UNIVERSITY OF EDUCATION, WINNEBA

USING GUIDED DISCOVERY METHOD TO MINIMIZE STUDENTS' DIFFICULTIES IN SOLVING WORD PROBLEMS INVOLVING LINEAR EQUATIONS IN ONE VARIABLE

SALIM IBRAHIM CHIBSAH MUHAMMAD

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DECLARATION

STUDENT'S DECLARATION

I, SALIM IBRAHIM CHIBSAH MUHAMMAD, declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

SIGNATURE:…………………………….

DATE:…………………………………….

SUPERVISOR'S DECLARATION

I hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of thesis as laid down by the University of Education, Winneba.

SUPERVISOR"S NAME: PROF. C. A. OKPOTI (Ph.D.)

SIGNATURE:…………………………….

DATE:…………………………………….

DEDICATION

To All My Teachers,

To My Father, Ibrahim Muhammad Zampuu

To My Mother, Abiba Gariba

To My Wife, Latifa Ateni Azure

And To My Daughter, Asifah Bint Salim

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May the almighty ALLAH to which there is no deity worthy of worship except him, the ever Living, the Cherisher, the Knower of the unseen, the Pure, the Bestower of Faith, the Overseer, the exalted in might, the Compeller, the Superior, the Creator, the Inventor, the Fashioner, the Sustainer of existence; to him belong the best names and exalted is Allah above whatever they associate with him be glorified for the guidance and protection throughout my stay at the University.

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ABSTRACT

The study was designed to examine whether guided discovery approach to teaching mathematical word problems would improve the performance of senior high school students of Jachie Pramso Senior High School in the Bosomtwe District of the Ashanti Region of Ghana. The research was to determine the impact of guided discovery learning on senior high school students' performance when solving word problem into mathematical statements or equations. The study adopted a mixed-method approach comprising qualitative and quantitative research methods as well as quasi-experimental design. The researcher used both purposive and simple random sampling techniques to select one hundred (100) participants: fifty (50) participants for the control group and fifty (50) participants for the experimental group. The data collection instruments used in the study was interview, pre-test and post-test. Analysis of data was carried out using descriptive statistics and independent samples t-test. The study found that 25 (25%) of the participants had conceptual difficulty in translating word problems into mathematical statements. Also, 27 (27%) of the participants had procedural difficulty in their quest to translate the word problems into mathematical statements or equations as contained in the pre-test. Moreover, few of the participants, 8(8%) experienced logical structure difficulty in translating word problems into mathematical statements or equations. It was noted that 10(10%) of the participants were not able to correctly contextualize the relationships involved in the word problems administered to them. Lastly, a critical examination of the scripts of the participants revealed that 30(30%) were not able to utilize the concept of variables in translating the word problems into mathematical statements or equations. The independent samples t-test analysis of the post-test scores for the experimental and control groups revealed that there was a statistically significant difference between the experimental group (M = 24.80; SD = 9.48) and the control group (M = 20.65; SD = 7.67). The estimated t-statistic was (t = 2.986; p = 0.005). This shows that the experimental group taught with the guided-discovery method outperformed the control group taught without the guided-discovery method. The analysis of the interview data indicated that guide-discovery method has contributed to the success of students‟ achievement in the word problems by arousing and sustaining the student‟s interest. The guided-discovery also made it easier for students to follow the instruction. GES should ensure that mathematics instructors get thorough in-service training in guided-discovery and its application in order to enhance good practices in the mathematics classroom. Pre-service teacher education should include considerable practice in fundamental and higher-order mathematical process abilities so that new teachers are more confident in their abilities to use guided-discovery method when teaching mathematics.

CHAPTER ONE

INTRODUCTION

1.1 Overview

Development in almost all areas of life is based on effective knowledge of science and mathematics and it is for this reason that the education system of countries concerned about development put a great deal of emphasis on the study of mathematics (Cinar, Pirasa, & Sadoglu, 2016). The main aim of teaching and learning mathematics in Ghanaian schools is to equip learners with the ability to describe, investigate and eventually solve mathematical problems through the use of their mathematical knowledge, concept, skills and techniques (Amponsah-Tawiah, 2020). The application of mathematics is evident in all fields of study and at the job market.

Suh, Matson, and Seshaiyer (2017) contended that, the world is becoming more mathematical, while Maass, Geiger, Ariza, and Goos (2019) commented that the role mathematics played in social, economic, technological and industrial development of any nation cannot be over emphasized. In line with this, Attard (2013), stressed on the need for strong mathematical foundation among students. Mathematics as explained by Yadav (2017) is the queen of all sciences, because it is the tool for all disciplines since no scientific advancement (practical applications and approach) could be achieved without it. Mathematics is accepted worldwide as a powerful tool for national development, yet most students perceive it to be abstract in nature and therefore do not commit themselves fully to the study of mathematics.

According to Abdulwahed (2017) with the advancement in the socio-economic and technological field, life of the individual is becoming more and more complex with a lot of problems, which the individuals and society have to face in the near future.

Therefore, the role of the school becomes increasingly important in developing scientific attitudes in students so that they will be able to solve their problems independently and adjust well in the future complex society (Jensen, Neeley, Hatch, & Piorczynski, 2017). The students need mathematics to develop the scientific thinking and aptitude, which is synonymous to mathematical thinking. Jacobs and Empson (2016) asserts that, if thinking is a way of improving understanding and extending control over the environment, mathematical thinking uses particular means to do this. Therefore, the need to have good methods of teaching the operations processes and dynamics of mathematics cannot be overemphasized.

According to Abdisa and Getinet (2012), guided discovery has rooted in the constructivist method of learning and teaching. This approach is oriented by the notion that learners make their own information not the information being transferred into the brains of learners (Abdisa & Getinet, 2012). The previous knowledge, the new knowledge timeliness, and the learner"s capability to comprehend the connections make the basis for the learners" construction of knowledge (Akinbobola, 2015). By this procedure, learners focus on modifying the present knowledge or establish novel knowledge. Common instructional strategies reflect learning experiences oriented by constructivism like inquiry-based, experiential learning, simulation-based, and problem-based learning (PBL) (Abdisa & Getinet, 2012). Numerous common features are shared within all constructivist instructional policies. Also, an experienced educator utilizing the PBL method may utilize guided discovery principles very well. The guided discovery approach refers to a learning process where the teacher does not directly expose the learning principal content. Instead, learners reveal them that make students active participants and the teacher a guardian within the learning procedure (Akinbobola, 2015).

Overall, the concept of constructivist teaching assumes that knowledge and skills acquisition are not based on rote learning and passively receiving information, although they include the participation of active learners via knowledge construction, as well as minds-on and hands-on activities (Akinbobola, 2015). The major characteristics of guided discovery are learning through the process of discovery and knowledge exploration and learner responsibility. In this way, the learner is helped to get proficiency over the required content for understanding. As a learner-centered approach, guided discovery learning engages the learners and their innate abilities in the process of discovering and learning (Forzani, 2014).

Moreover, word problem solving is the cornerstone of senior high school mathematics. Without the ability to solve problems, the usefulness and power of mathematical ideas, knowledge, and skills are severely limited (Nursyahidah, Saputro, & Rubowo, 2018). According to Verschaffel, Depaepe, and Van Dooren (2020), word problems in mathematics refers to the process where answer can be obtained by application of mathematical operations. Word problem refers to the everyday application of mathematics. The ability to interpret, translate and codify the everyday use of mathematics, often begin in word form or statement. The ability of a student to solve word problems is an important way of developing mathematical knowledge. Word problem is not a topic of itself in the mathematics syllabus, but every mathematics topic contains word problems (Kingsdorf & Krawec, 2014). The main purpose for teaching and learning mathematics is to develop the ability for students to solve a wide variety of both simple and complex mathematics problems in their daily lives (Kingsdorf & Krawec, 2014).

A student's ability to solve a mathematics problem, whether simple or complex, begins from learning to solve linear equations with one unknown. However, the researcher noticed with concern that word problems involving linear equations in one variable, generate among students a feeling of fear, anxiety, unease and insecurity. “I consider assignments and exercises on word problem involving linear equations in one variable as a punishment”, are common expressions how students feel or perceive about word problems.

According to Azure (2015) students are ill equipped from senior high schools to handle mathematics problems, because they are poorly taught. Arhin (2015), said, therefore, it is important that mathematics educators should work closely with the senior high schools in finding better alternative methods of instruction and discuss what should be included in the mathematics programmes. In furtherance, Arhin (2015) stated that due to poor teaching methods and lack of qualified mathematics teachers and public prejudices against mathematics, students view mathematics with apprehension. Therefore, the search for a better instructional method that should reduce such apprehension and aversion in students cannot be over emphasized.

According to Sumarna and Herman (2017), the learning difficulties among students which one observes as a teacher of mathematics raises many other questions that one might seek an answer from theories and methods of teaching.

According to Sumarna and Herman (2017), the learning difficulties among students which one observes as a teacher of mathematics raises many other questions that one might seek an answer from theories and methods of teaching. For example, although reflection on our own experience should suggest to us that learning cannot be achieved in a hurry, Acharya (2017), have this to say, that some students appear to learn slowly, some make very rapid progress and few even make outstanding progress given the opportunity to learn at their rate rather than the class rate. Therefore, it is only possible to accelerate the learning of mathematics for the majority of students when one uses the appropriate instructional method for each type of students.

Individual differences are very significant in many spheres of human activity. Some of us are barred from particular subject because of physical characteristics, like being too small, too heavy or having poor sight. Jailani, Sugiman, and Apino (2017), in discussing mathematicians drew attention to great differences in the kind of mathematical aptitude, which individuals have displayed. He further explained that, in the classroom, it might be that different learning environment and different styles are needed for different students. Therefore, any acceptable theory or method, which enables us to understand individual differences among our students, would be very valuable.

As has been suggested by Sumarna and Herman (2017), the learning environment might be an important factor in promoting and understanding mathematics. It might therefore be seen that the richer (encourage) the environment, the more efficient the learning. Hence, what constitutes a rich environment is a subject which is basically a creation of a human mind and in which the aim is to enable abstract arguments to take place through manipulation of symbols, appropriate learning techniques, learning materials and most of all understanding the needs of students within the rich environment.

Meanwhile, mathematics educators have constantly been criticized for the poor standards achieved by some students in mathematics in the Senior High School (Göransson, Hellblom-Thibblin, & Axdorph, 2016). The challenge in education today is to enhance effectively the teaching of students with diverse abilities and differing rates of learning in a way that will enable students to learn mathematical concepts while acquiring, positive attitudes, values and problem solving skills.

To overcome student"s challenges and difficulties, a variety of teaching and learning strategies have been advocated for use in mathematics classroom moving away from the teacher-centered approach to more student-centered approach. Thus guided discovery is one of the potential instructional strategies. Hence, this study is designed to use guided discovery Learning Approach to enhance students' ability to translate word problem involving linear equations to mathematical statement or equation.

Moreover, educators are often searching for strategies to support student engagement and success with solving application-based mathematics word problems. Studies have shown that interest in problems increases the level of students" performance (Rotgans & Schmidt, 2017; Bernacki & Walkington, 2018). My study is important to the field of education because it will provide insights into how the personalization of word problems shapes student interest in mathematics instruction while influencing overall performance.

When striving for equity in a mathematics classroom it is important to remove assumptions especially when creating application-based problems and making sure that all students can access the context of the situation presented. Invariably, for every word problem presented on an assignment or assessment there will be a percentage of students who cannot make connections because of personal history. The study aims to improve equity in a mathematics classroom to increase student identity in the class by creating word problems that are personalized to student interests. When mathematics class is primarily theoretical its connection to daily life disappears.

To overcome student‟s challenges and difficulties, a variety of teaching and learning strategies have been advocated for use in mathematics classroom moving away from the teacher-centered approach to more student-centered approach. Thus guided discovery is one of the potential instructional strategies. Hence, this study was designed to use guided discovery Learning Approach to enhance students' ability to translate word problem to mathematical statement or linear equations.

1.2 Statement of the Problem

Teaching at the senior high school for six years, the researchers‟ observed that many students consistently perform poorly in solving word problems. A particular example was a test conducted on five word problems to assess students' ability in word problem solving. Out of the 60 students who participated, only 6 representing 10% could answer all the questions correctly. In particular, 12 students representing 20% could answer two to three questions halfway, while 42 students representing 70% could not answer any question correctly. The results really showed that students have difficulties in solving word problems. One of the major difficulties students faced was how to develop linear equations from word problems hence the problem of the study.

According to Mensah, Okyere, and Kuranchie (2013), many students fail mathematics examination as a result of negative attitude towards mathematics, the results obtain in mathematics by students in both schools and public examinations is so alarming. Price, Kares, Segovia, and Loyd (2019), said that the attitudes exhibited by many people especially students and adults in the society towards mathematics need special attention for the young developing nation desirous to practicing scientific technology and industrial developments.

Studies have showed that high achievement in mathematics is related to low mathematics aversion level and low achievement in mathematics is related to high mathematics aversion level for secondary school student as observed by Sumarna and Herman (2017). However, the researcher observed that both students with high and low aversion level are taught by the same methods of teaching. This therefore, led the researcher to feel that there is need to find out an instructional method suitable to students with mathematics aversion, which can assist them in overcoming such aversion in mathematics. This study investigated into the impact that guided discovery method will have in minimizing students‟ difficulties in translating word problems involving linear equations into mathematical statement

1.3 Purpose of the Study

The purpose of this study is to investigate the effectiveness of using the guided discovery method in minimizing students' difficulties in solving word problems involving linear equations in one variable in Bosomtwe District in the Ashanti Region of Ghana

1.4 Research Objectives

The following research objectives guided the study to:

1. Examine the difficulties students experience in solving algebraic word problems.
2. Examine the extent to which the guided discovery method improves students' understanding of solving word problems into algebraic equations.

3. Determine the effectiveness of guided discovery approach of teaching and learning on students taught with guided discovery method and those without.

1.5 Research Questions

The following research questions underpinned the study:

1. What difficulties do students experience in solving algebraic word problems?
2. To what extent does the guided discovery method improve students' understanding of solving word problems into algebraic equations?
3. What is the effectiveness of guided discovery approach of teaching and learning on students taught with guided discovery method and those without?

1.6 Research Hypothesis

In order to achieve the third research objective, the following hypothesis was formulated and tested at 5% alpha level of significance.

H_0: There is no significant difference between the mean score of senior high school students taught with guided discovery method and those without.

1.7 Significance of the Study

The aim of this research is to provide information on the use of guided discovery as a pedagogical approach in the teaching of senior high school mathematical word problems at Jachie Pramso Senior High School in Bosomtwe District in the Ashanti Region of Ghana. The findings of the study would serve as a framework for effective teaching approach. The study's result would be invaluable to practicing mathematics educators, school administrators, curriculum planners, and mathematics trainers at universities in placing interventions that would result in high-quality teaching and learning experiences. The results would also be useful to policymakers, those interested in educational research and policy development, and other stakeholders in the field of education.

1.8 Delimitation of the Study

Delimitation describes the scope of a study or establishes parameters or limits for astudy. This study is delimited to the use of guided discovery approach to teaching mathematical word problems in the Bosomtwe District. Moreover, the study was delimited to all senior high schools in the Bosomtwe District in Ashanti Region of Ghana.

1.9 Limitation of the Study

The study was hindered by time constraints. The time constraints within the confines of this study did not allow for exploration beyond the scope of this study. Hence, the findings, inferences and deductions from this study could not be generalized to other senior high schools in other districts or municipalities other than the study area. Lack of adequate literature on guided discovery approach of learning in Ghana was a problem. This posed a challenge to the researcher. More so, financial resources also limited the study in terms of data collection.

1.10 Organization of the Study

This research is organised into five chapters. Chapter one presents the introduction to the study, comprising background to the study, statement of the problem, purpose and objectives of the study, as well as research hypothesis, significance of the study, delimitation of the study, limitation to the study and organization of the study. Chapter two presents review of related literature to the study. Chapter three concentrate on the methodology including an introduction to the chapter, research design, population, sample size and sampling technique, instrumentation, data collection and data analysis procedures as well as ethical considerations. Chapter four presents the research results and findings. Chapter five, the final chapter will present the summary of the findings of the study, conclusions and recommendations.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

This chapter establishes conceptual framework and empirical studies of the research. The conceptual framework is build, based on guided discovery approach to teaching and learning of mathematics. The review is centered on constructivist theory, theory of multiple representation, senior high school students‟ performance in mathematics, students‟ difficulty in learning word problem, eliminating difficulties in learning mathematics, methods of teaching SHS mathematics, impact of guided-discovery approach on performance of students in mathematics, linear equations with on variable, basic knowledge of solving equations with one variable, misconceptions about the concept of equality or equations, teacher role, social norms, and socio-mathematical norms, conceptual framework.

2.1 Constructivist Theory

Constructivism is a theory of knowledge that argues that humans generate knowledge and meaning from interaction between their experiences and their ideas (Charmaz, 2017). The basic idea in this theory is that learning is an active and constructive process with the learner viewed as an information constructor. In the constructivist classroom the teacher becomes a guide for the learner, providing bridging or scaffolding, helping to extend the learner‟s zone of proximal development. The student is encouraged to develop meta-cognitive skills such as reflective thinking and problem solving techniques. Independent learners are intrinsically motivated to generate, discover, build and enlarge their own frameworks of knowledge (Weimer, Fabricius, Schwanenflugel, & Suh, 2017).

Researchers such as Weimer, Fabricius, Schwanenflugel, and Suh (2017) and Brooks and Charmaz (2017) suggest that a constructivist approach to learning builds on the natural innate capabilities of the learner. From this perspective, the learner is viewed as an active, not passive person, actively constructing understanding through the use of authentic resources and social interaction (Fernando & Marikar, 2017). According to Weimer, Fabricius, Schwanenflugel, and Suh (2017), central to the notion of constructivism is the view that experience and knowledge are filtered through the learner‟s perceptions and personal theories. The focus of constructivism is on cognitive development and deep understanding in which learning is non-linear and learners are encouraged to freely and actively search for solutions which are necessary for a guided discovery approach of learning. Such an active learning coupled with deeper construction of meaning of knowledge is likely to promote retention, comprehension and high-level critical thinking skills which are attributes needed by the learners to improve their performance in mathematics. It is the desire of the researcher that mathematics educators or teachers create a meaningful learning experience for SHS students by creating an environment which supports investigation and problem solving through constructivist learning. It is rightly argued by Chuang (2021) that learners control their learning. This simple truth lies at the heart of the constructivist approach to education, that is, learners must be permitted the freedom to think, to question, to reflect, and to interact with ideas, objects, and others in other words, to construct meaning (Collins, Brown, & Newman, 2018; Fatimah, Rosidin, & Hidayat, 2022).

This constructivist learning that promotes deeper construction of meaning of knowledge is lacking in senior high school students in the research area during teaching and learning of mathematics. The conventional approach to teaching used by most Ghanaian mathematics teachers (Armah, Cofie, & Okpoti, 2018), does not offer senior high school students the opportunity to learn mathematics actively and thereby construct their own meaning of knowledge through thinking at the higher levels of Bloom"s Taxonomy of cognitive domain (application, analysis, synthesis, evaluation), this learning gap explains the recurring low performance by the students in mathematics and therefore this study examined how this learning gap might be filled through the use of a guided discovery approach in teaching mathematics. Recommending constructivist paradigm in teaching and learning mathematics, Fatimah, Rosidin, and Hidayat (2022) argue that constructivist learning paradigm should be considered as an alternative to transmission view since a fundamental goal of mathematics instruction is to help learners build structures that are more complex, powerful and abstract than those learners possess before instruction. This constructivist learning paradigm suggests that senior high school students in the study area learn best when learning is: active, self-directed, based on problems related to their experiences and perceived as relevant to their needs, and intrinsically motivated. Such learning is at the roots of constructivism and can take place in a social working environment that promotes sharing of knowledge and experiences gained. This manner of learning is best experienced through a guided discovery approach of learning mathematics in groups.

2.2 Theory of Multiple Representations

In the realm of education, Jerome Bruner offered many theories. Bruner's works in education were centered on cognitive development and psychology in education. Bruner (1966) learning strategy was based on logic-scientific processes. In his study, Bruner indicated that learners would better understand abstract concepts if a differentiated learning strategy was devised and implemented based on the learner's strengths and weaknesses. According to Bruner (1966) theory, which stressed representational difference, they maintained that each mode of cognition has three stages. These stages were enactive, iconic and symbolic. However, learning occurs through movement or activities in the enactive stage. The enactive stage includes manipulating a solid object and learning about its qualities. This step is demonstrated or represented by guiding students to use teaching and learning resources or material to discover representation of linear equations in the study. Through vivid representations, the iconic stage is aided in developing mental processes. Here, learning takes place through the use of visuals and iconography. For instance, investigating the qualities of a solid structure with photographs from mathematics textbooks is an iconic stage. This stage is perceived as watching a teacher demonstrates solutions of linear equations in a virtual setting using teaching and learning resources. Finally, the storage metaphor describes the symbolic stage (Bruner & Kenney,1965) in which information is stored in the form of codes or symbols: Learning is accomplished through abstract symbols. For instance, extracting linear equation in one variable embedded in a word problem using mathematical symbols is an example of the symbolic state of learning, according to Bruner (1966). This stage is interpreted as working with symbolic equations in a learning environment.

Moreover, Bruner (1966) work on representations in mathematics education has been viewed as multiple representations theory. Bennett, Inglis, and Gilmore (2019) and Dreher, Kuntze, and Lerman (2016) believed that multiple representation theory would explain how students learn abstract mathematical concepts through various mathematical representations. This view was shared by several other reformist mathematics educators, including Brantlinger (2014) as well as mathematics education organizations such as the National Council of Teachers of Mathematics

(NCTM) (2000). Furthermore, Kang and Liu (2018) emphasized that multiple representation theory can help students' cognitive processes in authentic, real-world issues and learning contexts. More so, some studies suggested that the best way to build learning settings that enhance conceptual understanding through multiple representation theory is to leverage technology (Samsuddin & Retnawati, 2018; Ott, Brünken, Vogel, & Malone, 2018).

According to Alacaci and McDonald (2012), technology provides various possibilities for learners to master abstract concepts in methods that were tailored to their particular learning styles and interests. Other studies (for instance, Allen & Trinick, 2021; Demetriou, Makris, Tachmatzidis, Kazi, & Spanoudis, 2019) called for the adoption of the multiple representation theory to establish the missing link between technology and mathematics education. Today, mathematics educators agree that multiple representation theory is a significant aspect of reformist mathematics education and that technology plays a vital role in accomplishing the reforms' desired objectives.

2.3 Benefits of Teaching and Learning with Multiple Representations

Kang and Liu (2018) mention that representation or illustration of mathematics concept need not be taught as though they are ends in themselves. Instead, they can be considered as useful tools for constructing understanding and for communicating information and understanding. If students simply complete assignments of constructing representations in forms that are already specified, they do not have opportunities to learn how to weigh the advantages and disadvantages of different forms or how to use those representations as tools with which to build their conceptual understanding. They go on to say that representations enhance the problem-solving ability and that students often construct meaning in forms that help them see patterns and perform calculations. The use of multiple representations with or without technology is one of the major topics in mathematics education that has gained importance in recent decades (Allen & Trinick, 2021). The significance of representing the solution of linear algebraic equations in multiple ways provides the same objective of more than one form. It is necessary to see how students use these representations. It is suggested that multiple representations provide an environment for students to abstract and understand major concepts Allen & Trinick, 2021) while constructivist theory suggests that we need to understand students‟ thinking processes in order to facilitate their learning in more empowering ways (Dreher, Kuntze, and Lerman (2016). Understanding students‟ thinking and their preferences while choosing a representation type for solving algebraic linear algebraic equations helps mathematics teachers gain insight into student thinking. Representations such as the do/undo flow chart and algebraic method are tools that provide the same information in more than one form.

The role of these tools or strategies is to represent solving linear algebraic equations using multiple concretizations of a concept, mitigate certain difficulties and to make mathematics more attractive and interesting (Anwar, Choirudin, Ningsih, Dewi, & Maseleno, 2019). Seto, Goh, Teh, and Chang (2020) emphasized that conceptual learning is maximized when children are exposed to a mathematical concept through a variety of physical contexts or embodiments. In other words, we should not expect that all students would perceive the same concept from one representation. Algebraic concepts have become a study of procedures and rules instead of exploration and concepts, which should lead to generalizations that justify the rules.

2.4 Benefits of Teaching with Multiple Strategies

Werts, Carpenter, and Fewell (2014) suggests that there is a possible trade-off in the initial stages of learning between the goal of the flexible use of multiple strategies and the goal of mastery of a standard algorithm. Schukajlow, Krug, and Rakoczy (2015) showed that prompting students to solve the same equation in different ways provides better results on items measuring students‟ strategic flexibility. By "student flexibility" we refer to the practice of allowing students to pursue multiple solution strategies within a given problem. Waalkens, Aleven, and Taatgen (2013) asked the question, "But does greater freedom mean that students learn more robustly?" They developed three versions of the same Intelligent Tutoring System (ITS) for solving linear algebraic equations that differed only in the amount of freedom given to students. The three conditions are (a) strict standard strategy, (b) flexible standard strategy and multi-strategy. The strict standard strategy adhered to a specific standard strategy, while the other two versions (flexible and multi) adhered to minor and major variations, respectively.

According to Waalkens, Aleven, and Taatgen (2013), with both the strict and flexible strategies, all equations had to be solved with a standard strategy that is widely used in American middle- school mathematics textbooks. They claimed that this standard strategy can solve almost all linear equations and is described as follows: First, use the distributive law to expand any term in parentheses. Second, combine constant terms and variable terms on each side of the equation. Third, move variable terms to one side of the equation and constant terms to the other side. And finally, divide both sides by the coefficient of the variable. The authors go on to say that students had the most freedom in the multi-strategy method because they could solve the linear equations with any strategy that progresses towards the goal of arriving at a solution.

For example, in the linear equation $2(x + 2) = 6$, students are allowed to divide both sides of the equation by 2 instead of using the distributive law to expand the term in parentheses, a step that is required in the two stricter methods. With the multi-strategy method, students have the most freedom because they can solve the equations with any strategy that progresses toward the goal of solving the equation. Waalkens, Aleven, and Taatgen (2013)"s study concluded that students improve their equation-solving skills. However, allowing minor or major strategy variations did not make a difference in learning gain, motivation, or perceived strategy freedom, compared to strictly enforcing a standard strategy with which students were familiar, without allowing any variations.

2.5 Senior High School Students' Performance in Mathematics

All over the world, stakeholders at the educational forefront are concerned about performance, in terms of students" mastery of concepts and skills in their experiences in formal educational settings. One of such concerns is associated with the performance of students in mathematics and other sciences. The performance of students in mathematics has been of grave concern to school administrator, teachers, parents and other stakeholder in educations. In Ghana, the performance of SHS students in mathematics is a key determinant of the progression of their education to the tertiary level. Ghanaian senior high students, after writing the West African Senior School Certificate Examination (WASSCE), are expected to obtain a grade not lower than A1 – C6 in core mathematics in order to progress to the tertiary level.

Available statistics from the West African Examination Council (WAEC) and other empirical studies suggests that between the periods 2015 to 2019, more than 50% of SHS leavers could not get admission to any tertiary institution in the country because they did not obtain the pass grade in mathematics in the WASSCE (Abreh, Owusu, & Amedahe, 2018; WAEC, 2018). Abreh, et al. (2018), investigated the trends in the WASSCE performance of SHS students in Ghana over a period of ten years (2007 – 2016) and the factors accounting for the abysmal performance. The study which employed the exploratory survey research design as method of enquiry sought to establish the trend in WASSCE performance over the period. The study also sought to identify some of the perceived factors influencing the trend in performance. The study participants, which consisted of the head teacher, one mathematics teacher, one science teacher and ten students each, were selected from a sample of 170 senior high schools selected across the country using proportionate stratified sampling technique. The instruments for data collection included data from WAEC and questionnaires for respondents. The findings from the analysis of data from WAEC, revealed that the trends in students" performance did not provide a definite pattern but the average performance of students over the period was below 50% and a high percentage of students acquired the lowest grade, F9. The analysis of data from the questionnaire instruments revealed a number of factors from the perspective of the respondents that contribute to the abysmal performance of SHS students in mathematics. Notable among these factors include; students" lack of interest in mathematics, entry characteristics, infrastructural deficit, and poor teaching methods employed by teachers.

The poor performance of SHS students in mathematics is not restricted to only Ghana. The situation in other jurisdictions is not quite different. A study conducted in Sierra Leone by Gegbe, Sheriff, and Turay (2015) on the factors contributing to the poor performance of students in their final external examination, WASSCE.

The study which explored school-based, demographic-based and student personal factors adopted a descriptive survey research design using a sample of 115 respondents. The respondents comprised of 100 WASSCE students, and 15 mathematics teachers selected randomly from five (5) purposively selected secondary schools in Kenema District.

The findings revealed among other factors that teachers‟ teaching strategies, low socio-economic background and lack of motivation of students in mathematics are some of the major causes of students‟ abysmal performance in mathematics. Specifically, the researchers found that about 70% of the teachers interviewed still use lecture method, which makes students passive listeners in the teaching and learning process. Other findings from the study indicated that participating students interviewed, enter senior high school (SHS) with a mean Basic Education Certificate Examination (BECE) aggregate of 5 in mathematics. This finding suggests that most students enter the SHS system with weak foundation in mathematics and that partly explained why there is still low performance in the WASSCE.

A similar study conducted in Nigeria also investigated the causes of poor performance in mathematics among public secondary school students in the Azare Metropolis of Bauchi State (Sa‟ad, Adamu, & Sadiq, 2014). The researchers used a larger sample of 300 high school students and 61 mathematics teachers, selected by proportionate stratified random sampling technique. They employed the descriptive survey design using a self-designed questionnaire involving two Likert-scale formats to collect data from the respondents. From the analysis of data using frequency and simple percentage, the researchers found that 85% of the respondents agreed that lack of motivation and negative attitudes of students toward the study of mathematics was a cause of poor performance in the subject among the students. The findings also revealed that 78% of the students were of the opinion that poor teaching strategies employed by teachers cause poor performance among public secondary school students. When asked what they think can remedy the situation, 83% were of the opinion that using appropriate methods of teaching, particularly methods that focus on the students can help in improving the performance of students in the subject. These results showed that majority of mathematics teachers still use inappropriate teaching methods which are teacher centered and does not encourage problem-solving in teaching and learning mathematics.

Beyond the boundaries of WASSCE, other studies conducted in across Africa point to poor performance of high school students amidst similar causes. Kariuki and Mbugua (2018), in a study conducted in Kenya, explored the factors contributing to students" poor performance in mathematics in the Kenya certificate of secondary education. The study also sought to establish the strategies that can be adopted to improve the performance of students. The study employed the descriptive survey method using 1876 respondents including 1718 secondary school students, 132 mathematics teachers and 26 head teachers selected from 26 secondary schools. From the analysis of questionnaires instruments, they found that a proportion of the teachers use the lecture method of teaching mathematics. The findings also revealed other factors such as lack of motivation to study mathematics, and low socio-economic status of students as contributing to the failure of students in mathematics.

Furthermore, VaraidzaiMakondo and Makondo (2020) conducted a case study to determine the causes of poor academic performance of high school students in Mathematics at ordinary level examination in Masvingo Province of Zimbabwe. The study adopted a case study design, where only students doing O‟ level in one high school in the province were purposefully selected for the study. The teachers who taught these students O‟ level mathematics were also among the respondents for the study. The main instrument for data collection was the use of questionnaires, which were physically administered by the researchers to have a wider view of the problem under study. Other instruments used by the researchers included document analysis, interviews and observation.

The findings after analyzing the data showed that a number of contributing factors to the poor performance of the students. These include; a significant number of the students lack interest in the subject as a result of fear, weak entry characteristics of students, negative attitudes of the community towards mathematics, students‟ inability to transfer knowledge from one concept to the other, all the teachers use question and answer and lecture method in teaching, among others. These findings are in line with earlier findings (Abreh et al., 2018; Gegbe et al., 2015; Mbugua et al., 2012; Sa‟ad et al., 2014) who in separate studies found similar factors contributing to the poor performance of high school students in mathematics. The researchers suggest that mathematics educators or teachers use varies strategies of teaching as well as problem solving strategies to improve students‟ performance in the mathematics.

Beyond the boundaries of Africa, the issue of abysmal performance of high school students in mathematics cannot be overemphasized. Many research studies conducted to ascertain the causes of this phenomenon found similar results as those found in Africa. For instance, Kalhotra (2013) conducted a study to identify the cause of failure in mathematics among high school students in India using a sample of 125 students who have failed their mathematics examination. The study employed descriptive survey design using questionnaire as main instrument for data collection. Among other findings, the study found that teachers use inappropriate teaching method, mostly teacher centered approaches to teaching and learning of mathematics. Most of the students‟ responses showed that they do not like their teachers‟ method of teaching. Majority of the students were also found to come from low socio-economic background. The authors suggested that teachers should be given certain refresher courses to expose them to appropriate methods of teaching to promote students‟ participation and interest in mathematics. This suggests that mathematics teachers all over the world still use teacher centered approaches, which do not encourage student thinking and problem solving, in teaching and learning of mathematics.

The implication of the findings from the studies described in the previous paragraphs is that the performance of students in general and high school students in particular in mathematics, all over the world, has been poor. This suggests that students lack conceptual, procedural and other key variables associated with teaching and learning of mathematics concepts. Word problems as an aspect of mathematics cannot be left out when we talk about the abysmal performance and lack of conceptual, procedural and metacognitive knowledge of students.

This claim was validated in a study conducted by Radmehr and Drake (2020) to explore final year high school and first year university students‟ metacognitive knowledge. The sample consist of 13 final year high school students, selected from scholarship and regular class using theoretical sampling, and 11 first year university students, selected using convenience sampling. The study employed case study design where interview was the main instrument for data collection, was based on the structure of metacognitive knowledge in Revised Bloom‟s Taxonomy.

The findings suggest among other things that the final year high school students lack knowledge particularly in the importance of knowing the rationale behind the theorems and formulas they learnt. The findings also revealed that the high school teachers of use more of procedural strategies in teaching unlike their university counterparts who use conceptual means of teaching. Moreover, with regard to students‟ metacognitive knowledge, the authors recommended that mathematics teachers should employ guided-discovery or problem-solving strategies in terms of developing strategies to identify how mathematical problems could be solved in order to develop students‟ metacognitive knowledge at both levels.

Research studies in the area of students‟ achievement in mathematics suggest that many factors account for the abysmal performance of high school students. For instance, a study conducted by Dlamini (2017), investigated the causes of South African high school students‟ abysmal performance in mathematics. The study which employed qualitative methods, analysed students‟ scripts already answered for their continuous assessment in mathematics, to explore the possible causes of the poor performance in mathematics. Three high schools in a district in South Africa were used to achieve the purpose of the study. The findings revealed several factors leading to poor performance of students from the analysis. These include; lack of knowledge and skills in algebra; lack of factorization skills, inability to multiply surds and exponents; lack of algebraic manipulation skills; incorrect reasoning; incompetence in functions, among others. The authors recommended modifications in the pedagogy employed by teachers to address the short falls identified in the study. Using a guided-discovery strategy could be a sure way of addressing these short falls. The current study investigates the use of guided discovery method to minimize students‟

difficulties in translating word problems involving linear equations into mathematical statement.

2.6 Students' Difficulty in Learning Word Problems

A Mathematical word problem is seen by many students as challenging content domain among in Senior High School mathematics domains. This is supported by Mohyuddin and Khalil (2016) who concluded that word problem is seen as a difficult subject among mathematics students and often they misunderstood the notion of function. Majority of students may only follow rules, that is, multiply out brackets, collect together like terms, look for common factors etc. In addition, students may have acquired learned procedures from prior discussion and instruction to use the substitution rule before doing the required expansion.

Jupri and Drijvers (2016) found that the majority of students included in their study have faced difficulty in using variables as generalized and changeable quantities. In addition, they found that students focus on or influence by arithmetic approach for items demanding an algebraic approach, practice "point-by-point or static way" of evaluating an independent variable of a function with the real domain. The ability to use variables as varying quantities showed a positive correlation with students‟ performance in word problems.

Haghverdi, Semnani, and Seifi (2012) research on the analysis of errors in word problems and found out that Errors displayed by students were conceptual and procedural; there were also errors of interpretation and linear extrapolation. Conceptual errors showed a failure to grasp the concepts in a word problem and a failure to appreciate the relationships in a problem. Procedural errors occurred when students failed to carry out manipulations or algorithms, even if concepts were understood. Interpretation errors occurred when students wrongly interpreted a concept due to over-generalization of the existing schema. Linear extrapolation errors occurred when students are not able to resolve mathematical word problem in one variable which seeks a linear statement or equation. The findings revealed that the participants were not familiar with basic operational signs such as addition, subtraction, multiplication and division relating to word problems. The participants demonstrated poor ability to simplify once they had completed grouping of variables. Therefore, it was recommended that mathematics educators should employ strategies that eliminate such errors to develop the students.

2.7. Difficulties in Understanding Mathematical Text

A lot of research has been carried out to study students‟ difficulties in understanding the mathematical text. Vondrová, Novotná, and Havlíčková (2019) analysed word problem-solving difficulties by considering the following aspects: The logical structure (the type of operation required the possible presence of extra or missing information); these components (the contextual relationships involved in the word problem); the syntactic component (the structural variables, the number of words, and position of the parts making up the problem). Furthermore, research in the field has expanded and developed. Novotná and Chvál (2018) and Samková and Tichá (2015) proposed an extensive literature review on the subject. In their work, an analysis of the results obtained in the last thirty years highlighted three main components that can cause difficulties in the process of "horizontal" mathematical procedures and therefore influence the choice of the solving process in word problems.

The process of solving word problems include the linguistic complexity of the problem text; the numerical complexity of the data being presented in the problem;

and the relationship between the linguistic and numerical components of the problem. However, these three dimensions are not exhaustive; sometimes there are very complex problems: for instance, the solver has to decode the problem by interpreting it into more straight forward problems; or there may be irrelevant or missing data that require the solver to decide what numerical and non-numerical data are needed for the resolution and soon. Therefore, solving word problems involves other difficulties because they cannot be solved with routine procedures, and the solver needs specific knowledge and expertise in the disciplinary domain, heuristics, and metacognitive strategies (Novotná & Chvál, 2018; Samkov & Tichá, 2015).

In furtherance, Vondrová, Novotná, and Havlíčková (2019) and Gasco, Villarroel, and Zuazagoitia, (2014) proposed three fundamental processes to be followed for correct implementation of a word problem-solving strategy. These processes include: The correct contextualization of the problem, with all its implications, the identification of all the information necessary for the resolution, the implementation of the resolution processes and the resulting mathematical operations.

In this study, we highlight that reading and interpreting a mathematical text does not only mean decoding the question but also implies the ability to find out relationships between the different parts of which it is composed. In this respect, Wijaya, van den Heuvel-Panhuizen, Doorman, and Robitzsch (2014) started from the assumption that there is a close link between "context" and "demand" followed by the focus of the problematic situation: the or the two components are connected, the greater will be the understanding. This connection is not always immediate; generally, the student faces a problem formulated by an individual different from him. For this reason, the solver is not the one who spontaneously asks himself the mathematical word problem following the reading of the "context", but it is imposed on him(Capone, Filiberti, & Lemmo, 2021).

2.8 Eliminating Difficulties in Learning Mathematics

Mathematics is widely recognized as a problematic subject that many parents find it difficult to help their children with mathematics at home. At a mathematics workshop, the parents brought their children to know why they were not coping with formal mathematics to their level of expectations. During unstructured dialogue with parents/facilitators and learners, it was discovered that fractions and rations accounted for the largest number of troubles for the children (Myers, et al., 2022).

More than half of the parents were of the opinion that most of what the children studied as part of the mathematics curriculum has no practical use. At the clinic, special card games were designed and used to teach concepts such as equivalent fractions. This was to help the children to learn while playing.

2.9 Students' Perceptions in Learning Mathematics

Mathematics perception is defined as how students personally view mathematics as a subject, and how they feel and think about learning the subject. Many students of mathematics have various opinions concerning the studying and the learning of mathematics. These views are due to perceptions. Researchers in the field of mathematics also believe that, students of mathematics hold various views in mathematics. A study seeks to determine the impact of perception of female students on the performance of the mathematics within secondary schools in Teso District (Wasike, Ndurumo, & Kisilu, 2013). The findings revealed that female student have negative perception and contribute to their poor performance in mathematics. A similar study revealed that the students have positive perception towards mathematics.

Though students see the learning of mathematics to be difficult, they are aware that mathematics is an important concept to their daily lives (Hagan, Amoaddai, Lawer, & Atteh, 2020). They added that, the perception of boarding towards the study of mathematics was positive and therefore influence the boarding female students to perform better than counterparts" day students.

On contrary Anhalt and Cortez (2016) concluded on their study to investigate the students" perception toward mathematics at university in Malaysia and concluded that perception of students towards matheamtics at the univeristy level is highly encouraging. Even though the above researchers have contrary view, we can see that, the above studies by (Wasik et al 2013; Hagan etal 2020) demostrated that at the High School level education, student have negative perception about the study of matheamtics and that negatively affect their performance in mathematics. Anhalt and Cortez (2016) finding also demsotrated that university students have positive perception about ma themtics studies. This may due to a lot of factors. Many of students used by Anhalt and Cortez (2016) were students who are studying mathematics at the university levels and therefore have interest in matheamtics already and they may have powsitive attitude towards the study of mathematics whereas the students sample at the High School may not necessary have interest in the field of matheamtics. The negative perception of students towards the study of mathematics is not different from the study conducted Salifu and Bakari (2022) which seek to investigate the relationship between students" perception, interest and matheamtics achievement using purely quantitative approach.the students emplyed questionnaires for measurings students" perception and interest and achievement test. The results revealed that student had students" interest and perception significantly predicted students achievement in matheamtics. Again a postive moderate and significant relationship was recorded between students perception and achievement in matheamtics.

Finally, weak positive significant correlation was recorded for the relationship between students‟ perception and interest towards mathematics. This study means that a student‟s perception is directly proportional to their achievements in mathematics. Other studies also suggested that students have positive perception in learning mathematics (Samuelsson & Samuelsson, 2016). This positive perception according Samuelsson and Samuelsson (2016) is due to the teachers method of teaching matheamtics and their personality.

Salifu & Bakari (2022) in a study indicated that the influence of students‟ perceptions towards mathematics showed that the teacher‟s method of teaching mathematics and his personality greatly accounted for the students‟ positive or negative perception towards mathematics. Ampadu also stressed that students‟ beliefs and perceptions have the potential to either facilitate or inhibit learning. Furthermore, Arthur, Asiedu-Addo, and Assuah (2017) investigated students‟ perception and its impact on Ghanaian students‟ interest in mathematics. The study established that 58.1% of total respondents agreed that students‟ negative perception of mathematics strongly influences their interest in mathematics as they progressed in their studies

2.10 Methods of Teaching Senior High School Mathematics

There are several methods of teaching in which teacher of mathematics can use in the classroom to presents mathematical facts, information, principles, skills or concepts to students. Some of the methods include: demonstration, discovery, discussion, project, laboratory, individualized, field trip and expository methods to mention but few.

Some of these methods which have their characteristic advantages and disadvantages as narrated by Sa"ad, Adamu, and Sadiq (2014), are specific for some situations and categories of students, while others can generally be apply to all categories of students.

Furthermore, Piaget, Bruner and Dienes each suggest that learning proceeds from the concrete to the abstract. It is believed that children should be actively informed in the learning process and opportunities for talking about their ideas should be provided. It is also believed that symbols and formal representation of mathematical ideas follow naturally from the concrete level, but only after conceptualization and understanding hence taken place (Surya & Syahputra, 2017). For the purpose of this study, the relevant teaching method amongst the above listed is guided discovery.

2.10.1 Guided Discovery Methods

The value of discovery has been the subject of debate and some disagreements among educational psychologists. As explained by Rahmawati, Sulisw进oro, and Prasetyo (2020) claimed to have established that guided discovery was the best method to promote the learning of certain rules. Aagesen, Konge, Brydges, and Kulasegaram (2020), argued that guided discovery only looked better because of what it had been compared with, usually-rote learning. He went further to claim that there was just no evidence that discovery of any kind was a more effective teaching method than meaningful exposition. Fauzi and Respati (2021) agree that guided discovery is important in promoting learning with young children, while Anggriani, Sarwi, and Masturi (2020) on the order hand agreed that active learning methods are more important for younger students than for elder ones. Yet guided discovering is quite popular with some teachers. They believed the students are better motivated by an active approach and perhaps by a challenge, but the teacher may justifiably step in at any time to ensure that the desired end point is reached.

Perhaps the most eloquent defender of learning by guided discovery is Rahmawati, Sulisworo, and Prasetyo (2020), who claimed that; first, guided discovery encouraged a way of learning mathematics by doing mathematics and encouraged development with view that mathematics is a process rather than a finished product. Secondly, in agreement with Ayu, Mudjiran, and Refnaldi (2022), guided discovery was seen as intrinsically rewarding for students, so that the teachers using guided discover methods should have little needs to use extrinsic form of reward. Thirdly, guided discovery learning, teaches students the techniques of discovery. Solving problems through guided discovery develops a style of problem solving or inquiry that serves any task that one may encounter. Finally, guided discovery learning results in better retention of what is learned because the student has organized his new information and know where to find the information when he needs it. Now, these points carry great weight, practical difficulties were acknowledged namely; that one could wait forever for students to discover, that the curriculum could not be completely open. Some students might even find their inability to discover extremely discouraging. It is of course, up to the teacher to make the kind of adjustment necessary to circumvent these difficulties. Such practical difficulties did not invalidate the case for active learning. Therefore, the effort of trying to use guided discovery methods will be worthwhile for what is to be achieved.

2.10.2 Impact of Guided Discovery Approach on Performance of Students in Mathematics

Teaching mathematics through a guided discovery approach, according to Suratno, Tonra, and Ardiana (2019), is a relatively new idea in the history of problem solving in the mathematics curriculum. In fact, because teaching mathematics through a guided discovery approach is a rather new approach, it has not been the subject of much research. Although less is known about the actual mechanisms learners use to learn and make sense of mathematics through a guided discovery approach, there is widespread agreement that teaching mathematics through a guided discovery approach holds the promise of fostering student learning (Amiyani & Widjajanti, 2019). One key question that needs addressing when talking about learning mathematics through a guided discovery approach is whether learners can explore problem situations and invent ways or employ alternative methods to solve problems.

Many researchers (e.g., Rahmawati, Sulisworo, & Prasetyo, 2020) have investigated learners" mathematical thinking. These researchers" findings indicate that learners can explore word problems and invent ways to solve the problems. For example, Rahmawati, Sulisworo, & Prasetyo (2020) found that many first-, second-, and third-grade learners were able to use invented strategies to solve a problem. They also found that 65% of the learners in their research sample used an invented strategy before standard algorithms were taught. By the end of their study, 88% of their sample had used invented strategies at some point during their first three years of school. They also found that learners who used invented strategies before they learned standard algorithms demonstrated better knowledge of base-ten number concepts and were more successful in extending their knowledge to new situations than learners who initially learned standard algorithms.

Recently, researchers (e.g. Sari, Gistituati, & Syarifuddin, 2019; Arifin, Wahyudin, & Herman, 2020) have also found evidence that middle school learners are able to use invented strategies to solve word problems.

Collectively, the aforementioned studies not only demonstrate that learners are capable of inventing their own strategies to solve word problems, but they also show that it is possible to use the learners‟ invented strategies to enhance their understanding of mathematics and hence their academic performance in mathematics.

One of the studies that have informed this study is the work of Kousar (2010). In a study, (Kousar, 2010) sought to determine the effect of a guided discovery approach on academic performance of mathematics learners at the secondary level. Secondary school learners studying mathematics were used as the population of the study. The learners of a grade 10 class of the Government Pakistan Girls High School Rawalpindi were selected as a sample for the study. Using a sample size of 48 learners Kousar (2010) equally divided them into an experimental group and a control group on the basis of an assessment he conducted. The experimental group was then taught over a period of six weeks based on a planned guided discovery approach. The control group continued with the instructional approach that they had prior to being identified as the control group. After the intervention, an assessment was used to see the effects of the intervention. A two-tailed t-test was used to analyze the data, which revealed that both the experimental and control groups were almost equal in mathematics knowledge at the beginning of the experiment. However the experimental group outscored the control group significantly on the assessment following the intervention.

2.11 Senior High School Algebra

According to Flower, McKenna, Muething, Bryant, and Bryant (2014), the concept of algebra encompasses a number of different perspectives. From geometrical views, for instance, Flower, et al.(2014) defined algebra as a concept that relates between symbolic and extensive magnitude; or also interpreted as written numerals and real numbers. Meanwhile, from the way it is taught, algebra can mean knowledge of finding unknowns through systematic procedures.

Although for some school of thoughts, the usefulness of algebra is not explicitly visible, the concept has been indeed crucial in many applied disciplines, such as, physics, economy, geometry, and computer science (Sugiarti & Retnawati, 2019). Its function as a language of mathematics would make it really required, especially in building mathematical models of life"s phenomena. Opoku, Tawiah, Agyei-Okyere, Osman, and Afriyie (2019) revealed that the invention of algebraic concepts has had immediate effects on the development of higher level mathematics, like calculus and analytic geometry.

Algebra in senior high schools is often called school algebra. School algebra is seen as a step to real algebra as well as the continuation of arithmetic learning (Egara, Eseadi, & Nzeadibe, 2022). Mainly, school algebra contains two aspects called procedural and structural algebra (Sugiarti & Retnawati, 2019). The procedural part covers computational-related topics; often this leads to the operational aspects which closely relate to arithmetical skills for students. Agyei, Agamah, and Entsie (2022) expressed that this part usually becomes an entrance for most people in their acquisition of algebraic knowledge. The second part, the structural, closely relates to the core concept of algebra itself. It focuses on the relationships among objects or quantities, rather than finding solutions of algebraic expressions. Looking deeper into the content, Moru and Mathunya (2022) mentioned four different conceptions that build school algebra. Each conception implies different roles and uses of variables, expressions, and tasks. These conceptions also determine how a sub- concept should be taught. Those conceptions are:

2.11.1 Algebra as a generalized arithmetic

This conception covers ways to state the relationships among numbers. In this case, variables are treated as pattern generalizers. The concept often becomes a basis for numeric formulas. The main tasks to approach this concept is translating numerical patterns and then making a generalization (Moru & Mathunya, 2022).

2.11.2 Algebra as a study of procedures for solving certain kinds of problems

This conception starts with a generalized formula. Symbols given in the formula become the focus of attention, i.e. the students are asked to determine the values represented in the symbols. Only certain numbers would satisfy the conditions of the formulas. In this case, the symbol, which is indeed the variable, usually represents a constant (Moru & Mathunya, 2022).

2.11.3 Algebra as a study of relationship among quantities

This conception discusses how an algebraic expression states interrelated components. This gives insights on how changes of certain values (quantities) affect the balanced situations. Unlike the previous conception, the variables in this conception are not constants. Instead, they have various values. Simply, the variables are either arguments or parameters (Moru & Mathunya, 2022).

2.11.4 Algebra as a study of structure

The last conception contains a high level skill in algebra working, which is, theorems and manipulations. It discusses how an expression could be stated without changing its values. The variables are treated purely as objects. They are not to be solved, neither to find nor to relate each other (Moru & Mathunya, 2022). These four conceptions are interrelated and are often used simultaneously in solving algebraic word problems. Unfortunately, in many curricula, these conceptions are often not treated proportionally, with a tendency to the procedures. As a consequence, the students tend to understand algebra as a set of rules and procedures that they have to memorize to be able to solve the problems. Summary of these four conceptions are given in table 2.1.

Table 2.1: Conceptions, uses of variables, and tasks building school algebra

Conceptions	Use of Variables	Tasks
Generalized arithmetic	Pattern generalizer	Translate, generalize
Study of procedures	Unknowns, constants	Solve, simplify
Relationship among quantities	Arguments, parameters	Relate
Study of structure	Arbitrary symbol	Manipulate, justify

2.12 Linear Equations with One Variable

One topic given to students in their early study of algebra in school is linear equations with one variable. The importance of this topic is viewed by Petersson, Sayers, and Andrews (2022) as a hallmark for students‟ algebraic proficiency in school. In many curricula, this topic mainly deals with solving two kinds of equations, namely, one-step and multi-step equations. This part will give an overview of how students usually deal with solving these kinds of equations and what knowledge they have to own to be able to solve problems on linear equations with one variable (Moyer, Robison, & Cai, 2018).

2.13 Strategies of Solving Linear Equations with One Variable

In investigations of students‟ learning on solving linear equations with one variable, researchers(Andrews & Öhman, 2019; Andrews, 2020; Linsell, 2009) found some strategies that students usually use to solve problems on linear equations with one variable, such as 1) guess and check, 2) counting techniques (known basic facts), 3) inverse operations, 4) working backwards then guess-and-check, 5) working backwards, then known facts, 6) working backwards, and 7) transformations.

Andrews and Öhman (2019) andAndrews (2020) named the first strategy trial-and-error substitution. This strategy simply requires students‟ recognition of letters as the representation of certain numbers in an algebraic expression and some sort of basic arithmetic skills. Here, the students should substitute any numbers and check whether the numbers fulfil the equation. Although this strategy is applicable to solve all kinds of equations, students should not really rely on it all the time, as they will have problems with relatively difficult questions, for example, ones that give fractional solutions.

The next two strategies, counting techniques and inverse operations, are often used to solve a one-step equation. Students who only rely on these strategies would not be able to solve the multi-step equation problems. The difference between these two strategies is observable when the students deal with problems involving a large number (Andrews & Öhman, 2019). Often, the students who used counting techniques struggle in it. Students‟ understanding of inverse operations would lead them apply the working backwards strategy to solve multi-step linear equations. Often, they combine this strategy with the other strategies after simplifying the expression into a one-step equation. The limitation of this strategy is when it deals with equations that involve variables in both sides. The last strategy, transformation, is often called the formal strategy. This strategy requires students to treat equation as objects. Thus, they can manipulate things in the equations, reformulate it, and then find the solution. Relying on this strategy will likely help students solve problems in any kind of equation.

In the study, Linsell (2009) found strong evidence that these strategies are indeed hierarchical. Thus, the development of students" strategies indicates their level of understanding of algebra. Given this range of strategies, some teachers strictly limit the students to a single approach to solve equations: the formal one. This decision has been proven to be ineffective to build up students" understanding and visions toward algebra concept (Andrews, 2020).

In an effort to introduce students to transformation strategy, many students found it difficult to understand equation as a structure. This failure, according to Andrews and Öhman (2019), can be recognized in three conditions, such as: 1) students" unsystematic and strategic errors when simplifying algebraic expressions, 2) students" neglecting to treat variables as objects, and 3) their misunderstanding of the equal sign. To anticipate this failure, basic knowledge should be given to students during their early study of algebra through word problems.

2.14 Basic Knowledge of Solving Linear Equations with One Variable

A study by Linsell (2009) mentioned four basic concepts that the students have to master to be able to solve any linear equation problem systematically; those are, 1) knowledge of arithmetic structure, 2) knowledge of algebraic notation andconvention,3) acceptance of lack of closure, and 4) understanding of equality.

Further, it was explained that this basic knowledge has a strong relationship with strategies that students can use to solve linear equation problems.

2.14.1 Knowledge of arithmetic structure

The closed relationship between arithmetic and algebra leads to a view of algebra as a generalized arithmetic. This view requires students to have a good understanding of arithmetical notions before doing algebra. Moreover, Andrews (2020) and Linsell (2009) emphasized that students" understanding of arithmetical structures has a dramatic effect on the most sophisticated strategy they are able to use when solving a linear equation problem.

The need for arithmetic in algebra is strengthened by other experts, like Galindo and Sonnenschein (2015), Ngo and Kwon (2015), and Outhwaite, Faulder, Gulliford, and Pitchford (2019), who believed that arithmetic should become an entrance for algebra learning. In addition, Andrews and Öhman (2019) stressed that intuitive precursors in arithmetic are absolutely needed to make students able to interpret as an addition of two objects; which is a higher level view of algebra. Therefore, it was suggested involving some arithmetical identities with some hidden numbers in introducing the concept of equation to students. This hidden number could be changed progressively from finger (cover up), box drawing, and then finally letters. Such a way of introducing algebra would enhance students" understanding of arithmetic in solving word problems.

2.14.2 Knowledge of algebraic notation and convention

This part is related to the use of symbols and the role of algebra as a language in mathematics. In school algebra, the use of symbols is crucial for students. Ngo and

Kwon (2015) explained that the symbols and notations that students produce during their algebra learning would be the basis for their reasoning ability.

A study by Andrews (2020) suggested that to help students" understanding of algebraic representations, algebra learning should be started on a concrete level, in which the students can produce and reflect on their own symbols against the true situation. This will make the symbols meaningful to them.

2.14.3 Acceptance of lack of closure

The concept of acceptance of lack of closure focuses on students" ability to hold them back from finishing an operation; it simply tells about manipulating algebraic expressions. This issue is often encountered in discussions on equivalence or on solving a high level algebra problem. Students" difficulties with this notion are revealed by Outhwaite, Faulder, Gulliford, and Pitchford (2019), recognizing the struggles of many students to judge an equivalence without finding the unknown.

Andrews and Öhman (2019) argued that teachers can help students build this knowledge by providing activities that develop the structure sense of students. The activity might give an image of the structures of two equivalent structures of expressions along with their decompositions and decompositions.

2.14.4 Understanding Equality

Discussions about equality in school algebra usually involved the equal sign and its meaning for students. Many students understand the equal sign as a signal for an answer, as they probably understand it in arithmetic. This becomes a problem, especially when students work with equations involving variables on both sides. Unfortunately, teachers often do not really pay attention to this problem (Leavy, Hourigan, & McMahon, 2013). Theodora and Hidayat (2018) presumed that this lack of attention could be the main cause of students‟ bad performance in algebra in general. In algebra, the equal sign must be seen as a relational rather than operational symbol; this becomes a pivotal aspect of understanding algebra (Theodora and Hidayat, 2018).

2.14.5 Promoting the View of Equations as Objects

Treating expressions in algebra as objects is crucial in studying word problems. In teaching, teachers usually give a direct suggestion that an equation is indeed like a balance pivoted about the equal sign without any visualization ((Theodora and Hidayat, 2018). This would hardly encourage students to imagine how it could happen and why they need to maintain a balance. Such questions need an indirect answer with a visual proof that can be done by presenting and experimenting with a real balance scale in the classroom. This idea is important because transferring the idea of transformation in algebra would require seeing an equation as an object to be acted on (Theodora and Hidayat, 2018).

2.14.6 Facilitating Understanding of Eliminations in Algebraic Operations

The second advantage of balancing activity is related with the first one, that is, to promote transformation strategy to students. Leavy, Hourigan, and McMahon (2013) found that the balance model is an effective tool for conveying the principles of transformation, because the principles applied to create a balance situation on a balance scale really suit the process of solving equations. In this case, the balance scale gives meaningful insights on eliminating the same terms from both sides of an equation to obtain the value of the unknown.

2.14.7 Increasing Representational Fluency

Moore, Miller, Lesh, Stohlmann, and Kim (2013) explained that providing students with the balance model gives them opportunity to come up with multiple representations. Having experience with real balancing stimulates students to show their understanding in drawings, verbal explanations, or even formal representation freely. Their experiences become the sources of reflections when they are going to represent their ideas.

Beside the advantages, several studies also reported limitations of the balance activity. The first limitation deals with the activity"s inapplicability to represent unknowns or expressions that involve negative numbers (Hill, Sharma, & Johnston, 2015). Another limitation is revealed by Hill, Sharma, O'Byrne, and Airey (2014) that the balance activity seems incapable, and even confuses students, to work with reversible operations. Those limitations imply the need to present other supporting activities to cover what the balance scale cannot.

2.15 Misconceptions about the Concept of Equality or Equations

Understanding the concept of equality is challenging for many students. Students enter the senior high school with the belief that the equal sign means they should write the final answer after completing necessary operations (Booth, McGinn, Barbieri, & Young, 2017), or the belief that it is a link to the next operation (Hill, Sharma, & Johnston, 2015). An equation is any algebraic expression of equality containing a letter (or letters)(Bush & Karp, 2013). For students, the knowledge of the equal sign (=) to represent equality of the two sides of the equation is minimal, if present (Andrews & Öhman, 2019). Students write the letter in the equation as the subject (stand-alone) or engage in guess work using specific values (Andrews & Öhman, 2019).

Many students seem to have more of an operational view of the equal sign rather than a relational view (Booth, McGinn, Barbieri, & Young, 2017). In their study, 56% of Grade 7 students" definitions of the equal sign were variations of the sign asking them to perform an operation, in contrast to the 36% who saw the sign as some form of equivalence. However, students" relational view of the equal sign improved from Grades 6 to Grade 8. This relational view is essential to understanding that the transformations performed in the process of solving an equation preserve the equivalence relation – an idea many students find difficult, and that is not an explicit focus of typical instruction.

Since word problems are literal, translations are sometimes done directly from the left to the right, leading to the formation of wrong symbolic notations and errors including situations involving inverse operations (Andrews & Öhman, 2019). Herscovics (2018) gave 150 first year engineering students the following question amongst others to write in algebraic notation.

Write an equation using the variables S and P to represent the following statement: There are six times as many students as professors in this university. Use S for the number of students and P for the number of professors.

Only 63 % of the students wrote the answer correctly. Of the 47 undergraduates who were taking college algebra as a course but were not science majors, only 43% obtained the right answer, with the rest answering incorrectly with $6S = P$. The students had read and translated into numbers and letters literally without regard to the reversal built into the question (Herscovics, 2018). One of the reasons that the students could have made in this error was that in their minds, they rightly saw the quantities of the two objects but were unable to establish equivalence. This is known as static comparison (Herscovics, 2018). High proportions of students using the letter as a label or having reversal errors on the same question have since then been identified. These were post-secondary students enrolled in algebra and calculus courses in colleges and universities, and pre-service teachers of various nationalities (Jeffery, Hobson, Conoyer, Miller, & Leach, 2018).

The use of mathematical terms and language may bring about misconceptions of operations that need to be performed. In the course of translation, sum and product are often misinterpreted, also resulting in reversal errors (Özerem, 2012). Baidoo (2019) found that only 35% of the 281 Year 9 students sampled could use symbols to represent the sentence " the number y is eight times the number z" with many writing it as $z = 8y$ instead of $y = 8z$. Also, less than 30% could correctly write "s is eight more than t" as many wrote $t = s + 8$ instead of $s = t + 8$. There was also an association of y with the number 8 while z and t became the subjects of the equation. Some students may wrongly generate expressions or inequalities as answers to word problems which rightly require construction of equations. These errors were named as "lack of equation" and "in equation" (Baidoo, 2019). Writing algebra involves using or interpreting the letter within a particular context without any extra meanings being read into it (Salsabila, Rahayu, Kharis, & Putri, 2019).

In conclusion, misconceptions arising both from the use of a letter as an object, label or word, and about the meaning of the equal sign make it difficult for many students to transit from arithmetic to introductory algebra, where letters are substituted and patterns emerge.

Inability to translate, perform inverse operations and develop suitable algebraic forms from word problems inhibits proper processing of questions and leads to errors before the computation and processing stages are reached (Barbieri, Miller-Cotto, & Booth, 2019).

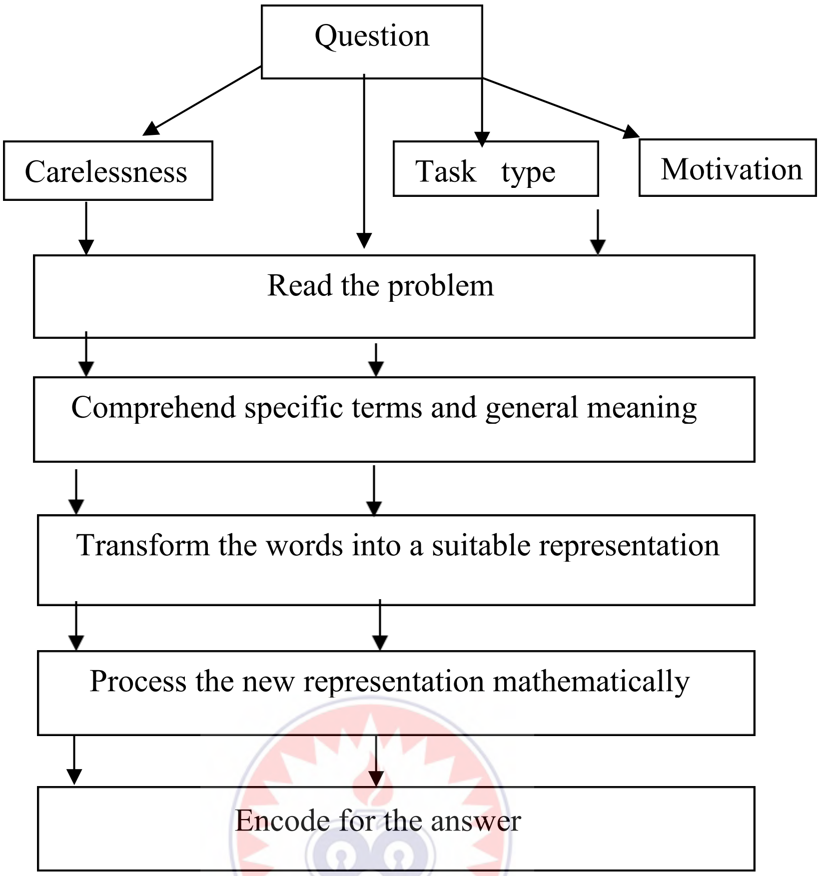
2.16 Teacher Role, Social Norms, and Socio-mathematical Norms

Efforts to reform teaching and learning in the mathematics classroom often deal with many aspects such as the teacher and classroom culture. Issues around the teacher‟s role have been central due to the learning shift from teacher-centered into students-centered. Issues about teachers‟ views on beliefs should have been a concern in the design of algebra learning. Putri, Dolk, and Zulkardi (2015) revealed that most teachers still hold traditional beliefs in algebra, which is central to results, rules, and procedures, as they were usually taught in their previous schools. This view should of course be repaired before they conduct the classroom.

Reforming teaching cultures will also have an effect on the classroom‟s atmosphere because students might not be accustomed to the new situation. However, Çakır and Akkoç (2020) argued that as long as teachers can facilitate a good social interaction between and among students, the students can easily adapt to the new norms. In mathematics, a shift in norms is also covered in a discussion called socio-mathematical norms. This notion focuses on the development of students‟ mathematical beliefs and values to become autonomous in mathematics (Çakır and Akkoç, 2020). Socio-mathematical norm will specifically discuss the mathematical issues that the students encounter during the class (Putri, Dolk, &Zulkardi, 2015).

In early algebra learning, investigating students" socio-mathematical norms is important, especially because the students are in transitions from arithmetical to algebraic thinking. A number of issues may be observed, such as students" views of a number of different strategies and different representations, completing a problem without finding the solutions, equal sign does not always separate operation and answers, and solving a complex problem by splitting it into simpler partitions. These ideas are probably new and different from what students encountered in arithmetic classrooms. To be able to help children understand this shift of values, a teacher must be prepared for several things, such as, students" theory building, students" misconceptions, the role of representations, and how to move from misconceptions to knowledge (Putri, Dolk, &Zulkardi, 2015).It is important to highlight that in the reformed mathematics classroom; the teacher should identify the students" limited knowledge and use it as a basis for their learning.

2.17 Conceptual Framework of the Study



Newman strategies adapted from Newman (1983b, p.2)

Figure 2.1: Conceptual Framework of the Study

In Figure 2.1, a word problem must be processed in stages to be able to arrive at the correct answer. Reading recognition: Reading of the question involves recognition of both the words and symbols in the given task. A reading error would stall the process of answering the question.

Comprehension: The second step entails students showing an understanding of specific terms and saying the questions in their own words. Ability to extract the core issues in a question is one of the most important skills in mathematics. If they cannot paraphrase what the question is about, they are unable to move any further and this is called a comprehension error.

Transformation: The word problem once understood is written in a mathematical form. The understanding derived from the question which is in a literal form now has to be transformed into other suitable and correct representations, depending on the requirements of the task. For this study, this would involve the symbolic form that would be needed to solve the question in algebra. An understanding of concepts and an identification of operation(s) and method(s) to be used is examined during this stage. If an individual is unable to identify the needed operation(s) it is called a transformational error.

Process skills: After successful transformation of the question, it is then processed using mathematical computation and conceptual understanding. Depending on the new representation form, the arithmetical operations and computations needed to process the question are carried out. If a person does not know, guesses, uses the wrong operations, calculations or procedures, it is termed a process skill error.

Encoding ability: At the last step, the solution has to be written in an acceptable form. The acceptability depends on what the question asked for, that is, an answer in symbols or words or a table. If a student is unable to write the answer in a form which is acceptable, it is called an encoding error.

Other errors exist that may affect an individual"s problem solving ability, though they are not directly related to the question being solved. The conceptual framework is classified as: careless errors, motivation errors and the task form. If an individual correctly carries out a step he missed during the first attempt of the question, it could be as a result of any one of the above three, which may occur at any of the stages.

2.18 Summary

The literature in this chapter has argued that a guided discovery approach to teaching mathematics is a learner-cantered approach to the teaching of mathematics. This approach is strongly supported by how students learn; constructivist views of knowing and learning that learners construct their own knowledge of learning by interacting cooperatively and collaboratively in small or large social groups through engaging actively in problem based tasks/activities. The teachers" subject content knowledge and how it can affect the desired learners" performance in the use of a guided discovery approach to teach mathematics has also been discussed in-depth. Various theories and studies aimed at providing guidelines for improving competencies and identifying that a guided discovery approach is suitable to improving senior high students" performance in translating word problems into mathematical statements or equations. Copiously discussed in the chapter are the concepts of algebra as associated with the solving of word problems in mathematics. The concepts of linear equation and equality symbol have been discussed. Finally, the researcher also examined at the role of the teacher in the mathematics classroom, as well as the conceptual framework of the study.

CHAPTER THREE

METHODOLOGY

3.1 Overview

This chapter discusses the methods and techniques used in carrying out the study. The chapter discussed the research design, research population, sample and sampling procedure, research instruments, validity, reliability, intervention, data collection procedure, data analysis procedure, and ethical considerations.

3.2 Research Design

According to Sileyew (2019), a research design is a set of guidelines and instructions used in conducting research. According to Turner, Cardinal, and Burton (2017), research design aims to achieve greater study control. The research design for the study would be a quasi-experimental design. In this design, the researchers would have a treatment group (students receiving instruction using the guided discovery method) and a control group (students receiving traditional instruction or another instructional method). The two groups would be compared to determine the effectiveness of the guided discovery method in minimizing students' difficulties in solving word problems involving linear equations.

The study used would use a mixed-method approach comprising quantitative and qualitative research approach. The quantitative approach was employed early in the study to gather quantitative data through the administration of pre-test and post-test items. This was then followed by the utilization of the qualitative approach to gather qualitative data of the study. Creswell (2014) indicates that since the problems addressed by social and health science are complex, using either qualitative or quantitative methods by themselves is inadequate to address the complex problems.

Tobi and Kampen (2018) agree that mixed-method research provides better inferences and minimises bias. The mixed-method is the best research paradigm called pragmatism because it eschews metaphysical concepts that have caused endless discussion and debate and present an efficient and applied research philosophy (Lucero et al., 2018; Guetterman & Fetters, 2018). The type of mixed method used was embedded research design. According to Turner, Cardinal, and Burton (2017). Embedded research design, first collects quantitative data and then collects qualitative data to help explain or elaborate on the quantitative results. Moreover, the researcher also employed the quasi-experimental design to test the hypothesis of the research. A quasi-experimental design involves a non-random assignment of participants in two groups, experimental and control groups (Turner, Cardinal, & Burton 2017; Creswell, 2014). The experimental group received the treatment (the guided-discovery teaching approach) whereas the control group did not. The control group was used to establish a baseline for reading achievement in this study. This design was used since the study was conducted in a classroom setting and was not possible to assign subjects randomly to groups. This study aimed at using guided discovery method to minimize students" difficulties in translating word problems involving linear equations into mathematical statement.

3.3 Research Population

According to Majid (2018), population refers to all the elements that meet the criteria for inclusion in a study. Furthermore, Asiamah, Mensah, and Oteng-Abayie (2017) referred to a population as the entire group of individuals to whom the findings of a study apply. Therefore, it is the group that the researcher wishes to explore. The study population comprised students at Jachie Pramso Senior High School in Bosomtwe District in the Ashanti Region of Ghana.

The school runs six academic programmes, namely Agricultural Science, Business Studies, General Arts, General Science, Home Economics and Visual Arts. The population of the school currently is 2,016 students, comprising 908 boys and 1108 girls. Six hundred and eighty eight (688) of these students are in Form 2 of which their age ranges between 15 to 21 years. The research was conducted in Form 2. The target population of the research consists of all the 688 second year students of Jachie Pramso Senior High School of which 320 were boys and 368 girls. The SHS2 year group has 16 classes made up of an Agricultural Science class, two Business classes, nine General Art"s classes, a General Science class, two Home Economics classes, and a Visual Arts class.

3.4 Sample and Sampling Procedure

According to Lohr (2021), a sample is a small group obtained from the accessible population. A sampling frame is a group of people drawn from a sample (Shannon-Baker, 2016). Moreover, Shaheen and Pradhan (2019) opine that purposive sampling is a sampling procedure where a researcher relies on his or her judgement to choose research subjects. It is a non-probability sampling method that occurs when subjects selected for a sample are done by the researcher's judgment.

Moreover, according to Etikan, Musa, and Alkassim (2016), a simple random sampling technique is a type of probability sampling whereby the researcher ensures that all the population members have equal chances of being selected for a study. Singh and Masuku (2014) explained that in a simple random sampling, each sample unit included has an equal chance of inclusion in the sample. Therefore, it provides an unbiased and better estimate of the parameters if the population is homogeneous. The researcher employed purposive and simple random sampling techniques to select the study sample.

The purposive sampling technique was used to select all Senior High School Form Two (SHS Form 2) classes. The school has two SHS Form 2 tracks, namely the Gold Track and Green Track, with six subject disciplines each for the tracks (General Arts, Agricultural Science Business, General Science, Visual Arts and Home Economics). After selecting all the SHS Form 2 classes in the school purposively, the simple random was used to select two in-tact classes for the study. Meanwhile, after selecting the two SHS Form 2 classes, the simple random was further used to determine the class assigned as the control group and the one assigned as the experimental group.

The rationale for using the purposive sampling technique to select all SHS Form 2 classes out of SHS Form 1 and SHS Form 3 classes was that SHS Form 2 classes were expected to have covered some prerequisites in algebra from the Form 1 mathematics content. Also, it was expected that the students had settled in the school as opposed to SHS Form 1 classes. The SHS Form 3 classes were not used in the study as they prepared for the West African Senior School Certificate Examination. Two in-tact classes (with 50 students each) from the school.

Table 3.1: Sample Distribution of the Study

Gender	Number of Participants	Percentage (%)
Male	48	48
Female	52	52
Total	**100**	**100**

From Table 3.1, 48 male students representing 48% and 52 female students representing 52% were sampled for the study. The information contained in the Table 3.1 revealed that the female students dominated the study.

3.5 Research Instruments

The study used three (3) instruments. The instruments were pre-test, post-test and interview guide.

3.5.1 Pre-test

The pre-test was used to determine the difficulties students have in solving word problems. The researcher designed the pre-test items to establish the learners' entry behaviour and initial knowledge of the intended learning areas. Also, it aimed to ensure that students were of the same relative ability in performance in the world problems. The questions were actually drawn from the past questions of the West African Senior School Certificate Examination (WASSCE). The concepts presented in the pre-test were on single variable. In designing the pre-test items, the age, mathematics fluency, and scope of the mathematics syllabus of senior high schools students were taken into consideration in order to ensure that all aspect of the algebra concepts as captured in the syllabus was explored in the pre-test.

3.5.2 Post-test

The students' post-test was administered to students after administering the intervention or the treatment. Specific tests evaluating the work done on each topic were given at the end. These were graded and eventually complied with at the end of the study.

3.5.3 Interviews guide

In Guetterman and Fetters (2018), interviews are one of the essential tools of qualitative research. Researchers often get better responses from interviews than other data gathering devices like the questionnaire. The researcher believes that the interview technique would give room for an in-depth probing that would provide a better knowledge of the participant's views and thinking processes about using guided discovery approach in learning word problems. The researcher prepared a comprehensive interview guide containing the questions which were posed to the participants during the interview session of the study. Eight (8) students were selected through the means of non-probability sampling from the experimental group for the interview. Each participant was allotted a minimum of 20 minutes to respond to the interview questions.

3.6 Validity

According to Vakili and Jahangiri (2018), validity establishes if the study tools accurately measure what they are designed to measure. The study instrument was submitted to a lecturer, who patiently went over it and made the required suggestions and revisions. Some lecturers from the Department of Mathematics Education of the University of Education of Winneba were consulted to validate the test items and determine the content and face validity of the items.

3.7 Reliability

The Split-Half method examined students' pre-and post-test scores' reliability. First, the pre-test and post-test were divided into two halves using the odd-even items, and the scores were associated or correlated. Based on Pearson's Product Moment Correlation, this resulted in an internal consistency of 0.89. This result was then compared to the tabulated dependability or reliability coefficient, which was found to be satisfactory at 0.8 by Vakili and Jahangiri (2018).

3.8 Intervention

The treatment start with writing of teaching guides like lesson plan and lesson notes for teaching guided discovery strategy. The content was developed according to the senior high school form two mathematics syllabus containing sets of objectives specifically outlined to the teachers, clear direction and kind of expected specific outcomes required from each topic. Again, the teaching guides encompassed a lot of suggestions, examples and counter examples that teachers follow in their presentation of lessons. For instance, the guided discovery approach emphasizes on asking leading questions on the specific tasks presented before the students by leading /guiding them until they discover the solution. One broad topic was used for the purpose of this study which includes world problems. The daily lesson plans were given to mathematics educators for face validation.

During the teaching and learning of word problems, teaching and tutorial questions were practiced in this study. The students in the experimental group of the study were taken through the principal procedure underlying word problems as demonstrated in the instruction of word problems (see Appendix A and B). Participants were also taken through how to extract equations from word problems. The students were given much time to explore on their own, with very little supervision from the researcher. This strategy aroused and sustained participants" interest in the teaching process. After the students became used to the word problems, the researcher introduced the word problems involving fractions. The concepts were treated one of the other at the pace of the learners. Throughout the intervention phase, group discussion was encouraged. In this manner, the researcher was crucial in teaching the concepts to the students. However, to stimulate students' cognitive processes, students were encouraged to share ideas consistently among themselves in groups on the questions provided to them. Throughout the tutorial sessions, students were taught using the guided discovery approach.

During these tutorial sessions, students were responsible for completing the tutorial questions independently, with the researcher acting as a facilitator. As a result, students were encouraged to work in groups to complete their tasks within the allotted time. The activities during the tutorial sessions followed the instructional cycle of activities, class discussions, and class exercises. However, the activities in this cycle were designed with a strong emphasis on reflection. Reflective exercises helped students enhance their thinking skills hence their metacognitive awareness in learning. In addition, the researcher ensured that the quality of reflection activities was tailored to affect and stimulate students' achievement and their learning awareness inclined for them to learn better.

Students in the control group, on the other hand, received their word problem instruction as usual. This group, like the experimental group, rehearsed teaching and tutorial questions. The teaching strategy in this group was based on conventional approaches of teaching word problems. The fluency of the mechanical portions, which entail mathematical procedures, was stressed. Students were exposed to a large number and sample of previous years' examination questions to recognize patterns of questions. To prepare them for the post-test, they were encouraged to memorize the patterns. In this group, the researcher taught word problems concepts without guided discovery approach and guided students to solve problems. In the tutorial classes, students in the control group completed their tutorial activities.

3.9 Data Collection Procedure

Prior to data collection, the researcher obtained permission from the school's headmaster and the Head of the Department of Mathematics and followed the school's policy for authorization to conduct research, including a confirmatory letter from the school. Furthermore, the researcher held a familiarization meeting with the teachers in the mathematics department and students to inform them of the study's aim and assure them that the data obtained would be used solely for the study's purposes.

3.9.1 The Pre-test and the Post-test

The following technique was used to collect data: a pre-test examination was given to the sample students to assess their knowledge of word problems in single variable and their difficulties in answering the problems. These difficulties allowed the researcher to respond to the first research question and, by extension, the study's initial objective. Secondly, the sampled students were taught word problems utilizing using guided discovery as a teaching medium. It took two weeks for the intervention to take place. After the researcher's intervention, a post-test was given to measure the learnt concepts. Before and after the intervention, a written test was administered. The purpose of the pre-test was to determine students' challenges with essential question involving word problems and their entering behaviour and basic knowledge of the desired learning areas of the study.

3.9.2 Interview guide

The researcher believes that the interview technique will allow for more in-depth questioning, resulting in a deeper understanding of the participant's ideas and thought processes regarding guided discovery in studying word problems. The interview data was gathered from eight (8) randomly chosen students for the experimental group. The interview took place immediately after the school hours, for three days and two hours every day. Three primary questions and relevant follow-up questions were included in the interview guide. The primary goal of the interview was to find out what the participants thought about using guided-discovery approach. A participant was allocated a maximum of 20 minutes for the interview. The researcher ensured that the atmosphere for the interview was friendly to enable the interviewees to relax in responding to the questions. Moreover, interviewees were advised to ask for the questions to be repeated if they did not understand the questions. Since the data gathered from the interview session was spoken words, the researcher transcribed the spoken words into text, giving the researcher the ability to qualitatively analyze the transcribed data.

3.10 Data Analysis Procedure

The study yielded both quantitative and qualitative data, including student scores on the pre-test and the post-test items. Inferential statistics were used to examine the achievement test results, which were used to answer research question three. The Statistical Package for Social Sciences (SPSS) software was used to analyze the data. The t-test was performed to determine whether there was a statically significant difference between the participants' pre-test and post-test scores. This was mostly accomplished by comparing the pre-test and post-test mean scores. Scores were analyzed using descriptive statistics, including percentages, mean, and standard deviation. The students' interview responses were transcribed and examined to answer the second study question.

3.11 Ethical Considerations

When carrying out this study, ethical procedures were followed. Permission to collect the data was sought from the school administration. The fact that the data is solely being used for research was stressed. Participants were also instructed not to use their real names on the pre-test or the post-test items but rather to use a study-specific identification number. This identification number was done to maintain the confidentiality and anonymity of the investigation. Furthermore, all information acquired during the survey was properly saved for research purposes only.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Overview

This chapter focuses on the discussion of results that emerged from the data gathered. The demographic information on the sample and the results are presented in this chapter captured under sub-headings which reflected the research questions. The following research questions underpinned the study:

1. What difficulties do students experience in solving algebraic word problems?
2. To what extent does the guided discovery method improve students' understanding of solving word problems into algebraic equations?
3. What is the effect of guided discovery approach of teaching and learning on students‟ performance in solving word problems?

4.1 Demographic Information

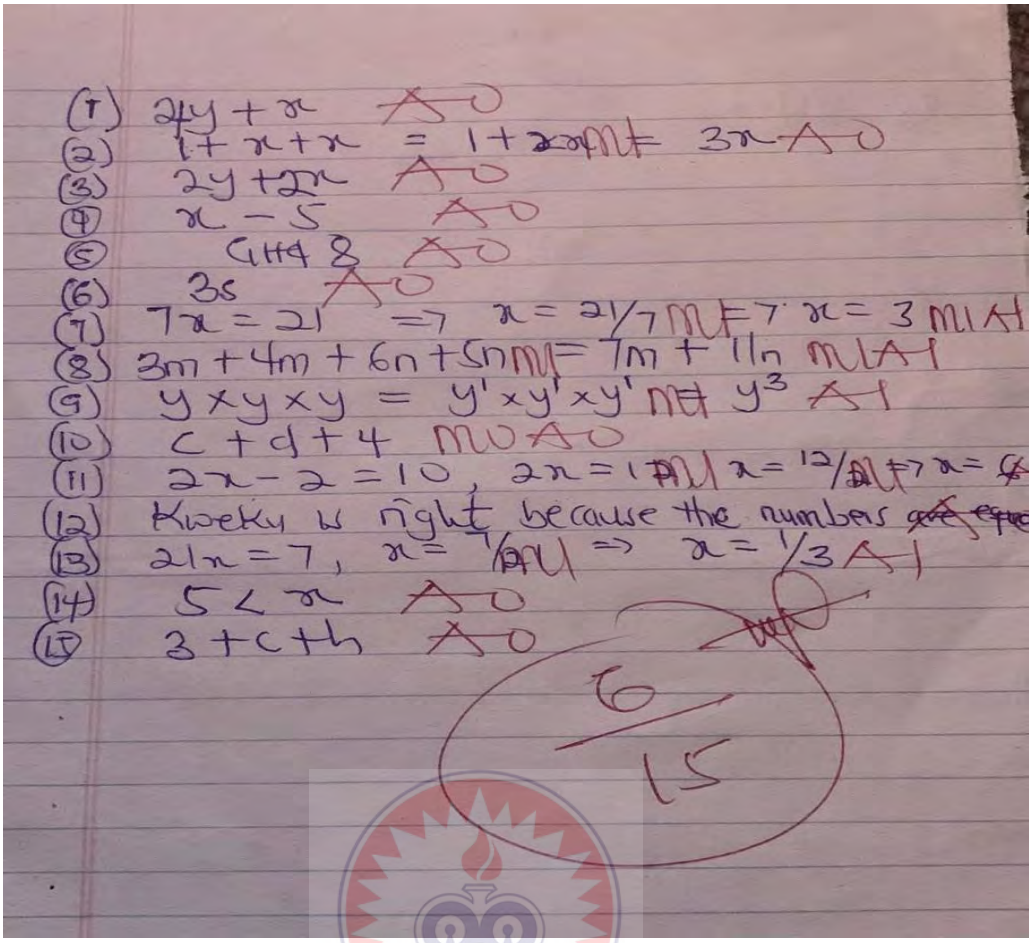
The information on the demographic characteristics of the respondents was presented using frequency counts and percentages.

Table 4.1: Demographic Characteristics of Respondents (N =100)

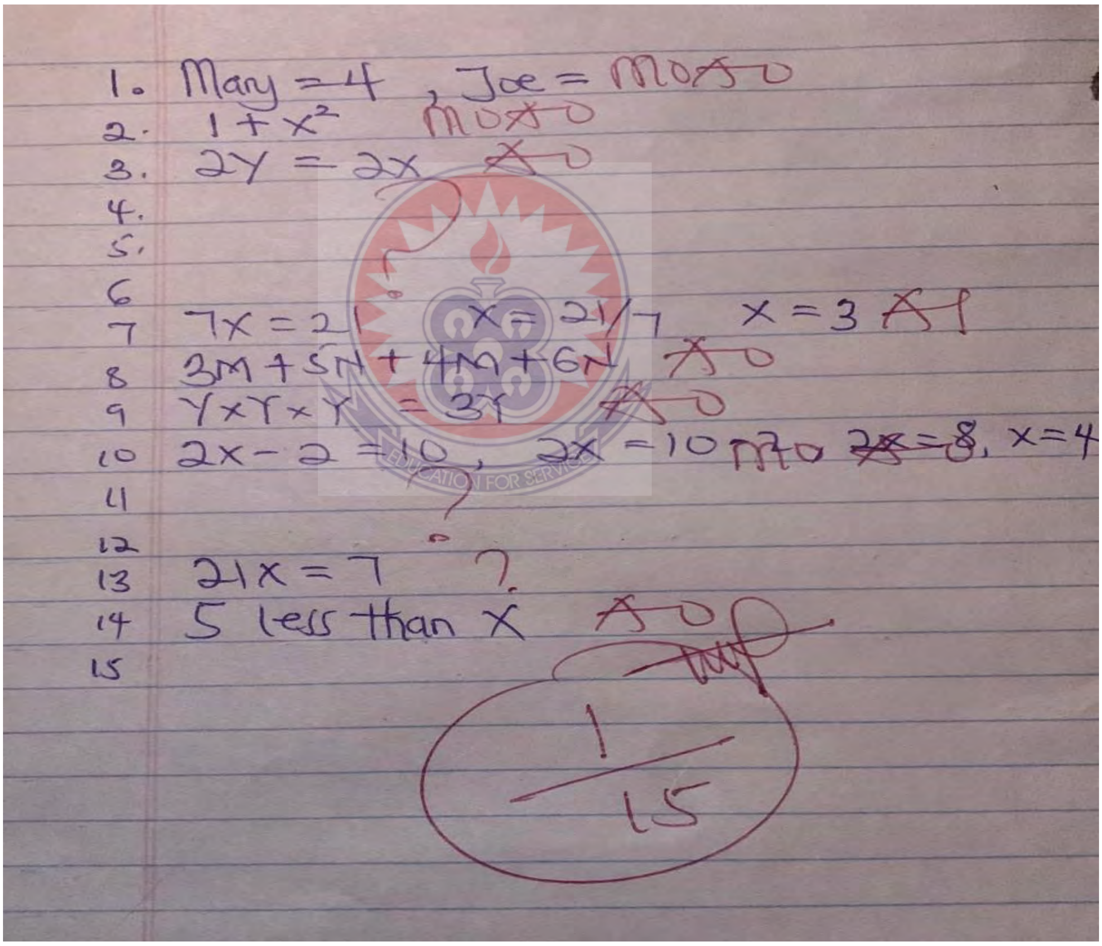
Variables	Categories	Number of Participants	Percentage (%)
Gender	Males	48	48.00
	Females	52	52.00
Total		100	100.0
Age	15 – 17	54	54.00
	18 – 20	32	32.00
	Above 21	14	14.00
Total		**100**	**100.0**

The result from Table 4.1 reveals the demographic characteristics of students sampled for the study. The statistics show that out of 100 student-participants, 48 (48%) of the respondents were males while 52 (52%) of them were females. The information contained in the Table 4.1 revealed that the female students dominated the study. Moreover, from Table 4.1, 54(54 %) of the student participants were between the ages of 15 and 17 years, 32 (32%) were between 18 and 20 years old while 14 (14%) were above 21 years. This information indicates that majority of the participants have their age falling in the 15-17 age category compared to the rest of the age category of the study.

This first research question of the study was to find out the difficulties the student-participants faced in solving algebraic word problems or questions. To accomplish this goal, the researcher devised an initial test or pre-test to determine the learners" entry behaviour and prior understanding of the intended learning areas in order to discover the difficulties or challenges students face in solving algebraic word problems or questions. All the pre-test items were drawn from the Ghanaian senior high schools mathematics syllabus. The pre-test questions on algebraic word problems were administered to the participants in both the control and the experimental groups of the study. A critical examination of the scripts of the participants revealed the difficulty they faced on the pre-test items. After the collection of the scripts of the pre-test, the researcher marked each of the pre-test items using a marking scheme prepared by the researcher (See Appendix E). While marking each script the researcher paid attention to systematic presentation of the solutions from the students.



The researcher also observed that some of the participants were able to solve all questions which involved equations correct but in terms of solving word problems into mathematical statements or equations for the given algebraic word problems and how participants conclude their final answers was the problem of the day . Each of the question was scored 1 mark. The total marks on the entire pre-test was 15 marks. The data generated by the pre-test items (see Appendix A) was used to answer this study question. With regard to the study, the operational definition of difficulty is the things that students find tough or difficult to do in solving algebraic word problems. The difficulties the student-participants experienced in the solving of word problems of the study were categorized into conceptual, procedural, logical structure, semantic component and difficulty of using variables. In the context of this study, conceptual difficulty refers to failure on the part of the sample of the study to grasp the concepts in the word problem. Also, procedural difficulty connotes the failure on the part of the student-participants to carry out manipulations or required algorithms to solve the word problems. More so, the logical structure refers to the type of operation required to carry out the resolution of the word problems of the study. Meanwhile, the semantic difficulty refers to the contextual relationships involved in the word problems while the difficulty of using variables in this study connotes the inability of the student-participants to use a quantity that is not static but varies and can take the place of any value.



Some participant did not attempt any word problem question at all with the exception of question 14 of which the participant had it wrong. Meaning the student lack knowledge of solving word problem into mathematical statement and equation in one variable. Some was also lacking the idea of grouping like terms and computing of figures.

The difficulties or challenges the student-participants faced in the pre-test have been tabulated and presented in Table 4.2.

Table 4.2: Difficulties Students Encounter in Translating Word Problems into Mathematical Statements or Equations (N = 100)

Difficulties	Number of Students	Percentage (%)
Conceptual	25	25.00
Procedural	27	27.00
Logical Structure	8	08.00
Semantic Component	10	10.00
Variable	30	30.00
Total	**100**	**100**

The information presented in Table 4.2 revealed that 25(25%) of the participants had conceptual difficulty in translating word problems into mathematical statements. The conceptual difficulty indicates that student-participants failed to grasp the concepts in the word problems. Hence, their inability to solve the word problems contained in the pre-test. Also, the results indicated that 27(27%) of the participants experienced procedural difficulty in their quest to translate the word problems into mathematical statements or equations as contained in the pre-test. This difficulty demonstrates that student-participants failed to carry out the required manipulations or algorithms in solving the problems administered to them during the pre-test phase of the study. Moreover, few of the participants, 8 (8%) experienced logical structure difficulty in translating word problems into mathematical statements or equations. This difficulty revealed that the student-participants were not able to carry out the required operation to solve the word problems in the pre-test items. It was noted that 10 (10%) of the participants were not able to correctly contextualize the relationships involved in the word problems administered to them. This difficulty was classified by the researcher as semantic component difficulty. Lastly, a critical examination of the scripts of the participants revealed that 30(30%) were not able to utilize the concept of variables in translating the word problems into mathematical statements or equations. This difficulty contributed to their low performance during the pre-test phase of the study. This indication was that, students were not able to interpret words like sum, difference, product of, less than, quotient, etc. In furtherance, the results in the students‟ scripts showed that the student-participants were not able to read and understand the word problems. They could not analyze and interpret the key words involved as well as translating them into mathematical statements and equations and solving the equations as well. This was evident in the scores they obtained in the pre–test. The pre–test was basically aimed at finding out the students‟ level of understanding and translating word problems into linear equations. The results from Table 4.3 showed that the experimental and control groups had mean scores of 17.72 and 17.48 respectively, with a mean difference of 0.24. The experimental group's minimum and maximum scores were 12 and 7 respectively, while the control's groups were 10 and 6. This observation indicated that the students were not able to interpret words like sum, more than, difference, less than, product, quotient, etc. For instance, students encountered problems with words like “sum”, “added to”, “increase by”, “more than” meaning „addition‟, while “decrease by”, “difference between”, “difference of”, “less than” also meant "subtraction". In addition to this, words such as "product of ", "times", "multiplied by" mean "multiplication" and words such as "out of", "quotient of", "per", "ratio of" also mean "division" were a problem to the students.

Moreover, considering the analysis on the scores obtained by the students in both the control and the experimental groups in the pre–test phase of the study, as shown in Tables 4.3, it can be deduced that the performance of the students before the intervention was very low. The results in Table 4.3 clearly showed the difference in the mean scores obtained by the students. This low performance by the students in the pre–test was due to the teaching strategy used in teaching the students and it took the lecture form of teaching. With this teaching strategy, the students did not have the chance of using their experience to create their own understanding, since lessons were delivered to them in an organized form previously. The students were used to only memorizing and imitating teachers and this did not give the students sufficient wisdom to survive independently, applying to the word problems as well as real world situations. As a result of this, students were not able to understand and translate the algebraic word problems into algebraic linear equations and solving the equations as well. The students also lacked cooperative learning and hence the average students could not help their low performing colleagues. The students could not analyze simple mathematical word problems and interpret key words, such as „sum" to mean „add", „difference" to mean „subtract", „product" to mean „multiply" and „quotient" to mean „divide"

4.3 To what extent would the guided discovery method improve students' understanding of solving word problems into algebraic equations?

The research question two sought to find out how the use of guided discovery learning strategy offer assistance to students understanding of solving word problem into numerical or mathematical statement. To answer this research question, an interview data was gathered from Eight (8) students who were selected through the means of non-probability sampling from the experimental group for the interview. The interview took place immediately after the school hours. Three primary questions and relevant follow-up questions were included in the interview guide which was posed to the participants in the experimental group to seek their views. The primary goal of the interview was to find out how the guided-discovery approach of instruction improved English reading and comprehension when translating word problems into numerical or mathematical statement. A participant was allocated a maximum of 20 minutes for the interview. The interview session afforded the researcher to elicit participants‟ views on the extent to which their understanding of word problems was improved by the guided-discovery approach of teaching and learning word problems. Since the data gathered from the interview session was spoken words, the researcher transcribed the spoken words into text, giving the researcher the ability to qualitatively analyze the transcribed data. The following illustrations demonstrate participant‟s responses to the questions posed to them.

When asked the question, have you ever been taught word problems with guided-discovery approach, All the interviewees remarked that:

> *"No, I have not been taught word problems with this teaching approach".*

When asked the question, to what extent did participation in learning word problems with this teaching approach (guided-discovery approach) helped or hindered your mathematics learning? Please explain.

The student indicated that:

> *"Learning word problems with this teaching approach did not hinder my ability to understand and translate word problems into mathematical statement or equation; rather, the activities that we were exposed to in the lesson enabled me to understand the English and Mathematical terminologies associated with word problems. Initially I found it difficult to answer word problems, now I can easily translate word problems into equations and solve them."*

The above response from the student suggests that the guided-discovery approach applied to the translating of mathematical word problems to mathematical statements or equations has contributed to the success of the student"s achievement in the word problems by arousing and sustaining the student"s interest. The guided-discovery approach of teaching has also made it easier for students to follow the concept of word problems with ease. That view was supported by another student who remarked that:

> *"Learning word problem with this new teaching style has made the whole word problem session fun for me because most of the concepts we were exposed were practically based and so it was easy for me to understand than compared to learning word problems without this teaching style. Initially, I found it difficult to use variables in translating word problems into equations. Now, I can confidently apply the variables."*

When the student was asked, will you recommend the use of guided discovery teaching strategy in learning mathematics? If yes Why? (What are the benefits of this teaching strategy?) Please elaborate.

The student indicated that:

> *"Our teachers should use this teaching style in teaching us abstract concepts that are difficult to understand so that we can appreciate mathematics and its application in other fields like engineering, banking and industries. When our teachers start using this teaching approach, the number of students who will pass the mathematics of West African Senior School Certificate Examination will increase."*

When the interviewee was asked, is there anything more you would like to add? The student indicated that:

> *"I was motivated to learn word problems because of the various activities you (the researcher) engaged us in. I was fortunate to be in the group that used the new teaching style to study word problems. I think that I will be able to apply the concepts in future".*

The above remark suggests that the guided-discovery method of teaching word problems has motivated the students in the experimental group to translate word problems into mathematical statements or equations and solving them efficiently and effectively.

When a student was asked, to what extent did participation in learning word problems with this teaching approach aided or hinder your mathematics learning? Please explain.

The student stated that:

> *"The first time we studied word problems, I did not understand, I became so sad because assignments and tests were administered on the word problems and I performed poorly. But now, I have understood every concept taught us so far with this teaching approach. I performed excellently in our second test because I have grasped the fundamental and salient concepts in word problems. I can now understand terminologies like* "sum", "added to", "increase by", "more than" "decrease by", "difference between", "difference of", "less than", "product of", "times", "multiplied by", "out of", "quotient of", "per", "ratio of", etc. *In our end of semester examination, when is see word problem questions, I will select and answer them with ease."*

The above revealed that the student"s fears concerning the study of word problems has been dealt with by the application of guided-discovery approach. This student will not have been confident to select and answer any word problem question during their semester examination if guided-discovery had not been used to aid students" understanding of translating word problems into mathematical equations and solving them.

Furthermore, the students were encouraged to learn mathematics as a result of the applicability of the guided-discovery approach, as evidenced by their comments. The findings from the interview revealed that guided-discovery lesson presentation of the word problem concept produced an environment that was favourable to practical activities and real-world connections. The post-test results confirmed that students who were taught with the guided-discovery method made substantial improvements from pre-test to post-test.

In furtherance, when a student was asked this question: Is there anything more you would like to add?

The student remarked that:

> *"In spite of the fact that the guided-discovery approach is capable of simplifying the concepts associated with word problems, it also involves lots of activities which make learning of difficult concepts easier. If we the students don't learn difficult concepts through many activities, it will be difficult to understand what we must learn."*

The above statement indicates that for students to be taught with guided-discovery efficiently and effectively, there must be lots of activities for the students to engage where majority of the learning is centred on the learners and the teacher becomes a guide or a facilitator of the lesson. Otherwise, it would be difficult to integrate the learner in the teaching process and the intended objectives of the lesson would not be achieved. Hence, they would not be able to appreciate the learning of mathematics.

In the context of the study, the control group refers to the group of students who did not receive the guided discovery method instruction. Since the study focused on evaluating the effectiveness of the guided discovery method in minimizing students' difficulties in solving word problems involving linear equations, the control group did not undergo any specific intervention related to the guided discovery method.

Typically, in experimental studies, the control group serves as a baseline or comparison group against which the experimental group (in this case, the group receiving the guided discovery method) is measured. The control group allows researchers to assess the impact of the intervention by comparing the outcomes of the experimental group to those of the control group.

Therefore, after the study, the control group would continue with their regular instruction or teaching methods as determined by the school or district. Their progress and performance in solving word problems involving linear equations would not have been influenced directly by the guided discovery method

4.4 What is the effectiveness of guided discovery approach of teaching and learning on students' performance in solving word problems?

This research question sought to determine what effect guided discovery approach of teaching and learning would have on senior high school students" mathematics achievement in translating word problems into mathematical statements or equations and solving them. The study tested the hypothesis that there is no significant difference between the mean score of senior high school students taught without guided-discovery method and those taught with guided discovery method. The post-test (See Appendix B) items administered to the control group and the experimental group generated quantitative data that was used to answer this question. The descriptive statistics of the pre-test and post-test scores are presented below.

Table 4.3: Descriptive Statistics of Pre-test for Experimental and Control Groups (N = 50)

Group	N	Mean	Stand Dev.	Maximum	Minimum
Experimental	50	17.72	7.24	12.00	7.00
Control Group	50	17.48	6.55	10.00	6.00

Table 4.3 shows that the experimental and control groups had mean scores of 17.72 and 17.48 respectively, with a mean difference of 0.24. The experimental group's minimum and maximum scores were 12 and 7 respectively, while the control's groups were 10 and 6. An independent sample t-test with a 95 per cent confidence interval was used to determine whether the difference in mean scores was statistically significant.

Table 4.4: Independent Samples T-test of Pre-test Scores

Group	N	Mean	Std. Dev.	t-value	Df	p-value
Experimental Group	50	17.72	7.24	0.388	98	0.887
Control Group	50	17.48	6.55			

The results of the independent samples t-test, as shown in Table 4.4, were that there was no statistically significant difference between the experimental group (M=17.72; SD = 7.24) and the control group (M = 17.48, SD = 6.55) on the pre-test scores of the two independent research groups. The estimated t-statistic, ($t = 0.388$; $p = 0.887 > 0.05$) indicates that both the experimental and the control groups had similar conceptual knowledge of the concepts of word problems prior to implementing the treatment (guided-discovery approach of teaching and learning word problems).

To answer research question three, the following hypothesis was developed, and an independent samples t-test with a 95 percent confidence interval was conducted to examine if there was a statistically significant difference in post-test scores between the experimental group being taught with guided-discovery approach of learning and the control group who were taught without guided-discovery approach.

H0: There is no significant difference between the mean score of senior high school students taught without guided discovery method and those taught with guided discovery method.

Table 4.5: Independent Samples t-test of Post-test Scores (N = 50)

Groups	N	Mean	Std. Dev.	t-value	Df	p-value	Eta Squared
Experimental Group	50	24.80	9.48	2.986	98	0.005	0.096
Control Group	50	20.65	7.67				

Table 4.5 shows that there was a statistically significant difference between the experimental group (M = 24.80; SD = 9.48) and the control group (M = 20.65; SD = 7.67). The estimated t-statistic was (t = 2.986; p = 0.005). This shows that the experimental group being taught with guided-discovery approach of learning outperformed the control group who were taught without guided-discovery approach. The eta squared value of 0.096 indicates that a medium effect size (Cohen, 1988), implying that the guided-discovery approach of learning (independent variable) accounted for 9.6% of the variance in the post-test scores (dependent variable). Consequently, the null hypothesis that there is no significant difference in post-test scores between the experimental group being taught with guided-discovery approach

and the control group who were taught without guided-discovery approach was rejected in favour of the alternative hypothesis that there is significant difference in post-test scores between the experimental group being taught with guided-discovery approach and the control group who were taught without guided-discovery approach. As a result, when compared to the conventional approaches, the results demonstrated that using guided-discovery of leaning in mathematical word problem instruction was effective.

4.5 Discussion of Results

This section focuses on the discussion of findings that emerged from the analysis of the data gathered. The findings of the study are discussed under the following sub-headings which reflect the objectives of the study.

4.5.1 Difficulties Students Experience in Solving Algebraic Word Problems

The findings from the study revealed that 25 (25%) of the participants had conceptual difficulty in translating word problems into mathematical statements. The conceptual difficulty indicates that student-participants failed to grasp the concepts in the word problems. Hence, their inability to solve the word problems contained in the pre-test. Also, the results indicated that 27 (27%) of the participants experienced procedural difficulty in their quest to translate the word problems into mathematical statements or equations as contained in the pre-test. This difficulty demonstrates that student-participants failed to carry out the required manipulations or algorithms in solving the problems administered to them during the pre-test phase of the study. Moreover, few of the participants, 8 (8%) experienced logical structure difficulty in translating word problems into mathematical statements or equations. This difficulty revealed that the student-participants were not able to carry out the required operation to solve the word problems in the pre-test items. It was noted that 10 (10%) of the participants were not able to correctly contextualize the relationships involved in the word problems administered to them. This difficulty was classified by the researcher as semantic component difficulty. Lastly, a critical examination of the scripts of the participants revealed that 30 (30%) were not able to utilize the concept of variables in translating the word problems into mathematical statements or equations. This difficulty contributed to their low performance during the pre-test phase of the study. This revelation indicated that, students were not able to interpret words like sum, difference, product of, less than, quotient, etc. In furtherance, the results in the students‟ scripts showed that the student-participants were not able to read and understand the word problems. They could not analyze and interpret the key words involved as well as translating them into mathematical statements and equations and solving the equations as well. This was evident in the scores they obtained in the pre–test. The pre–test was basically aimed at finding out the students‟ level of understanding and translating word problems into linear equations. The results from Table 4.3 showed that the experimental and control groups had mean scores of 17.72 and 17.48 respectively, with a mean difference of 0.24. The experimental group's minimum and maximum scores were 12 and 7 respectively, while the control's groups were 10 and 6. This observation indicated that the students were not able to interpret words like sum, more than, difference, less than, product, quotient, etc. For instance, students encountered problems with words like “sum”, “added to”, “increase by”, “more than” meaning „addition‟, while “decrease by”, “difference between”, “difference of”, “less than” also meant “subtraction”. In addition to this, words such as “product of “, “times”, “multiplied by" mean "multiplication" and words such as "out of", "quotient of", "per", "ratio of" also mean "division" were a problem to the students.

Moreover, considering the analysis on the scores obtained by the students in both the control and the experimental groups in the pre–test phase of the study, as shown in Tables 4.3, it can be deduced that the performance of the students before the intervention was very low. The results in Table 4.3 clearly showed the difference in the mean scores obtained by the students. This low performance by the students in the pre–test was due to the teaching strategy used in teaching the students and it took the lecture form of teaching. With this teaching strategy, the students did not have the chance of using their experience to create their own understanding, since lessons were delivered to them in an organized form previously. The students were used to only memorizing and imitating teachers and this did not give the students sufficient wisdom to survive independently, applying to the word problems as well as real world situations. As a result of this, students were not able to understand and translate the algebraic word problems into algebraic linear equations and solving the equations as well. The students also lacked cooperative learning and hence the average students could not help their low performing colleagues.

The students could not analyze simple mathematical word problems and interpret key words, such as „sum"to mean „add", „difference" to mean „subtract", „product" to mean „multiply" and „quotient" to mean „divide". These findings agree with Mohyuddin and Khalil (2016) thata mathematical word problem is seen by many students as challenging content domain among in Senior High School mathematics domains. Word problem is seen as a difficult subject among mathematics students and often they misunderstood the notion of function. Majority of students may only follow rules, that is, multiply out brackets, collect together like terms, look for common factors etc. In addition, students may have acquired learned procedures from prior discussion and instruction to use the substitution rule before doing the required expansion (Haghverdi, Semnani, & Seifi, 2012; Vondrová, Novotná, & Havlíčková, 2019; Novotná & Chvál, 2018; Samková & Tichá, 2015). The findings are in consonance with Jupri and Drijvers (2016) who in separte study found that the majority of students included in their study have faced difficulty in using variables as generalized and changeable quantities. In addition, they found that students focus on or influence by arithmetic approach for items demanding an algebraic approach, practice "point-by-point or static way" of evaluating an independent variable of a function with the real domain. The ability to use variables as varying quantities showed a positive correlation with students" performance in word problems.

In furtherance, the findings agree with Vondrová, Novotná, and Havlíčková (2019) and Gasco, Villarroel, and Zuazagoitia, (2014) who in separate studies found out that procedural errors occurred when students failed to carry out manipulations or algorithms, even if concepts were understood. Interpretation errors occurred when students wrongly interpreted a concept due to over-generalization of the existing schema. Linear extrapolation errors occurred when students are not able to resolve mathematical word problem in one variable which seeks a linear statement or equation. The findings revealed that the participants were not familiar with basic operational signs such as addition, subtraction, multiplication and division relating to word problems. The participants demonstrated poor ability to simplify once they had completed grouping of variables (Novotná & Chvál, 2018; Samkov & Tichá, 2015;

Wijaya, van den Heuvel-Panhuizen, Doorman, & Robitzsch, 2014; Capone, Filiberti, & Lemmo, 2021; Myers, et al., 2022).

4.5.2 The extent to which the guided discovery method improve students' understanding of solving word problems into algebraic equations?

Students‟ views on the application of guided-discovery approach to teaching and learning of word problems were investigated in the study. The results revealed that guided-discovery method has contributed to the success of students‟ performance in the learning of word problems by engaging the students, and arousing and sustaining the student‟s interest in the lesson. The guided-discovery method also made it easier for students to follow the word problem instruction.

According to the findings, the utilization of the guided-discovery approach to translating word problems into mathematical statements or equations and solving them has motivated students to study mathematics because of its numerous representation possibilities and the practical nature. These findings are in-line with the findings of Rahmawati, Sulisworo, and Prasetyo (2020) and Aagesen, Konge, Brydges, and Kulasegaram (2020) who in separate studies found that guided-discovery method of learning mathematics has the potential to pique and maintain students' interest while also improving their mathematics performance. They argued that guided-discovery method was the best method to promote the learning of certain rules or abstract concepts in mathematics. Therefore, in order to get more students taking up STEM courses at the university or tertiary levels, mathematics teachers must make their mathematics instructions lively and relating concepts to real-life situations and using appropriate teaching methodology to explain abstract mathematical concepts. This will enable students appreciate mathematics.

The findings also support the claims made by Fauzi and Respati (2021) that guided discovery is important in promoting learning with young children, while Anggriani, Sarwi, and Masturi (2020) on the order hand agreed that active learning methods are more important for younger students than for elder ones. The students must be actively engaged with abstract or concrete concepts for learning to occur in a mathematics classroom. Students' interests and accomplishment levels increased when teachers effectively integrated appropriate teaching method like the guided-discovery into the learning process. When the teaching method is an essential active element of the mathematical education process, it must be employed appropriately and judiciously to achieve learning outcomes. Using guided-discovery method in the mathematics classroom has a good impact on students' success and attitudes towards mathematics education (Ayu, Mudjiran, & Refnaldi, 2022; Suratno, Tonra, & Ardiana, 2019; Rahmawati, Sulisworo, & Prasetyo, 2020; Rahmawati, Sulisworo, & Prasetyo, 2020).

They stressed that judicious use of guided-discovery can support both the learning of mathematical methods and skills as well as the development of desired mathematical proficiencies such as problem-solving, reasoning, and justifying. The findings are also in consistent with those of Sari, Gistituati, and Syarifuddin (2019) andArifin, Wahyudin, and Herman (2020) who affirmed that utilizing guided-discovery method into mathematics instruction increases student motivation by virtue of its practical nature and visual representation of concepts. Therefore, students are able to retain abstract mathematical concepts which are indications that desired learning outcomes have taken place. Overall, the findings support the constructivists‟ theory of learning that humans generate knowledge and meaning from interaction between their experiences and their ideas (Charmaz, 2017). The basic idea in this theory is that learning is an active and constructive process with the learner viewed as an information constructor. By blending tenets of constructivist theory and that of the guided-discovery approach, the teacher becomes a guide for the learner, providing bridging or scaffolding, helping to extend the learner"s zone of proximal development while the learning process is centered on the learner (Weimer, Fabricius, Schwanenflugel, & Suh, 2017). Moreover, the findings are in sync with Kang and Liu (2018) who affirmed that representation or illustration of mathematics concept need not be taught as though they are ends in themselves.

Instead, they can be considered as useful tools for constructing understanding and for communicating information and understanding. If students simply complete assignments of constructing representations in forms that are already specified, they do not have opportunities to learn how to weigh the advantages and disadvantages of different forms or how to use those representations as tools with which to build their conceptual understanding. The representations enhance the problem-solving ability and that students often construct meaning in forms that help them see patterns and perform calculations. The use of multiple representations is one of the major topics in mathematics education that has gained importance in recent decades (Allen & Trinick, 2021). The significance of representing the solution of linear algebraic equations in multiple ways provides the same objective of more than one form. It is necessary to see how students use these representations. It is suggested that multiple representations provide an environment for students to abstract and understand major concepts (Allen & Trinick, 2021).

4.5.3 The Effectiveness of Guided-Discovery Approach of Teaching and Learning on Students' Performance in Solving Word Problems

The findings indicated that the experimental and control groups had mean scores of 17.72 and 17.48 respectively, with a mean difference of 0.24. The experimental group's minimum and maximum scores were 12 and 7 respectively, while the control's groups were 10 and 6. Moreover, the results of the independent samples t-test analysis showed that there was no statistically significant difference between the experimental group (M=17.72; SD = 7.24) and the control group (M = 17.48, SD = 6.55) on the pre-test scores of the two independent research groups. The estimated t-statistic, (t = 0.388; p = 0.887 > 0.05) indicates that both the experimental and the control groups had similar conceptual knowledge of the concepts of word problems prior to implementing the treatment (guided-discovery approach). Furthermore, an independent samples t-test analysis of the post-test scores for the experimental and control groups demonstrated that there was a statistically significant difference between the experimental group (M = 24.80; SD = 9.48) and the control group (M = 20.65; SD = 7.67). The estimated t-statistic was (t = 2.986; p = 0.005). This shows that the experimental group taught with guided-discovery method outperformed the control group taught without the guided-discovery method. The eta squared value of 0.096 indicates that a medium effect size (Cohen, 1988), implying that the conventional teaching method accounted for 9.6% of the variance in the post-test scores.

These findings support Rahmawati, Sulisworo, and Prasetyo (2020) who in separate study found that after using guided-discovery method to teach word problems in the first, second and third grades, many first-, second-, and third-grade learners were able to use invented strategies to solve a problem. They also found that 65% of the learners in their research sample used an invented strategy before standard algorithms were taught. By the end of their study, 88% of their sample had used invented strategies at some point during their first three years of school. They also found that learners who used invented strategies before they learned standard algorithms demonstrated better knowledge of base-ten number concepts and were more successful in extending their knowledge to new situations than learners who initially learned standard algorithms.

More so, the findings are in agreement with Kousar (2010) who sought to determine the effect of a guided discovery approach on academic performance of mathematics learners at the secondary level. The experimental group was then taught over a period of six weeks based on a planned guided discovery approach. The control group continued with the instructional approach that they had prior to being identified as the control group. After the intervention, an assessment was used to see the effects of the intervention. A two-tailed t-test was used to analyze the data, which revealed that both the experimental and control groups were almost equal in mathematics knowledge at the beginning of the experiment. However, the experimental group outscored the control group significantly on the assessment following the intervention.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Overview

The findings of the study are summarized in this chapter, together with conclusions, recommendations and area for further studies for improving senior high school mathematics instruction in the study area.

5.2 Summary of the Study

The study found that 25 (25%) of the participants had conceptual difficulty in translating word problems into mathematical statements. The conceptual difficulty indicates that student-participants failed to grasp the concepts in the word problems. Hence, their inability to solve the word problems contained in the pre-test. Also, the results indicated that 27 (27%) of the participants experienced procedural difficulty in their quest to translate the word problems into mathematical statements or equations as contained in the pre-test. This difficulty demonstrates that student-participants failed to carry out the required manipulations or algorithms in solving the problems administered to them during the pre-test phase of the study. Moreover, few of the participants, 8 (8%) experienced logical structure difficulty in translating word problems into mathematical statements or equations. This difficulty revealed that the student-participants were not able to carry out the required operation to solve the word problems in the pre-test items. It was noted that 10 (10%) of the participants were not able to correctly contextualize the relationships involved in the word problems administered to them. This difficulty was classified by the researcher as semantic component difficulty. Lastly, a critical examination of the scripts of the participants revealed that 30 (30%) were not able to utilize the concept of variables in translating the word problems into mathematical statements or equations. This difficulty contributed to their low performance during the pre-test phase of the study. This revelation indicated that, students were not able to interpret words like sum, difference, product of, less than, quotient, etc. In furtherance, the results in the students‟ scripts showed that the student-participants were not able to read and understand the word problems. They could not analyze and interpret the key words involved as well as translating them into mathematical statements and equations and solving the equations as well. This was evident in the scores they obtained in the pre–test. The pre–test was basically aimed at finding out the students‟ level of understanding and translating word problems into linear equations. The results from Table 4.3 showed that the experimental and control groups had mean scores of 17.72 and 17.48 respectively, with a mean difference of 0.24. The experimental group's minimum and maximum scores were 12 and 7 respectively, while the control's groups were 10 and 6. This observation indicated that the students were not able to interpret words like sum, more than, difference, less than, product, quotient, etc. For instance, students encountered problems with words like "sum", "added to", "increase by", "more than" meaning „addition‟, while "decrease by", "difference between", "difference of", "less than" also meant "subtraction". In addition to this, words such as "product of ", "times", "multiplied by" mean "multiplication" and words such as "out of", "quotient of", "per", "ratio of" also mean "division" were a problem to the students.

Moreover, considering the analysis on the scores obtained by the students in both the control and the experimental groups in the pre–test phase of the study, as shown in Tables 4.3, it can be deduced that the performance of the students before the intervention was very low. The results in Table 4.3 clearly showed the difference in the mean scores obtained by the students. This low performance by the students in the pre–test was due to the teaching strategy used in teaching the students and it took the lecture form of teaching. With this teaching strategy, the students did not have the chance of using their experience to create their own understanding, since lessons were delivered to them in an organized form previously. The students also lacked cooperative learning and hence the average students could not help their low performing colleagues.

The students could not analyze simple mathematical word problems and interpret key words, such as „sum"to mean „add", „difference" to mean „subtract", „product" to mean „multiply" and „quotient" to mean „divide". These findings agree with Mohyuddin and Khalil (2016) thata mathematical word problem is seen by many students as challenging content domain among in Senior High School mathematics domains. Word problem is seen as a difficult subject among mathematics students and often they misunderstood the notion of function. Majority of students may only follow rules, that is, multiply out brackets, collect together like terms, look for common factors etc. In addition, students may have acquired learned procedures from prior discussion and instruction to use the substitution rule before doing the required expansion (Haghverdi, Semnani, & Seifi, 2012; Vondrová, Novotná, & Havlíčková, 2019; Novotná & Chvál, 2018; Samková & Tichá, 2015). The findings are in consonance with Jupri and Drijvers (2016) who in separte study found that the majority of students included in their study have faced difficulty in using variables as generalized and changeable quantities. In addition, they found that students focus on or influence by arithmetic approach for items demanding an algebraic approach, practice "point-by-point or static way" of evaluating an independent variable of a function with the real domain. The ability to use variables as varying quantities showed a positive correlation with students" performance in word problems.

In furtherance, the findings agree with Vondrová, Novotná, and Havlíčková (2019) and Gasco, Villarroel, and Zuazagoitia, (2014) who in separate studies found out that procedural errors occurred when students failed to carry out manipulations or algorithms, even if concepts were understood. Interpretation errors occurred when students wrongly interpreted a concept due to over-generalization of the existing schema. Linear extrapolation errors occurred when students are not able to resolve mathematical word problem in one variable which seeks a linear statement or equation. The findings revealed that the participants were not familiar with basic operational signs such as addition, subtraction, multiplication and division relating to word problems. The participants demonstrated poor ability to simplify once they had completed grouping of variables (Novotná & Chvál, 2018; Samkov & Tichá, 2015; Wijaya, van den Heuvel-Panhuizen, Doorman, & Robitzsch, 2014; Capone, Filiberti, & Lemmo, 2021; Myers, et al., 2022).

Notwithstanding, the study findings suggest that the application of guided-discovery approach to teaching and learning of word problems were investigated in the study. The results revealed that guided-discovery method has contributed to the success of students‟ performance in the learning of word problems by engaging the students, and arousing and sustaining the student‟s interest in the lesson. The guided-discovery method also made it easier for students to follow the word problem instruction.

The findings indicated that students at Jachie Pramso Senior High School who were exposed to the guided discovery method exhibited improved problem-solving abilities. They were able to break down complex word problems into simpler components, identify the relevant information, and apply appropriate algebraic techniques to solve linear equations.

Reduced difficulties in translating word problems into equations, the study revealed that students at Jachie Pramso Senior High School often face challenges in translating word problems into algebraic equations. However, when the guided discovery method was employed, students gained a better understanding of the relationship between the given information and the variables, making it easier for them to write and solve equations accurately

According to the findings, the utilization of the guided-discovery approach to translating word problems into mathematical statements or equations and solving them has motivated students to study mathematics because of its numerous representation possibilities and the practical nature.

The study found that the guided discovery method promoted the development of critical thinking skills among students at Jachie Pramso Senior High School By exploring multiple strategies and considering various approaches, students became more adept at analyzing problems, evaluating potential solutions, and making informed decisions. These findings are in-line with the findings of Rahmawati, Sulisw249oro, and Prasetyo (2020) and Aagesen, Konge, Brydges, and Kulasegaram (2020) who in separate studies found that guided-discovery method of learning mathematics has the potential to pique and maintain students' interest while also improving their mathematics performance. They argued that guided-discovery method was the best method to promote the learning of certain rules or abstract concepts in mathematics. Therefore, in order to get more students taking up STEM courses at the university or tertiary levels, mathematics teachers must make their mathematics instructions lively and relating concepts to real-life situations and using appropriate teaching methodology to explain abstract mathematical concepts. This will enable students appreciate mathematics.

The findings demonstrated that students' at Jachie Pramso Senior High School confidence and motivation levels were positively influenced by the guided discovery method. As they experienced success in solving word problems, their self-belief grew, leading to a more positive attitude towards mathematics and a willingness to tackle more challenging tasks.

Guided discovery method enhances student engagement; the study found that implementing the guided discovery method in teaching word problems involving linear equations increased student engagement. Students at Jachie Pramso Senior High School were actively involved in the learning process, which enhanced their understanding and problem-solving skills

The findings also support the claims made by Fauzi and Respati (2021) that guided discovery is important in promoting learning with young children, while Anggriani, Sarwi, and Masturi (2020) on the order hand agreed that active learning methods are more important for younger students than for elder ones. The students must be actively engaged with abstract or concrete concepts for learning to occur in a mathematics classroom. Students' interests and accomplishment levels increased when teachers effectively integrated appropriate teaching method like the guided-discovery into the learning process. When the teaching method is an essential active element of the mathematical education process, it must be employed appropriately and judiciously to achieve learning outcomes. Using guided-discovery method in the mathematics classroom has a good impact on students' success and attitudes towards mathematics education (Ayu, Mudjiran, & Refnaldi, 2022; Suratno, Tonra, &

Ardiana, 2019; Rahmawati, Sulisworo, & Prasetyo, 2020;. They stressed that judicious use of guided-discovery can support both the learning of mathematical methods and skills as well as the development of desired mathematical proficiencies such as problem-solving, reasoning, and justifying.

The findings are also in consistent with those of Sari, Gistituati, and Syarifuddin (2019) andArifin, Wahyudin, and Herman (2020) who affirmed that utilizing guided-discovery method into mathematics instruction increases student motivation by virtue of its practical nature and visual representation of concepts. Therefore, students are able to retain abstract mathematical concepts which are indications that desired learning outcomes have taken place. Overall, the findings support the constructivists‟ theory of learning that humans generate knowledge and meaning from interaction between their experiences and their ideas (Charmaz, 2017). The basic idea in this theory is that learning is an active and constructive process with the learner viewed as an information constructor. By blending tenets of constructivist theory and that of the guided-discovery approach, the teacher becomes a guide for the learner, providing bridging or scaffolding, helping to extend the learner‟s zone of proximal development while the learning process is centered on the learner (Weimer, Fabricius, Schwanenflugel, & Suh, 2017). Moreover, the findings are in sync with Kang and Liu (2018) who affirmed that representation or illustration of mathematics concept need not be taught as though they are ends in themselves. Instead, they can be considered as useful tools for constructing understanding and for communicating information and understanding.

If students simply complete assignments of constructing representations in forms that are already specified, they do not have opportunities to learn how to weigh the advantages and disadvantages of different forms or how to use those representations as tools with which to build their conceptual understanding. The representations enhance the problem-solving ability and that students often construct meaning in forms that help them see patterns and perform calculations. The use of multiple representations is one of the major topics in mathematics education that has gained importance in recent decades (Allen & Trinick, 2021). The significance of representing the solution of linear algebraic equations in multiple ways provides the same objective of more than one form. It is necessary to see how students use these representations. It is suggested that multiple representations provide an environment for students to abstract and understand major concepts (Allen & Trinick, 2021).

In addition, the findings indicated that the experimental and control groups had mean scores of 17.72 and 17.48 respectively, with a mean difference of 0.24. The experimental group's minimum and maximum scores were 12 and 7 respectively, while the control's groups were 10 and 6. Moreover, the results of the independent samples t-test analysis showed that there was no statistically significant difference between the experimental group (M=17.72; SD = 7.24) and the control group (M = 17.48, SD = 6.55) on the pre-test scores of the two independent research groups. The estimated t-statistic, ($t = 0.388$; $p = 0.887 > 0.05$) indicates that both the experimental and the control groups had similar conceptual knowledge of the concepts of word problems prior to implementing the treatment (guided-discovery approach). Furthermore, an independent samples t-test analysis of the post-test scores for the experimental and control groups demonstrated that there was a statistically significant difference between the experimental group (M = 24.80; SD = 9.48) and the control group (M = 20.65; SD = 7.67). The estimated t-statistic was ($t = 2.986$; $p = 0.005$). This shows that the experimental group taught with guided-discovery method outperformed the control group taught without the guided-discovery method. The eta squared value of 0.096 indicates that a medium effect size (Cohen, 1988), implying that the conventional teaching method accounted for 9.6% of the variance in the post-test scores.

These findings support Rahmawati, Sulisw(Suliswo)ro, and Prasetyo (2020) who in separate study found that after using guided-discovery method to teach word problems in the first, second and third grades, many first-, second-, and third-grade learners were able to use invented strategies to solve a problem. They also found that 65% of the learners in their research sample used an invented strategy before standard algorithms were taught. By the end of their study, 88% of their sample had used invented strategies at some point during their first three years of school. They also found that learners who used invented strategies before they learned standard algorithms demonstrated better knowledge of base-ten number concepts and were more successful in extending their knowledge to new situations than learners who initially learned standard algorithms. More so, the findings are in agreement with Kousar (2010) who sought to determine the effect of a guided discovery approach on academic performance of mathematics learners at the secondary level. The experimental group was then taught over a period of six weeks based on a planned guided discovery approach. The control group continued with the instructional approach that they had prior to being identified as the control group. After the intervention, an assessment was used to see the effects of the intervention. A two-tailed t-test was used to analyze the data, which revealed that both the experimental and control groups were almost equal in mathematics knowledge at the beginning of the experiment. However the experimental group outscored the control group significantly on the assessment following the intervention.

5.3 Conclusion of Study

In conclusion, the study found that the guided discovery method is an effective approach for minimizing students' difficulties in solving word problems involving linear equations. The findings indicated that this method enhances student engagement, improves problem-solving abilities, reduces difficulties in equation translation, promotes critical thinking skills, and boosts confidence and motivation. To maximize the benefits of this approach, it is recommended to incorporate the guided discovery method into the curriculum, provide professional development for teachers, scaffold instruction gradually encourage collaboration and discussion, and offer feedback and support to students. By implementing these recommendations, educators can create a supportive learning environment that empowers students at Jachie Pramso Senior High School to excel in solving word problems involving linear equations.

Participants who were taught using the guided-discovery method outperformed their counterparts who were taught the same lesson using conventional strategies, according to the findings of this study. Finally, the findings revealed that integrating guided-discovery method into the teaching and learning of mathematics increases student motivation in the following ways: it induces learning practically, it arouses, motivates, sustains the interest of students in the teaching process, it presents concepts in a multi-dimensional way, it promotes real-life connections with concepts, it brings the students' immediate environment closer to the classroom, and it promotes retention through its visualizing qualities.

5.4 Recommendation of the Study

The study recommends that, Jachie Pramso Senior High School teachers used guided-discovery method frequently in the teaching and learning of word problems, Mathematics instructors at Jachie Pramso Senior High School must get thorough in-service training in guided-discovery and its application in order to enhance good practices in the mathematics classroom through Ghana Education Service (GES). Pre-service teacher education should include considerable practice in fundamental and higher-order mathematical process abilities so that new teachers are more confident in their ability to use guided-discovery method when teaching mathematics. Moreover, guided-discovery method of teaching and learning can be utilized to grab students' attention which has implications for mathematics teachers, particularly in schools where students' interest in mathematics is low. This teaching strategy or method can also be utilized to increase performance in low-performing schools, the areas which are under-resourced.

The government, through the Ghana Education Service (GES), should aim at using guided-discovery in teaching mathematics as much as feasible. It is further recommended that senior high school stakeholders consider periodic seminars/ workshops for mathematics instructors on the use of appropriate teaching method such as guided-discovery in teaching and learning mathematical concepts.

5.5 Areas for further studies

Long-term impact: Investigate the long-term effects of using the guided discovery method on students' problem-solving abilities beyond the immediate context of linear equations. Examine whether the skills and strategies developed through this approach transfer to other mathematical concepts and real-life problem-solving scenarios.

Differentiation: Explore how the guided discovery method can be tailored to meet the diverse learning needs of students. Investigate strategies for adapting the approach to address individual differences in students' prior knowledge, cognitive abilities, and learning styles.

Teacher professional development: Investigate the impact of providing professional development opportunities for teachers on implementing the guided discovery method effectively. Explore the training and support needs of teachers to enhance their instructional practices and improve student outcomes in solving word problems involving linear equations.

Technology integration: Examine the role of technology, such as educational software, simulations, or online platforms, in enhancing the guided discovery method for teaching and learning linear equations. Investigate how technology can be effectively integrated to provide additional support, practice, and feedback to students.

Comparative studies: Conduct comparative studies to evaluate the effectiveness of the guided discovery method in comparison to other instructional approaches for teaching linear equations. Compare the outcomes, engagement levels, and attitudes towards mathematics of students taught using guided discovery with those taught using traditional methods or other problem-solving approaches.

Generalization to other contexts: Explore the transferability of the guided discovery method to other schools, districts, or regions with similar or different demographic characteristics. Investigate how cultural, social, or economic factors may influence the effectiveness of the approach in different educational settings.

Assessment strategies: Investigate the development and use of appropriate assessment strategies to measure students' problem-solving skills and conceptual understanding in linear equations. Explore how the guided discovery method aligns with assessment practices and explore the use of alternative assessment formats that align with the goals of the approach.

These suggested areas of further study can help build upon the initial research on using the guided discovery method for solving word problems involving linear equations, contributing to a deeper understanding of the approach's effectiveness and its implications for mathematics education.

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APPENDICES

APPENDIX A

PRE-TEST ITEMS

Please answer the questions in the space provided after each question. Remember to show clearly all steps to arrive at the answer.

Mary has x oranges and Joe has four more oranges than Mary. How many oranges does Joe have?

Simplify as far as possible $1 + x + x$

Write in algebra: There are twice as many pencils as pens (let y be the number of pencils and x be the number of pens).

There is a u number of sweets in a packet. A girl has two packets of sweets and gives her friend five sweets. How many sweets does she have remaining?

A basket costs Eight Ghana Cedis and a bag costs c Ghana Cedis more than the basket. How much does the bag cost?

If s is the number of students and t is the number of tables, write in algebra: There are three students for every table.

Find the value of x: $7x = 21$

Simplify as far as possible $3m + 5n + 4m + 6n$

$y \times y \times y =$ ....

If d is the number of dogs and c is the number of cats, write in algebra: There are four more dogs than cats.

Find the value of x, $2x - 2 = 10$

Kofi has x bananas and Kweku has y bananas. Peter counts the number of bananas each of them have and finds they are the same. Kofi said you could write this as x=y, but Kweku said that x and $y$are different letters and so cannot be the same. Who do you think is correct?

Find the value of x, $21x = 7$..

What is the number that is five less than x?..

Write in algebra: There are three more caps than hats.

APPENDIX B

POST-TEST ITEMS

Please answer the questions in the space provided after each question. Remember to show clearly all steps to arrive at the answer.

Aku is 12 years older than Kofi. Next year, Aku will be 3 times older than Kofi. How old is Kofi?

Twice a number decreased by 22 is 48. Find the number.

Mary has *m* toys and John has three more toys than Mary. How many toys does John have?

Solve for x: $5x – 5 =20$

David is 14 years and his father is 37 years. In how many years will David‟s father be twice as old as David?

Find the value of x: $6x =24$

Emmanuel is 20 years of age and his brother is 12 years. How many years ago was Emmanuel three times as old as his brother?

Solve for x: $\frac{x}{2} - (5 - 6x) = \frac{2x}{5} + 52$

$\frac{2(3y-6)}{3} + \frac{3(2y+4)}{5} = 2\frac{5}{4}$. Solve for y.

There is a x number of pencils in a packet. A girl has three packets of pencils and gives her friend five pencils. How many pencils does she have remaining?

If b is the number of boys and g is the number of girls, write in algebra: There are three boys for every girl**.**

Simplify as far as possible $4z + 3p + 7z +2p$

8 times a number is less than 5 times the number. Find the number.

I‟m thinking of a number. If you multiply it by 6 and then add 7, you will get 55. What is my number?

The sum of two numbers is 30. The difference between ½ of one of the numbers and 1/3 of the other is 5. Find the two numbers

APPENDIX C

GUIDED DISCOVERY MANUAL MATERIALS

Word Problems

Solving Sums and Differences

Explain consecutive number

Students should be allowed to list some example of consecutive numbers

Give student some independent works on consecutive number

Solve some problems on word problems

Evaluate the lesson/Assignment

Word Problem on Product and Quotient of Whole Number

The idea of multiplying fraction with whole number using word problems

Use of concrete object like human age and height in word problems

Evaluation of the lesson/Assignment

Combinations of Sum and Difference in Word Problems

Students should solve variety of positive difference between products

Evaluate the lesson/Assignment

Word Problems Involve Fractions

Students should be independent use latter to solve expressions

The idea offsets

The use of fraction in word problems

Use of subtraction of fraction in word problems

Evaluate the lesson/Assignment

More Problems Involving Simple Algebraic Fractions

Idea of the use of latter fraction in word problems

Evaluate the lesson/Assignment

- 10 Further Word Problems Involving Fractions

Translate word problems involving fraction equation as well mathematics statement

Solve word problems involving fractions

Evaluate the lesson/Assignment

Solving Algebraic on Word Problems

The idea of fast reasoning

Relating the idea with algebraic expression

The use of chronological age

Evaluate the lesson/Assignment

Application of Word Problems from Everyday Situations

Solving problems from everyday situations

Formulating problems from everyday situations

Evaluate the lesson/Assignment

APPENDIX D

GUIDED DISCOVERY LESSON PLAN

Lesson I

Topics: Word problems in sum and difference

Objective: At the end of the lesson, the students will be able to solve word problems on sum and difference of whole number.

Previous Knowledge: students have learnt how to translate word problems into numerical expression and vice versa

Introduction: briefly ask students appropriate questions on relevant previous lesson such as: $\frac{(3+8)-4}{2}$ formulate word problems from thefollowing

Translate the following word problem into numerical expressions. Take away thirteen from the product of five and seven

Presentation:

Step I: ask the students to solve the problems.

The sum of four consecutive numbers is 58, Find the number? Ask: What are the numbers? The number must be one ahead each other.

Expected result: Let the numbers be $n, (n+1), (n+2), (n+3)$

The students discover that: $n+(n+1)+(n+2)+(n+3)=58$

$$4n = 58 - 6$$

$$n = \frac{52}{4} = 13$$

Since n = 13, therefore the four consecutive numbers are? Expected results are: 13, 14, 15 and 16.

b, find the sum of 12and9 Ask: what is the sum of 12 and9?

Then let the sum be n That is 12 + 9 = n

21 = n

c, Kofi bought 12 oranges from the market. His Daddy plucked 14 more Oranges for him from the garden, how many Oranges does Kofi now have?

Solution: Let the students lead the discussion given a variety ways in solving the problems.

What is the problem? State clearly: Kofi bought 12 Oranges

Daddy plucked 14 Oranges Kofi now has (12 + 14) Oranges Kofi then has 26oranges.

Ask: could we sum up that, the sum of a set of number is the result obtained when the numbers are added together?

Step II discuss and guide the students to solve problems involving both sum and difference.

Given: the difference between 8 and another number is 17.Findthe possible value, for the number.

Solution: **Expected workings areas follow:** Assume the other number to be x

X –8 =17

Add 8 to both sides X – 8 + 8= 1=>7+8

i. e x = 25

Alternatively 8 – x = 17

Add x to both sides 8 – x + x = 17+=>x 8 = 17 +x

Take 17 from both sides 8 – 17 = 17=>17 +x

x = -9 ∴

Thus the number could be:

Expected result 25 or-9

b. Safiya was given 42 groundnuts by her friends. She gave 20 of them to her brother, Faisal. How many groundnuts does Safiya have left?

She has (42 - 20)g/nuts left She has 22 g/nuts left

The students generalized by saying that the difference between two numbers is the result of subtracting one from the other. It is usual to subtract the smaller number from the larger. This gives a positive difference

Summary: the teacher briefly revises what he has done so far for the benefit of the weak ones.

Evaluation: the teacher evaluates the lesson by asking the following questions in order to be sure that they understood the lesson.

The sum of three consecutive numbers is 63. Find the numbers.

The difference between 12.6 and a number is 5.4. Find the two positive value of the number.

Assignment: the teacher asks students to solve the following:-

The sum of four odd numbers is 80. Find the number.

The difference between -3 and a number is 8. Find the two positive values for the number.

The sum of three consecutive odd integers is 72. What are the numbers?

Lesson II

Topics: Word problems on product and quotient of whole numbers

Objectives: At the end of the lesson, the students will be able to solve word problems on product and quotient of whole numbers.

Previous Knowledge: The students have already learnt how to solve word problems on sum and difference of whole numbers Introduction: The teacher introduces the lesson by asking students to solve word problems on sum and difference of whole numbers. For instance, the sum of three consecutive odd numbers is 72. What are the numbers?

Presentation:

Step I One third of number added to four-fifths of itself is equal to 17findthe number.

Solution: Let the students think of the number themselves

The discovered number is x One third of x = $\frac{x}{3}$

Four fifth of x = $\frac{4x}{5}$

Then the expected work is

$\frac{x}{3}$ + $\frac{4x}{5}$ = 17 allow the students to think of the LCM of 3.5and1 which is15.

Let them multiply the equation through by the LCM of (15) 15x $\frac{x}{3}$ +15 x $\frac{4x}{5}$ = 15 x 17

= 5x + 12x =255

This gives 17x = 255 Divide through by17 X = $\frac{255}{17}$ => x =15

Step II : give them another example

A man is four times as old as his son. In four years"time he will be three times as old. How old are they?

Solution: The students are expected to discover the age of the son to be y. then they are expected to give the father"s age to be 4 x y =4y.

In four years" time, their age will be Father = 4y +4

Son = y + 4

Then 4y + 4 = 3(y + 4)

Expound and collect the like terms.

4y + 4 = 3y + 12 = 4y – 3y = 12-4 = x =8 >

Therefore their ages are now Sun = y = 8

Father = 4y years = 4x8 years = 32 years.

Evaluation: Yesterday was Mrs. Badu"s birthday. She bought 32 sweet to share equally among her four (4) children. How many did she give each child?

Lead the Students through, by asking leading questions

Such as: How many sweets did Mrs. Badu buy? How many children does she have? She gave each child (32/4) sweets.

The teacher sees that they are able to solve the problem.

Summary: the teacher reviews the entire lesson briefly for all the students" to understand.

Assignment: the teacher uses these tests to measure student"s ability to discover new ideas

If I divide a number by 3, and I remove 12 from the result I got 29 as the answer, what number did I divide by3?

Miriam and Isa have 82 groundnuts between them Isa has 4 more than Miriam. How many has Isa?

The teacher should provide reading, research, or investigations in which ideas are explained independently.

Lesson III

Topic: The combination of product with sums and differences in word problems.

Objectives the students should be able to solve word problems combining products with sums and differences.

Previous Knowledge: the students have learnt how to solve word problems with the products and quotient of whole numbers

Introduction: the teacher introduces the lesson by asking questions from the students" relevant previous knowledge

Presentation:

Step I ask students variety of questions such as: Find the positive difference between 31 and the product of 4 and 14

What is the product of 4 and 14? Product of 4 and 14 = 4 x 14 = 56

What is the difference between 31 and56?

The difference between 31 and 56 = 56 – 31 = 25

Note that the problem is to find the differences between 31 and a product. Therefore, find the product first (4 x 14). 31 is equivalent to "positive difference between 31 and the product of the 4 and 14".

Step II:give them more examples. Find the product of the 11 and the positive difference between 4 and10

Ask: what is the positive difference between 4 and10?

The Positive difference between 4 and10= 10 – 4=6 What is the product of 11 and6?

Product of 11 and 6 = 11 x 6 = 66

Note that the problem is to find the product of 11 and positive differences therefore, find the difference between first

Hence 11 x (10 - 4) is equivalent to the "product of 11 and the positive difference between 4 and 10".

Consider the following example also.

Find the sum of .9 and the product of 1.7 and3

Sum = .9 + (1.7 x 3) 0.9 + 5.1 = 6.0

Find the product of $\frac{1}{7}$ and the sum of $\frac{3}{5}$ and $1\frac{1}{2}$

The product = $\frac{1}{7} \times (\frac{3}{5} + 1\frac{1}{2}) \Rightarrow \frac{1}{7} \times (\frac{3}{5} + \frac{3}{2}) = \frac{1}{7} \times (\frac{6}{10} + \frac{15}{10})$

= $\times \frac{1}{7}\ \frac{21}{10} = \frac{3}{10}$

Find the positive difference between the sum of 1.6 and 2 and the product of 7 and0.4

The differences = (1.6 + 2) - (7 x 0.4)

3.6 – 2.8 = 0.8

Teacher stimulated the thinking of the students using dialogues wherever they are solving the above problems on the B.B (chalk board) through questions and hints if possible.

Guesses, conjecture, trial and error will be used to search for ideas and to relate these new ideas to previous concept.

Summary: teacher review briefly the whole lesson to ensure that they have understood the concept.

Assignment:

The teacher then gives them the following work to do.

Find the product of 5 and the difference between 15 and17

Find the difference between the product of 0.6 and the sum of 0.6 and0.4

Find the sum of 29, the product of 2 and 9 and the difference between2 and9

Summary: the teacher reviews the lesson once more to help the weak ones.

Assignment: the following is the class activity and assignment.

Find one ninth of the difference between 49 and 13 (ans4)

Find one third of the sum of product of 12 and5.

Find one-eighth of the sum of 14, 15, and19.

Find the quarter of the difference between 17 and the square of 3

Find one third of the difference between 29 and the sum of 11 and 6

Lesson V

Topic: more word problems involving simple algebraic fractions

Objective: at the end of the lesson students should be able to solve word problems involving simple algebraic fractions.

Previous Knowledge: the students have learnt how to solve word problems involving fractions

Introduction: Introduces the lesson by considering problems involving algebraic fractions that could be written in the form of statements. For instance the average cost of certain number of books is #250.00. The total cost can be written as #250.00n if n is the number of books

Presentation:

Step I: consider this example, the average cost of a number ofpencilsis50k, if all the pencils cost #20.00, find the total number of pencils.

Solution: let the number of pencils be x

Average cost of pencils = Convent this to kobo $\frac{\#20}{x}$

#20/x = 20/x X 100

Hence 50= $\frac{20 \, x\, 100}{x}$

Multiply both sides by x 50x = 2000

Divide both sides by 50

$^{50x}/_{50}$= $^{2000}/_{50}$

= 40 ∴ x

There are 40 pencils on the whole Consider another example for students.

If the average weight is 30kg, find the number of the boys Solution: let the number of the boys be n

Total weight = 1200kg

Then the average weight = 1200/n That is 30 = 1200/n

Solve for n we have 30n = 1200

Then, $^{30n}/_{30}$= $\frac{120}{30}$ n = 40 there are 40boys.

The students should be able to deduce that when solving word problems or simple equations involving fractions, the following should be considered.

what is the unknown

Choose a latter that can represent unknown

3). Write the statement into an equation form

4). Solve the problems

Summary: go over the lesson briefly for the weaker ones to understand better.

Assignment:

A hunter kills x numbers of bush rats, the total weight of these is 48kg. What is the average weight of the animals in the terms of x (ans 48/xkg)

The total cost of a number of school bags is #56.00. Each bag costs#2.00.

a), write one equation connecting the #56.00 and #2.00 and the number of school bags (ans. 56/x = 2)

b), find the number of school bags (ans. 28)

a man but a shirt each for his sons at a total cost of #32.00. If the average cost of the shirt is #4.00 how many sons does the man have? (ans.8)

Lesson VI

Topic: Further word problems involving fraction equations

Objective: at the end of the lesson, students should be able to: translate word problems involving fraction equations as well as mathematical statements,

Solve word problem involving fractional operations.

Previous Knowledge: the students have learnt how to solve word problems involving simple algebraic fractions.

Introduction: The teacher introduces the lesson by asking relevant questions on the previous knowledge.

Presentation:

Step I: the teacher informs the students that in this lesson they will consider more difficult problems involving fractional algebraic equations.

Example1 a certain car covers 10 km at a certain average speed. If this average speed is reduce by 30km/h, the car takes the same amount of time to cover a distance of 6km. find the speed of the car in the first part of the journey.

Solution: the teacher guides the students to identify the key or essential statements in word problems and should be written out on the chalk board, then, their translation into symbolic statements can be done by bit.

Let the speed for the first part = a km/h

The speed for the second part = (a – 30km/h) Time taken by first car to cover 10km = 10/ah

Time taken by second car to cover 6km = 6km (a - 30)/h these times are the same, in both journeys

10/a = 6/(a - 30)

This equation can now be easily solved as follows Find the LCM of a and (a - 30)

LCM =a(a - 30)

Multiply both sides by a(a - 30) (a - 30) x 10 = 6 x a

In expanding we obtain

10a – 300=6a 10a – 6a =300 =>

4a = 300

a=300/4   a =75km/h∴

The speed in the first part of the journey is 75km/h

Example II: A boy circled 12km at a certain average speed.

He then increases his speed by 4km/h and takes the same time to travel 15km. find his speed for both parts of the journey.

Solution: the problem is to find the boy‟s speed. Let his speed for the first part of the journey be vkm/h

Then, his speed for the second part of the journey is V+ 4km/h (From the second sentence of the question)

Time taken for the first part = 12/Vh

Time taken for the second part = 15/(V + 4)h The two times are the same, thus

12/V = 15/(V = 4)

Multiply both sides by their LCM The LCM = V(V + 4)

12(v + 4) = 15v

12v + 48 = 15v

48 = 15v – 12v = 48 = 3v, v = 16.

His speed for the first part of the journey was 16km/h and for the second part20km/h

Activities: student should attempt the following.

When full, a car‟s petrol tank holds k litres

After using 15 litres, the remaining petrol is enough for the car to travel 34km.

Express the amount of petrol in terms of k (ans. K – 15litres)

Hence express the distance that the car travels in terms on one litre of petrol in terms of k (ans. 344km)k-5

If the car travel 80km litre, find the values of k (ans. K =58)

A man drives 146km at a certain average speed. He then increases this speed by 9km/h and takes the same time to travels the next164km. find his speed for the both parts of the journey (asn. 73 km,82km/h)

A man travels 29km on an open road at a certain average speed, in the city, he reduces his average speed by 42km/h and find that it took him the same time to cover 15km. find his average speed.

On the open road (ans.87km/h)

In the city (ans.45km/h)

Summary: The teacher explains to the students that in this lesson they have studied how to solve problems on simple equations involving fractions. The simple equations involve monomial and binomial denominators. He then guides them to conclude that the following steps may be necessary whenever they are solving word problems.

Find the LCM of all the denominators

Multiply each term in the equation by the LCM

Simplify and solve the resulting equations.

Lesson VII

Topic: Further word problem involving fractions.

Objective: At the end of the lesson, the students should be able to:

Translate word problems involving fractions into mathematical statements

Solve word problems involving fractions.

Previous Knowledge: They have learnt how to translate word problems involving fractions

Introduction: The teacher asks relevant question on the previous knowledge. Presentation:

Step I: the teacher discuss with class when solving problems on this kinds, what do we look for?

Choose a latter to stand for the unknown. Change the statement in question into algebraic expressions. After such discussion with the students, consider these examples.

Example1: 30 are divided by the sum of three (3) and x, if the result is 5, what is the value of x?

Solution: what is the sum of three and x? The sum of three and x is 3 + x =?

What is 30 divide by the sum of 3 and x to give the result to be 5 is = $5\frac{30}{3+x}$ Multiply both sides by (3 + x) (3 $\frac{30}{3+x}$ +x)x = 5(3 +x)

30 = 5(3 + x)

To expended the RHS and solve= 30 = 15+5x => Subtract 15from both sides

30 – 15 = 15 + 5x - 15

15 = 5x

Divide both sides by5 = = 3 =x x =3 => $\frac{15}{5}$ $\frac{5x}{5}$

Example2: A boy is 24 years younger than his father and in 2 years time, the sum of their ages will be 40

How old is his father and the son now?

Solution: let the father"s age be (x + 2) years, the son (x – 24 + 2) years respective.

But the sum of their ages is 40years, Thus: (x + 2) + (x – 24 + 2) =40

X +2 + x - 24 + 2 = 40

X + x + 2+2 – 24 = 40

2x – 20 = 40 = 2x = 40 + 20 => 2x = 60

2x=20 x =30. =>

Therefore the father is 30 and the son is (30 - 24) years or 6 years old.

Example3: A boy is older than his sister by three years, 5 years ago, the ratio of their ages was 4:3. Find their present ages.

Solution: let the boy"s age be x and the sister"s age be (x -3)

Five years ago, the boys age was (x-5) years and the sister was (x-3)-5 i.e (x - 8)

If the ratio of their age is4:3then = $\frac{x-5}{x-8}$ $\frac{4}{3}$

The LCM of (x - 8) and 3 = 3(x - 8) Multiply both sides by the LCM

We have 3(x - 8) x = 3(x - 8) x $\frac{x-5}{8}$ $\frac{4}{3}$

x

Expanding both sides 3x – 15 = 4x – 32

I.e 3x – 15 + 15 = 4x – 32 + 15

i.e 3x = 4x – 17

3x – 4x = 4x – 17 -4x

-x = -17 multiply both sides by -1

-1(-x)=-1(-17) x = 17 and x – 3=>14.

So the boy"s age is 17 years and his sister"s age is 14 years.

The teacher should emphasize this steps involving multiplication by (-1) and discourage students from saying "minus cancel minus"

Example4: A man is five years old than his wife, 4 years ago the ratio of theiragewas7:6. Find their present ages.

Solution: the problem is to find their ages, let the age of the man be y years. Then the age of his wife is y – 5 years. (Form the first sentence of the question)

Then, 4 years ago their ages were as follows Man"s age is (y - 4)years

Wife"s age is (y - 4) – 5 y – 9 years=>

Thus the ratio of their age 4 years ago was7:6 Multiply both sides by 6(y -9) being the LCM 6(y - 4) = 7(y -9)

Thus, 6y – 24 = 7y– 63 63 =>24 = 7y– 6y39 =y.

The man"s age is 39 years and his wife"s age is 34 years.

Summary: the teacher briefly reviews the lesson for proper understanding of the weak ones.

Activities: solve the following word problems.

The weight of a man and the son differ by 12kg and are in the ratio of9:6. Find the weight of each.(ans.Man"sweightis37kg,son"sownis15kg)

A woman is 25 years older than the son.

If the son is x years, find the age of them other.

Find their respective ages if the ratio of their ages 4 years ago was5:2 (ans.X+25,b,woman"s452/3son"s202/3)

The difference between the heights of a man and the son is 0.4cm. if the father is taller than the son and the ratio of that height is 4:3. Find the height of the son.

A man is thirty years older than his gardener.

I): If the gardener is x years old, how old is the man. (ans. (30+x)) II): Express the age of the man 6years ago in terms of x (ans. (24 +x))

III): Express the age of the gardener 6years ago in terms of x. (ans: x -6)

IV): 6years ago, the ratio of their ages was 5:3. Find their ages. (ans. Man‟s age is 81, gardener‟s age is 51).

Lesson VIII

Topic: Solving algebraic on word problems

Objective: At the end of the lesson the students should be able to: Solve word problems involving algebraic equations

Previous Knowledge: the students have learnt how to solve word problems involving algebraic fraction.

Introduction: the teacher asks questions from the students, previous knowledge. Presentation:

Step I: The teacher guide the students through two examples to illustrate word problem can be solved by algebraic method.

Example1: I think of a number I add 6 to it, the result is ten. What number did I think of?

Solution: Ask the student if the above problem can be express using fewer words? The expected result is, yes, it can be solve using fewer words.

Let the number be x

Add six the result is ten, can be translated into x + 6 = 10. Find x.

Example1: the sum of two numbers is 24, twice the first plus the second is26.Find the numbers

Solution: if x is the first number, then (24 - x) is the second number. But twice the first plus the second is 26.

Thus: 2x + (24 - x) =262x + 24 – 24 - x = 26

24 – X = 26 – 24 x = 2 =>

Therefore the numbers are 2 and (24 - 2) or 22.

Example2: the product of a certain number and 5 is equal to twice number subtracted from 20. Find the number

Let the number be x?

The product of x and 5 is 5x

Twice x subtracted from 20 is 20 – 2x

Thus: 5x = 20 – 2x

Adding 2x to both sides we have 7x = 20 divide both side by 7

X = 20/7= $^{6}/_{7}$

Example3: A rectangular field which is 50 meters wide, requires 260 meters of fencing how long is the field?

Solution: If the field is x meters long, then the total length of the fence around the field is (x + 50 + x +50)m

But, the total length of the fence is260meters.Thus: x + 50x + 50 =260

2x=160 x =80. =>

Therefore, the field is 80 meters long.

Activities: 1) A certain number is less than another by 3. If their difference is38,find the numbers. Ans, (20,17).

2): The sum of a number, three times the number, and five times the number is 171. Find the number. Ans (19)

3): I think of a certain number. If I multiply it by 6, add 8 to the product and double the sum, I will get 40 what is the number? Ans. Is(2)

4): the sum of two consecutive even numbers is 54. Find the number.

Ans. Is(26,28)

5): the sum of two consecutive odd numbers is 20. What are the numbers?

Ans. Is (103, 105).

Summary: the teacher reviews the lesson once more to help the weaker ones.

Lesson IX

Topic: Application of word problems from everyday situations Objective: At the end of the lesson, students should be able to

i): solve word problems from everyday situations. ii): Formulate word problem from everyday situations

Previous knowledge: the students have learnt how to sole word problems involving equations

Introduction: the teacher asks the students few questions base on the previous knowledge.

Presentation: the teacher presents the lesson in steps.

Step I): the teacher create situation such as: A girls hears that by selling a carton of milk a hawker gains #15.00.

Let the students generate problem or formulate a problem. For example;

Example I problem: if a girl needs #180.00 for her school fees and pocket money for nest term, how many cartons of milk should she sell during the holiday to get the#180.00?

Solution: refer to her #15.00 is gain from selling one carton.

#180.00 is the total gained from selling cartons = $\frac{\#180.00}{\#15.00}$12 carton.

Example 2 situation: A boy needs six 80 leaves exercise books, four biro pen two pencils a learner"s dictionary for school nest term.

Problem: if 80 Leaves exercise books cost #1:50 each, biro cost 40k each and pencil cost 20k each and learner"s dictionary cost #20.00 how much money does the boy needed for all the items?

Solution: 6(#1.5) + 40k(4) + (20kx2) +#20

= #9.00 + #1.60 + 40k + #20

=# 31 the boy needs#31.00.

Step II Assist and guide the students in creating situations and problems that come up daily in their lives. For example

Example II) situation: cost of living raised by 75%

Problem: last year our family experiences amounted to #400.00 a month. Now the cost of living has rising by 75%. If we wish to maintain the same living standard, how much extra do my parents need every month?

Solution: Extra money require per a month is75% of#400.00

I,e 75/100 x#400

= 75 x # 4.00 = #300.00.

The family needs extra #300.00 per month

Example2) situation: mother wants to prepare lunch most of the things she need are at home but she needs to buy 5 cups of rice and 3 cops of beans.

Problem: If the rice cost 75k per cup and beans cost 60k per cup, how much money does she need for five cups of rice and 3 cups of beans?

Solution: 5 x 0.75k + 3 x 0. 60k

#3.75k + #1.80 = #5.55k

Mother needs #5.55k

Activities: the teacher asks the student to formulate word problems for the following every day situation.

The cost of rice goes downing by25%

Daddy spends #500.00 every week on petrol.

A family has a dog and cat. It cost to feed the dog and cat #5 per day.

A man pays his labourers #240 per day

Ibrahim spends #15.00 every day on transport to work and back.

Lesson xiii

Topic: Application of word problems from every day situation.

Objective: the students at the end of the lesson should be able to: solve word problems from everyday situation.

Previous Knowledge: The students have learnt how to formulate and solve word problems from everyday situation.

Presentation:

Step 1: the teacher states some problems on the chalk board and asks all the students to discuss and come out with some facts.

Example I): square tiles, 30cm, are used to cover a floor. How many tiles are needed for a floor of 4.4m long and 3.8mwide?

Solution: length of room = 4.4m = 4.4 x 100cm =440cm.

Number of tiles 440/30 = 14 $\frac{2}{3}$

Thus, 15 liters are needed along each length of the room (the last tiles will be cut).

Width of the room = 3.8m = 3.8 x 100cm = 380cm

Number of tiles = 380/30 = 12 $\frac{2}{3}$

Thus, 13 tiles are needed across each width of the room.

The total number of the tiles needed = 15 x 13 = 195.

Example 2): A motorist travels 64km/h at a certain speed. She then decides to increase her speed by 10km/h to enable her to arrive at her destination 72km away on time. If the time taken is the same for both parts of the journey, find

a): Her speed for the first part of the journey b): Her speed for the second part of the journey

Solution: Let her speed for the first part of the journey be xkm/h Then, Her speed for the second part of the journey is (x + 10)km/h.

She takes $\frac{64}{x}$ km/hour for the first part of the journey and $\frac{72}{x+10}$ km/h for the second part of the journey

But, She takes the same time to do each part of the journey.

So, 64/x = 72/(x +10) a): 64(x + 10) =72x

Multiply both sides by the km x(x + 10) 64x + 640 = 72x

72x = 64x + 640

Add (-64x) to both sides

72x – 64x = 64x – 64x + 640

8x=640 8x/8 = 640/8; X =80

Her speed for the first part of the journey is 80km

b): Her speed for the second part of the journey is (80 + 10)km/h,thatis90km/h.

Example3: A farmer must deliver 1200 bushels of products to town. He has two loads the larger trucks breaks down, how many trips must the farmer make in the smaller truck if it started after the large truck broke down?

Solution: ask the class the following questions: How much is carried by the larger truck? How much is carried by the smaller truck? How do we solve the problem?

You are ready to choose a variable and solve the problem.

Let t be the number of trip for the smaller truck. In two trips the larger truck delivers 300 bushels.

Therefore 300 + 115t =1200

115t = 1200 – 300, 115t = 900 T = 900/115 = 7.8

The farmer makes eight trips in the smaller truck after the larger truck broke down.

Summary: briefly revise the lesson for the class.

Activities: let the students attempt the following on their own.

1): A farmer produces 7,315kg of rice from cultivating a certain number of hectares on a piece of farm land and 1,650kg from cultivating twice the number of hectares on another land. Find the number of hectares the farmer cultivated altogether if the average produce per hectare is 33kg

expected ans. Is (27) $\frac{1}{6}$

A distributor must mail 975 cassettes to the same address these cassettes must be shipped in either a large carton (holding 125 cassettes) or a smaller one (holding 75 cassettes). How many small cartons are necessary if there are only five large cartons available? Expected ans. Is (5 small cartons are needed)

3): A motor cyclist travels 48km at a certain speed. He then decides to decrease his speed by 3km/h to travels the remaining 42km. If five km is the same for both parts of the journey, find the speed for the first part of his journey?

4): A rectangular compound, 8.55m long by 5.89m wide is to be paved with the largest possible square tiles, which will fit in exactly. How many tiles will there be? (Hint: express 855 and 589 as products of prime numbers) expected ans. Is (1,395).

Topic: Application of word problems from everyday situations Objective: At the end of the lesson, students should be able to i): solve word problems from everyday situations.

ii): Formulate word problem from everyday situations Previous knowledge: the students have learnt how to sole word problems involving equations

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Ibrahim spends #15.00 every day on transport to work and back.

Lesson X

Topic: Application of word problems from every day situation.

Objective: the students at the end of the lesson should be able to: solve word problems from everyday situation.

Previous Knowledge: The students have learnt how to formulate and solve word problems from everyday situation.

Presentation:

Step 1: the teacher states some problems on the chalk board and asks all the students to discuss and come out with some facts.

Example1: A motorist drives from point A to point B a distance of 15kmin24 minutes. Where the road is good, he drives at the speed of 45km/h and at 30km/h where the road is rough. Find the number of kilometres of bad or rough road.

Solution: let the x km represent the good road and (15 – x), the bad road.

But, on x km road, the motorist goes at45km/h

Time taking on x km is $\frac{x}{45}$ km/hours and on (15 - x) km road, he goes at 30km/h

Hence, time taken is (15 -x)/30hours.

A total time taken on 15km road is 24 minutes. ($\frac{24}{60}$) hours = 2/5 hours.

$\therefore \frac{x}{45} + \frac{(15-x)}{30} = \frac{2}{5}$

Bring the students mind to the removal of the denominator through the lowest common multiples. The lowest common multiple of 45,30 and 5 is 90 that is

$\frac{x}{45} + \frac{(15-x)}{30} = \frac{2}{5} > 2x + 3(15 - x) = 2 \times 18$

2x + 45 – 3x = 36 collecting the like terms 45 – 36 = 3x – 2x that is 9 = x

Therefore x = 9.

The rough road is (15 - 9) km = 6km

Step II: the teacher revises all the formulas for finding areas of planes of difficult shapes

If possible write them out on the chalk board for the students. This will enable them to solve and understand the nest problem better.

Example I): square tiles, 30cm, are used to cover a floor. How many tiles are needed for a floor of 4.4m long and 3.8mwide?

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Number of tiles 440/30 = 14 $\frac{2}{3}$

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