UNIVERSITY OF EDUCATION, WINNEBA

PRIMARY SCHOOL TEACHERS' CONCEPTIONS AND PRACTICES OF MULTIPLE REPRESENTATIONS IN MATHEMATICS INSTRUCTION IN LOWER MANYA MUNICIPALITY IN THE EASTERN REGION OF GHANA

FELIX ASEMNOR

2018

UNIVERSITY OF EDUCATION, WINNEBA

PRIMARY SCHOOL TEACHERS' CONCEPTIONS AND PRACTICES OF MULTIPLE REPRESENTATIONS IN MATHEMATICS INSTRUCTION IN LOWER MANYA MUNICIPALITY IN THE EASTERN REGION OF GHANA

FELIX ASEMNOR

(8160030002)

A THESIS IN THE DEPARTMENT OF BASIC EDUCATION, FACULTY OF EDUCATIONAL STUDIES, SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES, UNIVERSITY OF EDUCATION, WINNEBA, IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF MASTER OF PHILOSPHY (BASIC EDUCATION) DEGREE

AUGUST, 2018

DECLARATION

STUDENT'S DECLARATION

I, FELIX ASEMNOR, declare that this thesis, with the exception of quotations and references contained in published works that have all been identified and duly acknowledged, is entirely my own original work, and that it has not been submitted, either in part or whole, for another degree elsewhere.

SIGNATURE: ……………………………….

DATE: ………………………………………

SUPERVISORS' DECLARATION

We hereby declare that the preparation and presentation of this work were supervised in accordance with the guidelines for supervision of thesis as laid down by the University of Education, Winneba.

PROF. MICHAEL JOHNSON NABIE (Principal Supervisor)

SIGNATURE: ………………………………….

DATE ………………………………………….

MR. NIXON SABA ADZIFOME (Co-Supervisor)

SIGNATURE ……………………………………

DATE ……………………………………………

ACKNOWLEDGEMENTS

I am grateful to God for helping me to complete this thesis. I would like to thank my supervisors Prof. Michael Johnson Nabie of the Department of Mathematics Education of the University of Education, Winneba and Mr Nixon Saba Adzifome of the Department of Basic Education of the University of Education, Winneba. The doors to my supervisors" offices were always open whenever I had any challenges with my research or writing. They consistently steered me in the right direction whenever I needed it. I am highly grateful to them. I also offer my sincerest gratitude to lecturers of the Department of Basic Education and the Department of Mathematics Education whose lectures and guidance provided the foundation for my thesis. I am immensely indebted to Prof. S. K Asiedu-Addo and Dr. Regina Caesar. I am also very grateful to all my Master of Philosophy colleagues, especially Samuel Ebo Arthur, Amadu Yenukwa, Patrick Kyeremeh and Beatrice Panyin, who were a great source of assistance, support and encouragement. I also thank my headteacher, Mr Kofi Teye and teachers in the Lower Manya Municipality for their support and cooperation. Finally, I must express my very profound gratitude to Mrs Mercy Asemnor (mother), Joyce Asemnor (sister), Eric Asemnor (brother), for providing me with unfailing support and continuous encouragement throughout my years of study. This accomplishment would not have been possible without them.

DEDICATION

This thesis is dedicated to my family.

TABLE OF CONTENTS

Contents **Page**

DECLARATION	ii
ACKNOWLEDGEMENTS	iii
DEDICATION	iv
TABLE OF CONTENTS	v
LIST OF FIGURES	ix
ABSTRACT	x
CHAPTER ONE: INTRODUCTION	1
1.0 Overview	1
1.1 Background to the Study	1
1.2 Statement of the Problem	5
1.3 Purpose of the Study	6
1.4 Objectives of the Study	6
1.6 Significance of the Study	7
1.7 Delimitation	8
1.8 Limitations	8
1.9 Organisation of the Study	9
CHAPTER TWO: REVIEW OF RELATED LITERATURE	10
2.0 Overview	10
2.1 Theoretical Framework	10
2.2 Definition of representations on the Lesh Multiple Theory	14
2.3 Origin of Multiple Representations in Mathematics Education	18
2.4 Constructivist View of Multiple Representations	34
2.5 Semiotic View on Multiple Representations	35
2.6 Kaput"s View on Multiple Representations	37
2.7 Janvier"s View on Multiple Representations	39

2.8 Multiple Representations in Instruction 42
2.9 Mathematical Content Knowledge 43
2.10 Teachers conceptions of Multiple Representations 45
2.11 Research on Teachers" Beliefs 47
2.12 Summary of the Literature Review 53

CHAPTER THREE: METHODOLOGY 55
3.0 Overview 55
3.1 Research Design 55
3.2 Study Area 57
3.3 Population 59
3.4 Sampling Technique and Sample Size 60
3.5 Research Instruments 62
3.5.1 Interview Guide 62
3.5.2 Observation 64
3.5.3 Document Analysis 64
3.6 Pilot Study 65
3.7 Data Collection Procedures 66
3.7.1 Conducting Interviews 67
3.7.3 Document Analysis 68
3.8 Establishing Trustworthiness 69
3.8.1 Credibility 70
3.8.2 Transferability 71
3.8.3 Dependability 71
3.8.4 Confirmability 72
3.9 Data Analysis 72
3.10 Ethical Consideration 73
3.11 Summary of Methodology 74

CHAPTER FOUR: RESULTS AND DISCUSSION 75

4.0 Overview 75

4.1 Introduction 75

4.2 Demographic Characteristics of the Respondents 76

4.3 Brief Description of Participants 77

4.3.1 Document Analysis 78

4.3.2 Results of the Observation 87

4.3.3 Results from Interviews 96

CHAPTER FIVE: SUMMARY OF FINDINGS, CONCLUSIONS AND RECOMMENDATIONS 105

5.0 Overview 105

5.1 Introduction 105

5.2 Conclusions 110

5.3 Recommendations 111

5.4 Suggestions for Further Research 112

REFERENCES 114

APPENDICES 132

APPENDIX A: Interview Schedule 132

APPENDIX B: Observation Checklist 134

APPENDIX C: Analysis of Syllabus and Pupils Textbook 135

APPENDIX D: Sample of Transcribed Interviews 136

APPENDIX E: Letter Of Introduction 140

LIST OF TABLES

Table **Page**

4.1: Characteristics of research participants 77

4.2: Multiple Representation in P3 Mathematics Syllabus and Textbook 80

4.3: Multiple Representations in P6 Mathematics Syllabus and Textbook 84

4.4: Brief Description of Lessons Taught 87

4.5: Observation of Teachers‟ classroom instructional practices 88

LIST OF FIGURES

Figure		Page
2.1:	Lesh‟s Multiple Theory	12
3.1:	Lower Manya Municipality Map	58

ABSTRACT

The purpose of the study was to explore primary school teachers" conceptions and use of multiple representations in the mathematics classroom in the Lower Manya Municipality in the Eastern Region of Ghana. The research was conducted in the Lower Manya Municipality in the Eastern Region of Ghana. The theoretical framework that underpinned the study was Lesh, Post, and Behr Multiple theory. The research design employed in the study was Case study. All primary three and six mathematics teachers constituted the population of the study. Cluster sampling, simple random sampling and purposive sampling technique were used to select five respondents from Basic three (P3) and six (P6) primary mathematics teachers from public schools in Lower Manya Municipality. Semi-structured interview guide and observation checklist were used to collect qualitative data. A document analysis sheet was used to examine the curriculum provisions for multiple representations. The qualitative data on the other hand was analysed using thematic analysis. From the analyses of the curriculum documents, it was found that Primary 3 (P3) syllabus had 32 out of 49 (65.31%) objectives whilst the Primary 6 (P6) syllabus had 38 out of 71 (53.52%) objectives presented using multiple representations. The textbook analysis showed that P3 textbook had 41 out of 45 (91.11%) subtopics whereas the P6 textbook had 51 out of 70 (72.86%) subtopics presented using multiple representations. These findings suggest that sufficient provision has been made in the curriculum documents to guide teachers to use multiple representations in their mathematics lesson delivery. The interview responses suggest that participants seemed to be aware of several multiple representations in mathematics teaching, although the term multiple representations were new to most of them. Evidence from classroom observations also suggest that the conceptions teachers hold about multiple representations are scarcely translated into practice. It is therefore recommended that in-service training programmes on multiple representations be organised for teachers on regular basis so that they will be well equipped with the needed skills and knowledge to adequately assist learners to construct their own understanding of mathematical concepts.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This chapter presents the background to the study, statement of the problem, purpose of the study, objectives of the study, research questions, significance of the study, delimitations, organization of the study and the definition of terms.

1.1 Background to the Study

According to Abudu and Fuseini (2014), education is an instrument for the achievement of national goals. In view of this, the 2018 budget of Ghana emphatically addressed the achievement of educational goals through climaxing the work on curriculum reforms that ranged from resource development to human capital development in teachers Continuous Professional Development. By so doing, strengthening a firm foundation in education. For any nation to develop a strong human resource background, there is the need to strengthen its educational institutions which train the brains, provide skills and open a new world of opportunities and possibilities to the nation (in this case the pupils). This explains why a huge chunk of the national budget is allocated to the Ministry of Education.

Besides, in order to make education meaningful and relevant to the society, it depends on how the curriculum is developed. The Ministry of Education (2011), with her Education for All policy and the curriculum premise that "all students can learn mathematics and that all need to learn mathematics" (p. ii), gives rise to high enrolment of children with different backgrounds, learning abilities, and learning styles in the mathematics classroom today. According to Nabie, Raheem, Agbemaka, and Sabtiwu (2016), the enrolment of pupils with such diverse characteristics in classroom settings poses challenges to mathematics teachers. Therefore, the mathematics teacher is challenged to provide equal learning opportunities and attention for children with different experiences, cognitive abilities, and learning styles. In view of this, the study sensitises teachers attention to the need to use multiple representations. To this effect, the Curriculum Research and Development Division (2012), agreed that the curriculum which teachers are to implement must embody pupils activities (project work, experiments, and investigations) which in other words talks about multiple representations to enable them own the knowledge they gain during learning processes and apply this knowledge in their day-to-day experiences.

According to Ampadu (2012), the standard of education has fallen due to poor performance in the Basic Education Certificate Examination due to weak foundation from the early stages. In Ghana, the available statistics of pupils" performance in mathematics from the National Educational Assessment (2016), report is also not encouraging. The majority of pupils who complete primary school do not achieve proficiency in core subject areas such as mathematics (Anamuah-Mensah, Mereku, & Ampiah, 2009).

The National Educational Assessment (2016), report reveals that the general performance of Ghanaian primary school children in mathematics is still well below acceptable levels. Analyses of pupil performance in mathematics revealed some noteworthy patterns. And these patterns in mathematics revealed that, both P4 and P6 pupils had difficulty with the higher order cognitive tasks involving Measurement (34% and 29% correct, on average, in P4 and P6 respectively) and Shapes and Space (38% and 39% correct). The highest score was in the P6 Data and Chance domain, where pupils scored 53% on average.

NEA (2016), report recommended that concerted effort needs to be made to ensure that when pupils reach P4, they have the foundational skills they need to complete the more advanced tasks of mid- and upper primary mathematics. In the early grades, pupils need to learn basic mathematics concepts and apply them as they attempt more complex operations and problem-solving tasks. In addition, pupils‟ mathematics skills at the end of primary 2 (P2) were lacking in the conceptual understanding needed to perform more difficult tasks at higher grades. Continued and more effective efforts to reduce these disparities are needed to ensure that the foundational skills are acquired by all pupils at all parts of the country. In the light of the above discussion, it is important for stakeholders in mathematics education to closely assess how the mathematics curriculum is being implemented in the classroom at the basic school level in terms of teachers‟ representations of mathematics concepts to aid understanding of individual pupils.

Teaching and learning of mathematics has been an issue of great concern to parents, government and other stakeholders due to the poor performance of pupils at the basic level. Ampadu (2012) asserts that, for some time now the mathematics curriculum in Ghana has been under intense scrutiny coupled with a number of restructuring and the introduction of new syllabus and teaching methods. This backdrop is due to the way mathematics is conceived, taught and learnt causing many pupils not to realise their full potentials and besides, not to realise the importance of the mathematics they learn at school since they are not able to apply what they have learnt to their real life situations. In response to this demand, researchers, educators and other stakeholders in the education sector have advanced educational arguments supporting the need for scientific evidence into the issue and the way forward. In Ghana, the government and other stakeholders in the education sector have introduced a number of initiatives to promote effective teaching and learning of mathematics with the aim of making the subject more enjoyable (Anku, 2008). For example, in 2003, the Ministry of Education (MoE), in collaboration with the Teacher Education Division (TED), reviewed the teacher education curriculum and upgraded all Initial Teacher Training Colleges (ITTC"s) to diploma awarding institutions with the aim of improving teachers" knowledge of content and pedagogical skills in the various subject areas. In addition, the Ministry of Education, in collaboration with other international agencies such as the Japan International Cooperation Agency (JICA), the United States Agency for International Development (USAID) and the Department for International Development (DFID), have shown enormous commitments by embarking on mathematics and science projects to improve the teaching and learning of mathematics and science at the basic, secondary, teacher training and tertiary levels. The latest of these initiatives was the introduction of a new mathematics curriculum in September 2012, which showed a paradigm shift in the teaching and learning of mathematics and other school curriculum subjects in the country.

In view of this, improving mathematics teaching and learning has been an issue of considerable concern to all. Consequently, the teaching and learning of mathematics has undergone a number of restructuring coupled with the introduction of new school curriculum and new teaching methods which factor in multiple representations to aimed at meeting the differentiated needs of learners. These initiatives aim at finding ways and means of empowering students to learn and do mathematics

1.2 Statement of the Problem

In Ghana, the issue of improving the teaching and learning of mathematics, especially at the primary level, has been the subject of national concern in recent times. There are lots of literature expressing divergent views on the need and the manner to carry out early educational programmes (Mereku, 2004; Eshun-Famiyeh, 2005).

An emerging theoretical view on mathematical learning is that using multiple representations in teaching mathematics can empower pupils learning and help develop deeper their understanding of mathematical relationships and concepts. Substantial research (Ampadu, 2012; Ozimanta, Akkoc, Bingolbali, Demir & Ergene, 2010; Tripathi, 2008) indicate that pupils taught with multiple representations demonstrate deeper understanding of mathematical relations and increase performances in problem solving tasks. The multiple representation approach has positive impact on pupil"s performance; however, its practice in the mathematics classroom varies as per individual teacher"s conception of the concept.

Studies have examined conceptions and practices areas (Ampadu, 2012; Park, 2013; Ahmed & Aziz 2009; Ozimanta, Akkoc, Bingolbali, Demir & Ergene, 2010; White & Pea, 2011) but literature discussing teachers conceptions and practices of multiple representations in teaching mathematics appears non-existent. Although studies consistently report teachers poor mastery of basic mathematical concepts (MOE, 2011; Nabie, Anamuah-Mensah, & Ngwan-Wara, 2010) and that teachers play a critical role in choosing and presenting mathematics in different forms (Bingolbali, 2011), it seems very little has been done to investigate teachers" conceptions and practices of multiple representations in teaching primary school mathematics. This study therefore sought to investigate primary school teachers" conceptions and practices of multiple representations in mathematics instruction in the Lower Manya Municipality of the Eastern Region of Ghana.

1.3 Purpose of the Study

Teachers‟ use of multiple representations, such as manipulatives, pictorial, symbolic, language, and realistic representations, in the mathematics classroom can enable pupils to build their conceptual understanding (Park, 2013). The purpose of the study was to explore primary school teachers‟ conceptions and use of multiple representations in the mathematics classroom. It was also to examine primary school teachers‟ conceptions and practices of multiple representations in mathematics instruction in the Lower Manya Municipality of the Eastern Region of Ghana.

1.4 Objectives of the Study

The objectives of the study were:

- To investigate Curriculum guidelines on multiple representations for teaching mathematics.
- To examine teachers‟ use of multiple representations in the teaching and learning of mathematics in the classroom.
- To identify the advantages and disadvantages of using multiple representations from the primary school teachers‟ perspective.
- To explore primary school teachers‟ conceptions of multiple representations for teaching mathematics.

1.5 Research Questions

The research questions that guided this study were:

1. What are the curriculum guidelines on multiple representations in the Primary School Mathematics Curriculum?
2. How do primary school teachers practice multiple representations in the teaching and learning of mathematics in the classroom?
3. What are the advantages and disadvantages of using the multiple representations in primary mathematics classroom?
4. What are primary school teachers" conceptions of multiple representations in mathematics teaching?

1.6 Significance of the Study

The findings of the study will enable teachers to realize the importance of multiple representations in the mathematics curriculum. The study would make teachers give much attention to the implementation of multiple representations in the mathematics classroom, so as to equip students with problem solving skills and strategies. This would then help pupils to handle real-life problem situations with much ease and also deal with future job careers with little or no difficulty.

This study will provides information for policy makers in education about teachers" conception and practices of multiple representations in the mathematics classroom. The results of the study can act as a guide to curriculum developers in planning and designing problem-solving through multiple means of representations to enriched Ghanaian primary mathematics curriculum in schools.

The findings of the study will reveals the various kinds of multiple representational strategies that primary school mathematics teachers in Lower Manya Municipality use and the relationship that exist between these multiple representational strategies and Mathematics teachers" job performance. This exposition will help the Lower Manya Municipality Educational Directorate to adapt workable measures in the use of multiple representations that are likely to improve teachers" job performance.

Finally, this study may inspire other researchers to embark on similar studies into multiple representations that improved job performance of teachers in public primary schools in other districts and municipalities. Such a study provides will provide more on multiple representations that contribute to job performance of teachers.

1.7 Delimitation

The study was delimited in both geographical and content wise. The study was conducted among public school primary mathematics teachers in the Lower Manya Municipality in the Eastern Region of Ghana. Although the municipality has forty-six public primary schools with a total of 92 teachers only five teachers were used. The scope of multiple representations cover all levels of pre-university education but the study focused only on public primary school mathematics teachers" conceptions and practices of the approach.

1.8 Limitations

One municipality out of the twenty-six districts in the region was selected for the study. The results of the study may therefore not be generalised to the entire region. The organisation and categorization of the data collected for analysis and discussions were the most demanding of the research design. It was particularly difficult to sieve all useful responses from the interviews and observations of teachers and analysis of syllabus and textbooks into categories for presentation and analysis. The categories identified in this study were therefore shaped by the researcher‟s perception, interpretation, and building of meaning of the data collected with guidance from supervisors.

1.9 Organisation of the Study

The study is organised into five chapters. Chapter One includes background, statement of the problem, purpose of the study, objectives of the study, research questions, significance of the study, delimitations, organisation of the study and definition of terms. Chapter Two discusses the conceptual framework and the review of related literature. Chapter Three deals with the research methodology, which includes the research design, population, sample and sampling technique, instrumentation, procedure for collection and analysis of data. Chapter Four presents the results and discussion. Chapter Five covers the summary of results, discussions, conclusion, recommendations and suggestions for further study.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.0 Overview

This chapter discusses the review of related literature. It includes, the theoretical framework, origin of multiple representations in mathematics education, views on multiple representations, multiple representations in mathematics curriculum and summary of literature review.

2.1 Theoretical Framework

According to Jacobs (2016), theoretical framework is a vantage point, a perspective or a set of lenses through which to view the research problem. As such, it can be regarded as a clarifying step in the research process. Cline (2011), with the above assertion agreed that, it sharpens the focus and consequently increases the clarity brought to the research problem. Creswell (2013), theoretical framework is a set of common concepts (or variables) and definitions that are formed into propositions or hypotheses to specify the relationship among the constructs. Theoretical framework essentially is theoretical structure that holds or supports a research work.

This study adapted the theoretical concepts of Lesh, Post and Behr (1987a), theory of multiple representations. Multiple representation refers to external embodiment of pupils internal conceptualisations (Lesh, Post & Behr, 1987a). According to Lesh, Post and Behr multiple representation theory, is a framework to represent the understanding of conceptual mathematical knowledge (Glancy & Moore, 2013). Philipp and Siegfried‟s (2015), view the theory as a clear picture holding deep and rich connections among ideas of representational fluency. These are evidence that shows that the use of multiple representations of mathematical objects is one way to build networks of mental constructions (Dick & Edwards, 2008). As an evidence of pupils understanding and as an end goal of mathematics instruction, fluency with multiple representations is a highly desired outcome of mathematics education.

National Council Teachers of Mathematics (2014), emphasize representations as one of the practices for understanding mathematical concepts. According to Lesh et al. (1987b), multiple representations are "external (and therefore observable) embodiments of students‟ internal conceptualizations" (p. 34). This theory suggests that if pupil understands any mathematical idea they have the ability of making translations between and within the modes of representations. The theory emphasizes student conceptual understanding through representing concepts in multiple forms and being able to translate within and among representations. According to NCTM‟s (2014) and Boston, Dillon, Smith, and Miller (2017), pupils who have deep understanding of a mathematical concept can easily provide many different forms of representations of the concept (manipulative, symbolic, pictorial, language, and realistic) and be able to describe how these representations relate to one another (translation). The Lesh multiple theory has been used to develop curriculum materials and classroom activities in order to help both teachers and students build a conceptual understanding of important mathematical ideas in the school mathematics curriculum (Cramer, 2003). In this study, the Lesh multiple theory was used as a conceptual framework to analyse teachers‟ conceptions and practices of multiple representations.

The Lesh multiple theory is a framework to represent the understanding of conceptual mathematical knowledge (Lesh & Doerr, 2003). It consists of multiple modes of representation: "(1) manipulative (concrete, hands-on models), (2) symbolic, (3) language, (4) pictorial, and (5) realistic (real-world or experienced contexts) representations" (Lesh, et al., 1987b, p. 38). The Lesh multiple theory emphasizes that the understanding of concepts lies in the ability of pupils‟ to represent mathematical concepts through the five different categories of representation, and in the ability to translate between and within multiple modes of representation (Cramer, 2003). This type of translation can support pupils‟ relational thinking and mathematical conceptual understanding.

Although distinct types of representational systems are important, the ability to translate among different modes of representation indicates deeper conceptual understanding within the system (Suh & Moyer-Packenham, 2007). Besides when confronted with new and complex ideas, it can be beneficial to have multiple external representations (e.g. text, pictures or equations) that provide different views on these ideas (Chamberlin & Moon, 2008; English, 2009). These potential benefits can be attributed to different functions that multiple external representations can fulfill (Ainsworth, 2006). A diagrammatic representation of Lesh, Post, and Behr (1987a), multiple representations is as shown in Figure 2.1

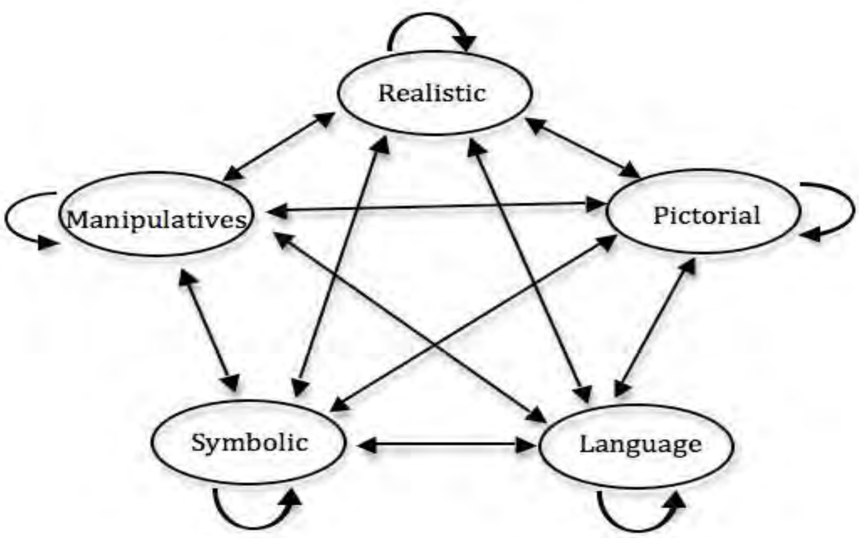

Figure 2.1: Lesh's Multiple Theory
Source: Lesh, Post, and Behr (1987a, p. 648).

A translation is the reinterpretation of a mathematical concept within the same representation or between different representations (Cramer, Monson, Wyberg, Leavitt, & Whitney, 2009). Researchers have argued that teaching via translating within and among multiple representations has a critical impact on developing students‟ abilities to understand mathematical concepts and constructs, and on developing more proficient problem solving skills (Cleaves, 2008; Cramer, 2003; Goldin & Shteingold, 2001; Kamii, Kirkland, & Lewis, 2001; Perry & Atkins, 2002). The Lesh Multiple theory has been used as a framework to analyse teachers conceptions and practices of multiple representations to develop an effective learning environment. Cramer (2003), also argues that it is important for teachers to experience using multiple representations and translate among these different modes of representations.

Although pupils‟ personal characteristics have a significant influence on their learning experiences, in the school the core of the interplay between the teacher and pupil and what is learnt is accredited to the teacher (Ampadu, 2010). Teachers are, and will continue to be, one of the most important educational influences on pupils‟ learning in general and mathematics in particular. Ahmed and Aziz (2009), claims pupils perceive teaching as providing a meaningful snapshot of what their teacher does, as their perceptions are "coloured by challenging and interesting experiences that allow them to observe learning and teaching behaviours more intimately" than their teachers (p. 19).The experience helps teachers become more aware of the weaknesses in the curricula they use, and thus supplement pupils‟ needs to learn mathematics. Implementing the Lesh Multiple Theory, in this way, has proven to be useful as a guide to develop the instruction of mathematics. This model promotes more teaching and learning through understanding concepts in the different representational forms and translating from one form to another.

2.2 Definition of representations on the Lesh Multiple Theory

The Lesh Multiple Theory includes five representations, such as manipulative, pictorial, symbolic, language, and realistic representations.

2.2.1 Manipulative representation

According to Park (2013) research has divided manipulative representations into physical and virtual manipulative ones. They have explored the impacts of using virtual manipulative with physical ones in the classroom.

2.2.1.1 Physical manipulative

Physical manipulative representations are concrete objects, which students use to explore mathematical ideas through the students" visual and tactile senses (McNeil & Jarvin, 2007). These representations are objects to be handled and arranged by pupils and teachers that are used to convey abstract ideas or concepts by modelling or representing their ideas concretely (NCTM, 2000). Manipulative include an array of items such as tens blocks, number cubes, 3-D models, and fraction circles.

2.2.1.2 Virtual manipulative

Virtual manipulative can be defined as interactive, web based visual representations of a dynamic object (Moyer, Bolyard & Spikell, 2002), while physical manipulative can be defined as concrete objects. That is, virtual manipulative can be interactive computer-based simulations or computer software that emulates physical manipulative by keyboard operation instead of physical action on three-dimensional objects (Suh, 2005). While virtual manipulative are often computerized versions of familiar physical manipulative, there is evidence that virtual tools offer additional benefits over the same tool in its concrete model (West, 2011). Virtual manipulative can be considered as a type of manipulative, but they are different from physical manipulative that have been called manipulative in previous studies up to now. Virtual manipulative might provide further advantages over physical manipulatives by eliminating some of the constraints they impose on the task.

Some computer manipulative may be more beneficial than any physical manipulative (Durmus & Karakirik, 2006). However, virtual manipulative representations should be used to give pupils opportunities to build mathematical knowledge, and these representations are often modeled after physical manipulative ones (Bouck & Flanagan, 2010; Moyer et al., 2002). For example, virtual manipulatives include a virtual geoboard with virtual rubber bands, Cuisenaire rods, virtual Base 10 Blocks, and algebra tiles. These virtual manipulatives are similar to physical manipulatives and often have the same names, but are presented in an interactive manner through an online format or a software environment (Bouck & Flanagan, 2010).

The use of virtual manipulatives in mathematics education is fairly recent (Suh, Moyer & Heo, 2005), and there have been few studies of their effectiveness in learning (Mildenhall, Swan, Northcote & Marshall, 2008; Steen, Brooks, & Lyon, 2006). These studies have found that using virtual manipulatives helps pupils improve their conceptual knowledge in mathematics (Reimer & Moyer, 2005; Steen et al., 2006; Suh et al., 2005; Suh & Moyer-Packenham, 2007). In addition, virtual manipulatives are designed to include multiple representations (Suh & Moyer-Packenham, 2007), such as pictures, drawings, letters, numbers, arithmetic operation signs, real-world contexts, and other representations. Suh, Moyer and Heo (2005), found that using virtual manipulatives helps pupils make connections between visual depictions and symbolic models. However, many teachers may not be using virtual manipulatives because they do not have much knowledge of how to use them for mathematics instruction; additionally, they may not understand how to use the technology (Mildenhall et al., 2008; Reimer & Moyer, 2005). Research recommends that more effective virtual manipulatives be developed for teaching and learning (Steen et al., 2006). Furthermore, teachers need to determine which activities are appropriate for their curriculum and their pupils‟ skill levels. Both virtual and physical manipulatives are only beneficial tools if teachers know how to integrate them into their teaching (Bouck & Flanagan, 2010).

2.2.2 Pictorial representation

According to Park (2013) and Huinker (2015), pictorial representations are visual representations such as drawings, graphs, tables, diagrams, and charts. These representations are drawn or are provided for pupils to read and interpret.

2.2.3 Symbolic representation

Park (2013), views symbolic representations as written with numbers or letters that pupils write or interpret to demonstrate an understanding of mathematical concepts or problems such as algebra formulas. In these representations, pupils and teachers symbolize language to express mathematical thoughts, including how to do problems. Huinker (2015), backs the above assertion that, symbolic representations records or work with mathematical ideas using numerals, variables, tables and other symbols. In this representation, pupils and teachers symbolize language to express mathematical thoughts including to do problems.

2.2.4 Language representation

According to Huinker (2015), language or verbal representations are the use of words and phrases to interpret, discuss, define or describe mathematical ideas, informal and formal mathematical language. Language representations are written or spoken language to explain or describe mathematical concepts, mathematical thoughts, or ways of solving problems without the use of a context (Park, 2013). In these representations, students and teachers can talk and write about mathematics using language that describes the concept.

2.2.5 Realistic representation

Huinker (2015), depicts realistic representations as mathematical ideas in everyday, real-world, or imaginary situations, using a variety of discrete and continuous measures (that is people, meters, yards). Park (2013), views realistic representations as real-life stories or situations, or experienced contexts that are related with mathematical concepts or problems. Park (2013), continuous by saying that, teachers and pupils use their knowledge of real-world contexts and experiences from their real lives to explain or understand mathematical concepts or mathematics problems. As an example of multiple representations, consider the notion of one-half. The symbolic representations include ½, 0.5, 50%, and 1:2. The language representation is, A whole is divided into two equal parts, consider only one of those parts. That is one-half of the whole. Physical manipulative representations could include items (e.g., counters) divided into two equal parts or a paper circle cut into two equal parts. There are virtual manipulative representations that have the same names as the physical manipulative ones, which are manipulated by a mouse, keyboard or touch screen. Pictorial representations include drawings of number lines (with the ½ space marked) or a circle that has one shaded part out of two equal parts. Finally, the real-world situation representation might be a contextual example where the idea of ½ is useful (e.g., one pizza equally shared between two people).

Backing the above assertion (Ayub, Ghazali & Othman, 2013; Bayazit, 2011), claim the need for using multiple perspectives in classroom learning, teachers use of instructional materials such as syllabus, pupil"s textbooks, teachers guide enable them present the content in a flexible manner and, at the same time, provide opportunities that allow pupils to use their dominant strengths and intelligences (Kennedy-Murray, 2016). This intends help pupils learn and retain information longer than other teaching approaches. The impact of this theory is to find out: Teachers" perceptions and practices of multiple representations in teaching mathematics. The researcher is of the view that, without theoretical knowledge, it is hard for teachers to learn and implement strategies and techniques needed to respond to pupils" thinking about subject content in ways that facilitate their learning.

2.3 Origin of Multiple Representations in Mathematics Education

Theorists on multiple representations have recommended that curriculum activities and materials be presented in multiple representations in order to develop effective learning environments for each student (Suh, 2007). The theory of multiple representations in understanding and manipulation of mathematical concepts has gained importance with Dienes" works. Dienes devoted a career to design materials for teaching mathematics and conducting experiments to enlighten certain aspects of mathematical concept acquisition. Diene was influenced by Piagetian theory and worked closely with Bruner on an experimental mathematics project (Resnick & Ford, 2012). In Dienes" works, the concept of multiple representations was named as Perceptual Variability Principle which means presenting the same conceptual structure in the form of as many perceptual equivalents as possible so that children could have the mathematical essence of an abstraction (Dienes, 1960). According to Dienes (1960), the concepts must be presented in multiple embodiments, that is to say, children should work with several different kinds of materials, all of which embody the concept of interest (Dienes, 1960).

Also according to Tripathi (2008), emphasized that using "multiple representations is like examining the concept through a variety of lenses, with each lens providing a different perspective that makes the picture concept richer and deeper" (p. 439). Using a variety of materials is designed to promote the use of abstract thinking (Gningue, 2006). For instance, when teaching fractions, teachers can teach fractions with various representations such as fraction circles, chips, pattern blocks, or dot paper models. These concrete representations help in building abstract conceptual understanding (Pape & Tchoshanov, 2001). Through teachers practices, students can make connections between those representations and the concepts of fractions with conceptual understanding. Multiple embodiments are viewed in the book of Resnick and Ford (2012), as a variety of environment in which the children could see the structure from several different perspectives and build up a rich store of mental images belonging to each concept. Dienes claimed that children are not accustomed to mathematical concepts in their daily life, and those concepts should be introduced to them within the realm of concrete experiences (Resnick & Ford, 2012). Due to this reason, he designed a set of mathematical materials called multi-base arithmetic blocks or Dienes blocks. They are made up of wood showing different base systems. He also cautions that using only mathematical materials would create a handicap for conceptualization of mathematical activities. Symbolization should also be placed in children‟s minds. He believes that as symbols are applied, mathematical concepts could be free from their concrete referents and be the new tools for creating new symbols (Lesh, Post & Behr, 1987a; Resnick & Ford, 2012).

In addition to the works of Dienes, Bruner made a significant contribution to the multiple representations theory. As cited in Resnick and Ford (2012), Bruner conducted studies in teaching cases with children. He examined the cognitive processes of children and how children represented the concepts mentally (Resnick & Ford, 2012). Bruner (1960), claims that mental development of children includes the construction of a model of the world in the child"s mind, an internalized set of structures for representing the world around them (Bruner, 1960). These structures have definite features, and in the course of development, they and the features that rule them alter in certain systematic ways (Bruner, 1960). When teachers are transmitting ideas to students, they sometimes face with problems of finding the language and the right pedagogy to carry their massage clear to their pupils as the others would intend to explain the same thing. In most cases the language and pedagogy that is used would not fit the child"s schema. Bruner (1960), argued that how past experience is coded and processed in child"s mind should be important so that it may indeed be relevant and usable in the present when needed. Such a system of coding and processing is what he called a representation. Bruner (1966), also argued that the importance of multiple forms of representations by stating that:

Any idea or problem or body of knowledge can be presented in a form simple enough so that any particular learner can understand it in a recognizable form. The ways in which humans mentally represented acts, objects, and ideas could be translated into ways of presenting concepts in classroom. And even though some students might be quite ready for a purely symbolic presentation, it seemed wise to present at least the iconic mode as well, so that learners would have mental images to fall back on in case their symbolic manipulations failed (Bruner, 1966).

Like Dienes, Bruner suggested that development of concepts involves successive restructurings of facts and relations, which came from children‟s interactions with and active manipulation of their environment (Bruner, 1966). He describes three modes of representation; namely, enactive, iconic, and symbolic. Enactive representation is a mode of representation through appropriate motor response (Cramer & Karnowski, 1995). Resnick and Ford (2012), illustrated enactive representation of Bruner as: what we are seeing in children who figure addition problems by tapping their fingers against chin in an obvious counting motion. Counting for these children may be represented as a motor act.

The second representation mode of Bruner is iconic, that is visualizing an operation or manipulation as a way of not only remembering the act but also recreating it mentally if it is necessary (Resnick & Ford, 2012). For instance, a child learning numbers between 1 and 10 might use the pictures of numbers arranged from 1 as a smallest picture to 10 as a biggest picture. Therefore, he could understand the numbers with reference to pictures of those numbers.

The last mode of representation is also the most abstract mode which is symbolic representation. In this mode of representation a symbol a word or a mark stands for something but does not look like that thing (Cramer & Karnowski, 1995). For example, numerals do not resemble their wordings (Resnick & Ford, 2012). According to Bruner, these three representational modes are developmental. The development of each mode is depending on the previous mode and after a long term practice with one mode, one can make transition to the next mode (Bruner, 1966).

2.3.1 Concepts of Multiple Representations

In the related literature, various definitions can be encountered related to the concept of multiple representations. The concept of multiple representations according to (Greer, 2009; Nathan, Alibali, Masarik, Stephens & Koedinger, 2010; Huinker, 2015) is the ability to use representations meaningfully to understand and communicate mathematical ideas and to solve problems. This ability in the above definition is the flexibility and fluency involved in adopting any of the modes which pupils makes meaning to. In fact, Collins (2011) challenged teachers to elevate the importance of multiple representations at the centre of classroom practice in mathematics and science. According to Nielsen (2016), multiple representation embodies the act of using two or more forms of representation to arrive at a solution. Representation of any system or process with representations such as diagrams, tables, equations, texts, graphics, animations, sounds and videos as two or more is expressed as multiple representations (Rosengrant, Etkina & Heuvelen, 2007). Similarly Ball, Thames, and Phelps (2008), also highlighted multiple representations as being part of the specialised content knowledge of mathematics unique to teaching. This specialised knowledge included selecting representations for particular purposes, recognizing what is involved in using a particular representation and linking representations to underlying ideas and other representations.

Although, definitions of multiple representation differ among various authorities it portrayed the same idea. In this research multiple representations refers to providing the same information in more than one form of external mathematical representation.

2.3.2 Types of Multiple Representations

Tripathi (2008,) emphasized that using "different representations is like examining the concept through a variety of lenses, with each lens providing a different perspective that makes the picture (concept) richer and deeper" (p. 439). Researchers and professionals who believe in multiple perspectives in learning (Nabie, Raheem, Agbemaka, & Sabtiwu, 2016; Bingolbali, 2011; Leikin, Levav-Waynberg, Gurevich & Mednikov, 2006; Ozgun-Koca, 2008) advocate multiple representations as a necessity as well as theoretical lens to developing mathematical understanding. According to Nabie et al. (2016), multiple perspectives to learning, enables pupils to develop better understanding of the basic mathematical ideas required for higher mathematics.

Several researchers identified representations in two forms. The first of these is as internal representations which are the personal mentally constructions of an individual, otherwise known as mental images. The second is as external representations which are open to inspection by others (Gilbert, 2008; Ainsworth 2008; Taba, 2009). The usage of multiple representations in mathematical learning is investigated in depth by Janvier who defined it as "understanding" a cumulative process mainly based upon the capacity of dealing with an "ever-enriching" set of representations (Janvier, 1987b, p. 67). There are two important key terms in a theory of representation that are; "to mean or to signify, as they are used to express the link existing between external representation (signifier) and internal representation (signified)" (Janvier, Girardon, & Morand, 1993, p. 81). External representations were defined as "acts stimuli on the senses or embodiments of ideas and concepts", whereas internal representations are regarded as "cognitive or mental models, schemas, concepts, conceptions, and mental objects which are illusive and not directly observed" (Janvier, et. al., 1993, p. 81).

Janvier, et al., (as cited in Porzio, 1994), classifies representations as external and internal. Internal representations concern most particularly mental images corresponding to internal formulations we construct of reality. External representations refer to all external symbolic organizations (symbol, schema, diagrams, etc.) that have as their objective to represent externally a certain mathematical „reality". (p. 109). Lesh, Post, and Behr (as cited in Bala, 2015), have said that external representations are the way by which mathematical ideas could be communicated and they are presented as physical objects, pictures, spoken language, or written symbols.

The research group headed by Janvier (1993), a recognized scholar in this field, expanded the idea of classification of representations. External representations act as stimuli on the senses and include charts, tables, graphs, diagrams, models, computer graphics, and formal symbol systems. They are often regarded as embodiments of ideas or concepts. According to them the nature of internal representations is more elusive, because they cannot be directly observed.

They affirm that important concepts in a representation theory are "to mean" or "to signify" (p. 81). Janvier et al. (1993), state that external representation, which they call signifier, and internal representation, called signified, should be linked. Cuoco (2001) affirms that:

> External representations are the representations we can easily communicate to other people; they are the marks on the paper, the drawings, the geometry sketches, and the equations. Internal representations are the images we create in our minds for mathematical objects and processes these are much harder to describe. (p. x)

According to Yee and Bostic (2014), also identified two categories of representations as symbolic and non-symbolic. Further explain symbolic representations as any representations using abstract symbols (internal) and numerical expressions. Non symbolic representations were coded as concrete model, pictorial, tabular, and mixed (external). The concrete model category characterized pupils employing manipulatives as the exclusive means for solving the problem. Pictorial approaches include diagrams and graphs while tabular representations characterized tables and charts. Mixed representation use indicated that a pupil used multiple representations during problem solving, which was further coded as the appropriate combination (e.g., pictorial-symbolic).

Goldin and Shteingold (as cited in Pal, 2014), expand the discussion on the types of representation arguing that, external systems of representation range from the conventional symbol systems of mathematics (such as base-ten numeration, formal algebraic notation, the real number line, or Cartesian coordinate representation) to structured learning environments (for example, those involving concrete manipulative materials or computer-based micro worlds). Also Internal systems, in contrast, include pupils‟ personal symbolization constructs and assignments of meaning to mathematical notations, as well as their natural language, their visual imagery and spatial representation, their problem-solving strategies and heuristics, and (very important) their affect in relation to mathematics.

2.3.3 Teacher use of Multiple Representations in the Mathematics Classroom

According to Fujita and Yamamoto (2011), mathematically-rich task should satisfy the three principles of "offering good mathematical content, purpose, and utility" (p. 250). These necessitate the essence of multiple representations (Valles, 2014) to minimize the difficulties teachers face. Below are some uses of multiple representations by teachers:

Research has suggested that using multiple representations influence learners positively on their conceptual understanding (Yilmaz, Argun & Keskin, 2009; Shumway, 2011; Budaloo, 2015). Studies of U.S. and Chinese teachers found differences in the kinds of representations used to teach elementary mathematics (Cai, 2004; Cai & Lester, 2005). Chinese teachers typically used symbolic representations, while U.S. teachers used verbal explanations and pictorial representations. Chinese students used more symbolic representations than their U.S. counterparts and had better scores on problem-solving tests. It is widely known that Chinese students outperform U.S. students in mathematics achievement (Hiebert, Gallimore, Garnier, Givvin, Hollingsworth, Jacobs & Stigler, 2003). It is not hard to imagine that teachers" choices of representations are a factor in students" levels of achievement. Thus, researchers have emphasized the importance of using mathematical representations to build mathematical knowledge and understanding. For example, (Greenes & Findell, 1999; Andrade, 2011), stated that pupils were able to interpret algebraic equations in various representations, such as pictorial, graphical or symbolic representations so that they developed mathematical reasoning in algebra. Meyer, 2001 and Yilmaz et al., 2009, argued in favour of the effectiveness of realistic mathematics education, which promotes the use of representation in middle-school algebra and progresses through levels of abstractions. Moreover, they argued that the first stage of mathematics activity should attach meaning to abstract ideas by using concrete experiences.

Moreover, multiple representations used by teachers shows their mathematical thinking and its bearing on pupils (Olkun & Toluk, 2004). This evaluate whether or not teachers understand the topic through their mathematical ideas bearing on their representational models. If teachers‟ do not vary their presentational modes then it presupposes that their understanding of the concept is minimal. Therefore, Goldin (2002), found that effective teachers pay attention to students‟ interactions with external representations in order to understand their internal systems of representation.

According to Pham (2015), the use of multiple representations gives students a chance to redefine the mathematical definitions or clear up misunderstanding that they have from the past. Teachers could help students understand mathematics concepts using multiple representations with the conjunction of guiding questions to unearth the talent in their pupils.

İpek and Okumuş (2012), examined which representations primary school mathematics teacher candidates used in the problem-solving stages and which problems they used with which representation. When it comes to teaching new topics experienced teachers use representations that students already know to teach new content, while novice teachers introduce new representations alongside new content. Since expect has been with pupils and conversant with majority of topics. In reality, expert teachers tend to use the same representations to teach multiple content topics. In addition, novice teachers often struggle to explain topics using representations because they are not familiar with the representations. Implications of this study suggest that it is important for students to understand the representations that teachers use, familiar representations can be useful for teaching new content, and one representation may be valuable for teaching multiple content topics. In view of these, teachers must be well versed in the representations they use to illustrate mathematical ideas.

Multiple representations use by teachers help pupils internalises abstract mathematical concepts (Puchner, Taylor, O‟Donnell & Fick, 2008). According to Park (2013), the theory of multiple representations, explains how teachers can use these representations to help students acquire more conceptual understanding. Research has shown that interaction among multiple representations plays a critical role in developing pupils mathematical conceptual understanding (Cramer, 2003). Suh (2007), assert that, multiple representations helps inform and differentiate classroom instruction for students‟ mathematical conceptual understanding because teachers can understand students‟ personal method of thinking. These are all strategies to help pupils gain experience with a plenty of representations (Bouck & Flanagan, 2010; Lamberty & Kolodner, 2002), as well as building and strengthening their conceptual understanding in abstract mathematical concepts (Suh & Moyer-Packenham, 2007).

Further, multiple representations used by teachers‟ brings more understanding (Tripathi, 2008). For example, a teacher might point (gesture) to beans (object) on a ten-frame (picture) while giving a verbal explanation. This suggests that presenting combinations of representations that include verbal explanations along with objects and pictures may be better than presenting visual representations in isolation. This idea supports the work of Abrahamson (2006), who suggested that teachers facilitate student discussions around mathematical representations and (Clements, 2000), who suggested that the three types of representations (concrete, pictorial, and symbolic) should be taught in parallel.

In sum, research has emphasized the importance of using multiple representations through the effort of teachers and interacting within and among the representations for students‟ mathematical conceptual understanding. Studies have found the effectiveness of using mathematical representations and translating within and among them in order to build mathematical conceptual understanding. However, there are a few studies about hindrances of using the representations for teaching and learning mathematics. Thus, teachers need to have positive practices about using the representations in order to implement them successfully in their classrooms. Research indicates that a greater understanding of teachers‟ beliefs is essential to improving instructional practices (Lumpe, Haney & Czerniak, 2000); this is because instructional practices are dependent on what individuals believe effective teaching entails (Speer, 2005). Research has also argued that changes in teachers‟ beliefs precede changes in their teaching practices in implementing a particular innovation that does not initially conform to their prior beliefs (Richards, Gallo & Renandya, 2001). However, few studies exist about teachers‟ beliefs with respect to using multiple representations, and their practices.

2.3.4 Empirical Studies on Multiple Representations in the Mathematics Classroom

The idea of using multiple representations has been effective in the teaching of mathematics (Cai, 2005), especially in the understanding of mathematical concepts and interpreting them from different points of view through the use of multiple representations (Cathcart, Pothier, Vance, & Bezuk, 2011; Hjalmarson, 2007). According to Tripathi (2008), the use of multiple representations in teaching mathematics is a strong instrument that eases the understanding of mathematical concept for pupils. Also, the use of multiple representations strengthens the understanding of students for learning how to form and solve problems in a mathematics course. Below are evidence of multiple representations to the above effect:

According to Villegas, Castro, and Gutierrez, (2009), their research with teacher candidates being educated in a mathematics department in Spain, examined their thinking ability during the process of solving problems and understanding mathematical concepts, as well as how they defined them with representations and their ability to transition from one representation to another. In this context, with the observation data presented as the representations used by teacher candidates, the transitions between representations, their way of thinking, and how often they were used was examined. At the end of research it was found that there was a significant and positive relation between the success of the problem solving of a candidate and their ability to use multiple representations.

Similarly, İpek and Okumuş (2012), examined which representations primary school mathematics teacher candidates used in the problem solving stages and which problems they used with representation. The data was collected from interviews done with 48 teacher candidates and also from multiple representations using a test. At the end of the research, it was found that teacher candidates, especially in the process of solving problems, use verbal representation more than algebraic, graphical and numerical representations. On the other hand, another important result was reached, especially in regard to the understanding of a problem: the candidates assumed difficulty forming suitable representations for a problem and doing transitions between representations.

Again, Monoyiou, Papageorgiou, and Gagatsis (2007), in their research examined the representations used in solving non-routine problems by primary school teachers in Cyprus. 20 teachers attended the research. With the data obtained at the end of the interviews, it was found that the teachers mostly preferred algebraic representations.

On the other hand, Herman (2007), examined the strategies of teacher candidates giving algebra lessons and their beliefs about multiple representations. In the study that was formed using the experimental design, the teacher candidates had ten weeks of training, and the differences between the pre-tests and post-tests were examined. Also, semi-structured interviews were done with the candidates as well. The candidates were asked to transform questions from the post-test, as well as six questions from the pre-test, into algebraic, graphical and table forms. At the end of the research, the results showed that teacher candidates preferred algebraic representations before training, whereas they also concentrated on graphical and table representations. Besides this, it was additionally found that the success of candidates in transitioning between representations increased after considering the results of the pre-test.

Delice and Sevimli (2010), in their research examined the representations that teacher candidates used in the solution of definite integral problems, their ability to transition between representations, and the relationship between the success of problem solving and the current representations. Forty-five mathematics teacher candidates applied for the study, which was designed according to a case study having a qualitative interpretive paradigm. In the study data was collected through a classical written test, an interview form, a representation preference and transition test, a participant observation form, and document analysis techniques. At the end of research it was found that the ability of teacher candidates to use multiple representations in the process of solving problems was not at the expected level. It was also found that the candidates wanted to use algebraic representation for the solution of all problems and that they were accustomed to using this representation.

Acquah (2011), study found the use of tasks on Adinkra symbols structured within the realm of problem-based learning, improved pre-service teachers" achievements in geometric transformations. Similarly, President"s Council of Advisors on Science and Technology (2012), in their report found that students in traditional lecture courses were twice as likely to drop out of college entirely compared with students taught using active learning techniques. English (2009), investigated students" outcomes as a result of implementing multiple representations with one class of seventh-grade students. The activity"s purpose was to support students to extend, explore, and refine ideas gained while solving previous modelling problems. Students" were presented with a situation about a summer reading program for secondary students as well as a data set, asked to determine an appropriate solution, and finally explained and justified their response in a verbal statement. Problem solvers worked in groups of three to four students over three consecutive 50-minute class periods. Participants created pictorial models, which helped them develop symbolically-oriented mathematical models. They also showed an ability to quantify elements of a context in order to solve the problem. The problem"s aspects included the length of a text and the text"s readability level. Students solved the open, complex, and realistic problem and the lesson provided evidence that it is possible to weave problem solving, mathematics, and other subject areas. English (2009), advocates that multiple representations and other similar problem-solving activities "should not be viewed as additional activities that add further load to an already crowded curriculum and overburdened teacher… they should be used to introduce, develop, consolidate, and enrich core concepts and processes" (p. 172-173).

In Preston and Garner‟s (2003) study, a mathematics educator and middle school teacher partnered to examine pupils‟ representation use to solve an open and complex word problem. Garner, a classroom teacher, asked her seventh-grade students to solve a word problem that drew on students‟ real-life experiences. Groups of three to five pupils worked collaboratively and then reported their result to the class. Garner indicated that the goal of the task was to give her pupils an opportunity to try different representations to mathematically model the problem and consider the benefits and limitations of each representation. Pupils used equations, graphs, charts, and wrote verbal statements that characterized the mathematical elements of the problem. The first three representations were more likely to help students solve it. During a whole-class discussion, pupils shared their mathematical models and offered their rationales for selecting their model.

Preston and Garner‟s (2003), analysis of pupils‟ work and engagement in the discussion indicated that pupils understood the text, crafted situation models, and created mathematical models. As a result of the task and instruction, pupils "began to explicitly express connections and offer their opinions as to the best representations for this activity" (p. 42). Participants recognized that choosing to employ one mathematical model over another gave them slight advantages in solving the problem. Allowing students to choose their own representation to solve a complex word problem provided a context for the entire class to examine ways of solving problems by using different mathematical models. This brief investigation into one teaching episode provides evidence that students are creative problem solvers, and when given rich tasks they are able to generate approaches that answer the question. Instruction should support students to learn non-symbolic representations and require them to give a mathematically appropriate rationale for using a specific representation to mathematically model a situation (Preston & Garner, 2003). In short, it was seen that studies related to multiple representations generally examine has a positive impact on pupils‟ learning in mathematics. When you take a close look at the above research studies it is always climax with success.

2.4 Constructivist View of Multiple Representations

Several theories emphasize the importance of multiple representations. Also, proponents of cognitive constructivism emphasize the benefits of using multiple representations of concepts and information (Coulson, Jacobson, Feltovich & Spiro, 2012). Multiple representations have also been investigated by constructivists. Their naming representational view of mind argues that representation is an active construction (Seeger & Werschescio, 1998; Von Glasersfeld, 1987). According to Vergnaud (1987), representation is an important element in the course of teaching and learning mathematics. This importance is appreciated for not only the use of symbolic systems is inevitable in mathematics, but also it is rich, varied, and universal. As a constructivist Goldin (2000), describes representational modes as a system that includes spoken and written symbols, static figural models and pictures, manipulative models, and real world situations. Representational system includes signs like letters, numerals and operations with signs like obtaining multi-digit numerals from single digit numerals or composing a word using letters.

According to him, a representational system has intrinsic (within itself) and extrinsic (with other systems of representation) structure. It is essential to provide a model consisting of interactions both within one representational mode and among the representational mode to enhance learning and problem solving in mathematics. Goldin (1998a), highlighted the close relationship between external and internal representations. Which he links Signifier as (External Representations) and Signified (Internal Representations).

In this outline, it can be realised that there is a close relationship between external and internal representations in terms of semiotic views. He identifies the external representational system as constructs to understand mathematics (Goldin, 1998a). They are easy to use, permit visualization, and universal. Internal representational system was defined by him as again constructs of mathematical behaviours (Goldin, 1998b). With the help of them how individual learn and conceptualize can be understood. He also added that the interaction between the two systems is not just essential, it is the whole point. The teacher plays a key role in blending this interaction for pupils. In order to trace this interaction, the child"s environment should be designed for obtaining all kinds of representations ranging from spoken language to mathematical symbols (Goldin, 1990).

2.5 Semiotic View on Multiple Representations

Semiotic"s deal with signs and actions of signs. In semiotic, there are three important terms, sign, interpretant, and object (Vile & Lerman, 1996). As cited in Klein (2003), Peirce indicated that; sign refers to symbols use to stand for something. If this sign has a meaning in somebody"s mind, it is called interpretant, and what this sign belongs to, is called object (Vile & Lerman, 1996). Klein (2003), combined semiotic view with multiple representations. According to him, representations are signs from which students learn something. He suggested that in mathematics curriculum the object from Peircian view can be considered the topic that should be taught, the sign is the representations used for teaching a topic, and interpretant is also signs that would help to conceptualize the object. Therefore, he defines a representation as a sign or combination of signs like (Janvier, 1998). For instance, a manipulative of an equation can be seen as a sign, its object is an equation, and its interpretant is accompanying text to this manipulative for equation.

External and internal representational systems from semiotic point of view were also examined by (Skemp, 1986). He argued that "a concept is a purely mental object" (p. 64). Since no one can have the ability of observing someone else‟s mind directly, means and indicators of the minds should be used for interpreting concepts. These are called external representations, according to Skemp. They are visible; and they should be mentally related with an idea. This idea is the meaning of that representation. He also claimed that an external representation is meaningless without its attached idea (Skemp, 1986).

Dufour-Janvier, Bednarz, and Belanger (1987), also clarified the internal and external representations by combining semiotic views. They claimed that the meaning of a signified refers to the internal representations which can be concerned more particularly "mental images corresponding to internal formulations which was a construction of reality" (Dufour-Janvier, et al., 1987, p. 110). External representations on the other hand refer to "all external symbolic organizations (symbol, schema, diagrams, etc.) that have as their objective to represent externally a certain mathematical reality" (Dufour-Janvier, et al., 1987, p. 110), which can be combined by the meaning of signifier.

2.6 Kaput's View on Multiple Representations

In addition to the earlier researchers mentioned, Kaput (1989, 1991, 1994), also proposed an important theory of understanding within multiple representational particularly in technology context. He distinguished between internal and external representations by stating that; the former is referred as mental structures and the latter as notation systems (Kaput, 1991). He defined those terms as follows; "Mental structures are means by which an individual organizes and manages the flow of experience, and notation systems are materially realizable cultural or linguistic artifacts shared by a cultural or language community" (Kaput, 1991, p. 55).

According to him, notation systems can be anything such as a mark on a paper or a sign on a computer screen, and they are used by the people to organize their mental structures (Kaput, 1991). He claimed that when someone is talking about notational system, its mental structure should also be considered. One cannot learn something from notational systems when those systems are told separately from mental structures (Kaput, 1989). He also supports the view of Von Glasersfeld (1987), who argues that; "A representation does not represent by itself, it needs interpreting and to be interpreted, it needs an interpreter" (p. 216). Kaput (1987), further stated that his semiotic-oriented definition by saying any particular representation should include five entities:

1. the represented world,
2. the representing world,
3. the aspects of the world being represented

4. the aspects of the representing world doing the representing
5. the relation between these two worlds.

Kaput (1994), echoes the constructivist view of representations by claiming that the act of representation is involved in the relation between the representing thing and the represented thing. In mathematics, the correspondence between the represented and representing world; or in other words, the signified and the signifier should be established for achieving a permanent and meaningful learning (Kaput, 1991). Responsibility lies on teachers‟ practices to build this interaction in the early years of children‟s mathematical activities. Kaput (1994), argued that internal representations are mental configurations that should be created and developed by the person himself. They are not observable, whereas external representations can be physical configurations and they can be observed, such as equations, pictures, or computer signs (Kaput, 1991). Kaput gave an explanatory example including internal and external representations. Sometimes an individual externalizes in his or her internal structures in physical form, by writing, speaking, manipulating the elements of some concrete system, and so on. For example consider the graph drawn in Cartesian coordinates by a person to represent an equation. The particular graph is not an isolated drawing from its equational context, its table configuration, and its several meanings (Kaput, 1989).

As it was stated before, Kaput (1994) is particularly interested in the representational role of technology. He believes that computer technology is a popular media to link the mathematical representations, such as graphs, tables, and formulae (Blanton & Kaput, 2003; Kaput, 1994). He states that usage of dynamic media makes the viewing of representations and performing the translations among representations easier

(Kaput, 1991). According to Kaput (1991), computers are the most helpful carrier for children to externalize their internal representations in multiple ways. Kaput stated that: "with more than one representation available at any given time, we can have our cake and eat it too, in the sense of being able to trade on the accessibility and strengths of different representations without being limited by the weakness of any particular one" (Kaput, 1991, p.70). As a result of his various research studies in using technology as a multiple representation supplier in mathematics education, Kaput concluded that students need to express the connected link between the external representations and they should be forced to generate new representational modes (Kaput, 1994).

2.7 Janvier's View on Multiple Representations

The usage of multiple representations in mathematical learning was investigated in depth by Claude Janvier who edited a book about the problems of representation in mathematical learning. He defined it as "understanding" a cumulative process mainly based upon the capacity of dealing with an "ever-enriching" set of representations (Janvier, 1987b, p. 67). Janvier (1987b) said that a representation "may be a combination of something written on paper, something existing in the form of physical objects and carefully constructed arrangement of idea in one"s mind" (Janvier, 1987b, p. 68). A representation can also be identified as a combination of three components: written symbols, real objects, and mental images. There are two important key terms in a theory of representation that are; "to mean or to signify, as they are used to express the link existing between external representation (signifier) and internal representation (signified)" (Janvier, Girardon, & Morand, 1993, p. 81). External representations were defined as "acts stimuli on the senses or embodiments of ideas and concepts", whereas internal representations are regarded as "cognitive or mental models, schemas, concepts, conceptions, and mental objects" which are illusive and not directly observed (Janvier, Girardon, & Morand, 1993, p. 81). He emphasized the external representations by further definitions. They include "some material organization of symbols such as diagram, graph, schema, which refers to other entities or „modelises" various mental processes" (Janvier, 1987a, p. 147). To him, a representation would be a sort of star-like iceberg that would show one point at a time. A translation would occur while going from one point to another when dealing external representations which were also named as schematization by Janvier (1987c). By a translation it was meant that "the psychological processes involved in going from one mode of representation to another; for example, from an equation to a graph" (Janvier, 1987c, p. 27).

Janvier (1987b), named the diagonal cells in which the translations between two same representational modes occur, like from tables to tables as transposition. The names given to the cells can be changed according to the "context in which a particular translation is achieved" (Janvier, 1987c, p. 27). He further mentioned that "transitional representations are pedagogical devices in order to clarify concepts in mathematics, with strengths and limitations that were explored" (Janvier, 1987b, p. 69).

There can be two kinds of different translations in Janvier"s view of representation: direct and indirect translations. Direct translations might be carried out from one representational mode to the other one without using any other kind of representational mode between this translation; for instance, from an equation to a representation of a table. On the other hand, a translation from an equation to a table can be conducted by making translations from an equation to a graph, and then to a table. In this case this kind of translation process is called as indirect translation (Janvier, 1998).

Another important point about the translation process is the source and target phenomenon. Any translation involves at least two modes of representations forms source and target. He claimed that "to achieve directly and correctly a given translation, one has to look at it from target point of view means which representation mode one would like to have after the translation and derive the results" (Janvier, 1987b, p. 68). Furthermore, Gagatsis and Shiakalli (2004), have found that these translation abilities are an important factor to improve students‟ problem-solving skills, and these abilities are related to student performance in mathematics. Therefore, these researchers have emphasized enhancing students‟ abilities to translate within and among mathematical representations. The cognitive processes of students might be changed with respect to being source or target of one representational mode. Therefore, teachers should design their instruction considering each representation either as a source or as a target (Janvier, 1987c).

Dufour-Janvier, et al. (1987), investigated the multiple representation theory in terms of students‟ usage of external representations in classrooms and their drawbacks. They claimed that using conventional representations as mathematical tools, rejecting one representation to another in a given mathematical situation, making translations from one representation to another, are expected from the learner in traditional mathematics. All these expectations presuppose that "the learner has grasped the multiple representations; that he knows the possibilities, the limits, and the effectiveness of each" (Dufour-Janvier, et al., 1987, p. 111). Moreover, before the learner is able to choose the appropriate mode of representation depends on the teacher‟s practices and the mathematical task as well. For instance, given an algebra problem to solve, the equation and the graph might not be giving equal access to the same information and possibilities. Hence, to meet all these expectations, instructional strategies should be improved in a way that they include variety of representations and are flexible to use translation processes in representations (Dufour- Janvier, et al., 1987).

2.8 Multiple Representations in Instruction

According to Valles (2014), teachers find it difficult in differentiating classroom instructions with multiple representations in their bid to meet the learning needs of pupils. In relation to that it has call for teachers to rely on prior knowledge, using what they have learnt in their formal schooling (Luo, Lo, & Leu, 2011). Regardless of individual differences, they use whatever means possible through their informed practices to make instructions realized.

Furthermore, Bestwick, Callingham, and Watson (2012), assert that beliefs are treated as knowledge by teachers. However, this may work in contrast to the expectations asked of a teacher. In efforts to provide deeper learning and broader learning for their students, it is believed that teachers will use multiple representations when possible in their instruction (Begg, 2011). But as Hill and Ball (2009), argue that "conventional content knowledge seems to be insufficient for skillfully handling the mathematical tasks of teaching" (p. 69).

Mitchell, Charalambous and Hill (2014), entreat prospective teachers to use multiple representations that informed their practices of effective method of instruction and as well as bringing (Green, Piel & Flowers, 2008), the desired mathematical concept to bear. In view of that National Council Teachers of Mathematics (2000), encouraged the use of multiple representations (e.g., manipulative) as part of a teacher"s instructional practice. Therefore, using multiple representations are not enough, if teachers do not have the mathematical knowledge nor the pedagogical knowledge required to use them in their lessons.

2.9 Mathematical Content Knowledge

According to Glossary of Education (2013), pedagogical content knowledge is an integration of teacher understanding that combines content (subject matter), pedagogy (instructional methods), and learner characteristics.

In recent years, teachers" knowledge of the subject that they teach has attracted increasing attention from policy makers (Oduro, 2015). Also internationally**,** the USA"s policy of "No Child Left Behind" which helps to provide pupils with high quality education, requires that teachers demonstrate competency through subject majors, certification, or other means.

When it comes to teachers" knowledge of mathematics scholars agree that teachers" own experience as students of elementary mathematics is not sufficient as a base to teach it (Science Conference Board, 2012; Valles, 2014). Hill, Sleep, Lewis and Ball (2007), state that while it is difficult to specify the knowledge required for the effective teaching of mathematics, simply possessing knowledge of mathematics is not sufficient. Over the years there has been growing focus internationally and on the command of content required for successful teaching (Schmidt, Tatto, Bankov, Blomeke, Cedillo, Cogan, L. & Santillan, 2007). These findings have inspired the attempt to characterize an effective teacher"s knowledge, noting that the scholarly literature on the subject repeatedly argues that the knowledge base of expert teachers is not only broader than that of inexperienced teachers, but that it is also more connected and integrated (Krauss, Baumert & Blum, 2008).

Beyond the relevance of strong content knowledge, several authors have argued that being successful mathematics teachers also requires a solid foundation in pedagogical content knowledge: that is, a type of professional knowledge that is used to teach the content of a particular branch of knowledge (Wilson, Floden & Ferrini-Mundy, 2002; Kuntze, Dreher & Friesen, 2015).

The content knowledge (CK) and pedagogical content knowledge (PCK) are strongly related but distinct entities (Turnuklu & Yesildere, 2007; Buschang, 2008). This allow for development and selection of tasks, the careful chosen of representations and explanations, the facilitation of productive classroom discussions, the interpretation of student responses, the emphasis on student comprehension and the quick and appropriate analysis of student mistakes and difficulties are all factors that embodies PCK. Research has shown that teachers" content knowledge is related to the mathematical quality of teachers" instructions and teaching style (Baumert, Kunter, Blum, Brunner, Voss, Jordan & Tsai, 2010; Charalambous, 2010; Hill & Ball, 2009; Hill, Blunk, Lewis, Phelps, Sleep & Ball, 2008).

An, Kulm, and Wu (2004) assert that between content, curriculum, and teaching, teaching knowledge is the basic component of pedagogical content knowledge. Park and Oliver (2008) further explain that PCK is modified by the teacher"s reflections on teaching as a whole and that the teacher"s understanding of students" misconceptions is the main factor that influences planning, conducting and evaluating teaching in PCK.

Using multiple representations for mathematical objects in the classroom is hence a key for fostering students" conceptual learning (Dreher & Kuntze, 2013). Accordingly, teachers need professional knowledge so as to foster their students" learning and they have to be able to use this knowledge descriptively to be able to teach effectively to meet the individual needs. Analysing mathematical content against the background of multiple representations is probably as important as ways of dealing with representations in the learning process.

Professional knowledge related to using multiple representations is a resource (Schoenfeld, 2012), for teachers, which is the base on which they can draw when they analyse content matter related to learning opportunities or the interaction with students in classroom situations. By analysing we understand an awareness driven, knowledge based process which connects the subject of analysis with relevant criterion knowledge related to using multiple representations.

This means in particular that teachers can identify and connect different representations of mathematical objects, but also think of examples of content in which the use of multiple representations plays an important role for example, for gaining in-depth conceptual insight or for simplifying problems (Kuntze, Lerman, Murphy, Kurz-Milcke, Siller & Winbourne, 2011). In both cases, the analysis needs an initial awareness of criteria linked with using multiple representations.

2.10 Teachers conceptions of Multiple Representations

According to Ahmed and Aziz (2009), teachers" conceptions of their teaching is a key aspect that reflects their effectiveness of mathematics teaching and learning and it also reinforces teacher"s decision making. Even though the curriculum has been planned to embody curriculum materials, this does not make teaching complete without the teachers practices coming into bear. Oduro (2015), assert that, teachers‟ practices have been found to have a powerful impact on teaching through processes such as the selection of content, styles of teaching, and modes of learning. Countries have gone through number reforms with the mere fact of restructuring the curriculum to factor the implementers (teachers). Handal and Herrington (2003), also argued that "successful curriculum change is most likely to occur when the curriculum reform goals relating to teachers‟ practice takes into account of the teacher‟s belief" (p. 65). Jurdak (as cited in Ampadu, 2012), argued that mathematics teachers‟ conception and what informed their practices influence the way they teach. Research evidence to the above assertions are as follows;

According to Teo (1997), investigation in to the beliefs of 16 teachers in Singapore, reported that teachers beliefs and conception about mathematics has an influence on the individual teacher‟s teaching. Similarly, Pepin in his comparative study also established that there is a direct relationship between the teacher‟s beliefs and their teaching practices. Also, Perkkila (2003) in his study involving Finish primary school teachers also revealed that teachers‟ recollection of their experiences and beliefs has great influence on their teaching. Perkkila further added that, the way a teacher teaches can be traced back to his/her school days how he/she experienced the teaching and learning of the subject. In all the above studies it was established that, although factors such as the demands of the mathematics curriculum and the national call for a change in the teaching and learning of mathematics impacts on teachers‟ teaching, the impact of the individual teacher‟s beliefs and experiences cannot be underestimated.

2.11 Research on Teachers' Beliefs

Beliefs play a key role in mathematics instruction which intend informed teachers. O„donovan (2015), proposed a simple definition for beliefs as "what people think (or hope) are true (or probably true)" (p. 307). According to Harvey (1986), beliefs are individual's representation of reality that has enough validity to guide thought and actions. Beliefs then become mental constructions of experience that guide people‟s thinking and actions.

Teachers‟ beliefs play a critical role in predicting their thinking, intentions, and practices in their classroom (Speer, 2005). Teachers differentiating classroom activities using multiple representations does not happen by chance, but are influenced by the individual teacher‟s beliefs about the subject. Further Hersh (as cited in Ampadu, 2012), claims that people perceive and understands the nature of mathematics base on how they think is taught and learned. Therefore Oduro (2015), established the importance of belief as a powerful impact on teaching through processes such as the selection of content, styles of teaching, and modes of learning.

Moreover, research has also established that teachers‟ beliefs varies from one geographical area to another, this happens when their conceptions tend to be consistent with the policies and cultural practices of a particular area of jurisdiction (Brown and Harris, 2009; Brown, Lake and Matters, 2011). These instructional practices which are externally driven are therefore brought to classroom, and what they bring is dependent upon what they believe effective pedagogy entails. The assertion that people act upon what they believe has been supported by studies tracing the relationship of beliefs to practice (Crawley & Koballa, 1992). The connection between teacher beliefs and practice has been well established (Cohen, 1990). Since new research-based perspectives often require radical changes in teachers" beliefs, research on teaching must consider teachers" beliefs in relation to the practice of new perspectives in their classroom (Crawley & Koballa, 1992; Roehrig & Kruse, 2005; Speer, 2005).

Other researchers argue that changes in teachers" beliefs precede changes in their teaching practices in implementing a particular innovation that does not initially conform to their prior beliefs (Richards, Gallo & Renandya, 2001). In addition to knowledge, it is necessary to consider beliefs to account for the differences between mathematics teachers. It is possible for two teachers to have very similar knowledge, but for one to teach mathematics with a problem solving orientation, whilst the other has a more didactic approach. Because of the potent effects of beliefs, like this, the model provides an extensive treatment of the mathematics teacher's beliefs.

Researchers claim beliefs consists of the teacher's system of beliefs, conceptions, values and ideology also referred as the teacher's 'dispositions' (Kuhs & Ball, 1986). The importance of teachers' beliefs and conceptions concerning subject matter has been noted by a number of authors, both for mathematics (Ernest, 1987; Underhill, 1987), and for other areas of the curriculum (Clark & Peterson, 1986). The argument is that such conceptions have a powerful impact on teaching through such processes as the selection of content and emphasis, styles of teaching, and modes of learning. In addition to subject matter related beliefs, the teacher's principles of education and views of its overall goals are also important (Wilson, Shulman & Richert, 1997).

Research on teacher beliefs on teaching and learning mathematics has become one of the most important domains in mathematics education research. Teacher beliefs shape the way in which a teaching method is implemented. Moreover, when teachers have negative beliefs about a new instructional approach, the approach is generally not effective for their students (Handal & Herrington, 2003). Research suggests that one obstacle to wide-scale implementation of multiple representations is many teachers‟ negative beliefs due to their unfamiliarity with them (Lloyd, 2002). However, it is still unclear as to how teachers‟ beliefs about teaching can be changed, and how belief changes can impact teachers‟ practices in teaching mathematics.

2.11.1 Curriculum and Teachers' Beliefs

The National Council of Teachers of Mathematics (NCTM, 2000) describes the framework for effective ways to structure mathematics classroom environments. These standards-based learning environments are based on communication, real-world relevance, cooperative learning, problem solving, discipline and subject connections, and an integration of technology. This has been widely accepted as an important structure for equitable and effective mathematics classrooms.

Richards (1998), claims that while teachers‟ belief systems shape the way they understand teaching and the priorities they accord to different dimensions of teaching, the thinking that teachers employ during the teaching process itself is also crucial to our understanding of the nature of teaching skills (p.73).

Series of research studies about relationships between curricula, such as standards-based curriculum, and teachers‟ beliefs have found that teacher beliefs influence effective learning environments and instructional practices (Handal & Herrington, 2003; Wilkins & Ma, 2003). It is only possible to understand how and why a teacher teaches in a particular way by understanding how the teacher interprets his/her teaching practices (Ampadu, 2012). Thus, more traditional beliefs have been associated with more traditional learning environments and practices (Stipek, Givvin, Salmon & MacGyvers, 2001). Ahmad and Aziz (2009), suggest that teachers‟ perception of their teaching and learning situations are important, as they reinforce teachers‟ decision-making about how to handle classroom situations. Since the teacher is the sole implementer, therefore will know how best to present or teach the subject (p.19).

Furthermore, studies of the curriculum have found that when teachers have beliefs that are compatible with curriculum change, the implemented curriculum is successful in their classrooms. However, when teachers have negative beliefs about the curriculum, the goals of the curriculum will be diluted (Handal & Herrington, 2003). The gap between the goal of the curriculum and teachers‟ beliefs seems to cause failure of curricular change in their classrooms (Cheung & Ng, 2000; Handal & Herrington, 2003).

Macnab and Payne (2003), believe that mathematics teachers are confident and have strong personal views when it comes to their perception and interpretation of their teaching practices. Ampadu (2012), confirmed that teachers‟ perceptions of their teaching are influenced by the curriculum recommendations. Even though teachers might have their own practices definitely are informed by the curriculum. According to Smith (1996), teachers‟ perceptions of their teaching practices have always supported policies and principles in the curriculum of which multiple representation is not an exception. Even though their actual teaching practices may vary or completely differ from the underlining principles of multiple representations.

In conclusion, based on the above assertion (Ampadu, 2012), teachers‟ opinions of their teaching are an important tool for measuring individual teaching styles. However, teachers‟ interpretations or perceptions of their teaching are influenced by the content of the national curriculum and associated initial training and continuing professional development.

2.11.2 Multiple Representations in the Mathematics Curriculum

According to Stabback (2016), curriculum in its simplest terms, is a description of what, why and when students should learn. The curriculum is not, of course, an end in itself. Rather, it seeks both to achieve worthwhile and useful learning outcomes for pupils (Guzey, Moore & Roehrig, 2010), and to realize a range of societal demands and government policies. It is in and through the curriculum that key economic, political, social and cultural questions about the aims, purposes, content and processes of education are resolved. The policy statement and technical document that represent the curriculum reflect also a broader political and social agreement about what a society deems of most worth (Hamilton, Lesh, Lester, & Brilleslyper (2008), that which is of sufficient importance to pass on to its children.

A good quality curriculum enables and encourages learning differentiation (Stabback, 2016), mathematical representations are of particular importance in helping students to advance their understanding of mathematical concepts and procedures, make sense of problems, and engage in mathematical discourse. The use of multiple representations allows students to draw on multiple sources of knowledge (Boston, Dillon, Smith & Miller, 2017). Drawing on multiple sources of knowledge acknowledges the mathematical, social, and cultural resources that pupils bring to mathematics. NCTM (2014), emphasized that, teachers who use this teaching practice effectively validate the resources that pupils bring to mathematics and connect instruction to pupils" experiences and interests. In other words, it provides space for teachers to adapt the curriculum to suit the students in their classes. It does not demand that each pupil learn the same content in the same way and in the same number of hours. It provides teachers with the flexibility to ensure that their treatment of the content is appropriate to their pupils‟ needs and capabilities.

Further (NCTM, 2014; Stabback, 2016), claim that in developing approaches to multiple representations, the curriculum and the pedagogy it promotes will acknowledge that pupils learn in different and individual ways, with their own learning styles and strategies. According to Bartell, Wager, Edwards, Battey, Foote and Spencer (2017), some pupils are effective and skilled listeners, others require visual stimulation, and others learn best through practical exercises. Researchers claim a good quality curriculum will encourage teachers to get to know their pupils individually and ensure that their teaching styles and their classroom behaviours are directed towards achieving the best learning outcomes for each of them (NCTM 2014; Strutchens, Quander & Gutierrez, 2011; Stabback, 2016).

2.11.3 Multiple Representations in the Ghanaian Mathematics Curriculum

In the Ghanaian perspective multiple representations in the primary school mathematics curriculum is not an exception. According to Ampadu (2012), the new curriculum is underpinned by constructivism and its advocates for variety of means to reach the classroom child in the teaching-learning process. Nabie, Raheem, Agbemaka, and Sabtiwu (2016), concurs with the above assertion that Constructivism is really a theoretical lens for Ghana‟s curriculum design and implementation process. Further explain that the new curriculum has made provision for multiplicity in content delivery to meet the differentiated Ghanaian classroom (that is, multiple presentations, solutions, learning abilities and styles. Ampadu (2012), highlighted some traces of multiple representations in the new mathematics curriculum as follows:

1. Create learning situations and provide guided opportunities for students to acquire as
2. Much knowledge and understanding as possible through their own activities.
3. Emphasise student-centred activities and communication.
4. Foster interest and self-confidence in the learning of mathematics by providing
5. Students with opportunities to explore various mathematical situations in their
6. Environment to enable them make their own observations and discoveries.
5. Apply various instructional practices to cater for individual students‟ needs.
6. Utilise concrete manipulatives to help students to compare, classify, analyse, look for patterns and spot relationships and draw their own conclusions.
7. Consider students‟ evaluation as an integral part of the teaching learning process and evaluation exercises should challenge students to apply their knowledge to issues and problems and engage them in developing solutions and increasing investigative skills.

In general, the new mathematics curriculum encourages teachers to tailor the teaching process to meet differentiated pupils in the classroom in order to achieve optimum pupils learning. The guidelines outlined in the new curriculum suggest that students‟ active participation in the teaching and learning process should be considered first by the teacher in his/her choice of any particular teaching method or activities for a particular topic.

2.12 Summary of the Literature Review

The theoretical framework of this study was based on Lesh‟s Multiple Representation theory. The researcher reviewed literature pertaining to primary school teachers conceptions and practices of multiple representations. In the review of literature it has been seen that multiple representations have provided the foundation for a variety of mathematics curricula and learning environments that illustrate the rich conceptual linkages among symbols, graphs, tables and situations. Also, the literature indicates that in terms of classroom mathematics delivery teachers have their own practices. Notwithstanding, teachers need to be guided by the curriculum to meet the differential needs of pupils.

CHAPTER THREE

METHODOLOGY

3.0 Overview

This chapter describes the general procedures employed in the collection and analysis of data. The chapter is organised under the following headings: research design, population, sample and sampling techniques, research instruments, data collection procedures, trustworthiness of data, and data analysis procedures.

3.1 Research Design

According to Creswell (2009), research design is the plan and procedure for research that span the decisions from broad assumptions to detailed methods of data collection and analysis. The study employed case study design as described by Merriam (2009). The study adopted the instrumental case study design within the Case Study design. According to Merriam is defined by its particular case features which focus on the study of a specific "situation, event, program, or phenomenon" (p.43). Creswell (2012) and Yin (2014), endorsed the use of case studies as appropriate for the exploration of a central phenomenon using a bounded system (case or cases). This design will be used to examine primary teachers‟ conceptions and practices of multiple representations in teaching mathematics in Lower Manya Municipality.

According to Simons (2009), case studies "allow deep probing and analysis as well as follow the life cycle of a case and can allow both generalizations and localized analysis" (p. 21). Case study is a practice design that seeks to gain an in-depth understanding of "one setting, or a single subject, a single depository of documents, or one particular event" (Bogdan & Biklen, 2007, p. 59). Case study research is used to conduct an in-depth investigation of an issue at a specific instance and location. According to Stake 1995; De Vos, Strydom, Fouche and Delport 1998, allude to three types of case studies, namely, the intrinsic, instrumental and collective case study. They further explain the types as follows; In an intrinsic case study, a researcher examines the case for its own sake. For instance, why does student A, age eight, fail to read when most children at that age can already read? However, in a collective case study, the researcher coordinates data from several different sources, such as schools or individuals. In an instrumental case study, the researcher selects a small group of subjects in order to examine a certain pattern of behaviour. Against this backdrop, I believe that the choice of an instrumental case study, for this study was the most appropriate since the purpose of this study was primarily to elaborate on existing theory as well as gain a better understanding of the concept of effective teaching which has wider implications for society. It is a method used to narrow down a very broad field of research into one easily researchable area. It is for this purpose that the Lower Manya Municipality of the Eastern region of Ghana was used for the study. Case study research design takes place in natural settings and enables the researcher to develop a level of detailed understanding about the individual or place.

According to Geertz (1973), case studies strive to portray what it is like to be in a particular situation, to catch the close up reality and thick description of participants live experiences of, thoughts about and feelings for a situation. It provides more realistic responses than a purely statistical survey. That is, whilst a statistical survey might show how much people do something, case study, on the other hand will determine why such a phenomenon is occurring and why it is so. The first foundation of the case study is the subject and its relevance for which you are deliberately trying to isolate a small study group, one individual case or one particular population (Shuttleworth, 2008).

The study was conducted in natural classroom setting where students and teachers interacted freely. Teachers and students were familiar with each other and naturally interact freely. The research design therefore enabled the researcher to gain an in-depth understanding of how teachers practice multiple representations with their students in the mathematical classrooms. The interactions of the researcher with the participants gave an opportunity for him to gain in-depth information concerning their practices. This design involves participants in data collection and seeks to build rapport and credibility with the individuals in the study (Patton, 2002).

Since the design of the study employed mostly qualitative approach in its data collection and analysis, based on the topic it was necessary to collect data from teachers for the interview, observation and do document analysis. In addition, the researcher gain in-depth understanding of the cases within this study by collecting multiple forms of data, such as interview data, observational data, and pertinent documents analysis (Creswell, 2012). The study provided useful quantitative data also for the study. As such, interviews, observations and analysis of document were used concurrently in the data collection process (Yin, 2008).

3.2 Study Area

The research was conducted in the Lower Manya Municipality (LMKM) in the Eastern Region of Ghana. According to the Ministry of Finance (2015) the Municipality is strategically located at the Eastern corner of the Eastern Region of Ghana and it lies between latitude 6.05N and 6.30N and longitude 0o08W and 0.20W with an altitude of 457.5m above sea level. The Municipality is bounded on the

North-West by Upper Manya Krobo District, on the North-east by Asuogyaman district, on the South-East by North Tongu District and on the South by Yilo and Dangme West District. The LMKM covers an area of 304.4 square kilometres, with a population density of 293.2 persons per square kilometre. The Lower ManyaMunicipality is the parent District, from which Upper Manya Krobo District was carved-out by Legislative Instrument 1842 on 1st November, 2007. The pictorial representation (map) of the municipality is presented in Figure 3.1.

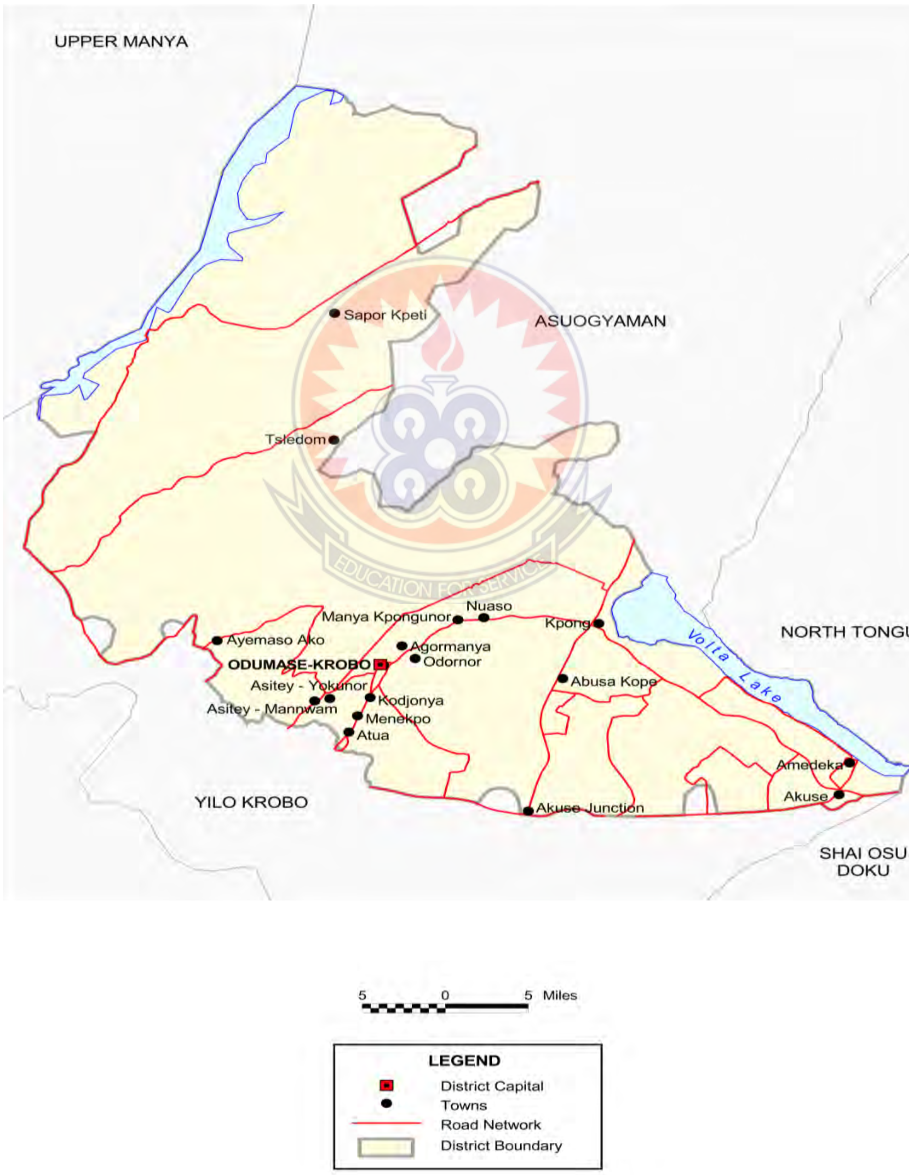

Figure 3.1: Lower Manya Municipality Map

Source: Ghana Statistical Service (2013), 2010 Population and Housing Census

The Education Directorate in the municipality is made up of eight (8) circuits namely: Middle Belt East, Middlle Belt West, Odumase A, Odumase B, Akuse, Kpong, Agormanya and Manya Kpongunor-Nuaso. The circuits in the municipality house 65 Kindergarten, 67 Primary and 60 Junior High Schools. The municipality also has four public senior high schools Krobo Girls‟ SHS, Akro S. H. Tech. School, Akuse S. H. Tech. School and Manya Krobo Senior High School and four private senior high schools namely: Modern Senior High, St. Annes Senior High and Vocational Institute, Paul Hans Vocational Institute, Bakhita Senior High School, Kpong Women‟s Vocational and King David Commercial College. The educational institutions are faced with several problems some of these are: inadequate and inequitable access to higher education, particularly after the basic level and for persons with special needs; poor educational infrastructure; poor supervision; lack of Information Communication Technology equipment, furniture and means of transport.

In the 2010 Population and Housing Census, a total population of 33,076 were attending school out of which 14.5% were at the Kindergarten level, 46.4% at the primary level, 20.8% at the Junior Secondary School/Junior high School level, and 10.5% at the SSS/SHS level. Only 1.7% was currently attending school at the tertiary level and less than 1% was attending vocational/technical/commercial schools. Approximately equal proportions of males and females were attending school at the basic level of education.

3.3 Population

According to Awanta and Asiedu-Addo (2008), population is the total number of subjects of the research that conforms to a clearly defined set of characteristics. Agyedu, Donkor, and Obeng (2013), also opine that the term "population refers to the complete set of individuals (subjects), objects or events having common observable characteristics in which the researcher is interested in studying" (p. 89). They further stated that, the population may be finite or infinite. A research population is a large well-defined collection of individuals having similar features (Castillo, 2009). Castillo differentiates between two types of population, the target population and accessible population.

The target population is the total group of subjects to which a researcher would like to generalise the results of a study and accessible population is the group of subjects that is accessible to the researcher for a study from which the study sample can be drawn (Castillo, 2009). The population for the study consisted of all primary mathematics teachers in the Eastern region of Ghana. The target population for this study consisted of ninety-two public primary school teachers in the Municipality. However, the accessible population of this research consisted of all primary three (3) and primary six (6) mathematics teachers in Lower Manya Municipality.

3.4 Sampling Technique and Sample Size

A sample consists of a carefully selected subset of the units that comprise the population (Awanta & Asiedu-Addo, 2008). Sampling is the technique of selecting a portion of the population to represent the entire population (Alhassan, 2006). According to Singh and Mazuku (2014), sampling refers to selection of a subset of individuals from within a population to estimate the characteristics of whole population.

Firstly, cluster sampling technique was used to categorized all the public primary schools into groups of five based on their enrolment. Cluster sampling is a sampling method where the entire population is divided into groups, or clusters, and a random sample of these clusters are selected (Singh & Mazuku, 2014). Cluster sampling is generally used when the researcher cannot get a complete list of the units of a population they wish to study but can get a complete list of groups or clusters of the population. In other words, cluster sampling was used in the study because it enabled the researcher to group respondents and give them equal opportunities.

Secondly, the researcher used simple random sampling techniques to select five primary schools from each cluster through paper folding. Simple random sampling was used because it provided space for each element and each combination of elements in the population to have equal probability of being selected as a part of the sample (Andale, 2015). Andale further opined that, simple random sampling is one of the simplest forms of random sampling, this method is one of the fair way to select a sample. As each member of the population has an equal probability of being selected, simple random sampling is the best-known probability sample.

Finally, purposive sampling technique was used to select five respondents (teachers) for the study. According to Creswell (2009), purposive sampling technique enables the researcher to reach the participants quickly and to use those participants to collect meaningful information for deeper understanding. In other words, purposive sampling enables the researcher to select individuals with requisite expertise and experiences that are central to the phenomenon under study. The reason for the selection was based on (1) familiarity with the geographical area; (ii) diversity, in terms of both deprived and well-endowed primary schools; and (iii) accessibility to the schools.

Marshall, Cardon, Poddar and Fontenote (2013, p.13) assert that the "choice of more than one case tends to dilute the overall analysis and when choosing multiple cases, researchers tend not to choose more than four to five". Also according to Hodges

(2011, p. 90) “the purpose of a qualitative study is not necessarily to predict or generalize and therefore the size of the sample chosen is more about saturation than representation, to obtain as much detail from each case rather than to generalize”. Rule, Davey and Balfour (2011), also declared that “the strengths of case study; include its ability to generate rich and thick description of phenomenon because they are manageable in terms of time, resources and number of sites and contexts, and have wide applicability and flexibility” (p. 302). Given the above considerations, the researcher purposively sampled five (5) practicing teachers comprising of 4 male and 1 female were selected from the selected based on their experience and willingness to participate in the study and bearing in mind that a small sample of information-rich individuals adequate enough to answer the research question is what matters in qualitative research (Johnson & Christensen, 2000).

3.5 Research Instruments

After a careful review of appropriate literature, it became necessary to collect data through observational guide, semi-structured interview schedule and documentary analysis. These instruments were used to offset the weaknesses of one instrument. Glenn (2009), said document analysis is often used in combination with other qualitative research methods as a means of triangulation.

3.5.1 Interview Guide

In qualitative research, interviews are often a predominant mode of data or information collection (De Vos, Delport, Fouche & Strydom, 2005). Thomas (2003), describes interviews as an effective means of eliciting responses from participants in a research. It usually involves a researcher orally asking questions for respondents or individuals to answer orally. Interviewing provides the researcher with greater flexibility and personal control. Interviews traditionally have been conducted face-to-face and one-on-one, with the researcher speaking directly with the interviewee at a time. Ary, Jacobs and Razavieh (2002), are also of the view that an interview is used to gather data on subjects" opinions, beliefs, and feelings about the situation in their own words. Ary et al. (2002), continue to argued that interview provide elaborated responses and a forum for sincere participation in the study. Mitchell and Jolley (2010), opine that there are three main types of interviews namely: structured, semi-structured and unstructured interview. They explained that structured interview is a type in which all respondents are asked a standard list of questions in a standard order. The semi-structured interview, like the structured interview is constructed around core standard questions. However, the interviewer may expand on any question in order to explore a given response in greater depth. Finally in the unstructured interview, the interviewers have objectives that they believe can be best met without an imposed structure. The interviewer is free to ask what he or she wants, how he or she wants to, and the respondent is free to answer how he or she pleases.

In this study, a one-on-one semi-structured interview guide was used to collect qualitative data on significant episodes since the study was meant to seek personalized information and depth of information that revealed the interviewee"s opinions on their conceptions and practices of multiple representation in the mathematics classroom. The interview was conducted for the respondents in their classrooms on different dates scheduled by both the researcher and the participants. This enabled the participants to express their views and concerns freely and explicitly. The interviews sought data on primary school teachers" conceptions of multiple representation in teaching mathematics as well as the advantages and disadvantages of using multiple representations in teaching primary mathematics.

3.5.2 Observation

According to Creswell and Clark (2007), observations refer to the process of taking field notes on the behaviour and activities of individuals at the research site. Gathering information by means of observation involves watching and listening to events, then recording what occurred. As direct observation, the researcher immediately sees and hears what is happening. Data obtained from observation are said to be imperative as it affords the researcher the opportunity to gather live data from live situations rather than at second hand (Padgett, 2004). This is because the researcher becomes the instrument and feels the reality of the situation or concept under investigation. Observation can be used to collect data on what is happening regarding a situation or to set in perspective data obtained by questionnaire or interviews (Robson, 1995). Since this study sought to examine and interpret, primary school mathematics teachers" conceptions and practices of multiple representations, observation was carried out because it provided the opportunity to observe what happens in relation to the use of multiple representations in the mathematics classroom. An observation checklist was used to collect data on how primary school teachers practice of multiple representations in teaching mathematics to help address research question 3.

3.5.3 Document Analysis

Merriam (2001) and Creswell (2003), contend that documentary analysis can be just as rich a data source as primary observation or interviews. Document analysis is a systematic procedure for reviewing or evaluating documents-both printed and electronic (computer-based and Internet-transmitted) material (Aforklenu, 2013). Like other analytical methods in qualitative research, document analysis requires that data is examined and interpreted in order to elicit meaning, gain understanding, and develop empirical knowledge (Glenn, 2009). Documents contain text (words) and images that have been recorded without a researcher‟s intervention. Glenn (2009) said document analysis is often used in combination with other qualitative research methods as a means of triangulation.

Documents that are used for systematic evaluation in a study take a variety of forms that these include: background papers; books and brochures; diaries and journals; attendance registers; and minutes of meetings among others. Documents reveal what people do or did and what they value. In this study, the documents reviewed included primary level three (3) and six (6) syllabi respectively as well as their textbooks for teaching and learning activities which portrays multiple representations. The analysis gave the researcher the opportunity to know the extent to which primary mathematics curriculum revealed traces of multiple representations as in research question 1.

3.6 Pilot Study

Awanta and Asiedu-Addo (2008) asserted that pilot-testing the instruments enabled the researcher to modify items that were difficult to understand, reduce ambiguities and incorporate new categories of responses that were identified as relevant to the study. Kumekpor (2002) concurs that, the main aim of the pre-testing the instruments was to enable the researcher find out the suitability of the responses in answering the research questions. Pre-interviews provide the researcher the opportunity to rehearse and gain prior knowledge to the appropriate technique that will be adopted during the actual interviews.

To determine the strength and weakness of the interview guide, it was pilot tested on two primary mathematics teachers in the Asuogyaman District of the Eastern Region. These teachers were selected based on their requisite expertise and experiences that are central to the phenomena under study. The researcher chose the district because it was deemed to have exhibited the similar characteristics as the district of interest to the researcher. The pilot-test helped the ensure validity of the observation and interview guides. Thus, the pilot test process was to determine the strengths and weaknesses of the instruments in the structure, wording, order and choice of items. It was also meant to help in the identification of variations of items to be responded to, meaning, item difficulty, and participants" interest and attention in responding to individual items, as well as to establish relationships among items and item responses (Mertens, 2010; Gay, Mills & Airasian, 2009). The responses were been scrutinized and peer-debriefed after listening to the recorded version (Merriam, 1998).

3.7 Data Collection Procedures

An application for permission to conduct this study in the Lower Manya Municipality of the Eastern Region of Ghana was sent to the Ghana Education Service Directorate in the municipality for permission to be granted to gain access to the schools and participants (see Appendix E). The letter stated the objective and purpose of the study and the need for the participants to give their consent to and co-operate with the researcher. This was done based on Creswell"s, (2003), advise that respecting the site where the research takes place and gaining permission before entering a site is paramount in research. In the same vain Kelley, Clark, Brown and Sitzia (2003), also asserted that these are the most important ethical issues in every research study. They further suggested that all information obtained should be used for the intended purpose.

To establish a close relationship with the teachers, each school Head convened a short meeting with the teachers to seek their maximum support. At the meeting, the researcher gave a brief overview of the study, addressed concerns teachers had about the study, and solicited teachers consent to participate in the study. The researcher assured them of confidentiality and their informed consent.

3.7.1 Conducting Interviews

The researcher interviewed five primary mathematics participating teachers on their practice of using multiple representations in teaching mathematics (see appendix A). Researcher personally conducted semi-structured interview to interviewees. Researcher first established rapport with the interviewee. Researcher then followed up immediately after each lesson observation with interviews. Normally we go outside when the classroom is noisy. Researcher started by welcoming the interviewees for availing themselves among the lot to partake in the study. Researcher then assured the participants the confidentiality of the study and under no circumstances would it be used aside it purpose. Researcher also enquired from them if they have something to say which the researcher could not address and that yielded no answer. I intern asked them the interview questions and in cases where there was misunderstanding researcher further explain. Each interview lasted 60 minutes and the researcher recorded all questions and their corresponding responses of each of the five interviewees.

3.7.2 The Observation Processes

Upon observing the participants provided the researcher the opportunity to come to terms with the realities of teachers‟ conceptions and practices of multiple representations in the classroom. The researcher observed five primary mathematics participating teachers on their practice of using multiple representations in teaching mathematics (see appendix B). Researcher personally observed the participant and on the day of observation researcher further established rapport with the teacher and the pupils through exchanging pleasantries in the classroom. The class later welcomed me and offered a sit behind the pupils. In the beginning of the lesson the teacher introduce me as a guest to be with them through the lesson delivery.

While observing teachers during lesson delivery, the researcher makes a tick on the checklist against any multiple representations being practiced. Each participant was observed once in an 80 minutes lesson. A participant observed per day to gain an in-depth idea about their practices. Observation notes which are the eyes, ears and perceptual senses for readers (Patton, 2002) were also taken to compensate the shortfalls of the observation checklist and also to provide further detail data to inform readers.

3.7.3 Document Analysis

According to Park (2013), multiple representations is achieved when one uses two or more of (1) manipulative (concrete, hands-on models), (2) symbolic, (3) language, (4) pictorial, and (5) realistic (real-world, or experienced context). Hence, the criteria used as guiding provision for the use of multiple representations in the mathematics syllabi for primary (3 and 6) and mathematics textbooks for primary (3 and 6) were (1) evidence of varying manipulative (concrete, hands-on models), (2) symbolic, (3) language, (4) pictorial, and (5) realistic (real-world, or experienced contexts). In this study syllabus and textbook were reviewed to look out for evidence of activities that depict multiple representations. For instance, in the syllabus, researcher sought out the specific objectives and the teaching and learning activities that portray multiple representations that were related to the learning objectives. The Pupils" textbooks were also examined to look out for number of sub-topics and its related teaching and learning activities that portrays multiple representations. The analysis gave the researcher the opportunity to know the extent to which the curriculum documents make provision for the practice of multiple representations in the classroom.

3.8 Establishing Trustworthiness

Morse, Barrett, Mayan, Olson, and Spiers (2008), assert that trustworthiness "without rigor, research is worthless, becomes fiction, and loses its utility" (p. 14). Oduro (2015), concurs that qualitative data production, analysis and interpretation are sometimes viewed as lacking in rigour due to the usually small samples utilised, which are normally not considered to be representative. The potential for bias lies in the fact that the "researcher brings a construction of reality to the research situation which interacts with other people"s constructions or interpretation of the phenomenon being studied" (Merriam, 1998, pp. 22 - 23). In any qualitative research project, four issues of trustworthiness that demand attention; include credibility, transferability, dependability, and confirmability (Morse, Barret, Mayan, Olson, & Spiers, 2008; Trochim, 2008). These terms have gained prominence in qualitative enquiry and have become the mainstay for evaluation of the overall significance, relevance, impact, and utility of completed research as elaborated below:

3.8.1 Credibility

According to De Vos, Delport, Fouche, and Strydom, (2005), credibility refers to the research being conducted accurately. Oduro (2015), also defined credibility as internal coherence of the data in relation to the findings, interpretations, and recommendations. Establishing credibility in any qualitative research is credible from the perspective of the participant. From this context the purpose of qualitative research is to describe or understand the phenomena of interest from the participants" point of view. The researcher made the teachers in the study to check the accuracy of facts. This crosschecking process was necessary to maintain reflexivity by encouraging self-awareness and self-correction. Apart from the peer reviews, some participants were asked to confirm the accuracy of my observations. They were also asked to comment on whether my interpretations were meaningful to them. This process provided participant validation of the findings.

Furthermore, researcher had frequent debriefing sessions with my supervisors on all the tools developed namely observation protocol and interview guides and in developing my ideas, emerging analysis and interpretations. According to Flick (2004), the term triangulation of data refers to combining data from different sources. This data may be collected from different places, at different times and with different people. Although the data may come from these different sources, the data can be verified against each other, thus leading to a process of triangulation. Data collected from the participants were verified. Transcripts of the interviews, observation guide, as well as the draft report were also made available to the participants for verification. They were allowed to offer comments on whether or not they feel the data were interpreted in a manner congruent with their own experiences. Gaining feedback on results from the participants increases credibility.

3.8.2 Transferability

Transferability is an "alternative to external validity or generalizability" (De Vos et al., 2005, p. 346). According to Oduro (2015), transferability refers to external validity or the extent to which findings from one study are applicable to another context. In the qualitative research paradigm, external validity is not a priority. Furthermore, generalisation is not a priority in case study methodology. According to Starman (2013), case study is the idea of representative sampling and statistical generalisations to a wider population should be rejected, and analytical induction should be chosen instead. However, others believe that case studies may be used to make generalizations, and the case study allows the researcher to recognize the similarities of the objects and issues in different contexts and by understanding the changes as they happen (Starman, 2013, p. 39). In the context of my study presented findings with „thick" descriptions of mathematics teachers" conceptions and practices multiple representations. I elucidated all the research process from data collection, context of the study to production of the final report. This richness of detail is to enable the reader to determine a judgment regarding transfer from researcher"s study to theirs.

3.8.3 Dependability

According to Cohen, Manion and Morrison (2007), dependability refers to the "process of identifying acceptable process of conducting the enquiry so that the results are consistent with the data" (p.120). It is "fulfilled if the research process is subject to triangulation, member checks and respondent validates" the data (p.120). As has already been stated above, during the process of data generation, aspects such as triangulation, member checks with the respondents and respondent validation was carried out on an on-going basis as a means to ensure the validity of the instrument.

3.8.4 Confirmability

Confirmability, according to De Vos et al. (2005), "replaces the traditional concept of objectivity, where the influence of the researcher is removed and the data itself is examined as objective" (p. 347). In qualitative studies there cannot be a situation of total objectivity as the researcher is integrated in the research context. However, in this study, efforts were made to remove situations of observer bias that influence participant responses. The researcher allowed respondents to express their views about the subject matter without any influence. Researcher also tried to be bias free by reporting using the exact wordings of the respondents to ensure confirmability.

3.9 Data Analysis

Analysing results of a case study tends to be more opinion based than statistical methods. The usual idea is to try and collate data into a manageable form and construct a narrative around it (Shuttleworth, 2008).

Data were collected from interviews, observations and analyses of documents to answer the research questions in the study. As stated earlier, the case study design employed qualitative and quantitative approaches in the research. These instruments yielded both qualitative and quantitative data. The document analysis yielded quantitative data.

Data analysis is the ordering and breaking down of data into constituent parts and performing of statistical calculations with the raw data to provide answers to the research questions which guide the research. Data analysis therefore refers to the systematic organization and synthesis of research data, and the testing of research hypotheses (Burns & Grove, 2003). The quantitative data was analysed using descriptive statistics. These documents were individually analysed using a coding criteria developed by the researcher to collect data on activities, words, statements, and phrases in the curriculum materials that reflected multiple representations to teaching mathematics. The three coded results were compared for consensus. Analysis of documents such as syllabi and pupils text books were calculated in percentages and the presentation and interpretation were done to reflect quantitative analysis.

The interview data was analysed using inductive thematic approach. Inductive thematic approach is the „bottom up" approach (Frith & Gleeson, 2004), in which themes identified are strongly linked to the data. The analysis began with the transcription of the tape-recorded data into text. This text was read to identify the emerging issues that were coded using open coding. This was to ensure that the codes are directly and strongly linked to the data set. Additionally, these codes were organised to come out with lower level themes in a more coherent manner and align similar ideas into corresponding themes, axial coding was chosen. This was because using axial coding ensured the clustering of emerging ideas into a coherent unit, allowing the themes to stand out. This, it is argued, will engender a deeper understanding of the research issues (Frith & Gleeson, 2004).

3.10 Ethical Consideration

Permission was sought from the municipal director of education to use Primary 3 and 6 Mathematics Teachers Classroom. Headteacher of schools involved were contacted with official letters of introduction from the University of Education, Winneba for their permission (appendix E). Participants consent were also be sought before conducting the interviews and observations. The researcher made participants aware that the purpose of the study is purely academic. The researcher also assured participants of anonymity and that the data collected would be treated with confidentiality. Assurance was given to participants that their participation in the study will be voluntary, and that they can withdraw from the study without any consequence.

3.11 Summary of Methodology

The study was conducted on primary school mathematics teachers of Lower Manya Municipality in the Eastern Region of Ghana. The research design employed by the researcher was Case Study. This discusses the design and the justification for the design of the study. It also described the sample and sampling technique used in drawing the sample, instruments and how they were validated and used for data collection. Ethical issues for the research were also added.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Overview

This chapter presents and discusses the results of the study. It covers an introduction to the chapter, the demographic characteristics of the respondents, the descriptive data analyses for each of the four research questions and discussion and an overall summary of the research findings.

4.1 Introduction

The purpose of the study was to investigate primary school teachers‟ conceptions and practices of multiple representations in teaching Mathematics in the Lower Manya Municipality in the Eastern Region of Ghana. Specifically, the study sought to: identify the curriculum guidelines for multiple representations in teaching mathematics. It also explored primary school teachers‟ conceptions of multiple representations in teaching mathematics; their use of multiple representations in the mathematics classroom, and their conceptions of the advantages and disadvantages of multiple representations in primary mathematics. The following research questions guided the study:

1. What are the curriculum guidelines on multiple representations in the Primary School Mathematics Curriculum?
2. What are primary school teachers‟ conceptions of multiple representation in teaching mathematics?
3. What are the advantages and disadvantages of multiple representations in primary mathematics?
4. How do primary school teachers practice multiple representations in the teaching and learning of mathematics in the classroom?

Data were collected using three instruments namely: interview guide, observation and document analysis. The data collected were analysed. The results of the data analysis are presented in this chapter. The results from the three instruments are presented to reflect to answer the research questions in three parts: first results from interview; second the results from observation and third the results from the documents analysis.

4.2 Demographic Characteristics of the Respondents

One-on-one semi-structured guided interview was used to collect qualitative data on public basic school mathematics teachers conception and practices of multiple representations in the Lower Manya Municipality in the Eastern Region of Ghana. Five (5) primary school mathematics teachers were interviewed for the study, using three stages of sampling, namely, cluster sampling, simple random sampling and lastly, purposive sampling techniques. At the beginning of each interview, an item on the interview instruments sought participants" demographic information. The demographic characteristics of the respondents centred on their gender, professional status, academic qualification, teaching experience and class. The demographic data was collected to determine the professional qualification and experience of the respondents, so as to offer an indication of their professional content knowledge and expertise for teaching primary mathematics. This would also give the reader a clearer picture of the kind of respondents used for this study as presented in Table 4.1.

Participants were identified by T1, T2, T3, T4, and T5 for anonymity, where T represent Teacher.

4.3 Brief Description of Participants

Table 4.1: Characteristics of Research Participants

Code	Gender	Professional Teacher	Academic Qualification	Teaching Experience (years)	Class
T1	M	Yes	Post-diploma	6	6
T2	M	Yes	Diploma	16	6
T3	M	Yes	Diploma	7	3
T4	M	Yes	Diploma	20	3
T5	F	Yes	Diploma	32	3

Teacher (T1) was a male teacher. Started teaching in 2012, after completing Teacher Training College. So far, he had taught for six (6), and happens to be the youngest teacher in this study. He has some experience from college, coupled with his relatively short stay in the profession and hopes to gained more. After college, he applied for further studies at the University of Education, Winneba. After two years, he was awarded a Post Diploma Certificate in Basic Education, which is his highest certificate.

Teacher (T2) attended Teacher Training College from 1999 to 2002. Highest certificate is Diploma in Basic Education. T2 has 16 years teaching experienced in the service.

Teacher (T3) enrolled in Untrained Teachers Diploma in Basic Education (UTDBE) programme for 4 years and was honoured with a Diploma in Basic Education Certificate in 2011. T3 has 7 years teaching experience as a trained teacher in the service.

Teacher (T4) is the second oldest in this research study. He has been in the service as a pupil teacher since 1998. He started as an untrained teacher and later enrolled in a distance education programme in 2012 for 4 years. After completion, he was awarded a Diploma in Basic Education Certificate. He has been in the service for 20 years.

Teacher (T5) enrolled at Teacher Training College Form 4 at Amedeka for 4 years and came out with Certificate. She then enrolled in a distance programme to pursue an educational programme at University of Cape Coast (2005-2008), after which she was awarded a Diploma in Basic Education. She ended her professional qualification at this level to make way for her kids to rise academically as well. She has been teaching for 32 years now and has gained so much experience.

As shown in Table 4.1, all participants are professional teachers (one bachelor degree holder and four diploma holders). Their teaching experiences ranged from six years to thirty-two years. Judging by their professional qualification and experience, they should all have had the requisite professional content knowledge and expertise for teaching primary mathematics.

4.3.1 Document Analysis

Research Question One: What are the curriculum guidelines for multiple representations in the Primary Mathematics Curriculum?

To respond to the first research question, the researcher analysed the mathematics curriculum documents to determine the provisions that guide the use of Multiple Representations in teaching mathematics in the primary mathematics curriculum. The primary mathematics syllabus for primary Class 3 and 6 and the Mathematics Textbooks for the two classes were assessed for contextual, visual, virtual, physical, realistic, pictorial and symbolic representations as evidence of text guiding multiple representations in the materials. According to Park (2013), multiple representations is achieved, when one uses two or more of these representations; (1) manipulatives (concrete, hands-on models), (2) symbolic, (3) language, (4) pictorial, and (5) realistic (real-world, or experienced contexts). Hence, the criteria used as guiding provision for the use of multiple representations in the mathematics syllabi for primary (3 and 6) and mathematics textbooks primary (3 and 6) were (1) evidence of varying manipulative (concrete, hands-on models), (2) symbolic, (3) language, (4) pictorial, and (5) realistic (real-world, or experienced contexts). NCTM (2014), describes their criteria of multiple representations as contextual, visual, verbal, physical, and symbolic.

For the implementation of the syllabus to be effective, alternative strategies, illustrations, and activities that guide the implementation process must be provided. Using Park (2013) criteria, Primary three (P3) and Primary six (P6) mathematics documents were examined to find out whether the mathematics syllabus and textbooks provided for multiple representations in their strategies, activities, conceptual illustrations and others. The results of the curriculum provisions for multiple representations in the P3 Syllabus specific objectives and the sub-topics in the Primary three mathematics textbook in use are presented in Table 4.2

Table 4.2: Multiple Representation (MR) in P3 Mathematics Syllabus and Textbook

Main Topics	Syllabus: Number of Objectives in Percentages	Syllabus: Number of Objectives with MR in Percentages	Textbook: Number of Sub-Topics in Percentages	Textbook: Number of Sub-Topics with MR in Percentages
Number and numerals	5 (10.2%)	4 (80%)	6 (13.3%)	6 (100%)
Addition and subtraction	5 (10.2%)	1 (20%)	5 (11.1%	3 (60%)
Length and area	3 (6.1%)	2 (67%)	3 (6.7%)	3 (100%)
Fraction I	5 (10. 2%)	5 (100%)	5 (11.1%)	5 (100%)
Coll. and handling data	2 (4.1%)	2 (100%)	2 (4.5%)	2 (100%)
Capacity and mass	4 (8.2%)	4 (100%)	3 (6.7%)	3 (100%)
Multiplication	7 (14.3%)	6 (86%)	6 (13.3%)	6 (100%)
Division	5 (10.2%)	2 (40%)	4 (8.9%)	2 (50%)
Shapes	5 (10.2%)	2 (40%)	4 (8.9%)	4 (100%)
Time and money	5 (10.2%)	2 (40%)	5 (11.1%)	5 (100%)
Fraction II	3 (6.1%)	2 (67%)	2 (4.5%)	2 (100%)
Total	49 (100.0%)	32 (65.31%)	45 (100.0%)	41 (91.11%)

Table 4.2 indicates that out of 49 specific objectives identified from the 11 main topics in the P3 mathematics syllabus, as many as 32 (65.31%) objectives were presented in more than one way. Only 17 (34.69%) objectives in the syllabus were not presented with multiple representations. This means that most topics provided direction to the teacher to engage in multiple representations. For example, a main topic, Fraction I, that has five (5) objectives has all five (5) objectives represented in multiple ways, based on its teaching and learning activities. The first objective in relation to teaching and learning activities provided two manipulative materials for representations (paper folding, Cuisenaire rods) and a pictorial representation (fraction charts) as stated.

> "*Lead pupils to use paper folding, fraction charts, Cuisenaire rods etc. to guide pupils to identify one out of eight equal parts as one-eighth*", and language representations "*Guide pupils to divide two or more wholes (up to five wholes) to find the number of eighths in two or more wholes*"

The second objective uses manipulative (materials) and symbolic (written numbers or letters) respectively as described.

> *"Guide pupils to use materials to illustrate one-eighth, write the symbol for one-eighth"*, *"Guide pupils to find that the 8 (denominator) in 1/8 represents the number of divisions of the whole and the 1 (numerator) represents the number of parts under consideration"*.

The third objective uses manipulative (paper folding) and pictorial (drawing number line) respectively

> *"Assist pupils to use paper folding and shading and let pupils identify multiplies of half, fourth and eighth and write their symbols"*, *"Guide pupils to locate multiples of half, fourth and eighth on the number line"*.

The fourth objective uses manipulative (countable objects), and language representations respectively

> *"Guide pupils to use materials to divide a whole into three equal parts, identify one-third and write its symbol", Guide pupils to divide objects into six equal parts and identify one part as one-sixth and write its symbol"*.

The last fraction objective uses manipulative representations (paper folding, Cuisenaire rod) and pictorial (number line, fraction chart) representations respectively

> *"Guide pupils to compare fractions with the same denominator (not greater than 8), using (i) paper folding (ii) Cuisenaire rods (iii) the number line (iv) fraction chart"*

The examples are all indicators of multiple representations since they involve two or more ways of teaching the same thing. In the syllabus, the topics that had the highest (100%) use of multiple representations are Fraction I, Collecting and Handling data and Capacity and Mass. The topics with the next highest (67% – 86%), use of multiple representation are Length and Area (67%), Fraction II (67%), Numbers and Numerals (80%) and Multiplication (86%). The data suggest there is sufficient provision in the P3 mathematics syllabus to encourage teachers to apply multiple representations in their mathematics lessons.

Table 4.2 also indicates that out of 45 sub-topics identified from the 11 main topics in the P3 mathematics textbook, as many as 41 sub-topics (91.11%) were presented in more than two ways. Only 4 sub-topics (8.89%) in the textbook were not presented with multiple representations. This means that majority of topics provided direction to the teacher to engage in multiple representations. For example, Number and Numerals has six (6) sub-topics all represented in multiple ways based on its teaching and learning activities. With the first sub-topic (Ones and tens) of Number and Numerals, teaching and learning activities were represented in pictorial "*cubes*" and symbolic representation "*10 ones = 1 ten*". The second sub-topic (Thousand as a unit) was represented in pictorial "*flat*" and symbolic representation "*10 hundreds = 1thousand"*. The third sub-topic (Place value) was represented in pictorial " *Place value chart* " and symbolic representations "*Arrange 3589 in place value chart*" . The fourth sub-topic (Numbers up to 10,000) was represented in manipulative or pictorial "*Abacus*", language representations "*Counting in thousands on the abacus*" and symbolic representations "*The number 4236 can be written as, 4 thousands + 2 hundreds + 3 tens + 6 ones. This means 4000 + 200 + 30 + 6 = 4236*". The fifth sub-topic (Numerals on a Number Line) was represented in pictorials "*Number line*" and symbolic representations "*Multiples of 100, multiples of 1000*". The sixth and final sub-topic (Comparing and ordering of whole numbers) was represented in symbols "*compare using >, < or =*" and language representation "*Compare 4897 and 7269*". These are all indicators of multiple ways of representations since it involves two or more ways of teaching the same thing.

From Table 4.2, all the topics (Numbers and Numerals, Length and Area, Fraction I, Collecting and Handling Data, Capacity and Mass (weight), Multiplication, Plane Shapes, Time and Money and Fraction II) had 100% multiple representations, except two of them, namely, Addition and Subtraction and Division, that had 60% and 50% respectively. The data suggest there is sufficient provision in the P3 mathematics textbook to encourage teachers to engage in multiple representations in their instructional delivery.

Similarly, the results of the analysis of specific objectives of topics in the syllabus and sub topics in the textbooks with multiple representations in primary class six (P6) mathematics document are as presented in Table 4.3.

Table 4.3: Multiple Representations in P6 Mathematics Syllabus and Textbook

Main Topics	Syllabus		Textbook	
	Number of Objective in Percentages	**Number of Objectives with MR in Percentages**	**Number of sub-topics in Percentages**	**Sub-topics with MR in Percentages**
Set of number	6 (8.5%)	2 (33.3%)	6 (8.57%)	5 (83%)
Operations on fraction	8 (11.3%)	5 (63%)	8 (11.43%)	5 (63%)
Numbers and numerals	4 (5.6%)	3 (75%)	4 (5.72%)	3 (75%)
Addition and subtraction	3 (4.2%)	2 (67%)	3 (4.29%)	3 (100%)
Decimal fractions and percentages	8 (11.3%)	2 (25%)	8 (11.43%)	5 (63%)
Measurements of length, capacity and mass	7 (9.9%)	5 (71.4%)	6 (8.57%)	4 (67%)
Ratio and proportion	5 (7.1%)	3 (60%)	5 (7.14%)	1 (20%)
Shape and space	5(7.1%)	5 (100%)	5 (7.14%)	4 (80%
Collecting and handling data	6 (8.5%)	3 (50%)	6 (8.57%)	4 (67%)
Multiplication and division	7 (9.9%)	1 (14.3%)	7 (10.00%)	5 (72%)
Investigations with numbers	3 (4.2%)	1 (33.3%)	3 (4.27%)	3 (100%)
Measurement of area and volume	2 (2.8%)	2 (100%)	2 (2.86%)	1 (100%)
Money	2 (2.8%)	2 (100%)	2 (2.86%)	3 (100%)
Chance	3 (4.2%)	0 (00.0%)	3 (4.27%)	3 (100%)
The number plane	2 (2.8%)	2 (100%)	2 (2.86%)	2 (100%)
Total	**71(100.0%)**	**38 (53.52%)**	**70 (100.00%)**	**51 (72.86%)**

Table 4.3 indicates that more than half of the specific objectives of the syllabus were treated in multiple ways. Out of the 71 specific objectives in the P6 syllabus, 38 representing 53.52% had teaching and learning activities presented in more than one way. A total of 33 specific objectives representing 46.48% of specific objectives in the P6 mathematics syllabus were not presented in multiple representations. This suggests that most topics provide direction to compel or motivate teachers to use multiple representations. For example, the topic Money, that has two (2) objectives has both objectives represented in multiple ways based on its teaching and learning activities. The topic"s first objective uses manipulative and realistic representations

"*Create a class shop where everyday items can be bought and sold and which allows the children to measure length, capacity and mass and use money to buy items and receive change* " and language representations "*Let pupil find (i) The sum of the cost of 3 or 4 items. (ii) Changes in the transactions"*. The second objective uses manipulative and realistic representations "*Guide pupils to solve problems involving profit and loss using selling price (S.P.) and cost price (C.P.) create a corner shop for pupils to buy and sell"*. These are all indicators of multiple ways of representations since it entails two or more ways of teaching the same thing. In addition, Measurement of Area and Volume, Shape and Space, Money and Number Plane all had 100% of their objectives presented with multiple representations.

Also, topics such as Numbers and Collecting and Handling Data, Numerals from 0-10,000,000, Ratio and Proportion, Addition and subtraction (sums 0-9,999,999) , Measurements of Length, Capacity and Mass and Operations on fractions had 50% - 75% of their objectives presented in multiple representations. Few topics such as Sets of Numbers, Decimal Fractions and Percentages, Multiplication and Division and Investigations with Numbers had14.3% - 33.35 of their specific objectives presented in multiple representations. Only one topic, Chance, had none (0%) of its three specific objectives with multiple representations.

In summary, the syllabus has 38 out of 71 (53.52%) of its objectives presented using multiple representation. Though this figure represents more than half, it indicates that quite a significant portion (46.48%) still remains to be adequately presented using multiple representations. However, it may be said that the syllabus to some extent has made provisions to motivate or compel teachers to practice multiple representations.

Table 4.3 also indicates that out of 70 sub-topics identified from the 15 main topics in the P6 mathematics textbook, 51 (72.86%) sub-topics were presented in more than two ways. About one-quarter of the sub-topics (27.14%) in the textbook were not presented with multiple representations. This means that majority of topics provided direction to the teacher to engage in multiple representations. For example, Addition and subtraction has three (3) sub-topics all represented in multiple ways based on its teaching and learning activities. For the first sub-topic (Adding 6 –and 7- digit numbers) of addition and subtraction, teaching activities was represented in symbols "*36 + 42 = 78*", language representation "*Find the sum of the following. Addends = sum*" and pictorial representation "*value chart*". The second sub-topic (Subtracting from 6- and 7- digit numbers) was represented in pictorial "*subtracting using the place value chart or abacus*" and language representation "*Find the difference using place value chart*". The third and final sub-topic (Word problems involving addition and subtraction) was represented in symbolic as well as language representation "*The population of Odikro district is 218, 926. There are 7, 489 people who do not attend school. How many people attend school?* ".

These are all indicators of multiple ways of representations since it borders on two or more ways of teaching the same thing. In summary, there were six (6) main topics, namely, Addition and Subtraction, Investigation with Numbers, Measurement of Area and Volume, Money, Chance and Number Plane, which had 100% multiple representation.

Topics such as Sets of numbers, Operations on fractions, Numbers and numerals, Decimal Fractions and Percentages, Measurement of Length, Capacity and Mass, Shape and space, Collecting and handling data, Multiplication and division had percentages ranging from 63% to 83% of objectives presented using multiple representations.

Lastly, Ratio and proportion, with five sub-topics, captured only one objective with multiple representations (20%). The data indicates that the textbook has made, to some extent, provision to motivate or compel teachers to engage in multiple representation practices.

4.3.2 Results of the Observation

Research Question Two: How do primary school teachers practice multiple representations in teaching mathematics?

In this study, observation was necessary because there was the need to see and understand how mathematics teachers teach multiple representations in real classroom setting. The five primary school mathematics teachers who were interviewed were also observed when they were having mathematics lessons. The classes, topics taught and specific objectives of the observed lessons are shown in Table 4.4.

Table 4.4: Brief Description of Lessons Taught

Teacher	Class	Topic	Specific Objective
T1	P6	Division	The pupil will be able to divide a 3-digit number by 1-digit number
T2	P6	Multiplication	The pupil will be able to multiply a 4-digit number by a 2-digit number
T3	P3	Division	The pupil will be able to illustrate division as making equal groups.
T4	P3	Division	The pupil will be able to illustrate division as making equal groups.
T5	P3	Division	The pupil will be able to illustrate division as making equal groups.

The observation was guided by an observation checklist (see Appendix B) that had 10 classroom instructional practices. The researcher observed and ticked any multiple representation indicator practised by the teacher in the lesson delivery process. The matrix of instructional practices used by teachers is as shown in Table 4.5.

Table 4.5: Observation of Teachers' Classroom Instructional Practices

Instructional Practices	T1	T2	T3	T4	T5
1. Topic/sub-topic can be taught through multiple representations.	Yes	Yes	Yes	Yes	Yes
2. Learners pre-informed of multiple representations in solving that problem.	No	Yes	No	No	No
3. Lead learners through only one representation in solving the mathematical problems.	Yes	Yes	Yes	Yes	Yes
4. Ask pupils to explore other forms of representations to solving the mathematical problems.	No	No	No	No	No
5. Lead learners through alternative multiple representations.	No	Yes	No	Yes	No
6. Assist pupils to give more examples of similar that can be solved using any of the multiple representations taught.	No	Yes	No	Yes	No
7. Create pupils" awareness on the need to know different multiple representations to solving problem.	No	Yes	No	No	No
8. Give learners the opportunity to solve problems using such	No	Yes	No	No	No
9. Motivate pupils to use the multiple representations identified.	No	No	No	No	No
10. Welcome all multiple representations used and award marks according to developed rubrics in exercise books.	None	Yes	None	None	None

Key

Yes : Instructional practice performed;

No: Instructional practice not performed; and

None: No evidence of performance of instructional practice.

Table 4.5 shows that two participants, T2 and T4, introduced learners to other approaches in presenting same lesson taught. Three participants, T1, T3 and T5 on the other hand, presented in only one representation. However, the materials within that domain were varied but without representing it in another way round. All the participants guided learners through a first approach but apart from T2 who pre-informed pupils that there are alternative ways to present the same calculation, the rest did not. Also, none encouraged their pupils to explore alternative approaches. However, T2 again created pupils‟ awareness on the need to know different approaches which would help them to verify their answers, but T1, T3, T4, and T5 could not met such criteria. Further, none motivated their pupils to use multiple representations.

Evidence from the classroom observations indicated that the conceptions teachers hold about multiple representations is hardly translated into practice. Generally, teachers‟ approach during their lesson delivery scarcely reflected their conceptions of multiple representations. Their multiple representation strategies or practices appeared accidentally as no conscious effort was made to aid learners in presenting problems in different ways. Although four (4) teachers taught Division, which has possibilities for using multiple representations, all of them treated the topic in only one way. The results generally suggest that teachers are not open to different presentations to mathematics problems (Bingolbali, 2011) and do not consciously motivate children, in whatever form, to explore alternatives in solving problems. This may be attributed to limited knowledge, skills and probably a mere lack of willingness of teachers to apply multiple representations.

Altogether, three research instruments (Document Analysis Sheet, Interview Guide and Observation Checklist) were used to examine the curriculum provisions for multiple representation, conceptions and practices of primary school teachers" use of multiple representations in the mathematics classroom in Lower Manya Municipality in the Eastern Region of Ghana.

From the analyses of the curriculum documents, it was found that concerning the provision for multiple representation, P3 syllabus has 32 out of 49 (65.31%) objectives whilst the P6 syllabus has 38 out of 71 (53.52%) objectives presented using multiple representation. The textbook analysis showed that P3 textbook has 41 out of 45 (91.11%) subtopics whereas the P6 textbook has 51 out of 70 (72.86%) subtopics presented using multiple representations. These findings suggest that sufficient provision has been made in the curriculum documents to guide teachers to use multiple representations in their mathematics lessons delivery.

The interview responses revealed teachers" varied interpretations and conceptions of multiple representation and their perceptions of its value. It further provided an insight into the challenges teachers experience in using multiple representations in the classroom. Generally, participants did seem to be aware of several multiple representations in mathematics teaching, although the term multiple representations were new to most of them.

Evidence from classroom observations also suggested that the conceptions teachers holds about multiple representations are scarcely translated into practice. Out of the five lessons observed, only one of them was adequately delivered using multiple representations.

Research Question Three – What are the advantages and disadvantages of using the multiple representations approach in primary mathematics classroom?

Research Question 3 focuses on how beneficial or not teachers perceive multiple representations to be. Do they see it as being very helpful, and therefore as learning approach to use all the time? Or on the other hand, do they think it is unnecessary and hence, should not be applied in the mathematics classroom? To answer this question, researcher asked the teachers whether they thought multiple representations are necessary in the classroom.

Responding to this question, all five respondents unanimously gave a definite positive response. They were all very emphatic that using multiple representations in the mathematics classroom is very necessary. T4 even went further to explain that if the teacher always uses only one way of teaching, some of the children may not understand certain concepts properly, so there is the need to vary the approach, so that a pupil who did not understand the first one, may get the second representation clearly. He added that even those *talented* ones who got the first approach, will understand it better when another representation is used. T5 also submitted that when a variety of methods are used, the children grasp the concepts better.

In order to better understand why they thought using multiple representations is necessary, researcher asked them how the use of multiple representations influenced their teaching of mathematics? Responding to the question, T1 submitted that it makes the teaching easy and also improves the understanding on the part of the pupils. As they interact with the materials, they also gain a lot of experience. *I think when you use it, there is positive outcome. Also, the success rate is higher when you use multiple representation in mathematics lessons*, he concluded. T2 on his part expressed that it helps him to present his lessons very well, so that at the end of the lesson, the pupils will be able to grasp what he intends to teach. It also guides him as a teacher, he added. He explained further that whenever, as a teacher, he teaches and the pupils understand whatever he is teaching very well, it builds confidence in the teacher. He believed that his success as a teacher is tied to the use of multiple representations. Since the children are introduced to a variety of activities, it makes them grasp the concepts better.

T3, however, did not seem to understand the question very well. Even after the researcher had intervened and explained the question further, he only still referred to how he transferred knowledge from other subjects to his mathematics lessons. He further expressed how this kind of method even helps him to prepare better and also, how he gets better participation from his pupils by employing this method.

It makes the teaching and learning process more effective, was the response of T4 to the same question. He stated further that it lets the pupils understand the lesson well and participate effectively as well. He also noted that whenever he uses „double representation‟, he realizes that the percentage of pupils who get stuck when working is much fewer. T5, responding to the question, expressed that using multiple representations makes her pupils understand her more, which in turn makes her happy at the end of the lesson, when her pupils have firmly grasped whatever she wanted to teach. She emphasized further that multiple representation is very good, because it makes the lesson or topic more meaningful to the pupils. From the various responses given, there is the strong indication that the use of multiple representations in their lessons has a positive influence on their teaching of mathematics.

Probing further to know their perceptions of how it benefits their pupils, I asked them whether they thought pupils benefit from the use of multiple representations or not, and why they thought so. T1 responded *Yes* to the above question and followed with the explanation that, *as pupils interact with these materials they don"tforget as they move ahead to the next class or even the next topic*. He also added that, it makes introducing a new topic easy. *Multiple representations make children involved in the learning activities*, he concluded.

Similarly, T2 said *Yes* and also subscribed to the positive effect of multiple representations on pupils. He stressed that they benefit a lot, because when the teacher uses several activities, it builds up their interest and they participate more in the lesson; and for that matter, the teacher is able to achieve his objectives for the lesson. T3, on the other hand, did not seem to understand this question at all. He said,

> *Ooh…100%, if you use it wisely… It should not be when starting the topic otherwise if you don't take time you will confuse them all, if you go to that subject directly...As I stated earlier, if I bring the multiplication method directly, they will become confused. So you will start doing the division before…*

Although he started by saying that the pupils benefit „100%" his further explanation as given above does not seem to address the substance of the question. From his submissions, it can easily be deduced that, despite the repeated interventions of the researcher, he still maintains his original conception of multiple representation as referring to multiplication. Hence, his confusion concerning all related questions under the term multiple representations. In his response to the question, T4 said, *Yes, because they get much understanding from it. Double representations help them learn to their understanding*. His response suggests that he believes the pupils benefit by gaining better understanding of concepts when he uses multiple representations.

T5 also gave a *Yes* response to the question. She was of the view that, pupils indeed benefit from the use of multiple representations, because for those who do not understand, she comes down to their level through the use of multiple representations. This, she said, makes even the weaker ones understand most of the things she teaches. She explained further, that she usually does not rely on the responses she gets from pupils when asked *Do you understand?* She rather assesses them through their classroom work as well as their facial expressions. So whenever she discovers anyone who is struggling, she comes down to their level to help them individually.

In order to establish their perceptions of the general benefits or otherwise of using multiple representation, I asked them what they thought are some advantages of using multiple representations.

It makes the class lively and develops pupils" interest in the lesson, T1 replied. He continued by suggesting that it also clears some doubt in some of these topics. He explained by saying that *when you teach without varying it presupposes it is not „normal", but when you chip in variety of means to clear those difficulties, pupils „go through" for better understanding*. T2, in his view, believed that it arouses the interest of the pupils and makes them participate fully in the lesson. He added that they understand the lesson better when one uses multiple representations. He also suggested that it also eliminates boredom. Responding to the same question, T3 opined that the main advantage of using multiple representation is that, it makes the pupils understand the topic better, especially when they have already understood the concept from a different subject. *…it lets them get the thing fast, fast*, he stressed.
On his part, T4 commented that, it makes teaching and learning to be practical and also helps the pupils to use the materials on their own.

T5 suggested that there are many advantages. However, after a brief lecture on how to apply the child-centred approach, she finally gave only two advantages. First, it makes pupils not quickly forget what they learn. Second, it makes the pupils understand better, whatever they learn. She further condemned rote learning, saying it does not make the pupils genuinely understand what they are learning. Having established the fact that they perceive multiple representations as beneficial, I now sought to find out whether they thought there are any disadvantages in using it, and if yes, why they thought so.

All of the respondents asserted that though using multiple representations is highly advantageous and to an extent, even necessary, there could be a few disadvantages of using it.

T1 gave an elaborate response to the question saying:

> *Yes, at times it takes a lot of time and causes the lesson to extend to other periods as well because of how the lesson is packed. Also, it sometimes makes the work complex, especially when the concept is not clear to pupils. For instance, if you are adding fractions, you may want pupils to shade, colour and fold, but sometimes, when you take them through the formula, it sticks alright. So at times, during exercises or even, in exams, they ask questions such as „whatdo we do, are we to draw and shade or use the formula? However, the kind of education system we have, most times we don"tdraw and shade. In the exams we have to use formula. So some of them begin to wonder why we used them at all.*

In the opinion of T2, one disadvantage could be the situation where the teacher may not have some of the materials that may be needed for the multiple representations. For instance, the case where the teacher wants to make a presentation using a computer and there are no computers to use. In such a situation, the teacher would obviously be handicapped.

In T3‟s view, he believes that sometimes, it makes the children not understand the teacher at all, so it really depends on how it is introduced or brought into the lesson. He stressed that there are many disadvantages, though he failed to specify further.

Yes, some of the children that are not fast enough in the learning, when you use the double representation, they get confused. Few pupils are able to embody several representations, suggested T4. He stated, however, that those who get confused are a very small minority, *say 2 out of 20 pupils*. He concluded by submitting that despite the disadvantages, the advantages far outweigh the disadvantages.

T5 was of the opinion that the multiple representations should not be overused. She explained further that sometimes, using the multiple representations amounts to „spoon-feeding‟ the pupils. She suggested that at times, there is the need to allow the pupils to think and come out with the answers for themselves. *One thing we have to realize is that these pupils have good things in them and the responsibility is on us as teachers to unearth such good things in them*, she concluded.

4.3.3 Results from Interviews

Teachers play a key role in the implementation of the mathematics curriculum. The decision to carry out or implement multiple representation problem-solving in the mathematics curriculum relies on the teacher‟s conception of what multiple representations is all about.

Research Question Four: What are primary school teachers' conceptions of multiple representations in teaching mathematics?

To answer this research question, teachers were asked, what they understood by multiple representations in teaching mathematics? Responding to this question, T1 said that it can be referred to as *multiple embodiment, where one uses different formulae of the same procedure to arrive at a solution*.

T2 simply put it that, *I'm not getting it*. He readily admitted his lack of understanding of the term and requested the help of the researcher, which request was granted. After the explanation, he submitted that he knows the concept, but just that the term sounded new.

T3 gave an elaborate response to the question saying,

> *I think like ... adding the things in order to get a particular answer. For instance, maybe you were given something and you want the answer. How are you going to get the answer exactly? You are supposed to use multiplication to get the actual answer. Maybe the things are many so counting 1, 2, 3, 4, 5, 6,... may waste your time. You may arrange the things in vertical and horizontal rows, then you multiply them. So using multiples, I can say is a very quick method for getting answers in a short time.*

Clearly, T3 had the wrong conception of multiple representations. The researcher then intervened with an explanation of multiple representations as varying the teaching activities to meet the differentiated needs of the pupils.

Just like T2, T4 and T5 were also not able to offer any specific definition or explanation of the term. However, upon the intervention of the researcher, they all claimed to have knowledge of the concept and even submitted that it is very important and has to be used as much as possible for pupils to get a better understanding of the lesson.

For instance, T5 elaborated on the concept with the following explanation:

> *... in the classroom, there are individual differences. What a child will grasp in a minute, another child will not be able to, because of varying differences in backgrounds where they come from. I'm sure you even witnessed some this morning. For some children, their parents teach them at home. In as much as possible, you have to come down to their level and even re-group them into ability groups. Therefore I give out work to them according to their ability groups and spend much time with the low ability groups. By so doing, I'm able to achieve my aim.*

The responses suggest that, apart from T1, all the respondents were not very familiar with the term multiple representations. Though these teachers had the concept of multiple representations, the term appeared to be new to them, until after the researcher"s intervention.

Probing further to find out the mode of application of the concept, the teachers were also asked how they used multiple representations in their lessons. In responding to this question T1 submitted that, supposing the lesson were to be on Addition or Subtraction, there are teaching and learning materials that go with them. So when one is teaching Addition, one could use bundles of sticks, bottle-tops (counters), Dienes blocks, place value chart, abacus and others. He further explained that, one has to bring these materials into the classroom, and allow the children to manipulate them if necessary so that they also follow the procedure the teacher demonstrates with them. He added that, after introducing the lesson, the pupils should be made to know that there are multiple ways of teaching the topic, after which the teacher would have to take them through one after the other.

T2 said that he uses the concept all the time. He claimed that he mostly uses real-life situations, which usually make the pupils understand the lesson very well, since it is within their setting, and have seen most of these things. He opined that the pupils understand the concepts best when it is related to their background. The responses from T1 and T2 indicate that they have a very good conception of multiple representations and they try to implement it in their lessons.

T3‟s response to the question is as follows:

> *Okay, in some of the topics, multiples will be good. A topic like division, for example, when asked to share, say, mangoes, sometimes they can say that since I am much older, I should take more. So you see, that will bring confusion. So when you use the multiple, it is sometimes easier. You take the number of pupils and you multiply it to the number of object given. So when you get it you know the particular answers that you are supposes to get. For instance, if there are 21 oranges, how many of the pupils are to share before they get 7? This thing is very difficult for the children to understand, so we use the multiplication method. So we say start saying the multiplication of the 7 until we get to the number of oranges given. Thus, 7 x 7 = 7, 7 x 2 = 14, 7 x 3 = 21. It means three pupils divide the 21 before one person gets 7. So the multiplication is very necessary but it is not for all the topics. It is not all the time.*

It is obvious from T3‟s detailed response that even after the intervention of the researcher concerning the meaning of the term, he still held on to his erroneous conception of multiple representations as implying using multiplication to solve problems. Though his submission indicates that he uses the concept of multiple representations, he somehow was unable to conceptualize the term correctly.

According to T4, when teaching a lesson on division for example, he uses diagrams depicting pupils sharing and also uses chalkboard illustration representing items pupils are sharing to arrive at the answer. Also, he uses bottle-tops (counters) and asks pupils to count and group according to dividend and divisor to arrive at the quotient.

T5, in response to the question, said she uses pebbles, bottle tops, etc, from pupils. Also, she puts them into groups according to their ability levels so that they can do different levels of work in the different groups. In so doing, she devotes enough time to the less ability group. She submitted that that is what she does to achieve the objectives of her lesson. When asked whether she uses it only when treating a new topic, she replied in the negative, and said she always uses that approach in all her lessons, especially for the lower ability groups.

From the responses of all the respondents, there is an indication that they all apply the principle of multiple representations in one way or the other, even though all of them, except T1, did not really understand the term multiple representation. This suggests that they practice it without knowing. In order to ascertain their understanding and mode of practice of multiple representations in their lesson delivery, the researcher asked the teachers to give some examples of multiple representation s that they use?

T1 gave the following examples with respect to teaching fractions: paper folding, shading and Cuisenaire rods. In counting and addition, he suggested that one can use the abacus, bundles of sticks and counters. Responding to the same question, T2 said at times he brings in real life situations, such as songs in teaching Mathematics. He was unable to give any further examples when researcher probed for more, and rather requested that I help him with some. So the researcher intervened.

T3, on the same question, said he usually borrows ideas from other subjects such as RME and Science. For instance, there is a topic in RME that involves division, which the pupils had already done. So when treating a similar topic in Mathematics, such as Division, all he does is to transfer the knowledge they have already gained in RME and apply it in his mathematics lesson. Similarly, he claimed that there is a topic in Science that deals with addition. So when treating Addition in Mathematics, he simply transferred that knowledge or understanding which the children had already gained to the current topic in Mathematics. This approach, he stressed, helps him a lot to make the lesson interesting and easily understandable for the pupil. From his response, it is obvious T3 was unable to give any specific example of multiple representations in a lesson. He only succeeded in describing how he transfers knowledge gained in other subject areas to his lesson.

In his response to the question, T4 said that depending on the topic, he sometimes uses word cards and counters. Also, if treating a topic like plane shapes, he usually prepares some cut-outs to enable them to pick what they have in the textbook. He added that sometimes, he could also write the names on the chalkboard and display some cards (cut-outs) on the table. He would then call pupils to come and pick the cards (cut-outs) that correspond to a particular name on the chalkboard. T5"s examples of materials used for multiple representations are manila cards, pictures and supplementary books.

4.4 Discussion

The study explored primary school teachers" conceptions and practices of multiple representations approach in teaching mathematics. The assumption was that teachers" use or non-use of multiple representations depends on their own conceptions, knowledge capacity of the curriculum content and its provisions. The curriculum materials and teacher"s conceptions are the key components that guide the teacher"s classroom practices. They provide information on how best teachers carry out multiple representations.

A critical analysis of the curriculum materials revealed that, specifically, out of 49 specific objectives identified from the 11 main topics in the P3 mathematics syllabus, as many as 32 objectives (65.31%) were presented in more than one way. Only 17 objectives (34.69%) in the syllabus were not presented with multiple representations. In the same vein, more than half of the specific objectives of the P6 syllabus were treated in multiple ways. Out of the 71 specific objectives in the P6 syllabus, 38 objectives, representing 53.52%, had teaching and learning activities presented in more than one way. A total of 33 specific objectives, representing 46.48% of specific objectives in the P6 mathematics syllabus, were not presented in multiple representations. This means that majority of topics in both the P3 and P6 syllabi provided direction to the teacher to use multiple representations in their lessons.

Also, out of 45 sub-topics identified from the 11 main topics in the P3 mathematics textbook, as many as 41 (91.11%) sub-topics were presented in more than two ways. As less as 4 (8.89%) sub-topics, in the textbook were not presented with multiple representation. Further, out of 70 sub-topics identified from the 15 main topics in the P6 mathematics textbook, 51 (72.86%) sub-topics were presented in more than two ways. As few as 19 (27.14%) sub-topics in the textbook were not presented with multiple representations. This means that in both textbooks, majority of topics provided direction to the teacher to apply multiple representations in the mathematics classroom. These findings support earlier studies (Stabback, 2016; Nabie, Raheem, Agbemaka, & Sabtiwu, 2016) that the curriculum had made provision for multiplicity in content delivery to meet the differentiated Ghanaian pupils.

The results also showed that the concept of multiple representations was perceived differently by different teachers. Specifically, out of the five teachers, four had a clear conception of multiple representations as solving problems in different or multiple ways. Yet, only one of them exhibited their conception of multiple representation in the delivery of the observed lesson. This observation may be as a result of the experience they themselves may have had in their school days, which is consistent with the view of Perkkila (2003) finding that the way teachers teach can be traced back to their school days, how they experienced the teaching and learning of the subject.

Enough provisions and guidelines to the use of multiple representations have been given in the mathematics curriculum documents and the teachers also perceive the use of the approach as advantageous. Multiple representations is seen as a means to respond to individual differences and an opportunity to provide for pupils to work in their comfort zones. Multiple representations aids in diagnosing children‟s strengths, weaknesses and their understanding of mathematics from different perspectives. Pupils also learn or develop alternatives strategies for solving problems.

A teacher who conceived multiple representation as using multiplication in teaching mathematics also saw its benefits as using multiplication for better understanding. The conception of multiple representation in terms of multiplication is a misconception. This suggests that even though some teachers seem to be aware of some benefits of multiple representations, the practice of the approach is limited because they probably lacked the relevant knowledge and skills for interpreting and enacting its practice. The results affirm the findings from earlier studies (Lloyd, 2002; Hill & Ball, 2009).

Evidence from classroom observations indicated that the conceptions teachers holds about multiple representations are hardly translated into practice. Out of the five teachers, majority of the teachers‟ ways of using multiple representation approach during their lesson delivery neither reflected their conceptions nor curiculum guidelines. Their multiple representations strategies or practices appeared to be a one-way approach since lessons were not differentiated to meet the individual pupils learning needs. Although four of the participants taught Division, none reflected multiple representations. The results generally suggest that teachers are not open to different solutions to mathematics problems and do not consciously motivate children, in whatever form, to explore alternatives in solving problems (Ampadu, 2012). This can be attributed to limited knowledge and skills of teachers on what it takes to teach through multiple representations.

CHAPTER FIVE

SUMMARY OF FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

5.0 Overview

This chapter presents a summary of the research findings, conclusions and recommendations of the study. Summary and conclusion involve significant issues established in the study while the recommendations are based on the findings.

5.1 Introduction

The study was to investigated primary school teachers‟ conceptions and practices of multiple representations in Lower Manya Municipality in the Eastern Region of Ghana. Specifically, the study sought to understand the curriculum guidelines on multiple representations on teaching mathematics, primary school teachers‟ conceptions of multiple representations, use of multiple representations for teaching mathematics and their perceived advantages and disadvantages of multiple representations in primary mathematics. The following overarching questions guided this study:

1. What are the curriculum guidelines on multiple representations in the Primary Mathematics Curriculum?
2. How do primary school teachers practice multiple representations in the teaching and learning of mathematics in the classroom?
3. What are the advantages and disadvantages of multiple representations in primary mathematics?
4. What are primary school teachers‟ conceptions of multiple representations in teaching mathematics?

The study adopted Lesh"s Multiple Representations Theory as the theoretical framework or foundation. Instrumental case study design was employed to address the purpose and research questions of the study. The target population for this study consisted of ninety-two (92) public primary school teachers in the Lower Manya Municipality. However, the accessible population of this research consisted of all primary three (3) and primary six (6) mathematics teachers in Lower Manya Municipality. A total of five (5) consented primary school mathematics teachers were selected from the sampled primary three (3) and primary six (6) classes for the study. The study further adopted; semi-structured interview guide, observation guide and document analysis to collect qualitative data. Descriptive statistics such as, percentages were employed to analyse aspects of the quantitative data in the documents analyses gathered from the syllabus and textbooks. The qualitative data on the other hand was analysed using thematic analysis.

What are Curriculum Guidelines on Multiple Representations in the Primary Mathematics Curriculum?

The results of the study indicate that the mathematics syllabus and textbooks have statements that reflect multiple representations. However, some of the statements lacked concrete direction and emphasis, and are insufficient to guide teachers to teach in multiple ways if teachers lack the mathematical lens to see beyond.

In the same vein, more than half of the specific objectives of the P6 syllabus were treated in multiple ways. Out of the 71 specific objectives in the P6 syllabus, 38 representing 53.52% had teaching and learning activities presented in more than one way. A total of 33 specific objectives representing 46.48% of specific objectives in the P6 mathematics syllabus were not presented in multiple representations. This suggests that most topics provide direction to compel or motivate teachers to use multiple representations. Further, out of 70 sub-topics identified from the 15 main topics in the P6 mathematics textbook, 51 (72.86%) sub-topics were presented in more than two ways. As few as 19 (27.14%) sub-topics in the textbook were not presented with multiple representations. This means that majority of topics provided direction to the teacher to engage in multiple representations. Ampadu (2012), affirmed the above assertion that, the new curriculum is truly underpinned by constructivism and its advocates for variety of means to reach the classroom child in the teaching-learning process. Nabie, Raheem, Agbemaka, and Sabtiwu (2016), concur with the above assertion that Constructivism is really a theoretical lens for Ghana"s curriculum design and implementation process. Though there are enough guidelines provided in the curriculum materials to implement multiple representations, some are vague for teachers to understand. Ampadu (2012), confirmed that teachers" perceptions of their teaching are influenced by the curriculum recommendations. The provisions in the curriculum documents are enough to back multiple representations to develop teachers Mathematical Knowledge for Teaching or Pedagogical Content Knowledge.

Primary School Teachers Practice of Multiple Representations in Teaching Mathematics in Lower Manya Municipality

In this study, observation was necessary because there was the need to see and understand how mathematics teachers teach multiple representations in real classroom setting. The five primary school mathematics teachers who were interviewed were also observed when they were having mathematics lessons.

According Perkkila (2003), mathematics teachers‟ perceptions of their classroom practices are consistent with their actual classroom practices. In this present study, the results showed that mathematics teachers‟ perceptions of their classroom practices were not wholly consistent with what they actually do. In conclusion, evidence from classroom observations indicated that the conceptions teachers holds about multiple representations are hardly translated into practice. Teachers‟ ways of using multiple representations approach during their lesson delivery neither reflected their conceptions of multiple representations. Their multiple representation strategies or practices appeared accidentally as no conscious effort was made to aid learners in presenting problems in different ways. Although four (4) teachers taught Division, which has possibilities for using multiple representations, all of them treated the topic in only one way. The results generally suggest that teachers are not open to different presentations to mathematics problems (Bingolbali, 2011) and do not consciously motivate children, in whatever form, to explore alternatives in solving problems. This may be attributed to limited knowledge, skills and probably a mere lack of willingness of teachers to apply multiple representations. In general, although most of the teachers professed that they used multiple representations in the classroom it found to be opposite.

Teachers' Perspectives of the Advantages and Disadvantages of Multiple Representations in Primary Mathematics in Lower Manya Municipality

The interview responses suggest that teachers conceptualized the use of the approach as highly advantageous as compared to the disadvantages. Multiple representations are seen as a means to respond to individual differences and an opportunity to make teaching easy and intents bring better understanding to pupils. Multiple representations aids in diagnosing pupil‟s strengths, weaknesses and their understanding of mathematics from different perspectives. Pupils also learn or develop alternatives strategies for solving problems. A teacher who conceived multiple representations as using multiplication in teaching mathematics also saw its benefits as using multiplication for better understanding. The conception of multiple representations in terms of multiplication is a misconception and lack of knowledge of the concept. This suggests that even though some teachers seem to be aware of some benefits of multiple representations, the practice of the approach is limited because they lacked the relevant knowledge and skills for interpreting and enacting its practice. The results affirm earlier studies (Nabie, Anamuah-Mensah & Ngwan-Wara, 2010), that some basic school teachers lack the knowledge capacity to teach.

What are Primary School Teachers' Conceptions of Multiple Representations in Teaching Mathematics in Lower Manya Municipality?

According to Ahmed and Aziz (2009), teachers‟ conceptions of their teaching is a key aspect that reflects their effectiveness of mathematics teaching and learning and it also reinforces teacher‟s decision making. Effective teaching is a function of teachers‟ knowledge. To explore teachers‟ conception of multiple representations, a few interview questions were asked under conception. These questions generated several views. Below are Summaries of the various responses are as follows:

The results showed that multiple representations was conceived differently by different teachers and some teachers have no idea of the concept. Specifically, T1 had a clear conception of multiple representations as solving problems in different or multiple ways, that is consistent with the view of (Leikin et al., 2007). A teacher viewed multiple representations as teaching for better understanding, a conception that reflects the ultimate goal of multiple representations. Another teacher misconceived multiple representations as using multiplication to solve a problem. This suggests that some teachers do not know what multiple representations is all about.

5.2 Conclusions

The main purpose of teaching and learning mathematics is to meet the varying learning styles of pupils to enable them solve mathematically related problems in the society. Therefore, the study sought to find out teachers way of thinking and what they do in the classroom whether there is any conformity in relation to differentiating mathematics teaching lessons to meet individual pupils" learning styles. Teachers teaching pupils in more than one way has prospects. The findings of this study suggest that sufficient provision has been made in the curriculum documents to guide teachers to use multiple representations in their mathematics lesson delivery.

Although some teachers in this study exhibited clear conceptions and knew the value of multiple representations in teaching, they hardly translated their conceptions and values into practice. Teachers do not make conscious effort to encourage multiple representations in their practice. The researcher concludes that although the curriculum document encourages the use of multiple representations for most of its specific objectives, its implementation is hindered by limited curriculum guidelines and teachers" know-how. If multiple representations is to be valued and practiced in the Ghanaian classroom, conscious efforts must be made to explicitly incorporate it in the curriculum documents to emphasise its use in the classroom and in teacher education. If teachers are well informed about multiple representations and have the skills to practice it, students would develop the mathematical skills and competencies for better understanding of mathematics concepts and solving of mathematical problems.

5.3 Recommendations

The findings of this study indicated that primary school Mathematics teachers in Lower Manya Municipality conceptions on multiple representations was less emphasize in their instructional practices. The study identified in a broader perspective the challenges of teachers not teaching multiple representations in mathematics to include inadequate teaching and learning materials, less time allocation to mathematics periods, inadequate knowledge of curricular materials. Based on these findings, the researcher recommends that:

1. Ghana Education Service in collaboration with the Ministry of Education and other stakeholders of Education should organise in-service programmes on multiple representations such as refresher course, short-term courses, workshops and seminars, so that primary school mathematics teachers will be well equipped with new skills and knowledge needed to assist learners construct their own understanding of topics. Furthermore, the stakeholders should ensure that primary school mathematics teachers are well equipped with resources for teaching and learning multiple representations. These resources should include teaching and learning materials such as manipulatives, pictorials, especially those that cannot be improvised. Provision of incentives and general improvement of condition of service of teachers will motivate teachers to do their best, since teaching mathematics using multiple representations has been identified as time consuming and as an approach demanding teacher resourcefulness.

2. The Teacher Training Universities and Colleges of Educations in Ghana should embark on training of primary school mathematics teachers on how to teach mathematics using multiple representations, that should include the various representations such as manipulative, symbolic, pictorial, language and realistic representations. The use of multiple representations could be incorporated into the teacher training programmes at both the diploma and degree levels to sensitise teachers‟ awareness about contemporary instructional practices.
3. Regular evaluation of the instructional process of primary school mathematics teachers by supervisors designated from within and outside the educational institution who are knowledgeable in multiple representations will ensure teachers‟ conformity to the use of this approach in the school setting. Additionally, teachers would be granted appropriate and timely assistance by these supervisors.
4. The Curriculum Research and Development Division of the Ghana Education Service should place much emphasis on the use of multiple representations through revision of the syllabus and textbooks, that will provide teachers with the ample time to teach mathematics using multiple representations.

5.4 Suggestions for Further Research

The results of the study have revealed primary school teachers conceptions and practices of multiple representations as less emphasis in instructional practices. The educational implications of the findings of this study calls for further investigation into the area of primary school Mathematics teachers‟ conceptions and practices. The following are recommended for further research:

1. It is suggested that a similar study be conducted in other districts or municipality in the Eastern Region and other region in Ghana. This would provide a basis for more generalisation of the conclusions to be arrived at about primary school Mathematics teachers.
2. Also, a study of how information and communication technology can promote the teaching of mathematics using multiple representation approach is recommended for further study.
3. Furthermore, studies be carried out to investigate factors that are hindering the effective implementation of multiple representations by Primary School mathematics teachers.
4. Finally, the Ghana Education Service and other stakeholders of education should take upon themselves to investigate teachers" prior school experiences, their on-going professional development, and their classroom practice.

REFERENCES

Abraham, D. (2006, November 02). Mathematical representations as conceptual composites: Implications for design. *In Proceedings of the Twenty-Eighhth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 464-466.

Abudu, A. M., & Fuseini, M. N. (2014). Influence of single parenting on pupils' academic achievement. *Educational Evaluation and Policy Analysis, 21*, 385-403.

Acquah, S. (2011). *Pre-service teachers' difficulties in learning geometric transformations and perception of factors inhibiting the development of their Mathematical knowledge for teaching: A Case Study of Two Colleges of Education* . Ghana: Unpublished master's theses. Department of Mathematics Education, UEW.

Aforklenu, D. K. (2013). *Junior high school students difficulties in solving word problems in Algebra in Tema Education Metropolis (PhD thesis).* Ghana: UEW.

Agyedu, G. O., Donkor, F. & Obeng, S. (2013). *Teach yourself research methods* . Amakom-Kumasi: Payless Publication Limited.

Ahmed, F. & Aziz, J. (2009). Students ' perception of the teachers' teachinh of literaure communicating and understanding through the eyes of the audience. *European Journal of Social Sciences, 7*(3), 17-26.

Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction, 16*(3), 183-198.

Ainsworth, S. (2008). The educational value of multiple -representations when learning complex scientific concepts. *In Visualization: Theory and Practice in Science Education*, 191-208.

Alhassan, S. (2006). *Modern approaches to research in educational administration for research students.* Kumasi: Payless Publications Ltd.

Ampadu, E. (2010). An exploratory study of Mathematics Teaching and Learning in Ghana. *Hearing Students Voices, Networks, 13*, 67-73.

Ampadu, E. (2012). Students' perceptions of their teachers teaching of Mathematics: The Case of Ghana. *International Online Journal of Educational Sciences, 4*(2).

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowlege of middle school mathematics teachers in China and the US. *Journal of Mathematics Teacher Education*, 145-172.

Anamuah-Mensah, J., Mireku, K. D., & Ampiah, J. G. (2009). *TIMSS 2007 Report: Findinds from IEA's TIMSS at Eighth Grade.* Accra: Adwinsa Publications (GH) Ltd.

Andale. S. (2015, June 26). *Probability sampling: definition, types, advantages, disadvantages*. Retrieved 14 03, 2018, from Statistics How To: htt://www.statisticshowto.com/probabability-sampling

Andrade, A. (2011). The clock project: gears as visual- tangible representations for mathematical concepts. *International Journal of Technology and Design Education, 21*(1), 93- 110.

Anku, S. E. (2008, April 17). *Revamp mathematics education in Ghana*. Retrieved 07 12, 2018, from Educational Journal: www.ghana.gov.gh/ghana/revamp mathematics education ghana

Ary, D., Jacobs, C, & Razavieh, A. (2002). *Introduction to research in education (6th Ed).* Belmont, CA: Wadsworth Publishing.

Assessment, N. E. (2016). *Finding report.* Accra: Assessment Service Unit.

Awanta, E. K., & Asiedu-Addo, S. K. (2008). *Essential statistical techniques in research for universities, colleges and research institutions.* Accra, Ghana: Salt and Light Publishers.

Ayub, A., Ghazali, M., & Othman, A. R. (2013). Preschool children's understanding of numbers from multiple representation perspective. *IOSR Journal of Humanities and Social Science,* (6).

Bala, P. (2015). Skills of using and transform multiple representations of the prospective teachers. *Procedia-Social and Behavioral Sciences, 197*, 582-588.

Ball, D. L., Thame, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special. *Journal of Teacher Education, 59*(5), 389-407.

Bartell, T., Wager, A., Edwards, A., Battey, D., Foote, M., & Spencer, J. (2017). Towards a framework for research linking equitable teaching with the standrds for mathematical practce. *Journal for Research in Mathematics Education, 48*(1), 7-21.

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A. Y., & Tsai, Y. M. (2010). Teachers' mathematical knowledge , cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*(1), 133-180.

Bayazit, I. (2011). Prospective teachers inclination to single representation and their development of the function concept. *Educational Research and Reviews, 6*(5), 436-446.

Begg, A. (2011). Mathematics 101: reconsidering the axioms. *International Journal of Mathematical Education in Science and Technology, 42*(7), 835-846.

Beswick, K., Callingham, R., & Watson, J. (2012). The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher Education, 15*(2), 131-157.

Bingolbali, E. (2011). Multiple solutions to problems in mathematics teaching: Do Teachers Really Value Them. *Australian Journal of Teacher Education, 36*(1), 18-31.

Blanton, M. L., & Kaput, J. J. (2003). Developing elementary teachers " Algebra eyes and ears". *Teaching Children Mathematics, 10*(2), 70.

Bogdan, R. C., & Biklen, S. K. (2007). *Qualitative research for education: An introduction to theories and methods.* Boston , MA: Laureate Education, Inc.

Boston, M., Dillon, F., Smith, M., & Miller, S. (2017). *Taking action: Implementing effective mathematics teaching practices in grade 9-12.* Wale: National Council of Teachers of Mathematics, Incorporated.

Bouck, E. C., & Flanagan, S. M. (2010). Virtual manipulatives: What they are and how teachers can use them. *Intervention in School and Clinic, 45*(3), 186-191.

Brown, G. T., & Harries, L. R. (2009). Unintended consequencesof using tests to improve learning: How improvement-oriented resources heighten conceptions of assessment as school accountability. *Journal of Multidisciplinary Evaluation, 6*(12), 68-91.

Brown, G. T., Lake, R., & Matters, G. (2011). Queensland teachers' conceptions of assessment: The impact of policy priorities on teacher attitudes. *Teaching and Teacher Education, 27*(1), 210-220.

Bruner, J. (1966). *Toward a theory of instruction* (Vol. 59). Massachusetts: Harvard University Press.

Bruner, J. S. (1960). On learning mathematics. *The Mathematics Teacher, 53*(8), 610-619.

Budaloo, V. R. (2015). *The use of visual reasoning by successful mathematics teachers: A study (PhD Thesis).* South Africa: University of KwaZulu-Natal, Edgewood. Retrieved from https://www.google.com.gh/search?q=Budaloo%2C+V.+R+(2015)&rlz=1C1VFKB

Burns, N., & Grove, S. K. (2003). *Understanding nursing research* (3rd ed.). Philadelphia: W.B Saunders Company.

Buschang, R. (2008). *Validating measures of mathematics teacher knowledge.* California, CA: Educational Research Association Annual Meeting, CERA, Rancho Mirage.

Cai, J. (2004). Why do USA and Chinese students think differently in mathematical problem solving? Impact of early algebra learning and teachers' beliefs. *The Journal of Mathematical Behavior, 23*(2), 153-167.

Cai, J. (2005). US and Chinese teachers' constructing, knowing and evaluating representations to teach mathematics. *Mathematical Thinkig and Learning, 2*, 135-169.

Cai, J. & Lester, J. F. (2005). Solution representations and pedagogical representations in Chinese and US classroom. *The Journal of Mathematical Behavior, 24*(3-4), 221-237.

Castillo, J. J. (2009). Convenience sampling. *Experiment resources*. Retrieved 01 07, 2018, from https://www.experiment-resources.com/convenience-sampling.html

Cathcart, W. G., Pothier, Y. M., Vance, J. H., & Bezuk, N. S. (2011). *Learning mathematics in elementary and middle schools: A learner- centered approach.* New Jersey: Peason Merrill Prentice Hall.

Chamberlin, S. A., & Moon, S. M. (2008). How does the problm based learning approach compare to the model- eliciting activity approach in mathematics. *International Journal for Mathematics Teaching and Learning, 9*(3), 78-105.

Charalambous, C. Y. (2010). Mathematical knowledge for teaching and task unfolding: An exploratory study. *The Elementary School Journal, 110*(3), 247-278.

Cheung, D., & Ng, P. H. (2000). Science teachers' beliefs about curriculum design. *Research in Science Education, 30*(4), 357-375.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought processes. *In Wittrock, M, C (Ed)*, 255-296.

Cleaves, W. P. (2008). Promoting mathematics accessibility through multiple representations jigsaws. *Mathematics Teaching in the Middle School, 13*(8), 446-452.

Clements, D. H. (2000). Concrete manipulatives , concrete ideas. *Contemporary Issues in Early Childhood, 1*(1), 45-60.

Cline, D. (2011). Logical structure, theoritical framework. *Centre for Excellence in Education*. Retrieved from http://education.astate.edu/dcline

Cohen, D. K. (1990). A revolution in one class: The case of Mrs. Oublier. *Educational Evaluation and Policy Analysis*, 311-329.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education.* London: Routledge.

Collins, A. (2011). Rpresentational competence: A commentary on the Greeno analysis of classroom practice. *In Theories of Learning and Studies of instructional Practice*, 105-111.

Coulson, R. L., Jacobson, M. J., Feltovich, P. J., & Spiro, R. (2012). Cognitive flexibility, contructivism, and hypertext: Random access instruction for advanced knowledge acquisition in ill-structured domains. *In Constructivism in Education*, 103-126.

Cramer, K. (2003). Using a translation model for curriculum development and classroom instruction: Models and modeling perspectives on mathematics problem solving. *Beyond Constructivism*, 449-464.

Cramer, K. A., Monson, D. S., Wyberg, T., Leavitt, S., & Whitney, S. B. (2009). Models for initial decimal ideas. *Teaching Children Mathematics, 16*(2), 106-117.

Cramer, K., & Karnowski, L. (1995). The importance of informal language in representing mathematical ideas. *Teaching Children Mathematics, 1*(6), 332-336.

Crawley, F. E. & Koballa, T. R. (1992). Attitude or behavior change in science education: Part 1-models and methods. *In a paper presented at the Annual Meeting of the National Association for Research in Science Teaching.* Boston, MA: National Association for Research in Science.

Creswell, J. W. (1999). *Research design: Qualitative and quantitative approaches.* London: Sage Publication.

Creswell, J. W. (2003). *Research design: Qualitative, quantitative and mixed methods approach* (2nd ed.). Thousand Oaks, CA: Sage Publication.

Creswell, J. W. (2009). *Research design: Qualitative , quantitative and mixed methods approaches.* Thousand Oaks, CA: Sage Publications.

Creswell, J. W. (2012). *Educational research: Planning, conducting, evaluating quantitative and qualitative research* (4th ed.). Boston: Pearson.

Creswell, J. W. (2013). *Research design: Qualitative, quantitative and mixed methods approaches* (4th ed.). Los Angeles, CA: Sage Publication.

Creswell, J. W., & Clark, V. P. (2007). *Designing and conducting mixed methods research.* Los Angeles: Sage Publication.

Cuoco, A. A. (2001). *The roles of representation in school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Curriculum Research and Development Division. (2012). *National Syllabus for Mathematics* . Ghana: Ministry of Education.

De Vos, A. S., Delport, C., Fouche, C. B., & Strydom, H. (2005). *Research at grassroots: For the social sciences and human service professions.* Pretoria: Van Schaik Publishing.

De Vos, A. S., Strydom, H., Fouche, C., & Delport, C. (1998). *Research at grassroot.* Pretoria: Van Schaik.

Delice, A., & Sevimli, E. (2010). An investigation of pre-services teachers' ability of using multiple representations in problem-solving success: The case of definite integral. *Educational Sciences : Theory and Practice, 7*(1), 137-149.

Dick, T. P., & Edwards, B. S. (2008). Multiple representations and local linearity. In G. (. I, *Research on technology and the teaching and learning of mathematics : Cases and perspectives* (255-275). Charlotte, NC: Information Age Publishing.

Dienes, Z. P. (1960). *Building up mathematics.* New York: Hutchinson Educational.

Dreher, S., & Kuntze, E. (2013). Pre-service and in-service teachers' views on the learning potential of tasks: Does specific content knowledge matter. *Proceedings of CERME*, 1596-1605.

Dufour-Janvier, B., Bednarz, N., & Belanger, M. (1987). Pedagogical considerations concerning the problem of representation. *Educational Journal*, 109-122.

Durmus, S., & Karakirik, E. (2006). Virtual manipulativesin mathematics education: A theoritical framework. *Turkish Online Journal of Educational Technology-TOJET*, 117-123.

Education, M. o. (2011). *National education assessment.* Accra: MOE.

English, L. D. (2009). Promoting interdisciplinarity through mathematical modelling. *ZDM, 41*(1-2), 161-181.

Ernest, P. (1987). Philosophy, mathematics and education. In P. Preece, *Philosophy and education* (28-36). Chicago: University Exeter.

Eshun-Famiyeh, J. (2005). Early number compentencies of children at the start of formal education. *African Journal of Educational Studies in Mathematics and Sciences, 3*(1), 21-33.

Flick, U. (2004). Triangulation in qualitative research. *A Companion to Qualitative Research, 3*, 178-183.

Fraenkel, J. R., & Wallen, N. E. (2003). *How to design and evaluate research in education.* New York: McGraw-Hill.

Frith, H., & Gleeson, K. (2004). Clothing and embodiment: Men managing body image and appearance. *Psychology of Men and Musculinity, 5*(1), 40-48.

Fujita, T., & Yamamoto, S. (2011). The development of children's understanding of mathematical patterns through mathematical activities. *Research in Mathematics Education, 13*(3), 249-267.

Gagatsis, A., & Shiakalli, M. (2004). Ability to translate from one representation of the concept of function to another and mathematical problem solving. *Educational Psychology, 24*(5), 645-657.

Gay, L. R., Mills, G. E., & Airasian, P. W. (2009). *Educational research: Competencies for analysis and applications.* Los Angeles, CA: Sage Publications, Inc.

Geertz, C. (1973). Thick description toward an interpretive theory of culture: The interpretation of cultures. *Selected Essays*, 3-30.

Ghana Statistical Service. (2013). *National Analytical Report.* Accra: Ghana Statistical Service.

Gilbert, J. K. (2008). Visualization: An emergent field of practice and enquiryin science education. *Theory and Practice in Science Education*, 3-24.

Glancy, A. W., & Moore, J. T. (2013). Theoritical foundations for effective STEM learning environments. *School Engineering Education Working Papers*. Retrieved 07 11, 2018, from http://docs.lib.purdue.edu/enewp/1

Glenn, B. (2009). Document analysis a qualitative research method. *Qualitative Research Journal, 9*(2), 27-40.

Glossary of Education. (2013). Pedagogical content knowledge. *Educational Journal*. Retrieved 06 14, 2018, from http://www.education.com/definition /pedagogical-content-knowledge.

Gningue, S. (2006). Students working within and between representations: An application of dienes's variability principles. *Learning of Mathematics, 26*(2), 41-47.

Golden, G. A. (1990). Epistemology, constructivism and discovery learning in mathematics. *Journal for Research in Mathematics Education, 4*, 31-210.

Golden, G. A. (1998a). Representational Systems, Learning and Problem Solving in Mathematics. *The Journal of Mathematical Behavior, 17*(2), 137-165.

Golden, G. A. (1998b). Discussions retrospective: The PME working group on representations. *Journal of Mathematical Behavior, 17*(2), 283.

Golden, G. A. (2000). Affective pathways and representation in mathematical problems solving. *Mathematical Thinking and Learning, 2*(3), 209-219.

Golden, G. A. (2002). Representation in mathematical learning and problem solving. *Handbook of International Research in Mathematics Education*, 197-218.

Golden, G., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. *The Roles of Representations in School Mathematics*, 1-23.

Green, M., Piel, J. A., & Flowers, C. (2008). Reversing education majors' arithmetic misconceptions with short term instruction using manipulatives. *The Journal of Educational Research, 101*(4), 234-242.

Greenes, C., & Findell, C. (1999). Developing students' algebraic reasoning. *Mathematical Reasoning in Grades K-12, 61*, 127-137.

Greer, B. (2009). Representational flexibility and mathematical expertise. *Educational Journal, 41*(5), 697-702.

Guzey, S. S., Moore, T. J., & Roehrig, G. H. (2010). Curriculum development for STEM integration: Bridge design on the white earth reservation. In M. K. L, *Handbook of curriculum development* (347-366). Hauppauge, NY: Nova.

Hamilton, E., Lesh, R., Lester, F. R., & Brilleslyper, M. (2008). Model-eliciting activities as a bridge between engineering education research and mathematics education research. *Advances in Engineering Education, 1*(2), 2-8.

Handal, B., & Herrington, A. (2003). Mathematics teachers' beliefs and curriculum reform. *Mathematics Education Research Journal, 15*(1), 59-69.

Harvey, O. J. (1986). Beliefs systems and attitudes towards the death penalty and other punishments. *Journal of Personality, 54*(4), 659-675.

Herman, M. (2007). What students choose to do and have to say about use of multiple representations in college algebra. *Journal of Computers in Mathematics and Science Teaching, 26*(1), 27-54.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J. J., & Stigler, J. (2003). *Teaching mathematics in seven countries.* Washington, DC: NCES Publication.

Hill, H. C., Blunk, M. L., Lewis, J. M., Phelps, G. C, Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430-511.

Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers' mathematical Knowledge: What knowledge matters and what evidence counts? In F. I, *Second handbook of research on mathematics teaching and learning* (111-155). Charlotte, NC: Information Age Publishing.

Hill, H., & Ball, D. L. (2009). The curious and crucial case of mathematical knowledge for teaching. *Phi Delta Kappan, 91*(2), 68-71.

Hjalmarson, M. (2007). Mathematical representations: Preparation and professional development of mathematics teachers. Retrieved 03 04, 2018, from http://mason.gmu.edu/gsalkind/portfolio/products/857Literature.pdf

Hodges, N. (2011). Qualitative research: A discussion of frequently articulated qualms. *Family and Consumer Sciences Research Journal, 40*(1), 90-92.

Huinker, D. (2015). Representational competence: A renewed focus for classroom practice in mathematics. *Wisconsin Teacher of Mathematics, 67*(2), 4-8.

Ipek, A. S., & Okumus, S. (2012). The representations of pre-service elementary mathematics: Teachers used in solving mathematical problems. *Gaziantep University Journal of Social Sciences, 11*(3), 681-700.

Jacobs, A. H. (2016). Using a theoritical framework of institutional culture to analyse an institutional strategy document. *Education as Change, 20*(2), 204-220.

Janvier, C. (1987a). Conceptions and representations: The circle as an example. In C. I, *Problems of representations in the learning and teaching of mathematics* (pp. 147-159). New Jersey: Lawrence Erlbaum Associates.

Janvier, C. (1987b). Representation and understanding: The notion of function as an example . In I. C. (Ed.), *Problems of Representations in the Learning and Teaching of Mathematics* (pp. 67-73). New Jersey: Lawrence Erlbaum Associates.

Janvier, C. (1987c). Representation system and mathematics. In In C. Janvier (Ed.), *Problems of Representations in the Learning and Teaching of Mathematics* (pp. 19-27). New Jersey: Lawrence Erlbaum Associates.

Janvier, C. (1998). The notion of chronicle as an epistemological obtacle to the concept of function. *The Journal of Mathematical Behavior, 17*(1), 79-103.

Janvier, C., Girardon, C., & Morand, J. (1993). Mathematical symbols and representations: Research ideas for the classroom. *High School Mathematics*, 79-102.

Johnson, B., & Christensen, L. (2000). *Educational research: Qantitative and qualitative approaches.* Boston: Allyn & Baon.

Kamii, C., Kirkland, L., & Lewis, B. A. (2001). Representation and abstraction in young children's numerical reasoning. T*he Roles of Representation in School Mathematics,* 24-34.

Kaput, J. J. (1987). *Representation systems and mathematics.* Hillsdale, NJ: Lawrence Erlbaum Associates Publishers.

Kaput, J. J. (1989). Linking representations in the symbol system of algebra. In K. S, *Research issues in the teaching and learning of algebra* (167-194). Reston, VA: National Council of Teachers of Mathematics.

Kaput, J. J. (1991). Notations and representations as mediators of constructive processes. In C. W, *Radical contructivism in mathematics education* (53-74). Springer: Dordrecht.

Kaput, J. J. (1994). The representational roles of technology in connecting mathematics with authentic experience. *Didactics of Mathematics as a Scientific discipline*, 379-397.

Kelley, K., Clark, B., Brown, V., & Sitzia, J. (2003). Good practice in the conduct and reporting survey research. *International Journal for Quality in Health Care, 15*(3), 261-266.

Kennedy-Murray, L. N. (2016). *Teachers' perceptions and practices of multiple intellegences: Theory in middle schools.* Walden: Walden University Scholar Works.

Klein, P. D. (2003). Rethinking the multiplicity of cognitive resources and curricular representations: Alternative to learning styles and multiple intelligences. *Journal of Curriculum Studies, 35*(1), 45-81.

Krauss, S., Baumert, J., & Blum, W. (2008). Secondary mathematics teachers' pedagogical content knowledge and content knowledge: Validation of the COACTIV constructs. *The International Journal on Mathematics Education, 40*(5), 873-892.

Kuhs, T. M., & Ball, D. L. (1986). *Approaches to teaching mathematics: Mapping the domains of knowledge, skills and disposition.* East Lansing: Michigan State University, Center on Teacher Education.

Kumekpor, T. K. (2002). *Research methods and techniques of social research.* Ghana: Sonlife Press & Services.

Kuntze, S., Dreher, A., & Friesen, M. (2015). Teachers' resources in analysing mathematical content and classroom situations: The case of using multiple representations. *Research in Mathematics Education*, 3213-3219.

Kuntze, S., Lerman, S., Murphy, B., Kurz-Milcke, E., Siller, H. S., & Winbourne, P. (2011). Professional knowledge related to big ideas in mathematics: An emperical study with preservice teachers. *Teacher Education Journal*, 290-306.

Lamberty, K. K., & Kolodner, J. L. (2002). Exploring digital quilt design using manipulatives as a mathematics learning tool. In *Proceedings of ICLS, 2*, 1-7.

Leikin, R., Levav-Waynberg, A., Gurevich, I., & Mednikov, L. (2006). Implementation of multiple solution connecting tasks: Do students' attitudes support teachers' reluctance. *Focus on Learning Problems in Mathematics, 28*(1), 1.

Lesh, R., & Doerr, H. M. (2003). *Foundations of models and modeling perspective on mathematics teaching, learning and problem solving.* Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., Post, T., & Behr, M. (1987a). Dienes revisited: Multiple embodiments in computer environment. *Development in School Mathematics Around the World*, 647-680.

Lesh, R., Post, T., & Behr, M. (1987b). Representations and translations among representations in mathematics learning and problem solving. *Problems of Representation in the Teaching and Learning of Mathematics*, 33-40.

Lloyd, G. (2002). Mathematics teachers' beliefs and experiences with innovative curriculum materials: Beliefs. *A Hidden Variable in Mathematics Education*, 149-159.

Lumpe, A. T., Haney, J. J., & Czerniak, C. M. (2000). Assessing teachers' beliefs about their science teaching context. *Journal of the National Association for Research in Science Teaching, 37*(3), 275-292.

Luo, F., Lo, J. J., & Leu, Y. C. (2011). Fundamental fraction knowledge of preservice elementary teachers: Across national study in the United States and Taiwan. *School Science and Mathematics, 111*(4), 164-177.

Macnab, D. S., & Payne, F. (2003). Beliefs, attitudes and practices in mathematics teaching: Perceptions of Scottish primary school students teachers. *Journal of Education for Teaching, 29*(1), 55-68.

Marshall, B., Cardon, P., Podder, A., & Fontenote, R. (2013). Does sample size matter in qualitative research: A review of qualitative interviews in research. *Journal of Computer Information Systems, 54*(1), 11-22.

Mathematics, N. C. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: NCTM.

McNeil, N., & Jarvin, L. (2007). When theories don't add up: Disentangling manipulative debate. *Theory into Practice, 46*(4), 309-316.

Mereku, K. D. (2004). Methods in Ghanaian primary mathematics textbooks and teachers classroom practice. *Research in Mathematics Education*, 6.

Merriam, S. B. (1998). *Qualitative research and case study applications in Education.* San Francisco, CA: Jossey-Bass Publishers.

Merriam, S. B. (2001). *Qualitative research and case study applications in education.* San Francisco, California: Jossey- Bass Publishers.

Merriam, S. B. (2009). *Qualitative research.* San Francisco,CA: Jossey-Bass.

Mertens, D. M. (2010). *Research and evaluation in education and psychology.* Los Angeles, CA: Sage Publications, Inc.

Meyer, M. R. (2001). Representation in realistic mathematics education. *The Roles of Representation in School Mathematics*, 238-250.

Mildenhall, P., Swan, P., Northcote, M., & Marshall, L. (2008). Virtual manipulatives on the interactive whiteboard: A preliminary investigation. *Australian Primary Mathematics Classroom, 13*(1), 9-15.

Ministry of Education. (2011). *National education assessment.* Accra: MOE.

Ministry of Finance. (2015). *The composite budget of the Lower Manya Municipality*. Retrieved 08 10, 2018, from http://www.google.com.gh

Mitchell, M., & Jolly, J. M. (2010). *Research design explained* (7 ed.). Belmont, CA: Wadsworth.

Mitchell, R., Charalambous, C. Y., & Hill, H. C. (2014). Examining the task and knowledge demands needed to teach with representations. *Journal of Mathematics Teacher Education, 17*(1), 37-60.

Monoyiou, A., Papageorgiou, P., & Gagatsis, A. (2007). Students' and teachers' representations in problem solving. *European Society for Research in Mathematics Education, 1*, 141-151.

Morse, J., Barret, M., Olson, K., & Spiers, J. (2008). Varification strategies for establishing reliability and validity in qualitative research. *International Journal of Qualitative Methods, 1*(2), 13-22.

Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives. *Teaching Children Mathematics, 8*(6), 372-377.

Nabie, M. J., Anamuah-Mensah, J., & Ngwan-Wara, E. I. (2010). Basic school teachers understanding of mathematical concepts. *International Journal of Pedagogy Policy and ICT in Education, 1*(1), 43-53.

Nabie, M. J., Raheem, K., Agbemeka, J. B., & Sabtiwu, R. (2016). Multiple solution approach: Conceptions and practices of primary school teachers in Ghana. *International Journal of Research in Education and Science, 2*(2), 333-344.

Nathan, M. J., Alibali, M. W., Masarik, D. K., Stephens, A. C., & Koedinger, K. R. (2010). *Enhancing middle school students representational fluency: A classroom base study.* Madison: Wisconsin Center for Education Research.

National Council Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: NCTM Publication.

National Council Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (Vol. 1). Reston, VA: NCTM Publication.

National Educational Assessment. (2016). *Finding report.* Accra: Assessment Service Unit.

Nielson, M. (2016). *Eighth-grade students use and justification of multiple representations*. Retrieved 05 16, 2018, from http://scholarworks.bgsu.edu/honorsprojects/324

O'donovan, R. (2015). Logical problems with teachers' beliefs research. *Proceedings PME, 39*(3), 305-312.

Oduro, E. O. (2015). Aesssessment in mathematics classrooms in Ghana: A study of teachers' practice (Doctoral dessertation). Retrieved 06 02, 2018, from http://sro.sussex.ac.uk/

Olkun, S., & Toluk, Z. (2004). Teacher questioning with an appropriate manipulative may make a big difference. Retrieved 04 21, 2018, from http://search.proquest.com/docview/61869322?accountid=1477

Olkun, S., & Tooluk, Z. (2004). *Teacher questioning with an appropriate manipulative may mmake a big difference*. Retrieved 04 21, 2018, from Issues in the Undergraduate Mathematics Preparation of School: http://search.proquest.com/docview/61869322?accountid=14771

Ozgun-Koca, S. A. (2008). Ninth grade students studying the movement of fish to learn about linear relationships: The use of video based analysis software in mathematics classroom. *The Mathematics Educator, 18*(1).

Ozimantar, M. F., Akkoc, H., Bingolbali, E., Demir, S., & Ergene, B. (2010). Preservice mathematics teachers use of multiple representations in technology rich environment. *Eurasia Journal of Mathematics, Science and Technology Education, 6*(1).

Padgett. (2004). *Coming of age: Theoritical thinking, social responsibility and a global perspective in qualitative research.* Belmont, CA: Wadsworth Publication.

Pal, M. (2014, April). *Making conceptual knowledge connections to clear misconceptions in fractions in primary classrooms.* Retrieved 05 12, 2018, from www.iosrjournals.org.

Pape, S. J., & Tchoshanov, M. A. (2001). The roles of representations in developing mathematical understanding. *Theory into Practice, 40*(2), 118-127.

Park, M. (2013). *Professional development and teacher change.* Minnesota: Graduate School.

Park, S., & Oliver, J. S. (2008). Revisiting the conceptualisation of pedagogical content Knowledge: PKC as a conceptual tool to understand teachers professionals. *Research in Science Education, 38*(3), 261-284.

Patton, M. Q. (2002). *Qualitative research and evaluation methods* (3rd ed.). Thousand Oaks, CA: Sage.

Perkkila, P. (2003). Primary school teachers' mathematics beliefs and teaching practices. *Third conference of the European society for research in mathematics education* (1-8). Kokkola: School of Halkokari.

Perry, J. A., & Atkins, S. L. (2002). It is not just notation: Valuing children's representations. *Teaching Children Mathematics, 9*(4), 196.

Pham, S. (2015). *Teachers perceptions on the use of mathematics manipulatives in elementary classrooms.* Toronto: Ontarion Institute.

Philipp, R. A., & Siegfried, J. M. (2015). Studying productive disposition: The early development of a construct. *Journal of Mathematics Teacher Education, 18*(5), 489-499.

Polit, D. F., & Beck, C. T. (2013). *Study guide for essentials of nursing research: Appraising evidence for nursing practice.* Philadelphia: Lippincott Williams & Wilkins.

Porzio, D. T. (1994). *The effect of differing technological approaches to calculus on students' use and understanding of multiple representations when solving problems.* (PhD thesis) Ohio: Ohio State University.

President's Council of Advisors on Science and Technology. (2012). *Producing one million additional college graduates with degrees in science, technology, engineering and mathematics.* Retrieved 05 17, 2018, from http://obamawhitehouse ,archves.gov/sites/default/files/microsites/ostp/pcast-engage-to-excell-final.pdf.

Preston, R. V., & Garner, A. S. (2003). Representation as a vehicle for solving and communicating. *Mathematics Teaching in the Middle School, 9*(1), 38.

Puchner, L., Taylor, A., O'donnell, B., & Fick, K. (2008). Teacher learning and mathematics manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School Science and Mathematics, 108*(7), 313-325.

Reimer, K., & Moyer, P. S. (2005). Third-graders learn about fractions using virtual manipulative: A class room study. *Journal of Computers in Mathematics and Science Teaching, 24*(1), 5-25.

Resnick, L. B., & Ford, W. W. (2012). *The psychology of mathematics for instruction.* Erlbaum: Lawrence Erlbaum Associates.

Richards, J. C. (1998). Teacher beliefs and decision making. *Beyond Training*, 65-85.

Richards, J. C., Gallo, P. B., & Renandya, W. A. (2001). Exploring teachers' beliefs and the processes of change. *PAC Journal, 1*(1), 41-58.

Robson, C. (1995). *Real world research: A resource for social scientists and practitioner researcher.* Great Britain, Padston: T.J. Press Ltd.

Roehrig, G. H., & Kruse, R. A. (2005). The role of teachers' beliefs and knowledge in the adoption of a reform based curriculum. *School Science and Mathematics*, 412-422.

Rosengrant, D., Etkina, E., & Van-Heuvelen, A. (2007). An overview of recent research on multiple representations. *Fourth AIP conference proceedings* (149-152). Pomana, Califonia: AIP Publication.

Rule, P., Davey, B., & Balffour, R. J. (2011). Unpacking the predominance of case study methodology in South African postgrduate educational research. *South African Journal of Higher Education, 25*(2), 301-321.

Schmidt, W. H., Tatto, M. T., Bankov, K., Blomeke, S., Cedillo, T., Cogan, L. M., & Santillan, M. (2007). *The preparation gap: Teacher education for middle school mathematics in six countries.* East Lansing: Michigan State University.

Schoenfeld, A. H. (2012). Toward professional development for teachers grounded in a theory of decision making. *ZDM, 25*(4), 457-469.

Science Conference Board (2012). *The mathematical education of teachers II.* America: American Mathematical Society.

Seeger, F., & Waschesco, U. (1998). *The culture of the mathematics classroom.* Cambrige: Cambridge University Press.

Shumway, J. W. (2011). *Number sense routines: Building numerical literacy everyday in grades.* Hawker, Brownlow: Stenhouse Publishers.

Shuttleworth, M. (2008). *Case study research design.* Retrieved 04 18, 2018, from htt://www.experiment-resources.com/case-studyresearch-design.html

Simons, M. (2009). *Case study research in practice.* Califonia: Sage Publication.

Singh, A. S., & Mazuka, M. B. (2014). Sampling techniques and determination of sample size in applied statistics research. *International Journal of Economics, Commerce and Management, 2*(11), 1-22.

Skemp, R. R. (1986). *The psychology of learning mathematics.* London: Penguin Books Ltd.

Smith III, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education*, 387-402.

Speer, N. M. (2005). Issues of methods and theory in the study of mathematics teachers' professed and attributed beliefs. *Educational Studies in Mathematics, 58*(3), 361-391.

Stabback, P. (2016). *What makes a quality curriculum? Current and critical issues.* France: UNESCO International Bureau of Education.

Stake, R. E. (1995). *The art of case study research: Perspective in practice.* London: Sage Publication.

Starman, A. B. (2013). The case study as a type of qualitative research. *Journal of Contemporary Educational Studies, 64*(1).

Steen, K., Brooks, D., & Lyon, T. (2006). The impact of virttual manipulatives on first grade geometry instruction and learning. *Journal of Computers in Mathematics and Science Technology*, *25*, 373-391.

Stipek, D., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education , 17*(2), 213-226.

Strutchens, M. E., Quander, J. R., & Gutierreez, R. (2011). Mathematics learning communities that foster reasoning and sense making for all high school students. *Focus in Hgh School Mathematics*, 101-114.

Suh, J. M. (2005). Third graders' mathematics achievement and representation preference using virtual and physical manipulatives for adding fractions and balancing equations. *Educational Journal*, 1-114.

Suh, J. M. (2007). It all together. *Teaching Children Mathematics, 14*(3), 163-169.

Suh, J., & Moyer-Packenham, P. (2007). Developing students' representational fluency using virtual and physical algebra balances. *Journal of Computers in Mathematics and Science Teaching, 26*(2), 155-173.

Suh, J., Moyer, P. S., & Heo, H. J. (2005). Examining technology uses in the classroom: Developing fraction sense using virtual manipulative concept tutorials. *Journal of Interactive Online Learning, 3*(4), 1-21.

Taba, K. S. (2009). *Learning at the symbolic level.* Springer: Dordrecht Publication.

Teo, W. L. (1997). *Espoused beliefs of Singapore teachers about mathematics and its teaching and learning.* Toronto: Ontorio Institution for Studies in Education.

Thomas, H. (2003). *The relationship between attitudes toward change and adoption of innovation.* Chicago, IL: American Education Research Association.

Tripathi, P. N. (2008). Developing mathematical understanding through multiple representations. *Mathematics Teaching in the Middle School, 8*, 4538-4.

Trochim, M. K. (2008). *Research method knowledge base*. Retrieved 07 20, 2018, from www.socialmethods.nrt/quaval.php

Turnuklu, E. B., & Yesildere, S. (2007). The pedagogical content knowledge in mathematicicss: Pre-service primary mathematics teachers perspectives in Turkey. *Issues in Undergraduate Mathematics Preparation of School Teachers, 1*, 1-4.

Twamasi-Ankrah, O. (2015). *Education expects' perceptions of the Ghanaian language policy and its implementation.* Lapland: Lapland University Press.

Underhill, R. G. (1987). Conceptualising mathematics teacher preparation research and programs. In *Conference of the international group for the psychology of mathematics education* (19-25). Montreal: Psychology of Mathematics Education Publication.

Valles, J. R. (2014). *Using multiple representations.* Mathitudes: Sage Publication.

Vergnaud, G. (1987). *Problems of representations in the teaching and learning of mathematics*. New Jersey: Lawrence Erlbaum.

Vile, A., & Lerman, S. (1996). Semiotics as a descriptive framework in mathematical dormains. *Proceeding of PME 20* (359-402). Valencia: PME Publications.

Villegas, J. L., Castro, E., & Gutierrez, J. (2009). Representation in problem solving: A case study with optimization prolems. *Electronic Journal of Research in Educational Psychology, 7*(1), 279-308.

Von Glsersfeld, E. (1987). *Learning as contructivity activity.* Pennsylvania: Erlbaum.

West, L. (2011). *Using physical and virtual manipulatives with eighth grade geometry students (PhD thesis).* Nebraska-Lincoln: Uniiversity of Nebraska-Lincoln.

White, T., & Pea, R. (2011). Distributed by design: On the promises and pitfalls of collaborative learning with multiple representations. *Journal of Learning Sciences, 20*(3), 489-547.

Wilkins, J. L., & Ma, X. (2003). Modeling change in students attitude toward and beliefs about mathematics. *The Journal of Educational Research, 97*(1), 52-63.

Wilson, S. M., Floden, R. E., & Ferrini-Mundy, J. (2002). Teacher preparation research: An insider's view from the outside. *Journal of Teacher Education, 53*(3), 190-204.

Wilson, S. M., Shulman, L. S., & Richert, A. E. (1997). Different ways of knowing: Representations of knowledge in teaching. *Educational Journal*, 104-124.

Yee, S. P., & Bostic, J. D. (2014). Developing a contextualization of students' mathematical problem solving. *The Journal of Mathematical Behavior, 36*, 1-19.

Yilmaz, R., Argun, Z., & Keskin, M. O. (2009). What is the role of visualization in generalization processes: The case of pre-service secondary mathematics teachers. *Humanity and Social Sciences Journal, 4*(2), 130-137.

Yin, R. K. (2008). *Case study research: Design and methods* (4th ed.). Thousand Oaks: Sage Publication, Inc.

Yin, R. K. (2014). *Case study research: Design and methods* (5th ed.). Thousand Oaks: Sage Publications, Inc.

APPENDICES

APPENDIX A

UNIVERSITY OF EDUCATION, WINNEBA

FACULTY OF EDUCATIONAL STUDIES

DEPARTMENT OF BASIC EDUCATION

Interview Schedule

Semi Structured Questions

Name of interviewee: ________________________________

Researcher: ____

Date: ______________________

Time: ______________________

Venue: ____________________

Start time: _________________

End time: __________________

Preamble:

Thank you for volunteering to participate in this study. This interview is to give you an option to express your view and experience about multiple representations. I want to assure you that the interview and anything you share will remain confidential. As I explained in the introductory letter, the study is about multiple representations conceptions and practices of primary school mathematics teachers‟. The questions that I will be asking will need you to describe, in as much detail as possible, your classroom experience. Please feel free to ask me to clarify what I mean at any time.

INTERVIEW SCHEDULE GUIDE FOR TEACHERS
SECTION A - BACKGROUND INFORMATION

Tell me about your teaching career very briefly, teacher training experience, further qualifications and teaching experience up to now.

SECTION B

1. In your own opinion, what is meant by multiple representations in teaching mathematics?
2. Do you think that multiple representations of mathematics concepts are necessary in the mathematics classroom?
3. Do you use multiple representations in your mathematics classroom?
4. How do you use multiple representations?
5. How often do you use multiple representations?
6. If not using: why do you not use multiple representations?
7. Do you find that it impedes your teaching? Why?
8. Will you be willing to try?
9. What are some of the examples of multiple representations that you use?
10. How does the use of multiple representations influence your teaching of mathematics?

11. Do you think that pupils benefit from the use of multiple representations? Why?
12. What do you think are some advantages of using multiple representations?
13. Do you think there are any disadvantages of using multiple representations? Explain.
14. Do you attribute your success to the use or non-use of multiple representations? Why? Thanks for your input.

APPENDIX B

OBSERVATION CHECKLIST FOR PRIMARY SCHOOL MATHEMATICS TEACHERS' CLASSROOM PRACTICES OF MULTIPLE REPRESENTATIONS (M.R.)

M.R. ACTIVITY	Yes	No	None
1. Topic/sub-topic can be taught through MR			
2. Learners pre-informed of multiple representations in solving that problem			
3. Lead learners through only one representation in solving the mathematical problems			
4. Ask pupils to explore other forms of representations to solving the mathematical problems			
5. Lead learners through alternative multiple representations			
6. Assist pupils to give more examples of similar that can be solved using any of the multiple representations taught			
7. Create pupils" awareness on the need to know different multiple representations to solving a problem			
8. Give learners the opportunity to solve problems using such multiple representations in their jotters or chalkboard			
9. Motivate pupils to use the multiple representations identified			
10. Welcome all multiple representations used and award marks according to developed rubrics in exercise books			

Key
MR: Multiple representation
Yes: Instructional practice performed;
No: Instructional practice not performed; and
None: No evidence of performance of instructional practice

APPENDIX C

ANALYSIS OF SYLLABUS AND PUPILS TEXTBOOK FOR MULTIPLE REPRESENTATIONS

Amount of multiple representations in primary school mathematics syllabus and textbooks for Basic three (3) and six (6)

	Syllabus		Textbook	
Main Topics	Number of Objectives	Number of Objectives with MR	Number of sub-topics	Sub-topics with MR

APPENDIX D

SAMPLE OF TRANSCRIBED INTERVIEWS

Transcription of Pilot Interview # 1 (Teye)

Date: Monday, 07-05-2018

Q: Tell me about your teaching career very briefly, teacher training experience, further qualifications and teaching experience up to now.

R: Starting from my college! I attended Ada College of Education from 2012 – 2015. This is my third year on the field. I began with R.M.E at the JHS. However, I started teaching mathematics here at the upper primary level somewhere last year. On teacher training experience, I learned so many things including pedagogy, how to motivate children to get the best out of them among others. I‟m currently a diploma holder and intend to further mu studies probably next year.

Q: In your own opinion, what is meant by multiple representations in teaching mathematics?

Multiple representations in teaching mathematics has to do with the methods and skills in teaching mathematics.

Q: Do you think that multiple representations of mathematics concepts are necessary in the mathematics classroom?
Yes, multiple representations is very necessary in the mathematics classroom

Q: Do you use multiple representations in your mathematics classroom?

For sure, I use because mathematics involves activities which requires multiple representations.

Q: How do you use multiple representations?

When explaining content, concept and the child is still not picking it, then I come in with another representation.

Q: How often do you use multiple representations?

Not all the time, because it‟s aimed at helping the child grasps the concept. If you teach a child and he/ she doesn‟t get it, you simply haven‟t done anything. So you need to bring in so many methods to help to the child. For example, when teaching addition and it‟s difficult for them to pick, you can bring in stones or bundles of sticks to get the idea.

Q: What are some of the examples of multiple representations that you use?
At times I use bundles of sticks, diagrams, formula among others things are examples of multiple approaches I use in the mathematics classroom. We also use charts (place value charts).

Q: How does the use of multiple representations influence your teaching of mathematics?

I may say multiple representations is very good. It helps a lot. Sometimes, you see the child nodding the head when such approaches are used which confirms that the child has grasped the concept.

Q: How does the use of multiple representations influence your teaching of mathematics?

Yes. As I said earlier on, without the use of multiple representations, it is just like pointing to an abstract object in the air for a child to view. Besides, children vary and are unique as well. Hence, they think differently. As a result, the use of multiple representations will provide level grounds to compete with others.

Q: What do you think are some advantages of using multiple representations?
Of course, there‟re some advantages of multiple representations; it makes teaching and learning faster, helps pupils understand topics/ concepts better, makes the classroom very lively and many more.

Q: Do you think there are any disadvantages of using multiple representations? Explain.

Yes, but not much….hmmm not much

Q: Do you attribute your success to the use or non-use of multiple representations? Why?

Yes. As I mentioned earlier that makes our teaching.

Transcription of Pilot Interviewee # 2 (Mamle)

Date: Tuesday, 08-05-2018

Q: Tell me about your teaching career very briefly, teacher training experience, further qualifications and teaching experience up to now.

I started as peer teaching in my community. And later after secondary school I had the opportunity to teach in one JHS school because they lack mathematics teachers. I taught for one and half year. I proceeded to training college afterwards. After college this is my first posting. I further did my post diploma program in Mathematics. There has been vast improvement as well in terms of my teaching method. Because as human beings the more you progress the more you add value to yourself.

Q: In your own opinion, what is meant by multiple representations in teaching mathematics?

Multiple representation is like in mathematics every day we have been doing mathematics, such as buying things at the market and also teaching. In a nutshell is like how we do things.

Q: Do you think that multiple representations of mathematics concepts are necessary in the mathematics classroom?

Oh why not very necessary! These days mathematics is taught using the traditional method, is no more. Now in modern dispensation there are numerous researcher to help improve pupils understanding in varying means.

Q: Do you use multiple representations in your mathematics classroom?

Yes, because you need not to go one way if not you are killing somebody especially in mathematics. The reason for varying it is that it could happen somebody is dull, at the end of the day that dull person will come to a stand point of going through all to get the answer. But from the beginning the teacher has to help such a pupil by varying the lesson to develop the concept.

Q: How do you use multiple representations?

When you are teaching and you realized that pupils are not getting what you want to teach does not have any impact then you chip in other representations. When you are starting a new topic, with that you have to adopt strategies for better understanding. Moreover, because it is a new topic you have to teach it in such a way to develop their interest. So is not always because the day is packed with exercises, marking and correction and the time frame will not be convenient to practice it all the time

Q: How often do you use multiple representations?

Not at all times.

Q: What are some of the examples of multiple representations that you use?

Sometimes fieldtrip, role play for instance a topic like profit and loss you could see that it is deals with money so you can role play whereby the pupils apportions roles of seller and a buyer. So one will pretend to sell and add some profit to the productions cost so when you sell there is some profit one make out of it. In the other way round when you sell below the production cost you will go at loss because it is below the production cost.

Q: How does the use of multiple representations influence your teaching of mathematics?

It enhance understanding, pupils performance improved.

Q: Do you think that pupils benefit from the use of multiple representations? Why?

Yes, because when you used the traditional method they‟re not getting it, till the multiple representation is used. Moreover learners benefit depending how one handle and explain to them. Well there is no way they will not understand.

Q: What do you think are some advantages of using multiple representations?

It brings understanding. Also help pupils to manipulate the materials themselves and improves their learning. They will applied it in their real life situation.

Q: Do you think there are any disadvantages of using multiple representations? Explain.

Sure, lapses come in when two or three pupils understand at the expense of others. The teacher has to teach all over again to develop their interest.

Q: Do you attribute your success to the use or non-use of multiple

representations? Why?

That is how I see it. Because it is yearning result when you use it pupils understand it that is okay but when you use it and pupils are not getting it. I will stop it, it don‟t also insist on pupils to on such things but allow them to make their own choices unless you are limited is to what to use. I think that not common too in mathematics. Thanks for your input.

APPENDIX E

LETTER OF INTRODUCTION

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF EDUCATIONAL STUDIES
DEPARTMENT OF BASIC EDUCATION
P. O. Box 25, Winneba, Ghana
beducation@uew.edu.gh
+233 (050) 9212015

Date: July 31, 2018

The Director
Municipal Education Directorate
Lower Manya Municipal Assembly
P. O. Box 49
Odumasi - Krobo, E/R

Dear Sir/Madam,

LETTER OF INTRODUCTION

I write to introduce to you, Mr. Felix Asemnor, a second year M.Phil student of the Department of Basic Education, University of Education, Winneba, with registration number 8160030002.

Mr. Felix Asemnor, is to carry out a research on the Topic *"Primary School Teachers' Conceptions and Practices of Multiple Representations in Lower Manya Municipality in the Eastern Region of Ghana"*

I would be grateful if permission is granted him to enable him carry out his studies in your Municipality.

Thank you.

DEPT. OF BASIC EDUCATION
UNIVERSITY OF EDUCATION
WINNEBA, GHANA

....
MR. KWEKU ESIA-DONKOH
(Ag. Head of Department)

www.uew.edu.gh