UNIVERSITY OF EDUCATION, WINNEBA

USING VAN HIELES' MODEL TO INVESTIGATE STUDENTS' DIFFICULTIES IN ROTATION: A STUDY AT NGLESHIE-AMANFRO SENIOR HIGH SCHOOL IN THE GA-SOUTH MUNICIPALITY.

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DECLARATION

Student's Declaration

I, Abagna Fuseini, declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

Signature: ……………………………

Date: ……………………………………

Supervisor's Declaration

I hereby declare that the preparation and presentation of this thesis was supervised in accordance with the guidelines for supervision of thesis as laid down by the University of Education, Winneba.

Name of Supervisor: Dr. Akayuure Peter

Signature: ……………………………..………

Date: ……………………………..…………

DEDICATION

This thesis is dedicated to my late mother, Mma Anumbi Atunta for her incessant love, counsel and prayers when she was alive. It really propelled me to pursue this postgraduate study.

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ABBREVIATION

MAT	Mathematics Achievement Test
MOESS	Ministry of Education, Science and Sports
NCTM	National Council of Teachers of Mathematics
SHS	Senior High School
TIMSS	Trends in International Mathematics and Science Study
WAEC	West African Examinatios Council
WASSCE	West Africa Senior Secondary Certificate Examinations

ABSTRACT

The aim of this study was to investigate and describe various difficulties which students demonstrate in the learning of rotation using the van Hieles‟ model of geometric thinking. The study made use of mixed-method approach in which qualitative data were used to assist in explaining and assigning reasons for quantitative findings. An achievement test on rotation was administered to 240 students who were randomly selected from Ngleshie Amanfro Senior High school for the study. Eight participants were then interviewed to examine the difficulties they encountered in answering questions on rotation at each level of van Hiele. Quantitative data was analysed descriptively using percentages, means and standard deviations, and inferentially with independent sample $t-test$. The qualitative data were transcribed into descriptive words according to van Heiles‟ levels to portray various difficulties of rotation encountered by students. The results showed that, students had more difficulties in deduction and abstraction than in analysis and visualisation. Also, majority of the students reached the van Hiele levels of visualization and analysis but only a few reached the abstraction and deduction levels. Also, there was no statistically significant difference between the mean scores of male and female students in the visualization, analysis and abstraction levels. However, there was statistically significant difference between the mean scores of males and females at the deduction level of van Hiele. The researcher recommends that teachers should examine or analyse students‟ difficulties in rotation using the van Hieles‟ levels. The study also recommends that teachers should encourage students to talk about geometric concepts relating to rotation so as to develop expressive language. Students should also be made to work with geometric models to enable them discover the properties themselves.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This chapter sets the study in context. It presents the background of the study, statement of the problem, the purpose of the study, objectives of the study as well as the educational significance and sets out the research questions guiding the study. The chapter further highlights the limitations and delimitations of the study and concludes by outlining the organization of the study.

1.1 Background to the Study

According to Olkun, Sinoplu, and Deryakulu (2005), one of the basic goals of teaching mathematics is to improve the students‟ geometric thinking levels. Thinking about geometry is significant in science, technology and in selecting a subject that will lead to a profession (Tahani, 2016). To help students realize their aim of developing mathematical reasoning abilities and also in promoting a deeper awareness of the real world, the National Council of Teachers of Mathematics (NCTM, 2000) suggested that reasoning about shapes should be used in coordinate and transformation techniques as well as the traditional skills that are embedded in rotation, reflection and translation. At the senior high level in Ghana, there has been consistent evidence (Fletcher & Anderson, 2012) regarding the inability of students to tackle questions requiring visualization of objects in space and geometric reasoning concerning mensuration, 3- dimensional problems and circle theorems. Of late however, there has been low performance in geometry from several assessment reports (Akayuure, Asiedu-Addo, & Alebna, 2016).

In Ghana, the Senior High School education policy is made up of three years during which students were made to sit for an external examination called West African Secondary School Certificate Examination. In this particular examination, every student writes mathematics as a core subject. Accoding to the report of Curriculum Research and Development Division (2010), it indicated that Plane Geometry, Mensuration, Algebra, Statistics and Probability, Vectors, Numbers and Numeration, Transformation in a Plane and Trigonometry are taught in all senior high schools as part of the Ghanaian curriculum. Meanwhile, the Chief Examiners Report stated clearly that candidates exhibit a poor understanding of a mathematical concept, they therefore identify a weakness in the concept of having similar triangles with proportional sides (WAEC report May/June 2007and 2009).

In another similar report, the Chief Examiner stated that candidates had issues with the concept of transforming a figure under anticlockwise rotation of 90° about a point. The report suggested teachers should find a way of reinforcing such concepts in students learning (WAEC report May/ June 2008). The report pointed to the fact that students might lack enough acquisition of geometric skills such as the ability to find a point and centre of rotation, finding the line and the order of rotational symmetry, identifying figure or shapes after transformation, using a given transformation to transform an object/image when given the coordinates, angles and shape. The findings of limited problem solving activities in textbooks, teacher incompetency, and lack of resource materials to engage and assess students were serious concerns (Nabie, Akayuure & Sofo, 2013). The result of analysis from the examination council also revealed that among the three concepts (Rotation, Reflection and Translation), students seemed to perform poorly specifically in the skills associated with Rotation.

The research work done by van Hiele (1986) with its roots in Piaget‟s work could be used to explain better the difficulty that was experienced by the students in the examination since it was focused primarily on five levels of geometric conceptualization. Van Hiele (1986) stated that there are two main reasons for the existence of levels.

- If students have not sequentially gone through the proposed five levels, then they cannot function adequately at any given level. Meaning they can perform a task at any level with no understanding.
- If the knowledge level of the instructor in terms of language or a teaching method is at variance with that of the students, a serious communication problem may occur and this may result in frustration and lack of understanding on the part of the students.

The above findings do have a great implication in the learning of transformation geometry. They do explain the reason why many Senior High School students are having problems with geometry learning. This study, therefore, investigates and describes various difficulties which might hinder Ghanaian students from learning rotation.

1.2 Statement of the Problem

A teacher‟s deficiency in a knowledge that was relevant in content and his skills would pose a major problem to him/her during the teaching of Senior High School geometry particularly rotation. For instance, according to a presentation made at the 2nd Speech and Prize Giving Day Celebration of the Apeguso SHS by (Mereku, 2012) , it came to light that:

- The curriculum content of SHS is still seen as memorization of concepts instead of understanding it in this 21st century which has come as a result of the external assessment requirements of the system (WAEC).
- Majority of teachers continue to use teacher-centred approaches because of large classes, and inadequate resources, facilities and training. This, in the long run, brings about a disparity between what should be implemented in the class and the curriculum.

Also, according to a research study, the van Hiele level of understanding reached by most Ghanaian students before entering Senior High School was lower than what most students at this stage reached in other countries in the study of geometry (Baffoe & Mereku, 2010). Also, one of the difficulties in learning by students was as a result of lack of appropriate learning strategies. Analysing the report from the examination council revealed that most students were not able to give the required solution when it came to solving a question in rotation. It was captured in the report that, students were not able to; determine the point and centre of rotation, determine the line and order of rotational symmetry, identify a figure after transformation when given the coordinates, angles and shapes (WAEC report May/June 2007and 2009).

My observation as a mathematics teacher at Ngleshie Amanfro Senior High School for the past ten years was that majority of students were very unsuccessful in solving problems on geometry especially rotation. Many solve problems on rotation algorithmically with little or no understanding. These problems had been manifested in many scripts that had come my way during end of term mathematics exams papers and WASSCE. Learners‟ poor performance might be attributed to a mismatch between the requirements of the curriculum and learners level of thinking as indicated by (Bleeker, 2011). Past research showed that the van Hieles‟ levels of learning geometry and transformation geometry can have implications for investigating students‟ difficulties and improving students‟ performance in transformation geometry (Ada & Kurtulus, 2010). Also, it can provide a framework on which geometry instructions could be structured and taught in schools (Ada & Kurtulus, 2010). However, this claim has not been comprehensively investigated in Ghana. Hence it deserves some exploration and investigation with students. This study therefore seeks to investigate the inherent difficulties in the area of transformation geometry (rotation) using the van Hieles‟ model.

1.3 Purpose of the Study

The purpose of the study was to investigate and describe students‟ difficulties in the learning of rotation using van Hieles‟ geometric thinking levels. The investigation would be focused on students‟ difficulties in:

- identifying the image of an object after rotation.
- using the concept of rotation to transform an image of an object when given the coordinates, angles and shape.
- describing geometric figures and their properties after rotating an object.
- discovering the properties of a figure in a given rotational transformation by locating centre, and angle of rotation.
- using rotational transformations to do proofs?

1.4 Objectives of the Study

The objectives of the study include:

1. to describe various difficulties of students" in rotation in Ngleshie Amanfro Senior High School with major emphasis on the first-four van Hieles" levels.
2. to determine the levels reached by the students in terms of the van Hieles" model concerning rotation.
3. to determine whether there is any difference between male and female in terms of the difficulties in rotation according to the van Hieles" levels

1.5 Research Questions

The objectives of this study will be realized by pursuing answers to the following questions:

1. What difficulties do Senior High School students" encounter in rotation on the first-four levels of van Hiele?
2. What are the levels reached in the van Hieles" model with respect to rotation?
3. Is there any significant difference between male and female senior high school students in terms of the difficulties in rotation according to the van Hieles" level?

In answering the third research question, the following hypothesis was formulated;

H_0: There is no significant difference between the male and female Senior High School student in terms of the difficulties in rotation in the van Hieles" level.

H_1: There is a significant difference between the male and female Senior High School student in rotation according to the van Hieles" level.

1.6 Significance of the Study

The findings of this study would help improve student"s performance in transformation geometry (Ada & Kurtuluş, 2010). It would also have the potential to change what teachers believe, in terms of the way rotation was to be presented in the class. Through this, teachers will see the need to stop looking for anyone correct way to teach rotation because every classroom was also different, every student was different, and finally, every moment of teaching was also different. Furthermore, it would help mathematics educators appreciate the difficulties of students in the study of rotation and subsequently lead to suggestions for improving teaching strategies. Finally, it would contribute to the improvement of existing knowledge in the mathematics curriculum.

1.7 Limitation

Even though we have sixty-eight public and private senior high schools in the Greater Accra region, only one that was Ngleshie Amanfro Senior High school was selected for the study in the Ga South Municipality and this had limited the scope of the research. The consequence of this was that generalization of the research findings was limited. This limitation was alleviated when students from Junior High Schools all over the sixteen regions of Ghana were randomly placed by the computerized placement system for which the school chosen was one of them. The sample used therefore represents the characteristics of any student in any part of the country who had spent at least two years in the school and was preparing to write his or her final WASSCE.

Furthermore, the Ghanaian Mathematics curriculums does not contain information regarding the van Hiele model hence further placing a limitation.

1.8 Delimitation of the Study

Transformation Geometry covers concepts such as reflection, translation, dilation and rotation. However, for this study, the emphasis was laid on only the concept of rotation. The coverage of this area in transformation geometry was as a result of the difficulties and low performances of students" in this area. Most importantly, the researcher was not concerned with coverage but difficulties of students within the concept of rotation. Also, the subject of this study would be a sample of the population of students in Ngleshie Amanfro Senior High School.

1.9 Definition of Terms

The following terms were used throughout the research report and they are defined here to establish a clearer and concise meaning.

Learning Transfer: refers to the degree to which an individual/learner applies previously learned knowledge and skills or concepts to new situations.

Van Hiele model: It is a model designed by Pierre van Hiele and his wife Dina van Hiele. It consists of three domains: levels of the model, characteristics of the model and the learning phases of the model. The model consists of five phases arranged hierarchically from the simple to the complex (Visualization, Analysis, Abstraction, Deduction and Rigor).

Geometric concepts acquisition: This is the students" learning of the concepts mentioned in a geometry unit Mathematical textbooks for form two students in senior high school. In this study, it refers to rotation as a concept in transformation geometry.

Mathematics: This simply implies the study of measurement, number and quantities.

Concepts: General idea about something, this involves understanding the components of a phenomenon.

Creativity: In an attempt to define creativity we look at personality trait of creative individuals. Such individuals are always thinking, always prepared to listen to others opinion, are critical of their work, are analytic and original; have adaptive flexibility, spontaneous flexibility, word fluency, the capacity to puzzled, they are motivated, confident, intellectually persistent and moral communication to work.

Curriculum: refers to experiences that are planned and are offered under the guidance of a school.

Syllabus: A document containing content which learners are expected to know before being examined?

Geometry education: Mathematics is an activity of solving problems concerning shapes, vision and location. Geometry education concern itself with theories, principles and methodologies in the teaching of geometry.

Shapes: Geometry shapes embedded in spatial objects and create an opportunity to move from two-dimensional perception and vice versa.

Vision: projections of reality from various vantage points are an important part of geometry.

Location: students have to be exposed to different systems for determining the position and how to use them appropriately.

Divergent thinking: Divergent thinking could be seen as reasoning that practices unanticipated and unusual responses. This may include cognitive processes such as critical thinking, analysis, synthesis, interpretation, conjecturing, and induction. Such thinking enhances creativity in students.

Problem-solving: May include working and making a drawing, create your problem, think of a similar problem that was solved successfully in attempting to solve a problem. Problem-solving is teacher-centred in the sense that the teacher can direct students at the said strategies.

The problem-centred approach: In the problem – centred approach, instruction begins with problems. It is from the solution of the problems that students acquire knowledge. In this approach, the students interpret the problem condition in the light of his repertoire of experience (knowledge and strategies previously assimilated). The teacher only provides the necessary scaffolding during this process.

Prospective final year students: students who might be considered to form the nucleus of year three in the school for the next academic year.

Transformational geometry: A subset of geometry in which students learn to identify and illustrate the movement of shapes.

Visualization ability: The ability to mentally manipulate, rotate, twist, or invert a pictorially presented stimulus object.

1.10 Organization of the Study

The study was organized systematically in five different chapters. In Chapter One, the background of the study, statement of the problem, the purpose of the study, objectives of the study, research questions, and significance of the study, delimitation, limitations of the study definition of terms and the organizational plan were presented. The theoretical framework and relevant literature review were presented in Chapter Two. The researcher described the research design and methodology in Chapter Three. Results and discussion were done in Chapter Four. Chapter Five consisted of a summary of key findings, conclusion and implications for practice, recommendations, and areas for further research.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

This Chapter focuses on varied views on what other authors have written concerning the topic under study. The literature review focused on the theoretical framework, geometry and transformation geometry, rationale for the inclusion of transformation geometry into the school curriculum, research on transformation geometry, spatial development as a pre-requisite for learning transformation geometry, the acquisition of language in developing geometrical understanding in terms of van Hieles" phases, studies related to van Hieles" model as well as gender differences in geometry at the secondary school level.

2.1 The Theoretical Framework

Of the range of theoretical work concerned with geometrical ideas, that of the van Hieles" "et al". are probably the most well-known.

The van Hieles" theory which was developed by Pierre Marie van Hiele and Dina van Hiele Geldof in the 1950s had been internationally recognized and had significantly affected the teaching of geometry in schools (Kekana, 2017). Abdallah and Zakaria (2013) asserted that the aim of the van Hieles" model was assisting learners progress from one level to the next which does not only reinforce their present understanding of geometry theories but also helped them to progress to further levels. The theory posits that students who are being taught at a van Hiele level higher than they have achieved or are ready for, do not attain the level of success in High School geometry that educators feel was necessary for further study of geometry, or for further study of other topics and subjects that depend on geometric knowledge.

Pierre designed the base model from the learning perspective and described in detail five ascending levels of geometric understanding, thought or development. These levels were originally numbered from zero to five (Kekana, 2017). The van Hieles‟ theory of learning was therefore, modelled into five levels which have been categorized according to the following;

2.1.1.1 Level 1 (Visualization and Recognition)

At this level, learners are expected to informally differentiate between the objects based on their properties or size. They can identify only the visual characteristics of the shapes, but may for example not make a distinction between a rhombus and a parallelogram (Muyeghu, 2008). In other words, they identify geometric figures by physical appearance and not through partial characteristics (Kekana, 2017). That is to say that in the first level, students can identify a transformation by change in the figure and motion „„visual approach‟‟ but are not able to provide its properties. The shape is judged only by its appearance.

2.1.1.2 Level 2 (Analysis or Description)

The second level (analysis) is predominantly descriptive in that students can identify particular properties of shapes, but not in a logical order. At this level, learners use visual perception and nonverbal thinking (Armah, 2015) Learners at this level do not identify the properties of geometric figures (Armah, 2015). This level, “analysis”, is achieved by students when they succeed in distinguishing and abstracting some of the properties of geometrical shape, though without establishing logical relationships between them.

2.1.1.3 Level 3 (Abstraction and relationship)

At this level, learners are focused on the individual facets of their engagement and familiarities with geometrical concepts that may lead to conceptual understanding (Xistouri & Pitta-Pantazi, 2013). Students perceive relationships between properties and between figures. Students can combine shapes and their properties to provide a precise definition as well as relate the shape to other shapes. That is logical implications and class inclusions, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood.

2.1.1.4 Level 4 (Deduction)

For a student at Level 4, properties of shapes become objects that are independent of the figure and the object itself. This stage corresponds to high school years. Students can use axiomatic structure at this stage and can construct proofs themselves within this system. A system structure starts to develop with axioms, definitions, theorems, results and assumptions. Students can work with abstract expressions about geometric properties and can make deductions based on logic rather than intuition (Ural, 2016). Students at Level 4 prove other theorems with deduction using previously proven theorems and axioms and can achieve reasoning processes through induction. They can recognize two different logical reasoning ways with the same theorem and differentiate between them (Ural, 2016).

2.1.1.5 Level 5 (Rigour)

At this level students learn that geometry needs to be understood in the abstract; see the "construction" of geometric systems. That is they can study geometry in the absence of concrete models (M. Lynn & Courtney M, 2010). Assisted by appropriate instructional experiences, the model asserts that the learner moves sequentially from the initial or basic level (visualization), where space is simply observed. The properties of figures are not explicitly recognized, through the sequence listed above to the highest level (rigour), which is concerned with formal abstract aspects of deduction. (M. Lynn & Courtney M, 2010).

2.1.2 Educational implication of the van Hieles' Model

Understanding these levels enables teachers to identify the general directions of students" learning and the level at which they are operating (Lim, 2011). The learning process in geometry covers many levels, but an appreciation of these levels still need to be emphasized during teaching in the classroom. The first-three levels involve the development of procedural fluency in geometry, whilst the last two display the development of conceptual understanding (Kilpatrick, Swafford, & Findell, 2001).

The van Hieles" theory is divided into two parts: the first part is the hierarchical sequence of the levels, which shows that each level must be fully developed by the student before proceeding to the next level. The second part is the development of intuition in students and the phases of learning that influence geometric learning. It is through the disregard of the hierarchical nature of the levels with the teacher and the students operating at different levels that account for much of the difficulties students have in the process of learning geometry (Evbuomwan, 2013). Pierre van Hiele observed that two persons who are reasoning at the different levels will not understand each other. The teacher and the other students who progressed to a higher level seem to speak the same language which cannot be understood by the student who have not yet reached that level. They might accept the explanation of the teacher, but the concept taught will not sink into their minds. The students themselves will feel perhaps they can imitate certain action, but they have no view of their activity until they have reached the new levels (van Hiele, 1986). van Hiele further explains that if a student is at the first levels and the teacher speaks on the second or even third level, the student will not understand the teacher. The teacher will think he had made it very simple and plain but the student acts as though the teacher was talking nonsense. At this point, the teacher feels helpless. Subsequently, the teaching process comes to a standstill (van Hiele, 1986). Van Hieles" levels provide teachers with a framework within which to conduct geometric activities by designing them with the assumptions of a particular level in mind and they can ask questions that are below or above a particular level (Lim, 2011; Van de Walle, 2004). The levels are also a good predictor of students" current and future performance in geometry. The results of these studies indicated that the van Hiele levels have been useful in studying the learning of plane geometry which is very closely related to transformation geometry. According to Jones (2002), „the Van Hieles" model of mathematical reasoning has become a proved descriptor of the progress of students" reasoning in geometry and is a valid framework for the design of teaching sequences in school geometry". Van Hieles" levels of geometrical thought will be the guiding principles for investigating students" difficulties in the learning of transformation geometry and for determining the level at which the average student in the sample operated.

2.1.3 Mode of numbering for van Hieles' Levels

Two different numbering systems were used to name the van Hiele levels of geometric understanding in past research, namely Level 1 to Level 5 and Level 0 to Level 4 (Clements and Battista, 1992; Crowley, 1987). Knight (2006) asserts that if mathematics teachers realized that learners did not have a complete conception even at the primitive level (recognition), they might need to identify a level before level 0 (recognition). In connection with the hands on application of the van Hieles" model, Huang, Liu and Kuo (2013) point out that these five levels of geometric thought could be stages that any learner could be at and these are not influenced by the students age and maturity. In a study to re-examine the van Hieles" theory of levels of geometric thinking, three van Hieles" levels were used instead of 5 levels (Duatepe & Ubuz, 2009) and (Gunham, 2014). In level 1(visualization), learners are expected to recognize objects globally and go through integration, free orientation and explication, bounded orientation as well as bounded information of phases of learning. In level 2 (descriptive), learners are expected to recognize objects by their geometric properties and go through all the phases mentioned in level 1. Finally, level three (theoretical) is the level of deductive reasoning in which learners prove the geometric relationship. The researcher will utilize Level 1 to Level 4 as adopted by (Muyeghu, 2008). As far as the numbering of the van Hieles" levels is concerned, Muyeghu (2008) for example condensed the van Hieles" levels to four and describe them in terms of adjectives namely visually, descriptive/analytic, abstract/relational and proof. A numbering system ranging from zero to five would accommodate learners who had not become proficient at van Hieles" initial level 0 (recognition) and the new level 0 would be referred to as pre-recognition (Muyeghu, 2008).

2.1.4 Piaget's theory of learning

The developmental stages of children"s cognition are one of the major contributions of the Piagetian theory. Piaget"s work on children"s quantitative development has provided Mathematics educators with crucial insights into how children learn mathematical concepts and ideas (Armah, 2015). From his observation of children, Piaget understood that children do have enough ideas. He argued that children were not limited to receiving knowledge from parents or teachers, but rather, they actively constructed their knowledge (Garner, 2008). The structures according to Piaget are mainly cognitive which enable people to process data by linking it with their previous knowledge and experience, determining relationships and patterns, identifying rules and forming principles that are abstract and relevant in different forms of applications (Garner, 2008). He also believed that the consequences of the change and transformation result in knowledge.

The implication of this was that children who were old, and even adults, process information in ways that are a feature for young children at the same developmental stage though they are yet to pass through later stages (Bobby, 2008). Development and learning are the two main themes in Piaget"s theory of learning. Development focuses on learners" capabilities and the learning focuses on the realization of such capabilities and the education within it is extrinsic (Baken, 2014).

2.1.4.1 Pre-school or sensory-motor stage (0-2 years)

This stage involves the use of motor activity without the use of symbols. Knowledge is limited in this stage because it is based on physical interactions and experiences. An additional characteristic of children at this stage is their ability to link numbers to objects e.g. one dog, two cats, three pigs (Armah. 2015). According to (Kendra, 2014) a child has an inherent tendency to organize its world as it develops. Object permanence develops at this stage whereby a child understands the objects, whether is hidden or visible. It is also at this stage that children only look at the world through their perspective. The idea of a simple closed curve is also important and helps to explain why very young children perceive shapes such as circles, squares and triangles as being essentially the same shape, particularly when they draw their own

(Armah, 2015). The child has physical interaction with his or her environment, builds a reality and how it works (Baken, 2014).

2.1.4.2 Pre-operational or intuitive stage (2 - 7 years)

At this stage, children start to represent spatial features through drawing and modelling (Armah, 2015). During this stage, intuitive mode of thought prevails, characterized by free association, fantasy and unique illogical meaning (Simatwa E. M., 2010). Children begin to use language; memory and imagination also develop. In the pre-operational stage, children engage in make-believe and can understand and express relationships between the past and the future. He develops an awareness of the conservation of mass, weight and volume. More complex concepts, such as to cause and effect relationships, have not been learned.

2.5.1.3 Concrete operations stage (7 - 11 years)

Intellectual development in this stage is demonstrated through the use of logical and systematic manipulation of symbols, which are related to concrete objects. Thinking becomes less egocentric with increased awareness of external events and involves concrete references. The child here was concerned with knowing only the facts and therefore becomes confused when faced with the relative, probabilistic nature of human knowledge (Simatwa E. M., 2010).

2.1.4.3 Formal operations stage (Adolescence - Adulthood)

At this stage of development, learners can visualize the concepts of area, volume, distance, translation, rotation and reflection. A learner should also be able to combine measurement concepts with projective skills (Armah, 2015). Piaget believed that intellectual development was a lifelong process, but that when formal operational thought was attained, no new structures were needed.

2.1.4.4 Implication of Piaget's Theory in terms of Learning

In our schools, Piaget‟s theory of cognitive development has far-reaching effects on curriculum development, planning, implementation, evaluation and instructional management. In developing the curriculum, therefore, adequate preparation has to be considered to meet the needs of each of the various stages of learning according to researchers.

The following can be considered when planning a lesson based on each of the stages:

- Provide concrete props and visual aids, such as models and/or timeline
- The use of examples that is common to facilitate the learning of more complex ideas, such as story problems in mathematics.
- Allow opportunities to classify and group information with increasing complexity; use outlines and hierarchies to facilitate assimilating new information with previous knowledge.
- Present problems that require logical analytic thinking; the use of tools such as "brain teasers" is encouraged Use visual aids and models.
- Making it possible for issues in terms of social, political, and cultural to be discussed. Concepts can be taught broadly instead of facts and subsequently situating it within a meaningful context which is relevant to the learner.

From the above-mentioned strategies, Piaget advises that traditional geometry should be learnt according to the stages of the intellectual development of the students. That means students‟ should be able to progress from one thinking level to the next one and instructions should be realized in a sequence corresponding to the cognitive development of the child (Piaget, 1971). In effect, one can incorporate Piaget`s theory in the classroom. Piaget takes a constructivist point of view and believes that learners are not passive in their knowledge. Piaget"s theory suggests that students need a curriculum that supports their cognitive development by learning concepts and logical steps cited from (Mwamwenda, 2009). He also suggests that children are only capable of learning specific material in specific stages of cognitive development. Piaget emphasizes that learning takes place as a result of the active engagement of learners, so teachers have to see to it that learners take an active role by participating in whatever is being taught and learned. Piaget"s theory acknowledges the individual difference in cognitive development. Teachers should arrange activities that learner"s intellectual development could absorb. Piaget shows that a child`s understanding is restricted by stages that he or she has reached and therefore teachers should take this into account as they teach children with different levels of intellectual developments (Mwamwenda, 2009).

2.5.1.6 Shortcomings in Piaget's theory of learning

Meanwhile, researchers during the 1960s and 1970's identified certain shortcomings in Piaget's theory. First, critics argue that by describing tasks with confusing abstract terms and using overly difficult tasks, Piaget underestimated children's abilities. Researchers have found that young children can succeed in simpler forms of tasks requiring the same skills. Second, Piaget's theory predicts that thinking within a particular stage would be similar across tasks. In other words, pre-school children should perform at the pre-operational level in all cognitive tasks (Simatwa, 2010). Research has shown diversity in children's thinking across cognitive tasks. Third, according to Piaget, efforts to teach children developmentally advanced concepts would be unsuccessful. Researchers have found that in some instances, children often learn more advanced concepts with relatively brief instruction. Researchers now believe that children may be more competent than Piaget originally thought, especially in their practical knowledge (Simatwa, 2010).

2.2 Geometry and Transformation Geometry

Geometry has been defined as the study of shape and space (Güven & Kosa, 2008). Geometer is the name given to any mathematician who works in the field of geometry. Earth measure is the original name for the word geometry and was first used for constructional and agricultural purposes. Gal & Linchevski (2010) researched and concluded that children prefer to rely on the visual aspect of the prototype than the use of verbal definition in a bid to classify and identify shapes. In the assignment of class membership, a child prefers to call upon the visual prototype rather than the verbal definition even though he/she holds both of them for a given geometric concept. The geometric and spatial knowledge students bring to school should be expanded by exploration, investigation, and discussion of shapes and structures in the classroom. Geometry is one of the core topics included in the modern mathematics syllabus that was approved by National Council for Teachers of Mathematics (NCTM) (Kekana, 2017).

To be self-proficient in describing, representing and navigating the environment students must be able to use geometric concepts in real-life situations since the knowledge of it goes beyond the skills required to manipulate it (Luneta, 2015). Transformation geometry at the Senior High level requires learners to do transformations, in which learners have to recognise, define and do transformations with points, line segments and simple geometric figures on a coordinate plane which focuses on the following reflections, translations, rotations and enlargement. It could also be represented by vector or coordinates of the image of the original shape. In the teaching and learning of transformation geometry, students are expected to carry out tasks involving, Reflection, Translations, Rotation and enlargement of an object. In doing this, students naturally or intuitively solve problems by manipulating a concrete object or drawing figures as requested (Evbuomwam, 2013). While there are many different kinds of geometric transformations, the focus of this study is the concept of rotation. A rotation is a type of rigid transformation where a figure is turned a specified angle and direction about a fixed point called the centre of rotation. The rotation turns the figure and all of the points on the figure through a specific angle measurement where the vertex of the angle is called the centre of rotation. For a description of rotation, three pieces of information are needed: the centre of rotation, the angle of rotation, and the direction of the rotation. Hollebrands (2003), identifies the provision of opportunities for students to think about important mathematical concepts, provision of a context within which students can view mathematics as an interconnected discipline and provision of opportunities for students to engage in higher-level reasoning activities using a variety representations as to the three important reasons why geometric transformations are studied in our school mathematics. In preparing teachers for the classroom, it is important to know in advance the difficulties students have when new concepts are supposed to be learnt (Hollebrands, 2004).

Transformation can lead students to explore students‟ geometrical experience, thought and imagination; and thereby enhance their spatial abilities and consequently helping them perform any abstract mathematical concepts of congruence, symmetry, similarity, and parallelism. By the end of JHS 3, it is abundantly clear from research that students should have enough knowledge of geometric transformations to succeed in any future mathematical studies at the tertiary level (Carraher & Schliemann, 2007;

NCTM, 2000). It is therefore not surprising that one of the aims of teaching Mathematics in Ghana is to develop an understanding of spatial concepts and relationships (CRDD, 2007). However, studies showed that students have difficulties in understanding the concepts and variations in performing and identifying transformations including translation, reflection, rotation and combinations of transformations of these types (Olson, Zenigami & Okazaki, 2008; Rollick, 2009). For example, Guven (2012), found that middle school students encounter difficulties in both executing and identifying transformations. Errors in execution came as a result of drawing images of reflections in the wrong orientation and out of scale. It was also concluded in the study that the majority of students have not developed an operational understanding of transformations likewise that of conceptual understanding. A study conducted shows identify that dynamic representations is a powerful tool that improves students" conceptual understanding from operational understanding (Guven, 2012). Current educational theories emphasize active involvement of students in teaching and learning of transformation geometry (Rotation) in particular, hence students are expected to find the point, angle, centre, symmetry, describe and turn any given figure through a given degree. In doing this student need to go through certain phases of learning that will end up giving them an impetus when they are confronted with the concept of rotation.

2.3 Rationale for the Inclusion of Transformation Geometry into the School Curriculum

Transformation geometry serves as a tangible reason why learners have an early conceptualization of vectors and can therefore afford an outstanding example of comparing mathematics with the outside world through the notion of isometric transformation (Kekana, 2017). For example, when learners look at themselves in the mirror, they will need to understand that the image formed is due to reflection through the mirror. They will also need to know that the object and its image are the same in terms of shape and size. Besides, tessellations are an example of decorative arts that occur around as in nature and which are the product of Islamic civilization (Kekana, 2017). The making of patterns and the movement of objects are examples of transformation geometry. In a study, it came to the fore that transformation geometry provides an ample opportunity for learners to develop their spatial visualization skills and having a sense of reasoning ability in geometry (Sarah & Jayaluxmi, 2012). In the traditional Euclidean geometry, many students experienced difficulty writing proofs, and most students were unsuccessful involving geometrical problems (Evbuomwan, 2013).

According to Hollebrands (2003), there are three important reasons to study geometric Transformations in school mathematics:

- It provides opportunities for students to think about concepts in mathematics that are essential for example functions and symmetry.
- It provides a context within which students can view mathematics as an interconnected discipline.
- It provides opportunities for students to engage in higher-level reasoning activities using a variety of representations.

Transformation can lead students to explore the abstract concepts in mathematics such as symmetry, congruency, similarity, and parallelism; enrich students‟ geometrical experience, thought and imagination; and thereby enhance their spatial abilities (Guven, 2012).

From my viewpoint, with regards to the importance and purpose of transformation geometry highlighted above, it implies that if transformation geometry skills were to be achieved through a geometric course, therefore, upon finishing the course, students should be able to understand, appreciate and use transformation geometry to solve personal and societal problems. As a result, students‟ difficulties in geometry and rotation in transformation geometry call for concern on the part of researchers and the mathematics community to investigate how these geometries are being taught and learned by students.

2.4 Research on Transformation Geometry

In 2003, Edwards identified a misconception about rotations. She found out that instead of seeing rotation as mapping all the points of the plane around a centre point, the students in her study had a hard time seeing rotation as occurring „at a distance‟ from the object" instead of seeing the shape to be sliding towards a given centre point and then turn around it (Edwards, 2003). Transformation geometry as a topic makes it possible for one to assess how skills and abilities can be merged in algebraic and geometric ideas. This topic in the mathematics curriculum also provides a context for combining algebra and geometry and encourages visual as well as an analytic approach. The approaches when combined, would be more prevalent with SHS 3 learners because by then learners would have been exposed to both analytical and visual techniques (Sarah & Jayaluxmi, 2012). Ada and Kurtulus (2010) conducted a comparative study to investigate learners‟ misconceptions and errors concerning two aspects of transformation geometry namely translation and rotations. With regards to translation, the results of the study revealed that 75% of the learners who took part in the study obtained correct answers; 3% of the learners got wrong answers; whereas 9% made technical mistakes. With regards to the rotation, 35% the leaners‟ got the correct answer to the technical question; nonetheless, 55% of them used true rotation transformation. Comparing learners" percentages in terms of translation and rotation, the indication is that learners understood translation better than they understood rotation (Ada & Kurtulus, 2010).

According to Ministry of Education, Science and Sports (MOESS, 2007), the essence of geometry instruction is to enable students to develop logical and divergent reasoning in problem-solving situations and in their everyday mathematical communication processes. In elementary geometry lessons, Jones (2002) also noted that shapes and space are taught to foster the learning of higher mathematics such as mechanics, vector and mensuration. Given the above, many countries are concerned about how teachers teach or how students learn aspects of geometry in the basic school mathematics curriculum (Gunhan, 2014; Golan, 2011; Boakes, 2009; Martin, Mullis & Foy, 2008). In the Ghanaian mathematics curriculum, Geometry is treated as either a course (Institute of Education, 2005) or one of six strands of mathematics at the higher levels. From a study of the primary school level, Geometry is treated as Shape and Space and occupies approximately 17% of six major content areas covered in the mathematics teaching syllabus. The rationale for treating shape and space is to give emphasis to pupils" early development of spatial visualization and mental rotation abilities and to enable them "organize and use spatial relationships in two or three dimensions, particularly in solving problems" (MOESS, 2007) and for progress in learning higher mathematics (Akayuure, Asiedu-Addo, & Alebna, 2016).

There have been concerns about weak geometric knowledge among pre-tertiary students in Ghana of late and this stems from the fact that students have very weak spatial abilities. Several assessment reports have indicated that students" performance in geometry has been generally low. Report from Trends in International Mathematics and Science Study (TIMSS) in 2003, 2007 and 2011 indicated that JHS 3 pupils in Ghana, perform the lowest in geometry among countries which participated (Gunhan, 2014; Mullis, Martin, Foy & Stanco 2012). At the senior high level, there has been consistent evidence (Fletcher & Anderson, 2012) regarding the inability of candidates to tackle questions requiring visualizing figures or objects in space in addition to reasoning in geometry with 3D (3dimensional problems), mensuration and circle theorems in core Mathematics. A study done by (Mwamwenda, 2009; Tahani, 2016) pointed out that concepts and their interconnections are the difficult areas facing students during learning.

A study has shown that, teachers have observed that many young children have numerous misconceptions about geometry (Özerem, 2012). According to a study in South Africa, the experimental group in the post-test, the percentage of the number of learners at level 0 decreased from 56% to 26%, the percentage of the number of learners at level 1 increased from 26% to 35% and the percentage of the number of learners at level 2 increased from 17% to 38%. While in the control group in the post-test, the percentage of the number of learners at level 0 decreased from 56% to 47%, the percentage of the number of learners at level 1 increased from 25% to 32% and the percentage of the number of learners at level 2 increased from 18% to 20%. The significant improvement in the performance of the experimental group having more learners at level 2 than at level 0 and level 1 in the post-test suggests that the van Hiele theory-based instruction for the experimental group had a more positive effect than those in the control group (Jogymol & Kuttickattu, 2016). In a study, involving discussion of geometry proof problem in class, a teacher was supposed to do oral presentation of the formal proof with body movements whilst students were made to watch, listen, jot notes, and think as the presentation continuous. Through this, students are supposed to learn by imitating many parts of the teachrs, movement during the instruction.

In sampled studied class, when construction activities are used, they involve developing new ideas and connecting these with students' existing ideas. To perceive what a teacher sees as a geometric situation in terms of a student‟s achievement of higher-level then he or she should be at a particular van Hiele level. If a learner bypasses any of the level then there is bound to be a misconception or errors in students thinking level in the van Hiele model. A teacher should get students to explain how they come to their answers or rules so that she/he can analyze the faulty interaction between the students' negative ideas and the concept that it purports to drive at (Özerem, 2012).

Todri (2004) A research carried out from the national exams of quality control which were done in Jordan showed that performance of 82% of the students in the geometry field was moderate and that of 60% was lower than cited from (Tahani, 2016). A study by Ada and Kurtulus (2010) on 126 University students revealed that there are various challenges about the teaching and learning of geometry. Their analysis came up with certain mistakes that were made by the students in the area of their geometry course. The analysis was based on students‟ performance in two-dimensional transformation geometry and exploration of the mistakes made by the students. The result of the analysis showed that these students did not understand how to apply the rotational transformation. Algebraic meaning of translation and also of rotation was mostly understood by students but unfortunately did not seem to appreciate the meaning of geometric concepts. In a similar study, Hollebrands (2003) investigated the nature of students understanding of geometric transformations, which included translation, reflections, rotation and dilations, in the context of the technological tool, the Geometer‟s Sketchpad. The researcher implemented a seven-week instructional unit on geometry transformations within an Honours Geometric class and came up with an analysis of students‟ understanding in the area of concept of transformations as a function during the study. The analysis suggested that students‟ understanding of key concepts including domain, variable and parameters, and relationships and properties of transformation were critical for supporting the development of deeper understandings of transformations.

Perham (1976) investigated factors that contributed to the difficulties encountered by students in performing rigid transformation tasks. The direction of the transformation task revealed that children seemed to be able to perform vertical or horizontal transformation but not tasks over the diagonal. Perham‟s study with first-grade children revealed that children had some understanding of slides before the unit of instruction but not of flips or turns of any type. After instructions, diagonal transformations that included slides were not performed correctly. Outcomes from these studies mentioned above suggested that students are faced with difficulties such as the inability to perform transformation vertically, horizontally and diagonally. Those observations call for intervention from all the stakeholders in the teaching and learning of transformation geometry. Students and teachers can use technology to also access a wide range of tools in mathematics education concerning transformation geometry in the final analysis.

2.5 Spatial Development as a Prerequisite for Learning Transformation Geometry

Spatial abilities are often categorized into spatial visualization and spatial orientation cited from (Akayuure, Asiedu & Alebna, 2016). There are two ways of looking at figures and recognizing what they stand for: the natural and the mathematical (Duval, 2011). One important issue in the learning of geometry in primary and secondary school is to identify the figural units which can be discriminated in any constructed figure. According to Duval (2011), visualization ability in geometry is closely related to the ability to recognize all figural units that can be mathematically relevant.

The work of a pair of Dutch researchers, namely Pierre van Hiele and Dina van Hiele-Geldof went into much of the current thinking about the development of geometric thinking in students. Their model of geometric thinking identifies five levels of development through which students pass when assisted by appropriate instruction

- Visual recognition of shapes by their appearances as a whole (level 0)
- Analysis and description of shapes in terms of their properties (level 1)
- Higher "theoretical" levels involving informal deduction (level 2)
- Formal deduction involving axioms and theorems (level 3)
- Work with abstract geometric systems (level 4). Cited from (Geddes & Fortunato, 1993).

Since students differ in abilities, teachers should therefore, present instructions in a manner that take this into account during teaching and learning. Furthermore, in a geometry class, gifted students rely on symbolic thinking while those less gifted should visualize the problem in a problem-solving situation. Certainly, visualization does not harm the gifted students but if left out of the curriculum, it limits the chance of success in geometry problem solving of the less gifted child (Kirby & Schofield, 1991).

In summary, students ought to be aware of the presence of geometry in all human endeavours particularly in art and other structures that are built by man. Realizing that geometry and geometric applications are all around them hence through the study will appreciate how the applications are done which would culminate in their appreciation of the role of geometry in life. Artisans such as carpenters use triangles for structural support while scientists make use of geometric models of molecules which give clues to the understanding chemical and physical properties. Finally, merchants also use traffic-flow diagrams to plan and display the placement of their stock. These and many, many more examples should leave no doubt in students" minds as to the importance of the study of geometry when there is a deep sense of spatial development cited from (Geddes & Fortunato, 1993).

2.6 The Acquisition of a Language in Developing Geometric Understanding In terms of van Hieles' phases

It can be argued that knowledge of students" levels of geometric reasoning is essential for effective teaching (Luneta K., 2015). Van Hieles" views on education reflect levels that depict a particular model. In this model, van Hiele asserted that a student must progress through each of the levels of thought as coming out of instruction which was put into five phases of learning. The model of van Hiele which organized the students" learning phases of geometry subjects in a sequential system is as follows:

Information: Regarding the first learning phase, van Hiele says, "The researcher will pose a question that will be well-known language symbols, in which the context he wants to use becomes clear". Teachers need to introduce and use appropriate words and symbols that will introduce a very current concept. Teaching a subject to students must aid them to discover specific information by asking them some questions. Since the question will be directly related to what they know, it will then attract their attention to the information that should be learnt. A clear instance is where teachers could be made to ask questions such as: What is a square and rectangles are and the similarities between these forms of figures? The whole image of these geometric shapes is the major pre-occupation of the teacher"s goal towards getting the students to know and also identify these figures.

Guided orientation: In this second learning phase, van Hiele explains that students need to use the new language they have been introduced to, although it may not be completely understood, using the language or symbols appropriately within carefully chosen tasks, the student will begin to understand the language and symbols that go with the concept being learnt. Activities presented to students in a structured form will help them identify and voice out their understanding of the new concepts in geometry which have been introduced in the information phase. That is students explore the objects of instruction in carefully structured tasks such as folding, measuring, or constructing.

Explicitation: In the explanation phase, students" new knowledge is formed through experience and knowledge. They explain and state their views about the geometry structure based on the observations that will be explained carried out before Crowley (1987). Students use their own words to describe what they have learned about a given geometric topic. This is where the teacher introduces relevant mathematical terminologies.

Free Orientation: **S**tudents now understand and make connections among the relationships they see and have worked with on tasks, and the students "now know the relevant language symbols". Students are comfortable speaking of and using language and language symbols appropriately for the geometric concept they have been studying. Students clarify and recognize their thoughts and understanding of geometric concepts by talking about them and using the language specifically related to the concepts. Complex tasks can be solved by students through a series of steps and ways Crowley (1987).

Integration: In the final phase, called integration, students review and summarize what they have learnt to make a novel overall view about a network of objects and their relationship (Crowley, 1987; Serow, 2008). Students summarize and integrate what they have learned, developing a new network of objects and relations. A student may need to cycle through some of the five phases more than once with a particular topic. A student operating with the van Hiele model cannot achieve one level of understanding without mastering all the previous levels.

Language structure is a critical factor in the movement through the van Hiele level – from global (concrete) structures (level 0) to visual geometric structures (level 1-2) to abstract structures (level 3-4). Van Hiele noted that many failures exist in teaching geometry as a result of a language barrier. The teacher in using the language of a higher level than expected by a student at a particular level makes it difficult for students to be understood what is being taught. Burger and Shaughnessy (1986) have found this consequence to be true within their research when studying the discussion between teachers and their students. Thus, students must acquire the language of the level the learning activities are presented at before they can even comprehend the discussion or instruction the teacher is engaging them in; only in this way can students be conversant about the material and concepts at that level. For example, a student at level 2, abstraction, may regard a rhombus as a special parallelogram, but students at lower van Hiele levels cannot understand this concept.

In other countries and also that of the United States, research has supported this view only with an exception. Mathematically talented students seem to skip levels, primarily because they develop logical reasoning skills in ways other than through Geometry as it is expected of them. The thought of most high school geometry teachers at the fourth or fifth levels of van Hiele has given them a clue that most students begin a high school geometry course at the first or second level. The teacher needs to remember that although the teacher and the student may both use the same word; they may interpret it quite differently. For example, if a student is at the first level, the word "square" brings to mind a shape that looks like a square, but little else. At the second level, the student thinks in terms of the properties of a square, but may not know which ones are necessary or sufficient to determine a square. The student may feel that to prove that a figure is a square; all the properties must be proved. According to Usiskin (1982), the important characteristics of this theory are the following:

Fixed order, meaning that the order through which the students" progress through the thought level is invariant, therefore a student cannot be at one level without passing through the other level.

Adjacency, meaning that, at every level, what was intrinsic in the preceding level becomes extrinsic in the current level.

Distinction, meaning that each level has its own set of linguistic symbols and own network of relationship connecting those symbols.

Separation, in the sense that two persons who reason at different levels cannot understand each other.

Crowley (1987) described the distinctive characteristics of the five levels of the van Hiele model as follows:

The model is sequential in that a learner cannot function at a higher level without first progressing through the thought processes of all previous levels. Progress from one level to the next is not through biological development but rather on instruction. The linguistic symbols of each level are unique, that is each level is regarded as having its language, and learners on different levels cannot understand one another. The intrinsic characteristics of one level become the extrinsic object of study of the next. The mismatch between the levels of instruction and the level at which a student is functioning may restrict the desired level.

Van Hieles‟ theory implied that for effective learning to take place students must actively experience the objects of the study in a manner that is appropriate in terms of contexts, discussion and reflection. The use of lecture and memorization as the key components of instruction will not promote effective learning according to the van Hiele theory. Teachers should provide their students with appropriate experiences and the opportunities to discuss them. Assessing students‟ levels of thought and providing instruction at those levels will rather improve student‟s levels of understanding. The teacher should provide experiences organized according to the phases of learning to develop each successive level of understanding (Soon, 1989). From the account of the van Hieles‟ model, it can be said that the geometric understanding of students will be improved through teaching. This can be achieved if teacher‟s instruction is organized in such a way that, it takes learners thinking ability into account whilst new works are introduced. Alex and Mammen (2016) suggests that the levels have proved a useful tool in identifying the problems in students‟ understanding of certain geometrical concepts; secondly, in evaluating the structure or development of geometric content in secondary school textbooks and thirdly, in guiding the development of syllabi. The van Hiele theory is particularly relevant in the Ghanaian schools, where mathematics remains a problematic learning area. Also, Atebe (2008) posits that the theory offers a model of teaching that teachers can apply to promote their learners‟ levels of understanding of geometry (Jogymol & Kuttickattu, 2016)

2.6.1 Why the van Hieles' model is selected over that of Piagets' theory

Though the van Hieles‟ model takes its root from that of Piaget‟s work (Colignatus, 2014), it is also a theory of its own. Two main ways in which van Hieles‟ theory deviates from that of Piaget are:

It empirically defines and develops the levels of abstraction in the understanding of mathematics and to defend the notion of a link that is independent of students‟ chronological age (Colignatus, 2014). The development theory proposed by Piaget fails to take account of learning, and fear that the developmental stages (the pre-operational and concrete operational stages) are not enough to enable geometrical concepts to be understood.

Language is a key component of van Hieles" theory since there is a sense of inspiration that is drawn from Vygotsky"s theory (Knight, 2006). The fact that van Hieles" model was designed in the light of general development theories, there appear to be other reasons that have guided the choice of this model being selected. First, I wanted to select a model that would be suited for the research and had already been tested and/or validated by several authors. Second, the model also needed to reflect the content that is supposed to be covered in the study programmes (Yildiz, Aydin, & Kogce, 2009). Third, it was needed to determine the progress of teaching/learning with some precision and illustrate the main phases through which students must pass to progress in geometry (Marchand, 2009).

2.7 Studies Related to Van Hieles' Model

The van Hieles" levels of understanding provide a valuable aid in the assessment of learner performance. Gunhan (2014) investigated a small scale study with six Grade 8 learners (3 girls and 3boys); getting their training at a community basic school that was indiscriminately nominated from among schools of a reasonable socioeconomic prominence. The results enabled the classification of performance according to the van Hieles" levels of thinking, and in doing so created an improved understanding between teachers and leaners. Thus the use of van Hieles" might enhance teaching and learning of transformation geometry. Halat (2006) compares the attainment of van Hieles" levels of learning in Grade 6, in which 273 leaners were separated into two sets; 123 learners were classified as control group were the instruction was facilitated using the traditional methods and 150 learners were classified as the treatment group was taught according to the reform-based curriculum. The findings revealed the following;

- Level 1: 27.6% (control group) vs. 17.3%(treatment group)
- Level 2: 52% (control group) vs 65.3% (treatment group)
- Level 3: 20.4% (control group) vs 17.4% (treatment group)

The indication from the results was that learners from both groups progressed by the levels; however, learners did not reach the fourth (abstraction) and the fifth level (rigour) (Kekana, 2017).

A study examined the van Hieles‟ levels of pre-service basic and mathematics teachers and the van Hiele Geometric Test (VHGT) was directed to accumulate data about the geometric thinking of learners. Pre and post-tests were directed towards all participating learners. The results of the first sample consisting of all learners taking the 400 level geometry courses revealed that out of 18 learners who wrote the pre-test, the mean was found to be 2.895, a standard deviation of 0.658 and t = -7.324. the post-test scores revealed that out of 12 learners who wrote the test, the mean was to be 3.077, standard deviation 0.862 and t = -3.860. These results show that the van Hieles‟ level of learners in the 400 level course was statistically lower than the level 4 (deduction) (Kekana, 2017).

Soon (1989) investigated the van Hiele‟s levels of achievements in transformation geometry of secondary school students and the existence of the hierarchy of a van Hiele level of understanding of transformation geometry. An interview and observation technique was used to collect data from a group of about 20 students within the age range of 15 to16. The result of the investigation indicated that the levels as exemplified by the task did form scales. This seemed to support the existence of a hierarchy in terms of the van Hiele‟s level related to transformation geometry. The study further revealed that students could recognize transformations easily, but they had problems in describing transformations. According to the findings, in terms of tasks for each of the concept strand, students were more successful for tasks in reflection and least for enlargement. Students in the study generally did not know the rigour of proofs. Analyses from the interview indicated that students did proofs by giving particular examples. This suggested to the researcher that students‟ response to the interview reveal rote learning (Soon, 1989).

Similarly, the Chicago Project was fashioned to test the ability of van Hieles‟ theory to describe and predict the performance of students in secondary school geometry (Usiskin, 1982). Approximately 2900 students from six different states in the USA were involved in this study. Four tests were administered in this project, they included:

- A multiple-choice test that was used to test prerequisites of high school geometry administered as pretest and posttests.
- Multiple choice test associated with the van Hiele levels was also administered as pre-and post-test this was.
- A proof-writing ability test was administered after a year of high school geometry and finally
- A Post-test was given as a standardized geometry test on geometry achievement.

Certain few concepts were also looked at during the development of the van Hiele level test in other to predict an overall van Hiele level. From the investigation, the study revealed that some students were able to answer questions set at a higher level, yet failed to answer correctly lower-level questions.

Mayberry (1982) studied the van Hiele"s level of geometry thought of undergraduate pre-service teachers. He looked at the hierarchical nature of the van Hiele levels. The study developed test items corresponding to the van Hiele model on seven concepts in geometry which include, „square, circle, isosceles triangle, right triangle, parallel lines, similar figures and congruent figures". The items were validated by thirteen (13) mathematicians and mathematics teachers. They were then revised and administered to nineteen (19) pre-service elementary teachers through interviews. The result of the study confirmed that the van Hieles" levels formed hierarchy and her students could be assigned a level. However, there was no consensus across concepts implying that students could be at different levels for various concepts (Mayberry, 1982). In a similar study, Denis (1987) also investigated the relationships between Piagetian stage of development and van Hiele level of geometry thought among Puerto Rican adolescents. His study showed that van Hiele levels are hierarchical among subjects in the formal operational stage of development. Denis also found no consensus across concepts in the van Hiele levels.

Denis (1987) and Mayberry (1982) studies greatly favoured the van Hiele model in the study of geometry. The Hierarchical nature of the van Hiele"s levels exists and the levels appear to be useful in explaining student"s thinking processes in geometry. The van Hiele"s theory explains the behaviour of students in learning and provides guidelines to diagnose the 36 difficulties experienced by students in solving geometry problems (Denis, 1987). However, Burger and Shaughnessy (1986) recommended using the model for the investigation of students" responses on other mathematics topic and suggested its use in the study of geometry transformation. Hoffer (1981) wrote a textbook entitled: Geometry, a model of the universe for Secondary School Students. This textbook is based on the van Hiele"s structure. It was written for a one-year course emphasizing investigation and activities in the first semester and preparing students to work in a deductive system (van Hiele level 3) in the second semester. An informal study with one class using a traditional textbook and a second class (experimental) using the Hoffer materials was conducted. The results of the investigation revealed that students in the experimental class learned better geometry and could write proofs better than those in the traditional class where a well-established text was used (Soon, 1989). Other similar studies by Battista and Clements (1992), investigated the van Hiele model in geometry learning under the environment of logo with eight-year-olds and seven graders.

Battista and Clements (1992) used the synthesis of Piaget"s and the van Hiele theory and the logo environment to develop instructional activities and assessment to observe eight-year pupils" geometric conceptualization. The dynamic features of the logo in term of turtle paths and movements enhanced students understanding as they analyzed the movements of the turtle in forming shapes and facilitate their learning of concepts such as angle, line segment and their interrelationships. One of their findings was that the logo environment helped the students to make the transition from the visual to the descriptive thought level of van Hiele"s hierarchy.

However, as described earlier, Soon (1989) investigated the extent to which van Hieles" theory supported the hierarchical levels in the learning of concepts in transformation geometry. Nevertheless, the study pays less emphasis in explaining the effect which these levels have on students learning of concepts in transformational geometry. From the foregoing, the investigator began the present study to substantiate and share more light on the extent to which students" level of visualization, abstraction, analysis and deduction can enhance students learning of the concept of Rotation in transformation geometry

2.8 Gender Differences in Geometry at the Secondary School Level

Many of the research findings showed that sex differences in mathematics are varied at middle school levels. Evidence on when sex differences in perceptions of competence in mathematics start to occur are not entirely consistent. Several studies have found that there is a sex difference between boys and girls in learning geometry. For instance, according to Armstrong (1981), thirteen-year-old girls performed better at computation and spatial visualization than boys. Peterson & Fennema (1985) conducted a study using the 1978 National Assessment of Educational Progress (NAEP) results to examine the sex-related difference in mathematics performance. They found that males significantly outperformed females in the area of geometry. This study also reported that there was no significant difference in mathematical performance between male and female students ages 9 and 13; however, there was a significant difference in the achievement of a 17-year-old male and female students. 17-year-old male students‟ performance exceeded that of 17-year-old female students at every cognitive level. Their findings provide very important insights for research to explore what causes the gap in achievement between male and female students as their ages increase.

Battista (1990) examined high school students‟ gender and geometry performance. In his study, male students scored significantly higher than female students on a geometry problem-solving test. The greatest difference between males‟ and females‟ geometry scores occurred for students whose nonvisual reasoning scores were much greater than their visual reasoning scores; the smallest difference occurred when the visual solution score was much greater than the nonvisual solution score. He found that males and females differed in geometry performance but not in preferences for solution methods. Similarly, Mayer and Massa (2003) also concluded that there were no significant gender differences in students" preferences for solution methods. This result supports the finding of Maccoby & Jacklin (1974) stating that adolescence males showed greater performance than females on items measuring spatial visualization skills. However, female students at ages 9 and 13 scored higher than male students on numeration skills. Likewise, according to Fennema & Sherman (1978), variables, such as mathematics as being a male domain, confidence in learning mathematics, attitudes toward success, spatial visualization, mathematics computation, comprehension, application, problem-solving, verbal ability, usefulness, effective motivation, parental involvement and teachers were vital in student achievement about sex differences in mathematics. Among these variables, they identified two significant sex-related effective variables, which were confidence in learning mathematics, and mathematics as a male domain.

However, others are claiming that there is no difference between the sexes in geometry. For example, Armstrong (1981) expressed the view that there was no difference in the achievement of boys and girls at the sixth grade level in the skills of measurement applications, geometry applications, and probability/statistics. This was in line with the claim of Fennema & Sherman (1978) who found that there were no statistically significant sex-related differences in spatial visualization and that there was no significant sex-related difference in motivation between boys and girls in mathematics. These results support the argument of Ryan & Pintrich (1997). According to a study the problem-solving abilities of boys and girls at age, 13, were nearly equal but slightly favoured boys. Moreover, 13-year-old girls began a high school mathematics program with the same skills as boys. However, this phenomenon had changed by the end of high school. This indicates that gender differences in geometry performance are evident in some countries; however, other countries showed no gender difference in geometry performance (Neuschmid, Barth, & Hastedt, 2008). Thus, there are no conclusive findings regarding gender, preferences, and performance. A study indicates that boys‟ mean score is numerically higher than that of the girls. The analysis of covariance (ANCOVA), however, indicates that this difference is not statistically significant in terms of the van Hiele levels in geometry between boys and girls, [$F(1, 149) = 2.446$; $p > .05$]. While it seems that there is again favouring boys based on their levels, it is not statistically significant, hence, no gender differences were found in the study (Erdogan, 2006).

2.9 Difficulties in Learning Geometry among Elementary Learners

Development in reasoning, practices and materials for instruction and the processes in mathematics can be explained as some of the difficulties learners encounter in learning or studying geometry according to Idris (2007). Walker, Winner, Hetland & Goldsmith (2011) stated that learners who can perceive ideas visually have an advantage when it comes to reasoning and making a good judgment in the geometrical analysis.

Many studies have focused on investigating individuals' understanding and difficulties in transformational geometry concept, at all levels of education (Portnoy, Grundmeier, & Graham, 2006). Moyer and Dumais (1978) argued against piglet‟s position that children‟s spontaneous intuitive structures are built in close correspondence with structures that mathematicians have developed. He investigated the compatibility between the mathematical organization of transformations and the cognitive structure of four to eight-year-old children. In a study, children were interviewed while solving nine tasks of translations, reflections, and rotations of marked circles (Xistouri & Pitta-Pantazi, 2013). According to the study, children do not classify geometric transformations as translations, reflections and rotations and therefore, the relative difficulty between the three is meaningless. This is because the difficulty does not lie within the mathematical nature but the cognitive nature of transformation. The study further stated that children perform scanning procedures to compare the figure to its image, and what determines the difficulty is the degree of the discrepancy between the two images (Xistouri & Pitta-Pantazi, 2013).

The largest study concerning students" understanding of transformational geometry concepts as part of an assessment of mathematics learning in British school children directed by (Hart, Brown, Kuchemann, Kerslake, Ruddock & McCartney, 1981). It was carried out by the concepts in secondary mathematics and science group of Chelsea College in London. In the rotations and reflections research, a total of 1026, 13-to 15 – year olds were given a 52 item paper and pencil test. The test consisted of three parts: single reflections, single rotations and combinations of reflections and rotations cited from (Xistouri & Pitta-Pantazi, 2011). For reflections, the basic task was to sketch the result of a reflection over a mirror line, shown in various orientations on either a grid or plain background. In the rotations test were asked to sketch the images of various figures after counterclockwise rotations of a quarter turn. They were also asked to find centres of rotation. In the final section, two types of questions addressed composite transformations. In the first task, children had to find unknown transformations which are followed by a rotation, moved a shape onto its image. In the second task, students were asked to draw the image of a given shape after applying two sequential transformations. Then to draw a mirror line or centre of rotation for the equivalent single transformation cited from (Xistouri & Pitta-Pantazi, 2011). Nearly all students experienced some success perfuming single reflections and rotations; however, most students experienced some success performing single reflections and rotations; however, most students had a difficult time performing combinations of transformations.

Xistouri and Pitta-Pantazi (2011) report that while students‟ understanding of translations and reflections are equally difficult; rotations seem to be more difficult. Many factors have been put forward to explain why the learning of geometry is difficult. Some of these factors are the language of geometry, visualization abilities, and ineffective instruction. Poor reasoning skills are also another area of concern among secondary school students. Many students are unable to extract necessary information from given data and many more are unable to interpret answers and make conclusions. (Asiedu-Addo, Aseman, & Oppong, 2017).

Traditional approaches in learning geometry emphasize more on how much the students can remember and less on how well the students can think and reason. Thus, learning becomes forced and seldom brings satisfaction to the students (Baffoe & Mereku, 2010). Luneta (2008) defines errors as „simple symptoms of the difficulties a student is encountering during a learning experience‟. These difficulties have been rectified in some Western countries and few African nations who have used van Hieles‟ nodel of learning in geometry effectively to improve the performance of students in geometry. Knowledge of students‟ difficulties was essential and teachers should provide opportunities for students to display their difficulties as these would be essential stepping stones for effective instruction.

2.10 Summary of the Literature Review

In studies that used the van Hiele theory to identify students" misconceptions (Atebe, 2008; Alex & Mammen, 2014; Luneta, 2015), it proved to be a useful framework for extracting, measuring, understanding and addressing students" difficulties with school geometry. In the context of this study, the van Hiele theory w0uld play an important role in that it would help diagnose some of the causes of the difficulties displayed by the students in that questions would be set to cater for particular levels of development of students" reasoning in geometry. It was due to this that the researcher hopes to apply the model in his study to be able to identify the inherent difficulties there was in the study of transformation geometry within the context of rotation. This was because educators are constantly concerned with the poor performance of learners in transformation geometry, Ghana has not been an exception. The van Hieles" model would, therefore, be useful in analyzing the performance of Senior High School students" difficulties in transformation geometry especially in the area of rotation due to its hierarchical nature.

Students have been exposed to visualizing objects in nature and also in designs, and have developed their intuitive understandings of rotations as regard to transformation geometry. The principal aim of this study was to find the weaknesses of prospective third-year Senior High School students at transformation geometry questions specifically rotation. The results of this research would therefore be a pointer to some factors that could explain why learners experience difficulties with transformation geometry in NASEC in the Ga-South municipality. Finally, through these difficulties emanating from students, teachers would be able to design an appropriate strategy to drastically reduced if not eliminate the inherent deficiencies.

CHAPTER THREE

METHODOLOGY

3.0 Overview

The study sought to find out the use of van Hieles" model to investigate and describe various difficulties which students may have in the learning of transformation geometry, in particular, the concept of rotation. In pursuance of the purposes stated above, the following research questions were formulated to guide the study:

1. What difficulties do senior high school students" encounter in rotation on the first-four levels of van Hiele?
2. What are the levels reached in the van Hieles" model with respect to rotation?
3. Are there any significant difference between male and female Senior High School students in terms of the difficulties in rotation according to the van Hieles" level?

H_0: There is no significant difference between the male and female Senior High School student in terms of the difficulties in rotation according to the van Hieles" level.

H_1: There is a significant difference between the male and female Senior High School student in rotation according to the van Hieles" level.

This chapter discusses the research methodology adopted for the study. The methodology is expressed in terms of the research design, population, sample and sampling procedure, research instruments and data collection procedure. Issues concerning the research instruments, ensuring the validity and reliability of research instruments, and data analysis techniques are also discussed.

3.1 Research Design

The researcher chose the Explanatory Sequential Design to allow the researcher to explain initial quantitative results and support it with significant qualitative results. (Creswell and Clark, 2007). This allows the researcher to combine both quantitative and a qualitative methods in a single study. Although the main method is qualitative which deals with unearthing students difficulties through interviews, quantitative components were included as evidenced in the paper and pencil test. According to Creswell and Clark (2007) using a mixed-method approach is considered to be appropriate to gain a more comprehensive picture of the phenomena being studied and greater accuracy in the research findings. The important thing to note is that the mixed method research does not look at research from one angle; it tends to investigate the knowledge of both what is happening and how or why things happen (Lu, 2008). In a qualitative study, depth and detail are captured by interviews, observations, and documents. The qualitative method also helped to assist in explaining and assigning reasons for quantitative findings (Fife-Schaw, 2012).

A self-designed Student Mathematics Achievement Test (SMAT) was used for the test. This was done hand in hand with input from my supervisor. An interview section was also administered in line with the van Hieles" levels. The written test was used to show students strength and weakness and classify students according to their level of understanding rotation. This was done about visualization, description, analysis and deduction as outlined in the van Hieles" model. To further maintain validity, triangulation was used to confirm and compare the result from these two data sources. The responses from the written test, as well as the interviews with the recorded versions, were fully analysed to bring out a common pattern of difficulties.

3.2 Population

The target population for the study was all prospective third-year Senior High School (SHS) students" in the Ga South Municipality of the Greater Accra Region. These were students that were being prepared to write the WASSCE so as to get access into the tertiary institutions.

3.3 Sample and Sampling Procedure

The sample size had 240 students comprising 119 females and 121 males. According to krejcie and Morgan (1970), in a population of 650, a researcher would be allowed a sample size of 240. These were all prospective third-year students who had already learnt rotation in transformation geometry which meant that even the students who constituted the sample knew Geometrical Transformation and by extension rotation. A random sampling technique was used to select the students from a population of 650 prospective third year students in Ngleshie Amanfro Senior High School. This ensured that bias was eliminated while giving equal opportunities to each sample point selected. The sample units in the population were selected by a random process, using a random number generator so that each participant in the population had the same probability of been selected for the study.

In addition to this, purposive sampling was also employed in the selection of the school since this technique may be defined as choosing individuals or institutions based on specific purpose in answering research questions (Kekana, 2017). To ensure fairness in the selection of the sample a quota system was also used to select the students representing the various programs in the school. Hence a ratio of 80, 60, 40, 40, and 20 representing students from General Arts, Home Economics, General Science, Business and Visual Arts programs in the school. Students partaking in the interview were selected based on their performance from the group of students that took part in the written test. Only students that performed poorly (i.e. those whose overall performance was 5 marks and below out of 21 marks) in the written test were interviewed. To ensure equal representation of students in terms of sex and ability for the interview, eight (8) students consisting of 4 males and 4 females were randomly selected from the sample. There were a total of 20 students that were preliminarily selected from those prospective third-year students in the school who were pencilled to be interviewed. The male and female students were then assembled and given an overview of the interview. The reason for this was to ensure that every participant (student) that was finally chosen for the interview must be willing and not forced to participate in the interview. Finally, eight volunteers were interviewed.

3.4 Instrument for Data Collection

Considering the nature of research questions been examined, the instruments used for the collection of data were a transformational geometry test developed by Soon (1989) and an Interview guide. These instruments were used together to answer all the research questions.

3.4.1 Test on rotation

Soon (1989) determined van Hiele-like levels for learning in transformation geometry as shown in Table 1. Table 1 also shows a modified version of the researcher to take care of how the rotation questions were framed.

Table 1: Levels of understanding in transformation geometry and how questions were framed by the researcher following the van Hieles' levels.

Levels	Characteristics of student determined by Soon (1989)	Characteristics of students according to the researcher
Level 1	➢ Identifies transformation by the changes in the figure; (a) in simple drawings of figures and images; and (b) in pictures of everyday application ➢ Identifies transformation by performing the actual motion, names discriminates the transformation. ➢ Names or labels transformation using standard and / non-standard names and labels appropriately. ➢ Solve problems by operating on changes in figures or motion rather than using properties of the changes.	➢ Identify transformation in groups by the change in the figure. ➢ Identify transformation by actual motion ➢ Name or label transformation using standard and non-standard names or labels appropriately. E.g. flips, slides and turns.
Level 2	➢ Uses the properties of the changes to draw the pre-image or image of a given transformation. ➢ Discover the properties of changes to figures resulting from specific transformation. ➢ Use appropriate vocabulary for the properties and transformation. ➢ Can locate the axis of reflection, a centre of rotation, translation vector and centre of enlargement. ➢ Relate transformations using coordinates. ➢ Solve problems using known properties of transformations.	➢ Analyze any given transformation using appropriate vocabulary as it relates to transformation geometry. ➢ Discover properties and new images after transformation. ➢ Locate angle and centre of rotation.
Level 3	➢ Perform composition of simple transformations. ➢ Describe changes to states (pre-image, image) after composite transformations. ➢ Represents transformation using coordinates and matrices. ➢ Given initial and final states, can name a simple transformation. ➢ Given initial and final states, can decompose and recombine a transformation as a composition of simple transformations.	➢ Rotate any given figure through a given degree. ➢ Interrelate the properties of the figure and b its image. ➢ Perform composition of simple transformation involving rotation.
Level 4	➢ Gives geometric proofs using a transformational approach. ➢ Gives proof using the coordinates and matrices. ➢ Think through multistep problems and gives reasons for problems.	➢ Give geometric proof using a rotational approach.
Level 5	➢ Understands-associative, commutative, inverse, identity concerning a composite transformation operation ➢ Identifies groups of transformations. ➢ Proves or disproves subset of transformation from group structure.	

Soon (1989), after diagnosing the level and applying a Guttman Scologram analysis, indicated that they have a hierarchical structure. As can be seen from Table 1, level 5 was seen as a way beyond the performance levels of Senior High School students as such only level 1 to level 4 would be relevant and was considered in the study.

3.4.1.1 Student Mathematics Achievement Test (SMAT)

This was done in the form of the worksheet which was made up of questions related to diagrams and those without reference to any diagram. This enabled the students to communicate their mathematical ideas involving rotation in writing and also helped them to provide further information regarding their thought processes with regards to rotation. In developing the test, the van Hieles" levels and its characterization were crucial and formed a focal point on which the test was developed.

The content of the test was developed in such a way that each question was tied into each van Hieles" level. To ensure this, a template of a matrix of level by concept was adopted from (Soon, 1989). The original version of the test includes four questions at the first level, ten questions at the second level, nine questions at the third level and eight questions at the fourth level (Guven, 2012). However in this study, five questions were based on van Hiele level 1, seven questions on van Hiele level 2, four questions on van Hiele level 3 and five questions on van Hiele level 4 (see Table 2). This matrix was useful in that it ensured that all the levels of learning of van Hiele which were visualization, analysis, abstraction, deduction and their characterizations were adequately represented in the test questions. Some questions were developed by the researcher and some taken from past WAEC core mathematics questions whilst others were sourced from the transformational geometry test developed by (Soon, 1989). The selections of the questions were proportionate in order of the levels corresponding to that of van Hieles‟ model. It was based on their ability to solicit students‟ difficulties with regards to aspects of identifying, rotating, twisting, inventing an image, showing similarities and differences between two transformed objects in rotation.

Table 2: Matrix of level by the concept that was used for developing test items.

Van Hieles' Levels	**Concept (Rotation)**
Levels	Number of questions
1	5 questions
2	7 questions
3	4 questions
4	5 questions
Total	**21 questions**

The choice of questions in each category was to generate additional information on students thinking processes in which one to two questions might not be adequate. It was also to allow students to provide detailed or alternative responses to questions they might not have provided sufficient information on. The contents of the test were developed to correspond to each anticipated difficulties which were associated with visualization, analysis, abstraction and deduction as outlined in the van Hieles‟ model of learning geometry (see Table 1).

Students were classified as having difficulties at a level if they fail to answer questions as prescribed by the performance indicator from Table 3.

Table 3: Performance Indicators that Shows Students' Difficulty at each Level of van Hieles' Model

Levels of achievement	Performance indicator
Basic Level 1 (Visualization)	• Through a simple picture, students can identify transformation (rotation) by changes in the figure. • Students can name or label transformation using standard or non- standard name. e.g. flip, turn and slides
Level 2 (Analysis/ Description)	• Use the properties of change to draw the pre-image or image of the given transformation. • Discover property of change to a figure due to rotated figure. • Able to locate the centre, angle and direction of rotation. • Relate the rotated image by using coordinates.
Level 3 (Abstraction)	• Perform composition of the simple transformation of rotation. • Interrelate the properties of change to figure due to rotation. • Given the initial and final state, can name the single transformation.
Level 4 (Deduction)	• Perform rotational geometry proofs using a transformational approach. • Think through and give reasons in a multi-step problem.

3.4.2 Face-to-face interview

The second method was the interview section that addressed the same content as the test, but which aimed to elicit qualitative responses that could shed light on the test results. In these interviews, students were asked questions similar to those in the questionnaire, but this time they were required, individually to explain, for instance, why certain answers were provided in the written test. The expectation was that having completed the form two mathematics topics, they would be able to talk about their understanding and thus operate at level 3 of Van Hiele"s theory, which requires them to "constantly justifies their reasoning" (Lim, 2006). The researcher divided the participants into four groups of two students. The groups were randomly sampled from the cohorts of students whose performance was not the best. The interviews were conducted over four days and lasted about 25 minutes on average. This was to enable the researcher not to unnecessarily delay the students since they were all day students. The interview was an alternate method of collecting survey data which asked direct questions and recorded respondents‟ answers (Maduekwe & Esiobu, 2011).

An interview guide based on a structured model was designed and used for a sample of 8 students out of the 240 students for the research. During the interviews, learners within the sample were asked to explain their solutions to each question as they revisited their tasks by being provided with a plain sheet of paper, pencil and a pen. The paper, pencil and a pen were to allow them to express their opinion through writing if they so wish. They were prompted where necessary to clarify their thinking as well as to ascertain the strategies they were using. The interviews were audio-taped and then transcribed by the researcher. The analysis of the interviews was carried out in conjunction with the learners‟ written responses which formed the basis of the interview questions. The analysis was based on the difficulties students had in terms of visualizing, analyzing, describing, and using deduction in solving problems as it relates to the concept of rotation in transformation geometry. The data generated during the interview session gave credence to the data coming from the written test. The interview was done immediately after school and this was to allow for good observation not only of verbal but also non-verbal data to be seen and taken note of (Sarah & Alan, 2009).

3.5 Validity

Validity refers to the extent to which the research instruments are effectively authentic or truthful. It is a demonstration that a particular research instrument measures what it purports to measure (Mushquash & Bova, 2007; Williams, 2014). "Validity is the extent to which all the evidence that has been gathered supports the intended interpretation of test scores for the proposed purpose" (AERA, 2008). Validity may also be defined as the appropriateness, meaningfulness, correctness and usefulness of any deductions that are obtained through the use of an instrument validity measures that were taken in this study were based on these conceptions and notions of validity (Kekana, 2017).

3.5.1 Validity of the student Mathematics achievement test

The study used a content analysis technique in which each question was analysed according to the content it contained (students difficulties). Kerlinger (1986) emphasizes that content analysis is a method for studying and analysing communication in a systematic, objective and quantitative manner to determine the levels of variables that have been achieved. A student"s answer, in this case, becomes an indication of his ability to communicate freely with transformation geometry (rotation) examination questions. The variable to be measured was their responses (associated difficulties) against the correct answers. The analysis made inferences to the communication (student's answers) by systematically and objectively identifying specific characteristics of the student's difficulties in the answers. To be able to carry out this type of validity test on the instrument it was ensured that the following were considered for the study:

- Some tasks in the worksheet (test) were chosen on the basis that they have been previously used by Soon (1989) to describe students‟ levels of achievements using the van Hiele levels of learning in transformation geometry such as questions 1.2 and that of 4.
- Geometry curriculum, as well as the textbooks of the students of SHS, was looked into to give an insight into what learners are expected to learn.
- The adequacy of the final content of the test instrument was based on the collective opinion of colleagues‟ mathematics tutors on the field and my supervisor based on their professional assurance (Sangoseni, Hellman & Hill, 2013).
- Some questions were taken from past WAEC core mathematics papers such as question 2.2 was adopted from core mathematics paper (2009).

Finally, the outcome of the results from the test was discussed with the respondents to allow them to know that whatever scores that have been used in the research are the true reflection of what was obtained from them. This was referred to as "member checking" by (Lincoln & Guba, 1985).

3.6 Pilot Study

Polit, Beck and Hungler (2001), referred to a pilot study as so-called feasibility studies which are "small scale versions, or trial runs, done in preparation for the major study. Notwithstanding this, a pilot study can also be a pre-testing or „trying out‟ of a particular research instrument (Baker, 1994). One of the advantages of conducting a pilot study is that it might give a warning about where the main research project could fail, where research protocols might not be followed, or whether proposed methods or instruments are inappropriate or too complicated. A pilot survey was first carried out to a similar group of prospective ihird year students in a different Senior High School. The pilot study audience was similar in gender groupings of males and females (7 males, 7 females). The pilot study was a representative of a group similar to the actual study group. These students were selected with input from their mathematics tutors in the school using the criteria that (a) they would be responsive and (b) they would represent a good cross-section of students from the school. In this regard, a letter was written to the head of the school to seek his or her consent. To maintain confidentiality, a numbering system was used to correlate each name with a numbered survey.

The pilot study aimed to give the researcher an insight into whether the intended questions to be given to students would yield the desired data that would be needed to answer the research questions. To determine to what extent they understood the question and to decide whether some of the contents of the questions should be reconstructed or not, the researcher assessed the time it took students to complete the task and other difficulties such as language, meaning, and choice they have to make (Thomas, 2003). After the pilot study for the written test, the researcher administered an interview to these same groups of students the next day using the same interview protocol that was meant for the actual study school. Responses from the written test and interview went through a similar analysis to determine students‟ difficulties and also to check and ascertain the reliability of the instrument.

3.7 Reliability

It refers to the extent to which a measuring instrument, a questionnaire, a test yields the same results on repeated applications (Armah, 2015). Creswell and Clark (2007) also explained that the reliability of an instrument was seen as the degree to which the instrument measured accurately and consistently what it was intended to measure

3.7.1 Reliability of the test

According to William (2006), reliability is the consistency or dependability of the measurement; or the extent to which an instrument measures the same way each time it is used under the same condition with the same subjects. Because of this, the test items were carefully selected after much deliberation between me, colleagues and my supervisor. The recommendation of my colleague"s mathematics tutors and my supervisor"s assessment of the test and interview instrument were key in the final determination of what constituted a very reliable content for the study. The language that was used was sufficiently basic for most of the respondents to understand.

The researcher calculated Kuder-Richardson (KR-20) reliability coefficient after the pilot test with Microsoft excel version 2013 using the formula

$r_{KR20} = \left(\frac{k}{k-1}\right)\left(1 - \left(\frac{\sum pq}{\sigma^2}\right)\right)$ where,

r_{KR20} --- Reliability coefficient for the test of internal consistency (Kuder-Richardson formula 20)

k --------- Number of test items

p --------- Proportion of the test takers who pass an item

q --------- Proportion of the test takers who fail an item (i.e.$1 - p$)

σ^2 ------- Variation of the entire test

The outcome of the test recorded $r_{KR20} = 0.81$ as the coefficient for the test of internal consistency. The result simply indicates how reliable the test items hold together. This was consistent with the findings of Kökçü and Demirel (2020) as they indicated that a KR-20 reliability coefficient that is equal or greater than 0.70 is considered as adequate for the reliability of the test scores.

To ensure reliability in the interview data collected, interview questions were structured in line with the test questions. The content of the interview was a follow up on questions asked in the written test. An interview protocol was used to ensure that the same questions were given to each interviewee (see Table 4).

Table 4: An interview protocol followed to unearth the students' thought processes based on the various levels

Levels	
Basic level 1 (Visualization); Visually identify an image of initial and final state after rotation.	• Which among these transformations represent rotation and why do you say so? • Can you demonstrate a figure and its image after a rotation using objects given to you? • Which of the images is a true representation of the rotation of $-90°$ and why?
Level 2 (Analysis/ Deduction); Identify geometric figures after transformation.	• Students will be made to find the centre and angle of rotation. • Explain how the image came about. • Explain how a particular image is obtained etc.
Level 3 (Abstraction); use rotation to transform an object when given the coordinates, angles and shape.	• Students will be asked to describe the transformation. • Students will be made to describe how the rotation will be achieved. • Students will be made to explain how his or her solution was achieved.
Level 4 (Deduction); Use transformation (rotation) to do proofs.	• Students will be made to show how a triangle is congruent to its image. • Students will be made to use transformation to explain how an object is rotated to its image.

The same audio-tape was used for all participants to ensure fairness in the quality of the instruments used. All the interviews took place at the end of the lessons in the school library to avoid noise and disturbance of any kind. The participants were free in answering the questions as the interview was one-on-one without other people in the library.

Furthermore, confidentiality was ensured and the researcher recorded the interview for confirmability, describe the data using student identity for trustworthiness, followed the van Hiele criteria to ensure replication, and interpreted the interview data based on his personal experience as a teacher for over 10 years. The researcher also admits that any bias in interpretation was as a result of his perspective and limitation (Cope, 2014; Zohrabi, 2013).

3.8 Data Collection

After the pilot test, the researcher appended all the necessary corrections in the instrument. The student Mathematics Achievement Test (SMAT) was then administered to the 240 sampled students under the supervision of the researcher and other mathematics teachers present. The researcher created a reliable and uninterrupted assessment environment for participants as they respond to the items. This was to observe individual differences and participation among the participants as well as elicit factors such as fear and panic that might reduce their interest and passion which may skew their responses and affect their true individual academic reflection. After the achievement test, the researcher interviewed 8 students.

3.9 Data Analysis

Looking at data in the general sense while it does not proceed in a straight forward fashion it is the activity of making sense of, interpreting and theorizing data that signifies a search for general statements among categories of data (Schwandt, 2007). From this notion, it was inferred that data analysis requires some sort or form of the logic applied to research. In this regard, Best and Khan (2006) posit that the analysis and interpretation of data represent the application of deductive and inductive logic to the research.

3.9.1 Quantitative data analysis

Data were analysed descriptively using means, standard deviations and percentages. An independence sample t-test was also used to test for the significant gender difference in achievement along with the van Hieles‟ geometric thinking levels. Frequency and percentage were calculated from students responses on the SMAT items and used to answer research question one and two. These test statistics were used because the research questions were descriptive. Also, an independence sample t-test was used to compare the means of difficulties in transformation geometry using the van Hieles‟ level between two unrelated groups (i.e. male and female). Independent samples t-test was used to compare the means of mathematics achievement scores between the male and female students in the various level of van Hiele. An independent sample t-test was used to test the first null hypothesis. The purpose was to determine whether there were statistically significantly difference between male and female students‟ score in the various van Hieles‟ levels.

Test of the Assumptions of the t-tests

The *t*-tests are parametric tests and therefore some assumptions need to be met before they are used to analyze any quantitative data. The data that were collected in this study warranted the use of independent sample *t*-test. The reason is that the scores from the achievement test were treated as continuous data segregated by gender. Another assumption that was met before the t-test was used, was homogeneity of variance as shown in Table 5.

Table 5: Homogeneity of variance test for running Independent sample t-test

		Levene's Test for Equality of Variances	
		F	Sig.
Visualisation	Equal variances assumed	.001	.973
	Equal variances not assumed		
Analysis	Equal variances assumed	.194	.660
	Equal variances not assumed		
Abstraction	Equal variances assumed	.772	.380
	Equal variances not assumed		
Deduction	Equal variances assumed	3.860	.051
	Equal variances not assumed		

The results in Table 5 reveal that the variances are equal since the p-values recorded are all greater than the alpha value of .05. This suggests that the homogeneity of variance assumption for running independent sample *t*-test was not violated.

3.9.2 Qualitative data analysis

The audiotape interviews that were obtained during the interview was analysed in terms of the difficulties experienced by individual students in a particular question or cluster of questions. The responses were transcribed and presented in descriptive words. The interview questions and its analysis were focused on the extent to which students can visualize, describe, analyse, abstract relation, and deduction. A student was considered having difficulties in a particular level if he/she failed to meet the performance indicator as prescribed already. The multiple data that arose from the written test, interview regarding students‟ verbatim quotes and notes from paper and pencil were all analysed.

3.10 Ethical Considerations

Ethics are generally considered to deal with beliefs about what is right or wrong, proper or Improper, good or bad (McMillan & Schumacher, 2001). It is the responsibility of the researcher to ensure that ethical standards are adhered to. In terms of ethical considerations, the researcher ensured that he has received informed consent from every respondent before the interviews or the written test and informed the respondents of the goals of the study and what I hoped to achieve. Also, the researcher was open and honest with the respondents and that the respondents were not misled during the study. Information obtained from the respondents remained. Due to this, codes were generated for all students starting from S1M, S2F, S3M…, etc. M, F and S were used to represent male, female and student respectively whereas 1, 2, 3 etc. represented the students in their ordered form. No physical or mental discomfort to the respondents was experienced in the study. Moreover, the data collection schedule was at their convenience in other to ensure that their normal classroom lessons are not disturbed.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.0 Overview

The study seeks to use both quantitative and qualitative approach to investigate and identify students‟ difficulties in transformation geometry, in particular, the concept of rotation by applying the van Hieles‟ model. The researcher, therefore, used the data obtained from both the written test and the interview for the analysis and interpretation. Three research questions guided the study. These questions are:

1. What difficulties do Senior High School students‟ encounter in rotation on the first four levels of van Hiele?
2. What are the levels reached in the van Hieles‟ model with respect to rotation?
3. Are there any significant difference between male and female Senior High School students in terms of the difficulties in rotation according to the van Hieles‟ level?

4.1 Research Question 1: What difficulties do senior high school students' encounter in rotation on the first four levels of van Hiele?

To investigate SHS students‟ difficulties in rotation concerning the levels of van Hiele, participants were made to answer essay type questions on Visualization, Analysis, Abstraction and Deduction. To answer the research question 1, the researcher examined the students‟ difficulties from the way they presented their answers to the questions step-by-step. The difficulties identified were the ability to:

i. identify and name a figure that has gone through rotation by its motion using a standard or non-standard name (Visualization)
ii. discover the properties of a figure and its images after rotation and use these properties to analyze a rotation (Analysis)

iii. use rotation to transform an object when given the coordinates, angles and shape (Abstraction)

iv. use the transformation of rotation to do proofs (Deduction).

The next four sections present the descriptive statistics on the difficulties students faced in rotation with exhibits of some students‟ worked samples.

Table 6 shows some types of difficulties and performance on the items in the achievement test measuring (Visualization).

Table 6: Distribution of some types of difficulties students faced in solving rotation under visualization stage (Level 1)

Factor (Difficulty)	Sample size	Not Attempted		Not Correct Answer		Correct Answer	
		N	%	N	%	N	%
identifying transformation by change in the figure	240	2	0.8	51	21.3	187	77.9
identifying the image of X after a transformation of -90°	240	2	0.8	78	32.5	160	66.7
identifying the quadrant in which triangle FGH is located	240	0	0	49	20.4	191	79.6
identifying the polygon that are congruent to polygon 1	240	0	0	28	11.7	212	88.3
naming a transformation using standard or non-standard name	240	0	0	54	22.5	186	77.5
Average	240	1	0.4	52	21.7	187	77.9

Source: Fieldwork, 2018.

Table 6 indicates that out of the 240 respondents who took part in the study, 2 representing 0.8% of the students did not attempt item under visualization stage which required respondents‟ ability to identify transformation by a change in the figure. On this item, 51 students representing 21.3% were not able to recognize the transformation by a change in the figure. Similarly, students were required to identify the image of X after a transformation of -90° and the quadrant in which triangle FGH is located, 78 (32.5%) of the students were not able to identify the image of X after a transformation and likewise that of and 49 (20.4%) were not able to identify the quadrant in which triangle FGH is located. Also, concerning students‟ ability to name a transformation using standard or non-standard name, only 77.5% of the students had correctly provided an answer to the question. Averagely, these findings indicated that 52 students representing 21.7% had difficulties associated with visualization. This suggests that quite a few numbers of the students had difficulties with the visualization stage under van Hiele levels.

To substantiate students‟ difficulties, students were interviewed with the view of diagnosing their difficulties. These students were asked various questions as contained in the interview protocol in Table 4. Questions asked at the Basic level focused on students‟ ability to visually identify and name a rotation by its motion. Example of questions asked included:

1. Which among these transformations represents rotation and why do you say so?
2. Can you demonstrate a figure and its image after a rotation using objects given to you?
3. Which of the image in question 1.2 (see Appendix B) is a true representation of A after a rotation of -90°and why?

Details of all questions used could be found in Table 4.

The above questions are tied to question one in the test, and also the research question one. Its purpose was to solicit a further response from students with the view of identifying their difficulties. These questions were presented to students by using a physically manipulated triangular cardboard to represent a figure and its image after rotation, translation and reflection as contained in the written test question 1.1 (see Appendix B). Each student was requested to identify the transformed triangular figure by naming the transformation. Only two of the eight students interviewed were able to identify and name the image of the figure and therefore satisfy for the attainment of the visualization level. All other students failed to answer correctly question 1.1 and 1.2. Further probing was done to find out how each of these students perceived the rotated figure and its image. Students were given a triangular card to manipulate geometric figure of various size and asked to use them to describe from their understanding what is rotation and how does one know that a figure and its image are rotated. Six of these students had difficulties in doing this. They gave an example of a translation and reflection instead of rotation. It did appear that they had mistaken the concept of translation and reflection for rotation. They did not really know the differences between rotated figures and translated and reflected figure respectively. I asked more probing questions to find out why this difficulty has occurred by asking them to describe what happens when a figure is rotated. A direct quote from one student with name S51F (see Appendix C) is presented below.

Interviewer: Each of the diagrams represents a different transformation of the triangle PQR. Which among the transformation represent a rotation? The student pointed at the reflected figure C amongst all the transformations presented on the table.

Interviewer: Why do you select diagram C?

Student S51F: *It moves from this place to this place. (This students' demonstration indicated that he was referring to one point of the plane to another on diagram C).*

Interviewer: So are you saying that any figure that moves from place A to place B is rotation?

Student S51F: *Yes*

Interviewer: Pointing at one of the transformed images, why don‟t you choose this image instead of choosing the one you did?

Student S51F: *Because this image looks the same as this one and I use the rule (x, y) → (x, -y) (pointing at the reflected one).*

Student S94M: *I selected 'A' because it was opposite and similar to the object (also pointing at a translated figure).*

Interviewer: In question 1.2 why did you choose C as a true representation of „A‟ after rotation of -90^{o}?

Student S130M: *I chose C because it is opposite to A.*

Student S39M: *The position is C*

Student S 51F: *B*

Interviewer: Why do you say so?

Student S51M: *Because when A is rotated, it comes to B*

Interviewer: Then through how many degrees will that be?

Student S51M: *Straight line*

Interviewer: What do you understand by rotation through negative 90^{0}?

Student S115F: *Sir, this one I don't know.*

Interviewer: Can you describe for me what happens to an image when it is rotated?

Student S110F: *Sir, rotation the way I understand it is the object going round to become the image.*

Interviewer: Once again look at the various shapes of polygons labelled 1, 2, 3 and 4. Which of them is congruent to polygon 1?

Student S67F: *This one* **(***pointing at polygon 4) on display.*

Interviewer: what do you understand by the word congruent?

Student S67F: *Congruent means an object which is the same shape but small.*

The above responses from students provided evidence that some students at this stage of learning still had problems with visualization. The van Hiele"s basic level of learning demands a lot of visualization on the part of students. This set of students interviewed failed to identify and name transformation. They had difficulties in differentiating between rotation, reflection and translation hence they perceived a reflected or translated figure also as a rotation. Another important discovery was made in question 1.2. Students were requested to mentally identify the position of figure A after a rotation of -90°. It was discovered that some of these students had difficulties in finding the image of A after a rotation. Example of student response is given in Appendix C. The origin of this problem may be that students are not competent enough with the task which involves angle and its measurements as they find it difficult to measure angles when requested to do so. The amount of a rotated figure is measured in angles, and students" ability to visually know how many degrees a figure and its image was rotated is also a prerequisite at the van Hiele"s Basic level from the responses of students above. When it came to responses of questions 1.3, 1.4.1, and 1.4.2 students were clearly out of touch with what was expected of them. Some of the responses such that of S100F and S110F are found in Appendix C Figure III. The above students" responses demonstrated that students had difficulties in identifying an image after a rotation through a given degree.

Furthermore, students were requested to respond to items in the achievement test to test students‟ ability to discover the properties of a figure and its images after rotation and use these properties to analyze a transformation of rotation (Analysis). Table 7 shows the difficulties they encountered at the analysis stage.

Table 7: Distribution of some types of difficulties students faced in solving rotation under analysis stage (Level 2)

Factor (Difficulty)	Sample size	Not Attempted		Not Correct Answer		Correct Answer	
		N	%	N	%	N	%
Locating the centre of rotation	240	8	3.3	61	25.4	171	71.3
Locating the angle of rotation	240	6	2.5	65	27.1	169	70.4
discovering the type of transformation between an object and its image	240	4	1.7	79	32.9	157	65.4
determining the property of a change in a figure due to a rotated figure	240	2	0.8	97	40.4	141	58.8
relating a rotated image by using coordinates	240	3	1.3	59	24.6	178	74.2
assigning reasons to question 2.2.1	240	6	2.5	88	36.7	146	60.8
determining an equal distance of a point and its image from the origin after rotation	240	17	7.1	74	30.8	149	62.1
determining the property of a change in a figure due to a rotated figure	240	9	3.8	57	23.8	174	72.5
Average	240	7	2.9	72	30.0	161	67.1

Source: Fieldwork, 2018.

Table 7 shows that out of the 240 respondents who took part in the study, 8 (3.3%) and 6 (2.5%) of the students did not attempt items under analysis stage which required respondents‟ ability to locate the centre of rotation and locate the angle of rotation respectively. On these items, 61 (25.4%) and 65 (27.1%) were not able to locate the centre of rotation and locate the angle of rotation respectively. Likewise, students were required to discover the property of a change in a figure due to a rotated figure and to relate a rotated image by using coordinates, 97 (40.4%) and 59 (24.6%) of the students were not able to solve the questions correctly. Also, with respect to students" ability to discover equal distance of a point and its image from the origin after rotation as well as discover the property of a change in a figure due to a rotated figure, 74 (30.8%) and 57 (23.8%) had difficulties in solving for the questions correctly. On average, these findings indicated that 74 students representing 30.9% had difficulties associated with the analysis stage. These implied that more students had difficulties with the analysis stage under van Hiele levels as compared to the visualization stage.

Consequently, to validate the difficulties students had under analysis stage, follow up questions to question 2 from the written test revealed that most students appeared to have difficulties in finding the centre and angle of rotation. They were also unable to integrate these properties when requested to analyze a rotation. This was how student S130M analyzed a rotated figure and its image during the interview. Questions used during this section of the interview are available in Appendix A.

Interviewer: Referring to question 2 in the written test. Explain how you find the centre and angle of rotation in question 2.1.1

Student S130M: *The figure is rotated through an anticlockwise direction.*

Student S130M was further probed to provide a clearer details, He was asked to describe this clockwise direction. He only used movement and direction in his description, but when it came to explaining the centre and angle of rotation, he retorted by saying "I don"t know". This was common among all responses from students. During the interview of all the eight students namely S51F, S39M, S130M, S31M, S115F, S100F, S67F, S85M, S94M and S11OF, S85M, S94M and S110F were able to verbally tell the direction of movement of the rotated figure, by locating the centre and angle of rotation from the figure and its rotated image whilst the rest could not. These students gave no explanations for their decision when requested to do so. Some responses of S31M, S100F and S130M are captured in Appendix C solutions to Figure IV.

In question 2.2.1 students were required to locate the exact image and coordinates of a point after a rotation. The eight students interviewed had difficulties in locating the exact images of a figure after rotation.

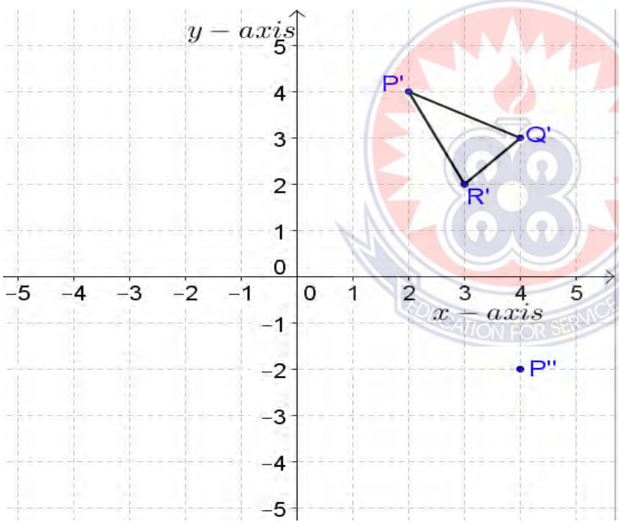

Figure 1: Rotation about the origin

In Figure 1, students were requested to find the coordinate of P““‟after triangle P‟‟, Q‟‟, R‟‟ has undergone a clockwise rotation of 90°. They were requested to find the coordinates of P““‟and explain how they got it. Students S31M and S115F were able to complete this task but could not explain themselves well with the appropriate words. Students S51F and S85M responses are given in Appendix C, Figure V. One noticeable and interesting thing during the interview is that some students seemed to have difficulties in providing the correct answer at one stage but do very well at another stage. The above responses from the written test also corroborated the students‟ response during the interview. It also demonstrated in question 2.2 once again that students have difficulties in visualizing images of objects. Students are unable to find the position of a new image after rotation.

The above question was followed by question 2.2.4 in the test. This question tested students‟ ability in using correct word and properties of rotated figure and its image in analyzing a rotation. Again this was how student S115F responded to the interview question.

Interviewer: Can you explain your choice of option (e) which says each angle of the figure is congruent to the corresponding angle of the image in the question asked in question 2.2.4

Student S115F: *From my understanding of congruent I think when a figure is rotated; it should be the same as its image figure.*

Interviewer: What do you mean when you say the same?

Student S115F: *They are equal*

The above conversation also allowed the interviewer to reinforce the same concept by using some manipulated object in his explanation. This gave the students two different ways of understanding the question at hand. From the above interview conversation, it can be concluded that these students have difficulties in using the correct words and properties of a rotated figure when analyzing a rotation. This difficulty was similar to all students interviewed. It could also be conclude that these students have difficulties with the meaning of the term similar and congruent figures and use the word congruent to mean a similar figure. From the van Hieles‟ concept, at level 2 prescribed that students should be able to note the difference between two figures with the view of pointing out their similarity and differences concerning rotation. However, this characteristic was lacking from the students interviewed.

A response to this question was also captured under Appendix C, Figure V.

Also, students were requested to respond to items in the achievement test to test students‟ ability to use rotation to transform an object when given the coordinates, angles and shape (Abstraction). Table 8 shows the difficulties they encountered at the abstraction stage.

Table 8: Distribution of some types of difficulties students faced in solving rotation under abstraction stage (Level 3)

Factor (Difficulty)	Sample size	Not Attempted		Not Correct Answer		Correct Answer	
		N	%	N	%	N	%
Performing a simple transformation of rotation	240	6	2.5	104	43.3	130	54.2
Stating the required angle when a figure returns to its original position after rotation	240	9	3.8	127	52.9	104	43.3
Describing a transformation that maps an object to its image	240	25	10.4	99	41.3	116	48.3
Appreciating the congruency of a figure	240	14	5.8	104	43.3	122	50.8
Average	240	13	5.4	109	45.4	118	49.2

Source: Fieldwork, 2018.

Indications from Table 8 shows that 104 (43.3%) and 127 (52.9%) of the students had difficulties solving questions under abstraction stage correctly which required respondents‟ ability to perform a simple transformation of rotation and ability to state the required angle when a figure returns to its original position after rotation respectively. In the same view, students were required to describe a transformation that maps an object to its image and appreciate the congruency of a figure, 99 (41.3%) and 104 (43.3%) of the students did not solve the questions correctly showing that they had difficulties with geometrical concepts under transformation on abstraction stage. On average, the results from Table 8 established that 109 students representing 45.2% had difficulties associated with abstraction stage. These infer that 109 students had difficulties solving questions concerning abstraction stage under geometrical transformation analysis stage under van Hiele levels.

However, to further determine students" difficulties as they apply the concept of rotation to transform a figure under abstraction stage of van Hiele levels, concerning question 3.1.1 and 3.1.2, students were interviewed to;

- rotate a manipulated cut out shape on a table through 90^{o} anticlockwise.
- explain or demonstrate through how many angles in degrees clockwise or anticlockwise one can rotate a figure so that it can fit exactly into the original figure.

Only two students S110F and S85F were able to get these questions correctly. They both knew that when an object turned 360° it then returned to its original position. However, other students such as S39M, S51F, S67F, S100F, S31F and S94M gave varied responses which were completely wrong. Students" responses in the written test are available in Appendix C Figure VI. Their explanations also show that they do not know the implication when a figure is rotated several times. To probe further on these difficulties during the interview, a manipulated rectangular shapes cut to size were provided and each of them was requested to use this shape to explain and demonstrate what they were thinking or how they did arrive at the answer. This was how S31F, S94M and S100F responded to the interview.

Interviewer: Using the shape on the table, demonstrate why you say that your answer to the question is 270°?

Student S31F: *Because it moves in this direction and when you count them, it will amount to 270°*

Interviewer: Can you show me how you counted it? (The student moved the rectangular figure round and round and failed to stop when it made a complete rotation).

Student S94M: *I have to rotate the figure three times or more.*

Interviewer: Are three angles meaning 360° that you wrote here in your written test?

Student S94M: *Yes*

Interviewer: Now rotate the figure twice and show me where it will be

Student S94M: *It will meet here.*

From his demonstration, this student rotated the figure round before it could fit exactly onto the original.

Interviewer: What is the measurement of movements in degrees?

Student S100F: *I do not know.*

Interviewer: Let me put it this way. What term is given to a body that goes round another body once?

Student S100M: *Revolution*

Student S100M could only manage to give a relevant response after being guided to do so. Judging from my interaction with the students above, it became obvious that these set of students were having difficulties with angle measurement that exceed 90°. They also have a problem of manipulating object mentally. The different movement and answers given by students were clear evidence that they had a different perception of what the question required of them and their inability to visualize, resulted to their difficulties in carrying out task relating to abstract relation which could have been aided by visualization. These students also had some language difficulties. Language associated with geometry and transformation geometry is crucial for children to acquire a more complete understanding of geometry concepts (Pickreign & Capps, 2000).

From the written test and interviews it was revealed that students show little or no understanding of geometry terms used. For example, some students do not know the meaning of mapped unto, congruency and horizontal. When students used such words, they did not depict what the students were explaining. Some students also used their term such as fit and moving around. This deficiency resulted in students‟ inability to apply geometric terminology when describing a rotated figure and its image. In question 3.2.1 students were requested to describe a transformation and how they arrived at their answer. Students‟ response to these questions revealing their difficulties could also be seen in Appendix C, Figure VI.

Six students interviewed were not able to describe the transformation that mapped quadrilateral ABCD onto A‟B‟‟C‟‟D‟ in question 3.2.1 as a rotation.

Interviewer: Can you use the terms centre and angle of rotation to describe the image and its figure in question 3.1?

Student S115F: *As for rotation, anyway is very difficult for me.*

Interviewer: I want you to use the centre and angle of rotation in your description

Student S115F: *I don't know how to use it.*

This was how another student answered the same question during the interview

Student S130M: *Clockwise movement and it is a rotation.*

Since these groups of students had problems with finding centre and angle of rotation in the previous questions and level 1 earlier, this also contributed to their inability to also use these specifics to describe any given rotated figure. From Appendix C, Figure VIII, it illustrates some responses of S51F and S39M from the written test.

Finally, students were requested to respond to items in the achievement test to test students" ability to use the transformation of rotation to do proofs (Deduction). Table 9 shows the difficulties they encountered at the deduction stage.

Table 9: Distribution of some types of difficulties students faced in solving rotation under deduction stage (Level 4)

Factor (Difficulty)	Sample size	Not Attempted		Not Correct Answer		Correct Answer	
		N	%	N	%	N	%
Stating the type of triangle after a rotation of +70 degrees	240	10	4.2	135	56.3	95	39.6
Giving reasoning to question 4.1.1	240	17	7.1	157	65.4	66	27.5
Proving that triangle XYZ is congruent to triangle XY'Z'	240	31	12.9	166	69.2	43	17.9
Stating whether a figure and its image are congruent	240	20	8.3	112	46.7	108	45.0
Using diagram to the answer in question 4.2.1	240	32	13.3	156	65	52	21.7
Average	240	22	9.2	145	60.4	73	30.4

Source: Fieldwork, 2018.

Results from Table 9 indicate that 135 (56.3%) and 157 (65.4%) of the students had difficulties solving questions under deduction which required respondents" ability to state the type of triangle after a rotation of +70 degrees and ability to state reasons for indicating that of the triangle after a rotation of + 70 degrees respectively. Students were also required to prove that triangle XYZ is congruent to triangle X"Y'Z', 166 (69.2%) and 112 (46.7%) of the students did not solve the questions correctly showing that their understanding geometrical concepts under transformation on deduction stage. On average, the results from Table 9 indicated that 145 students representing 60.5% had difficulties associated with deduction stage. These infer that 145 students had difficulties with deduction stage of van Hiele level under geometrical transformation.

Finally, a further probe to find out the extent of these difficulties resulted in me eliciting more information from these students. A follow-up interview question to question 4 in the written test reveals that students do not understand what proofs entail. Generally, when a figure is rotated, students could not tell why they think a figure and its image is a rotation. None of the students interviewed was able to demonstrate that a rotated figure and its image were congruent. They also failed to use some concept like the size of the angle, the preservation of shape, size of shape and length in their arguments to show that both triangles in question 4 were congruent. Below are responses from some of the students interviewed.

Interviewer: I want you to use rotation to prove that triangle XYZ is congruent to X Y"Z'.

Student S115F: *By looking at the point X, it means that by rotation, triangle XYZ is congruent to X Y' Z' and also the two triangles are the same.*

Interviewer: Does it mean to you that any two figures without having a meeting point and are the same is not congruent?

Student S115F: *Yes.*

Interviewer: Good, but tell me why you think these figures are the same

Student S115F: *I look at the way they look like.*

Student S31M also had no idea when asking initially. But after some few minutes when I tried to reframe the question, she started drawing and making some sketches. In conclusion, she could not also tell why both figures were congruent. Students S39M, S115F and S130M response to these questions can be found in Appendix A, figure VIII.

Interviewer: Why you think that this figure is congruent to this one or why you think they are the same.

Student S31M: *They are both triangles.*

Sketches of students S130M and S100F to the written test are given in Appendix A, figure IX.

The interviews with students test score analysis indicated that students had difficulties at the various levels of van Hieles.

For the visualisation level, they had difficulties in differentiating between rotation reflection and translation, finding the image of an object after a rotation, relating angles and its measurements in rotation and identifying an image after a rotation through a given degree.

At the analysis level, students had difficulties in integrating a figure and its properties in rotation, finding the position of a new image after rotation, appreciating the usage of the word congruency and similarity of a figure in rotation and differentiating between two figures with a view of pointing out their similarities and differences in rotation.

With regards to abstraction level of van Hiele, students had difficulties in knowing the implication when a figure is rotated several times, measuring an angle that exceed 90°, using a language to express a thought in rotation and applying geometric terminology when describing a rotated figure and its image.

Finally, at the deduction level of van Hiele, Students had difficulties in describing why a figure and its image is a rotation, demonstrating that a rotated figure and its image are congruent and understanding concepts such as the size of angle, preservation of shape, size of shape and length in their arguments.

However, these difficulties increased as the levels increase, theefore in summary, the findings from both descriptive report and interview report showed that majority of students had difficulties in abstraction and deduction level as compared to the visualization and analysis stage. These results are congruent to the findings of Usiskin (1982); Hoffer (1981); Atebe and Schafer (2010); Baffoe and Mereku (2010) who found out that most African students at the high school were not able to solve a variety of geometric problem and that most of the students encountered difficulty in reaching the abstraction and deductive levels. Also, Xistouri and Pitta-Pantazi (2011) found out that students had difficulties in understanding rotation as against translations and reflections.

4.2 Research Question 2: What are the levels reached in the van Hieles' model with respect to rotation?

To examine the level SHS students reached using the van Hieles" model in transformational geometry of rotation concerning the levels of van Hiele, participants were made to answer questions under Visualization, Analysis, Abstraction and Deduction.

Table 10 shows students" attaining level on the items in the achievement test, measuring their ability to identify and name a figure that has gone through a transformation of rotation by its motion using a standard or non-standard name (Visualization).

Table 10:Distribution of Students who reached the Visualization Stage (Level 1) in Solving Rotation

Factor	Sample size	Attempted		Correct Answer	
	N	N	%	N	%
Identifying transformation by a change in the figure.	240	238	99.02	187	77.9
Identifying the image of X after a transformation of -90^{o} .	240	238	99.02	160	66.7
Identifying the quadrant in which triangle FGH is located.	240	240	100.0	191	79.6
Identifying the polygon that is congruent to polygon 1.	240	240	100.0	212	88.3
Naming a transformation using standard or non-standard name	240	240	100.0	186	77.5
Averagely	**240**	**239**	**99.6**	**187**	**77.9**

Source: Fieldwork, 2018.

Table 10 revealed that out of the 240 respondents who took part in the study, 238 representing 99.02% of the students do attempt item under visualization stage which required respondents‟ ability to identify transformation by the change in the figure. On this item, 187 students representing 77.9% were able to recognize the transformation by the change in the figure. Similarly, students were required to identify the image of X after a transformation of -90^{o} and the quadrant in which triangle FGH is located, 160 (66.7%) and 191 (79.6%) of the students were able to identify the image of X after a transformation and the quadrant of triangle FGH respectively. Also, with respect to students‟ ability to name a transformation using standard or non-standard name and identify the polygon that was congruent to polygon 1, 186 (77.5%) and 212 (88.3%) of the students had correctly provided an answer to the question. Averagely, these findings indicated that 187 students representing 78.0% were able to provide answers correctly to the items associated with visualization. These suggested that quite a large number of the students had reached the visualization stage under van Hiele levels.

Additionally, students were requested to respond to items in the achievement test to test students‟ ability to discover the properties of a figure and its images after rotation and use these properties to analyze a transformation of rotation (Analysis). Table 11 shows the attaining level of students at the analysis stage.

Table 11: Distribution of Students who reached Analysis Stage (Level 2) in Solving Rotation

Factor	Sample size	Attempted		Correct Answer	
	N	N	%	N	%
Locating the centre of rotation.	240	232	96.7	171	71.3
Locating the angle of rotation.	240	234	97.5	169	70.4
Ability to discover the type of transformation between an object and its image.	240	236	98.3	157	65.4
Ability to discover the property of a change in a figure due to a rotated figure.	240	238	99.2	141	58.8
Ability to relate a rotated image by using coordinates.	240	237	98.8	178	74.2
Ability to assign reasons to question 2.2.1.	240	234	97.5	146	60.8
Ability to discover equal distance of a point and its image from the origin after rotation.	240	223	92.9	149	62.1
Discovering the property of a change in a figure due to a rotated figure	240	231	96.3	174	72.5
Averagely	**240**	**233**	**97.1**	**161**	**67.1**

Source: Fieldwork, 2018.

Observations from Table 11 portray that out of the 232 (96.7%) and 234 (97.5%) of the students attempted items under analysis stage which required respondents" ability to locate the centre of rotation and locate the angle of rotation respectively. On these items, 171 (71.3%) and 169 (70.4%) were able to locate the centre of rotation and locate the angle of rotation respectively. Similarly, students were required to discover the type of transformation between an object and its image and to relate a rotated image by using coordinates. A total of 157 (65.4%) and 178 (74.2%) of the students did solve the questions correctly showing that they understand the concepts very well. Furthermore, with regards to students" ability to discover equal distance of a point and its image from the origin after rotation as well as discover the property of a change in a figure due to a rotated figure, 149 (62.1%) and 174 (72.5%) of students had shown their understanding for solving the questions correctly. On the whole, the outcomes from Table 11 confirmed that 159 students representing 66.3% had no difficulties associated with the analysis stage. These implied that 159 students had reached an analysis stage under van Hiele levels.

Table 12: Distribution of Students who reached Abstraction Stage (Level 3) in Solving Rotation

Factor	Sample size	No Attempt		Correct Answer	
	N	N	%	N	%
Ability to perform a simple transformation of rotation.	240	234	97.5	130	54.2
Ability to state the required angle when a figure returns to its original position after rotation.	240	231	96.3	104	43.3
Ability to describe a transformation that maps an object to its image.	240	215	89.6	116	48.3
Ability to appreciate the congruency of a figure	240	226	94.2	122	50.8
Averagely	**240**	**227**	**94.4**	**118**	**49.2**

Source: Fieldwork, 2018.

Indications from Table 12 show that 130 (54.2%) and 104 (43.3%) of the students had no difficulties solving questions under abstraction stage correctly, which required respondents" ability to perform a simple transformation of rotation and ability to state the required angle when a figure returns to its original position after rotation respectively. Also, students were required to describe a transformation that mapped an object to its image and appreciate the congruency of a figure, 116 (48.3%) and 122 (50.8%) of the students did solve the questions correctly showing that they understood geometrical concepts under transformation on abstraction stage. On average, the results from Table 12 established that 118 students representing 49.2% had no difficulties associated with abstraction stage. These infer that 118 students had reached abstraction stage under geometrical transformation analysis stage under van Hiele levels.

Table 13: Distribution of Students who reached the Deduction Stage (Level 4) in Solving Rotation

Factor	Sample size	Attempted		Correct Answer	
	N	N	%	N	%
Stating the type of triangle after a rotation of +70 degrees.	240	230	95.8	95	39.6
Giving reasoning to question 4.1.1.	240	223	92.9	66	27.5
Ability to prove that triangle XYZ is congruent to triangle XY'Z'.	240	209	87.1	43	17.9
Stating whether a figure and its image are congruent.	240	220	91.7	108	45.0
Using a diagram to the answer in question 4.2.1		208	86.7	52	21.7
Averagely	**240**	**218**	**90.8**	**73**	**30.4**

Source: Fieldwork, 2018.

Results from Table 13 indicate that 95 (39.6%) and 66 (27.5%) of the students had no difficulties solving questions under deduction which required respondents" ability to State the type of triangle after a rotation of +70 degrees and ability to give reasons on it respectively. Students were also required to prove that triangle XYZ was congruent to triangle X'Y'Z', 43 (17.9%) and 108 (45.0%) of the students solve the questions correctly showing that their understanding on geometrical concepts under transformation on deduction stage. On average, the results from Table 13 established that 67 students representing 30.3% had no difficulties associated with deduction stage. These infer that 67 students had reached a deduction stage of van Hiele level under geometrical transformation analysis stage.

The findings from the analysis showed that most students did not perform well in the VHGT. In a nutshell, indications from Table 10, 11, 12 and 13 showed that averagely 187 (78.0%), 159 (66.3%), 118 (49.2%), and 67 (30.3%) of the students reached visualization, analysis, abstraction and deduction levels respectively under van Hiele geometrical transformation level. Table 10, 11, 12 and 13 can further be summarised in Figure 2 below.

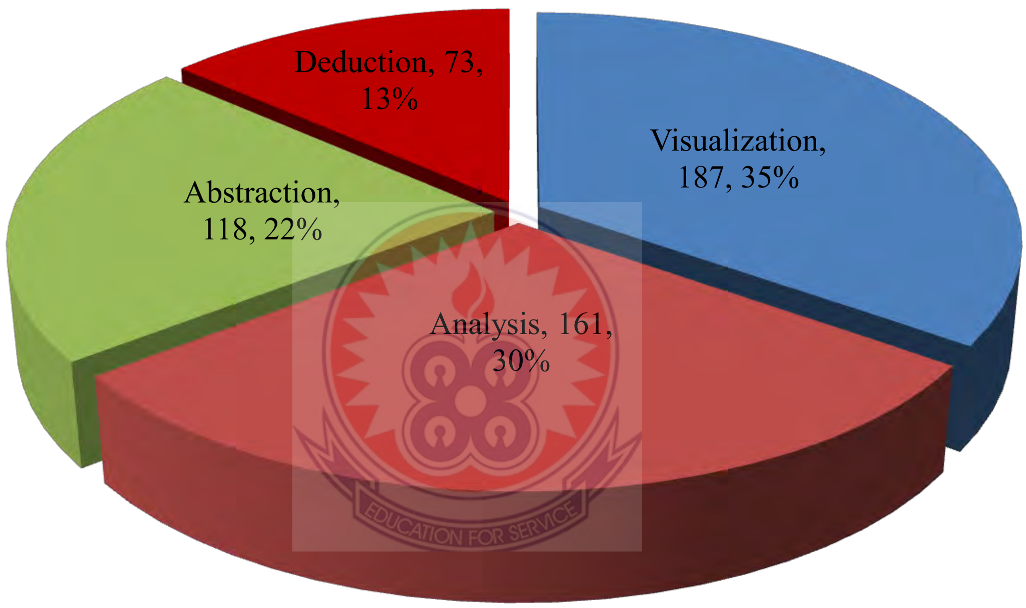

Figure 2: Summary of Students' Levels reached in the van Hieles' Model

The analysis in Figure 2 revealed a converse relationship between attainment and the level. This reflected in the summary analysis as the attainment level of the student in visualization is 35% out of the overall items. Also, 30% have reached the analysis level, 22% have reached abstraction, and 13% have reached deduction.

These findings align with Atebe and Schafer (2010) who reported that students were not able to solve questions under deduction level. To support this finding, Baffoe and Mereku (2010) also stated that majority of the students had not reached third and four-level that is abstraction and deduction but reached the first and second levels of the Van Hieles" Geometric thinking levels, that is the Visualization and Analysis level. The number of students who reached levels 3 and 4, i.e abstraction and deductive levels show that most students were not able to classify and generalize by attributes and develop proofs using axioms and definitions. The findings in the study showed that students who reached the abstraction and deductive levels could not state whether a figure and its image were congruent. Similarly, it was observed that none of the geometry activities in the SHS textbooks could be described as one at the Van Hieles" Geometric Thinking model for level 4 (Deductive) where students could prove theorems deductively and establish interrelationships among networks of theorems. This implies that less attention is given to the van Hieles" model of Geometric Thinking level.

4.3 Research Question 3: Is there any significant difference between male and female students' in terms of the difficulties in rotation using the van Hieles' level?

In answering the third research question, the results obtained in the achievement test were examined and compared for the two groups of students i.e male and female under each level of van Hiele. The results are presented in Table 14.

Table 14: Independent sample t-test results showing gender differences in mean achievement on the difficulties in rotation under the van Hieles' levels.

van Hiele	Gender	N	Mean	Std. Deviation	T	Df	P
Visualization	Female	121	3.94	1.422	0.470	238	0.639
	Male	119	3.86	1.380			
Analysis	Female	121	5.30	2.691	0.205	237	0.838
	Male	119	5.41	2.615			
Abstraction	Female	121	1.87	1.602	0.911	235	0.363
	Male	119	2.07	1.544			
Deduction	Female	121	1.26	1.499	2.661	230	0.008*
	Male	119	1.78	1.627			

Source: Fieldwork, 2018.

The differences in Df was as a result of those who did not attempt the questions under each of the level. Hence 2, 3, 5 and 10 students did not answer questions on rotation with respect to Visualisation, Analysis, Abstraction and Deduction stages of van Hiele.

The result in Table 14 therefore, indicated that there was no statistically significant difference between the mean scores of males (M = 3.86, SD = 1.38) and females (M = 3.94, SD = 1.42) under visualization level thus (t (238) = 0.470, p =0.639); males (M = 5.41, SD = 2.62) and females (M = 5.30, SD = 2.69) under analysis level thus (t (238) = 0.205, p = 0.838), males (M = 2.07, SD = 1.54) and females (M = 1.87, SD = 1.60) under abstraction level thus (t (235) = 0.911, p = 0.363). The results in Table 14 showed that the females achieved better mean score in the visualization and analysis levels than their male counterparts. However, at abstraction level, male students achieved better mean score than their female counterparts. The results indicated that the difference in the mean scores was fortuitous since there was no evidence to suggest that any significant differences existed between males and females in the visualization level, analysis level and abstraction level. These results suggested that the performance of both male and female students of the visualisation level, the analysis level and abstraction level were almost the same.

On the contrary, the results in Table 14 indicate that there was statistically significant difference between the mean scores of males (M = 1.78, SD = 1.63) and females (M = 1.26, SD = 1.50); t (230) = 2.66, p = 0.008) under the deduction level of van Hiele. This reveald that males achieved better in the deduction level than their female counterparts because the difference in the mean scores was not due to chance since there was enough evidence to conclude that significant differences existed between males and females in the deduction level under van Hiele. This finding is in line with the claim of Fennema & Sherman (1978) who found that there were no statistically significant sex-related differences in spatial visualization. Also, Neuschmid, Barth and Hastedt (2008) reported that gender differences in geometry performance were evident in some countries; however, other countries showed no gender difference in geometry performance.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Overview

This chapter provides a summary of the entire study. Also, the research methodologies that were used to arrive at the findings of the study were presented as well as the key findings. In light of the findings of the study, conclusions were drawn and recommendations made. Suggestions for further studies were also presented in this chapter.

5.1 Summary

The main aim of this study was to investigate and describe various difficulties which students have in the learning of rotation with respect to the van Hieles" model of geometric thinking levels. The investigation was focused on analyzing in a broader context how students visualize an image after rotation, use the concept of rotation to transform an image when given the coordinates, angles and shape, describe geometric figures and their properties after transformation through rotation, discover the properties in a given rotational transformation by locating centre, and angle of rotation and use rotational transformations to do challenges encountered in integrating practical work into physics lessons and how they can be addressed. In all, three research questions and one research hypothesis were formulated and tested using descriptive and inferential statistical tools respectively.

The study adopted both quantitative and qualitative research approaches. Moreover, the qualitative data and results were used to assist in explaining and assigning reasons for quantitative findings. In all, the achievement test was administered to 240 students from the school selected for the study after seeking permission from the head of the department. Thus, a response rate of 100% was obtained. The data collected were analysed using frequency and percentages, means and standard deviations and independent-sample t-test. The IBM SPSS Statistical software was used to obtain the results of the study. Based on the results of the study, the following key findings were obtained.

5.2 Major Key Findings

With regards to research question one, the findings from the study show that 52 students representing 21.7% had difficulties associated with visualization. It was also observed that 74 students representing 30.9% had difficulties associated with the analysis level. The study also portrayed that 109 students representing 45.2% had difficulties associated with abstraction level. Finally, 145 students representing 60.5% had difficulties associated with deduction level. These difficulties at the various levels of van Hiele can be linked to the findings of Xistouri and Pitta-Pantazi (2011) who reported that while students" understanding of translations and reflections are equally difficult; rotations seem to be more difficult. Hence the degree in difficulties as the levels proceeds in the van Hieles" model.

Concerning research question one, it was found out that 187 students representing 78.0% were able to reach the visualization level under rotation in transformation geometry. This finding was in tune with Walker, Winner, Hetland & Goldsmith (2011). According to these reseachers, learners who can perceive ideas visually have an advantage when it comes to reasoning and making a good judgment in the geometrical analysis. Also, the outcome of the study shows that 159 students representing 66.3% had no difficulties associated with the analysis level. This implies that 159 students had reached an analysis level under van Hiele levels under rotation in transformation geometry. Also, the results from the study established that 118 students representing 49.2% had no difficulties associated with abstraction level under rotation in transformation geometry. This implied that 118 students had reached abstraction level under rotation in transformation geometry. Finally, 67 students representing 30.3% had no difficulties associated with deduction level. This implied that 67 students had reached the deduction level of van Hiele level under rotation in transformation geometry.

Finally, to answer research question three, the results from the study showed that there was no statistically significant difference between the mean scores of males and the females since there was no evidence to suggest that any significant differences existed between males and females in the visualization level, analysis level and abstraction level. These results suggest that the performance of both male and female students at the visualisation level, the analysis level and abstraction level were almost the same. This findings finds favour with that of (Erdogan, 2006). His study also indicated that boys" mean score is numerically higher than that of the girls. The analysis of the covariance (ANCOVA), however, indicates that this difference is not statistically significant in terms of the van Hiele levels in geometry between boys and girls hence no gender differences were found in the study. Also, it was found out that there was a statistically significant difference between the mean scores of males and females under the deduction level.

5.3 Conclusion

Based on the findings of the study, the majority of the students demonstrated that they could visualize by identifying a rotated figure and its image but have difficulties in finding the properties of the new image after rotation. Students provided different explanations on the properties of rotated figures as their views in geometric understanding changes. This change takes time and the growth could be encouraged but it could not be rushed through by the teacher. It takes time for students to conceptualize the properties of rotated figures as an important aspect of their description.

It was observed that Level two and level three concepts were difficult for some students. This is the level which represents a new and important way of organizing thinking which usually does not come naturally to students. However, it represents an important part that is often overlooked in the chain of events when moving to formal deduction in the learning of rotation. Students" ability to connect in terms of minimum properties at level three concepts represents a further development in understanding beyond the concept of the class enclosure. For such activities to be meaningful, students would need to be familiar with the properties of figures and their relationships.

5.4 Recommendations

The following general recommendations are proposed to help deal with the difficulties students encounter in the area of transformation geometry (rotation) to help improve the geometrical pedagogical practices and mathematical performance of learners. Transformation geometry of rotation has the reputation of being just a set of tricks and students see no point in studying transformations. To encourage students in the study of transformation, teachers should learn how to use transformation geometry not as a system of describing motion but as constructing a rule which allows one shape to be mapped onto another (Burke, Stagl, Salas, Pierce, & Kendall, 2006). Moreover, teachers should examine or analyse students" difficulties in rotation using the van Hieles" levels. This will help them to detect the difficulties at each level so as to provide the appropriate interventions or strategies to address it and promote smooth progress at that level.

Teachers should, therefore, encourage students to talk about geometric concepts relating to rotation and discover the properties themselves to develop expressive language. In a classroom situation, students should also be made to work with geometric models to enable them discover the properties themselves. Teachers should implement this by asking students to describe a figure, rather than just to select a name for it from the list. Students" understanding of key concepts such flips, turns, glides, similarity, congruency, angle of rotation and the relationships of the properties of rotation, in particular, is critical for supporting the development of deeper understandings of transformations of rotation (Hollebrands, 2003).

Mathematics tutors should be encouraged to rely on hands-on activities using manipulative concrete materials. Manipulative materials or teachers specifically designed materials can according to Driscoll, Confrey and Martz (1986) show the way to conceptual understanding. They also provide experience in which students can transfer their understanding smoothly from one concept to another. One way of letting the lower-achieving students concentrate on the learning of transformation geometry is to use information communication technology (ICT). For example, students could be given many congruent shapes placed at a different location in the plane, and the task would be to find a way through the transformation of rotation to move an original shape to each of the congruent shapes. This could be a game for two or more students where one student instructs the computer to perform a transformation and the other has to find out which one it was. If repeated, this would help students get a feel for what the image looks like. For example, if it is "flipped over" it has to be a reflection or glide, otherwise a rotation or translation (Wesslen & Seipel, 2005).

The Secondary School transformation geometry curriculum should be appropriate for the various thought levels. The curriculum should require students to explain and justify their ideas. It should also encourage students to refine their thinking.

In conclusion, the present study adds the following to the field of transformation geometry education:

- It has employed van Hieles" theory of geometry learning to describe and analyze students" difficulties in transformation geometry within the context of Rotation.
- It has also suggested guidelines for classroom practice that can contribute to improved teaching and learning of transformation geometry.

Teachers do rely heavily on texts for their daily instructions. To bring about these changes, textbook and teachers" education that is focused on the van Hieles" model is recommended.

This should be done because students need to be assured that their readiness for transformation geometry is related to their previous experience and instruction and that lack of readiness is not a reflection on their intelligence. The result of the investigation is an indication that the van Hiele"s model of development in geometry can serve as a useful frame of reference when analyzing student"s thinking processes in geometry tasks. The conclusions are also synonymous with some conclusions reached by Soon (1989) and Ada and Kurtulus (2010).

5.5 Area for Further Research

The researcher suggested that the scope of the study should be extended to include larger number of Senior High schools so that the picture would be clear about the kind of difficulties students encounter with regard to rotation using the van Hieles‟ model. Also, due to the differences noted among the different kinds of students, it indicates that the proposed method of evaluation of the van Hieles‟ levels was coherent and should be researched further.

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APPENDIX A

INTRODUCTORY LETTER

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF SCIENCE EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION
P. O. Box 25, Winneba, Ghana
math@uew.edu.gh
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July 5, 2017

TO WHOM IT MAY CONCERN

LETTER OF INTRODUCTION

I write to introduce to you the bearer of this letter ABAGNA FUSEINI, a postgraduate student in the University of Education, Winneba who is reading for a Master of Philosophy degree in Mathematics Education.

As part of the requirements of the programme he is undertaking a research titled – *Using Van Hiele's Level to investigate difficulties of students' in Rational Transformation Geometry in Ghana: Case Study of Ngleshie-Amanfrom Senior High Secondary School*

He needs to gather information to be analysed for the said research and he has chosen to do so in your institution. I would be grateful if he is given the needed assistance to carry out this exercise. Thank you.

Yours faithfully,

Prof. D. K. Mereku
Head of Department

APPENDIX B

MATHEMATICS ACHIEVEMENT TEST

Learner Code **Time 1HR 30MINS**

This is a non-evaluative assessment. Your performance in this test has nothing to do with your CASS marks. The assessment is designed to help identify difficulties students encounter in the area of transformation (rotation). Through it will help your teacher appreciate and come to terms with what you really miss about rotation and be able to design an appropriate methodology when teaching the concept.

Instructions

1. Answer all questions by ensuring that the steps you take in arriving at a particular are of importance rather than the correct answer.

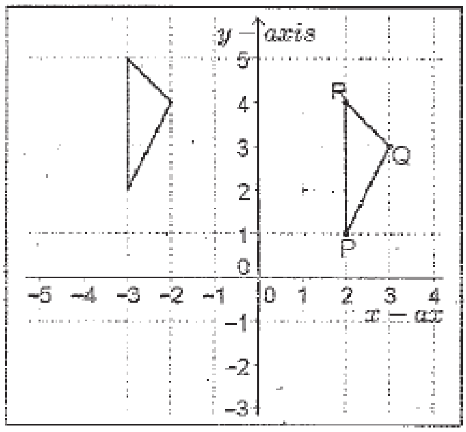
2. You are allowed to make any marks on the question paper and show all steps you took in arriving at the answer.

Question 1 (Basic level)

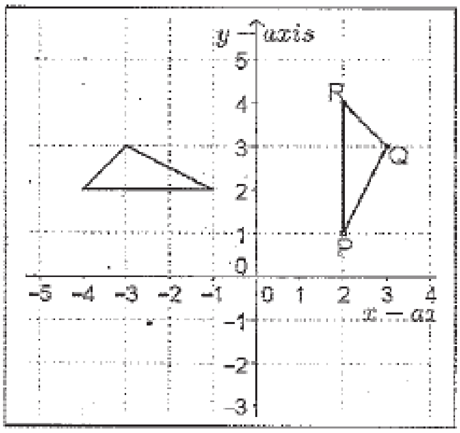
Consider each of the diagrams below that represents a transformation of triangle PQR.

Which among the transformation represent a rotation?

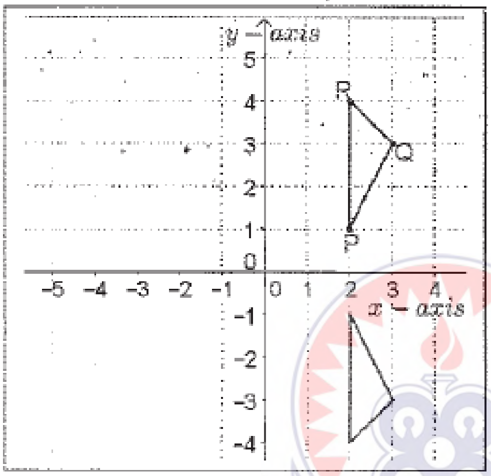
A



B

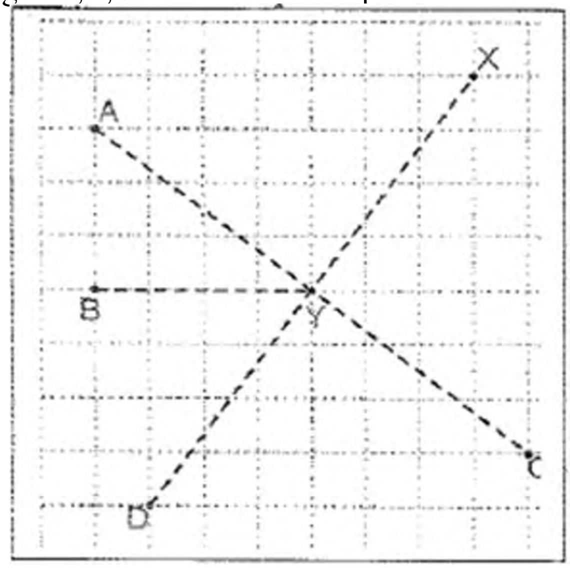


C

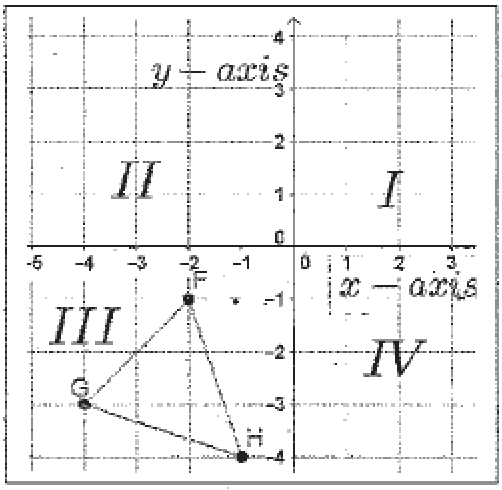


Answer.....................................

1.2 In the figure below, X has been rotated $-90°$ about the point Y as centre. Which of the following images A,B,C and D is a true representation of X after a rotation of $-90°$?

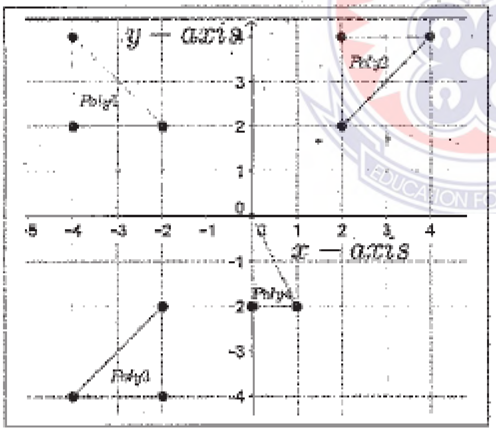


1.3 Identify in which quadrant I, II, III and IV will the image of $\Delta\, FGH$ be located if rotated through 90° about the origin.



Answer................

1.4 Consider the diagram below.



1.4.1 Identify the polygons that are congruent to polygon 1--------------------------------

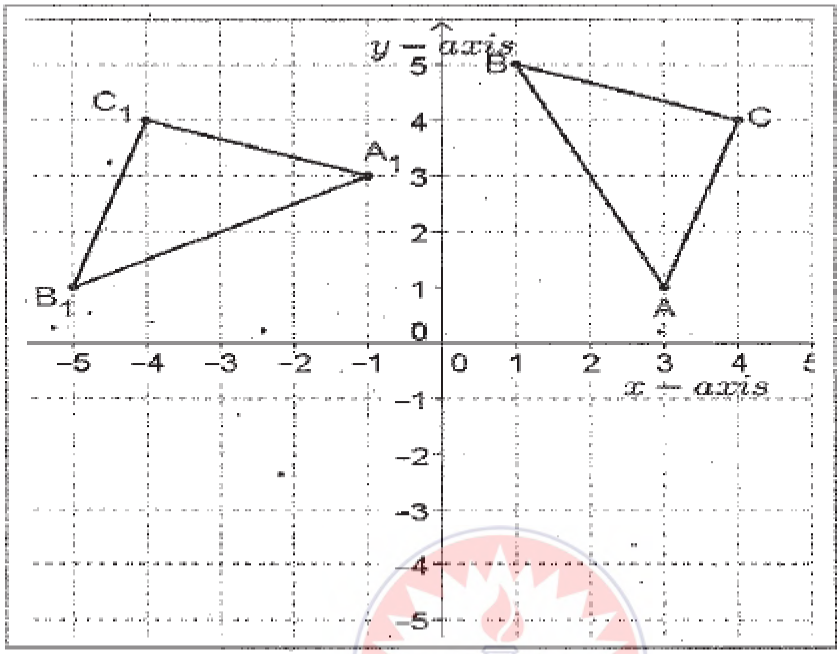
----

1.4.2 Name the type of rotation from polygon 1 to polygon 2 -----------------------------

----

Question 2 (Level 2).

2.1.1 A figure (ABC) and its image ($A_1B_1C_1$) after rotation is given below, locate or draw the following



i) Centre of rotation--

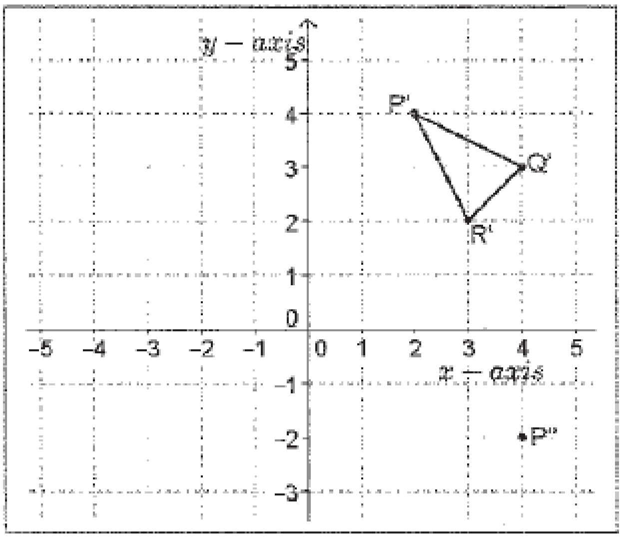
ii) Angle of rotation ---

2.1.2. What type of rotation is represented above? ---

2.1.3. What can you say about the image $A_1B_1C_1$ as compared with the preimage ABC? ---

----

The triangle P‟Q‟R‟ below has been rotated about the origin (0,0) through an angle of 90 degrees clockwise to map onto triangle P‟‟Q‟‟R‟‟.



2.2.1 Write down the coordinates of P‟‟

2.2.2 Explain how you got P‟‟?

2.2.3 What do you know about the length of OP‟ and OP‟‟?

2.2.4 Indicate whether the following properties correctly describes the transformation on question 2 or not by using either correct or not correct.

a. All angles with vertex P formed by a point and its image do not have the same measurement------------------------

b. Each figure is congruent to its image figure ----------------------------

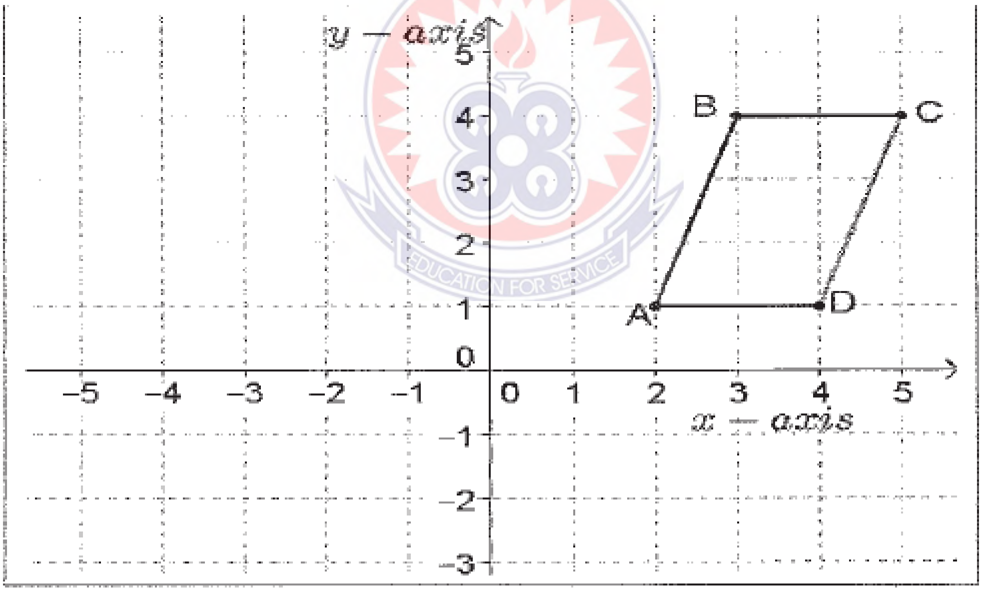
c. Orientation of the figure is different from its image figure-----------------------

d. A point and its image are both the same distance from the centre of rotation---------

e. Each angle of the figure is congruent to the corresponding angle of the image figure----

Question 3 (Level 3)

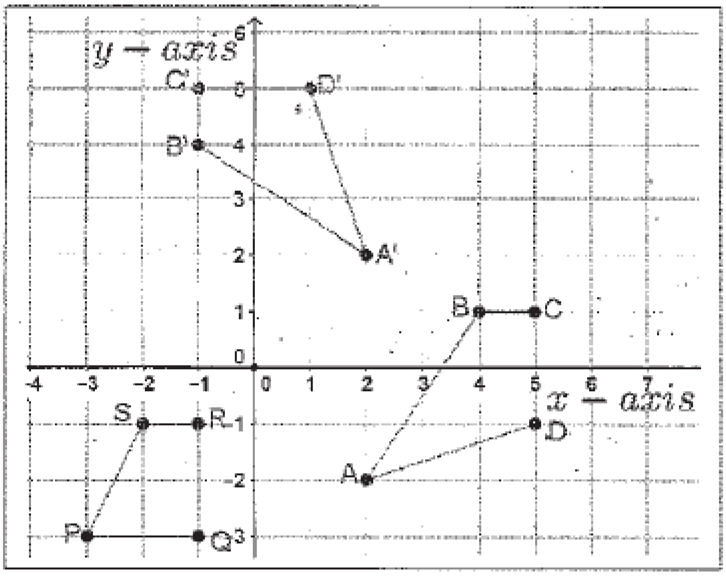
On the grid below, x and y axes have been labeled.



3.1.1 Rotate ABCD through 90° (anticlockwise), about (0,0) and label the image PQRS where $A \to P, B \to Q, C \to R$ and $D \to S$.

3.1.2 Through how many angles in degrees anticlockwise can you rotate the same figure so that it can fit exactly on to the original figure?

Consider the diagram below.

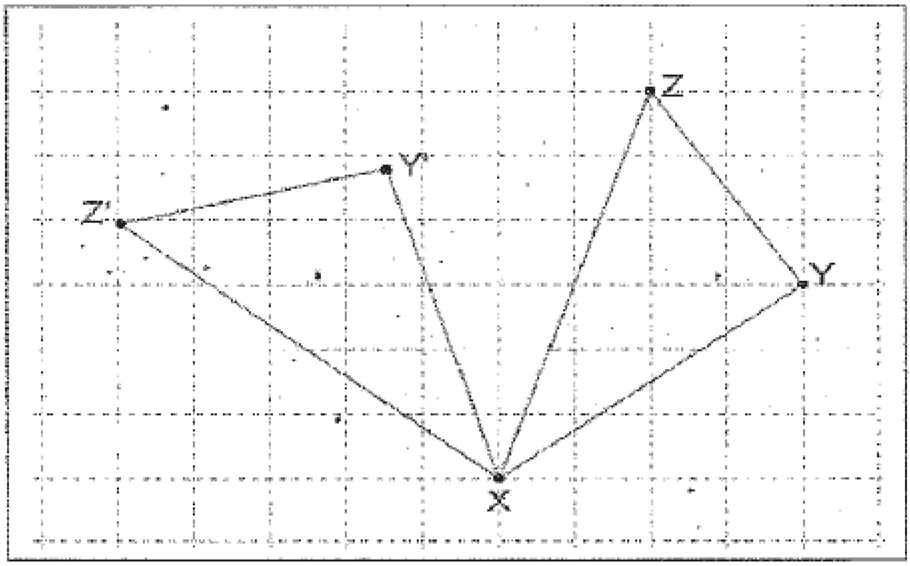


3.2.1 Describe fully the transformation that maps quadrilateral ABCD onto that of of

A‟B‟‟C‟‟D‟?

3.2.2 Is the quadrilateral ABCD congruent to that of PQRS? ---------------------------

Question 4(Level 4)



The triangle XYZ has been rotated through +70 degree about X. XY‟Z‟ is the image of XYZ after a rotation.

4.1.1 What type of triangle is XYZ? ……………………………………………

4.1.2 Give a reason for your answer to question 4.1.1

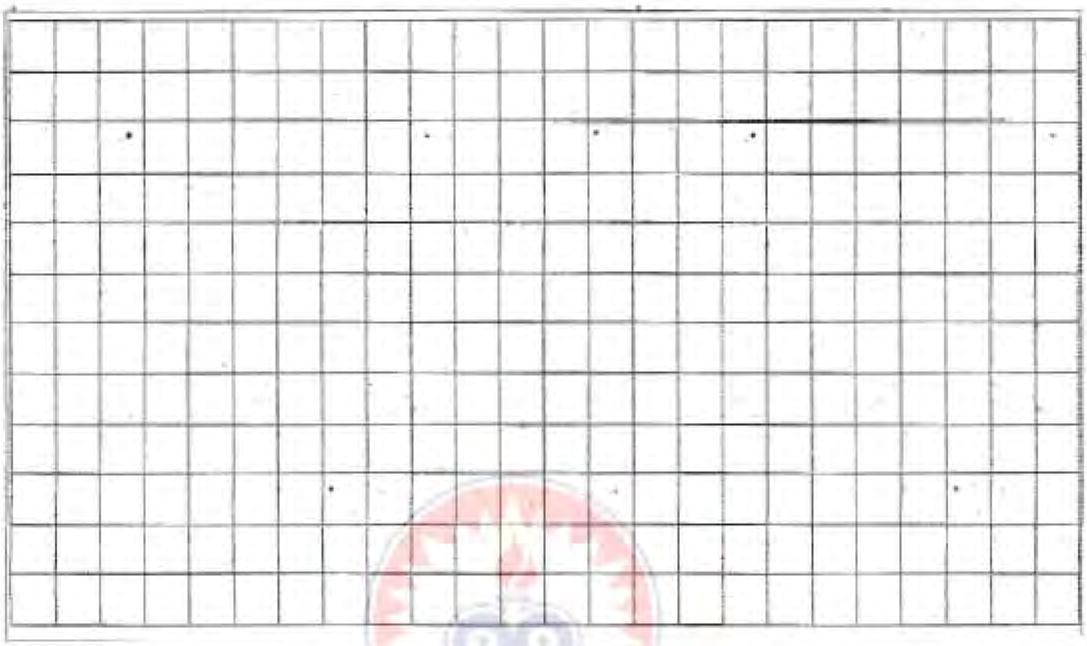
4.1.3 Use rotation to prove that triangle XYZ is congruent to triangle XY‟Z‟.

--- ---

---.

4.2.1 Consider the statement: When a figure is rotated, the figure and its image are congruent, do you agree?. Yes/No

4.2.2 Use an appropriate diagram to explain your answer your reasoning



APPENDIX C

MARKING SCHEME OF THE ACHIEVEMENT TEST

Question 1 (Level 1)

1.1 B

1.2 A

1.3 IV

1.4.1 Polygon 2 and polygon 3

1.4.2 Clockwise rotation of 90^0 or anticlockwise 270^0 **5marks**

Question 2 (level 2)

2.1.1

(i) Origin i.e. (0,0)

(ii) 90^0

2.1.2 Anticlockwise rotation

2.1.3 Image $A_1B_1C_1$ is congruent to the preimage ABC

2.2.1 (4, -2)

2.2.2 I got p^{11} by switching x and y positions and making y - negative i.e. using the rule for anticlockwise 270^0 about the origin. $(x, y) \rightarrow (y, -x)$.

2.2.3 Length OP^1 is equal to the length OP^{11}

2.2.4 a. correct

b. correct

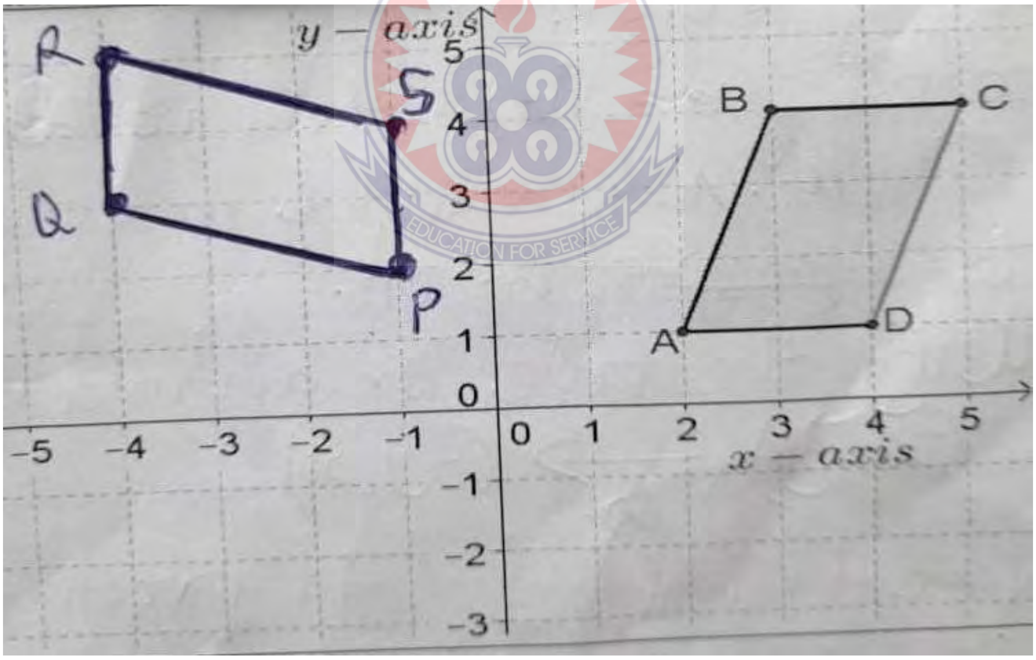
c. not correct

d. correct

e. correct **8marks**

Question 3 (level 3)

(i) 3.1.1 Anticlockwise 90^0 rotation of ABCD to PQRS



3.1.2 360^0 or 1 revolution

3.2.1 Using the tracing paper: the shape and centre of rotation is traced. The tracing paper is held down with a pencil on the centre of the rotation. It is then rotated through an anticlockwise 90^0. Afterward the image $A_1B_1C_1D_1$ is then traced on the new position on the graph sheet.

OR

Using the rule for anticlockwise rotation of 90^0 (x, y) → (-y x)

OR

- Place the nod part of the protractor on the vertex of the angle.
- Extend a line from the origin to A.
- Measure the angle where the other side crosses the number line.
- Take the length of OA to OA^1 and make a mark.

Repeat the process to each of the following vertex B, C and D. Draw the figure with the points marked.

3.2.2 No **4marks**

Question 4 (level 4)

4.1.1 Isosceles triangle

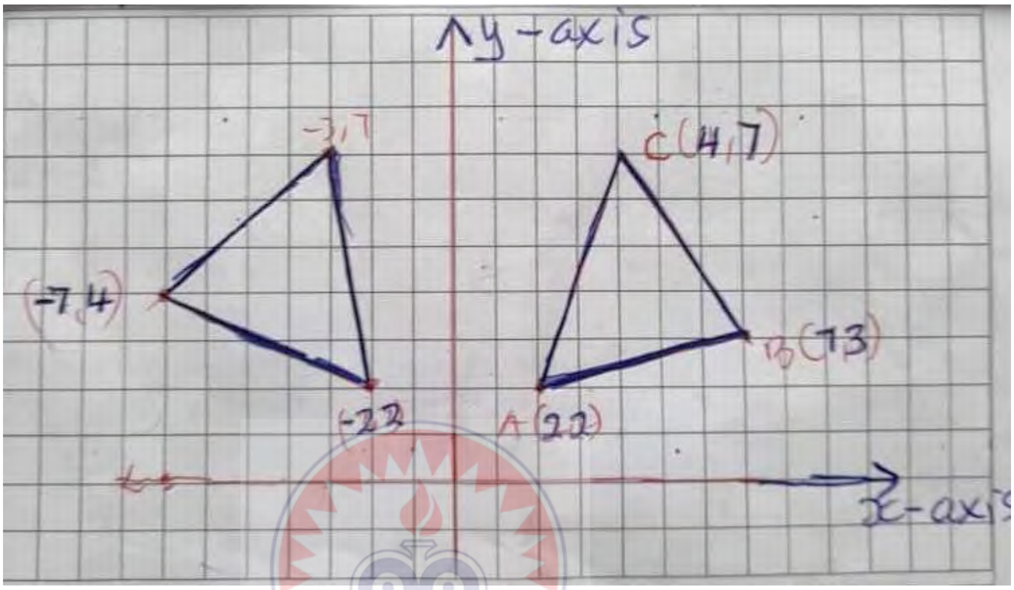
4.1.2 Length X Y ≠ length X Z ≠ length Z Y

4.1.3 ΔXY^1Z^1 is the image of ΔXYZ with centre of rotation at X. $<ZXY = <Z^1XY^1$. Also $/XY/ = /XY^1/$, $/XZ/ = /XZ^1/$ and $/YZ/ = /Y^1Z^1/$. Since $<ZXZ^1 = <YXY^1 = 90^0$ by measuring and through anticlockwise direction. It stands to reason that ΔXYZ and

$\Delta XY^{1}Z^{1}$ are congruent because from the above, each of the lengths were preserved, angles were preserved, each of the lines were mapped unto one another.

4.2.1 Yes

4.2.2 Sample of diagram explaining reasoning behind 4.2.1



4 marks

APPENDIX D

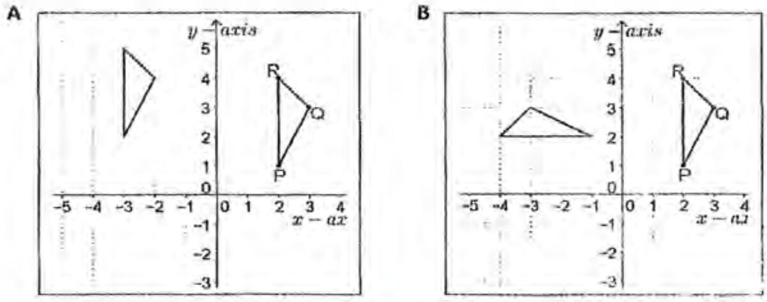
SOLUTIONS GIVEN BY STUDENTS IN THE MAT AND INTERVIEW

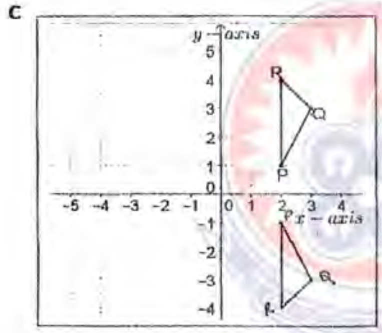
Question 1 (Basic level 1)

1.1. Consider each of the diagrams below that represents a different transformation of triangle PQR.

S51F Which among the transformation represent a Rotation?

Fig. I





If R(4, 2) P(1, 2) Q(3, 1) Then rotation will be 90° clockwise. We would the arrive at R = {-4, 2} P = {-1, 2} Q = {-3, 1}

Answer.........C X..........

S130M 1.2. In the figure below, X has been rotated -90 ° about the point Y as centre. Which of the following images A, B, C and D is a true representation of X after a rotation of -90 °?

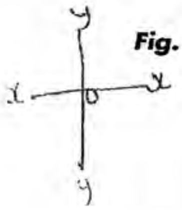
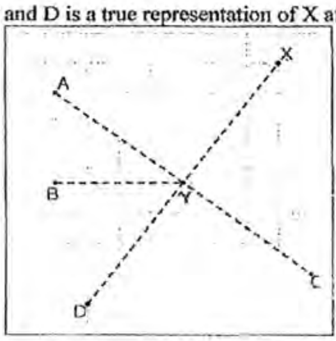


Fig. II



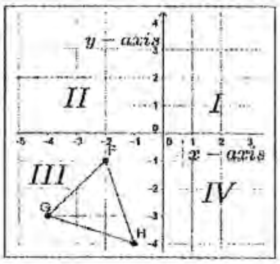
Answer.......A..... X (C)

When x is rotated -90° which means anticlockwise ...

2

1.3 Identify in which quadrant I, II, III and IV will the image of Δ FGH be located if rotated through 90° about the origin.

S67M



Answer.......III....

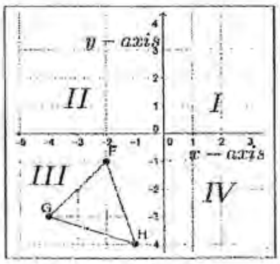
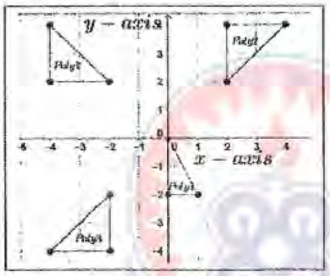


Fig. III

Answer.....I....... ×

1.4 Consider the diagram below.



S100F

1.4.1 Identify the polygons that are congruent to polygon1 ....

....right-angle....triangle....×..

1.4.2 Name the type of rotation from polygon 1 to polygon 2 ....

....right-angle....triangle..

S110F

1.4.1 Identify the polygons that are congruent to polygon1 ....(2,4)........(0,-2)

(4,2)......(-2,-4)...

1.4.2 Name the type of rotation from polygon 1 to polygon 2 ....I polygon...

is ~~anti~~ ~~anti~~ anticlockwise rotation.........

polygon is a rotation of

Question 2 (Level 2)

2.1.1 A figure (ABC) and its image ($A_1B_1C_1$) after rotation is given below, locate or draw the following.

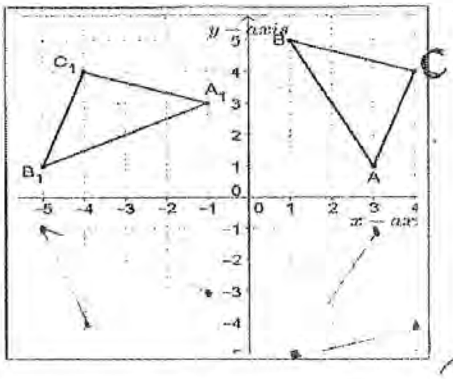


Fig. IV

S130M

(i) Centre of rotation ...~~~~ anticlckwise ...... rotation

(ii) Angle of rotation ...the...image...A1, B1, C1 rotation on the image A2 B2, C2

2.1.2. What type of rotation is represented above? anticlckwise rotation

2.1.3. What can you say about the image $A_1B_1C_1$ as compared with the preimage ABC?

The image A1, B1, and C1 has a compliment of one

S31M

(i) Centre of rotation ..... Clockwise

(ii) Angle of rotation ...... ~~90°~~ 270°

2.1.2. What type of rotation is represented above? Clockwise Rotation

2.1.3. What can you say about the image $A_1B_1C_1$ as compared with the preimage ABC?

The image of A B C has been rotated from 1st quadrant to second quadrant at the angle of 270° as compared to image A, B, C

P = (2, 4)
Q = (4, 3)
R = (3, 2)

S100F

(i) Centre of rotation (y, x) → (y, -x)

(ii) Angle of rotation ... ~~270°~~ 90°

2.1.2. What type of rotation is represented above? rotation through ~~270~~ 90° about origin

2.1.3. What can you say about the image $A_1B_1C_1$ as compared with the preimage ABC?

The image is the reflex of the preimage ABC.

The triangle $P'\ Q'\ R'$ below has been rotated about the origin (0.0) through an angle of 90 degrees clockwise to map onto triangle $P''\ Q''\ R''$.

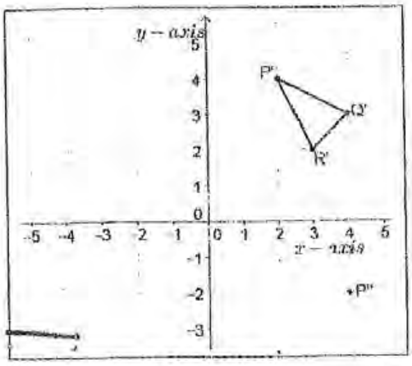


Fig. V (a)

4

S51F

2.2.1. Write down the coordinates of P'' ~~(-2,4)~~ (-4, -2) 1

2.2.2 Explain how you got P''?

When an angle rotates 90° clockwise, the "x" value becomes negative. So therefore to get P'' the "x" value in the P' must be affected with a negative sign. X

2.2.3 What do you know about the length of OP' and OP''?

The length of OP' and OP'' can be measured by using magnitude and direction. That is, OP'' = OP' X ⓪

S85M

2.2.1. Write down the coordinates of P'' (4, -2) 1

2.2.2 Explain how you got P''?

P'' was drawn through the rotation through 180° clockwise about the origin. And the coordinates of P'' were obtained by tracing from where ~~the~~ point P'' to the x and the y axises. ?

2.2.3 What do you know about the length of OP' and OP''?

The length of OP' is the length or measure from the origin to the point named P' and OP'' is from origin to P''. ?

S115F

2.2.4 Indicate whether the following properties correctly describes the transformation on question 2 or not by using either Correct or not correct

a. All angles with vertex P formed by a point and its image do not have the same measure. Correct X

b. Each figure is congruent to its image figure. Correct ✓

c. Orientation of the figure is different from its image figure. Correct ✓

d. A point and its image are both the same distance from the centre of rotation. Not correct X

e. Each angle of the figure is congruent to the corresponding angle of the image figure. Not correct X

Question 3 (Level 3)

On the grid below, x and y axes have been drawn and labeled.

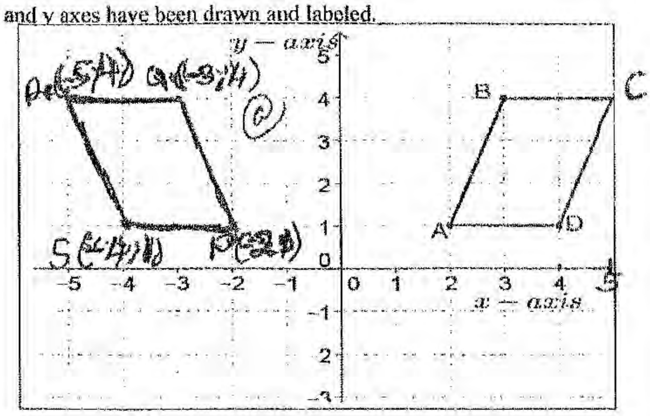


Fig. VI

3.1.1. Rotate ABCD through 90° (anticlockwise), about (0, 0) and label the image PQRS where A → P, B → Q, C → R, and D → S.

3.1.2. Through how many angles in degrees anticlockwise can you rotate the same figure so that it can fit exactly onto the original figure?

S39M 180°

S51F Rotation through 90° clockwise

S67F 300° or − 300°

S100F Two angles in degrees anticlockwise ?

S31F 270° anticlockwise

S94M 3 angle anticlockwise

3.2.1

Consider the diagram below.

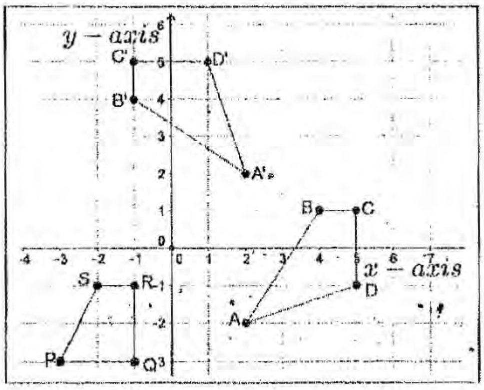


Fig. VII

551F

3.2.1 Describe fully the transformation that maps quadrilateral $ABCD$ onto that of $A'B'C'D'$

Original image = A=(2,2)

B=(4,1)

C=(6,1)

D=(5,-1) (X)

Transformation = A'=(2,2)

B'=(-1,4)

C'=(-1,5)

D'=(1,5) (X)

539M

3.2.1 Describe fully the transformation that maps quadrilateral $ABCD$ onto that of $A'B'C'D'$

The transformation that maps quadrilateral is that part of the diagram was drawn and later the other half was added to the diagram which made it full and that is how the transformation that maps quadrilateral ABCD onto that of A'B'C'D' X (0)

Question 4 (Level 4)

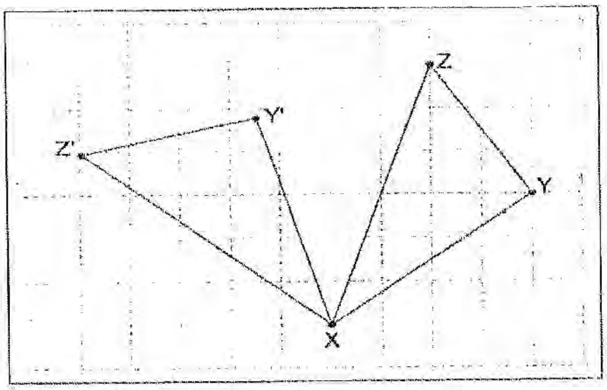


Fig. VIII

The triangle XYZ has been rotated through +70 degree about X. XY'Z' is the image of XYZ after rotation

S39M

4.1.1 What type of triangle is XYZ?

XYZ is a Right angle triangle (0)

4.1.2 Give a reason for your answer to question 4.1.1

Because it has been rotated about the point x. x makes 90° up to reflect Y and Z'

4.1.3 Use rotation to prove that triangle XYZ is congruent to triangle XY'Z'

(x,y) → (-x,y) (0)

Rotation through -90° or anticlockwise 90° above x

where the shapes and size are the same

∴ if (x,y) → (-x,y) Hence ∆XYZ is congruent (0) to ∆ XY'Z'

S115F

4.1.1 What type of triangle is XYZ?

Right angle triangle x (0)

4.1.2 Give a reason for your answer to question 4.1.1

Because triangle XYZ has hypotenuse, opposite, adjacent (0)

4.1.3 Use rotation to prove that triangle XYZ is congruent to triangle XY'Z'

We rotate it clockwise

Angle X = (-X, X) → X' = (-X, -X) → X''' = (X, -X)

Y = (-Y, Y) → X' = (-Y, -Y) → Y''' = (Y, -Y) (1)

Z = (-Z, Z) → Z'' = (-Z, -Z) → Z''' = (Z, -Z)

S130M

4.1.1 What type of triangle is XYZ?

Equilateral triangle x (0)

4.1.2 Give a reason for your answer to question 4.1.1

Because it is three sided figure x (0)

4.1.3 Use rotation to prove that triangle XYZ is congruent to triangle XY'Z'

XYZ is congruent to triangle XY'Z' x (0)
because they have the same of image.