UNIVERSITY OF EDUCATION, WINNEBA

EFFECT OF USING GEOGEBRA ON VAN HIELE'S GEOMETRIC THINKING LEVELS OF SENIOR HIGH TECHNICAL SCHOOL STUDENTS' ATTAINMENT OF GEOMETRY

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DECLARATION

Student's Declaration

I, Ansah Susan, declare that this Thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

Signature:……………………………………………

Date:………………………………………………….

Supervisor's Declaration

I hereby declare that the preparation and presentation of this Thesis was supervised in accordance with the guidelines for supervision of Thesis as laid down by the University of Education, Winneba.

Name of Supervisor: Mr. Michael Edmund Amppiah

Signature:……………………………………………

Date:………………………………………………….

DEDICATION

This thesis is dedicated my dearest husband Mr.Felix Kafui Amegbe- Ansah and my cherished daughter Deladem Ansaba Amegbe for their love, support and encouragement.

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ABSTRACT

The study explored the effect of using GeoGebra on students' performance of geometry, the effectiveness of the use of GeoGebra on students' van Hiele level of geometric thinking, effect of using GeoGebra on students' motivation to learn geometry and students' perceptions of using Geogebra in the learning of geometry. The study utilized mixed method approach involving one group pretest-posttest pre-experimental design. A total of eighty (80) participants were used in the study. Simple random sampling procedure was used to select a sample size of eighty (80) SHS 2 students. Geometry Achievement Test (GAT) was used to explore the effect of GeoGebra on students learning performance of geometry. Van Hiele Geometry Test (VHGT) was used to explore the effectiveness of using GeoGebra on students Van Hieles' geometric think levels. Semi structured interview guide was used to collect data on how GeoGebra motivates students to learn geometry. Questionnaires were used to collect numeric data on students' perception of using GeoGebra in learning geometry. Inferential statistics of one-way ANOVA and paired sample T-test were used to test the hypotheses. Findings reveal that the use of GeoGebra had positive effects on students learning performance in geometry and students are motivated. The findings also reveal that most students did not perform well in the pre-VHGT item test as compared to the post-VHGT item test. The study concluded that, using GeoGebra to teach geometry improved students performance in geometry. Moreover, the use of GeoGebra on students van Hiele level of geometric thinking was effective because majority of the students 95.0% ($F = 76$) obtained more than half of the marks allotted to the test while 5.0% ($F = 4$) had the total marks allotted to the test after the use of GeoGebra. It also motivated students because it took away dullness thus, making learning easier and fascinating. Again students had positive perception of using GeoGebra to learn geometry. The hypotheses concluded that there was a statistically significant difference at the $p < .005$ level in the level means of students van Hiele geometric thinking levels after GeoGebra instruction and $p < .005$ level in the mean pre-VHGT and post-VHGT scores of senior high technical school students'. The study recommends that: The government should endeavour to equip senior high technical schools with functional computer laboratories. Mathematics teachers should incorporate GeoGebra and other Mathematics software in the teaching of Mathematics concepts.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This chapter sets the study in context. It presents the background of the study, statement of the problem, purpose of the study, objectives of the study as well as the educational significance and the research questions guiding the study. The Chapter further highlights the delimitations and limitations of the study, defines certain key terms and concludes by outlining the organization of the dissertation.

1.1 Background to the Study

Education is a systematic process through which a man acquires knowledge, experience, skill, and sound attitude. It makes an individual civilized, refined, cultured and educated (Singh, 2019). According to Singh (2019) by education he means an all-round drawing out of the best in child and man- ‗body, mind and spirit'. In this context education is certainly a means of all round development of man. The formal system of secondary education has mainly three components i.e. teacher, students and curriculum. The curriculum of secondary education exits mainly five subjects i.e. Languages (L1 & L2), social science, science and mathematics (Singh, 2019). Mathematics has an important place in school education from the early grades to the tertiary level. It is the numerical and computation part of one‗s life and knowledge. It helps people to give exact interpretations to their various ideas and conclusions (Singh, 2019). According to (Singh, 2019), the science of numbers and their operations, interrelations, combinations, generalizations and abstractions and of space configurations is called mathematics. So the knowledge of mathematics is helpful in the day to day life of people. Mathematics is divided into various branches such as algebra, trigonometry, geometry, statistics, set theory and calculus.

Geometry plays a significant role in primary and secondary schools mathematics curricula in Ghana and other countries. It provides a rich source of visualization for understanding arithmetical, algebraic, and statistical concepts (Fabiyi, 2017). Fabiyi (2017) is of the view that geometry provides a complete appreciation of the world we live in. Geometry appears naturally in the structure of the solar system, a geological formation, rocks and crystals, plants and flowers, and even in animals. It is also a major part of the synthetic world such as art, architecture, cars, machines, and virtually everything humans create. In the same vein, studies revealed that geometry is applicable and relevant to employment in everyday life and other subjects in the curriculum such as science, arts, and technology (Mehta,2018). Also, geometry is used to develop students' spatial awareness, intuition, visualizations and to solve practical problems and so on (Fabiyi, 2017).

Plane Geometry in the senior high schools mathematics curriculum covers areas such as polygons, Pythagoras theorem as well as circle theorems including tangents (MOE, 2010). The study of plane geometry is also beneficial in the development of students' representational and problem solving skills as well as application of the knowledge gained in other areas of mathematics and in real world situation. According to Singh (2019), geometry is a branch of mathematics that deals with points, lines, angles, surfaces and solids. Geometry is visual and dynamic in nature. Therefore, it requires visualizing abilities in the teaching-learning process. There are basically two objectives of geometry learning, which are to develop logical thinking skill and to develop spatial intuitions that refer to how one views space and area in the real world (Abdullah & Zakaria, 2013). Thus, it is important for students learning geometry, to be able to imagine, construct and understand construction of shape in order to connect them with related facts (Mwingirwa, 2016).

In spite of the importance of geometry, a lot of concerns have been raised about the level of students‘ understanding of geometry in Ghanaian schools (Mereku, 2010). Sunzuma, Masocha and Zezekwa (2013) argue that factors responsible for students‘ difficulty in learning geometry include: lack of background knowledge, poor reasoning skill in geometry, geometric language comprehension, lack of visualizing abilities, teachers‘ method of teaching, non-availability of instructional materials and lack of proof by students. My experience both as a student and a Mathematics teacher in one of the senior high technical school in Abura Asebu Kwamankese district indicates that many mathematics teachers hold negative perception in integrating ICT tool in teaching mathematics lesson especially geometry, thus most of the mathematics teachers employ the traditional methods in teaching. These negative perceptions are evidenced in the deprecating comments often made about geometry. My observations of mathematics teachers using ICT tools such GeoGebra in teaching geometry are that of panic, worry and lack of self-confidence.

Learning Geometry has been identified as an area of Mathematics that poses various problems for many secondary school learners‘ (Binti, Tay & Lian, 2003). Many student fail to develop an adequate understanding of geometrical concepts and to demonstrate reasoning and problem solving skills (Khoo & Clements, 2001). Many researchers point to different difficulties that students face while learning geometry. These difficulties include: students‘ lack of coordination in their views of three-dimensional objects (Clements & Battista, 2007; cited in Oladosu, 2014); inability to use theoretical statements in deductive reasoning and to recognise visually relevant geometrical properties (Laborde, 2003); challenges in learning the appropriate language required for understanding and discussing geometric principles (Swindal, 2000); issues in relation to how students extract information from objects and form both natural and formal concepts (Battista, 2009); and challenges related to measurement and deductive proof, linking chains of reasoning, and understanding definitions in geometry (Oladosu, 2014). These difficulties centre on the meanings that students take out of the learning they experience in and out of the geometry classroom. Fredua-Kwarteng and Ahia (2004) indicated that Ghanaian students have internalized the false belief that Mathematics learning including geometry requires an innate ability or the ―brains of an elephant". My own concerns have been agitated by the following questions:

1. Why do students find it difficult in learning geometry?
2. What kind of instructional tool do mathematics teachers use in teaching geometry?
3. How do mathematics teachers use this instructional tool to motivate students to learn geometry? These questions have bothered me for some time now.

However, the current teaching and learning practice in classroom do not reflect the importance of geometry in the lives of students, and the emphasis that is supposed to be given to geometry topics in the mathematics curriculum (Abdullah & Zakaria, 2013). According to (Abdullah & Zakaria, 2013), in terms of teachers teaching practice and attitude, more often teachers who teach mathematics use the blackboard to explain certain theorems, definitions, and concepts, and to show the solutions for the related problems. Students are commonly fed methods and algorithms, which are then memorized without them actually understanding the concepts. Students often fail to develop the visualization and exploration skills required for geometrical concepts, problem-solving skills and geometry reasoning (Battista, 2007; Idris, 2006). Geometry learning should emphasise hands-on and mind-on approaches (Abdullah & Zakaria, 2013).

Though there have been some significant improvements in the performance of students in senior high schools, the overall performance in core mathematics at the West African Senior School Certificate Examination (WASSCE) has been low, with about 60% obtaining poor grades (i.e. D7, E8 and F9). Performance in core mathematics, the pass rate has been fluctuating over the years. It decreased from 32% in 2006 to 29% in 2009, jumped to 44% in 2011 and decreased to 37% in 2013 (MoE, 2014). The national and international reports show that Ghanaian students perform poorly in higher order thinking problems. According to Anamuah-Mensah, Mereku and Asabere-Ameyaw (2004), the overall performance of students from Ghana on the TIMSS 2003 mathematics tests was very low. Ghana obtained low mean scale scores of 276 in mathematics, placing the nation last but one of the overall results (i.e. placing 45th out of 46 participating countries). According to the TIMSS 2003 report, as cited in Anamuah-Mensah, Mereku and Asabere-Ameyaw (2004), compared to other African countries that took part in the examination, the performance of Ghana was one of the lowest. They argued that the Ghanaian students‘ inability to reach the higher benchmarks calls for the need to assist students to build a sound grounding in the mastery of basic knowledge and skills needed to solve more cognitively demanding problems. For the 2007 TIMSS, they reported that there had been a little improvement in mathematics achievement and yet Ghana‘s performance remained low by comparison to the quality of mathematics and education in other countries surveyed in the TIMSS. Students' areas of greatest weakness in mathematics were in algebra, measurement and geometry

According to National Council of Teachers of Mathematics (NCTM, 2000), the use of technology has been an essential tool for teaching and learning mathematics at all grade levels as it improves students' skills in decision making, reasoning and problem solving. The use of technology in mathematics education not only help students construct their visual representations of mathematics ideas and concepts, summarise and analyse data, but also enables students to investigate every area of mathematics, such as geometry, algebra and statistics (NCTM, 2000). NCTM (2008) emphasises that, the use of technology in education is essential for teaching and learning of mathematics and therefore all schools should have necessary technological substructure and equipment for the active use of educational technologies in mathematics education. Moreover, the Ministry of Education, Youth and Sports (MOEYS) and Ghana Education Service (GES) (2002) are of the view that integrating technology in classroom instruction guarantee greater motivation, improves good questioning skills, encourages initiatives and independent learning, develops problem solving capabilities increase focus time on task and improves social and communication skills. Similarly, Ochkov and Bogomolova (2015), reported on the use of computer software and internet for teaching mathematics. They pointed out that advanced mathematical computer programs allow using a fresh approach to the teaching of mathematics in schools and universities, taking into account the attraction of students to computers by means of graphics and animation. As such, one can significantly increase the understanding of students of the basic concepts and theorems of mathematics.

Despite the impact of educational technology and strong advocacy for the need to utilize ICT in the teaching and learning of mathematics, classrooms in Ghana are still characterized by traditional method of teaching. The traditional method is the teaching approach characterised by lecture/oral exposition. This teaching approach is more of teacher-centred rather than learner centred. With the dominance of traditional methods in Mathematics instruction in Ghana coupled with students‘ learning difficulty in geometry, one probable approach for enhancing instruction and student learning could be implementing realistic instructional method such as the use of GeoGebra. GeoGebra is one of the educational technology tools used in mathematics instruction and other subjects. According to Bwalya (2019) GeoGebra is useful as a supportive tool in the teaching and learning of mathematics. GeoGebra shows positive impact on students‘ engagement; increase the amount of students‘ interactions with teachers; increase achievement in geometry, transformations and trigonometry; increase test scores; and benefit students who struggle with visualisation (Bwalya, 2019).

In the mathematics classroom, the use of GeoGebra helps students and teachers to explore the mathematical ideas and concepts and the association of these ideas and concepts with real life examples, thus resulting in permanent and effective learning in mathematics and higher mathematics achievement (Mwingirwa, 2016). GeoGebra motivates learners to approach Mathematics with an experimental method (Tay & Mensah-Wonkyi, 2018).

While geometry is a crucial sub-discipline in the field of mathematics, most students have difficulties with school geometry (Ozkan & Oner, 2019). One of the explanations for these difficulties with learning geometry is the lack of instruction that is designed based on students‘ van Hiele levels of geometric thinking, proposed by the two Dutch mathematics educators (Dina van Hiele-Geldof & Pierre van Hiele) in the late 1950s (Ozkan & Oner, 2019). The van Hiele model described five sequential levels of geometric thinking (visual, analysis, informal deduction, deduction, and rigor) that students go through when becoming proficient in geometry (van Hiele, 1999). Several studies confirmed that the van Hiele levels of geometric thinking scheme were a valid indicator of the achievement in school geometry (e.g., Burger & Shaughnessy, 1986; Senk, 1989; Usiskin, 1982). Not only did the van Hieles focus on describing students' cognitive development regarding geometry but also suggested teaching strategies to support this development. Instruction that supports the development of the van Hiele levels of geometric thinking should consist of five learning phases, which are inquiry, direct orientation, explication, free orientation, and integration. Students can pass through one level to the next if instruction based on these phases is provided (Ozkan & Oner, 2019).

Results of a great deal of studies have shown that GeoGebra has significant effect on Van Hiele geometry understanding level of students. For instance, Kutluca (2013) found out from his study that GeoGebra instruction employed on the experimental group was better on increasing Van Hiele geometry thinking levels of students than traditional approach of teaching circle. He indicated that GeoGebra helped students in creating their own geometric shapes, testing and constructing their own knowledge. GeoGebra, as both teaching and learning tool, also helped the teachers to change their classroom to an investigative environment whereby students were actively involved in the instructional process. More so, students learning under such environment were able to contribute their thoughts at ease, argue the results with colleagues and make their individual understanding about Geometry (Kutluca, 2013). It is obvious from Kutluca's (2013) study that when GeoGebra is fully utilised in the classroom, it will enhance better teaching and learning.

Also, Bhagat and Chang (2015) used quasi-experimental research design to survey ~~the~~ effect of using GeoGebra, on student's Mathematics attainment in learning

Geometry" among fifty students divided into an experimental and a control group. The experimental group was taught using GeoGebra while students in the control group were instructed through traditional teaching approach. They observed difference in the mean achievement scores of students taught with GeoGebra and that of students taught with traditional method. It was again revealed that students' cognitive and visualization skills improved tremendously. Again, GeoGebra facilitated the learners in the demonstration of mathematical ideas in diverse ways, which can influence students to learn Mathematics. It is clear from the study of Bhagat and Chang (2015) that teaching and learning Geometry with GeoGebra, helped students to improve their reasoning, visualization skills and representation of mathematical concepts in diverse ways. Notwithstanding, researches indicated that GeoGebra has a positive effect on students' mathematics achievement on geometry concepts covered in the mathematics curriculum (Bilgici & Selçik, 2011; Doktoroğlu, 2013; İçel, 2011).

Reviewing the studies above, it is seen that the previous studies of GeoGebra on Van Hiele geometry understanding levels of students conducted have yield positive effects. Therefore this study aim at delving into the effect of using GeoGebra on Van Hiele geometric thinking levels of senior high technical school students' learning attainment of geometry.

1.2 Statement of the Problem

The problem confronting the research is the persistent mass failure in geometry questions at the secondary school level. Senior high technical school have difficulties identifying properties of shapes, identifying similarities and differences among shapes and solving problems relating to concepts of shapes. Many students are quite unsuccessful in geometry. According to Tay & Mensah-Wonkyi (2018), most senior high school students are unable to construct, visualize and justify geometrical concepts due to traditional approach of teaching and learning process in Ghanaian classrooms. This method of teaching makes students passive listeners and deficient in geometrical analysis and reasoning (Mereku, 2010). For this reason, students are not encouraged to discuss, interact with each other and to explore the content collaboratively, and repeatedly fail to build the exploration and visualization skills demanded for geometrical ideas, geometry reasoning and problem-solving skills (Battista, 2009).

Adegun and Adegun, (2013) stated that students in general have difficulties in solving geometry tasks and their performance is always poor in the senior high school mathematics exercises or tests. For example, in the fall only 52% of the students could calculate the area of a square given its sides (Usiskin, 1987). Usiskin (1987) further states that of learners who enrolled in geometry, only 63% of them are able to correctly identify triangles that are presented along with properties. Therefore learners' performance in identifying common geometric shapes is a matter of concern in many countries. This is supported by Clements and Battista (2007) who showed that learners' performance in dealing with properties of shapes, visualisation of shapes and applications was poor. For instance, only 10% of grade 7 learners could find the area of a square and less than 90% of them could identify a triangle. It is of concern that the number of secondary school learners who enrol in geometry is relative small compared to other subjects in schools.

However, Adolphus (2011) found out that many students fail to understand the major geometrical concepts and leave mathematics classes without acquiring the basic skills.

The WAEC Chief Examiners' Report (2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017 and 2018), confirmed that candidates had weaknesses in mensuration, construction, and circle theorems. All these make students perform woefully in the examinations (Mifetu, 2019). In addition, students perform poorly in WASSCE and this is shown in Table 1.1

Table 1.1: percentage mean of students obtaining grades A1-C6 in WASSCE

Year	2006	2007	2008	2009	2011	2012	2013	2014	2015
Qualifying rate (%)	32	25	26	29	44	50	36.8	34.2	25.3

Source: (MoE, 2012, p. 22; 2013, p. 61; GNA, 2015, March 16; Giovanni, 2015, August 10, Blog).

Although the mean performance of the students in the WASSCE improved from 2008 to 2012, that of 2013 dropped. This poor performance of students in mathematics has been a thing of concern to mathematics educators, parents and governments (Adolphus, 2011). The WAEC Chief Examiner's Report (2005) suggested that students' performance in mathematics could be improved through meaningful and proper teaching strategies. The integration of the Computer in the classroom especially with Mathematics software like GeoGebra could enable students to produce quick calculations and assist them in abstracting Mathematical concepts.

There have been concerns raised about the levels of students' geometric thinking in Ghanaian schools, especially at the basic and secondary school level (Anamuah-Mensah and Mereku (2005); Anamuah- Mensah, Mereku and Asabere-Ameyaw, (2008); Baffoe and Mereku, (2010). In Addition, the West African Examination Council (WAEC) Chief Examiners annual reports for the SSSCE & WASSCE from 2003 to 2006 observed that candidates were weak in Geometry of circles and 3-dimensional problems. According to their reports, most candidates avoided questions on 3- dimensional problems, where they attempted geometry questions; only few of the candidates showed a clear understanding of the problem in their working. Students' mathematical competencies have been closely linked to their levels of geometric understanding (Van Hiele, 1986; French, 2004). The van Hiele theory has been applied to many curricula to improve geometry classroom instruction in many developed nations but in Ghana, the literature appears to suggest that there has been little investigation involving the van Hiele theory. A number of studies have been carried out to investigate students understanding in mathematics in Ghana (Anamuah-Mensah & Mereku, 2005; Anamuah-Mensah, Mereku & Asabere-Ameyaw, 2008; Baffoe & Mereku, 2010). These studies have reported nothing but the abysmal performance of students especially in the field of geometry. The study by Baffoe and Mereku (2010) specifically sought to find out the stages of the van Hiele levels of understanding Ghanaian students reach in the study of geometry before entering senior high school. Results from the study indicated that the stage of the van Hiele level of understanding reached by most (i.e. over 90%) Ghanaian students before entering senior high school is lower than what most students at this stage reach in other countries in the study of geometry.

Despite the widespread application of the Van Hiele theory to improve mathematics curricula in many Western countries, only a few have utilized this model in an African context. My literature research indicates that there has been little investigation involving using Geogebra on students Van Hiele geometric thinking level in Ghana. And as far as I have been able to ascertain, very few studies have applied GeoGebra as an instructional tool on students Van Hiele geometric thinking level to determine the level of geometric conceptualization of Ghanaian high school students. In acknowledging the difficulties by Ghanaian students with geometry, and affirming the relevance of GeoGebra on Van Hiele model in ameliorating these difficulties, Baffoe and Mereku, (2010) for example, asserted that unless we embark on a major revision of the primary school geometry curriculum along Van Hiele lines, it seems clear that no amount of effort at the secondary school will be successful.

Personal experience as a mathematics teacher had shown that the conditions available for Ghanaian students at the senior high technical school level does not allow them to explore geometric concepts and shapes prior to the course in geometry. It would seem necessary first to determine the van Hiele geometric thinking levels of senior high school students and the kind of instruction tool teachers' use in teaching geometry. I observed that mathematics is one of the most poorly taught, widely hated and abysmally understood subject in the technical schools and students run away from the subject. Since mathematics at the Tertiary level builds on the knowledge and competencies developed at the SHS level, it is important for teachers to use effective instructional tool such as GeoGebra in teaching mathematics especially geometry and to identify the geometric thinking levels of students leaving senior high school. However, the assessment tools used in our classrooms do not provide comprehensive description of our students' geometric thinking levels in order for teachers to plan appropriate interventions. Tay and Mensah-Wonkyi (2018) pointed that, challenges teachers faced in teaching Geometry is due to lack of resources to teach Geometry, its abstract nature and inability of students to visualize geometrical images. Therefore, it is the aim of this study to determine the effects of using GeoGebra as an instructional tool on van Hiele's geometric thinking levels of senior high technical school students in learning attainment of geometry. This will enable students visualize geometric images in GeoGebra interface and discover properties about geometry.

1.3 Purpose of the Study

The purpose of the study was to investigate the effect of using GeoGebra on Van Hiele geometric thinking levels of senior high technical school students' learning attainment of geometry.

1.4 Objectives of the Study

The objectives of the study sought to;

(1). Explore the effect of using GeoGebra on senior high technical school students' performance in geometry.

(2). Explore the effectiveness of using GeoGebra on senior high technical school students' van Hiele level of geometric thinking.

(3). Explore the effect of the use of GeoGebra on senior high technical school students' motivation to learn geometry.

(4). Examine senior high technical school students' perception of using GeoGebra in learning of geometry.

1.5 Research Questions

The following questions were formulated as a guide to the study.

(1). What is the effect of using GeoGebra on senior high technical school students' performance in geometry?

(2). How effective is the use of GeoGebra on senior high technical school students' van Hiele level of geometric thinking?

(3). How does the use of GeoGebra motivate senior high technical school students' to learn geometry?

(4). What are the senior high technical school students' perceptions of using GeoGebra in learning of geometry?

1.6 Hypotheses

In order to answer research question 2, the following hypotheses below was formulated and tested:

H_0 : There is no statistically significant difference in senior high technical school students van Hiele geometric thinking levels after GeoGebra instruction.

H_0 : There is no statistically significant difference between the pre-VHGT and post-VHGT scores of senior high technical school students‘.

1.7 Significance of the Study

The finding of this study would provide information to teachers about students‘ understanding and learning processes when using the GeoGebra in relation to the geometry topic in mathematics. Findings of this research can as well be used as a suggestion about technology use in mathematics classrooms. In this way, it can help students by providing them with permanent and effective learning of mathematics. Therefore, this study will enable teachers to identify different pedagogies in teaching concepts in geometry and how to enable their students understand as well as apply the concepts in other areas of their studies. This creativity when extended by mathematics teachers to other topics in mathematics will help improve student‘s performance in the subject tremendously.

The findings of this study will also provide information on the importance of the Van Hiele Model of Learning in Geometry as an alternative tool to supplement other assessment procedures used in Ghana. This will also enable curriculum developers and teachers to help students in the Central Region of Ghana specifically on how the use of the Van Hiele Model of Learning in Geometry improves students‘ geometric thinking. Again, the findings of the study would provide relevant literature to other researchers who wish to research into the use of GeoGebra in teaching concepts in geometry. In addition, the study would serve as base for organizing in-service training courses for teachers who teach mathematics at the senior high schools. The approach that will be adopted by the researcher will serve as the basis for those who will in future wish to research more into the problem.

1.8 Delimitations of the Study

In order to work successfully within the limited time frame available, the study was limited to senior high technical school students in Abura Asebu Kwamankese in the Central Region of Ghana. The focus of the study was also restricted to only SHS 2 students because the first year students had not been taught topics in geometry in the SHS mathematics syllabus.

1.9 Limitations of the Study

One major problem faced initially was the difficulty in having access to the students in responding to the instrument since they were always busy with their class teachers. The distance between the schools was far, so a lot of travelling was done in collecting data, so the instrument was not responded to at the same time. At times the intended students who were to respond to the instrument do not come to school and they have to be chased several times. This had extended the time projected for the completion of the study.

Finally, the insufficient duration of this study produces an obvious limitation. The three consecutive lessons in this study definitely cannot achieve a very convincing result. A more lengthy study may produce persuasive results in examining the effectiveness of using GeoGebra on van Hiele geometric thinking levels of students.

Nevertheless, it could not have any significant effect on the data collected for the study.

1.10 Operational Definition of Terms

The following terms are used throughout the research report and they are defined here to establish a clearer and concise meaning.

GeoGebra: In this study, GeoGebra is a kind of computer software that enables students and teachers to visualize geometric figures and shapes, explore geometric relationships and concepts, which enhance the teaching and learning of geometry concept.

Traditional Methods: The study refers to traditional methods as students being passive learners and note-takers in the learning environment.

Learning Attainment: In this study, learning attainment refers to the experience students have in using GeoGebra to solve geometry problems in the classroom.

Perception: In this study, perception refers to what students belief and have in mind about the instructional tool (GeoGebra) used in teaching geometry which motivate him/her to learn the geometry concept.

Van Hiele Geometric Thinking Levels: In this study, a van Hiele geometric thinking level refers to how students reason about shapes and other geometric ideas in a hierarchy manner.

Learner Performance: According to Bell (2004), learners‘ performance refers to how well the learner meets standards set by teachers. Learners‘ performance refers to the ability of a learner to demonstrate knowledge by participating in class work and homework, writing test, making presentation and participating in discussion (Wesslen

& Maria, 2005). In this study, learner performance means a concept in learning closely- related to that of academic performance.

1.11 Organization of the Study

The study was organized systematically in five chapters. In Chapter One, the background of the study, statement of the problem, purpose of the study, objectives of the study, research questions, and significance of the study, delimitation, and limitations of the study, operational definition of terms and the organizational plan were presented. The theoretical framework and relevant literature review were presented in Chapter Two. The researcher described the research design and methodology in Chapter Three. Results and discussion were done in Chapter Four. Chapter Five consisted of summary of the study and key findings, conclusion and implications for practice, recommendations, and areas for further research.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

This chapter primarily focused on varied views on what other authors have written concerning the topic under study. The literature review focused on the theoretical framework of the study, technology usage in mathematics education, perception of using GeoGebra in learning of geometry, teaching geometry with GeoGebra, van Hieles' geometric thinking levels of students' with technology in geometry, effectiveness of GeoGebra on students achievements, students motivation to learn geometry and research gap.

2.1 Theoretical Framework of the Study

The study was underpinned by APOS Theory and Van Hiele Theory.

2.1.1 APOS theory

The APOS Theory was developed by Dubinsky and McDonald (2001). This theory was developed in line with constructivist theories, advocating that an individual needs to construct the necessary cognitive structures in order to make sense of mathematical concepts. APOS Theory arose out of an attempt to understand the mechanism of reflective abstraction, introduced by Piaget (1985) to describe the development of logical thinking in children, and extend this idea to more advanced mathematical concepts (Dubinsky & McDonald, 2001). This work has been carried on by a small group of researchers called a Research in Undergraduate Mathematics Education Community (RUMEC) who have been collaborating on specific research projects using APOS Theory within a 5 broader research and curriculum development framework. The framework consists of essentially three components: a theoretical analysis of a certain mathematical concept, the development and implementation of instructional treatments (using several non-standard pedagogical strategies such as cooperative learning and constructing mathematical concepts on a computer) based on this theoretical analysis, and the collection and analysis of data to test and refine both the initial theoretical analysis and the instruction. This cycle is repeated as often as necessary to understand the epistemology of the concept and to obtain effective pedagogical strategies for helping students learn it. The theoretical analysis is based initially on the general APOS theory and the researchers' understanding of the mathematical concept in question. After one or more repetitions of the cycle and revisions, it is also based on the fine-grained analyses described above of data obtained from students who are trying to learn or who have learned the concept. The theoretical analysis proposes, in the form of a genetic decomposition, a set of mental constructions that a student might make in order to understand the mathematical concept being studied.

The APOS Theory states that an individual mathematical knowledge is his or her tendency to respond to perceived mathematical problem situations and their solutions by reflecting on them in a social context and constructing or reconstructing mathematical actions, processes and objects and organising these in schemas to use in dealing with the situations (Dubinsky, 1994). In reference to these mental constructions the theory was called APOS Theory (Dubinsky & McDonald, 2001). The ideas arise from an attempts to extend the level of collegiate mathematics learning the work of J. Piaget on reflective abstraction in children's learning. According to the theory, individuals tend to deal with mathematical situations by constructing mental actions which they transform into processes and objects, as well as the organization of schemas in their attempts to make sense of problems and to be able to solve presented situations (Dubinsky & McDonald, 2001). The APOS theory was presented by Dubinsky and McDonald (2001) and it consist of four components summarized below:

1. Action: Transformation of objects perceived by an individual in reaction to stimuli. An action requires that each step be taught and performed explicitly. An example can be of a student finding an equation to link the relationship between the face, edges and vertices of shapes in geometry but not being able to perceive the relationship without the equation. This is referred to as the action stage where the student can only perceive and react to external stimuli in the form of what is taught or learnt.

2. Process: Occurs when an individual repeats the action stage. As the student continues to repeat and reflect on the action, even in the absence of external stimuli, the action becomes interiorized in the mind to become a mental structure. The mental structure is referred to as a process. The student can now construct mental processes with regards to the transformations and shifts that can be applied to the basic shapes. A student at this stage is now able to apply the information learnt previously during the process of solving problems.

3. Object: The action stage and the process of constructing mental structures help the student to view action and process in totality, not individual entities leading to transformations of one‘s imaginations. The student encapsulates the process into a cognitive object. For example, in shapes the student can now confront questions of a higher order that draw upon the mental structures formed during the action and process stages.

4. Schema: The result of actions, processes and objects, being organized in order to form a clear framework. When solving mathematical problems, a learner should be in a position to decide on the appropriate schema to use. This is only possible if the student has constructed clear and coherent schemas. For instance, in geometry students are only able to solve higher order questions if they have been able to create their own understanding of concepts without always relying on external stimuli. This study focused on the effects of using GeoGebra on Van Hiele‘s geometric thinking levels of senior high technical school students learning attainment of geometry.

The four components, action, process, object, and schema have been presented here in a hierarchical, ordered list. This is a useful way of talking about these constructions and, in some sense, each conception in the list must be constructed before the next step is possible. In reality, however, when an individual is developing his or her understanding of a concept, the constructions are not actually made in such a linear manner. APOS Theory can be used directly in the analysis of data by a researcher. In very fine grained analyses, the researcher can compare the success or failure of students on a mathematical task with the specific mental constructions they may or may not have made. If there appear two students who agree in their performance up to a very specific mathematical point and then one student can take a further step while the other cannot, the researcher tries to explain the difference by pointing to mental constructions of actions, processes, objects and/or schemas that the former student appears to have made but the other has not. The theory then makes testable predictions that if a particular collection of actions, processes, objects and schemas are constructed in a certain manner by a student, then this individual will likely be successful using certain mathematical concepts and in certain problem situations.

Several studies that are guided by APOS theory have been carried out locally and elsewhere in the world. Demir (2012) studied learners‘ concept development and understanding of sine and cosine functions in a study conducted at pre-university level (VWO) at a Dutch secondary school in Amsterdam with a class of 24 learners whose ages ranged from 16 to 17. The study investigated a new theoretical and educational approach. Results showed that the new approach, which was based on the implemented learning curve, was effective in promoting understanding of trigonometric functions. Brijall and Maharaj (2009) cited in Jojo (2011), used APOS theory when they investigated fourth-year undergraduate teacher trainee students' understanding of the two fundamental concepts, monotonicity and boundedness of infinite real sequences at a South African University. As conclusion to their study, they found that structured worksheets promoted group work and created an environment that is conducive to abstract thinking and that the learners were able to use symbols, language and mental images to make constructions of internal processes during the process of understanding the monotonicity and boundedness of sequences.

In consonance with APOS theory, the researcher believed that the technology (GeoGebra) can help students construct mental actions which they can transform into processes and objects, and organization of schemas, thereby constructing an understanding of mathematical knowledge. The mathematical understanding will eventually translate into improved achievement in mathematical exercise. The APOS theory will help teachers to use the technology (GeoGebra) that could stimulate students to go through the series of actions and processes so as to objectively construct their own schemas. Students continue to go back and forth as they construct their own knowledge based on the experience provided by the technology (GeoGebra). Moreover, the APOS theory helps to promote the development of an inquisitive mind which seeks to explore and achieve a deeper understanding of the concepts being learnt. Thus, applying this theory in this study will help teachers to effectively utilise GeoGebra as an instructional tool in teaching geometry to enhance students understanding in learning geometry.

2.1.2 Van Hiele theory

The van Hiele Theory was developed by two Dutch mathematics educators in separate doctoral dissertations at the University of Utrecht in 1957, Pierre Marie van Hiele, and his wife Dina van Hiele-Geldof (Akgul, 2014). The van Hieles‘ were disappointed with learners‘ low level of knowledge in geometry and were also concerned about their own failure to communicate ideas successfully during their time as mathematics teachers (Kekana, 2016). They were two Dutch teachers who experienced challenges with regard to their learners‘ lack of understanding of geometric concepts, which explains their interest in investigating ways that could help learners to understand geometric concepts better (Kekana, 2016).

In 1957 the van Hieles‘ proposed a five level scale according to which learners could be assisted to progress from one level to the next and they described the geometric thinking at each level. Pierre and Dina took different angles in their respective research studies: Pierre designed the base model from a learning perspective and described in detail five ascending levels of geometric understanding, thought or development. These levels were originally numbered from zero to four (Kekana, 2016) and were described using abstract nouns: Level 0 is Visualisation or recognition; Level 1 is analysis; Level 2 is informal deduction or abstraction; Level 3 is formal deduction; and Level 4 is rigor (Knight, 2006). Dina‘s research was done from a teaching perspective and focused on the process of helping learners to progress by describing five teaching phases, the first phase is inquiry, the second phase is direct orientation, the third phase is explication, the fourth phase is free orientation and the fifth phase is integration (Pusey, 2003). These teaching phases each corresponded with a learning level, or a level of development in geometric thinking. The combined model later became known as _the van Hiele‘ levels‘ for short.

Teaching based on the van Hieles‘ model is widely acclaimed as being effective to motivate learners and to create a better environment for teaching and learning of geometry (Abu & Abidin, 2013).

The van Hieles‘ theory states that learner‘s progress sequentially from one level to the next level by working through instructional activities that are applicable in terms of dialectal and mission for their level of understanding (Connoly, 2010). The theory has been applied to explain why many students have difficulty with the higher order cognitive processes, particularly proof, required success in high school geometry (Akgul, 2014). The van Hieles theorized that students who have trouble are being taught at a higher van Hiele level than they are at or ready for. The theory outlines the hierarchy of levels through students‘ progress as they develop geometric ideas. Put it differently, the van Hiele model explains the stages of human geometric reasoning. The theory also offers a remedy: go through the sequence of levels in a specific way (Akgul, 2014). Van Hiele Levels are sequential and progress from one level to another depends more on the content and method of instruction than on age or biological maturation. The van Hieles‘ theory has three aspects: the existence of levels of understanding, properties of the levels and the movement from one level to the next (Usiskin 1982; Knight 2006; Vojkuvkova, 2012).

2.1.2.1 Existence of levels: According to the theory, there are five levels of understanding in geometry. These levels are described by the van Hieles‘ in various places in both general and behavioural terms. Summary of general descriptions and examples are;

Level 1: (recognition)

At this level students use visual perception and nonverbal thinking. They recognize geometric figures by their shape as ―a whole" and compare the figures with their prototypes of everyday things (―it looks like door”), categorize them (―it is / it is not a…”). They use simple language. They do not identify the properties of geometric figures.

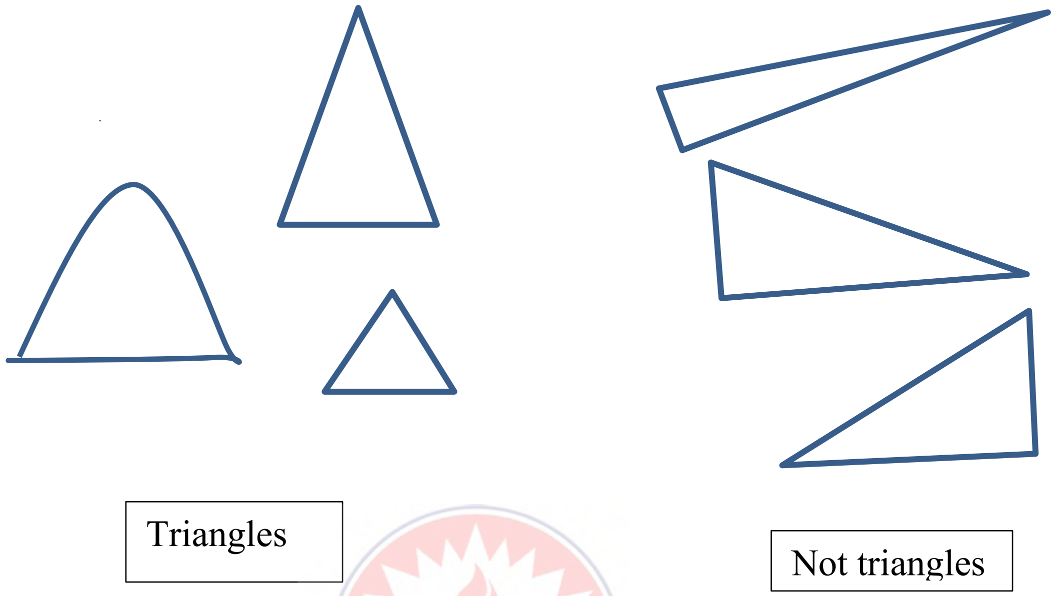


Figure 2.1: Students at level 1 categorise triangles

The student can learn names of figures and recognizes a shape as a whole. (Squares and rectangles seem to be different)

Level 2: (analysis)

At this level student start analysing and naming properties of geometric figures. They do not see relationships between properties, they think all properties are important (= there is no difference between necessary and sufficient properties). They do not see a need for proof of facts discovered empirically. They can measure, fold and cut paper, use geometric software etc.

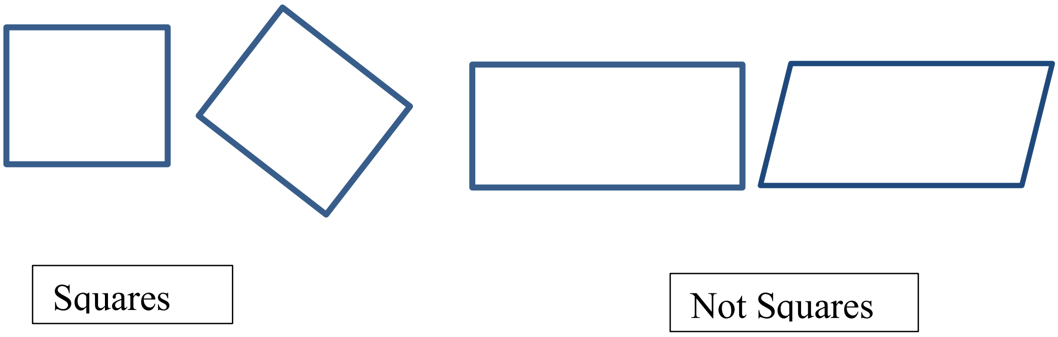
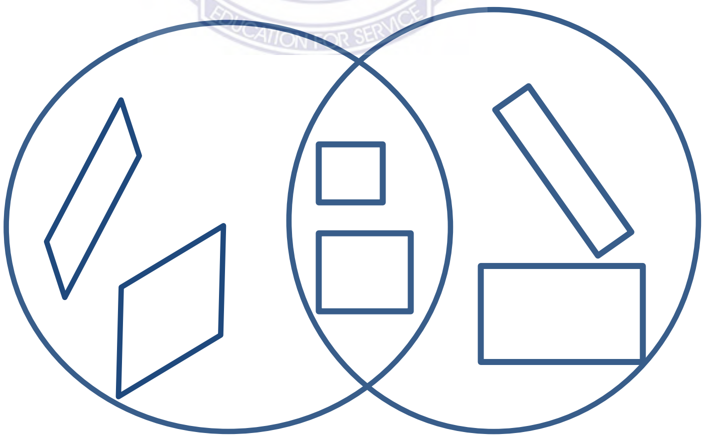


Figure 2.2: Students at level 2 identify only one of the properties of squares

The student can identify properties of figures. (Rectangles have four right angles)

Level 3: Abstraction (Informal deduction or Ordering or Relational)

At this level students perceive relationships between properties and figures. They create meaningful definitions. They are able to give simple arguments to justify their reasoning. They can draw logical maps and diagrams. They use sketches, grid paper, etc. The student can logically order figures and relationships but does not operate within a mathematical system (Simple deduction can be followed but proof is not understood).



Logically Ordered relationships

Figure 2.3: Student at Level 2 can draw a logical map of parallelograms

Pierre van Hiele wrote: ―My experience as a teacher of geometry convinces me that all too often, students have not yet achieved this level of informal deduction.

Consequently, they are not successful in their study of the kind of geometry that Euclid created, which involves formal deduction.

Level 4: (deduction)

The student understands the significance of deduction and the roles of postulates, theorems and proof. (Proofs can be written with understanding)

Level 5: (rigor)

The student understands the necessity for rigor and is able to make abstract deductions (Vojkuvkova, 2012).

2.1.2.2 Properties of levels: It is inherent in the van Hiele theory that, in understanding geometry, a person must go through the levels in order. We call this fixed sequence property of the levels.

Property 1: (fixed sequence)

A student cannot be at van Hiele level *n* without having gone through level *n-1*.

Property 2: (adjacency)

At each level of thought what was intrinsic in the preceding level becomes extrinsic in the current level.

Property 3: (distinction)

Each level has its own linguistic symbols and its own network of relationships connecting those symbols.

Property 4: (separation)

Two persons who reason at different levels cannot understand each other

Property 5: (Attainment)

The learning process leading to complete understanding at the next level has five phases – information, guided orientation, explanation, free orientation, integration, which are approximately not strictly sequential (Vojkuvkova 2012).

To exemplify these properties, consider the student who remarks to a geometry teacher, ―I can follow a proof when you do it in class but I cannot do it at home". This student may be at level 3 while the teacher is operating at level 4. Property 4 indicates that the student cannot understand the teacher and property 3 explains why there is no understanding, because the teacher is using objects (propositions, in the case of proof) and a network of relationships (proof itself) which the student does not understand the proof used in this way. If the student is at level 3, then the student's network consists of simple ordering of propositions and property 2 indicates that these orderings, intrinsic at level 3 become extrinsic at level 4 (Simbarashe, 2017).

2.1.2.3 Movement from one level to the next

Van Hieles believed that cognitive progress in geometry can be accelerated by instruction. The progress from one level to the next one is more dependent upon instruction than on age or maturity. They gave clear explanations of how the teacher should proceed to guide students from one level to the next. However, this process takes tens of hours.

2.1.2.3.1 Information or inquiry

Students get the material and start discovering its structure. The teacher holds a conversation with the pupils, in well-known language symbols, in which the context he wants to use becomes clear. (A teacher might say: ―This is a rhombus. Construct some more rhombi on your paper.")

2.1.2.3.2 Guided or directed orientation

Students deal with tasks which help them to explore implicit relationships. The teacher suggests activities that enable students to recognize the properties of the new concept. The relations belonging to the context are discovered and discussed. (A teacher might ask: What happens when you cut out and fold the rhombus along a diagonal? Along the other diagonal?)

2.1.2.3.3 Explanation or explication

Students formulate what they have discovered, and new terminology is introduced. They share their opinions on the relationships they have discovered in the activity. The teacher makes sure that the correct technical language is developed and used. The van Hieles thought it is more useful to learn terminology after students have had an opportunity to become familiar with the concept. (A teacher might say: ―Here are the properties we have noticed and some associated terminology for the things you have discovered. Let us discuss what these mean: The diagonals lie on the lines of symmetry. There are two lines of symmetry. The opposite angles are congruent. The diagonals bisect the vertex angles.")

2.1.2.3.4 Free orientation

Students solve more complex tasks independently. It brings them to master the network of relationships in the material. They know the properties being studied, but they need to develop understanding of relationships in various situations. This type of activity is much more open-ended. (A teacher might say: ―How could you construct a rhombus given only two of its sides?" and other problems for which students have not learned a fixed procedure.)

2.1.2.3.5 Integration

Students summarize what they have learned and keep it in mind. The teacher should give to the students an overview of everything they have learned. It is important that the teacher does not present any new material during this phase, but only a summary of what has already been learned. (A teacher might say: ―Here is a summary of what we have learned. Write this in your notebook and do these exercises for homework.") (Vojkuvkova 2012).

One of the first major studies on the van Hiele Theory was performed by Usiskin (1982, as cited in Akgul, 2014). Usiskin developed a multiple-choice test to measure students' van Hiele Geometric Thinking Levels and this test has been widely used by other researchers. Usiskin developed this test to find out if the test could predict students' achievement in geometry. He tested 2900, 10th graders and looked for a correlation between their van Hiele Geometric Thinking Levels and Geometry Achievement. The study results indicated that there was a moderately strong correlation (r=.64) between the subjects' Geometry Achievement and van Hiele Geometric Thinking Level. The study results also revealed that the students were generally at Level 0 or Level 1, hence, most of the students were not ready for high school geometry.

Thus, applying the van Hiele theory to this study provides the researcher with a thoughtful a framework within which to conduct geometric activities. Since, the van Hiele theory does not specify content or curriculum but can be applied to most activities and these activities are in sequential levels (Vande Walle, 2001). This helped the researcher to design most geometric activities at a particular level and then be raised or lowered by means of the types of questioning and guidance provided. The van Hiele theory helped the researcher to understand that to arrive at any level above level 0, students must move through all prior levels. To move through a level means that one has experienced geometric thinking appropriate for that level and has created in one's own mind the types of objects or relationships that are the focus of thought at the next level.

In addition, the van Hiele‘s theory is a developmental approach to instruction, which demands that we listen to children and begin where we find them. The van Hiele theory highlights the necessity of teaching at the child‘s level. However, almost any activity can be modified to span two levels of thinking, even within the same classroom (Vande Walle, 2001). Thus, listening to students and beginning from where we find them helps teachers respect the responses and observations made by students that suggest a lower level of thought while encouraging and challenging students to operate at the next level. This helps teachers to remember that it is the type of thinking that students are required to do that makes a difference in learning, not the specific content.

2.1.3 Fusion of van Hiele and APOS theories for this study

This study jointly used Van Hiele and APOS theories to investigate the effect of using GeoGebra on van Hiele‘s geometric thinking levels of senior high technical school students‘ learning attainment of geometry. Although the theories were propounded at distinctively different times (Van Hiele theory, published in 1957 and APOS theory, published in 1984), they have since made similar contributions to the field of educational instruction and hence form the bases of this study.

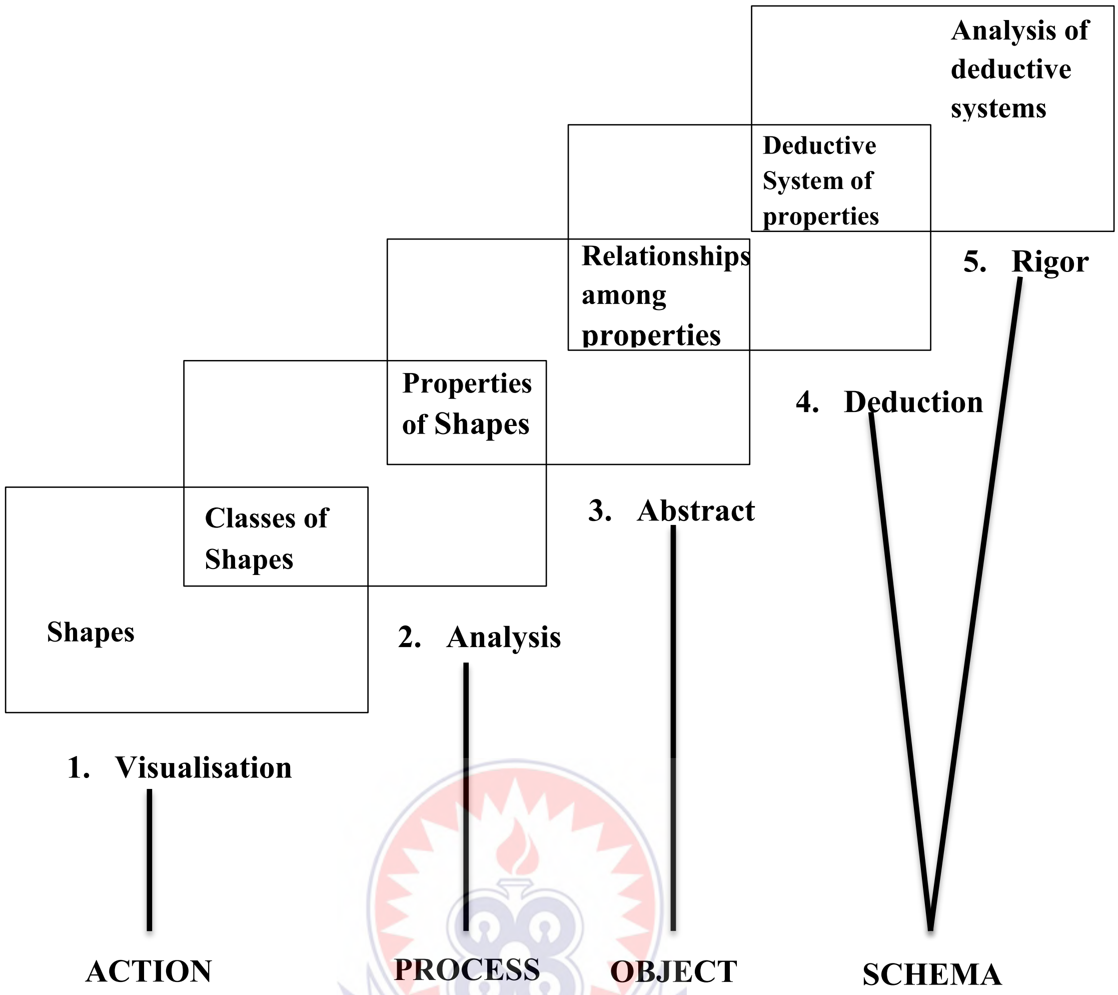


Figure 2.4: Theoretical framework: Fusion of Van Hiele and Apos theories

As illustrated in Figure 2.3, the five Van Hiele levels and the four APOS levels overlap, for example, although actions are directly related to visualisation, they are also related to the analysis level, and this overlapping applies to all the other levels. In some cases, at each Van Hiele level it is possible to achieve all four APOS theory levels, but for the purpose of this study, the relationship between Van Hiele theory and APOS theory is as shown in Figure 2.3. Both APOS and Van Hiele theories are rooted in the learning theory of constructivism, in which learning is viewed as an active, contextualised process of constructing knowledge rather than the acquisition of knowledge (Devries & Zan, 2003). These two theories were deliberately selected as the joint theoretical framework because of their relevance to the teaching and learning process of geometry; the Van Hiele theory was used as a framework to analyse the learners' levels and/or stages that they go through when engaged in geometry problem-solving and APOS was used as the general guideline to the research process.

2.2 Technology Usage in Mathematics Education

The use of technology in the learning environment not only helps education for maintaining in accordance with the necessities of the era, but also provides individuals with opportunities for growing adequately (Ersoy, 2003). The power of new technologies as one of the strongest forces in the contemporary growth and evolution of mathematics and math teaching are technology and technological advances which obviously affect how we learn and teach mathematics (Goldenberg, 2000). Moreover, the traditional methods used in classrooms remain insufficient in terms of meeting all the criterion of a quality teaching and learning of mathematics (Alakoç, 2003). It is the common viewpoint of educators that the existing problems related to the teaching cannot be solved by using the traditional teaching methods (Aktüment & Kaçar, 2003). Akgul (2014) stated that, the role of instruction is crucial in teaching and learning geometry. The more systematically structured the instruction, the more helpful it will be for middle school students to overcome their difficulties and to increase their understanding of geometry. Hence, the common opinion of many researchers, mathematics teachers, and studies focus on the notion that the novelties in mathematics education and technology integration into mathematics education support students' understanding of mathematics, and they suggest the use of technology in mathematics classrooms (Hollebrands, 2003).

Furthermore, the mathematics education researchers have a parallel interest in investigating the effect of technology on learning and teaching mathematics, and the curriculum. Technology tools provide powerful range of visual representations which help teachers to focus students' attention to mathematical concepts and techniques (Zbiek, Heid, Blume & Dick, 2007). Thus, technological tools, such as Computers, Graphic Calculators, Interactive White Boards, Web-Based Applications, and Dynamic Mathematics/Geometry Softwares have started to be widely used in mathematics classroom and many studies investigated to determine the effectiveness of technology in mathematics education (Baki, 2001; Borwein & Bailey, 2003; Doğan, 2012; Ersoy, 2003; Hollebrands, 2003; Koehler & Mishra, 2005; Lester, 1996; NCTM, 2000). Technology use not only plays a crucial role in mathematics education, but also helps mathematics educators to better capture the attention of the students and provide students with better understanding of mathematics and mastering the mathematical concepts (Khouyibaba, 2010). However, the integration of technology in the learning and teaching of mathematics requires special attention in many respects (Iranzo, 2009). Technology environments allow teachers to adapt their instruction and teaching methods more effectively to meet their students' needs (NCTM, 2008). Computers are one of the mainly used technologies in learning environments. The purpose of giving computers place in the learning environments is to grow productive, creative, successful, critical thinker, problem solver and adequate individuals in order to improve certain knowledge, skill and attitude. Thus, all of these goals may be fulfilled by utilizing the computers in the teaching learning process (Aktümen ve Kaçar, 2003). Ersoy (2003) conducted a study on the use of computers and calculators in teaching and learning mathematics to contribute in developing strategies and developments in mathematics teaching process. The results of his study showed that the students need to understand how to use technology tools in their learning experiences. When integrated properly into the teaching and learning process, computers improve student proficiency in mathematics. Through different software applications, computers reduce the cognitive load of mathematical learning (Kozma, 1987; Liu & Bera, 2005). As a supportive tool, interactive mathematics computer programs such as Geometer's Sketchpad (Jackiw, 1995) and virtual modeling and visualization tools also provide students with dynamic multiple representations and support their understanding as they interact with concepts in a variety of ways (Flores, Knaupp, Middleton, & Staley, 2002; Drier, Harper, Timmerman, Garofalo & Shockey, 2000). Additionally, students can develop and demonstrate deeper understanding of mathematical concepts and are able to cope with more advanced mathematical contents in technology-enriched learning environments than in ‗traditional' teaching environments (NCTM, 2008). Students can benefit in different ways from technology integration into everyday teaching and learning. New learning opportunities are provided in technological environments, potentially engaging 20 students of different mathematical skills and levels of understanding with mathematical tasks and activities (Hollebrands, 2007). By the help of the visualization of mathematical concepts and exploring mathematics in multimedia environments, students' understanding in a new way can be fostered. Laborde, Kynigos, Hollebrands and Strasser (2006) summarized technology use in mathematics education as following;

> *"(…) Research on the use of technology in geometry not only offered a window on students' mathematical conceptions of notions such as angle, quadrilaterals, transformations, but also showed that technology contributes to the construction of other views of these concepts. Research gave evidence of the research and progress in students conceptualization due to geometrical activities (such as construction activities or proof activities) making use of technology with the design of adequate tasks and pedagogical organization. Technology revealed how much its shapes mathematical activity and led researchers to revisit the epistemology of geometry" (Laborde et al., 2006, p. 296).*

2.3 Perception of Using GeoGebra in Learning of Geometry

Amissah and Agbeke (2015) defined perception as a process of building on our ill-defined and incomplete sensory experiences. Perception is any act or process of knowing objects, facts and truths whether by sense, experience or by thought; it is awareness of consciousness. According to Twum (2016), perception involves an interaction or transaction between an individual and his environment; the individual receives information from the external world which in some ways modifies his experience and behaviour.

Perception is a process by which an individual absorbs sensory information from the environment and utilizes such information as a means of interacting with the environment (Ibibo & Tubona, 2019). It is a way individuals perceive things around them that define their character and attitude. How students perceive a particular lesson shapes their goals and reflect on their outlook. Perception on a school subject by a student is a determinant of whether that student is happy or grossly hate that subject and this implies that students formulate their own opinion on which side to swing in terms of taking decision on a particular lesson (Ibibo and Tubona, 2019). According to Twum (2016) perception could be influenced by varieties of factors. These are cultural values, personal attitudes, expectation attitudes, expectation and motivational states.

Ibibo and Tubona (2019) investigated Students' Perception and Performance across ability levels on GeoGebra Software usage in Learning of Circle Geometry. The study was guided by two research questions and two null hypotheses which were tested at .05 alpha level. The result showed that students had a positive perception on the use of GeoGebra software for the teaching and learning of circle geometry and there was no significant difference between the perception of the male and female students on the use of GeoGebra software for the teaching and learning of circle geometry. The result also revealed that students of all ability levels benefitted from the use of GeoGebra software in the teaching and learning of circle geometry. It was concluded that the use of GeoGebra software to teach circle geometry improved students' performance of all ability levels and students have a positive perception of the use of GeoGebra software.

Adeleke, Fajemidagbai and Akanmu (2018) investigated the perceptions of secondary school students towards the use of GeoGebra instructional package in learning linear equations. The focus was on ease of use and usefulness of GeoGebra instructional package. Other variables investigated include students' attitude and behavioural intentions towards the usage of GeoGebra. Four research hypotheses were generated and tested at 0.05 alpha level of significance. The study was a developmental research involving training of a group of senior secondary school one (SS1) students and determining their perceptions about GeoGebra. Questionnaire adapted from technology acceptance model was used to collect data. The study revealed that the relationships between student's perception towards the ease of use and usefulness of instructional package, ease of use of GeoGebra instructional package and student's attitude, and student's attitude and behavioral intention of student about the use of GeoGebra instructional package were positively high (β=0.901, 0.811,0.842 &0.871) respectively. Based on the results, it was concluded that the attitude of students towards the use of GeoGebra instructional package depend on their perceptions on its usefulness and its ease of use. Finally, it was recommended that GeoGebra package should be integrated into the teaching and learning of mathematics in secondary schools.

According to Santosh (2015), ICT based tools like computer; laptop, calculator, GeoGebra etc. allow students to use graphics, images and text together, to demonstrate their understanding of mathematical concepts. So by using ICT tools such as GeoGebra in geometry lesson, students visualize the problem which helps to understand the problem and leads to a change of positive perception towards geometry lesson. Hence, the study intend to explore senior high technical school students‘ perceptions on the use of GeoGebra in learning geometry.

2.4 Effectiveness of GeoGebra on Students' Achievement

There are studies that focus on GeoGebra as a tool in teaching and the great effects of this software on students‘ achievement. Like the work of Arbain and Shukor (2015), titled: ‗the effects of GeoGebra on students‘ achievement‘, which underlines the changes coming with GeoGebra for students. The study examines the effectiveness of using GeoGebra system on Mathematics learning among 62 students in Malaysia. The outcomes demonstrate that students have positive opinions regarding their learning, including having better learning achievements by using the system (Arbain & Shukor, 2015).

In another study titled: ‗Introducing Dynamic Mathematics Software to Mathematics Teachers: The Case of GeoGebra‘, Preiner (2008) conducted a research study which provided instructions for achieving the goal of providing more successful introductory material for professional development with dynamic mathematics software, by identifying impediments teachers face when being introduced to this new technological tool. The researcher suggested that dynamic mathematics software is a very important learning tool for teaching students. These two resources provide good insight into the potential and active role of dynamic mathematics software like

GeoGebra for students as well as teachers. However, in neither of these literatures, is the visual design aspect of mathematics or UI and UX discussed.

In short, the advantage of GeoGebra software in learning and teaching mathematics is that students can have more engagement with the subject by using this technology. Most students in the 21st century are competent in using computers as such they can receive great support from technology in their learning. The use of the internet and smart devices creates a productive platform in which students can communicate, which can motivate them to simultaneously share and gain quite valuable knowledge and understanding.

In many specific studies by Korenova (2012), Yenilmez (2009) and Ozdamli, Karabey and Nizamoglu (2013) as well as Hohenwarter, Hohenwarter and Lavicza (2009), the results confirm the positive role of computers in assisting learners to understand complex concepts in mathematics, which can result in enhancing students' self-confidence and motivation. Prodromou (2014) claimed that GeoGebra software has a very constructive effect on college students' achievement in the area of statistics. The author argues that students have a remarkably positive attitude towards GeoGebra. Hence, the study intent to explore the effectiveness of GeoGebra on senior high technical school students' performance in geometry.

2.5 Teaching Geometry with GeoGebra

In teaching mathematics, Computer Algebra System (CAS) and Dynamic Geometry Software (DGS) are very well known and 'trendy'. CAS is used in teaching algebraic topics and DGS is used in teaching geometric topics. In CAS, the main focus is on the manipulation of expressions. In DGS the correlations among lines, circles and lines and the visual manipulations of shapes and forms are used as a teaching tool for various mathematical concepts (Hadadi, 2018). There is also 25 software available to teach and learn mathematics that is used as a tool to ease problem solving (Hadadi, 2018). Some of these software that have been developed to assist teaching and learning of mathematics include, GeoGebra, Geometer's Sketchpad and Mathematica. In this study, GeoGebra is used as an instructional tool to teach geometry. GeoGebra can be defined as an effective and important tool in establishing relationship between geometry and algebra concepts in elementary mathematics since it proved its capability and potential in mathematics education (Hohenwarter & Jones, 2007). Students can explore mathematics alone or in groups and the teacher tries to be a guide in the background who gives support when students need help. The students' results of their experiments with GeoGebra constitute the basis for discussions in class so that teachers can have more time to concentrate on fundamental ideas and mathematical reasoning (Akugul, 2014).

GeoGebra was created by Markus Hohenwarter in 2001/2002 as part of his master's thesis in mathematics education and computer science at the University of Salzburg in Austria. Supported by the Austrian Academy of Science he was able to develop the software as a part of his PhD project in mathematics education (Majerek, 2014). Meanwhile GeoGebra received many international awards, and was translated by mathematics instructors and teachers all over the world to more than 25 languages (Majerek, 2014).

In Hohenwarter and Preiner (2007), Hohenwarter the creator of GeoGebra explains the software as follows:

> *"Students can practice, do homework, prepare for their lessons and revise from home. It also supports multiple languages and is a great asset for classrooms that have multilingual learners. As it is an open source, its users can communicate worldwide with other users. They*

can create and share their contributions or use templates provided with the ability to customize to their needs using GeoGebra Wiki tool. There is a user forum where they can share ideas and discuss questions" (Hohenwarter & Preiner, 2007, p.126).

He also adds that: Since GeoGebra joins dynamic geometry with computer algebra, its user interface contains additional components that can‘t be found in pure dynamic geometry software. At first, the software offers two views of each object. The algebraic representation corresponds to the textual component, whereas the graphical representation adds the visual component mentioned in this principle. Secondly, a dynamic construction protocol can be opened and placed next to the graphics window. It contains the name, definition, command, and algebraic expression for each object used in the construction and provides a navigation bar to go through the construction process step-by-step. The current construction step is highlighted within the construction protocol while the corresponding object appears in the graphics window of GeoGebra. However, in order to use GeoGebra, students and teachers are required to master a certain range of basic computer skills. A screenshot from GeoGebra window is presented in Figure 2.4 below.

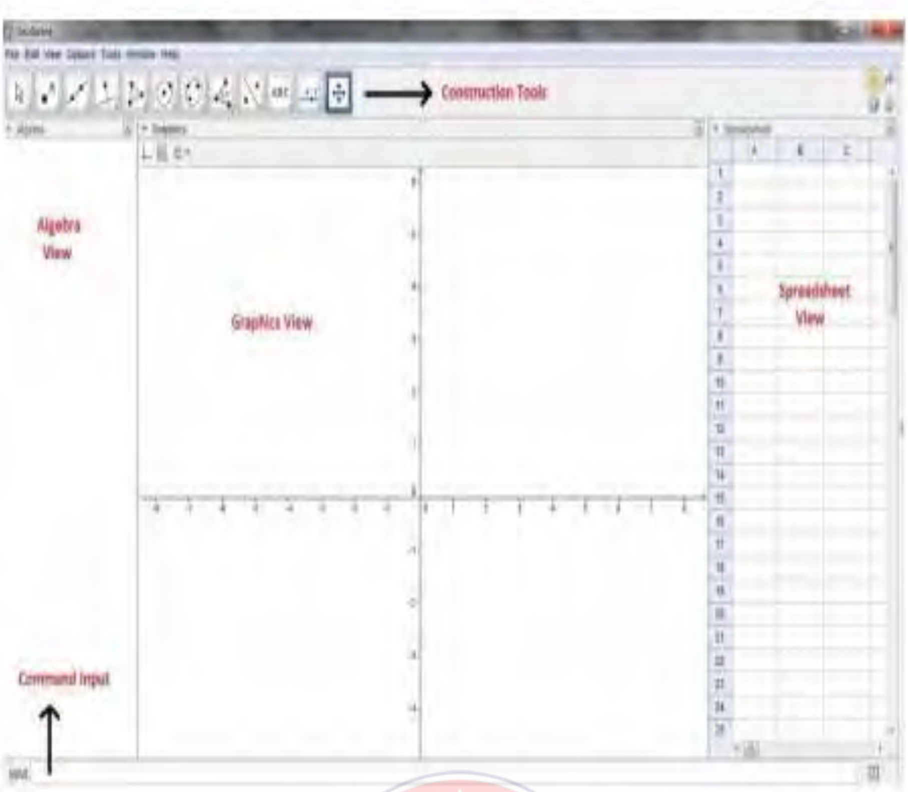


Figure 2.5: Screenshot from GeoGebra user interface

Suppose we want to construct a circle described on the triangle (Figure 2.5). We know that the centre of the circle must be an interception of bisectors. Radius of the circle is the sector from intersection of two bisectors to one of the vertex. Construction of a circle is performed in the following steps:

1. draw any triangle ABC,
2. construct two bisectors of any two sides,
3. find the interception of bisectors and mark it by D,
4. draw a circle with center in D and radius $\overline{DA}$.

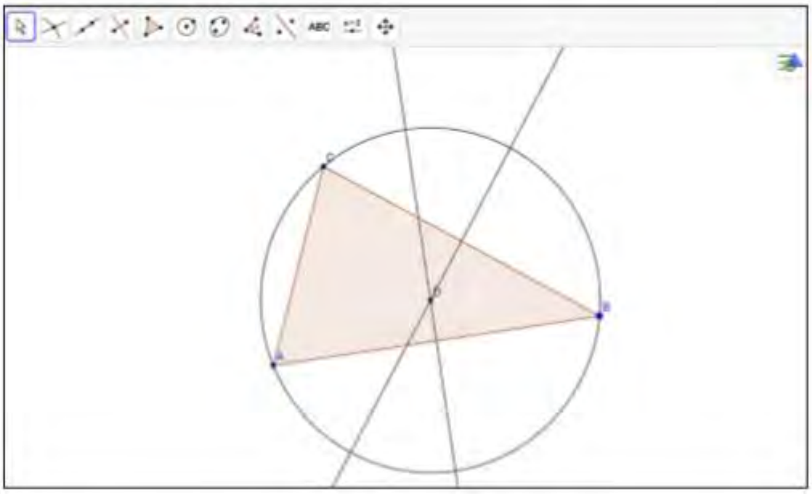


Figure 2.6: Circle described on a triangle

2.6 Van Hieles' Geometric Thinking levels of Students with Technology in Geometry

Van Hiele states that all students progress in geometrical thinking through five sequential and hierarchical levels named as the levels of Recognition, Analysis, Order, Deduction, and Rigor (Akugul, 2014). Vande Walle (2001) described the van Hiele geometric thinking levels of students as follows:

Level 1: Visualization

The objects of thought at level 1 are shapes and what they "look like." Students recognize and name figures based on the global, visual characteristics of the figure—a gestalt like approach to shape. Students operating at this level are able to make measurements and even talk about properties of shapes, but these properties are not thought about explicitly. It is the appearance of the shape that defines it for the student. A square is a square ―because it looks like a square." Because appearance is dominant at this level, appearances can overpower properties of a shape. For example, a square that has been rotated so that all sides are at a 45° angle to the vertical may not appear to be a square for a level 1 thinker. Students at this level will sort and classify shapes based on their appearances—―I put these together because they all look sort of alike." The products of thought at level 1 are classes or groupings of shapes that seem to be ―alike" (Vande Walle, 2001).

Level 2: Analysis

The objects of thought at level 2 are classes of shapes rather than individual shapes.

Students at the analysis level are able to consider all shapes within a class rather than a single shape. Instead of talking about this rectangle, it is possible to talk about all rectangles. By focusing on a class of shapes, students are able to think about what makes a rectangle a rectangle (four sides, opposite sides parallel, opposite sides same length, four right angles, congruent diagonals, etc.). The irrelevant features (e.g., size or orientation) fade into the background. At this level, students begin to appreciate that a collection of shapes goes together because of properties. Ideas about an individual shape can now be generalized to all shapes that fit that class. If a shape belongs to a particular class such as cubes, it has the corresponding properties of that class. ―All cubes have six congruent faces, and each of those faces is a square." These properties were only implicit at level 1. Students operating at level 2 may be able to list all the properties of squares, rectangles, and parallelograms but not see that these are subclasses of one another that all squares are rectangles and all rectangles are parallelograms. In defining a shape, level 2 thinkers are likely to list as many properties of a shape as they know. The products of thought at level 2 are the properties of shapes.

Level 3: Informal Deduction

The objects of thought at level 3 are the properties of shapes.

As students begin to be able to think about properties of geometric objects without the constraints of a particular object, they are able to develop relationships between and among these properties. ―If all four angles are right angles, the shape must be a rectangle. If it is a square, all angles are right angles. If it is a square, it must be a rectangle." With greater ability to engage in ―if-then" reasoning, shapes can be classified using only minimum characteristics. For example, four congruent sides and at least one right angle can be sufficient to define a square. Rectangles are parallelograms with a right angle. Observations go beyond properties themselves and begin to focus on logical arguments about the properties. Students at level 3 will be able to follow and appreciate an informal deductive argument about shapes and their properties. ―Proofs" may be more intuitive than rigorously deductive. However, there is an appreciation that a logical argument is compelling. An appreciation of the axiomatic structure of a formal deductive system, however, remains under the surface. The products of thought at level 3 are relationships among properties of geometric objects.

Level 4: Deduction

The objects of thought at level 4 are relationships among properties of geometric objects.

At level 4, students are able to examine more than just the properties of shapes. Their earlier thinking has produced conjectures concerning relationships among properties. Are these conjectures correct? Are they ―true"? As this analysis of the informal arguments takes place, the structure of a system complete with axioms, definitions, theorems, corollaries, and postulates begins to develop and can be appreciated as the necessary means of establishing geometric truth. At this level, students begin to appreciate the need for a system of logic that rests on a minimum set of assumptions and from which other truths can be derived. The student at this level is able to work with abstract statements about geometric properties and make conclusions based more on logic than intuition. This is the level of the traditional high school geometry course. A student operating at level 4 can clearly observe that the diagonals of a rectangle bisect each other, just as a student at a lower level of thought can. However, at level 4, there is an appreciation of the need to prove this from a series of deductive arguments.

Level 5: Rigor

The objects of thought at level 4 are deductive axiomatic systems for geometry.

At the highest level of the van Hiele hierarchy, the object of attention is axiomatic systems themselves, not just the deductions within a system. There is an appreciation of the distinctions and relationships between different axiomatic systems. This is generally the level of a college mathematics major who is studying geometry as a branch of mathematical science. The products of thought at level 5 are comparisons and contrasts among different axiomatic systems of geometry (Van de Walle, 2001).

Abdullah and Zakaria (2013) argue that the interventions using the GeoGebra can be applied in classrooms in order to positively and effectively improve students' thinking Levels and help students achieve better level of geometric understanding.

Ozczkir and Cakiroglu (2019) conducted a study on effects of dynamic geometry activities on seventh graders' learning on area of quadrilaterals. The purpose of the study was to investigate the effects of mathematics instruction supported by dynamic geometry activities on seventh grade students' achievements in the area of quadrilaterals, based on their van Hiele geometric thinking levels. The study followed a nonrandomized control group pretest and posttest research design. Participants of the study were 76 seventh grade students. Students in the experimental group worked in a learning environment supported by dynamic geometry software while other students worked in their conventional settings. The results of the study indicated that there is a significant interaction between the effects of method of teaching and van

Hiele geometric thinking levels on students' achievement levels in area of quadrilaterals. In addition, mathematics instruction supported by dynamic geometry activities has significant effects on seventh grade students' achievement in the area of quadrilaterals. Moreover, the results revealed that students in the dynamic geometry supported instruction group received significantly higher scores in the area of quadrilaterals than students in the traditional instruction group when students are at second level of van Hiele geometric thinking. Thus, this study investigate the effects of using GeoGebra on van Hieles' geometric thinking level of senior high technical school students.

2.7 Students' Motivation to Learn Geometry

In the effort to improve students cognition and affective outcomes in Mathematics and/or school learning, educational psychologists and Mathematics educators, have continued to search for variables that could be manipulated in favour of academic gains. Several personal and psychological variables have attracted researchers in this area of educational achievement. However, motivation seems to be gaining more popularity and leading other variables (Tella, 2007).

Motivation is an individual's internal status toward something. It has power to enhance the strength of the relationship between the input and the output of human behaviour. Motivation refers to the reasons for directing behaviour towards a particular goal, engaging in a certain activity, or increasing energy and effort to achieve the goal (Liu & Lin, 2010). It is assumed that, in this research, senior high technical school student will express positive emotions when doing hands-on activities in groups. Based on the notion that hands-on activities as integrated in the GeoGebra support student-centred instructions, the researcher assumed that senior high technical school should be stimulated to interact with each other for discussions and sharing of ideas.

A research conducted by Rosnaini, Mohd, and Ismail (2009) on development and evaluation of a computer aided instructions (CAI) G-Reflect, on Students' achievement and motivation in learning mathematics in Malaysia. The results from t-test showed that there was a significant difference in the mean scores obtained ($t(67) = 10.162$, $p \leq 0.05$). The treatment group was found to perform better in the test compared to the Control group. In terms of motivation, results from the questionnaire showed that the students from the treatment group were highly motivated in learning mathematics.

Moreover, Tella (2007) pointed out that, in making instruction interesting in learning Mathematics, there is the need to use methods and materials which will make the learning of Mathematics active and investigative as much as possible. Such methods also must be ones that take into account, learner's differences and attitudes toward Mathematics as a subject. Examples could be the use of concrete materials and other instructional devices, which are manipulated. Also, geometry exercises in the form of various pencil and paper activities should be used. This study illustrates the way in which instruction based on the GeoGebra can change the value of a task, increase student self-effectiveness, and improve student worth. In line with this study, the researcher employed activities based on the GeoGebra into geometry lesson to explore its effects on students' motivation to learn.

2.8 Research Gap

As observed from the foregoing literature review section, several studies have been conducted in various parts of the country that have shown the effect of GeoGebra on van Hieles' geometric thinking level of students' attainment of geometry. The chapter discussed the theories supporting the study, APOS theory and van Hieles' geometric thinking levels of students. It looked at the technology usage in mathematics education, students' perceptions of using GeoGebra in learning geometry, teaching geometry with GeoGebra and van Hiele geometric thinking levels of students with technology in geometry. Discuss the effectiveness of GeoGebra on students' performance and effects of GeoGebra on students' motivation to learn geometry. The current study aimed at generating knowledge on the effects of GeoGebra on van Hieles' geometric thinking levels of senior high technical school students learning attainment of geometry. Thus, the current study research aimed at bringing the gap and establishing further knowledge on the effectiveness of GeoGebra on students' achievement.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

This chapter presents the methods used in order to investigate the effect of using GeoGebra on van Hieles‘ geometric thinking levels of senior high technical school students‘ attainment of geometry. The focus was on mathematics teachers and students who use GeoGebra in the classrooms. The research was planned to respond to the questions and hypotheses formulated in the study. The plan was expressed in terms of the research approach, research design, population, sample and sampling procedure, research instruments, pilot testing, validity and reliability of research instruments, the intervention procedure, data collection procedure, data analysis procedure and ethical considerations.

3.1 Research Approach

The study followed a mixed methods research approach. Ivankova, Creswell and Clark (2007) defined mixed method research as a procedure for accumulating, scrutinising and merging together quantitative and qualitative data at some point so as to comprehend a problem even better. According to Kekana (2016) mixed method research does not look at research from one angle; it tends to investigate the knowledge of both what is happening and how or why things happen. According to Ivankova et al. (2007), there are four main reasons why researchers might want to do a mixed method research. The first reason might be that the researchers want to explain quantitative outcomes with successive qualitative outcomes. The second reason might be that researchers want to use qualitative records to acquire a first-hand theory that is consequently quantitatively verified. The third reason might be that researchers want to compare quantitative and qualitative data scenarios that will be trustworthy. The fourth reason might be that researchers want to augment their research with a complementary data set; it may be quantitative or qualitative (Ivankova et al., 2007). For this study, the researcher used both quantitative and qualitative outcomes to answer the research questions and the hypotheses.

The use of both qualitative and quantitative (mixed research) methods was as a result of both approaches having weaknesses (Teye, 2012). Mixed method approach is the use of both qualitative and quantitative methods in a single research. Advantages of using mixed method in a single research includes the following; it helps one method to complement the other, it helps in expansion, there is development of method, one is able to seek convergence and corroboration, and then for the purpose of initiation (Johnson & Onwuegbuzie, 2004). There are some shortcomings with using the mixed method strategy despite its advantages. Johnson and Onwuegbuzie (2004) has noted that, merging both quantitative and qualitative approaches in a single research can be time consuming and expensive. Also, there is difficulty in finding an experienced researcher to integrate the two methods. Moreover, analysing both data and interpreting conflicting results can be very challenging. Notwithstanding, the study adopted the mixed method strategy in collecting detailed information due to the complexity of issues regarding the use of GeoGebra on van Hieles‘ geometric thinking levels of senior high technical school students learning attainment of geometry.

Quantitatively, geometry achievement tests (GAT) were used to determine the effect of using GeoGebra on students‘ learning performance in geometry, Van Hieles‘ geometric thinking level tests (VHGT) were used to determine the effectiveness of using GeoGebra on students' geometric levels and questionnaire was used to gather information on students' perception of using GeoGebra in learning of geometry. Qualitatively, interview guide was used to solicit information from students in order to understand how GeoGebra motivate them to learn geometry, as well as gaining more insight on the use of GeoGebra in teaching geometry to senior high technical school students. The use of both methods helped in dealing with all the needed information in the research problem. The mixed method approach provided a comprehensive information and understanding into the effect of using GeoGebra on van Hieles' geometric thinking levels of senior high technical school students' learning of geometry.

3.2 Research Design

A research design is the plan that describes the conditions and procedures for collecting and analysing data (McMillan, & Schumacher, 2014). In this study, one group pretest-posttest pre-experimental design was employed. It is a design in which it cannot be made an exhaustive control of the context variables (Blas, 2013). In one group pretest-posttest pre-experimental design participants are exposed to treatment and measured afterwards to see if there were any effects (Baumgartner, Strong & Hensley, 2002). This study used one group pretest-posttest pre-experimental design because there is no control for comparison. The researcher put one group and used pre-test and post-test to see the results of the test.

The one-group pretest-posttest design usually involved three steps as follows:

1. Administering a pre-test measuring the dependent variable;
2. Giving the experimental treatment to the subjects and
3. Administering a post-test measuring the dependent variable. Differences attributed to application of the experimental treatment are evaluated by

comparing the pretest and posttest scores (Ary, 2010). This is illustrated in Table 3.1

Table 3.1: Illustration of one group pre-test and post-test design

Pre-test	Independent Variable (Treatment)	Post-test
$\boldsymbol{Y_1}$	X	Y_2

In this study, the procedures of one group pretest-posttest pre-experimental design were:

1. Administering a pre-test before applying GeoGebra with a purpose measuring the van Hieles‘ geometric thinking levels of students and students learning performance in geometry.
2. Applying treatment in teaching geometry by using GeoGebra on the van Hieles‘ geometric thinking levels of students and students learning performance in geometry.
4. Administering a post-test after applying GeoGebra with a purpose measuring the van Hieles‘ geometric thinking levels of students learning attainment of geometry and students learning performance in geometry.
5. Comparing the scores of pre-test and post-test of van Hieles‘ geometric thinking levels of students and students learning performance in geometry.

(1). The research design of the study is summarized in Table 3.2

Table 3.2: Research design of the study

Class	Pre-test	Independent Variable (Treatment)	Post-test
SHS 2 Students	GAT	GeoGebra	GAT
	VHGT	GeoGebra	VHGT

***Key: VHGT = van Hiele Geometric Thinking Level Test and GAT = Geometry Achievement Test*.**

Table 3.2 shows that the researcher used only one class in this research. During the experiments, the researcher had two tests (GAT and VHGT). The GAT was conducted before and after students were taught using GeoGebra. The results were compared to know the effect of GeoGebra on students learning performance in geometry. Finally, The VHGT was conducted before students were taught using GeoGebra on van Hieles' geometric thinking level and students taught after using GeoGebra on van Hieles' geometric thinking level. Then both of students' score were compared to find out if there is a significant difference. By applying the treatment was to know whether the scores are increasing or not. VHGT Pre-test and post-test were given to measure if there were significant difference between scores before and after the students were taught by using GeoGebra on students van Hieles' geometric thinking levels. However, one group pretest-posttest pre-experimental design helped the researcher focused on one class. Thus, investigate the effect of using GeoGebra on van Hieles' geometric thinking levels of senior high technical school students learning attainment of geometry.

3.3 Population of the Study

McMillan and Schumacher (2014) define population as a group of individuals or events from which a sample is drawn and to which results can be generalized. The target population for the study was all students in the senior high technical school in the Central region of Ghana. However, there is only one senior high technical school in Abura Asebu Kwamankese district in the central region of Ghana. The accessible population of the study was form 2 senior high technical school students in Abura Asebu Kwamankese district in the Central region of Ghana. The population is made up of 546 males and 64 females. SHS 2 students were considered due to their relatively long period of stay in the school and as such must have experienced some technology usage in learning mathematics as compared to the first year students. Besides, the SHS 2 students would soon enter SHS 3, so there is the need to help them overcome their learning difficulties in mathematics especially geometry before they sit for the final examination.

3.4 Sample and Sampling Procedure

A sample is the group of subjects from whom data are collected; often representative of a specific population (McMillan & Schumacher, 2014). Gay, Mills and Airasian (2012) refer to sampling as the process of choosing individuals from a population, usually in such a way that the selected individuals represent the larger group from which they were selected. In this study simple random sampling procedure was used to select a sample size of eighty (80) SHS 2 senior high technical school students. The students in this study were taught mathematics on daily basis by their teachers. They were randomly selected by assigning each student a number. Numbers 1 (yes) and 2 (no) were put in a box. SHS 2 student who pick the number 1(yes) was chosen to participate in the study. This gave a total of 80 participants. Random sampling is done when processing the entire dataset is unnecessary and too expensive in terms of response time or usage of resources (Kekana, 2016). Random sampling provides all persons or events with equal opportunity of being chosen (Onwuegbuzie & Collins, 2007). It was found to be convenient and less time consuming in choosing SHS 2 students randomly.

3.5 Research Instruments

The study used Geometry Achievement Test (GAT), van Hieles' Geometry Test (VHGT), interview guide and questionnaire to collect data.

Geometry Achievement Test (GAT) was used to explore the effect of GeoGebra on students learning performance of geometry. Van Hiele Geometry Test (VHGT) was used to explore the effectiveness of using GeoGebra on students Van Hieles' geometric thing levels, semi-structured interview guide was used in this study to collect data on how GeoGebra instruction motivates students to learn geometry and questionnaires were used to collect numeric data on students' perception of using GeoGebra in learning geometry.

3.5.1 Geometry achievement test (GAT)

The Geometry Achievement Test (GAT) was a test aimed at exploring the effect of GeoGebra on students learning performance in geometry. The GAT was used to test students' understanding of the angles, property of parallel lines, interior and exterior angle theorems, polygons etc. Also students were required to investigate (through geometrical construction) and discover the properties of these shapes. Such investigation and discovery lead them to construct their own conjectures about the properties of the shapes and the relationships between these properties.

3.5.1.1 Development of geometry achievement test (GAT)

The Geometry Achievement Test (GAT) was developed by the researcher (See Appendix B). It was constructed in line with the content of the curriculum for SHS 2 under plane geometry 1 which includes: angles, property of parallel lines, interior and exterior angle theorems, polygons etc. to reveal strengths and weaknesses in students‘ mathematical abilities. The Geometry Achievement Test was presented in the form of a structured questionnaire which was issued to the students. A structured questionnaire allows the researcher to seek answers from the respondents within a given range of responses (Cohen et al., 2000). This means that the respondents are constrained to select an answer or group of answers ―from a fixed list of answers provided" (Cohen et al., p.295).

This instrument Geometry Achievement Test consisted of ten (10) multiple choice items with options A-E having four distractors and one correct option. The researcher scored the instrument immediately after its administration and each correct option was scored four (4) marks while any wrong option was scored zero (0). Part B of the GAT consist of two (2) items where the students were expected to provide written responses. The possible scores of the GAT ranged between 0 and 80. This instrument was used as a pretest one week before the beginning of the study to determine students learning performance in geometry. Also, GAT was administered as a posttest to the students one week after the intervention was completed. This enabled the researcher to compare the pretest and the posttest scores in order to ascertain the effectiveness of the intervention.

3.5.2 The Van Hiele Geometric thinking level test (VHGT)

The VHGT which was developed by Usiskin (1982), under the Cognitive Development and Achievement Secondary School Geometry (CDASSG) special programme was used to explore the effectiveness of using GeoGebra on students van Hieles‘ geometric thinking level in this study. In order to access GeoGebra as an instructional tool on students‘ geometric thinking levels, the van Hiele Geometry Test (VHGT) was administered to the students as the pre-test and post-test during a single class period. The test consists of 20- items multiple choice questions representing four van Hiele levels (See Appendix C). The second part of the VHGT was a test consisting of 2 items where participants were expected to provide written responses. This was designed to further explore the problem-solving abilities of the students. These items included some commonly found in texts and examination papers set for these students. Table 3.3 shows the characteristics of van Hiele‘s Geometry Test.

Table 3.3 Characteristics of van Hiele's geometry thinking level test

Questions	Levels	Features
1-5	1	It is about visual form. It aims to determine whether the students recognize the shape by looking at the shape of the figure
6-10	2	It is concerned with the Characteristics of the forms and on the one hand it aims to show that the students do not know the forms and on the other hand they do not know the Characteristics of the forms.
11-15	3	It determines whether students can recognize the relationships between forms. They identify students who respond correctly to questions in this group and have proven that they have knowledge of axioms
16-20	4	It is the question of reasoning and logical deduction. In these questions, it is determined whether the students are at a level of understanding and writing.

Source: Salifu et al., (2020)

The study used the grading system developed by Usiskin (1982) for assigning the various levels. Usiskin reported that a student can score 0 as the minimum mark and a maximum of 15 points (1+2+4+8 points) from the VHGT. The grading key for Van Hieles‘ Geometry Test as developed by Usiskin is shown below:

- If at least three questions (between 1 and 5) are answered correctly: 1 point
- If at least three questions (between 6 and 10) are answered correctly: 2 points
- If at least three questions (between 11 and 15) are answered correctly: 4 points
- If at least three questions (between 16 and 20) are answered correctly: 8 points

Zero point is scored if a student gets 2 out of 5 corrects answers. For a student to pass from one level to another, then the students' needs to answer correctly at least three of previous level questions in order to be assigned a level. For instance, a student who was able to correctly answer three questions from 1 to 5, correctly answer two questions from 6 to 10, correctly answer three questions from 11 to 15, gets 1 point from first level, 0 point from second level, 4 points from third level respectively making a total of 5 points. Even though Van Hiele's level 3 criterion was met by this student, he cannot be placed in Van Hiele's level 3 because the student failed to answer correctly at least three of second level questions (Salifu et al., 2020).

The rationale for the VHGT is based on the notion that students' understanding of geometry can be described largely by their relative positions in the van Hiele scale of geometric thinking levels (Atebe, 2009). As with the CDASSG van Hiele test (see Usiskin, 1982), the VHGT was designed to determine the van Hiele levels of the participating students. Thus, the instrument was to assign students to the various levels of geometric thinking so as to determine how GeoGebra was effectively used in relation to the students' van Hiele levels. Hence, measuring specific skills, such as ordering the properties of the figures; identifying and comparing the figures and deduction; constitute geometric thinking levels of students.

3.5.3 Focused group discussion

According to Cohen et al. (2000, p.267) an interview is ―an interchange of views between two or more people on a topic of mutual interest". It sees the centrality of human interaction for knowledge production and emphasises the social context of research data. In addition, McMillan and Schumacher (2001, p.41) state that ―interviews enable participants to discuss their interpretations of the world in which they live and to express how they regard situations from their own point of view". It could be argued that an interview is a two-way conversation initiated by the interviewer for the specific purpose of obtaining research-relevant information. Focused group discussion is a type of in-depth interview accomplished in a group, whose meetings present characteristics defined with respect to the proposal, size, composition, and interview procedures (Mishra, 2016).

In a focused group discussion, the interviewer facilitates the discussion and creates an environment that promotes the communication of different perceptions and points of view

This information focuses on content specified by the research aims of systematic description, prediction or explanation. It involves the gathering of data through direct verbal interaction between individuals.

The focused group discussion was used in this study to collect data on how GeoGebra instruction motivates students to learn geometry. The interview guide helped maintain the focus of each interview and at the same time allowing the teachers the flexibility to provide alternative and detailed responses to the questions (Opie, 2004). Interviews were seemingly vital as the student respondents openly voiced their opinions, beliefs and views (Nieuwenhuis, 2007) tied to the learning of geometry.

Also, the researcher used the group discussion to gather data on the effect of the GeoGebra lessons in supporting students' attitudes and performance in geometry**.** This source of data was used to analyze students' reflections and views on the role of GeoGebra in supporting their motivation and student-centered learning.

The respondents of the interview were given the freedom to express their views on the lessons learnt as there were no fixed questions. The researcher asked questions that pertains to the study as opportunities arose, then listened closely to participants' responses for clues as to what question to ask next, or whether it was necessary to probe for additional information. In all a total of 20 students who were sampled for the study were randomly selected to participate in the interview. This data were therefore used to answer research question 3. The interview was done at the end of the intervention.

3.5.4 Questionnaire

A questionnaire is a written instrument that contains a series of questions or statements called items that attempt to collect information on a particular topic (Agyedu, Donkor & Obeng, 2013). There are many ways of classifying questionnaire items. However, the two broad categories are: i) Open-ended or semi-structured questionnaire, this type require the respondents to construct or write a response, from a word to several paragraphs. ii) Closed-ended or structured questionnaire requires the respondent to make a choice by ticking, checking or circling the one they wish. The structured questionnaire may be in the form of dichotomous response items (say – yes or no), multiple-choice items (say – 0-5, 6-10, 11- and above), rating scale items (say – strongly disagree, disagree, etc.), among others (Agyedu, Donkor & Obeng, 2013).

3.5.4.1. Development of questionnaire

In this study, the researcher used questionnaire to collect numeric data on students' perceptions of using GeoGebra in learning geometry. The respondents were limited to a list of options from which they were to choose one as a respond to each item. Specifically, the questionnaire contained 16 closed-ended items and divided into two main sections A and B. The first section of the instrument asked the respondents to provide demographic information. The second section was made up of two main parts. The first part was questionnaire for students consisting 2- points Likert scale type ranging from 1 to 2 (1 = Agree and 2 = Disagree). Items under this section were to explore students' perceptions on the use of GeoGebra in learning geometry (See Appendix A).

3.6 Pilot Testing

Before the study was carried out, the items on the Geometry Achievement Test (GAT), interview guide, the van Hieles' geometric thinking level and the questionnaire were tested to avoid ambiguity and to test for validity and reliability. This was done through a pilot study that was carried out prior to the actual collection of the data. A pilot study can be defined as a small scale version or trial run in preparation for a major study (Polit & Beck, 2004). Such a trial run may have various purposes such as testing study procedures, validity of tools, estimation of the recruitment rate and an estimation of parameters such as the variance of the outcome variable to calculate sample size (Arain, Campbell, Cooper, & Lancaster, 2010). The current study tested whether the items in the questionnaire, observation schedule and the van Hieles' geometric thinking level test were valid and reliable as recommended by (Welman & Kruger, 2000), who stated that a pilot study is needed to detect possible flaws in measurement procedures and is also valuable to identify unclear or ambiguous items in a questionnaire. In this study, the pilot tests were carried out in senior secondary technical high school at Abura Asebu kwamankese district in the Central of region of Ghana. It was carried out by administering questionnaire, Geometry Achievement Test, van Hieles' geometric thinking level test to eighty (80) SHS 2 students and out of the eighty (80) SHS 2 students twenty (20) were randomly selected to conduct the interview. Thus, eighty (80) participants were involved in the pilot study. The purpose of the pilot study was to test the validity and reliability of the research instruments. It provided some insights that made the researcher modify and make necessary amendments to the instruments.

3.7 Validity and Reliability of Research Instruments

According to Kekana (2016), there are two important questions which need to be asked when it comes to a study. Firstly, to what extent will the results be appropriate and meaningful (validity). Secondly, to what extent will the results be free from errors (reliability)?

3.7.1 Validity of research instruments

Validity denotes the magnitude to which a computing instrument calculates what it is meant to calculate (Di Fabio & Maree, 2012; Muyeghu, 2008). Validity may also be defined as the appropriateness, meaningfulness, correctness and usefulness of any deductions that are obtained through the use of an instrument (Lu, 2008).

After developing the questionnaire, a group of graduate students from the University of Education, Winneba and other mathematics teachers from some secondary schools in Abura Asebu Kwamankese, were requested to carefully and systematically scrutinize and assess the questionnaire for its relevance. The feedback from the graduate students and mathematics teachers were factored into the final preparation of the questionnaire. Issues such as length of the items and general format of the questionnaire were some of the concern pointed out to the researcher during the pilot stage.

The interview guide was cross checked and corrections made by the researcher‘s supervisor. Some M.Phil. Mathematics Education students also read through the observation schedule and made suggestions that were incorporated before use.

The van Hiele geometric thinking level tests and Geometry Achievement Test was approved by the head of mathematics department in the senior high technical school at Abura Asebu Kwamankese and mathematics teachers in order to check whether it was appropriate for the students‘ level. The study supervisors also made inputs before the test could be administered to the students. The secondary mathematics curriculum and other relevant core mathematics textbook were used for questions to make certain that the questions were consistent and applicable to the level of the students.

3.7.2 Reliability of research instruments

Creswell (2010) explained reliability of an instrument as the degree to which the instrument measures accurately and consistently what it was intended to measure.

To check the reliability of the questionnaire, Geometry Achievement Test the van Hiele geometric thinking level test items, Cronbach‘s Alpha co-efficient was used. Cronbach Alpha (α) was computed from a sample of eighty (80) responses that were gathered from the pre-testing. The choice of Cronbach alpha (α) co-efficient was made on the merit of views of Mitchell (2004) who contended that Cronbach Alpha is used when measures have multi-scored items. This exercise helped to correct any ambiguities that were detected and other items that will not be relevant to the research. Cronbach‘s alpha was established for each of the questionnaires and test items. The values of, Cronbach‘s alpha of 0.71 (Geometry Achievement Test) and 0.76 (students‘ questionnaires) were obtained. Therefore, the instrument was considered reliable and appropriate to collect the relevant data to answer the questions posed. Also Fraenkel and Wallen (2000) posited that ―for research purposes a useful rule of thumb is that reliability should be at .70 and preferably higher" (p.17). With this, the instrument was said to be of good quality capable of collecting useful data for the study. The queries that came out of the item analyses were catered for. All these actions were taken to ensure that the instruments were capable of collecting quality and useful data for the study.

In order to ensure reliability during the interview, the same audio-tape was used with all the SHS 2 students. This measure ensured that no data was distorted due to the use of selective memory. The researcher remained uninvolved in all cases so as to avoid interfering with the study and avoiding any element of bias. The researcher focused on her role as a listener in all the interview process.

3.8 Intervention Procedure

To assess the effectiveness of using GeoGebra on van Hieles' geometric thinking levels of senior high technical school students learning of geometry and the effects of GeoGebra on students learning performance in geometry, the researcher used Geometry Achievement Test (GAT) and the van Hiele Geometric Thinking Test (VHGT) in accordance with the research design. Pre-test and post-test based on Geometry Achievement Test (GAT) and Van Hiele Geometric Test (VHGT) were administered to the selected SHS 2 students before and after the implementation of the designed activities. Before the intervention students were introduced to the GeoGebra software and shown how to use it with no particular emphasis on the topic of geometry. This was done during two extra lessons just after they had written the pre-test.

The students were guided to review their prior knowledge on the topic. They were then taken through sub-topics like calculating angles at a point, using the types of angles to calculate angles at a point and stating and using the properties of parallel lines. The students were also taken through the exterior angle theorem and how to state and use the exterior angle theorem to find the value of missing angles in a triangle in the ICT lab using the GeoGebra and a worksheet which provided instructions to carry out the activities in the first and second week. The students were guided to solve more examples using the GeoGebra and assignments were given afterwards. After the treatment, the post-test was administered to students. The duration of the treatments including the tests were four (4) weeks.

3.8.1. Treatment

Treatment was GeoGebra software as an instruction tool. The GeoGebra software as an instructional tool refers to the teaching approach involving both the teacher-led demonstrations and students‘ hands-on activities. Lessons were held in the computer laboratory where student explored the concepts of geometry using computer. Students were taught geometry by using the GeoGebra software with worksheets which were designed by the researcher according to activities in senior high school students‘ Mathematics curriculum. Lesson plan was designed to help ensure that classroom instruction followed the curriculum aims and objectives of the topic (geometry) treated.

Since most of the students were not familiar with the GeoGebra, a preparatory instruction was given in order to familiarize students to GeoGebra. The activity sheets included directions to the use of the GeoGebra. Students were given the opportunity to explore and manipulate geometric figures and objects such as angles, triangles, parallel lines and other theorems according to the directions stated. The researcher gave feedback on the students‘ errors and guided them with their questions during the activities. The researcher served as a facilitator during the lessons and also offered assistance to the students when they faced difficulties with the computer as well as the software. The purpose was to make the activities in the syllabus interactive dynamic activities. Therefore, similar activities to the curriculum were designed. After completion of each dynamic geometry activity, the researcher gave feedbacks on students' errors and started discussion sessions for outcomes of the activity and students' work (See Appendix D). After the treatment, VGHT and GAT post-test was administered to the selected SHS 2 students. The treatment period lasted two (2) weeks, four (4) hours per week.

3.9 Data Collection Procedure

A letter of introduction was collected from the Department of Mathematics Education in University of Education, Winneba. The researcher went to the school and introduced herself to the headmaster of the school, sought his permission and cooperation and briefed the mathematics teachers and SHS 2 students on the purpose of the exercise. Permission was sought from the head of the ICT department in the senior high technical school to use the laboratory for the teaching of the intended Geometry lessons. The GeoGebra software was installed in all the computers and the researcher's computer was projected over the screen for students to see and draw. The Worksheets were also printed out to facilitate the group activity. Each student was assigned computer. The researcher explained her purpose to them and assured them of confidentiality. She emphasized that the respondents should write neither their names nor the names of their various schools when responding to the questionnaire. This was done to cater for anonymity.

Administration of the GAT and VHGT

The GAT and VHGT pre-test was administered personally to the participants during the sixth and seventh week in the second semester within the 2018/2019 academic year. The post-test (which was the same GAT and VHGT) was personally administered to the participants during the eighth and ninth week after the participants have been taken through the treatment in the second semester within the 2018/2019 academic year. After the intervention, the researcher compared the results of GAT to determine whether the use of GeoGebra had yield positive results or not. Also the researcher compared the results of VHGT to determine whether there is any significant difference between using GeoGebra on van Hieles' geometric thinking level of senior high technical school students.

Conducting of Interviews

After the intervention and with support of the cooperative teacher some students were interviewed on how the GeoGebra instruction motivated them to learn geometry (see Appendix D Interview guide). This source of data helped to analyse students' reflections and views on the role of GeoGebra in supporting their motivation, interactions and participations in class discussions, student-centered learning activities, conceptual understanding and problem solving strategies. Their answers then were compared with the other data sources. Therefore, these data helped in answering the effect of the use of GeoGebra on senior high technical school students' motivation to learn geometry.

Administration of the Questionnaire

The researcher personally administered the questionnaires to the respondents. After the explanation of the questionnaire by the researcher, the respondents were later allowed to independently give their responses to the items with little supervision by the researcher. The respondents in each case were given thirty (30) minutes to answer the questions. The researcher went round to collect the work immediately the time given had elapsed. The researcher adopted this approach to eliminate adulterated responses of respondents. The questionnaire was collected on the same day after the stipulated time.

3.10 Data Analysis Procedure

Muyeghu (2008) defined data analysis as a stage of describing data in significant terms. Data analysis in mixed methods investigation takes place within both the quantitative and qualitative data collection processes. Qualitative data analysis is a process of checking for patterns in the data, constructing and testing conjectures, asking questions and seeking more data (Mouton, 2001).

Respondents‘ demographic information was analysed using frequency tables and percentages. For the purpose of answering the first research question concerning the effect of using GeoGebra on learning performance, frequency tables, percentages and charts were used to analyse the data on the Geometry Achievement Test (GAT). The van Hieles‘ Geometry Tests results of using GeoGebra on van Hieles‘ geometric thinking levels of students were analysed using frequency tables, percentages and bar charts. Paired sample t-test was used to answer research question 1 and hypothesis 2.

Data collected were analysed by ―thick description" after the researcher had read the transcribed interviews and identified categories of responses that answered the research questions. The researcher reported all events that emanated from the study by describing and interpreting the outcomes. In this study, patterns that emerged from interview data were described so that one can make meaning from the data. Again For the purpose of answering the research question concerning students‘ perceptions of using GeoGebra in learning geometry, frequency tables and percentages were used to analyse the data.

3.11 Ethical Considerations

It is essential in any research, more especially when human participants are involved to adhere to strict ethical requirement (Maree, 2007). The following ethical considerations were adhered to;

Voluntary Participation: Voluntary participation means that participants were never forced to take part in the study. Participants were informed that participation in the study was completely voluntary and that they could withdraw from the study at any time without prejudice.

Informed Consent: According to Babbie (2014), researchers are expected to obtain consent from all those who are directly involved in the research, before collecting data. The aim of informed consent is to show respect to the participants and make them feel free to make independent decisions without fear of negative consequences. The researcher ensured that participants had access to relevant information prior to signing the consent form. The participants were asked to sign consent forms for the observation schedule and for tape recording of the lesson delivery.

No Harm to the Participants: The researcher ensured that there was no harm to participants by clearly explaining what would be involved in this study. The researcher also guarded against asking questions that could embarrass or endanger the participants.

Anonymity and Confidentiality: Participants were assured of confidentiality and anonymity. Participants were informed that only the researcher and her supervisors would have access to the recordings and the transcripts. Anonymity was ensured so that participants cannot be identified with the responses. The researcher used pseudonyms instead of the participants' real names. The participants were also informed that neither their names nor their departments' would be mentioned in the research report.

Deception: The researcher ensured that all participants were aware that the research was conducted as part of his academic studies. The participants were provided with the researcher' and her supervisor's contact details in the eventuality that they needed more clarity or information regarding the study.

Ethics Clearance: The researcher obtained the Ethics clearance from the said department in University of Education, Winneba (South-Campus). The proposal for this study was also approved by the Department of Mathematics Education. The researcher ensured that she conduct the study in an ethical manner.

CHAPTER FOUR

RESULTS AND DISUSSION

4.0 Overview

This chapter presents and discusses the findings of the study. The study aims at investigating the effects of GeoGebra on van Hieles' geometric thinking levels of senior high technical school students' attainment of geometry. The analysis was based on Geometry Achievement Test (GAT), van Hieles' Geometric Test (VHGT), interview guide and the responses given to the questionnaire. Information obtained was presented in the form of tables and figures where appropriate. The data analysis has been presented in four main parts: the first part deals with the demographic data, the second part deals with the analysis of research findings and hypotheses testing, the third parts deals with the discussion of research findings and the fourth parts deals with the summary of research findings.

4.1 Analysis of Demographic Data

Here, SHS 2 students were asked to indicate their gender, programme offered and mathematics lesson with technology. The results are presented in Tables 4.1- 4.3.

Gender of SHS 2 Students

Item 1 on the questionnaire sought to find out the gender of SHS 2 Students. The result is presented in Table 4.1.

Table 4.1: Gender of SHS 2 students

Gender	Frequency	Percentage%
Female	30	37.5%
Male	50	62.5%
Total	**80**	**100%**

Source: Field Data (2019)

Results from Table 4.1 shows that 30 (37.5%) of participants who took part in the study were females while 50 (62.5%) were males. This means that more male students participated in this study than female students.

Programme Offered by SHS 2 Students

Item 2 on the questionnaire sought to find out the programme offered by SHS 2 Students. The result is presented in Table 4.2.

Table 4.2: Programme offered by SHS 2 students

Programme Offered	Frequency	Percentage%
Electrical Engineering Technology.	15 (all males)	19%
Mechanical Engineering Technology.	13 (all males)	16%
Fashion Designing and Construction.	14 (all females)	18%
Agricultural Mechanization Technology.	13 (2 females)	16%
Welding and Fabrication	13 (2 females)	16%
Hospitality and catering.	12 (all females)	15%
Total	**80**	**100%**

Source: Field Data (2019)

Results from Table 4.2 shows that 15 (19%) of the SHS 2 students offered Electrical Engineering technology, followed by 14 (18%) offered fashion designing and construction, 13 (16%) offered Agricultural mechanization technology, 13(16%) offered mechanical engineering technology, 13 (16%) offered welding and fabrication and 12 (15%) offered Hospitality and Catering. This means that more of the Electrical Engineering technology students participated in this study. Thus, SHS 2 were randomly sampled for the study.

Mathematics Lesson with Technology

Item 3 of the questionnaire sought to find out whether SHS 2 students have learnt mathematics lessons with technology. The results are presented in Table 4.3

Table 4.3: Mathematics lessons with technology

Mathematics lesson with technology	Frequency	Percentage %
Yes	69	86%
No	11	14%
Total	**80**	**100%**

Source: Field Data (2019)

Results from Table 4.3 shows that 69 of the SHS 2 students representing 86% had learnt mathematics lessons with technology whiles 11of the SHS 2 students representing 14% had not learnt mathematics lessons with technology. The result clearly shows that majority of the SHS 2 students in this study that had learnt mathematics lessons with technology.

4.2 Research Question 1: What are the effects of using GeoGebra on senior high technical school students' performance in geometry?

This research question sought to explore the effect of using GeoGebra on senior high technical school students' learning performance in geometry. The researcher used Geometry Achievement Test (GAT) to assess the effectiveness of using GeoGebra on students learning performance in geometry. The GAT was in two groups, namely; pre-GAT and post-GAT.

4.2.1 Analysis of the Pre-Geometry Achievement Test (GAT)

A pre-Geometry Achievement Test was administered to the selected sample comprising of 80 students. This was to help assess students learning performance in geometry. The pre-test was also to reveal students' performance in geometry before the treatment. The performance was categorized into 8 categories: 1 – 10, 11 – 20, 21 – 30, 31 – 40, 41 – 50, 51 – 60, 61 – 70, and 71 – 80. The results of the test indicate that 3 students scored between 1and 10 in the test, 11 students scored between 11 and 20, 10 students scored between 21 and 30, 40 students scored between 31 and 40, 10 students scored between 41 and 50, 6 students scored between 51 and 60, none of the students scored between 61 and 70 and 71 and 80 with a mean score of 33.125. The results are presented in Table 4.4.

Table 4.4: Students performance in pre-geometry achievement test (GAT)

Marks Range (%)	Frequency (f)	Class Mid-point (x)	fx
1 – 10	3	5.5	16.5
11 – 20	11	15.5	170.5
21 – 30	10	25.5	255
31 – 40	40	35.5	1420
41 – 50	10	45.5	455
51 – 60	6	55.5	333
61 – 70	0	65.5	0
71 – 80	0	75.5	0
	$\sum f = 80$		$\sum fx = 2650$

Source: Field Data (2019)

$$\therefore \text{Mean score} = \frac{\sum fx}{\sum f} = \frac{2650}{80} = 33.125$$

4.2.2 Analysis of the post-geometry achievement test (GAT)

After the use of GeoGebra as an instructional tool, a post-Geometry Test was administered to the sample students. The post-test was to reveal students learning performance in geometry after the treatment. The performance was categorized into 8: 1 – 10, 11 – 20, 21 – 30, 31 – 40, 41 – 50, 51 – 60, 61 – 70, and 71 – 80. The results of the test indicate that none of the students scored between 1and 10, 11 and 20, 21 and 30 and 31 and 40 in the test, 20 students scored between 41 and 50, 21 students scored between 51 and 60, 36 students scored between 61 and 70 and 3 students scored between 71 and 80 with a mean score of 58.25. The results are presented in Table 4.5 and figure 4.2

Table 4.5: Students' performance in post-geometry achievement test (GAT)

Marks Range (%)	Frequency (f)	Class Mid-point (x)	fx
1 – 10	0	5.5	0
11 – 20	0	15.5	0
21 – 30	0	25.5	0
31 – 40	0	35.5	0
41 – 50	20	45.5	910
51 – 60	21	55.5	1165.5
61 – 70	36	65.5	2358
71 – 80	3	75.5	226.5
	$\sum f = 80$		$\sum fx = 4660$

Source: Field Data (2019)

$\therefore$ Mean score $= \frac{\sum fx}{\sum f} = \frac{4660}{80} = 58.25$

4.2.3 General comparison of pre and post-gat scores of students

The comparison of the pre-test and post-test scores of the Geometric Achievement Test (GAT) is presented in Table 4.6.

Table 4.6 compares the pre-test and post-test scores of the GAT of students.

Marks	Pre-Test	Post-Test
1 – 10	3	0
11 – 20	11	0
21 – 30	10	0
31 – 40	40	0
41 – 50	10	20
51 – 60	6	21
61 – 70	0	36
71 – 80	0	3

Source: Field Data (2019)

The results of Table 4.6 indicated that, out of 80 students who took the pre and post geometry achievement test (GAT). Three (3) of students scored between 1and 10 in the pre-test while none of the students scored between 1and 10 in the post-test, followed by 11 students scored between 11 and 20 in the pre-test while none of the students scored between 11 and 20 in the post-test, 10 of the students scored between 21 and 30 in the pre-test while none of the students scored between 21 and 30 in the post-test. Also, 40 students scored between 31 and 40 in the pre-test while none of the students in the post-test scored between 31 and 40.

Again, 10 of the students scored between 41 and 50 in the pre-test but increased to 20 students who scored between 41 and 50 in the post-test, 6 of the students scored between 51 and 60 in the pre-test while 21 of the students scored between 51 and 60 in the post-test, none of the students scored between 61 and 70 in the pre-test while 36 of the student scored between 61 and 70 in the post-test and none of the students scored between 71 and 80% in the pre-test while 3 of the students scored between 71 and 80 in the post-test. The post-GAT scores indicate much improvement in students learning performance in geometry as compared to the pre-GAT scores.

Table 4.7 shows the descriptive statistics of the pre and post-GAT scores of students.

Table 4.7: Descriptive statistics of pre and post-GAT scores of students

Test	N	Minimum	Maximum	Mean	Std. Deviation
Pre-test	80	4	60	31.83	11.843
Post-test	80	42	80	57.73	9.604

Source: Field Data (2019)

Results of Table 4.7 shows that, the minimum score the senior high technical school students obtained in the pre-test was 4 out of 80, while the maximum score was 60 out of 80. In the post-test, the minimum score was also 42 out of 80, while the maximum score was 80. The mean score of the senior high technical students in the pre-test was 31.83(SD=11.843), while that of the post-test was 57.73 (SD=9.604).

This appears that, the overall performance of the senior high technical school students‘ on the pre-test was generally low. This is an indication that in the post-test, every students learning performance in geometry had increased. These improvements might be due to the use of the GeoGebra instruction.

To determine whether the improvement was significant, a paired samples t-test statistic was conducted on the pre- and post-test scores in the Geometry Achievement Test (GAT). The results of the paired samples t-test (Table 4.8) of students in the pre-test mean score (*M = 31.83, SD = 11.843*) and post-test mean score (*M = 57.73, SD = 9.604*) GAT were found to be statistically significant at $t\ (80) = -32.467,\ p = 0.000 < 0.05$.

Table 4.8: Paired sample T-tests of pre and post-geometry achievement test GAT)

TEST	N	Mean	Std. Dev.	Mean Difference	t-value	df	Sig. (2-tailed)
Post-Test	80	57.73	9.604	25.9	-32.467	79	.000
Pre-Test	80	31.83	11.843				

4.3 Research Question 2: How effective is the use of GeoGebra on senior high technical school students' van Hiele level of geometric thinking?

This research question sought to find out the effectiveness of using GeoGebra on senior high technical school students van Hieles‘ level of geometric thinking. The researcher used the Van Hiele Geometry Test (VHGT) to find out the effectiveness of using GeoGebra on students van Hieles‘ level of geometric thinking. The VHGT were of two groups, namely; VHGT pre-test and VHGT post-test.

4.3.1 Analysis of the Pre-VHGT

A pre-test was administered to the selected sample comprising of 80 students. This was to help unravel students' difficulties in geometry so as to plan instructions suitable to their level of understanding. The pre-test was also to reveal students van Hieles' geometric thinking levels before the treatment.

Table 4.9 presents the overall students' performance on each item of the section A in the pre-test. As can be seen in Table 4.9, each van Hiele Level (VHL) had five items with five multiple choice options. However, some students did not choose any of the options for some items. This made the researcher to include an additional option (a ―blank" option) in the table. For each item, the number in bold corresponds to the right option and also represents the total number of students who answered that item correctly.

Table 4.9: VHGT pre-test: section an item analysis for each level per student

Level 1	**Choice Items**	**1 F (%)**	**2 F (%)**	**3 F (%)**	**4 F (%)**	**5 F (%)**
	A	**22(27.5%)**	0(0%)	0(0%)	36(45.0%)	2(2.5%)
	B	4(5.0%)	40(50.0%)	18(22.5%)	**2(2.5%)**	16(20.0%)
	C	30(37.5%)	10(12.5%)	**40(50.0%)**	18(22.5%)	0(0%)
	D	8(10.0%)	**18(22.5%)**	4(5.0%)	8(10.0%)	**62(77.5%)**
	E	10(12.5%)	4(5.0%)	18(22.5%)	12(15.0%)	0(0%)
	Blank	6(7.5%)	8(10.0%)	0(0%)	4(5.0%)	0(0%)
Level 2	Choice Items	6	7	8	9	10
	A	**52(65.0%)**	4(5.0%)	14(17.5%)	32(40%)	4(5.0%)
	B	24(30.0%)	40(50.0%)	12(15.0%)	2(2.5%)	6(7.5%)
	C	2(2.5%)	**30(37.5%)**	**42(52.5%)**	**22(27.5%)**	22(27.5%)
	D	2(2.5%)	2(2.5%)	6(7.5%)	16(20.0%)	**28(35.0%)**
	E	0(0%)	4(5.0%)	6(7.5%)	8(10.0%)	20(25.0%)
	Blank	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)
Level 3	Choice Items	11	12	13	14	15
	A	**34(42.5%)**	**42(52.5%)**	16(20.0%)	**46(57.5%)**	0(0%)
	B	22(27.5%)	10(12.5%)	**28(35.0%)**	10(12.5%)	16(20%)
	C	8(10.0%)	10(12.5%)	8(10.0%)	12(15.0%)	24(30.0%)
	D	2(2.5%)	4(5.0%)	6(7.5%)	8(10.0%)	**34(42.5%)**
	E	14(17.5%)	12(15.0%)	20(25.0%)	2(2.5%)	6(7.5%)
	Blank	0(0%)	2(2.5%)	2(2.5%)	2(2.5%)	0(0%)
Level 4	Choice Items	16	17	18	19	20
	A	12(15.0%)	**34(42.5%)**	**26(32.5%)**	**38(47.5%)**	8(10.0%)
	B	22(27.5%)	22(27.5%)	18(22.5%)	22(27.5%)	8(10.0%)
	C	20(25.0%)	8(10.0%)	22(27.5%)	14(17.5%)	8(10.0%)
	D	8(10.0%)	8(10.0%)	12(15.0%)	0(0%)	4(5.0%)
	E	**18(22.5%)**	8(10.0%)	2(2.5%)	6(7.5%)	**52(65.0%)**
	Blank	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)

Source: Field Data (2019) **N = 80**

The figures in ***bold*** *represent the total number (**n**) (%) of students who answered that item correctly.*

Results from Table 4.9 reveal performance of students van Hieles' geometric thinking levels at the pre-test stage. Level 1 of the VHGT shows that, 22 (27.5%), 18 (22.5%), 40 (50.0%), 2 (2.5%) and 62 (77.5%) of the students managed to correctly answer items 1, 2, 3, 4 and 5 respectively. Majority of the students 62 (77.5%) had item 5 correctly. Exactly half of the students 40 (50.0%) had item 3 correctly. More than half of the students performed poorly in items 1 (22 (27.5%)), 2 (18 (22.5%)) and 4 (2 (2.5%)). This indicates that most of the students were not able to score most of the items in level 1 (visualization stage).

Concerning Level 2 of the VHGT, 52 (65.0%), 30 (37.5%), 42 (52.5%), 22 (27.5%) and 28 (35.0%) of the students respectively had items 6, 7, 8, 9 and 10 correctly. More than half of the students 52(65.0%) and 42 (52.5%) respectively had item 6 and 8 correctly. Majority of the students scored a low mark in item 7 (30 (37.5%)), item 9 (22 (27.5%)) and item 10 (28 (35.0%)). This indicates that in the level 2 (analysis stage), the students' performances were not satisfactory because it is only items 6 and 8 that more than half of the students were able to score.

Moreover, Level 3 of the VHGT shows that, 34 (42.5%), 42 (52.5%), 28 (35.0%), 46 (57.5%) and 34 (42.5%) of the students correctly answered items 11, 12, 13, 14 and 15 respectively. More than half of the students 42(52.5%)) and 46 (57.5%) had item 12 and 14 correctly. Majority of the students performed poorly in item 11 (34 (42.5%)), item 13 (28 (35.0%)) and item 15 (34 (42.5%)) respectively. This indicates that in the level 3 (ordering stage), the students could not score well because it is only items 12 and 14 that more than half of the students were able to score.

Finally, in Level 4 of the VHGT, 18 (22.5%), 34 (42.5%), 26 (32.5%), 38 (47.5%) and 52 (65.0%) of the students correctly had items 16, 17, 18, 19 and 20 respectively.

More than half of the students 52(65.0%)) had item 20 correctly. Majority of the students scored a low mark in item 16 (18 (22.5%)), item 17 (34 (42.5%)), item 18 (26 (32.5%)) and item 19 (38 (47.5%)) correctly. This indicates that in the level 4 (deduction stage), the students performances were abysmally poor because it is only items 20 that more than half of the students were able to score.

4.3.2 The overall scores of students in the Pre-VHGT item test

There were 20 items that were used to assess the students‘ levels with each item allotted with one mark. Table 4.9.1 shows the general performance of students in the 20 pre-test items.

Table 4.9.1: Total scores obtained by students in Pre-VHGT by cumulative frequency

Score	Number of students (F)	Cumulative (F)	Percentage (%)	Cumulative Percentage (%)
5	10	10	12.5%	12.5%
6	15	25	18.8%	31.3%
7	10	35	12.5%	43.8%
8	14	49	17.5%	61.3%
9	16	65	20.0%	81.3%
10	5	70	6.3%	87.5%
11	3	73	3.8%	91.3%
12	4	77	5.0%	96.3%
13	3	80	3.8%	100.0%

Source: Field Data (2019)

Results in Table 4.9.1 shows that 81.3% (F = 65) of the students obtained less than half of the total score, 6.3% of the students (F = 5) scored half of the total marks allotted to the test while 12.5% (F = 10) obtained more than half of the total marks allotted to the test. In spite of the low performance of the students in at the pre-test, no student scored zero with only ten of the students obtaining the minimum mark of 5.

Interestingly, the highest mark scored in the test was 13 out of the 20 and three students obtained that. Moreover, no students could score marks above 14. This indicates that the general performance of the SHS 2 students in the pre-VHGT Item test was very weak.

4.3.3 Performance on van Hiele geometry pre-test – section B

Table 4.9.2 summarizes the overall performance of students' in the section B part of the van Hiele Geometry pre-test. There were 2 test items. The responses of students who demonstrated good knowledge and provided the right responses for the items were described as correct. Responses of students who attempted items but did not get the total marks allotted per test item were described as partially correct, while the responses that exhibited lack of knowledge about the items were described as completely wrong. However, few students' did not attempt some of the items at all; these were described as ―blank".

Table 4.9.2: Students Van Hiele geometry pre-test: section B item analysis

	Correct	Partially Correct	Completely Wrong	Blank
Item	**F (%)**	**F (%)**	**F (%)**	**F (%)**
1	29 (36.3%)	33 (41.3%)	15 (18.8%)	3 (3.8%)
2	28 (35.0%)	30 (37.5%)	20 (25.0%)	2 (2.5%)

Source: Field Data (2019) **N = 80**

Results in Table 4.9.2 shows that, 29 students representing 36.3% had item 1 correct, followed by 33 students representing 41.3% had item 1 partially correct, 15 students representing 18.8% had item 1 completely wrong and 3 students representing 3.8% had items 1 blank. The performance of students in item 1 was average because students who had item 1 completely wrong were not up to one-fourth of the number of students who took the test.

In item 2, 28 students representing 35.0% had item 2 correct, followed by 30 students representing 37.5% had item 2 partially correct, 20 students representing 25.0% had item 2 completely wrong and 2 students representing 2.5% had items 2 blank. The performance of students in item 2 was not encouraging because students who had item 2 completely wrong were up to one-fourth of the number of students who took the test. Hence, the performance in item 2 was below average.

4.3.4 Levels reached by students' in the van Hiele Geometry Pre-Test

Table 4.9.3 shows the van Hiele levels of geometric thinking attained by the students after the van Hiele Geometry pre-test.

Table 4.9.3: Students van Hiele levels attained in the pre-VHGT

Levels	Number of students (F)	Percentage (%)
0	10	12.5
1	24	30
2	36	45
3	10	12.5
4	0	0
Total	**80**	**100**

Source: Field Data (2019)

Table 4.9.3 indicate that, 12.5% (n = 10) of the students could not reach any of the levels, 30% (n = 24) of the students reached the Visualization (level 1), while 45% (n = 36) reached the Analysis (level 2) of the Van Hiele Geometric thinking levels. Furthermore, 12.5% (n = 10) reached the Ordering (level 3),with None of the students reaching the level 4 of the Van Hiele's Geometric thinking levels as 0% (n = 0). Students who did not reach any of the levels of Van Hiele Geometric thinking means that the students could not meet the criteria for attaining VHGT level, that is the students could not answer three (3) questions correctly from the items 1 to 5.

Again, it can be seen that 12.5% (n = 10), students reached the Ordering stage (level 3); this is an indication that out of 80 students only ten (10) of the students could reach the levels 1, 2 and 3. This means that only ten (10) students could perform in level 3, where students can logically order the properties of shapes. Finally, none (n = 0) of the students reached the Deduction stage (level 4) of the Van Hiele Geometric thinking levels. This indicates that none of the students was able to meet the criteria 3 of 5 correct suggested by Usiskin (1982) in all the levels, that means no students could answer 3 items correctly in questions items; 1 to 5, 6 to 10, 11 to 15, 16 to 20. It shows that, at this level a student understands the significance of deduction. Even though Van Hiele�663s level 3 criterion was met by student at the pre-test, they cannot be placed in Van Hieles‘ level 3 because the student failed to answer correctly at least three of second level questions. Hence, the van Hiele Levels of geometric thinking attained by the students after the van Hiele Geometry pre-test was level 2.

4.3.5 Analysis of the post-VHGT

After the GeoGebra was applied to students as an instructional tool, the van Hiele Geometry Test was again administered to the students. This was to determine the effectiveness of the use of GeoGebra on students‘ geometric thinking Levels. Table 4.9 presents the overall students‘ performance on each item of the section A in the van Hiele Geometry post-test. As can be seen in Table 4.9 each van Hiele Level (VHL) had five items with five multiple choice options. However, few students did not choose any of the options for some items. This made the researcher include an additional option (a blank option) in the table.

Table 4.10: VHGT post-test: section an item analysis for each level per student

Level 1	**Choice Items**	**1 F (%)**	**2 F (%)**	**3 F (%)**	**4 F (%)**	**5 F (%)**
	A	0(0%)	4(5.0%)	8(10.0%)	2(2.5%)	20(25.0%)
	B	**78(97.5%)**	0(0%)	0(0%)	**44(55.0%)**	2(2.5%)
	C	0(0%)	4(5.0%)	**70(87.5%)**	8(10.0%)	14(17.5%)
	D	2(2.5%)	**72(90.0%)**	0(0%)	22(27.5%)	**38(47.5%)**
	E	0(0%)	0(0%)	0(0%)	2(2.5%)	4(5.0%)
	Blank	0(0%)	0(0%)	2(0%)	2(2.5%)	2(2.5%)
Level 2	Choice Items	6	7	8	9	10
	A	4(5.0%)	4(5.0%)	**48(60.0%)**	2(2.5%)	8(10.0%)
	B	**48(60.0%)**	0(0%)	12(15.0%)	4(5.0%)	6(7.5%)
	C	16(20.0%)	4(5.0%)	6(7.5%)	**58(72.5%)**	**56(70.0%)**
	D	10(12.5%)	8(10.0%)	10(12.5%)	2(2.5%)	2(2.5%)
	E	0(0%)	**64(80.0%)**	4(5.0%)	14(17.5%)	6(7.5%)
	Blank	2(2.5%)	0(0%)	0(0%)	0(0%)	2(2.5%)
Level 3	Choice Items	11	12	13	14	15
	A	8(10.0%)	6(7.5%)	**42(52.5%)**	**30(37.5%)**	10(12.5%)
	B	12(15.0%)	**62(77.5%)**	6(7.5%)	14(17.5%)	**56(70.0%)**
	C	**50(62.5%)**	4(5.0%)	6(7.5%)	20(25.0%)	8(10.0%)
	D	0(0%)	2(2.5%)	0(0%)	0(0%)	6(7.5%)
	E	8(10.0%)	6(7.5%)	24(30.0%)	14(17.5%)	0(0%)
	Blank	2(2.5%)	0(0%)	2(2.5%)	2(2.5%)	0(0%)
Level 4	Choice Items	16	17	18	19	20
	A	16(20.0%)	24(30.0%)	**38(47.5%)**	**22(27.5%)**	**52(65.0%)**
	B	18(22.5%)	16(20.0%)	20(25.0%)	18(22.5%)	10(12.5%)
	C	**32(40.0%)**	6(7.5%)	16(20.0%)	14(17.5%)	12(15.0%)
	D	4(5.0%)	**22(27.5%)**	4(5.0%)	8(10.0%)	6(7.5%)
	E	8(10.0%)	10(12.5%)	0(0%)	18(22.5%)	0(0%)
	Blank	2(2.5%)	2(2.5%)	2(2.5%)	0(0%)	0(0%)

Source: Field Data (2019) **N = 80**

NB: *The figures in* **bold** *represent the total number (**n**) (%) of students who answered that item correctly.*

Results from Table 4.10 reveal performance of students‘ van Hieles‘ geometric thinking levels at the post-test stage. Level 1 of the VHGT shows that, 78 (97.5%), 72 (90.0%), 70 (87.5%), 44 (55.0%) and 38 (47.5%) of the students managed to correctly answer items 1, 2, 3, 4 and 5 respectively. Majority of the students 78 (97.5%), 72 (90.0%), 70 (87.5%), respectively had item 1, 2 and 3 correctly. More than half of the students 44 (55.0%) had item 4 correctly. 38 (47.5%) students had item 5 correctly. This indicates that most of the students were able to score most of the items in level 1 (visualization stage).

Concerning Level 2 of the VHGT, 48 (60.0%), 64 (80.0%), 48 (60.0%), 58 (72.5%) and 56 (70.0%) of the students correctly had items 6, 7, 8, 9 and 10 respectively. More than half of the students respectively answered item 6, 7, 8, 9 and 10 correctly. This indicates that in the level 2 (analysis stage), the students‘ performances were above average because more than half of the students were able to score all the items.

Furthermore, Level 3 of the VHGT shows that, 50 (62.5%), 62 (77.5%), 42 (52.5%), 30 (37.5%) and 56 (70.0%) of the students correctly had items 11, 12, 13, 14 and 15 respectively. More than half of the students 50 (62.5%), 62 (77.5%), 42 (52.5%) and 56 (70.0%) respectively had item 11, 12, 13 and 15 correctly. Students scored a low mark in item 14 (30 (37.5%)). This indicates that in the level 3 (ordering stage), the students performances were impressive because more than half of the students were able to answer item 11, 12, 13 and 15 respectively correct.

Finally, in Level 4 of the VHGT, 32 (40.0%), 22 (27.5%), 38 (47.5%), 22 (27.5%) and 52 (65.0%) of the students respectively answered items 16, 17, 18, 19 and 20 correctly. More than half of the students 52(65.0%)) had item 20 correctly. Majority of the students scored a low mark in item 16 (32 (40.0%)), item 17 (22 (27.5%)), item

18 (38 (47.5%)) and item 19 (22(27.5%)) correctly. This indicates that in the level 4 (deduction stage), the students performances were not impressive because it is only items 20 that more than half of the students were able to score.

4.3.6 The overall scores of students in the post-VHGT item test

There were 20 items that were used to assess the students‘ levels with each item allotted with one mark. Table 4.10.1 shows the general performance of students in the 20 pre-test items.

Table 4.10.1: Total scores obtained by students in post-VHGT by cumulative frequency

Score	Number of students (F)	Cumulative (F)	Percentage (%)	Cumulative Percentage (%)
12	11	11	13.8%	13.8%
13	10	21	12.5%	26.3%
14	10	31	12.5%	38.8%
15	10	41	12.5%	51.3%
16	12	53	15.0%	66.3%
17	10	63	12.5%	78.8%
18	7	70	8.8%	87.5%
19	6	76	7.5%	95.0%
20	4	80	5.0%	100.0%

Source: Field Data (2019)

Results in Table 4.10.1 show a minimum mark of 12 and maximum mark of 20. Majority of the students 95.0% (F = 76) obtained more than half of the marks allotted to the test, while 5.0% (F = 4) had the total marks allotted to the test. This indicates that the general performance of the SHS 2 students in the post-VHGT Item test was very impressive hence above average remarks.

4.3.7 Performance on van Hiele geometry post-test. Section B

Table 4.10.2 summarizes the overall performance of students' in the section B part of the van Hiele Geometry post-test. There were 2 test items. The responses of students' who demonstrated good knowledge and provided the right responses for the items were described as correct. Responses of students' who attempted items but did not get the total marks allotted per test item were described as partially correct, while the responses that exhibited lack of knowledge about the items were described as completely wrong. However, few students' did not attempt some of the items at all; these were described as "blank".

Table 4.10.2: Students Van Hiele geometry post-test: section B item analysis

Item	Correct F (%)	Partially Correct F (%)	Completely Wrong F (%)	Blank F (%)
1	44 (55.0%)	33 (41.3%)	3 (3.8%)	0 (0%)
2	35 (43.8%)	39 (48.8%)	5 (6.3%)	1 (1.3%)

Source: Field Data (2019) **N = 80**

Results in Table 4.10.2 shows that, 44 students representing 55.0% had item 1 correct, followed by 33 students representing 41.3% had item 1 partially correct, 3 students representing 3.8% had item 1 completely wrong and none of the students representing 0% had items 1 blank. The performance of students in item 1 was impressive because only 3students had item 1 completely wrong.

In item 2, 35 students representing 43.8% had item 2 correct, followed by 39 students representing 48.8% had item 2 partially correct, 5 students representing 6.3% had item 2 completely wrong and 1 student representing 1.3% had items 2 blank. The performances of students in item 2 were encouraging. The general performance of students in item 1 and 2 indicates that, they demonstrated a better understanding of the geometric concepts covered in the test.

4.3.8 Levels reached by students in the van Hiele geometry post-test

Table 4.10.3 shows the van Hiele levels of geometric thinking attained by the students after the van Hiele Geometry post-test.

Table 4.10.3: Students van Hiele levels attained in the post-VHGT

Levels	Number of students (F)	Percentage (%)
0	0	0
1	14	17.5
2	23	28.75
3	40	50
4	3	3.75
Total	**80**	**100**

Source: Field Data (2019)

Table 4.10.3 indicate that, 0% (n = 0) of the students could not reach any of the levels, 17.5% (n = 14) of the students reached the Visualization (level 1), while 28.75% (n = 23) reached the Analysis (level 2) of the Van Hiele Geometric thinking levels. Furthermore, 50% (n = 40) reached the Ordering (level 3). 3.75% (n = 3) of the students reaching the level 4 of the Van Hiele‘s Geometric thinking levels. Students who did not reach any of the levels of Van Hiele Geometric thinking means that the students could not meet the criteria for attaining VHGT level, that is the students could not answer three (3) questions correctly from the items 1 to 5. Again, it can be seen that 50% (n = 40) students reached the Ordering stage (level 3); this is an indication that out of 80 students only fourteen (14) of the students could reach the levels 1, 2 and 3. This means that fourteen (14) students could perform in level 3, where students can logically order the properties of shapes. Finally, 3.75% (n = 3) of the students reached the Deduction stage (level 4) of the Van Hiele Geometric thinking levels. This indicates three (3) out of 80 students were able to meet the criteria 3 of 5 correct suggested by Usiskin (1982) in all the levels, that means three of the students could answer at least 3 items correctly in questions items; 1 to 5, 6 to 10, 11 to 15, 16 to 20. It shows that, at this level a student understands the significance of deduction and the role of postulates, axioms, theorems and proofs. These are important geometric knowledge which students need to study in geometry related courses at the tertiary level. This shows the sequential order of Van Hiele‘s level 3 criterion because the students were able to answer correctly at least three of second level questions. Hence, the van Hiele Levels of geometric thinking attained by the students after the van Hiele Geometry post-test was level 3 and 4

4.3.9 Analyses of hypotheses

Hypothesis 1

H_0 : There is no statistically significant difference in senior high technical school students van Hiele geometric thinking levels after GeoGebra instruction.

H_1 : There is statistically significant difference in senior high technical school students van Hiele geometric thinking levels after GeoGebra instruction.

The first research hypothesis sought to find out whether or not there was a statistically significant difference in senior high technical school students van Hiele geometric thinking levels after GeoGebra instruction in Senior High Technical School. The One-Way Analysis of Variance (ANOVA) was used in the analysis. The ANOVA is used to determine whether there are any statistically significant differences between the means of three or more independent groups. With regards to this study, there were four independent groups regarding students van Hiele geometric thinking levels such as level 1, level 2, level 3 and level 4. Therefore, the means of these independent groups were compared in order to find out whether any differences existed between these independent groups on students van Hiele geometric thinking levels after GeoGebra instruction. Result is illustrated in Table 4.11.

Table 4.11: One-way ANOVA analysis of students van Hiele geometric thinking levels after GeoGebra instruction

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	2975.468	1	2975.468	14.079	.001
Within Groups	7608.111	36	211.336		
Total	**10583.579**	**37**			

Source: Field Data (2019) ****Significant @ 0.05 level***

Results in Table 4.12 shows that the statistical test is the *F* ratio and it can be seen that the *F* ratio is 14.078 and the *p* value of the *F* ratio is .001. Since, the *p* value of .001 is less than the alpha level of .05; it implies that there is a statistically significant difference among the level means of students van Hiele geometric thinking levels after GeoGebra instruction.

Table 4.11.1: Post-hoc analysis of students' van Hiele geometric thinking levels before and after GeoGebra instruction

Test	N	Mean	Std. Deviation	Std. Error	95% Confidence interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Pre-test	80	33.50	14.006	3.132	26.95	40.05	2	62
Post-test	80	51.22	15.110	3.561	43.71	58.74	22	78

Source: Field Data (2019) **Significant @ 0.05 level**

The Post hoc analysis on table 4.12.1 indicates that the mean of students van Hiele Geometric Thinking Levels before GeoGebra instruction was 33.50 while the mean of students van Hiele Geometric Thinking Levels after GeoGebra instruction was 51.22. Hence, the mean difference of performance of students van Hiele Geometric Thinking

Levels before and after GeoGebra instruction was 42.36 ((33.50 + 51.22) ÷2) .The result show that the mean difference was significant at 0.05 level. Since the mean performance of students van Hiele Geometric Thinking Levels after GeoGebra instruction is higher than those students van Hiele Geometric Thinking Levels before GeoGebra instruction and the difference is significant, it then follows that the mean difference in the performance after GeoGebra instruction is significantly higher than the one before the GeoGebra instruction. The difference is therefore generalizable.

Hypothesis 2

H_0 : There is no statistically significant difference between the pre-VHGT and post-VHGT scores of senior high technical school students.

H_1 : There is statistically significant difference between the pre-VHGT and post-VHGT scores of senior high technical school students.

The second research hypothesis sought to find out whether or not there was a statistically significant difference in Van Hiele Geometry Tests (VHGT) scores of senior high technical school students. The paired samples t-test was used to test the null hypothesis that there was no significant difference in the two tests. The results obtained for the t-test analysis is presented in in Table 4.12.

Table 4.12: Paired sample T-test of pre and post VHGT scores of students

TEST	N	Mean	Std. Dev.	Mean Difference	t-value	df	Sig. (2-tailed)
Post-Test	80	15.43	2.375	7.48	-16.654	79	.000
Pre-Test	80	7.95	2.140				

Source: Field Data (2019) ***Significant @ 0.05 level***

Results in Table 4.13 shows the paired samples t-test results in Table 4.12 shows that the pre-test mean score (M=7.95; SD=2.140) and post-test mean score (M=15.43;

SD=2.375) were found to be statistically significant at t= -116.654; df=79; p<0.05. Therefore, the null hypothesis that there is no statistical significant difference between the pre-VHGT and post-VHGT scores of senior high technical school students was rejected.

4.4 Research Question 3: How does the use of GeoGebra motivate senior high technical school students' to learn geometry? This research question sought to explore the effect of the use of GeoGebra on senior high technical school students' motivation to learn geometry.

The researcher used focused group discussion to solicit students' views on how GeoGegra motivate them to learn geometry. Twenty students were randomly selected and put into five groups, namely; Group 1, 2, 3, 4 and 5. Each group contain four students.

The researcher asked the students, how does the use of GeoGebra motivate you to learn geometry? Below were the students' responses

Group 1 commented that:

> *The use of GeoGebra software has helped us a lot. With the use of the GeoGebra tools, we can now construct angles and we are able to find the interior and exterior angles. Again, GeoGebra has increased our time and interest to learn Geometry, it has also helped us to know more about geometry and how to solve questions under it. Thus, GeoGebra has increased our performance in Mathematics.*

In addition, Group 2 was of the view that:

> *The use of GeoGebra software has motivated us to learn geometry. The GeoGebra software is easy to use and it has helped us to construct the angles on our own. Hence helps us to develop our conceptual understanding of geometry.*

Also, Group 3 said that:

> *The use of GeoGebra software has enhanced their understanding in geometry. The software has helped us to solve mathematical calculation and lessen our burden in our mathematical calculation. We are also motivated to learn geometry because with the GeoGebra we can learn on the PC, it is easy to draw and measure angles.*

Moreover, Group 4 commented that:

> *GeoGebra has made Geometry easy and interesting. GeoGebra has increased our interest and time to learn Geometry. As we practice we get motivated. The process of using the GeoGebra Software was like rotating the angles clockwise and anticlockwise, which excite us to learn more of the angles. For instance, we got to know the difference between interior and exterior angles and how they are formed. "If we rotate the angles in the clockwise direction, we get the interior angle and if we rotate in anti-clockwise direction we get the exterior angle.*

Finally, Group 5 also said that:

> *We find Geometry lesson interesting and free from fear and confusion because we can freely share ask questions and share our experiences with other group members and the teacher. Also, during the group activities our friends explain some things we don't understand to us.*

Interview responses revealed that the students found the lessons interesting and easy to understand. The students suggested that, the GeoGebra software should be employed in most lessons because it made them active in the teaching and learning process, took away dullness and also made learning easier the practical investigations as integrated in the GeoGebra lesson. The students also said they liked the manipulative and concrete nature of the GeoGebra tools.

4.5 Research Question 4: What are the senior high technical school students' perceptions of using GeoGebra in learning geometry?

This research question sought to find out senior high technical school students‘ perceptions of using GeoGebra in learning geometry. The result is presented in table 4.13

Table 4.13: Students' Perception of using GeoGebra in learning geometry

Statement	A		D	
	F	%	F	%
I feel confident when I do geometric activities by using GeoGebra software.	75	94%	5	6%
I can think creatively and critically when using GeoGebra software.	60	75%	20	25%
GeoGebra software helps increase my performance in mathematics class.	65	81%	15	19%
I am excited when asked to explore the GeoGebra software.	76	95%	4	5%
I am happy if the mathematics teachers use the GeoGebra software in teaching mathematics especially geometry	78	98%	2	2%
GeoGebra allows me to visualize and manipulate geometric concepts.	79	99%	1	%
I was able to make logical connections between geometric theorems using GeoGebra.	51	64%	29	36%
I was engaged in the learning process using GeoGebra.	69	86%	11	14%

Source: Field Data (2019) N = 80

Key: ***A*** *= agree,* ***D*** *= disagree,* ***F*** *= frequency and* ***%*** *= percentage*

Results from Table 4.13 shows that 75(94%) agreed to the statement that, they feel confident when they do geometric activities by using GeoGebra software while 5(6%) disagreed to the statement. In addition, 60(75%) agreed that, they can think creatively and critically when using GeoGebra software while 20(25%) disagreed. Again, 65(81%) agreed that, GeoGebra software helps increase their performance in mathematics class while 15(19%) disagree. Concerning students being excited when asked to explore the GeoGebra software, 76(95%) agreed while 4(5%) disagreed. Moreover, 78(98%) agreed that, they are happy when their teachers uses the GeoGebra software in teaching mathematics while 2(2%) disagreed. 79 (99%) agreed that, GeoGebra allows them to visualise and manipulate geometric concepts while 1(1%) disagreed. Furthermore, 51(64%) agreed that, they were able to make logical connections between geometric theorems using GeoGebra while 29(36%) disagreed. Finally, 69(86%) agreed that, they are engaged in the learning process using

GeoGebra while 11(14%) disagree. This reveals that students had positive perception towards GeoGebra for the learning of geometry.

4.6 Discussion of Research Findings

What are the effects of using GeoGebra on senior high technical school students' performance in geometry?

The first research question sought to explore the effect of using GeoGebra on senior high technical school students‘ performance in geometry. The pre-GAT findings as presented in table 4.4 (p. 76) indicate that 3 students scored between 1and 10 in the test, 11 students scored between 11 and 20, 10 students scored between 21 and 30, 40 students scored between 31 and 40, 10 students scored between 41 and 50, 6 students scored between 51 and 60, none of the students scored between 61 and 70 and 71 and 80 with a mean score of 33.125. The finding implies that the performance of students in geometry was still very low. These findings agree with Charles and Lynwood (1990) as cited in Kabutey (2016), who argued that poor performance in senior high school geometry has traditionally been high and this has been ascribed to various causes such as the difficulty of the subject, others have to blame it to ineptitude or laziness on part of the student. While others have held that students lose interest in geometry because of its abstract nature which they regard as having no practical value. They argue that demonstrative geometry is not easiest subject to learn. Similarly, Rukangu (2000) conducted a study on pupils development of spatial ability on Mathematics and found out that 67% did not enjoy learning spatial concepts because they are confusing, abstractly demanding a lot of thinking and difficultly to understand.

Moreover, the post-GAT findings from table 4.5 (p.77) indicate that none of the students scored between 1and 10; 11 and 20; 21 and 30; 31 and 40 in the test, 20 of the students scored between 41 and 50, 21 of the students scored between 51 and 60, 36 of the students scored between 61 and 70% and 3 students scored between 71 and 80 with a mean score of 58.25. Prodromou (2014) claimed that GeoGebra software has a very constructive effect on college students' achievement in the area of statistics. The author argues that students have a remarkably positive attitude towards GeoGebra.

Finally, table 4.6 (p.78) compared the pre-test and post-test results of the GAT of the students. In the post-GAT the results showed an improvement in students learning performance in geometry. The post-GAT group had higher scores (M = 57.73, SD = 9.604) than those in the pre-GAT group (M = 31.83, SD =11.843).This is an indication that in the post-test, the use of GeoGebra had positive effects on students' performance in geometry. These findings concur with Arbain and Shukor (2015) studies on ‗The effects of GeoGebra on students' achievement', which underlines the changes coming with GeoGebra for students. The study examines the effectiveness of using GeoGebra system on Mathematics learning among 62 students in Malaysia. The outcomes demonstrate that students have positive opinions regarding their learning, including having better learning achievements by using the GeoGebra Software.

Again, a paired samples t-test statistic presented in table 4.8 (p. 80) was conducted on the pre- and post-test scores in the Geometry Achievement Test (GAT) to determine whether the improvement was significant showed that there was significant difference in their mean scores for the pre-test ($M = 31.83, SD = 11.843$) and post-test mean score ($M = 57.73, SD = 9.604$) at $t\ (80) = -32.467, p = 0.000 < 0.05$.

How effective is the use of GeoGebra on senior high technical school students' van Hiele level of geometric thinking?

This research question sought to find out the effectiveness of using GeoGebra on senior high technical school students' van Hiele level of geometric thinking. In order to answer this question, the researcher used VHGT to solicit information from the students.

The findings indicates that in Level 1 of the VHGT, 22 (27.5%), 18 (22.5%), 40 (50.0%), 2 (2.5%) and 62 (77.5%) of the students managed to correctly answer items 1, 2, 3, 4 and 5 in the pre-test in Table 4.9 (p. 82) while 78 (97.5%), 72 (90.0%), 70 (87.5%), 44 (55.0%) and 38 (47.5%) of the students managed to correctly answer items 1, 2, 3, 4 and 5 respectively in the post-test in Table 4.10 (p. 88). Concerning Level 2 of the VHGT, 52 (65.0%), 30 (37.5%), 42 (52.5%), 22 (27.5%) and 28 (35.0%) of the students correctly had items 6, 7, 8, 9 and 10 respectively in the pre-test in Table 4.9 while 48 (60.0%), 64 (80.0%), 48 (60.0%), 58 (72.5%) and 56 (70.0%) of the students correctly answered items 6, 7, 8, 9 and 10 respectively in the post-test in Table 4.10.

Moreover, in Level 3 of the VHGT, 34 (42.5%), 42 (52.5%), 28 (35.0%), 46 (57.5%) and 34 (42.5%) of the students respectively had items 11, 12, 13, 14 and 15 correctly in the pre-test in Table 4.9 while 50 (62.5%), 62 (77.5%), 42 (52.5%), 30 (37.5%) and 56 (70.0%) of the students correctly had items 11, 12, 13, 14 and 15 respectively in the post-test in Table 4.10.

Finally, in Level 4 of the VHGT, 18 (22.5%), 34 (42.5%), 26 (32.5%), 38 (47.5%) and 52 (65.0%) of the students had items 16, 17, 18, 19 and 20 respectively correct in the pre-test in Table 4.9 while 32 (40.0%), 22 (27.5%), 38 (47.5%), 22 (27.5%) and

52 (65.0%) of the students had items 16, 17, 18, 19 and 20 correctly in the post-test in Table 4.10.

This indicates that most students did not perform well in the pre-VHGT Item test and therefore could not solve the Levels 3 and 4 items as compared to the post-VHGT Item test. Students who attempted the questions used incorrect working procedures in their effort to solve the items in the pre-VHGT Item test. This resulted in some students arriving at various answers. Furthermore, students inability to solve the Ordering and Deductive levels questions agree with the findings of Atebe and Schafer (2010); Baffoe and Mereku (2010) who stated that students ‗weaknesses had obstructed the progress of mapping the steps appropriately to finding the solution. After the treatment period (the use of GeoGebra), it was observes that students were able to solve the ordering and Deductive level questions, this coincides with Bwalya (2019) who concludes that GeoGebra is one sure solution to the poor performance in questions involving geometric concepts as it enhances understanding which is the key ingredient to good mathematics learning and hence improved performance in the area of geometry at secondary school level.

Also the analysis of levels reached by students on the Van Hiele‗s Geometric thinking levels showed that, majority of the students had not reach any level or reached the first and second levels of the Van Hiele‗s Geometric thinking levels, that is the Visualization and Analysis level in the pre-test in table 4.9.3 (p 86)as compare to table 4.10.3(p 92) in the post-test. The number of students who reached levels 3 and 4, that is ordering and deductive levels shows that most students were not able to categorize and generalize by attributes and develop proofs using axioms and definitions in the pre-test as compared to the post-test. The findings in the study showed that students who reached the Ordering and Deductive levels could classify and generalize by attributes and develop proofs using axioms and definitions.

It can be concluded that, the use of GeoGebra on students van Hiele level of geometric thinking was effective because majority of the students 95.0% (F = 76) obtained more than half of the marks allotted to the test while 5.0% (F = 4) had the total marks allotted to the test after the use of GeoGebra. The general performance of students after the use of GeoGebra in items 1 and 2 indicate that, students demonstrated better understanding of the geometric concepts covered in the test. After the use of GeoGebra three (3) out of 80 students were able to meet the criteria 3 of 5 correct suggested by Usiskin (1982) in all the levels, that means three of the students could answer 4 items correctly in questions items; 1 to 5, 6 to 10, 11 to 15, 16 to 20. An indication that, student at this level understands the significance of deduction and the role of postulates, axioms, theorems and proofs. These are in fact fundamental geometric knowledge which students need to study in geometry related courses at the tertiary level. This shows the sequential order of Van Hiele‘s level 3 criterion because the student were able to answer correctly at least three of second level questions. Hence, van Hiele Levels of geometric thinking attained by the students after the use of GeoGebra was level 3 and 4.

Senior High Technical School Students Van Hiele Geometric Thinking Levels After GeoGebra Instruction

In the first hypothesis, one-way ANOVA was used to determine whether or not there was a statistically significant difference in senior high technical school students van Hiele geometric thinking levels after GeoGebra instruction. Findings from Table 4.11(p. 94) indicate that, the *F* ratio (14.079) is significant (p = .001) at the .05 alpha level. This implies that there is a statistically significant difference among the level means of students van Hiele geometric thinking levels after GeoGebra instruction. It is concluded that there is a statistically significant difference at the $p < .005$ level in the level means of students van Hiele geometric thinking levels after GeoGebra instruction [F (14.079) = 4.113, p = 0.001]. Therefore, the null hypothesis which stated that there is no statistically significant difference in senior high technical school students van Hiele geometric thinking levels after GeoGebra instruction is rejected. This is in line with the study of Ahmad and Rohani (2010) which discovered that the independent-t test comparing the post-test results of the two groups showed that there was a significant difference between mean performance scores of the control group (M=54.7, SD= 15.660) compared to GeoGebra group (M= 65.23, SD= 19.202; t(51) = 2.259, p = .028 < .05). This finding indicated that students who had learned Coordinate Geometry using GeoGebra was significantly better in their performance compared to students who did not.

A post-hoc test of Least Significant Difference presented in table 4.11.1 (p. 95) was used to test the significance of students van Hiele Geometric Thinking Levels after GeoGebra instruction and the result showed that the mean difference is significant at the .05 level. The result concurs with Emaikwu, Iji and Abari (2015) in their study that there is no significant difference between the pre-test and post-test performance of students taught Mathematics with the use of GeoGebra software". The study revealed that the students in experimental group gained higher scores in their post-test performance than the pre-test performance. By implication, there was significant difference between the pre-test and post-test performance of students taught Mathematics with the use of GeoGebra software.

Van Hiele geometry tests (VHGT) scores of senior high technical school students

In the second hypothesis paired samples t-test which was used to test whether a significant difference exist, disclosed that pre-test mean score (M=7.95; SD=2.140) and post-test mean score (M=15.43; SD=2.375) were found to be statistically significant at t= -116.654; df=79; $p<0.05$.

This implies that there is a statistically significant difference among the mean pre-VHGT and post-VHGT scores of senior high technical school students. Therefore, the null hypothesis which stated that there is no statistically significant difference between the pre-VHGT and post-VHGT scores of senior high technical school students' is rejected.

How does the use of GeoGebra motivate senior high technical school students' to learn geometry?

This research question sought to explore the effect of the use of GeoGebra on senior high technical school students' motivation to learn geometry.

The findings reveals that students exhibited high levels of eagerness and attention due to the systematic nature of the GeoGebra software coupled with the manipulative and concrete nature of the teaching and learning materials. For instance, one of the groups even remarked that:

> *"They find Geometry lesson interesting and free from fear and confusion because they am free to ask questions from group members and the teacher. Also, during the group activities their friends explain some things they don't understand to them".*

Students found the freedom in expressing their ideas without being bound by any rules and definite answers. This is in line with the study by Siew and Chong (2014) who found that useful concrete manipulative materials integrated with the GeoGebra in learning geometry is able to help develop better interest and creativity.

Responses from the students reveal that the GeoGebra motivates them to learn geometry by eliminating dullness and making learning easier and fascinating. These findings confirm to what Smiešková and Barcíková, (2014) said, that technology coupled with well-planned hands-on investigations promotes student-centred learning, reduces memorization and also motivates students by providing a better learning environment.

Finally, the responses resonates strongly with what Fisch, Lesh, Motoki, Crespo and Melfi (2010) cited in Colgan (2014) said that experiential learning connect deeply with learner‘s passions and interests, making learning profoundly personal. By adopting engaging tools in the classroom, teachers may be able to transform feelings about learning and Mathematics by changing the focus from teaching facts and skills to building positive relationships between learners and Mathematics. The way that learners feel about Mathematics profoundly influences what they do with it and how they reflect on it, which in turn influences how knowledge grows and connects.

What are the senior high technical school students' perceptions of using GeoGebra in learning of geometry?

The fourth research question sought to find out senior high technical school students‘ perceptions of using GeoGebra in learning of geometry. In order to answer this question, the researcher used structured questionnaire to solicit information from the students.

The findings presented in table 4.13 (p 99) indicated that, 75(94%) agreed that, they feel confident when they do geometric activities by using GeoGebra software. In addition, 60(75%) agreed that, they can think creatively and critically when using GeoGebra software. Again, 65(81%) agreed that, GeoGebra software helps increase their performance in mathematics class. Concerning students being excited when asked to explore the GeoGebra software, 76(95%) agreed. Moreover, 78(98%) agreed that, they are happy when their teachers uses the GeoGebra software in teaching mathematics. 79 (99%) agreed that, GeoGebra allows them to visualise and manipulate geometric concepts. Furthermore, 51(64%) agreed that, they were able to make logical connections between geometric theorems using GeoGebra. Finally, 69(86%) agreed that, they are engaged in the learning process using GeoGebra.

From the responses from the students, the researcher can deduces that, majority of the respondents perceptions of using GeoGebra in learning geometry were; GeoGebra allows them to visualise and manipulate geometric concepts (99%), they are happy when their teachers use the GeoGebra software in teaching mathematics (98%), they were excited when asked to esxplore the GeoGebra software (95%) and they feel confident when they do geometric activities by using GeoGebra software (94%). The findings suggest that GeoGebra software enable students to explore the geometric environment and thereby engaging them in the teaching and learning process. This finding is supported by (EU, 2013; Arbain & Shukor, 2015) that the use of GeoGebra to engage students in learning enable them to think at a higher level that leads to better learning achievement.

From the foregoing, it can be concluded that, students‘ perceptions of using GeoGebra in learning of geometry are; they feel confident when they do geometric activities by using GeoGebra software, they can think creatively and critically when using GeoGebra software, GeoGebra software helps increase their performance in mathematics class, they are happy when their teachers uses the GeoGebra software in teaching mathematics, they were able to make logical connections between geometric theorems using GeoGebra and they are engaged in the learning process using GeoGebra.

4.7 Summary of Findings

The first findings on the effect of using GeoGebra on students learning performance in geometry reveals that there was an improvement in students learning performance in geometry. The post-GAT group had higher scores (M = 57.73, SD = 9.604) than those in the pre-GAT group (M = 31.83, SD =11.843).This shows that the use of GeoGebra had positive effects on students learning performance in geometry.

The second findings from Van Hiele Geometry Test (VHGT) on the effectiveness of using GeoGebra on students' van Hiele level of geometric thinking indicates that, most of the students did not perform well in the pre-VHGT Item test and therefore could not solve the Levels 3 and 4 items as compared to the post-VHGT Item test. Students who attempted the questions used wrong working processes in their attempt to solve the items in the pre-VHGT Item test. After the treatment period (the use of GeoGebra), it was observes that students were able to solve the ordering and Deductive level questions. Majority of the students 95.0% (F = 76) obtained more than half of the total marks allotted to the test while 5.0% (F = 4) had the total marks allotted to the test after the use of GeoGebra. The general performance of students after the use of GeoGebra in item 1 and 2 indicates that, students demonstrated a better understanding of the geometric concepts covered in the test. Van Hiele Levels of geometric thinking attained by the students after the use of GeoGebra was level 3 and 4. Thus, the use of GeoGebra on students van Hiele level of geometric thinking was effective.

The study tested the following hypotheses

H_0 : There is no statistically significant difference in senior high technical school students van Hiele geometric thinking levels after GeoGebra instruction.

H_0 : There is no statistically significant difference between the pre-VHGT and post-VHGT scores of senior high technical school students.

The third findings on how GeoGebra motivate students to learn geometry shows that majority of the students were motivated to learn geometry through the use of the GeoGebra software. They added that the GeoGebra took away dullness thus, making learning easier and attractive. The findings further revealed that majority of the students were of the view that, in the geometry lessons, the researcher made them very active participants, provided opportunities for asking questions and made them learn from one another in a series of guided group activities. Thus, the students formed a central position in the lesson delivery.

The fourth research findings from the students‘ questionnaire on the use of GeoGebra in learning of geometry indicates that, 75(94%) agreed that, they feel confident when they do geometric activities by using GeoGebra software. In addition, 60(75%) agreed that, they can think creatively and critically when using GeoGebra software. Again, 65(81%) agreed that, GeoGebra software helps increase their performance in mathematics class. Concerning students being excited when asked to explore the GeoGebra software, 76(95%) agreed. Moreover, 78(98%) agreed that, they are happy when their teachers uses the GeoGebra software in teaching mathematics. 79 (99%) agreed that, GeoGebra allows them to visualise and manipulate geometric concepts. Furthermore, 51(64%) agreed that, they were able to make logical connections between geometric theorems using GeoGebra. Finally, 69(86%) agreed that, they are engaged in the learning process using GeoGebra. This reveals that students had positive perception of the use of GeoGebra in learning of geometry.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Overview

This chapter comprises of the summary of the study, emphasizing on the major findings. It discusses the conclusion, recommendations as well as suggestions for future research.

5.1 Summary of the Study

Geometry learning is problematic to senior high technical school students, as they fail in developing the appropriate understanding of geometric concepts and in acquiring the geometric problem solving skills. Most of the students have difficulties identifying properties of shapes, identifying similarities and differences among shapes and solving problems relating to concepts of shapes. The purpose of the study was to investigate the effect of using GeoGebra on Van Hiele geometric thinking levels of senior high technical school students‘ attainment of geometry. In order to achieve this purpose, the following questions were set as a guide to this study.

1. What are the effects of using GeoGebra on senior high technical school students‘ learning performance in geometry?
2. How effective is the use of Geogebra on senior high technical school students‘ van Hiele level of geometric thinking?
3. How does the use of GeoGebra motivate senior high technical school students‘ to learn geometry?
4. What are the senior high technical school students‘ perceptions of using Geogebra in learning of geometry?

The finding of the study would contribute to all secondary school teachers when looking at the effect of utilizing technology (GeoGebra) for teaching mathematics, especially geometry. This study would also provide information to teachers about students' understanding and learning processes when using the GeoGebra in relation to the geometry topic in mathematics. From the literature review, it was observed that several studies have been conducted in various parts of the country that have shown the effect of GeoGebra on van Hieles' geometric thinking level of students learning attainment of geometry. Most of the study's results had proven to be effective when using GeoGebra as an instructional tool. Thus, the current study aimed at generating knowledge on the effects of Geogebra on van Hieles' geometric thinking levels of senior high technical school students learning attainment.

The study followed a mixed methods research approach. This approach provided a comprehensive information and understanding on the problem study. In this study, one group pretest-posttest pre-experimental design was employed. This study used one group pretest-posttest pre-experimental design because there is no control for comparison. Simple random sampling procedure was used to select a sample size of eighty (80) SHS 2 senior high technical school students.

The instruments used in this study were Geometry Achievement Test (GAT), van Hieles' Geometry Test (VHGT) and interview guide and questionnaire. Responses from the Geometry Achievement Test (GAT) and Van Hiele Geometry Test (VHGT) were analysed using frequency tables, percentages and bar charts. Responses from the interview guide were analysed qualitatively using thematic approach. Each questionnaire responses were analysed descriptively using frequency tables and percentages. Inferential statistics of one-way ANOVA and Paired sample T-test were used to test the hypotheses using the Statistical Package for the Social Sciences (SPSS) Software version 22.

5.2 Major Findings

The findings of the study are summarized and presented under the four sub-headings in line with the research questions.

5.2.1 Research question 1: What are the effects of using GeoGebra on senior high technical school students' performance in geometry?

Findings on the effect of using GeoGebra on students' performance in geometry reveals that there was an improvement in students learning performance in geometry. The post-GAT group had higher scores (M = 57.73, SD = 9.604) than those in the pre-GAT group (M = 31.83, SD =11.843).This shows that, the use of GeoGebra had positive effects on students learning performance in geometry.

5.2.2 Research question 2: How effective is the use of GeoGebra on senior high technical school students' van Hiele level of geometric thinking?

Findings on how effective is the use of Geogebra on senior high technical school students' van Hiele level of geometric thinking, indicate that most students did not perform well in the pre-VHGT Item test and therefore could not solve the Levels 3 and 4 items as compared to the post-VHGT Item test. Students who attempted the questions used wrong working processes in their attempt to solve the items in the pre-VHGT Item test. After the treatment period (the use of GeoGebra), it was observed that students were able to solve the ordering and Deductive level questions. Majority of the students 95.0% (F = 76) obtained more than half of the total marks allotted to the test while 5.0% (F = 4) had the total marks allotted to the test after the use of GeoGebra. The general performance of students after the use of GeoGebra in item 1 and 2 indicates that, students demonstrated a better understanding of the geometric concepts covered in the test. Van Hiele Levels of geometric thinking attained by the students after the use of GeoGebra was level 3 and 4. Thus, the use of GeoGebra on students van Hiele level of geometric thinking was effective.

The null hypothesis which stated that there is no statistically significant difference in senior high technical school students van Hiele geometric thinking levels after GeoGebra instruction was rejected in the first hypothesis because *p* value of .001 was less than the alpha level of .005.

Findings from the second hypothesis indicate that, pre-test mean score (M=7.95; SD=2.140) and post-test mean score (M=15.43; SD=2.375) were found to be statistically significant at t= -116.654; df=79; p<0.05.Thus it was tested that there is a statistically significant difference among the mean pre-VHGT and post-VHGT scores of senior high technical school students‘. Therefore, the null hypothesis which stated that there is no statistically significant difference between the pre-VHGT and post-VHGT scores of senior high technical school students‘ was rejected.

5.2.3 Research question 3: How does the use of GeoGebra motivate senior high technical school students' to learn geometry?

Findings on how GeoGebra motivate students to learn geometry shows that majority of the students were motivated to learn geometry through the use of the GeoGebra software. They added that the GeoGebra took away dullness because they are actively involved in the teaching and learning process.

5.2.4 Research question 4: What are the senior high technical school students' perceptions of using GeoGebra in learning of geometry?

Findings of students' perception on the use of GeoGebra in learning of geometry reveals that, majority of the participants agreed that, they feel confident when they do geometric activities by using GeoGebra software, they can think creatively and critically when using GeoGebra software, GeoGebra software helps increase their performance in mathematics class. They are excited when asked to explore the GeoGebra software, they are happy when their teachers uses the GeoGebra software in teaching mathematics, GeoGebra allows them to visualise and manipulate geometric concepts, they were able to make logical connections between geometric theorems using GeoGebra and they are engaged in the learning process using GeoGebra. This reveals that students had positive perception of the use of GeoGebra in learning of geometry.

5.3 Conclusion

The purpose of the study was to nvestigate the effect of using GeoGebra on Van Hiele geometric thinking levels of senior high technical school students' attainment of geometry. The following conclusions were drawn based on the findings of the study:

- The use of GeoGebra had positive effects on students learning performance in geometry. Also, the use of GeoGebra on students van Hiele level of geometric thinking was effective because Majority of the students 95.0% (F = 76) obtained more than half of the total marks allotted to the test while 5.0% (F = 4) had the total marks allotted to the test after the use of GeoGebra. The general performance of students after the use of GeoGebra indicated that, students demonstrated a better understanding of the geometric concepts covered in the test. In addition, it helped students to attain level 3 and level 4 of van Hiele Levels of geometric thinking.

The first hypothesis concluded that there is a statistically significant difference at the $p < .005$ level in the level means of students van Hiele geometric thinking levels after GeoGebra instruction [F (2975.468) = 14.018, p = 0.001]. Therefore, the null hypothesis which stated that there is no statistically significant difference in senior high technical school students van Hiele geometric thinking levels after GeoGebra instruction was rejected.

The second hypothesis concluded that there is a statistically significant difference at the $p < .005$ level in the mean pre-VHGT and post-VHGT scores of senior high technical school students'. Therefore, the null hypothesis which stated that there is no statistically significant difference between the pre-VHGT and post-VHGT scores of senior high technical school students' was rejected.

The responses from the interview revealed, the senior high technical school students were motivated to learn geometry through the use of the GeoGebra software. It was observed that the GeoGebra software aroused the students' interest and eagerness to learn due to the manipulative nature of the software, this made them participated fully during the lessons

Finally, senior high technical school students had positive perception towards the use of GeoGebra in learning geometry because majority of the students agreed that, the use of the software has improved their confident when doing geometric activities, they can also think creatively and critically when using the GeoGebra software. Thus the use of GeoGebra leads to enhanced achievement in students in mathematical exercises.

5.4 Recommendations

The results obtained from this study raised a number of issues of importance and interest to students, parents, educational authorities as well as the general public

The following recommendations were drawn based on the findings of the study:

(1). The study recommends that, heads of senior high schools and other educational stakeholders' should organise in- service training for mathematics teachers to equip them with the required skills on how to utilise GeoGebra for effective teaching and learning of geometry, as well as mathematics lessons in general.

(2). Also senior high school Mathematics curriculum should capture different models of geometry teaching more especially GeoGebra as an instructional tool. Similarly, Mathematics teachers and students should be encouraged by the head of the department to use the GeoGebra in teaching and learning of geometry concepts as well other concepts in mathematics.

(3). Furthermore, seminars/workshops should be organized at the at the regional and national levels of education for senior high technical school Mathematics teachers on the use of appropriate technological tools such as GeoGebra in the teaching and learning of mathematical concepts by experts from the universities or higher level of technology knowledge. This is because the application of GeoGebra in teaching and learning requires in-depth skills and competency on the part of teacher.

(4). Finally, the study recommends that, the ministry of education should endeavour to equip senior high technical schools with functional computer laboratories. Mathematics teachers should incorporate GeoGebra and other Mathematics software in the teaching and learning of Mathematics.

5.5 Suggestions for Further Studies

Future study employing this study design may consider using the model to investigate other areas of geometry such as three-dimensional figures, circle theorems and coordinate geometry. A study in this area can also be done to involve more senior high technical school in the Country to obtain a general picture of the effects of using GeoGebra on students van Hiele geometric thinking levels.

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APPENDICES

APPENDIX A

Questionnaire for SHS 2 Students

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF SOCIAL SCIENCES
DEPARTMENT OF MATHEMATICS EDUCATION

The purpose of this questionnaire is to investigate the effect of GeoGebra on van Hieles' geometric thinking levels of senior high technical school students learning attainment of geometry. Your responses will be helpful in planning and design possible solutions for the problem. The information you provide in this questionnaire is only for academic purpose.

Please be honest and as objective as you can. Tick (√) the appropriate response as applicable to you and fill in the blank spaces where answers are not supplied. Confidentiality in respect of whatever information you give is fully assured.

Thanks for your cooperation.

SECTION A

STUDENTS' BIO-DATA

Please tick (√) where appropriate

1. Gender

a. Male ( )

b. Female ( )

2. Programme Offered

a. Electrical Engineering Technology ( )

b. Mechanical Engineering Technology ( )

c. Fashion Designing and Construction ( )

d. Agricultural Mechanization Technology ( )

e. Welding and Fabrication Technology ( )

f. Hospitality and catering Technology ( )

3. Years of Learning Mathematics Lessons with Technology

a. 1 - 2 years ( )

b. 3 - 4 years ( )

c. 5 years and above ( )

SECTION B

STUDENTS PERCEPTION OF USING GEOGEBRA IN LEARNING GEOMETRY

What are the senior high technical school students‘ perceptions of the use of GeoGebra in learning geometry? Please indicate your level of agreement or disagreement on the statements below. It has been rated in the form A = Agree and D = Disagree. Tick (√) as appropriate.

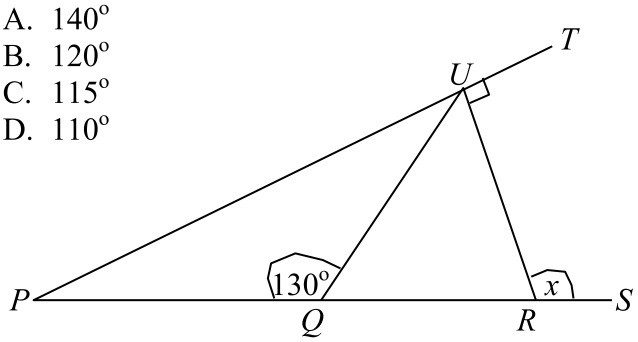
N/S	Statement	A	D
1	I feel confident when I do geometric activities by using GeoGebra software		
2	I can think creatively and critically when using GeoGebra software		
3	GeoGebra software helps increase my performance in mathematics class		
4	I am excited when asked to explore the GeoGebra software		
5	I am happy if the mathematics teachers use the GeoGebra software in teaching mathematics especially geometry		
6	GeoGebra allows me to visualize and manipulate geometric concepts		
7	I was able to make logical connections between geometric theorems using GeoGebra		
8	I was engaged in the learning process using GeoGebra		

APPENDIX B

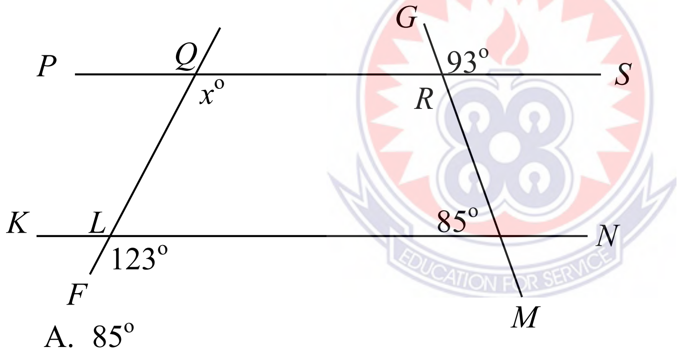
Students' Learning Performance in Geometry

Answer all questions in this section.

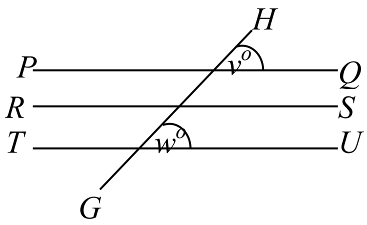
1. In the diagram *PQRS* and *PUT* are straight lines. $\angle RUT = 90^{o}$ and $\angle PQU = 130^{o}$. If $|PQ| = |QU|$. Find the value of x
   A. 140^{o}
   B. 120^{o}
   C. 115^{o}
   D. 110^{o}



2. In the diagram, *FLQ*, *GRM*, *PQRS* and *KLMN* are straight lines. $\angle FLM = 123^{o}$, $\angle LMR = 85^{o}$, $\angle GRS = 93^{o}$ and $\angle LMR = x^{o}$. Find x

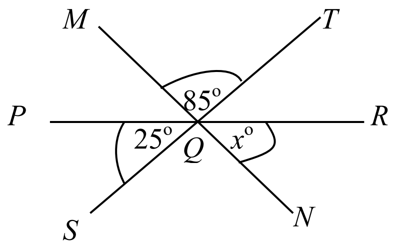


   A. 85^{o}
   B. 105^{o}
   C. 123^{o}
   D. 125^{o}



3. *PQ*, *RS*, *TU* and *GH* are straight lines, *PQ*//*RS*//*TU*. What kind of angles are v and w?
   A. corresponding
   B. alternate
   C. vertically opposite
   D. adjacent

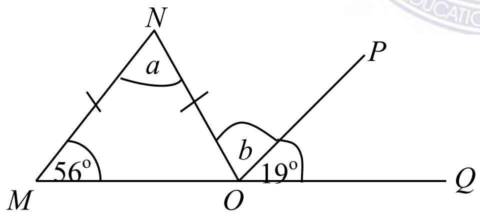
PQR, *SQT*, *MQN* are straight lines ∠*PQS*=25° and ∠*MQT*=85°. Use the information to answer questions ***4 and5***



4. Find the value x

   A. 25°
   B. 70°
   C. 85°
   D. 110°
5. What kind of angles are *Q* and 85°
   A. corresponding
   B. alternate
   C. vertically opposite
   D. adjacent

In the diagram *MNO* is a triangle in which |*MN*|=|*NO*|, ∠*NMO*=56°, ∠*POQ*=19° and *MOQ* is a straight line. Use the information to answer questions **6**, ***7 and 8***

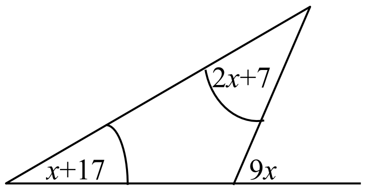


6. What is the value of the angle marked *a*?

   A. 28°
   B. 68°
   C. 78
   D. 56°

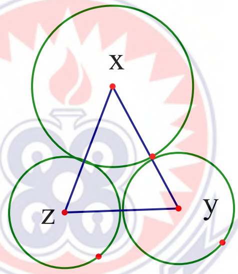
7. What type of angle is marked *b*?
   A. Acute angle
   B. Complex angle
   C. Obtuse angle
   D. Reflex angle

8. What type of triangle is *MNO*
   A. Equilateral triangle
   B. A scalene triangle
   C. Isosceles triangle
   D. Obtuse triangle



9. Find the value of $9x$ in the diagram
A. 18^{o}
B. 36^{o}
C. 54^{o}
D. 60^{o}

The figure below shows three circles which touch each other. x, y and z are centres of the circles. The circle with centres y and z have equal radii.



10. Which of the following statements is true of triangle XYZ?
    A. Equilateral
    B. Obtuse angles
    C. Scalene
    D. Isosceles

ANSWERS
1. C
2. C
3. A
4. B
5. C
6. B
7. A
8. C
9. D
10. C

UNIVERSITY OF EDUCATION, WINNEBA
Department of Mathematics Education

VAN HIELE GEOMETRY TEST (VHGT)

The VHGT for SHS 2 students

GEOMETRY TEST

Dear Student,

I am an M.Phil Mathematics Education student of the University of Education, Winneba. This research study is being conducted to enable me write my thesis. Please answer the questions as accurately as possible. The answers to these questions are for educational purposes and are in no way meant for individual or personal assessment. Your answers will be treated as strictly confidential. Thank you for your co-operation.

1. **Program offered**: ……………………………………………………………………………..
2. **Sex:** 1. [M] [F]

GUIDELINES:

i Do not start until you are told to do so.
ii While you are waiting, please fill the appropriate information in the spaces below.
iii This paper consists of **OBJECTIVE TEST**. You are expected to **answer all** the questions on this paper. **OBJECTIVE TEST will last for 30 minutes.**

Instructions for OBJECTIVE TEST

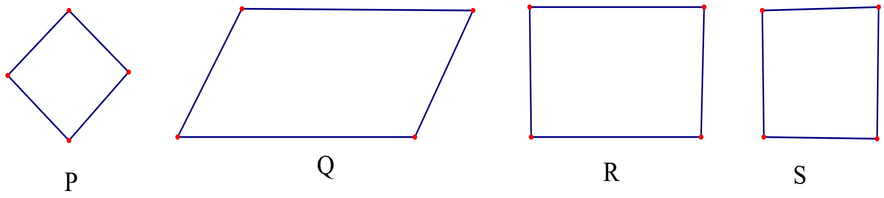
iv **OBJECTIVE TEST**, consisting of **20 multiple- choice** questions.
v Each question is followed by **five** options lettered **A to E**. There is only one correct answer to each question. Circle the correct option for each question. **Give only one answer to each question.**

NOTE: The **diagrams** in this test are **not necessarily** drawn to scale.

VAN HIELE GEOMETRY TEST (VHGT)

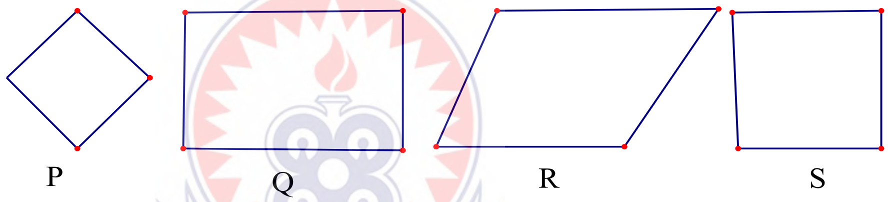
PRE-TEST

1. Which of these quadrilaterals are trapeziums?



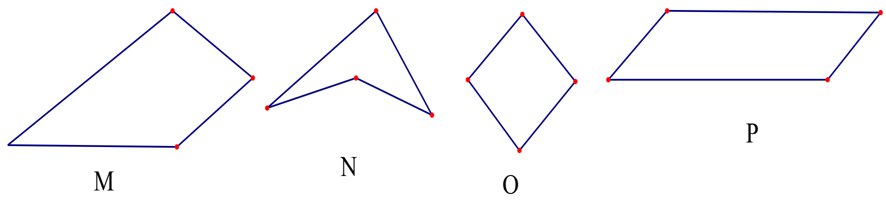
A. All
B. S only
C. Q, R and S only
D. R and S only
E. P, R and S only

2. Which of these quadrilaterals are parallelograms?



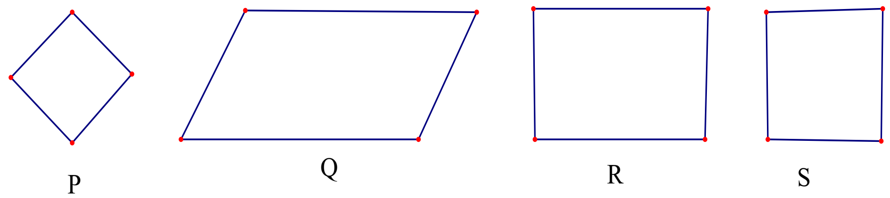
A. All
B. R only
C. Q, R and S only
D. P,Q and Sonly
E. Q and S only

3. Which of these shapes are kite?



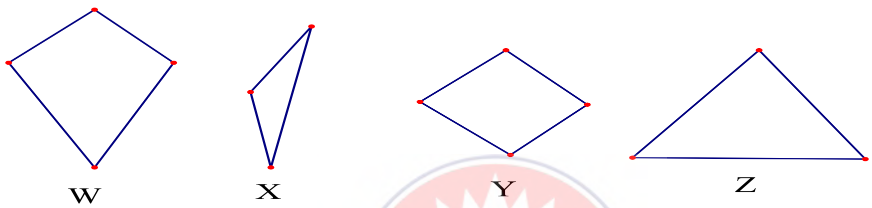
A. None of these are kites
B. O and P only
C. M, N and O only
D. P only
E. M and O only

4. Which of these shapes are rhombuses?



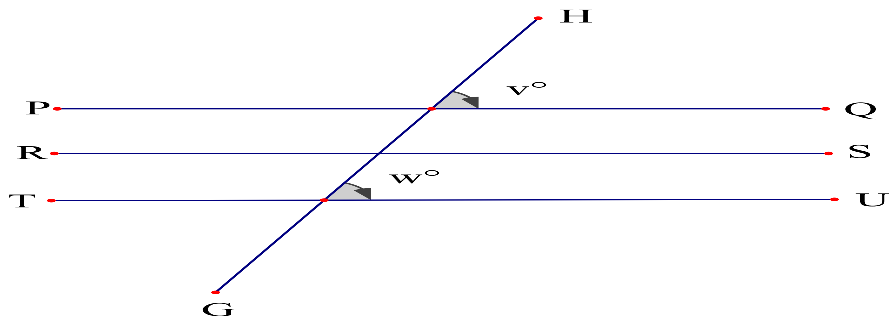
A. None of these are rhombuses
B. P and S only
C. P only
D. S only
E. P and R only

5. Which of these plane figures are triangles?



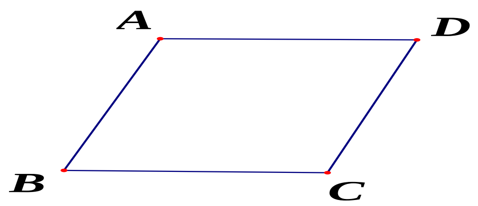
A. All
B. Z only
C. W and Z only
D. X only Z
E. X only

6. $\overline{PQ}, \overline{RS}, \overline{TU}, \overline{GH}$ are straight lines. $\overline{PQ} \| \overline{RS} \| \overline{TU}$. What kind of angles are v and w?



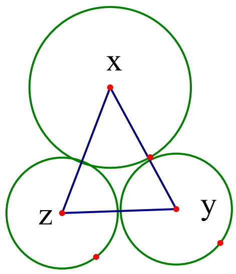
A. Corresponding angles
B. Alternate angles
C. Vertically opposite angles
D. Adjacent angles
E. Interior angles

7. ABCD is a rhombus. Which relationship is true in all rhombuses?



A. $\overline{AC}$ and $\overline{BC}$ have the same length
B. $\overline{AB}$ and $\overline{DC}$ are perpendicular.
C. $\overline{AC}$ and $\overline{BD}$ are perpendicular
D. $\overline{DB}$ and $\overline{DC}$ have the same length
E. Angle A is larger than angle

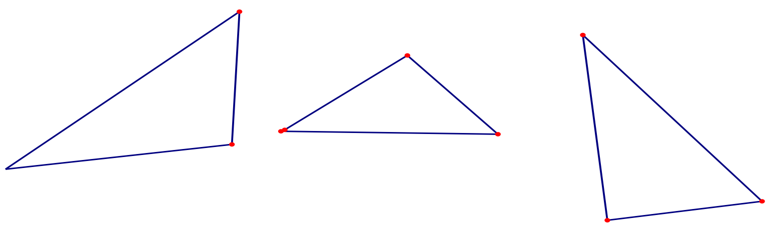
8. The figure below shows three circles which touch each other. x, y and z are centres of the circles. The circle with centres y and z have equal radii.



Which of the following statements is necessarily true of triangle XYZ?

E. Equilateral
F. Obtuse angles
G. Isosceles
H. Scalene
I. Right angle

9. A scalene triangle is a triangle with all the three sides different in length. Three examples are given below.



Which of (A) – (D) is not true of every scalene triangle?

A. The measures of all the interior angles are not the same.
B. Scalene triangle has no line of symmetry.
C. Each angle bisector is a line of symmetry.
D. Each angle bisector does not bisect the opposite side perpendicularly.
E. All of (A) – (D) are not true.

10. Which of the following is not a property of a parallelogram?
A. The opposite sides are equal.
B. The diagonals bisect each other.
C. The opposite angles are equal.
D. The diagonal does not bisect each other.
E. Opposite sides are not parallel.

11. The set R= {rhombuses} and T= {rectangles} are subset of U= {quadrilaterals}. If S= R∩T, what is the set S?
A. Parallelogram
B. Polygons
C. Squares
D. Kites
E. Trapeziums

12. Which of (A)-(D) is not true in every square?
A. All of (B)-(E) are true in every rectangle.
B. There are four right angles.
C. All four sides are equal.
D. The diagonals have equal length.
E. The diagonals bisect each other.

13. Which is **true**?
A. All properties of rectangles are properties of all parallelograms.
B. All properties of squares are properties of all rectangles.
C. All properties of squares are properties of all parallelograms.
D. All properties of rectangles are properties of all squares.
E. None of (A) – (D) is true.

14. Consider the following statements.

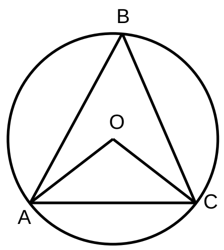
Statement M: In ΔPQR, ∠P and ∠R are congruent.

Statement N: ΔPQR is Isosceles triangle.

Which is true?

A. If **N** is true, then **M** is true
B. If **M** is true, then **N** is false.
C. If **N** is false, then **M** is true.
D. **M** and **N** cannot both be true.
E. **M** and **N** cannot both be false.

15. In the diagram below, O is the centre of the circle. AC is a chord and B is any point on the circumference. Which relationship is true?

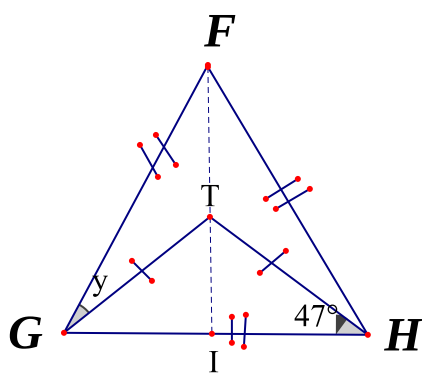


A. ΔAOC is isosceles.
B. AB and BC have equal measure.
C. ∠ AOC = ∠ ABC.
D. OA and OC have equal measure
E. (A) and (D), are true.

16. What do all squares have that some rectangles **do not** have?
A. Opposite sides are parallel.
B. Diagonals are equal in length.
C. Opposite sides are equal in length.
D. Opposite angles have equal measure.
E. None of (A) – (D).

17. Examine the three properties of a plane figure
Property R: It has diagonals of equal length.
Property S: It is a square.
Property T: It is a rectangle.
Which is true?
A. S implies T which implies R.
B. R implies S which implies T.
C. R implies T which implies S.
D. T implies R which implies S.
E. T implies S which implies R.

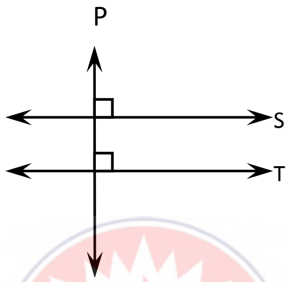
18. In the diagram FGH is an equilateral triangle and GTH is an isosceles triangle. If ∠GHT =47°, what is the value of ***y*,** given reason to support your answer



A. $y = 13°$ ($\angle$FGH $=$ $\angle$GHF $=$ HFG $= 60°, TGH = GHT = 47°, y + 47° = 60°$).
B. $y = 73°$(($\angle$FGH $+$ $\angle$GHF $+$ HFG $= 180°, y + 47° + 60° =$ $180°, sum\ of\ interior\ angle\ of\ a\ triangle\ FGH$).
C. $y = 47°$ ($\angle$FGH $=$ $\angle$GHF $=$ $\angle$HFG $= 60°, \angle TGH = \angle GHT = 47°$).
D. $y = 43°$ ($\angle TGH = \angle GHT = 47°, 47° + 47° + \angle HTG = 180°, \angle ITG = \frac{1}{2}\angle HTG$)
E. $y = 13°$ ($60° - 47°$).

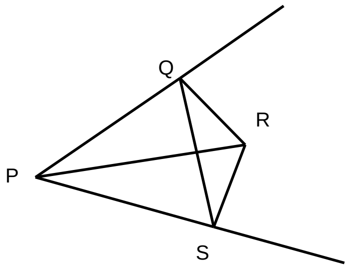
19. Study these three statements and answer the question that follows.
i. Two lines perpendicular to the same line are parallel.
ii. A line that is perpendicular to one of two parallel lines is perpendicular to the other.
iii. If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines S and P are perpendicular and lines T and P are perpendicular.



Which of the above statements could be the reason that line S is parallel to line T?
A. i only
B. ii only
C. iii only
D. Either ii or iii
E. Either i or ii

20. In the diagram, line PQ is adjacent to line PS and line QR is adjacent to line RS with diagonals PR and QS.

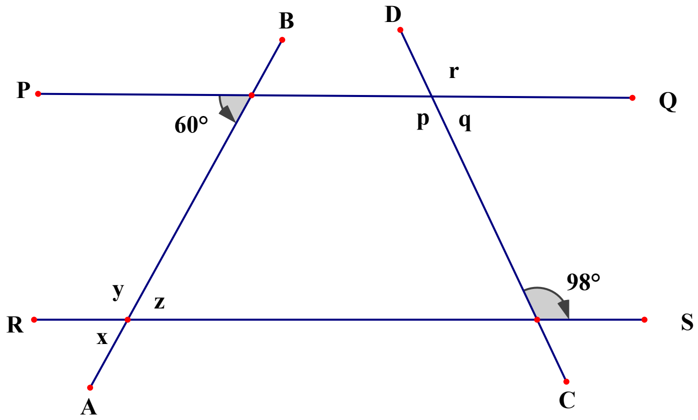


Which of these is **not true**?
A. PQRS is a kite.
B. PQ and PS have the same measure.
C. PR bisects angles QPS and angle QRS.
D. PR intersects QS at right angles.
E. QS is perpendicular to PS.

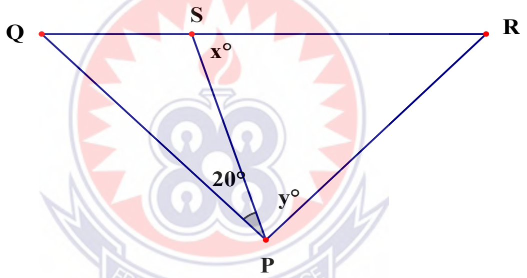
SECTION B

Answer all two (2) questions in this section by clearly showing working

1. In the diagram, $\overline{PQ}$ and $\overline{RS}$ are parallel and $\overline{AB}$ and $\overline{CD}$ are transversals. Calculate the values of the marked angles, giving reasons for your answers.

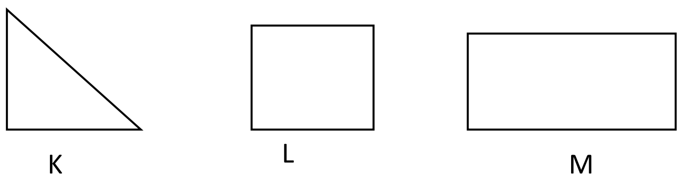


2. In the equilateral triangle $\angle$ PQR shown in the figure below, S is a point on $\overline{QR}$ such that ∠QPS =20°, SPR = y° and RSP =x°. Find the values of x° and$y$°.

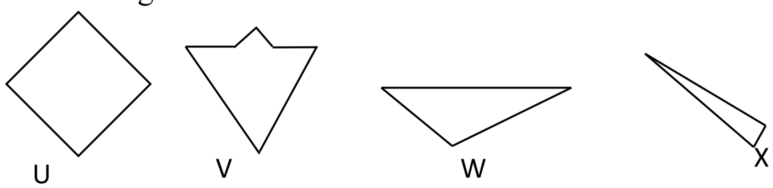


POST-TEST

1. Which of these are squares?
A. K only
B. L only
C. M only
D. L and M only
E. All are squares



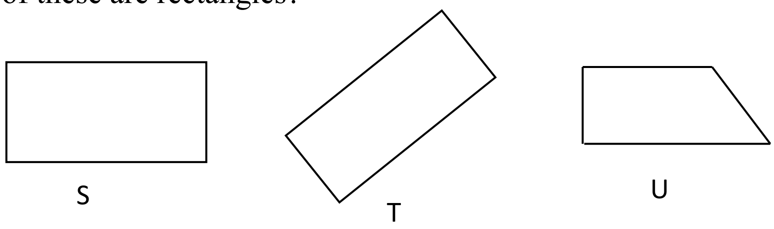
2. Which of these are triangles?



A. None of these are triangles
B. V only

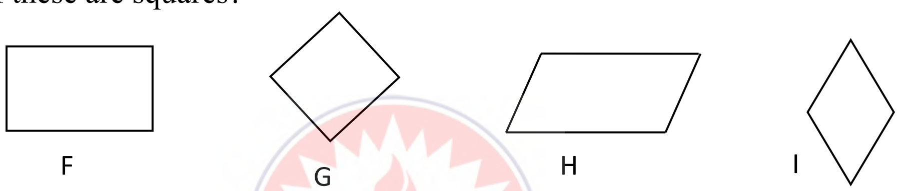
C. W only
D. W and X only
E. V and W only

3. Which of these are rectangles?



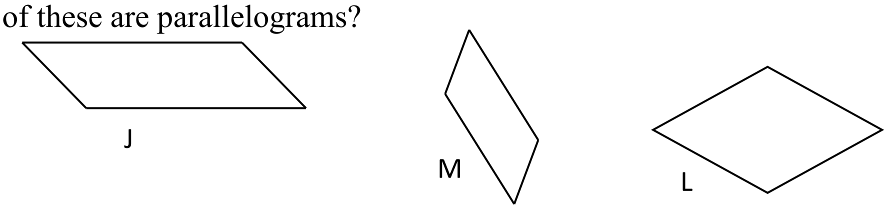
A. S only
B. T only
C. S and T only
D. S and U only
E. All are rectangles

4. Which of these are squares?



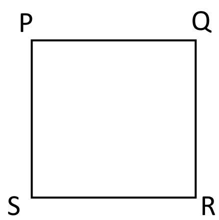
A. None of these are squares
B. G only
C. F and G only
D. G and I only
E. All are squares

5. Which of these are parallelograms?



A. J only
B. L only
C. J and M only
D. None of these are parallelograms
E. All are parallelograms

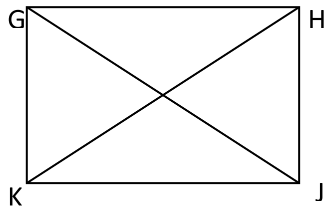
6. PQRS is a square
Which relationship is true in all squares?



A. $\overline{PR}$ and $\overline{RS}$ have the same length
B. $\overline{QS}$ and $\overline{PR}$ are perpendicular
C. $\overline{PS}$ and $\overline{QR}$ are perpendicular

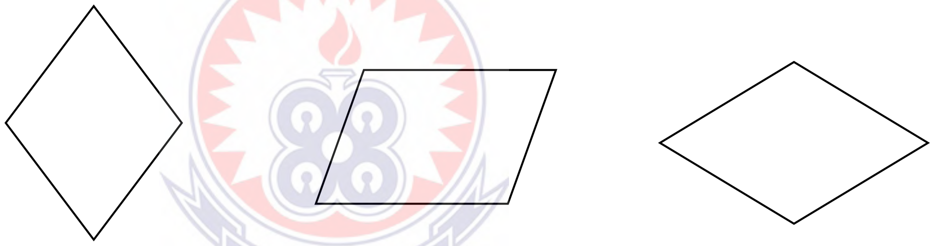
D. $\overline{PS}$ and $\overline{QS}$ have the same length
E. Angle Q is larger than angle R

7. In a rectangle GHJK, $\overline{GJ}$ and $\overline{HK}$ are the diagonals.



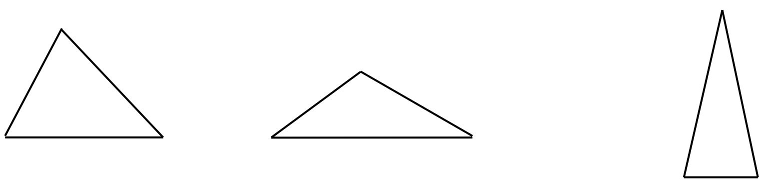
Which of (A) – (D) is not true in every rectangle?
a. There are four right angles.
b. There are four sides.
c. The diagonals have the same length.
d. The opposite sides have the same length.
e. All of (A) – (D) are true in every rectangle.

8. A rhombus is a 4- sided figure with all sides of the same length.
Here are three examples.



Which of (A) – (D) is not true in every rhombus?
A. The two diagonals have the same length.
B. Each diagonal bisects two angles of the rhombus.
C. The two diagonals are perpendicular.
D. The opposite angles have the same measure.
E. All of (A) – (D) are true in every rhombus.

9. An isosceles triangle is a triangle with two sides of equal length. Here are three examples



Which of (A)-(D) is true in every isosceles triangle
A. The three sides must have the same length
B. One side must have twice the length of another side
C. There must be at least two angles with the same measure

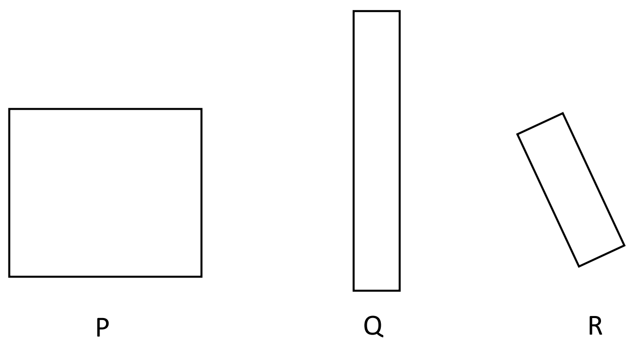
D. The three angles must have the same measure
E. None of (A)- (D) is true in every isosceles triangle

10. What do all rectangles have that some parallelograms do not have?
A. Opposite sides are parallel
B. Diagonals are equal in length
C. Opposite sides are equal in length
D. Opposite angles have equal measure.
E. None of (A) – (D)

11. Here are two statements
Statement 1: Figure F is a rectangle
Statement 2: Figure F is a triangle
Which is correct?
a. If 1 is true, then 2 is true
b. If 1 is false, then 2 is true
c. 1 and 2 cannot both be true
d. 1 and 2 cannot both be false
e. None of (A)-(D) is correct

12. Here are two statements.
Statement S: $\triangle ABC$ has three sides of the same length
Statement T: in $\triangle ABC$, $\angle B$ and $\angle C$ have the same measure
Which is correct?
a. Statement S and T cannot both be true
b. If S is true, then T is true
c. If T is true, then S is true
d. If S is false, then T is false
e. None of (A)-(D) is correct

13. Which of these can be called rectangles?



A. All can.
B. Q only
C. R only

D. P and Q only
E. Q and R only

14. Which is true?
A. All properties of rectangles are properties of all squares.
B. All properties of squares are properties of all rectangles.
C. All properties of rectangles are properties of all parallelograms.
D. All properties of squares are properties of all parallelograms.
E. None of (A) – (D) is true.

15. What do all rectangles have that some parallelograms do not have?
A. Opposite sides equal
B. Diagonals equal
C. Opposite sides parallel
D. Opposite angles equal
E. None of (A) – (D)

16. Here are three properties of a figure.
Property D: It has diagonals of equal length.
Property S: It is a square.
Property R: It is a rectangle.

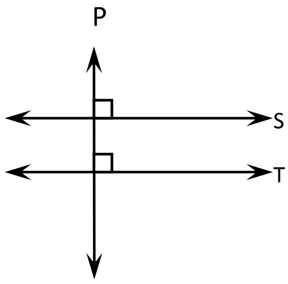
Which is true?
A. D implies S which implies R.
B. D implies R which implies S.
C. S implies R which implies D.
D. R implies D which implies S
E. R implies S which implies D.

17. Here are two statements.
I: If a figure is a rectangle, then its diagonals bisect each other.
II: If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?
A. To prove I is true, it is enough to prove that II is true.
B. To prove II is true, it is enough to prove that I is true.
C. To prove II is true, it is enough to find one rectangle whose diagonal bisect each other.
D. To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.
E. E. None of (A)-(D) is correct.

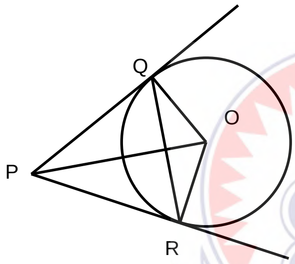
18. Examine these three sentences.
1. Two lines perpendicular to the same line are parallel.

2. A line that is perpendicular to one of two parallel lines is perpendicular to the other
3. If two lines are equidistant, then they are parallel.
In the figure below, it is given that lines S and P are perpendicular and lines T and P are perpendicular. Which of the above sentences could be the reason that line S is parallel to line T?



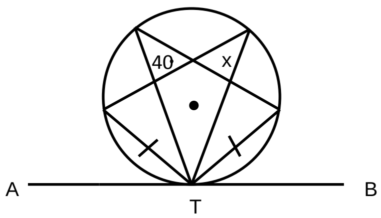
A. (1) only
B. (2) only
C. (3) only
D. Either (1) or (2)
E. Either (2) or (3)

19. In the diagram, lines PQ and PR are tangents to the circle at Q and R respectively. O is the centre of the circle.



Which of these is **not true**?
A. PQOR is a kite.
F. PQ and PR have the same measure.
G. PO bisects angles P and O.
H. QR intersects PO at right angles.
I. QR is perpendicular to PQ.

20. In the diagram, ATB is a tangent. Three students, **F**, **G** and **H** were asked to find the value of **x**, giving a reason to support their answers.



Here are their answers alongside their reasons.
Student F: x = 40° (*angles subtended by equal chords*)
Student G: x = 40° (*angles in the same segment of a circle*)

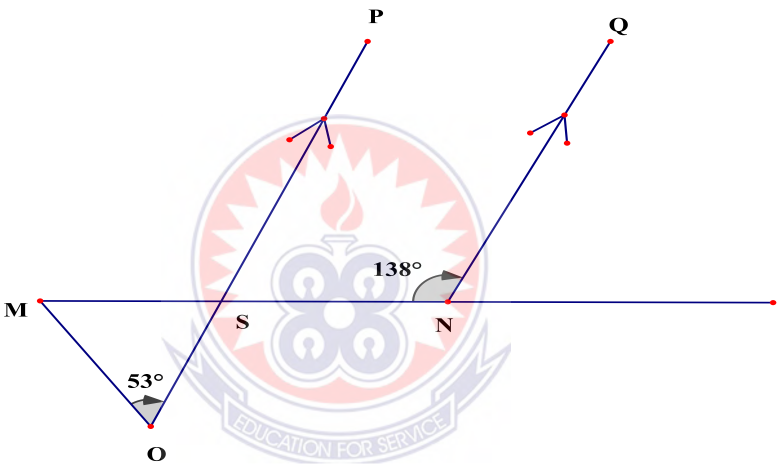
Student H: x = 40° (*angle between a tangent and a chord equals angle in the alt. seg.*)
Which of these students gives the **correct reason** for their answers?

A. F only
B. G only
C. H only
D. F and H only
E. All of F, G and H.

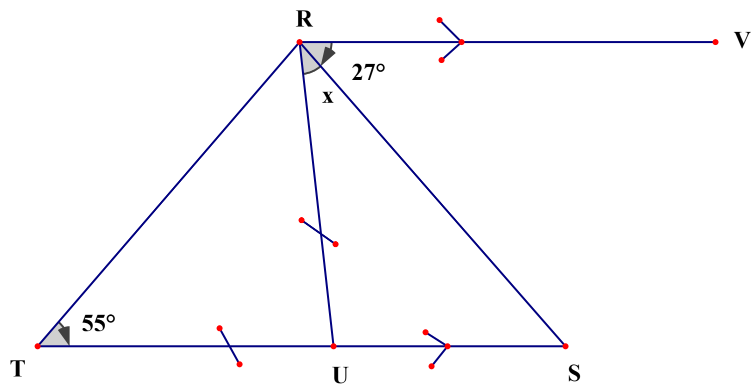
SECTION B

Answer all two (2) questions in this section by clearly showing working

1. In the diagram, $\overline{OP}\ ||\overline{NQ}$, MSN is a straight line, $\angle\text{MOP} = 53°$ and $\angle\text{SNQ} = 138°$. Find $\angle\text{OMS}$.



2. In the diagram below, $\overline{RV}$ ||, $\overline{TS}$ and /, $\overline{RU}$/ =, $\overline{TU}$//. $\angle\text{SRV} = 27°$ and $\angle\text{RTU} = 55°$. Find the value of$x$.



MARKING SCHEME (PRE-TEST & POST-TEST)

PRE-TEST
Marking Scheme for the VHGT – Section A

1. A
2. D
3. C
4. B
5. D
6. A
7. C
8. C
9. C
10. D
11. A
12. A
13. B
14. A
15. D
16. E
17. A
18. A
19. A
20. E

SECTION B
QUESTION 1

$z = 60^0$ [Alternate interior angles] $\boxed{MB1}$

$p = 98^0$ [Alternate interior angles] $\boxed{MB1}$

$\boxed{MB1}$

$x = z = 60^0$ [Vertically opposite angles]

$r = p = 98^0$ [Vertically opposite angles] $\boxed{MB1}$

$\boxed{B1}$

$y + z = 180^0$ [Angles on a straight line]

$y + 60 = 180^0$ $\boxed{M1}$

$\boxed{A1}$

$y = 120^0$

$p + q = 180^0$ [Supplementary angles] MB1

$98^0 + q = 180^0$ M1

$q = 82^0$ A1

QUESTION 2

Since Δ PQR is equilateral triangle,

$\angle QPR = \angle PRQ = \angle RQP = 60^0$ M1B1

But

$\angle QPR = 20^0 + y$ M1

$60^0 = 20^0 + y$ M1

$y = 40^0$ A1

In ΔSPR,

$\angle RSP + \angle SPR\angle + PRS = 180^0$ [Sum of interior angle of ΔSPR M1B1

$x + y + 60^0 = 180^0$ M1

$x + 40^0 + 60^0 = 180^0$ M1

$x = 80^0$ A1

POST-TEST

1. B
2. D
3. C
4. B
5. D
6. B
7. E
8. A
9. C
10. D

11. C
12. B
13. A
14. A
15. B
16. C
17. C
18. D
19. D
20. A

SECTION B

QUESTION1
METHOD 1

$\angle SRV = +\angle RST\angle = 27^0$ [Alternate angles, RV|| TS] $\boxed{M_1B_1}$

$\angle RTU = \angle \mathrm{TRU} = 55^0$ [Base angle of isosceles ΔRTU] $\boxed{M_1B_1}$

$\angle RTS + \angle RTU\angle TRS = 180^0$ [Sum of angles in a triangle]…… (1) $\boxed{M_1B_1}$

But $\angle TRS = x + \angle TRU = x + 55^0$ $\boxed{M_1}$

Thus from (1), $27^0 + 55^0 + (x + 55^0) = 180^0$ $\boxed{M_1}$

$x + 137^0 = 180^0$ $\boxed{M_1}$

$x = 43^0$ $\boxed{A_1}$

METHOD 2 (ALTERNATIVE METHOD)

$\angle SRV = +\angle RST\angle = 27^0$ [Alternate angles, RV|| TS] $\boxed{M_1B_1}$

$\angle RUT = x + \angle RST\angle = x + 27^0$ [Exterior angles ΔRSU] $\boxed{M_1B_1}$

$\angle RTU = \angle \mathrm{TRU} = 55^0$ [Base angles of isosceles ΔRTU] $\boxed{M_1B_1}$

$\angle RTU + \angle \mathrm{TRU} + \angle \mathrm{RUT} = 180^0$ [Sum of angles in a triangle] $\boxed{M_1B_1}$

$55^0 + 55^0 + (x + 27^0) = 180^0$ $\boxed{M_1}$

$\mathrm{x} + 137^0 = 180^0$

$\mathrm{x} = 43^0$ $\boxed{A_1}$

APPENDIX C

Treatment Period

LESSON PLAN FOR WEEK ONE

Subject: Core Mathematics

Topic: Plane Geometry I

Sub-Topic: Angles at a point

Duration: 120 minutes

Target group: SHS 2 students

Relevant Previous Knowledge:

1. Students are familiar with basic computer operations.
2. Students can use mouse and keyboard as inputs and to monitor corresponding outputs on the screen.
3. Students can draw lines and triangles.

Teaching and Learning Materials

1. The lesson was carried out in a computer laboratory where the GeoGebra Software was installed on the computers for students to use.
2. Mathematical set and calculators
3. Worksheets were available for small group of students thus activities in the lesson were designed alongside with the students' worksheets.

Objectives

By the end of the lesson, the student will be able to:

i. Construct and measure angles formed by two straight lines.

Advanced Preparation

The researcher presented a brief description about the interactive geometry software and its usage in the teaching and learning of geometry and other mathematics topics. The researcher also demonstrated to the students how the software works in a geometry classroom. The students were given sets of instructions by the researcher to follow and apply them on their desk top computers. The instructions were based on the GeoGebra introductory book downloaded from the official website of GeoGebra. The researcher gave the students an opportunity to explore the software and draw various geometric structures whilst the students continue to work with GeoGebra under the instructions and support of the researcher.

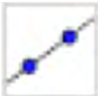
Activity one

The researcher assists the students by organizing them in small group to explore and draw at least three different types of angles and measure the angles enclosed. Students were encouraged to explore the software to find the technical names of the angles drawn using the software.

i. Open GeoGebra window on your desktop

ii. Select the point tool 

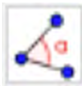
from the tool bar to create a point anywhere in the construction area.

iii. Select the Line tool  from the tool bar and create a line AB anywhere, by clicking to create the points A and B that line AB passes through.

iv. Select the Move tool  and use it to move point.

v. At the point A or B, draw another line say AC or BC using the line to create an angle.

vi. Rename and label the three points ABC in clockwise direction.

vii. Measure the angle formed by the two lines using the angle tool .

viii. Click on the three points that form the angle or the two lines that form the angle in a clockwise direction to measure the angle.

ix. The researcher discusses with the students what happens if the angle is measured in anticlockwise direction.

EXPECTED RESULTS FROM DISCUSSIONS

1. When students follow the instructions carefully, the will observed that some of the angles are less than 90^0, exactly 90^0 , and above 90^0.
2. The exterior angle will be displayed when the angles are measured in anticlockwise direction.

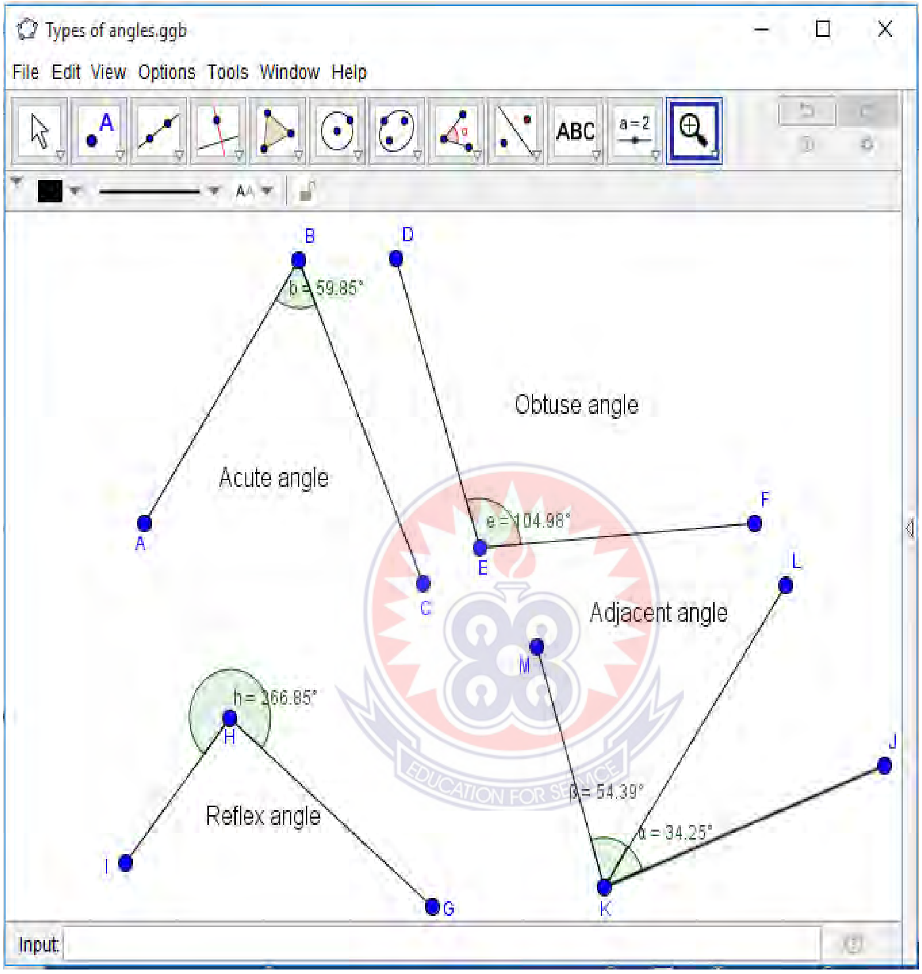


Figure 1: Diagrams showing the types of angles

Conclusion

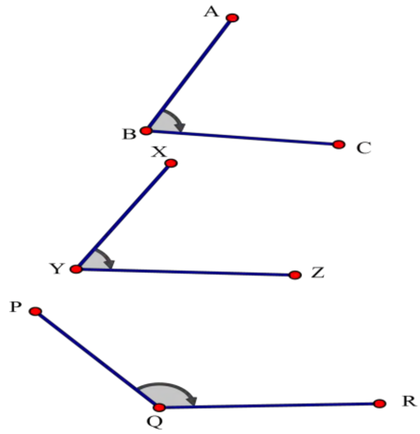
Discuss with students their observation after going through the activities and offer the necessary clarification where necessary.

Evaluation Exercises

Students should answer the following questions to evaluate the lesson.

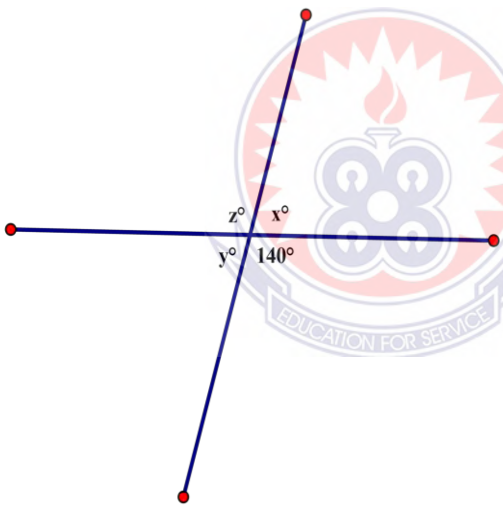
Question 1.

Name the angles below



Question 2.

Determine the sizes of the marked angles in the diagram below



LESSON PLAN FOR WEEK TWO

Subject: Core Mathematic

Topic: Plane Geometry I

Sub-Topic: Properties of Angles Formed by Parallel Lines and Their Transversal

Duration: 120 minutes

Target group: Form 2 Electrical Engineering Technology (EET) students

Teacher: The researcher

Relevant Previous Knowledge:

1. Students are familiar with basic computer operations.
2. Students can use mouse and keyboard as inputs and to monitor corresponding outputs on the screen.
3. Students have learnt the concept of lines and how to measure angles in the traditional lesson
4. Students have learnt the concept of parallel lines in the traditional lesson
5. Students can perform various arithmetic operations

Teaching and Learning Materials

4. The lesson was carried out in a computer laboratory where the GeoGebra Software was installed on the computers for students to use.
5. Mathematical set and calculators
6. Worksheets were available for small group of students thus activities in the lesson were designed alongside with the students' worksheets.

Objectives

By the end of the lesson, the student will be able to:

i. Construct, identify and use the properties of parallel lines.

ii. Determine the relationships between the angles formed by two parallel lines and a transversal.

Advanced preparation

Researcher reviews students' previous knowledge on angles and their properties, and then asks students questions on the definition of parallel lines. The researcher further discuss with students parallel lines and their properties. Researcher ensures that students have an understanding of parallel lines and can construct them.

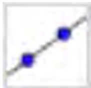
Activity Two

In small groups, the researcher assists the students to draw parallel lines and a transversal and encourage the students to explore to discover the relationship between angles formed by the parallel lines and the transversal.

i. Open GeoGebra new window on desktop for activity 2.

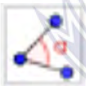
ii. Check the menu ―Options" | ―Labeling" | ―New Points Only".

iii. Using the point tool  locate a point *A*anywhere in the construction area.

iv. Construct a line segment AB from A using the line tool 

v. Using the line tool Construct another line AC which intersect the point construction area.

vi. Using the Parallel-Line tool (pull-down from the line tool), construct a line through point C and parallel to AB.

vii. using the angle tool  measure the angles formed by the two parallel lines and the transversal

EXPECTED RESULTS FROM DISCUSSIONS

When the instructions are carefully followed, students will discover that:

1. Pairs of angles whose interior lie between two parallel lines, but on opposite sides of the transversal are congruent (equal).
2. Pairs of angles whose interior lie outside two parallel lines, but on opposite sides of the transversal are congruent (equal).
3. Two non-adjacent angles whose interior lie on the same side of the transversal such that one angle lie between the parallel line and the other angle lie on the outside of one of the parallel lines.
4. Angles between any two parallel lines and a given transversal are supplementary.

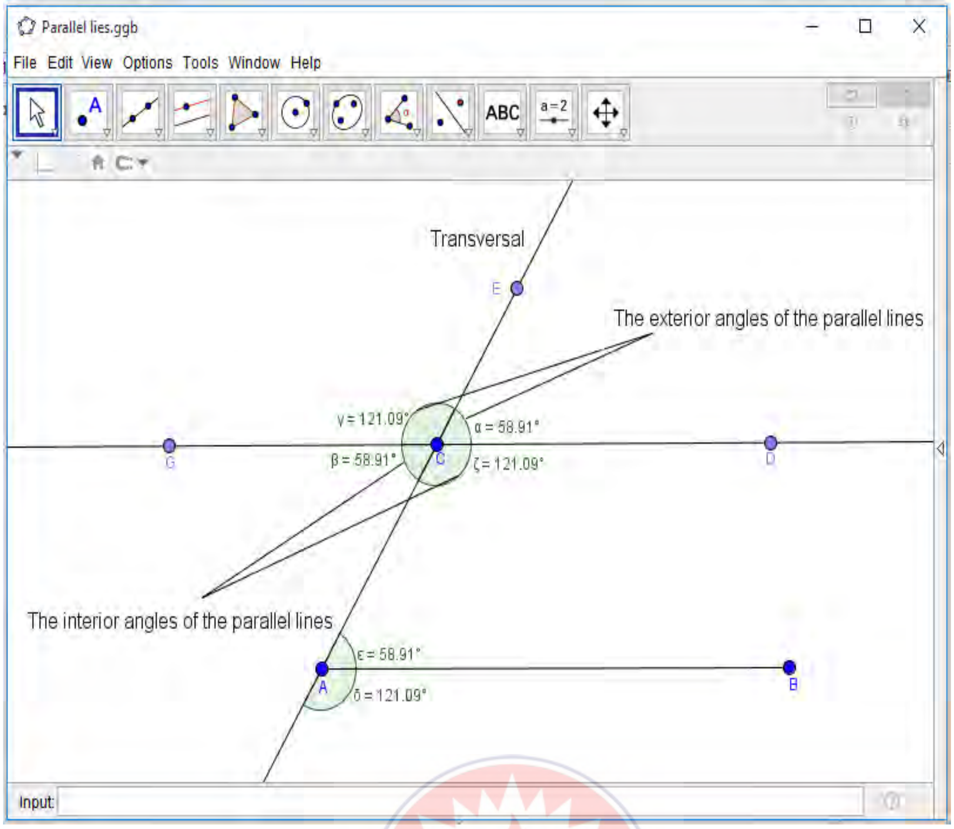


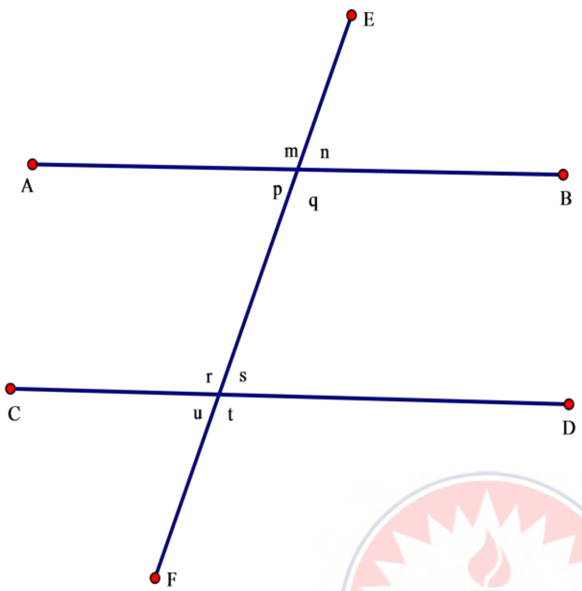
Figure 2: Diagram showing properties of parallel lines

QUESTION 1

In the diagram, AB and CD are parallel

Lines and EF is a transversal.

Find the marked angles giving reasons.



Expected Answers

1. Vertically opposite angles

$m = q, n = p, r = t,$ s=u

2. Alternate interior angles

$$p = s, q = r$$

3. Alternate exterior angles

$$m = t, n = u$$

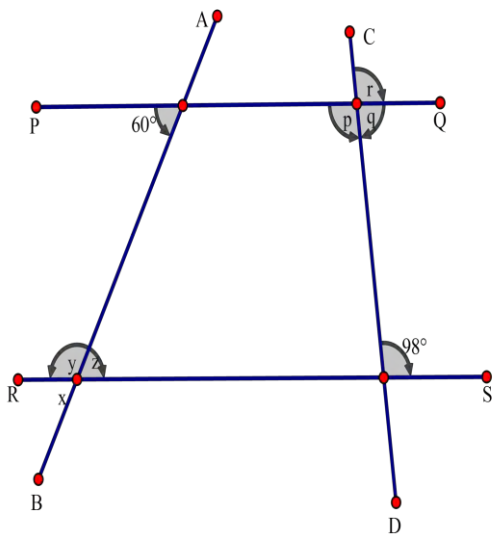
4. Corresponding angles

$$m = r, n = s$$

$$p = u, q = t$$

QUESTION 2

In the diagram, PQ and RS are parallel lines and AB and CD are transversals. Calculate the value of the marked angles giving reasons for your answers.



$z = 60^0, p = 98^0$(Alternate interior angles).

$x = 60^0, p = 98^0$(Vertically opposite angles)

$y + z = 180^0$ (Angles on a straight line)

$$y + 60^0 = 180^0$$

$$y = 120^0$$

Also $p + q = 180^0$

$$98^0 + q = 180^0$$

$$q = 82^0$$

LESSON PLAN FOR WEEK THREE AND FOUR

Subject: Core Mathematic

Topic: Plane Geometry I

Sub-Topic: The Interior and Exterior Angles Theorem

Duration: 120 minutes

Target group: Form 2 Electrical Engineering Technology (EET) students

Relevant Previous Knowledge

1. Students are familiar with basic computer operations.
2. Students can use mouse and keyboard as inputs and to monitor corresponding outputs on the screen.
3. Students can draw lines and triangles.
4. Students can identify the various types of triangles
5. Students can perform various arithmetic operations

Teaching and Learning Materials

7. The lesson will be carried out in a computer laboratory where the GeoGebra Software will be installed on the computers for students to use.
8. Mathematical set and calculators
9. Worksheets will be available for small group of students thus activities in the lesson will be designed alongside with the students' worksheets.

Objectives

By the end of the lesson, the student will be able to:

i. Construct at least a triangle and use it to verify the interior and exterior angle theorems.

ii. Apply the concepts discovered in solving related problems.

Advanced Preparation

The researcher introduces the lesson by asking students to draw any triangle on a sheet of paper and indicate all the angles in the triangle and measure the angles in the triangle. The researcher further encourages the students to draw different triangles and measure the angles and come out with their findings.

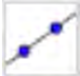
Activity three

1. In small groups, the researcher guides the students to discover the various types of triangle and their properties using the software.
2. The researcher also assist the students to explore the software to verify the interior and exterior angle theorems.

Construction of equilateral triangle

i. Open GeoGebra new window on desktop for the following activities;

ii. Check the –Options” | –Labeling” | –New Points Only” menu.

iii. Use the Segment tool  (pull down from the Line tool ) to construct segment AB.

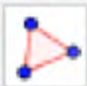
iv. Use the Compass tool  (pull down from the Circle tool ) to construct a circle with center at A and passing through B.

v. Use the Compass tool  to construct a circle with center at B and passing through A such that the length of side BC will be dependent on the length of the base side AB.

vi. Use the Point tool 

or the Intersect tool  (pulled down from the Point tool 

) to construct point C at an intersection of the two circles.

vii. Use the Polygon tool  to construct triangle ABC (click on points A, B, C, A).

viii. Use the angle tool  to construct angle ACB, angle CBA and angle BAC.

ix. Click on the three vertices of the triangle in a clockwise direction to define each angle.

EXPECTED RESULTS FROM DISCUSSIONS

When the instructions are carefully followed, students will discover that:

1. All the three sides of the triangle are congruent (the same length as each other).
2. All the three angles stay congruent (the same size as each other).
3. The triangle has three lines of symmetry that bisect each angle and are equal in length.

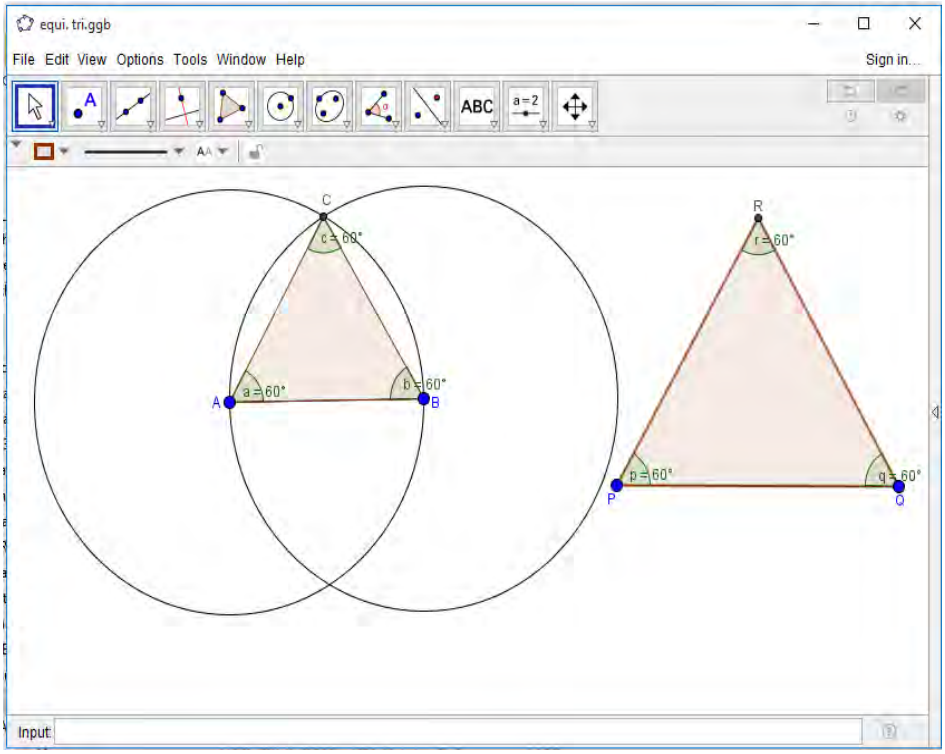
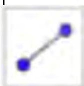
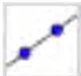


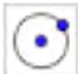
Figure 3: diagram showing an equilateral triangle

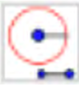
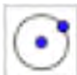
Construction of Isosceles Triangle

i. Open GeoGebra new window on desktop for the following activities:

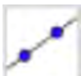
ii. Check the ―Options” | ―Labeling” | ―New Points Only” menu.

iii. Use the Segment tool  (pull down from the Line tool ) to construct segment AB.

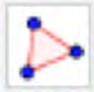
iv. Use the Compass tool  (pull down from the Circle tool ) to construct a circle with center at A and passing through B.

v. Use the Compass tool  (pull down from the Circle tool ) to construct a circle with center at B and passing through A.

vi. Use the Point tool  or the Intersect tool  (pulled down from the Point tool ) to construct point C at an intersection of the two circles.

vii. Using the perpendicular bisector tool pulled down from the line tool , construct a perpendicular bisector of line AB.

viii. Use the Point tool  to locate a point C on the perpendicular bisector of line AB.

ix. Use the Polygon tool  to construct triangle ABC (click on points A, B, C, A).

x. Use the angle tool  to construct angle ACB, angle CBA and angle BAC.

xi. Click on the three vertices of the triangle in a clockwise direction to define each angle.

EXPECTED RESULTS FROM DISCUSSIONS

When the instructions are carefully followed, students will discover that:

1. Two sides of the triangle are congruent (the same length as each other).
2. Two of its interior angles stay congruent (the same size as each other).
3. It has one line of symmetry that bisect the base angle at right angle and the angle opposite the base.

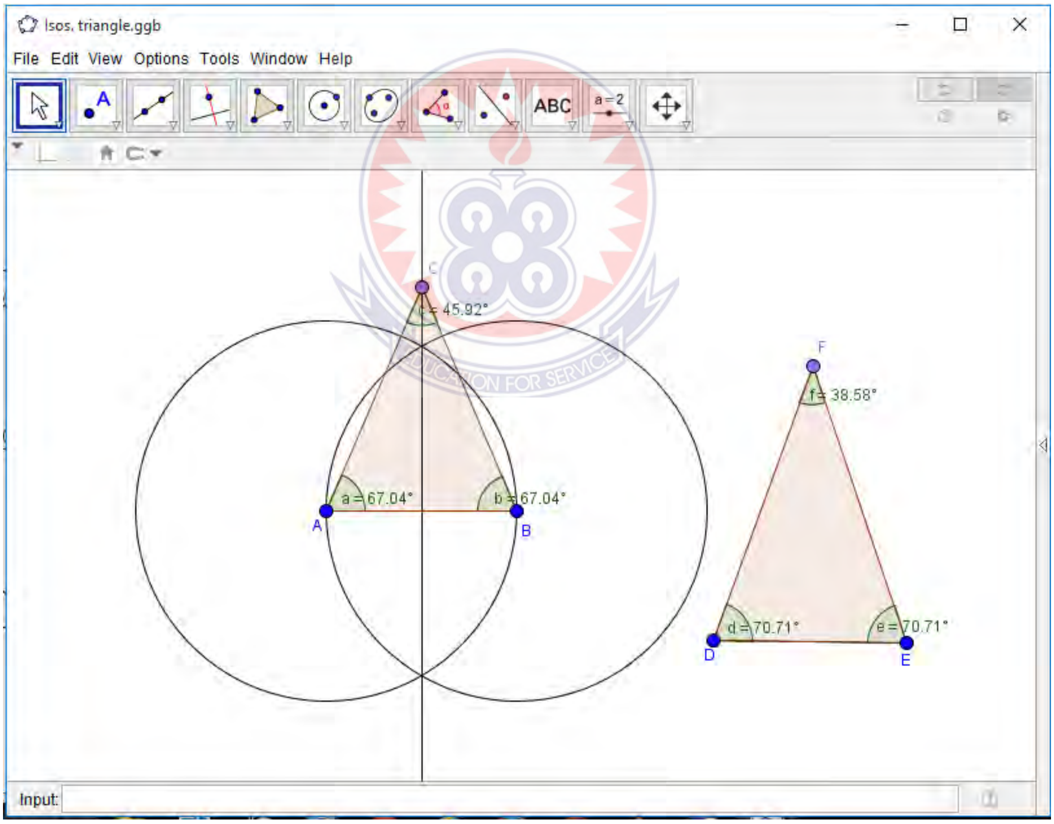
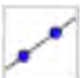


Figure 4: A diagram showing isosceles triangle

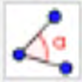
Activities for the Interior and exterior angle theorems

i. Use the line tool  to draw line AB.

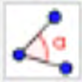
ii. Use the point tool  to construct point D and F on line AB.

iii. Use the ray tool draw down from the line tool to construct ray AC.

iv. Use the point tool to construct point E on ray AC.

v. Use the angle tool  to construct angle ACB, angle CBA and angle BAC.

vi. Click on the three vertices of the triangle in a clockwise direction to define each angle.

vii. Use the angle tool  to construct angle FAC, angle ECB and angle DBC in anticlockwise direction.

EXPECTED RESULTS FROM DISCUSSIONS

When the instructions are carefully followed, students will discover that:

1. Interior angles are measured in a clockwise direction using the software.
2. Exterior angles are measured in anticlockwise direction.
3. Sum of two interior angles is equal to an exterior angle.

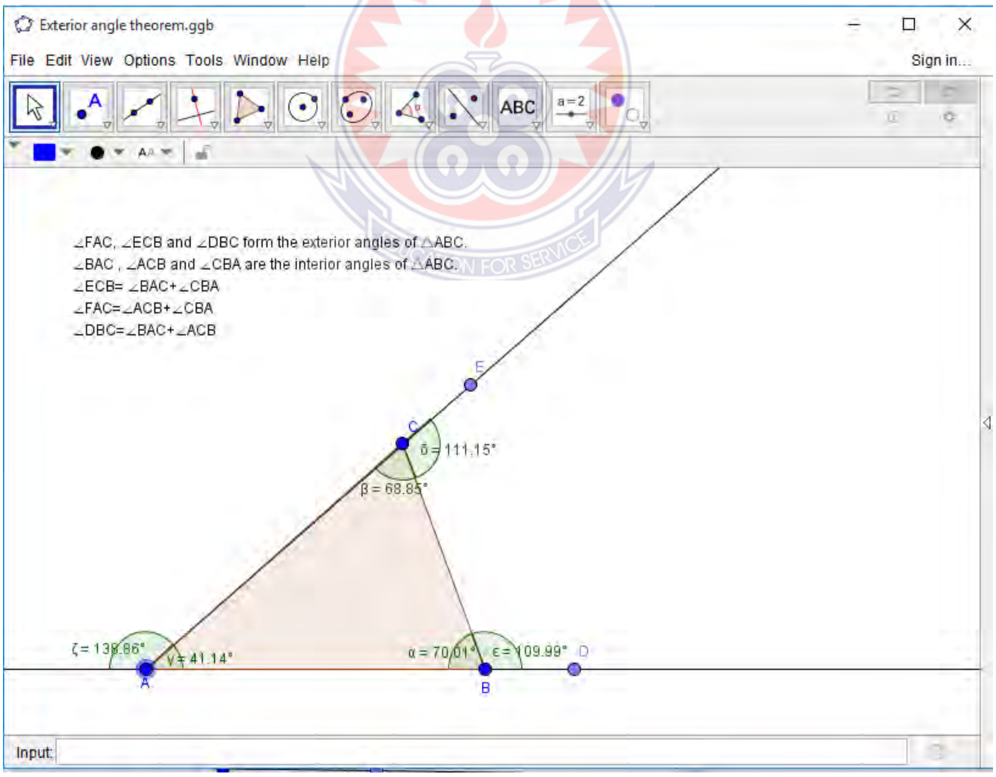
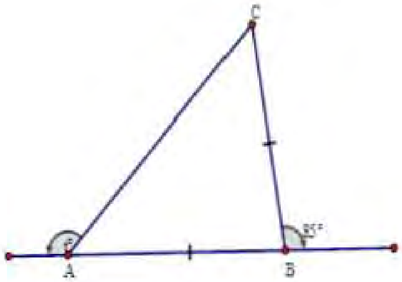


Figure 5: A diagram showing the exterior angle theorem

Evaluation

QUESTION 1



Expected Answer

Since Δ ABC is Isosceles,

$< BAC = < ACB$

$2 < BAC = 85^0$ (Sum of interior opposite angles = the exterior angle.

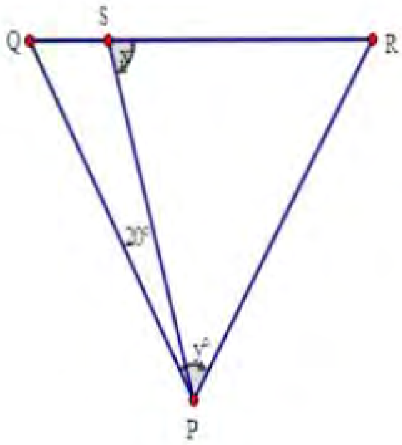
$< BAC = 42.5^0$

Also $t^0 + < BAC = 180^0$

$t = 137.5^0$

QUESTION 2

In the equilateral triangle PQR shown in the figure below, S is a point on QR such that $< QPS = 20^0$, $< SPR = y^0$ and $< RSP = x^0$. Find the values of x^0 and y^0



Expected Answer

Since Δ is an equilateral triangle, $< QPR = 60^0$

$< QPR = < PRQ = < RQP = 60^0$

But $< QPR = 20^0 + y^0$

$y = 40^0$

In ΔSPR,

$< RSP + < SPR + < PRS = 180^0$ (Sum of interior angles of a triangle)

$x + 40 + 60 = 180^0$

$x = 80^0$