UNIVERSITY OF EDUCATION, WINNEBA

CONCEPTIONS AND MISCONCEPTIONS IN ADDITION OF UNLIKE FRACTIONS AMONG STUDENTS IN WETO CIRCUIT PUBLIC JUNIOR HIGH SCHOOLS IN AFADZATO SOUTH DISTRICT OF GHANA

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DECLARATIONS

Student's Declaration

I, Adolph Atsu Yao Anku, hereby declare that this thesis is the result of my original research and that no part of it has been presented for another degree in this university or elsewhere.

Signature: ....

Date: ....

Supervisors' Declaration

We hereby declare that the preparation and presentation of the thesis were supervised per the guidelines on supervision of the thesis laid down by the University of Education, Winneba.

Name: PROF. PETER AKAYUURE (Principal Supervisor)

Signature: ....

Date: ....

Name: PROF. CLEMENT ALI (Co-Supervisor)

Signature: ....

Date: ....

DEDICATION

To my family, especially my wife, Mrs. Bernice Lucy Ametefe-Anku and my daughters, Scholastica Seyram Anku, Queenstar Sedinam Anku and Fortune Semenyo Anku.

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GLOSSARY/ABBREVIATIONS

CoHBS	Conference of Heads of Basic School
CRDD	Curriculum Research Development Division
HRI	Horizon Research International
NaCCA	National Council of Curriculum and Assessment
WAEC	West Africa Examination Council
ZPD	Zone of Proximal Development

ABSTRACT

The purpose of this study is to explore the conceptions and misconceptions of the learning of addition of unlike fractions among students in public junior high schools in Afadzato South District of Ghana. The study used the mixed method approach with sequential explanatory research design. Questionnaires were administered to a sample of 120 students in the Afadzato South District who were randomly selected after which 10 teachers were purposefully selected to be interviewed. The descriptive statistics and thematic analysis were used in the study. The study revealed that students have a positive but moderate conception of the addition of unlike fractions. It also revealed that, students do not even understand the basic concepts of fractions as they have the misconception that fractions are always lesser than 1. It also revealed that, teachers use facts and procedures to solving fractional problems, rather than building a deeper understanding. This hinders students" ability to continue to understand more abstract, rational ideas. It is therefore recommended that the instruction needs is to be focused on, more than simply abstract forms of teaching.

CHAPTER ONE

INTRODUCTION

1.0 Background to the Study

Mathematics is a process that involves calculation, and necessitates a thorough comprehension of concepts. It is a lifelong learning process that entails the conceptualization of prior information, the development of varied abilities, and the mastering of fundamental mathematical concepts (Sarwadi & Shahril, 2014). Theories and notions are formed not only through learning, but also through experiments, survey research, and observations that help to reinforce knowledge (Mohyuddin & Khalil, 2016).

According to Essuman, Korda, and Essigyan (2021), mathematics is primarily thought, as taught as an abstract subject that rationally requires methodical thinking. As a result, there are frequently certain misconceptions about mathematics in the teaching and learning process, which act as a barrier for students while addressing mathematical problems. The definition of a misperception is an incorrect understanding of something (Kawulich, Garner, & Wagner, 2009). Meanwhile, the errors could be the result of a misunderstanding. Carelessness, difficulty reading or interpreting the questions, and a lack of expertise are all possible causes of errors (Mohyuddin & Khalil, 2016). Mistakes and misconceptions, according to Dhlamini and Kibirige (2014), are related, but they are of two different concepts. However, the systematic and unsystematic mistakes are the two sorts of mistakes. Systematic errors are frequently the result of a misconception that allows students to repeat mistakes in a systematic manner over time until they acquire new knowledge. Unintentional or the unsystematic mistakes are displayed without the students' knowledge, and they are not likely to repeat such a misunderstanding, and students can remedy them on their own.

Students' mistakes might be used to measure their comprehension of topics, problems, or methods. The outcomes of the analysis enable teachers to identify difficulties and express their understanding of whether they are dealing with misunderstandings or mistakes.

Fractions in all of their "variations" or sub-constructs are important concepts in the school curriculum that can be interpreted in a variety of ways depending on the situation. Fractions are used for more than just dividing a pizza into pieces. Secondary school pupils' maths performance may be significantly enhanced if learners got a better knowledge of fractions, according to Siegler, Duncan, Davis-Kean, Duckworth, Claessens, Engel, Susperreguy and Chen (2012). Precision and accuracy are required in fields such as architecture, medicine, chemistry, engineering, and technology. This means that understanding the many meanings of fractions, as well as fractional computations, is crucial.

Learners come into contact with fractions informally, such as when sharing sweets, dealing with cards for a card game, or baking/cooking (using ingredients). Despite the fact that learners are exposed to fractions in their daily lives, it is not until they are introduced to fractions in elementary school that they truly encounter abstract thinking. In order to acquire basic mathematics skills, the focus in lower grades is on addition, subtraction, multiplication, and division of whole numbers. Fractions appear in subjects like algebra, geometry, probability, and trigonometry in higher grades. Fractions are taught in a variety of ways in schools, from elementary to secondary. If students want to succeed in higher grades, they must have a solid comprehension of fractions' meanings. Only through being exposed to fractions through instruction will students get a better understanding of them.

Students learn more effectively when they participate actively in the learning process, establishing connections, generalizing, and solving issues. Despite the difficulties in grasping the notion, fractions are extremely important in everyday operations. For example, when cooking at home, one estimates the various ingredient ratios to use in order to save waste. Also, when the knowledge of fraction is well equipped, it will help an individual in securing jobs in areas such as engineering where percentages are used, finance where ratio and proportions are used, and in catering where the measuring of recipes as in addition, subtraction, multiplication and division as aspects of fractions are used. Mathematically, rich fractions have an important place in the learning of algebraic subjects, one of the areas of advanced learning (Redmond, 2009; Smith, 2002). In the elementary or basic school curriculum, fractions are one of the most basic but rarely understood concepts in mathematics. Another school of thought maintains that fractions are one of the most difficult disciplines or content areas in mathematics to teach due to their cognitive complexity (Smith, 2002). Students can understand simple problems, but they struggle with more sophisticated fractional notions (Askew & Ebbutt, 2000). Students memorize the rules, formulas, algorithms, and phrases rather than trying to understand the reasoning behind fractional operations (Murray & Newstead, 1998). Hands-on activities, rather than memorizing, are one of the answers to these issues that develop higher-level thinking abilities.

1.1 Statement of the Problem

Fractions are difficult for both students and teachers, resulting in poor performance in basic and advanced mathematics, which has an impact on adult numeracy, which is essential for functioning in the world (Orpwood, Schollen, Leek, Marinelli-Henriques, & Assiri, 2011). At practically and in almost all levels, students have difficulty in answering issues involving fractions and, as a result, in problem-posing. In mathematics, the concept of fractions has always been regarded as a difficult topic for students to grasp. The majority of students struggle to think about fractions as numbers. They usually think of it as a simple computation (division) or as a complicated set of two numbers written on top of each other (Weinberg, 2001). That is why those students employ certain abstract rules to solve fractions problems without fully comprehending the rules' implications. The fact that rational numbers can be expressed in a variety of ways is most likely why people find them difficult to comprehend (parts of a whole, ratios, quotients). According to research, the majority of students find fractions to be too tough and complex to see and relate to in computations; they also find them difficult to relate to in their daily life.

Mathematical learning is a method of mastering mathematical computations and procedures that requires building on existing knowledge and combining distinct abilities and basic concepts (Ashlock, 2001; Sarwadi & Shahrill, 2014). As a result, rather than focusing on memorizing of rules and processes using drill and practice techniques, teachers must emphasize the importance of basic mathematical concepts at the early stages of learning. Students build on previous information to construct their mathematical knowledge, which means that any misconceptions they develop when learning mathematics may impair their future understanding of related mathematical ideas (Vamvakoussy & Vosniadou, 2010), As a result, it is critical that such errors be discovered as early as possible in order to aid students in correcting them and allowing for future learning of related, more complicated topics.

Mathematical errors and misconceptions were researched by some scholars, who discovered that pupils' mathematical faults hampered their academic progress in mathematically related topics in the future (Sarwadi & Shahrill, 2014). Students' comprehension of decimals was robust, according to Vamvakoussy and Vosniadou (2010), who concluded that students' grasp and conceptualization of decimals was robust and that students found it difficult to make the connection between decimals and fractions. Because fractions are so important in people's daily lives, it's important to learn them (i.e., calculating tips for purchases, discounts, and sharing).

Despite the importance of fractions and teachers' efforts to teach them, the Ghanaian educational system continues to see poor performance in fraction computations at the junior high school level. According to the 2018 Basic Education Certificate Examination Chief Examiners report in Mathematics, one of the applicants' problems was in analysing expressions using fractions, according to the West African Examination Council and KBSL (2020). Since candidates or students from the Afadzato South District in Ghana's Volta Region also took part in this external examination, which was conducted by the West Africa Examination Council, a unique and independent examining body, the issue of poor performance in the concept, fraction has become a major concern. As a result, the researcher took key interest in investigating the conceptions, errors, and misconceptions about learning addition of unlike fractions in Weto Circuit public junior high schools in Ghana"s Afadzato South District.

Furthermore, this research aims to provide insight on the factors that contribute to, or causes students" misconceptions about addition of unlike fractions. Finally, the researcher suggests activities and strategies that can be utilized to help students better grasp fractions and lower the level of difficulty they encounter while performing fraction calculations, especially in adding unlike fractions.

1.2 Purpose of the Study

The purpose of this study was to explore the conceptions and misconceptions in addition of unlike fractions among students in Weto Circuit Public Junior High Schools in Afadzato South District of Ghana.

1.3 Research Objectives

The study sought to:

1. Identify Junior High School pupils" conceptions of addition of unlike fractions.
2. Explore Junior High School pupils" common errors and misconceptions in adding unlike fractions.
3. Identify the factors that causes errors and misconceptions in addition of unlike fractions among Junior High School pupils.
4. Find out the measures that can be used to help improve Junior High School pupils" conceptions and misconceptions in additions of unlike fractions.

1.4 Research Questions

The study was guided by the following research questions.

1. How do Junior High School pupils conceptualize addition of unlike fractions in junior high schools?
2. What common errors and misconceptions do Junior High School pupils make in the learning of addition of unlike fractions?
3. What are the factors that causes errors and misconceptions of addition of unlike fractions among Junior High School pupils?
4. What measures can be used to help improve Junior High School pupils" conceptions and misconceptions of additions of unlike fractions?

1.5 Significance of the Study

The study is aimed to delve into the conceptions and misconceptions of learning addition of unlike fractions in public junior high schools in Afadzato South District in Volta Region of Ghana.

The findings of the study will assist JHS pupils to identify some of the misconceptions they have about addition of unlike fractions and also how to correct these misconceptions.

The findings of this research will help inform the Ministry of Education to develop a comprehensive strategy to equip existing junior high schools with the necessary teaching and learning facilities that will promote teaching and learning of addition of unlike fractions and for that matter mathematics as a whole at that level.

Furthermore, the findings of this research will add to the already existing knowledge that policymakers and other educational stakeholders possess concerning JHS duration. These findings will also serve as reference material for future researchers.

1.6 Delimitations

Delimitations have to do with the scope of the research. There are many public junior high schools in the Afadzato South District in the Volta Region of Ghana. The scope of this study would therefore be limited to only one circuit out of the nine circuits in the district due to several constraints such as time and finance. The Weto Circuit is, therefore, to be used for the study. This is where the researcher finds himself as a worker (teacher). No private school was involved in the study. This is because the performance of learners in mathematics is quite encouraging. The scope of this study will also be delimited to the conceptions and misconceptions in learning addition of unlike fractions. The study also only covers junior high schools, one and two learners.

This is where the topic is intensively taught and learned as enshrined in the mathematics syllabus (NaCCA, 2019).

1.7 Limitations

Limitations of any particular study concern potential weaknesses that are usually out of the researcher‟s control, and are closely associated with the chosen research design. Only limited number of respondents were involved in the study because there was not enough money for the researcher to print more questionnaire and interview guides, to elicit for a wide range of responses from the respondents for the study.

Also, adequate time for the study was not realized. This was seen in a situation where the researcher was teaching, performing administrative roles as the Head teacher of one of the sampled schools and at the same time conducting the study. Here, sometimes the researcher is being invited for Conference of Heads of Basic Schools (CoHBS) meeting at the District Education Directorate, which is very far from where the study is been conducted. By the time he comes back, then he is tired and no academic work could be done, thereby reducing the time frame set for the completion of the study. Again, some teacher respondents were feeling reluctant or unwilling to respond to the items on the interview guide when been interviewed. This is because some of them feel we are on the same level of education before the researcher decided to further his education, resulting into interviewing or probing them for relevant data for my study.

1.8 Organisation of the Study

This study contains five chapters namely: introduction, literature review, methodology, findings and discussions; and conclusions and recommendations. Chapter One has presented the different descriptions of fractions and their relevance. Also contained in Chapter One is the introduction which encompasses the background of the study, statement of the problem, the purpose of the study, research objectives, research questions, significance of the study and delimitations. Chapter Two of the study focuses on reviewing literature that will be relevant to the study. Chapter Three provides the methodology used in the study. It looks at the methods the researcher used to gather data and to address the research questions that guided the study. Issues covered include the research design, the population, the sample and sampling procedures, the instruments used to gather the data, and the data collection procedure. It also presents the various procedures adopted by the researcher in analysing the data gathered. Chapter Four contains the research findings and analyses the data collected and report salient findings determined from the study. The researcher used data tables, and statements to effectively present these findings. Chapter Five provides a summary of the whole study. The findings of the study and the conclusions drawn from the findings are provided. Recommendations for policy and practice, and suggestions for further studies are also provided.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

This chapter examines the literature on basic school students' misconceptions about adding unlike fractions in the mathematics classroom. This chapter has been divided into three main sections: theoretical, conceptual, and empirical. The theoretical framework encapsulates the theory that underpins or supports the research. The conceptual framework addresses themes such as fraction conceptions and misconceptions, fraction addition conceptions and misconceptions, addition of unlike fractions conceptions and misconceptions, empirical examination of misconceptions of unlike fractions, and a chapter summary.

2.1 Theoretical Framework

Mathematics, according to Sarwadi and Shahril (2014), is a method of measuring that requires a full understanding of concepts. It is a process of lifelong learning that includes the conceptualization of earlier information, the development of varied abilities, and the mastering of fundamental mathematical concepts. Experiments, survey research, and observations all contribute to the formation of theories and conceptions, which help to improve awareness (Mohyuddin & Khalil, 2016). As a result, mathematics instructors are expected to employ a variety of strategies during the teaching and learning process in order to assess student‟s progress and improve instruction.

Albert Bandura's social cognitive theory served as a rock-solid foundation. Ideas from cognitivist, constructivist, and socio-cultural theories on how children learn mathematics and can be tested to ensure improved learning, according to the sociocultural constructivist (Legacy, 2010). According to socio-cultural constructivists, the most recent approaches or methods for teaching fraction addition in the classroom have moved away from the idea of the teacher being the centre of attention in the classroom and toward the idea of the teacher involving the learners in every teaching and learning activity.

Learners are seen as actively creating knowledge and understanding through cognitive processes (Piaget, 1954) within a social and cultural context (Greenfield, 2009; Vygotsky and Cole, 1978), and within a social and cultural context (Bransford, Brown, & Cocking, 2000); as building new knowledge on what they already know (Bransford, Brown, & Cocking, 2000); and as developing the metacognitive skills needed to regulate their learning (Bransford, Brown, & Bruner, 1985; Vygotsky, 1978). These learning and development understandings have consequences for the employment of the proper technique in the teaching and learning of fraction addition in the classroom.

Vygotsky and Cole‟s (1978) work clarify the sociocultural components of this idea. The role of the social context in the development of knowledge is emphasized in this theory. Students build knowledge and comprehension in an area over time in an interactive social setting under the leadership of a more experienced individual, such as teachers, according to Vygotsky and Cole (1978).

Studies in cognitive theory (Bruning, Schraw, & Norby, 2011; Piaget, 1965) have shown the relationship between different cognitive abilities and mathematical abilities. Cognitive theory views the learner as a thinking individual actively processing information to increase the breadth and depth of his or her knowledge. The learner participates in the process of knowledge acquisition and integration. From this perspective, mathematics instructions based on the cognitive theoretical principles should be authentic and real (Yilmaz, 2011). The teacher is expected to provide a rich classroom environment with activities that foster explorations and investigations for meaningful learning. Environments that provide for explorations engage the learners in mental and social conversations as well as practice. This requires the availability of instructional materials through which students become active constructors of their own knowledge from interactive experiences in a meaningful context. This means mathematics instructional materials should provide for demonstrations, illustrative examples, and constructive feedback for students to develop their mental models learning.

The cognitive approach to learning mathematics focuses on making mathematical knowledge meaningful and helping learners to relate new concepts to existing ones. Cognitivism place emphasis on learning by problem solving as a recursive process, whereby a problem is interpreted by assigning it to existing internal representations or schema (Klinger, 2009). Although individuals develop their personal schema through varied experiences, the schema must always be activated for learning new experiences. To activate and utilize schema for learning, the learner must be made aware of the relevant previous knowledge and strategies to bridge to pre-requisite skills to the new learning objectives (Yilmaz, 2011). The cognitive approach has been shown to yield superior learning outcomes for more experienced learners. This implies that, mathematics teachers should present ideas based on existing mathematical schema in learners.

The ultimate goal of learning mathematics is to understand and apply the concepts and skills in addressing everyday problems in our society. Skemp (1976; 1989) describe understanding as assimilating into the appropriate cognitive schema. Piaget explained cognitive schema in terms of assimilation and accommodation. When the learner uses existing schema to make sense of environmental stimuli or mathematical experiences, the learner is assimilating that experience. In other words, assimilation is how one interprets classroom or life experiences. Accommodation is the adjustment of the internal models after interpreting classroom or life experiences. Learners, therefore, use their schema or mental frameworks to build their internal senses of reality. From the cognitivist perspective, it is important for mathematics teachers to create conditions and strategies for learners that facilitate connections between new mathematical ideas and their prior knowledge in the long-term memory for understanding.

Yilmaz (2011) listed six (6) methods as the most distinctive methods of teaching based on a cognitive perspective on learning. These are: cognitive apprenticeship, reciprocal teaching, anchored instruction, inquiry learning, discovery learning and problem-based learning.

The method of cognitive apprenticeship helps the learner understand concepts and procedures under the guidance of the teacher who is seen as an expert in the field. Cognitive apprenticeship is based on Vygotsky‟s ZPD and involve five (5) processes namely: modelling, coaching, articulation, reflection, and exploration. *Modelling*-The teacher performs the task for students to see. This provides learners with the opportunity to develop conditionalized knowledge. *Coaching*-Students perform the task while the teacher observes and provide hints, clues, feedback, and help when the need arises. *Articulation*-Students are made to think aloud on what they do as they perform a given task and justify their actions. By making students externalize their actions and strategies, the teacher can determine their misconceptions or use of inappropriate strategies; *Reflections*- As students complete their tasks, they are encouraged to look back and compare their actions with their teacher or peers to evaluate their performance; *Exploration*- Opportunities are provided for students to identify a problem, formulate a hypothesis and explore opportunities for its resolutions. This provides the child space for independent thinking.

Cognitive learning strategies

Cognitive learning can be seen as an approach to learning in which the learner uses the brain to construct knowledge that is meaningful to him or her. Learning strategies can be classified into three (3) as: Cognitive, Metacognitive (organizing learning), and Social/Affective strategies (involves interactions).

Cognitive strategies

Cognitive strategies can be described as memorized strategies intended to develop the learner‟s thinking skills for problem solving. In Suharno‟s (2010) view, a cognitive strategy is "an organized internal competence which can lead to the students in their learning process, that is, thinking process, problem solving and decision making. It enables the student to think systematically and critically" (p. 62). The aim of using cognitive strategies in learning mathematics is to develop the cognitive processes of the learner. Cognitive strategies make thinking process unique and thereby giving the student executive control of his or her actions and are deployed by learners to be successful. Successful learners understand or commit what they learn permanently to memory. Based on Kennedy, et al. (2004) assertion that "children must understand whatever they are learning if the learning is to be permanent" (p. 30), cognitive strategies in the mathematics classroom include strategies that enhance committing what is learned to long term memory. Some cognitive strategies include: making mind maps, visualizations, associations intended to make learners make mental connections of what is learnt. Hamzeh‟s (2014) study of teaching strategies used by the Jordan mathematics teachers using a questionnaire that covers the behavioural, cognitive and affective domains, the cognitive strategies were the second highly used strategies in the mathematics classroom. Of the 15 cognitive strategies in the instrument, eight (8) were highly used and the remaining seven (7) were moderately used. The highly used cognitive strategies include teachers:

1. Connecting lesson parts together;
2. Encouraging learners to verify information and facts before giving judgement;
3. Moving from abstract to examples;
4. Teaching learners to plan, observe, and evaluate their teaching activities;
5. Giving opportunities for learners to generate new ideas;
6. Using problem solving strategy in the teaching situations; and
7. Giving opportunities for learners to question and investigate among others.

A critical look at these strategies suggests that cognitive strategies are essentially actions taken to engage the learner‟s mind acquire and process information to learn. From your experiences as a learner with different mathematics teachers, what strategies can you add to this list? Cognitive strategies are consciously employed to regulate thought processes in order to solve problems. However, different teachers tend to use different strategies to help their children learn mathematics. The strategy a teacher may develop is a function of several variables including the knowledge base of the teacher and the cognitive ability of the teacher.

According to cognitivist theorists, the transfer of information into long-term memory occurs as either assimilation or accommodation. Assimilation occurs when the information is changed to fit into existing cognitive structure. During accommodation, the existing cognitive structure is changed to incorporate the new information. Mathematics teachers se cognitive strategies that will facilitate this information process for easy learning.

Learning is explained as a social mechanism and the basis of human intelligence in society or culture by Vygotsky's sociocultural theory of human learning. Vygotsky's theoretical framework revolves around the idea that social interaction is crucial to a child's cognitive development. Everything, according to Vygotsky, is taught on two levels. Contact with others comes first, followed by incorporation into the person's mental framework. Each function in a child's cultural development occurs twice: first on a social level, then on a personal level; first between individuals (inter-psychological), then within the kid (intra-psychological). This is true of voluntary attention, conceptual memory, and idea generation as well. All of the higher roles manifest as genuine human interactions (Vygotsky & Cole, 1978).

The concept that cognitive development ability is limited to a zone of proximal development is the second aspect of Vygotsky's theory (ZPD). This area of study in which the student is cognitively equipped but need more support and social connection in order to fully develop (Bruner, 1985). Scaffolding can be provided by an instructor or a more experienced colleague to aid the learner's emerging comprehension of knowledge domains or sophisticated skill development. Collaborative learning, conversation, modelling, and scaffolding are examples of strategies that improve analytical knowledge and abilities while also encouraging deliberate learning. Vygotsky and Cole‟s (1978) views were articulated by Ash and Levitt (2003), who argued that learners learn best in a collaborative manner with teachers in a social setting, rather than as individuals. Teachers and students collaborate in this way to ensure that the learning goals or objectives are met (Ash & Levitt, 2003). Teachers' roles in this regard are to act as a link between students and learning objectives, as well as to offer the necessary support to aid in the achievement of the goal (Black & Wiliam, 2009; Walqui & van Lier, 2010).

This suggests that the youngster can execute a task independently to a certain extent before requiring the support of a more experienced individual to finish it. This causes a void in the child's ability to learn a specific skill. The "zone of proximal development," as defined by Vygotsky and Cole (1978), is the developmental gap between what a youngster can achieve independently and what the individual can do with the help of a more competent person (ZPD). This puts the teacher in a position to swiftly correct any misunderstandings regarding a particular idea. It also evaluates an interactive exercise in which the child can ask and answer questions throughout the learning process. According to Heritage (2010), effective teaching and learning includes the function of interaction between and among teacher-student(s) and students-students, as well as collective activity in the learning process, from a socio-cultural perspective. According to Heritage, effective learning is multifaceted, involving both instructors and students in a collaborative effort to increase learning and achieve the intended goal within a community of practice. Teachers and students work together to categorize this activity as they respond to a learning indicator (Leahy, Lyon, Thompson, & Wiliam, 2005).

According to the sociocultural constructivist perspective, intelligent thoughts require metacognition, or self-monitoring of learning and thinking, and that learning should allow students to develop these metacognitive skills (Shepard, 2000). As a result of its function in guaranteeing effective learning and increased pupil performance in today's society, this idea has been embraced. As a result, the theories will give a framework for mathematics teaching and learning, allowing researchers to better understand the poor performance of students in the Afadzato South District, particularly in mathematics.

2.2 The Concept of Fractions

Fractions, according to Copeland (1967), are symbols or numerals that indicate a collection of numbers known as fractional numbers, and a fraction can be thought of as a fractured portion of a larger whole. Fractions were viewed by this said researcher as (i) part of a whole, (ii) parts of a group of objects as well as portions of a single unit, (iii) division indicators, (iv) comparison indicators, and (v) numerals. D' Augustine (1968) agreed with Copeland (1967) and stated a fraction in the form a/b, where a and b are whole numbers and a and b are the numerator and denominator, respectively. Other authors (Collier & Lerch, 1969; Fellr & Phillips, 1972; Gerber, 1982; Kinney, Marks & Puidy, 1965; May, 1970) backed up Copeland's fraction definitions and qualities.

However, according to Fehr & Phillips (1972) and Gerber (1982), distinguishing between the term‟s “fraction” and "fractional numbers" is problematic. Even though it is not suitable to provide the definition of fractional numbers to the child during his early intuitive discoveries, D' Augustine (1968) maintained that the basic definition should always play a role in the teacher's presentation. May (1970) went on to say that the true meaning of fractional numbers can't be taught until a student's understanding has progressed beyond halves and fourths. Several authors (Beavers, 1985; Hoelzle, Hutchison & Streeter, 1995; May, 1970; Williams & Shuard, 1988; Reisman, 1977) shared D‟Augustine's (1968) ideas on numerators and denominators definition. The denominator, on the other hand, tells us how many parts a unit or a whole has been divided into, whereas the numerator tells us how many parts of a unit are used. These authors employed circular, rectangular, and triangular diagrams to show various numerators and denominators. Too often, fraction terminology is reinforced before students comprehend any of the basic principles, according to Beardslee & Jerman (1978) and Paling (1982). They claimed that children are frequently perplexed by the terminology. They reasoned that introducing the phrases 'numerator' and 'denominator' would be easier to learn if the teachers used the terms frequently rather than forcing the "little children" to use them.

Fractions were introduced as part of a whole and as part of a set in the Ghana Mathematical Series, Primary Mathematics, Pupil's Book One to Three (NaCCA, 2019) mathematics textbooks for basic schools. Shaded congruent portions of geometrical forms, illustrations, and the number line are all used extensively to illustrate the concept of fractions. The lack of the term‟s „numerator‟ and „denominator‟ in these publications is noteworthy. The Ghana Mathematic Series, Primary Mathematics, Pupil's Book Four to Six, revises and emphasizes the concept of fractions as a part of the whole and a set. The number line is used extensively in the Ghana Mathematics Series, Junior Secondary School, Pupil's Book One to consolidate the concept of fractions as parts of a whole. The terms 'numerator' and 'denominator' were used without definition in this textbook and the Pupil's Book Four, Book Five and Book Six (NaCCA, 2019).

2.2.1 Equivalent Fractions

Two fractions that reflect the same fractional number are said to be equivalent or equal, according to some writers (Byrne, 1966; Gerber, 1982; Hutchison & Streeter, 1995). If a/b is comparable to c/d, then a/b = c/d if and only if a/d= b/c was stated. The equivalency rule was used to achieve this. Other authors (Blevins, Hanson, Podraza & Prall, 1969; Brown & Webber, 1963; Kinney, Marks, & Puidy, 1965) selected the term "the equality of fractions" to describe the concept. The equality concept, often known as the equivalent rule, was explained with illustrations. Although Ganoe, Grossnickle, Perry, and Reckzeh (1983) prefer the word "equal fraction" to emphasize that two fractional numerals relate to the same number, both expressions can refer to the same concept. Fehr & Phillips (1972) agreed with Ganoe et al (1983), stating that distinguishing between the logical differences of equal and equivalent has no real significance when teaching fraction equivalency. 1/2 and 2/4 may be referred to be equivalent by the teacher because they represent the same number.

However, Beardslee & Jerman (1978) recommended that while explaining equivalent fractions, it is important to clarify between the concepts of equal and equivalence so that children are not confused. It was feared that if youngsters saw a/b = c/d, they would assume that a = c and b = d. Some textbooks utilized the symbol "," to indicate equivalence, but most textbooks use the "=" sign to reduce symbolism. The region model (the most commonly used model) displays comparable fractions by comparing regions of equal area, which is one of the causes for the confusion between equal and equivalence. Through the use of geometric shape, the notion of equivalent fraction was presented to learners in the Ghana Mathematics Series, Primary Mathematics, Pupil's Book Three to Six. The subject was covered in Pupil's Books 5 and 6, with a few illustrations. The equality test, which states that a/b =c/d is true only if ad =bc, was stated. With coloured geometrical forms and the number line, the Ghana Mathematics Series, Junior Secondary School, Pupil's Book One consolidated the concept of equivalent fractions. It ended with the fundamental premise of fractions: a/b = (ma)/(mb) if „m" is a whole number and b and a are counting numbers. Some researchers, "Titters (Pothier & Sawada, 1990: Rowan, Payne & Towsley, 1990; Vance, 1992) advised that the notion develops slowly over time, anticipating the challenges teachers could experience in teaching equal fractions. Because no present textbook contains appropriate developmental work on concepts, teachers were recommended to provide many opportunities for pupils to explore and create important adjustments to textbooks.

2.2.2 Addition of Fractions Involving Like and Unlike Denominators

Several authors (Beavers, 1985; Booth, Dossey, Randull & Smith, 1992; Hoelzle, Hutchison & Streeter, 1995; Shuard & Williams, 1988) described how to add fractions with like and unlike denominators. The numerators should be added and the sum placed over the same denominator to add two fractional values with the same denominators. To add fractional numbers with distinct denominators, we should first describe the fractions as equivalent fractions with similar denominators using the least common multiple ideas. The numerators of the resulting fractions should be put together, and the result should be placed over the common denominator. When necessary, the student is reminded to simplify the resulting fraction. In every case, worked examples were provided to demonstrate how the concepts were applied. Beavers (1985) and Gerber (1982), on the other hand, used more diagrammatic images to explain these principles. Gerber (1982) established the notion that if a/b and c/b are two fractional integers, then a/b+c/b = ((a+c))/b for the addition of fractions with the same denominators. The researcher also argued that a/b+c/b = ad/bd + bc/bd = ((ad+bc)/bd for fractional quantities with distinct denominators, such as a/b and c/b. Several other authors (Bennett & Nelson, 1998; Blevins, Hanson, Podraza, & Prall, 1969; Brown & Webber, 1963; Byrne, 1966; Demana & Leitzel, 1984; Dessart & Suydam, 1978; Dumas & Howard, 1966; Lake & Newmark, 1977) agreed with Gerber's viewpoint (1982). The general ideas were explained, along with illustrations.

Dessart and Suydam (1978) questioned if such a formal definition would be adequate as an algorithm technique for youngsters when adding and subtracting fractions, especially when the numerators and denominators are different. Even if the kid does not need to find a least common denominator, it was suggested that the evident disadvantage is the increased number of errors in expressing the final answer to the lowest term. Teachers should perform early problem-solving work involving addition and subtraction of fractions with comparable denominators on an exploratory level, using manipulative materials, drawings, and visual models, according to Ganoe, Grossnickle, and Reekzeh (1983).

D'Augustine (1968) agreed with Ganoe et al. viewpoints (1983). The researcher claims that using a number line has an advantage over the majority of other models we may use. The researcher again claimed that the number line may be easily changed for fractional number sums less than or equal to one, as well as sums greater than one. In the Ghana Mathematics Series, Primary Mathematics, Pupil's Book Three, the number line and shaded geometric shapes were utilized to introduce the concept of addition of fractions with like denominators to children (NaCCA, 2019). Diagrams are used to illustrate worked instances. By way of further elaboration, the concept is not consolidated in the Pupil's Book Four. Pupils were given practice exercises on adding fractions with like and unlike denominators. In the Pupil‟s Book Five, the addition of fractions with similar and unlike denominators is addressed. There are no diagrammatic illustrations in the worked examples. The least common multiple is a notion that is used to rename fractions into comparable fractions with common denominators. The sum of the numerators of these equivalent fractions is laid over the common denominator.

The definition for the least common denominator and its application are described in the Ghana Mathematics Series, Primary Mathematics, Pupil's Book Six, under the topic „Addition and Subtraction of Rational Numbers.‟ Following the discussion, students were given practice exercises involving like and unlike denominators. Pupils were given practice exercises on adding fractions with like and unlike denominators. In the Pupil's Book Five, the addition of fractions with similar and unlike denominators is addressed. There are no diagrammatic illustrations in the worked examples. The least common multiple is a notion that is used to rename fractions into comparable fractions with common denominators. The sum of the numerators of these equivalent fractions is laid over the common denominator. The definition for the least common denominator and its application are described in the Ghana Mathematics Series, Primary Mathematics, Pupil's Book Six, under the topic Addition and Subtraction of Rational Numbers. Following the discussion, students were given practice exercises involving like and unlike denominators.

Pupils are not provided worked examples to study. With a few worked examples, the Ghana Mathematics Series, Junior Secondary School, Pupil's Book One (NaCCA, 2019) covered solely addition of fractions with unlike denominators. The least common multiples approach was utilized to rename the given fractions into equivalent fractions with common denominators. The equivalent fractions' numerators were put together, and the result was placed over the shared denominator. According to Kinney, Marks, and Puidy (1965), careful education is essential in the addition of fractions to avoid errors such as adding both numerators and denominators. They claim that such issues can be avoided by carefully selecting situations in which students name sums using items and fractional numbers are related to whole numbers while working with students. figuring out the technique Kinney et al. were supported by certain writers (Copeland, 1967; Fehr & Philips, 1972; Reisman, 1977).

The use of a useful pedagogical device to help students avoid mistakes like adding both numerators and denominators was highlighted. This was accomplished by writing the denominator as a word, i.e. (a) 2 fifths + I fifth and (b) I half + I third. This device, it was suggested, might serve as a constant reminder and check for the learner to change the fractions to be added so that they have the same term before adding. Copeland (1967) went on to say that presenting the numbers as numerators and the words as denominators in a vertical format helps to develop the idea of adding the measurements (numerators) and thinking of the denominators as the unit of measure. It's also worth noting that, regardless of what professors say about how fractions should be added, children continue to assume that something fundamentally proper about the 'top + top' method over the 'bottom + bottom one, according to Howard (1991). The researcher observed that children are unable to reason through the meaning of adding fractions on their own and instead rely on rote learning or habit. The researcher came to the conclusion that students struggle to understand the '+' procedure for adding fractions because it does not fit into their previous arithmetic operations schemas. The diagram below shows the concept mapping of fraction.

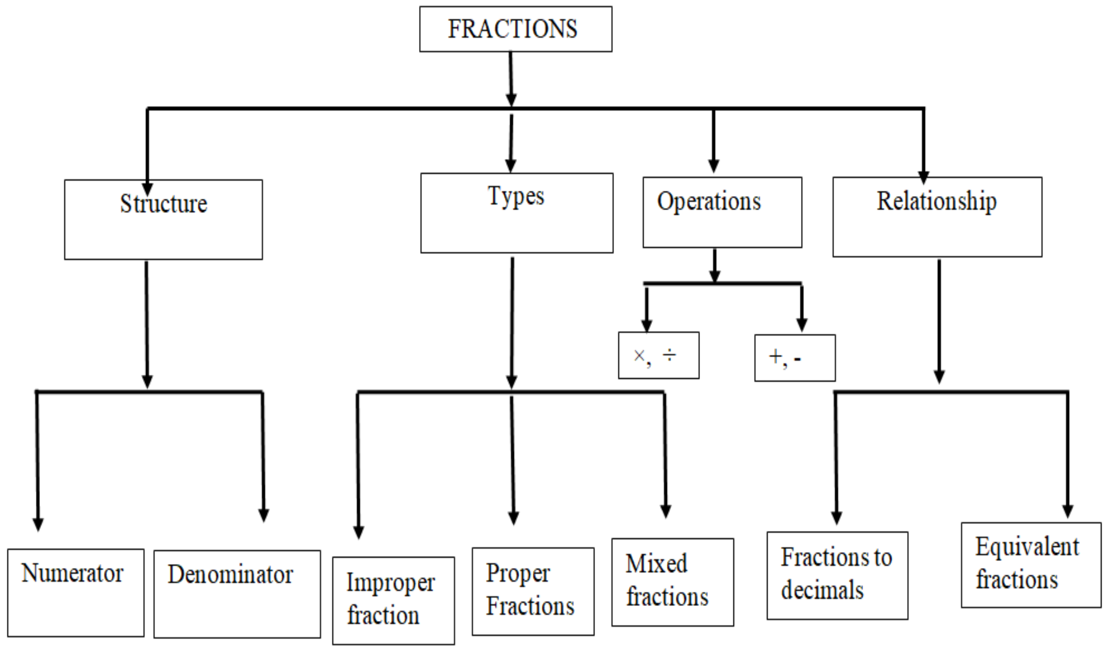

Figure 2.1: Mind Mapping of fraction.

The below diagram also depicts the Concept map of Adding Unlike Fractions.

Conceptual Framework

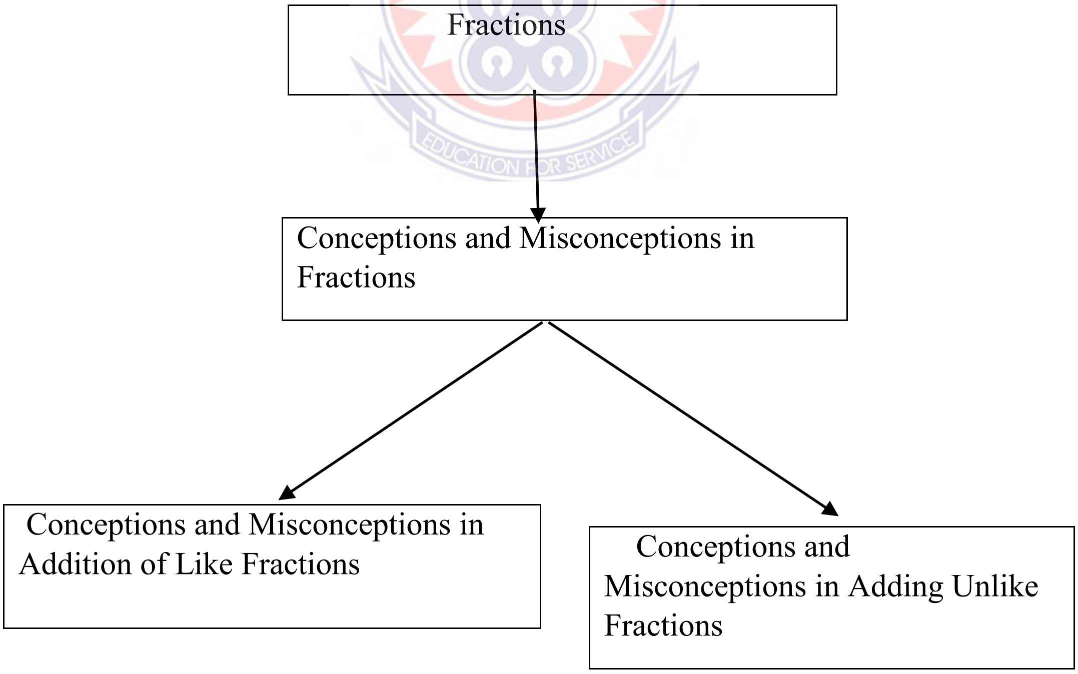

Figure 2.2: Concept mapping of Adding Unlike Fractions

2.3 Mathematical Mistakes and Misconceptions

The terms "mistake" and "error" were employed in a variety of settings. This chapter delves into the definitions of error and misunderstanding, as well as the link between the two. To clarify the words, examples of students' arithmetic blunders and misconceptions from the literature are supplied. Misunderstanding has a variety of definitions in the literature. Hammer (1996) uses terminology like preconception, alternate conceptions, and naive conceptions to describe misperception. A student's "misconception" is a false belief that leads to a series of blunders (Smith, diSessa, & Roschelle, 1993, p.205). The apparent gaps between students' concepts and the beliefs of consistent experts are also referred to as a misperception (Zembat, 2007). Misconceptions, according to Hammer (1996), include: are strongly held, stable cognitive structures,

i. differ from expert conceptions,
ii. affect in a fundamental sense how students understand natural phenomena and scientific explanations; and
iii. must be overcome, avoided, or eliminated for students to achieve expert understanding (Hammer, 1996).

The wrong application of a law, over- or under generalization, or a distinct perspective on the circumstance are all examples of misconceptions (Drew, 2005). The term "mistake" is understood in a variety of ways in the literature, in addition to being a misunderstanding. A divergence from precision is referred to as a mistake, slip, mishap, or inaccuracy (Luneta & Makonye, 2010). Riccomini (2005) defines unsystematic errors as "accidental and atypical incorrect responses." These inaccurate responses are simply correctable by students. On the other hand, systematic errors result in the same inaccurate replies being offered over and over again. These false answers are constructed and generated in a systematic manner across location and time. In addition to Riccomini's classification, Tirosh (2000) separated student errors into three categories: algorithmically based mistakes, intuitively based mistakes, and mistakes based on structured knowledge. Among the algorithmically dependent errors were mistakes in arithmetic operations. A common student blunder, for example, was the following multiplication operation. Students' notions and assumptions about mathematical entities, as well as the mental models they employed to represent mathematical concepts and processes, were the source of these inaccuracies, (Even & Tirosh, 2008). 'Multiplication always makes things larger, and division always makes them smaller,' was an example of a blunder. Last but not least, there were inaccuracies that were based on formal data. Formal knowledge included axioms, definitions, theorems, and proofs (Fischbein, 1994).

Mistakes were made due to a lack of understanding relevant to this organized knowledge (Tirosh, 2000). Consider the mistaken idea that division was commutative, resulting in' ='= 2. Errors can happen for a variety of reasons. Carelessness, a misunderstanding of symbols or language, a lack of relevant knowledge or expertise connected to the mathematical topic/learning objective/concept, a lack of comprehension or incapacity to check the response given, or a misinterpretation could all be contributing factors to the errors (Drew, 2005). Despite the fact that errors and myths were linked, they were distinct. Students' replies were previously split into two categories: correct and incorrect, obscuring the underlying logical defects of their errors (Smith, diSessa, & Roschelle, 1993). Researchers, on the other hand, concurred that prejudices were to blame for the inaccuracies (Drew, 2005; Zembat, 2007). They went on to claim that learners' misconceptions were intuitively rational, and that corrective training may be motivated by them. Mistakes are obvious, such as in written text or student speech. On the other side, misconceptions are frequently hidden. Misconceptions can mask inaccurate replies when correct answers are offered by mistake. The following section will identify the students' mathematical errors and misconceptions, as well as provide examples to further demonstrate the relationship between errors and misconceptions.

2.4 Conceptions, Errors and Misconceptions of Fractions

Fractions have long been seen as a difficult concept for pupils to master in mathematics. The majority of children have difficulty visualizing fractions as numbers. They frequently consider it to be a simple computation (division) or a complex collection of two integers written on top of each other (Weinberg, 2001). As a result, some pupils answer fractions problems using sophisticated abstract rules without fully appreciating the rules' implications. The fact that rational numbers can be stated in a multitude of ways is most likely why people struggle to understand them (parts of a whole, ratios, quotients). When it comes to equations, the majority of pupils perceive fractions to be too difficult to understand and solve to envisage and put into practice in their daily lives.

Mathematical learning is a process of acquiring mathematical computations and procedures that combines multiple abilities and basic concepts while building on existing knowledge (Ashlock, 2001; Sarwadi & Shahrill, 2014). As a result, rather than focusing on drill and practice methods for memorizing of rules and processes, teachers must emphasize the relevance of basic mathematical principles in the early stages of learning. Students build on prior information to construct their mathematical knowledge; as a result, any misconceptions they have while learning mathematics can affect their future learning of related mathematical topics (Vamvakoussy &

Vosniadou, 2010). As a result, such misunderstandings must be discovered as soon as possible so that students can be guided in correcting them. Some typical misconceptions concerning fractions in teaching and learning are as follows:

Students make errors as a result of carelessness, misinterpretation of symbols and text, a lack of relevant experience or expertise related to the mathematical subject, learning objectives, and a lack of understanding or inability to verify the answers given, according to Vamvakoussy and Vosniadou (2010). Sarwadi and Shahrill (2014) investigated mathematical errors and misconceptions and found that students' mathematical errors impeded their future academic performance in mathematically related courses. The students' understanding of decimals was strong. According to Vamvakoussy and Vosniadou (2010), students' grasp and conceptualization of decimals were strong, but they struggled to make the connection between decimals and fractions. Because fractions are so significant in people's daily lives, it's critical to understand them (i.e., calculating tips for purchases, discounts, and sharing). The researcher investigates the numerous fraction myths claimed by college students. Furthermore, the goal of this study is to refute widespread misconceptions concerning fractions. The pupils' fraction understanding as well as their fraction mistake patterns are studied.

Mack (1990) investigated how pupils' understanding of fractions evolved over time, in terms of how they employed informal information and the impact of rote processes knowledge. In this study, the researcher used eight sixth-grade children with weak fractions skills. The teaching content was derived from subjects in conventional textbook fraction chapters by the researcher. Furthermore, the necessity of students' informal fraction estimating expertise for education was underlined.

According to the researcher, both pupils entered the lesson with misconceptions about fractions and a plethora of informal knowledge. Students came up with alternate algorithms based on their formal understanding following the class, said by the investigator or the researcher. Furthermore, the researcher argues that all students' informal knowledge enabled them to determine the units in real-life difficulties; nonetheless, students had difficulty recognizing the symbolic and concrete unit specified. She advocated for as many tangible linkages as possible between fraction symbols and pupils' informal knowledge.

Brown and Quinn (2006) investigated student error patterns when applying fraction concepts and performing fraction operations in order to give teachers with an explanation for students' common mistakes. A 25-item questionnaire was distributed to 143 primary algebra classes. According to the researcher, pupils were confused about the correct algorithmic strategy when it came to things that might be applied to a definition precisely. Furthermore, they claimed that knowing fraction operations was unrelated to student fraction operations. According to them, student responses, for example, demonstrated that students did not understand the relative magnitude of fractions or the multiplicative operator. Finally, they came to the conclusion that pupils lacked knowledge with basic fraction ideas, and that the findings demonstrated students' lack of fluency with fraction computation.

Students‟ understanding of fractions and fraction operations was found to be lacking in both of these investigations. There have also been numerous reports in Turkey on students' perceptions of fractions and their difficulties with them. In the next three trials, the researchers looked at the students' beliefs and misconceptions concerning fractions.

Haser and Ubuz (2002) investigated students' perceptions of fractions when completing word problems. In this sample, 122 fifth-grade elementary children were given ten-word problems to solve. Finally, they came to the conclusion that pupils lacked knowledge with basic fraction ideas, and that the findings demonstrated students' lack of fluency with fraction computation.

The findings of these two investigations demonstrated that students' understanding of fractions is hindered by students' lack of fluency with fraction computation. Students could not understand the part-whole link, according to the researchers, and were ignorant of the operation's consequent unit. Students also have misconceptions regarding fractions, according to them. The students in the study, for example, were unaware that the numerator and denominator of a fraction must both be natural numbers. According to the study, students had difficulty performing mixed number computations with more than one whole. Also, pupils were discovered to be mixing fractional parts and wholes, as well as picking inappropriate operations, due to their inability to grasp the issue effectively. These results were identical to those previously reported.

Finally, they came to the conclusion that pupils lacked knowledge with basic fraction ideas, and that the findings demonstrated students' lack of fluency with fraction computation.

Pesen (2007) investigated the misconceptions that underpin popular fractions errors among third-graders, and these two studies revealed that students' comprehension of fractions is as a result of, lack of fractional language in teaching the concept. This was seen when 131 pupils from 11 different primary schools were given a diagnostic test with 24 items. At the end of the investigation, the researcher concluded that pupils made common mistakes while breaking the entire into equal pieces. Furthermore, he claimed that children had difficulty dividing circular forms into equal parts as compared to rectangular ones, and that they misplaced and swapped the numerators and denominators. Finally, they came to the conclusion that pupils lacked knowledge with basic fraction ideas, and that the findings demonstrated students' lack of fluency with fraction computation.

These two investigations found that pupils' understanding of Pesen was poor. The researcher also claimed that, despite having difficulty reading fractional numbers, children could write fraction numbers that belonged to a model. To summarize, the part-whole relationship is the most difficult feature of fractions for pupils, as evidenced by the studies above (Haser & Ubuz, 2002; Pesen, 2007). The following paragraph summarizes the findings of a study on pupils' fractions problems. Alacac (2009) explored students' misconceptions about fractions and their difficulty with them, as well as the reasons behind these misconceptions. Finally, they came to the conclusion that pupils lacked knowledge with basic fraction ideas, and that the findings demonstrated students' lack of fluency with fraction computation.

These two investigations found that pupils' understanding of Pesen was poor. According to him, students' understanding of the complete definition, fraction idea, fraction comparison, improper fraction units, and fraction computations was restricted. The pupils' initial obstacle is the concept as a whole. Students think that two fractional numbers that are the same always depict the same quantity; nevertheless, by utilizing different-sized wholes, two fractions might refer to different-sized fractional parts. Kofi eats half of a pizza in one question, while Adwoa eats half of another pizza in another. According to Kofi, he consumes more pizza than

Adwoa. Finally, the researchers here, came to the conclusion that pupils lacked knowledge with basic fraction ideas, and that the findings demonstrated students' lack of fluency with fraction computation.

These two investigations found that students' understanding of Pesen Adwoa stated that they both consume the same quantity of pizza. Which is the correct answer? The findings of this question revealed that the majority of pupils lacked conceptual understanding of the entire idea. Kofi had eaten more pizza than Adwoa, according to only a quarter of the students. The second issue is with the concept of fractions. Students didn't comprehend that fractional parts are equal shares or pieces of the same size. Finally, students struggled with fraction comparison. Finally, they came to the conclusion that pupils lacked knowledge with basic fraction ideas, and that the findings demonstrated students' lack of fluency with fraction computation.

These two investigations found that pupils' understanding of Pesen was poor. They reasoned that fractions with larger numerators and denominators would be larger. Another issue was determining the unit of an improper fraction. Furthermore, because the numerator and denominator were perceived as independent integers, pupils struggled to compute fractions. Finally, intuitions about fractions and language challenges with fractions produced problems with fractions (Alacac, 2009). Finally, they came to the conclusion that pupils lacked knowledge with basic fraction ideas, and that the findings demonstrated students' lack of fluency with fraction computation.

Finally, these trials demonstrated that pupils had logical flaws, inaccuracies, and misconceptions concerning fractions, according to the results. In this study, the knowledge of prospective primary mathematics teachers towards elementary students' faults was investigated. Students' algorithmically based errors, intuitively based errors, and errors based on formal fraction information were all questioned about by the prospective teachers. In addition, their comprehension of the causes of these errors was tested.

2.5 Conceptions, Errors and Misconceptions of Additions of Unlike Fractions

Finally, they came to the conclusion that pupils lacked knowledge with basic fraction ideas, and that the findings demonstrated students' lack of fluency with fraction computation.

Misconceptions are erroneous perceptions that lead to misunderstandings and misinterpretations. They're brought on by 'naive theories,' which block learners' ability to reason logically. Misconceptions occur in many different sizes and shapes. According to a correct knowledge of money, the value of coin currency, for example, is unrelated to its size. However, some Pre-K children have fundamental misconceptions about money and the value of coins. Some kids believe nickels are more valuable than dimes because they are larger. Finally, they came to the conclusion that pupils lacked knowledge with basic fraction ideas, and that the findings demonstrated students' lack of fluency with fraction computation. Misconceptions are misinterpretations and misunderstandings. Some elementary and even middle school pupils believe that 1/4 is larger than 1/2 since 4 is more than 2.

Furthermore, it is a prevalent fallacy that multiplication always results in an increase in a number. This makes learning how to multiply a positive number by a fraction less than one challenging for children. According to Ojose (2015), misconceptions persist "in part due to students' overriding drive to make meaning of the instruction that they get" (p. xii). For example, the rules for adding fractions with like and unlike denominators are slightly different. As they advance from adding fractions with similar denominators to adding fractions with different denominators, students must make sense of the varied scenarios and make improvements. Learners commonly suffer cognitive conflicts and dissonance as a result of the change, according to Ojose (2015), because it requires them to unlearn what they previously knew. Because school mathematics is so important, it is crucial to understand how myths manifest.

The rules can appear to alter from one definition to the next from the perspective of a pupil. Now, 0.4 + 0.7 equals 1.1 (one decimal place) when decimals are added by addition, whereas 0.4 × 0.7 equals 0.28 when decimals are multiplied (Two decimal places). Students may have misconceptions about the difference between addition and multiplication with decimals. Another feature of mathematics is that some incorrect procedures and calculation errors can lead to accurate answers, which may be one of the reasons why students adhere to them. When you split 1/9 by 1/3, for example, you obtain 1/3. Students could divide the numerators to obtain 1 and then divide the denominators to obtain 3 when confronted with this problem, resulting in a 1/3 as correct solution (through a mathematically incorrect method). When this happens, it is the classroom teacher's obligation to recognize and correct the misconception. In general, understanding the nature of a misperception and its source aids teachers in devising strategies for providing effective instruction to students.

Students' awareness of fraction magnitudes and fraction arithmetic skills are especially important in comprehending one another (Bailey, Hansen, & Jordan, 2016). There are five sub-constructs in the learning of fractions, according to Pantziara and Philippou (2011): part-whole, ratio, quotient, measure, and operator. They claim that pupils in fourth and fifth grades who grasp both fractional ideas and methods perform better than those who only comprehend one. Students appear to be given too much training on the part-whole sub-construct at the expense of others, particularly measurement.

Students appear to find fraction problems with variables to be the most difficult. Fourth-grade pupils struggle with the symbolic form of fractions and fraction vocabulary, according to Lewis, Gibbons, Kazemi, and Lind (2015). Because they only wanted to deal with whole numbers, the same students struggled when asked to segment whole quantities into fractional ones. Students would frequently divide a whole into parts without regard for the size of the components (Lewis, Gibbons, Kazemi, & Lind, 2015). "Errors on fraction computation issues may represent misapplication of whole number rules to fractions," Hansen, Jordan, and Rodrigues (2017) write (p. 46). The goal of Fonger, Tran, and Elliot's (2015) study is to "explore children's informal techniques and knowledge of fractions by looking at their inventions, interpretations, and connections inside and across numerous fractions representations," according to Fonger, Tran, and Elliot (2015). (p. 4). Children "frequently indicate discrepancies or mismatches when thinking across representation kinds, sometimes creating a hindrance to their problem-solving process," according to their interviews with children in grades 2-6. (p. 13). They believe that children grasp fraction problems better when they are presented with circumstances that are relevant to their earlier experiences.

When cross-multiplication of fractions was not required, several pupils displayed a distinct misunderstanding. In the context of comparing measurements, students also had differing understandings of fractions. Students interpreted the measurement task as an operator, the part-whole meaning, and the measurement meaning separately, according to the authors, rather than all three having the same innate meaning.

According to the study, pupils who struggled with arranging fractions on a number line in 7th and 8th school also failed with algebra skills in 9th grade (Mou et al., 2016). According to the study's authors, putting a greater emphasis on fraction magnitudes in early grades will help students do better in later mathematics classes. Understanding fractions is essential for developing number awareness, which is the foundation for further mathematics courses. The authors conclude that educators should select instructional targets for middle school mathematics education, such as fraction magnitude and numerator/denominator relationships.

Though fractions are primarily taught in elementary and middle school, algebraic fractions use the same fraction concepts. According to Makonye and Khanyile (2015), pupils who had persistent fraction misconceptions in 10th grade "handled mathematical topics as though they were distinct chunks of information" (p. 55). Makonye and Khanyile (2015) discovered that students were able to overcome their misconceptions about the simplification of algebraic fractions by interviewing them, creating interview questions that required conceptual rather than instrumental understanding of fractions, and encouraging students to provide a reason for each calculation.

Early specific instruction targeting magnitude understanding and arithmetic skills "is more likely to produce more persistent effects on these same specific skills than early support that targets mathematics achievement more broadly will produce on general mathematics achievement measures," according to Bailey, Hansen, and Jordan (2016). (p. 516). They also come to the conclusion that students' grasp of fraction magnitude and arithmetic skills do not transfer until later in the learning process. "Persistent difficulty with fraction operations throughout elementary/early middle school are connected with low mathematics achievement at the end of sixth grade," according to Hansen, Jordan, and Rodrigues (2015). (p. 53).

Despite the fact that fractions are taught in school throughout elementary and early middle school, Hansen, Jordan, and Rodrigues (2015) found that many students made little gain in their comprehension of fractions during this time. Because misconceptions about fractions continue in this way, intervention must begin early in the educational process. If no action is taken, those beliefs will resurface when students enrol in higher-level mathematical courses, causing long-term worries about mathematics ability.

According to Steinke (2017), 23% of pupils enrolled in developmental mathematics had trouble comprehending fractional concepts like part-whole cohabitation. The growth of fraction understanding, and consequently higher-level mathematical accomplishment, is dependent on understanding of part-whole coexistence (Steinke, 2017; Bailey et al., 2016). Recognizing that elementary and middle school students struggle to understand various fractional concepts, and that university developmental mathematics students continue to struggle with fractional concepts, the question of how much University developmental mathematics students struggle with fraction understanding and fraction operations remains unanswered. As a result, this study looks at which fractions, ratios, and proportions concepts are more or less successful for developmental mathematics students.

A study by Aksoy and Yazlik (2017) indicated the following errors committed by pupils:

1. Student Errors Related to Ordering in Fractions
2. Student Errors Related to the Relationship between Compound Fractions and Complex Fractions
3. Student Errors related to Showing Fractions on Number Line
4. Student Errors Related to Addition and Subtraction with Fractional
5. Student Errors Related to Multiplication-Division Transactions with Fractions (from Part to Whole, from Whole to Part)
6. Student Mistakes Related to Modelling Equivalent Fractions and Operations That Require Expressing Equivalent Fractions
7. Student Errors Related to Operations That Require Problem Solving with Fractions

2.6 Mitigating Errors and Misconceptions of Addition of Unlike Fractions

Educators must first address the most common misconceptions about fraction learning in order to enhance their classrooms. According to Isiksal and Cakiroglu (2011), one of the most common fallacies concerning fraction learning is that students try to apply principles and assumptions from whole number operations to fractional operations. The outcome of multiplying two whole numbers together, for example, is bigger than the sum of the two whole numbers. Students multiply fractions with the expectation that the result will exceed the variables. However, multiplying two-unit fractions, such as 1/2 and 1/4, yields 1/8, which is less than either of the factors 1/2 and 1/4. When pupils divide two whole numbers, the quotient is frequently taught to be less than the dividend. When splitting fractions, this isn't always the case. For instance, 2/1/4 equals 8, which is greater than both 2 and 1/4. Because division is frequently connected with portioning, students are taught that the quotient of a division problem should be a whole integer. This is erroneous. Students grew to rely on rote memorization for these types of algorithms rather than learning the bigger concept, as evidenced by the pattern. Furthermore, according to Wu (2008), fractional division and multiplication are two of the most challenging abstract ideas for adolescents to master.

Fractional division has been an issue in fractional learning due to the complexity of the division algorithm (Isiksal & Cakiroglu, 2008; Isiksal & Cakiroglu, 2011; Newstead & Murray, 1998). The invert-and-multiply method is used to teach students fractional division. However, they are unaware that the divisor is the only number that must be reversed. The pupils were perplexed as to why this occurred. The actual issue, according to Newstead and Murray (1998), is that students misunderstand the aim of the query. For example, in problem 6/1/3, students had to figure out how many one-thirds there are in six, which was not always evident (Newstead and Murray, 1998). Students may be able to answer a specific division problem using the technique, but according to Isik and Kar (2012), they have trouble adapting these division problems to real-world circumstances. Students who can't apply these scenarios to division issues show that they handle problems in a systematic manner. Another issue that pupils have is seeing a fraction as a part of a larger whole. Students did not see the fraction as a quotient relationship between two whole numbers, according to Newstead and Murray (1998). They saw the numerator and denominator as two separate numbers, which made comparing many fractions challenging. The magnitude of the fraction was calculated using the numerator or denominator's biggest values rather than the relationship as a whole. When given the choice, students tended to pick the fraction with the bigger denominator, whether or not it was correct.

Fractions have a numerical value and can be plotted on a number line alongside whole numbers (Shaughnessy, 2011). Number lines are introduced in elementary school. Students won't be able to appreciate fraction magnitude if they cannot figure out where fractions go on a number line. Also, without the part-whole principle, operations with fractions were said to be a challenge (Newstead & Murray, 1998). Because they are dealing with the numerator and denominator independently, students applied the values of the numerators and denominators to the issues. Students struggled with fraction multiplication for identical reasons. Instead of multiplying the numerators and denominators, the students added them together (Isiksal & Cakiroglu, 2011). These misunderstandings have their roots in the way fractions are taught in the classroom (Isik & Kar, 2012, Isiksal & Cakiroglu, 2008; Isiksal & Cakiroglu, 2011; Newstead & Murray, 1998, Shaughnessy, 2011; Wu, 2008). Many math teachers rely on teaching knowledge, laws, and procedures in the classroom (Isiksal & Cakiroglu, 2008). This approach of instruction, on the other hand, overlooks the various learning demands of pupils in a classroom. Teachers can use a variety of strategies, including specific examples, according to Isiksal and Cakiroglu (2011). According to Newstead and Murray, when the precise examples were expanded to real-life settings, students were able to imagine the fraction and procedures utilizing fractions (1998). The teacher can tell that children understand fraction operations when they can articulate them in words (Isiksal & Cakiroglu, 2011).

1. Teachers should urge students to utilize particular terminology when describing fractions, according to Bay-Williams (2013). Instead of the word "divide," the act of separating a unit into sections is described as "partitioning." The following are some of the most common misconceptions students have about learning unlike fractions.

2. Students have difficulty answering fractional division and multiplication issues because they rely on rote methods like invert-multiply.
3. Students struggle with the concept of a fraction as a component of a whole and how to relate this to the fraction's magnitude.
4. Rather than developing a deeper understanding, teachers depend on teaching facts and processes to solve fraction problems. This makes it difficult for students to understand more abstract, rational concepts.

Teachers should research current research on how to incorporate fractions into lessons in order to increase students' understanding of unlike fractions. Prior studies addressing fractional definitions and misconceptions should be explored to better serve the diversity of student learning.

One of the most typical challenges when working with fractions, according to research, is the disconnect between conceptual and procedural knowledge (Aksu, 1997; Brown & Quinn, 2006; Gabriel, Coche, Szucs, Carette, Rey, & Content, 2012; Hecht, Close, & Santisi, 2003; Stafylidou & Vosniadou, 2004). Logical knowledge of fractions refers to the domain's underlying principles, which in this case apply to rational numbers. Procedural understanding of fractions focuses on the capacity to solve a problem through rote memorization rather than profound insight.

It was discovered that students' procedural knowledge was more frequent than their conceptual knowledge. Students' skills to compare, estimate, do fundamental operations, solve equations, and solve fraction word problems were impaired as a result of this. Several studies have examined students' conceptualizations of the part-whole relationship (Brown & Quinn, 2006; Hecht, Close, & Santisi, 2003; Stafylidou & Vosniadou, 2004). Stafylidou and Vosniadou (2004) devised a questionnaire to assess fourth, sixth, seventh, and eighth-grade students' conceptual understanding of fractions.

The 200 participants were asked to compare and order multiple fractions from smallest to greatest. The researchers came to the conclusion that the 200 respondents saw the numerator and denominator of a fraction as two separate numbers. The fraction was no longer regarded a single rational number as a result. As a result, pupils assumed that when the numerator or denominator was larger, the proportion was greater (Stafylidou & Vosniadou, 2004). Brown and Quinn (2006) discovered that the numerator and denominator both had the same comprehension gap. They asked 143 ninth and tenth grade students to equate fractions with various numerators and denominators.

They discovered that when comparing fractions, respondents who noted the relationship between the numerator and denominator were able to establish a common denominator and decide which fraction was greater. Respondents who recognized the relationship between the numerator and denominator showed a solid understanding of rational numbers. (Stafylidou & Vosniadou, 2004; Brown & Quinn, 2006).

Students' comprehension of the part-whole connection influenced operations involving fractions, such as addition, subtraction, multiplication, and division (Aksu, 1997; Brown & Quinn, 2006; Gabriel et al., 2012; Hecht, Close & Santisi, 2003; Stafylidou & Vosniadou, 2004). Brown and Quinn (2006) discovered that students had the most difficulty determining the least common denominator when adding and subtracting fractions; instead, they just subtracted the numerators and denominators from each other. When multiplying two fractions together, these ninth and tenth-grade respondents attempted to cross multiply; they struggled with grasping unlike fractions.

Students had trouble visualizing the part-whole relationship because of their difficulties understanding the numerator and denominator relationship (Hecht, Close, & Santisi, 2003). Hecht, Close, and Santisi (2003) used shading to examine the part-whole relationship. The 105 fifth-graders were given a shaded polygon and instructed to write the fraction that represented the shaded portion of the figure. The pupils were also required to reverse the process and darken the figure using the fraction indicated. Respondents regarded fractions as plucking a portion from an entire pie, according to the study. The pie represented the entire unit, and the piece represented the portion that was being studied.

The overarching concept was what enabled the respondents to see the fraction as a number. The authors discovered that failure to understand a fraction as a numerical number resulted in errors when addressing various sorts of problems (Hecht et al., 2003; Stafylidou & Vosniadou, 2004). According to Stafylidou and Vosniadou (2004), the 200 students who took part in their study would have benefited from a higher emphasis on unit and partitioning exercises.

According to various studies, students have the most difficulty with word problems using fractions (Aksu, 1997; Brown & Quinn, 2006; Gabriel et al., 2012; Hecht et al., 2003). Aksu administered a three-part test to 155 sixth-grade students in 1997. Fractions as a definition, fraction operations, and fraction problem-solving were the three sections of the test. The problem-solving section was where the kids performed the worst (Aksu, 1997). All three experiments revealed the strongest link between the concept and problem-solving exams. The students' view of the part-whole relationship was shown to be inadequate in the study, which had a direct impact on their ability to solve word problems.

Brown and Quinn (2006) looked into the cause of the error. The students' inaccuracy in completing word problems was linked to their attempts to imitate techniques learned in class, even if the procedures had no precedent. Pictorial depictions better reflected these concerns, although the pupils were hesitant to draw illustrations. The use of images depicts the relationship between the various components of a bigger whole (Brown & Quinn, 2006). Students can comprehend fractions' mental awareness with the help of physical manipulatives and learning-by-doing exercises (Brown & Quinn, 2006; Gabriel et al., 2012; Hecht et al., 2003).

Gabriel et al. (2012) conducted research to investigate if manipulatives were effective. For ten weeks, an experimental group played five different games, while a control group of the same size was given basic operating guidance. Among the games played were Memory, War, Old Maid, Treasure Hunt, and Blackjack. 40 decks of cards were used in these games. Four alternative decks and difficulty levels were created with varying fractions on each card. Through the game of War, the participants got a better comprehension of fraction magnitude comparisons. Each player was handed a deck of cards to turn over one at a time. The person who had the highest fraction face up won the hand and was able to collect all of the cards into their pile. The players needed to know the magnitudes of fractions to determine the winners in each round.

After the intervention, students in the study group improved their conceptual knowledge more than those in the control group (Gabriel et al., 2012). One game that helped responders understand the concept of a rational number as well as the magnitudes of various fractions was war. Respondents in the control group improved their procedural awareness of fractions. Students were receiving direct instruction from the instructor, according to the investigators, and as a result, they just repeated the steps they were taught. Using hands-on games, students' mood was enhanced, and they were able to comprehend logical numbers (Gabriel et al., 2012).

Students improve their performance in the classroom if assignments are more participatory and hands-on, according to Hecht et al., especially at the basic level. Classroom instruction has a direct impact on the development of fraction understanding in the primary grades (Brown & Quinn, 2006). Brown and Quinn came to the conclusion that unit rectangles, number lines, and other physical manipulatives had a beneficial impact on the success of certain responders over others.

Unlike fractional conceptions, some ancient civilizations were modified till present times. In the curriculum, many representations such as drawings, symbols, and manipulatives have been used. Several studies have been conducted to examine student misconceptions about various fraction problems. The purpose of this study was to look into how students learned to add unlike fractions in public junior high schools in Ghana's Afadzato South District.

Researchers have also examined ways that teachers can employ to improve pupils' math achievements (Suydam & Higgins, 1977; Grouws & Cebulla, 2000). Coercive materials, according to Suydam and Higgins (1977), lead to bigger accomplishment improvements than not employing them. The eclectic approach should be applied in the classroom for teaching mathematics, according to Cockroft (1982). Math should be taught and learnt in an activity-oriented way, with students actively engaged in the session, according to Bird (1985). According to studies, providing physical facilities such as demonstration rooms, as well as implementing effective teaching tactics such as offered activities, project work, and field excursions, are some of the strategies that can be used to increase students' interest in mathematics (Grouws & Cebulla, 2000; Asafo-Adjei, 2001).

Using calculators in math class, allowing students to explore and generate new information, and having whole-class discussion after individual and group work all boost student accomplishment, according to (Grouws & Cebulla, 2000). According to Nabie, Akayuure, and Sofo (2013), students' productivity can be boosted by introducing problem-solving and investigations into mathematics teaching and learning. Teachers should address and correct pertinent misconceptions about fraction arithmetic among students; address and correct pertinent misconceptions about fraction arithmetic among students (Stafylidou & Vosniadou, 2004).

2.7 Empirical Studies of Fraction Conceptions, Errors, and Misconceptions

Students' misconceptions regarding fractions were investigated by Soylu and Soylu (2005), who discovered that 5th graders exhibit misconceptions about ordering, addition, subtraction, and multiplication infractions. Students operate fractions by examining the numerator and denominator independently and using previously taught rules to the succeeding rules, particularly when collecting and multiplying fractions, according to the findings of the study. Students neglected the norm of splitting into equal parts when it came to fraction definitions and indicators, and they kept to their natural-number operational habits when it came to multiplication, addition, and subtraction operations in fractions, according to Haser and Ubuz (2002).

In Stafylidou and Vosniadou (2004) study, 37.5% of fifth-graders thought of fractions as two different natural numbers. Children were led to assume that the value of a fraction grows as the numerator or denominator value increases, or that the value of a fraction reduces as the numerator or denominator value decreases, due to this misunderstanding. The majority of 5th-grade children had misconceptions about fraction ordering, addition, subtraction, and multiplication, according to Biber, Tuna, and Aktas (2013). In this study, the researchers uncovered three separate fallacies concerning fraction addition. The numerator and denominator must be added individually, expansion must be applied only to the numerator and not to the denominator, and the expansion coefficient must be added to the numerator and denominator.

Biber et al. (2013) also uncovered two different myths about fraction multiplication. The first is when a learner uses additional rules while multiplying fractions, such as multiplying only the numerator and not multiplying the denominator. The first fraction's numerator is multiplied by the second fraction's denominator, and the first fraction's denominator is multiplied by the second fraction's numerator. In his study of the learning challenges of 3rd-grade primary school pupils and the reasons behind their frequent blunders, Pesen (2008) revealed that some of the kids had difficulty dividing a whole on the number axis into mating parts. In this study, it was revealed that some students have difficulty understanding a /b, the symbolic representation of a fraction, as an odd number on the number line, with the numerator and denominator regarded as separate numbers. Karaagac and Kose (2015), Students, on the other hand, could not grasp the concept that the total expressed in fractions would alter depending on the dependent whole, and therefore the part-whole relationship in fractions did not fully develop by the end of their study with 7th graders. Similarly, Kocaoglu and Yenilmez found that pupils did not have a part-whole link and had trouble understanding the challenges in their study with 5th graders.

2.8 Chapter Summary

"One of the most basic and also one of the most difficult human activities is teaching" (Pesen; 2008, p34). The expertise of teachers is a critical component in improving teaching. Teachers' and prospective teachers' mathematical material skills for teaching was investigated. The findings of these studies revealed that prospective teachers had little knowledge of mathematical material for teaching mathematics.

In elementary school mathematics, fractions are one of the most important fields that are mathematically rich, cognitively complex, and difficult to teach Smith (2002). Several studies have looked at students' struggles with fractions and popular myths about them (Brown & Quinn, 2006; Haser & Ubuz, 2002; Pesen; 2008). The findings of these tests, on the other hand, showed that students had a poor understanding of fractions and held many assumptions regarding fraction concepts. As a result, teachers must be familiar with students' common conceptions and misconceptions about fractions to improve students' conceptions and correct misconceptions.

Student learning, according to Pesen (2008) may be influenced not only by teachers' content awareness, but also by the relationship between teachers‟ knowledge of students, their learning, and strategies for enhancing the learning. Furthermore, according to Pesen (2008) mathematical skills for teaching included prospective teachers‟ knowledge and comprehension of students' common mistakes and misconceptions, as well as teachers‟ responses to students' incorrect answers. However, as previously noted, few research studies have focused on prospective teachers' awareness of students' mistakes and the causes of these mistakes. There are very few inquiries in Turkey into prospective teachers‟ experience of students' errors. Furthermore, in previous studies, researchers tended to concentrate on only a few aspects of fractions, such as fraction multiplication and division, and fraction definition. This research, on the other hand, looks at students' errors in all fractions topics rather than concentrating on one in particular. As a result, this study aimed to look into prospective elementary mathematics teachers‟ awareness of elementary students' fractions mistakes and their proposed strategies for correcting them.

CHAPTER THREE

METHODOLOGY

3.0 Overview

This chapter explains with the various methods and procedures used in gathering data. Therefore, the research design, the population, the sample and sampling procedure, the instruments used for data collection, piloting of instruments, data collection procedure, and data analysis procedure are presented.

3.1 Philosophical Underpinning of the Study

The research is underpinned by the pragmatic paradigm. In social research, the term "paradigm" is used to refer to the philosophical assumptions that guide actions and define the worldview of the researcher (Lincoln, Lynham, & Guba, 2011). Kuhn (1970) used the term paradigm to discuss the shared generalizations, beliefs, and values of a community of specialists regarding the nature of reality and knowledge. Each paradigm has a different perspective on the axiology which refers to beliefs about the role of values and morals in research, ontology thus the assumptions about the nature of reality. Epistemology is assumptions about how we know the world, how we gain knowledge, the relationship between the knower and the known, methodology has to do with shared understanding of best means for gaining knowledge about the world and Rhetoric means shared understanding of the language of research (Creswell, 2009).

As a research paradigm, pragmatism is based on the proposition that, researchers should use the philosophical and/or methodological approach that works best for the particular research problem that is being investigated (Creswell & Clark, 2011), It is often associated with mixed methods or multiple methods, where the focus is on the consequences of research and on the research questions rather than on the methods. Pragmatism as a research paradigm refuses to get involved in the contentious metaphysical concepts such as truth and reality. Instead, it accepts that, there can be single or multiple realities that are open to empirical inquiry (Creswell & Clark, 2011). A major underpinning of pragmatist philosophy is that knowledge and reality are based on beliefs and habits that are socially constructed (Yefimov, 2004). Pragmatists generally agree that all knowledge in this world is socially constructed, but some versions of those social constructions match individuals" experiences more than others (Freebody, 2006), This research therefore resorted to the use this pragmatic paradigm as basis for the researcher"s position on knowledge and reality.

3.2 Research Design

The study used the mixed method approach with sequential explanatory as the research design. This approach allows triangulation of both qualitative and quantitative research strategies to elicit relevant information from research respondents (Cohen, et al., 2011; Creswell, 2009). Creswell, (2013), identified three different approaches to mixed methodology; namely concurrent, sequential and conversion methods. This study adopted the sequential approach where the quantitative phase (numbers) is followed by the qualitative phase (personal experience) (Creswell, 2013); where the qualitative findings are used to contextualise the quantitative data (Creswell, Plano-Clark, Gutmann and Hanson, 2003). Qualitative data can also enhance and enrich the findings (Taylor & Trumbull, 2005) and, help generate new knowledge (Stange, 2006).

3.3 Population

Cohen et al (2004) explain that a population is a group of elements or cases, whether individuals, objects or events, that conforms to specific criteria which can then be generated to the population. Here, the target population was all public Junior High School One (1) and Two (2) Students in Afadzato South District and all the mathematics teachers in the district that teach these students. Accessible population will be all public JHS one and two mathematics teachers and all public JHS one and two students in Have and Weto Circuits. These are circuits that are closer to each other in terms of geographical location. Burns and Grove (2001) described sampling criteria as the characteristics or attributes essential for membership in the target population.

3.4 Sample and Sampling technique

The target population for the study was 1,506 respondents whereas the accessible population was 462 respondents. Purposive sampling technique and a simple random sampling technique were used to select seventeen (17) students each from all the six (6) public junior high schools‟ grades One and Two; and eighteen (18) students from one public junior high school grades one and two; and all the mathematics teachers that teach these students in Weto Circuit. Therefore, the sample size is 10 teachers and 120 students totalling 130 respondents sampled for the study.

3.5 Data Collection Instruments

The researcher used questionnaire and interview guide in the data collection process in this study. The questionnaire was used to collect quantitative data from the student respondents while the interview guide was used to collect qualitative data from the teacher respondents.

3.5.1 Questionnaire

A questionnaire is a research instrument that consists of a set of questions to collect information from a respondent. It is a list of questions or items used to gather data from respondents about their attitudes, experiences, or opinions. Questionnaires can be used to collect quantitative and/or qualitative information. It includes specific questions with the goal to understand a topic from the respondents' point of view. Questionnaires typically have closed-ended, open-ended, short-form, and long-form questions. It comes with different types such as: *Structured Questionnaires*-These questionnaires have fixed, predefined questions with specific response options. They are ideal for quantitative research, providing standardized data for easy analysis. *Unstructured Questionnaires*-Open-ended questions in unstructured questionnaires allow respondents to express their thoughts freely, providing detailed qualitative insights. *Semi-Structured Questionnaires*-These questionnaires balance structured and unstructured formats, with both closed-ended and open-ended questions that provide quantitative and qualitative data. *Dichotomous Questionnaires*-They present questions with only two response options – typically „yes" or „no". They are straightforward and yield easy-to-analyse data. *Multiple Choice Questionnaires*-Participants choose from a list of predefined options. They are versatile and suitable for various topics, allowing respondents to select the most relevant answer. *Likert Scale Questionnaires*-Likert scale questions are used to measure attitudes and opinions through a series of statements where respondents indicate their level of agreement or disagreement on a scale.

The questionnaire was administered to the students to ascertain the conception of learners" and their misconception on the addition of unlike fractions. The study adapted the questionnaire of Mdaka (2011) which was originally developed by

Horizon Research, Inc. (HRI). Questionnaire was made up of two parts and its structure consisted of series of semi-structured questions. Part, one elicited the background data of the respondents. Part two elicited information about learners" conceptions about the misconceptions of learners in addition of unlike fractions.

The researcher chose questionnaire because all the respondents can read, understand and respond appropriately to the questions contained in it. It also provided the researcher the opportunity to generate numerical values needed to test for the quantitative data. Questionnaires have some advantages including the fact that they are cheap and can be used to gather data from a large population. One limitation of it is that, respondents may skip some of the questions or may refuse to return the questionnaire. Some of the respondents may misconstrue some of the questions thereby affecting the findings of the study. To overcome this weakness, the researcher explained key items to respondents and employed their frank responses.

Before the administration of the questionnaire to chosen respondents, the questionnaire guide was first piloted in a school in Have Circuit, which were not part of the sampled size in the research. The piloted study resulted a Cronbach alpha value of 0.82. As prescribed by Nunnaly and Bernstein (1994), the general convention is to strive for reliability values of 0.7 or higher. This was done to ascertain reliability and validity of the questionnaire guide.

3.5.2 Interview

Kerlinger (2003), observed that more people are willing to communicate orally than in writing and therefore, provided data more readily and in-depth. According to Kerlinger (2003), the advantage of the interview technique is that it enables the respondents to enlighten the researcher about unfamiliar aspects of the setting and situation.

An interview is a conversation for gathering information. It refers to a one-on-one conversation between an interviewer and an interviewee. The interviewer asks questions to which the interviewee responds, usually providing information. That information may be used or provided to other audiences immediately or later. Also, an interview involves an interviewer, who coordinates the process of the conversation and asks questions, and an interviewee, who responds to those questions. Interviews can be conducted face-to-face or over the telephone.

This study used the face-on-face interview. An interview guide was developed to guide the interviews in collecting the data with a field notebook and a recorder. An interview guide is a document that enables organizations to structure the way they conduct their candidate interviews. An interview guide is a list of the high level topics that you plan on covering in the interview with the high level questions that you want to answer under each topic. It helps interviewers to know what to ask about and in what order and it ensures a candidate experience that is the same for all applicants. The researcher develops an interview guide in advance to refer to during the interview (or memorizes in advance of the interview). An interview guide is a list of questions or topics that the interviewer hopes to cover during the course of an interview.

Interviewing particularly is an effective technique for collecting data about the life experience of respondents in a phenomenological study (Van den Berg, 2005, Vagle, 2014). Interviews were conducted with a semi-structured approach reflecting on the framework presented in the literature. Semi structured interviews are defined by Merriam (2009) as interviews in which “the questions are more flexibly worded and consist of a mix of more or less structured questions", allowing the researcher to respond to and be shaped by the situation as it unfolds, and fully explore the emerging worldview of the respondent. Fontana and Frey (2000) comment that this is "one of the most powerful ways in which we try to understand our fellow human beings …" Freebody (2006, p194), also notes that „interviews can offer insight into individuals‟ constructed worlds and the ways they present these constructions …". It is clear that exploring personal perspectives requires the use of interviews, both in focus groups and individually, as it provides the research with the anecdotal and contextual evidence that is vital to an understanding of these interactions (Seidman, 2013).

One-on-one interviews are primarily used with respondents who are "not hesitant to speak, who are articulate, and who can share ideas comfortably," (Creswell, 2017). The researcher constructed the semi-structured interview guide under the guidance of the research question. The interview guide developed for the study, which is Appendix B had 2 sections with 20 items. Under the first section, there were nine (9) questions or items concerning general information on mathematics teachers selected for the study. The next section, had eleven (11) questions on measures to use to improve errors and misconceptions of students on addition of unlike fractions. Prior to the study, a pilot interview was performed to determine whether changes were appropriate to the planned interview protocol. The teachers chosen for the pilot interview were not part of the selected respondents in the research. The semi-structured interview format of the study used an interview guide to help steer the interview in the right direction to remain close to the topic.

Merriam (1998) notes that the precise wording of interviews is not determined in advance, but the fundamental questions of the interview are predetermined. Allowing the respondents, the opportunity to address the questions of the interview as they chose, the interviewer allowed respondents and any new ideas or subjects that could emerge to be properly followed up (Corbin & Morse, 2013). One-on-one interviews were conducted with the interviewer to ensure confidentiality and preserve each respondent 's comfort levels, protecting their privacy. The one-to-one format also allowed each respondent to express their personal views on caring practices in mathematics that support the learning of mathematics by the student. Yin (2009) argues that while interviews can be biased due to the opinions of the researcher, the interview guide allowed the researcher to concentrate on the particular subject being studied. Merriam (1998) recommends that the interviewer will maintain the "respectful, non-judgmental, and non-threatening" environment of the interview (p. 85) so that respondents will feel comfortable sharing their stories. In a quiet environment that was conducive to individual interviews, the interviews were carried out. The location depended on availability at each school and on each respondent 's level of comfort. Each interview was approximately 30-45 minutes and allowed extended time flexibility.

3.6 Validity of Instruments

In every research work, it is paramount to ensure the validity and reliability of the instrument used to collect the data. Joppe (2000), defined validity as “when a research measures that which it was intended to measure or how truthful the research results are” (p.1). To ensure face validity, the extent to which an instrument measures what it is supposed to measure, content validity, and format of the instrument, the self-developed instruments would be subjected to expert validation (Lafaille & Wildeboer,

1995) by my supervisors. Based on their comments, the necessary corrections were made to improve the validity of the instruments.

3.7 Reliability of Instruments

Reliability is explained as the extent to which results are consistent over time and are an accurate representation of the total population under study (Joppe, 2000). Joppe further explained that if the results of a study can be reproduced under a similar methodology, then the research instrument is considered to be reliable. To determine the reliability of my research instruments, the instrument was piloted among pupils who would not be part of the selected sample.

3.7.1 Trustworthiness

In order to assess the soundness of qualitative research as an alternative to more conventional quantitatively-oriented standards, Creswell (2012) suggested four criteria: credibility, transferability, dependability and confirmability. Credibility means establishing that, from the perspective of the respondents, the findings of the study are credible. Transferability refers to the degree to which it is possible to generalize the findings or transfer results to other settings. Dependability stresses the need for the researcher to account for the ever-changing context in which the study occurs. All the respondents were assured of their anonymity. Again, the respondents were given the chance to pull out if they so wished. Finally, all authors cited have been duly referenced at the reference section.

3.7.2 Credibility

Since the purpose of the study was to describe or understand caring from the respondents" perspectives, the respondents were the only ones who could legitimately judge the credibility of the results. Member checks, or showing research material to the people on whom the research was done is the most crucial technique for establishing credibility (Lincoln & Guba, 1989). The respondents can indicate their agreement or disagreement with the way the researcher has represented them. For member checks, the researcher allowed the teachers to read and review my transcripts and observations of them to make any corrections or additions deemed necessary to accurately represent their views. It was crucial for this particular study that the respondents agreed with the data results from the study.

3.7.3 Confirmability

The unique perspective brought to the study by the researcher could cause bias or distortion of the data collection or analysis. If the results of the study needed to be confirmed by others, interpretive validity (Polit & Beck, 1999) was helpful in producing unbiased and consistent results. Interpretive validity is the degree that the respondents‟ viewpoints, thoughts, intentions, and experiences are accurately understood and reported by the researcher. The researcher used the strategy of reflexivity and actively engaged in critical self-reflections to avoid biases or predispositions. The researcher controlled and monitored biases by writing memos to stay aware of personal assumptions. The researcher documented the procedures for checking and rechecking the data throughout the study leaving a "paper trail" of my actions. The researcher also actively searched for and described any negative instances that contradicted prior observations.

Ongoing feedback from supervisors also helped with data analysis. For triangulation, the data was examined for similarities within each teacher across the interview and observations. In other words, the researcher determined, for each teacher, if nominators‟ descriptions of the teacher, the teacher‟s self-report (via the interview), and researcher"s observation were consistent. The greatest emphasis was placed on the consistency between the teachers" self-reports about caring and the researcher"s observations of the teachers" caring. The researcher qualitatively determined to what extent each teacher actually did what she reported to do.

3.8 Ethical Consideration

These are laid down principles and guidelines for conducting studies in an ethically appropriate manner which requires researchers to obtain approval from the ethics committee or equivalent and the respondents (Halai, 2006). Based on this premise, the ethical considerations suggested by Creswell (2012) for conducting quantitative research will be adopted for this study. That is confidentiality, anonymity and permission from authorities. Permission was sought from the Department of Basic Education and School of Graduate Studies, University of Education, Winneba.

3.9 Data Collection Procedure

With an introductory letter from my Head of Department, I took a verbal permission from the District Director of Ghana Education Service, Afadzato South District, upon showing her my introductory letter from the University"s Head of Basic Education Department. From here, the District Director of Education, Afadzato South District introduced me to the sampled schools under her jurisdiction. In each school visited, the purpose of the study was communicated to the respondents after taking pleasantries with the school head and teachers. They were also, assured of their anonymity and the keeping of their responses confidential. Due respect was given to the study population.

3.10 Data Processing and Analysis

The instruments used to collect data were the questionnaire and interview guide. The questionnaire which was on a four-point Likert scale was coded into Statistical Product for the Social Sciences, (SPSS) version 26.0. Items were coded as follows: "Strongly Disagree" (1), "Disagree" (2), "Agree" (3), and "Strongly Agree" (4). Data from the interview were analysed thematically. Also, data were tabulated question by question, in recognisance of percentage and frequency counts of the respondents who provided correct answers and those with incorrect answers. Follow-up probing interviews were done to gain insight of learners" understanding which led to the misunderstanding and errors, which were categorized as either conceptual, application, carelessness or procedural errors. Teachers" interviews too were analysed through grouping responses that is similar and compared to get more clarity on their perspectives. The answered questionnaire guides by respondents were analysed in response to aim to identify misconceptions and errors learners display when adding unlike fractions. The researcher did attempt to account for the diversity in the data with the developed categories of errors, which describes all the research findings as Noddings (2013) put it.

3.11 Chapter Summary

This section runs a summary discussion of Chapter three. It tells how the research is organized and conducted. Here the research design provides the necessary guidance and direction toward a specific task a researcher is aiming to assess. This study has relied on the mixed-method approach hence using sequential explanatory design to determine the conceptions and misconceptions in addition of unlike fractions among students in Weto Circuit Public Junior High Schools in Afadzato South District of Ghana. Questionnaires were administered to a sample of 120 students in the Afadzato

South District who were randomly selected after which 10 teachers were purposefully selected to be interviewed. The descriptive statistics and thematic analysis were used in the study.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Overview

This chapter discusses the results of the study. The results and discussion of the study are presented in line with the research questions and hypotheses that steered the study. Certainly, the results and discussion of the study are presented under the following sub-headings; the personal information of respondents, the conceptions of students in the addition of unlike fractions, misconceptions of students in the addition of unlike fractions, factors that cause the errors and misconceptions of addition of unlike fractions, causes of the misconceptions of unlike fractions, measures to use in helping to improve learners‟ conceptions and misconceptions in adding unlike fractions.

4.1 Demographics of Respondents

The study sought for background information of the respondents which were relevant to the study. These included their gender, age, and form. The results are provided in Table 4.1.

Table 4.1: Biographical Data of Students

Details	Category	Frequency	Percentage (%)
Gender	Male	64	53.3
	Female	56	46.7
	Total	**120**	**100.0**
Age	10- 13	48	40.0
	14- 16	62	51.7
	17 and above	10	8.3
	Total	**120**	**100**
Form	JHS 1	58	48.3
	JHS 2	62	51.7
	Total	**120**	**100**

Source: Field Data, 2020

Results from Table 4.1 show that 64 (53.3%) students who participated in the study were males with the remaining 56 (46.7%) being females. This shows that more males than females participated in the study.

Significantly, however, the findings of the study represent the ideas from both gender groups. Also, the age distribution of the students who participated in the study was such that most of them were within a young age. The students who were below 13 years were 48 (40.0%) of the respondents, 62 (51.7%) of the respondents were between the ages of 14-16 years while the remaining 10 (8.3%) of the respondents were 17 years old and above. The table 4.1 also revealed that 58 (48.3%) of the respondents were in JHS 1 and the remaining 62 (51.6%) of the respondents were in JHS 2. This indicates that most of the respondents were yet to get to almost the final year and might have had a lot of experiences with fractions.

The study also employed for the biographic and educational information from the teacher respondents. The table 4.2. revealed this information, as below:

Table 4.2: Demographic Information of Teachers

Respondents	Age (Yrs.)	Gender	Academic Qualification	Professional Qualification	Teaching Experience (Yrs.)
Teacher 1	45	Female	1st Degree	Bachelor of Education	12
Teacher 2	34	Male	1st Degree	Bachelor of Education	7
Teacher 3	35	Male	Master"s degree	Master of Education	5
Teacher 4	38	Female	1st Degree	Bachelor of Education	9
Teacher 5	30	Female	1st Degree	Bachelor of Education	4
Teacher 6	54	Male	Diploma	Diploma in Basic Education	18
Teacher 7	37	Male	1st Degree	Bachelor of Education	6
Teacher 8	34	Male	1st Degree	Bachelor of Education	3
Teacher 9	36	Male	1st Degree	Bachelor of Education	6
Teacher 10	34	Male	1st Degree	Bachelor of Education	5

Source: Field data 2020

Results from Table 4.2 shows that, all 10 were certificated Basic school teachers in the district. Respondents" experiences ranged from three (3) to eighteen (18) years and were teaching in Junior High Schools 1 and 2. Eight of the respondents had their first degree in Basic Education and one had a second (master) degree in Education. Only one had a diploma in Basic Education. These teachers in the district are professionals and are expected to have in-depth knowledge about the teaching of Mathematics specifically fractions. From the table, eight of the respondents were below 40 years whiles the remaining two of the respondents are above the age of 40 years. This means that any intervention that will be put to enhance the mathematics caring practices of teachers in mathematics instructions will be very relevant since most of them will still be in the service for at most twenty years.

4.2 Research Question 1

How do Junior High School pupils conceptualize addition of unlike fractions in Junior High Schools?

This question examined how students or learners conceive the concept of the addition of unlike fractions in the Junior High School Mathematics classroom. It required respondents to indicate their conception or conceptualization of fractions and specifically, on the addition of unlike fractions. Tables 4.3 to 4.5 present the results on the conceptualization of addition of unlike fractions among Junior High School learners.

Table 4.3: Conceptions of Students in the Addition of Unlike Fractions

	STATEMENT	SD (%)	D (%)	A (%)	SA (%)	M (STD)
1	All fractions are always part of a whole number	48 (40.0)	17 (14.2)	26 (21.7)	29 (24.2)	2.3 (1.2)
2	Addition of fractions makes numbers bigger, and subtraction makes them smaller	16 (13.3)	14 (11.7)	40 (33.3)	50 (41.7)	3.0 (1.0)
3	When adding fractions, you add only the numerator	28 (23.3)	30 (25.0)	33 (27.5)	29 (24.2)	2.5 (1.1)
4	Addition of fractions is done using the LCM of the denominators.	15 (12.5)	13 (10.8)	33 (27.5)	59 (49.2)	3.12 (1.04)
5	The larger the denominator the smaller the fraction regardless of the numerator.	38 (31.7)	20 (16.7)	46 (38.3)	16 (13.3)	2.31 (1.02)
6	The larger the denominator, the bigger the fraction. If the numerator is the same.	20 (16.8)	27 (22.7)	41 (34.5)	31 (26.1)	2.6 (1.1)
7	Numerator and denominator are separate values, two separate whole numbers	26 (21.7)	15 (12.5)	59 (49.2)	20 (16.7)	2.6 (0.9)
8	Fractions are not always lesser than 1	29 (24.2)	25 (28.0)	25 (28.0)	41 (34.2)	2.7 (1.2)

Source: Field Data, 2020

Key: SD= Strongly Disagree; D= Disagree; A= Agree; SA= Strongly Agree; M= Mean; STD= Standard Deviation.

Results from Table 4.3 show that 48 (40.0%) of the students strongly disagreed that all fractions are always part of a whole number, 17 (14.2%) of them also disagreed with this assertion, with 26 (21.7%) of them agreeing that all fractions are part of a whole number and the remaining 29 (24.2%) students strongly agreed. A mean score of 2.3 confirms that, on average, students have the conception that all fractions are part of a whole. Also, a standard deviation of 1.2 shows that the students had very similar conceptions regarding what a fraction is. Table 4.3 also showed that 16 (13.3%) of the respondents strongly disagreed that, the addition of fractions makes numbers bigger, and subtraction makes them smaller, 14 (11.7%) of the respondents disagreed with this statement and they are of the view that addition of fractions does not necessarily make numbers bigger, and subtraction makes them smaller but 40 (33.3%) of the respondents agreed to this while the remaining 50 (41.7%) strongly disagreed to this assertion.

Concerning the addition of fractions, 28 (23.3%) of the respondents strongly disagreed that, whenever you are adding fractions, you add only the numerator and 30 (25.0%) of the respondents disagreed with this statement. Also, it is clear from Table 4.3 that, 33 (27.5%) of the respondents agreed that only numerators are added whenever you are adding fractions, and the remaining 29 (24.2%) strongly agreed with this statement.

The respondents had a similar conception that, the addition of fractions is done using the Least Common Multiple (LCM) of the denominators (Mean = 3.12); the larger the denominator the smaller the fraction, regardless of the numerator also obtained a mean value of (2.31). Again, the respondents came out that, the larger the denominator, the bigger the fraction, if the numerator is the same, with a mean score of (2.6); numerator and denominator are separate values, two separate whole numbers (2.6); and finally, fractions are not always lesser than 1 (2.7).

An overall mean score of 2.8 and a standard deviation of 1.1 indicate a positive conception in the addition of fractions.

4.3 Research Question 2

What common errors and misconceptions do JHS pupils make in the learning of addition of unlike fractions?

In order to answer this research question, the semi-structured questionnaire guide was used to elicit responses from the student"s participants. The Table 4.4 below explicitly explain the results from the administration of the questionnaire.

Table 4.4: Misconceptions of Students in the Addition of Unlike Fractions

	STATEMENT	SD (%)	D (%)	A (%)	SA (%)	M (STD)
1	All fractions are lesser than 1	47 (39.2)	38 (31.7)	19 (15.8)	16 (13.3)	2.0 (1.0)
2	All fractions are always part of 1, never bigger than	31 (25.8)	29 (24.2)	32 (26.7)	26 (21.7)	2.6 (1.8)
3	Properties of whole numbers can be applied to fractions.	14 (11.7)	18 (15.0)	62 (51.7)	26 (21.7)	2.8 (0.9)
4	The numbers in numerator and denominator should be compared separately rather than considering the whole fraction.	19 (15.8)	24 (20.0)	42 (35.0)	34 (28.3)	3.0 (2.9)
5	Operation rules for natural numbers can be applied to operations with fractions.	14 (11.7)	21 (17.5)	52 (43.3)	33 (27.5)	2.9 (1.0)
6	The value of the fraction increases when either the numerator or the denominator increase.	37 (30.4)	19 (15.8)	30 (25.0)	34 (28.3)	2.5 (1.2)
7	Add only the whole numbers when dealing with mixed fractions	33 (27.5)	26 (21.7)	27 (22.5)	34 (28.3)	2.5 (1.2)
8	During addition of fractions, add the same denominators	33 (27.5)	19 (15.8)	27 (22.5)	41 (34.2)	2.6 (1.2)
9	During addition of fractions, add the same numerators	29 (24.2)	17 (14.2)	24 (20.0)	50 (41.7)	2.8 (1.2)
10	When adding fractions, you add the numerator and the denominator.	33 (27.5)	10 (8.3)	29 (24.2)	48 (40.0)	2.8 (1.4)

Source: Field Data, 2020

Key: SD= Strongly Agree; D= Disagree; A= Agree; SA= Strongly Agree; M= Mean; STD= Standard Deviation

Results from Table 4.4 indicated that 47 (39.2%) of the respondents strongly disagreed with the assertion that, all fractions are lesser than 1, 38 (31.7%) of them disagreed but 19 (15.8%) of them agreed to the statement with 16 (13.3%) of them strongly agreeing to the fact that all fractions are lesser than 1. A mean score of 2.0 indicates that at least half of the respondents believe that not all fractions are lesser than 1.

The Table 4.4 also showed that 31 (25.8%) of the respondents strongly disagreed that all fractions are always part of 1, never bigger than 1, these results were further affirmed by 29 (24.2%) of the respondents who also disagreed with this statement. But 32 (26.7%) of the respondents agreed whiles 26 (21.7%) of the respondents strongly agreed that every fraction is part of a 1 and never bigger than 1. A mean and standard deviation score of 2.6 (1.8) indicates that over of the respondents disagreed with this statement and the standard deviation score indicates the data points are spread out over a large range of values.

Table 4.4 further showed that 14 (11.7%) of the respondents strongly disagreed that properties of whole numbers can be applied to fractions,18 (15.0%) of the respondents disagreed with this but 62 (51.7%) of the respondents agreed to this statement with the remaining 26 (21.7%) of them strongly agreeing. A mean score of 2.8 indicates most of the respondents agreed with this misconception.

Also, Table 4.4 revealed that 19 (15.8%) of the respondents strongly disagreed that the numbers in numerator and denominator should be compared separately rather than considering the whole fraction, and 24 (20.0%) of the respondents disagreed with this. Again, it is clear that, 42 (35.0%) of the respondents agreed that the numbers in numerator and denominator should be compared separately rather than considering the whole fraction, with the remaining 34 (28.3%) of them strongly agreeing to this statement. A mean score of 3.0 indicates that, when 4 of the respondents are chosen at random, 3 of them agreed to this statement.

Table 4.4 also showed that 37 (30.4%) of the respondents strongly disagreed that the value of the fraction increases when either the numerator or the denominator increase, and 19 (15.8%) of the respondents disagreed with this statement but 30 (25.0%) of the respondents agreed that the value of the fraction increases when either the numerator or the denominator increase whiles 34 (28.3%) of the respondents strongly agreed to this statement. The respondents had a similar misconception about the addition of whole numbers when dealing with mixed fractions 2.5 (1.2), adding the same denominators when adding fractions = 2.5 (1.2).

Finally, the Table 4.4 revealed that 33 (27.5%) of the respondents strongly disagreed with the assertion that when adding fractions, you add the numerator and the denominator, 10 (8.3%) of the respondents disagreed with this statement, but 29 (24.2%) of the respondents agreed that when adding fractions, you add the numerator and the denominator and finally the remaining 48 (40.0%) of the respondents strongly agreed to this statement. A mean score of 2.8 indicated that 3 out of 4 of the respondents agreed to this assertion. This confirms the findings of Murray and Newstead (1998) as they indicated in their study that students applied the values of the numerators and denominators to the problems since they are handling the numerator and denominator separately. An average mean of 2.7 indicates that the teachers have a moderate conception about the addition of unlike fractions.

In addition to elicit more responses to be able to answer the research question two (2) here, an interview was conducted for the teachers that participated in the study. The interview was conducted for JHS Mathematics teachers to get an in-depth understanding of junior high school students" misconceptions and why they make such errors, resulting in misconceptions in the learning of addition of unlike fractions. Ten (10) mathematics teachers were interviewed. Thematic analysis of interview data located JHS students" misconception of addition of unlike fractions in two (2) domains namely: (i) treating fractions as whole numbers and (ii) cross addition of the fractions. The following are the results from the interview, which ascertain or affirm the two main themes derived.

The misconceptions of Addition of Unlike Fractions

When Teacher 1 was asked: Do you think students encounter some problems when adding unlike fractions? She responded that:

> *"Oh yeah, they have a lot of problems paaa"*

The researcher further asked, "what are their main problems during the addition of unlike fractions.

Teacher 1 maintained that:

> *"They lack the concept of finding the LCM and adding two-digit numbers, they sometimes think getting the LCM is just by multiplying the denominators which are not always so"*

Continuing, she added that:

> *"Hmmmm due to this, they find it difficult adding simple, unlike fractions. And even for the addition of like fractions koraaa they find it difficult."*

Teacher 6 also stated that:

> *"They keep on adding numerator to numerator and denominator to denominator"*

He further indicated that:

> *"Look let's say for example if you tell them to add* $\frac{1}{3}+\frac{1}{2}$*, you will be surprised that they will tell you the answer is* $\frac{2}{5}$ *which is wrong"*

When Teacher 7 was asked the same questions, he indicated that:

> *"These our pupils erh…… hmm, they see the numerator and the denominator in a fraction as separate and they cannot even tell that the numbers are related. They fall into a misconception by considering the natural number values whiles ranking fractions"*

These responses go in line with the findings of Isiksal and Cakiroglu (2011), as they indicated that one key misconception that students have in the addition of unlike fractions is that they tend to treat fractions as whole numbers.

Teacher 8 was asked the same question by the interviewer and the respondent indicated that:

> *"The students try to apply the knowledge they had in the addition of like fractions to unlike fractions which is a big misconception, u will see that they will add numerators and choose any of the denominators. For example, if you ask them to add something like* $\frac{2}{3}+\frac{1}{4}$ *then tend to add the numerators 2+1=3 and choose any of the denominators like 4 so will be like* $\frac{3}{4}$ *which is very wrong."*

Finally, the Teacher 8 again added that:

> *"Learners at times do cross addition for a given problem. For instance, in the above example, they add the numerator of the first fraction to the denominator of the second fraction as in 2+4=6 and 3+1=4, after which they decide to write any of them over the bigger denominator".*

This supports the findings of Isik and Kar (2012) as they indicated in their study that, learners have difficulty in the right application of concepts they have learnt previously to current one they are studying.

4.4 Research Question 3

What are the factors that causes errors and misconceptions of addition of unlike fractions among JHS pupils?

This question examined the various factors that influence the JHS pupils to have misconceptions and make basic errors in the addition of unlike fractions. It required respondents to indicate what they think is the cause of their misconception of fractions and specifically, the addition of unlike fractions. Table 4.5 present the results.

Table 4.5: Factors that Causes Errors and Misconceptions of addition of unlike fractions among JHS pupils.

	STATEMENT	SD (%)	D (%)	A (%)	SA (%)	M (STD)
1	The student does not understand that fractions are numbers as well as portions of a whole.	22 (18.3)	23 (19.2)	29 (24.2)	46 (38.3)	2.8 (1.1)
2	Student thinks that mixed numbers are larger than improper fractions	26 (21.7)	18 (15.0)	40 (33.3)	36 (30.0)	2.7 (1.1)
3	Students have restricted their definition and think fractions have to be less than 1.	44 (36.7)	32 (26.7)	25 (20.8)	19 (15.8)	2.2 (1.1)
4	Students counts fractional parts as they count whole number	30 (25.0)	27 (22.5)	40 (33.3)	23 (19.2)	2.2 (1.1)
5	Teachers fail to use the right approach in teaching fractions.	40 (33.3)	32 (26.7)	14 (11.7)	34 (28.3)	2.4 (1.2)
6	The student has restricted his definition of fractions to one type of situation or model, such as part/whole with pieces	24 (20.2)	13 (10.9)	45 (37.8)	37 (31.1)	2.8 (1.1)
7	Misapplication of rules for comparing whole numbers in fraction situations	19 (16.0)	26 (21.8)	55 (46.2)	19 (16.0)	2.6 (0.9)
8	Misapplication of additive ideas when finding equivalent fractions	23 (19.3)	24 (20.2)	45 (37.8)	27 (22.7)	2.6 (1.0)
9	Students add fractions, generalizes the procedure for multiplication of fractions by adding the numerators and adding the denominators	24 (20.2)	25 (21.0)	27 (22.7)	42 (35.3)	2.8 (1.3)
10	Students think that dividing by one-half is the same as dividing in half	26 (21.8)	24 (20.2)	32 (26.9)	37 (31.1)	2.7 (1.1)

Source: Field Data, 2020

Key: SD= Strongly Agree; D= Disagree; A= Agree; SA= Strongly Agree; M= Mean; STD= Standard Deviation

Responses from Table 4.5 indicated that 22 (18.3%) of the respondents strongly disagreed that, the student does not understand that fractions are numbers as well as portions of a whole, and 23 (19.2%) of them disagreed with this statement whiles 29 (24.2%) of the respondents agreed to this statement with the remaining 46 (38.3%) of the respondents strongly agreeing that student does not understand that fractions are numbers as well as portions of a whole. A mean score of 2.8 indicates more than half of the respondents agreed to the fact that students or learners do not understand that fractions are numbers as well as portions of a whole, a standard deviation of 1.1 indicates that, the data points are spread out over a large range of values.

Also, 26 (21.7%) of the respondents strongly disagreed that students think that mixed numbers are larger than improper fractions while the remaining 18 (15.0%) disagreed to this statement as they are of the view that most learners have the view that mixed numbers are larger than improper fractions. Table 4.5 also revealed that 44 (36.7%) of the respondents strongly disagreed that students have restricted their definition and think fractions have to be less than 1 and 32 (26.7%) of the respondents disagreed with this statement but 25 (20.8%) of the respondents agreed and the remaining 19 (15.8%) of the respondents strongly agreed to this statement.

The respondents indicated similar responses about students counts fractional parts as they count whole number = 2.2 (1.1), failure of teachers to use the right approach= 2.4 (1.2); misapplication of rules for comparing whole numbers in fraction situations = 2.6 (0.9); misapplication of additive ideas when finding equivalent fractions= 2.6 (1.0). These results from the Table 4.5 goes in line with the findings of Isiksal and

Cakiroglu (2011), as they indicated in their study that students try to apply rules and assumptions from whole number operations to fractional operations.

Table 4.5 further indicated that 26 (21.8%) of the respondents strongly disagreed that students think that dividing by one-half is the same as dividing in half and 24 (20.2%) of the respondents disagreed with this statement but the remaining 69 (58.0%) of the respondents agreed to it. A mean score of 2.7 indicated that more than half of the respondents agreed to this statement. It is not surprising students have this misconception because it was stated in Wu (2008) article‟s that, fractional division and multiplication are the two most difficult abstract concepts for students to grasp at the age of adolescence. And this leads to them having difficulty applying these concepts problems to real-world situations (Isik & Kar, 2012). Policy makers should draft policies to motivate or guide teachers to teach practically using the relevant teaching aids to make lessons interesting and stay away from the abstract form of teaching, also lessons should be related to real life situations.

An interview was conducted, in addition to the questionnaire administered to students, in responding to Research Question three (3).

Causes of the Misconceptions of Unlike Fractions

To answer the third research question in depth, the researcher further interviewed the teacher respondents to ascertain the causes of these misconceptions among their students when it comes to the addition of unlike fractions. The responses from the respondents were put in three themes namely, (i) generalization of observations that are valid in integers for fractions, (ii) inattentiveness of students in class and (iii) bad approaches to the teaching of fractions.

(i) Generalization of Observations that are Valid in Integers for Fractions

In an attempt to generate a response to the theme derived above, the researcher asked the respondents this question: "How do learners relate some of the concepts learnt earl on to a new concept"?

The following below were some of the responses from the respondents in relation to or answering the question asked.

Teacher 2 indicated that:

> *"These students confuse whole numbers with fractions, they add fractions as if they are adding whole numbers. That is why they add numerators and add denominators during the addition of unlike fractions."*

Teacher 5 too was asked the same question and this was what he had to say about it

> *"I think one of the key causes of these misconceptions is the fact that students do not really grasp the concept of fractions as they treat fractions like normal counting numbers"*

This finding was supported by the results obtained by Kavramasi (2003), which stated that many students either at the primary or secondary level who still do not understand the concept of fractions failed to complete the breakdown well and even add, unlike fractions. Also, Vamvakoussy and Vosniadou (2010), concluded that students' understanding and conceptualization of decimals and integers was robust and that students found it difficult to make the relation between decimals, integers and fractions. Pesen (2007) also investigated the misconceptions that underpin popular fractions errors and found out that, students made common mistakes when splitting the whole into equal parts at the end of the study.

(ii) Inattentiveness of students in class

Another theme drawn out of the interview data was the lack of attention paid by students in the classroom during Mathematics lessons. In an attempt by the researcher to deduce the above theme, the researcher asked respondents this question: "How do learners behave during Mathematics lessons, and for that matter during the teaching and learning of addition of unlike fractions"?

The following are the responses of some of the interviewees (teachers) in response to the question asked here.

Teacher 4 answered by saying that:

> *"Look the truth is these students are not serious at all in class, if you are teaching, they will not be paying attention ooo. Because I don't know why they will be making these basic and simple mistakes"*

Teacher 1 also responded that:

> *"I think one of the key causes of these misconceptions is that they don't pay attention in class. Because the way I take my time to be teaching them that they still make these errors is very disturbing. They can be quiet looking at you whiles teaching and they will still make their errors".*

Teacher 1‟s response goes in line with the findings of Hansen (2006) as he defined these errors as, errors made by students as a result of carelessness, misinterpretation of symbols and text, a lack of relevant experience or expertise related to the mathematical subject, learning objectives, and a lack of understanding or inability to verify the answers given.

The researcher went further and asked the same question to all the teacher respondents, and this was what Teacher 10 answered in affirming the actual causes of these misconceptions and this was what Teacher 10 stated:

> *"Teaching mathematics in these areas is very difficult. In fact, teaching here as a whole is not easy since students always come to school very tired and they will be sleeping in class, makes it hard for them to concentrate in class. That account for these misconceptions*

> *because if they really paid attention like they will not make these mistakes"*

(iii) Bad Approaches to The Teaching of Fractions

Bad approaches to the teaching of Fractions were one of the themes derived from the transcribed interview data.

The researcher asked that: "How do teachers teach the learners, the concept in adding unlike fractions"?

The following are the responses from the interviewees:

Teacher 3 responded that:

> *"To be honest I am a teacher but the way some teachers teach fractions is very bad, so the students come from their previous class with a lot of misconceptions that they find it difficult to let it go. So, I think the way fractions are taught are part of the reasons why students have these misconceptions".*

Teacher 7 also admitted that:

> "*Most teachers teach fractions in the abstract form, per my observation if they make it practical, the students will not have these wrong ideas about unlike fractions. I think students will understand things better if fractions are taught with the right approach to involve the students."*

This goes in line with the findings of Alacac (2010) as in this study suggested that, an early and hasty transition to the representation of fractions in the classroom with abstract symbols without dependence on student experience and a basic conceptual framework leads to misconceptions. The origins of these misunderstandings can be traced back to the way fractions are taught in the classroom (Isik & Kar, 2012, Isiksal & Cakiroglu, 2008; Isiksal & Cakiroglu, 2011; Murray & Newstead, 1998, Shaughnessy, 2011; Wu, 2008). In the classroom, many math teachers rely on teaching information, laws, and procedures (Isiksal & Cakiroglu, 2008). So, Bay-Williams (2013) recommended that teachers encourage specific language use describing fractions and use the right approach to help students understand the concepts.

4.5 Research Question 4

What measures that can be used to help improve JHS pupils" conceptions and misconceptions of additions of unlike fractions?

In an attempt to find out the measures needed to be put in place to assist learners to clear predetermined conceptions and misconceptions, the researcher interviewed the mathematics teachers selected for this study. Their responses were put into two themes, these are: (i) practicalizing teaching of unlike fractions, and (ii) using the appropriate Teaching and Learning Materials (TLMs) and pedagogies.

Measures that can be used to improve learners' conceptions and misconceptions in adding unlike fractions, among JHS Pupils.

The study further sought to find out the measures that can be adopted to help solve this problem, thus doing away with these misconceptions on the part of the learners, and by improving on the way the learners conceptualize addition of unlike fractions, teachers were interviewed to gather data to help answer this question and the following were some of their responses.

1. Practicalizing Teaching of Unlike Fractions

The researcher in an attempt to find out suitable responses to the Research Question 4, the researcher asked: "Based on the causes of these misconceptions in adding unlike fractions by learners, what are some of the measures that can be put in place to remedy the situation and also to help improve on the conception in adding unlike fractions"?

In responding suitably to the question raised here, Teacher 1 stated that:

> *"I think students will be free of these misconceptions if, from the basic level, fractions are practically taught for them to actually grasp the concept".*

Teacher 1 further reiterated that:

> *"Students often understand concept paaa if it is practical, abstract teaching of any topic in mathematics makes it difficult for the students ooo and even we teachers".*

Teacher 5 also said that:

> *"Hmmmm, I think the best way to teach like and unlike fractions is by using manipulatives to make the lesson interesting and practical. Abstract teaching doesn't promote effective understanding so I think we should make our teachings practical".*

This confirms the findings of Tydings (2014) as in this study, it is indicated that instruction needs to focus on more than simply showing students symbolic fraction expressions.

Also, the provision of physical facilities such as demonstration rooms; adoption of effective teaching strategies such as provided activities, project work, and field trips are some of the strategies that can be used to improve pupils' interests in mathematics, according to studies (Grouws & Cebulla, 2000; Asafo-Adjei, 2001).

2. (i) Using appropriate instructional pedagogies.

In an attempt to come out with the second theme in answering the Research Question 4, the researcher asked that: "What other means do you think, we(teachers) are to bring on board in remedying the misconception in adding unlike fractions and also help to improve the conception they have in this same concept"?

With this, Teacher 7 indicated that:

> *"I think for students to really understand the concept of fractions and specifically, addition of unlike fractions, teachers should take their time to teach the concept very well."*

Teacher 7 further continued that:

> *"Teachers need to use manipulatives in their teaching of fractions because the hands-on approach helps paaa. The students learn well if you involve them."*

Teacher 4 also, when asked the same question that was asked Teacher 7, Teacher 4 said that:

> *"The teachers need to take their time during lesson delivery and first revise what students learnt in the previous class since learners build on their previous knowledge."*

Students construct their mathematical knowledge through building on previous knowledge; as a result, any misconceptions they develop while learning mathematics can have an impact on their future learning of related mathematical concepts (Vamvakoussy &Vosniadou, 2010).

For solving the mistakes of students, Eroğlu (2012), recommended strategies of verbal explanations, area models, daily-life examples, repetition of preliminary knowledge, teaching standard solutions, asking leading questions, using easy examples, using opposite examples, exercises and practices, leading students to notice their mistakes and increasing students" motivation. Also, Bird (1985) added that mathematics should be taught and learned in an activity-oriented manner, with students actively participating in the lesson. Provision of physical facilities such as demonstration rooms; adoption of effective teaching strategies such as provided activities, project work, and field trips are some of the strategies that can be used to improve pupils' interests in mathematics, according to studies (Grouws & Cebulla, 2000; Asafo-Adjei, 2001).

(ii) Using Appropriate Teaching and Learning Materials (TLMs)

During the interview, the researcher asked about some of the measures that can help improve the conceptualisation of adding unlike fractions among the learners. Teacher 3 indicated to this that:

> *"Hmmm for me I think the use of teaching and learning materials will help learners to fully understand the concept fractions and specifically, unlike fractions, we all know how some learners do not like maths, so using the teaching materials will motivate them to learn."*

When Teacher 7 was asked similar question, the following was the respondent"s response:

> *"It will be easier for students to understand unlike fractions if teachers use manipulatives during their fraction lessons.*

The researcher further probed to find out which manipulative does the respondent here think will be appropriate? This was the response from Teacher 7:

> *"Oh, for manipulatives in the teaching of fractions, I think the best ones will be fractional charts, to show parts of a whole, or can use oranges, card boards and even Cuisenaire rod."*

Their responses were in line with the findings of Brown and Quinn (2006) as they concluded that unit rectangles, number lines, and other physical manipulatives positively influenced the success of some of the respondents versus others. Unlike fractional concepts transformed some ancient civilizations until modern-day times. Different representations including pictures, symbols, and manipulatives have been utilized in the curriculum. In affirming this response, Higgins (1977) also indicated that, coercive materials lead to greater achievement gains than not using them.

4.6 Discussion of Findings

Misconceptions among learners is not a novel situation, the study revealed that JHS pupils in the Afadzato South district had several misconceptions about the addition of unlike fractions, and some of these misconceptions revealed in the study are when learners think all fractions are lesser than 1, when they add only the numerator during the addition of unlike fractions and also pupils tend to think that the value of fraction increases as either the numerator or denominator increases. These misconceptions are similar to findings of several authors like Murray and Newstead (1998) as they indicated in their study that students applied the values of the numerators and denominators to the problems since they are handling the numerator and denominator separately.

Several factors affect the pupils understanding with respect to the addition of unlike fractions. The study revealed that, teachers teaching fractions in abstract is a cause or a factor that led to these misconceptions, just as Alacac (2010) stated in his study that, an early and hasty transition to the representation of fractions in the classroom with abstract symbols without dependence on student experience and a basic conceptual framework leads to misconceptions. Learners" inattentiveness in class was also revealed as a contributing factor to the misconception of addition of unlike fractions among JHS pupils.

The study also revealed that, students have a lot of misconception when it comes to the addition of unlike fractions, some of these misconceptions are pupils treating fractions like whole numbers as they add numerators to numerator and denominator to denominator.

Measures taken to help mitigate this misconception of addition of unlike fractions, teachers are to use adequate Teaching and Learning Materials (TLMs) in teaching, lesson should not be taught in abstract form but in symbolic forms by using real life examples, physical materials. As it was explained in the study of Brown and Quinn (2006) as they concluded that unit rectangles, number lines, and other physical manipulatives positively influenced the success of some of the respondent"s s versus others. Finally, teachers should take time in explaining the concept of fractions to learners. Studies have revealed that mathematics teachers teach the subject in abstract form (Essuman et al, 2021), this does not help in better understanding of concept by the learner.

CHAPTER FIVE

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

5.0 Overview

This chapter provides a summary of the study, the conclusions drawn from the findings of the study, and recommendations made from the conclusions of the study.

5.1 Summary of the Study

The purpose of this study is to explore the conceptions and misconceptions of the learning of addition of unlike fractions in public junior high schools in Afadzato South District of Ghana. Specifically, it sought to find out the various conceptions and misconceptions that JHS students have in the addition of unlike fractions.

The study used the mixed-method approach with sequential explanatory as the research design and used two instruments for data collection namely questionnaire and interview. The population of the study included all JHS students and teachers in the Afadzato South District of Ghana.

A sample of 10 teachers and 120 students were involved in the study. The researcher used the purposive sampling technique which is a non-probability sampling technique and the simple random sampling technique, which is a probability sampling technique to select 120 students and 10 teachers, totaling 130 respondents selected for the interview.

Descriptive statistics were employed in presenting the results of the study. Descriptive statistics such as frequency counts, percentages, means, and standard deviations were used to report the conceptions, misconceptions of learners. The interview data was also analyzed thematically.

5.2 Key Findings

Research Question 1

The study revealed that students have a positive but moderate conception of the addition of unlike fractions, it also revealed that, students do not even understand the basic concepts of fractions as they are of the view that fractions are lesser than 1. It was also evident in the study that, students treated fractions like how they treat the whole numbers, therefore, wrongly applying the knowledge they have gained from the previous class or lesson on the addition of whole numbers**.**

Research Question 2

The study revealed that, the fractions as taught in abstract by teachers is one of the leading causes or factors that leads to these errors and misconceptions committed by pupils, just as an early and hasty transition to the representation of fractions in the classroom with abstract symbols without dependence on student experience and a basic conceptual framework leads to misconceptions. Learners‟ inattentiveness in class was also revealed as a contributing factor to the misconception of addition of unlike fractions among JHS pupils.

Research Question 3

In this regard, the study further revealed that, learners have a lot of problems with regards to additions of unlike fractions as they lack the concept of finding the Least Common Multiple (LCM) and adding of two digits‟ numbers, they sometimes think getting Least Common Multiple (LCM) is by just multiplying the denominators which are not always so.

The study also revealed the following as some of the key causes of students" misconceptions, students do not pay attention in class and this led to them not being able to fully understand the concept of fractions thereby making it difficult for them to appropriately add, unlike fractions.

Another reason that the respondents indicated as one of the causes of the misconceptions of unlike fraction is, the teachers do not use the right approach in teaching, as it was evident that they teach the concept of fractions in abstract and symbolic form, failing to make it practical in real-life situations. Teachers rely on teaching facts and procedures to solving fractional problems, rather than building a deeper understanding. This hinders students" ability to continue to understand more abstract, rational ideas.

Research Question 4

Based on the data collected it was evident that, to solve these misconceptions of learners, teachers should make teaching and learning of fractions practical, by using an appropriate instructional pedagogy and also use appropriate teaching and learning materials to motivate students to learn and better explain the concepts of fractions and specifically, addition of unlike fractions.

5.3 Conclusions

Based on the findings, the following conclusions were made:

1. Junior High School pupils" conceive addition of unlike fractions to be difficult. This however, affect their performance in mathematics.
2. Several misconceptions about the addition of unlike fractions, and some of these misconceptions revealed in the study are when learners think all fractions are lesser than 1. Also, it is conceived that students applied the

values of the numerators and denominators to the problems since they are handling the numerator and denominator separately.

3. Mathematics teachers teaching fractions in abstract way is a cause or a factor that led to these misconceptions, just as Alacac (2010) stated in his study that, an early and hasty transition to the representation of fractions in the classroom with abstract symbols without dependence on student experience and a basic conceptual framework leads to misconceptions. Learners" inattentiveness in class was also revealed as a contributing factor to the misconception of addition of unlike fractions among JHS pupils.
4. The use adequate Teaching and Learning Materials (TLMs) in teaching mathematics lessons should not be taught in abstract form but in symbolic forms by using real life examples, physical materials. Also, teachers should take time in explaining the concept of fractions to learners.

5.4 Recommendations

Based on the key findings, it is recommended that,

1. Mathematics teachers should teach mathematical concepts with clear understanding to the students, clearing every misconception and demystifying the wrong application of the knowledge gained on fractions.
2. Mathematics teachers should teach mathematical concept with concrete objects before moving to the abstract.
3. Education directorate in the Afadzato South District should organize workshops for mathematics teachers by highlighting what causes the errors and misconceptions in addition of unlike fractions among Junior High School pupils.

4. Mathematics teachers should make teaching and learning of fractions practical, by using an appropriate instructional pedagogy and use appropriate teaching and learning materials to motivate students to learn and better explain the concepts of fractions and specifically, addition of unlike fractions.

5.5 Suggestions for Further Studies

This study is not comprehensive. It is recommended that this study should be replicated in other areas of the country to find out if the findings of the study persist in those areas. It is further recommended that a study should be conducted to ascertain learners‟ misconceptions on subtraction of unlike fractions.

The researcher also suggests that, other researchers can conduct similar studies in finding the misconceptions of learners on multiplication of fractions.

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APPENDICES

APPENDIX A

Questionnaire for Students

UNIVERSITY OF EDUCATION, WINNEBA

SCHOOL OF GRADUATE STUDIES

DEPARTMENT OF BASIC EDUCATION

AN EXPLORATION INTO THE CONCEPTIONS AND MISCONCEPTIONS OF ADDITION OF UNLIKE FRACTIONS IN PUBLIC JUNIOR HIGH SCHOOLS IN THE AFADZATO SOUTH DISTRICT OF GHANA

This questionnaire is being used to gather information on the conceptions and misconceptions of addition of fractions. The information is being collected as part of a Master‟s Thesis. It is therefore strictly for academic purposes. I will be grateful to have you take part in the study by answering the questions as honestly as possible.

Please be assured that the information you provide will be kept confidential.

Thank you.

Instruction: Tick √ the appropriate bracket [ ] representing your response to the question or statement or write your response in the blank spaces where necessary.

SECTION A: Demographic Characteristics

1. Gender: Male [ ] Female [ ]
2. Age: 10- 13 [ ] 14- 16 [ ] 17 and above [ ]
3. Form: JHS 1 [ ] JHS 2 [ ]
4. School type: [ ] Private [ ] Public

SECTION B

CONCEPTIONS OF STUDENTS IN THE ADDITION OF UNLIKE FRACTIONS

	STATEMENT	SD	D	A	SA
1	All fractions are always part of a whole number				
2	Addition of fractions makes numbers bigger, and subtraction makes them smaller				
3	When adding fractions, you add only the numerator				
4	Addition of fractions is done using the LCM of the denominators.				
5	The larger the denominator the smaller the fraction regardless of numerator.				
6	The larger the denominator, the bigger the fraction. If				
	the numerator is the same.				
7	Numerator and denominator are separate values, two separate whole numbers				
8	Fractions are not always lesser than 1				

Key:
SD = Strongly Disagree, D=Disagree, A= Agree, SA= Strongly Agree.

SECTION C

MISCONCEPTIONS OF STUDENTS IN THE ADDITION OF UNLIKE FRACTIONS

	STATEMENT	SD	D	A	SA
1	All fractions are lesser than 1				
2	All fractions are always part of 1, never bigger than				
3	Properties of whole numbers can be applied to fractions.				
4	The numbers in numerator and denominator should be compared separately rather than considering the whole fraction.				
5	Operation rules for natural numbers can be applied to operations with fractions.				
6	The value of the fraction increases when either the numerator or the denominator increase.				
7	Add only the whole numbers when dealing with mixed fractions				
8	During addition of fractions, add the same denominators				
9	During addition of unlike fractions, add the same numerators				
10	When adding fractions, you add the numerator and the denominator.				

Key:
SD = Strongly Disagree, D=Disagree, A= Agree, SA= Strongly Agree.

SECTION D

FACTORS THAT CAUSE THE ERRORS AND MISCONCEPTIONS OF ADDITION OF UNLIKE FRACTIONS

	STATEMENT	SD	D	A	SA
1	Student does not understand that fractions are numbers as well as portions of a whole.				
2	Student thinks that mixed numbers are larger than improper fractions				
3	Students have restricted their definition and thinks				
	fractions have to be less than 1.				
4	Students counts fractional parts as they count whole number				
5	Teachers fail to use the right approach in teaching fractions.				
6	Student has restricted his definition of fractions to one type of situation or model, such as part/whole with pieces				
7	Misapplication of rules for comparing whole numbers in fraction situations				
8	Misapplication of additive ideas when finding equivalent fractions				
9	Students add fractions, generalizes the procedure for multiplication of fractions by adding the numerators and adding the denominators				
10	Students think that dividing by one-half is the same as dividing in half				

Key:
SD = Strongly Disagree, D=Disagree, A= Agree, SA= Strongly Agree.

APPENDIX B

UNIVERSITY OF EDUCATION, WINNEBA
SCHOOL OF GRADUATE STUDIES
DEPARTMENT OF BASIC EDUCATION
AN EXPLORATION INTO THE CONCEPTIONS AND

MISCONCEPTIONS OF ADDITION OF UNLIKE FRACTIONS IN

PUBLIC JUNIOR HIGH SCHOOLS IN THE AFADZATO SOUTH

DISTRICT OF GHANA

INTERVIEW FOR TEACHERS

SECTION I: General Information

i. Age……………………

ii. Gender…………………

iii. Academic qualification………….............

iv. Professional qualification…………………..

v. How many years have you taught as a basic school teacher?

vi. At what school are you presently teaching?

vii. How many years? ....

viii. What grade do you teach? …………………………

ix. How many students do you have on roll?

SECTION II: Measures to mitigate Errors and Misconceptions of students on addition of unlike fractions

1. Do you think students encounter some problems when adding unlike fractions?

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2. If Yes, why?

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3. Have you observed some mistakes or errors, that your students display when working with the addition of unlike fractions?

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4. If yes, can you give some examples?

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5. What do you think are the possible causes of these errors?

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6. Do you think pupils have misconception in the addition of unlike fractions?

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7. If yes, can you give examples of such misconceptions?

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8. What are the possible causes of the misconception held by pupils?

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9. Do you think there is a reason why pupils commit these errors?

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………

10. If yes what is the reason?

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11. In your view, what do you think are the possible ways to help eliminate or reduce these errors?

……………………………………………………………………………
……………………………………………………………………………
……………………………………………………………………………

APPENDIX C

Letter of Introduction

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF EDUCATIONAL STUDIES
DEPARTMENT OF BASIC EDUCATION
P. O. Box 25, Winneba, Ghana
beducation@uew.edu.gh
+233 (050) 9212015

Date: April 7, 2021

The District Director
Afadzato South District Education Directorate
Golokwati -- V/R.

Dear Sir/ Madam,

LETTER OF INTRODUCTION

I write to introduce to you Mr. Adolph Atsu Yao Anku, a second year M. Phil student of the Department of Basic Education, University of Education, Winneba, with registration number 200026566.

Mr. Adolph Atsu Yao Anku is to carry out a research on the Topic ***"An Exploration into the Conceptions and Misconceptions of Learning Addition of Unlike Fractions in Public Junior High Schools in the Afadzato South District of Ghana".***

We would be grateful if permission is granted him to carry out his studies in the District.

Thank you.

Yours faithfully,

DEPT. OF BASIC EDUCATION
UNIVERSITY OF EDUCATION
WINNEBA, GHANA

PROF MRS. SAKINA ACQUAH (PHD)
(Head of Department)

www.uew.edu.gh