could then be explored in detail, qualitatively and quantitatively.

Teachers who are not used to balanced model during teaching of equations should try as much as possible to employ the use of the model as part of their instructional material in their planning and delivering of lessons as teaching of principles in linear equation should always involve cooperative and discovery learning to make it more practicable as possible. This also makes the pupils to relate what they have learnt to the environment but not rather think in abstract.

Mathematics teachers are therefore encouraged to explore cooperative-instructional strategies using the balance model in the teaching and learning of linear equations in one variable as this method eventually shifts learning from a teacher-centred to a learner-centred approach. By encouraging cooperative-instructional methods in the teaching of linear equations using the balance model, the teacher is able to act as a facilitator of learning and not a transmitter of knowledge as pupils construct their own mathematical meanings and could therefore encourage and facilitate learning in the mathematics classroom.

The balance model could also serve as a link between concrete and abstract symbolic representations of mathematical ideas. Also, the use of models based on children‟s experiences makes teaching and learning of mathematics more meaningful.

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# APPENDIX A

## Pre-Test Questions

Answer all questions

Solve for x in the following linear equations

1. $5x = 15$
2. $3x + 1 = 13$
3. $5x + 2 = x + 18$
4. $7x + 1 = 4x + 7$
5. $4(x + 3) = 20$
6. $3(2x - 2) = 4(x + 3)$

# APPENDIX B

## MARKING SCHEME FOR PRE-TEST QUESTIONS

| Q.NO | SOLUTION | MKS | DETAILS |
|---|---|---|---|
| 1. | 5x = 15 | | |
| | $\frac{5x}{5} = \frac{15}{5}$ | M2 | Divide both sides by 5 |
| | x = 3 | A1 | Correct answer only |
| 2. | 3x + 1 = 13 | M1 | Grouping like terms. |
| | 3x = 13 – 1 | M1A1 | Simplify the right hand side |
| | 3x = 12 | M1 | Divide both sides by 3 |
| | $\frac{3x}{3} = \frac{12}{3}$ | | |
| | x = 4 | A1 | Correct answer only |
| 3. | 5x + 2 = x + 18 | M1 | Group like terms |
| | 5x – x = 18 – 2 | M1A1 | Simplify both sides |
| | 4x = 16 | M1 | Divide both sides by 4 |
| | $\frac{4x}{4} = \frac{16}{4}$ | | |
| | x = 4 | A1 | Correct answer only |
| 4. | 7x + 1 = 4x + 7 | M1 | Grouping like terms. |
| | 7x – 4x = 7 – 1 | M1A1 | Simplify both sides |
| | 3x = 6 | M1 | Divide both sides by 3 |
| | $\frac{3x}{3} = \frac{6}{3}$ | | |
| | x = 2 | A1 | Correct answer only |

| | | | |
|---|---|---|---|
| 5. | $4(x + 3) = 20$ | M1 | Expand to remove the bracket |
| | $4x + 12 = 20$ | M1A1 | Grouping like terms. |
| | $4x = 20 - 12$ | M1 | Simplify both sides |
| | $4x = 8$ | M1 | Divide both sides by 4 |
| | $\frac{4x}{4} = \frac{8}{4}$ | | |
| | $x = 2$ | M1 | Correct answer only |
| 6. | $3(2x - 2) = 4(x + 3)$ | M1 | Expand to remove the bracket |
| | $6x - 6 = 4x + 12$ | M1A1 | Grouping like terms. |
| | $6x - 4x = 12 + 6$ | M1 | Simplify both sides |
| | $\frac{2x}{2} = \frac{18}{2}$ | M1 | Divide both sides by 4 |
| | $x = 9$ | | |
| | | M1 | Correct answer only |

# APPENDIX C

## Post-Test Questions

Answer all questions

Solve for x in the following linear equations

1. $7x = 14$
2. $2x + 1 = 9$
3. $8x + 1 = 2x + 7$
4. $5x + 2 = 2x + 17$
5. $3(x + 4) = 24$
6. $5(2x - 2) = 4(x + 3)$

## APPENDIX D

## MARKING SCHEME FOR POST-TEST QUESTIONS

| Q.NO | SOLUTION | MKS | DETAILS |
|---|---|---|---|
| 1. | 7x = 14 | | |
| | $\frac{7x}{7} = \frac{14}{7}$ | M1 | Divide both sides by 5 |
| | x = 2 | A1 | Correct answer only |
| 2. | 2x + 1 = 9 | M1 | Grouping like terms. |
| | 2x = 9 – 1 | M1A1 | Simplify the right hand side |
| | 2x = 8 | M1 | Divide both sides by 3 |
| | $\frac{2x}{2} = \frac{8}{2}$ | | |
| | x = 4 | A1 | Correct answer only |
| 3. | 8x + 1 = 2x + 7 | M1 | Group like terms |
| | 8x – 2x = 7 – 1 | M1A1 | Simplify both sides |
| | 6x = 6 | M1 | Divide both sides by 4 |
| | $\frac{6x}{6} = \frac{6}{6}$ | | |
| | x = 1 | A1 | Correct answer only |
| 4. | 5x + 2 = 2x + 17 | M1 | Grouping like terms. |
| | 5x – 2x = 17 – 2 | M1A1 | Simplify both sides |
| | 3x = 15 | M1 | Divide both sides by 3 |
| | $\frac{3x}{3} = \frac{15}{3}$ | | |
| | x = 5 | A1 | Correct answer only |

| | | | |
|---|---|---|---|
| 5. | 3(x + 4) = 24 | M1 | Expand to remove the bracket |
| | 3x + 12 = 24 | M1A1 | Grouping like terms. |
| | 3x = 24 – 12 | M1 | Simplify both sides |
| | 3x = 12 | M1 | Divide both sides by 4 |
| | $\frac{3x}{3} = \frac{12}{3}$ | | |
| | x = 4 | M1 | Correct answer only |
| 6. | 5(2x – 4) = 4(x + 4) | M1 | Expand to remove the bracket |
| | 10x – 20 = 4x + 16 | M1A1 | Grouping like terms. |
| | 10x – 4x = 16 + 20 | M1 | Simplify both sides |
| | 6x = 36 | M1 | Divide both sides by 4 |
| | $\frac{6x}{6} = \frac{36}{6}$ | | |
| | x = 6 | M1 | Correct answer only |