UNIVERSITY OF EDUCATION, WINNEBA

EFFECT OF NUMBER PATTERNS ON JUNIOR HIGH SCHOOL LEARNERS' TRANSITION FROM ARITHMETIC TO ALGEBRA: THE CASE OF AGONA WEST MUNICIPALITY

EMMANUEL ADOKOH

202140554

2024

UNIVERSITY OF EDUCATION, WINNEBA

EFFECT OF NUMBER PATTERNS ON JUNIOR HIGH SCHOOL LEARNERS' TRANSITION FROM ARITHMETIC TO ALGEBRA: THE CASE OF AGONA WEST MUNICIPALITY

EMMANUEL ADOKOH

(202140554)

A Thesis in the Department of Basic Education, School of Education and Life-Long Learning, submitted to the School of Graduate Studies, in partial fulfilment of the requirement for the award of the degree of Master of Philosophy (Basic Education) in the University of Education, Winneba

JUNE, 2023

DECLARATION

Student's Declaration

I, Emmanuel Adokoh, declare that this thesis with the exception of quotations and references contained in published works which have all been identified and duly acknowledged is entirely my own original work, and it has not been submitted, either in part or whole for another degree elsewhere.

Signature…………………………………………

Date……………………………………………....

Supervisor's Declaration

I hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of thesis as laid down by the University of Education, Winneba.

Supervisor's Name: Prof. Clement A. Ali

Signature…………………………………………

Date……………………………………………....

DEDICATION

To my brothers: Mr. Isaac Paintsil, Mr. Richard Benya and Mr. Charles Baah

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to the Almighty God for how far he has brought me.I would also like to express my sincere gratitude to my supervisor Dr. Clement A. Ali, Department of Basic Education for his unconditional supports and invaluable guidance throughout the research which made possible to complete my research work.

I would again like to express my deepest gratitude to Dr. Vivian N. A. Acquaye Ag: Graduate Coordinator, Department of Basic Education for her supportive and encourageable suggestions for my thesis.

Likewise, I would also like to offer my profound gratitude to Mr. Aikins Addae Lecturer, Department of Basic Education for the most constructive instructions and insights for the completion of this research. I would like to manifest my heartily gratitude to all lecturers, Department of Basic Education, for motivating me with creative ideas with study materials and guidelines. Similarly, I want to express my special gratitude to all the 2020-2021 Mphil students of the Department of Basic Education, Winneba for their supports. I wish to convey my especial thanks to my brothers and sisters for providing the required helped to complete the thesis.

Finally, I am thankful to Mr. Samuel Tatsi, Mr. Mustapha K. Tahiru, Mr. Alexader Afakpor, Mr. Richmond Amprah, Mr. Samuel Amo snr, and all Mphil students in 2020-2021 academic year, friends and well-wishers for supporting and motivating me throughout the research work who directly and indirectly standby me.

TABLE OF CONTENTS

Content **Page**

LIST OF TABLES

Table **Page**

ABSTRACT

This study sought to determine the effect of number patterns on junior high school learners' transition from arithmetic to algebra. A quasi-experimental non-equivalent group's pre-test–post-test control group design was employed, involving two intact classes comprising one hundred (100) Basic Seven (7) learners. Tests, including both pre-test and post-test assessments, were used to collect data on the learners. The pre-test assessed the learners' initial performance, identifying strengths and weaknesses to inform necessary interventions, while the post-test measured the effectiveness of using number patterns in facilitating the transition from arithmetic to algebra. Data analysis using mean, standard deviation, and independent samples t-test revealed that number patterns positively impacted learners' academic performance in transitioning from arithmetic to algebra, with results showing a significant statistical difference at $\alpha = 0.05$. Consequently, it is recommended that teachers adopt number patterns to enhance learners' academic performance during this critical transition phase.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This chapter consists of the background to the study, statement of the problem, purpose of the study, objective of the study, research questions, significant of the study, delimitation and organization of the study.

1.1 Background to the Study

Mathematics is a culturally shared study of patterns and language for everyday life, a central part of human communication and a means of articulating patterns, relationships, rationality and aesthetic (Jaworski, 2006). Mathematics is the bedrock of science and technology (Haruna, 2014). This makes it clear that, mathematics forms the foundation on which science and technology rest. In this regard, Anthony and Walshaw (2014) once asserted that the main purpose of teaching and learning mathematics subject should be to develop the ability of learners and connect any mathematical problems to the learners' real-world situations. On the other hand, mathematical ability involves effective thinking with conceptual understanding, thus learners need to be taught to think logically along with practicing numerical problems (Mahmood, et al., 2012).

Traditionally, mathematics education is on the grounds of arithmetic then algebra approach. More explicitly, learners are first expected to gain procedural fluency for arithmetic in elementary grades; then, they face algebra, mostly based on a procedural approach in the middle grades (Blanton et al., 2007). Parallel with this approach, algebra does not appear in the Ghana mathematics curriculum as a learning domain for elementary grades (MoE, 2018). However, the transition from concrete arithmetic thinking to increasingly abstract algebraic reasoning, which is required in secondary school and later tertiary, became a hurdle for learners' mathematics learning (Bekdemir & Isik, 2007; Carpenter et al., 2000; Knuth et al., 2016). This problem led educators and mathematics education researchers to consider the "deep, long-term algebra reform" (Kaput, 1999). Kaput (1999) described a route to that reform as "infusing algebra throughout the mathematics curriculum from the very beginning of the school". Teachers can provide learners with a more sophisticated algebra background, which involves solid understandings and experiences for middle grades and high school, by placing algebra in the curriculum from kindergarten onward (NCTM, 2000). This new approach is currently known as "Early Algebra."

Early algebra does not mean serving common algebraic concepts and procedures addressed in the middle grades to the elementary student's earlier (Carraher et al., 2008). Besides, early algebra is not an attempt to make the elementary curriculum bigger (Kaput et al., 2008). Early algebra is a way of thinking to provide learners opportunities to generalize relationships and mathematical facts by delving into the concepts already in the curriculum to provide a deep and coherent mathematical understanding (Blanton et al., 2007).

Algebra is vital importance since it function as a gatekeeper for later mathematics courses (Ferrini-Mundy, McCrory, Senk & Marcus, 2005).It is the gatekeeper of the foundation of mathematics (National Council of Teachers of Mathematics, 2000).In other words, algebra has significant and pivotal roles in mathematics as a whole. It plays the foundational role in mathematics (Yarkwah, 2017).This implies that, mathematics falls greatly on algebra. A good foundation in algebra will positively affect the performance of learners in mathematics and the opposite is also true. Algebra has applications in almost all the other areas of mathematics (Yarkwah, 2017).

Increasingly, algebra is the focus of mathematics discussions in schools and districts across the United States. Policymakers, professional organizations, and researchers emphasize the importance of developing algebraic reasoning at increasingly earlier ages. The National Mathematics Advisory Panel (2008) has issued initial reports stating that learners need to develop understanding of concepts, problem-solving skills, and computational skills related to algebra in kindergarten. In 2006, the National Council of Teachers of Mathematics published the Curriculum Focal Points for preschool through Grade 8 Mathematics, which emphasizes connections to algebra as early as kindergarten and promotes the development of algebraic reasoning across the elementary and middle school grades. Finally, mathematicians and mathematics educators are speaking up about the need to increase teachers' awareness and abilities for teaching algebra across the grades.

The knowledge acquired in algebra affects mathematics performance in general. This makes it difficult for one to excel in mathematics, when he or she has a weak foundation in algebra. The evidence that Algebra form the basis for many content concepts in mathematics is clearly seen in areas such as Sets, Length and Area, Shapes and Space, Relation, Algebraic Expressions, Linear Equation and Inequalities, Areas and Volume and Angle. Looking at the Junior High School Mathematics Curriculum, Algebraic Expressions are supposed to be taught in Form 1, Form 2 and Form 3 (GES, 2007). This rings a bell on the crucial advantages for basic school pupils to get control over algebra content and its application in other basic school mathematics contents.

Teachers good repertoire of knowledge in algebra has the potential of affecting learners achievement in mathematics (Yarkwah, 2017), and as consequence affects science and technology and hence national development.

This study is situated on the fact that, the knowledge of teachers greatly affects learners learning (Wilmot, 2009; Yarkwah, 2017). This established that, teachers' algebra knowledge affects learners' algebra learning (the algebra knowledge they acquire), hence their performance in mathematics. Research has also established that, student algebra knowledge affects significantly their general mathematical knowledge hence their general performance in mathematics, this is because mathematics falls significantly on algebra (Yarkwah, 2017). In reference to this assertion, one can say that, the algebra teaching knowledge of mathematics teachers affect the general performance of learners in mathematics. In view of this, the algebra teaching knowledge level of basic school teachers should be monitored to positively affect learners' algebra knowledge, hence their performance in mathematics at the basic school level.

Teachers can help learners make the transition by developing their algebraic thinking early on. Much of the difficulty that learners encounter in the transition from arithmetic to algebra stems from their early learning and understanding of arithmetic. Too often, learners learn about the whole number system and the operations that govern that system as a set of procedures to solve addition, subtraction, multiplication, and division problems. Teachers may introduced number properties as ''truths'' or axioms without developing learners' deep conceptual understanding. To provide rich and explicit instruction to learners in early algebraic thinking, teachers should clearly model what they want learners to be able to do. Learners must understand variables and constants, decomposing and representing word problems algebraically, symbol manipulation, and functions to develop algebraic thinking.

When we think about algebra in the curriculum, we often think of a separate area of mathematics concerned with symbols and equations, such as $3x + 7y - 2 = 30$. Mathematics curriculums often reinforce the notion of separateness by identifying algebra as a distinct strand with such subtopics as patterning, data analysis, simple functions, and coordinate systems. However, arithmetic and algebra are not mutually exclusive areas of mathematical study.

Basic algebra, as opposed to modern or abstract algebra, extends learners' understanding of arithmetic and enables them to express arithmetical understandings as generalizations using variable notation. Much of the difficulty that learners encounter in the transition from arithmetic to algebra stems from their early learning and understanding of arithmetic.

The National Council of Teachers of Mathematics has attempted to bridge the gap between arithmetic and algebra by embedding algebraic reasoning standards in elementary school mathematics. From in grades 3 to 5, algebra is embedded with number and operations as one of the three main focal points; beginning in grade 6, algebra is the predominant topic. However, it is not always clear how to develop learners ' algebraic thinking as they learn about numbers, operations, properties of numbers, data display and analysis, and problem solving.

The role of algebra in mathematics is so sensitive that, proper attention needs to be given to it study and understanding (Ball, 2003). Algebra as a foundation course, serves as a concierge, posing varying opportunities for entry into advanced mathematics course for ground work for college and for the world of work. Algebra plays a key role in building the mathematical foundation of young people who desire to pursue mathematics at the higher levels of education. Research tells us that success in algebra is a factor in many other important student outcomes. Emerging researches suggest that, student who start an algebra curriculum in the early grade tend to do better in the subject in secondary school level (Knuth , Stephens, Blanton & Gardiner, 2016). The knowledge of teachers greatly affects learners performance (Wilmot, 2009; Yarkwah, 2017).

In effect, it is what teachers have that they pass on to their learners. If a teacher has a weak foundation in a subject matter, he or she will pass same or similar knowledge to his or her learners and noting higher than his or her own knowledge can be communicated. In this regard, as mathematics educators, we must consider the type of mathematical knowledge basic school teachers need to provide all learners with reasonable prospect to lean algebra (Yarkwah, 2017). We can never dispute the fact that teachers need a profound understanding of the mathematics they teach to help learners to perform as expected on the field **(**CBMS, 2001**).** Research is packed with fact that teachers` content knowledge is often thin and insufficient to provide instruction for learners in today`s classrooms (Ball 1988, 2003; Ball &Bass, 2000; Ma, 1999; Stacey, et al., 2001). This generally affects the overall performances of learners in this subject where these problems exist and mathematics is no exception. Over the years, basic school pupils have demonstrated weak control over algebra related contents in the Basic Education Certificate Examination (B. E. C. E). Therefore, this study sought to find the effect of number patterns on junior high school learners' transition from arithmetic to algebra.

1.2 Statement of the Problem

Algebra is a fundamental component of mathematics education, yet many learners face significant challenges in mastering it. Research and teaching experiences indicate multiple difficulties that students encounter in learning algebra, stemming from factors such as teaching methods, teachers' styles, and a lack of practical learning experiences (Kilpatrick & Izsak, 2008). Learners often struggle with understanding the equal sign, transitioning from arithmetic to algebraic conventions, and grasping the abstract nature of algebraic structures.

The West African Examination Council Chief Examiner's reports from 2010 to 2018 have consistently highlighted weak foundations in algebra among pupils who sit for the Basic Education Certificate Examination (B.E.C.E), negatively impacting their overall mathematics performance. Specific difficulties include working with variables, performing arithmetic operations involving positive and negative signs, expanding brackets, solving word problems involving fractions, and correctly applying the BODMAS rule. These persistent issues suggest that traditional teaching methods are insufficient in addressing the needs of students in understanding algebra.

Observations of the teaching and learning of algebra reveal significant conceptual, perceptual, theoretical, and practical gaps, particularly in the transition from Number Patterns to Algebra. Teachers often lack the pedagogical knowledge to effectively integrate number pattern teaching approaches, and textbooks fail to adequately incorporate these methods. Additionally, there is a shortage of resources to support innovative teaching practices.

Recent studies suggest that number pattern teaching methods can play a crucial role in enhancing students' understanding of algebra (Rittle-Johnson et al., 2019; Yarkwah, 2017). Despite these findings, there is a lack of empirical research specifically examining the impact of number pattern teach o find out the difference in scores among pupils taught with Number Pattern and without Number Patterns. Therefore, this study seeks to investigate the effect of number patterns on junior high school learners' transition from arithmetic to algebra.

1.3 Purpose of the Study

This study sought to determine the effect of number patterns on junior high school learners' transition from arithmetic to algebra in the Agona West Municipality.

1.4 Research Objectives

The following research objectives were formulated to guide this study:

1. To find out the difference in scores among pupils taught with Number Pattern and pupils taught without Number Patterns.
2. To explore the difference in scores among pupils taught with arithmetic-algebra connection and without arithmetic-algebra connection using number patterns.
3. To determine the difference in performance among pupils taught with algebraic reasoning and without algebraic reasoning using number patterns.

1.5 Research Questions

The following research questions were derived from the objectives to guide this study:

1. What is the difference in performance among pupils taught with Number Pattern and pupils taught without Number Patterns?

2. What is the difference in performance among pupils taught with arithmetic-algebra connection and without arithmetic-algebra connection using number patterns?
3. What is the differences in performance among pupils taught with algebraic reasoning and without algebraic reasoning using number patterns?

1.5.1 Research Hypotheses

The following hypotheses were tested at 0.05 level of significance.

H_{01}. There is no statistically significance difference in performance pupils taught with Number Pattern and without Number Patterns.

H_{02}. There is no statistically significance difference in performance between pupils taught transitioning from arithmetic-algebra using number patterns and pupils taught without using number patterns.

H_{03}. There is no statistically significance difference in performance between pupils taught algebraic reasoning using number patterns and pupils taught without number patterns.

1.6 Significance of the Study

It is anticipated that the findings of the study would shed light on the benefits of using number patterns in mathematics learning of arithmetic and algebra in particular. The study would also attempt to bridge the gap between theoretical and practical side of using number patterns in teaching transition from arithmetic to algebra. Moreover, the findings of this study may be helpful to curricular designers and GES to develop teaching materials and resources which suit various ways of teaching and match student's level of achievement in Mathematical in general. Furthermore, this study may help teachers by facilitating their role as well as learners by helping them absorb the concept of Mathematics quite easily and smoothly. Finally, this study may encourage other researchers to conduct further studies on the same topic, which will enrich both the local and international literature.

1.7 Delimitations

This study was conducted among learners in the Agona West Municipality of Ghana. Although the Municipality has a lot of basic schools, only two were selected for the study. The selection of the schools depended on what the researcher wants to know, the purpose of the research, what makes the study credible and what could be done with the available resources at hand (Patton, 2002). The population for the study was delimited to a sample size of 100 learners in the selected schools. Even though other approaches could be used for the same purpose, the researcher adopted the quantitative approach.

1.8 Organization of the Study

This research is presented in five chapters. Chapter one, presents the background to the study, statement of the problem, purpose of the study, research objective, research questions, significance of the study and organization of the study. Chapter two is the review of related literature to the study. This comprises the theories and concepts as well as empirical evidence of what other people have already discovered or written on the topic. Chapter three deals with the methodology consisting of the research design, population and sample selection, research instruments. Chapter four deals with analysis of data collected. Chapter five deals with summarizing the findings based on the data, drawing up conclusion and necessary recommendations for further study.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.0 Overview

This chapter discussed a review of available researched documents of previous studies done by recognised authorities on the issue under study. The thesis was reviewed in two parts, namely the theoretical/conceptual framework and the empirical review. Relevant literature was reviewed in the following areas:

1. Theoretical Framework for the Study
2. Conceptual Framework
3. Gender differences in performance in Mathematics
4. Motivation
5. Number Patterns
6. Significance of Teaching Patterning
7. Significance of Learning Patterning
8. Patterning and Mathematical Skills
9. Patterning and Mathematical Achievement
10. Arithmetic and Building on student's understanding of Arithmetic
11. The Arithmetic-Algebra Connection
12. Teaching Arithmetic and Algebra
13. Meaning and Importance of Algebra
14. Fostering Learners' Algebraic Reasoning
15. Challenges in Learners' Algebraic Reasoning
16. Summary

2.1 Theoretical Framework

A theoretical framework is the structure that can hold or support a theory of a research study. According to Abend, (2008), and Swanson and Chermack (2013), the theoretical framework introduces and describes the theory that explains why the research problem under study exist. Researchers therefore use theories to guide them in their studies to observe and generate new ideas. In providing a theoretical framework to underpin a study, researchers therefore fall on theories. In this thesis, the theory of Structure of Observed Learning Outcomes (SOLO) was mainly used to form the basis of the thesis. The literature was reviewed from research articles, journals, books, and includes empirical studies.

2.1.1 Theory of Structure of Observed Learning Outcomes

Structure of Observed Learning Outcomes (SOLO) offers a structured outline for the learners to use to build their learning and thinking. It motivates learners to ponder where they are presently in terms of their level of understanding, and what they must do to progress. Solo Taxonomy is a systematic way that describes how learners' understanding build from easy to difficult while learning different tasks or subjects. The Solo Taxonomy can be used to enhance the quality of learning within the classroom teaching and provide a systematic way of developing deep understanding (Damopolii, 2020).

Student learning can be guided in ways that promote deep learning. SOLO describes levels of progressively complex understanding, through five general stages that are intended to be relevant to all subjects in all disciplines. In SOLO, understanding is conceived as an increase in the number and complexity of connections learners make as they progress from incompetence to expertise. Each level is intended to encompass and transcend the previous level. The SOLO taxonomy was created by carefully analyzing student responses to assessment tasks (Biggs and Collis, 1982; Collis and Biggs, 1986), and has been validated for use in a wide range of disciplines (Hattie and Brown, 2004).

In developing SOLO, Biggs and Collis (1982) took into account many factors that affect student learning, such as: learners' prior knowledge and misconceptions, motives and intentions regarding education, and their learning strategies. The result is a construct that has both quantitative and qualitative dimensions. The first level of SOLO is really a stage of ignorance that exists outside of the taxonomy. The next two stages (uni-structural and multi-structural) are both levels of surface understanding, in which knowledge (usually concrete knowledge) accrues in greater quantity. No increase in quantity in the number of facts or ideas known can result in depth of learning. Depth comes with a qualitative change in how ideas are understood in connection with other ideas. These connections are connected to increasing abstraction, so the last two levels of SOLO are characterized not only by the integration and connection of knowledge but also, necessarily, by increased abstraction. Such qualitative change is cognitively challenging, but Biggs and others who have written about SOLO advise to remember that the later levels of SOLO aren't necessarily more "difficult" than previous levels; after all, remembering a vast number of discrete facts can be quite difficult. That doesn't make it useful, either.

Hattie and Brown (2004) contend that depth is not the same as difficulty perhaps it is this confusion that explains why so many questions posed by teachers do not require learners to use higher-order thinking skills but instead require a greater attention to details. At the pre-structural level, you haven`t yet entered the learning cycle. The learning cycle is the sequence of stages from uni-structural to relational, in which your understanding grows and deepens. You may need to go through various levels within the learning cycle multiple times as new ideas are brought in, but the goal is for you to leave the learning cycle eventually by reaching the extended abstract stage (Panizzon, 2003; Pegg, 2003; Levins and Pegg, 1993; Pegg, 1992; Campbell et al, 1992).

2.2 Conceptual Framework of the Study

Conceptual framework is a framework that consists of concepts that are placed within a logical and sequential design (Naizaro, 2012). The following conceptual framework enables the researcher to clarify concepts and propose relationships among the concepts in a study.

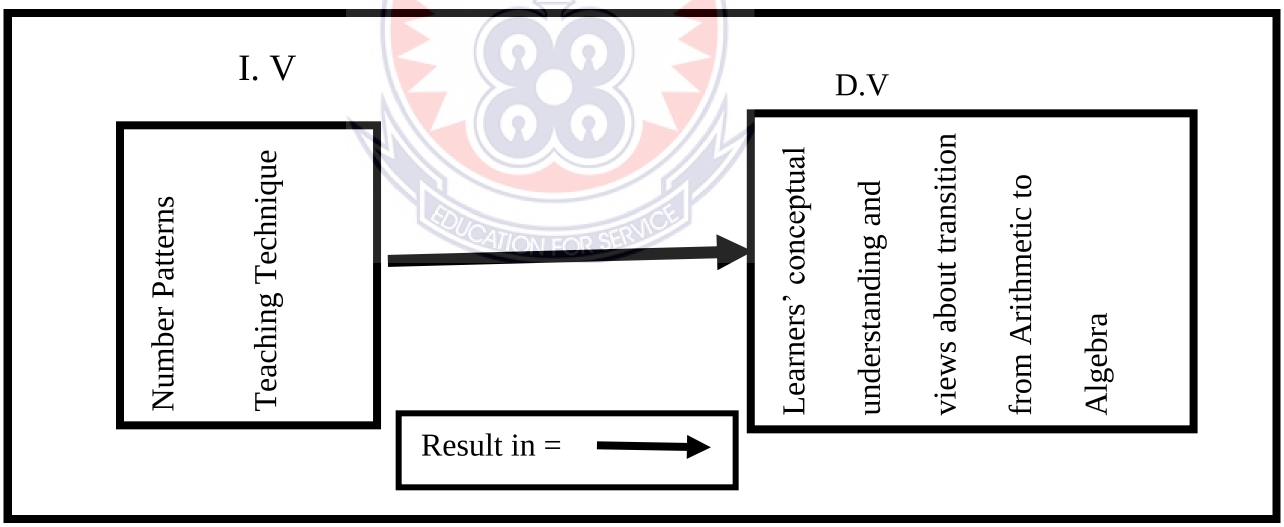


Fig 1. Transition from Arithmetic to Algebra (Researcher's own construct)

On Figure 1, the independent variables are the methods, techniques and skills in using Number patterns. The dependent variables are the learners' conceptual understanding of Algebra. When teaching and learning start from Arithmetic to Algebra, it will result in improvement in mathematics performance. Apart from Number Patterns, there are other moderating variables. These variables include gender, motivation, structure of patterns, and teaching and learning patterning. These variables are briefly described below.

2.3 Gender Differences in Performance in Mathematics

Gender differences in mathematics performance have been a contentious issue in educational domain and research documents show great discrepancies among boys and girls in school mathematics (Sprigler and Alsup 2003). Also, according to Janson (1996), Mullis' study (as cited in Manoah, Indoshi and Othuon, 2011), male advantage in mathematics performance is a universal phenomenon, while early research (Fennema and Sherman, 1976) indicated that males out performed females in mathematics achievement at the junior high and senior high school levels. These earlier studies added that there were also significant differences in achievement towards mathematics between the boys and girls. Gallagher and Kaufman (2006) recognized that the achievement and interest of boys are higher than the girls. They however explained that they do not know the main cause of these differences. In spite of massive research evidence for male's superiority in mathematics achievement, some research findings do not support the difference between the two genders in mathematics achievement.

Also, Springler and Alsup (2003) referred to researcher indications that showed no gender differences on the mathematically reasoning ability at elementary level. Findings from longitudinal study about gender differences in mathematics show that there is no difference among boys and girls in mathematics achievement, (Ding, Song and Richardson, 2007). This study shows that growth trend in mathematics between the two genders was equivalent during the study times. According to Mullis, Martin, Gonzales, & Chrostowski, (2004) the recent international study conducted by IEA showed that, on average across all countries, there was essentially no difference in achievement between boys and girls at either the eighth or fourth grade. Over the last three decades, diverse theories and frameworks have been developed and many have tried to identify factors that influence mathematics performance in order to reduce gender inequality in mathematics achievement (O'Connor-Petruso & Miranda)'s study (as cited in Campbell 2005). Geary (2000) has argued that research evidence shows that gender differences in mathematics achievement are due to various factors such as biological factors, mathematics learning strategies, sex hormones on brain organization, and symbolic gender. O'Connor-Petruso & Miranda's study (as cited in Campbell 2005) have shown that gender differences in mathematics achievement become apparent at the secondary level when female learners begin to exhibit less confidence in their mathematics ability and perform lower than the males on problem solving and higher-level tasks.

In Ghana, Eshun (1999, 2000) observed a higher achievement of males than females in mathematics at the secondary school level. Wilmot (2008) even showed that in Ghana, the difference in mathematics achievement between boys and girls begins or becomes apparent at the sixth grade. According to Asante (2010) the Ghanaian government in realization of the significant role of mathematics to nation building, has made the subject compulsory at the basic and secondary levels. This was aimed at ensuring the inculcation of mathematics literacy and the associated equipment with logical and abstract thinking needed for living, problem solving and educational furtherance. Therefore, there is no gain saying that gender is a significant variable in this study.

2.4 Motivation

According to Huber (2006), “the concept of motivation is derived from the Latin word "movere", which means to move”. Motivation is an action word that influences every aspect of our daily lives. Whether in athletics, academics, business, industry, entertainment or any other Endeavour, motivation is fundamental in the level of success an individual attains. We are either motivated or not motivated to perform certain tasks. Psychologists assert that motivation activates behavior and propels an individual forward toward achieving goals or needs. Motivation describes the process that energizes and stimulates human behavior toward reaching specific goals (Huber, 2006). In essence, motivation propels an individual to act and continue until a goal or need is met (Slavin, 2006).

According to Slavin (2006), motivation is a psychological force that moves a person into action to attain preset goals or the satisfaction of certain needs. Ormrod (2008) also defines motivation as an internal state that arouses us to action, pushes us in a particular direction and keeps us engaged in certain activities (Ormrod, 2008). However, according to Fianu (2005), the word motivation is derived from ‘motive’ which means an impulsion or urge to attain some goal-object. He stressed that motivation implies direction, that is, goals toward which behavior is oriented as a result of physiological conditions, interest, attitudes and aspirations.

Nevid (2013), sees motivation as the term that refers to factors that activate direct and sustain goal-directed behavior. Motives are the ‘whys of behavior, the needs or wants that drive behavior and explain what we do. We do not actually observe a motive; rather, we infer that one exists based on the behavior we observe. Motivation arouses interest. Interest is the mother of attention and attention is the mother of learning.

Supportably, motivation may be broadly interpreted to mean the ‘whys of behavior’ (Amissah, Samuel & Sam-Tagoe, 2009). They see motivation as a contested concept with no agreed-upon single definition. With this, they opine that motivation is a process that initiates, guides and maintains goal -oriented behaviour. In the view of O'Donnell et al (2009), motivation involves any force that energizes and directs behavior. To them, energy means that behaviour is strong, intense, and full of effort whereas direction is focused on accomplishing a particular goal or outcome.

Thus, motivation is multidimensional, stimulating individual’s thoughts, investigation, discovery and conclusion. Motivation, therefore, is the heart of the learning process which teachers should try to implore effectively so as to enhance interest of learners and understanding.

2.5 Structure of Patterns

The structure of patterns can vary not only in terms of attributes, but in complexity as well. The most common type of patterning skills revolves around learners manipulating patterns either by copying, extending, or creating. Research has shown that copying a pattern is the easiest skill for learners with 71% of kindergarten learners able to complete this task. Expanding a pattern was seen as more difficult with a 48% success rate. Creating a new color pattern found 21% of learners successful while over 45% were unable to create even the most basic pattern (Reid & Andrews, 2016). There are three types of patterns that are significant in early mathematics: repeating patterns, spatial structural patterns, and growing patterns (Papic et al., 2011) Repeating patterns are patterns that have a recognizable unit of repeat that occurs continuously and contains more than one variable (Hutchinson, 2011).

Repeating patterns are often the first types of patterns that early childhood learners encounter (Collins & Laski, 2015) and are the easiest type of pattern for early learners to master (Gadzichowski, 2012). The simplest version of this type of pattern involves two alternating items and is typically represented as ABAB (Papic et al., 2011). Visual repeating patterns do not involve requiring the student to have any additional math skills such as numeracy or operations (Collins & Laski, 2015). Repetitive patterns provide a sense of organization to a child when items do not seem to be able to be classified (Bock et al., 2018). Spatial structural patterns are typically composed of geometrical shapes or other mathematical shapes such as grids or blocks. The pattern may include features such as spacing, size, or number (Papic et al., 2011). These patterns are organized in such a way that they are easily recognizable; for example, the pattern could consist of stars within a rectangle (Hutchinson & Pournara, 2011). Growing patterns are characterized by items that ascend or descend in a systematic manner, such as numbers that increase or decrease as the pattern progresses (Bock et al., 2018).

Growing patterns can be spatial patterns (Papic et al., 2011) since these types of patterns are also frequently associated with patterns of spatial structures, such as shapes that increase in size (Hutchison & Pournara, 2011). Growing patterns can incorporate different groups of shapes, objects, or numbers (Geist et al., 2012) and are considered important to the development of skip counting because they emphasize the importance of increases and decreases (Gadzichowski, 2012). It is important that teachers incorporate many different types of patterns into their mathematics curriculum (Papic et al., 2011) as patterning instruction must advance beyond alternating repeating patterns (Gadzichowski, 2012).

Expanding beyond basic patterns to more complex patterns is crucial to developing mathematical knowledge. A fundamental aspect of mathematics is understanding systematic increases, so providing opportunities for learners to work on growing patterns is very important. A study of first graders determined that learners do not necessarily expand their understanding to advanced patterns, especially those which consist of more elements, so instruction on more complex patterns will prove to be very beneficial (Gadzichowski, 2012). The structure of each type of pattern can vary. A pattern can have a repeating unit such as alternating colors (e.g. red-green-red-green) or a repeating rule such as adding two to the previous numeral (e.g. 2-4-6-8). Patterns can also be composed of a growing relationship as in the example of doubling the last number (e.g.1-2-4-8-16).

Patterns can utilize colors, shapes, sizes, numerals, letters, or other symbols. The complexity of patterns can also vary from simple alternating color or shape patterns to more complicated ones that involve mathematical operations (Collins & Laski, 2015) or multiple attributes. First-grade learners were able to recognize patterns that consisted of numbers, letters, and other various objects with the same accuracy as significantly easier items such as colors or shapes. Researchers have determined that learners are able to apply the same rule to patterns, even when the patterns are presented in different dimensions (Gadzichowski, 2012).

2.6 Teaching Patterning

Patterns are considered to play an important role in early mathematics education

(Tsamir et al. 2017) and are typically introduced in early childhood classrooms. Within the past ten years' researchers have undertaken the study of patterning and how it is linked to working memory, executive function, fluid reasoning, spatial skills, and reading in addition to mathematical achievement. While this research is important, this literature review will narrow the focus to conclusions that speak to the effect of patterning on mathematical skills and achievement. Patterning skills are taught as part of early childhood math curriculums because most educators feel patterning has value (Kidd et al., 2013). The Iowa Early Learning Standards (2018) states that children should develop an understanding of patterns and be able to recognize, create, and extend both simple and complex patterns in a variety of settings, including nature. In contrast, the Common Core (2010) does not include any standards that are related to patterning in grades kindergarten through second grade.

The Common Core mentions numerical patterning as a standard in grades three through five and again briefly as a math practice skill. The omission of patterning as a Common Core standard in kindergarten through second grade was based on insufficient research evidence to support the inclusion of patterning at the time the standards were written (Rittle- Johnson et al., 2019). Many educators support the implementation of patterning in early mathematics education and the inclusion of patterning as a specific standard within the Common Core (Zippert et al., 2020). Although not all researchers see the merit in teaching patterning (Burgoyne et al., 2017), recent studies have garnered evidence that connects patterning and mathematical achievement. The contradiction of the importance of teaching patterning skills in recent years has led researchers to study patterning in order to ascertain the role patterning plays with learning mathematical concepts and how patterning affects achievement. Research studies seek to answer the question of whether understanding patterning leads to an increase in mathematical academic performance (Kidd et al., 2013). Current research findings contribute to the idea that patterning is connected with mathematics knowledge (Fyfe et al., 2017). As a result, researchers have concluded that patterning and its effect on mathematical knowledge warrants additional study to ascertain the relationship between the two. Recent studies have yielded information not only as to how teaching patterning relates to future mathematical achievement, but also to specific mathematical achievement such as numeracy, calculations, and algebra (Zippert et al., 2020).

2.7 Learning Patterning

Research has shown a connection between early knowledge in patterning and later mathematical achievement. The next step is to ascertain which methods or strategies prove most beneficial for increasing future mathematical skills (Collins & Laski, 2015). It is vitally important that teachers understand how teaching strategies should be used in early math education and which are best for promoting patterning skills that lead to future mathematical achievement (Clements & Sarama, 2011). In order to determine these skills, one must first understand what constitutes a pattern. Papic, Mulligan and Mitchelmore (2011) stated that the structure of a pattern is a basic repetition that incorporates various features.

The general consensus within math education is that the key component of pattern is that of repetition (McGarvey, 2013). Patterns are often described by using abstract labels such as the letters of the alphabet with each letter representing a different component of the pattern, for example ABAB; patterning is the ability to recognize and maneuver these patterns (Schmerold et al., 2016). Patterns are predictable and have a structure or rule that govern them (McGarvey, 2013); however, research has determined that the dimension by which the pattern is presented does not influence the result (Gadzichowski, 2012).

2.8 Patterning and Mathematical Skills

Researchers have determined that when learners are taught patterning skills, they are better able to complete patterning tasks than those learners who were not instructed in patterning (Kidd et al., 2013). In addition, when young learners are able to compare two different patterns that are made with the same materials, the result is a higher level of reasoning (Tsamir et al., 2017). A long-term study by Papic, Mulligan and Mitchelmore (2011) compared two groups of preschoolers. Over the course of six months one group received a pattern-based intervention while the comparison group did not. The intervention group outperformed the non-intervention group in patterning skills at the end of the study. More importantly, the learners who received the patterning intervention continued to do better with repeating patterns one year later in kindergarten and again in first grade, when compared to the non-intervention group.

In addition, the researchers reported that the learners in the intervention group were able to successfully complete tasks that were not part of the intervention and performed better on an assessment that measured numeracy, which is the understanding of whole numbers and included counting. Patterning instruction can have a positive effect not only on patterning skills, but in mathematics as well (Schmerold et. al., 2016). Kidd et al. (2013) divided first graders into four intervention groups for an additional fifteen minutes of direct instruction each day in one of four areas: patterning, reading, math, and social studies. The group that received the additional patterning instructions improved their patterning scores when compared to the other three groups. The patterning intervention group also scored higher than the other groups on an assessment that measured mathematical concepts. Consequently, the researchers concluded that academic achievement is bolstered by patterning instruction.

In addition to advancing patterning skills, patterning instruction has been shown to have an effect on the development of numeracy skills. Learners who receive patterning instruction performed at a higher level on an assessment that measured numeracy than those who did not receive patterning interventions (Papic et al., 2011). Rittle-Johnson et al. (2019) noted that a child's ability to pattern is a good indicator of general math knowledge and numeracy skills. Although learners do not need to have a knowledge of numbers to complete patterning tasks, patterning was a predictor of numeracy knowledge and educators should use patterns as a method for promoting numeracy. Not all researchers agree that patterning ability is the best indicator of mathematical achievement. Nguyen et al. (2016) conducted a study to ascertain what skills were important for future mathematical success. The study evaluated preschoolers in the fall and spring on counting, number recognition, patterns, measurement, and geometry.

A similar test geared for third through fifth graders was given to measure the proficiency of the math skills of fifth grade learners. The study concluded that although geometry, patterning, and measurement were predictive of mathematical achievement, counting and numeracy skills were considered to be a better indicator of mathematical achievement in fifth grade. Pattern knowledge in preschool has been shown to predict general math knowledge and numeracy skills in kindergarten, specifically the ability to count to 100. Pre-kindergarten learners with higher repeating patterning scores were 27 times more likely to be able to count to by the time they completed kindergarten than their peers who were not as proficient in completing repeating patterns (Zippert et al., 2020). Researchers have concluded that patterning knowledge influences numeracy skills because numbers follow patterns (Zippert et al., 2020). Numeracy is enhanced by patterning due to the predictable sequence found in numbers: when learners learn to find patterns, counting and calculation skills are enhanced (Fyfe et al., 2017). Patterning promotes additional counting skills such as skip counting, especially by twos and fives, since counting sequences contain the numerals zero to nine and the numbers repeat over again with each decade thus creating a pattern (Rittle-Johnson et al., 2019). In addition, patterning skills have been found to be predictive of calculations skills, such as addition and subtraction (Fyfe et al., 2017). Early learners who are able to complete complex repeating patterns demonstrate advanced knowledge of addition and subtraction computation skills at the end of first grade. In addition, the ability to repeat patterns abilities yielded an understanding of addition and subtraction strategies other than counting (Luken et al., 2014). Patterning is connected to calculation skill because of the predictable sequences that learners develop over time while identifying and describing sequences of numbers. Examples of predictable sequences include adding two even numbers together to produce an even number, adding nine to a number of results in an answer that is ten more than the given number minus one, and the next number in a sequence is one plus the previous number (Fyfe et al., 2017).

Another key area of research with regards to patterning is the effect of pattern instruction on algebraic skills. Algebraic thinking involves generalization, which is the idea that there is a consistent relationship that remains the same (Papic et al., 2011) Pre-algebraic thinking is developed when learners search for the relationship between random objects or symbols (Pasnak et al., 2019). Lee et al. (2011) researched the effects of patterning on algebraic word problems with nine- and ten-year-old learners in Singapore. The study hypothesized that patterning skills influenced algebraic skills. The results indicated that learners who are proficient with number patterns did better with algebraic reasoning most likely due to the idea that patterns and algebra utilize the same thought processes. The evidence compiled in the study was able to show a connection between patterns and algebraic skill; however, the study was not able to ascertain which components of patterning were most important for algebraic problem solving. However, the study also concluded that although patterning plays a significant role in algebraic proficiency, updating and computational abilities played a greater role in predicting algebraic performance. Analyzing the structure of repeating patterns leads early learners to think in terms of how items are related, thus increasing early algebra skills (Plessis, 2018). Learning to recognize the rules and relationships that lie within a pattern is especially important with regards to algebraic reasoning. Learners who learn and apply rules when given a new pattern demonstrate an understanding of how the parts of a particular pattern are related; this knowledge forms a foundation for algebraic reasoning (Gadzichowski, 2012).

2.9 Patterning and Mathematical Achievement

Patterning positively predicts general math knowledge (Rittle-Johnson et al., 2019). Mulligan, Oslington, and English (2020) concluded that learners who received an intervention in patterning demonstrated significant growth in awareness of pattern and structure when compared to learners who did not receive the additional intervention. When the groups were compared one year later, the difference between the two groups was even greater. The impact of the patterning instruction had a positive influence on the first-grade skills of the intervention group, and their teachers reported that the intervention learners were able to explain and make connections between mathematical concepts. The study did note that this intervention was more intense and encompassed more classroom time then a typical classroom mathematics instruction. An unexpected key finding was that learners were able to develop patterning and structure skills that were above the normal curriculum expectations in kindergarten and appeared to support early algebraic thinking. Rittle-Johnson et al. (2017) concluded that early knowledge of patterning elicited higher math achievement several years later.

The researchers synthesized that working with patterns helps learners learn to identify underlying rules and develop an awareness of spatial skills. Although both studies concurred on the importance of patterning to future mathematical achievement, Rittle-Johnson et al.'s study found patterning to be the most influential factor, not numeracy or counting. Schmerold et al. (2016) researched the correlation between patterns, executive function, and mathematics with first graders, finding a significant relationship between patterning and mathematical achievement. Patterning skills were more highly related to mathematical achievement than to executive function.

Patterning skills are predictive of math skills (Zippert et al., 2020). This study suggest that older learners would also benefit from patterning instruction and that more complex patterns result in more progress in mathematical skills. Research has determined that patterning does play a predictive role in current and future mathematical achievement

(Rittle-Johnson et al., 2019). The importance of patterning in early learning is important because patterns lead to the discovery and understanding of concepts in mathematics (Kandir et al., 2018). The impact that patterning has on mathematical skills such as numeracy (Zippert et al., 2020), calculations (Fyfe et al., 2017) and algebra (Plessis et al., 2018) has led researchers to conclude that patterning does have a positive impact on future mathematics performance (Rittle-Johnson et al., 2017).

Patterning skills have emerged as a quintessential component of math education and research needs to continue to focus on its relationship to mathematics, especially the nature of how patterns are understood, and what training is needed in order for patterning skills transfer to mathematical knowledge (Burgoyne et al., 2019) and to clarify the role patterning plays in regard to a student's skill (Rittle-Johnson et al., 2019) The next step is to examine how learners best learn patterning skills in order to maximize the positive effect patterning has on future mathematics achievement (Tsamir et al., 2017)

Arithmetic is often used as a synonym for math, but there is a difference between arithmetic math and mathematics. In general, arithmetic math is math that deals with the numbers themselves, whereas mathematics is more about the theories of numbers. When a person starts learning math, they will begin with arithmetic math and then go on to learn more about advanced mathematics. It is crucial to have this foundation and to understand what arithmetic math is before moving to more advanced topics. Arithmetic is working with the numbers themselves. This includes counting, adding, subtracting, multiplying, and dividing. It also includes fractions, positive and negative numbers, the order of operations, sequencing, and more. Basically, arithmetic math is how the numbers work together to get an answer to a problem.

Learning arithmetic math is usually the beginning of a person's math education starting with the basics, although there are far more advanced components to arithmetic a person can delve into later. According to the arithmetic definition, arithmetic starts with learning how to count, then progresses through adding, subtracting, multiplying, dividing, sequences, and more detailed topic as a huge area of math, this is the foundation for more advanced mathematics. Mathematics, on the other hand, includes more advanced problems.

2.11 Building on Learners' Understanding of Arithmetic

Modern school algebra relies on a more extensive and technical symbols apparatus. As learners learn to manipulate variables, terms and expressions as if they were objects, it is easy for them to lose sight of the facts that the symbols are about quantities. In the content of arithmetic, learners have only learned to use symbols to notate numbers and to encode binary operations, usually carried out on time. Algebra, not only introduces new symbols such as, letters, and expressions but also new ways of dealing with symbols. Without guidance from institution, learners face great difficulty in adjusting to the new symbolism. So Bhaskara's precept that algebra is about insight into quantities and their relationship and not just the use of symbols is perhaps even more relevant to the learning of modern school algebra. What do learners carry over from their experience of arithmetic that can be useful in the learning of algebra? Do learners obtain insight into quantitative relationships of the kind of Bhasahara is possibly referring to through their experience of arithmetic, which can be used as a starting point for an entry into symbolic algebras? Of course, one cannot expect such insight to be sophisticated.

We should also expect that learners may not be able to symbolize their insight about quantitative relationships because of their limited experience of symbols in the content of arithmetic. Fujii and Stephens, (2001) found evidence of what they call learners relational understanding of numbers and operations in the content of arithmetic tasks. In a missing number sentence like 746 + ____-262 = 747, learners could find the number in the blank without calculation. They were able to anticipate the results of operating with numbers by finding relations among the operands. Similar tasks have also used by others in the primary grades (Van den Heuvel-Panhuizen, 1996). Missing number sentence of this kind are different from those of the kind 13 + 5 = _______+ 8, where the algebraic element is limited to the meaning of the ''='' sign as a relation that ''balances'' both sides. Relational understanding as revealed in the responses to the former kind of sentence lies in anticipating the result of operations without actual calculation. Fujii and Stephens (2001) argue that in these tasks although learners are working with specific numbers as ''quasi-variable.''

Learners relational understanding as described by Fujii and Stephens (2001), is a form of operational sense (Slavit, 1999), limited perhaps to specific combinations of numbers. The learners' performance on these tasks needs to be contrasted with the findings of other studies. For example, Chaiklin and Lesgold (1984) found that without recourse to computation, learners were unable to judge whether or not 685 – 492 + 947 + 492 – 685 are equivalent. Learners are not consistent in the way they parse expressions containing multiple operation signs. It is possible that they are not even aware of the requirement that every numerical expression must have a unique value. It is likely, therefore, that learners' relational understanding is elicited in certain context, while difficulties with the symbolism overpowers such understanding in other contexts.

Can their incipient relational understanding develop into a more powerful and general understanding of quantitative relationships that can form the basis for algebraic understanding, as suggested by Bhaskara? For this to be possible, one needs to build an idea of how symbolization can be guided by such understanding. In a later study, Fujii and Stephens (2008), explored learners' abilities to generalize and symbolize relational understanding. They used learners' awareness of computational shortcuts (to take away six, take away ten and add four) and developed tasks that involved generalizing such procedures and using symbolic expression to represent then.

Other efforts to build learners' understanding of symbolism on the basis of their knowledge of arithmetic have taken what one may describe as an inductive approach, with the actual process of calculation supported by using a calculator (Liebenberg et al. 1999; Malara & Iaderosa 1999). In these studies, learners worked with numerical expressions with the aim of developing an understanding of the structure by applying operation precedence rules and using the calculator to check their computation. These efforts were not successful in leading to an understanding of structure that could then be used to deal with algebraic expressions because of over-reliance on computation or because of interpreting numerical and algebraic expressions in different ways. The findings suggest that an approach where structure is focused more centrally and is used to support a range of tasks including evaluation of expressions, as well as comparison and transformation of expressions, may be more effective in building a more robust understanding of symbolic expressions.

2.12 The Arithmetic-Algebra Connection

As we remarked earlier, learning algebra involves learning to read and use symbols in new ways. These new ways of interpreting symbols need to build on and amplify learners' intuition about quantitative relationships. The view that algebra is the foundation of arithmetic, held by Indian mathematicians, entails that learners need to interpret the familiar symbols of arithmetic also in new ways. The literature on the transition from arithmetic to algebra has identified some of the differences in the way symbols are used in arithmetic and algebra: the use of letter symbols, the changed interpretation of key symbols such as the "=" sign, and the acceptance of unclosed expressions as appropriate representations not only for operations but also for the result of operations (Kieran, 2006). An aspect related to the last of the changes mentioned that we wish to emphasize is the interpretation of numerical and algebraic expressions as encoding the operational composition of a number.

The use of expressions to stand for quantities is related to the fact that, while in arithmetic one represents and thinks about one binary operation, in algebra we need to represent and think about more than one binary operation taken together. As learners learn computation with numbers in arithmetic, they typically carry out a single binary operation at a time. Even if a problem requires multiple operations, these are carried out singly in a sequence. Consequently, the symbolic representations that learners typically use in arithmetic problem-solving contexts are expressions encoding a single binary operation. In the case of formulas, the representation may involve more than one binary operation, but they are still interpreted as recipes for carrying out single binary operations one at a time. They do not involve attending to the structure of expressions or manipulating the expressions. Indeed, one of the key differences of the arithmetic approach to solving problems, as opposed to the algebraic, is that learners compute intermediate quantities in closed numerical form rather than leaving them as symbols that can be operated upon. And these intermediate quantities need to be thought about explicitly and must be meaningful in themselves (Stacey & Macgregor, 2000). The representational capabilities of learners need to be expanded beyond the ability to represent single binary operations before they move on to algebra.

In the traditional curriculum, this is sought to be achieved by including a topic on arithmetic or numerical expressions, where learners learn to evaluate expressions encoding multiple binary operations. However, learners' work on this topic in the traditional curriculum is largely procedural, and learners fail to develop a sense of the structure of expressions. As discussed earlier, learners show relational understanding in certain contexts, but in general have difficulty in interpreting symbolic expressions. One problem that arises when numerical expressions encode multiple binary operations is that such expressions are ambiguous with respect to operation precedence when brackets are not used. At the same time, one cannot fully disambiguate the expression using brackets since the excessive use of brackets distracts from the structure of the expression and is hence counter-productive. Learners are, therefore, taught to disambiguate the expression by using the operation precedence rules. The rationale for this, namely, that numerical expressions have a unique value is often left implicit and not fully grasped by many learners. Even if the requirement is made explicit, learners are unlikely to appreciate why such a requirement is necessary.

The transformation rules of algebra are possible only when algebraic expressions yield numerical expressions with a unique value when variables are appropriately substituted. Thus, disambiguating numerical expressions is a pre-condition for the use of rules of transformations that preserve the unique value of the expression. Since learners are yet to work with transformations of expressions, they cannot appreciate the requirement that numerical expressions must be unambiguous with regard to value. In the traditional curriculum, learners' work with numerical expressions is limited and is seen merely as preparatory to work with algebraic expressions. How does one motivate a context for work with numerical expressions encoding multiple binary operations? Student tasks with such expressions need to include three interrelated aspects representational, procedural (evaluation of expressions), and transformational.

To fully elaborate these aspects, we need to interpret expressions in a way different from the usual interpretation of an expression as encoding a sequence of such operations to be carried out one after another, a sequence determined by the visual layout in combination with the precedence rules. The alternative interpretation that learners need to internalize is that such expressions express or represent the operational composition of a quantity or number. In other words, the expression reveals how the number or quantity that is represented is built up from other numbers and quantities using the familiar operations on numbers. This interpretation embodies a more explicit reification of operations and has a greater potential to make connections between symbols and their semantic referents.

The idea of the operational composition of a number, we suggest, is one of the key ideas marking the transition from arithmetic to algebra. Let us illustrate this idea with a few examples: (i) the expression $500 - 500 \times \frac{20}{100}$ may indicate that the net price is equal to the marked price less the discount, which in turn is a fraction of the marked price, (ii) the expression $5 \times 100 + 3 \times 10 + 6$ shows the operational composition expressed by the canonical representation of a number (536) as composed of multiunit which are different powers of ten, (iii) the expression $300 + 0.6t$ may indicate cell phone charges as including a fixed rent and airtime charges at a fixed rate per unit of airtime. In examples (i) and (iii), the operational composition refers back to quantities identifiable in particular situations, while in example (ii) abstract quantities are put together or "operationally composed" to yield the number 536.

It is important to preserve both these senses in unpacking the notion of operational composition. By operational composition of a quantity, we mean information contained in the expression such as the following: what are the additive part quantities that a quantity is composed of? Are any of these parts scaled up or down? By how much? Are they obtained as a product or quotient of other quantities? The symbolic expression that denotes the quantity simultaneously reveals its operational composition, and in particular, the additive part quantities are indicated by the terms of the expression.

A refined understanding of operational composition includes accurate judgments about relational and transformational aspects. What is the relative contribution of each part quantity (each term) as indicated by the expression? Do they increase or decrease the target quantity? Which contributions are large, which small? How will these contributions change if the numbers involved change? How does the target quantity change when the additive terms are inverted, that is, replaced by the additive inverse of the given term? What changes invert the quantity as a whole? What are the transformations that keep the target quantity unchanged? If additive parts are themselves composed from other quantities, how do we represent and understand this?

The idea of the operational composition encoded by an expression is similar to the idea of a function but is more general and less precise. Looking at an expression as a function has a more narrow focus: how does the target quantity vary when one or more specific part quantities are varied in a systematic manner while retaining the form of the operational composition? When expressions are compared and judged to be equivalent, we judge that different operational compositions yield the same value. However, the idea of operational composition may play a role in developing the understanding of functions. When we interpret expressions as encoding operational composition, we are not restricted to algebraic expressions. In fact, numerical expressions emerge as an important domain for reasoning about quantity, about relations and transformations, and for developing a structure-based understanding of symbolic representation through the notion of operational composition. The pedagogical work possible in the domain of numerical expressions as a preparation for algebra expands beyond what is conceived in the traditional curriculum.

Numerical expressions emerge as a domain for reasoning and for developing an understanding of the structure of symbolic representation. When learners' tasks focus on numerical expressions as encoding operational composition, attention is drawn to the relations encoded by the expression. Learners are freed from the need to unpack the expression as a sequence of operations, fixed by a set of operation precedence rules. In the teaching approach that we developed, we emphasized ways of working with expressions that attend to the structure of expressions and are broadly aimed at developing an insight into quantitative relationships that must accompany working with symbols. A simple numerical expression like 5 + 3 is usually interpreted as encoding an instruction to carry out the addition operation on the numbers 5 and 3. In changing the focus to operational composition, the first transition that learners make is to see the expression as "expressing" some information about the number 8. This information can be expressed verbally in various ways: 8 is the sum of 5 and 3, 8 is 3 more than 5, etc. Other expressions such as 6+2 or 2×4 contain other information about the number 8, i.e., they encode different operational compositions of the number 8. Starting from this point, learners move on to expressions with two or more operations of addition and subtraction.

Each expression gives information about the number which is the "value" of the expression, and reveals a particular operational composition of the number. What grounding concepts can scaffold learners' attempts to study and understand the operational composition revealed in an expression? The basic level of information is contained in the terms or the additive units of the expression. Simple terms are just numbers together with the preceding "+" or "–" sign. Positive terms increase the value of the number denoted by the expression and negative terms decrease the value. Additive units are dimensionally "homogenous," and can be combined in any order.

This shift in perspective subtly turns attention away from procedure towards structure. In order to evaluate an expression, learners do not need to work out and implement a sequence of binary operations in the correct order. Rather, to determine the value of the expression, they may combine simple terms in any order, keeping in view the compensating contributions of positive and negative terms. The concept of negative terms provides an entry point into signed numbers as encoding increase or decrease, which is one of the three interpretations of integers proposed by Vergnaud cited in Fuson (1992).

The approach of combining simple terms in any order, affords flexibility in evaluating an expression or in comparing expressions that is critical to uncovering structure. Thus, learners may cancel out terms that are additive inverses of one another; they may gather together some or all of the positive terms or the negative terms and find easy ways to compute the value of the expression by combining terms. Since the identification of additive units namely, terms, is the starting point of this approach, we have described this approach elsewhere as the "terms approach" (Banerjee & Subramaniam 2008). Identifying the additive units correctly is one of the major hurdles that some learners face. This is indicated by the frequency of such errors as "detachment of the minus sign" ($50 - 10 + 10 = 30$), and "jumping off with the posterior operation" ($115 - n + 9 = 106 - n$ or $106 + n$) (Linchevski & Livneh 1999).

Although these errors are often not taken to be serious, they are widespread among learners and impede progress in algebra. Not having a secure idea about the units in an expression and not knowing how they combine to produce the value may enhance the experience of algebra as consisting of arbitrary rules. In working with transformations of expressions, some studies indicate that visual patterns are often more salient to learners than the rules that the learners may know for transforming expressions (Kirshner and Awtry 2004), suggesting that visual routines are easier to learn and implement than verbal rules. One advantage with the "terms approach" is the emphasis on visual routines rather than on verbal rules in parsing and evaluating an expression. Terms were identified in our teaching approach by enclosing them in boxes. In fact, the rule that multiplication precedes addition can be recast to be consistent with visual routines. This is done by moving beyond simple terms, which are pure numbers with the attached + or − sign, to product terms. In expressions containing "+," "−," and the "×" operation signs, learners learn to distinguish the product terms from the simple terms: the product terms contain the "×" sign. Thus, in the expression $5 + 3 \times 2$ the terms are $+5$ and $+3 \times 2$. In analyzing the operational composition encoded by the expression, or in combining terms to find the value of the expression, learners first identify the simple and the product terms by enclosing them in boxes.

The convention followed is that product terms must be converted to simple terms before they can be combined with other simple terms. Thus, the conventional rule that in the absence of brackets multiplication precedes addition or subtraction is recast in terms of the visual layout and operational composition. Product terms are the first of the complex terms that learners learn. Complex terms include product terms, bracket terms (e.g., $+(8-2\times3)$) and variable terms (e.g., $-3 \times x$). The approach included both procedurally oriented tasks such as evaluation of expressions and more structurally oriented tasks, such as identifying equivalent expressions and comparing expressions. As remarked earlier, one of the main features of the approach evolved only after the initial trials the use of the idea of terms in the context of both procedurally and structurally oriented tasks. In the earlier trials, the use of the idea was restricted to structurally oriented tasks involving comparison of expressions, and the operation precedence rules were used for the more procedurally oriented tasks of evaluating expressions. By using the "terms idea" in both kinds of tasks, learners began to attend to operational composition for both evaluating and comparing expressions, which allowed them to develop a more robust understanding of the structure of expressions. By supporting the use of structure for the range of tasks, this approach actually blurred the distinction between structural and procedural tasks.

The view that understanding quantitative relationships is more important than just using symbols and the idea that algebra provides the foundation for arithmetic are powerful ideas whose implications we have tried to spell out. We have argued that symbolic expressions, in the first instance, numerical expressions, need to be seen as encoding operational composition of a number or quantity rather than as a set of instructions to carry out operations. We have also pointed to the importance, from a perspective that emphasizes structure, of working with numerical expressions as a preparation for beginning symbolic algebra.

2.13 Teaching Arithmetic

Numbers occur everywhere in our day-to-day life. ‘’Numbers are in everything’’, said Pythagoras, an ancient Greek mathematician. If we understand more about numbers, then we know more about mathematics. ‘’Mathematics is the queen of sciences, and the theory of numbers is the queen of mathematics’’, said Carl Friedrish Gauss, a German mathematician. Numbers possess very nice properties and the properties will help us to solve problems of other sciences. ‘’God created the natural numbers and all the rest is the work of art’’, exclaimed Kronecker. Natural numbers were introduced as counting numbers and other numbers were developed from them to fulfill our requirements (Sathiskumar & Ratnaliken, 2003). Direct or indirect, we use arithmetic in our day-to-day life. The literacy rate is calculated by the 3R’s namely, ‘Read’, ‘wRite’ and ‘aRithmetic’.

From this, we can conclude that arithmetic plays an important role in our daily life. For successful life one must know the values of arithmetic.

The part of mathematics that deals with numbers and counting or calculation is known as arithmetic. For a person belonging to any profession, knowledge of arithmetic is essential to cope up with the needs of daily life (Shatini-Wadhwa, 2008). A good command of arithmetic is essential for the skilled mechanic, for the up-to-date farmer, for the progressive professional man, for the efficient house wife and for the brilliant learners. Therefore, arithmetic has a greater value in day-to-day life.

Arithmetic is the science of number and the art of computation. Historically, arithmetic developed out of a need for a system of counting. It is considered to be essential for efficient and successful living. The need of a good command of arithmetic by a house-wife, by a modern farmer, by a successful merchant, by a skilled worker, and by a progressive professional man; is too obvious any discussion. Also, its utilitarian, cultural and disciplinary values are too obvious to need any argument at this stage. The teaching of arithmetic has to fulfil two major responsibilities (Shatini-Wadhwa, 2008).

1) The inculcation of an appreciative understanding of our number system and an intelligent proficiency in its fundamental processes;
2) The socialization of number experiences

2.13.1 Major Objectives of the Teaching of Arithmetic

The two major objectives of teaching of arithmetic are;

i. To develop the ability to perform various number-operations skillfully
ii. To provide variety of experiences, which will enable the learners to apply quantitative procedure effectively in society

2.13.2 General Rules for Teaching of Arithmetic

The following rules are to be followed in teaching of arithmetic, which will help the learners in understanding the subject in a better way.

1) Before teaching a lesson, the teacher should explain to the learners the general principles underlying the topic
2) Learners can be allowed to work independently
3) Enough drill can be given
4) Teacher should do correction work very carefully
5) An effort be made to develop orderliness, accuracy and speed
6) Learners be given enough training in oral mathematics
7) The interest of learners should not be allowed to diminish
8) Learners be encouraged to do their written work systematically
9) Learners should be encouraged to write figures neatly and correctly
10) They should be encouraged to use a simple, understandable and correct language
11) The teacher should follow the maxim of proceedings from simple to complex
12) The value of orderliness should be taught to the learners (Shatini-Wadhwa, 2008)

2.13.3 Aim of Teaching of Arithmetic

The following are the aims of teaching arithmetic

i. To develop mathematical thinking in the learners
ii. To arouse interest in the quantitative side of the world
iii. To provide the knowledge of the practical utility of arithmetic in everyday life
iv. To prepare learners to study higher mathematics

v. To give accuracy and facility in simple computations of the fundamental processes (Shatini-Wadhwa, 2008).

2.13.4 Importance of Numbers in Teaching of Arithmetic

Numbers is the basic element of arithmetic. Without number nothing is possible in arithmetic (Tyagi, 2008). For this reason, arithmetic teaching should start with numbers. At the infant stage, the children may be taught about numbers, their different combinations, method of numeration, etc. after this they will be able to proceed further.

2.14 Teaching of Four Fundamental Rules

1. **Teaching of Addition**

The teaching of addition should start at pre-primary stage and should continue up to primary stage. (Kulbir-Singh & Sidhu, 2006). For this first of all the learners should be trained in counting of numbers and then they should be trained in the combination of numbers. Afterwards the learners can be given training in addition of one digit, then two digits and so on. In teaching of additions, the teacher should proceed from simple to complex. The teacher can make use of figures and other concrete object to explain the process.

Addition is simple, a process of counting in forward direction. If a child has to add two and three, the child may be asked to represent the integers two and three with the help of lines as under: || **(+)** ||| **=** ||||| then ask the child to count all the lines. He/she will count up to 5. Then ask him/her to write as under: 2 + 3 = 5. Similarly, he/she can be given a good practice in the process. In the same way method of adding the numbers of two or more digits can be taught. If we are to find the sum 0f 287 and 465, then first of all arranged the given numbers in columns of hundred, tens and ones. The learners can be then told the method of 'carry over' if the sum of integers in a particular column is more than 9

H	T	O
1	1	Carry Over
2	8	7
+4	6	5
7	5	2

Step I:

Adding integers in the column of ones, we get, 7 + 5 = 12ones

12ones = 1ten, 2ones

Write 2 in one's place and carry over 1 ten to ten's column to add with tens.

Step II:

Adding tens, we get 1 + 8 + 6 = 15 tens

15 tens = 1 hundred, 5 tens

Write 5 in tens place and carry over 1 hundred to hundred's column to add with hundreds.

Step III:

Adding hundreds, we get 1 + 2 + 4 = 7 hundred.

Hence the sum is 7 hundred, 5 tens 2 ones = 752. Similarly, the sum of two numbers going up to thousands or more than that can be found.

2. **Teaching of Subtraction**

In subtraction, we teach the learners to take out numbers from one another. Only smaller number can be taken out from a bigger number. The teaching of subtraction should begin at pre-primary stage and should not go beyond primary stage. As in teaching of adding so also in teaching subtraction teacher should proceed from simple to complex. In the beginning, the teacher should take help of concrete things such as balls, coloured pieces of woods, beeds, etc. The system of borrowing and returning may also be explained and the method of changing coins may also be explained (Shatini-Wadhwa, 2008).

Process of Subtraction

Subtraction is simple a process of counting in the backward direction. Suppose the child is to subtract 3 from 5. Ask him/her to write integer 5 and the integer 3 below it with sign (-). Represent the integer 5 in the form of lines. Cross the lines equal to the integer, which is to be subtracted and then write the remainder in the following way:

5-3 = 2

Similarly, for the numbers consisting of two, three, four digits, etc.

52 -21 = 31

Subtract separately the integer of each column and repeat the process. Then, question arise what to do it the integer of upper row in a particular column is less than the lower integer of the same column. This type of subtraction is done by the method of ‘’borrowing’’ as under: suppose we are to subtract 1732 from 5620.

Solution: Arrange the numbers as below:

Borrow	Th	H	T	O
		1	1	10
	(4)	(15)	(11)	
	5	6	2	0
	-1	7	3	2
	3	8	8	8

Since it is not possible to subtract 2 ones from 0 ones. Hence, we borrow a ten from ten's column.

Step I:

10 ones – 2 ones = 8 ones

Hence write 8 in one's place.

Step II:

We cannot subtract 3 tens from 1 ten left with us, so we borrow 1 hundred (10 tens) from 6 hundred, leaving 5 hundred and making 10 + 1 = 11 tens.

Hence, 11 tens – 3 tens = 8 tens

Write 8 in tens place.

Step III:

We are left with 5 hundreds. We cannot subtract 7 hundred from 5 hundred, so we borrow 1 thousand (10 hundreds) from 5 thousand, leaving 4 thousands and making 10 + 5 = 15 hundreds. Hence 15 – 7 = 8, thus write 8 in hundreds place.

Step IV:

Finally, one thousand is subtracted from thousand. i.e 4 – 1 = 3, thus write 3 in place of thousands.

Hence 5620 – 1732 = 3888

3. **Teaching of Multiplication**

It is a short method of ‘’addition of equal quantities’’. In this method, there is addition of more than one group. It is quick method of addition. It helps and simplifies the process of addition. It therefore, advisable to teach it at the primary stage of education and then take it in more difficult and complex form at junior high school or pre-secondary stage of education. In the beginning, the learners should be made to learn the multiplication tables of 2’s, 5’s and 10’s. then they should be made to learn multiplication tables of 4’s, 8’s, 3’s, 6’s, and 9’s. in the end, the multiplication of 7’s should be taught. Learning of multiplication tables is very helpful in future life.

Process of Multiplication

Suppose 1 is to be multiplied by 2, draw 1 time two lines and count them. If 2 is to be multiplied by 2, draw two lines two times and count them. Similar, proceed for others cases. In multiplication you should always remember that the product of a number and zero is equal to zero.

Step in Teaching of Multiplication

To teach multiplication successfully, the teacher should take up the following steps:

1) Teacher should proceed from simple to complex
2) First of all, multiplication of one digit may be taught and then the multiplication of digit that have ‘’carry-over’’

3) While teaching multiplication, the knowledge of multiplication of learners should invariable be put to test.

4) While teaching multiplication, the learners should be explained that in multiplication any of the digits may be used as a multiplier and the result shall be same.

   For example, 7 × 8 = 56. Similarly, 8 × 7 = 56

5) While teaching multiplication, the teachers should determine the difficulties and the drawbacks of the learners. If certain basic difficulties are removed, it shall be possible to teach them the process quickly and scientifically.

Checking the Result of Multiplication

Suppose two numbers have been multiplied and their result obtained, the problem is ‘’does there exist any method by which it can be ascertained that the result obtained is correct’’. Suppose, for example, that the number 9573 and 3058 have been multiplied and the result obtained is correct? To check the results following method may be adopted. Draw two lines intersecting, one another in the form of a cross.

i. Add all the digits of numbers which are being multiplied and reduced the sum to a number lying between zero and nine.

Thus, 9 + 5 + 7 + 3 = 24 and 2 + 4 = 6

3+ 0 + 5 + 8 = 16 and 1 + 6 = 7

Write 6 on one side of cross and 7 on other side of cross.

ii. Multiply now 6 and 7. The result obtained is 42. Add the digits of this number i.e, 4 + 2 = 6

iii. Lastly add digits of the result obtained. Thus 2 + 9 + 2 + 7 + 4 + 2 + 3 + 4 = 33 and 3 + 3 = 6. This number 6 is equal to the result to the number 6 obtained by adding 4 + 2 = 6. Hence multiplication is correct. To strengthen the knowledge and experiences it is advisable to give them more practice and drill in multiplication tables.

Teaching of Division

It is easy and shortened way of subtraction. It is reserve of multiplication while teaching division following things have to be given due consideration:

1. The learners be discouraged from using note method and they should be encouraged to discover and assimilate the rules. For this they be given repeated exercises.
2. Teaching of division should be done with the help of concrete problems.
3. Learners be made to practice in different situations which will be quite helpful in strengthening their knowledge and experience.
4. To start with division without ‘’carry over’’ should be taught and then complex ones with ‘’carry over’’ be taken up.
5. Learners be encouraged to write down the answer in the form that they may write down the dividend and the remainder.

Process of Division

Suppose we want to divide 15 by 5, here we have to see how many times 5 can be subtracted from 15.

```
 15
 -5    one time
 10
 - 5   two times
 5
 -5    three times
 0
```

It shows 5 can be subtracted from 15 repeatedly three times. Thus, quotient of 15 dividend by 5 in 3. Quotient means the number of times 5 can be subtracted repeatedly from 15. The number 15 is known as dividend and 3, the divisor. This means 15 ÷ 5 = 3 Suppose we divide 10 by 3. In this case 3 can be subtracted 3 times, from 10 and 1 left over at the end. In this case 1 is known as remainder. It is written as 10 ÷ 3 = 3 – 1.

2.15 Algebra

Algebraic reasoning includes analyzing, generalizing, and noticing structure in mathematical patterns (Dougherty et al., 2015) and two of the National Council of Teachers of Mathematics (NCTM) eight mathematical practices align with this definition of algebraic reasoning. One of the mathematical practices is to look for and make use of structure and another practice is to look for and express regularity in repeated reasoning (NCTM, 2014).

Algebraic reasoning can be encouraged by eliminating the numbers from the task thus inviting learners to generalize the pattern. Challenging learners to generalize mathematical patterns leads them to a transition from the concrete to the abstract. This generalization elicits discussing key algebraic concepts like the common difference, which speaks to the recursive nature of this pattern (Lannin, 2004). This way of thinking motivates learners to take an opportunity to "look at" and "look through" a problem to find patterns in its mathematical structure. In other words, learners start analyzing and noticing structure with concrete numbers until they independently begin to recognize the abstraction or generalization of the mathematical pattern.

2.16 Meaning and Importance of Algebra

According to Blanton and Kaput (2005), algebra is the activity of doing, thinking and talking about mathematics from a generalized and relational perspective. A generalized perspective is an awareness that mathematical conjectures or properties of equality hold true for all instances of a similar nature, while a relational perspective includes an understanding that the mathematical properties or definitions can apply to all expressions, both numeric and abstract. Dougherty et al. (2015) describe an algebra skillset as being able to analyze quantitative relationships, generalize, model, justify or prove, predict, problem solve, and notice structure" (Dougherty et al., 2015). Because algebra can cover many topics and is very difficult to define, this study will focus on generalizing by analyzing and noticing structure of the mathematical patterns. Through this process, there will also be evidence of problem-solving, prediction and discussing relationships but these algebraic ideas will weave in and out of learners' discussions as they try to generalize mathematical patterns. Algebraic reasoning is a combination of mathematical thinking with formal algebraic procedures so learners can conceptualize mathematical patterns (Kriegler, 2007; Lannin, 2003). The ability to see mathematical structure gives learners confidence to focus on the big picture rather than procedural skills (Dougherty et al., 2015). Learners with algebraic reasoning skills can initiate the problem-solving process independently and have confidence in their own reasoning skills, which will equip them to go beyond answer-getting and have the abilities to analyze and understand mathematical patterns (Green, 2014). Algebraic reasoning includes informal algebraic reasoning alongside formal algebraic procedures (Kriegler, 2007).

If we want learners to understand the algebraic concepts, they must see and recognize the underlying structure which means algebra should be learned from a structural and conceptual approach (Kieran, 1992). Algebra should not be a series of routine computations or comfortingly familiar processes, but a thought-provoking experience, not seen as rules to be memorized but as a way of problem-solving that makes sense (Green, 2014). The use of generalization tasks that elicit algebraic reasoning lead learners through the informal and formal reasoning processes, which will also be called the stages of generalization (Mason, 1996).

2.17 Fostering Learners' Algebraic Reasoning

Generalizing patterns valuably bridges the gap between the concrete and the abstract (Tabach et al., 2008). Once these connections are made learners' algebraic understanding deepens and they learn to appreciate the power of the abstract (Windsor & Norton, 2011). According to Radford (1996) generalizing is not confined to just looking for mathematical patterns but is also one of the key tools to finding scientific knowledge and day-to-day knowledge as well.

Researchers agree algebraic reasoning through generalization includes both algebraic reasoning and algebraic symbolism. For algebraic reasoning to be more than just generalization, it needs to be paired with formal algebra to justify and validate the generalization (Kieran, 1992). Blanton and Kaput (2005) and Lannin (2003) refer to algebraic reasoning as the learners' activities of generalizing about the mathematical relationships and establishing those generalizations through a verbal explanation. However, they also stress the importance of eventually expressing these generalizations in formal algebraic ways.

Generalization is defined by the Center for Algebraic Thinking (2020) into stages: seeing, saying, recording, and algebraic representation. These stages are a summary of Mason's (1996) work on generalization: Taking time to state your rule in simple language helps you to find a formula. By inviting children to express their rules out loud, you create an opportunity for children to exercise their powers of observation and description to an audience, to show tolerance and appreciation of other people's struggles to express themselves, and to encounter other ways of seeing and thinking which might be useful in the future. Mason's (1996) stages are represented as articulating (saying), getting a sense of, and manipulating (seeing) concrete numbers during the problem-solving process until learners arrive at an understanding (getting a sense of) of the mathematical pattern. Once learners have a solid understanding of the mathematical pattern, they will have confidence to explain the generalization in their own words (recording). Then learners will begin to translate their verbal generalizations into symbolic forms (algebraic representation).

2.18 Challenges in Learners' Algebraic Reasoning

When learners are given generalization tasks to encourage algebraic reasoning, finishing

The activity provided is not the point but rather situating the student into recognizing the important and not-so-important mathematical patterns, which will be helpful in future mathematics courses (Mason, 1996). Learners encounter challenges attempting to find those mathematically helpful patterns. Learners naturally discuss and visualize when reasoning algebraically through tasks involving generalizing; however, justifying mathematical patterns is not natural (Lannin, 2006).

Learners should be encouraged to provide reasons for their explanations of the mathematical patterns for deeper reflection. This mathematical reflection encourages learners to construct valid arguments to defend their generalization or critique the generalizations of others. According to Lannin (2006), agreeing or disagreeing with peers' generalizations does not come naturally. Learners need practice being able to justify their own answers and why their classmates may have made errors. Classroom conversations about what the correct answer is and the possibility that several different answers could all be correct will help learners transition into formal algebraic procedures.

When reasoning algebraically through tasks that involve generalizing, learners encounter challenges translating their verbal generalization into a symbolic form. If learners have already recognized the mathematical pattern but are not comfortable using symbols to write an algebraic expression such as $8+ 2(n - 1)$, then it is understandable they will not make this transition (Quinlan, 2001). Using this symbolic expression would mean learners have to know they are letting n represent the length of an unknow number $(n - 1)$ represent a number one less than the unknown number and that $2(n - 1)$ is representing two times a number that is one less than the unknown number. It is easy to see why learners get lost in this process. Their lack of confidence can get them stuck trying only concrete, numerical strategies, like guess and check, rather than looking for an understanding of how the pieces of the given symbolic expression point to the mathematical pattern represented (Lannin, 2004). Classroom discussions of the verbal generalizations cause reflection on mathematical structure and development of confidence (Mason, 1996), but learners struggling with formal algebra need help accessing reasoning tools that will help them make the transition from informal to formal abstraction.

2.19 Summary

Algebra is the activity of analyzing, noticing structure, and generalizing mathematical patterns. Through these mathematical reflections, learners will be transitioning through the stages of generalization to form an abstraction of the pattern. The stages of generalization explain the process from manipulating concrete numbers until an understanding of the pattern can be verbalized and then translated to a symbolic form. Although there are challenges to developing algebraic reasoning through tasks involving generalizing and a risk in taking time to foster a deeper and richer understanding of the underlying mathematical structure, the risk is worth it. Using generalization tasks to encourage algebraic reasoning bridges the gap between arithmetic and algebra. There is no right or wrong way to achieve generalizations but rather power in the ownership of discovery and creativity that each student can call his or her own.

CHAPTER THREE

METHODOLOGY

3.0 Overview

This chapter discusses the methodology employed in the collection and analysis of data. The chapter is organized under the following headings: research paradigms, research approach, research design, population, sample and sampling technique, research instruments, data collection procedures and data analysis procedures.

3.1 Research Paradigms

A paradigm is a cluster of belief and dictates which scientists in particular discipline use to influence what should be studied, how research should be conducted and how results should be interpreted (Bryman, 2008). This study would adopt the research philosophy of positivism. Positivism posits that reality exists irrespective of knowledge of its existence (Grix, 2004) and perceives the social world as something revealed rather than constructed through research. Collins (2010) argued that positivism depends on quantifiable observations that lend themselves to statistical analysis. Positivist, therefore, subscribes to the application of natural science methods and practices which could be applied to research in social sciences (Denscombe, 2010). In view of this, social scientist can apply the natural sciences general principles and understanding of what is referred to as knowledge and the methods to knowledge creation (Potter, 2000). The advocates of this paradigm believe real knowledge can be gained through observation and experiments.

Positivists usually select a scientific method for knowledge production (Rahi, 2017). Positivism is also referred to as Scientific Method, Empirical Science, Post

Positivism and Quantitative Research (Rahi, 2017). Levine, Sober and Wright (2010) explained that reality stays stable of constant in positivism, and that can be viewed or represented by an objective. The same happens to the social world, for positivists. Since reality is context-free, various researchers working a given phenomenon at different times and location will converge at the same conclusion. Positivist methodology is very much focused on experimentation. Hypotheses about the casual relation between phenomena are brought out in propositional or question form. Empirical data is gathered and then analyzed and expressed in the form of a theory that describes the independent variable's effect on the dependent variable.

According to Rehman and Alharti (2016), the approach to analysis data is deductive; first, a hypothesis is suggested, and either accepted or rejected based on the results of statistical analysis. To Cohen, Manion and Morrison (2007), the objective is to measure, control, predict, constrict laws and ascribe causality. In practice, all these arguments culminate into what is often referred to as quantitative research.

3.2 Research Approach

This study employed a quantitative design in order to address the research questions. The quantitative approach is based on positivism and follows the principles and assumptions of research in the natural science. Quantitative research can be understood as a research strategy that places emphasis on "quantification in the collection and analysis of data (Bryman, 2008). Bryman (2012) defines quantitative research as a strategy that focuses on quantification of data in terms of their collection and analysis. Quantitative research is formal, objective, rigorous, deductive approach, and systematic strategies for generating and refining knowledge to problem solving (Kivunja & Kuyini, 2017). It is an approach involving the use of numbers, and it is a means for testing theories by examining the relationship between and among variables (Creswell, 2004).

This variable is usually measured on instruments, so that numerical data can be analysed statistical procedures. The strategy to use quantitative approach is to make it easy for researcher to make broad generalizations out of the findings, based on the characteristics of the sample scientifically drawn from its population (Agbesinyale, 2003). The researcher felt it necessary to use quantitative approach because it tries to achieve objectivity in their research as it is one of most important pillars of generating knowledge through research (Creswell, 2009). Quantitative research employs 'measurement' as the ''most precise and universally accepted method for assigning quantitative values to the characteristics or properties of objects or events for the purpose of discovering relationships between variables under study (Koul, 2011).

The approach employs several statistical tools that allow for essay aggregation, categorization and comparison of research data. It makes it easy for researchers to make broad generalizations out of the findings, based on the characteristics of the sample scientifically drawn from its population (Agbesinyale, 2003). In other words, quantitative approach holds assumption about testing theories deductively, building in protections against bias, controlling for alternative explanations, and being able to generalize and replicate the findings (Creswell, 2009). Quantitative approach was driven by investigators with the need to quantify data. Since then quantitative research has dominated both local and western cultural as the research method to create new knowledge. This method was originally developed in the natural sciences to study natural phenomena (Joppe, 2016). In quantitative research a variable is a factor that can be controlled or changed in an experiment (Wong, 2014). It deals with quantifying and analyzing variables in order to get results. It is strictly positivistic, objective, scientific, and experimental. It should be used when a highly structured research design is needed and can be naturally imposed on the experiment being conducted. According to (Creswell, 2014), the researcher needs to be totally objective; is not part of what he/she observes, and does not bring his/her own interests, values, or biases to the research, and although the phenomena being captured may be complex, they can be broken down and assigned some types of numerical value.

Quantitative research methods deal with numbers and anything those are measurable in a systematic way of investigation of phenomena and their relationships. It is used to answer questions on relationships within measurable variables with an intention to explain, predict and control a phenomenon (Bryman, 2012). In quantitative research, researchers decide what to study, asks specific and narrow questions, collects quantifiable data from participants, analyzes these numbers using statics, and conducts the inquiry in an unbiased and objective manner. At present two-third research article are published by the use of quantitative data, which are highly valid and provide high level of research quality. The analysis of information from large samples almost inevitable requires quantitative methods (Yilmaz, 2013). According to Babbie (2015), statistical, mathematical or computational techniques are applied to obtain accurate results in quantitative research. Recently this type of research is widely used in business studies, natural science, mathematics science and social sciences.

The quantitative research data are collected through closed-ended questionnaires. The type of data is in numerical form, such as statistics, percentages, graphs, etc. The data are used to develop and employ models based on the form of mathematical models, theories, and hypotheses to obtain the desired result. According to Kumar (2019), a research hypothesis is an empirically testable statement that is generated from a proposition, which is clearly stated relation between independent and dependent variables. Creswell (2019) posited that the findings from quantitative research are to develop and use mathematical models, theories and hypotheses/propositions pertaining to phenomena. Collecting test scores of student’s transitions from arithmetic to algebra for the content strands could easily be done with quantitative measures.

3.3 Research Design

According to Amoani (2005), research design is an arrangement of conditions for collecting data which will be relevant to the researcher in the most economical manner. The study employed a quasi-experimental non-equivalent pre-test, post-test design. A quasi-experiment is an empirical study used to appraise the causal effect of an intervention (treatment) on a target population from which participants are not randomly assigned in groups (Creswell, 2014). Quasi-experimental research shares similarities with the traditional experimental design or randomized controlled trail, but it specifically lacks the element of random assignment to treatment or control. Instead, quasi-experimental designs typically allow the researcher to control the assignment to the treatment condition, but using some criterion other than random assignments.

Quasi-experiments are subject to concerns regarding internal validity, because the treatment and control groups may not be comparable at baseline. In other words, it may not be possible to convincingly demonstrate a causal like between the treatment condition and observed outcomes. This is particularly true if there are confounding variables that cannot be controlled or accounted for. With random assignment, study participants have the same chance of being assigned to the intervention group or the comparison group. As a result, difference between groups on both observed and unobserved characteristics would be due to chance, rather than to a systematic factor related to treatment (e.g., illness severity). Randomization itself does not guarantee that groups will be equivalent at baseline. Any change in characteristics post-intervention is likely attributable to the intervention.

3.4.1 Advantage of Quasi-Experimental Design

Quasi-experimental designs are used when randomization is impractical and/or unethical. So, they are typically easier to set up than true experimental designs, which require random assignment of subjects. Additionally, utilizing quasi-experimental designs minimizes threats to ecological validity as natural environments do not suffer the same problems of artificiality as compared to a well-controlled laboratory setting. Since quasi-experiments are natural experiments, findings in one may be applied to other subjects and settings, allowing for some generalizations to be made about population (Scott, 2012).

Also, this experimentation method is efficient in longitudinal research that involves longer time periods which can be followed up in different environments. Other advantages of quasi experiments include the idea of having any manipulations the experimenter so chooses. In natural experiments, the researchers have to let manipulations occur on their own and have no control over them whatsoever. Also, using self-selected groups in quasi-experiments also takes away to chance of ethical, conditional, etc. concerns while conducting the study (Scott, 2012).

However, quasi-experimental design are subject to contamination by confounding variables. The lack of random assignment in the quasi-experimental design method may allow studies to be more feasible, but this also poses many challenges for the investigator in terms of internal validity (Dinardo, 2008). Robson, Shannon, Goldenhar and Hale (2001), opined that this deficiency in randomization makes it harder to rule out confounding variables and introduces new threats to internal validity. Because randomization is absent, some knowledge about the data can be approximated, but conclusions of causal relationships are difficult to determine due to a variety of extraneous and confounding variables that exist in social environment.

Moreover, threats to internal validity means causation still cannot be fully established because the experiment does not have total control over extraneous variables. Disadvantages also include the study groups may provide weaker evidence because of the lack randomness. According to Wager, Stefan and Athey (2018), randomness brings a lot of useful information to a study because it broadens results and therefore gives a better representation of the population as a whole. Using unequal groups can also be threat to internal validity. If groups are not equal, which is sometimes the case in quasi experiments, then the experimenter might not be positive what the causes are for the results. These loopholes have been addressed in the subsequent sections of this chapter.

3.5 Setting

The study was carried out at Agona West Municipal in the Central Region of Ghana.

Agona Swedru is a small city in southern coastal part of Ghana, located about 65 miles north of Winneba. The city is the headquarters of a large district and is a home to about 70 thousand people. Agona Swedru is a commercial settlement, with a few large shopping centers and markets. There is a new large governmental hospital in the city. The choice of the schools is based on: Familiarity with Agona Swedru and easy access to the schools.

3.6 Population

Generally, a population is a set of elements, objects, people of which the researcher is interested in investigating in a given geographical area. According to Bryman (2008), it is the universe of units from which the sample is selected. Gerrig and Zimbardo (2000) see population as the entire set of individuals to which generalizations will be made based on an experimental sample.

3.7 Target Population

The part of the general population left after its refinement is termed target population, which is defined as the group of individuals or participants with the specific attributes of interest and relevance (Bartlett et al., 2001; Creswell, 2003). The target population is more refined as compared to the general population on the basis of containing no attribute that controverts a research assumption, context or goal (Asiamah, Mensah & Oteng-Abayie, 2017). The target population for this study was all basic seven (7) pupil's public schools in the Circuit B which is made up of ten (10) public basic schools in the Agona West Municipality of the Central Region of Ghana. In all 200 basic seven pupils' public basic schools were in the circuit B in the Agona West Municipality.

3.8 Accessible Population

The accessible population is reached after taking out all individuals of the target population who will or may not participate or who cannot be accessed at the study period (Bartlett et al., 2001). It is the final group of participants from which data is collected by surveying either all its members or a sample drawn from it. It represents the sampling frame (Bartlett et al., 2001), it intention is to draw a sample from it. The accessible population for this study consisted of basic (7) pupils' public school in circuit B in the Agona Swedru Municipality of the Central Region of Ghana.

3.9 Sample and Sampling Techniques

Sampling, according to Neuman (2014), is the selection of a small set of cases or units from a large pool, which allows a researcher to generalize findings to the population. Sampling is crucial in this study as it enables more accurate measurements, saves time and money, and is essential when the population consists of infinitely many or various members (Kothari, 2004). To ensure the study's validity and reliability, a stratified sampling technique was used.

Stratified sampling involves dividing the population into distinct subgroups or strata and randomly selecting samples from each stratum to ensure representation across key subgroups (Scott, 2012). For this study, the population was divided into two strata based on the schools within Circuit B: Methodist Junior High School and Anglican Junior High School. These schools were chosen to represent the experimental and control groups, respectively. In each school, basic seven pupils were selected to form the experimental and control groups. The experimental group comprised 52 pupils (37 boys and 15 girls) from Methodist Junior High School, while the control group included 48 pupils (28 boys and 20 girls) from Anglican Junior High School. Thus, the total sample size was 100 pupils. According to Castillo (2010), a sample size of 22 or more is considered large in statistical analysis, allowing for relevant discrete statistical and inferential analysis to be conducted. This stratified sampling technique ensures that the findings can be generalized to other subjects and settings while minimizing threats to ecological validity.

3.10 Research Instrument

Data for this study were collected using a pre-test and a post-test

3.10.1 Pre-Test

A pre-test was designed to investigate the equivalence of the experimental and control groups. This was administered to the learners in both the experimental and control group prior to the experiment. If the means of the performances of the two groups do not differ significantly, it can be assumed that the two groups are comparable. lLearners from each group were given 30 minutes to complete the pre-test. (See Appendix B for Pre-Test questions)

3.10.2 Post-Test

A post-test was designed and administered at the end of the experiment to pupils in both the experimental and control groups. If the mean performance of the experimental group is statistically significantly different from the mean performance of the control group, it can be assumed that the performance of pupils must have been influenced by the use of number pattern. Learners from each group were given 60minutes to complete the post-test. The same evaluation scheme that was used for the pre-test was used to evaluate each question in the post-test. (See Appendix C)

3.10.3 Treatment

The number patterns approach was applied to the experimental group whereas a traditional method of instruction was applied to the control group throughout this study. The number patterns refer to the teaching approach involving both the teacher-led demonstration and learners' hands-on activities using number patterns for examples a pattern includes adding two even numbers together to produce an even number, adding nine to a number results in an answer that is ten more than the given number minus one, and the next number in a pattern is one plus the previous number. This means that the treatment in the experimental group was affected by collaborative learning unlike the control group where all lessons were taught using teacher-centered approach.

3.10.4 Pilot Testing

The instrument was field tested after it was improved based on the outcome of the first test, advice from professionals to meet standard and later approved by professionals including my supervisor from the Department of Basic Education in the University of Education, Winneba. The test was administered to the selected basic school pupils in the Gomoa Central District. In all 64 basic school pupils participated in the pilot test.

3.11 Reliability of the Test

At the end of the pilot testing exercise of the instrument on 64 basic school pupils, the total scores obtained by each of the pupils ranged between 0-27 out 35 items. Although, 60 minutes was allocated to the answering of the pilot test instrument, it took them approximately 45 minutes to complete the 35 items on the instrument. A re-test was conducted and the reliability was calculated on the scores of 30 of the participants and the correlation was 0.801. Again, the reliability of the pilot test was calculated using the KR-20 formula and Cronbach's alpha was found to be 0.798. These estimates agree with Nyurmally and Bernstein (1994) and Vaaske's (2008) recommendation that reliability coefficients in the .65-.80 range are 'adequate' and acceptable. The difficulty and discrimination indices were also calculated to affirm the validity of the instrument. The pilot test enabled the researcher to effect few changes including the duration for answering the 35 items on the instrument. The reliability coefficient of the final instrument was 0.808.

3.12 Validity of the Test

According to Ary, Jacobs and Razavieh (1990), the term validity refers to the extent to which an instrument measures what it intends to measure. Validity addresses the following two questions (De Vos, 2002). What does the research instrument measure? What do the results mean? The core essence of validity is captured nicely by the word accuracy. From this general perspective, a researcher's data are valid to the extent that results of the measurement process are accurate. The following process was implemented to ensure the validity of the research instrument:

The validity of the tests was also established by two experienced mathematics lecturers who were also mathematics examiners as they reviewed the face validity, content, clarity, construct validity, correctness and standard of questions with regard to the pupil's level. Pilot testing is very helpful as it makes a researcher aware of any possible unforeseen problems that may emerge during the main investigation (Ntsohi, 2005). Based on the opinions and comments I got from the teachers and lecturers and the pilot testing, the instruments were amended. Therefore, after the wide consulting of experts, incorporating their opinions and comments as well as pilot testing, it may be concluded that the instruments portray the desired level of construct validity.

3.13 Threat to Internal Validity

Internal validity is an important construct that is useful in research design and psychological testing and is considered the "basic minimum without which any experiment is uninterpretable (Campbell & Stanley, 1963). There are a variety of factors that may impact internal validity that can be related to the researcher (test administrator), research participant (test taker) and the environment in which the research (test) is conducted. These threats to internal validity basically decrease the likelihood that the results of the experiment are due to relationships among/between the independent and dependent variables.

3.13.1 Maturation Threat

The maturation threat can operate when biological or psychological changes occur within subjects and these changes may account in part or in total for effects discerned in the study. This describes the impact of time as a variable in a study. If a study takes place over a period of time in which it is possible that participants naturally changed in some way (grew older, became tired), then it may be impossible to rule out whether effects seen in the study were simply due to the effect of time.

3.13.2 Testing Threat

The testing threat may occur when changes in test scores occur not because of the intervention but rather because of repeated testing. A testing threat is the tendency for most people to perform better on a task after having experienced the same or a similar task in the past. Some potential explanations for testing threat include: learning (e.g., test takers could develop strategies for some questions), practice (i.e., familiarity with the test format), social desirability (i.e., participants' answers may change because they learn what was socially expected after the first testing), and attitude polarization (i.e., attitudes become more extreme after people have been asked to think about them). This is of particular concern when the researcher administered identical pre-tests and post-tests.

3.13.3 Instrumentation Threat

Instrumentation can be a threat to internal validity because it can result in instrumental bias (or instrumental decay). Such instrumental bias takes place when the measuring instrument (e.g., a measuring device, a survey, interviews/participant observation) that is used in a study changes over time. Instrumentation becomes a threat to internal validity when it reduces the confidence that the changes (differences) in the scores on the dependent variable may be due to instrumentation and not the treatments (i.e., the independent variable). It sometimes helped the researcher to think about instrumental bias arising either because of the use of a physical measuring device or the actions of the researcher.

3.13.4 Regression to the Mean

Statistical regression or regression towards the mean can be a threat to internal validity because the scores of individuals on the dependent variable may not only be the due to the natural performance of those individuals, but also measurement errors or chance. When these scores are particularly high or low i.e., they are extreme scores, there is a tendency for these scores to move i.e., regress towards the mean i.e., the average score: so an individual with an extremely high score during the pre-test measurement of an experiment gets a lower score on the post-test measurement; and vice versa, with an individual with an extremely low score during the pre-test measurement getting a higher score on the post-test measurement.

3.13.5 Differential Selection

The selection threat is of utmost concern when subjects cannot be randomly assigned to treatment groups, particularly if groups are unequal in relevant variables before treatment intervention. In the quasi-experimental research, the researcher needed to make sure that the groups are equivalent before you start or there could be difference between the treatment and control groups (i.e., before any interventions are made), which may explain the differences in scores on the dependent variable.

3.13.6 Attrition

Attrition, withdrawals or dropouts is a problematic when there was differential loss of subjects from comparison groups subsequent to randomization, resulting in unequal groups at the study's end. Experimental mortality became a threat to internal validity when the number of dropouts across the comparison groups (i.e., the treatment group and control group) is different.

3.14 Minimization to Threat to Validity

There was a feasible time within the evaluation questions, reducing the amount of time between the pretest and posttest that limit maturation threats. The same duration for pre-test and post-test was given to the control group and the experimental group also the structure of the question was the same as their scoring key. Instrumentation threats was reduced or eliminated by making every effort to maintain consistency at each test. Selecting of participants based on extreme performance or scores was avoided. Intact class was used with a nonequivalent comparison group. Good tracking system with detailed contact information was used to check for pupil's dropout from the study by using their school register log book etc. also project identity was created for pupil's identification.

Treatments/Interventions

This study employed two different treatments. The treatment for the experimental group was the use of number pattern while the control group was taught and by traditional method. However, the material to be taught and learnt was the same for the two groups; it was the modes of delivery that were different. The content material on the topic was given to two mathematics teachers for review. This was done to ensure that the content conformed to what has been prescribed by the junior high school syllabus. The teaching for both groups lasted for four weeks. During the four weeks project, the activities and contents were the same for both groups.

Scheme of work for the Implementation Stage

Week	Week ending	Day	Intervention	Activity	Duration/Time
1	10-06-22	Monday 06/06/22		General pre-test	60 minutes 4:00 - 5:00pm
		Thursday 09/06/22	1	Predicting Subsequent Number in a Pattern.	60 minutes 4:00 - 5:00pm
2	17-06-22	Monday 13/06/22	2	Identify an Error in a pattern	60 minutes 4:00 – 5:00pm
3		Thursday 16/06/22	3	Identify the relation or rule in a pattern	60 minutes 4:00 – 5:00pm
4	24-06-22	Monday 20/06/22	4	Determining the algebraic rule for a numerical pattern.	60 minutes 4:00 – 5:00pm
		Thursday 23/06/22	5	Determining a number when given the algebraic rule	60 minutes 4:00 – 5:00pm
	01-07-22	Monday 27/06/22		General post test	60 minutes 4:00– 5:00pm

Week One: General Pre-Test

Day 1 : Monday

Date : 6th June, 2022

Time : 4:00pm to 5:00pm

Duration : 60 minutes

A pre-test was designed to investigate the equivalence of the experimental and control groups. This was administered to the learners in both the experimental and control group prior to the experiment. If the means of the performances of the two groups do not differ significantly, it can be assumed that the two groups are comparable. Learners from each group were given 30 minutes to complete the pre-test. (See Appendix B for Pre-Test questions)

Exercises

Q1: Which numerical pattern follows the rule ''subtract 2, then multiply by 3,'' when starting with 5?

a) 5, 7, 21, 69

b) 5, 2, 4, 1, 2

c) 5, 15, 13, 39, 37

d) 5, 3, 9, 7, 21, 19, 57

Q2. The first number in a pattern is 5. The pattern follows the rule ''Add 3''. What are the next 4 numbers in the pattern?

a) 1, 4, 7, 10

b) 5, 10, 15, 20

c) 8, 11, 14, 17

d) 3, 6, 9, 13

Q3. Which is the next ordered pair in the pattern? (10, 5), (8, 4), (6, 3)

a) (2, 4)

b) (2, 5)

c) (4, 2)

d) (5, 2)

Q4. John used the rule ''double the number'' to create the pattern 3, 6, 12, 24 … Which pair of numbers is part of the pattern?

a) 36, 72

b) 48, 96

c) 96, 144

d) 96, 192

Q5. Which statement about the third corresponding terms in both Pattern X and Pattern Y is true

Pattern X: start at 10 and add 5

Pattern Y: start at 10 and add 10

a) Both number values will be 20.

b) Both number values will be less than 20.

c) Both number values will be greater than 20.

d) The T value will be 20 and the X value will be more than 20

Q6. Which is the missing ordered pair in the pattern? (3, 9), (5, 15), (?, ?), (9, 27)

a) (6, 9)

b) (6,18)

c) (7, 10)

d) (7, 21)

Q7. A number pattern is shown as 100, 95, 110, 105, 120 … If this pattern continues, what number will come next?

a) 100

b) 115

c) 120

d) 130

Q8. What is the rule for the following pattern? 10, 7, 4, 1

a) Add 4

b) Add 8

c) Multiply by 2

d) Subtract 3

Q9. Select the missing number. 55, 54, 53, 52,__________

a) 56

b) 51

c) 50

d) 55

Q10. Continue the pattern. AB, ABB, ABBB

a) ABAB

b) AAB

c) ABBBB

Intervention One: Predicting Subsequent Number in a Pattern

Day 2 : Thursday

Date : 9th June, 2022

Time : 4:00pm to 5:00pm

Duration : 60 minutes

Starter

Pupils recite the multiples of two to create the idea of predicting a subsequent number in a pattern.

Activity

To be able to extend a give number, we should study to understand the rule of the pattern. The rule in a pattern serve as a guide. For instance 3, 6, 9, 12, 15 … From the pattern, the number increase by 3. To find the next term or number after 15, add 3 to get 18. That is 15 + 3 = 18, 18 + 3 = 21, 21 + 3 = 24, and 24 + 3 = 27. The next four terms of the pattern are 18, 21, 24, and 27. The rule for the pattern is ‘’add 3’’ or 3 must be added to the previous number or term to generate the next in the pattern.

Example 1:

Write the next 4 terms for the pattern. 2, 4, 6, 8, 10, 12 …

Solution

Observing the pattern, the numbers increasing by 2. To get the next term, 2 is added to a number to get the next number. Therefore, the pattern is 2, 4, 6, 8, 10, 12, 14, 16, 18, and 20.

Exercise

Extend the patterns by three terms and write the rule

1. 4, 8, 12, 16, …
2. 1, 3, 5, 7, 9, …
3. 20, 21, 22, …
4. 60, 63, 66, 69, …
5. 15, 20, 25, 30…

Week 2: Intervention Two: Identify an Error in a pattern

Day 1 : Monday

Date : 13th June, 2022

Time : 4:00pm to 5:00pm

Duration : 60 minutes

Starter

Pupil noticed that when you multiply one even number and an odd number the product is an even number. For instance $2 \times 3 = 6,\ 6 \times 5 = 30,\ 8 \times 9 = 72$ etc.

Activity

Errors in patterns occur when the wrong element or term is written. Before an error, in a pattern could be identified, the rule of the pattern should be understood.

Example 1

Kofi put pens into five groups. He put 3 pens in group1, 6 pens in group 2, 9 pens in group 3, 13 pens in group 4 and 15 pens in group 5. His sister, Ama noticed an error in the grouping. Identify the error and describe it.

Solution

Draw a table for the groups and number of pens in each group.

Group	1	2	3	4	5
Number of pens	3	6	9	13	15

From the table, the first three terms (3, 6, 9), the rule for the pattern could be described as ‘’3 must be added to the previous term’’. If the rule is ‘’add 3’’, then the fourth term should be 9 + 3 = 12. Kofi should put 12 pens in the fourth group but 13 pens. Therefore, the new table is

Group	1	2	3	4	5
Number of pens	3	6	9	12	15

Exercise

Identify the errors in the pattern and correct them. Complete the pattern

1.

Number of cars	1	2	3	4	5	6
Number of tyres	5	10	15	19		

2.

A	1	2	3	4	5	6	7
B	1	4	9	15	25		

3.

Triangle	1	2	3	4	5	6
Number of lines	3	5	7	10		

Intervention Three: Identify the relation or rule in a pattern

Day 2 : Thursday

Date : 16th June, 2022

Time : 4:00pm to 5:00pm

Duration : 60 minutes

Activity

Example

a) Copy and complete the table for hexagons

Hexagon (*x*)	1	2	3	4	5	6
Number of rods (*y*)	6	11	16			

b) Write the relation as ordered pairs

c) What is the rule of the relation

Solution

a)

Hexagon (*x*)	1	2	3	4	5	6
Number of rods (*y*)	6	11	16	21	26	31

b) (1, 6), (2, 11), (3, 16), (4, 21), (5, 26), 6, 31)

c) From the table above, there is an increase of 5 for each value of y plus 1

1 hexagon = $5 \times 1 + 1 rods$

2 hexagon = $5 \times 2 + 1 rods$

3 hexagon = $5 \times 3 + 1 rods$ etc.

Week 3: Intervention 4: Determining the algebraic rule for a numerical pattern

Day 1 : Monday

Date : 20th June, 2022

Time : 4:00pm to 5:00pm

Duration : 60 minutes

Activity

An algebraic rule for a numerical pattern can be describe by stating the algebraic rule or formula. This rule will serve as a definition for the relation pattern.

Example

What is the algebraic rule for this relation pattern?

X Y

1 ---→ 1

2 ---→ 3

3 ---→ 5

4 ---→ 7

5 ---→ 9

Solution

By inspection each member is related onto twice itself minus 1. Therefore if x stands for any number then, the rule for the relation pattern is x ---→ $2x - 1$.

$x = 1$ ---→ $2(1) - 1 = 1$

$x = 2$ ---→ $2(2) - 1 = 3$

$x = 1$ ---→ $2(3) - 1 = 5$

$x = 1$ ---→ $2(4) - 1 = 7$

Intervention 5: Determining a number when given the algebraic rule

Day 2 : Thursday

Date : 23rd June, 2022

Time : 4:00pm to 5:00pm

Duration : 60 minutes

Activity

Under this intervention, each number is substituted into the given algebraic rule to obtain the relation pattern.

Example

A relation pattern is defined by the algebraic rule $x \dashrightarrow 2x + 1$. If the number is {0, 1, 2, 3} find the relation pattern of the ordered pairs using the algebraic rule.

Solution

Using the algebraic rule find the ordered pair of the relation pattern

$x \dashrightarrow 2x + 1$

$0 \dashrightarrow 2(0) + 1 = 1$

$1 \dashrightarrow 2(1) + 1 = 3$

$2 \dashrightarrow 2(2) + 1 = 5$

$3 \dashrightarrow 2(3) + 1 = 7$

Therefore, the relation pattern is {1, 3, 5, 7}

Week 4 : General Post-Test

Day 1 : Monday

Date : 27th June, 2022

Time : 4:00pm to 5:00pm

Duration : 60 minutes

A post-test was designed and administered at the end of the experiment to pupils in both the experimental and control groups. If the mean performance of the experimental group is statistically significantly different from the mean performance of the control group, it can be assumed that the performance of pupils must have been influenced by the use of number pattern. Learners from each group were given 60minutes to complete the post-test. The same evaluation scheme that was used for the pre-test was used to evaluate each question in the post-test. (See Appendix C)

3.16 Data Collection Procedures

Permission was sought from the Heads of the schools to conduct the research in their schools. Consent was also sought from the class teachers. The pre-test was administered to both the control and experimental groups after permission had been given. There were two different treatment patterns that were applied during the experiment. The control group was taught through the traditional approach while the number pattern was used for the experimental group. Right after the teaching of the control and experimental groups which lasted for four weeks, the post-test was given to both groups.

3.17 Data Analysis Method

Data were analyzed using descriptive statistics such as means and standard deviation were used to describe the general performance of learners in both groups in the pre-test and post-test. The independent sample t-test was used to find whether the performance of the learners within each group improved or not while the effect size (eta statistics square) was used to determine the magnitude of improvement in each group. The purpose was to determine whether there was statistically significant difference between each student score in the pre-test and post-test.

3.18 Test Analysis

During the week, one group of learners was instructed in number pattern (Experimental group) and another group in the Control group. Both groups were studied on their retaining of transition from arithmetic to algebra concepts instructed in the both Experimental group and Control group during the week. To guarantee the two groups were equivalent and to decrease the possible for a type 1 error, a Levene's Test for Equality of Variances was conducted using the data from the Mathematic Achievement Test (MAT). The fallouts signposted that the groups did not differ significantly as $p>0.05$.

To determine if the treatment, in this case the use of the number pattern in transition from arithmetic to algebra teaching affects a difference in student number and algebra, an independent samples t-test was performed using MAT scores at end of the week. An independent samples t-test was piloted on the post-test scores and the treatment group exposed a significant difference from the control group with $p<0.05$ in that the difference in scores between the Experimental group and the Control group was significant. It was emphatically demonstrated that the difference between the scores at beginning and at the end of the two groups were at the in favor of student instructed in number pattern.

An inferential statistical test was best appropriate for examining the data for the research questions. Through a t-test, groups mean scores were associated to determine if a significant variance exists between the two groups. From a statistical viewpoint an independent data item is categorical and the dependent variable continuous, analysis of both population means was used to determine an answer to the research questions. The achievement scores (pre-test and post-test within each group (Control and Experimental) were analysed using mean and standard deviation while the two groups were compared using independent sample t-test. To employ the t-test, four major assumptions had to be met. (Pallant, 2005).

To ensure normality, the Kolmogorov-Smirnov test was run. According to Pallant (2005), a non-significant result (significant value greater than 0.05) indicates normality. To satisfy the assumption of independence, the achievement test scores must either be taken from respondents in two separate groups or from specified respondents at different times (Morgan, Leech, Gloeckner, & Barrett, 2004). In the current study, each group (Control and Experimental) had its student's contributing achievement scores at different times/days. Also, scores were collected from learners who happened to be in two separate group (Control and Experimental). Thus, the study met the assumption of independence (Pallant, 2005). Homogeneity of variances, is the next assumption to be satisfied. According to Pallant (2005), a significant value greater than 0.05 indicates that the assumption of equal variances is not violated.

Again, Levene's Test of Equality of Variances is the last assumption to be satisfied. According to Pallant (2005), a significant value greater than 0.05 indicates that the assumption of equal variances is not violated. It can be concluded that all this assumption was not violated and thus, the independent sample t-test can be run to compare the means of the Post-test scores of learners in the Control and Experimental groups.

3.19 Ethical Consideration

The following ethnical consideration were considered for the study

3.19.1 Informed Consent

The researcher believe that informed consent implies the agreement to participate in research after learning about the study, including possible risks and benefits. This implies that participates must be aware of what the research entails and how they are going to benefit from the research. The schools signed the consent forms on behalf of the respondents. The participants were told about the general nature of the study as well as about any potential harm or risk that the study may cause.

3.19.2 Confidentiality

Confidentiality is explained as not disclosing information from participates in any way that might identify that individual or that might enable the individual to be traced. The researcher used coding abstracted data with unique identifiers rather than names.

3.19.3 Harm to participants

The researcher in this study made sure that participates were not exposed to physical, psychological and emotional harm. Sufficient information was provided to participate so that they could make informed decision. Data was not disclosed to any other person without the consent of participates. The researcher carried out a thorough risk/benefit analysis

CHAPTER FOUR

RESULTS, DISCUSSION AND FINDINGS

4.0 Overview

The aim of this study was to investigate the effect of number pattern on junior high school learners' transition from arithmetic to algebra. Three main research questions were raised namely: The following research questions were the research questions:

1. What statistical differences in scores are there among pupils taught with Number Pattern and without Number Patterns?
2. What statistical differences in scores are there among pupils taught with arithmetic-algebra connection and without arithmetic-algebra connection?
3. How are the differences in performance among pupils taught with algebraic reasoning and without algebraic reasoning?

This chapter presents the results of the data analyzed in the study and interpretation of results. The results were organized and presented using tables, figures, descriptive statistics (mean and standard deviation) and inferential statistics. The findings of the pre-test are presented first followed by the findings from the post-test.

4.1 Demographic Background of Junior High School Student

An equal number of learners with respect to gender were considered for the sake of gender equality and therefore 100 learners were selected for the study. The Table 1 below shows the gender of participants and their percentages.

Table 1: Gender of Participants

Gender	Frequency	Percentage
Male	58	58%
Female	42	42%
Total	**100**	**100%**

Field Work: 2022

Table 1 shows that 58% of the participants were male and 42% of the participants were female. The above data show that the male were more than the female.

Table 2: Age of Participants

Age	Frequency	Percentage
11	20	20%
12	28	28%
13	30	30%
14 and above	22	22%
Total	**100**	**100%**

Source: Field Work 2022

Table 2 show that 20 pupils representing 20% of the participants were within the age of 11 years, 28 pupils representing 28% were within the ages of 12 years, 30 pupils representing 30% were within the age of 13 and 22 pupils representing 22% were within the age group of 14 and above showing that the study included young junior high school learners who could use the knowledge to influenced their teaching and learning of Arithmetic and Algebra.

Table 3: Previous School Attended of Participants

Previous Sch.	Frequency	Percentage
Public	67	58%
Private	42	42%
Total	**100**	**100%**

Source: Field Work 2022

From the table 3 above, it shows that 67% attended public basic school and 33% attended private basic school and this reveals real interest of the participants in the study

Pre-Test

Pre-test is an initial measurement in order to identify any problem before an experimental treatment is administered and subsequent measurements are taken? Before post-tests was undertaken a general pre-test was conducted. The same questions were administered to both groups (control and experimental groups) one week before the intervention in order to check whether the two groups were of comparable ability before the interventions. Table 4 shows the descriptive statistics for pre-test for the two groups.

Table 4: Pre-Test Results

Groups		N	Mean	Std. Deviation
Pre-Test	Experimental Groups	52	27.54	4.812
	Control Groups	48	27.63	4.684

Source: Field Work 2022

Table 4 shows the descriptive statistics for the pre-test results for the two groups. The experimental group pre-test mean (M = 27.54; SD = 4.812), is slightly lower than that of the control group pre-test mean (M = 27.63; SD = 4.684). The experimental group had mean slightly lower than the control group and also recorded a slightly higher standard deviation in scores obtained.

Statically significance difference in scores were check using independent sample t-test to compare the mean achievement scores of the pre-test scores of the control and experimental groups. The test was test as 5% level *of significance.* The effect size statistics was also computed for an independent sample t-test to find out the strength of the differences in performance of both the experimental groups and control groups

The achievement of learners in the pre-test is shown in the Table 5.

Table 5: Independent Samples Test for Pre-Test Results

Test	Group	Mean	Std. Dev	T	Df	Sig.
Pre-test	Experimental	27.54	4.812	-.091	98	0.928
	Control	27.63	4.684			

Source: Field Work 2022

Table 5 shows an independent sample t-test conducted to compare experimental group and control group. The results indicate that there was no statistically significant difference in scores for experimental group (M = 27.54; SD = 4.812) and control group (M = 27.63; SD = 4.684); t (98) = -.091, p =.928). The magnitude of the difference in the means was very small (eta square = 0.008). According to Cohen (1988), the eta square of 0.01 is considered small effect; 0.06 is considered moderate effect and effect size of 0.14 is considered large effect.

Research Question 1: *What are the differences in performance among pupils taught with Number Pattern and without Number Patterns?*

In this section the findings of the post-intervention test (post-test), which was administered after the intervention, are presented and used to address the research questions 1

Tables 6: Pupils Taught with Number Pattern and without Number Patterns

	Groups	N	Mean	Std. Deviation
Post-Test	Experimental Groups	52	35.13	1.596
	Control Groups	48	29.23	4.099

Source: Field Work (2022)

Table 6 shows the descriptive statistics for the post-test results for the two groups. The experimental group post-test mean (M = 35.13; SD = 5.885), is slightly higher than that of the control group post-test mean (M = 29.23; SD = 7.741). The experimental group had mean slightly higher than control group and recorded a slightly lower standard deviation in scores obtained.

Research Question 2: *What are the differences in performance among pupils taught with arithmetic-algebra connection and without arithmetic-algebra connection using number patterns?*

In this section the findings of the post-intervention test (post-test), which was administered after the intervention, are presented and used to address the research questions two.

Table 7: Pupils Taught with and without Arithmetic-Algebra Connection

Groups		N	Mean	Std. Deviation
Post-Test	Experimental Groups	52	38.15	5.085
	Control Groups	48	22.83	4.991

Source: Field Work 2022

Table 7 shows the descriptive statistics for the post-test results for the two groups. The experimental group post-test mean (M = 38.15; SD = 5.085), is higher than that of the control group post-test mean (M = 22.83; SD = 4.991). The experimental group had mean slightly higher than control group and recorded a slightly lower standard deviation in scores obtained. Signifying that the experimental group performs far better than the control group.

Research question 3: *What are the differences in performance among pupils taught with algebraic reasoning and without algebraic reasoning using number patterns?*

In this section the findings of the post-test, which was administered are presented and used to address the research questions three.

Table 8: Pupils Taught with and without Algebraic Reasoning

Test	Groups	N	Mean	Std. Deviation
Post Test	Experimental Groups	52	65.69	9.411
	Control Groups	48	50.46	9.631

Source: Field Work (2022)

Table 8 shows the descriptive statistics for the post-test results for the two groups. The experimental group post-test mean (M = 65.69; SD = 9.411), is higher than that of the control group post-test mean (M = 50.46; SD = 9.631). The experimental group had mean higher than control group signifying that the experimental group performs far better than the control group.

Test of hypothesis

The hypothesis were to confirm whether the mean scores of the Post-Test are statistically significant or not The independent sample t-test was run to compare the mean achievement scores of the post-test scores of the control and experimental groups. The hypothesis was test as a 5% level of significance. The effect size statistics was also computed for an independent sample t-test to find out the strength of the differences in performance of both the experimental groups and control groups.

H_{01}. There is no statically significance difference in performance among pupils taught with Number Pattern and without Number Patterns.

Table 9: Independent Samples Test

Test	Group	Mean	Std. Dev	t	df	Sig.
Post-test	Experimental	35.13	1.596	9.248	98	0.000
	Control	29.23	4.099			

Source: Field Work (2022)

Table 9 shows an independent sample t-test conducted to compare the performance among the experimental group and control group. The results indicate that there was statistically significant difference in performance for experimental group (M = 35.13; SD = 1.596) and control group (M = 29.23; SD = 4.099); t (98) = 9.248, p = 0.000). The magnitude of the difference in the means was very large effect (eta square = 0.466).

According to Cohen (1988), the eta square of 0.01 is considered small effect; 0.06 is considered moderate and effect size of 0.14 is considered large effect.

H_{02}. *There is no statically significance difference in performance among pupils taught with arithmetic-algebra connection and without arithmetic-algebra connection using number patterns.*

Table 10: Independent Samples Test

Test	Group	Mean	Std. Dev	t	Df	Sig.
Post-test	Experimental	38.15	5.085	15.187	98	0.000
	Control	22.83	4.991			

Table 10 reveals the on the differences in performance among pupils taught with arithmetic-algebra connection and without arithmetic-algebra connection using number patterns. Results from Table 10, indicated that there is a statistically significant difference in the experimental groups and control groups. This is observed from the results as (M = 38.15; SD = 5.085) was found for experimental groups; (M = 22.83; SD = 4.991) was found for control groups; t (98) = 15.187, p< 0.05, (p = 0.000). This means that there is difference in the experimental groups and control groups. Therefore, the researcher rejects the null hypothesis. Hence, the result is statistically significant. The magnitude of the difference in the means was very large effect (eta square = 0.701).

H_{03}. There is no statically significance difference among pupils taught with algebraic reasoning and without algebraic reasoning using number patterns

Table 11: Independent Sample Test

Test	Group	Mean	Std. Dev	T	Df	Sig.
Post-test	Experimental	65.69	9.411	7.997	98	0.000
	Control	50.46	9.631			

Source: Field Work (2022)

Table 11 shows the results on the difference in the effect of number patterns on teaching and learning transition from arithmetic to algebra. Results from Table 11, indicated that there is statistically significant difference in the effect of number patterns. This is observed from the results as (M = 65.69; SD = 9.411) was found for experimental groups in the effect of number patterns on teaching transition from arithmetic to algebra; (M = 50.46; SD = 9.631) was found for the control groups; t (98) = 7.997, p <0.05, (p = 0.000). This means that there is statistically significant difference in the effect of number patterns on teaching and learning transition from arithmetic to algebra. Therefore, the researcher rejects the null hypothesis. Hence, the result is statistically significant. The magnitude of the difference in the means was very large effect (eta square = 0.394). According to Cohen (1988), the eta square of 0.01 is considered small effect; 0.06 is considered moderate and effect size of 0.14 is considered large effect.

4.2 Discussion of Results

Before post-tests was undertaken a general pre-test was conducted, the data was analyzed using independent sample t-test, while statistically inference was taken at 0.05 alpha levels (95% confidence interval). The results are displayed in Tables 4 and 5. The results show that there is no statistically significant difference in pre-test marks of experimental group and control group with eta square of 0.008 showing small effect size of the strength of the difference in performance. On the basis of these findings, it was concluded that there was no statistically significant difference between the performances of learners exposed to number pattern compared to learners taught without number pattern. This finding totally corroborates with findings of Aburime (2007) whose study groups in pre-test did not reveal significant difference.

In the first research question, the data was analyzed using mean and standard deviation. The results is displayed in Tables 6. The experimental group post-test mean was slightly higher than that of the control group post-test mean. This result is similar to the studies of Aburime (2007), Akkus (2004), Battle (2007), Doias (2013), Gurbuz (2010), Gurbuz and Toprak (2014), and Ogg (2010) whose findings indicated that learners who used manipulatives performed better than their counterparts who were taught without any manipulative.

In the second research question, the data shows that the experimental group post-test mean was higher than that of the control group post-test mean. The experimental group had mean slightly higher than control group and recorded a slightly lower standard deviation in scores obtained. On the basis of these findings, it was concluded that the experimental group perform better than that of the control group who were exposed to number pattern compared to learners taught without number pattern. This study is in line with Saraswati et al. (2016), where they also found that learners' post-test results were far better than their pre-test.

In the research question three the experimental group post-test mean is higher than that of the control group post-test mean. The experimental group had mean higher than control group and recorded a slightly lower standard deviation in scores obtained. The results of the third research question did not contradict the bulk of literature. The effect of increased achievement in transition from arithmetic to algebra has confirmed the assertion that some forms of number pattern activities have the ability to improve intellectual performance, spatial reasoning and other learning enhancing related skills (Gurbuz & Toprak 2014). Although Gurbuz and Toprak targeted 58 7th grade learners were used as the sample; had Experimental and Control groups; and researched into designing, implementing and evaluating activities that enable 7th grade learners to make transition from arithmetic to algebra. The outcomes of the current study also agree with that of Aburime (2007), Akkus (2004), Battle (2007), Doias (2013), Gurbuz (2010), Gurbuz and Toprak (2014), and Ogg (2010).

The effects of number pattern integrated arithmetic to algebra lesson had on the attitudes of learners towards transition from arithmetic to algebra as well as their performance in transition from arithmetic to algebra were not by chance. SOLO taxonomy by (Biggs and Collis, 1982; Collis and Biggs, 1986) underpinned the entire lesson structure. For instance, Damopolii, (2020) posit that the Solo Taxonomy can be used to enhance the quality of learning within the classroom teaching and provide a systematic way of developing deep understanding. Thus, the researcher adopted different strategies such us group assignment, individual presentations as well as inclusion of number pattern into the lesson to broaden the range of talents, skills and abilities.

The findings of the study, particularly the significant improvement in the experimental groups following the intervention, can be analyzed through the lens of the Structure of Observed Learning Outcomes (SOLO) taxonomy. The SOLO taxonomy, developed by Biggs and Collis (1982), describes levels of increasing complexity in students' understanding of subjects. It categorizes learning outcomes into five levels: pre-structural, uni-structural, multi-structural, relational, and extended abstract.

Initially, both the experimental and control groups demonstrated similar abilities, as indicated by the lack of a statistically significant difference in their pre-test scores. This suggests that, prior to the intervention, the students were predominantly at the same SOLO level, likely within the pre-structural or uni-structural stages, where understanding is either lacking or involves simple, isolated aspects of the content.

After the introduction of the intervention, significant improvements were observed in the experimental group. This aligns with the students' progression through the SOLO levels:

1. ‘Multi-structural Level:’ Initially, students might have begun to recognize multiple elements of algebraic concepts independently. The intervention likely helped them identify and understand these discrete components, moving them from the uni-structural to the multi-structural level.

2. ‘Relational Level:’ As the intervention progressed, students in the experimental group started to integrate these separate elements into coherent structures. They began to see relationships between arithmetic and algebra through the use of number patterns, indicating a shift to the relational level. This is where students can combine facts and procedures into a meaningful whole, demonstrating a deeper understanding.

3. ‘Extended Abstract Level:’ For some students, the intervention might have further propelled their understanding to the extended abstract level. Here, students not only integrate and apply their knowledge but also generalize it to new and complex situations. They could potentially apply algebraic reasoning to novel problems, reflecting the highest level of understanding in the SOLO taxonomy.

The statistically significant difference between the experimental and control groups, with a p-value of 0.000, underscores the effectiveness of the intervention. The very large effect size indicates that the intervention had a profound impact on the students' learning outcomes. This substantial improvement suggests that the teaching method effectively moved students through the SOLO levels more rapidly and comprehensively than traditional methods.

The findings support the use of structured, pattern-based approaches in teaching algebra, as these methods align well with the progression of understanding outlined in the SOLO taxonomy. By systematically building on students' existing knowledge and encouraging the integration of new concepts, educators can facilitate deeper, more meaningful learning experiences. This approach not only helps students grasp algebraic concepts more effectively but also equips them with the cognitive tools to apply their knowledge in broader contexts.

In conclusion, the significant improvement in the experimental group's performance can be attributed to their progression through the SOLO levels, facilitated by the structured intervention. This highlights the importance of adopting teaching methods that align with cognitive development theories, ensuring that students develop a comprehensive and relational understanding of complex subjects like algebra.

Chapter Summary

This chapter highlighted the findings of the study. The study revealed that Basic Seven (7) learners of the two schools had misconceptions about Number and Algebra as a mathematics content area. To most of them, Number and Algebra are not related. This view affected their general attitude towards transition from arithmetic to algebra. When number pattern were used learners were familiar with, learners' attitude toward number and algebra and its related topics was enhanced so much that, their views about transition from arithmetic to algebra were changed positively.

Beyond the positive effect such as number pattern had on learners' general attitude, their performance in the transition from arithmetic to algebra tests was enhanced. The gains for participants of transition from arithmetic to algebra lessons supported with number pattern was more than that for learners who took part in transition from arithmetic to algebra lessons without the integration of number pattern.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Overview

This chapter summarizes the study and major findings of the study. It also looks at the conclusions drawn out of the main issues concerning the results and further makes recommendations.

5.1 Summary

This study was conducted to investigate the effect of number pattern on junior high school student's transition from arithmetic to algebra in some selected schools in Agona West Municipality in the Central Region of Ghana. Three research questions and three hypotheses were developed to guide the study. The three research questions aimed at addressing the three research objectives, while the three hypotheses aimed at addressing the statistically significant between the research questions.

The study employed quantitative approach. The quantitative approaches were adequate to exhaustively answer the research questions and to test the hypotheses. To provide an appropriate understanding of the effects of the teaching approach on learners' transition from arithmetic to algebra and achievement, quasi-experimental non-equivalent pre-test post-test control group was adopted. In all 52 and 48 Basic Seven (7) learners were used for the Experimental and Control groups respectively. For data collection, teacher made transition from arithmetic to algebra achievement test (pre-test and post-test) were used for the quantitative data. Scores from the transition from arithmetic to algebra achievement tests were processed and analysed using SPSS software.

Descriptive statistics such as mean scores and standard deviations were used to describe each of the research questions. Independent sample t-test was used to find out if there was differences in the mean scores of the Control and Experimental groups.

5.2 Key findings

The key findings outline the answers the research questions and the results of the hypothesis of the study. The findings on the performance of the learners revealed that at the initial stage both the experimental groups and control groups have similar ability before the treatment because there was no statistically significant difference in the pre-test for experimental groups and control groups as such, any differences in performance could be attributed to the interventions.

Majority of the learners in the experimental groups improve significantly when the intervention was introduced when teaching transition from arithmetic to algebra with number pattern showing that there was a statistically significant difference between the Experimental groups and Control groups at confidence interval of p- value of 0.000 the magnitude of the difference in the mean considered to be very large effect.

5.3 Conclusion

1. (a) Basic seven (7) learners have perceptions that arithmetic and algebra are isolation and cannot be linked. Hence, they made little or no progress when solving algebra problems using number patterns

(b) Since the number patterns influence the perception of Basic 7 learners toward algebra, their general achievement scores were affected. This implies that, Basic 7 learners' participation in transition from arithmetic to algebra lessons supported with number patterns affect their attitudes.

2. Teaching strategies that target the affective domain of Basic 7 learners have higher chances of enhancing positive attitudes that could result in achievements irrespective of how underachieving the learners may be.

3. Teachers should use variety of teaching strategies including number patterns and physical activities to motivate learners to practice and increase their confidence in recalling their basic number facts.

5.4 Recommendations

The recommendations made in this study were based on the study findings.

1. A majority of the study school's mathematics teachers are not trained to use number pattern in their teaching and assessment. Thus, they need to be trained to use number pattern as tools in mathematics classroom in order to have confidence in incorporating into their mathematics programs in the Agona West Municipality.
2. Teachers in the Agona West Municipality should try as much as possible to teach both arithmetic and algebra together but not in isolation. This may help learners to transition smoothly between number patterns and algebra
3. Mathematics teachers in the Agona West Municipality should employ activities outside the classroom which have influence on teaching and learning of Algebra. This may help learners to apply Algebra in their everyday activities.

5.5 Areas for Further Studies

It is suggested that, further study should be conducted on the effect of:

1. Number pattern at the various level of education in Ghana.
2. Unavailability of teaching and learning resources on the arithmetic and algebra should be conducted.

REFERENCES

Abend, G. (2008). The Meaning of Theory. San Francisco, CA: Berrett-Koehler Publishers.

Aburime, F. E. (2007). How manipulatives affect the mathematics achievement of students in Nigerian schools. *Educational Research Quarterly*, 31(1), 3-15.

Agbesinyale, D. (2003). *Research design: qualitative, quantitative and mixed methods approaches, (2nd ed).* London: Sage Publication.

Akkus, O. (2004). *The effects of multiple representations-based instruction on seventh grade students' algebra performance, attitude toward mathematics, and representation preference* [Unpublished doctoral dissertation]. Middle East Technical University.

Amissah, P. K. A., Frimpong, S. O. & Sam-Tagoe, J. (2009). *Psychology of Human Development and Learning.* Accra. Emmpong press.

Amoani, F. R. (2005). *Research Methodology.* An Overview. Winneba: University of Education.

Anthony, T. & Walshaw, D. (2014). The link between middle school mathematics course placement and achievement. *Child Development, 85(5)*, 1948-1964.

Ary, T. J., Jacobs, N., & Razavieh, Y. A. (1990). Best practice in problem analysis. In P. Harrison and A. Thomas (Eds.), *Best Practice in School Psychology VI (pp. 87-98)*. Washington, DC: National Association of School Psychologists.

Asante, K. O. (2010). Sex differences in mathematics performance among senior high students in Ghana. Retrieved July 27, 2011, from http://www.faqs.org/periodicals.

Asiamah, H., Mensah, Y., & Oteng-Abayie, R. (2017). *Understanding Psychology 7th* E. New York: McGraw Hill Higher Education.

Babbie, E. (2015). *The practice of social research* (7th Ed.). New York: International Thomson Publishing Company.

Ball, D. L. (1988a). *Research on teaching mathematics: Making subject matter knowledge part of the question.* East Lansing, MI: National Center for Research on Teacher Education.

Ball, D. L. (2003a). *Mathematical proficiency for all students: Toward a strategic research and developmental program in mathematics education.* Santa Monica, CA: RAND.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In Boaler (Ed.) *Multiple perpectives on the teaching and learning of mathematics (pp. 83-104).* Westport, CT: Ablex.

Bartlett, A. L., Britner, S. L., & Pajares, F. (2001). A comparative study of the self-efficacy beliefs of successful men and women in mathematics, science, and technology careers. *Journal of Research in Science Teachinh, 45(9),* 1036-1058.

Battle, T. S. (2007). *Infusing math manipulatives: The key to an increase in academic achievement in the mathematics classroom*. https://files.eric.ed.gov/fulltext/ED498579.pdf

Bekdemir, A. & Isik, K. (2007). T*he learning and teaching of algebra ideas, insights, and activities* 1st ed. New York: Routledge

Biggs, J. and Collis, K. F. (1986). *"Using the SOLO taxonomy"*. Set: Research Information for Teachers, 2 (4).

Biggs, J., & Collis, K. (1989). Towards a model of school-based curriculum development and assessment using the SOLO taxonomy. *Australian journal of education, 33(2), 151-163.*

Blanton, M. L., Boggan, M., Harper, S., & Whitmire, A. (2007). *Algebra and the elementary classroom: transforming thinking transforming practice.* Portsmouth: Heinermann.

Bock, A. M., Cartwright, K. B., McKnight, P. E., Patterson, A. B., Shriver, A. G., Leaf, B. M., Mohtasham, M. K., Vennergrund, K. C., & Pasnak, R. (2018). Patterning, reading, and executive functions. Frontiers in Psychology, 9, 1802. https://doi.org/10.3389 fpsyg.2018.01802

Bryman, A. (2008). *Social Research Methods, (3rd ed.),* New York: Oxford University Press.

Bryman, A. (2012). *Quantity and quality in social research*. London: Rutledge

Bryman, A. (2012). *Social Research Methods, (3rded.),* New York: Oxford University Press.

Burgoyne, E., Trigueros, M., and Ursini, S. (2019). Research on the learning and teaching of algebra, in *the second Handbook of Research on the Psychology Mathematics Education*, Ă. Gutiḗrrez, G. C. Leder, and P. Boero, Eds. (Rotterdam: Sense Publishers) pp. 73-108.

Burgoyne, K., Witteveen, K., Tolan, A., Malone, S., & Hulme, C. (2017). Pattern understanding: Relationships with arithmetic and reading development. *Child Development Perspective*, 11 (4), 239-244. https://doi.org/10.1111/cedep.12240 CA: SAGE Publications.CA: SAGE Publications.

Cai, M. K., & Moyer, A. F. (2008). Algebra A challenge at the Crossroads of Policy and Practice. *Review of Educational Research, 81(4),* 453-492.

Campbell J. R. (2005). Connecting Mathematics achievement to parental influence. St. John's University. Retrieved January 19, 2011 from http//www.stjohns.edu.

Campbell, G. & Stanley, A. (1963). Student characteristics, pre-college, college, and environmental factors as predictors of majoring in and earning a STEM degree: An analysis of students attending a Hispanic Serving Institution. *American Educational Research Journal*, *46(4)*, 924-942.

Campbell, K. (1992). Volume measurement and intellectual development. *Journal of Structural Learning and Intelligent Systems, 11*.

Carpenter, T. P., & Franke, M. L. (2000). Developing algebraic reasoning in the elementary school: Generalization and proof. In H. Chick, K. Stacey, J. Vincent & J. Vincent (Eds.), *Proceedings of the 12th ICMI Study Conference: The Future of the Teaching and Learning of Algebra (Vol. 1, pp. 155-162).* Melbourne, Australia.

Carraher, M., Perry, L., & Brombacher, A. (2008). Students' Early Grade Understanding of the Equal Sign and Non-standard Equations in Jordan and India. *International Journal of Research in Education and Science, 4(2*), 656- 669. Retrieved from: https://doi.org/10.21890/ijres.432520.

Castillo, J.J. (2010). *Research Population.* Retrieved August 18, from http//www.

Center for Algebraic Thinking (2020). Examining early algebraic thinking: insights from empirical data *Educ. Stud. Math*. *98 57-76.*

Chaiklin, S., & Lesgold, S. (1984). *Prealgebra student's knowledge of algebraic tasks with arithmetic expressions*. Paper presented at the annual meeting of the American Research Association.

Clements, D., & Samara, J. (2011). Early childhood mathematics intervention. *Science 333*, 968-970. https://doi:10.1126/science.1204537

Cohen, L., Manion, L. and Morrison, K. (2007). *Research methods in education*. New York: Routledge.

Collins, J. (2010). *Educational research: an introduction, (8th Ed.)* Boston: Pearson International Edition

Collins, M. & Laski, E. (2015). *Preschoolers' strategies for solving visual pattern tasks*. Early Childhood Research Quarterly 32, 204-214.

Collis, K. & Biggs, J. (1982). Towards a model of school-based curriculum development and assessment using the SOLO taxonomy. *Australian journal of education, 33(2), 151-163.*

Common Core Curriculum (2019). *Mathematics Curriculum for junior high schools (JHS 7-9)*, Accra: Ministry of Education

Conference Board of the Mathematical Sciences (CBMS). (2001). Issues in Mathematics Education: Vol. 11. *The mathematical education of teachers. Providence, RI: American Mathematical Society, in cooperation with Mathematical Association of American.*

Creswell, J. (2014). Research design: Educational Research 4thed.). Thousand Oaks,

Creswell, J. W. (2004). Research design: Educational Research 4thed.). Thousand Oaks,

Creswell, J. W. (2009). *Educational research: planning conducting and evaluating quantitative and qualitative research*. New Jersey: Pearson Education.

Creswell, J. W. (2017). Designing and conducting mixed methods research. London: Sage

Creswell, J. W. (2019). *Research design: Qualitative, quantitative and mixed methods approaches (4th ed.)*. London: Sage Publications.

Damopolii, I. (2020). The effectiveness of Inquiry-based learning to train students' thinking skill based on SOLO taxonomy. *In Journal of Physics: Conference Series (Vol. 1567, No. 4, p. 042025)*. IOP Publishing.

Denscombe, M. (2010). *A good research guide for small scale social research projects*. Buckingham: Open University Press.

Dinardo, J. (2008). *"natural experiments and quasi-natural experiments".* The New Palgrave Dictionary of Economics. Pp. 856-859.

Ding, M., Song, S., & Richadson E. (200)7. *Closing the expectations gap: Fourth annual 50-state progress report on the alignment of high school policies with the demands of college and careers*. New York: Achieve Inc.

Doias, E. D. (2013). *The effect of manipulatives on achievement scores in the middle school mathematics class* [Ed.D. dissertation, Lindenwood University].

Dougherty, N., Lee, L., Jupri, A. & Drijvers, P. (2015). Generalization of patterns: The tension between algebraic thinking and algebraic notion *Educational Studies in Mathematics **49*** 379-402

Eshun, B. A. (1999). The pattern of mathematical achievement of secondary school students in Ghana:*Journal of Science and Mathematics Education. (2nd ed.)*, (1). (pp. 22-33).

Fennema, E., & Sherman, J. A. (1976). Fennema-Sherman mathematics attitude scales: instrument designed to measure attitudes towards the learning of mathematics by males and females. *Catalogue of Selected Documents in Psychology*. (6th ed.), (1). (pp. 31).

Ferrini-Mundy, J., McCrory, R., Senk, S. L., & Marcus, R. (2005). *Knowledge for algebra teaching.* A paper presented to the American Educational Research Association (AERA) Annual Meeeting in Montreal, Canada, on April 14, 2005.

Fianu, E. (2005). *Principles and Methods of English Language Teaching*. Koforidua: Saama production.

Fujii, J., & Stephens, M. (2008). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21(2), 33–49.*

Fujii, T., & Stephens, M. (2001). Fostering an understanding of algebraic generalization through numerical expressions: The role of quasi-variable. In H. Chick, K. Stacey, & J. Vincent (Eds.). *Proceedings of the 12th ICMI Study Conference: The future of the Teaching and Learning of Algebra* (Vol. 1, pp. 258-264). Melbourne, Australia: The University of Melbourne.

Fyfe, E. R., Evans, J.L., Matz, L.M., Hunt, K. M., & Alibali, M. W. (2017). *Relation between patterning skill and differing aspects of early mathematical knowledge*. Cognitive Development 44, 1-11.

Gadzichowski, K. M. (2012). *Patterning abilities of first grade children*: Effects of dimension and type. Creative Education 3(5), 632-635.

Gallagher, A. M., & Kaufman, J.C. (2006). Influential factors causing the gender differences in mathematics' achievement scores among Iranians' Eight graders based on TIMSS 2003 data. Retrieved June 13, 2010 from http//www.iea.ne/fileadmin/user-upload.

Geary, D. C. (2000). Influential factors causing the gender differences in mathematics' achievement scores among Iranian Eight graders based on TIMSS 2003 data. Retrieved June 13, 2010, from http//www.iea.ne/fileadmin/user-upload.

Geist, K., Geist, E., & Kuznik, K. (2012). *The patterns of music: Young children learning mathematics through beat, rhythm, and melody*. Young Children (67), 74-79.

Ghana Education Service. (2007). *Teaching syllabus for mathematics,* (JHS 1-3). Accra: Ghana.

Green, P. (2014). Patterning abilities of first grade children: effect of dimension type. *Creative Education 3*(5), 632-635.

Gurbuz, R., & Toprak, Z. (2014). Designation, implementation and evaluation of activities to ensure transition from arithmetic to algebra. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 8(1),* 178-203. https://doi.org/10.12973/ nefmed.2014.8.1.a8

Haruna, I., U. (2014). Mathematics Education for National Development Values and Attitudes in the Socio-Culture Context of the Nigerian Society. *International Journal of Novel Research in Education and Learning. 1* (1), 10-14.

Hattie, J.A.C. and Brown, G.T.L. (2004). *Cognitive Processes in as TTle*: The SOLO Taxonomy. AsTTle Technical Report #43, University of Auckland/Ministry of Education.https:/link.gale.com/apps/doc/A329301783/AONEu=sliowa_main&sid=AONE&xid=ff533123https:/link.gale.com/apps/doc/A329301783/AONEu=sliowa_main&sid=AONE&xid=ff533123

Huber, D. L. (2006). *Leadership and nursing care management (3rd Ed.)*. Philadelphia: Saunders Elsevier.

Hutchinson, E. & Pournara, C. (2011). Pre-school children's understanding of mathematical patterns. *South African Journal of Childhood Education,* 1(2), 92–111.

Iowa Department of Education (2018). Iowa early learning standards. Retrieved September 16, 2020 from https://educateiowa.gov/sites/files/ed/documentsIowaEarlyLearningStandards 3rdEdition12.20.18_508.pdf

Janson, S. (1996). Influential factors causing the gender differences inmathematics achievement scores among Iranians' Eight graders based on TIMSS 2003 data. Retrieved June 13, 2010, from http//www.iea.ne/fileadmin/user-upload.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education, 9, 187 - 211.*

Joppe, D. E. (2016). *Doing research in the real world*. London: Sage Publications Ltd

Kandir, A., Colak, F. G., Aktulun, O.U. (2018). The effect of pattern-based mathematics education program (PMEP) on 61-72-month-old preschoolers' early academic and languages skills. *Education Research and Reviews 13*(22), 735-744.

Kaput, J. J. (1999). Developing Algebraic Thinking in Earlier Grades:. In C. E. Greene, *Algebra and Algebraic Thinking in School Mathematics.* (pp. 169 - 179). National Council of Teachers of Mathematics

Kaput, J. J. (2005). Algebraic thinking in The Early Grades: What is it? *Math Edu*. **8** 139-151.

Kaput, J. J., Kepner, H., Lindquist, M., & Reys, R. (2008). What is algebra? What is algebraic reasoning?, *in Algebra in the Early Grades*, J. J. Kaput, D. W. Carraher, and M. L. Blanton, Eds. (New York: Taylor & Francis Group, LLC) pp. 8-17.

Kidd, J. K., Carlson, A. G., Gadzichowski, K. M., Boyer, C. E., Gallington, D. A., & Pasnak, R. (2013). Effects of patterning instruction on the academic achievement of 1st-grade children. *Journal of Research in Childhood Education, 27(2), 224-228.*

Kieran, C. (1992). Algebraic thinking in the early grades: What is it? *The Mathematics Education **8*** 139-151

Kieran, C. (2003). *The Twentieth-century Emergence of the Canadian Mathematics Education Research Community*. In G. S. Kilpatrick (Eds.), a History of School Mathematics (pp. 1701 - 1778). National Council of Teachers of Mathematics.

Kieran, C. (2006). Fostering relational thinking while negotiating the meaning of the equals sign. *Teaching Children Mathematics, 13(2),* 111-117.

Kilpatrick, J. &. Izsak, A. (2008). History of Algebra in the School Curriculum. In C. E. Greene (Ed.), *Algebra and Algebraic Thinking in School Mathematics (pp. 3 - 18). National Council of Teachers of Mathematics.*

Kirshner, D., & Awtry, T. (2004). Visual salience of algebraic transformations. *Journal for Research in Mathematics Education, 35(4),* 224-257

Kivunja, M. & Kuyini, J. (2017). *Research methodology: a step-by-step guide for beginners*. London: SGE: Publications Ltd

Knuth, E. J., McNeil, N. M., Grandau, L., Alibali, M. W., Stephens, A. C., Hattikudur, S., & Krill, D. L. (2016). Middle-school students' understanding of the equal sign: The book they read can't help. *Cognition and Instruction, 24*(3), 367-385.

Kothari, M. (2004), *Recursive partitioning for heterogeneous causal effect." Proceedings of the National Academy of sciences 113, (27), 7353-7360*

Koul, L. (2011). *Methodology of educational research, (4th ed.)* New Delhi: Vikas Publishing House PVT LTD.

Kriegler, F. (2007). Analysis of Secondary School Students' Algebraic Thinking and Math-Talk Learning Community to Help Students Learn *International Conference on Mathematics and Science Education (ICMScE)* **895** 012054 (Bandung: Journal of Physics: Conf. Series)

Kulbir-Singh, G. & Sidhu, P. (2006). *''The teaching of Mathematics''*, Sterling Publishers Pvt. Ltd.

Kumar, H. (2019). *Naturalistic inquiry*. Beverly Hills, Califonia: Sage.

Lannin, J. K. (2004). Generalization and justification: The challenge of introducing algebraic reasoning through patterning activities *Mathematical Thinking and Learning **7*** 231-258

Lannin, J. K. (2006). Bridging arithmetic and algebra: Evolution of a teaching sequence. In O. Figueras et al. (Eds.), *International Group of the Psychology of Mathematics Education: Proceedings of the Joint Meeting of PME 32 and PME-NA XXX (PME29)* (Vol. 2, pp. 121-128). Morelia, Mexico.

Lannin, J. K.. (2003). Algebra for 8th graders: evidence on its effects from 10North Carolina districts. *Journal of Human Resources, 50(1),* 159-188.

Lee, K., Ng, S. F., Bull, R., Pe, M. L., & Ho, R. H. M. (2011). Are patterns important? An investigations of the relationships between proficiencies in patterns, computation, excutive functioning, ad algebraic word problems. *Journal of Educational Psychology, 103*(2), 269-281. https://doi.org/10.1037a0023068

Levine, P., Sober, G., & Wright, P. (2010). *A matter of record*. Cambridge: policy.

Levins, A. & Pegg, C. (1993). Third graders' interpretations of equality and the equal symbol. *Educational Studies in Mathematics, 35(2*), 153-187.

Liebenberg, R., Linchevski, L., Sasman, M. C., & Olivier, A. (1999). Focusing on the structural aspects of numerical expressions. In J. Kuiper (Ed.), *Proceedings of the 7th Annual Conference of the Southern African Association for Research in*

Mathematics and Science Education (SAARMSE) (pp. 249-256). Harare, Zimbabwe

Linchevski, M. & Livneh, R. (1999). From an operational to a relational conception of the equal sign. Third graders' developing algebraic thinking. *Focus on learning Problems in Mathematics, 30(1),* 61-80.

Luken, M., Peter-Koop, A., & Kollhoff, S. (2014). Influence of early repeating patterning ability on school mathematics learning. In P. Liljedahl, C. Nico, S. Oesterle, & D. Allan (Eds.), *Proceedings of 38th Conference of the International Group of the North American Chapter of the Psychology of Mathematics Education.* Vol. 4 (pp. 137-144). Vancouver, Canada: PME.

Ma J. (1999). *Survey research and analysis:* Application in parks, recreation and human dimension. State College, PA: Venture.

Mahmood, A., Fauzi, M., & Mohammad, Y. (2012). A Conceptual Framework for Mathematical Ability Analysis through the Lens of Cultural Neuroscience: 56, pp. 175 - 182. *Retrieved From: https// doi.org/ 10.1016/ j. sbspro.2012.09.644.*

Malara, N., & iaderosa, R. (1999). *The interweaving of arithmetic and algebra: some questions about syntactic and structural aspects and their teaching and learning*. In I. Schwank (Ed.). Math-magic: Book 3 (2006). New Delhi: National Council of educational Research and Training.

Mason, G. (1996). Algebraic Reasoning in Solving Mathematical Problem Based on *Learning Style MISEIC **947*** 0120661(Surabaya: Journal of Physics: Conf. Series)

McGarvey, L. M. (2013). Is it a pattern? Teaching and learning patterns in early childhood mathematics. *Teaching Children Mathematics*, 19(9), 564-571.

Ministry of Education (2018). *Mathematic curriculum for Basic 7_9.* Ghana: Accra

Morgan, A. G., Leech, L. N., Gloeckner, W. G., & Barrett, C. K. (2004). *SPSS for Introductory Statistics: Use and Interpretation (2ne ed.)*. London: Lawrence Erlbaum associates.

Mulligan, J., Oslington, G. & English, L. (2020).*Supporting early mathematical development through a "pattern and structure" intervention program*. ZDM Mathematics Education 52, 663–676. https://doi.org/10.1007/s11858-020-01147-9

Mullis, I.V.S., Martin, M. O., Gonzales, E. J., & Chrostowski, S. J. (2004). TIMSS 2003 international mathematics report. Findings from IEA's trends in international mathematics and science study at the fourth and eighth grades. Chestnut Hill, MA: Boston College. Retrieved July 27, 2011, from http://www.iea.nl/publications_list.html.

Naizaro, L. (2012). The progressive development of early embodied algebraic *thinking Math. Educ. Res. J.***26** 257-277.

National Council for Educational Statistics (2014). National assessment of educational progress. *Retrieved from http://nces.ed.gov/nationsreportcard*

National Council of Teachers of Mathematics (2000). *Principle and standards for school mathematics.* New York: National Council of Teachers of Mathematics, Inc.

National Council of Teachers of Mathematics (2014). *Principle to action: ensuring mathematics success for all*. Reston, VV: National Council of Teachers of Mathematics

National Mathematic Advisory Panel. (2008). *The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

Neuman, J. Z. (2014). Supporting the out-of-school time workforce in fostering intentional social and emotional learning. *Journal of Youth Development, 15 (1), 239–265. https://doi.org/10.5195/jyd.2020.807*

Nevid, G. (2013). http://www.psychology.about.com (Retrieved on 10th January, 2017).

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C., & Spitler, M. E. (2016). Which preschool mathe*?. Early Childhood Research Quarterly,*matics competencies are most predictive of fifth grade achievement 36, 550-560. https://doi.org/10.1016/j.ecresq.2016.02.003

Ntsohi, J. H. (2005). Examining Hetrogeneity in the Effect of Taking Algebra in Eighth Grade. *The journal of Educational Research, 106(4),* 251-268.

Nunnally, J. C., & Bernstein, I. H. (1994). *Psychometric theory (3).* New York: McGraw-Hill

O'Donnell, A. M, Reeve, J. & Smith, J. K. (2009). *Educational psychology: Reflection for action.* (3rd Ed.). United States of America. John Wiley and sons, Inc.

Ogg, B. (2010). What is the impact of math manipulatives on student learning? *Master research project of Ohio University.*

Ormrod, J. E. (2008). *Human learning (5thEd.)*. Upper Saddle River: Pearson Prentice Hall.

Pallant, J. (2005). SPSS survival manual: *A step by step guide to data analysis using SPSS*

Panizzon, H. (2003). Mobile learning and student congnitive: A systematic review of PK-12 research using Bloom's Taxonomy. Bristish Journal of Education Technology,50(2), 684-701.

Papic, M., Mulligan, J., & Mitchelmore, M. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education, 42(3), 237 269.*

Pasnak, R., Gagliano, K., Righi, M., and Kidd, J. (2019). Teaching early literacy, mathematics, and patterning to kindergartners. *Psychology, 10,* 1493-1505. Doi: 10.4236/psych. 2019. 1011098

Patton, G. (2002). Algebraic Thinking: A problem solving approach in *Shaping the Future of Mathematics Education: proceedings of the 33rd Annual Conference of the Mathematics Education Group of Australasia* pp. 665-672.

Peg, C. (2003). Assessing knowledge of mathematical equivalence: A construct modeling approach. *Journal of Educational Psychology*, 103(1), 85-104. DOI: 10.1037/a0021334

Plessis, J. (2018). Early algebra: Repeating pattern and structural thinking at foundation phase. South African Journal of Early Childhood Education, 8 (2). http://dx.doi.org/ 10.4102/ sajce.v8i2.578

Potter, G. (2000). *The philosophy of social science*: new perspectives. London: Sage Publication.

Quinlan, E. (2001). How children view the equal sign. *Mathematics Teaching*, 92, 13-15

Raford, R. M. (2010). Three phase ranking frameworks for assessing conceptual understanding in algebra using multiple representations. *College Student Journal, 45(2*), 219.

Rahi, L. (2017). *Researching real-world problems: a guide to methods of inquiry*. London: Sage Publications.

Rehman, M. & Alharti, C. (2016). '*Interpretism and generalization*'. Sociology, 34, 2, pp. 209-224

Reid, K., & Andrews, N. (2016). Fostering understanding of early Numeracy development. Australian Council for Educational Research (ACER). https://research.acer.edu .au/ monitoring learning/29

Rittle-Johnson, B., Fyfe, E. R., Hofer, K. G., & Farran, D. C. (2017). Early math trajectories: low-income children's mathematics knowledge from ages 4 to 11. Child Development, 88(5), 1727–1742. https://doi.org/10.1111/cdev.12662

Rittle-Johnson, B., Zippert, E.L., Boice, K. L. (2019). The roles of patterning and spatial skills in early mathematics development. *Early Childhood Research Quarterly. DOI: 10.1016 j.ecresq.2018.03.0*

Robson, T., Shannon, G., Goldenhar, P. & Hale, J. (2001). "The Effectiveness of facilitated Monitoring a Longitudinal Quasi Experiment". *Journal of Vocational Behavior **54**(3): 483-502*

Sarantakos, T. J. (2008). Best practices in problem analysis. In A. Thomas and J. Grimes (Eds.), *Best practices in School Psychology V (pp. 159-176).* Washington, DC: National Association of School Psychologists.

Saraswati, B., Fyfe, E. R., Loehr, A. M., & Miller, M. R. (2016). Beyond numeracy in preschool: adding patterns to the equation. Early Childhood Research Quarterly, 31, 101–112. https://doi.org/10.1016/j.ecresq.2015.01.005

Sathiskumar, G. & Ratnaliken, D. (2003). ‘*’Teaching of Arithmetic’’*, Common Wealth Publishers.

Schmerold, K., Bock, A., Peterson, M., Leaf, B., Vennergrund, K., & Pasnak, R. (2016). The relations between patterning, executive function, and mathematics. *The Journal of Psychology Interdisciplinary and Applied. http://dx.doi.org 10.1080/00223980.2016.1252708*

Scott, J. (2012*). A matter of record*. Cambridge: Polity.

Shatini-Wadhwa, K. (2008). ‘*’Modern methods of teaching mathematics’’*, Karan Paparbacks, India.

Slavin, D. (2006). *The Educational Psychology: Theory into Practice*. Eaglewood Cliff, N. J.: Prentice Hall.

Slavit, D. (1999). The role of operation sense in transitions from arithmetic to algebraic thought. *Educational Studies in Mathematics, 37(3),* 251-274.

Springler, D. M., & Alsup, J. K. (2003). Mathematical reasoning: Analogies, metaphors and images. Teaching children mathematics. Retrieved May 4, 2010, from http//www.highbeam.com

Stacey, K., Helme, S., Steinle, V., Baturo, A., Irwin, K., & Bana, J. (2001). Preservice teachers' knowledge of difficulties in decimal numeration. *Journal of mathematics Teacher Education, 4(3),* 205-225.

Stacy, K., & Macgregor, M. (2000). Learning the algebraic method of solving problem. *Journal of Mathematics Behaviour*, 18(2), 149-167.

Swanson, R. A., & Thomas J. Chermack, T. J. (2013). *Theory Building in Applied Disciplines*. San Francisco, CA: Berrett-Koehler Publishers.

Tabach, S., Katz, L., & Malara, N. (2008). Algebra, with *Arithmetic and Mensuration from the Sanscrit of Brah-megupta and Bhascara*. London: John Murray

Tsamir, P., Tirosh, D., Levenson, E., Barkai, R., & Tabach, M. (2017). Repeating patterns in kindergarten: Findings from children's enactments of two activities. Educational Studies in Mathematics 96 (1).

Tyagi, K. (2008). US Math Performance in Global Perspective: How Well Does Each State Do at Producing High-Achieving Students? PEPG Report No.: 10-19. *Program on Educational Policy and Governance, Harvard University*

Van den Heuvel-Panhuizen, M. (1996). *Assessment and Realistic Mathematics Education*. Utrecht, the Netherlands: CD-*B* Press Utrecht University.

Vlassis, J. (2002). The balance model: *Hindrance or support for the solving of linear equation with one known. Education Studies in Mathematics, 49(3), 341-359.*

Wager, Stefan, and Susan Athey (2018), "Estimation and inference of heterogeneous treatment effects using random forests." *Journal of the American Statistical Association 133 (523), 1228*-1242,

West African Examination Council (2010). Chief Examiners' Report. Accra:

West African Examination Council (2011). Chief Examiners' Report. Accra: WAEC Press.
West African Examination Council (2012). Chief Examiners' Report. Accra:

West African Examination Council (2013). Chief Examiners' Report. Accra: WAEC Press.

West African Examination Council (2014). Chief Examiners' Report. Accra: WAEC Press.

West African Examination Council (2015). Chief Examiners' Report. Accra: WAEC Press.

West African Examination Council (2017). Chief Examiners' Report. Accra: WAEC Press.

West African Examination Council (2018). Chief Examiners' Report. Accra: WAEC Press.

Wilmot, E. M. (2008). An investigation into the profile of Ghanaian high school mathematics teacher's knowledge for teaching algebra and its relationship with student's performance. Retrieved July 26, 2011, fromhttp://www.grin.com/en/doc.

Wilmot, E. M. (2009). Thinking about Learning and Teaching Sequences for the Addition and Subtraction of Fractions. In C. Bruce (Chair), *Think Tank on the Addition and Subtraction of Fractions*. Think Tank conducted in Barrie, Ontario.

Windsor, W. & Norton, B. (2011). Early algebra and mathematical generalization ZDM *Mathematics Education **40*** 3-22

Wong, G. A. (2014). *"Quasi-Experimental Designs" Journal of the American academy of Child & Adolescent Psychiatry*. 39 (6): 794-796

Yarkwah C. (2017). *An investigation into Senior High School MATHEMATICS Teachers' Knowledge for Teaching Algebra*. An Unpublished doctoral thesis submitted, Department of Mathematics and I.C.T. Education, University of Cape Coast.

Yilmaz, F. (2013). *Case study research design and methods, (3rd ed.).* New York: Sage Publication

Zimbardo, R. (2000). Research methodology: A step-by-step guide for beginners. London: Sage.

Zippert, E. L., Douglas, A.-A., & Rittle-Johnson, B. (2020). Finding patterns in objects and numbers: repeating patterning in pre-k predicts kindergarten mathematics knowledge. Journal of Experimental ChildPsychology,200.https://doi.org/10.1016 j.jecp.2020.104965

APPENDIX A

INTRODUCTORY LETTER

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF EDUCATIONAL STUDIES
DEPARTMENT OF BASIC EDUCATION
P. O. Box 25, Winneba, Ghana
+233 (050) 9212015
beducation@uew.edu.gh

Date: April 27, 2022

Municipal Educational Directorate
Agona West Municipal Assembly
Agona Swedru

Dear Sir/Madam,

LETTER OF INTRODUCTION

We forward to you, a letter from Mr. Emmanuel Addoko, a second year M.Phil student of the Department of Basic Education, University of Education, Winneba, with registration number 202140554.

Mr. Emmanuel Addoko, is to carry out a research on the Topic ***"The Effects of Number Pattern on Junior High School Students' Transition from Arithmetic to Algebra.***

We would be grateful if permission is granted him to carry out this study in the Municipality.

Thank you.

Yours faithfully,

....

MR. NIXON SABA ADZIFOME
(Ag. Head of Department)

DEPT. OF BASIC EDUCATION
UNIVERSITY OF EDUCATION
WINNEBA, GHANA

www.uew.edu.gh

APPENDIX B

Pre-Test Questions

Q1: Which numerical pattern follows the rule ''subtract 2, then multiply by 3,'' when starting with 5?

a) 5, 7, 21, 69

b) 5, 2, 4, 1, 2

c) 5, 15, 13, 39, 37

d) 5, 3, 9, 7, 21, 19, 57

Q2. The first number in a pattern is 5. The pattern follows the rule ''Add 3''. What are the next 4 numbers in the pattern?

a) 1, 4, 7, 10

b) 5, 10, 15, 20

c) 8, 11, 14, 17

d) 3, 6, 9, 13

Q3. Which is the next ordered pair in the pattern? (10, 5), (8, 4), (6, 3)

a) (2, 4)

b) (2, 5)

c) (4, 2)

d) (5, 2)

Q4. John used the rule ''double the number'' to create the pattern 3, 6, 12, 24 … Which pair of numbers is part of the pattern?

a) 36, 72

b) 48, 96

c) 96, 144

d) 96, 192

Q5. Which statement about the third corresponding terms in both Pattern X and Pattern Y is true

Pattern X: start at 10 and add 5

Pattern Y: start at 10 and add 10

a) Both number values will be 20.

b) Both number values will be less than 20.

c) Both number values will be greater than 20.

d) The T value will be 20 and the X value will be more than 20

Q6. Which is the missing ordered pair in the pattern? (3, 9), (5, 15), (?, ?), (9, 27)

a) (6, 9)

b) (6,18)

c) (7, 10)

d) (7, 21)

Q7. A number pattern is shown as 100, 95, 110, 105, 120 … If this pattern continues, what number will come next?

a) 100

b) 115

c) 120

d) 130

Q8. What is the rule for the following pattern? 10, 7, 4, 1

a) Add 4

b) Add 8

c) Multiply by 2

d) Subtract 3

Q9. Select the missing number. 55, 54, 53, 52,__________

a) 56

b) 51

c) 50

d) 55

Q10. Continue the pattern. AB, ABB, ABBB

a) ABAB

b) AAB

c) ABBBB

APPENDIX C

Post-Test Questions

Q1: Which numerical pattern follows the rule ''subtract 2, then multiply by 3,'' when starting with 5?

a) 5, 7, 21, 69

b) 5, 2, 4, 1, 2

c) 5, 15, 13, 39, 37

d) 5, 3, 9, 7, 21, 19, 57

Q2. The first number in a pattern is 5. The pattern follows the rule ''Add 3''. What are the next 4 numbers in the pattern?

a) 1, 4, 7, 10

b) 5, 10, 15, 20

c) 8, 11, 14, 17

d) 3, 6, 9, 13

Q3. Which is the next ordered pair in the pattern? (10, 5), (8, 4), (6, 3)

a) (2, 4)

b) (2, 5)

c) (4, 2)

d) (5, 2)

Q4. John used the rule ''double the number'' to create the pattern 3, 6, 12, 24 … Which pair of numbers is part of the pattern?

a) 36, 72

b) 48, 96

c) 96, 144

d) 96, 192

Q5. Which statement about the third corresponding terms in both Pattern X and Pattern Y is true

Pattern X: start at 10 and add 5

Pattern Y: start at 10 and add 10

a) Both number values will be 20.

b) Both number values will be less than 20.

c) Both number values will be greater than 20.

d) The T value will be 20 and the X value will be more than 20

Q6. Which is the missing ordered pair in the pattern? (3, 9), (5, 15), (?, ?), (9, 27)

a) (6, 9)

b) (6,18)

c) (7, 10)

d) (7, 21)

Q7. A number pattern is shown as 100, 95, 110, 105, 120 … If this pattern continues, what number will come next?

a) 100

b) 115

c) 120

d) 130

Q8. What is the rule for the following pattern? 10, 7, 4, 1

a) Add 4

b) Add 8

c) Multiply by 2

d) Subtract 3

Q9. Select the missing number. 55, 54, 53, 52,__________

a) 56

b) 51

c) 50

d) 55

Q10. Continue the pattern. AB, ABB, ABBB

a) ABAB

b) AAB

c) ABBBB

d) ABBAB

SECTION B

1. Study the following patterns and describe, in words, the rule that is used to generate each term.

Hence, draw the next four terms of each pattern.

(i) □, O, Δ, □, O, …

2. Study the following patterns and describe, in words, the rule that is used to generate each term.

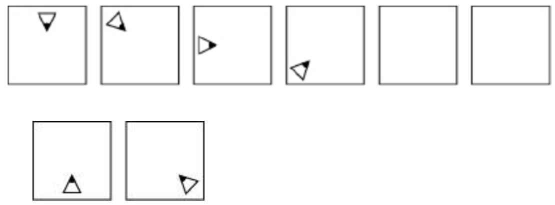
Hence, write out the next four terms of each pattern

(i) 5, 7, 9, 11, …

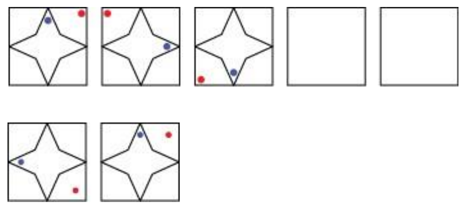
(ii) 24, 12, 6, 3, …

(iii) 10, 5, 0, − 5, − 10, …

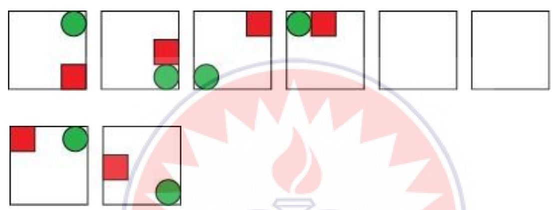
3. In your exercise book, draw the next two terms of the following patterns:



(ii)



(iii)



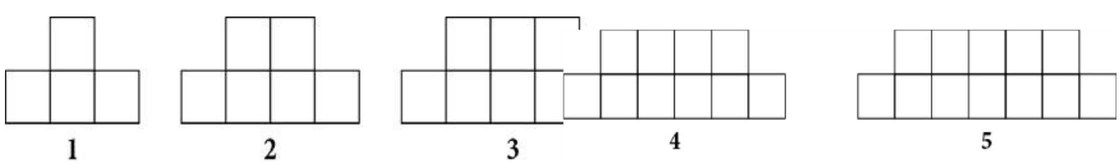
4. Find the missing terms of the following linear (arithmetic) patterns:

(i) 3, 8, 13, __, 23, 28, __, __, 43

(ii) −20, −17, __, __, −8, −5, __, __, __

5. A sequence is as follows: −15, −8, −1, 6, … (i) Prove that the sequence is arithmetic.

6. The following diagram shows a pattern of blocks:



Draw the next two patterns.

7. The following diagram shows the second and fourth patterns in a sequence.

Each pattern is made from toothpicks. Draw the first and third steps of the pattern.

			
Pattern 1	Pattern 2	Pattern 3	Pattern 4

8. Identify the error in the following pattern; 2,4,6,8,10,11,14,16

9. Describe the rule for the pattern

1 ⟶ 1, 2 ⟶ 4, 3 ⟶ 9, 4 ⟶ 16, 5 ⟶ 25.

10. Correct the error in the following pattern 2, 4, 6, 8, 10, 11, 14, 16