UNIVERSITY OF EDUCATION, WINNEBA

# THE EFFECT OF VHPI ON PRE-SERVICE TEACHERS' GEOMETRIC THINKING AND MOTIVATION TO LEARN GEOMETRY

ROBERT BENJAMIN ARMAH

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**A Thesis in the Department of MATHEMATICS EDUCATION, Faculty of SCIENCE EDUCATION, submitted to the School of Graduate Studies, University of Education, Winneba in partial fulfillment of the requirements for the award of the Degree of Master of Philosophy in Mathematics Education.**

**JUNE, 2015**

# DECLARATION

## STUDENT'S DECLARATION

I, Robert Benjamin Armah, declare that this Thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

SIGNATURE:…………………………………………

DATE:…………….………

## SUPERVISOR'S DECLARATION

I hereby declare that the preparation and presentation of this Thesis was supervised in accordance with the guidelines for supervision of Thesis as laid down by the University of Education, Winneba.

NAME OF SUPERVISOR: DR. P. O. COFIE

SIGNATURE:…………………………………………

DATE:…………….………

## ACKNOWLEDGEMENTS

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# DEDICATION

This thesis is dedicated to my two elder brothers, Mr. Samuel Ebenezer Armah and Mr. Ishmael Kojo Armah for their love, counsel and financial assistance that propelled me to pursue this postgraduate study.

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# ABSTRACT

This study investigated the effect of the van Hiele Phase-based Instruction (VHPI) on Pre-service Teachers' (PTs) geometric thinking in terms of the van Hiele Levels and how the VHPI can motivate PTs to learn geometry. It further investigated how College of Education (CE) Mathematics tutors facilitate geometry teaching and learning consistent with the van Hiele Levels of geometric thinking. The study utilized mixed method approach involving quasi-experimental design in which two Colleges of Education (CsE) were randomly assigned as control and experimental groups. A sample of 150 PTs and 5 CE Mathematics tutors from the two CsE were randomly selected for the study. Van Hiele Geometry Test (VHGT) was administered to all PTs as pre-test. VHPI was used to teach PTs in the experimental group after which 9 of them were interviewed while the control group was instructed by traditional instruction. The same test was then given to the PTs as post-test. Observations were also done on geometry instruction of 5 CE Mathematics tutors. Chi-square results found that PTs in both groups showed increment in their post-VHGT as compared to the pre-VHGT. However, PTs in the experimental group achieved better levels of geometric thinking compared to those in the control group. Findings also revealed that VHPI highly motivated PTs to learn geometry by eliminating dullness and making learning easier and fascinating. However, much of the geometry teaching and learning strategies of CE Mathematics tutors are not structured in a way that support the development of geometric thinking as described in the van Hiele theory. In conclusion, VHPI was found to promote effective geometry teaching and learning.

# CHAPTER ONE

# INTRODUCTION

## 1.0 Overview

This opening Chapter One sets the study in context. It presents the background of the study, statement of the problem, purpose of the study, objectives of the study as well as the educational significance and sets out the research questions guiding the study. The Chapter further highlights the delimitations and limitations and concludes by outlining the organization of the dissertation.

## 1.1 Background of the Study

In Ghana, College of Education (CE) geometry is in two different aspects; content aspects, which is studied in first year and methodology aspects, studied in second year. Geometry forms a considerable amount of the content of Senior High School (SHS) and CE mathematics curricula and has a separate subject status in the CE curriculum in Ghana. It is a branch of mathematics that deals with the study of shape and space. Without spatial ability, students cannot fully appreciate the natural world (Güven & Kosa, 2008; Yegambaram & Naidoo, 2009). In order to conceptualize and analyze not only physical but also imagined spatial environments, geometry provides for a complex network of interconnected concepts which require representation systems and reasoning skills (Alex & Mammen, 2012). Geometry is in every part of the school Mathematics curriculum in any country running through to College and University level. Since most educational fields use a spiraling curriculum, the concepts are re-visited throughout the classes advancing in Levels of difficulty. Characteristically in the early years, learners identify shapes and solids, use problem solving skills, deductive reasoning, understand transformations, symmetry and use spatial reasoning. In High School, there is a great deal of focus on analyzing properties of two and three dimensional shapes, reasoning about geometric relationships and using the coordinate system. The study of geometry offers many foundational skills and helps to build the thinking skills of logic, deductive reasoning, analytical reasoning and problem solving (Russell, 2014). Moreover, High School geometry builds on Elementary School geometry which is usually based on measurement and the informal development of the basic concepts required in geometry at the High School level. The topics on measurements of perimeter, of area, and of volume which are revisited in the High School curriculum provide excellent opportunities for further applications of algebraic concepts in geometry (Dindyal, 2007). As a result, ensuring students' understanding of geometry is crucial.

However, a lot of concerns have been raised about the level of students' understanding of geometry in Ghanaian schools. My experience both as a student and a Mathematics tutor in one of the Colleges of Education (CsE) in Ghana indicates that many Pre-service Teachers (PTs) hold negative perceptions about geometry. These negative perceptions are evidenced in the deprecating comments often made about geometry. My observations of PTs attitude towards geometry are that of panic, worry and lack of self-confidence. This is supported by Fredua-Kwarteng and Ahia (2004) who indicated that Ghanaian students have internalized the false belief that Mathematics learning including geometry requires an innate ability or the "brains of an elephant". My own concerns have been agitated by the following questions:

- How do students come to hate geometry?
- What geometrical ideas or competencies do students come to CE with?
- What motivates students to learn geometry? These issues have bothered me for some time now.

Following the elevation of Teacher Training Colleges to diploma awarding institutions in Ghana, the Mathematics curriculum has been planned in such a way that PTs take only content courses for the first year and both content and methods courses for the second year. The third year is exclusively for teaching practice and project work writing (Teacher Education Division (TED), 2008; Abudu & Donkor, 2014). Geometry is one of the areas in the course structure which is included in the course outlines of both content and methods. Geometry in the first year content course covers areas such as lines and angles, polygons, congruent and similar triangles, geometrical constructions including loci, circle theorems, two and three-dimensional shapes, movement geometry, co-ordinate geometry and equation of a circle. In the method course, Geometry covers areas such as developing ideas about shape and space, teaching measurements, teaching geometrical constructions and teaching rigid motion (Institute of Education, University of Cape Coast (UCC), 2005).

The CE Mathematics course outlines provide for the teaching of content, dealing with the subject matter of Mathematics and methodology aspects that deal with the pedagogical skills of the subject matter. The choice of the content in the course outlines is based on needs of the PTs and the learners they would be expected to teach in the basic schools after their training. Moreover, the selection of the respective topics was also based on the assumption that the PTs had had a sound foundation in Mathematics in the basic concepts in their first and second cycles of education. This is supported by the fact that the admission requirements demand at least a pass in Mathematics at the West African Senior School Certificate Examination (WASSCE). By the end of their three year pre-service programme, the PTs are expected to have a sound knowledge and foundation in Mathematics and geometry in particular to teach it effectively (Etsey, 2004).

In order to obtain the full benefits of geometry in the CE Mathematics curriculum, classroom instructions should aim at enhancing PTs' geometric thinking. Improving PTs' geometric thinking Levels is one of the major aims of the CE Mathematics education in Ghana (Institute of Education, University of Cape Coast (UCC), 2005). This is because geometry is linked to many other topics in Mathematics, particularly measurement and is used daily by architects, engineers, physicists, land surveyors and many more professionals (Sunzuma, Masocha & Zezekwa, 2013; Russell, 2014). Also, other Mathematical concepts which run very deeply through modern Mathematics and technology, such as symmetry, are most easily introduced in a geometric context. Whether one is designing an electronic circuit board, a building, a dress, an airport, a bookshelf, or a newspaper page, an understanding of geometric principles is required (Yegambaram & Naidoo, 2009). Teaching geometry at the CE level should be done in ways that promotes geometric thinking.

However, it is observed that the teaching and learning culture of Mathematics including geometry in Ghanaian schools have the following characteristics which have contributed to the Mathematics underachievement of Ghanaian students: students accept whatever the teacher teaches them. The teacher is the sole authority of mathematical knowledge in the classroom, while the students are mere receptors of mathematical facts, principles, formulas, and theorems. Thus, if the teacher makes any mistakes the students would also make the same mistakes as the teacher made. It is believed that the teaching culture in most Ghanaian Mathematics classroom has significantly shaped the Mathematics learning culture highlighted above (Fredua-Kwarteng & Ahia, 2004).

It seems to be that, there is something wrong with the way Mathematics including geometry is learnt and taught in Ghana. This is because of late the performance of students in both internal and external Mathematics examinations has remained consistently poor. The Trends in International Mathematics and Science Study (TIMSS) (Anamuah-Mensah & Mereku, 2005; Anamuah-Mensah, Mereku & Asabere-Ameyaw, 2008), revealed that Ghanaian students' performance in the subject was abysmally low with mean scores far below the international average. The reports affirmed that students' performance in geometry was the lowest in the five domains the test covered. In Addition, the West African Examination Council (WAEC) Chief Examiner's annual reports for the West African Senior School Certificate Examination (WASSCE) from 2008 to 2011 observed that candidates were weak in 2 and 3-dimensional geometrical problems. Also, the chief examiner's annual report for CsE End-of-Second Semester Mathematics Examination in geometry, from the years 2008 to 2010 state that generally, the performance of candidates in the content part was weak and their presentations of solutions to most of the content questions were poor and majority of them had problems solving questions involving the concepts of exterior and interior angles of polygons and their properties, among other concepts. In 2011 and 2012, the examiner's report once again revealed candidates' lack of adequate knowledge in geometry and application of geometric concepts.

It is established that learners' difficulty with school Mathematics generally and geometry in particular is caused largely by teachers' failure to deliver instruction that is appropriate to the learners' geometric level of thinking (van Hiele, 1986). The van Hiele Phase-based Instruction (VHPI) therefore places learners at the center of learning and has interventions that can effectively improve learners' geometric thinking Levels. Integrating hands-on investigations with manipulative concrete materials in the VHPI can help learners to build concrete concepts, and also provide learners ample opportunities to unleash their own creativity as they explore geometric concepts (Zhang, 2003 cited in Siew & Chong, 2014).

Most importantly, PTs are being trained to teach at the basic level, as a result, if measures are not put in place to address their insufficient knowledge and poor performance in geometry, it may create great difficulties for the Ghanaian Mathematics education, especially at the basic level. Instead of memorizing properties and definitions, it is suggested that PTs should be given opportunities to work with concrete manipulative materials, drawings and symbolic notations in order to progress tremendously through the Levels of geometric understanding as defined in the van Hiele framework. This calls for an alternative teaching approach where the VHPI can be used to improve PTs' geometric thinking Levels and problem-solving skills.

## 1.2 Statement of the Problem

PTs have difficulties identifying properties of shapes, identifying similarities and differences among shapes and solving problems relating to concepts of shapes. Current teaching and learning practices in the classroom do not reflect the importance of geometry in the lives of students, and the emphasis that is placed on it in the

Mathematics curriculum. Teacher training is still bound to the traditional approach, which is teacher-centered. The teacher centered traditional teaching method turns students into passive listeners, which makes students deficient in geometrical analysis and reasoning. In this method of teaching, most students resort to memorization as they are not given chances of problem solving, using information, that is, reforming the knowledge and they are not provided with hands-on activities that will actively engage them and help them use their thinking skills effectively (Duru, 2010; Siew, Chong & Abdullah, 2013).

Although PTs have not done enough geometry at the SHS level, the geometry topics being treated at CE (Institute of Education, University of Cape Coast (UCC), 2005) are not so different from those at the SHS (Ministry of Education, 2010). Their abysmal performance may be probably due to the ineffective teaching methods and lack of appropriate use of teaching resources in the traditional Ghanaian classrooms.

The van Hiele model has been applied to many curricula to improve geometry classroom instruction in many developed nations but in Ghana, the literature appears to suggest that there has been little investigation involving the van Hiele model. Thus, very little studies have applied the van Hiele model to determine the level of geometric conceptualization of Ghanaian CE students and also to improve geometry instruction. Meanwhile, there is evidence that many students in Ghana encounter severe difficulties with school geometry (Baffoe & Mereku, 2010). In an attempt to seek a teaching strategy that can improve PTs' achievement in geometry, this study investigated the effect of the van Hiele Phase-based Instruction (VHPI) on PTs' achievement in geometry which is taught in Ghanaian CsE.

## 1.3 Purpose of the Study

The purpose of the study is to investigate the effect of the VHPI on PTs' geometric thinking in terms of the van Hiele Levels and how the VHPI can motivate PTs to learn geometry. The study also aimed at investigating how CE Mathematics tutors facilitate geometry teaching and learning consistent with the van Hiele Levels of geometric thinking.

## 1.4 Objectives of the Study

This study was guided by the following objectives:

- To explore the effectiveness of VHPI in improving PTs geometric thinking Levels.
- To explore the effect of the use of VHPI on PTs motivation to learn geometry and;
- To use the van Hiele theory in investigating geometry teaching strategies employed by CE Mathematics tutors in Ghana.

## 1.5 Research Questions

In pursuance of the objectives stated above, this study seeks answers to and is structured around three main research questions. These are as follows:

- To what extent does the use of VHPI improve PTs geometric thinking Levels?
- How does the use of VHPI motivate PTs to learn geometry?
- To what extent do CE Mathematics tutors facilitate geometry teaching and learning consistent with the van Hiele Levels of geometric thinking?

In answering the first research question, the hypothesis below was formulated for the study;

$H_0$: There is no significant difference in the geometric thinking Levels between the control and experimental groups in the post-test.

## 1.6 Significance of the Study

This study is unique and significant as it represents, as far as the researcher has been able to ascertain, an attempt to investigate the effectiveness of the VHPI in Ghanaian CsE. The measure and description of Ghanaian PTs' van Hiele Levels of understanding in geometry shall be of great interest to Mathematics teacher educators, assessment developers and PTs' curriculum developers.

As the school curriculum is a major factor in shaping the quality of education (CRDD, 2007), the findings that will come out of this study can be used to help curriculum developers and CE tutors on how to use the van Hiele model in order to improve the geometric thinking Levels of students.

In this research it is expected that PTs will be engaged in mathematical communications where they explain their ideas clearly and also follow each others' reasoning rather than just the tutor's instruction. As a result, the researcher is optimistic that the use of the VHPI will not only improve PTs geometric thinking Levels and achievements in geometry but would also offer PTs enhanced opportunities of varied forms of mathematical communications which are absent in other forms of teaching approaches such as the tutor-centred approach.

Moreover, the research would help dispel PTs' general negative perceptions about Mathematics and geometry in particular which would influence their teaching of Mathematics positively at the basic level after completing their programme. Also, the study would add new knowledge to Mathematics education as well as serve as a reference material at the library for Mathematics educators and the general public.

In summary, the van Hiele model can provide a framework on which geometry instructions can be structured and taught in schools: teachers could, for example, attempt first to raise their learners' van Hiele geometric thinking Levels through the instructional phases of the van Hiele model in order to improve the mathematical performance of their learners. Also, the study will aid Ghanaian PTs to overcome some barriers they encounter in using their informal strategies in the process of acquiring more sophisticated strategies. It is hoped that this will in turn sustain interest in geometry as PTs progress to higher classes and make them derive the full benefit of having a good knowledge of geometry.

## 1.7 Delimitation

CE Geometry covers a wide area of studies including two and three dimensional figures. However, for the purpose of this study, emphasis was laid on only two dimensional figures. The coverage of this area in geometry was as a result of the difficulties and low performances of PTs in this area of geometry as discussed in the background of the study. Even though PTs find other areas of geometry difficult, because time within which to complete this study is limited these areas were not covered. Most importantly, the research was not concerned with coverage but knowledge.

## 1.8 Limitations of the Study

From a total of 38 public CsE in Ghana, only 2 CsE were selected for this study and this has limited the scope of the research. The consequence of this was that, generalization of the research findings was limited. This limitation was alleviated when students from SHS all over the ten regions of Ghana were admitted into these 2 CsE. This has enriched the sample used for the study in terms of PTs' abilities, cultural and social backgrounds. The sample used therefore represents the characteristics of Ghanaian PTs in any part of the country who had spent at least a year studying geometry in the College.

Furthermore, the non-existence of information regarding the van Hiele model in the Ghanaian Mathematics curriculum was also a limitation. The researcher was unable to draw from local examples and knowledge.

Finally, the insufficient duration of this study produces an obvious limitation. The three consecutive lessons in this study definitely cannot achieve a very convincing result. A more lengthy study may produce persuasive results in examining the effectiveness of the VHPI.

## 1.9 Organization of the Study

The study was organized systematically in five different chapters. In Chapter One, the background of the study, statement of the problem, purpose of the study, objectives of the study, research questions, and significance of the study, delimitation, limitations of the study and the organizational plan were presented. The theoretical framework and relevant literature review were presented in Chapter Two. The researcher described the research design and methodology in Chapter Three. Results and discussion were done in Chapter Four. Chapter Five consisted of summary of key findings, conclusion and implications for practice, recommendations, and areas for further research.

# CHAPTER TWO

# LITERATURE REVIEW

## 2.0 Overview

This chapter primarily focuses on varied views on what other authors have written concerning the topic under study. The literature review focused on the theoretical framework of the study, some issues concerning geometry in school Mathematics curriculum, pre-service teachers' Mathematics education, effect of the VHPI on students' geometric thinking Levels, students' motivation to learn geometry as well as the constructivist approach to instruction.

## 2.1 Theoretical Framework: The van Hiele Theory

In the field of geometry, the best and most well-defined theory for students' levels of thinking is based on the van Hiele theory (Crowley, 1987; Dindyal, 2007; Yegambaram & Naidoo, 2009). The theory enables insight into why many students encounter difficulties in their geometry courses, particularly with formal proofs. The van Hiele theory comprises three main aspects, namely: Levels of geometric thinking, properties of the Levels and phases of learning (the van Hiele Phase-based Instruction) which offers a model of teaching that teachers could apply in order to promote their learners' levels of understanding in geometry (Usiskin, 1982; van Hiele, 1986; Crowley, 1987). In this study, these aspects of the van Hiele theory were utilized to explore PTs' geometric thinking Levels and the dominant patterns of geometry classroom instruction in CsE in Ghana.

The van Hiele theory originally consists of five sequential and hierarchical discrete Levels of geometric thought namely: *Recognition, Analysis, Order (Informal*

*Deduction), Deduction,* and *Rigor* (Usiskin, 1982; Alex & Mammen, 2012). There are two different numbering schemes that are commonly used to describe the van Hiele Levels: Level 0 through to 4, and Level 1 through to 5. Originally the van Hieles numbering scheme used Level 0 through to 4, however, Americans (Usiskin, 1982; Mason, 1998) and van Hiele's (1986; 1999) more recent writings make use of the Level 1 through to 5 numbering scheme instead. This according to Mason (1998) allows for a sixth Level, Pre-recognition Level (i.e. Level for learners who have not yet achieved even the basic Level 1) to be called Level 0. This study will use the Level 1 to 5 numbering scheme to allow utilization of Level 0. The geometrical framework by the van Hieles has made a significant impact on motivating students and in creating a better environment for the teaching and learning of geometry (Malloy, 2002; Abu & Abidin, 2013). Each of the five Levels describes the thinking processes used in geometric contexts. As one progresses from one Level to the next, the object of one's geometric thinking changes. The van Hiele Levels can be described as follows:

### 2.1.1 Level 1: Recognition (or visual level)

At this level learners use visual perception and nonverbal thinking. They recognize figures by appearance alone (van Hiele, 1986; van Hiele, 1999; Vojkuvkova, 2012). Learners at this level do not identify the properties of geometric figures (Yazdani, 2007).When learners call a figure a square, they react to the whole figure and not to its right angles, equal side lengths, and equal diagonal lengths. For example, learners at this level can recognize certain squares very easily because they look like the outline of a window or frame (Figure 2.1, left). However, they do not call the second shape in Figure 2.1 a square because it does not look like the outline of a window or frame (Halat, 2008).

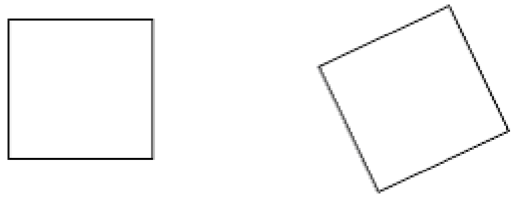


**Figure 2.1: Two Perspectives of a Square.**

### 2.1.2 Level 2: Analysis (or descriptive level)

At this level, "figures are the bearers of their properties. A figure is no longer judged because it looks like one but rather because it has certain properties" (van Hiele, 1999, p. 311). Students start analyzing and naming properties of geometric figures but they do not understand the interrelationship between different types of figures, and they also cannot fully understand or appreciate the uses of definitions at this level (Mason, 1998; Yazdani, 2007; Vojkuvkova, 2012). They would correctly identify only the second and fourth shapes in Figure 2.2 as parallelograms. Though at this level the students are capable of recognizing various relationships among the parts of the figures, they do not recognize any relationship between squares and rectangles or rectangles and parallelograms; students perceive properties of one class of shapes empirically, but cannot relate the properties of two different classes of shapes. For example, students would not see rectangles or squares as parallelograms because they do not see one set of figures as a subset of another (Halat, 2008).

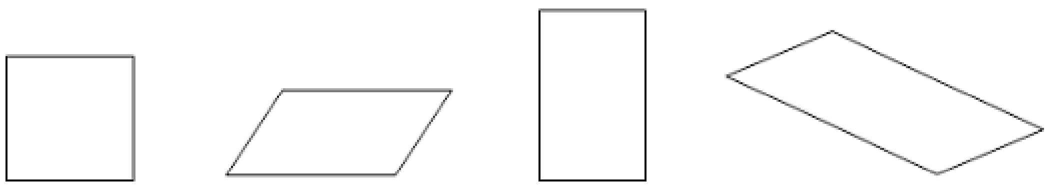


**Figure 2.2: Examples of Parallelograms.**

### 2.1.3 Level 3: Order (or informal deduction level)

At this level, students are able to "see the interrelationship between different types of figures" (Breyfogle & Lynch, 2010, p. 234). Students can create meaningful definitions and give informal arguments to justify their reasoning at this level. Logical implications and class inclusions, such as squares being a type of rectangle, are understood (Mason, 1998; Yazdani, 2007; Halat, 2008). Simple inference can also be made, for example, in a quadrilateral, opposite sides being parallel necessitates opposite angles being equal (Crowley, 1987). Students are able to make simple deductions and may be able to follow formal proofs but do not understand the significance of working in an axiomatic system and are not able to construct proofs meaningfully on their own at this level (Yazdani, 2007).

### 2.1.4 Level 4: Deduction

At this level students can give deductive geometric proofs. They understand the role of definitions, theorems, axioms and proofs (Yazdani, 2007; Vojkuvkova, 2012). Students at this level can supply reasons for statements in formal proofs (Halat, 2008). Learning by memorization is minimized and students see the possibility of developing a proof in more than one way. For example, given three properties about a quadrilateral, a student could logically deduce which statement implies which about the quadrilateral (Breyfogle & Lynch, 2010). It is the view of Schwartz (2008), that since high school geometry curricula are built on geometric proofs, this is the level of development that high school students need to be at in order to understand high school geometry.

### 2.1.5 Level 5: Rigour

Students at this level "understand the formal aspects of deduction, such as establishing and comparing mathematical systems" (Mason, 1998, p. 5). Here, students learn that geometry needs to be understood in the abstract; see the "construction" of geometric systems. Students at this level should understand that other geometries exist and that what is important is the structure of axioms, postulates, and theorems (Crowley, 1987; Breyfogle & Lynch, 2010). For example, students at this level are able to establish that the locus of all points equidistant from a fixed point is a circle in Euclidean geometry, whereas, the same locus is a square in Taxicab geometry (Krause, 1986 cited in Atebe, 2008).

## 2.2 The van Hiele Theory: A Historical Perspective

Throughout the history of the modern educational system, there have always been students who have had difficulties and thus, fallen behind others in the field of Mathematics especially in geometry. This has encouraged teachers to experiment with new methods of teaching in an attempt to understand and correct this imbalance. However, again and again these teachers met little success in the field of geometry. The problem concerning the teaching and learning of geometry was identified in the 1950s by two Dutch Mathematics educators, Pierre van Hiele and Dina van Hiele-Geldof, who due to their frustrations investigated possible reasons that could have created such problems in their classrooms. Pierre van Hiele and Dina van Hiele-Geldof were a husband-and-wife team of Dutch Mathematics educators who did research in the late 1950s on thought and concept development in geometry among school children. As a result of many years of teaching experience, they noticed with disappointment the difficulties that their students had in learning geometry (van Hiele, 1986; Atebe &

Schäfer, 2008). The van Hiele theory of geometric thought emerged from the separate doctoral works of Dina van Hiele-Geldof and Pierre van Hiele, which were completed simultaneously at the University of Utrecht, Netherlands in 1957 (Uziskin, 1982).

The van Hieles recognized that in most of the Mathematics curriculum, the geometry content is relatively at a higher level and difficult for students to learn. Moreover, students lack experiences and abilities to do this kind of geometry. As a result, the van Hieles focused on categorizing students' thinking ability in geometry by Levels and also providing an ideal instructional methodology which will help students move from a lower level to a higher level. In their research, each of them took a different angle; Dina's dissertation was about a teaching experiment and in that sense was more prescriptive regarding the ordering of geometry content and learning activities of students. She therefore strove to explain from a teaching perspective how to help children make progress with the Levels, and described five teaching phases within each level. Pierre's dissertation mainly tried to explain why students experienced problems in geometry education; in this respect it was explanatory and descriptive. He was responsible for coming up with the model and describing these Levels in more details (De Villiers, 2004). Since Dina died shortly after finishing her dissertation, it was her husband, Pierre who clarified, amended, and advanced the theory.

The works of the van Hieles was slow in gaining international attention. In the years 1958-1959, Pierre wrote three papers (two in English, one in Dutch but translated into French) that received little attention in the West, but were applied in curriculum development by the Soviet academician Pyshkalo in 1968. It was not until the 1970s that a North American, Izaak Wirszup, began to write and speak about the model. At about the same time, Hans Freudenthal, the van Hieles' professor and mentor from the University of Utrecht, called attention to their works in his "massive" book, *Mathematics as an Educational Task* in 1973. Through Freudenthal and the Soviets, the work of the van Hieles came to the attention of Wirszup, who was the first to speak about the van Hiele theory on this side of the Atlantic in 1974 and later published his speech in 1976 (Uziskin, 1982; Crowley, 1987).

The van Hiele theory has now gained a lot of international attention. One of the major research works using the van Hiele framework was by Usiskin (1982) at the University of Chicago in the United States of America (USA). Usiskin developed a test to measure learners' van Hiele Levels of reasoning. Based on Usiskin's work, the van Hiele theory has become the most influential factor in the American geometry curriculum (Van de Walle, 2001). There has since been an increased interest in the van Hieles' contributions as a significant amount of research in school geometry has focused on the van Hiele Levels of thinking. Consequently, several researchers have applied the theory to improve geometry instruction (Dindyal, 2007; Yazdani, 2007; Breyfogle & Lynch, 2010; Alex & Mammen 2012; Siew, Chong & Abdullah, 2013). It is indeed the wide and diverse use of the van Hiele theory that legitimizes it as a good yardstick with which to survey the geometry terrain of this study.

## 2.3 Properties of the van Hiele Levels

The van Hieles made certain observations about the nature of the Levels of thinking in geometry (that is, properties of the Levels) and their relationships to geometry classroom instruction. Crowley (1987, p. 4) indicated that "these properties are particularly significant to educators because they provide guidance for making instructional decisions". In order to understand how an individual student progresses from one van Hiele Level to the next, early researchers who have studied and employed the van Hiele theory and, certainly the van Hieles themselves, identified the following properties as pertaining to the Levels:

### 2.3.1 Property 1: Fixed Sequence/Hierarchy

It is inherent in the van Hiele theory that, in understanding geometry, a person must go through the Levels in order. This is referred to as the fixed sequence property of the Levels. A student cannot operate, with understanding, at van Hiele Level $n + 1$ without having gone through Level $n$ (Usiskin, 1982; van Hiele, 1986; Gutierrez, 1992). This simply implies that without passing through the low Levels, a student cannot move into the higher Levels. As a result, for effective geometric thinking, all Levels must be taken step by step in order for students to advance tremendously.

### 2.3.2 Property 2: Linguistic Character

Language plays a significant role in the learning process. Each of the van Hiele Levels of thought is characterized by its own linguistic symbols and network of relations. Learners reasoning at different Levels speak different languages and the same term is interpreted differently (Gutierrez, 1992; Mason, 1998). Consequently, a relation that is "correct" at one level may be modified at another level. For example, a figure may have more than one name (class inclusion); a square is also a rectangle and a parallelogram. A student at Level 1 does not conceptualize that this kind of nesting can occur. This type of notion and its accompanying language, however, are fundamental at Level 2. As a result, learners on different Levels cannot understand one another (Crowley, 1987). For this reason and for effective geometry instruction, teachers should use Level-appropriate terminology, symbols, or general language in their geometry instructional practices.

### 2.3.3 Property 3: Adjacency

At each Level, concepts that were implicitly understood at one Level become explicitly understood at the next Level. For example, at Level 1 a student recognizes that a rectangle is a rectangle because it looks like one (that is, only the form of a figure is perceived). However, at Level 2 a rectangle looks like a rectangle because it has two pairs of parallel sides that are the same length and four right angles (here, the figure is of course, analyzed and its components and properties are discovered) (Usiskin, 1982; Crowley, 1987).

### 2.3.4 Property 4: Advancement/Ascendancy

Advancement or progress from one Level to the next is more dependent on instructional experience than on age or biological maturation (van Hiele, 1986; Crowley, 1987; Mason, 1998; van Hiele, 1999). Some methods enhance progress, whereas others impede or even prevent movement between Levels, however, no method of instruction allows a student to skip a level (Mason, 1998; van Hiele, 1999). Van Hiele (1999) himself lays more emphasis on this property when he indicates that:

> Instruction intended to foster development from one level to the next should include sequences of activities, beginning with an exploratory phase, gradually building concepts and related language, and culminating in summary activities that help students integrate what they have learnt into what they already know (p. 311).

### 2.3.5 Property 5: Mismatch

If learners are at one Level and instruction is at a different Level, the desired learning and progress may not occur. In particular, if the teacher, instructional materials, content, vocabulary, and so forth are at a higher level than the learner, the learner will not be able to follow the thought processes being used (Crowley, 1987). Usually, if a teacher tries to teach at a level of thought that is above a student's level the student will try to memorize the material and may appear to have mastered it, but the student will not actually understand the material. Students may easily forget material that has been memorized, or be unable to apply it, especially in an unfamiliar situation (Mason, 1998).

## 2.4 The van Hiele Phase–based Instruction (VHPI)

As indicated in property 4, the van Hieles emphasized that cognitive progress in geometry can be accelerated by instruction. According to them, an effective process of learning geometry is not the same as the teaching and learning process of other topics in Mathematics such as numbers, algebra and probability. As a result, the method and organization of geometry instruction, as well as the content and materials used, are important areas of pedagogical concern. In order to deal with these issues, the van Hieles proposed that the learning process leading to complete understanding at the next higher level has five phases namely: information/inquiry, guided/directed orientation, explicitation, free orientation, and integration. The approach used in these five phases provides a structured lesson (Usiskin, 1982; van Hiele; 1986; Crowley, 1987).

### 2.4.1 Phase 1: Information/Inquiry

This phase of learning involves a discussion between the teacher and the student concerning a geometric topic. The teacher holds a conversation with the student, in well-known language symbols, in which the context he wants to use becomes clear. This two-way teacher-student interaction is essential in understanding certain geometrical shapes such as making observations, asking questions and understanding the vocabulary for that particular geometrical shape (Vojkuvkova, 2012; Abu & Abidin, 2013). For example, the teacher may ask students, "What is a rhombus? A square? A parallelogram? How are they alike? Different? Do you think a square could be a rhombus? Could a rhombus be a square? Why do you say that? . . ." (Crowley, 1987, p. 5). According to Crowley (1987) the purpose of these activities is twofold: (1) the teacher learns what prior knowledge the students have about the topic, and (2) the students learn what direction further study will take.

### 2.4.2 Phase 2: Guided/Directed Orientation

In this phase, the teacher guides the students to explore the object of instruction by assigning carefully structured but simple tasks such as folding, measuring, or constructing that the teacher has carefully sequenced (Crowley, 1987; van Hiele, 1999). The teacher sees to it that students explore specific concepts. These activities should gradually reveal to the students the structured characteristic of a particular level. Thus, much of the material should be short tasks designed to elicit specific responses. For example, simple activities involving folding, reflecting and measuring using a shape such as a rhombus may be assigned to the students. The students are expected to observe things about the angles, sides, and diagonals. The teacher should allow students to use their own language, but should occasionally bring in right terminologies (Pegg, 1995 cited in Atebe, 2008).

### 2.4.3 Phase 3: Explicitation

In this phase, the students describe what they have learned about the topic using their own language (Crowley, 1987; Mason, 1998; van Hiele, 1999). Progressing the rhombus example in phase 2, students may discuss with each other as well as with the teacher what figures and properties emerged in the activities above. Vojkuvkova (2012, p. 74) also indicated that in this phase "students formulate what they have discovered, and new terminology is introduced. They share their opinions on the relationships they have discovered in the activity". The teacher ensures that the accurate and appropriate terminology is developed and used (Crowley, 1987; Mason, 1998; van Hiele, 1999). It is the view of Vojkuvkova (2012) that the van Hieles thought it is more useful to learn terminology after students have had an opportunity to become familiar with the concept.

### 2.4.4 Phase 4: Free Orientation

In this phase students know the properties being studied, but they need to develop understanding of relationships in various situations. As a result, students in this phase solve more complex tasks independently which brings them to master the network of relationships in the material (Vojkuvkova, 2012). The tasks designed here are to provide the students with problems that are open-ended and have multi-path solutions. In other words, the tasks here can be completed in several ways (Crowley, 1987; Mason, 1998; van Hiele, 1999; Vojkuvkova, 2012).

### 2.4.5 Phase 5: Integration

Students now have a clear sense of purpose and can review and summarize what they have learned with the goal of forming an overview of the new network of objects and relations (Crowley, 1987; Mason, 1998; Vojkuvkova, 2012). The teacher should provide to the students an overview of everything they have learned. It is imperative that the teacher does not present any new material during this phase, but only a summary of what has already been learned (Vojkuvkova, 2012).

After Phase 5, students advance to the next higher level of thought. The new domain of thinking replaces the old, and students with the help of teachers are ready to repeat the phases of learning at the next level (Yazdani, 2007). The basic idea is that students need to cycle (and, for many, re-cycle) through the phases of learning while developing their understanding at each level (Breyfogle & Lynch, 2010).

## 2.5 Piaget's Cognitive Development Theory

Jean Piaget, a Swiss biologist, philosopher, and behavioural scientist developed one of the most significant theories in cognitive psychology. The developmental stages of children's cognition is one of the major contributions of the Piagetian theory. Piaget's work on children's quantitative development has provided Mathematics educators with crucial insights into how children learn mathematical concepts and ideas (Lutz & Huitt, 2004; Ojose, 2008). From his observation of children, Piaget understood that children do have enough ideas. He argued that children were not limited to receiving knowledge from parents or teachers, but rather, they actively constructed their own knowledge. Piaget's work provides the foundation on which constructivist theories are based (Wood, Smith & Grossniklaus, 2001).

In studying the cognitive development of children and adolescents, Piaget identified four major stages: sensorimotor, pre-operational, concrete operational and formal operational. The first stage suggested by Piaget is the sensorimotor stage. In general, this stage lasts from birth to about two years of age. At this point intelligence is based on physical and motor activity, but excludes the use of symbols. Children cannot predict reaction, and therefore must constantly experiment and learn through trial and error (Wood, Smith & Grossniklaus, 2001; Lutz & Huitt, 2004). An additional characteristic of children at this stage is their ability to link numbers to objects e.g. one dog, two cats, three pigs (Piaget, 1977 cited in Ojose, 2008). The end of this stage is marked by the immature use of symbols and language development that signals the progression to the second stage (Lutz & Huitt, 2004).

The second stage, called pre-operational, lasts from about two years of age until approximately seven. Children at this stage are able to mentally represent objects and events, and at this point in development, memory and imagination are developed (Lutz & Huitt, 2004). In the view of Piaget, children at this stage acquire representational skills in the area of mental imagery, especially language and can use these representational skills only to view the world from their own perspective (Silverthorn, 1999).

According to Piaget the reaching of the third stage, the concrete operational, is confirmed by a child's ability to demonstrate logically integrated thought, and the typical age span for this stage is from seven to eleven. At this stage, children use their senses in order to know; they can now consider two or three dimensions simultaneously instead of successively. For example, in the liquids experiment, if the child perceives the lowered level of the liquid, he also perceives the dish is wider, seeing both dimensions at the same time. Additionally, *seriation* and *classification* are the two logical operations that develop during this stage (Piaget, 1977 cited in Ojose, 2008) and both are essential for understanding number concepts. Seriation is the ability to order objects according to increasing or decreasing length, weight, or volume. Classification, on the other hand, involves grouping objects on the basis of a common characteristic (Ojose, 2008).

The last stage is labeled the formal operational stage of development and is the period from adolescence through adulthood. At this stage, intelligence is shown through the logical use of symbols related to abstract concepts. Adolescents can think about multiple variables in systematic ways, can formulate hypotheses, and think about abstract relationships and concepts (Wood, Smith & Grossniklaus, 2001; Lutz & Huitt, 2004). Piaget considered this as the ultimate stage of development, and stated that although the children would still have to revise their knowledge base, their way of thinking was as powerful as it would get. However, Lutz & Huitt (2004) argued that most students have not attained the formal operational stage by the time they get out of high school, let alone at age 15 when Piaget states that most young people should have attained it.

**2.5.1 Piaget's Theory and the Development of Space and Geometry Concepts**

Piaget conducted studies on children's conception of geometry most importantly measurement and also studies on children's conception of spatial concepts. This later motivated empirical studies which investigated children's understanding of area and volume. Piaget and other theorists suggested three Levels of children's geometric development. These are:

- Topological: that is, the whole of geometric shapes, for example, connectedness, enclosure, not taking size into account,

- Projective: that is, observing an object from different angles (3-dimentional representations),
- Euclidean: metric or 2-dimensional representations, that is, taking the distance, direction, angle, and length of an object into account (Wen-Tung & Chin-Hsiang, 2010).

Research by Piaget and Inhelder suggest that early spatial conceptions are topological in nature. That is, children at ages 3 to 4 are at the topological level, in which they represent figures as enclosure or openness regardless of Euclidean geometry, such as side lengths, angles, and sizes of shapes (Davis & Hyun, 2005; Wen-Tung & Chin-Hsiang, 2010; Way, 2014). Children at ages 4 to 6 are in the transition stage; it is not until ages 6 to 8 that they perceive the concept of Euclidean geometry (Wen-Tung & Chin-Hsiang, 2010).

The observations gathered by Piaget and Inhelder led them to propose four stages of development in spatial thinking. These stages can be explained as follows:

**2.5.1.1 Sensorimotor Stage (0-2 years)**

Piaget and Inhelder are of the view that, at the beginning of the sensorimotor stage the children's perspective of the world including space is *idiocentric*. That is, they see the location of an object in space to be in relation to their own body (e.g. the ball is within my reach). As they become more mobile, they see the object in relation to its surroundings (e.g. the ball is under the table). Piaget believes that young children see objects in a topological sense, whereby the objects they see are not fixed in shape (Rynhart, 2012). The idea of a simple closed curve is also important and helps to explain why very young children perceive shapes such as circles, squares and triangles as being essentially the same shape, particularly when they draw their own (Way, 2014).

### 2.5.1.2 Pre-operational Stage (2-7 years)

At this stage, children start to represent spatial features through drawing and modeling. Their topological thinking is evident in their drawings. For example, in the drawing of a duck in Figure 2.3, done by a five-year-old, the sky and the ground are represented as separate objects; there is no comprehension of the horizon. Both eyes are drawn on one side of the head because, to the child, the important feature is that they are inside (enclosed within) the head shape (McNally, 1975 cited in Way, 2014). As is characteristic around this age, the child does not yet possess the type of thinking that can be described by Projective Geometry, and which would allow him or her to imagine the other side of the duck.

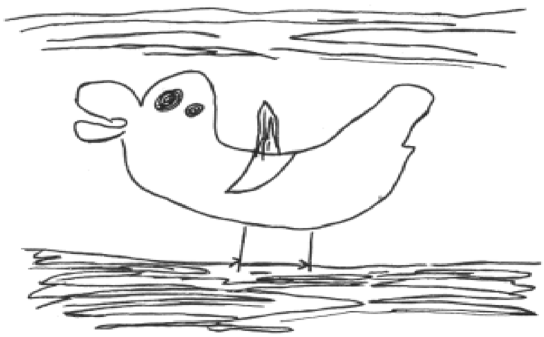


**Figure 2.3: Drawing of a Duck by a five-year old**

### 2.5.1.3 Concrete Operational Stage (7-12 years)

From this stage onwards, Piaget suggests that through the child's interaction with the world, further development of geometric space is actually built upon previously held spatial conceptions. In other words earlier perceptions are not rejected but continuously revised and undergoes transformations as the child learns more about the world. Between the ages of 7 and 12 the child is more able to apply projective geometry in his or her thinking or view of the world. The child is able to further understand the placement of objects in relation to each other and take into account vertical and horizontal relationships (Rynhart, 2012; Way, 2014).

#### 2.5.1.4 Formal Operational Stage (12 year to adulthood)

At this stage of development, learners are able to visualize the concepts of area, volume, distance, translation, rotation and reflection. A learner should also be able to combine measurement concepts with projective skills (Rynhart, 2012).

Piaget strongly felt that the understanding of spatial concepts described by the types of geometry (i.e. topological, projective and Euclidian) is learnt sequentially, from one stage to another. There are other researchers who disagree on this point. They feel that thinking continues to develop and become more integrated over time (Rynhart, 2012; Way, 2014).

## 2.6 The Relationship between van Hiele's Theory and Piaget's Cognitive Development Theory

Before the van Hiele theory, Jean Piaget also described the nature of children's thinking and learning in the area of spatial and geometric concepts. Quite a number of van Hiele's views are in agreement with Piaget's theory of cognitive development and Piaget's comments on Mathematics education. Van Hiele himself indicated that, "Piaget's point of view was that 'giving no education is better than giving it at the wrong time.' We must provide teaching that is appropriate to the level of children's thinking" (van Hiele, 1999, p. 310). Even though van Hiele (1986) indicated that his work has roots in the theories of Piaget, he disagreed with certain aspects of Piaget's theory. Van Hiele believed that the theory of Piaget was one of development and not one of learning, and that, Piaget's theory is structured using stages of cognitive development associated with specific chronological ages. Following the footsteps of Piaget, the van Hieles also identified five hierarchical, sequential and discrete Levels of geometric development that are dependent on a learner's experience. However, the relationship between these two theories is interesting because they both included a study about learning geometry and both propose some form of hierarchical structure. Though there has been little research conducted on the issues of similarities and differences between Piaget's and van Hiele's theories, below are a few similarities as well as differences.

### 2.6.1 Similarities and Differences between van Hiele's Theory and Piaget's Cognitive Development Theory

In a very similar way, both Piaget's and van Hiele's theories suggest that students must pass through lower Levels of geometric thought before they can attain higher Levels and that this passage takes a considerable amount of time. The van Hiele theory advocates that instruction should facilitate students' gradual progress through lower Levels of geometric thought before they begin a proof-oriented study of geometry. If formal proof is dealt with prematurely, it can lead students only to attempt at memorization and to confusion about the purpose of proof. This is simply because students cannot bypass Levels and achieve understanding. Moreover, both theories suggest that students can understand and explicitly work with axiomatic systems only after they have reached the highest Levels in both hierarchies. As a result, the explicit study of axiomatic systems is unlikely to be productive for the vast majority of students in high school geometry (Battista & Clements, 1995).

Another similarity between Piaget's and van Hiele's theories according to Clements and Battista (1992) is that, both theories emphasize the role of the students in actively constructing their own knowledge, as well as the non-verbal development of knowledge that is organized into complex systems. This type of learning makes students to not only learn facts, names, or rules, but a network of relationships that link geometric concepts and processes and are eventually organized into schemata. This emphasizes the importance of students passing through Levels of thinking.

Despite these similarities, there have also been significant differences between the two theories. The two theories differ on their opinion of how learners progress along the stages or Levels. Piaget proposed that progress is largely dependent on biological changes and that higher Levels of thinking are innate and realized when children become aware of them. He also stressed that teachers can only usefully provide information when the learner is ready for it. In other words, he claimed that the two processes are separate and that educational instruction does not always coincide with developmental stage (Rynhart, 2012). In contrast with Piaget's theory, van Hiele theory argues that development is not dependant on age but rather on experience and the quality of instruction (Uziskin, 1982; van Hiele, 1986). In other words, Piaget's theory is age dependent whilst van Hiele's theory is dependent on systematic instruction.

According to Rynhart (2012), another difference between the two theories is the order in which children learn about space. Piaget claimed that the child's understanding of space began from topological notions to projective (3-dimensional) representations and finally metric (2- dimensional or Euclidean) representations. That of van Hiele differs whereby he placed Euclidean geometry first, followed by projective geometry and finally topology.

In the movement among Levels or stages, Piaget's theory suggests that the movement among stages is dependent on activity while the van Hiele theory suggests that the movement through the Levels of thinking is dependent on language. In the view of van Hiele, transition from one level to the next is not a natural process; it takes place under influence of a teaching and learning program. Van Hiele emphasized the importance of knowing the level of language acquisition of the child as it is an important prerequisite for developing an effective method of instruction by the teacher. This significance of language on the development of spatial concepts was also confirmed by other prominent researchers from the Soviet Union, including Luria and Vygotski (Rynhart, 2012). It can therefore be said that the two theories differ in that the van Hiele theory attempts to help teachers improve instruction methods by describing Levels of thinking for students, whereas Piaget's theory is focused more on descriptions of the progression and maturity of thinking. This implies that the van Hiele theory informs instruction while Piaget's theory informs development.

Furthermore, Battista and Clements (1995) indicated that Piaget's theory describes how thinking in general progresses from being non-reflective and unsystematic, to empirical, and finally to logical-deductive. On the other hand, the van Hiele theory deals particularly with geometric thought as it develops through several Levels of sophistication under the influence of a school curriculum.

The van Hiele theory has been chosen by the researcher in surveying the geometry terrain of this study. Uziskin (1982, p. 7) argues that "with attempts to apply the theory in geometry curricula in countries as diverse as the Netherlands, the Soviet Union, and the United States, the theory is obviously seen as both widely and easily applicable". In implementing instruction based on the van Hiele framework, teachers have two major tasks. First, teachers need to know and understand the van Hiele Levels of their students, and second, they need to help their students progress through these Levels in preparation for the axiomatic, deductive reasoning that is required in high school geometry (Malloy, 2002). Additionally, the van Hiele theory also includes an outline on instructional methods which is very useful in the sense that it gives Mathematics instructors insights about the ways in which instructional decisions can be made to promote students' growth and development in spatial and geometric reasoning.

## 2.7 Importance of Geometry in the School Mathematics Curriculum

Geometry is an essential area in the school Mathematics curriculum throughout history. It has had great importance in people's lives, originating with the need of human beings to specify quantities, to measure figures, land and earth, and make maps. In order to represent and solve problems in topics of Mathematics like trigonometry and in daily life situations, sound geometry knowledge is necessary (Sunzuma, Masocha & Zezekwa, 2013). Thus, focusing on geometry first in the Mathematics curriculum is crucial. Young students have a stronger background in geometry due to their surroundings and objects that they play with during infancy. Fostering these strengths is critical in developing important Mathematics concepts at the elementary level (Yegambaram & Naidoo, 2009; Furner & Marinas, 2011). Reys (2009) cited in Furner & Marinas (2011) discusses the following aspects as to why we teach geometry:

- Geometric knowledge, relationships, and insights are useful in everyday life and are connected to other mathematical and school subjects - it surrounds us.
- Geometry helps us to represent and describe in an orderly manner the world in which we live.
- Children are naturally interested in geometry and find it intriguing and motivating.
- Children's spatial abilities frequently exceed their numerical skills and tapping these strengths fosters an interest in Mathematics and improves number understanding and skills later on when starting with geometry first.

In support of these reasons, Furner and Marinas (2011) suggested that often it may be paramount to start teaching young children geometry first and not numbers as numbers are considered to be more abstract and difficult to learn. Focusing on geometry, one of the most concrete branches of Mathematics first, can benefit students' whole view of Mathematics and their attitudes towards learning it. Geometry has been defined as the study of shape and space (Güven & Kosa, 2008). Spatial reasoning is important in other curriculum areas as well as Mathematics, Science, Geography, Art, Design and Technology (Jones, 2002). It is therefore not surprising that one of the aims of teaching Mathematics in Ghana is to develop an understanding of spatial concepts and relationships (CRDD, 2007). Jones (2002) stated that:

> Geometry appeals to our visual, aesthetic and intuitive senses. As a result it can be a topic that captures the interest of learners, often those learners who may find other areas of Mathematics, such as number and algebra, a source of bewilderment and failure rather than excitement and creativity (p. 122).

Geometry is a natural place for the development of students' reasoning and justification skills, culminating in work with proof in the secondary Levels. Geometric modeling and spatial reasoning offer ways to interpret and describe physical environments and can be significant tools in problem solving. Also, geometric ideas are useful in representing and solving problems in other areas of Mathematics such as algebra, calculus, trigonometry and also in real-world situations, so geometry should be integrated with these other areas. Geometric representations can also help students make sense of area and fractions; histograms and scatter plots can give insights about data; and coordinate graphs can serve to connect ideas in geometry and algebra (National Council of Teachers of Mathematics (NCTM), 2009). Jones (2002) also argues that the study of geometry contributes to helping students develop the skills of visualization, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof. In his view, geometric representations can also be used to help students make sense of other areas of Mathematics: fractions and multiplication in arithmetic, the relationships between the graphs of functions (of both two and three variables), and graphical representations of data in statistics.

It is therefore of great importance to include geometry in the Ghanaian Mathematics curriculum for students to master as part of their experience with Mathematics. By doing so, students will have an extensive range of options in choosing careers and fields of study to help boost development through such areas as architecture and design, engineering, and various aspects of construction work. Though geometry is seen as a significant branch of Mathematics, there have been many difficulties in teaching and learning it.

## 2.8 Problem of Teaching and Learning Geometry in Ghana

Geometry is one of the major areas in Mathematics where students' performance has been abysmal. Research has shown that many students are having difficulties in learning geometry and showing poor performance in Mathematics classrooms (Baffoe & Mereku, 2010; Alex & Mammen, 2012; Abu & Abidin, 2013). According to Strutchens, Harris and Martin (2001), students learn geometry by memorizing geometric properties rather than by exploring and discovering the underlying properties. In the view of Gal and Lew (2008), if students learn geometry by merely memorizing the definitions, they would not be able to perform well in higher level task and thus, they may simply make decision wrongly based on their own examples. Geometry knowledge learned in this way is shallow and incomplete. As a result, over the years, students are frequently found to have a number of misconceptions in geometry.

A number of factors have been put forward to understand the major problems of teaching and learning geometry, these include geometry language, lack of instructional aid and ineffective instruction (Adolphus, 2011). First, some misconceptions in learning geometry could be as a result of inappropriate terminology and language (Keuroghlian, 2013 cited in Siew, Chong & Abdullah, 2013). For example, students' were puzzled by the word 'right angle' as the angle opens to the right, therefore, there is a 'left angle' if it is opened to the left (Mack, 2007 cited in Siew, Chong & Abdullah, 2013). In the view of Ampiah (2010), language is the most important tool in the teaching/learning process. The choice of the language of instruction employed in school is of maximum importance. Initial instruction in the learner's first language improves learning outcomes. The importance of its effective use in basic education cannot therefore be underestimated. In Ghanaian basic education there have been two languages: L1 being the child's vernacular and L2 being English language which has a much extensive use in education.

The official policy regarding these two languages in education, for many years has been the use of vernacular as a medium of instruction as well as one of the subjects to be studied at lower primary (Primary 1 to Primary 3), while English language is a subject. Starting from primary 4 onwards English language becomes the medium of instruction as well as a subject. This policy appeared to work almost perfectly until it began to generate a controversy between policy makers and language professionals. Policy makers now regard this policy as unworkable and they believe it has been the cause of lowering of standards in basic education (Ampiah, 2010). But most importantly, it is crucial to ensure that the proper geometric terminologies are used by both the teachers and students when teaching about geometric shapes and concepts. This will go a long way to address the issue of language barriers in students who use English as a second language.

Moreover, "the quality and availability of learning materials strongly affect what teachers can do" (Ampiah, 2010, p. 4). For a course like geometry which is the bedrock of engineering and technological development, the issue of adequate physical facilities cannot be over-emphasized. Physical facilities such as models will help grasp the idea of geometry which seems to be abstract and difficult. It is the facilities in terms of infrastructure, equipment and materials that enhance understanding and afford the students the opportunity to acquire the necessary knowledge (Adolphus, 2011). Solarin (2001) cited in Sunzuma, Masocha and Zezekwa (2013) observed on a general note, that secondary schools lack facilities and equipment for teaching. According to him, such a situation where teachers are forced to discuss theoretically, practical aspect of the subject (geometry) is not good enough. Most importantly, to obtain the rich geometrical knowledge and a sound background in geometry, Etsey (2004) indicated that it is necessary for school and College authorities to provide the necessary resources and materials needed to teach the subject so that PTs gain adequate content knowledge and professional skills before graduating from the CsE.

Furthermore, an additional problem of geometry learning is the issue of ineffective classroom instruction. De Villiers (2012) posited that:

> Traditionally most teachers and textbook authors have simply provided students with ready-made content (definitions, theorems, proofs, classifications, and so on) that they merely have to assimilate and regurgitate in tests and exams. Traditional geometry education of this kind can be compared to a cooking and bakery class where the teacher only shows students cakes… (p.13).

This unfortunate situation is not different from what is occurring in Ghanaian schools. Fredua-Kwarteng and Ahia (2004) indicated that in most Ghanaian schools students learn Mathematics by listening to their teacher and copying from the chalkboard rather than asking questions for clarifications and justification, discussing, and negotiating meanings and conjectures. Consequently, students learn Mathematics as a body of objective facts rather than a product of human invention. It is sad to note that even during practical geometry lessons which should be activity-based and involve some hands-on investigations, this ineffective and poor way of instruction is still employed.

It is the view of Chen (2013) that although students have learned how to calculate the surface area, volume, and the trigonometric functions, conic sections with space vector concepts, they may not be capable of integrating the mathematical knowledge to design the final geometric work. Therefore, it is necessary to develop learning activities to guide students in creative design. These activities have to meet students' learning interest as well as taking into account the effect of individual learning goals. This approach according to Chen (2013) will first develop some problems by researchers and teachers to integrate geometric concepts, and then group students in learning activities to the construction of the different products involving geometric knowledge in the works. Similarly, Etsey (2004) believed that the teaching of Mathematics including geometry must be hierarchical. In his view, complex tasks must be divided into subtasks to make the performance of the complex task easy. In teaching Mathematics and geometry topics in particular, the student must be taken upwards through a hierarchy starting from sub-skills which are within the learners' previous competence. At each level the learner puts together two or more of his/her existing skills to achieve the new skill. In summary, the teaching and learning of geometry should involve more hands-on activities that will stimulate curiosity and encourage exploration. This will enhance students' conceptual understanding of geometric concepts.

## 2.9 The Teaching of Shape and Space in Schools

According to the Professional Development Service for Teachers (PDST), fundamental facts regarding shape and space have been divided into four sections:

- fundamental facts associated with two-dimensional (2-D) shapes;
- fundamental facts associated with three-dimensional (3-D) shapes;
- fundamental facts associated with symmetry; and
- fundamental facts associated with spatial awareness (PDST, 2014).

Spatial awareness is an intuitive feel for shape and space. It involves the concepts of geometry, including an ability to recognize, visualize, represent, and transform geometric shapes. It also involves other, less formal ways of looking at two- and three-dimensional space, such as paper-folding, transformations, tessellations, and projections. Spatial sense is necessary for understanding and appreciating the many geometric aspects of our world. Insights and intuitions about the characteristics of two-dimensional shapes and three-dimensional figures, the interrelationships of shapes, and the effects of changes to shapes are important aspects of spatial sense. Students develop their spatial sense by visualizing, drawing, and comparing shapes and figures in various positions (Ontario Curriculum, Mathematics, 2005; Yegambaram & Naidoo, 2009).

Traditionally, elementary school geometry instruction has focused on the categorization of shapes and at the secondary level, it has been taught as the key example of a formal deductive system. By virtue of living in a 3-D world, learners enter school with a significant amount of intuitive geometric knowledge. However, most Mathematics teachers teach three-dimensional shapes using just text books and the chalkboards. Worksheets of 3-D shapes are displayed as a 2-D shape. The apparent lack of depth of teaching 3-D shapes has a lasting impression on the minds of the learners. The learner's perception of 3-D shapes is that of it being a 2-D shape. This negative approach employed by most Mathematics teachers in teaching 3-D figures using 2-D medium has a negative impact on the geometrical understanding of learners later in their study of geometry (Yegambaram & Naidoo, 2009).

In determining geometric properties of 3-D figures, visual perception plays a major role, and in recalling and describing 3-D figures, image formation is a key component. Over the years, much concern has been expressed about the limitations of traditional approaches to the teaching and learning of geometry and specifically of spatial abilities. Most current curriculum materials and even computer-based systems address the investigation of 3D geometry by requiring students to operate within 2-D representations of 3-D geometry with barely any hands-on activities. This is not an effective approach of investigating human 3-D spatial ability. The limitations of these representations hinder the development of many of the 3-D spatial abilities identified in most geometry classrooms. In most geometry lessons, the main emphasis is put on 2-D geometry, while 3-D geometry rather stays in the background. While the teaching of 3-D shapes and drawing in 3-D geometry have been neglected for decades, ordinary routine calculations dominate 3-D geometric activities. Spatial-visual skills are frequently and extensively avoided (Christou, Jones, Pitta, Pittalis, Mousoulides & Boytchev, 2007).

Yegambaram and Naidoo (2009) further indicated that different ways in which learners interact in physical space may be distinguished. These include:

1. Observing spatial objects in a discriminating way, that is, two and three dimensional figures.
2. Determining distances, elevations, area and volumes.
3. Designing spatial objects and configurations, for example, gardens, furniture arrangements, furniture, buildings and artistic designs.
4. Representing spatial configurations with plane drawings.
5. Interpreting plane representations of spatial configurations.

They argued that traditional school geometry has attempted to address the first three aspects, but is singularly lean on the rich domain of geometrical ideas pertaining to aspects (4) and (5), considering that a great deal of our interaction in physical space involves dealing with 2-D representations of this space.

## 2.10 Pre-service Teachers' Mathematics Education

Asante and Mereku (2012) have indicated that "the low standard of Mathematics proficiency among pupils and students alike has persisted for decades and test information has consistently indicated problems in the way students learn" (p. 23). In 2003 and 2007 Ghanaian JHS2 students participated in the Trends in International Mathematics and Science Study (TIMSS) assessment. The abysmal performance of the students in TIMSS 2003 placed Ghana at the 45th position on the overall Mathematics achievement results table (Anamuah-Mensah, Mereku & Asabere-Ameyaw, 2004). Though in TIMSS 2007 there was an improvement in performance over that of 2003, the students' scores were lower than those obtained by all participating African countries (Anamuah-Mensah, Mereku & Asabere-Ameyaw, 2008). It is therefore necessary to look at the kind of training and preparation being given to PTs in Mathematics. This is because "the issue of providing quality education to pupils is directly related to the quality of teachers in the system" (Ampiah, 2010, p. 3).

The situation is not too different among SHS students. The level of performance of most SHS students in Ghana has not been encouraging and as a result majority of students admitted into CsE have low grades in Mathematics (Akyeampong, 2003 cited in Adu-Yeboah, 2011). This may negatively affect the calibre of trained teachers being produced by the CsE because most students admitted have a weak background in Mathematics. Asante and Mereku (2012) argue that teachers need to know and understand the topics and procedures that they teach because it is for this reason that Teacher Education and policy makers of Teacher Education have designed the Diploma in Basic Education (DBE) Mathematics course to include Content Knowledge. Therefore, with the weak Mathematics background of most PTs, a lot rest on CsE

Mathematics tutors to transform them into competent Mathematics teachers for the duty ahead.

The key institutions that train teachers for basic schools are the CsE. The CsE operates the semester system. PTs are taken through well planned series of courses to equip them to teach all subjects in the basic school curriculum. The Mathematics curriculum and even the entire curriculum of CsE are structured to comprise of Content and Methodology. The method courses are offered in the second year of the programme by which time the trainees had already been taken through the basic Mathematics content course. This apparently is to improve on students' content knowledge before being introduced to pedagogy in the second year. The pattern of development of CsE has been in response to national demands in general and changes at the basic school level in particular. Currently PTs go through four semesters in College, and one academic year of internship in a Basic School. Trainees spend the first two years in College to study subject matter and methods courses, and in third year, they go out to schools to have a year-long teaching practice. This is commonly termed, "IN-IN-OUT" system (Etsey, 2004; Adu-Yeboah, 2011; Asante & Mereku 2012).

The content aspect of geometry for PTs offering Science programme is taught in first year first semester under the course titled "Geometry". The topics expected to be treated within that semester with their respective durations include lines, angles and polygons (2 weeks), congruent and similar triangles (1 week), geometrical constructions including loci (2 weeks), circle theorems (2 weeks), two and three-dimensional shapes (2 weeks), co-ordinate geometry (2 weeks) and equation of a circle (2 week). Pre-service teachers offering non-science programmes undertake their geometry content course in first year second semester under the course titled "Geometry and Trigonometry". This course includes all the topics listed under "Geometry" for the science PTs but does not include equation of a circle, congruent and similar triangles. These topics have been replaced with Pythagorean Theorem (1 week), simple trigonometric ratios (1 week), movement geometry and vectors (2 weeks) (Institute of Education, University of Cape Coast (UCC), 2005).

In second year, the methodology aspect of geometry is treated under "Methods of Teaching Primary School Mathematics" and "Methods of Teaching Junior High School Mathematics" in the first and second semesters respectively. The specific areas and their respective durations are "Developing ideas about shape and space" (including plane and solid shapes, identification of faces, edges, vertices and the number of each of these in each solid shape, relationship between the number of faces, edges and vertices and nets of solids) (2 weeks), "Teaching measurements" (2 weeks), "Teaching geometrical constructions" (2 weeks) and "Teaching rigid motion" (2 weeks) (Institute of Education, University of Cape Coast (UCC), 2005).

Although the DBE programme in CsE places a great deal of emphasis on content and methodology to ensure a sound mathematical knowledge and pedagogic content knowledge, the time allocations for the various topics seem inadequate with regard to the weak mathematical and geometric backgrounds of most PTs. Due to the inadequate time allocation, most College tutors rush to cover all the topics mechanically in order to finish on time for end-of-semester examinations rather than striving for comprehensive students learning. This approach to teaching only leads to rote-learning. Sakyi (2014) emphasized that "there is also the need to shift away from rote-learning and the current exam-centred educational system in Ghana, which produces robotic products who cannot solve problems or apply their knowledge to creative endeavours" (p. 4).

Apart from the inadequate time allocations for the various Mathematics topics, it seems the CsE are ill-equipped in terms of teaching and learning materials. The researcher's observation from a few CsE indicates that there are no Mathematics textbooks written purposely for PTs. Most College tutors and students depend heavily on books that are written specifically for SHS students and few pamphlets written by College Mathematics tutors as well as what tutors write on the chalk or marker boards. Moreover, the CsE lack Teaching and Learning Materials (TLMs) especially for the teaching of geometry. In the view of Adu-Yeboah (2011) the few materials that are available are used in a way that fails to engage with why and how they work to produce understanding. What may be needed are new TLMs and resources which call for more critical engagement with teaching and learning resources for learning Mathematics especially geometry. It must be noted that "the extent of teacher trainees' exposure to and understanding of curricular materials, including textbooks shapes their level of effectiveness in teaching school Mathematics" (Ma, 1999 cited in Adu-Yeboah, 2011, p. 2).

In summary, there is clearly an emphasis in the CE curriculum on Mathematics content knowledge which takes up a considerable amount of time in the programme. However, conditions at CsE as discussed in this section do not allow for full achievement of the general aims and objectives of teacher education. Reformers need to have another look at how some of these contents can be developed and taught in ways which enables PTs to understand deeply how the concepts might be taught using strategies and resources that convey deep meaning. Introducing mathematical investigations might be one way of shifting this focus (Adu-Yeboah, 2011).

## 2.11 Effect of the VHPI on Students' Geometric Thinking Levels

Apart from using the van Hiele model to identify students' Levels of geometric thinking, the model can also be used for planning effective geometry instruction. Abdullah and Zakaria (2013) argue that the interventions using the VHPI can be applied in classrooms in order to positively and effectively improve students' thinking Levels and help students achieve better level of geometric understanding. In the view of Crowley (1987) the VHPI is the way for teachers to provide more opportunities for students to experience geometry units or topics associated with the van Hiele model. The purpose of this section is to look at the effects of the VHPI on the development of learners' geometric thinking Levels.

Erdogan and Durmus (2009) investigated the effects of instruction based on the VHPI on 142 senior PTs' geometric thinking Levels. There were eight classes of senior PTs, two of them were randomly assigned as experimental groups which were instructed with the VHPI and the other two were randomly assigned as control groups which were instructed with traditional instruction. An instruction consistent with geometric thinking Levels of van Hiele model was applied to experimental groups whereas traditional method was applied to control groups throughout the study. The activities applied in experimental groups were carried out with a method in which the concepts of discussion, group work, collaborative learning approaches were implemented in a related web in accordance with geometric thinking Levels of van Hiele whereas, in control groups, the activities were applied with traditional approaches in which the students follow the instruction the teacher gives and active participations are not promoted.

Significant difference was found between the pre-test and post test scores of van Hiele Geometry Test of experimental groups. However, when the pre-test and post-test scores of van Hiele Geometry Test of PTs in control group were taken into consideration, it was seen that there was no significant difference between the results. It can be claimed that, while instruction given with traditional method did not improve the geometric thinking Levels of PTs, instruction consistent with VHPI has a positive effect on the geometric thinking Levels of PTs.

Siew and Chong (2014) also conducted a study to determine the effects of VHPI on learners' creativity using tangrams activities among grade three primary school learners. A total of 144 Grade three learners took part in the study that employed a pre-test and post-test single group experimental design. The learners were taught two-dimensional geometry and Symmetry through the van Hiele's five phases of learning using tangrams. Paired samples t-tests which compared the mean scores of pre- and post- test indicated significant differences in mean scores between pre- and post- test. After a series of statistical analysis, the finding revealed that creativity can be fostered through the instruction using tangrams which was based on VHPI. The authors further indicated that hands-on activities enable students to develop knowledge and properties of polygons and their creativity. The study showed that the tangram, when integrated with VHPI is able to foster learners' creativity in geometric lessons.

In his study of the correlation between students' level of understanding geometry according to the van Hiele Model and students' achievement in plane geometry, Yazdani (2007) involved one hundred and sixty nine students in an experimental study. A pre-test was administered to all participants at the beginning of the semester. The pre-test consisted of the following two selected response assessment instruments: The "Plane Geometry National Achievement Test" and the "van Hiele Geometry Test". The same battery of tests was employed as a post-test and was administered to the participants after 6 weeks of instruction using the VHPI. A measure of the linear relationship between students' level of understanding geometry according to the van Hiele Model and students' achievement in geometry found a correlation coefficient of 0.8665 for the post-test. The results indicated that there was a strong positive correlation between the advancement of the van Hiele level of understanding geometry and achievement in geometry. The VHPI has significant implications for teaching geometry. It is therefore suggested that educators responsible for geometry instruction and professionals in charge of teacher training programs incorporate the principles upon which the VHPI is based into instructional and curricular design.

### 2.12 Students' Motivation to Learn Geometry

In the effort to improve students cognition and affective outcomes in Mathematics and/or school learning, educational psychologists and Mathematics educators, have continued to search for variables that could be manipulated in favour of academic gains. Several personal and psychological variables have attracted researchers in this area of educational achievement. However, motivation seems to be gaining more popularity and leading other variables (Tella, 2003 cited in Tella, 2007). Motivation is an individual's internal status toward something. It has power to enhance the strength of the relationship between the input and the output of human behaviour. Motivation refers to the reasons for directing behaviour towards a particular goal, engaging in a certain activity, or increasing energy and effort to achieve the goal. (Kleinginna & Kleinginna, 1981 cited in Liu & Lin, 2010). It is assumed that, in this research, PTs will express positive emotions when doing hands-on activities in groups. Based on the notion that hands-on activities as integrated in the VHPI support student-centred instructions, the researcher assumed that PTs should be stimulated to interact with each other for discussions and sharing of ideas.

Halat, Jakubowski and Aydin (2008) compared motivation of sixth-grade students engaged in instruction using reform-based curriculum with sixth-grade students engaged in instruction using a traditional curriculum. Van Hiele Theory-based curriculum was a geometry curriculum in which the authors designed teaching materials based on educational theories, in particular the van Hiele theory. Traditional curriculum was a regular Mathematics curriculum in which the authors did not implement the characteristics of the van Hiele theory in their presentation of geometry. The study engaged 273 sixth- grade Mathematics students, 123 in a control group and 150 in the treatment group. The researchers used a questionnaire and the Course Interest Survey (CIS), administered to the students before and after a five-week of instruction. The study showed that there was a statistically significant difference in motivation between the groups favouring the treatment group. In other words, the reform-based curricula designed on the basis of van Hiele theory, compared to a traditional one, had more positive effects on students' overall motivation in learning geometry at the sixth grade level. The use of real-life applications, group practices, and hands-on activities play important roles in students' motivation to learn geometry. The authors indicated that if

Mathematics teachers pay more attention to reform-based curricula, and prepare their geometry lessons under the guidance of the van Hiele theory, they could be more successful in motivating their students toward their courses.

Smiešková and Barcíková (2014) also investigated the motivation to learn geometry among a group of high school visual arts students. They intended to deal with the problem of lack of students' motivation in geometry and one of their aims was to show students a cross curricular relation between Mathematics and visual arts. The study was done in a school where there was low patronage of Mathematics classes and also lack of students interest in learning geometry. Due to these problems the researchers tried to find a way to motivate students to learn geometry. One geometry lesson was prepared based on the VHPI with specific activities fitting the first and second Levels of the van Hiele model of geometric thinking. During the lesson, students had the opportunity to explore and also observe the geometrical shapes in their environment. Even students who were not interested in Mathematics participated in these activities and they enjoyed it. The use of the VHPI in the teaching of geometry made the subject more interesting and served as an element of motivation to the students which encouraged them to learn the subject.

Moreover, Tella (2007) pointed out that, in making instruction interesting in learning Mathematics, there is the need to use methods and materials which will make the learning of Mathematics active and investigative as much as possible. Such methods also must be ones that take into account, learner's differences and attitudes towards Mathematics as a subject. Examples could be the use of concrete materials and other instructional devices, which are manipulated. Also, geometry exercises in form of various pencil and paper activities should be used. This study illustrates the way in which instruction based on the VHPI can change the value of a task, increase student self-effectiveness, and improve student worth. In line with this study, the researcher employed activities based on the VHPI into geometry lesson to explore its effects on students' motivation to learn.

## 2.13 The Constructivist Approach to Instruction

Adopting a constructivist approach to teaching geometry is beneficial. In the view of Gujarati (2014), one approach is based on the van Hiele model in which a student progresses through each level of geometric thought as a result of instruction that is organized into five phases of learning. As discussed earlier, the van Hieles recommended five phases for guiding students from one level to another on a given topic. The phases of learning advocated by the van Hieles motivate students to learn geometry by means of hands-on investigations. This helps students exploit problem-solving strategies that, when combined with concrete experiences, yield higher order thinking skills. In other words the van Hiele theory is a cognitive one that supports constructivist beliefs of Mathematics education. The rationale of this section therefore, is to situate the van Hiele theory in the context of a much broader theory of education that places emphasis on learners as active participants in knowledge generation in the teaching–learning process.

The constructivist view of learning in the classroom, can point towards a number of different teaching practices. Generally, the constructivism concept means encouraging students to use active techniques such as experiments and real-world problem solving to create more knowledge and then to reflect on and talk about what they are doing and how their understanding is changing. The teacher makes sure that he understands the students' pre-existing conceptions, and guides the activity to address them and then build on them. Constructivism modifies the role of the teacher to that of a facilitator who helps students to construct knowledge rather than to reproduce a series of facts (Khalid & Azeem, 2012).

In a constructivist learning environment, the teacher guides the students through problem-solving and inquiry-based learning activities with which students put together and test their ideas, draw conclusions and inferences, group and convey their knowledge in a collaborative learning environment. One of the most important strengths of the constructivist approach to instruction is that it transforms the student from a passive recipient of information to an active participant in the learning process. Always guided by the teacher, students construct their knowledge actively rather than just mechanically ingesting knowledge from the teacher or the textbook. The task of the instructor is to translate information to be learned into a format appropriate to the learner's current state of understanding (Khalid & Azeem, 2012).

Bruner (1966) cited in (Khalid & Azeem, 2012) states that a constructivist theory of instruction should address four major aspects: (1) predisposition towards learning, (2) the ways in which a body of knowledge can be structured so that it can be most readily grasped by the learner, (3) the most effective sequences in which to present material, and (4) the nature and pacing of rewards and punishments. This clearly indicates that there are many aspects of the van Hiele theory that are consistent with constructivist ideas about teaching and learning. Van Hiele (1999) emphasized these aspects in his phases of learning when he clearly stated that:

> throughout these phases the teacher has varied roles: planning tasks, directing children's attention to geometric qualities of shapes, introducing terminology and engaging children in discussions using these terms, and encouraging explanations and problem-solving approaches that make use of children's descriptive thinking about shapes (p. 316).

Van Hiele further indicated that in helping learners develop from one Level to the next, instruction should include sequences of activities, starting with an exploratory phase to gradually building concepts and related language and concluding in summary activities that assist students integrate what they have learned into what they already know (van Hiele, 1999). For effective geometry instruction, the van Hiele model as located in the constructivist cognitive model must be adopted most of the time.

# CHAPTER THREE

# METHODOLOGY

## 3.0 Overview

The study sought to find out the effect of the VHPI on PTs' geometric thinking Levels and motivation to learn plane geometry. It also investigated the extent to which CE Mathematics tutors facilitate geometry teaching and learning consistent with the van Hiele Levels of geometric thinking. In pursuance of the purposes stated above, the following research questions were formulated to guide the study:

- To what extent does the use of VHPI improve PTs geometric thinking Levels?
- How does the use of VHPI motivate PTs to learn geometry?
- To what extent do CE Mathematics tutors facilitate geometry teaching and learning consistent with the van Hiele Levels of geometric thinking?

In answering the first research question, the hypothesis below was formulated for the study;

$H_0$: There is no difference in the geometric thinking Levels between the control and experimental groups in the post-test.

This chapter discusses the research methodology adopted for the study. The methodology is expressed in terms of the research design, population, sample and sampling procedure, research instruments and data collection procedure. Issues considering the research instruments, ensuring the validity and reliability of research instruments, the intervention procedure and data analysis techniques are also discussed.

## 3.1 Research Design

Pursuant to the purpose of this study, the researcher employed the mixed method approach with quasi-experimental design as a strategy of enquiry. The mixed method as a methodology focuses on collecting, analyzing, and mixing both quantitative and qualitative data in a single study or series of studies (Creswell, 2003; Creswell, 2006). Qualitative methods were used in the study in order to provide a more profound understanding of the effect of the van Hiele model on the variables that were investigated. The central premise of the mixed method research is that "the use of quantitative and qualitative approaches in combination provides a better understanding of research problems than either approach alone" (Creswell, 2006, p. 5). The purpose of this strategy is also to use qualitative data and results to assist in explaining and assigning reasons for quantitative findings (Fife-Schaw, 2012). A quasi-experimental study is an empirical study used to estimate the causal impact of an intervention on its target population. It takes place in a real life setting as opposed to only a laboratory setting (Vanderstoep & Johnson, 2009; Hashemi, 2014).

According to Shadish, Cook and Campbell (2002) quasi-experiments continue to be frequently utilized by researchers today for three primary reasons:

- to meet the practical requirements of funding, school administrators and ethics.
- to evaluate the effectiveness of an intervention when the intervention has been implemented by educators prior to the evaluation procedure having been considered.
- when researchers want to dedicate greater resources to issues of external and construct validity.

The procedure of pre-test post-test, two group design under quasi-experimental design was used. Working with this design, the researcher had a control group to compare with an experimental group. Both groups took a pre-test and post-test, however, only the experimental group received treatment by the VHPI. The pre-test provided the researcher with some idea of how similar the control and experiment groups were before the treatment.

## 3.2 Population

The target population for the study was second year PTs and Mathematics tutors in all teacher training institutions in Ghana now designated as Colleges of Education (CsE). PTs are students being trained to become first time professional teachers. The ages of PTs in CsE in Ghana are between 18 and 30 years. Almost every ethnicity in Ghana is found in each CE. PTs in Ghana are trained with the same curriculum and run the same academic calendar.

The PTs in the CsE have studied Mathematics in Ghana at the SHS level and have all passed the West African Senior School Certificate Examination (WASSCE), which tests among other things their ability in geometry. However, majority of them enter the CsE with very low grades in Mathematics. As a matter of fact, PTs have similar characteristics in terms of multiple ethnicities, age differentials, admission requirements and the institution that oversees their certification. Furthermore, most of the tutors who handle PTs in these CsE are products of the two teaching Universities in Ghana; University of Education, Winneba (UEW) and the University of Cape Coast (UCC).

### 3.3 Sample and Sampling Procedure

In this study, non-probability sampling, particularly convenience sampling was one of the sampling procedures used. This sampling procedure was used to select two CsE, A and B (pseudonyms for the Colleges) in the Ashanti Region and Greater-Accra Region of Ghana respectively. Convenience sampling was used because of logistic and financial constraints as well as ease of accessibility. PTs in these CsE are admitted from all over the ten regions in Ghana. The researcher is of the view that the two CsE were ideal for this study because the researcher was able to monitor the progress of the PTs during the intervention period.

Simple random sampling is the second sampling technique that the researcher employed in selecting participants in this study. This ensured that bias was eliminated while giving equal opportunities to each sample point selected. The sample units in the population were selected by a random process, using a random number generator so that each participant in the population had the same probability of been selected for the study. The sample comprised of 150 second year PTs, 75 in the experimental group (in College A) and the other 75 in the control group (in College B). Five College Mathematics tutors were also randomly sampled for observation, with three tutors coming from College A and two tutors from College B.

### 3.4 Research Instruments

Considering the nature of research questions been examined, the instruments used for the collection of data were van Hiele Geometry Test (VHGT), Interview and Observation.

### 3.4.1 The van Hiele Geometry Test (VHGT)

Following the development of the van Hiele theory of the Levels of thought in geometry, experts and professional bodies have since developed achievement tests that can be used to measure the attainment of the van Hiele Levels among learners. One such test is the Cognitive Development and Achievement in Secondary School Geometry (CDASSG), which is widely used in the United States (Usiskin, 1982). The VHGT items (see Appendix A) used in assessing the second year PTs' van Hiele Levels was adopted from that of Usiskin (1982). The test was used as both the pre-test and post-test for this study. The initial reasoning to develop this test was to determine the van Hiele Level of Understanding Geometry of a student. Each item was written as a means to identify the behaviours, using quotes from the van Hieles, of the students at each Level. This is a well-known geometry test which has been used in several Masters and PhD works (Usiskin, 1982; Burger & Shaughnessy, 1986; Knight, 2006; Atebe, 2008; Baffoe & Mereku, 2010) since it was developed. Consequently, the CDASSG van Hiele Test has also been used in studies to investigate the reasoning stages of pre- and in-service elementary school teachers in geometry (Halat & Şahin, 2008), to assess the readiness for geometry in a group of mathematically talented secondary school students (Alex & Mammen, 2012) and in exploring the existence of a relationship between students' van Hiele Levels of understanding geometry and their achievement in plane geometry (Yazdani, 2007).

The VHGT is a 25-item multiple choice test and is organized sequentially in blocks of five Items. Items 1-5 measure student understanding at Level 1, Items 6-10 measure student understanding at Level 2, Items 11-15 measure student understanding at Level

3, Items 16-20 measure student understanding at Level 4, and Items 21-25 measure student understanding at Level 5 (Knight, 2006).

Part B of the VHGT was a test consisting of 3 items where participants were expected to provide written responses. This was designed to further explore the problem-solving abilities of the PTs. These items included some commonly found in texts and examination papers set for these learners. Item 1 required the PTs to calculate a missing value in a given geometrical shape; item 2 also required the PTs to find the surface area of a geometric figure; and item 3 required the PTs to write a complete proof of a theorem in geometry giving reasons.

**3.4.1.1 Administration and Grading of the VHGT**

The VHGT was used, both as the pre-test and post-test of the study. The test was meant to be written by all second year PTs who were participating in the study. All participants' answer sheets from VHGT were read and scored (see Appendix B) by the researcher. Scoring of the part A of the VHGT was done as indicated below;

First grading method: Each correct response to the 25-item multiple-choice test was assigned 1 point. Hence, each student's score ranges from 0–25 marks.

Second grading method: the second method of grading the VHGT was based on "3 of 5 correct" success criterion suggested by Usiskin (1982). By this criterion, if a PT answered correctly at least 3 out of the 5 items in any of the 5 subtests within the VHGT, the PT was considered to have mastered that level. Using this grading system developed by Usiskin (1982), the PTs were assigned weighted sum scores in the following manner:

- 1 point for meeting criterion on items 1-5 (Level-I, Recognition);
- 2 points for meeting criterion on items 6-10 (Level-II, Analysis);
- 4 points for meeting criterion on items 11-15 (Level-III, Ordering);
- 8 points for meeting criterion on items 16-20 (Level-IV, Deduction);
- 16 points for meeting criterion on items 21-25 (Level-V, Rigor).

Thus, the maximum point obtainable by any PT was 1 + 2 + 4 + 8 + 16 = 31 points. The method of calculating the weighted sum makes it possible for a person to determine upon which van Hiele Level the criterion has been met from the weighted sum alone. For example, a score of 7 indicates that the learner met the criterion at Levels I, II and III (i.e.1+ 2 + 4 = 7). The second grading system served the purpose of assigning the learners into various van Hiele Levels based on their responses.

According to Usiskin (1982), there are two different cases that can be used in assigning Levels to students namely, the Classical Case and the Modified Case. The study employed the modified case in assigning Levels to PTs. Usiskin (1982) posited that;

> The assigning of Levels in either the classical or modified case requires that a student's responses satisfy Property 1 of the Levels, i.e., that the student at level n satisfy the criterion not only at that level but also at all preceding Levels. Thus a student who satisfies the criterion at Levels 1, 2 and 5, for instance, would have a weighted sum of 1 + 2 + 16 or 19 points, would have no classical van Hiele Level, but would be assigned the modified van Hiele Level 2. A student who satisfies the criterion at Level 3 only would not be assigned either a classical or modified van Hiele Level. Neither of these students would be said to fit the classical van Hiele model (p.25).

The first case, identified as the Classical case, is based on there being five distinct Levels. The second case, identified as the Modified case, is based on four distinct Levels. The decision to use the Modified Case to identify the van Hiele Level of the test subjects was based on the fact that the "modified van Hiele Levels fit more students more consistently than the classical van Hiele Levels" (Usiskin, 1982, p. 42) also it gives a higher percentage of subjects that could be analyzed. Working with the modified van Hiele Levels, the weighted sums and their corresponding van Hiele Levels are as shown in Table 3.1.

**Table 3.1: Modified van Hiele Levels and their Weighted Sums**

| Levels | Corresponding Weighted Sum |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 3 |
| 3 | 7 |
| 4 | 15 |

For the Part B, each of the 3 items was assigned 10 points (see Appendix B). Thus, PTs' scores ranged between 0 and 30 marks.

### 3.4.2 Interview

The nature of interviews used in qualitative research is in a range or continuum (namely, unstructured, semi-structured and structured). Participant observers tend to support the use of in-depth unstructured interview instruments, which are open-ended. These only outline the topics to be discussed with participants.

Unstructured interviews allow researchers to focus the respondents' talk on a particular topic of interest, and also allow researchers the opportunity to test out their preliminary understanding, while still allowing for ample opportunity for new ways of seeing and understanding to develop (Robert Wood Johnson Foundation, 2008). The researcher therefore used unstructured interview to seek participants' views on how the VHPI motivates them to learn geometry.

There were no predetermined questions and the respondents were given the freedom of response. The researcher asked questions pertinent to the study as opportunities arose, then listened closely to participants' responses for clues as to what question to ask next, or whether it was necessary to probe for additional information. In all a total of 9 PTs in the experimental group participated in the interview.

### 3.4.3 Observation

The third data collection technique employed was classroom observation. An observation schedule was adapted from Muyeghu (2008) based on the van Hiele model. The schedule was constructed by taking into account the findings of researchers on the knowledge and skills PTs and tutors need to master and facilitate at the various van Hiele Levels. This schedule was developed based on these activities. Muyeghu (2008) considered the role of the teacher at the Visual Level as well as at the Analysis Level thus activities at the Visual Level (Level 1) and at the Analysis Level (Level 2) were shown in his schedule. These activities were evaluated against the teacher's practice to determine the teaching strategies that selected teachers used to facilitate the development of geometric thinking at the van Hiele Levels 1 and 2 (Muyeghu, 2008). However, for the purpose of this study, the researcher adapted the observation schedule by extending the Levels to include Levels 3, 4 and 5. The schedule assessed the teaching skills of the tutors on a three-point scale. They are weak, moderate and strong. There was an additional column for any comment that the researcher wished to make. This data collection technique helped the researcher to determine the extent to which the selected CE Mathematics tutors facilitate geometry learning and teaching consistent with the van Hiele Levels of geometric thinking.

## 3.5 Validity and Reliability of Instruments

Validity refers to the extent to which the research instruments are effectively authentic or truthful. It is a demonstration that a particular research instrument in fact measures what it purports to measure (Mushquash & Bova, 2007; Williams, 2014). Validity measures that were taken in this study were based on these conceptions and notions of validity.

The researcher consulted the geometry curriculum as well as some Mathematics books of the PTs in order to validate the research instruments (test items). This helped to gain insight into what the PTs are expected to learn so that the researcher could develop the instruments accordingly. The main focus of the test items was to explore and spell out the van Hiele Levels of geometric understanding of the PTs. Thus, only questions on the PTs understanding of geometry were asked. After constructing the test items, the researcher's supervisor was consulted to cross check them.

Test items were also given to some CE tutors to cross check and contribute to the geometric content areas that were been tested in this study in order to further ensure that the content that were chosen was within the prescribed domain of the study for the PTs concerned. Moreover, forty PTs were randomly selected from one of the second year classes in a sister CE (St. Louis CE) and were asked to answer the test items, mainly to detect lack of clarity in the phrasing of the questions, and to give indication for time needed for its completion. The answered test items were scored using the rubric. This helped to refine these instruments. The observation schedule was also cross checked and corrections made by the researcher's supervisor. Some M.Phil. Mathematics

Education students also read through the observation schedule and made suggestions that were incorporated before use.

After administering the tests and the marks scored, the researcher returned to the various Colleges and discussed with PTs their scores. This made PTs to be convinced that the scores accurately represented their abilities in these learning areas. The process of validity just described is what Lincoln and Guba (1985) cited in Shenton (2004) referred to as member checking, a process that has the advantage of putting the respondent on record as having said or done certain things and having agreed to the correctness of the researcher's records of them.

Reliability on the other hand means dependability or consistency. It refers to the extent to which a measuring instrument; a questionnaire, a test yields the same results on repeated applications (Williams, 2014). It means the degree of dependability of a measuring instrument. In this study, the split-half method was used to check the reliability of the instruments. The spilt-half method requires the construction of a single test consisting of a number of items. These items are then divided or split into two parallel halves (usually, making use of the even-odd item criterion). Participants' scores from these halves were correlated using the Spearman-Brown formula used in reliability testing. The value of the reliability coefficient was 0.75. This value indicates a high degree of reliability of the instrument.

### 3.6 Intervention

The van Hiele Phase-based Instruction (VHPI) was applied to the experimental group whereas traditional method of instruction was applied to the control group throughout this study. As indicated by Yazdani (2007), the van Hiele model is applied by first identifying the learners' level of thinking before designing the instruction for their particular level in order to assist them advance to the next higher level. The lessons (see Appendix C) taught in the experimental group was therefore planned (based on the VHPI) after administration of the pre-test so that PTs' initial van Hiele Levels were taken into consideration. This helped the researcher to plan instruction accordingly for the PTs' particular level.

The lessons taught in the experimental group were carried out with a method in which the concepts of discussion, group work, hands-on investigations and collaborative learning approaches are implemented in a related web in accordance with geometric thinking Levels of van Hiele. There were 75 PTs in the experimental group, 25 PTs each from the Science, General and French classes (forming 3 different classes) so that all three respective programmes ran by the College were fairly represented. In each class, PTs were put in groups of five and each group nominated a leader who collated their responses and presented it to the researcher for assessment. As advocated by van Hiele, the lessons were activity-based with PTs working collaboratively using concrete and manipulative materials integrated in the VHPI and the researcher playing the role of a facilitator. The researcher was available to offer explanations and clarifications to the whole class or any group needing assistance. The import of the group work was to create a platform for the PTs to discuss, interact and work collaboratively to make discoveries.

PTs in the experimental group were taken through 3 different lessons which were taught during the third, fourth and fifth weeks in the second semester of the 2014/2015 CsE academic calendar. The topics treated covered properties of angles formed by parallel lines and their transversal, properties of quadrilaterals and lastly, the relationship between quadrilaterals (see Appendix C). In line with the VHPI, Phase 1 (Information/Inquiry Phase) of each of the 3 lessons involved reviewing PTs' previous knowledge on the various topics and further holding a conversation with the PTs concerning the topic, in well-known language symbols making the context clear. This helped the researcher learn what prior knowledge the students have about the topic, and also informed PTs what direction further studies will take (Crowley, 1987).

In Phase 2; Guided Orientation Phase, PTs were given hands-on activities (see Appendix C) that allow them to become familiar with the many properties of the geometric concept. The researcher guided PTs to carefully explore the objects used in the instruction by assigning carefully structured but simple tasks such as folding, measuring and constructing that the researcher has carefully sequenced. In this phase, the PTs were expected to observe features such as angles, sides, diagonals etc. The researcher allowed PTs to use their own language, but occasionally introduced right terminologies. In Phase 3; Explicitation Phase, PTs expressed in their own words what they have discovered in the previous phase. The role of the researcher here was to introduce relevant geometrical terminologies. PTs shared their opinions on the relationships they have discovered in the hands-on activities.

In Phase 4 which is Free Orientation Phase, PTs were asked to solve more complex tasks independently. These tasks were designed to provide the PTs with problems that are open-ended and have multi-path solutions. Some of the tasks were more complex and required more free exploration to find solutions. This helped the PTs to master the network of relationships in the material. Lastly, in Phase 5 which is the Integration Phase, PTs had a clear sense of purpose and could review and summarize what they have learned with the goal of forming an overview of the new network of relations. The researcher provided the PTs with an overview of everything they have learned. The researcher did not present any new material during this phase, but only a summary of what has already been learned.

In the control group, the PTs were taught the same topics (as in the experimental group) but with traditional approaches in which the learners strictly follow the instruction the teacher gives and active participations are not promoted. The PTs were not given any hands-on activities to help them explore geometrical concepts and form their own knowledge. In other words, geometry instruction was not in line with the VHPI but mainly in lecture format and therefore instruction was tutor-centered. The 75 PTs in the control group were put in two groups with 37 PTs in one group and 38 in the other group. Their lessons were taught in the sixth, seventh and eight weeks in the second semester of the 2014/2015 CsE academic calendar.

### 3.7 Data Collection Procedure

The researcher visited the 2 CsE in the latter part of the first semester. Letters of introduction were obtained from the Head of Mathematics Department of University of Education, Winneba to be given to the CsE to enable the researcher visit these Colleges. The researcher met the Heads of these institutions and briefed them on the purpose of the study and craved for their cooperation for the field work. The researcher later visited participants in the 2 CsE and administered the pre-test (the VHGT) during the tenth week in the first semester of the 2014/2015 CsE academic calendar.

After the treatment (VHPI), unstructured interview was granted to 9 PTs in the experimental group. This was to seek views on how the VHPI motivates PTs to learn geometry. The post test (which was the same VHGT) was administered a week after the treatment. Also, geometry instruction of 5 College Mathematics tutors were observed in order to investigate the extent to which CE Mathematics tutors facilitate geometry teaching and learning consistent with the van Hiele Levels of geometric thinking. These tutors were observed teaching plane geometry, 3 of the tutors were in College A and the other 2 in College B. Each tutor was observed once for duration of 2 hours.

### 3.8 Data Analysis

The study used the mixed method approach which employed quasi-experimental as a strategy of enquiry. It generated both quantitative and qualitative data. Research question one generated mainly quantitative numerical data in the form of tests from the PTs. Thus, the researcher employed both descriptive and inferential data analysis. The descriptive data analysis was used in an attempt to understand, interpret and describe the experiences of the PTs in terms of their Levels of geometric conceptualization. The descriptive data analysis helped to organize and describe the data by investigating how the scores were distributed on each construct, and by determining whether the scores on different constructs were related to each other. In specific terms, various descriptive statistics such as frequency distribution, charts and measures of central tendency were used to analyse, describe and compare the quantitative data in this study.

Statistical Package for Social Sciences (SPSS) version 21 was also used for the statistical analysis of the quantitative data. Specifically, the Independent samples t-test statistical procedure at 95% confidence level was used to compare the PTs' conceptual understanding in geometry at the pre-test. Paired-samples (dependent) t-test statistical procedure at 95% confidence level was also used to compare the mean scores of PTs (control and experimental groups) in the pre-test and post-test. The chi-square test was used to test the significance of differences in the geometric thinking Levels between the control and experimental groups. The null hypothesis that there will be no significant difference in the geometric thinking Levels between the control and experimental groups in the post-test was formulated and tested at 0.05 level of significance.

For the qualitative data, the process of data analysis involved contextualization, where research findings were interpreted with reference to data gotten from interviews and observations (Mertler & Charles, 2005). Data collected were analysed by "thick description" after the researcher had read the transcribed interviews and identified categories of responses that answered the research questions. The researcher reported all events that emanated from the study by describing and interpreting the outcomes. In this study, patterns that emerged from interview and observation data were described so that one can make meaning from the data.

# CHAPTER FOUR

# RESULTS AND DISCUSSION

## 4.0 Overview

The study seeks to use both quantitative and qualitative analysis to find out the effect of the VHPI on Ghanaian PTs' geometric thinking Levels and how it motivates PTs to learn geometry. It also finds out the extent to which CE Mathematics tutors facilitate geometry teaching and learning consistent with the van Hiele Levels of geometric thinking. The researcher therefore implemented an intervention based on the VHPI to address the PTs' difficulties in learning plane geometry. Three research questions guided the study. These questions are:

- To what extent does the use of VHPI improve PTs geometric thinking Levels?
- How does the use of VHPI motivate PTs to learn geometry?
- To what extent do CE Mathematics tutors facilitate geometry teaching and learning consistent with the van Hiele Levels of geometric thinking?

The findings of the study and discussion of the findings are presented in three sections according to the research questions.

## 4.1 The van Hiele Geometry Test (VHGT) Results

In this section, the results of the van Hiele geometry test are presented. The same test was carried out twice as pre-test and post-test in the entire study. First of all, before carrying out the VHPI in the experimental group and the traditional instruction in the control group, all the PTs in the sample took a van Hiele Geometry Test as a pre-test of the study. This was made up of two parts; section A which was made up of a 25 multiple-choice items and section B which was made up of three items where PTs were expected to provide written responses. Secondly, after the treatment, the same test was administered to the participating PTs once again as the post-test of the study.

### 4.1.1 The van Hiele Geometry Pre-test Results from the Control and Experimental Groups

A pre-test was administered to the selected sample comprising of 150 PTs (75 PTs in the control group and the other 75 in the experimental group). This was to help unravel PTs difficulties in plane geometry so as to plan instructions suitable to their level of understanding. The pre-test was also to reveal how similar the control and experimental groups were before the treatment.

Table 4.1 presents the overall PTs' performance on each item of the section A in the pre-test. As can be seen in Table 4.1, each van Hiele Level (VHL) had five items with five multiple choice options. However, some PTs did not choose any of the options for some items. This made the researcher include an additional option (a "blank" option) in the table. For each item, the number in bold corresponds to the right option and also represents the total number of PTs who answered that item correctly. In this section the participants' overall performance on the items in the five subtests are discussed.

**Table 4.1: Van Hiele Geometry Pre-Test: Section A Item Analysis for Each Level per Group**

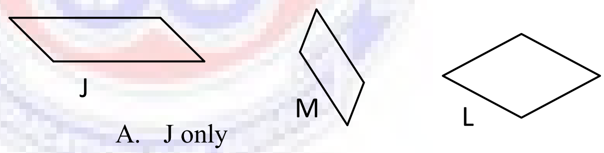
| | | Control Group | | | | | Experimental Group | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Level 1 | Choice items | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| | A | 0 | 1 | 6 | 4 | 7 | 0 | 0 | 7 | 3 | 7 |
| | B | **71** | 0 | 1 | **49** | 0 | **68** | 0 | 1 | **50** | 0 |
| | C | 0 | 12 | **63** | 5 | 35 | 0 | 13 | **62** | 5 | 38 |
| | D | 3 | **58** | 0 | 15 | **29** | 4 | **60** | 0 | 15 | **26** |
| | E | 1 | 1 | 4 | 1 | 3 | 3 | 0 | 4 | 1 | 3 |
| | Blank | 0 | 3 | 1 | 1 | 1 | 0 | 2 | 1 | 1 | 1 |
| Level 2 | Choice items | 6 | 7 | 8 | 9 | 10 | 6 | 7 | 8 | 9 | 10 |
| | A | 10 | 11 | **20** | 3 | 8 | 8 | 12 | **20** | 4 | 10 |
| | B | **21** | 1 | 6 | 2 | 6 | **25** | 1 | 5 | 2 | 4 |
| | C | 32 | 7 | 20 | **54** | 18 | 31 | 8 | 23 | **53** | 18 |
| | D | 7 | 2 | 9 | 4 | **29** | 7 | 2 | 8 | 2 | **29** |
| | E | 1 | **49** | 13 | 11 | 6 | 0 | **47** | 12 | 13 | 4 |
| | Blank | 4 | 5 | 7 | 1 | 8 | 4 | 5 | 7 | 1 | 10 |
| Level 3 | Choice items | 11 | 12 | 13 | 14 | 15 | 11 | 12 | 13 | 14 | 15 |
| | A | 15 | 16 | **8** | **10** | 6 | 17 | 19 | **7** | **14** | 7 |
| | B | 19 | **34** | 0 | 3 | **31** | 16 | **33** | 2 | 0 | **29** |
| | C | **13** | 8 | 0 | 25 | 3 | **15** | 7 | 2 | 26 | 3 |
| | D | 11 | 5 | 8 | 5 | 15 | 9 | 4 | 6 | 4 | 16 |
| | E | 14 | 10 | 58 | 28 | 15 | 14 | 9 | 57 | 28 | 18 |
| | Blank | 3 | 2 | 1 | 4 | 5 | 4 | 3 | 1 | 3 | 2 |
| Level 4 | Choice items | 16 | 17 | 18 | 19 | 20 | 16 | 17 | 18 | 19 | 20 |
| | A | 19 | 34 | 14 | 36 | **19** | 19 | 32 | 16 | 36 | **17** |
| | B | 16 | 7 | 12 | 14 | 10 | 14 | 6 | 14 | 14 | 12 |
| | C | **11** | **12** | 14 | 10 | 12 | **13** | **14** | 14 | 10 | 13 |
| | D | 7 | 13 | **18** | **3** | 19 | 5 | 13 | **18** | **3** | 18 |
| | E | 8 | 3 | 8 | 5 | 7 | 10 | 4 | 6 | 5 | 8 |
| | Blank | 14 | 6 | 9 | 7 | 8 | 14 | 6 | 7 | 7 | 7 |
| Level 5 | Choice items | 21 | 22 | 23 | 24 | 25 | 21 | 22 | 23 | 24 | 25 |
| | A | 20 | 15 | 32 | 6 | 8 | 20 | 19 | 38 | 8 | 8 |
| | B | **8** | 4 | 15 | 9 | 22 | **9** | 9 | 9 | 6 | 20 |
| | C | 12 | 13 | 4 | 6 | 25 | 14 | 11 | 5 | 9 | 25 |
| | D | 9 | 26 | **8** | 38 | **5** | 9 | 19 | **7** | 36 | **7** |
| | E | 13 | **9** | 10 | **8** | 8 | 8 | **7** | 9 | **9** | 6 |
| | Blank | 13 | 8 | 6 | 8 | 7 | 15 | 10 | 7 | 7 | 9 |

*The figures in bold correspond to the right options and also represent the total number of Pre-service Teachers who answered that item correctly.
n control = 75.
n experimental = 75.

### 4.1.2 Performance on Subtest 1: van Hiele Level 1

The PTs performed well only in the first four items of subtest 1. Table 4.1 shows that 71 (94.67%), 58 (77.33%), 63 (84%), 49 (65.33%) of the PTs in the control group managed to correctly answer items 1, 2, 3 and 4 respectively, compared to item 5, 29 (38.67%). Similarly, 68 (90.67%), 60 (80%), 62 (82.67%), 50 (66.67%) of the PTs in the experimental group correctly answered items 1, 2, 3 and 4 respectively, compared to item 5, 26 (34.67%) which was not very encouraging. Figure 4.1 is an item from Subtest 1. The correct answer for this item is choice D. Table 4.1 shows that only 29 (38.67%) and 26 (34.67%) of PTs in the control and experimental groups respectively, had this item correct, that is, knew that all the given quadrilaterals can be referred to as parallelograms. This clearly indicates that 95 (63.33%) of the PTs who participated in this research study lack knowledge of "class inclusion".

Item 5: Which of these are parallelograms?



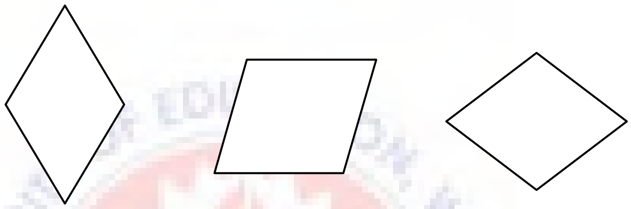

A. J only
B. L only
C. J and M only
D. All are parallelograms
E. None of these are parallelograms

**Figure 4.1: Sample Item in Subtest 1**

### 4.1.3 Performance on Subtest 2: van Hiele Level 2

PTs did quite well on items 7 and 9. Out of the 75 PTs in the control group, 49 (65.33%) and 54 (72%) respectively answered items 7 and 9 correctly, similarly, of a total of 75 PTs in the experimental group, 47 (62.67%) and 53 (70.67%) respectively answered items 7 and 9 correctly. However, PTs performance on items 6, 8 and 10 was not satisfactory. From a total of 75 PTs in the control group only 21 (28%), 20 (26.67%) and 29 (38.67%) of the PTs were able to answer questions on these items respectively. Also, of the 75 PTs in the experimental group only 25 (33.33%), 20 (26.67%) and 29 (38.67%) of the PTs were able to answer questions on these items respectively.

Item 8: A rhombus is a 4- sided figure with all sides of the same length. Here are three examples.



Which of (A) – (D) is not true in every rhombus?

A. The two diagonals have the same length.
B. Each diagonal bisects two angles of the rhombus.
C. The two diagonals are perpendicular.
D. The opposite angles have the same measure.
E. All of (A) – (D) are true in every rhombus.

**Figure 4.2: Sample Item in Subtest 2**

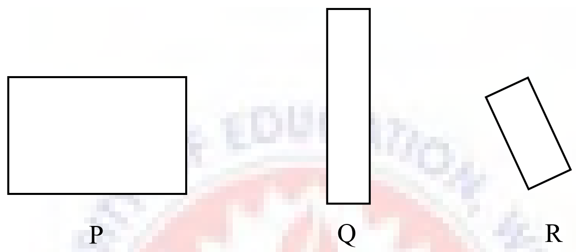
Figure 4.2 is an item from Subtest 2. The correct answer for this item is choice A. Table 4.1 shows that only 20 (26.67%) and 20 (26.67%) of PTs in the control and experimental groups respectively, had this question correct. A total of 73.33% of the PTs in both groups answered this item wrongly. This shows PTs' lack of knowledge about the properties of a rhombus.

### 4.1.4 Performance on Subtest 3: van Hiele Level 3

Subtest 3 is about learners knowing the interrelationship between different types of figures. The performance of the PTs for Subtest 3 was generally not encouraging. Table

4.1 shows that 13 (17.33%), 34 (45.33%), 8 (10.67%), 10 (13.33%) and 31 (41.33%) of the PTs in the control group correctly answered items 11, 12, 13, 14 and 15 respectively. Similarly, 15 (20%), 33 (44%), 7 (9.33%), 14 (18.67%) and 29 (38.67%) of the PTs in the experimental group also answered items 11, 12, 13, 14 and 15 correctly in that order, which was not encouraging. The performance of PTs on item 13 was abysmally poor. This item is presented in Figure 4.3.

Item 13: Which of these can be called rectangles?




A. All can.
B. Q only
C. R only
D. P and Q only
E. Q and R only

**Figure 4.3: Sample Item in Subtest 3**

The correct choice for item 13 in Figure 4.3 is A. However, only 8 (10.67%) and 7 (9.33%) of PTs in the control and experimental groups respectively, had this item correct. This clearly shows that majority (90%) of the PTs did not know that rectangles have common properties with squares, in order words all squares are rectangles. This implies that PTs have difficulties in understanding “class inclusion”.

### 4.1.5 Performance on Subtest 4: van Hiele Level 4

Subtest 4 is about learners being able to give deductive geometric proofs, understanding the role of definitions, theorems, axioms and proofs. Learners at this Level should be able to supply reasons for statements in formal proofs. This is the Level of development that high school students need to be prior to completion of high school. However, the performance of the PTs for Subtest 4 was generally very poor. Table 4.1 shows that 11 (14.67%), 12 (16%), 18 (24%), 3 (4%) and 19 (25.33%) of the PTs in the control group managed to correctly answer items 16, 17, 18, 19 and 20 respectively. Similarly, 13 (17.33%), 14 (18.67%), 18 (24%), 3 (4%) and 17 (22.67%) of the PTs in the experimental also answered items 16, 17, 18, 19 and 20 correctly in that order, which was abysmally poor. This generally indicates that the PTs have difficulties understanding simple deductive geometric proofs, understanding the role of definitions, theorems, axioms and proofs.

#### 4.1.6 Performance on Subtest 5: van Hiele Level 5

Subtest 5 is about learners being able to work in a variety of axiomatic systems that is, being able to study non-Euclidean geometries comparing different systems and also seeing geometry in the abstract. Table 4.1 indicates that 8 (10.67%), 9 (12%), 8 (10.67%), 8 (10.67%), and 5 (6.67%) of the PTs in the control group managed to correctly answer items 20, 21, 22, 23, 24 and 20 respectively. Similarly, 9 (12%), 7 (9.33%), 7 (9.33%), 9 (12%), and 7 (9.33%) of the PTs in the experimental group also correctly answered items 20, 21, 22, 23, 24 and 20 respectively.

Even though some PTs were able to answer some items in subtests 4 and 5 correctly, no PT (in both control and experimental groups) attained Level 4 and Level 5 in the pre-test.

### 4.1.7 Performance on van Hiele Geometry Pre-Test – Section B

Table 4.2 summarizes the overall performance of PTs in the section B part of the van Hiele Geometry pre-test in both the control and experimental groups. There were 3 test items; item 1 was on triangles, properties of parallel lines and transversal, item 2 was on area of two-dimensional shapes while item 3 was a short proof on congruent triangles. Seventy five PTs each from the control and experimental groups wrote the test. The responses of PTs who demonstrated good knowledge and provided the right responses for the items were described as correct. Responses of PTs who attempted items but did not get the total marks allotted per test item were described as partially correct, while the responses that exhibited lack of knowledge about the items were described as completely wrong. However, few PTs did not attempt some of the items at all; these were described as "blank".

**Table 4.2: Van Hiele Geometry Pre-Test: Section B Item Analysis for Both Groups**

| | Control Group | | | | Experimental Group | | | |
|---|---|---|---|---|---|---|---|---|
| | Correct | Partially Correct | Completely Wrong | Blank | Correct | Partially Correct | Completely Wrong | Blank |
| Item | N(%) | N(%) | N(%) | N(%) | N(%) | N(%) | N(%) | N(%) |
| 1 | 23(30.67) | 26(34.67) | 22(29.33) | 4(5.33) | 21(28) | 29(38.67) | 22(29.33) | 3(4) |
| 2 | 6(8) | 49(65.33) | 17(22.67) | 3(4) | 9(12) | 46(61.33) | 18(24) | 2(2.67) |
| 3 | 7(9.33) | 15(20) | 51(68) | 2(2.67) | 6(8) | 18(24) | 48(64) | 3(4) |

*n control = 75
n experimental = 75

The results in Table 4.2 show that the PTs performed well only in the first item. Majority of the PTs (26) representing 34.67% had item 1 partially correct while 23 PTs representing 30.67 had item 1 correct in the control group. In the experimental group, 29 PTs representing 38.67% had item 1 partially correct while 21 PTs representing 28% had item 1 correct. The performance of PTs in item 2 was not encouraging; out of a total of

75 PTs in the control group only 6 PTs representing 8% answered this item correctly. Similarly, in the experimental group, only 9 PTs representing 12% answered this item correctly. Again, PTs performance in item 3 was extremely poor; in the control group, majority (51) PTs representing 68% had this item completely wrong. Similarly, in the experimental group, 48 PTs representing 64% had this item completely wrong. This again revealed PTs difficulties in understanding simple deductive geometric proofs, understanding the role of simple definitions, theorems, axioms and proofs.

### 4.1.8 Levels Reached by PTs in the van Hiele Geometry Pre-Test

In Figure 4.4, the van Hiele Levels of geometric thinking attained by the PTs after the van Hiele Geometry pre-test in both the control and experimental groups are presented in a bar chart.

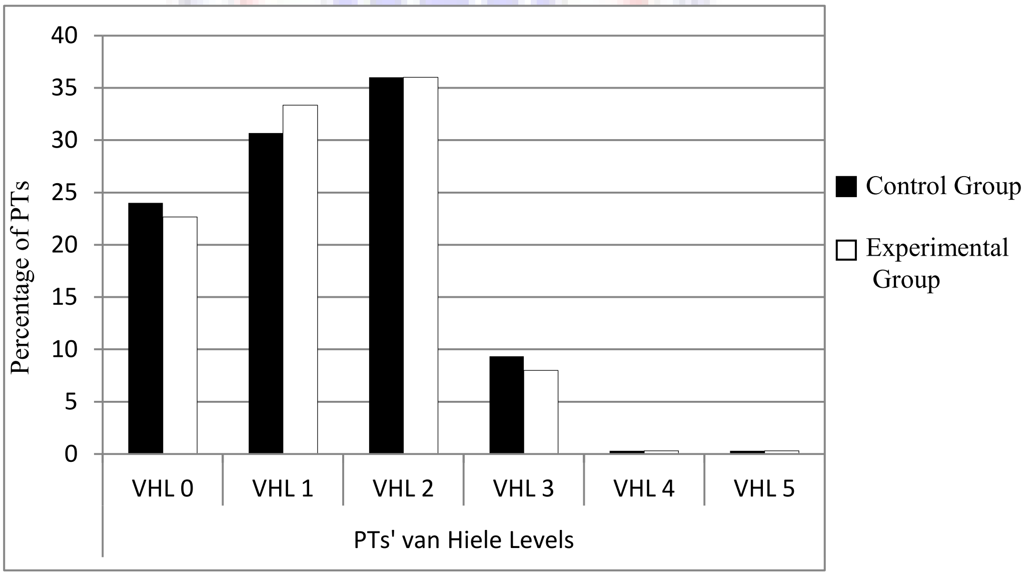



**Figure 4.4: A Bar Chart showing PTs van Hiele Levels of Geometric Thinking in the Pre-test**

As shown in Figure 4.4, 24% and 22.67% of PTs attained VHL 0 (i.e. the Pre-recognition Level or Level for those who have not yet attained any van Hiele Level) in the control and experimental groups respectively. For VHL 1, 30.67% and 33.33% of

PTs in the control and experimental groups attained that Level respectively. 36% of PTs each in the control and experimental groups attained VHL 2. In addition, 9.33% and 8% of PTs attained VHL 3 in the control and experimental groups respectively. However, no PT attained VHL 4 and 5.

#### 4.1.9 General Comparison of Pre-test Scores of Control and Experimental Groups

In general, pre-test results revealed no significant difference between the control and experimental groups. Table 4.3 compares the pre-test results of PTs within the control and experimental groups. The Minimum score PTs obtained in the control group was 20%, while the Maximum score was 60%. In the experimental group, the Minimum score was also 18.18%, while the Maximum score was 58.18%. The mean score of PTs in the control group was 34.25%, while that of the experimental group was 33.31%. Again, the control group recorded a standard deviation of 4.18 while the experimental group recorded a standard deviation of 3.74. This is an indication of how similar the control and experimental group were before the treatment.

**Table 4.3: Means, Standard Deviations, Minimum and Maximum Pre-test Scores for Control and Experimental Groups**

| Group | N | Mean | Stand Dev | Maximum | Minimum |
|---|---|---|---|---|---|
| Control | 75 | 34.25 | 4.18 | 60 | 20 |
| Experimental | 75 | 33.31 | 3.74 | 58.18 | 18.18 |

Independent samples t-test statistic was also conducted on the pre-test scores in both control and experimental groups. The results of the independent samples t-test on the participants' scores in the pre-test are presented in Table 4.4.

**Table 4.4: Independent Samples t-test of Pre-test of Control and Experimental Groups**

| Groups | N | Mean | Std. Dev. | t-value | df | p-value |
|---|---|---|---|---|---|---|
| Control | 75 | 18.84 | 4.18 | 0.803 | 148 | 0.423 |
| Experimental | 75 | 18.32 | 3.74 | | | |

Table 4.4 presents the results of the independent-samples t-test performed on the pre-test scores of the 150 randomly selected PTs of the two independent groups (i.e. control and experimental groups). As can be seen in Table 4.3, comparison of the mean scores would suggest that the control group performed better (mean = 34.25%) than their counterparts in the experimental group (mean = 33.31%). To test whether the difference in mean scores between the control and experimental groups was statistically significant, independent-samples t-test was performed. The results of this test (Table 4.4) revealed that there was no statistically significant difference in mean scores between the control group ($M = 18.84, SD = 4.18$) and experimental group ($M = 18.32, SD = 3.74$) conditions; $t\ (148) = 0.803,\ p = 0.423 > 0.05$. These results suggest that both the control group and the experimental group were almost at the same level of conceptual understanding of geometry before the start of the intervention.

### 4.1.10 Analysis of the VHGT According to the van Hiele Levels

The chi-square test was further used to investigate whether there was a statistically significant difference in the van Hiele Levels of geometric thinking between the control and experimental groups before the start of intervention. Some PTs did not attain any van Hiele Level (VHL), the study utilized the 1 to 5 van Hiele numbering scheme to allow utilization of Level 0 (i.e. the Pre-recognition Level) for these categories of PTs. Since no PT attained Level 4 and Level 5 in the pre-test, only the first three van Hiele Levels were measured. The investigation therefore involved a 2 × 4 design where van

Hiele Level (VHL) was measured on four Levels (i.e. VHL 0, VHL 1, VHL 2 and VHL 3) and Group was measured on two Levels (i.e. Control and Experimental). Table 4.5 presents the 2 × 4 contingency table showing the actual and expected counts as well as within group and within van Hiele Level percentages of the distribution. As can be seen from the table there are only slight differences in the distribution. For example, in the control group 18 PTs were classified as being at VHL 0 but less (i.e. 17.5) were expected, whereas, in the experimental group 17 PTs were classified as being at VHL 0 when more (i.e.17.5) were expected. Also the actual count for PTs in VHL 1 in the control group was 23 with an expected count of 24.0, whilst in the experimental group, there were 25 PTs in VHL 1 with an expected count of 24.0. The actual count for PTs in VHL 2 in the control group was 27 with an expected count of 27.0, similarly, in the experimental group, there were 27 PTs in VHL 2 with an expected count of 27.0. For the VHL 3 category, 7 were from the control group although 6.5 were expected, similarly, another 7 came from the experimental group when 6.5 were expected.

**Table 4.5: Contingency table (Cross tabulation) of Group and van Hiele Level (for Pre-test)**

| | | | Group | | Total |
|---|---|---|---|---|---|
| | | | Control | Experimental | |
| Van Hiele Level | VHL0 | Count | 18 | 17 | 35 |
| | | Expected Count | 17.5 | 17.5 | 35.0 |
| | | % within van Hiele Level | 51.4% | 48.6% | 100.0% |
| | | % within Group | 24.0% | 22.7% | 23.3% |
| | | % of Total | 12.0% | 11.3% | 23.3% |
| | VHL1 | Count | 23 | 25 | 48 |
| | | Expected Count | 24.0 | 24.0 | 48.0 |
| | | % within van Hiele Level | 47.9% | 52.1% | 100.0% |
| | | % within Group | 30.7% | 33.3% | 32.0% |
| | | % of Total | 15.3% | 16.7% | 32.0% |
| | VHL2 | Count | 27 | 27 | 54 |
| | | Expected Count | 27.0 | 27.0 | 54.0 |
| | | % within van Hiele Level | 50.0% | 50.0% | 100.0% |
| | | % within Group | 36.0% | 36.0% | 36.0% |
| | | % of Total | 18.0% | 18.0% | 36.0% |
| | VHL3 | Count | 7 | 6 | 13 |
| | | Expected Count | 6.5 | 6.5 | 13.0 |
| | | % within van Hiele Level | 53.8% | 46.2% | 100.0% |
| | | % within Group | 9.3% | 8.0% | 8.7% |
| | | % of Total | 4.7% | 4.0% | 8.7% |
| | Total | Count | 75 | 75 | 150 |
| | | Expected Count | 75.0 | 75.0 | 150.0 |
| | | % within van Hiele Level | 50.0% | 50.0% | 100.0% |
| | | % within Group | 100.0% | 100.0% | 100.0% |
| | | % of Total | 50.0% | 50.0% | 100.0% |

* Van Hiele Level and Group Cross tabulation

The chi-square test was performed to ascertain whether these differences are enough to be significant. Table 4.6 presents the results of the chi-square test for PTs pre-test scores and as can be seen from this table the test reveals that there is no statistically significant difference in the van Hiele Levels of geometric thinking between the control and experimental groups at the pre-test ($\chi^2 = 0.189; df = 3; p > 0.05$).

**Table 4.6: Results from Chi-square Tests (for Pre-test)**

| Chi-Square Tests | | | |
|---|---|---|---|
| | Value | df | Asymp. Sig. (2-sided) |
| Pearson Chi-Square | .189[a] | 3 | .979 |
| Likelihood Ratio | .189 | 3 | .979 |
| N of Valid Cases | 150 | | |

* 0 cells (.0%) have expected count less than 5. The minimum expected count is 6.50.

### 4.1.11 The van Hiele Geometry Post-test Results from the Control and Experimental Groups

After the VHPI was applied to PTs in the experimental group and the traditional mode of instruction (tutor-centered approach) to PTs in the control group, the van Hiele Geometry Test was again administered to all PTs in both groups. This was to determine the effect of the use of VHPI on PTs' geometric thinking Levels.

Table 4.7 presents the overall PTs' performance on each item of the section A in the van Hiele Geometry post-test. As can be seen in Table 4.7 each van Hiele Level (VHL) had five items with five multiple choice options. However, few PTs did not choose any of the options for some items. This made the researcher include an additional option (a blank option) in the table. For each item, the number in bold corresponds to the right option and also represents the total number of PTs who answered that item correctly. In this section the participants' overall performance on the items in the five subtests are discussed.

**Table 4.7: Van Hiele Geometry Post-Test: Section A Item Analysis for Each Level per Group**

| | | Control Group | | | | | Experimental Group | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Level 1 | Choice items | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| | A | 0 | 0 | 8 | 4 | 8 | 0 | 0 | 2 | 1 | 3 |
| | B | **72** | 0 | 1 | **50** | 0 | **75** | 0 | 0 | **68** | 0 |
| | C | 0 | 14 | **61** | 7 | 32 | 0 | 2 | **72** | 1 | 10 |
| | D | 2 | **58** | 0 | 11 | **29** | 0 | **73** | 0 | 5 | **60** |
| | E | 1 | 2 | 5 | 1 | 6 | 0 | 0 | 1 | 0 | 2 |
| | Blank | 0 | 1 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| Level 2 | Choice items | 6 | 7 | 8 | 9 | 10 | 6 | 7 | 8 | 9 | 10 |
| | A | 9 | 9 | **19** | 5 | 6 | 2 | 4 | **62** | 04 | 6 |
| | B | **23** | 1 | 5 | 0 | 11 | **61** | 0 | 0 | 0 | 3 |
| | C | 32 | 9 | 25 | **53** | 15 | 9 | 3 | 9 | **73** | 10 |
| | D | 7 | 3 | 9 | 4 | **31** | 3 | 0 | 1 | 0 | **48** |
| | E | 1 | **49** | 11 | 12 | 6 | 0 | **68** | 3 | 0 | 4 |
| | Blank | 3 | 4 | 6 | 1 | 6 | 0 | 0 | 0 | 0 | 4 |
| Level 3 | Choice items | 11 | 12 | 13 | 14 | 15 | 11 | 12 | 13 | 14 | 15 |
| | A | 18 | 18 | **11** | **10** | 4 | 13 | 13 | **56** | **48** | 1 |
| | B | 21 | **33** | 0 | 2 | **31** | 11 | **42** | 0 | 0 | **58** |
| | C | **13** | 8 | 1 | 23 | 4 | **32** | 6 | 0 | 10 | 1 |
| | D | 9 | 5 | 6 | 7 | 17 | 7 | 3 | 3 | 2 | 6 |
| | E | 12 | 9 | 56 | 30 | 14 | 10 | 8 | 16 | 14 | 9 |
| | Blank | 2 | 2 | 1 | 3 | 5 | 2 | 3 | 0 | 1 | 0 |
| Level 4 | Choice items | 16 | 17 | 18 | 19 | 20 | 16 | 17 | 18 | 19 | 20 |
| | A | 17 | 36 | 15 | 34 | **19** | 16 | 29 | 9 | 32 | **41** |
| | B | 15 | 6 | 11 | 16 | 12 | 9 | 6 | 10 | 12 | 6 |
| | C | **13** | **12** | 16 | 10 | 9 | **22** | **21** | 11 | 10 | 8 |
| | D | 13 | 12 | **17** | **3** | 20 | 11 | 9 | **33** | **11** | 11 |
| | E | 9 | 2 | 10 | 7 | 9 | 9 | 4 | 10 | 5 | 6 |
| | Blank | 8 | 7 | 6 | 5 | 6 | 8 | 6 | 2 | 5 | 3 |
| Level 5 | Choice items | 21 | 22 | 23 | 24 | 25 | 21 | 22 | 23 | 24 | 25 |
| | A | 17 | 16 | 29 | 4 | 8 | 21 | 21 | 41 | 8 | 9 |
| | B | **9** | 6 | 13 | 8 | 22 | **8** | 9 | 7 | 6 | 21 |
| | C | 13 | 11 | 7 | 4 | 24 | 14 | 10 | 4 | 10 | 27 |
| | D | 9 | 28 | **8** | 42 | **5** | 9 | 18 | **9** | 34 | **7** |
| | E | 18 | **7** | 12 | **9** | 9 | 8 | **7** | 7 | **11** | 6 |
| | Blank | 9 | 7 | 6 | 8 | 7 | 15 | 10 | 7 | 6 | 5 |

* The figures in bold correspond to the right option and also represent the total number of Pre-service Teachers who answered that item correctly.
n control = 75
n experimental = 75

### 4.1.12 Performance on Subtest 1: van Hiele Level 1

PTs in the experimental group performed extremely well in all the items of subtest 1 as compared to their counterparts in the control group. Table 4.7 shows that 72 (96%), 58 (77.33%), 61 (81.33%), 50 (66.67%) and 29 (38.67%) of the PTs in the control group managed to correctly answer items 1, 2, 3, 4 and 5 respectively. These results were not so different from those recorded in the pre-test. However, 75 (100%), 73 (97.33%), 72 (96%), 68 (90.67%) and 60 (80%) of the PTs in the experimental group correctly answered items 1, 2, 3, 4 and 5 respectively. This clearly indicates that the problem of lack of knowledge of "class inclusion" had been alleviated in the experimental group. Compared to the pre-test, the results obtained by PTs in the experimental group for subtest 1 were far better.

### 4.1.13 Performance on Subtest 2: van Hiele Level 2

Again, the performance of PTs in the control group in subtest 2 of the post test was not so different from that obtained in the pre-test. Out of the 75 PTs in the control group, 23 (30.67%), 49 (65.33%), 19 (25.33%), 53 (70.67%) and 31 (41.33%) of the PTs in the control group managed to correctly answer items 6, 7, 8, 9 and 10 respectively. However, in the experimental group there was an extremely better performance. Out of a total of 75 PTs in the experimental group, 61 (81.33%), 68 (90.67%), 62 (82.67%), 73 (97.33%) and 48 (64%) of them correctly answered items 1, 2, 3, 4 and 5 respectively. PTs in the experimental group have gained good knowledge of the properties of a rhombus.

### 4.1.14 Performance on Subtest 3: van Hiele Level 3

The performance of PTs in the control group for Subtest 3 was still not encouraging. Table 4.7 shows that 13 (17.33%), 33 (44%), 11 (14.67%), 10 (13.33%) and 31 (41.33%) of the PTs in the control group correctly answered items 11, 12, 13, 14 and 15 respectively. However, 32 (42.67%), 42 (56%), 56 (74.67%), 48 (64%) and 58 (77.33%) of the PTs in the experimental group correctly answered items 11, 12, 13, 14 and 15 respectively, which was very encouraging.

### 4.1.15 Performance on Subtest 4: van Hiele Level 4

Again, the performance of PTs in the control group in subtest 4 of the post test was not so different from that obtained in the pre-test. Out of the 75 PTs in the control group, 13 (17.33%), 12 (16%), 17 (22.67%), 3 (4%) and 19 (25.33%) of the PTs in the control group managed to correctly answer items 16, 17, 18, 19 and 20 respectively. In the experimental group, 22 (29.33%), 21 (28%), 33 (44%), 11 (14.67%) and 41 (54.67%) of the PTs answered items 16, 17, 18, 19 and 20 respectively, which was satisfactory, compared to the pre-test results for subtest 4. This indicates that there has been an improvement in the understanding at van Hiele Level 4 among PTs in the experimental group.

### 4.1.16 Performance on Subtest 5: van Hiele Level 5

Table 4.7 indicates that 9 (12%), 7 (9.33%), 8 (10.67%), 9 (12%) and 5 (6.67%) of the PTs in the control group managed to correctly answer items 20, 21, 22, 23, 24 and 25 respectively. Similarly, 8 (10.67%), 7 (9.33%), 9 (12%), 11 (14.67%) and 7 (9.33%) of the PTs in the experimental also correctly answered items 20, 21, 22, 23, 24 and 25 respectively. However, no PT in either group attained van Hiele Level 5 in the post test.

### 4.1.17 Performance on van Hiele Geometry Post-Test – Section B

Table 4.8 summarizes the overall performance of PTs in the section B part of the van Hiele Geometry post-test in both the control and experimental groups.

**Table 4.8: Van Hiele Geometry Post-Test: Section B Item Analysis for Both Groups**

| | Control Group | | | | Experimental Group | | | |
|---|---|---|---|---|---|---|---|---|
| | Correct | Partially Correct | Completely Wrong | Blank | Correct | Partially Correct | Completely Wrong | Blank |
| Item | N(%) | N(%) | N(%) | N(%) | N(%) | N(%) | N(%) | N(%) |
| 1 | 25(33.33) | 25(33.33) | 22(29.33) | 3(4) | 60(80) | 11(14.67) | 4(5.33) | 0(0) |
| 2 | 9(12) | 47(62.67) | 17(22.67) | 2(2.67) | 31(41.33) | 40(53.33) | 4(5.33) | 0(0) |
| 3 | 7(9.33) | 14(18.67) | 50(66.67) | 4(5.33) | 58(77.33) | 12(16) | 3(4) | 2(2.67) |

* n control = 75
n experimental = 75

Results in Table 4.8 indicate that majority of the PTs (60 representing 80%) in the experimental group had item 1 correct as compared to 25 PTs representing 33.33% in the control group. Again, 31 PTs representing 41.33% in the experimental group had item 2 correct as compared to 9 PTs representing 12% in the control group. The results on item 3 indicated that majority (58 representing 77.33%) of PTs in the experimental group had item 3 correct as compared to 7 PTs representing 9.33% in the control group. Results from the section B of the van Hiele Geometry post-test (Table 4.8) revealed that the PTs in the experimental group demonstrated a better understanding of the geometric concepts covered in the test.

### 4.1.18 Levels Reached by PTs in the van Hiele Geometry Post-Test

The bar chart in Figure 4.5 provides a visual confirmation of the van Hiele Levels of geometric thinking attained by the PTs after the van Hiele Geometry post-test in both the control and experimental groups.

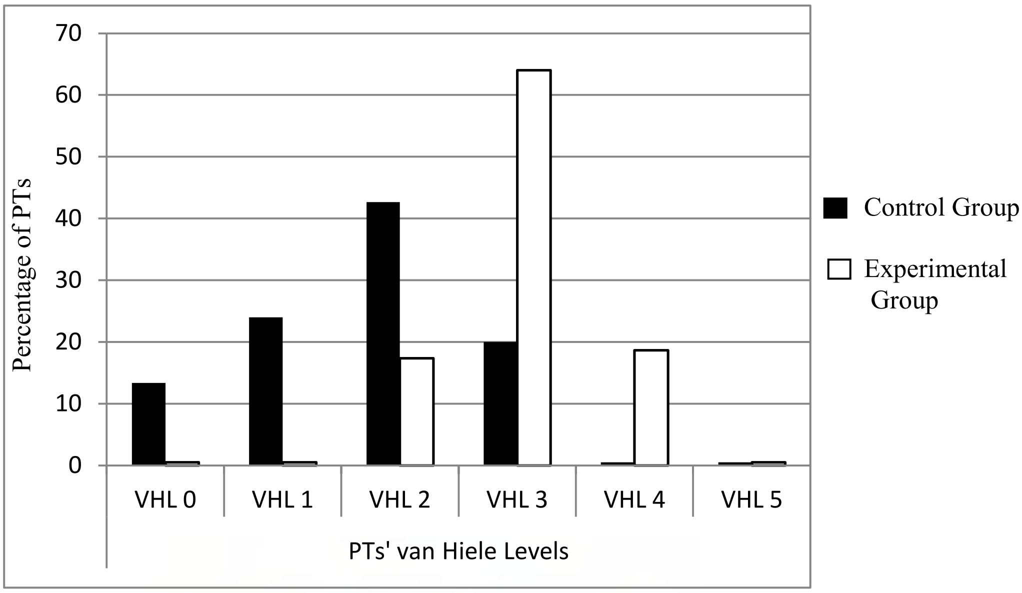



**Figure 4.5: A Bar Chart showing PTs van Hiele Levels of Geometric Thinking in the Post-test**

The post test indicated a huge improvement in the van Hiele Levels of geometric thinking among PTs in the experimental group. In other words, the PTs in the experimental group attained higher van Hiele Levels of geometric thinking as compared to their counterparts in the control group. As shown in Figure 4.5, 13.33% of PTs attained VHL 0 (i.e. the Pre-recognition Level or Level for those who have not yet attained any van Hiele Level) in the control group. However, no PT attained VHL 0 in the experimental group. 24% and 0% of the PTs attained VHL 1 in the control and experimental groups respectively. In addition, 42.67% and 17.33% of the PTs attained VHL 2 in the control and experimental groups respectively. 20% of PTs in the control group attained VHL 3 as compared to 64% in the experimental group. Significantly, 18.67% of PTs in the experimental group attained VHL 4 as compared to 0% in the control group. This indicates a significant improvement in the geometric thinking Levels among PTs in the experimental group. However, no PT attained VHL 5.

### 4.1.19 General Comparison of Pre-test and Post-test Scores of Control and Experimental Groups

Table 4.9 compares the pre-test and post-test results of the PTs within the experimental group. In the experimental group the results showed an improvement in PTs' understanding of geometry in the post-test. The minimum score PTs obtained in the pre-test was 18.18%, while the maximum score was 58.18%. However, in the post-test, the minimum score was 41.82%, while the maximum score was 78.18%. The mean score of PTs in the pre-test was 33.31%, while that of the post-test was 55.71%, an increase of 22.40%. This is an indication that in the post-test, every PT's performance had increased in the experimental group. These improvements might be due to the use of the van Hiele Phase-based Instruction (VHPI).

**Table 4.9: Means, Standard Deviations, Minimum and Maximum Pre-test and Post-test scores of Experimental Group**

| Test | N | Mean | Stand Dev. | Maximum | Minimum |
|---|---|---|---|---|---|
| Pre-test | 75 | 33.31 | 3.71 | 58.18 | 18.18 |
| Post-test | 75 | 55.71 | 4.56 | 78.18 | 41.82 |

Within the control group the results was not so different from those recorded in the pre-test. Table 4.10 compares the pre-test and post-test results of PTs within the control group. The minimum score PTs obtained in the pre-test was 20%, while the maximum score was 60%. In the post-test, the minimum score was 25%, while the maximum score was 67.63%. PTs mean score in the pre-test was 34.25%, while that of the post-test was 36.28%.

**Table 4.10: Means, Standard Deviations, Minimum and Maximum Pre-test and Post-test scores of Control Group**

| Group | N | Mean | Stand Dev | Maximum | Minimum |
|---|---|---|---|---|---|
| Pre-test | 75 | 34.25 | 4.18 | 60 | 20 |
| Post-test | 75 | 36.28 | 4.30 | 67.63 | 25 |

Paired samples t-test statistic was conducted on the pre- and post-test scores in both control and experimental groups. The results of the paired samples t-test (Table 4.11) of PTs in the control group indicated that there was no significant difference in their mean scores for the pre-test ($M = 18.84, SD = 4.182$) and post-test ($M = 20.25, SD = 4.305$) conditions; $t (74) = -0.445, p = 0.658 > 0.05$. However, in the experimental group, there was a significant difference in their mean scores for the pre-test ($M = 18.32, SD = 3.739$) and post-test ($M = 30.79, SD = 4.294$) conditions; $t (74) = -30.776, p = 0.000 < 0.05$.

**Table 4.11: Paired Sample t-tests of Pre-and Post-tests of the Two Groups**

| | N | Mean | Std. Dev. | Mean Difference | t-value | df | p-value |
|---|---|---|---|---|---|---|---|
| Pre-test – Post-test (Control Group) | 75 | 18.84 | 4.182 | 0.103 | -0.445 | 74 | 0.658 |
| | | 20.25 | 4.305 | | | | |
| Pre-test – Post-test (Experimental Group) | 75 | 18.32 | 3.739 | 12.467 | -30.776 | 74 | 0.000 |
| | | 30.79 | 4.294 | | | | |

### 4.1.20 Research Question 1: To what extent does the use of VHPI improve PTs geometric thinking Levels?

Research question one essentially focused on the effectiveness of the use of the van Hiele Phase-based Instruction (VHPI) in improving PTs geometric thinking in terms of the van Hiele Levels. In answering this research question, the hypothesis below was formulated for the study;

$H_0$: There is no significant difference in the geometric thinking Levels between the control and experimental groups in the post-test.

The chi-square test was again used to investigate whether there was a significant difference in the van Hiele Levels of geometric thinking between the control and experimental groups after the post-test. The investigation involved a 2 × 2 design where van Hiele Level (VHL) was measured on two Levels (i.e. VHL0,1&2 and VHL3&4) and Group was measured on two Levels (i.e. Control and Experimental). Table 4.12 presents the 2 × 2 contingency table showing the actual and expected counts as well as within group and within van Hiele Level percentages of the distribution. As can be seen from the table there are differences in the distribution. For example, in the control group 60 PTs were classified as being at VHL0,1&2 when a lot less (i.e. 36.5) were expected, whereas, in the experimental group 13 PTs were classified as being at VHL0,1&2 when more (i.e. 36.5) were expected. For the VHL3&4 category, 15 were from the control group but a lot more (i.e. 38.5) were expected, while, 62 came from the experimental group when 38.5 were expected.

**Table 4.12: Contingency Table (Cross Tabulation) of Group and van Hiele Level (for Post-test)**

| | | | Group | | Total |
|---|---|---|---|---|---|
| | | | Control | Experimental | |
| Van Hiele Level | VHL0,1&2 | Count | 60 | 13 | 73 |
| | | Expected Count | 36.5 | 36.5 | 73.0 |
| | | % within van Hiele Level | 82.2% | 17.8% | 100.0% |
| | | % within Group | 80.0% | 17.3% | 48.7% |
| | | % of Total | 40.0% | 8.7% | 48.7% |
| | VHL3&4 | Count | 15 | 62 | 77 |
| | | Expected Count | 38.5 | 38.5 | 77.0 |
| | | % within van Hiele Level | 19.5% | 80.5% | 100.0% |
| | | % within Group | 20.0% | 82.7% | 51.3% |
| | | % of Total | 10.0% | 41.3% | 51.3% |
| | Total | Count | 75 | 75 | 150 |
| | | Expected Count | 75.0 | 75.0 | 150.0 |
| | | % within van Hiele Level | 50.0% | 50.0% | 100.0% |
| | | % within Group | 100.0% | 100.0% | 100.0% |
| | | % of Total | 50.0% | 50.0% | 100.0% |

*Van Hiele Level and Group Cross tabulation

The chi-square test was performed to ascertain whether these differences are enough to be significant. Table 4.12 reveals that only 19.5% of PTs in the control group attained VHL3&4, whereas 80.5% of PTs in the experimental group attained VHL3&4. Table 4.13 presents the results of the chi-square test for PTs post-test scores and as can be seen from this table the difference highlighted above was statistically significant ($\chi^2 = 58.949$; $df = 1$; $p < 0.05$). Hence the null hypothesis that there is no significant difference in the geometric thinking Levels between the control and experimental groups in the post test was rejected.

**Table 4.13: Results from Chi-square Tests (Post-test)**

| | Value | df | Asymp. Sig. (2-sided) | Exact Sig. (2-sided) | Exact Sig. (1-sided) |
|---|---|---|---|---|---|
| Pearson Chi-Square | 58.949[a] | 1 | .000 | .000 | .000 |
| Continuity Correction[b] | 56.467 | 1 | .000 | | |
| Likelihood Ratio | 63.607 | 1 | .000 | .000 | .000 |
| Fisher's Exact Test | | | | .000 | .000 |
| N of Valid Cases | 150 | | | | |

*0 cells (0.0%) have expected count less than 5. The minimum expected count is 36.50.

Computed only for a 2x2 table

## 4.2 Research Question 2: How does the use of VHPI motivate PTs to learn geometry?

In order to answer this question, the researcher used unstructured interview for some PTs in the experimental group. After the intervention, nine PTs were randomly selected to participate in an interview. Interview responses revealed that the PTs found the lessons interesting and easy to understand. The nine PTs suggested that the hands-on investigations as integrated in the VHPI should be employed in most lessons because it took away dullness and also made learning easier. The nine PTs also said they liked the manipulative and concrete nature of the teaching and learning materials.

Furthermore, the PTs exhibited high levels of eagerness and attention due to the systematic nature of the VHPI coupled with the manipulative and concrete nature of the teaching and learning materials. One of the PTs even remarked that "*I find this Geometry lesson interesting and free from fear and confusion because I am free to ask questions from group members and the teacher. Also, during the group activities my friends explain some things I don't understand to me*". PTs found the freedom in expressing their ideas without being bound by any rules and definite answers. This is in line with the study by Siew and Chong (2014) who found that useful concrete manipulative materials integrated with the VHPI in learning geometry is able to help develop better interest and creativity.

Moreover, results from the post-test and also further interrogation showed that the PTs understood very well, the topics that were taught by the researcher using the VHPI and that learning by memorization was reduced drastically. For instance, when one of the PTs was asked whether the use of the VHPI made understanding of concepts better and easier, she said "*In fact, for the first time, I understand these concepts better and learning has been made easier because now I don't need to do "chew and pour" as I used to*". PTs gained an interest and motivation in making new shapes besides having broadened their understanding about geometric shapes.

Responses from these PTs reveal that the VHPI motivates them to learn geometry by eliminating dullness and making learning easier and fascinating. These findings confirm the literature that the VHPI coupled with well planned hands-on investigations promotes student-centered learning, reduces memorization and also motivates students by providing a better learning environment (Halat, Jakubowski and Aydin, 2008; Smiešková & Barcíková, 2014).

Again, the responses resonates strongly with what Fisch, Lesh, Motoki, Crespo and Melfi (2010) cited in Colgan (2014) said that experiential learning connect deeply with learner's passions and interests, making learning profoundly personal. By adopting engaging tools in the classroom, teachers may be able to transform feelings about learning and Mathematics by changing the focus from teaching facts and skills to building positive relationships between learners and Mathematics. The way that learners feel about Mathematics profoundly influences what they do with it and how they reflect on it, which in turn influences how knowledge grows and connects.

In this study, it is revealed that if PTs are given the requisite training in geometry through the VHPI, they would be motivated to learn geometry and would be able to teach the subject diligently at the basic level later when they have completed CE.

### 4.3 Research Question 3: To what extent do CE Mathematics tutors facilitate geometry teaching and learning consistent with the van Hiele Levels of geometric thinking?

Classroom observations were carried out in order to investigate how the participating CE Mathematics tutors facilitated geometry teaching and learning consistent with the van Hiele Levels. Each tutor was observed once in a CE first year class. The classroom observation protocol (see Appendix D) that was designed incorporated elements of the van Hiele Levels. The researcher was particularly interested in observing the structure of lessons, introductions, presentations, evaluations and classroom management of lessons with particular reference to the criteria that surround the van Hiele Levels.

The themes discussed in this section emerged from the classroom observations. They also represent some of the characteristic teaching elements of the van Hiele Levels. The researcher reports on the observations according to the following emergent themes:

- Observations regarding displaying of shapes
- Observations regarding the use of language to describe shapes
- Observations regarding providing hands-on activities for PTs
- Observations regarding guiding PTs to establish the properties of a shape
- Observations regarding guiding PTs to analyze properties of geometric shapes and interrelationship between different types of shapes
- Observations regarding the development and usage of accurate terminology
- Observations regarding guiding PTs to create, compare and contrast different proofs
- Observations regarding guiding PTs to work in different geometric or axiomatic systems

### 4.3.1 Observations Regarding Displaying of Shapes

Among the five CE Mathematics tutors observed, only one of them had teaching aids such as readymade geometric shapes. This was a female Mathematics tutor (tutor A). These shapes were triangles and parallelograms (squares, rectangles and rhombuses). It was also noted that apart from tutor A, none of the tutors provided examples of the properties of shapes. During the lessons, most of the Tutors emphasized that the PTs had been taught such shapes as triangles and parallelograms at their previous Levels of education (JHS and SHS) thus, they were already familiar with them and therefore there was no need to display such shapes to them again. This approach, from my observation did not help the PTs to visualize the shapes even though they were familiar with them.

Although PTs had been taught such shapes during their Mathematics lessons at the JHS and SHS Levels, it was observed that during tutor A's lessons PTs were stimulated and motivated through a visualization approach; PTs were able to visualize shapes to develop geometric thinking. In addition, tutor A also provided some routine problems on the shapes and asked her PTs to solve using the area and perimeter formula rather than referring to the properties in general. This strategy is consistent with van Hiele Level 1, as PTs know the perimeter and area formulae from their previous Levels of education (JHS and SHS).

In summary, most of the tutors did not display a variety of different ready-made geometric shapes to their classes as should be done in line with the van Hiele Level 1.

### 4.3.2 Observations Regarding the use of Language to Describe Shapes

In line with the van Hiele Level 1, observations were made to determine whether the tutors used informal as well as formal language to describe shapes, and whether the tutors encouraged PTs to describe in their own words the properties of a typical shape. It was observed that the tutors frequently used both informal and formal language to describe shapes. The following are some examples:

Tutor A

> The tutor asked PTs to describe a scalene triangle in their own words. One of the PTs answered "in a scalene triangle all the parts have different measure".

Tutor E

> The tutor asked PTs to describe a rectangle. One PT answered "a rectangle is a figure with opposite sides equal and also parallel"

This shows that tutors encourage learners to describe concepts in their own words using informal language which is regarded as Level 1 geometric thought within the van Hiele theory (van Hiele, 1986; Yazdani, 2007).

### 4.3.3 Observations Regarding Providing Hands-on Activities for PTs

In line with van Hiele Level 2, observations were carried out to determine whether tutors provided hands-on activities to PTs requiring them to focus on the properties of shapes and to use vocabulary appropriately. Observations showed that none of the tutors provided hands-on activities to their PTs. The tutors taught concepts of shapes theoretically using the chalk or marker boards and teaching was greatly dominated by the tutors as confirmed in the literature (Fredua-Kwarteng & Ahia, 2004; Yegambaram & Naidoo, 2009; De Villiers 2012; Khalid & Azeem, 2012). Once again, from the classroom observations, the assumptions of the tutors was that the PTs had been taught concepts of shapes at their previous Levels of education (JHS and SHS) thus, there was no need for any hands-on activities.

### 4.3.4 Observations Regarding Guiding PTs to Establish the Properties of a Shape

Teaching learners how to empirically establish the properties of a typical shape is regarded as Level 2 geometric thought within the van Hiele theory. It is easier and logical to establish properties of a typical shape by first undertaking hands-on activities on those shapes (van Hiele, 1986; Vojkuvkova, 2012). Since tutors did not provide any hands-on activities on geometrical shapes to PTs which required them to focus on the properties of the shapes, it was observed that tutors could not logically guide PTs to empirically establish the properties of a typical shape.

### 4.3.5 Observations Regarding Guiding PTs to Analyze Properties of Geometric Shapes and Interrelationship between Different Types of Shapes

Van Hiele (1999) posited that at van Hiele Level 3:

> Students use properties that they already know to formulate definitions, for example, for squares, rectangles, and equilateral triangles, and use them to justify relationships, such as explaining why all squares are rectangles or why the sum of the angle measures of the angles of any triangle must be 180 (p. 311).

In order to effectively help learners analyze properties of geometric shapes and see the interrelationship between different types of shapes, appropriate activities should be designed for them (Crowley, 1987; Mason, 1998; van Hiele, 1999). It was expected that tutors provide hands-on activities to PTs however, they did not. Therefore, guiding PTs to analyze properties of geometric shapes and interrelationship between different types of shapes was difficult for the tutors and most of them did not achieve this.

### 4.3.6 Observations Regarding the Development and Usage of Accurate Terminology

Observations were made to determine whether tutors ensured that the accurate and appropriate geometric terminology is developed and used. Classroom observations revealed that most of the tutors ensured the development and usage of accurate geometric terminology. Below are examples by some tutors:

Tutor A

> The tutor drew an isosceles triangle on the board and asked PTs to name and describe it. One PT answered "it's a triangle with two equal parts and two equal angles". The tutor asked the PT to use the appropriate terminology and describe

the triangle well. Another PT answered "it's an isosceles triangle; it has two opposite sides and angles equal". The tutor then emphasized that "it's an isosceles triangle and the equal angles are called base angles".

Tutor C

The tutor asked PTs to define perpendicular lines. One PT answered "they are lines that form an angle of 90°". The tutor then emphasized that "yes, they are lines that meet at 90° or right angles".

These techniques are consistent with the van Hiele theory and operate at van Hiele Levels 2 and 3.

### 4.3.7 Observations Regarding Guiding PTs to Create, Compare and Contrast Different Proofs

At van Hiele Level 4 learners should be able to construct proofs, understand the role of axioms and definitions, and also know the meaning of necessary and sufficient conditions (Yazdani, 2007). Mathematics course outline for CE Science PTs indicate that they are to be taught "congruent and similar triangles" where they are to use criteria or postulates to prove whether triangles are congruent or not. Also, under the topic "circle theorems", all PTs (both Science and Non-science) are to do some simple proofs. However, it was observed that two of the tutors involved in this study who taught Science PTs only discussed the various types of triangles and their properties on the topic "congruent and similar triangles". No deductive geometric proofs were carried out to facilitate understanding of van Hiele Level 4.

### 4.3.8 Observations Regarding Guiding PTs to Work in Different Geometric or Axiomatic systems

According to Crowley (1987), at van Hiele Level 5 "the learner can work in a variety of axiomatic systems, that is, non-Euclidean geometries can be studied, and different systems can be compared. Geometry is seen in the abstract" (p. 3). Observations made revealed that the selected tutors did not present instructions to facilitate learning at this level.

The van Hiele Level 5 (Rigor) is the least developed in the original works of the van Hieles and has received little attention from researchers. Majority of high school geometry courses are taught at Level 3, as a result, most research has also concentrated on the lower Levels.

## 4.4 Discussion of Major Findings

In this research the purpose was to investigate the effect of the VHPI on PTs' geometric thinking in terms of the van Hiele Levels and how the VHPI motivates PTs to learn geometry. The research also aimed at investigating how CE Mathematics tutors facilitate geometry teaching and learning consistent with the van Hiele Levels of geometric thinking. The findings indicated that the VHPI as a mode of instruction provided PTs with new learning experiences in geometry lessons. These learning experiences included: allowing PTs to work in groups and interact with each other. This offered the PTs enhanced opportunities of varied forms of mathematical communications which are absent in other forms of teaching approaches such as the tutor-centered approach.

The findings from the chi-square analysis indicated that there was no significant difference between PTs' geometric thinking Levels in the control and experimental groups at the pre-test (i.e. before the start of the intervention) at $p > 0.05$. This showed that both the control and the experimental groups were at the same level of geometric thinking before the intervention. However there was a significant difference between

PTs' geometric thinking Levels in the control and experimental groups at the post-test (i.e. after the intervention) at $p < 0.05$ favouring PTs in the experimental group; 19.5% of PTs in the control group attained VHL3&4, as compared to 80.5% of PTs in the experimental group who attained VHL3&4. This finding agrees with the findings of Yazdani (2007) and Erdogan and Durmus (2009) who reported that instruction consistent with VHPI has a positive effect on the geometric thinking Levels of learners.

Again the analysis of the pre-test scores from the independent-samples t-test showed that there was no significant difference between PTs conceptual understanding of plane geometry in experimental and control groups. However, analysis of results from the pre-test and post-test on the performance of PTs in both experimental and controls groups in the paired samples t-test statistic indicated that, that of the experimental group was significant at $p < 0.05$. Consequently, further analysis of both group means differences showed a higher mean difference of 12.467 in the experimental group as against 0.103 in the control group in the post-test. The difference in means between both groups comparatively was a pointer to the result that the experimental group performed better than the control group in the post-test. This could be attributed to the use of the VHPI in the teaching of PTs in the experimental group. These results are also consistent with that of Siew and Chong (2014) who used VHPI to determine its effects on learners' creativity and geometric understanding using tangrams activities and reported that the VHPI is able to foster learners' creativity and geometric understanding.

The results also showed that majority of the PTs were motivated to learn geometry through the use of the VHPI. They added that the VHPI took away dullness thus, making learning easier and fascinating. The findings further revealed that majority of the PTs were of the view that in the three geometry lessons, the researcher made them very active participants, provided opportunities for asking questions and made them learn from one another in a series of guided group activities. Thus, the PTs formed a central position in the lesson delivery. Furthermore, the findings revealed that much of the geometry teaching and learning strategies of Mathematics tutors in the selected CsE in Ghana are not structured in a way that support the development of geometric thinking as described in the van Hiele theory.

# CHAPTER FIVE

## SUMMARY, CONCLUSION AND RECOMMENDATIONS

### 5.0 Overview

This chapter provides the summary of the study and the major findings. It highlights the conclusion of the study and implications for practice. It further outlines some recommendations and avenues for future research.

### 5.1 Summary of Study

The study investigated the effect of the VHPI on PTs' geometric thinking in terms of the van Hiele Levels and their motivation to learn geometry. The study further investigated how CE Mathematics tutors facilitate geometry teaching and learning consistent with the van Hiele Levels of geometric thinking.

The study was guided by the following research questions:

- To what extent does the use of VHPI improve PTs geometric thinking Levels?
- How does the use of VHPI motivate PTs to learn geometry?
- To what extent do CE Mathematics tutors facilitate geometry teaching and learning consistent with the van Hiele Levels of geometric thinking?

The general approach chosen for the study was a mixed method approach which employed a quasi-experimental research design as a strategy of enquiry. The model of quasi-experimental design used was the pre-test post-test, two group design. Test (VHGT), Interview and Observation were the instruments used to collect data.

The population was made up of PTs and Mathematics tutors in all CsE in Ghana. The sample comprised of 150 second year PTs, 75 in the experimental group (in College A) and the other 75 in the control group (in College B). Five College Mathematics tutors were also randomly sampled for observation, with three tutors coming from College A and two tutors from College B. The results of the different data sources: the pre-test and post-test (VHGT), Interview, and Observation were combined to answer the research questions. In particular, each research question was looked at from all relevant data sources.

## 5.2 Major Findings

The findings of the study are summarized and presented under the three sub-headings in line with the research questions.

### 5.2.1 Research question 1: To what extent does the use of VHPI improve PTs geometric thinking Levels?

Findings from the pre-test and post-test showed that PTs in the experimental group made a significant progress in developing higher geometric thinking Levels. Findings in this research question further indicated that there was a significant difference between the results of the pre-test and post-test in terms of developed solution methods and scores among PTs in the control and experimental groups favouring the PTs in the experimental group. Thus, the engagement of PTs in the experimental group in the VHPI showed a rather remarkable performance in their post-test.

### 5.2.2 Research question 2: How does the use of VHPI motivate PTs to learn geometry?

Responses from the Interview with the PTs revealed that the PTs liked the geometry lessons which employed the VHPI because it took away dullness and asserted that it made learning of geometry easier and fascinating. From further interrogation and also discussions and interactions during the geometry lessons, it was noticed that the VHPI raised PTs' interest and eagerness towards geometry concepts as they actively engaged in hands-on investigations with manipulative concrete materials. PTs gained an interest and motivation in making new shapes besides having broadened their understanding about geometric shapes. The results from the interview indicated that the use of the VHPI during the geometry lessons has increased and sustained PTs' interest more to learn geometry. The VHPI motivated PTs to learn geometry by taking away dullness and making learning of the geometry concepts easier and free from nervousness.

### 5.2.3 Research question 3: To what extent do CE Mathematics tutors facilitate geometry teaching and learning consistent with the van Hiele Levels of geometric thinking?

The findings obtained from the classroom observations indicated that much of the geometry teaching and learning strategies of Mathematics tutors in the selected CsE in Ghana are not structured in a way that support the development of geometric thinking as described in the van Hiele theory. Generally, tutors did not provide any hands-on activities for the PTs in their respective geometry lessons. This teaching approach did not encourage participation of the PTs as advocated by the van Hieles and also did not support the development of thinking and learning consistent with the van Hiele model. Though PTs are to do some simple deductive geometric proofs as indicated in their geometry course outlines, no deductive geometric proofs were carried out by the tutors to facilitate understanding of van Hiele Level 4. Consequently, the tutors did not choose materials and tasks which targeted the key concepts and procedures under consideration. Teaching was therefore not motivating, enthusiastic and challenging. The findings suggest that much of the teaching strategies employed by the selected CE tutors in this study generally, did not support learning at the various van Hiele Levels.

## 5.3 Conclusion

In this study, PTs in the experimental group were given instruction according to the VHPI while PTs in the control group were instructed by traditional instruction. PTs in both groups showed increment in their post-VHGT as compared to the pre-VHGT. However, PTs in the experimental group achieved better levels of geometric thinking compared to those in the control group. In other words, it can be claimed that the VHPI has a positive effect on PTs geometric thinking Levels. The hands-on activities using concrete manipulative materials provided an equal support for every PT to eventually achieve an enhanced conceptual understanding of the geometric concepts taught. This finding is in line with earlier studies on the effect of the use of the VHPI in the teaching and learning of geometry (Yazdani, 2007; Erdogan & Durmus, 2009; Siew & Chong, 2014). These studies indicated that the use of the VHPI has a number of suggestions for improving teaching and learning of geometry in Ghanaian classrooms. The use of the VHPI has the potential of helping PTs improve their geometric reasoning while enhancing their understanding of geometry concepts to enable them teach the subject diligently at the basic level later when they have completed CE.

In this study, one of the most significant findings was that the VHPI motivates PTs to learn geometry by eliminating dullness and making learning easier and fascinating. The findings from the Interviews revealed that PTs enjoyed the geometry lessons with the VHPI and they largely developed conceptual understanding regarding the geometric concepts, due to the hands-on activities together with the high Levels of motivation during lessons. This is an encouraging phenomenon, because researches (Siew & Chong, 2014; Deci & Ryan, 1985 cited in Colgan, 2014) have shown that when learners are positive about and engaged with Mathematics, they are more motivated to learn, accept new ideas and try more challenging tasks. This, in turn, leads to the development of improved self-esteem, confidence, perseverance, creativity and performance.

However, the study revealed that CE tutors gave instruction according to the traditional method; teaching was greatly dominated by the tutors and no hands-on activities were provided for PTs to explore. When the pre-test and post-test scores of van Hiele Geometry Test of PTs in control group were taken into consideration, it was seen that there was no significant difference between the results. Thus, it can be claimed that instruction given with traditional method does not improve the geometric thinking Levels of PTs. This finding is also consistent with other researches (Yazdani, 2007; Erdogan & Durmus, 2009). Alternatively, integrating the VHPI into geometry lessons greatly improves learners geometric thinking Levels, motivates learners and also contributes to learners' positive perception towards geometry learning.

CE Mathematics tutors need to move away from traditional forms of lesson delivery especially in geometry content and put emphasis on hands-on activities using concrete manipulative materials as integrated in the VHPI so that geometrical concepts can be grasped easily. The findings in this study indicate that the use of the VHPI has the ability to improve PTs' geometric thinking Levels at the CE level in Ghana. If government wants to improve Ghanaian students' Mathematics performance in international and national examinations like TIMSS, BECE, WASSCE and DBE examinations then the use of the VHPI is imperative. The information generated from this study therefore is available for policy makers, teachers and other stakeholders to help improve students' geometric thinking Levels in Ghana through the use the VHPI.

## 5.4 Recommendations

From the findings of this study, it is recommended that;

- Geometric thinking Levels of PTs should be determined and instructions should be applied based upon these Levels. In view of this, CE Mathematics tutors should revise their instructional methods to utilize the VHPI in planning and delivering lessons. This would address the finding that teaching approaches adopted by CE tutors in the teaching of two-dimensional figures were not effective to aid PTs' understanding of the concepts.

- CE Mathematics tutors should be encouraged to use teaching learning materials in enhancing and developing the spatial orientation of PTs. This suggestion was made in view of the finding that there was lack of effective use of teaching learning materials by CE tutors in teaching two-dimensional figures.

- CE Mathematics tutors should be encouraged to provide PTs with hands-on activities using manipulative concrete materials for discovering the properties of simple geometric shapes in different orientations. In addition to this, experiences related to geometry and guidance for developing geometric thinking Levels should be provided for PTs. This recommendation was made to deal with the finding that CE tutors taught concepts of shapes theoretically using the chalk or marker boards and teaching was greatly dominated by the tutors.

- Curriculum developers, text book writers and policy makers need to look into the van Hiele model for clues on how to improve learner achievement in Mathematics in general and geometry in particular. This is in view of the finding that the model can be used to improve learners' conceptual understanding of geometry and geometric thinking Levels in Ghana.

## 5.5 Areas for further research

The educational implication of the findings of this study calls for further research involving the van Hiele model in Ghana. The following are suggested for further research:

- The research concentrated on investigating only two-dimensional figures using the van Hiele model. A study can be undertaken on using the model to investigate other areas of geometry such as three-dimensional figures, circle theorems and coordinate geometry.

- A study in this area can be done to involve more CsE to obtain the general picture of how the VHPI improves PTs' geometric thinking Levels in Ghana.

# REFERENCES

Abdullah, A. H. & Zakaria, E. (2013). The Effects of van Hiele's Phase-Based Instruction Using the Geometer's Sketchpad (GSP) on Students' Levels of Geometric Thinking. *Research Journal of Applied Sciences, Engineering and Technology*. 5(5): 1652-1660.

Abu, M. S. & Abidin, Z. Z. (2013). Improving the Levels of Geometric Thinking of Secondary School Students using Geometry Learning Video based on van Hiele Theory. *International Journal of Evaluation and Research in Education (IJERE)*. 2(1): 16-22.

Abudu, A. M. & Donkor, A. K. (2014). The In-In-Out Programme of Teacher Education in Ghana: The Perception of Implementers. *International Journal of Academic Research in Education and Review*. 2(2): 32-48.

Adolphus, T. (2011). Problems of Teaching and Learning of Geometry in Secondary Schools in Rivers State, Nigeria. *International Journal of Emerging Sciences*. 1(2): 143-152.

Adu-Yeboah, C. (2011). *Teacher Preparation and Continuing Professional Development in Africa (TPA). Learning to teach Reading and Mathematics and its influence on Practice in Ghana*. Retrieved September 30, 2014 from https://www.sussex.ac.uk/webteam/gateway/file.php?name=report-ghana-1july2011.pdf&site=320.

Alex, J. K. & Mammen, K. J. (2012). A Survey of South African Grade 10 Learners' Geometric Thinking Levels in Terms of the van Hiele Theory. *Anthropologist*. 14(2): 123-129.

Ampiah, J. G. (2010). *Quality Basic Education in Ghana: Prescription, Praxis and Problems*. Paper Delivered at the Experience Sharing Seminar. Erata Hotel Accra: Ghana.

Anamuah-Mensah, J. & Mereku, D. K. (2005). Ghanaian Junior Secondary School two students abysmal Mathematics Achievement in TIMSS 2003: A consequence of the Basic school Mathematics. *Mathematics Connection*. 5(1): 1-11.

Anamuah-Mensah, J., Mereku, D. K. & Asabere-Ameyaw, A. (2004). *Ghanaian Junior Secondary School Students' Achievement in Mathematics and Science: Results from Ghana's participation in the 2003 Trends in International Mathematics and Science Study*. Accra: Ministry of Education Youth and Sports.

Anamuah-Mensah, J., Mereku, D. K., & Asabere-Ameyaw, A. (2008). *Ghanaian Junior Secondary School Students' Achievement in Mathemtics and Science:Results from Ghana's participation in the 2007 Trends in International Mathemtics and Science Study*. Accra: Ministry of Education Youth and Sports.

Asante, J. N. & Mereku, D. K. (2012). The Effect of Ghanaian Pre-service Teachers' Content Knowledge on their Mathematical Knowledge for teaching Basic School Mathematics. *African Journal of Educational Studies in Mathematics and Sciences*. 10: 23-37.

Atebe, H. U. (2008). *Students' van Hiele Levels of Geometric Thought and Conception in Plane Geometry: A Collective Case Study of Nigeria and South Africa.* Unpublished PhD. Thesis, South Africa: Rhodes University.

Atebe, H. U., & Schäfer, M. (2008). Van Hiele Levels of geometric thinking of Nigerian and South African Mathematics learners. *Proceedings of the 15th Annual Conference of the Southern Africa Association for Research in Mathematics, Science and Technology Education (SAAMSTE),* Grahamstown, South Africa.

Baffoe, E. & Mereku, D. K. (2010). The van Hiele Levels of understanding of students entering Senior High School in Ghana. *African Journal of Educational Studies in Mathematics and Sciences.* 8: 51-61.

Battista, M. T. & Clements, D. H. (1995). Geometry and proof. *Mathematics Teacher*. 88(1): 48-54.

Breyfogle, M. L. & Lynch, C. M. (2010). Van Hiele Revisited. *Mathematics Teaching in the Middle School.* 16(4): 232-238.

Burger, E. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele Levels of development in geometry. *Journal for Research in Mathematics Education.* 17(1): 31–48.

Chen, W. H. (2013). *Teaching Geometry through Problem-Based Learning and Creative Design.* Proceedings of the 2013 International Conference on Education and Educational Technologies. Tunghai University: Taiwan, R. O. C.

Christou, C., Jones, K., Pitta, D., Pittalis, M., Mousoulides, N. & Boytchev, P. (2007). *Developing Student Spatial Ability with 3-Dimensional Applications*. Retrieved July 30, 2014 from http://www.elica.net/download/papers/devstudspatabil3d.pdf.

Clements, D. H. & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on Mathematics teaching and learning* (pp.420–464). New York: Macmillan Publishing Company.

Colgan, L. (2014). *Making Math Children Will Love: Building Positive Mathitudes to Improve Student Achievement in Mathematics*. Retrieved January 26, 2015 from http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/WW_Making Math.pdf.

Creswell, J. (2003). *Research Design: Quantitative, Qualitative and Mixed Methods Approaches.* Thousand Oaks, CA: SAGE Publications.

Creswell, J. (2006). *Understanding Mixed Methods Research.* Thousand Oaks, CA: SAGE Publications.

Crowley, M. L. (1987). The van Hiele Model of development of geometric thought. In M. M. Lindquist, & A. P. Shulte (Eds.), *Learning and teaching geometry, K-12, 1987 Yearbook* (pp. 1-16). Reston, VA: National Council of Teachers of Mathematics.

Curriculum Research and Development Division of GES (CRDD). (2007*). Mathematics Syllabus for Junior High School*. Accra: Ghana Publishing Corporation.

Davis, G. A. & Hyun, E. (2005). A Study of Kindergarten Children's Spatial Representation in a Mapping Project. *Mathematics Education Research Journal.* 17(1): 73-100.

De Villiers, M. (2004). Using dynamic geometry to expand Mathematics teachers' understanding of proof. *International Journal of Mathematical Education in Science and Technology.* 35(5): 703–724.

De Villiers, M. (2012). *Some Reflections on the van Hiele Theory*. Retrieved April 8, 2014 from http://frink.machighway.com/~dynamicm/vanhiele-reflection.pdf.

Dindyal, J. (2007). The Need for an Inclusive Framework for Students' Thinking in School Geometry. *The Montana Mathematics Enthusiast. 4(1)*: 73-83.

Duru, A. (2010). The experimental teaching in some of topics geometry. *Educational Research and Review.* 5 (10): 584-592.

Erdogan, T. & Durmus, S. (2009). The effect of the instruction based on van Hiele model on the geometrical thinking Levels of preservice elementary school teachers. *Procedia Social and Behavioral Sciences.* 1: 154–159.

Etsey, Y. K. A. (2004). Opportunity to Learn First Year Mathematics in Teacher Training Colleges in Ghana. *African Journal of Educational Studies in Mathematics and Sciences.* 2(2): 65-80.

Fife-Schaw, C. (2012). *Research Methods in Psychology*. Retrieved October 12, 2014 from http://www.sagepub.com/upm-data/46877_Breakwell_Ch04.pdf.

Fredua-Kwarteng, Y. & Ahia, F. (2004). *Confronting National Mathematics Phobia in Ghana (Part 2).* Retrieved January, 31, 2014 from http://www.ghanaweb.com/GhanaHomePage/NewsArchive/artikel.php?ID=61432.

Furner, J. M. & Marinas, C. A. (2011). *Geoboards to Geogebra: moving from the concrete to the abstract in geometry*. Retrieved February, 4, 2014 from http://archives.math.utk.edu/ICTCM/VOL23/S088/paper.pdf.

Gal, H. & Lew, H. C. (2008). *Is a rectangle a parallelogram-Towards A bypass of van Hiele level 3 decision making*. Proceedings of the 11th International Congress on Mathematical Education. Monterrey, Mexico.

Gujarati, J. (2014). Examining the van Hiele Theory: Strategies to Develop Geometric Thought. *Strategies for Successful Learning.* 7(2): 1-4.

Gutierrez, A. (1992). Exploring the Links between van Hiele Levels and 3-Dimensional Geometry. *Structural Topology.* 18: 31-47.

Güven, B. & Kosa, T. (2008). The effect of dynamic geometry software on student Mathematics teachers' spatial visualization skills. *The Turkish Online Journal of Educational Technology*. 7(11): 100-107.

Halat, E, Jakubowski, E. & Aydin, N. (2008). Reform-Based Curriculum and Motivation in Geometry. *Eurasia Journal of Mathematics, Science & Technology Education*. 4(3): 285-292.

Halat, E. & Şahin, O. (2008). Van Hiele Levels of Pre- and In- Service Turkish Elementary School Teachers and Gender Related Differences in Geometry. *The Mathematics Educator.* 11(1/2): 143-158.

Halat, E. (2008). In-Service Middle and High School Mathematics Teachers: Geometric Reasoning Stages and Gender. *The Mathematics Educator.* 18(1): 8–14.

Hashemi, S. B. (2014*). Research Methods and Analysis. Analytic Experimental Studies. Quasi Design*. Retrieved October 13, 2014 from http://www.doctorsforafghanistan.com/_articles/Baha_Hashimi/Lecture%2010%20Quasi%20Study%20Design.pdf.

Jones, K. (2002). Issues in the Teaching and Learning of Geometry. In: Linda Haggarty (Ed), *Aspects of Teaching Secondary Mathematics: perspectives on practice* (pp.121-139). London: RoutledgeFalmer.

Khalid, A. & Azeem, M. (2012). Constructivist Vs Traditional: Effective Instructional Approach in Teacher Education. *International Journal of Humanities and Social Science.* 2(5): 170-177.

Knight, K. C. (2006). *An investigation into the van Hiele level of understanding geometry of pre-service elementary and secondary mathematics teachers*. Unpublished master's thesis. University of Maine, Orono, ME.

Liu, E. Z. F. & Lin, C. H. (2010). The Survey Study of Mathematics Motivated Strategies for Learning Questionnaire (MMSLQ) For Grade 10–12 Taiwanese Students. *The Turkish Online Journal of Educational Technology*. 9(2): 221-233.

Lutz, S., & Huitt, W. (2004). Connecting cognitive development and constructivism: Implications from theory for instruction and assessment. *Constructivism in the Human Sciences.* 9(1): 67-90.

Malloy, C. (2002). *The van Hiele Framework.* Retrieved January 13, 2014 from http://tian.terc.edu/empower_readings/Malloy_van%20Hiele%20Framework.pdf

Mason, M. (1998). The van Hiele Levels of geometric understanding. In L. McDougal (Ed.). *The professional handbook for teachers: Geometry* (pp. 4–8). Boston: McDougal-Littell/Houghton-Mifflin.

Mertler, C. A. & Charles, C.M. (5th Ed.) (2005). *Introduction to educational research.* New York: Pearson Education, Inc.

Ministry of Education (2010). *Teaching Syllabus for Core Mathematics (Senior High School 1-3).* Accra: Ministry of Education.

Mushquash, C. J. & Bova, D. L. (2007). Cross-Cultural Assessment and Measurement Issues. *Journal on Developmental Disabilities*. 13(1): 53-65.

Muyeghu, A. (2008). *The use of the van Hiele theory in Investigating Teaching Strategies used by Grade 10 Geometry teachers in Namibia*. Unpublished Masters Thesis. Rhodes University, Grahamstown, RSA.

National Council of Teachers of Mathematics (NCTM). (2009). *Guiding Principles for Mathematics Curriculum and Assessment.* Retrieved February, 2, 2014 from http://www.nctm.org/uploadedFiles/Math_Standards/NCTM%20Guiding%20Principles%206209.pdf.

Ojose, B. (2008). Applying Piaget's Theory of Cognitive Development to Mathematics Instruction. *The Mathematics Educator.* 18(1): 26–30.

Ontario Curriculum, Mathematics. (2005). *Geometry and Spatial Sense.* Retrieved July 30, 2014 from http://teachingmathliteracy.weebly.com/geometry-and-spatial-sense.html.

Professional Development Service for Teachers (PDST). (2014). *Shape and Space Manual: A Guide to Teaching and Learning in Irish Primary Schools.* Dublin: Ireland.

Robert Wood Johnson Foundation (2008). *Unstructured Interviews*. Retrieved March 2, 2015 from http://www.qualres.org/HomeUnst-3630.html.

Russell, D. (2014). *What is Geometry?* Retrieved January 14, 2014 from http://math.about.com/od/geometry/a/whygeom.htm.

Rynhart, P. (2012). *Piaget and Spatial Concepts.* Retrieved July 24, 2014 from http://proactiveplay.com/piaget-and-van-hiele-compare-contrast-and-my-thoughts/.

Sakyi, K. A. (2014). *Towards Quality Delivery of Science and Mathematics Education in Schools and Colleges in Ghana.* Retrieved September 28, 2014 from http://www.ghanaiandiaspora.com/wp/wp-content/uploads/2014/07/TOWARDS-TEACHING-IMPROVEMENT-OF-SCIENCE-MATHEMATICS-EDUCATION.pdf

Schwartz, J. E. (2008). *Elementary Mathematics Pedagogical Content Knowledge: Powerful Ideas for Teachers*. Retrieved March 2, 2014 from http://www.education.com/reference/article/why-people-have-difficulty-geometry/

Shadish, W., Cook, T., & Campbell, D. (2002). *Experimental and quasi-experimental designs for generalized causal inference.* Boston: Houghton Mifflin Company.

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information*. 22: 63–75.

Siew, N. M & Chong, C. L. (2014). Fostering Students' Creativity through van Hiele's 5 Phase-Based Tangram Activities. *Journal of Education and Learning*. 3(2): 66-80.

Siew, N. M., Chong, C. L. & Abdullah, M. R. (2013). Facilitating Students' Geometric Thinking through van Hiele's Phase-Based Learning Using Tangram. *Journal of Social Sciences.* 9(3): 101-111.

Silverthorn, P. (1999). *Jean Piaget's Theory of Development.* Retrieved July 24, 2014 from http://members.iinet.net.au/~aamcarthur/4_Mar_2008_files/Piagets_Theory_of_Development.pdf.

Smiešková, E. & Barcíková, E. (2014). *Motivation to Geometry at High School of Visual Arts.* Retrieved January 26, 2015 from http://www.nmk.fpv.ukf.sk/2014/proceedings/20_smieskova_barcikova_fulltext.pdf.

Strutchens, M. E., Harris, K. A., & Martin, W. G. (2001). Assessing geometric and measurement understanding using manipulatives. *Mathematics Teaching in the Middle School.* 6(7): 402-405.

Sunzuma, G., Masocha, M. & Zezekwa, N. (2013). Secondary School Students'Attitudes towards their Learning of Geometry: A Survey of Bindura Urban Secondary Schools. *Greener Journal of Educational Research.* 3(8): 402-410.

Teacher Education Division (TED). (2008). *Teacher Trainees' Handbook (Revised Edition)*. 3 Year Diploma in Basic Education for Teacher Training Colleges. Accra: GES, TED.

Tella, A. (2007). The Impact of Motivation on Student's Academic Achievement and Learning Outcomes in Mathematics among Secondary School Students in Nigeria. *Eurasia Journal of Mathematics, Science & Technology Education.* 3(2): 149-156.

University of Cape Coast (UCC), Institute of Education (2005). *Three- Year Diploma in Basic Education Mathematics Syllabus*. Cape Coast: University of Cape Coast-Institute of Education.

University of Cape Coast, Institute of Education (2008). *Chief Examiner's Report on the 2008 Three- Year Post-Secondary Teacher Training College Diploma in Basic Education Mathematics II (GEOMETRY & TRIGONOMETRY) Examination*. Cape Coast: University of Cape Coast, Institute of Education.

University of Cape Coast, Institute of Education (2009). *Chief Examiner's Report on the 2009 Three- Year Post-Secondary Teacher Training College Diploma in Basic Education Mathematics II (GEOMETRY & TRIGONOMETRY) Examination*. Cape Coast: University of Cape Coast, Institute of Education.

University of Cape Coast, Institute of Education (2010). *Chief Examiner's Report on the 2010 Three- Year Post-Secondary Teacher Training College Diploma in Basic Education Mathematics II (GEOMETRY & TRIGONOMETRY) Examination*. Cape Coast: University of Cape Coast, Institute of Education.

University of Cape Coast, Institute of Education (2011). *Chief Examiner's Report on the 2011 Three- Year Post-Secondary Teacher Training College Diploma in Basic Education Mathematics II (GEOMETRY & TRIGONOMETRY) Examination*. Cape Coast: University of Cape Coast, Institute of Education.

University of Cape Coast, Institute of Education (2012). *Chief Examiner's Report on the 2012 Three-Year Post-Secondary Teacher Training College Diploma in Basic Education GEOMETRY Examination*. Cape Coast: University of Cape Coast, Institute of Education.

Usiskin, Z. (1982). *Van Hiele Levels and achievement in secondary school geometry: Cognitive development and achievement in secondary school geometry project.* Chicago: University of Chicago Press.

Van de Walle, J. A. (2001). *Geometric Thinking and Geometric Concepts. In Elementary and Middle School Mathematics: Teaching Developmentally*. (4th Ed.). Boston: Allyn and Bacon.

Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education.* Orlando: Academic Press.

Van Hiele, P. M. (1999). Developing Geometric Thinking through Activities that Begin with Play. *Teaching Children Mathematics*. 6: 310–316.

Vanderstoep, S. W. & Johnson, D. D. (2009). Research Methods for Everyday Life: *Blending Qualitative and Quantitative Approaches*. San Francisco: Jossey Bass.

Vojkuvkova, I. (2012). The van Hiele Model of Geometric Thinking. *WDS'12 Proceedings of Contributed Papers*. 1: 72–75.

Way, J. (2014). *The Development of Spatial and Geometric Thinking: 5 to 18.* Retrieved July 24, 2014 from http://nrich.maths.org/2483.

Wen-Tung, H. & Chin-Hsiang F. (2010). *Exploring Geometric Cognition of Young Children*. Retrieved July 2, 2014 from http://ir.meiho.edu.tw/bitstream/987654321/1147/1/%E9%A6%AC%E4%BE%86%E8%A5%BF%E4%BA%9E%E7%A0%94%E8%A8%8E%E6%9C%83_Exploring+Geometric+Cognition+of+Young+Children.pdf.

West African Examination Council. (2008). *West African Senior School Certificate Examinations Chief Examiner's Report for Core Mathematics.*

West African Examination Council. (2009). *West African Senior School Certificate Examinations Chief Examiner's Report for Core Mathematics.*

West African Examination Council. (2011). *West African Senior School Certificate Examinations Chief Examiner's Report for Core Mathematics.*

Williams, L. (2014). *Identifying and Describing Procedures for Observation and Measurement*. Retrieved September 30, 2014 from http://msccounsellingyear1term2.weebly.com/uploads/5/4/3/5/5435893/unit_5_2014_part_1_identifying_and_describing_procedures_for_observation_and_measurement.pdf.

Wood, K. C., Smith, H. & Grossniklaus, D. (2001). *Piaget's Stages of Cognitive Development*. In M. Orey (Ed.), Emerging perspectives on learning, teaching, and technology. Retrieved June 28, 2014 from http://epltt.coe.uga.edu/index.php?title=Piaget%27s_Stages.

Yazdani, M. A. (2007). Correlation between Students' level of Understanding Geometry According to the van Hieles' Model and Students' Achievement in Plane Geometry. *Journal of mathematical Sciences & Mathematics Education*. 2(1): 40-45.

Yegambaram, P. & Naidoo, R. (2009). *Better learning of geometry with computer.* Retrieved January 16, 2014 from http://atcm.mathandtech.org/EP2009/papers_full/2812009_17080.pdf.

# APPENDICES

## APPENDIX A – PRE-TEST AND POST-TEST ITEMS

GEOMETRY TEST

Dear Student,

I am an M.Phil Mathematics Education student of the University of Education, Winneba. I am conducting a research study to enable me write my thesis. Kindly answer the questions as accurately as possible. The answers are for educational purposes and are in no way meant for individual or personal assessment. Your answers will be treated as strictly confidential. Thank you for your co-operation.

Name of Student:…………………………………………………………………………

Name of College:…………………………………………………………………….....

Class:…………………………………………………………………………………...

Directions

Do not open this test booklet until you are told to do so.

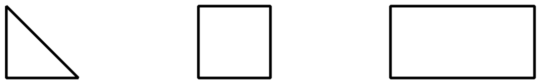
This test is in two parts, sections A and B. Answer all questions in both sections. There are 25 multiple choice questions in section A, circle the right option using a pencil. If you want to change an answer, completely erase the first answer. You will have 55 minutes to complete this test.

Wait until the researcher says that you may begin.

# VAN HIELE GEOMETRY TEST

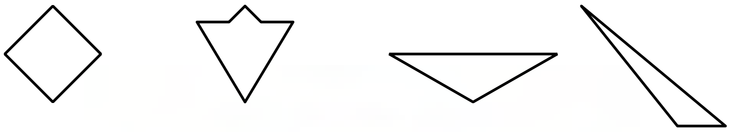
1. Which of these are squares?

A. K only
B. L only
C. M only
D. L and M only
E. All are squares



K L M

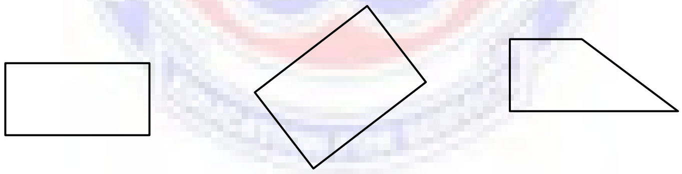
2. Which of these are triangles?



U V W X

A. None of these are triangles.
B. V only
C. W only
D. W and X only
E. V and W only

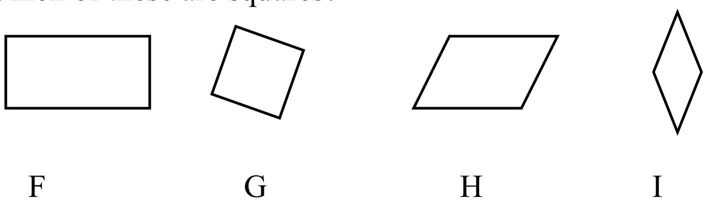
3. Which of these are rectangles?



S T U

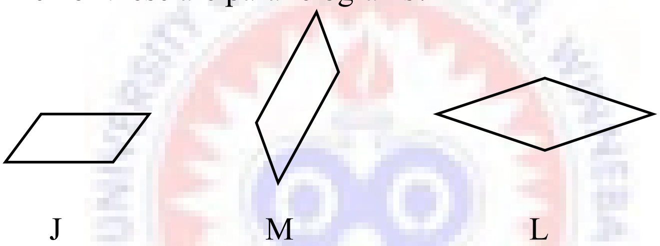
A. S only
B. T only
C. S and T only
D. S and U only
E. All are rectangles.

4. Which of these are squares?




A. G and I only.
B. G only
C. F and G only
D. All are squares
E. None of these are squares.

5. Which of these are parallelograms?

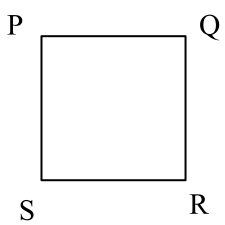



A. J only
B. L only
C. J and M only
D. All are parallelograms.
E. None of these are parallelograms.

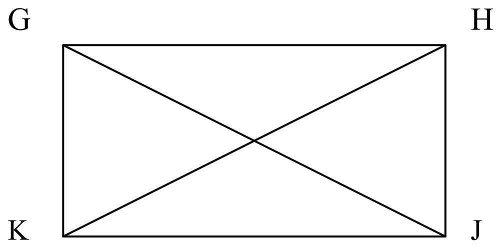
6. PQRS is a square.

Which relationship is true in all squares?

A. $\overline{PR}$ and $\overline{RS}$ have the same length.
B. $\overline{QS}$ and $\overline{PR}$ are perpendicular.
C. $\overline{PS}$ and $\overline{QR}$ are perpendicular.
D. $\overline{PS}$ and $\overline{QS}$ have the same length.
E. Angle Q is larger than angle R.



7. In the rectangle GHJK, $\overline{GJ}$ and $\overline{HK}$ are the diagonals.

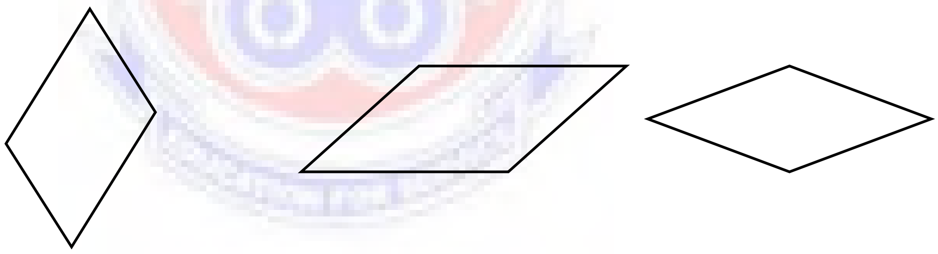



Which of (A)-(D) is not true in every rectangle?

A. There are four right angles.
B. There are four sides.
C. The diagonals have the same length.
D. The opposite sides have the same length.
E. All of (A)-(D) are true in every rectangle.

8. A rhombus is a 4-sided figure with all sides of the same length.

Here are three examples.

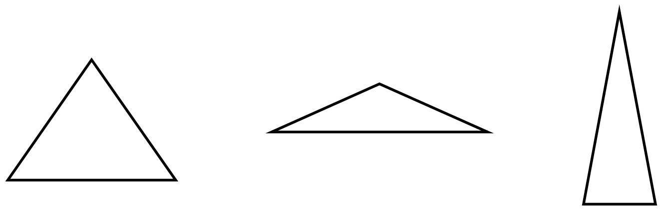


Which of (A)-(D) is not true in every rhombus?

A. The two diagonals have the same length.
B. Each diagonal bisects two angles of the rhombus.
C. The two diagonals are perpendicular.
D. The opposite angles have the same measure.
E. All of (A)-(D) are true in every rhombus.

9. An isosceles triangle is a triangle with two sides of equal length.

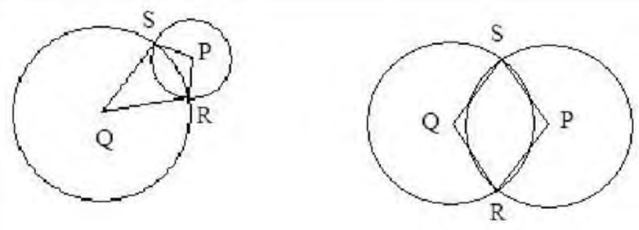
Here are three examples.



Which of (A)-(D) is true in every isosceles triangle?

A. The three sides must have the same length.
B. One side must have twice the length of another side.
C. There must be at least two angles with the same measure.
D. The three angles must have the same measure.
E. None of (A)-(D) is true in every isosceles triangle.

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.




Which of (A)-(D) is not always true?

A. PRQS will have two pairs of sides of equal length.
B. PRQS will have at least two angles of equal measure.
C. The lines $\overline{PQ}$ and $\overline{RS}$ will be perpendicular.
D. Angles P and Q will have the same measure.
E. All of (A)-(D) are true.

11. Here are two statements.

Statement 1: Figure F is a rectangle.

Statement 2: Figure F is a triangle.

Which is correct?

A. If 1 is true, then 2 is true.
B. If 1 is false, then 2 is true.
C. 1 and 2 cannot both be true.
D. 1 and 2 cannot both be false.
E. None of (A)-(D) is correct.

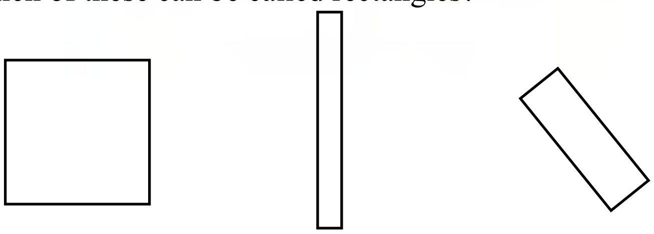
12. Here are two statements.

Statement S: ΔABC has three sides of the same length
Statement T: In ΔABC, ∠B and ∠C have the same measure.

Which is correct?

A. Statement S and T cannot both be true.
B. If S is true, then T is true.
C. If T is true, then S is true.
D. If S is false, then T is false.
E. None of (A)-(D) is correct.

13. Which of these can be called rectangles?



P Q R

A. All can.
B. Q only
C. R only
D. P and Q only
E. Q and R only

14. Which is true?

A. All properties of rectangles are properties of all squares.

B. All properties of squares are properties of rectangles.

C. All properties of rectangles are properties of all parallelograms.

D. All properties of squares are properties of all parallelograms.

E. None of (A)-(D) is true.

15. What do all rectangles have that some parallelograms do not have?

A. Opposite sides equal

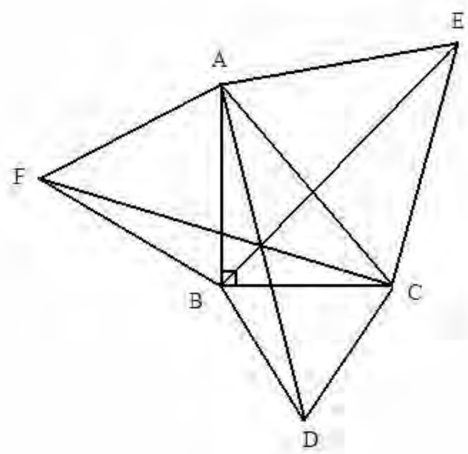
B. Diagonals equal

C. Opposite sides parallel

D. Opposite angles equal

E. None of (A)-(D)

16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.




From this information, one can prove that AD, BE, and CF have a point in common. What would this proof tell you?

A. Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

B. In some but not all right triangles, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

C. In any right triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

D. In any triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

E. In any equilateral triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.
Property S: It is a square.
Property R: It is a rectangle.

Which is true?

A. D implies S which implies R.
B. D implies R which implies S.
C. S implies R which implies D.
D. R implies D which implies S.
E. R implies S which implies D.

18. Here are two statements.

I: If a figure is a rectangle, then its diagonals bisect each other.

II: If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

A. To prove I is true, it is enough to prove that II is true.
B. To prove II is true, it is enough to prove that I is true.
C. To prove II is true, it is enough to find one rectangle whose diagonal bisect each other.
D. To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.
E. None of (A)-(D) is correct.

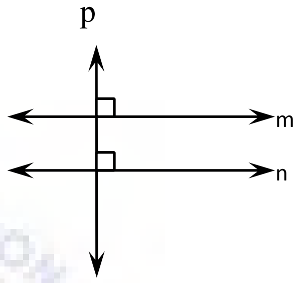
19. In geometry:

A. Every term can be defined and every true statement can be proved true.
B. Every term can be defined but it is necessary to assume that certain statements are true.
C. Some terms must be left undefined but every true statement can be proved true.
D. Some terms must be left undefined and it is necessary to have some statements which are assumed true.
E. None of (A)-(D) is correct.

20. Examine these three sentences.

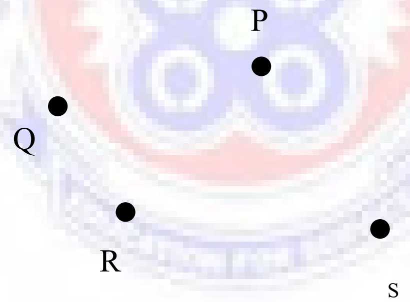
1. Two lines perpendicular to the same line are parallel.
2. A line that is perpendicular to one of two parallel lines is perpendicular to the other
3. If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?




A. (1) only
B. (2) only
C. (3) only
D. Either (1) or (2)
E. Either (2) or (3)

21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R and S, and the lines are {P,Q}, {P,R}, {P,S}, {Q,R}, {Q,S}, and {R,S}.




Here are how the words "intersect" and "parallel" are used in F-geometry. The lines {P,Q} and {P,R} intersect at P because {P,Q} and {P,R} have P in common.

The lines {P,Q} and {R,S} are parallel because they have no points in common.

From this information, which is correct?

A. {P,R} and {Q,S} intersect.
B. {P,R} and {Q,S} are parallel.
C. {Q,R} and {R,S} are parallel.
D. {P,S} and {Q,R} intersect.
E. None of (A)-(D) is correct.

22. To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?

A. In general, it is impossible to bisect angles using only a compass and an unmarked ruler.

B. In general, it is impossible to trisect angles using only a compass and a marked ruler.

C. In general, it is impossible to trisect angles using any drawing instruments.

D. It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.

E. No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

23. There is a geometry invented by a mathematician J in which the following is true:

The sum of the measures of the angles of a triangle is less than 180°.

Which is correct?

A. J made a mistake in measuring the angles of the triangle.

B. J made a mistake in logical reasoning.

C. J has a wrong idea of what is meant by "true".

D. J started with different assumptions than those in the usual geometry.

E. None of (A)-(D) is correct.

24. The geometry books define the word rectangle in different ways. Which is true?

A. One of the books has an error.

B. One of the definitions is wrong. There cannot be two different definitions for rectangle.

C. The rectangles in one of the books must have different properties from those in the other book.

D. The rectangles in one of the books must have the same properties as those in the other book.

E. The properties of rectangles in the two books might be different.

25. Suppose you have proved statements I and II.

I: If p, then q.

II: If s, then not q.

Which statement follows from statements I and II?

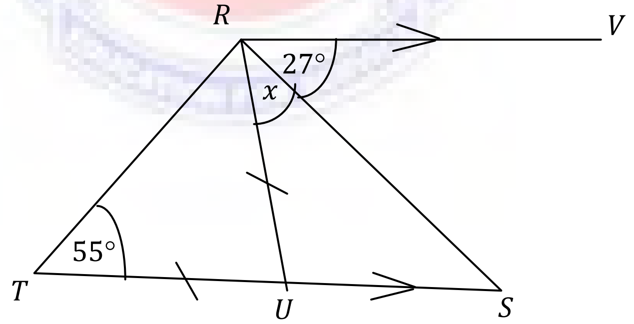
A. If p, then s.
B. If not p, then not q.
C. If p or q, then s.
D. If s, then not p.
E. If not s, then p.

## SECTION B

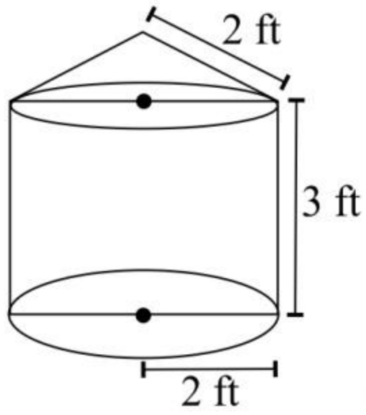
**Answer all three (3) questions in this section by clearly showing working**

1. In the diagram below, $RV//TS$ and $RU = TU$. $\angle SRV = 27°$ and $\angle RTU = 55°$.

Find the value of $x$. You are to show your workings, giving a reason for each step.



2. Kwabena stores his toys in a container that has a cylindrical body and a conical lid, as shown below.




Find;

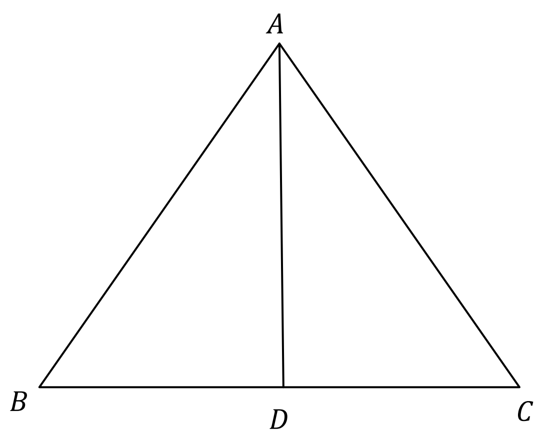
(i) the surface area of the base of the container

(ii) the surface area of the cylindrical body (excluding the base and top)

(iii) the surface area of the conical lid

(iv) If Kwabena wants to cover the entire exterior portion of the container with paper. How much paper, in square feet, would he need?

3. Given that AB = AC and $\overline{AD}$ bisects $\overline{BC}$, show, giving reasons that $\Delta ABD$ and $\Delta ACD$ have equal corresponding angles and equal corresponding sides. What then can you say about the two triangles?



## APPENDIX B - MARKING SCHEME (PRE-TEST & POST-TEST)

**Marking Scheme for the VHGT – Section A**

1. B
2. D
3. C
4. B
5. D
6. B
7. E
8. A
9. C
10. D
11. C
12. B
13. A
14. A
15. B
16. C
17. C
18. D
19. D
20. A
21. B
22. E
23. D
24. E
25. D

## SECTION B

### QUESTION 1

**METHOD 1**

$\angle SRV = \angle RST = 27^\circ$ [Alternate angles, $RV \parallel TS$] $M_1\ B_1$

$\angle RTU = \angle TRU = 55^\circ$ [Base angles of isosceles $\Delta RTU$] $M_1\ B_1$

$\angle RST + \angle RTU + \angle TRS = 180^\circ$ [Sum of angles in a triangle]................(1) $M_1\ B_1$

But $\angle TRS = x + \angle TRU = x + 55^\circ$ $M_1$

Thus, from (1), $27^\circ + 55^\circ + (x + 55^\circ) = 180^\circ$ $M_1$

$\Rightarrow x + 137^\circ = 180^\circ$ $M_1$

$\Rightarrow x = 43^\circ$ $A_1$

**METHOD 2 (ALTERNATIVE METHOD)**

$\angle SRV = \angle RST = 27^\circ$ [Alternate angles, $RV \parallel TS$] $M_1\ B_1$

$\angle RUT = x + \angle RST = x + 27^\circ$ [Exterior angles of $\Delta RSU$] $M_1\ B_1$

$\angle RTU = \angle TRU = 55^\circ$ [Base angles of isosceles $\Delta RTU$] $M_1\ B_1$

$\angle RTU + \angle TRU + \angle RUT = 180^\circ$ [Sum of angles in a triangle] $M_1\ B_1$

$\Rightarrow 55^\circ + 55^\circ + (x + 27^\circ) = 180^\circ$ $M_1$

$\Rightarrow x + 137^\circ = 180^\circ$

$\Rightarrow x = 43^\circ$ $A_1$

### QUESTION 2

(i) Surface area of the base of the container = $\pi$ ×square of the radius of the base ($r$)

$= \pi \times 2ft \times 2ft = 4\pi\ sq.ft$ $M_2\ A_1$

(ii) Surface area of the cylindrical body = $2 \times \pi \times$ radius of the base ($r$) × height of the cylinder ($h$)

$= 2 \times \pi \times 2ft \times 3ft$

$= 12\pi\ sq.ft$ $M_2\ A_1$

(iii) Surface area of the conical lid = $\pi \times$ radius of the top (r) × slant height of conical lid (l)

$= \pi \times 2ft \times 2ft$

$= 4\pi \; sq.ft$ $M_1 \; A_1$

(iv) Amount of paper in square feet needed to cover the entire exterior portion of the container $= 4\pi \; sq.ft + \; 12\pi \; sq.ft + 4\pi \; sq.ft$

$= 20\pi \; sq.ft$ $M_1 \; A_1$

**QUESTION 3**

$AB = AC$ [Given] $B_1$

$BD = CD$ [Line AD bisects line BC] $B_1$

$AD = AD$ [ AD is common to both triangles] $B_1$

$\angle ABD = \angle ACD$ [Base angles of isosceles triangle] $B_1$

$\angle BAD = \angle CAD$ [Line $AD$ bisects $\angle BAC$] $B_2$

$\angle ADB = \angle ADC$ [Line $AD$ is perpendicular to line $BC$] $B_2$

Therefore, the two triangles, $\Delta ABD$ and $\Delta ACD$ are congruent. $A_2$

# APPENDIX C – LESSON PLANS

**LESSON PLAN 1**

**Subject: Geometry**

**Topic: Properties of Angles Formed by Two Parallel Lines and Their Transversal**

**Duration of lesson: 120 minutes**

**Target group: CE Level 200**

**Tutor: Armah Robert Benjamin**

- **Relevant Previous Knowledge:**

PTs are familiar with concept of lines and how to measure angles. PTs have also learnt parallel lines during the traditional lessons. PTs can solve equations or expressions.

- **Teaching and Learning Materials:**

Mathematical sets, cut-outs of cardboards, pair of scissors, and masking tape.

- **Learning Objectives:**

By the end of the lesson the PT should be able to:

- Identify some basic properties of parallel lines.
- Discover the relationships between the angles formed by two parallel lines cut by a transversal.

**Phase 1: Information/inquiry**

Researcher reviews PTs' previous knowledge on angles and their properties. The researcher then asks PTs questions on the definition of parallel lines. The researcher further holds a conversation with the PTs concerning parallel lines and their properties, in well-known language symbols making the context clear. Researcher ensures that PTs have an understanding of parallel lines and can construct them.

**Phase 2: Guided/Directed Orientation**

**Activity: 1**

PTs in each group were given a rectangular cardboard cut-out, secured by taping them to a table and were guided to draw two diagrams of two parallel lines and a transversal forming eight angles each as can be seen below;

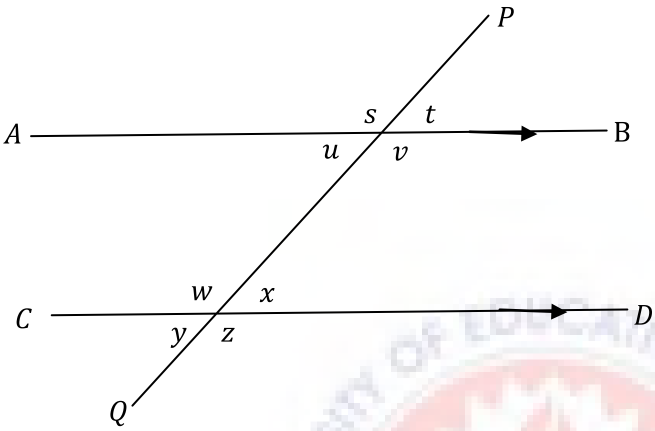



*Diagram* 1

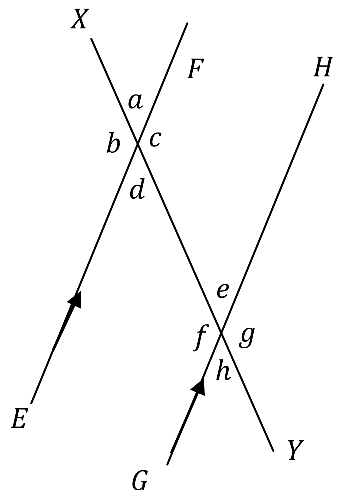



*Diagram* 2

Names of the various pairs of angles were discussed with the PTs; the researcher gave PTs the standard definition of vertical opposite, corresponding, alternate and co-interior angles and pointed them out. For example, angles $u$ and $t$, $v$ and $z$, $v$ and $x$, and $u$ and $x$ are vertical opposite, corresponding angles, co-interior angles and alternate angles respectively.

**Activity 2**

PTs were guided to trace and cut out the various angles, place some on each other to establish, for example, the congruency of vertical opposite, alternate and corresponding angles. They also placed some side by side for their vertices to meet to establish the relationship between co-interior angles as well as adjacent angles. These were done as follows;

- PTs trace and cut out the alternate angles $u$ and $x$ found in diagram 1. Researcher then asks PTs: "Do these angles form a particular shape?" Response should be they form a Z - shape.
- Repeat the above using diagram 2. Have PTs cut out angles $d$ and $e$. Ask: "Do these form a particular shape?" Should resemble an N-shape.
- Next have PTs trace and cut out the co-interior angles $v$ and $x$ in diagram 1. Ask: "What type of shape do they form?" Should resemble a C shape. Repeat with angles $u$ and $w$. Ask: "Do they form a similar shape?" These two form a backwards C shape.
- Have the PTs trace and cut out the corresponding angles $v$ and $z$ in diagram 1. Ask: "Do these angles form a particular shape?" Should resemble an F shape. Repeat with angles $u$ and $y$. Ask: "Do these angles form the same type of shape?" They form an F shape that is turned backwards. Repeat with other examples of corresponding angles from diagram 1 and 2 and they all resemble an F shape that has been flipped or turned a certain way.

**Phase 3: Explicitation**

In this phase, the PTs were asked to describe what they have learned about the topic using their own language. PTs were also asked to come out with the discoveries they made from the hands-activities.

**Phase 4: Free Orientation**

Once all relationships were covered, the researcher incorporated measurements of the angles emphasizing which angles are congruent and which are supplementary. PTs at this point were asked to discover, independently, the values of the unknown angles from the diagram 3;

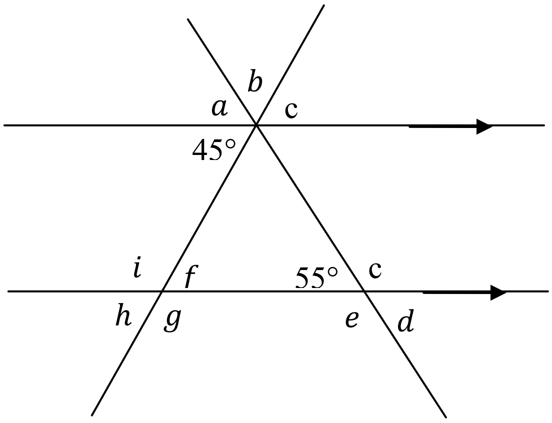



*Diagram* 3

Researcher also asked PTs to discuss the relationships of angles in the diagram below;

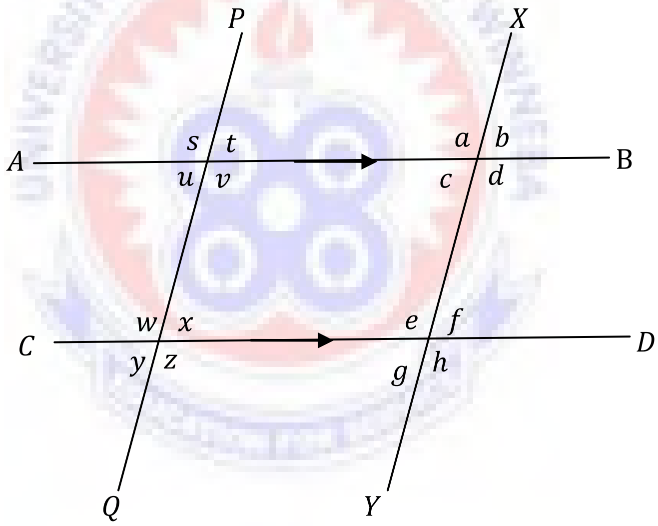


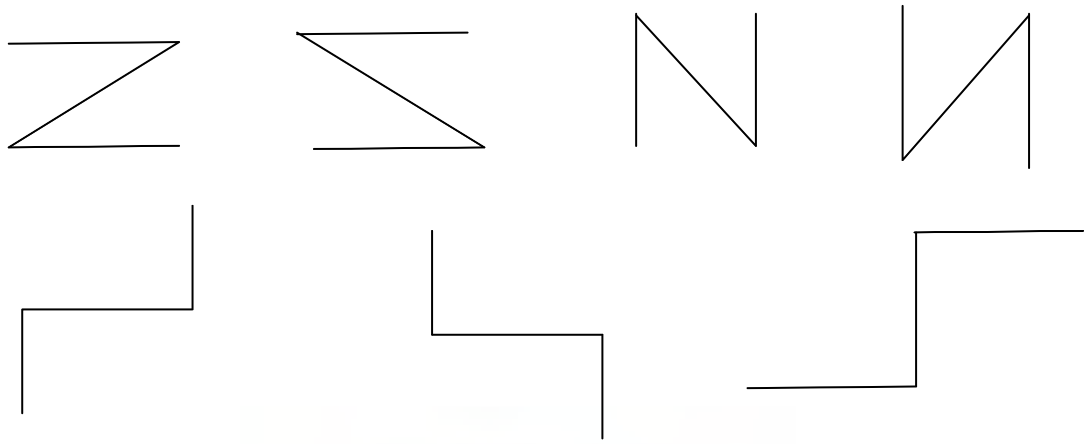

*Diagram* 4

**Phase 5: Integration**

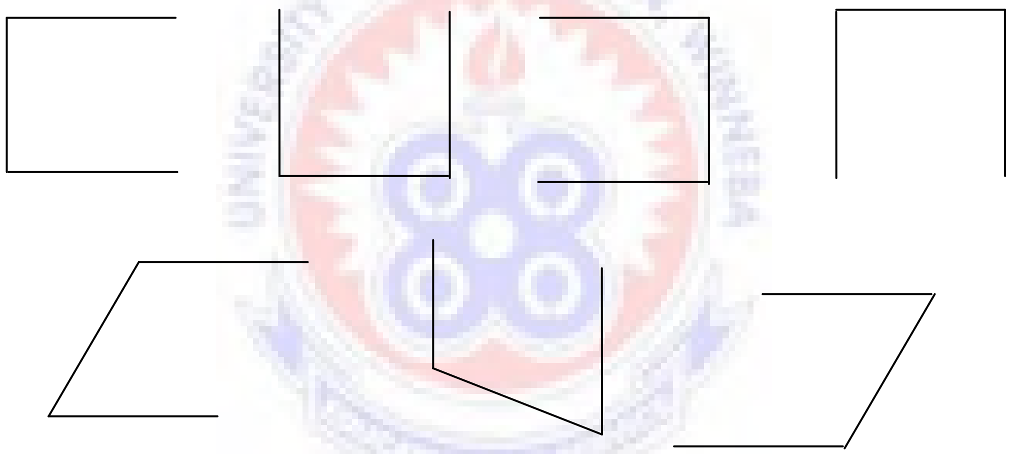
Lesson overview

Diagram 5

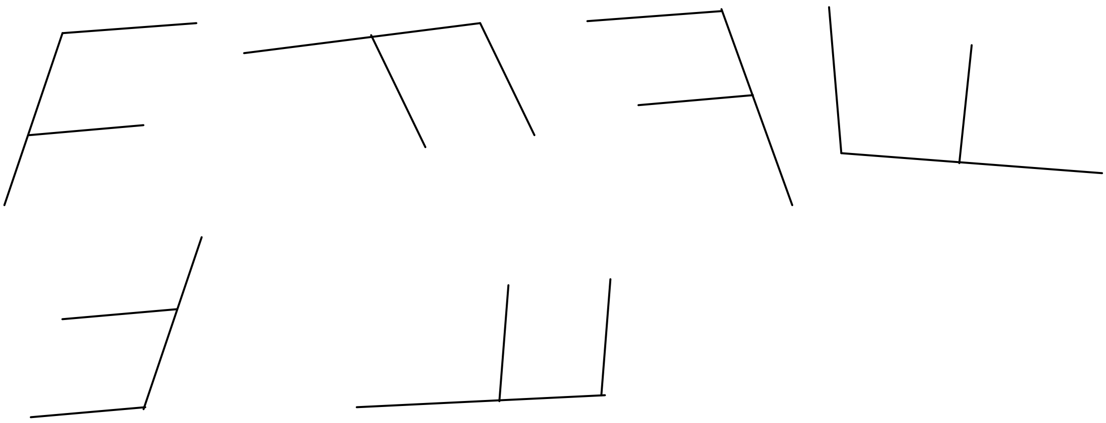
Alternate Angles Shapes



Co-interior Angles Shapes



Corresponding Angles Shapes



**LESSON PLAN 2**

**Subject: Geometry**

**Topic: Properties of Quadrilaterals**

**Duration of lesson: 120 minutes**

**Target group: CE Level 200**

**Tutor: Armah Robert Benjamin**

- **Relevant Previous Knowledge:**

PTs are familiar with concept of triangles. PTs have also learnt Quadrilaterals in their traditional lessons.

- **Teaching and Learning Materials:**

Mathematical sets, papers, pair of scissors, computer and projector (for tutor) for displaying diagrams.

- **Learning Objectives:**

By the end of the lesson the PT should be able to:

- Discover properties of Quadrilaterals (squares, rectangles, rhombuses and parallelograms).
- Use relationships among sides and angles of Quadrilaterals (squares, rectangles, rhombuses and parallelograms).
- Use relationships among diagonals of Quadrilaterals.

**Phase 1: Information/inquiry**

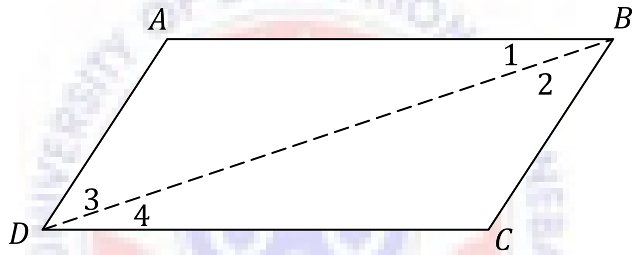
The researcher reviewed PTs' previous knowledge on types of triangles and their properties. The researcher further held a conversation with the PTs concerning triangles and their properties, in well-known language symbols making the context clear. This was to begin the activities with a sort of informal work that gives PTs an idea of the topic.

**Phase 2: Guided/Directed Orientation**

**Activities:**

PTs in their various groups were given a work sheet showing diagrams of the various quadrilaterals (square, rectangle, parallelogram and rhombus) and guided through activities as follows;

Draw a diagonal to the parallelogram $ABCD$ by connecting $AC$ to form $\Delta ABC$ and $\Delta CDA$ as shown below;




Guide PTs to use properties of parallel lines to indicate all angles on the diagram. With $AB \parallel DC$ and $AD \parallel BC$, guide PTs to show that $AB = DC$ and $AD = BC$, $\angle DAB = \angle DCB$ and $\angle ABC = ADC$ as follows;

$\angle ABD = \angle BDC$ [Alternate angles, $AB \parallel DC$]

$\angle DBC = \angle BDC$ [Alternate angles, $AD \parallel BC$]

$AC$ is a common side

$\therefore \Delta ABC \cong \Delta CDA$

$\therefore AB = CD$ and $BC = DA$

Thus, it has been shown that, $\angle ABD = \angle BDC$ and $\angle DBC = \angle BDC$, therefore

$\angle ABC = \angle ABD + \angle BCD = \angle ADB + \angle BDC = \angle ADC$, Also

$\angle DAB = \angle DCB$

Now guide PTs to summarize the properties of a parallelogram;

- Both pairs of opposite sides are parallel.
- Both pairs of opposite sides are equal in length.
- Both pairs of opposite angles are equal.
- Both diagonals bisect each other.

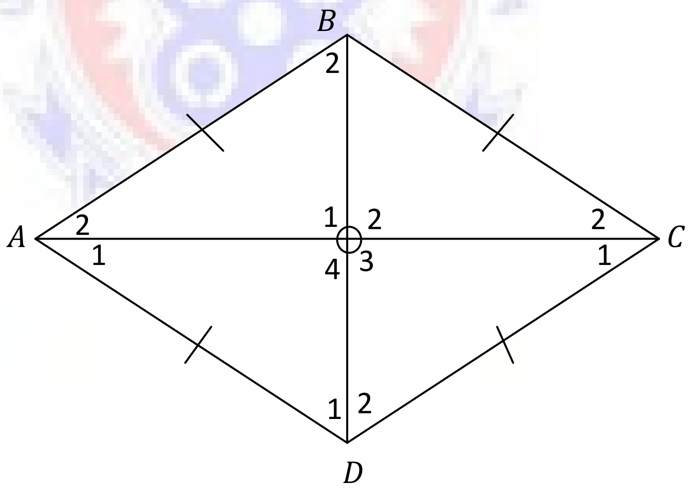
In a similar way, guide PTs in their various groups to discover the special properties of squares, rectangles and rhombuses.

**Phase 3: Explicitation**

In this phase, the PTs were asked to describe what they have learned about the topic using their own language. PTs were asked to come out with the discoveries they made from the hands-activities (that is the properties of special Quadrilaterals).

**Phase 4: Free Orientation**

Now, PTs independently solve the example below;




$ABCD$ is a rhombus. Show that:

1. the diagonals bisect each other perpendicularly;
2. the diagonals bisect the interior angles.

**Phase 5: Integration**

Lesson overview.

**LESSON PLAN 3**

**Subject: Geometry**

**Topic: Relationships between Properties of Quadrilaterals**

**Duration of lesson: 120 minutes**

**Target group: CE Level 200**

**Tutor: Armah Robert Benjamin**

- **Relevant Previous Knowledge:**

PTs are familiar with concept of triangles. PTs can discover the properties of quadrilaterals.

- **Teaching and Learning Materials:**

Mathematical sets, papers, pair of scissors, computer and projector (for tutor) for displaying diagrams.

- **Learning Objectives:**

By the end of the lesson the PT should be able to:

- Define and classify special types of Quadrilaterals.

**Phase 1: Information/inquiry**

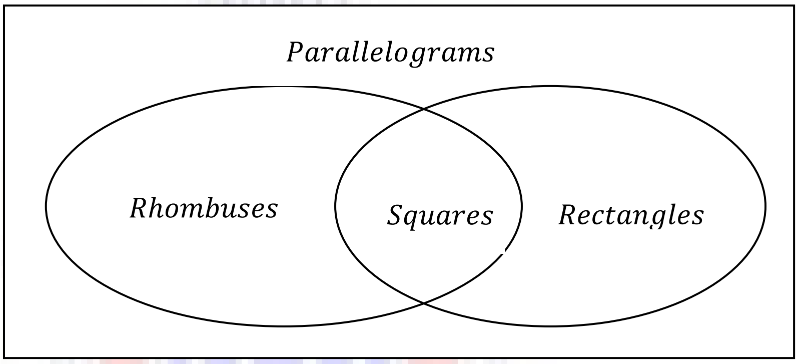
Tutor reviews PTs' previous knowledge on properties of Quadrilaterals (squares, rectangles, rhombuses and parallelograms). The tutor further holds a conversation with the PTs concerning Quadrilaterals (squares, rectangles, rhombuses and parallelograms) and their properties, in well-known language symbols making the context clear.

**Phase 2: Guided/Directed Orientation**

**Activities**

PTs in their various groups are guided to examine the special Quadrilaterals very carefully and use a Venn diagram to show the relationships that exist among them. The researcher guided PTs to generally establish the following using the appropriate terminologies as follows:

- Parallelograms, rhombuses, rectangles and squares all have two pairs of parallel sides, so parallelograms are the largest set.
- Rhombuses have four congruent sides, so they are equilateral and rectangles have four congruent angles, so they are equiangular.
- Squares are both equilateral and equiangular, so they have the characteristics of rhombuses and rectangles and hence belong to both groups.




Relationship among Special Parallelograms

**Phase 3: Explicitation**

In this phase, the PTs were asked to express in their own words what they have discovered in the previous phase. PTs were asked to come out with the discoveries they made from the hands-activities.

**Phase 4: Free Orientation**

Now, PTs are asked to answer the following questions independently;

1. Which of these are true and which are false? Explain why in each case.

   i. All squares are rectangles

ii. All squares are rhombuses

iii. All squares are parallelograms

iv. No rectangles are rhombuses

v. No rectangles are parallelograms

vi. No rectangles are squares

vii. Some parallelograms are rectangles.

2. Draw a shape using these conditions. If the task is impossible, say why.

i. A rectangle that is *not* a rhombus.

ii. A rectangle that is a rhombus.

3. Draw Venn diagrams to show the relationships between the following sets (in some cases you may need to include an extra set):

   i. Squares and rectangles.

   ii. Parallelograms, rhombuses, and squares.

   iii. All quadrilaterals, rhombuses and parallelograms.

**Phase 5: Integration**

Lesson overview.

## APPENDIX D - OBSERVATION SCHEDULE FOR TUTORS BASED ON THE VAN HIELE THEORY

Subject:……………………………………
Date……………..
Lesson starts:……………………………. Lesson ends:…………………………
Teacher observed…………………………..
Observer:…………………………….
Level: 100
Topic:………………………………………………..
Objective(s):………………………………………………………………………

| Activities | Level | Weak | Moderate | Strong | Comments |
|---|---|---|---|---|---|
| Tutor displays a variety of different readymade geometric shapes to the class. | 1 | | | | |
| Tutor asks learners to list examples of shapes in the outside world. | 1 | | | | |
| Tutor uses informal language to describe shapes. | 1 | | | | |
| Tutor asks learners to construct shapes according to their appearance. | 1 | | | | |
| Tutor asks learners to solve routine problems by operating on the shapes rather than referring to the properties in general. | 1 | | | | |
| **Activities** | **Level** | **Weak** | **Moderate** | **Strong** | **Comments** |
| Tutor introduces a typical topic on properties of shapes. | 2 | | | | |
| Tutor asks learners to list properties of shapes. | 2 | | | | |
| Tutor asks learners to construct shapes according to their properties. | 2 | | | | |
| Tutor teaches learners how to solve a certain problem by using known properties of shapes. | 2 | | | | |
| Tutor provides hands-on activities to learners which require them to focus on the properties of the shapes. | 2 | | | | |
| **Activities** | **Level** | **Weak** | **Moderate** | **Strong** | **Comments** |
| Tutor asks learners to describe what they have learned about the topic using their own language | 3 | | | | |
| Tutor guides learners to analyze properties of geometric shapes and to understand the interrelationship between different types of shapes | 3 | | | | |

| | | | | | |
|---|---|---|---|---|---|
| Tutor guides learners to make simple inferences e.g. in a quadrilateral, opposite sides being parallel necessitates opposite angles being equal. | 3 | | | | |
| Tutor ensures that the accurate and appropriate terminology is developed and used | 3 | | | | |
| Tutor guides learners in doing problem solving, including tasks in which properties of shapes are important components | 3 | | | | |
| **Activities** | **Level** | **Weak** | **Moderate** | **Strong** | **Comments** |
| Tutor designs tasks that provide the learners with problems that are open-ended and have multi-path solutions. | 4 | | | | |
| Tutor guides learners to create, compare, and contrast different proofs. | 4 | | | | |
| Tutor guides learners to supply reasons for statements in formal proofs | 4 | | | | |
| **Activities** | **Level** | **Weak** | **Moderate** | **Strong** | **Comments** |
| Tutor guides learners to work in different geometric or axiomatic systems | 5 | | | | |