UNIVERSITY OF EDUCATION, WINNEBA

TEACHER-TRAINEES' MATHEMATICS BACKGROUND IN RESPONSE TO COLLEGE TUTORS' INSTRUCTIONAL STRATEGIES IN MATHEMATICS

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Declaration

STUDENTS' DECLARATION

I, Kofi Ashiboe-Mensah, declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

SIGNATURE: ……………………………………

DATE: ……………………………………………

SUPERVISOR'S DECLARATION

We hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of dissertation as laid down by the University of Education, Winneba.

PRINCIPAL SUPERVISOR: PROF. SAMUEL ASIEDU-ADDO

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Dedication

To my wife and my three lovely daughters

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Abstract

The study investigated the effect of constructivism on teacher-trainees' performance in mathematics at the colleges of education. A purposive sampling technique was used to select the Volta Region and three colleges of education from the region. Furthermore, a convenient sampling technique was adopted to select six hundred and forty-one (641) 2019/2020 third-year teacher-trainees from the three colleges. The instrument used for the collection of data was carefully developed and structured from reviewed literature to measure CA, IC, PUFM and CGI as basis to determine constructivism. The instrument was validated and found to be reliable when factor analysis was conducted using the principal component analysis method. The binomial test, descriptive statistics analysis and composite scores of the mean likert-scale response indicated that instructional coherence which supported constructivist theory of learning was mostly used among the studied instructional strategies by the college tutors to support teacher-trainees mathematics learning. The indication was that tutors' use of instructional coherence instruction guaranteed constructivist theory which is a solid foundation to teacher-trainees' performance in mathematics at the colleges of education. Finally, partial least squares-structural equation modelling (PLS-SEM) results revealed that CGI had the highest path coefficient (β =-0.161) as the major construct affecting TTP and IC as the highest path model value of β =0.302 affecting CONST.

Key Words: *Constructivism, Cognitive Activation (CA), Instructional Coherence (IC), Profound Understanding of Fundamental Mathematics (PUFM), Cognitive Guided Instruction (CGI), Teacher Quality (TQ), Self-Determination Theory (SDT), Relevant Previous Knowledge (RPK), Teacher-Trainees' Performance (TTP).*

CHAPTER 1

INTRODUCTION

1.0 Overview

This chapter discusses the background to the study, the statement of the problem, purpose of the study, objectives of the study, research questions, and significance of the study, delimitation of the study and the organization of the study.

1.1 Background of the Study

Teachers are critical factors in students' mathematics learning as their knowledge in the contents, pedagogy and beliefs have significant impact on instructional strategies and the role teacher-trainees' play to acquire mathematical knowledge and skills (Mapelo & Akinsola, 2015). The indication here is that teachers' memories of mathematics from their school years are a major driving factor that affects their mathematics beliefs, content knowledge and performance in pedagogy. The study therefore investigated the instructional strategies used to teach teacher-trainees in the colleges of education and how much knowledge and skills they acquire to teach pupils at the basic schools for a solid mathematics foundation. Most people still believe that mathematics is about computation, however, computation is just a tool for understanding structures, relationships and patterns of mathematical concepts which produce solutions to non- linear complex problems (Saritas & Akdemir, 2009). Mathematics is the queen of science, the language of nature, and the bedrock of national development; a subject without which a nation cannot advance scientifically and technologically (Alutu & Eraikhuemen, 2004). Akinoso (2011) also states that it is the foundation for science and technology and a tool for nation building. Mathematics consist of magnitude and numbers that are very useful in all subject areas which include science, engineering, technology and the humanities from which industrial development takes off (Anigbo, 2016).

Consequently, competency in mathematics is very important to every individual and nations in domestic and business pacts, scientific works, technological innovation, problem-solving and decision making in diverse circumstances of life. It may be in consideration of these and other vital usefulness that it is a core and compulsory subject at all levels of education as contained in the nations' educational policies, especially at the basic and secondary levels of education. Some programmes at the universities also offer some courses in mathematics that play critical roles in the intellectual and social development of the students. It is one subject that is mostly feared among learners in schools (Ashcraft & Faust, 1994; Akinoso, 2011). And the factors that contribute to this nature of the subject may include teachers' attitude, teachers' content knowledge and pedagogical skills, students' negative perception due to lack of interest, government's inability to create the necessary learning environment and so on. In addition, teachers' inability to teach the subject competently and in an interesting way to prepare the student for the task ahead cannot be ruled out. This lack of students' interests which is mostly generated by the way teachers teach the subject creates the tendency for students to respond to mathematical concepts with little or no self-confidence, negative feelings and worry. Literature has shown that these underachievement and poor performances in mathematics are determined by the way the subject is taught. However, there is an indication that poor performance cannot be exclusively attributed to the culture of teaching and learning but also includes the unavailability of resources, students' socio-economic background and their superficial notion about the subject coupled with parental and societal beliefs (Christmas, Kudzai & Josiah, 2013). Rivera-Batiz and Marti (1995) conducted a multiple regression analysis and concluded that high student population in a class also affects mathematics performance negatively.

Students are averse to mathematics because of the normal ways of teaching where teachers insist on using certain rigid skills which discourage students from trying to invent new ways for themselves (Minsky, 2008). In this regard, abstract concepts and principles are often presented first and later illustrated with examples that may be far removed from the students' personal experiences and/or interests. Memorization of facts and algorithmic problem solving skills are stressed, rather than conceptual understanding with its relationship to the things around the learner. The new information which is transferred to the student is assumed to fall into a preexisting framework with all the proper connections automatically displacing any other ideas and the interests the students may already have (Carey, 1986 cited in Vander Kooi, 2006). However, the work of mathematics teachers is to ensure that every student receives the highest quality of instruction to understand mathematical concepts. This is possible when mathematics teachers have other instructional strategies rather than the passive traditional teaching methods which ensure the introduction of mathematical concepts through the use of simple real-life problems, games and plays to motivate the child to learn and understand the subject.

Another widely given explanations to why students do not learn mathematics includes the inadequacy of teachers' mathematics content knowledge and lack of rigorous certification that is required with insufficient pedagogical competencies (Hare, 1999). In addition, students' poor achievement is attributed to several other factors such as inability of not having time to study the subject, fear of figures, poor learning environment, peer-pressure and lack of parental guide. Another factor that hinders the learning of mathematics according to Ismail et al., (2014) is that teachers are always using less challenging problems to prevent students from the possibility of becoming demotivated in learning the subject. The possible explanation for this is that students are unable to cope with high-order thinking tasks as they are already accustomed to being spoon-fed by their teachers for as long as they can remember. Therefore, many students in mathematics classes wait for their teachers to give solution to a problem in an attempt to avoid embarrassment due to failures or wrong answers. These serious issues that need immediate attention and solutions are in the domain of the mathematics teachers. To this end, the American Council of Education [ACE] (1999) believes that a strong foundation in college-level mathematics and professional competence in practice are necessary for good teaching.

The erroneous impression many people have about mathematics is that it is largely connected to intelligence and talent, which is why passing the subject poses challenges to students. These experiences make students to escape from mathematics and give up their intentions and desires because of protracted failures, while others continue to learn the subject because of the interest and joy they have in learning. More importantly, most individuals are being forced to learn the subject because of its requirement in many disciplines. Individuals should rather see mathematics as a path that helps students go a step forward in life to meet their desired goals (Durmaz & Akkus, 2016). In this situation, one is tempted to know how mathematics becomes a source of gratification for some students and a source of fear and worry for others.

Mathematics teaching and learning have been a problem even when Ghana had the best educational achievement in Africa (Ahia & Fredua-Kwateng, 2004). It is a problem that students, parents, teachers, education authorities and governments are continually grappling with, as it is offered almost at all levels and as a requirement for gaining admission into the next level of most educational ladder. However, evidence suggests that few students develop conceptual understanding of the subject (Howie, 2001; Resource National Training, 1989) while many students are unable to use it in situations outside the classroom context (Boaler, 1998). Generally, students see mathematics as an abstract subject and therefore struggle to find its relevance in their lives, hence do not pay attention to lessons (Uyangor, 2012). This assertion is due to the fact that mathematics has often been taught using textbook questions, quizzes and tests, which are not creative and activity-oriented (Lamer & Mergendoller, 2010). Asare and Nti (2004) interacted with mathematics teachers in Ghanaians schools about mathematics instructions and came up with the fact that mathematics is taught without focusing on the students. If this assumption is extended to the colleges of education, one will be wondering what caliber of diploma teachers are produced to teach mathematics in the basic schools. Hence the reason that prompted the researcher to research into the instructional strategies that college tutors use in teaching mathematics. In this connection, the Ontario Ministry of Education (2005) states that the focus on the study of mathematics education has shifted from content-knowledge towards process skills and ability to apply same to real world situations.

In 2002, a Presidential Committee on Education recommended a critical review and approach to making teacher education relevant to the development of the child. This statement resonates with the positions of Adegoke (2003) and Benneh (2006) who indicated that the mission of Ghana's teacher education is to provide a comprehensive programme through pre-service and in-service training that would produce competent, committed, and dedicated basic school teachers to improve the quality of teaching and learning of mathematics. However, in Ghana, mathematics teaching is characterized by transmission and command models where students are not encouraged to pose questions or engage in hands-on and problem-solving activities in order to attain both conceptual and procedural understanding of the subject (Fredua-Kwarteng, 2005; Appiah, 2010). Furthermore, most of Ghanaian students' inadequate conceptual understanding of mathematics at the basic level is partly due to how the subject was taught (Baffoe & Mereku, 2010). Consequently, the performance in geometry of Junior High School (JHS) students in Ghana, before entering Senior High School (SHS), is lower than the performance of most students at this age in other countries such as Singapore (Baffoe & Mereku, 2010). Accordingly, Ghanaian students lack appropriate cognitive learning strategies because teachers often use inappropriate instructional strategies in the classrooms hence giving rise to the learning problems. Based on research findings, it was indicated that the teaching and learning of mathematics at the basic level should involve more hands-on activities to engage the students (Baffoe & Mereku, 2010) because the rigid school curriculum for mathematics at the basic level does not afford the learners to apply concepts to everyday life and by extension it inhibits the study of mathematics at the tertiary level especially in the colleges of education (Minsky, 2008; Ali, 2019). However, the Ghanaian mathematics curriculums for the Junior and Senior High Schools encourage teachers to emphasize constructivism model in their lessons.

In a period when quality education is a concern for educational institutions that dominate national and international discourse, teacher education with quality is a priority with the responsibility that teacher performance is of highest interest towards achieving excellent educational agenda (Asare & Nti, 2014). Therefore, the need for teachers' understanding of the subject-matter and their pedagogical orientations and decisions to enable them ask pertinent questions, select appropriate tasks, effectively evaluate learners' understanding and make relevant curricular choices is very critical (McDiarmid, Ball & Anderson, 1989). Hence, understanding mathematics has to do with attitude and the kind of motivation received from teachers, parents and other individuals and the teachers' ability to adopt the appropriate instructional strategies to ensure that the learners understand mathematical concepts. Also, the problem of students' negative perception about mathematics and their conclusion that the subject is difficult can be resolved when teachers encourage them through lesson activities. However, since the teacher on the other hand may also suffer in the hands of incompetent teachers whilst in school, they may not have the pedagogical skills to transform the learners to become good mathematics learners. Even though higher qualification in mathematics and effective pedagogical skills acquired by the teacher may ensure good mathematical instruction leading to students' high achievement in the subject, poor attitudes from the teacher and the students render mathematics teaching and learning meaningless. Meanwhile, the mathematical foundation of every educational structure stems from the basic schools whose teachers are mostly from the colleges of education. The implication therefore, is that if the content knowledge and pedagogical skills of the college of education diplomats are not strong then there will be numerous problems associated with the pupils' mathematics performance.

Following this, a section of the Ghana-Vision 2020 policy document (NDPC, 1996) and the Sustainable Development Goal 4 (SDG4) document (UNGA, 2015) are designed to upgrade the quality of teacher-trainees and to ensure equitable quality education to promote life-long learning opportunities for all in a bid to substantially increase of the supply of quality teachers by 2030. Furthermore, the Teacher Training Colleges are upgraded to the Colleges of Education (tertiary) to improve the quality of teachers (Colleges of Education Act, 2012, Act 847) (T-TEL, 2016). It is therefore the desire of every nation including Ghana to produce quality teachers who will lay good educational foundation for nation building. Accordingly, the teacher is to adopt and expose students to the modern and innovative techniques of teaching and learning that promote critical thinking and problem solving. The college tutors are to therefore view teacher-trainees as active constructors of knowledge who are able to create learning contexts that are learner-centered through collaboration with their peers. This strategy enhances the teacher-trainees' mathematical competency in the use of several pedagogical strategies such as project-based, inquiry-based, and problem-based techniques which are embedded in constructivism to meet the diverse needs of the students.

To build an industrial economy there is the need for a strong mathematics culture (Akinoso, 2011). However, the false presentation of mathematics concepts were internalized and passed down the generation by teachers, school administrators, parents and community members making the offering of STEM programmes at the tertiary institutions a mirage. Therefore, the government of Ghana in 2010 saw STEM programmes as tools for economic development and hence formulated a strategic plan for the composition of student numbers in tertiary institutions. The 2010-2020 Education Strategic Plan (ESP) of the Ministry of Education targets 60% of students in public tertiary institutions to enroll in science, technology, engineering and mathematics (STEM) disciplines and 40% in the humanities (ESP Report, 2010). However, retrospective available statistics indicate that during the 2007/08 academic year, the enrolment ratio stood at 38% for STEM disciplines and 62% for humanities in public universities and 32% and 68% for STEM and humanities disciplines respectively in the technical universities and polytechnics (ESP Report, 2010). The 2012/2013 academic year showcased that tertiary students pursuing programmes in the Sciences, constitute 36% (Mathematical Sciences– 8%; Natural Sciences– 6%; Applied Sciences-22%) and 64% for the Arts and Social Sciences (NAB, Tertiary Education Statistics Report, 2015). Furthermore, the 2018-2030 Strategic Plan of the Ministry of Education again put the ratio of Science to Humanities enrolment at 40:60 as at 2019 in the tertiary institutions as against the policy of 60:40 in favour of the sciences. At Ho Technical University, over the past six years ending 2017/2018 academic year, students' enrolment stood at 61.2% for the humanities and 38.8% for the engineering, mathematics and statistics and applied sciences. With respect to data collected from the field, 27% of the teacher-trainees offered science programmes whilst 73% offered the humanities at the colleges of education as at 2018/2019 academic year. It is common to find in literature that STEM education and learning opportunities are enhanced by strong mathematics background (Alfieri, Higashi, Shoop, & Schunn, 2015; Hefty, 2015; Magiera, 2013; Smith et al., 2013). It is therefore a fact that for candidates to enroll in STEM programmes in tertiary institutions, one needs to have a credit pass in mathematics at the junior and senior high school levels (Boaler, 2008; Connes, 2005). This is not achievable as students have difficulties in understanding mathematics concept. Consequently, the low intake in STEM programmes in the universities, polytechnics and colleges of education is mostly due to poor performance in mathematics and sciences at the basic and senior secondary schools levels (Shearman, 2012). Accordingly, Ghana Statistical Service [GSS] (2013) stated that technological and engineering industries and organizations are very low in the country. It is in this direction that the National Science, Technology and Innovation Policy, 2017 – 2020, (2017) of the Ministry of Environment, Science, Technology and Innovation (MESTI) stated that the strength of the nation's Science, Technology and Innovation (STI) hinges on the quality of training in mathematics and science given to learners in first and second cycle schools. MESTI for this reason indicated that the production of critical mass of young students will now be prepared for STEM programmes for the universities and Colleges of Education. This is one of the reasons that prompted the researcher to conduct the study because for any successful economy, particularly in today's quest for knowledge-based economies, science, technology and engineering are basic requisites (Ghanaian Times, March 22, 2016). The quality of education and training for students offering STEM programmes in tertiary institutions determine the quality of trained and skilled personnel that will be needed to build the nation's STI capacity. Countries that are making substantial progress in socio-economic development invest a lot in reducing illiteracy and improving access to higher and further education especially in STEM programmes (Dzidonu, 2003). To corroborate this statement, President Obama, when addressing the “STEM Challenge” forum in the USA, noted that “strengthening STEM education is vital to preparing students to compete in the 21st century economies and therefore called for the need to recruit and train mathematics and science teachers to support the nation’s students” (White House Press Release, September 27, 2010). President Obama went on to say that without high-quality, knowledge-intensive jobs and innovative enterprises that lead to the discovery of new technology, economies will suffer and citizens will face a lower standard of living. The demand for scientists and engineers are rising, yet countries are faced with discouraging statistics in the number of students pursuing STEM programmes, because of the disappointing mathematics and science scores that place students’ performance below many industrialized nations (Mohammed, 2015). These statements are very illuminating, because they capture the fundamental point that a strong mathematics interest is a prerequisite to building an industrial culture in any nation (Ahia & Fredua-Kwateng, 2004).

Notwithstanding the need for quality teachers, research evidence in Ghana suggests that new trained teachers are ill-prepared to handle the new direction of the curriculum that was put in place as part of the 1974 Educational Reforms in the Primary and Junior High Schools (MOE, 1996; Pecku, 1998). The educational reforms consequently reduced the number of schooling years from 17 to 12 years from primary to secondary levels. The average age at which the majority of students wrote their matriculation examinations has also reduced from 23 to 17 years. This trend has greatly contributed to the decline of students’ academic performance especially in mathematics as only few of them are capable of understanding the scope as well as the complexity of the content prescribed by the syllabuses (Mereku, 1999). Teacher education was therefore accused of failing to prepare teacher-trainees for the reality of the teaching profession. From the accumulated experience and observations, and in addition to the above assertion, Ahia and Fredua-Kwateng (2004) trace the problem of mathematics education, to include but not limited to, four major causes viz, historic, culture of mathematics teaching and learning and language of instruction.

Historic

The overall aversion to mathematics learning that permeates educational institutions is due to the negative seed sowed over the years by our colonial masters, such that Ghanaian policymakers could not implement any policy to change the pandemic (Mereku, 1999). The religious factors introduced by colonial masters have increased the dislike for mathematics teaching and learning leading to teachers having negative beliefs, inadequate content knowledge, incompetent teaching skills and poor ability to make decisions when teaching other subjects. Our colonial masters who established formal education in Ghana believed that Africans were not capable of understanding mathematics and science because the subject requires abstract thinking abilities which Africans do not possess. The colonial masters therefore did not lay down any strong mathematics culture in Ghanaian schools because they thought that Africans were not capable of establishing technological industries. These European merchants rather trained Africans to become teachers with strong emphasis in Latin, History, Geography, English and Christianity (Ahia & Fredua-Kwateng, 2004). The notion here is that the Ghanaian trained-teachers helped the European merchants with language translation to facilitate their businesses (Antwi, 1992; McWilliam & Kwamena-Poh, 1975). This unfortunate situation has made educational stakeholders not to lay emphasis on mathematics education and hence the beginnings of the woes of mathematics underachievement such that people in authorities begin to think that mathematical knowledge is generic and not a learnable school subject. However, everyone is capable of learning mathematics to whichever level he/she deems fit if only he/she receives the needed support (Piaget, 1968).

At an International Mathematics Seminar in Kuwait in 1986, it was pointed out that the 'western' curriculum, which was designed in a particular historical and cultural context for a few, has not only been forced upon all in recent years but also exported to countries across the world including Ghana, Nigeria and Sierra-Leone (Howson & Wilson, 1986). In addition, there was the emergence of two curriculum projects involved in carrying out mathematics innovation in Ghana in 1961 (Hawes, 1979), namely African Mathematics Programme (AMP), designed for basic schools and later changed to Ghana Mathematics Series (GMS) and Joint School Project (JSP) for senior high school (Mereku, 1999). The two series lacked continuity because the textbooks were developed under different philosophies, contexts and style of presentation of content materials. These observations suggest that the school mathematics curriculum unintentionally acquired a universal status which unfortunately led many countries across the world to view the subject as formal (Mereku, 1999). Meanwhile, countries such as USA, Germany, Sweden, Canada, Finland, Japan, Singapore, Hong Kong and South Korea have long overhauled their mathematics curriculum through best practices to localize its contents and delivery

(Sakyi, 2014). It was therefore not surprising that most Ghanaian students experienced difficulty in learning the subject.

Culture of Mathematics Teaching

The culture of teaching contributes to students' aversion to mathematics (Saritas & Akdemir, 2009). Thus, if students are not learning mathematics as effectively as teachers and policymakers want, then one may say that teachers' content knowledge, instructional skills, beliefs, decisions and actions that are brought to bear on the learning of the subject are not positive. Following from this analogy, weak mathematics teachers transmit mathematical difficulties to their students and vice versa. These put many students off and make them math-phobic, hence these students would not want to pursue the subject anymore, leading to the shortage of mathematics teachers in our schools and consequently allowing unqualified and incompetent mathematics teachers to teach the subject.

Akyeampong (2003) cited in Asare and Nti (2014) reflected on a number of approaches used in teaching mathematics courses in the colleges of education. These include

- teacher-centered - where tutors lecture their supposed passive students who they consider as novices.
- student-centered teaching- where students play active roles and are engaged in classroom discussions and debates on relevant issues.
- questions and answers approach- in which case, tutors and students mainly asked questions and students' answers are used to further develop lessons.

- discovery learning process- where students are sent out to explore knowledge by themselves (inductive learning).
- brainstorming method- where students are given the opportunity to think critically over a topic and come out with responses or soutions.
- project-based method- where students are asked to undertake workrelated problems and report to the class through presentations.
- problem-based learning- where students will be given problems to solve and find answers to them.

In addition to the aforementioned, role-plays and demonstration, simulation methods, educational visits and field experiences and deductive and inductive methods, expository teaching process, drills, teacher-led discussions and case studies are also used in the delivery of contents (Ghana Education Service, TED, 2004). All the above strategies except for teacher-centered technique clearly support constructivist theory where students are involved in constructing their own understanding of the subject matter with the teacher's support. However, teacher-centered pedagogy is what is dominant in the colleges of education in which teacher-trainees are regarded as "empty vessels" (tabular rasa) with little or no knowledge or experience in the teaching and learning process (Lewin & Stuart, 2003). Even though, the Revised Mathematics Syllabus (2014) for the three year diploma in basic education for the colleges of education does not contain any specific mode of teaching mathematics, three key features such as establishing judicious balance between theoretical knowledge and teaching skills, training teachers to be facilitators of learning and producing teachers who are creative researchers which point to constructivism were mentioned. However, after a subsequent revision in 2019, constructivism has explicitly featured n the mathematics syllabus of the colleges of education.

A study by ODA/GES (1993) indicated that in the colleges of education, approaches to teaching and learning have been largely teacher-centered, emphasizing lectures, dictation and recall of notes. This method of teaching has become an entrenched culture and change-resistant because new approaches are perceived as more time-consuming. In addition, this approach which favours the examination cultures requires 'chewing and pouring' of textbook knowledge without sufficient demand on critical thinking to acquire applicable skills. Learning was therefore heavily examination-oriented where teacher-trainees were largely the passive recipient of 'content' and 'theory' while methodology and practical teaching strategies were largely ignored (ODA/GES, 1993). In his study Akyeampong (1997), finds the use of learning aids and materials in the colleges of education to be often non-existent. Even though the use of student-centered, interactional approaches was introduced in science, mathematics, English language, technical skills and education, their impact has been very minimal such that many tutors are still not applying the activity-based teaching methodology advocated for teacher education programmes. This is so because the tutors often see these methods as more demanding than the 'chalk and talk' approach with which they are more familiar. Since students pass their examinations via the 'chalk and talk' approach they see little reason to change their teaching methods. This is a typical case of examination requirements to promote the use of a certain kind of instructional approach.

Inadequate textbooks, lack of technology and teachers' negative attitudes, poor motivation, and undesirable beliefs among others are also factors that affect mathematics teaching. Even though there were new approaches to teaching mathematics, developed by psychologists and the introduction of new contents into mathematics syllabus which has very little connection to real life application, the constructivist approaches were not easily adopted (Mereku, 1999). In his curriculum analysis study, Mereku (1995) revealed that though there was information about the introduction of curriculum materials that suggest discovery teaching methods, the teaching activities did not encourage the use of such teaching skills because teachers were not aware of the underlying structures of these approaches. Meeting students' mathematical needs in the next millennium requires that teachers adhere strictly to the teaching standards that the syllabus stipulates. Thus, all students should have the opportunity to study a style of mathematics appropriate to them as groups and as individuals emphasizing what every student is capable of doing at a particular stage so as make him comfortable in mathematics classroom. This will consequently eliminate the norm where every student in a particular mathematics class will be made to learn the same concept even if he/she is not capable of understanding the concept.

Culture of Mathematics Learning

To Anker (2004), learning results from the summed interactions of reflections, documentation and mentoring. Consequently, learners are to remember what is to be learnt by considering previous knowledge, writing down notes and receiving guides from their instructors. Stimulating the interest of mathematics learning in students at all levels is very critical in this current dispensation. While the traditional lecture method of giving exercises and drills help students to memorize facts and formulas in order to solve mathematical problems, it does not help them to learn and attain an in-depth understanding of what is required in the new knowledge settings (Armah, 2017). A cognitive research supports the notion that when one has a deep understanding of a subject matter, the individual is able to transform the factual information into usable knowledge (McTighe & Seif, 2011) hence putting that theoretical concept into practice. In furtherance to this, knowledge transfer occurs when the learner understands underlying concepts and principles that can be applied to problems in new contexts (Armah, 2017). On the other hand, when knowledge is obtained by rote, it is rarely transferred because the acquired knowledge is discrete and fragmented.

Further research suggests that pre-service elementary and secondary school teachers often do not have fundamental understanding of school mathematics (Cooney, Shealy & Avold, 1998; Ma, 1999; Simon, 1993; Akinsola, 2003). In particular, the research states that primary school teachers are incompetent in mathematics contents and pedagogy mainly due to very short pre-service teacher training periods (Mahmood, 2002; Akinsola & Adjiboye, 2009). Teachers' insufficient subject content knowledge is not surprising since these teachers themselves are products of primary and secondary schools, where research has shown that they rarely developed a deep understanding of mathematical content when they were in school (Ball & McDiarmid, 1990; Boaler, 1998). According to Hodgen (2003), teachers' inadequacy in mathematical content knowledge is more than simply ensuring that they acquire satisfactory subject-matter, meanwhile the improvement of instructional strategies which are policymakers' concern is about maximizing student learning (National Commission on Teaching and America's Future [NCTAF], 1996).

Though natural aptitude for learning mathematics helps, it must be put on record that everybody has the capacity to learn the subject to a large extent for intellectual, technological, vocational and life purposes. This is possible when students retain what they are taught and assessed same after every lesson through class exercises, tests, quizzes or examinations and are finally able to apply the concepts learnt to everyday life. It is therefore important to ensure that students understand the objects of mathematics that is presented to them during lessons, bringing to mind how the subject is perceived by both the students and the teacher. Students' perception about and interest in mathematics and the frequency of mathematics examinations taken coupled with teachers' interest in the students are related to mathematics achievement (Guvendir, 2016). The indication therefore is that students' philosophical thought about the subject is a factor in understanding mathematical concepts as may be presented by the teacher. In effect, students who enjoy learning mathematics are very fortunate as the subject often shapes lives. For this reason, mathematics is a powerful, important and useful tool in all manner of jobs and in everyday life.

Language of mathematics instruction

According to the National Syllabus for Mathematics of the Ministry of Education (2012), mathematics is a logical, reliable and growing body of concepts which makes use of specific language and skills to model, analyze and interpret the world. It provides a means of communication that is powerful, concise and precise. Mathematics terminology and vocabulary and the level of proficiency of English among teachers and pupils are pre-requisites in learning mathematics. The proficiency in English will enable learners to discover, adapt, modify and innovate to communicate ideas to resolve challenges. Critical reflection may be developed to enable learners to think outside the box as they share these ideas through the use of their own words to explain ideas and to record their thoughts through symbols, diagrams and models. The official medium of instruction in the lower primary is the children's first language (GES, 2012). However, mathematics teachers are encouraged at this level to sometimes combine the Ghanaian language with English language when teaching the subject because there are no readily available vocabularies in Ghanaian languages for some mathematical terms, diagrams and symbols. Unless they have the vocabulary to talk about division, perimeter, capacity, etc, they cannot make progress in understanding the various areas of mathematical knowledge. To this end, teachers of mathematics must ensure that children understand mathematical vocabulary through any means including asking and answering questions during lessons, carrying out mathematics tasks through cycles of oral work of reading and writing. In this regard, it is important to ask questions in variety of ways so that children who do not understand a concept for the first time may subsequently pick up the meaning at a later stage. Accordingly, one should not use only questions that require recall of facts but questions which require a higher level of thinking to promote good dialogues and interactions which will eventually make learners to begin to develop complex answers in explaining their thinking in their own words. An important reason is that mathematical language is crucial to children's development of thinking.

When teaching mathematics, teachers must be aware of the fact that their construction of knowledge through the words they speak is likely to be different from the pupils' understanding. This is because in teaching, the words used are those of the teachers' with meaning from the teachers, and pupils on hearing the teachers' words interpret them according to their own understanding, thereby creating a misconception in the class. In addition to the words spoken by the teacher, pupils try to interpret gestures, facial expressions, voice pitches, and hand movements and so on to understand concepts. When pupils are not able to interpret all these to the satisfaction of the teacher, communication breaks down where the teaching and learning process is halted. Through the use of language and social interaction, individual knowledge can be challenged and new knowledge constructed. It is therefore crucial for teachers to realize how mathematics learning is linked to language, social interaction and cultural context (Cakir, 2008). In his own experience, mathematics has been taught using language as if the language itself bore little relation to the acquisition of mathematical concepts. There is therefore a substantial literature that addresses the ways in which language and social structures impinge on the learning and teaching of mathematics (Austin & Howson, 1979; Orton, 1987; Pimm, 1987).

Although, the major purpose of teaching is to provide an opportunity for the learner to construct knowledge, it is still not too clear how teachers use language to facilitate knowledge construction (Cakir, 2008). These connections establish the importance of language to drive a conceptual change. Accordingly, Vygotsky (1978) states that learners develop mathematical understanding when they communicate and express mathematical ideas through languages (Uznadze, 1986), thereby regarding language as a mediation tool to help learners to enhance performance. The work of Vygotsky (1978) gained increased recognition in mathematics education because the development of a child's intelligence results from social interactions, co-operative activity and communication (Sutherland, 1993). The idea behind effective learning is therefore seen between or among people with different levels of mathematical knowledge and understanding through the use of language (O'Neil, 2011). In this connection, Ismaila et al., (2014) mentioned language and communication as one of the five main areas that constitute effective mathematics teaching. If cognitive structures are innate and merely fixed, are teachers using language to activate these innate cognitive structures (Chomsky, 1975)? Mathematics is a logical, reliable and growing body of concepts, which makes use of language to model, analyze and interpret the world (National Syllabus for Mathematics for JHS, 2012).

1.1.1 Performance of students in Mathematics

Knowledge in mathematics, especially at basic and secondary school levels is seen as an issue in education, particularly within the mathematics community (Ball, 1990) by policymakers, mathematics educators and students (Alexander, Rose, & Woodhead, 1992). In Ghana, poor performance in mathematics at the basic and secondary levels has attracted a lot of attention from the government, mathematics educators, educational researchers, curriculum designers, parents, and employers and the call for immediate solutions. This is evident of how teaching and learning of the subject takes place at all the educational levels including tertiary institutions especially in the colleges of education whose graduates teach at the basic level (Fredua-Kwarteng, 2005; Appiah, 2010).

Rote-learning and exam-centered assessment strategies which produce mechanical graduates who cannot solve real-life problems or apply acquired knowledge and skills to creative activities has become the order of classroom teaching and learning. These attitudes do not empower students to reflect deeply on mathematical meaning to the environment and world of work because mathematics concepts are learnt abstractly (Sakyi, 2014). The learning of complex formulae in mathematics put many learners off as they do not see the immediate applications of these concepts in the real world. Mathematics achievement of students in the United States of America (USA), when compared with the performance of students in other high achieving countries, (e.g. Singapore, Japan, Germany) leads one to deduce that there is the need to improve mathematics education at all levels (Ball, 2003).

The 2007 TIMSS reported that US fourth-grade students' average mathematics score was lower than eight Asian and European countries that are considered high achieving countries. Additionally, TIMSS has shown that in the USA students spend a large amount of time during mathematics instruction by reviewing the materials they already learned, and focus mostly on practice and procedures rather than developing a conceptual understanding in mathematical lessons (Stigler & Hiebert, 2009). When videos of teachers' instruction from TIMSS were analyzed, the USA's motto for mathematics instruction was classified as "learning terms and practicing procedures", whereas Germany's motto was classified as "developing advanced procedures", and Japan's motto was classified as "structured problem solving" (Stigler & Hiebert, 2009). So, it was common for students to share multiple solution strategies in a typical Japanese classroom since high achieving countries frequently used problem solving approach with an emphasis on conceptual understanding in mathematics lessons. Therefore, the results of TIMSS have revealed the need to improve school mathematics in the USA (Hiebert et al., 2003).

The importance of mathematics and science in today's technological society provides a context for comparison (Anamuah-Mensah & Mereku, 2005). TIMSS is a series of studies undertaken once every four years by the International Association for the Evaluation of Educational Achievement (IEA) to examine students' achievement in science and mathematics (Anamuah-Mensah & Mereku, 2005). TIMSS uses the international average and international benchmarks in reporting students' achievement in science and mathematics to describe achievement in a test. TIMSS assessment is organized around content and cognitive dimensions. The content domain includes number, algebra, measurement, geometry and data whilst the cognitive domains involve understanding of facts and procedures that enable them to solve non-routine problems (Anamuah-Mensah & Mereku, 2005). The benchmarks which represent the range of performance shown by students internationally are: a) Advanced international benchmark– 625 points; b) high international benchmark – 550 points; c) intermediate international benchmark 475 points; and d) low international benchmark 400 points (Anamuah-Mensah & Mereku, 2005; Appiah, 2010).

Ghana participated in TIMSS in 2003 and 2007 to compare its educational potential with other countries that participated (Appiah, 2010). The analyses indicate that the performance of Ghanaians JHS 2 pupils in mathematics was among the lowest in Africa and the World. In 2003, out of 46 countries that participated, Ghana took the 44th position with a mean score of 276 points far below the international mean score of 467 points and international benchmark of 400 points (AnamuahMensah & Mereku, 2005). In 2007, Ghana's mean score was 316 points, far below the international mean score of 500 points and also below the lowest international benchmark score of 400 points (Appiah, 2010). In the foregoing, Ghana can learn from Singapore which performed very well in 2003 and 2007 TIMSS. The reason is that that Singaporeans National Curriculum emphasizes problem solving skills which are being carried out practically through meaningful communication or oral work coupled with group discussions, presentations, and investigative works for mathematical thinking (San, 2000).

Despite serious attention paid to the study of the subject by all stakeholders in Ghana and elsewhere, students still do not perform well in mathematics examinations, rendering some of them not able to proceed to the next level of education (Sarfo, Eshun, Elen & Adentwi, 2014). Performance in mathematics by students of basic and secondary schools has become a continuous worry to parents, educators and governments in recent times as this fact came out vividly when the average pass rate of grades 1-6 for WASSCE and BECE candidates in mathematics for the past five years ending 2017 is 21.6% and 61.0% respectively in the Volta Region. These figures are lower than the national averages of 72.6% and 30.1% for

WASSCE and BECE respectively (WAEC, 2019). Consequently, one may enquire about the teacher-trainees' mathematical content-knowledge before entering the colleges of education. The WASSCE results in mathematics of the participants for which they were admitted into the colleges of education indicated that on the average 50% of the of the students obtained grade D, 36% obtained grade C, 9% obtained grade B whilst 5% obtained grade A. In addition, the performance in mathematics course code FDC 122 by participants in this study indicate that 3.9% had grade A, 18.6% had grade B, 37.1% had grade C, 29.8% had grade D and 10.7% had grade E (UCC, 2019). Majority of these teacher-trainees will be responsible for the teaching of mathematics at the basic level, a source of worry for effective mathematical conceptualization by basic pupils. Contextual factors such as ineffective teaching might account for this poor performance (Asabere-Ameyaw & Mereku, 2009).

The Chief Examiners of WAEC report that teachers must explain basic mathematical concepts to students with practice and drills through exercises and assignments, guide students to learn using relevant previous knowledge, use concrete materials in every lesson as they make mathematics lessons very practical and relating same to real life and to provide feedback on their performances. Candidates were also required not to resort to the use of formula they did not understand but learn how to use mathematics formulae from basic or first principles (WAEC, 2018). In furtherance to this assertion, mathematics curricula are designed with the recognition that the subject is not only a collection of concepts and skills to be mastered but involves processes that help individuals to develop the ability to explore, conjecture, solve problems and reason logically whilst students construct their own understanding of mathematics concepts (Ministry of Education, 2007). The National Core Mathematics Standard has therefore been designed to cater for students to take responsibility of their own learning with activity-oriented classrooms which bring creativity and application through students who works co-operatively in groups to develop problem-solving strategies.

1.1.2 T-TEL's Interventions to Teacher Education

To deliver quality education in Ghana's schools, teachers need to be equipped with modern and new professional skills to enable them enter classrooms with confidence. Today's schoolchildren have a range of needs and teachers must employ different approaches to ensure that children learn well and be prepared for the modern-day world. Tutors at the Colleges of Education are therefore the pivot around which these children will change the dynamics of education in the modern world (TTEL, 2016). Consequently, tutors of the Colleges of Education have received training under the Transforming Teacher Education and Learning (T-TEL), a Government of Ghana programme supported by the British government's UK aid, and designed to improve the quality of teaching and learning by equipping the next generation of teachers with the skills they need to prepare pupils to succeed at school and in the modern world. Various workshops were therefore organized by T-TEL in all 45 public colleges of education in Ghana for four consecutive years. T-TEL's work with the college tutors have impacted teacher education beyond the classroom, by transforming the way college principals, administrators, tutors and teacher-trainees approach their professional academic activities. Professional Development Coordinators (PDCs) whose roles supported the facilitation of the weekly professional development sessions in the colleges using T-TEL materials were appointed. Receiving regular training from T-TEL as trainers, PDCs ensured that their tutor colleagues made the most out of the professional development training, in order to improve their lessons with their teacher-trainees.

Before the start of the T-TEL programme, PDCs and tutors have seen that pupils' low performance or failures at the basic education level were attributed to two reasons; inadequate teacher preparation and training and ineffective classroom lessons delivery (T-TEL, 2016). According to them, they have little control over the teaching strategies because they merely implemented the Ministry of Education policy and curriculum instead of playing a role in shaping the curriculum. As a result, the tutors have considered it the government's responsibility to produce high performing teachers through quality training in effective pedagogy for basic education. Another consequence of this failure according to the tutors is that most of them have stuck to the traditional methods of lecturing, as teacher-trainees are also not learning effectively. Some tutors also believed that just completing the college syllabuses and preparing students for exams and not concentrating on the teaching methodologies to challenge teacher-trainees beyond traditional approaches to learning modern methods of teaching is enough in teacher education. Having realized the inability of the college tutors and teacher-trainees to acquire this pedagogical knowledge and skills, the Government of Ghana through T-TEL offered professional development sessions with the college tutors to help inject new set of approaches into their teaching, especially when supervising teacher-trainees' on- and off- campus teaching practices. T-TEL has consequently, equipped college tutors with a number of teaching strategies using games, role-plays, storytelling, pair work, small group activities, 'talk for learning' and 'Think-Pair-Share' to promote quality learning among the teacher-trainees. Through these workshops, tutors increased their teaching skills and learnt modern methodologies, such as questioning techniques and group work which drastically increased teacher-trainees' participation in class, strengthened their communication and information gathering skills, and created a more positive and inspiring learning environment in the classrooms. These also helped teacher-trainees to learn effectively to re-shape their attitudes as well as deepen their experiences of teaching practices. The use of role-play as a creative approach promotes learning among teacher-trainees whilst the 'talk for learning' teaching strategy completely changed the understanding of tutors about teacher-trainees because they are now given the opportunity to talk, initiate and express themselves much better than before. The 'Think-Pair-Share' method created a much better teacher-trainees' response, thereby allowing a teacher-trainee in a group to first think about a problem before pairing up with colleagues to discuss the solution and finally sharing same with the whole class. This increases teacher-trainees' analytical skills, confidence, speaking proficiencies and communication skills hence improving learning dramatically. There is now attitudinal change amongst teacher-trainees as they willingly answer and ask questions and contribute to class and group discussion without fear of being teased or intimidated by their classmates during the learning process. These interventions also allow teacher-trainees to participate in discussions during lessons, as they now confidently and productively give response to issues such as lesson planning and presentations and assessment of learning outcomes. T-TEL's approaches in bringing clarity to instructional strategies that were earlier on regarded as wastes of time are now impacting on the teaching and learning approaches as teaching has become more practical and teacher-trainees' interests in the subjects are increasing dramatically. Consequently, college tutors prepare teacher-trainees for improved classroom teaching and good interaction and participation of pupils in lessons.

Indeed, a recent T-TEL survey shows that tutors' use of student-focused teaching methods in college classrooms have increased by 60% among males and by 90% among females. In addition, the training helped tutors to improve teaching practice supervision strategies which encompass interactions with teacher-trainees to increase their levels of learning and teaching during their studies at the College (TTEL, 2016). For instance, teacher-trainees now keep a Teaching Practice Journal, which allow better analysis of experiences and mentorship progress by reviewing, reflecting and improving on lesson delivery. Tutors' training on effective use of questioning helped them to address teacher-trainees' learning needs when teaching courses in pedagogy. Teacher-trainees can now distinguish between good and bad teaching skills and teach confidently because they have established good relationships with and respect the views of their pupils during teaching practice sessions. From the experiences acquired over the four years, some tutors made the following comments:

Tutor's Comment 1

One key challenge to teacher education is the inability for teacher-trainees to receive quality teaching practice skills before graduating from college, leaving them ill-prepared for the demands of a full and busy classroom. Tutors' general approach towards teacher-trainees' understanding of concept was to point out their mistakes and intimidate them into working hard. Assessments of teacher-trainees' strengths and weaknesses are focused mainly on content knowledge rather than on how the teacher-trainees will use their content knowledge to guide and facilitate pupils' learning at the basic level. T-TEL therefore created learning environments for improving teaching practices as we met every week to share concrete and useful ideas and skills across the college curriculums.

Tutor's comment 2

Prior to T-TEL's interventions, tutors did not have positive or productive relationships with the teacher-trainees. The teacher-trainees now share their learning experiences and views about lessons freely while respecting the views of others. When I look back to my teaching days, I can see that my teaching skills and knowledge in managerial and communication skills have dramatically improved. The same is true for my colleagues.

Tutor's Comment 3

Before the introduction of this professional development training, I used the lecture method during lesson delivery, where I taught abstractly without using questioning, games and storytelling to demonstrate concepts to my students. My lecturing approaches reduced by 30% as I now talk less and allow teacher-trainees to talk, participate actively in class, and work together while I facilitate the lessons. I used to see group work as a waste of time in class, and did most of the talking and writing on the white board as my students passively copied notes from the board. To a large extent, T-TEL broadened my horizons, giving me the opportunity to interact with international education specialists for example. I am excited by my progress, and more committed than ever to sharing my new teaching and learning knowledge and experiences with all teacher educators in Ghana.

Tutor's Comment 4

The attitudes of my colleagues, teacher-trainees and mine have changed regarding teaching and learning as our knowledge in pedagogy and communication skills has deepened. My students have embraced interactive and participatory learning as they see me display these teaching qualities and have encouraged them to also adopt these teaching strategies during their teaching practices. Before these methods were introduced, just like me, teacher-trainees were not using teaching and learning materials in their lessons. About 80% of teacher-trainees are now engaged in quality teaching due to change in attitudes because of group work and use of teaching materials. I also observed that through this training, 70% of my colleagues were motivated and therefore challenged teacher-trainees to improve their teaching skills through the use of creative approaches including group work.

Tutor's Comment 5

I am now very confident about my lesson delivery as I have mastered lots of teaching strategies that made teacher-trainees understand difficult lessons through the adoption of creative approaches which include questioning and 'talk for learning' which influenced the way teacher-trainees teach during their on- and off-campus teaching practices. This gives me confidence that they will make successful teachers. As I look forward to receiving more training from T-TEL, I am confident that my professional knowledge and skills in pedagogy and the overall quality of teaching and learning will continue to improve. Mentors of teacher-trainees have provided positive feedback about the teacher-trainees, indicating a great improvement in classroom interaction with and lesson participation by pupils.

Tutor's Comment 6

I have improved my professional knowledge and skills as a tutor, especially in methodology, which my students desperately need. For instance, I spend about 10% more on lesson preparation before each class. I am also thrilled to see that my students have improved on interaction and participation in class by 15% over the previous year. I hope to sustain these professional gains through continuous practice with my teacher-trainees.

Tutor's Comment 7

The teaching strategies of T-TEL have really helped to introducing group work to address large class size problems. Teacher-trainees who would not normally contribute to class discussions now have the opportunity to share their views in smaller groups which are interactive and participatory. Due to all these, my students have noticed the difference in my teaching and now attend classes regularly and on time.

1.2 Problem Statement

Less than 30% of WASSCE candidates got above 50% mark in elective mathematics (Survey, 2018). In addition, the national pass rate of core mathematics between 2013 and 2017 at the WASSCE level indicates that 30.1% of the candidates passed as against 21.6% in the Volta Region and at the BECE, 72.6% of the candidates passed as against 61.0% in the Volta Region (WAEC, 2019).

Furthermore, Ghana's participation in TIMSS in 2003 and 2007 indicate a poor performance of JHS 2 pupils in mathematics as one of the lowest in Africa. Finally, the results of candidates admitted into colleges of education in 2017, had breakdown of mathematics performance at WASSCE as 50% had grade D, 36% had grade C, 9% had grade B and 5% had grade A (CoE, 2019). Meanwhile, the mathematics syllabuses of basic and secondary schools indicate constructivist theory of learning with the concept of scaffolding, inclusion and differentiated teaching models (Ministry of Education, 2007 & 2019). In addition, the teaching strategies of mathematics in the colleges of education give priority to student-centered, problem solving, decision making, critical and reflective thinking and mentoring approaches as well as emphasis on practical and tutorial sessions (Colleges of Education Mathematics Curriculum, 2019) which explicitly relates to constructivism. Now, with all these instructional strategies why are pupils at the basic schools and students at the secondary schools performing poorly in mathematics?

Constructivism emphasizes that knowledge is a product of one's cognitive act by building on previous knowledge that supports effective teaching and allows one to move to new knowledge (Lerman, 1996). At a training session for teachers in the Volta region, where the researcher was the resource person, it was revealed that teachers do not have sufficient knowledge about the concept of constructivism when a survey was conducted at one of the training sessions. The survey gathered that out of a total of 138 teachers who responded to the questionnaire, only 35 respondents stated that they understand the term, constructivism. However, 75.3% of the 35 participants could not explain constructivism. The meaning therefore is that most teachers, including mathematics teachers at the basic schools have no idea about constructivist theory of learning. So, the poor performance in mathematics is confirmed by Saritas & Akdemir (2009) who state that poor academic achievement rate at all levels is as a result of ineffective instructional strategies and methods, teacher incompetency in education, and lack of motivation and concentration in learning. Besides, mathematics Chief Examiners' reports of WAEC (2014) over the years reiterated the point of students having difficulty in understanding fundamental mathematics and advised that teachers adopt effective instructional strategies in delivering mathematics lessons.

1.3 Purpose of the Study

The purpose of the study was to investigate the instructional strategies that predicts constructivism and used by tutors in the colleges of education to teach mathematics to improve teacher-trainees' performance vis-à-vis their mathematics background.

1.4 Objectives of the Study

In order to address this body of instructional strategies in mathematics, the objectives of this study was to build a model that explained and identified the critical factors affecting teacher-trainees' mathematics performance using smart-PLS version 3.0. The following objectives therefore guided the study:

1. To examine the mathematics background of teacher-trainees in the colleges of education.
2. To determine which instructional strategies college tutors use and mostly use in teaching mathematics.
3. To investigate the effect of teacher quality on the instructional strategies.

4. To examine the relationships between the instructional strategies and constructivism.
5. To assess the contribution of constructivism to teacher-trainees' performance in mathematics.

1.5 Research Questions

The researcher accomplished this study through the following questions:

1. What is the mathematics background of teacher-trainees in the colleges of education?
2. Which instructional strategies do college tutors use and mostly use in teaching mathematics?
3. What is the effect of teacher professional practice on tutors' instructional strategies?
4. What are the relationships between the instructional strategies and constructivism?
5. How do other constructs and constructivism affect teacher-trainees' performance in mathematics?

1.6 Significance of the Study

The result of any good teacher educational system is to produce quality teachers who will inculcate into learners' skills and values, which will enable them to become successful in their lives and be useful to society. For any dynamic society, there is the need for college tutors to develop and implement effective instructional strategies that will greatly respond to these needs. These research findings are to help college tutors redirect their teaching strategies which will enable teacher-trainees to be responsible for their own learning in mathematics as they are consciously exposed to constructivism. The findings in this research are to enable the Colleges of Education design new educational activities for the teaching and learning of mathematics in order to produce competent teachers for the teaching of mathematics at the basic schools. These findings will subsequently support pupils to have interest in learning mathematics that will enable them offer STEM programmes in the tertiary institutions. Consequently, science, engineering and technological drives will be improved for national development.

1.7 Delimitation of the Study

The study covered three colleges of education (Akatsi College of Education, Peki College of Education and St. Francis College of Education) in the Volta Region where participants were mainly third year teacher-trainees who were due to complete in 2019/2020 academic year. The study was directed to the third years because they offered more mathematics courses than the first and second years' teacher-trainees. The Volta Region was selected because the students performed poorly in mathematics at the basic and senior high school levels according to the national league table.

1.8 Organization of the Research

Chapter one itemizes the reasons that inspired the researcher to conduct the research. This included the background to the research, statement of the problem, purpose and objectives of the study, research questions, as well as the significance and limitation of the study. Chapter two discussed the relevant literature that underpinned the research to create new knowledge on the subject of instructional strategies in relations to constructivism that enables students to learn independently using the three learning theories of Piaget (1978), Ausubel (1968) and Vygotsky (1986). The research methodology which is Chapter three is about the population, sampling technique, data collection instruments, and the procedure for collecting the data. The chapter also mentioned the validity and reliability of the instrument used to collect the data. Chapter Four discussed the results from the analysis and the findings of the research where offered inferences were made from the sample to the population. Chapter Five summarized the major findings, conclusions, recommendations and suggested future study.

CHAPTER 2

LITERATURE REVIEW

2.0 Overview

This chapter discusses the literature review which consists of the theoretical and framework of constructivism which is explained by Piaget, Ausubel and Vygotsky theories of learning and operationalized by the conceptual framework of Cognitive Activation (CA), Profound Understanding of Fundamental Mathematics (PUFM), Instructional Coherence (IC) and Cognitive Guided Instruction (CGI) instructional strategies with teacher quality and its determinants. The chapter also discussed the characteristics of Instructional Strategies and mathematics education.

2.1 Theoretical Framework

This study was anchored on constructivist theory of learning which emphasizes that knowledge is a product of one's cognitive act by building on previous knowledge that allows one to move to new knowledge (Lerman, 1996). Constructivism influences academic instruction by encouraging discovery, hands-on, experiential, collaborative, project-based and task learning. It is rooted in cognitive psychology and an approach to education that lays emphasis on the ways knowledge is created while exploring the world. As a theory of learning, constructivism is relevant in this study as the researcher wished to establish how teachers teach mathematics using manipulatives and employing activities to help learners learn that there are many ways to solve mathematical problems through discussion among themselves and working cooperatively to solve mathematics problems. This is because the constructivism philosophical paradigm is an efficient tool that yields many benefits when implemented in carrying out research in diverse fields of study as well as in understanding teaching and learning activities at any educational level (Adom, Yeboah & Ankrah, 2016). Constructivism is an approach to learning and holds the view that learners actively construct their own knowledge which is determined by the experiences they acquired (Elliott et al., 2000) rather than passively imbibing information from their instructors. The students are therefore expected to experience the environment and reflect on them, build their own representations and incorporate new information into their pre-existing knowledge. Constructivism therefore, is a philosophical viewing platform of learning where new knowledge is acquired when learners construct and connect mathematical concepts from their own ideas and understanding through activities in an acceptable environment (Cakir, 2008). One reason for the broad and intuitive process that has enhanced the growth of constructivism as an epistemological commitment and instructional model is due to the fact that it includes aspects of Piagetian, Ausubelian and Vygotskian learning theories and instructional strategies which agreed that knowledge is not acquired automatically but acquired when the learner constructs his own understanding through personal activities with or without the help of a facilitator (Cakir, 2008). These connections establish the importance of prior knowledge or existing cognitive frameworks, as well as the use of relevant information and language to drive this conceptual change.

According to Mattar (2018) constructivism is defined as an educational instruction that comprises numerous and diverse instructional strategies that help students to construct their own understanding of concepts using previous knowledge and/or experience. Obviously, a learner's prior knowledge impacts the process of incorporating new ideas and concept into already developed and existing structures. To this end, Siemens (2004) describes the central principles of constructivism as knowledge that emerges from an individual's learning network when connections are made between and/or among concepts, opinions and perspectives that are retrieved through various means. In constructivism, knowledge acquisition is through a personal construct such that one's inner practicality is dependent on personal experiences and based on prior knowledge which comes through interactions with people, personal ideas, and adaptations to differences perceived in the environment. It is in response to these difficulties and others that constructivism is seen as the way in which a learner will come to know it rather than the teacher telling him what he needs to know. This knowledge that is coming from the learner is the principle of constructivism, which has a conceptual meaning and associated with problem-based learning, inquiry-based learning, competency-based training, learner-focused learning technique, andragogy, and project-based education. By this, students learn through challenges to construct their own ideas because they remember what is being taught through their active involvement in the learning process leading them to connect mathematical concepts. Thus, students are capable of inventing their own concepts and ideas and linking same to what they already know. This personal "meaning- making" theory of learning is called constructivism. In other words, when learners do not make their own mental constructs, they consequently make conceptual errors or have misconceptions or are not able to interpret the material being studied. However, if learners are aware of what they do and can express themselves in connection with their experiences, and that of others then understanding take place, which is lasting and easy to remember. To succeed in this personal concept construction, the learners need the guidance of competent teachers or knowledgeable peers in order to avoid building misconceptions and inadequate mathematical concepts if they know the "how and why" of the procedure (Ma, 1999). The theory of constructivist requires a move from a purely individual knowledge construction to one in which the social processes of discussion and negotiation have a significant role (Pimm, 1987; Austin & Howson, 1979; Orton, 1987). This brings about a situation in which a person speaking might ensure that all information needed for understanding a concept is provided. Therefore, teachers must be aware that when teaching mathematics, the construction of knowledge is likely to be different from each learner in the class because of language. This is so because, in teaching, the words used in the lesson are those of the teachers' with meanings from the teacher. The learners in hearing the teacher's words interpret them according to their individual understanding. It is therefore crucial for teachers to realize how mathematics learning is linked to language, social interaction and cultural context.

The theory of constructivism is generally believed to have contributed to the teaching and learning of mathematics to the effect that most of the mathematics curricula in the USA and the United Kingdom are based on the principles of constructivism (Jaworski, 1991, 1994). These reforms are aimed at preparing young students in the current generation to face the globalized economy in a knowledge-based society with an information-rich curriculum to understanding mathematical concepts (Wong, Han & Lee, 2004). In all these, policymakers, mathematics educators and researchers have concluded that mathematics curriculum reforms are centered on the aims, pedagogy and assessment strategies, based on theoretical principles, such as constructivism (Eggleton, 1995; Frykholm, 1995; Gregg, 1995; Knapp, & Peterson, 1995; Watson, 1995 Fan, 2003; Zheng, 2004, Chen, 2010).

Consequently, the underlying philosophy of reform-oriented mathematics curricula has undergone a substantial shift from absolutist to fallibilist view, from teacher-centered to learner-focused view and from behaviourist to social constructivist view (Davenport, 2000; Gregg, 1995; Chen & Leung, 2013; HerbelEisenmann, Lubienski & Id-Deen, 2006; Smith, 1996) and are in line with international trend of student-centered, problem-based, enquiry-based, activity-based, project-based and competency-based learning (Chen, 2010; Fan, 2003; Xie, 2007; Zheng, 2004). Therefore, these features of constructivism espoused by Ernest (1989) bring to the fore the link between constructivism and instructional strategies that are rooted in the theories of Jean Piaget (1978), David Ausbel (1986) and Lev Vygotsky (1968). The transition from conventional learning theories to constructivism can be associated with the move from the works of Piaget (Piaget & Inheler, 1969) to the works of Vygotsky (1962). In this connection, learning theories of mathematics describe the learning process that provide important frameworks for instructional design for educators to create learning environments that empower learners to get the most from instructional experiences (Grassian & Kaplowitz, 2009). So, mathematics teaching which involves the consideration of several learning theories such as Piaget (1978), Ausubel (1968) and Vygotsky (1986) make it possible to construct an educational model (Steiner, 1990) which has stable foundations that can be implemented flexibly (Godino, 1991). Dewey (1916) also suggested that reconstruction or reorganization of one's experience increases the ability to understand mathematics concepts and unearth subsequent experiences using constructivism. It is in this regard that Furingghetti, Matos and Menghini (2013) establish some dimensions in mathematics education that ensure the possibility of studying theoretical teaching models that is concerned with promoting mathematical thinking and the psychological-cognitive theories of teaching and learning propounded by Piaget (1970), Vygostky (1978) and Ausubel (1986) among others.

2.1.1 Piaget Theory of Genetic Epistemology

This cognitive theory of learning, which is a developmental constructivism (Romberg, 1969) maintains that learners acquire number concepts and operations by construction from the inside and not by internalization from other persons (Kammi & Lewis, 2009). In this instance, adults use mental patterns to guide learner's thought and behaviour and interpret new experiences in relation to their existing concepts in order for the learner to construct personal knowledge. Piaget (1968) pointed out that every normal child is capable of good mathematical reasoning if attention and care are directed to activities of his interest and if emotional inhibitions that will give him a feeling of inferiority are minimized or eliminated. He believed that the amount of time each child spends in each stage varies depending on the environment in which learning is taking place (Kamii, 1982). Consequently, when a learner meets conditions in which his existing scheme cannot explain new information, then either the existing schemes must change for new ones to be adjusted so as to fit into the existing scheme. This concept of assimilation is the ability of the individual to absorb new information by fitting features of the environment into internal cognitive structures. The concept of accommodation enables the individual to modify internal cognitive structures to conform to new information in order to meet the demands of the environment. A balance of these two is maintained through equilibration, as the individual organizes the demands of the environment in terms of previously existing cognitive structures. Children consequently move from one stage of cognitive development to another through the process of equilibration to understand an underlying concept (Slavin, 1988). Staff (1998) cited in Ellington (2002) indicates that the child has the capacity to develop understanding of mathematical concepts if guided. Froebel (1902) who pioneered early childhood educational reform believed that every child possesses at birth, full educational potential, such that an appropriate educational environment is necessary to encourage the child to learn, grow and develop in an optimal manner (Mangal, 2008).

However, the age at which a child enters each stage varies according to each child's hereditary, environmental characteristics and mentorship. This implies that older children, and even adults, who have not passed through a particular stage handles information in a manner that is characteristic of a young child at that very developmental stage (Eggen & Kauchak, 2000). In this instance Piaget (1978) advocates that children have difficulty with particular concepts because there is too rapid passage from the qualitative nature of a problem to the quantitative or mathematical formulation. The numbers and quantities used to teach the children should be meaningful to them at various stages in order to encourage mathematical reasoning. Conditions that can help the child to search for understanding of a concept are the use of active methods that permit the child to explore new knowledge or concept to be learned, rediscovered or reconstructed spontaneously and simply not be told to him. In this case, the role of the teacher is that of a facilitator, guide and/or organizer who creates situations and activities that present solutions to problems to the child. In Piaget's argument, he noted that children who achieve certain knowledge spontaneously through free investigations retain them and later acquire the methodologies and procedures that serve them for the rest of their lives, and as a result stimulates their curiosity without the risk of losing them. In this instance, the teacher provides examples and counterexamples that lead the child's ability to reflect on and reconsider hasty solutions. In so doing, teachers understand the levels at which children function and try to ascertain their cognitive levels so as to adjust their teaching accordingly because not all children in a class operate at the same level. By emphasizing methods of reasoning, the teacher provides critical direction so that each child can discover concepts through investigation to ascertain meaningful learning. He therefore maintains that every child should be encouraged to self-check, approximate, reflect and reason while the teacher studies the child's work to understand his thoughts (Ojose, 2008). He also asserted that providing various mathematical representations acknowledges the uniqueness of students and provide them with multiple paths for making mathematical ideas meaningful.

Clarification requires the child to identify and analyze the elements in a problem, to decode information that is needed to solve them. By encouraging the child to bring out the relevant information from problem statements enhances the child's mathematical understanding of inductions and deductions and makes mathematical inferences. Furthermore, the child evaluates his solutions using standards to judge the adequacy or otherwise of the problem, leading to formulating hypotheses about future events and ascertaining if one's problem solving technique is correct or wrong. Finally, the child applies the mathematical concepts to real-life situations (Ojose, 2008).

According Piaget (1968), children are usually grouped chronologically by age, even if their levels of development differ significantly as well as the rate at which each individual child passes through each stage (Weinert & Helmke, 1998). These differences may depend on maturity, experience, culture, environment and the ability of the child to grasp information (Papila & Olds, 1996). The sensorimotor stage (which begins from year zero to two), enables the child to develop mental and cognitive characteristics through the teachers' solid mathematical foundation and provision of activities that incorporate counting to enhance the child's conceptual development of numbers. Mathematical concepts are therefore built when teachers give the child ample time to interact with the environment without restriction, but in a safe and organized manner (Martin, 2000). During the preoperational stage (2-7 years) the child develops language and speaking ability, symbolic thought, and egocentric perspective and thinks in one direction such that he is unable to reverse thoughts, hence putting the child in a problem solving mode of similarities and differences. The teacher at this point elicits conversation from the child and the teacher encourages him to innovate varieties of ways to do a thing, therefore concluding the mechanism of the child's thought processes (Thompson, 1990). The Concrete Operations stage which is between 7 and 11 years featured a significant cognitive growth through the child's language development, and acquisition of basic skills through hands-on activities for experience and cognitive development (Burns & Silbey, 2000). The child utilizes the mind to consider the multi-dimensions of an object concurrently by making abstract ideas to becoming concrete as a means of understanding mathematical concepts. In this regard, serration and classification are the two logical operations the child developed to understand number concepts (Piaget, 1977). Serration is the ability to put objects in either ascending or descending order whilst classification involves grouping objects on the basis of a common characteristic. The child is capable of forming propositions and deducing possible outcomes at the Formal Operations Stage (11-15 years) by constructing his own mathematical concepts. The child naturally begins to develop abstract thought patterns where reasoning is performed using pure symbols without understanding the concept behind the data. Reasoning skills at this stage refer to the mental process that is involved in the generalization and evaluation of logical arguments to include clarification, inference, evaluation, and application (Anderson, 1990).

2.1.2 Ausubel Theory of Meaningful Learning

According to Ausubel (1968), prior or existing knowledge is of major importance for learners to acquire new conceptual knowledge meaningfully through hierarchical structure of concepts and preposition. Thus, meaningful learning is obtained from networking of facts or concepts when the new learning fits into an existing concept which is easily understood, learned and retained (Slavin, 1988). Concept mapping is the theory of meaningful learning when new information is incorporated into an existing concept and undergoes further changes and growth in the child (Novak, 1998). It is a useful structure for negotiating ideas through brainstorming in order to stimulate students to relate the new information to the previous. Its structure integrates new learning into prior ideas that a learner has through mental images, and created by words or thoughts which help the child to focus on a topic in a lesson. It is a representation that enables students organize, relate and explain mathematical concepts explicitly or implicitly. It also represents relationships between the main and related ideas which are denoted in levels of abstraction; thus the main idea is placed at the top and related ideas at the bottom. Furthermore, Ausubel as a constructivist ensures that learning takes place when learners make connections between or among ideas throughout their personal learning (Dunaway, 2011). As a constructivist-related process of connecting information, it consequently rest in diversity of opinions by the learners to develop and maintain these connections in order to facilitate continuous learning. Siemens (2004) relates this principle to constructivist theory as the knowledge that emerges from an individual's learning network when connections are made between or among concepts. Ausubel learning theory suggests that effective instruction requires the teacher to choose relevant topics to teach and to provide the means in helping students to relate new information to concepts they already possess (Slavin, 1988). In this instance, the teacher makes teaching and learning meaningful when the background and interest of the child is known. The teacher therefore develops strategies to help the child to assimilate and accommodate the new information during the learning process. This support for students by the teacher is done through scaffolding and mediation in which the child does most of the activities during the learning process and the teacher acts as the facilitator to reinforce the new learning that is to be acquired. Scaffolding refers to the use of variety of instructional techniques aimed at moving learners progressively towards stronger understanding and ultimately to greater independence in the learning process. It involves breaking up the learning tasks, experiences and concepts into smaller parts and then providing learners with the support they need to learn each part. Ausubel therefore maintained this position very clearly when he stated:

> 'If I had to reduce all educational psychology to just one principle, I would say this: The most important single factor

influencing learning is what the learner already knows. Ascertain this and teach him/her accordingly' (Ausubel, Novak & Hanesian, 1978).

There are three requirements in Ausubel theory that is relevant to knowledge acquisition. The first is that students build mental pictures of what they already know in order to match them to the new information in which concepts are analyzed at different stages. The second is the relevant learning material which students use to construct significant concepts and the plans that are related to the knowledge to be obtained. Lastly, the learner must choose to learn meaningfully through conscious and deliberate means and relate the new knowledge to the existing knowledge.

Distinguishing between meaningful and rote learning, Ausubel (1968) indicates that meaningful learning takes place when information are broken into parts for understanding since there is a relationship between the new and previously acquired knowledge while rote learning happens when information are wholly memorized without breaking them into parts or relating them to prior knowledge. In this instance, new information are not retained for a long period because it is random, discrete, verbatim and non-substantive and cannot be integrated into any new or previous ideas for intellectual understanding, therefore providing difficulties in showing patterns of recall. On the other hand, when students learn meaningfully, they retain the information much longer because there is a relationship between the new and previously acquired knowledge. In this learning, the cognitive structure is clear and it facilitates the retention of new content. Consequently, information which students learnt meaningfully is applied in a variety of ways to solve unfamiliar problems through knowledge transfer. In doing so, the new information is cognitively fitted into a larger pattern with the learner having relevant ideas or appropriate concept that relates to what he already knows (Ausubel & Robinson, 1969). Accordingly, advance organizer is explicitly used in organizing activities which strengthens students' cognitive understanding for knowledge retention in which new knowledge are remembered because it is related to the previous knowledge (Ausubel, 1968).

2.1.3 Vygotsky Theory of Concept Formation

The work of Vygotsky (1978) which gained increased recognition in mathematics education community, states that the development of a child's intelligence results from social interactions, co-operative activities and communications (Sutherland, 1993). The idea behind effective learning is therefore seen in social interaction between or among people with different levels of mathematical knowledge and understanding through the use of language (O'Neil, 2011). The theory of concept formation is therefore a powerful theory which discovers how an individual constructs a new mathematical concept by bridging the gap between the individual's mathematical knowledge in the classroom and the body of socially approved environment. The theory consequently focuses extensively on group learning rather than individual learning in mathematics lessons (Van der Veer & Valsiner, 1994). One central idea of this theory is the concept of mediation which discusses learning that is mainly focused on interactions between the teacher and the students or among the students. Learning is therefore more meaningful, easy, manageable, effective and efficient when the child's learning is mediated and scaffolded by the teacher or by a knowledgeable adult or peer (Denhere, Chinyoka & Mambeu, 2013). Jaworski (1994) explains the idea of scaffolding as teachers' ability to offer relevant teaching strategies to guide the learner to acquire a particular mathematical skill. Wood (1988) also provides additional insight to scaffolding which he calls 'contingent instruction' by teachers when they pace the amount of assistance learners are given on the basis of moment-to-moment understanding of a mathematical concept. In mediation which is central to socio-cultural theory, the teachers' role enhances students' learning when materials are well selected to shape the students' experiences (Williams & Burden, 1997). As a great significance to socio-cultural theory, Kozulin (2002) also indicates that mediation is effective if a knowledgeable adult or peer is involved in enhancing learners' performances.

In developing students' conceptual understanding of mathematics, recent instructions fluctuate between emphases on mastering algorithms at the expense of conceptual understanding. However, in a complete disregard for algorithms under the catchphrase of meaningful conceptual learning, Vygotsky (1986) provides the foundation for integrating these two aspects, because algorithms appear in curriculum in a conceptual form. To elucidate this point, teachers must introduce and guide students to the realistic representations that accurately link actions to objects; thus helping students to distinguish between part and whole in a variety of mathematical relationships. For this reason, Vygotsky Learning Theory (1986) emphasizes the organization of mathematical knowledge systematically where each new principle is always connected to previously learned material. While Vygotsky (1986) regards change of the environment as the paradigm shift constructivism is a paradigm of a person's activity in adapting to the environment (Glasersfeld, 1995). Factoring out all the numerous actions influencing natural development, Vygotsky

(1986) suggested active concept formation through exploration, and at the same time being aware of the cultural ramifications. One of the primary agents of this formative process is the use of symbolic tools appropriated by the student to develop conceptual reasoning in learning. The theory is therefore central to the sociocultural character of the learner during the learning process especially when different types of knowledge are acquired.

The culture of scientific reasoning which is different from everyday understanding has become very apparent in explaining Vygotsky theory. At this juncture questions which are first formulated in the context of science teaching started getting answers from culture which is not usually seen in the classroom. This means that students are perceived as individuals who possess natural tasks of perception, memory, and problem solving techniques that may be used to support learning in the classroom. Culture therefore appeared as an informative content of the curriculum that is external to the process of classroom learning. As to whether students should receive knowledge from the teacher in a ready-made form or whether in an actively and independently constructed form, Vygotsky (1986) considers educational development as a source and process rather than a consequence of enhancing cognitive learning.

Some researchers (e.g., Rogoff, 1990) perceived apprenticeship as a typical Vygotskian educational model in which Cobb (1996) indicate that systematic classroom learning and everyday apprenticeship are different types of sociocultural perspectives and activities which are linked to concept formation. While classroom learning is aimed at developing students' systematic (scientific) concepts, the apprenticeship leads to the development of everyday concepts that are experientially and practically rich in a given context, yet often incompatible with scientific notions (Karpov, 2003). In this connection, Vygotskian theory argues that the apprenticeship type of learning uses existing cognitive abilities of the student. This type of model is seen in our environment where emphasis is placed on the fact that students are required to go on attachment or internship after an academic work in the school in order to integrate the classroom learning to hands-on activities at the work place.

In a bid to solidifying constructivism, Vygotsky (1986) indicate the difference between spontaneous concepts formed through practical experience with independent thinking from the home and scientific concepts taught in school by the teacher (Moll, 1990). Therefore there is the connection of the scientific concept to systematic and hierarchical knowledge acquisition as different from the non-systematic and unorganized knowledge gained from everyday experience. So, there is emphasis on the fact that what students learn at school influences the concepts that are acquired through everyday experience from the home and vice versa. By extension, spontaneous concepts grow and change under the influence of instruction received from scientific environment with the notion that scientific concept is also developed when it is incorporated into everyday concepts (How, 1993). In acquiring a scientific concept, thinking must move up towards abstraction and generalization but the nonscientific concept move towards concretizing learning. So, it is the combination of these two that guides the principle of constructivism. The difference here is that the acquisition of the scientific knowledge takes place in a hierarchical system whilst acquisition of everyday experience happens in a non-coherent system. This coherent system which is defined as the distance between the real development level of the child to solve problem independently and the potential developmental level of the child to solve problems through the guidance of a knowledgeable adult or peer is called the Zone of Proximal Development (Murray, & Arroyo, 2002). In this connection, Vygotsky (1978) opined that the ZPD is the existing developmental space that the child uses to attain the next level through the use of mediating environmental tools, facilitated by a capable adult or peer. ZPD should therefore be invoked when a major change is taking place from one psychological age period to another for students' understanding. ZPD is also used in any situation in which collaborative or assisted learning produces a dynamic process of cognitive change in the child (Chaiklin, 2003). On a more practical plane, ZPD is often used as a theoretical framework for developing a variety of potential instructional and assessment techniques (Lidz & Gindis, 2003). Consequently, ZPD is used as a space of interaction between scientific and spontaneous concepts to identify and assess the psychological functions of the child, dominating the instructional improvement of the teacher and learners' understanding of mathematics concepts.

In a nutshell, the theoretical framework for the constructivist theory hinges on Piaget's notion that learning is individually constructed with the help of a knowledgeable adult or peer, Vygotsky's idea of encouraging student-centered approach and active learning and Ausubel's view that teachers should build on what the learner already knows.

2.2 Conceptual Framework

Closely related to learning theories, are instructional strategies, which focus on how to structure teaching in order to facilitate learning (Wiggins & McTighe, 1998). These connections consequently establish the importance of prior knowledge or existing cognitive frameworks as well as the use of relevant study tools to drive this conceptual change. Similarly, different models of learning can inform instruction in useful ways (Grassian & Kaplowitz, 2009).

A conceptual frame is the structure which the researcher used to best explain the progress of the phenomenon to be studied (Camp, 2001). It is the description of a given system that illustrates the key relationships between and among the variables with the purpose of facilitating the understanding of ideas in accessible terms. The conceptual framework linked the concepts from the relevant literature to the learning theories to promote and systemize the espoused knowledge (Peshkin, 1993). The framework, therefore illustrated what the researcher expected to find out from the research, including how the variables being considered might relate to each other. From the preceding assertions, conceptual framework is an analytical tool with several variations and contexts that helps to organize ideas and make distinctions in the research. The model therefore explained the relationships between the latent variables and their related manifest variables. The developed model had a total of 52 variables which were finalized from literature and categorized into four (4) exogenous latent construct mediators as cognitive activation (CA), profound understanding of fundamental mathematics (PUFM), instructional coherence (IC) and cognitive guided instruction strategies (CGI) (Ittner et al., 1997) which measured the endogenous latent variable, teacher-trainee performance (TTP). So, based on Drenger et al (2012) and Carson et al (2008) the researcher developed the model (fig 2.1) to examine the effect of teacher quality on teacher-trainees' performance through the mediator variables of CA, PUFM, IC and CGI because . Based on the National Core Standard of Mathematics Curriculum (2012) CA, PUFM, IC and CGI constructivist-related teaching model are considered to explore teacher-trainees' performance in mathematics. The constructivist theory is not only espoused by Piaget, Ausubel and Vygotsky theories of learning but also operationalized by the exogenous variables of CA, PUFM, IC and CGI to define constructivism and explain the endogenous variable of teacher-trainees' mathematics performance (Sarstedt et al., 2017a, 2017b). They state that several types of data could be used to generate indices as a measure of performance. For instance, Ittner et al., (1997) operationalized strategy with four indicators as the ratio of research and development to sales, the market-to-book ratio, the ratio of employees to sales and the number of new product or service introductions.

Studies on factors affecting teacher-trainees' academic performance have used various techniques and methods to explain the relationships between particular variables. Therefore, in this study, CA, PUFM, IC, and CGI are considered as the instructional strategies which agreed that knowledge is not acquired automatically but acquired when the learner constructs his own understanding with or without the help of a facilitator to enhance performance.

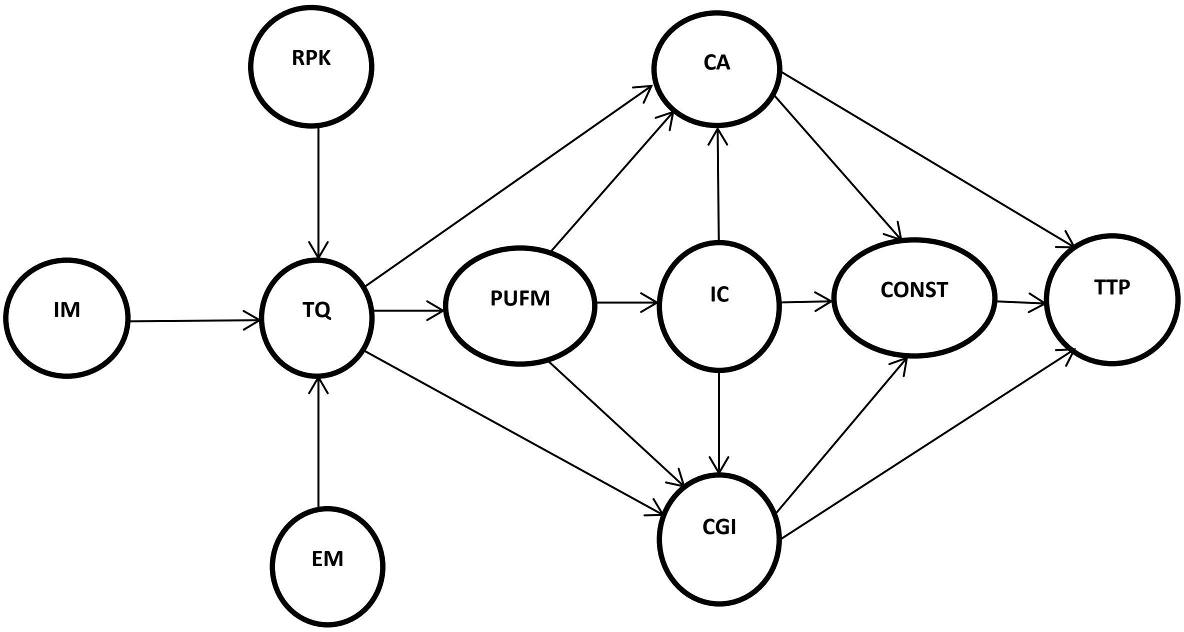

Fig. 2. 1

2.2.1 Cognitive Activation (CA)

Cognitive Activation as one type of learning strategies that a teacher introduces to his students by encouraging them to think more deeply in order to find solutions to problems and focusing on the method rather than focusing on the answer adopts the constructivism principle. Also as a type of instructional strategy in constructivism that promotes mathematics achievement among learners, teachers stimulate students in the learning processes through guidance and to perform challenging tasks as they use their existing knowledge to think extensively to solve problems (Klieme, Pauli & Reuser, 2009). Giving problems to learners with no immediate obvious method for solutions encourage them to reflect on the problems that require thinking for an extended time. In addition to this, the strategy asks learners to use their own procedures for solving complex problems, explaining how they solve the problems and why they chose that method is very essential in constructivism. Furthermore, learners applying their own method in a range of situations while the teacher uses their suggestions as a basis for planning subsequent lessons has become very pragmatic in constructing one's own idea. Creating a learning community where pupils are able to learn from their mistakes, encourages them to identify how mathematics concepts can be applied in a range of situations. Encouraging a culture of exploratory talk in the classroom, where learners consider a range of possible solutions to problems and identify and analyze for themselves what they need to learn has become a panacea to critical thinking. Further support for these findings come from Organization for Economic Operation and Development's (OECD) International Survey of Teaching and Learning (TALIS) which notes that in fostering cognitive activation, teachers need to use deep and challenging content (Mayer, 2004; Brown, 1994). It is a fact that argumentation and non-routine problem solving develop learners' ability to make connections between and among mathematical concepts, procedures, ideas and representations (Hiebert & Grouws, 2007; OECD, 2012).

The Programme for International Student Assessment (PISA) identifies cognitive activation as one of several instructional strategies that supports the development of mathematical literacy and exposes learners to activities in mathematics lessons (OECD, 2013) as well as giving challenging tasks to students for them to use their existing knowledge in the instructional process (Klieme, Pauli & Reusser, 2009). These are prominent influences that guide students' activities in the classroom, where the teacher helps the students to engage in higher-level thinking to solve problems (Klieme et al., 2009; Lipowsky et al., 2009; Hiebert & Grouws, 2007; Mayer, 2004; Reusser, 2006). This consequently stresses the instructional process for understanding (Cohen, 1993, Pauli, Reusser, & Grob, 2007) and provides challenging tasks using students' existing knowledge, ideas, and experiences to explore new concepts and asks stimulating questions (Lipowsky et al., 2009) in order to excite the learning process (Klieme et al, 2009). Cognitive activation is composed of specific aspects of teacher-student relationship that solicit positive and constructive teacher feedback; a positive approach to correcting students' errors and misconceptions by a caring teacher (Fauth, Decristan, Reiser, Klieme & Buttner, 2004) which subsequently promotes students' motivation in a mathematics classroom. Cognitive activation as a mathematical instruction has a direct effect on students' performance (Pitkaniemi & Hakkkinen, 2012), however, (Lipowsky et al., 2009) concluded that not all students benefit to the same degree from cognitive activation instruction because the process subsequently has greater potential for those students who are more interested in mathematics. That is, a serious student who is desirous of making strides in understanding mathematics concepts will employ all the available resources and connect them appropriately to excel. However, a student who does not have the interests will be limited in expanding his horizon. Assumptions in cognitive activation therefore is that students are at equal ability level and that they are all having background of equal measure of economic, social and cultural status.

Studies have shown that cognitive activation is linked to factors such as classroom climate and management for student achievement (Lipowsky et al., 2009; Hugener et al., 2009; Rakocxy, Klieme, Buergermeister & Harks, 2008; Klieme et al., 2009). For instance, research findings suggest that students' high academic achievement is linked to safe and orderly climate (Reynolds et al., 1996) and normal temperatures (Hanner, 1974), good school setting (Saritas & Akdemir, 2009) and standard school buildings (Cash, 1995). A meta-analysis by Seidel and

Shavelson (2007) concluded that a more cognitive activated instruction with a more supportive classroom climate have effect of stimulating students' interest and transforming their existing interest towards mathematics learning. So, cognitive activation becomes effective when students take advantage of the available environment to understand mathematical concepts.

In summary, cognitive activation is about exposing learners to instructional strategies that encourage them to think more deeply in order to find solutions to problems and to focus on the method they use to get the answer rather than simply focusing on the answer. Activities in cognitive activation include summarizing, questioning and predicting possible ideas and linking new information to those they already have.

2.2.2 Profound Understanding of Fundamental Mathematics (PUFM)

One of the major findings of qualitative studies on mathematics instruction is the collection of instructional strategies and pool of differentiated mathematical illustrations and clarifications available to teachers. So, in comparing teachers in China and the United States, Ma (1999) showed that profound understanding of fundamental mathematics is reflected in a broad collection of pedagogical strategies over a range of mathematical topics. The breadth, depth, and flexibility of Chinese teachers' understanding of mathematics afford them a broader and more varied range of strategies to represent and explain mathematics than what is available to their colleagues in the United States. Studies in which teachers were presented with examples of critical classroom events revealed that an insufficient understanding of mathematical content limits their capacity to explain and represent the content to students in a way that make sense; a shortfall that cannot be balanced by pedagogical skills (Baumert et. al, 2010). The teachers should therefore help students to develop high level of mathematical proficiency when they teach to make students realize why certain procedure works. The teacher should also possess deeper understanding of the conceptual and procedural knowledge of the topic to be treated. Ball (1990) and Ma (1999) demonstrated this understanding for multiplication and place values; Borko et al. (1992) and Simon (1993) did this for division; Even (1993); Stein, Baxter & Leinhardt (1990) and Heaton (2000) exemplified this for patterns and functions; and Putnam, Heaton, Prawat, & Remillard (1992) showed it for geometry where these concepts are taught from first principles. Given their case studies, Putnam et al. (1992) concluded that the efforts of teachers with a limited conceptual understanding of mathematical topics fell short of providing students with powerful mathematical experiences. In conclusion, Ma stipulated that mathematics teachers must not only teach students how to solve mathematics problems but also teach them to learn mathematics by knowing the 'why' during the problem-solving stage. Furthermore, learning mathematics requires a deep understanding of the concepts with the ability to produce effective solutions to ill-structured problems (Saritas & Akdemir, 2009). This understanding of fundamental mathematics with respect to the basic operations of addition, subtraction, multiplication, division and place value, espoused by Ma (2010) is deep, broad and exhaustive. This principle is in consonance with Bloom's taxonomy which emphasis higher order questions, affording students to think outside the box; thus ensuring that they don't concentrate on recalling answers to questions. To this end, mathematical thinking is what mathematics students do such that they are able to 'see' how an answer to a problem is obtained.

Students develop procedural confidence but often lack deep conceptual understanding to solving problems or make connections between two or among more mathematical concepts to solve problems (MacMath, Wallace & Chi, 2009). This strategy that conceptualizes teachers' ability to teaching differently argue that teacher effects on students' achievement are driven by their capacity to understand and use subject matter knowledge accurately in carrying out the task of deep teaching (Ball, 1990; Shulman, 1986; Wilson, Shulman & Richert, 1987; Akinsola, 1999; Akinsola, 2003). To fully promote mathematics learning, Ma (2010) mentioned that teachers must feel confident and comfortable in what they teach. Teachers, therefore do not only need to calculate correctly, but also know how to use diagrams, symbols and manipulative to represent mathematical concepts and procedures in order to provide students with explanations from basic mathematics principles and procedures to solve complex real problems (Hill, Rowan & Ball, 2005). PUFM therefore is the ability of students to understand mathematics at the foundation level by knowing the 'why' and the 'how' of concepts. The implication here is that graduates from the colleges of education must acquire this fundamental knowledge from their college tutors in order to teach effectively at the basic schools.

According to Ma (2010), PUFM strategy consists of four features that enable teachers to teach deep, broad and exhaustive to the understanding of students. Firstly, teachers must be able to connect mathematical topics that relates to other topics in mathematics so that students can identify these connections and build new knowledge on what they already know. Furthermore, teachers and teacher-trainees are to develop the skills of linking the new learning to the past, correlating new learning in one area to the other among different subjects or areas (e.g. Mathematics and Physics), connecting ideas within the branches of the same area (e.g. Zoology and Botany in Biology) and relating new learning to real life happenings or situations as found in mathematics for Rotation, Enlargement in transformation. The second feature of PUFM has to do with multiple perspectives where teachers are supposed to approach mathematics teaching in variety of ways, appreciating different aspects to espousing mathematical ideas and using various methods to solve problems and consequently knowing the advantages and disadvantages of the alternative methods. This is an indication that students have complete knowledge of the subject matter. For example, if students are given a problem in addition, one may use words to explain the solution whilst another may use figures in explaining same. Thus, using various strategies to solve a problem and explaining one's thinking about the 'why' to the solution provides a deeper understanding of mathematical concepts. Thirdly, teachers must understand that elementary mathematics consists of basic principles which recur throughout mathematics learning, thereby creating a solid foundation on which future learning of new concepts is built. In this instance, teachers are to encourage students to explore ideas to solve problems as opposed to simply calculating the answer for the students. Longitudinal Coherence as the fourth feature ensures that what is taught today becomes the base for future knowledge. No matter how fragmented that knowledge may be, current mathematics teaching must be built on students' previous knowledge. This is done through bringing together fragmented knowledge that struggling students may have and using them as foundation on which future learning occurs. In this regard, teachers with PUFM strategy are not limited to the knowledge that should be taught in a certain grade but rather should be willing to revisit previous' years learning in order to meet the present needs of their students.

2.2.3 Instructional Coherence (IC)

Instructional coherence as a highly valued strategy in quality mathematics instruction (Chen & Li, 2009; Anthony & Ding, 2011) is described as the interconnectivity of mathematical concepts in a lesson (Hiebert et al., 2003). Wang and Murphy (2004) explain instructional coherence as a link between structured content and classroom activities. It is a constructivist concept that describes how teaching consists of related sequences or activities, showing which aspects of the current subject-matter or related issues are linked to previous ones and understood by the students in the teaching process (Pitkaniemi & Hakkkinen, 2012). This concept is related to Ausubel's idea of previous instructional knowledge that is brought to bear on current lesson in order to help the learner form a concept map.

A research conducted by Pitkaniemi and Hakkkinen (2012) indicate that students' performance in a lesson after instructional coherence was adopted, yielded positive results in students' understanding of mathematical concepts. Findings from another research conducted by Chen and Li (2010) suggest that a coherent instruction and teachers' perception of coherent knowledge support the teachers' effort of effective teaching. In another development, Cai, Ding and Wang (2014) referred to interconnected mathematical concepts which gradually deepens students' thinking as real coherence. In this instance, the lesson moves from easy to difficult, from simple to complex and from specific to abstract as the teacher allocates reasonable time including plans to accommodate students' diverse needs.

Additionally, Badreddine & Buty (2011) conducted a research on 7th grade students in a physics class which focused on teaching sequence and reported a positive effect on student learning that is derived from making explicit links among the past, present and future of the content within the instructional discourse. However, one of the notions that teacher-trainees hold about their educators is the lack of coherence between the teachers' instructional practices and their perception about teaching mathematics in schools (Rojas & Chandia, 2016). In this regard, they indicated that coherence is perceived when teacher-trainees notice the characteristics of the teaching model that the teachers profess which are sometimes not consistent. This considers the degree to which learning opportunities are organized conceptually with available logistics to achieve educational goals are very paramount to instructional coherence (Tatto, 1996).

With this concept, two types of coherence are identified; conceptual coherence which explains the professional perspectives of those who work with teachers and structural coherence which is associated with the design of learning opportunities (Hammerness, 2006). The conceptual coherence has to do with how college tutors relate with each other during lessons in order to adopt similar teaching strategies that will cut across the school curriculum. Coherent curriculum and the teacher's perception of knowledge therefore support this realization of comprehensible instruction where teachers emphasize the underlying structures and knowledge connections that are embodied in these activities and focuses on challenging students' thinking. The structural coherence definition emphasizes the idea that coherence requires the examination of learning opportunities where students' ideas fit into what is being taught for deeper understanding (Grossman, Hammerness, McDonald & Ronfeldt, 2008) if they connect mathematical concepts

(Pitkaniemi & Hakkkinen, 2012). On this note, instructional coherence is seen as an important feature of teaching strategies in mathematics classrooms with empirical support that links teaching and learning. There is therefore a positive effect on lessons with high level clarity and coherence on students' perception of learning activities which develop their competence, hence justified as instructional quality (Seidel, Riimmele & Prenzel, 2005).

In summary, instructional coherence as a constructivist model provides a framework of teachers' strategies in examining the sequence in mathematics classrooms by integrating activities through consistent instruction for students' understanding. In basic terms, instructional coherence in mathematics involves the connection of mathematical concepts in which teachers implement connectivity of their lessons to what students already know in order to generate new knowledge.

2.2.4 Cognitive Guided Instruction (CGI)

CGI is a primary level inquiry and constructivist-based approach to teaching mathematics that was developed at the Wisconsin Center for Education Research, Madison, Wisconsin (Carpenter et al, 1999) informing the teacher to fuse their cultural mindset and that of the learners into mathematics concepts (Carpenter, et al., 1998). This is an instructional strategy that identifies how values and beliefs of students and teachers inform organized teaching and learning, enabling the teacher to anchor the development of students' mathematical thinking to his instructional practices, knowledge and beliefs (Carpenter, Fennema, Franke, Levi & Empson, 2000). Thus, the learning of procedural skills does not have to come before problem solving, suggesting that teachers who participate in CGI help their students to achieve gains in problem-solving to improve their students' abilities to communicate mathematical ideas. Affirming this compatibility, a study conducted by Hankes' (1998) shifted teachers from being culturally-insensitive to culturally-responsive mathematics instructors which is found to have positive effect on student achievement. Subsequently, CGI enhances teachers' knowledge about students' growth in each stage through a series of professional development experiences (Carpenter, Fennema, Peterson, Chiang & Loef, 1989). CGI therefore affords teachers the opportunity to plan mathematics instruction based on the learners' level and cultural understanding thereby guiding them towards greater mathematical reasoning and concept mastery (Sencibaugh, Sencibaugh & Bond, 2016). In other words, CGI provides teachers with opportunities and capacity to understand how learners develop mathematical thinking and plan instructions that guide students to learn mathematical skills with understanding. In this regard, learners' capacity to learn should be considered as teachers make instructional decisions based on the learners' informal knowledge of solving problems without the teacher's instruction. The teacher's role therefore is to build a learning environment that enables learners to construct their own knowledge rather than the teacher trying to transmit knowledge, hence constructivism.

CGI differs from other projects such that students' thinking is used as a context for teachers to enhance understanding of mathematics instruction (Carpenter et al., 1996). Therefore, the goal of CGI is not to show teachers the representations that they can directly teach to their students, but to help teachers understand the ways students naturally solve problems, even if those methods are not the most efficient ways (Carpenter et al., 1999). Franke and Kazemi (2001) state that understanding the sequence of how children develop problem solving techniques enables teachers to pose problems that challenge their thinking. In CGI, learners solve problems that they can succeed in and not the teachers asking the students to do something that they cannot do even though some may go on to solve different problems that are difficult. This approach is culturally sensitive because students are given responsibilities that they are able to succeed in or good at or have the talents to do.

Cultural probing interviews with students enable teachers to reflect on the teaching strategies to adopt even though it is believed that every individual has a gift and different ways to do things. That way of teaching and learning is cultural because children can solve problems in ways that make sense to them as there is not just one way of solving problems. CGI is culturally sensitive because it is students' own thinking as they solve problems themselves. In this way, the students make sense out of what they think and do, giving them confidence, joy, success and hence making them feel good about mathematics. This excitement is evident when the teacher guides them, and not telling them; when the teacher is letting them find their own answers, and draw their own conclusions. This therefore brings adequate cooperation and group effort among the students, making every learner encouraged to work together. This kind of instruction is not trying to decide who the best student is and who is not; this is not where they compete but desirous of contributing to the group for completion and success. Consequently, learners share their strategies, thoughts, enthusiasm, experiences and eagerness for their satisfaction and success. Sharing all these attributes with others enable learners to master and understand mathematical concepts because development of a child proceeds from socio-centric to egocentric. Children are therefore, excited in sharing when they solve problems by themselves as these shared ideas are shaped and sharpened by those with whom the sharing is actualized. This philosophy of sharing is culturally sensitive as it is traced back to creation.

Before shifting from this discussion of cultural strategies, and having confirmed the similarities between them, it is interesting and important to reflect on the philosophical foundations of the difference between CGI and the cultural way of teaching and learning. Whereas the cultural way (spontaneous or non-coherent) of learning is grounded deeply on spiritual, traditional and personal values, the principle of CGI is derived from the integration of research-based knowledge, constructed by educators within a structured and responsive environment. So, culturally it is believed that learners are capable of solving problems independently because that is what humans are created to do; thus constructing one's own understanding. In a research conducted by Hankes (1998) to investigate the compatibility of culture and cognitive learning, one participant of the study has this to say:

> ''Initially, I just followed the workbook manual, ensuring that children did what they were supposed to do in the workbook, and I always wondered why they were struggling. Initially, it took about 35 minutes to present a lesson, and did little worksheets, but now lots of work was done and intertwined with activities including discussions under CGI. Since the introduction of CGI, one is learning to be more appreciative of mathematics than before because as I saw the children having fun with mathematics, it made me to challenge them more. I now inspired these children and it was nice to see them enjoyed mathematics which gave me the indication that every child in the class felt successful. I remembered

> not being successful at mathematics at first because of fear, but the biggest difference was that I had a better understanding of the subject and how children also developed and went through the stages to understand mathematics concepts. Originally, when children could not understand mathematics concept, I thought it was their fault but now, I know that there are different developmental stages in solving problems and it was just that they were not at that level''.

According to Hankes (1998), teachers continue to teach the way they teach even though they are aware that their learners are confused and do not understand the mathematical concepts. This is because teachers really don't understand children's developmental level of thinking as far as mathematics is concerned. Meanwhile, CGI gives students and teachers the freedom to think critically and solve problems as they are now on equal grounds. In this case, it is agreed that teachers do not have the right answers to problems all the time, so all have to figure it out. Teachers' lack of knowledge in mathematics and about children's mathematical thinking forced them to rely solely on textbook for instruction without making any personal inputs. This reveals the way these teachers were taught mathematics whilst in school and have never developed any understanding of mathematics concepts. CGI therefore seems very natural where lessons are designed from what intrigues learners and teachers to incorporate their own ideas in lessons to give joy in doing mathematics.

One of the most important factors in developing learners' competence in mathematics is the attitude of the teacher (Meyer & Koehler, 1990). This attitude should develop culturally responsive instruction is exhibited in cognitive guided instruction where teachers possess deep cultural knowledge, which is latent, community-based, socially created and integral to each student's life. In addition, the teachers and the learners need to have the cultural and mathematical content knowledge through thoughtful reflection for cultural compatibility. This informs the teacher to deploy CGI to teach in a culturally responsive way. Accordingly, while knowledge about how the learner thinks is important, the teachers' content and pedagogical knowledge are also very critical to the environmental learning culture. So, content and pedagogical knowledge in conjunction with learners' knowledge allow the teacher to design outlines for mathematical tasks. It is therefore rational to expect that teachers will feel successful when their learners perform well irrespective of whether or not the students come from a historically disadvantaged school or home. Similarly, it is expected that teachers would be frustrated and unsuccessful when learners do not perform well.

CGI has a philosophy of constructivism in which teacher has the knowledge about the types of problems, solutions and strategies to help develop children's cognitive understanding of concepts. A CGI classroom is where new knowledge is built on what children know and not on stated objectives by the teacher. As captured above, experienced CGI teachers is about the teacher making instructional decisions based on their knowledge about individual child's thinking (Fennema, 1992) such that while classroom learning is aimed at developing students' systematic (scientific) concepts, apprenticeship leads to the development of everyday concepts that are experientially and practically rich in a given context (Karpov, 2003). For these instructional strategies to be used successful in mathematics delivery, there are some foundations that needed to be established.

These are Relevant Previous Knowledge (RPK), Self-Determination Theory (SDT), and Teacher Quality (TQ).

2.2.5 Relevant Previous Knowledge (RPK)

Mathematics lessons must begin with connection to prior knowledge which students and teachers must possess as they enter into a lesson. This knowledge influences what students learn considerably and the task they perform in mathematics lessons that is presented by the mathematics teacher. Previous knowledge is the knowledge the learner and the teacher already have before meeting new information during a lesson. Therefore, learners' understanding of a text is improved by activating previous knowledge before dealing with the new information. Consequently, it is important to consider prior knowledge in a new lesson because in responding to a question or solving a problem, one needs to bring forward some information into working memory to communicate an answer or explain a problem for further understanding. In activating this prior knowledge, students and teachers connect the new knowledge they learn and teach respectively to retain more information. In essence, new knowledge sticks better when it is connected to previous knowledge. To enhance memory retention, learners must connect what is being learnt to what has been known earlier for smooth understanding of mathematical concepts. Good learners constantly try to make sense out of what they learn by seeing how it fits into what is already known. This prior knowledge includes skills acquisition, beliefs, attitudes and academic, cultural and personal experiences which influence how students attend to, interpret, and organize in-coming information.

2.2.6 Self-Determination Theory (SDT)

Every individual including policy makers, educators and researchers have been examining factors that may have meaningfully and consistently lead to mathematics achievement. One of these factors is the effect of motivation as students' basic psychological needs towards the construction of new concepts. Recent empirical studies outline high levels of clarity of teachers' instruction and how it is structured as important components of effective classroom management (Waldis, Grob, Paul & Reusser, 2010; Gruehn, 2000). This is a salient predictor of students' interest in mathematics classes (Ntoumanis, 2008). Research related to self-determination theory (Deci & Ryan, 2002) revealed that effective classroom management enhances students' experience of intrinsic satisfaction and thus facilitates students' interest (Val°as & Søvik; Kunter, Baumert & ¨oller, 2007; Gruehn, 2000). To be motivated means moving self or someone to do something which he may not have done ordinarily. And a person who has no push or stimulus to act is deemed to be unmotivated, whereas anyone who is energized or activated towards an end is considered motivated (Ryan & Deci, 2000). Students who are highly motivated take studies more seriously, accept challenges, participate in classroom activities and consider teachers' recommendations and as a result have high academic achievement (Pajares & Schunk, 2001; Wolters & Rosenthal, 2000). These actions enable them to display their real potential (Eggen & Kauchak, 1997). Using cognitive activation to buttress this point, Turner and Mayer (2004) focused on "challenge" as a source of motivation in mathematics instruction as they argue that combinations of challenging instruction with positive affection and support from the teacher are necessary to cultivate motivation among students during lessons. A committed and knowledgeable teacher will do everything possible to get his students work hard, based on the activities he rolls out in the classroom. Students' motivation therefore plays a key role in mathematics education (Gelman & Greeno, 1989; Hannula, 2006; Middleton & Spanias, 1999; Singh, Granville & Dika, 2002; Walker & Guzdial, 1999) and mathematics achievement is related to both intrinsic and extrinsic motivational factors.

Extrinsic motivation is when students engage in learning for external rewards such as promotion to the next level, approval from teachers, parents and peer etcetera. (Mueller, Yankelewitz & Maher, 2012). Extrinsic motivation is related to an individual aligning the external forces, provided by social setting to advance his personality, supported by internal regulations and acting in the context of external causality to his advantage (Ryan & Deci, 2000a). Extrinsically motivated individual is somewhat controlled by others or by the environment and sometimes become indifferent to oneself (Ryan & Deci, 2000a) and this is very reflective on mathematics learning. With this motivation, the individual acts with the propensity of anticipating a reward or avoiding punishment, an apparent locus of causality (Hayamizu, 1997). Middleton and Spanias (1999) in their work also stated that extrinsically motivated students do not necessarily have the joy of owning the mathematics they learn but rather focus on praise from teachers, parents and peers for a sense of belonging whilst avoiding punishment or embarrassment. Several researchers (e.g. Deci, 1971; Deci, Koestner & Ryan, 1999; Kruglanski, Friedman & Zeevi, 1971; Lepper, Greene & Nisbett, 1973; Lepper & Henderlong, 2000) contend that extrinsic motivation sometimes undermine existing intrinsic motivation. Despite the negative ideas of extrinsic motivation, it triggers the intrinsic motivation rather than undermining it and that it has a positive effect especially when students have low levels of intrinsic motivation (Brophy, 2004; Cameron, 2001; Lepper, Corpus & Lyengar, 2005).

With respect to intrinsic motivation, the individual acts for internal satisfaction such as interest, curiosity and enjoyment in line with his/her own values and internal regulation (Chirkov, Vansteenkiste, Tao, & Lynch, 2007) and has two crucial factors as competence and autonomy (Deci & Ryan, 1985). Referring to a research conducted by Pajares (1996), intrinsic motivation leads to self-efficacy which is a clear predictor of students' academic performance in mathematics (Alliman-Brissett & Turner, 2010; Mousoulides & Philipou, 2005). But this intrinsic motivation is not achievable if the extrinsic factor which the teacher provides through challenging exercises with encouragement to the learner does not play a role. In support to this statement, Middleton, Littlefield and Lehrer (1992) found out that students' intrinsic motivation to learn mathematics is highly influenced through classroom activities that the teacher designed. However, Zhu and Leung (2011) argued that intrinsic motivation by nature originates from within the individual and does not mostly depend on external influences. Nonetheless, intrinsically motivated students must not be discouraged by more complex problems (Middleton & Spanias, 1999) however they must spend more time on tasks and be more persistent and confident to use varieties of challenging strategies to solve mathematical problems (Lepper, 1998; Lepper & Henderlong, 2000).

Boaler (1999) reported a relationship between meaningful mathematical tasks and student intrinsic motivations, and stated that when students are given the opportunity to engage in meaningful mathematical tasks that maintain their cognitive integrity, they do not only tolerate mathematical work, but express pleasure and satisfaction in them. Accordingly, students who are highly motivated intrinsically to study mathematics increase their achievement rate through determination and confidence even in the face of disappointments (Lehmann, 1986; Pokay & Blumenfeld, 1990). Middleton, Littlefield and Lehrer (1992) however, conclude that mathematics activities must be difficult enough so that students are not bored, but the tasks must allow for a high degree of success given appropriate effort by the student.

Self-Determination Theory (SDT) is based on personality traits and motivation, and it is the interaction between individuals and society for personal achievement. SDT is the ability of people to internalize external forces in their social environment for adaptation through various means and regulations to positively promote their personality and the society. Accordingly, the theory shares three universal and innate psychological needs that have positive and negative results when an individual is supported or slowed down during interaction respectively (Ryan & Deci, 2000a).

The first psychological need is relatedness. It is about the individual's feeling as to whether he/she is being loved by those he/she cares about and has relationship with (Vlachopoulos & Michailidou, 2006). This need is sustained when the individual is accepted by the social environment in which he finds himself and when his self-assurance and emotional support are guaranteed, with the help and suggestions coming from people (Ntoumanis, Edmunds & Duda, 2009). This is consequential to an educational setting where mutual respect, help and trust are crucial for the support of this need (Deci et al., 2001). So, if there is a positive relationship between the teacher and the students in a mathematics class, then there will be learning. The mathematics teacher must create the atmosphere to ensure student-teacher and student-student relationships which are acceptable for learning.

Crosnoe et al. (2010) conducted a study and stated that children with differing mathematics skills prior to primary school showed different but parallel trajectories of mathematics learning throughout their primary school years. When they were enrolled in inference-based instructional class, those who initially had the lowest skills narrowed the learning gaps when they had positive relationship with their teachers who gave them challenging tasks and with their peers with whom they had some discussions. However, they did not show the same progress when they were in a class that focused on basic skills instruction or/and when they were in inference-based classroom and had negative relationships with their teachers. Every mathematics teacher should therefore ensure a positive relationship by engaging his students in activities in order to improve mathematics achievement. Competence is the second psychological factor an individual needs to undertake activities which confidently brings about an effective and desired performance. Competence is the acquisition of knowledge, skills and ability which a student has to accomplish tasks. Therefore a competent student will lead societal advancement by applying knowledge and skills to better the lot of his/her mathematical achievements. Students' attitude in acquiring knowledge and skills is vital in this instance where they need to adopt positive thinking to what is to be achieved when they are self-determined (Ryan, & Deci, 2000). The third psychological factor is autonomy that propels the feeling of an individual to make choices in order to perform mathematical activities convincingly without pressure (Deci et al., 2001). Autonomous people excel by their own actions, whereas a regulated individual feels timid to external forces (Gagné, 2003). Buff, Reusser, Rakoczy and Pauli (2011) in their study stated that students' experiences, backgrounds, cognitive and motivational consequences during mathematics instructions affect learning which are more cognitively activated, and better able to deal with learning targets. However, in a study conducted by Hugener et al. (2009), a discovery pattern of cognitive autonomy by students led to students' inability to understand mathematical concepts but positive when students are guided with emotional support.

Schoenfeld (1985) in his study perceives that students' mathematical understanding and the ability to solve problems depend on a) the availability of resources that serve as the students' foundation for basic mathematical knowledge b) students' ability to select the necessary resources to solve problems and have control over them c) students' need to possess a set of broad based problem-solving techniques that make them succeed in the learning process and d) students' capacity to bring their belief systems that have positive bearing on problem situations. All four of these categories must be taken into account when explaining students' attitude to learning mathematics. However, if all these four principles are missing in a mathematics class, learners create anxiety or fear towards the subject. Fear is a reflex that one feels when there are real threats whereas; anxiety appears when there is a threat against one's inner world (Bağcı, 2008). Mathematics anxiety is one that is felt about mathematical contents (Yenilmez & Özabacı, 2003) and appears as a consequence to negative emotions on autonomous nerve system

(Erktin, Dönmez, & Özel, 2006). Consequently, anxiety has negative influence on learning when it occupies the short and long term memories (Aydın, Delice, Dilmaç & Ertekin, 2009). If mathematics anxiety becomes chronic, it can result in desperation, hopelessness and desertion which may limit choices for the individual as it becomes an obstacle to his or her success in life (Durmaz & Akkuş, 2016).

2.2.7 Teacher Quality (TQ)

There has been growing emphasis on teaching quality in recent years across various cultures and perspectives where state laws (e.g. Ghana) are being established to direct specific content knowledge for entry into teaching degrees and diplomas with more disciplined courses taken within programmes (Lowrie & Jorgensen, 2015). In this sense, the teacher with strong disciplined knowledge and sound disposition towards teaching is the most important variable affecting student performance in mathematics (Hattie, 2009). Among multiple factors within schools, teacher quality is so extraordinarily important to the lives of students and that teachers do matter most when it comes to school improvement and student learning (Stronge, 2010). Fobih, Akyeampong and Koomson (1999) assert that a significant part of problems confronting pupil's low academic performance has to do with teacher instructional quality and professional commitment. Their view is consistent with the assertion of Akyeampong and Lewin (2002) that the content of teacher education programme in Ghana might be lacking in producing teachers capable of improving quality of basic education.

Shulman (1986) has been instrumental in crafting two clear differences in what teachers need to know in teaching; these are what to teach (content-knowledge) and how to teach it (pedagogical knowledge). In his broad study of factors relating to student achievement, Hattie (2009) describes quality teachers as those who challenge their pupils with problems in different contexts and ask them to apply what they have learned to new contexts. At the 1985 American Educational Research Association meeting, Shulman (1986) went further than the general standpoint of educational psychology and emphasized the importance of domain-specific processes of learning and teaching indicate that teachers do not only need the subject-matter knowledge but also need knowledge in pedagogy and students' interests and backgrounds (Bransford et al., 2000). In this context, educational research has distinguished three core dimensions of teacher knowledge; these are content knowledge (CK), pedagogical content knowledge (PCK), and generic pedagogical knowledge (GPK) (Baumert et. al, 2010). Various authors (e.g., Grossman, 1995; Sherin, 1996; Shulman, 1987) have added to and further specified these core components of teachers' professional knowledge.

In a research on teaching and teacher education, there is a shared understanding that domain-specific and generic pedagogical knowledge are important determinants of instructional quality that affect students' learning and motivational development (Bransford, Darling-Hammond, & LePage, 2005; Bransford, Derry, Berliner, & Hammerness, 2005; Grossman & McDonald, 2008; Grossman & Schoenfeld, 2005; Hiebert, Morris, Berk, & Jansen, 2007; Munby, Russell, & Martin, 2001; Reynolds, 1989). Nevertheless, few empirical studies have assessed different mechanisms of teachers' knowledge directly and used them to measure instructional quality and student outcomes (Fennema et al., 1996; Harbison & Hanushek, 1992; Hill, Ball, Blunk, Goffney, & Rowan, 2007; Hill, Rowan, & Ball, 2005; Mullens, Murnane, & Willet, 1996; Rowan, Chiang, & Miller, 1997). If teachers are to prepare group of students for challenging tasks, such as outlining problems, finding information, integrating ideas, synthesizing materials, creating diverse solutions, learning on their own and working cooperatively, then teachers require substantial knowledge and completely specialized skills different from what they may already have (Darling-Hammond, 1997).

The quality of a teacher is estimated on how much the students understand what the teacher teaches (Remesh, 2013). However, research has it that negative dispositions to mathematics teaching stems from the teachers' personal experiences when they were in school with a growth cycle of adverse perceptions that is strengthened throughout their school-life (Ball, 1990; Mayers, 1994). In addition, teachers' attitudes to, and beliefs and confidence in teaching mathematics which vary considerably (Biddulph, 1999; Swars, 2005; Bursal, 2010; O'Neill, & Stephenson, 2012), are often influenced by how they were taught in school (McNeal & Simon, 2000; Hattie, 2009). Numerous studies have also revealed that teachers' attitudes and beliefs influence their thinking and behaviour, most importantly with their teaching practices and instructional methods (Wilson & Cooney, 2002; Philipp, 2007) with the beliefs that they mostly influence the effect on their instructional practices (Wilkins, 2008). Teacher content-knowledge and beliefs about mathematics alongside their pedagogical practices have been identified as key factors in quality mathematics education (Lowrie & Jorgensen, 2015). Following this revelation, they suggest that there should be attempts to address these dispositions, by building confidence and competences in teacher-trainees (Carroll, 1994; Mayers, 1994; Schuck, 1996) because teachers' content knowledge and beliefs about mathematics is directly connected to their instructional choices and procedures (Brophy, 1990; Brown, 1985; National Council of Teachers of Mathematics, 1989; Thompson, 1992; Wilson, 1990a). In addition, Geliert (1999) reported that in mathematics education, it is acknowledged that the teacher's philosophy of mathematics has a significant influence on how to structure mathematics classes. Consequently, teachers' understanding of mathematical concepts has great influence on their instructional strategies hence the effect on student learning. One important attributes of strong disciplined knowledge is when Mandeville and Qiduan (1997) found that the strength of teaching lies in the capacity of the teacher to create networks of knowledge between and among mathematical concepts so as to build strong connections for learners' understanding. Conversely, teachers with poor content-knowledge and pedagogical skills tend to take structured teaching approaches where skills are taught in isolation (Lowrie & Jorgensen, 2015).

Emerging changes in mathematics curricula calls for mathematics teachers to acquire special training to monitor their own skills for continued and efficient performance at any stage (Remesh, 2013). Therefore mathematics teachers should teach the subject based on their philosophical references that inform their instructional strategies to include choice of texts, programming, delivery and assessment. It is also required that effective teachers must have mathematical content knowledge, pedagogical skills, commitment and practice, knowledge about students' interests, needs, beliefs, attitudes, and culture worldview (Ahia & Fredua-Kwarteng, 2004). These are necessary to be used as basis of theorizing the culture of teaching mathematics in the Colleges of Education. In this regard, mathematics pedagogy which is theoretically based must be validated practically so as to enable teacher-trainees achieve instructional knowledge for their professional pursuit. Asare and Nti (2004) added that quality teachers must add value to themselves by attending some professional training programmes to become efficient, innovative, versatile and competitive in their practice of teaching and learning. Thus, teacher-trainees must participate in professional development training workshops apart from receiving higher qualification in mathematics. When these professional strategies are conveniently pursued, the tendencies of teacher-trainees adopting good pedagogical skills from their mentors/tutors are guaranteed.

The art of teaching does not only involve a simple transfer of knowledge from one person to another but it is a complex process that facilitates the sharing of knowledge (Remesh, 2013). Teaching can therefore be described as an activity of sharing knowledge, skills, experiences, attitudes and values between facilitators and their learners and ultimately among the learners. Effective teaching is regarded as the provision of stimulus to the psychological and intellectual growth of the learner through the process of teachers attending to students' needs, experiences and feelings, and helping them to learn a particular thing. The teaching process involves the teacher, learners and curriculum that contains the knowledge, facts, information and skills to be acquired. By this, the learner makes a deliberate attempt to learn and the teacher respects the learners' cognitive integrity and freedom of choice of what is to be learnt. Teaching is therefore an activity aimed at unearthing the students' latent talent which brings about meaningful learning through teaching methods that are professionally and pedagogically acceptable.

In a broader view, teaching is creating situations to facilitate learning by motivating learners to have interest in what is being taught and discussed in the classroom (Tamakloe, Amedahe & Atta, 2005). When the teacher teaches, it is expected that the student learns, as teaching and learning play complementary roles. As Farrant (1980) puts it, teaching and learning go together as they are like opposite sides to the same coin. So, Lefrancois (1988) stated that effective teaching happens when there is attainment of instructional objectives by the learners and for them to function successfully in the schools and communities through the acquisition of skills to transforming self and the social environment. To this, econometric analysis suggests that some teachers are dramatically more effective than others, and that these differences have lasting effects on student learning (Rinkin, Hanusahek & Kain, 2005; Larson, 2002).

Educational production function studies measured teachers' mathematical knowledge directly to determine student performance (Mullens, Murnane & Willet, 1996; Rowan, Chiang & Miller, 1997). To this, Mapalelo and Akinsola (2015) stated that production function studies have sought to measure students' achievement through the courses their teachers have taken and the degree attained per certificated examinations. Quantitative research on teacher competence is anchored almost exclusively on representations such as certificates obtained and mathematics course work completed in schools (Cochran-Smith & Zeichner, 2005). When certification in a subject is assessed and correlated with student achievement in the same domain, findings tend to indicate a positive relationship, especially for mathematics (Goldhaber & Brewer, 2000). In this direction, Goldhaber & Brewer (2000), Monk (1994) and Rowan et al. (1997) indicate that higher teacher qualifications tend to be connected to better student achievement at secondary and tertiary levels, particularly in mathematics. Nonetheless, these findings show that the number of courses taken in a teaching subject is inconsistent across school subjects but generally positive for mathematics, such that a teacher who is exposed to and took more mathematics courses during the university-based stage of teacher training appears to have positive effects on secondary students' learning achievements.

Monk (1994) reported that the more teachers interact with students coupled with their prior knowledge, the better the subject matter of teaching pedagogy. Beyond these methodological issues, other potential findings from this research are that teacher preparations and job experiences are good proxies for teacher knowledge and teaching skills which mostly help students to learn. Intervention studies also show that enhancement of mathematical content knowledge support high quality instruction (e.g., Fennema & Franke 1992; Swafford, Jones, & Thornton, 1997). To this Lipowsky et al. (2009) found two features of teachers' instructional quality as cognitive activation and classroom management which have positive effect on mathematics learning.

2.3 Characteristics of Instructional Strategies

Instructional strategies in mathematics have been a problem even when Ghana had the best educational achievement (Ahia & Fredua-Kwateng, 2004). Akyeampong (2003) cited in Asare & Nti (2014) reflected on a number of strategies used in teaching mathematics but only the teacher-centered method is used in the colleges of education as against learner-centered which is linked to the constructivist principles.

Instructional strategies, according to this study, are teaching techniques used when the teacher selects the appropriate methods to assist learners to learn independently and constructively in order to accomplish desired learning tasks. From the foregoing, instructional strategy is the extent to which teaching and learning goals are shared by everyone involved in educating prospective teachers. For the sake of this study CA, PUFM, IC and CGI were considered as the instructional strategies which are operationalized to constructivism.

The learner-focused view of teaching mathematics is essentially built on the constructivist view because it centers on the students' active involvement in learning mathematics through explorations in a bid to formalize one's own ideas (Kuhs & Ball, 1986). This problem-based teaching strategy requires that the teacher plays the role of a facilitator and stimulator for the students through posing interesting and challenging questions in a defined situation for investigations so as to make students think and uncover inadequacies or otherwise in their own learning. Though, teachers realize the importance of this teaching strategy, they hardly take into account the particular needs of individual students because of large class sizes and the broad content coverage in the curriculum with inadequate time frame (Wang & Cai, 2007). It is therefore important for mathematics teachers to understand the actual interest of their learners, and stand on this existing knowledge to integrate learning activities that have real significance for each learner (Mattar, 2010). This idea resonates well with Ausubel's learning theory.

Active approach to teaching and learning mathematics occurs when students think critically and practice what they learn to gain knowledge and experience.

Learners are consequently encouraged to set up globally accepted academic standard for knowledge transfer and demonstrate how this knowledge helps them to unearth skills that will enable them solve real life problems for socio-economic growth. In this learning strategy, the students are supposed to understand the concept of doing, thereby constructing their own meaning as they solve problems. Taking students through meaningful learning activities such as class exercises, home and class assignments, project works, questioning and answering, discussing, explaining, debating or brainstorming with the teacher giving feedback make them to understand mathematical concepts. As the teacher introduces the learner to a new topic and demonstrates how concepts are interconnected, the learner gains knowledge through hands-on-approach that is geared towards adventurism; linking instructions received to personal experience and skills. With this, the learner has the tendency to discuss the knowledge acquired with their mates, enabling them to retain the information. The strategies also help learners to get feedback on their incomplete understandings and are encouraged to fix them through the help of the teacher or their peers. It also gives the teacher feedback on what learners understand and who needs help. Therefore, cooperative learning is an instructional strategy that helps students with different ability levels to work in teams on problems and projects using variety of learning activities to enhance the understanding of concepts for positive interdependence, individual accountability and overall academic achievements (Fredericks, 2005). To this end, group formation supports the learners to help each other to learn faster than the teacher would have done with the whole class.

For inductive learning, the teaching methodology challenges learners to develop working hypothesis to problems from a set of observed instances. This strategy enables students to deepen their understanding of the content and develop their inferences and evidence-gathering skills when they are first presented with mathematics problems which allow them to notice a pattern and come up with the correct conclusions in their own words; a foundational process of constructivism and higher order thinking (Silver, Dewing & Perini, 2012). Inductive learning in mathematics is a teaching skill that focuses on the student's personal construction and understanding of knowledge which is directly linked to social constructivist view rooted in students' active participation in learning so as to discover and establish ideas personally (Thompson, 1992). The National Ministry of Education (2001) expressed that mathematics teaching should be built on students' existing knowledge, experiences and cognitive development. To this, teachers should stimulate the students' interest in learning and providing sufficient opportunities to engage them in mathematical activities. For this to be successful, the role of the teacher changes from being a transmitter of knowledge to a facilitator of developmental efforts; from being the authority in the classroom to organizers, guides and collaborators so as to make students to become masters of mathematics learning. Therefore a combined instructional strategies of PUFM, CGI, CA and IC define constructivism and assess teacher-trainees' mathematics performance in this study.

It is an acceptable fact that one principal goal of mathematics learning at the primary, secondary and tertiary level is to develop students' reasoning capacities, analysis and visualization. So, in today's technological era, learning should not be perceived as a clear and linear process, but rather as a complex process of problem solving, with the need to approaching issues from multi-dimensional perspectives (Sakyi, 2014). To this end, mathematics teaching and learning must exclusively be supported by teachers' content knowledge and pedagogical skills, provision of material resources, good students' socio-economic background, maximum students' perceived understanding and positive parental and community beliefs about the subject. However, among all these factors, the research looked into how constructivism through instructional strategies leads to teacher-trainees' performance in mathematics at the colleges of education.

2.4 Mathematics Education

In observing mathematics lesson, Ismaila et al., (2014) mentioned five main areas that constitute effective mathematics teaching. These are teacher knowledge, mathematical language and communication employed by the teacher, the mathematical tasks the teacher roles out, learning organization deployed by the teacher and an ethics of care coming from the teacher. Therefore, mathematics teachers have to be well-prepared for lessons, well-versed in the teaching dynamics and thoroughly support the changes in the curriculum and instructional strategies that comes with good interactive abilities such as asking and answering questions. The teacher must not only be an explainer but justify every explanation by the 'how and why' the procedure works and ask questions that enable students to develop procedures themselves. To this, Suffolk (2007) implied that explaining a concept is teacher-centered and posing of questions to students is student-centered. This is to say that students must not always be told to do a thing, but teachers must ask questions that will enable students to think critically and bring out innovative ideas in a mathematics class. However, Wong (2007) maintained that the teacher needs to be the key figure in the mathematics classroom and not the students because it is the teacher who designs the learning activities, which is fundamental to understanding a mathematical concept. Consequently, the teacher is expected to lead the class and its learning activities (Tsang et al., 2014). As Khalid (2009) puts it, the effective teacher decide what aspect of a task is to be highlighted that will promote students' creativity, how to organize and coordinate the work of students such that they learn from each other, what questions to ask to bring out new ideas having varied levels of expertise and how to support students without taking over the process of thinking for them.

As prerequisites for an effective mathematics lessons, it is suggested that teachers must be knowledgeable and competent in the subject matter, possess good pedagogical skills especially in questioning, and have a good relationship with their students (Kani et al., 2014; Omar et al., 2014; Omar et al., 2014; Salam & Shahril, 2014; Shahril, 2009, 2013a, 2013b; Shahrill & Clarke, 2014; Shahrill, Kani & Nor, 2013, Shahrill & Munda, 2014). Furthermore, as change in the real world is inevitable, it is therefore vital for mathematics teachers to constantly learn and update their instructional practices so as to promote and equip students with the required mathematical understanding to meet the challenges of the 21st century. In his broad study of factors relating to student achievement, Hattie (2009) describes quality teachers as those who challenge their pupils with problems in different contexts and ask them to apply what they have learned to new contexts.

Teachers' mathematical knowledge and beliefs together with their pedagogical practices have been recognized as key factors in quality mathematics education (Lowrie & Jorgensen, 2015). This is what defines mathematics education such that there are considerable debates about focusing on mathematical content and pedagogical knowledge. Consequently, Shulman (1986) has been instrumental in creating two clear distinctions about teaching; thus what teachers needed to know (content) and how to teach the content (pedagogy). Teachers' mathematical content knowledge was seen to be a critical factor in students' success (Hill et al., 2005). Correspondingly, studies have also focused on teacher-trainees' knowledge (Goulding et al., 2002; Zevenbergen, 2005) which impact on their capacity for quality teaching and mathematical understanding. A comprehensive account of teachers' strong content knowledge reveals that the strength of teaching lies in the teachers' capacity to create networks of knowledge between mathematical concepts in order to build strong connections for learners through the teachers' delivery (Mandeville & Qidan, 1997). Conversely, teachers' poor content knowledge leads to taking a structured teaching approach such that lessons are taught in isolation. This is what instructed Ma (1999) to reveal that Chinese teachers had more sophisticated understanding of mathematics content that their US counterparts when she investigated North American teachers' inability to explain the processes of invert and multiply when dividing two fractions, but their Chinese counterparts did this well.

With regards to how mathematics should be taught, Kuhs and Ball (1986) cited in Chen and Leung (2013) identify learner-focused as one of the dominants and distinctive views about how the subject must be taught and learnt. Collier (1969), as cited in Seaman et al., (2005) described in their study that the responsibility of mathematics teachers' understanding and beliefs help them shift from an authoritarian, and teacher-dominated classroom, to a democratically learner-centered classroom and also shift from a lesson emphasizing formal mathematical content to a lesson emphasizing the creative and investigative nature of mathematics. Similarly, there have been debates in academic circles where conservatives have been arguing for the basic instructions accompanied with drill and practice as against the reformist arguing for investigative approaches (Ernest, 1991). In this connection, the selected instructional strategies of Cognitive Activation (CA), Instructional Coherence (IC), Profound Understanding Fundamental Mathematics (PUFM) and Cognitive Guided Instruction (CGI) for this study are suitable for mathematical instructions as they indicate different aspects of the constructivist model.

2.5 Summary

Constructivism is the ability of a learner constructing his own understanding of concepts. Understanding mathematical concept is possible when learners learn from stage to stage through the connection of ideas, relying on previous knowledge, and taking parts in activities with the teacher playing a key role of facilitating all these processes. Constructivism is therefore explained by Piaget, Ausubel and Vygotsky theories of learning and operationalized by CA, PUFM, IC and CGI. The constructivist theory has become very important teaching strategy in mathematics classrooms of the colleges of education because it offers teacher-trainees the opportunities of cooperative and collaborative learning. By extension, this research explains and connects the constructivist theory to the conceptualized instructional strategies.

CHAPTER 3

METHODOLOGY

3.0 Overview

This chapter described the research procedures and techniques employed by the researcher to answer the research questions as informed by the conceptual model. This included the research design, population, sample and sampling techniques of the study, instruments used to collect data and the collection procedures which largely depended on the research questions. The reliability and validity tests were conducted to ensure that the items in the questionnaire actually defined and measured the constructs. The chapter finally suggested the software to be used to analyze the data.

3.1 Research Design

The research design essentially was the researcher's plan to illustrate the procedure to investigate the instructional strategies that colleges of education tutors adopt in teaching mathematics (Burns, 1997; Cohen, Manion, & Morrison, 2000). The current study employed a survey design in which data were collected from third year teacher-trainees of three colleges of education as participants of the sample frame for the purpose of approximating the characteristics of the population (Jaeger, 1997). Thus, the researcher was interested in the opinion of teacher-trainees in the Volta Region about the instructional strategies being used by their tutors in the colleges of education to teach mathematics. As the research questions guided the selection of the research method (Creswell & Miller, 2000; Merriam, 1998), a cross-sectional data were collected using a questionnaire with the intent of generalizing the sample to the population (Fowler, 2008). Thus, data were collected one time from the sample that was drawn from a predetermined population (Fraenkel & Wallen, 2000). Questions related to the issues were asked the sample of the population from which answers were obtained to draw conclusions and make recommendations on the research. The survey provided both qualitative and quantitative description of opinions of the population by studying the sample.

3.2 Population

At the end of the 2018/2019 academic year, there were 55,189 teacher-trainees in the 48 colleges of education in Ghana (NCTE, 2019). There are three private colleges of education with a population of 7,094 and 45 public colleges of education with a population of 48,095 (NCTE, 2019). There are seven (7) public colleges of education in the Volta Region as at 2019 with a total population of 6,638 (Appendix B) for the 2018/2019 academic year and consisted of teacher-trainees who were admitted on the basis of their performance in six subjects including core mathematics at the West African Senior School Certificate Examination (WASSCE). Teacher-trainees in the Volta Region was selected as the target population from among the ten regions as at 2019 because of the extremely poor academic performances in mathematics (Cohen, et al. 2000) as shown on league tables with respect to various indicators (Appendices D and E). Also other national league tables included the provision of basic amenities in Metropolitan/Municipal/District Assemblies of the Local Government Ministry (Appendix C), the national results in BECE (Appendix D) and WASSCE (Appendix E) mathematics, and the categorization of Senior High Schools (Appendix F) in Ghana. The narrations of these assertions are detailed as follows:

a) According to the Ghana's District Assembly League Table (2017) on strengthening social accountability for National Development, the Volta Region was the worst performing region among the previously 10 regions using six (6) indicators of Education (BECE Pass Rate), Health (skilled delivery of health care), Security (Police Personnel coverage), Governance (Functional Organizations), Sanitation (Open defecation) and Water (Rural water coverage) as seen in Appendix C. This assertion was based on the research which suggested that students' high academic achievement is linked to safe and orderly environment (Reynolds et al., 1996), good school features (Harner, 1974) and standard school buildings (Cash, 1995).

b) In the case of performance in mathematics, the Volta Region had the 8th position out of the previously 10 regions on the National League Table for BECE from 2013 to 2017 with an average pass rate of 61.0% which was below the average national pass rate of 72.6% as shown in Appendix D. In the case of WASSCE, Volta Region placed 7th position during the same period with an average pass rate of 21.6% which was below the average national pass rate of 30.1% (WAEC, 2019) as indicated in Appendix E.

c) Senior High Schools in Ghana are categorized as grades A, B, C, D, E, F and

G with grade A, being the most endowed schools and G the less endowed. Criteria for the categorization of the schools include but not limited to good academic performance, quality of teachers in terms of qualification and experiences, students' pass rate, infrastructure in terms of spacious classroom blocks, dormitories, science and computer laboratories, library, dining hall, assembly hall, staff common room, staff accommodation and offices, and sports field among others (NDPC, 2005 cited in Higgins, 2009). The Volta region had 148 Senior High Schools out the 1633 in Ghana; thus 9% of the total population. On the average, category D schools are the highest in each region with a total of 583 forming 35.5% of the total schools in Ghana (GES, 2015) as can be seen in Appendix F. There are more category D schools in the Volta Region than any other region with a record of 58.8% of the total schools in the region hence its selection as the population.

d) Even though the Gross Enrolment Rate (GER) for Senior High Schools was high (26.6%) in the Volta Region when compared to the Upper East (19.5%), Northern (22.0%) and Upper West (22.9%) Regions, the pass rate of 25.6% for WASSCE mathematics in the Volta Region except for the Northern Region (15.5%) was lower than that of the Upper East (30.1%) and Upper West (33.2%) regions (Higgins, 2009). These reasons as stated above have therefore constituted the basis for selecting the Volta Region as the population for the study. Constituting 12% of the total teacher-trainees in Ghana, the target population of seven colleges of education in the Volta Region was made up of 2,878 females (43.4%) and 3,760 males (56.6%) as shown in Table 3.1 (NCTE, 2019).

Table 3. 1: Target Population

S/N	Colleges of Education	2018/2019 Population				
		Female		Male		
		N	%	N	%	TOTAL
1	Akatsi	396	31.4	867	68.6	1,263
2	Dambai	219	28.7	545	71.3	764
3	E.P. Amedzofe	200	33.4	399	66.6	599
4	Jasikan	550	46.3	639	53.7	1,189
5	Peki	367	42.0	507	58.0	874
6	St. Francis	371	31.6	803	68.4	1,174
7	St. Theresa	775	100	0	0	775
	Total	**2,878**	**43.4**	**3,760**	**56.6**	**6,638**

Source: NCTE 2019

3.3 Sample and Sampling Techniques

After conducting face validity on the items, the questionnaire was administered to the teacher-trainees of two piloted colleges of education; E.P. College of Education, Amedzofe (Public) and Holy Spirit College of Education (Private), Ho with population sizes of 404 and 509 respectively. The third year teacher-trainees were the participants with their numbers as 131 for E.P. College of Education and 120 for Holy Spirit College of Education. Out of these numbers, a convenient sampling technique was employed to collect data from 112 and 109 respondents with response rates of 85.5% and 90.8% for Amedzofe and Holy Spirit respectively. For a questionnaire to be reliable, it is required that identical respondents, at least with similar background should get the same score while respondents with different backgrounds should get completely different scores as it is difficult for two people to be fully equal or unequal (Field, 2009). The available population of 3,311 which was drawn from three colleges of education consisted of 1,134 females making 34.2% and 2,117 males making 65.8%. These are Akatsi College of Education which had total of 1,263 teacher-trainees, contributing 38.1% to the available population, Peki College of Education which had 874 teacher-trainees contributing 26.4% to the available population whilst St. Francis College of Education also had 1,174 teacher-trainees, contributing 35.5% to the available population (Table 3.2).

Table 3. 2: Available Population

College of Education	Female		Male		Total	
	N	%	N	%	N	%
Akatsi	396	31.4	867	68.6	1263	38.1
Peki	367	42.0	507	58.0	874	26.4
St. Francis	371	31.6	803	68.4	1174	35.5
Total	**1,134**	**34.2**	**2,177**	**65.8**	**3,311**	**100.0**

Source: NCTE, 2019

These colleges were considered because they offered specific programmes which include General, Technical and Science/Mathematics to ensure the relevance of the study. In this regard, Akatsi College of Education offer Technical, Science/Mathematics and General Programmes, St. Francis College of Education offer Science/Mathematics and General Programmes, whilst Peki College of Education offer only General programme, (Diploma in Child Education). Purposive sampling technique was therefore adopted in selecting these three colleges because of the varying programmes they offer. This was a non-random sampling technique which had no underlying theories, but a deliberate choice of selecting participants due to the special cases in terms of programmes the colleges offer (Bernard, 2002 cited in Etikan, 2016). This special characteristic was to assist with the relevance and appropriateness of the research because the information was statistically "saturated" (Padgett, 1998) and fulfilled most of the concerns of the available population.

The participants' adequate responses to the items in the questionnaire were due to the assumption that the sample frame was homogeneous with the selected sample unit likely to behave like any other sample from the target population (Gobo, 2004). In addition, the human and material resources, programmes policy direction, curriculum content, supervision and certification of graduates from the colleges of education come from a single source, thus the University of Cape Coast. It was therefore envisaged that there would not be any difference in the results if different sample frames with similar characteristics were selected from the target population for the study. In social studies, representativeness is often a practical matter, hardly ever an outcome of statistical procedures, which are often difficult to implement because social significance of samples were considered instead of a statistical logic (Gobo, 2004). Therefore, social sciences research stipulates that there is no homogeneity of sample units if sample size is obtained by means of probability because no human being acts in the same manner (Gobo, 2004). Consequently, the results of the research were generalized to the population even though a non-probabilistic sampling technique was adopted (Rothman, Gallacher & Hatch, 2013). The available sample size of 1,064 were third year teachers-trainees in the three colleges of education as indicated in Table 3.3 with female numbers of 392 constituting 36.8% and their male counterpart of 672 constituting 63.2%.

Table 3. 3 : Available Sample

Colleges of Education	Female N	Female %	Male N	Male %	TOTAL N	TOTAL %
Akatsi	164	42.7	222	57.8	384	36.1
Peki	112	39.4	172	60.6	284	26.7
St. Francis	116	29.4	278	70.6	394	37.0
Total	**392**	**36.8**	**672**	**63.2**	**1,064**	**100.0**

Source: Sampled Colleges of Education

Akatsi College of Education contributed 384 (36.1%) teacher-trainees to the available sample, Peki College of Education contributed 284 (26.7%) teacher-trainees to the available sample and St. Francis College of Education contributed 394 (37.0%) teacher-trainees to the available sample. The convenient sample of 842 was those present in the halls at the time of administering the questionnaire. This sample was considered in the study because the participants studied mathematics in their first and second years therefore had enough knowledge on the teaching and learning of mathematics in the colleges and were therefore readily prepared to respond to the items in the questionnaire. Additionally, they were the last batch of the diploma programme teacher-trainees of the colleges of education. The average response rate was 76.1% as indicated in Table 3.4.

Table 3. 4: Response Rate

Colleges of Education	Available Sample (N)	Administered Questionnaire (N)	Questionnaire Returned (N)	Response Rate (%)
Akatsi	384	301	216	71.8
Peki	284	219	179	81.7
St. Francis	394	322	246	76.4
Total	**1,064**	**842**	**641**	**76.1**

The female and male participants' contributions to the convenient sample of 641 were 238 (37.1%) and 403 (62.9%) teacher-trainees respectively. In this sample, Akatsi College of Education contributed 216 (33.7%) participants, Peki College of Education contributed 179 (27.9%) participants and St. Francis College of Education contributed 246 (38.4%) participants as shown in Table 3.5.

Table 3. 5: Convenient Sample

Colleges of Education	2018/2019 Convenient Sampling					
	Female		Male		Total	
	N	%	N	%	N	%
Akatsi	76	35.2	140	64.8	216	33.7
Peki	82	45.8	97	54.2	179	27.9
St. Francis	80	32.5	166	67.5	246	38.4
Total	**238**	**37.1**	**403**	**62.9**	**641**	**100.0**

Source: Various Colleges of Education

3.4 Research Instrument

A research instrument refers to the particular method of collecting data to respond to research questions or hypothesis by use of a questionnaire (Yusoff, 2019). The use of the questionnaire indicates the appropriateness and the representativeness of its items to the targeted constructs. Whilst its relevance refers to the purpose of the assessment, the representativeness refers to the degree to which its items are related to the targeted construct (Yusoff, 2019). The research instrument was carefully developed and structured from reviewed literature (Hunter 2012; Adjei, Pinkrah, & Denanyoh 2014; Denanyoh, Adjei & Nyemekye, 2015) at the backdrop that there are no measurement scales that measured tutors' instructional strategies with respect to Cognitive Activation (CA), Instructional Coherence (IC), Profound Understanding of Fundamental Mathematics (PUFM) and Cognitive Guided Instruction (CGI) from the perspectives of the teacher-trainees in mathematics education from the colleges of education (Bastos et al., 2014). However, the found measurement scales had open-ended items with varying definitions of constructs which were specifically graded and coming from different underlying cultures hence were not relevant and purposeful to this study.

Using the self-designed questionnaire, the researcher collected data from teacher-trainees about the instructional strategies adopted by their mathematics tutors in the colleges of education. This data was later reduced by critical review by the researcher and other experts in the discipline and with the help of factor analysis (Field, 2005). The questionnaire consisted of sections A and B with open-ended and closed-ended items respectively. The instrument has 21 open-ended response items in Section A (Appendix A) which included participants' gender, age, grades obtained in mathematics at BECE, WASSCE and Colleges of Education, choice of programme at the SHS and Colleges of Education, and the year of completion of SHS. There was also a question about participants' name of District Assembly where they come from because the researcher's intent was also to connect the towns in which participants went to basic and senior high schools to the colleges in the Volta Region. Respondents were also asked to rate their mathematics teachers at both JHS and SHS levels in order to evaluate the impact that their teachers had on them whilst studying mathematics. Other questions included participants' first choice of profession and why the decision to be trained as a teacher and whether they have the interest to teach mathematics. Finally, this section ended with whether participants had an idea about the term "constructivism". The reason for asking the participants about their knowledge on constructivism became very vital to this study because constructivism has been widely accepted as one the best teaching model that supports learners' understanding of concepts (Jaworski, 1991, 1994).

The twenty-one (21) open-ended items which solicited responses from participants discovered the meaning they give to the teaching and learning of mathematics and their personal mathematical experiences. However, these qualitative data were analyzed quantitatively (Bogdan & Biklen, 2003; Denzin & Lincoln, 2003). According to Stake (1995) qualitative data was justified in section A because the nature of the research questions required exploration of participants' views and the meanings they have about themselves and that of their tutors in mathematics lessons and not that of the researcher nor what the literature says (LeCompte & Schensul, 1999; Hatch, 2002; Creswell, 2009). Even though the researcher quantified and interpreted what participants said, knew and understood about mathematics, the interpretations were not disconnected from the researcher's background, past experiences and prior understanding of mathematical concepts (Marshall & Rossman, 2006; Creswell, 2009). Furthermore, the qualitative model in section A employed a holistic view of the social phenomena and reported multiple viewpoints that were being examined, identifying key factors involved, as well as describing information that evolved from the data (Armah, 2018). The researcher was not however, restricted to a rigid cause-effect relationship between and among the factors, but had the freedom to identify and develop the interactions in the given situation (Hatch, 2002; Marshall & Ross man, 2006).

The closed-ended items in Section B of the questionnaire were measured using a 5-point likert-scale type from strongly disagree of '1' point to strongly agree of '5' point. This consists of 107 closed-ended response items in respect of conceptualized instructional strategies of Cognitive Activation (CA) which was explained by 28 items, Instructional Coherence (IC) which was explained by 23 items, Profound Understanding of Fundamental Mathematics (PUFM) which was explained by 31 items, and Cognitive Guided Instruction (CGI) which was explained by 25 items. These instructional strategies according to this research were pointers to constructivism. There were also 131 items in this section which served as foundation items for Teacher Quality (TQ) constructs. These items defined Self-Determination Theory (SDT) which was explained by internal and external motivation and Relevant Previous Knowledge (RPK). In order to ensure the reliability and validity of the questionnaire and to reduce the measurement errors of the items, the researcher considered several indicator variables in the questionnaire on a plot base (Cortina, 1993; Field, 2009).

With a postpositive worldview of research, the researcher employed a quantitative approach to collect data in section B where primary numerical data which were coded to respond to closed-ended items, systematically investigated the instructional strategies that tutors use to teach mathematics in the colleges of education (Schoonenboom & Johnson, 2017). Consequently, quantitative data were obtained and measured along a 5-point likert-scale to indicate how much of these variables were present in mathematics lessons (Fraenkel & Wallen, 2000). So, the researcher tried to find the relationship among the latent variables which were measured and analyzed using statistical procedures. This research consequently discovered the meaning teacher-trainees give to instructional strategies that they experienced during mathematics lessons in the colleges of education (Bogdan & Biklen, 2003; Denzin & Lincoln, 2003).

3.5 Scale Validation

In this study, the researcher developed a pre-determined set of items as data collection tool in order to measure the instructional strategies that tutors use to teach mathematics in the colleges of education (Kember & Leung, 2008; Wong, Ong & Kuek, 2012). To avoid responses that were socially acceptable and to receive information about attitudes with its related aspects, the researcher solicited for participants' degree of agreement to the items in the questionnaire about the instructional strategies tutors use in mathematics lessons (O'Keefe, 2000). Due to the fact that responses received from participants may not be reliable and valid because of divergent views and also for items that may not completely measure the actual constructs, reliability and validity tests were conducted (Ratray & Jones, 2007) using pilot samples. Ultimately, this measurement required a tool, whose validity and reliability are necessary and sufficient in the research process (Kember & Leung, 2008). The internal consistency of the items was therefore measured using the Cronbach's Alpha coefficient, which is the average correlation between the indicators of a given construct (Ravand & Baghaei, 2016). The external consistency was measured using the Cohen's Kappa's interrater technique from two sets of piloted samples of E.P College of Education, Amedzofe and Holy Spirit College of Education, Ho as they responded to the same instruments (Singleton & Straits, 2010; McHugh, 2012). The Cohen's Kappa interrater technique illustrated the extent to which responses to the items by participants in the two sets of the piloted samples were equivalent or otherwise (Last, 2001; Rothman, Greenland & Lash, 2008; Wong, Ong & Kuek, 2012). The statistic for the interrater scale is the Cohen's Kappa which was obtained by calculating the ratio of the likert-scale responses of the two sets of piloted samples for the study (McHugh, 2012), whilst the Cronbach's Alpha was measured from analyzing the data using the 2.0 version of SPSS software. A statistic close to 1.0 for both Cronbach's Alpha and Cohen's Kappa indexes indicate a high internal and external consistencies of the items; hence a perfect reliability of the questionnaire. However, the calculated Chronbach's Alpha value of 0.7 or above was considered highly reliable (De Vellis, 2003; Kline, 2005; Cohen, Manion & Morrison, 2007) and the Cohen's Kappa statistic whose ratio is close to 1.0 is deemed to be very high (McHugh, 2012). The Cohen's Kappa statistic required equivalent responses from respondents which were demonstrated by assessing interrater reliability in which reference is made to the consistency with which respondents make equivalent judgments (Liang et al., 2014; Erdvik, Øverby & Haugen, 2015; Deniz & Alsaffar, 2013). Therefore, to ensure that the measurement tool actually measured the intended constructs and provided consistent responses, the interrater reliability was considered (McHugh, 2015).

Validity expressed the degree to which the instrument measured what it purported to measure with several varieties which included face validity, construct validity, and content validity and consequently categorized as internal and external (Last, 2001; Rothman, Greenland, & Lash, 2008; Wong, Ong & Kuek, 2012). Internal validity referred to the accuracy of the scores obtained which actually quantified what it was designed to measure whereas external validity refers to the accuracy of the scores that described the population from which the study sample was drawn (Wong, Ong & Kuek, 2012).

The face validity test of the questionnaire was established by two mathematics tutors from the piloted colleges of education who understood the research topic. The tutors ensured that the items defined the constructs through the refinement and/or removal of double-barreled, confusing, leading and weak survey items. The items were matched to the corresponding constructs and it was concluded that they measured the traits of interest. Personnel from the Office of the Quality Assurance of Ho Technical University where the researcher happened to be the Head also went through the questionnaire and ascertain the items' effectiveness for each constructs in the questionnaire. Lastly, the main and co-supervisors to this study agreed that the items explained the constructs of the study. Thus, all these experts looked at the items in the questionnaire and agreed that they were valid measures of the constructs which were rated on the face of it (Bölenius et al., 2012; Sangoseni, Hellman & Hill, 2013).

Construct validity indicates how well the items measured the operational definition of the latent constructs and actually reflects the theoretical meaning of the concept (Bornstedt, 1977; Ratray & Jones, 2007). Factor analysis was conducted to ensure the validity of the constructs in the questionnaire using Principal Component Analysis (PCA) method. This was to estimate the sample adequacies and the factor loadings of the data by extracting important variables from the large data with the aim of retaining as much information as possible about the constructs (Field, 2009). PCA method of factor analysis has the presumption that all variances within the dataset are shared (Costello & Osborne, 2005; Field, 2009; Tabachnik & Fidell, 2001; Rietveld & Van Hout, 1993). Consequently, the dataset could be reduced when the sample adequacy measured by Kaiser Meyer Olkin (KMO) is above 0.7 and the factor loadings measured by the communalities (coefficient of determination) are 0.6 or above (Field, 2009).

The relevance of questionnaire advocated by Davis (1992) as frequently used was determined by establishing the content validity (Polit & Beck, 2006 & 2007) which supported its strength. Content validity, is defined as the degree to which items of research instruments are relevant to and representative of the construct for a particular research purpose (Cook & Beckman, 2006; Haynes & Kubany, 1995). Accordingly, content validity is the degree to which an instrument has an appropriate sample of items to define the constructs that are being measured (Polit & Beck, 2004). There is therefore the general agreement for the definition that content validity refers to how closely items are put together to adequately provide operational definition of a construct (Rodrigues et al., 2017). It played a major role in establishing that items in the questionnaire interpret the constructs (Fitzpatrick, 1983). The content validity test for this study was established using six (6) panels of experts to examine how the theoretical constructs were well represented and operationalized (Bhattacherjee, 2012). The experts fully assessed and measured the construct to rationally analyze and review all the items for readability, clarity and comprehensiveness and came to some level of agreement as to which items should be included in the final questionnaire (Fitzpatrick, 1983; Bhattacherjee, 2012; Yusoff, 2019) which was administered to the study sample.

3.6 The Pilot Study

The questionnaire was first administered to 131 and 120 participants of E.P. College of Education, Amedzofe and Holy Spirit College of Education, Ho respectively on a pilot basis to ensure reliability using the Cronbach's Alpha and Cohen's Kappa interrater techniques (McHugh, 2015). Studies with a significant number of participants as in this study presented results with small margins of errors (Bastos, Duquia, González-Chica, Mesa & Bonamigo, 2014). For the sake of accuracy, all items in the questionnaire were positively written because people do not express the same opinion when they evaluate a negatively phrased item in a questionnaire (Kamoen, Holleman & Van den Bergh, 2007). When the reliability and validation coupled with factor analysis were completed the resultant questionnaire was then administered to the sample of the study.

With a face-to-face interview through a pen- and-paper questionnaire, the items were read out to the participants of the two piloted Colleges (de Leeuw, Hox & Dillman, 2008) by the researcher. The reading of the items which took not less than one hour was to explain and clarify some items in a bid to ensuring that participants were on the same level of understanding. In addton, it will enable them to accurately respond to the items, complete the questionnaire in good time and to reduce social desirability bias (Kaminska & Foulsham, 2013). This mode of reading the items to the respondents which motivated and kept their attention ensured a smooth flow of information as well as provided responses to their verbal and non-verbal cues (de Leeuw, Hox & Dillman, 2008).

More often, survey items ask people to reveal the unpleasant sides of them which sometimes have to do with participants' behaviours that are not accepted or approved by society. Social desirability bias is one of the recognized types of measurement error when respondents provide answers to questions which are more socially acceptable than their true attitude or behaviour (Kaminska & Foulsham, 2013). In one breadth, respondents understand the questions so well, come out with the correct answer, but report different answers that make them look good. This behaviour from respondents is to avoid embarrassment which leads to underreporting or over-reporting. In another development, respondents misreport a response subconsciously, either due to lack of knowledge, lack of effort in reading the items meticulously or lack of concentration on the items that are being read. In this regard, effortless response to items may lead to respondents reporting pleasant or unpleasant behaviours and attitudes instead of the actual whilst reading the items in a fast mode without understanding. All these may lead to inaccurate reporting. In order to solve this problem, the principles of accurate wordings of the items in the questionnaire were constructed with the utmost care so as to minimize the social desirable bias (Holbrook & Krosnick, 2010).

3.7 Reliability

The questionnaire which was developed with a total of 255 items was reviewed to 242 items in respect of four (4) instructional strategies to predict constructivism and three (3) constructs that predicted teacher quality. The reduction in the items by 13 was as a result of critical study of the items when twenty (20) participants were first selected to respond to the items. Distributed to two piloted samples of E.P. College of Education, Amedzofe (Public) and Holy Spirit College of Education, Ho, (Private), the internal and external consistencies of the data were calculated to measure the reliability of the questionnaire using the Cronbach's Alpha and the Cohen's Kappa indexes respectively as indicated in Table 3.6.

Table 3. 6: Reliability of Questionnaire (Constructivism Predictors)

Instructional	**Cronbach's Alpha**				**Cohen's**
Strategies	**Amedzofe**	**Holy Spirit**	**Agreement Ratio**	**Combined**	**Kappa**
CA	0.924	0.935	0.988	0.932	0.924
PUFM	0.937	0.921	0.983	0.916	0.927
IC	0.935	0.901	0.964	0.941	0.946
CGI	0.924	0.937	0.986	0.936	0.935
Mean	**0.930**	**0.924**	**0.980**	**0.931**	**0.933**

Combined Sample = Amedzofe & Holy Spirit items with total respondents of 221

According to Table 3.6, the Cronbach's Alpha values of each instructional strategy for both colleges were higher than the acceptable index of 0.7 (De Vellis, 2003; Kline, 2005; Cohen, Manion & Morrison, 2007) with the mean values of 0.930 and 0.924 for Amedzofe and Holy Spirit Colleges of Education respectively. The two colleges had a mean agreement ratio of 0.980 which was close to 1.0 (McHugh, 2012). These values were with respect to the internal consistency of the items for the instructional strategies. For further analysis of the reliability of the questionnaire, the responses to the items of the questionnaire from the two pilot samples were combined, giving a total sample size of 221. The reliability for this combined sample was also measured for each instructional strategy with Cronbach's Alpha values ranging between 0.932 and 0.941 with a mean value of 0.931. The Cohen's Kappa index which measured the external consistency of the items with an acceptable value of 0.7 (McHugh, 2012) had interrater reliability index between 0.924 and 0.997 for the instructional strategies with a mean value of 0.933.

Table 3. 7: Reliability of Constructivism Construct

Endogenous	**Cronbach's Alpha**				**Cohen's**
		Construct	**Ratio**		
Amedzofe	**Holy Spirit**	**Agreement**	**Combined**	**Kappa**	
Constructivism	**0.903**	**0.825**	**0.914**	**0.774**	**0.997**

Measuring the internal consistency of the items that define constructivism, Cronbach's Alpha value for Amedzofe's and Holy Spirit were 0.903 and 0.825 respectively with the two results agreeing to a ratio of 0.914. However, the Cronbach's Alpha value of the combined items was 0.774 whilst the Cohen's

Kappa statistics which measured the external consistency of the items for constructivism was 0.997 as seen in Table 3.7.

Table 3.8 shows the internal and external consistencies of items that measured the predictor items of teacher quality.

Table 3. 8: Reliability of Predictors of Teacher Quality Construct

TQ Predictors	Cronbach's Alpha				Cohen's
	Amedzofe	Holy Spirit	Agreement Ratio	Combined	Kappa
SDT_IM	0.878	0.911	0.964	0.904	0.970
SDT_EM	0.950	0.972	0.977	0.964	0.963
RPK	0.906	0.850	0.938	0.893	0.922
Mean	**0.911**	**0.911**	**0.960**	**0.920**	**0.952**

Combined Sample = Amedzofe & Holy Spirit items

SDT_IM - Self-Determination Theory (Internal Motivation)

SDT_EM - Self-Determination Theory (External Motivation)

RPK - Relevant Previous Knowledge

In a like manner, the Cronbach's Alpha values of the questionnaire with respect to teacher quality predictors for the two colleges had a mean reliability agreement ratio of 0.960 with agreement ratios of 0.964, 0.977 and 0.938 for SDT_IM, SDT_EM and RPK respectively. For the combined items, Cronbach's Alpha values for the predictor constructs were 0.964 for SDT_IM, 0.964 for SDT_EM and 0.893 for RPK with a mean reliability value of 0.920. The Cohen's Kappa indexes measuring the external consistency of the items of the two colleges had values of

0.970, 0.963 and 0.922 for SDT_IM, SDT_EM and RPK respectively with a mean value of 0.952 as indicated in Table 3.8.

Table 3. 9: Reliability of Teacher Quality Construct

Exogenous Construct	Cronbach's Alpha				Cohen's Kappa
	Amedzofe	Holy Spirit	Agreement Ratio	Combined	
TQ	**0.952**	**0.973**	**0.978**	**0.963**	**0.960**

The internal consistency of the items that defined teacher quality had Cronbach's Alpha values of 0.952 for Amedzofe and 0.973 for Holy Spirit with an agreement ratio of 0.978, where the external consistency of the two samples recorded a Cohen's Kappa value of 0.960 as in Table 3.9.

3.8 Construct Validity

Construct validity which is the most valuable and difficult test, measured the meaning of the instrument as administered to the participants (Drost, 2011; Wong, Ong & Kuek, 2012). This was conducted using factor analysis when the responses to the items were entered into SPSS software version 2.0. With this analysis, construct validity of the questionnaire was tested (Bornstedt, 1977; Ratray & Jones, 2007) when all items together with their responses represented the underlying construct (Fitzpatrick, 1983). Thus exploratory factor analysis detected the factors that lie beneath the dataset which were based on the correlations between the variables (Field, 2009; Tabachnik & Fidell, 2001; Rietveld & Van Hout, 1993). The trustworthiness of factor analysis which depended on sample size also depended on factor loadings such that the coefficients of determination of the variables were all above the acceptable level of 0.6 (McCallum et al., 1999 cited in Field, 2015). Therefore to conduct construct validity, factor analysis using the Principal Component Analysis (PCA) method was considered in order to reduce the items for each construct and still explain the construct that is to be measured. For each instructional strategy which is deemed to predict constructivism (CONST) and for each Teacher Quality (TQ) predictor, the determinants of the R-matrices should be greater than 0.00001, the KMO values is to be higher than 0.7, the Cumulative Rotation Sums of Squared Loadings (CRSSL) explained aree more than 50% of the variances and the factor loadings are more than 0.6 as shown in Tables 3.10 and 3.11.

Table 3. 10: Indexes for Instructional Strategies (Constructivism Predictors)

Instructional Strategies	**Amedzofe**				**Holy Spirit**			
	DRM	**KMO**	**CRSSL (%)**	**FL**	**DRM**	**KMO**	**CRSSL (%)**	**FL**
CA	0.037	0.877	59.0	0.711	0.003	0.884	57.1	0.666
PUFM	0.000	0.907	65.2	0.677	5.88E05	0.930	64.8	0.707
IC	0.008	0.867	61.2	0.697	0.005	0.892	59.6	0.682
CGI	0.007	0.876	57.2	0.673	0.017	0.878	58.1	0.697
		0.882			**0.0063**	**0.896**		
Mean	**0.013**		**60.7**	**0.690**			**59.9**	**0.688**

DRM- Determinant of R-Matrix

KMO- Kaiser Meyer Olkin (Sample Adequacy)

CRSSL- Cumulative Rotation Sums of Squared Loadings

FL- Factor Loadings

All the indexes for Amedzofe and Holy Spirit in respect of the instructional strategies satisfied the accepted criteria. For the determinant of the R-matrices, the mean values of 0.013 and 0.0063 for Amedzofe and Holy Spirit respectively were higher than 0.00001 accepted values. The sample adequacy (KMO) values for each instructional strategy were above the acceptable threshold of 0.7 with mean values of 0.882 for Amedzofe and 0.896 for Holy Spirit. The total distributions of the variance over the extracted factors (CRSSL) for Amedzofe and Holy Spirit had mean percentages of 60.7 and 59.9 respectively, such that respective CRSSL values for each instructional strategy for the two colleges were more than 50%. With regards to the factor loadings for each instructional strategy, all the values were higher than 0.6 with their mean values of 0.690 and 0.688 for Amedzofe and Holy Spirit respectively. The indexes for the piloted colleges therefore satisfied all the conditions for conducting factor analysis (Field, 2009).

Table 3. 11: Indexes for Constructivism

Endogenous Construct	**Amedzofe**				**Holy Spirit**			
	DRM	**KMO**	**CRSSL (%)**	**FL**	**DRM**	**KMO**	**CRSSL (%)**	**FL**
Constructivism	0.008	0.897	54.0	0.660	0.003	0.886	61.9	0.695

As seen from Table 3.11, the constructivism construct of the two piloted samples satisfied all the conditions for factor analysis such that the determinant of the R-matrix was 0.008, the sample adequacy was 0.897, the total distributions of the variance over the extracted factors (CRSSL) was 54.0%, and factor loadings was 0.660 for Amedzofe. In the case of Holy Spirit, R-matrix was 0.003, the sample adequacy was 0.886, the total distributions of the variance over the extracted factors (CRSSL) were 61.9%, and factor loading was 0.695. All these values satisfied the conditions for factor analysis.

Table 3. 12: Indexes for Teacher Quality Predictors

TQ Predictors	Amedzofe DRM	Amedzofe KMO	Amedzofe CRSSL (%)	Amedzofe FL	Holy Spirit DRM	Holy Spirit KMO	Holy Spirit CRSSL (%)	Holy Spirit FL
SDT_IM	0.007	0.869	58.2	0.691	0.002	0.907	60.2	0.698
SDT_EM	0.001	0.910	62.3	0.707	0.000	0.917	69.8	0.749
RPK	0.024	0.883	62.2	0.726	0.091	0.767	63.1	0.733
Mean	**0.011**	**0.887**	**60.9**	**0.708**	**0.031**	**0.864**	**64.4**	**0.727**

For Table 3.12, the indexes with respect to the constructs that predicted Teacher Quality were also higher than the accepted threshold. The mean determinant of the R-matrices for Amedzofe and Holy Spirit were 0.011 and 0.031 respectively. The mean sample adequacy value for Amedzofe was 0.887 and that for Holy Spirit was 0.864. The variances for Amedzofe and Holy Spirit were distributed over the extracted factors by 60.9% and 64.4% respectively with factor loadings of 0.708 and 0.727 assigned to each of the colleges in that order.

Table 3. 13: Indexes for Teacher Quality

Exogenous Construct	Amedzofe DRM	Amedzofe KMO	Amedzofe CRSSL (%)	Amedzofe FL	Holy Spirit DRM	Holy Spirit KMO	Holy Spirit CRSSL (%)	Holy Spirit FL
TQ	**0.004**	**0.897**	**57.8**	**0.698**	**1.40E-05**	**0.908**	**67.3**	**0.732**

In ensuring factor analysis for teacher quality constructs, Table 3.13 showed that all criteria for conducting factor analysis were satisfied for both piloted samples such that the determinant of the R-matrix of 0.004 was greater than 0.00001, the sample adequacy (KMO) of 0.897 was higher than 0.7 threshold, the CRSSL value of 57.8% was more than 50% and factor loadings was above the 0.6 accepted value for Amedzofe. With respect to Holy Spirit, the R-matrix determinant was 1.40E-05 which was higher than the accepted value of 1.0E05, the sample adequacy (KMO) of 0.908 was above 0.7, CRSSL of 67.3% was more than 50% and factor loadings of 0.732 was above 0.6. From all these analysis, the questionnaire for measuring the constructs for the study was reliable and valid.

3.9 Comparative Analysis of Combined and Average Samples

In this section, the researcher adopted a strategy to compare the combined sample to the averaged sample. The combined sample was obtained when the responses to the items by respondents from Amedzofe and Holy Spirit were combined before factor analysis was conducted. The averaged sample was obtained when factor analysis was conducted on each pilot separately and average values for each construct was determined.

Table 3. 14: Items Reduction for Predictors of Constructivism

Instructional Strategies	Original Items	No. of Items Retained			Ratios	
		Combined	Amedzofe	Holy Spirit	Reduction	Agreement
CA	28	14	9	12	0.50	0.75
PUFM	27	15	15	14	0.56	0.93
IC	24	13	10	11	0.54	0.91
CGI	25	10	11	10	0.38	0.91
Total/Mean	**104**	**52**	**45**	**47**	**0.50**	**0.96**

According to Table 3.14, a total of 104 items were originally distributed among the four instructional strategies. Fifty-two (52) items were retained for the combined sample resulting in a mean reduction ratio of 0.50 when factor analysis was conducted. The reduction ratio of the combined items was based on the retained items to the original items whilst the agreement ratio was based on the ratios of the likert scale responses to the items of the two piloted samples of Amedzofe and Holy Spirit Colleges of Education. With regards to Amedzofe and Holy Spirit, a total of 45 and 47 items were respectively retained when factor analysis was conducted giving a high mean agreement ratio of 0.96. The agreement ratio of 0.75 was the lowest and 0.93 was the highest with respect to the instructional strategies for the two colleges.

Table 3. 15: Items Reduction for Constructivism

Endogenous Construct	Original Items	No. of Items Retained			Ratios	
		Combined	Amedzofe	Holy Spirit	Reduction	Agreement
Constructivism	24	11	12	11	0.46	0.92

With respect to constructivism, the 24 original items were reduced to 11 with a ratio of 0.46 in favour of the combined sample whilst 12 and 11 items were retained for Amedzofe and Holy Spirit respectively giving an agreement ratio of 0.92 as indicated in Table 3.15.

Table 3. 16: Items Reduction for Teacher Quality Predictors

Instructional Strategies	Original Items	No. of Items Retained			Ratios	
		Combined	Amedzofe	Holy Spirit	Reduction	Agreement
SDT_IM	16	10	12	12	0.63	1.00
SDT_EM	43	7	12	11	0.16	0.92
RPK	17	10	9	7	0.59	0.78
Total/Mean	**76**	**27**	**33**	**30**	**0.36**	**0.91**

With respect to the constructs measuring Teacher Quality, a total of 76 items were reduced to 27 after factor analysis was conducted for the combined sample with a mean reduction ratio of 0.36. With a mean agreement ratio of 0.91 for Amedzofe and Holy Spirit, the original items were reduced to 33 and 30 respectively as seen in Table 3.16.

Table 3. 17: Items Reduction for Teacher Quality

Exogenous Construct	Original Items	No. of Items Retained			Ratios	
		Combined	Amedzofe	Holy Spirit	Reduction	Agreement
TQ	37	10	12	14	0.27	0.92

From Table 3.17, the original items that defined teacher quality were 37. For the combined sample, these items were reduced to 10 and for Amedzofe and Holy Spirit the items were reduced to 12 and 14 respectively with an agreement ratio of 0.92.

Having done all these, a comparative analysis between the combined and averaged samples was conducted to select the better sample to ensure construct validity and content validity of the questionnaire. Indicators such as Cronbach's Alpha, Cohen's Kappa, KMO, Factor Loadings and likert-scale responses were examined for the two samples. For the combined sample, the researcher put the responses to the items from Amedzofe and Holy Spirit together and calculated the Cronbach's Alpha, Cohen's Kappa, KMO, Factor Loadings and likert-scale response values. In respect of the averaged sample, the means for Cronbach's Alpha, Cohen's Kappa, KMO, Factor Loadings and likert-scale response of the two respective samples were calculated. These values were shown in Table 3.18 below.

Table 3. 18: Comparison of Combined and Averaged Variables

I S	Mean Likert-Scale		Mean Factor Loadings		Mean Cronbach Alpha		Mean KMO	
	Combined	Average	Combined	Average	Combined	Average	Combined	Average
CA	3.703	3.706	0.597	0.714	0.932	0.930	0.883	0.830
PUFM	3.843	3.780	0.669	0.751	0.912	0.917	0.888	0.820
IC	3.770	3.760	0.606	0.730	0.941	0.918	0.915	0.874
CGI	3.684	3.657	0.602	0.721	0.936	0.931	0.906	0.849
Mean	**3.750**	**3.726**	**0.619**	**0.729**	**0.930**	**0.924**	**0.898**	**0.843**

IS- Instructional Strategies

The results in Table 3.18 revealed that except for the Factor Loadings for the averaged sample, the combined sample showed higher values for mean likert-scale of 3.750, Mean Cronbach's Alpha of 0.930 and Mean KMO of 0.898 than the mean values of 3.726, 0.924, and 0.843 for averaged sample respectively. More importantly, the combined sample size of 221 was larger than the individuals sample sizes. With these revelations, the researcher considered the combined sample of the two colleges as the better option for further analysis of reliability and validity.

From Table 3.19, common themes were determined to represent the items that loaded onto the same factors for each instructional strategy in order to ensure what the survey was measuring. Finally, the items which loaded onto the same factors were aggregated during the final data analysis phase (Field, 2009). The results in Table 3.19 were obtained when factor analysis was conducted on the combined sample using the Principal Component Analysis method.

Table 3. 19: Items extracted for Combined Variables

Instructional Strategies	Factor 1		Factor 2	
CA	**Themes**	**Retained Variables**	**Themes**	**Retained Variables**
	Exploring mathematical ideas	1, 2, 3, 5, 9, 10, 12, 21, 22, 23, 25	Developing mathematical skills	7, 8, 15, 17, 18, 19, 20, 27
PUFM	Foundation to Mathematical concepts	56, 35, 36, 59, 58, 46, 49, 37	Understanding mathematics concepts	52, 42, 53, 39, 44, 55, 43, 45
IC	**Continuity in personal learning**	61, 66, 67, 64, 71, 79, 78, 72, 73	Continuity in classroom activities	65, 62, 81, 84, 80, 83, 81, 69, 74, 75.
CGI	**Personal experience for mathematical input**	95, 94, 88, 92, 101, 97	Use of culture and environment for mathematics learning	86, 103, 93, 104, 106, 109, 91

Table 3. 20: Indicators for Combined Variables under each factor

Instructional Strategies	KMO	Mean Likert-Scale	Factor Loadings		Cronbach's Alpha	
			Factor 1	Factor 2	Factor 1	Factor 2
CA	0.902	3.732	0.643	0.645	0.884	0.850
PUFM	0.936	3.821	0.681	0.636	0.892	0.875
IC	0.900	3.943	0.647	0.653	0.850	0.850
CGI	0.862	3.682	0.633	0.688	0.820	0.800
Mean	**0.900**	**3.795**	**0.659**	**0.656**	**0.862**	**0.844**

With a mean sample adequacy of 0.902 and mean likert-scale response of 3.732 in Table 3.20, items 1, 2, 3, 5, 9, 10, 12, 21, 22, 23 and 25 loaded onto Factor 1 which represented "Exploring mathematical ideas" and items 7, 8, 15, 17, 18, 19, 20 and 27 loaded onto Factor 2 which represented "Developing mathematical skills" in respect of Cognitive Activation as seen in Table 3.19. The factor loadings and the Cronbach's Alpha values for Factor 1 were 0.643 and 0.884 respectively and that for Factor 2 were 0.645 and 0.850 respectively for cognitive activation (CA) as indicated as in Table 3.20.

In the case of Profound Understanding of Fundamental Mathematics (PUFM), items 56, 35, 36, 59, 58, 46, 49 and 37 loaded onto Factor 1 under the theme "Foundation to Mathematical concepts" and items 52, 42, 53, 39, 44, 55, 43 and 45 loaded onto Factor 2 under the theme "Understanding mathematics concepts" as in Table 3.19. With a sample adequacy of 0.936 and mean likert-scale response of 3.821 as in Table 3.20, Factor 1 had a factor loading of 0.681 and a Cronbach's Alpha value of 0.892 and Factor 2 had a factor loading of 0.636 and a Cronbach's Alpha value of 0.875.

Whilst the Instructional Coherence (IC) had items 61, 66, 67, 64, 71, 79, 78, 72, and 73 under Factor 1 with the theme "Continuity in personal learning", items 65, 62, 81, 84, 80, 83, 81, 69, 74 and 75 were under Factor 2 with the theme "Continuity in classroom activities" as shown in Table 3.19. Factor 1 had a factor loading of 0.647 and Cronbach's Alpha value of 0.850 and Factor 2 had a factor loading of 0.653 and Cronbach's Alpha value of 0.850. Furthermore, with a sample adequacy value of 0.900, the mean likert-scale response for this instructional strategy was 3.943 as indicated in Table 3.20.

Finally, Cognitive Guided Instruction (CGI) had two factors. Items 95, 94, 88, 92, 101 and 97 loaded onto Factor 1 which had the theme "Personal experience for mathematical input" and items 86, 103, 93, 104, 106, 109 and 91 loaded onto Factor 2 with "Use of culture and environment for mathematics learning" as the theme in Table 3.19. With a mean likert-scale response value of 3.682, the factor loading for Factor 1 was 0.663 and the corresponding Cronbach's Alpha value was 0.820. The factor loading for Factor 2 was 0.688 with the corresponding Cronbach's Alpha value of 0.800 and KMO value of 0.862. All these are shown in Table 3.20. All the calculated indexes for the combined items satisfied the conditions for conducting factor analysis as indicated in Table 3.21 with the correlation matrices indicated in Appendix M. The reliability of the questionnaire which was calculated using Cronbach's Alpha was also found for the various instructional strategies with respect to the combined sample.

Table 3. 21: Indexes for Constructivism Predictors

Instructional Strategies	DRM	KMO	CRSSL (%)	FL	CA
CA	0.007	0.902	51.35	0.670	0.914
PUFM	0.000	0.936	59.20	0.683	0.938
IC	0.002	0.900	56.00	0.681	0.911
CGI	0.014	0.862	60.01	0.733	0.882
Mean	**0.006**	**0.900**	**56.6**	**0.692**	**0.911**

From Table 3.21, with a mean value of 0.006, the determinants of the R-matrices (DRM) of the instructional strategies that predicted constructivism were all greater than 0.00001. The sample for the analysis was adequate with KMO values for each instructional strategy greater than 0.7 with a mean value of 0.900. The total distributions of the variance over the extracted factors (CRSSL) were more than 50% with a mean percentage value of 56.6. The Cronbach's Alpha (CA) indexes whose mean value was 0.911 and higher than 0.7 had values ranging between 0.882 and 0.938 for the instructional. Finally, the factor loadings (FL) for the instructional strategies had values higher than the threshold of 0.6 with a mean value of 0.692.

Table 3. 22: Index for Constructivism

Endogenous Construct	DRM	KMO	CRSSL (%)	FL	CA
Constructivism	**0.014**	**0.873**	**56.10**	**0.708**	**0.886**

The endogenous construct of constructivism, which measured teacher-training performance, had all its indexes higher than the acceptable value, hence satisfied the criteria for conducting factor analysis as indicated in Table 3.22. The Cronbach's Alpha (CA) value of 0.886 indicated that the items for constructivism were internally consistent hence the questionnaire was reliable. Table 3.23 exhibited the various indexes for each statistic in the case of teacher quality predictors.

Table 3. 23: Indexes for Teacher Quality Predictors

TQ Predictors	DRM	KMO	CRSSL (%)	FL	CA
SDT_IM	0.010	0.871	61.1	0.712	0.891
SDT_EM	0.057	0.892	56.2	0.748	0.869
RPK	0.041	0.885	54.7	0.682	0.877
Mean	**0.036**	**0.883**	**57.3**	**0.714**	**0.879**

According to Table 3.23, the determinants of the R-matrices (DRM) for the foundations to the instructional strategies had each of the predictors higher than the threshold value of 0.00001 with a mean value of 0.036. The sample adequacy (KMO) value for each teacher quality predictor was higher than the threshold of 0.7 with a mean value of 0.883. In the case of Cumulative Rotation Sums of Squared Loadings (CRSSL), the total distributions of the variance over the extracted factors for each of the predictors were all higher than 50% with mean value of 57.3%. The factor loadings (FL) registered a mean value of 0.714 for all of the foundation to the instructional strategies with each of them higher than the acceptable value of 0.6. Finally, the Cronbach's Alpha (CA) for the predictors had a mean value of 0.879 with values hovering between 0.869 and 0.891 for each predictor.

Table 3. 24: Teacher Quality Index for Combined Sample Items

Exogenous Construct	DRM	KMO	CRSSL (%)	FL	CA
TQ	**0.020**	**0.869**	**62.95**	**0.744**	**0.874**

The exogenous construct of teacher quality which measured the mediator constructs of CA, PUFM, IC and CGI, had all their indexes higher than the standard values and therefore qualified to undergo factor analysis. In addition, the items that defined teacher quality were internally consistent with a value of 0.874 as seen in Table 3.24.

3.10 Content Validity

The items of the combined sample that were retained were later sent to experts in the field of mathematics education to ascertain as to whether the items in the questionnaire actually explained the constructs, hence the content validation to calculate the content validity index (CVI) as in Appendix G. The selection of individuals to critique and review the questionnaire was based on their expertise and knowledge in the research topic under study. To calculate the content validity index, a validation form was sent to six (6) experts through the mail with the provision of clear instructions and definitions of the constructs that were to be measured (Yusoff, 2019). The experts reviewed the items under each construct and provided scores for each item to indicate the items' relevance or otherwise to the constructs, on a 4-point ordinal scale with the labels as advocated by Davis (1992);

where 1=not relevant, 2=somewhat relevant, 3=quite relevant, and 4=highly relevant. In the calculation, a choice scale of 3 or 4 by the experts attracted a score of 1 for analysis by the researcher and a choice scale of 1 or 2 by the experts attracted a score of 0 for analysis by the researcher (Waltz & Bausell, 1981). Two types of content validity index (CVI) were then calculated; thus content validity index of individual items (I-CVI) and the content validity index of the overall (S-CVI) scales (Lynn, 1986) as shown in Table 3.25 and Table 3.26. This instrument's content validation was done through a non-face-to-face approach with respect to an acceptable S-CVI and I-CVI values of 0.83 (Polit & Beck, 2006; Polit et al., 2007). In this study, the acceptable value of 0.83 for I-CVI was considered.

Table 3. 25: Content Validity Report on Predictors of Constructivism

Predictors of CONST	**Expert 1**	**Expert 2**	**Expert 3**	**Expert 4**	**Expert 5**	**Expert 6**	**AER**	**I-CVI**	**S-CVI**
CA	1.00	0.93	1.00	0.71	0.79	0.86	5.29	0.88	0.64
PUFM	1.00	1.00	1.00	0.80	0.87	0.80	5.47	0,91	0.73
IC	1.00	0.92	0.92	0.77	1.00	0.92	5.54	0.92	0.62
CGI	1.00	0.70	1.00	0.90	0.80	0.90	5.30	0.90	0.30
Averages	**1.00**	**0.89**	**0.98**	**0.80**	**0.86**	**0.87**	**5.40**	**0.90**	**0.57**

AER – Average Expert Response

I-CVI- Individual-Content Validity Index

S-CVI- Sum-Content Validity Index

The I-CVI of the instructional strategies for each expert ranges between 0.88 for CA and 0.92 for IC with a mean value of 0.90 which was above the threshold of 0.83. However, the S-CVI which was 0.57 according to the table was the overall average of the universal agreement for all the experts for each instructional strategy with its value less than the 0.83 acceptable levels. Therefore, with regards to this study, the content validity index was calculated using the I-CVI as indicated by Davis (1992). So, the predictors of constructivism were content valid.

Table 3. 26: Content Validity Report on Constructivism Construct

Endogenous Constructs	**Expert 1**	**Expert 2**	**Expert 3**	**Expert 4**	**Expert 5**	**Expert 6**	**AER**	**I-CVI**	**S-CVI**
CONST	1.00	0.91	1.00	0.82	1.00	0.91	5.64	0.94	0.64

The I-CVI of constructivism construct for each expert ranges between 0.82 and 1.00 with a mean value of 0.94 which was above the threshold of 0.83 (Davis, 1992). However, the S-CVI was 0.64 according to Table 3.26 which was not considered in this study.

Table 3. 27: Content Validity Report on Teacher Quality Predictors

redictor s of TQ	**Expert 1**	**Expert 2**	**Expert 3**	**Expert 4**	**Expert 5**	**Expert 6**	**AER**	**I-CVI**	**S-CVI**
SD_IM	1.00	1.00	1.00	0.70	0.70	0.80	5.20	0.87	0.40
SD_EM	1.00	1.00	1.00	0.86	1.00	0.86	5.71	0.95	0.71
RPK	1.00	1.00	0.90	0.90	0.90	0.90	5.60	0.93	0.60
Averages	**1.00**	**1.00**	**0.97**	**0.82**	**0.87**	**0.85**	**5.50**	**0.92**	**0.57**

The content validity for the predictor constructs of teacher quality had the I-CVI for each expert ranging between 0.82 and 1.00 with a mean value of 0.92 which was above the threshold of 0.83 (Davis, 1992). According to Table 3.27, the I-CVI constructs that define teacher quality was 0.92 with S-CVI value of 0.57. However, the S-CVI which was 0.57 according to the table was the overall average of the universal agreement for all the experts for each predictor construct with its value less than the 0.83 acceptable levels as indicated in Table 3.27 and therefore not considered in the study.

Table 3. 28: Content Validity Report on Teacher Quality Construct

Constructs	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6	AER	I-CVI	S-CVI
TQ	1.00	0.79	1.00	0.79	0.79	0.79	5.14	0.86	0.43

For teacher quality, the experts agreed on 86% of the items as relevant according to the I-CVI even though the S-CVI value was 0.43 which was less than the agreed value of 0.83. Content validity estimates focused on the extent to which the experts are consistent in their responses to the items with respect to the rating scale. A method of confirming content validity when there are multiple experts is to measure the internal consistency of the extracted items using the Cronbach's Alpha coefficient and others (Waltz et al., 2005). These coefficients shown in Table 3.29 are confirmed by Total Variance Explained and Scree Plots as seen in Appendices O and P.

Table 3. 29a

Constructivism Constructs	R-Matrix Det.	KMO	Cronbach's Alpha		Factor Loadings	
			Factor 1	Factor 2	Factor 1	Factor 2
CA	0.006	0.903	0.884	0.850	0.643	0.645
PUFM	0.000	0.936	0.892	0.875	0.681	0.636
IC	0.001	0.900	0.850	0.850	0.647	0.653
CGI	0.008	0.862	0.820	0.800	0.633	0.688

Table 3. 30b

Teacher Quality Constructs	R-Matrix Det.	KMO	Cronbach's Alpha		Factor Loadings	
			Factor 1	Factor 2	Factor 1	Factor 2
SDT-IM	0.013	0.871	0.891	0.882	0.610	0.615
SDT-EM	0.130	0.892	0.839	0.842	0.622	0.625
RPK	0.028	0.885	0.825	0.822	0.601	0.599

Table 3. 31c

Exogenous Constructs	R-Matrix Det.	KMO	Cronbach's Alpha		Factor Loadings	
			Factor 1	Factor 2	Factor 1	Factor 2
Constructivism	0.029	0.873	0.798	0.805	0.665	0.617
Teacher Quality	0.001	0.869	0.803	0.817	0.634	0.621

3.11 Retained items for Constructs

From Tables 3.29, all the indicator values of the constructs were higher than the acceptable values of 0.7 for KMO, 0.6 for factor loadings and 0.7 for Cronbach's Alpha; hence the questionnaire was reliable and valid. These are seen in the correlation matrices in Appendices M. The final number of items administered to the study sample were 14 for CA, 15 for PUFM, 13 for IC, and 10 for CGI; all making a total of 52 items. With respect to teacher quality predictors, the final number of items for each these predictors were 10 for relevant previous knowledge (RPK), 10 for internal motivation (IM) and 7 for external motivation (EM). Lastly, the number of items that defined constructivism and teacher quality was 11 and 14 respectively. The following items which consequently explained the various constructs were therefore administered to the study sample of the third year teacher-trainees of Akatsi, Peki and St. Francis Colleges of Education.

3.11.1 Instructional Strategies (Constructivism Predictors)

Cognitive Activation (CA)

CA is the ability of students to espouse their own theory to solve problems through tutors' support and encouragement.

Cognitive Activation (CA)

Ability of students to espouse their own theory to solve problems through tutors' support and encouragement

N/S/N	O/S/N	ITEMS
1	5	I was encouraged by my tutor to reflect on problems that require thinking for extended time in order to solve the problems.
2	1	My tutor approached mathematics teaching in variety of ways which supports my understanding of mathematical concepts.
3	22	Good organization of learning activities by my tutor gave me a high level of clarity in the learning objectives.
4	21	My tutor considered what I know and not my ignorance to teach mathematics.
5	2	My tutor appreciated different aspects of espousing mathematical ideas.
6	3	My tutor's teaching skills enabled me to solve mathematics problems.
7	12	My tutor empowered me to use fruitful discussions as a way of discovering problem solving techniques.
8	23	My tutor represented the subject in varied ways to respond to my needs.
9	17	My tutor developed effective instructional processes to enable me understand mathematics.
10	8	My tutor expected me to always explain why I chose particular

		methods in solving problems.
11	7	My tutor always wanted me to explain how I solve mathematics problems.
12	15	I was allowed to predict possible ideas to solving mathematical problems.
13	19	My tutor adopted positive approaches to correcting my errors and misconceptions.
14	20	I am made to focus on the methods that were used to get the answers than simply focusing on the answer itself.

N/S/N- New Serial Numbers of Items

O/S/N- Old Serial Numbers of Items

Instructional Coherence (IC)

Teachers' capacity to ensure smooth flow of mathematical concepts in every lesson on the basis of connecting the old to the new

N/S/N	O/S/N	ITEMS
1.	84	My tutor made connections between the mathematical theories and practices.
2.	82	My tutor used activities that focused on challenging my mathematical thinking.
3.	83	My tutor was always consistent in managing mathematics classes.
4.	75	I noticed a smooth flow in all deliveries of mathematics concept by my tutor.
5.	74	Tests and examinations always reflected the objectives of lessons.

6.	80	My tutor demonstrated practical support that linked mathematics teaching to learning.
7.	81	My tutor exhibited continuity in contents of the learning process.
8.	65	My tutor portrayed the subject as a collection of dynamic and continuous knowledge.
9.	64	The contents of a lesson reflect the stated objectives.
10.	61	My tutor used all necessary links to enable me understand mathematics concepts.
11.	67	My tutor consistently communicated with me using mathematical language.
12.	68	My tutor's combination of the curriculum and his positive perception about mathematics improve my understanding of the subject.
13.	73	My tutor's instructional practices informed me about how I am expected to teach mathematics to pupils.

Profound Understanding of Fundamental Mathematics (PUFM)

PUFM is the ability of teachers to build basic mathematical foundation from first principles for students' understanding.

N/S/N	O/S/N	ITEMS
1.	34	34. My tutor is clear in his mathematical knowledge and thoughts.
2.	35	35. My tutor represented mathematics content in a way that made me to understand.
3.	50	50. My tutor had the ability to carry out tasks of deep mathematics teaching.

4.	47	My tutor had broader approaches to explain mathematics concepts.
5.	33	My tutor had the capacity to explain mathematics contents to me.
6.	46	My tutor's mathematical understanding afforded him a more varied ways to represent mathematics concepts to me.
7.	49	My tutor did not only calculate correctly but also explain to me what it takes to get correct answers to problems.
8.	59	The teaching model my tutor adopted helped me to understand mathematics concept.
9.	36	My tutor taught mathematics from basic or first principles.
10.	42	My tutor explained to me the 'how' of solving mathematics problems.
11.	39	My tutor demonstrated the basic principles that underlie basic mathematics operations.
12.	40	40. My tutor established the basic principles underlying patterns and functions.
13.	43	My tutor clarified the 'why' of solving mathematics problems to me.
14.	48	My teacher used my previously disjointed knowledge to assist me to understand a particular mathematics topic.
15.	45	My tutor explained the breadth, depth, and flexibility of any mathematics topic.

Cognitive Guided Instruction (CGI)

Balancing knowledge acquired culturally with that obtained through scientific methods or classroom activities.

N/S/N	O/S/N	ITEMS

1.	94	My tutor's content knowledge and pedagogical skills in conjunction with my knowledge allowed him to design interesting lesson plans in mathematics.
2.	95	My tutor's incorporation of his own ideas in presenting lessons made me to enjoy learning mathematics.
3.	88	My tutor understood that solving problems depended on my developmental stages.
4.	86	My tutor fused my cultural mind-set into mathematics concepts
5.	93	My tutor made use of practical inputs during mathematics lessons
6.	91	My tutor presented lessons with lots of activities which were intertwined with discussions.
7.	106	My tutor's instructions were integral to everyday life.
8.	107	My tutor combined cultural environment and mathematical content for instructions.
9.	104	My tutor's mathematics lessons were always related to the community or environment.
10.	108	My tutor took me through thoughtful mathematics reflection for cultural compatibility.

3.11.2 Foundation to Instructional Strategies (Teacher Quality Predictors)

Relevant Previous Knowledge (RPK)

Previous knowledge or experience used to facilitate understanding of current mathematical concept

N/S/N	O/S/N	ITEMS

1.	16	My tutor showed me the interconnection among mathematics concepts to deepen my mathematical understanding.
2.	17	My tutor related mathematical concepts to everyday life.
3.	13	My tutor has a concept of connectivity to teach the subject dynamically.
4.	4	My tutor reminded me to understand basic elementary mathematics principles which recur throughout mathematics learning.
5.	15	My tutor established that current subject-matter is linked to previous ones.
6.	11	My tutor linked new information to old ones that I already have.
7.	8	My tutor gave me challenging tasks in which I applied my previous knowledge to solve.
8.	10	My tutor used my existing knowledge and ideas to explore new mathematics concepts.
9.	9	My tutor provided me with challenging tasks using my existing experiences.
10.	7	My tutor used my fundamental knowledge as basis for planning subsequent lessons.

SDT-Internal Motivation (SD_IM)

Theory that enables students to learn liberally with personal interest

N/S/N	O/S/N	ITEMS
1.	41	My belief systems have positive bearing on my ability to understand mathematics.
2.	40	My mathematical achievement is due to the motivation I have from within.
3.	46	Learning mathematics is in line with my own values and internal regulation.
4.	39	I have control over solving mathematical problems.
5.	44	Internal satisfaction made me to enjoy mathematics.
6.	42	My attitude towards mathematics is positive.
7.	38	I have good perceptions about mathematics.
8.	29	I have interest and joy in learning mathematics.
9.	30	I learn mathematics for internal satisfaction.
10.	32	I have self-satisfaction whenever I learn mathematics.

SDT-External Motivation (SD_EM)

Theory that enables students to learn with support from other people or factors

N/S/N	O/S/N	ITEMS
1.	50	My tutor gave me enough confidence using variety of challenging activities that made me to understand mathematics.
2.	57	I am motivated to learn mathematics because there is positive learner-tutor relationship.

3.	53	My mathematics learning is sustained because I accepted the available social environment.
4.	66	I always considered my tutor's recommendations as source of encouragement to understand mathematics.
5.	60	My tutor's mathematics activities gave me confidence for desired learning outcomes.
6.	88	A favourable environment was created for me to learn from my mistakes.
7.	69	Classroom activities adopted by my tutor influenced my mathematics understanding.

3.11.3 Exogenous Constructs

a. Constructivism

Educational instruction that comprises numerous and diverse instructional strategies that help students to construct their own understanding of concepts

N/S/N	O/S/N	ITEMS
1.	16.	I had the opportunity to identify what I needed to learn.
2.	15.	I was instrumental in constructing my own mathematical ideas.
3.	19.	I have the ability to make connections among mathematical concepts and procedures.
4.	17.	I identified how mathematics concepts are applied in different situations.
5.	20.	I summarized lessons to indicate my understanding of mathematical concepts.
6.	18.	I think critically to enhance the understanding of mathematical

		concepts.
7.	8.	I have high academic achievement because I display my mathematical potential.
8.	4.	I possess a set of broad learning techniques to solve mathematics problems.
9.	7.	I make personal choices to perform mathematical activities convincingly without pressure.
10.	6.	My acquisition of knowledge and skills help me to accomplish mathematics tasks.
11.	9.	Solving simple mathematical problems encourage me to learn.

b. Teacher Quality

Teacher's expertise to deliver quality instructions for effective learning by students

N/S/N	O/S/N	ITEMS
1.	15	My tutor asked stimulating questions in order to excite me to learn mathematics.
2.	14	My mathematics tutor understood what he was about to teach.
3.	22	My tutor's used effective pedagogical skills to teach mathematics to my understanding.
4.	16	My tutor encouraged me to analyze mathematics problems.
5.	36	My tutor's competences in mathematics assured me of my profession as a teacher.
6.	35	My tutor did not continue to teach anytime he was confused in a lesson.
7.	3	My tutor has a sound outlook towards the teaching of mathematics.

8.	7	My tutor's content-knowledge in mathematics was connected to my learning strategies.
9.	8	My tutor is knowledgeable about my developmental stage in learning mathematics.
10.	9	My tutor prepared me to work in groups during mathematics assignments.
11.	2	My tutor has a good understanding of my mathematical knowledge.
12.	12	My tutor's mathematical beliefs have mostly influenced my learning strategies.
13.	5	My tutor knew about my interests so he taught effectively.
14.	6	Knowing my background informed my tutor about how to teach.

The above reliable and validated items in the questionnaire were substantiated by factor analysis conducted as in Appendices G, H, and O.

3.12 Data Collection

To examine the instructional strategies that college tutors use in teaching mathematics, primary data were collected from the respondents through the use of structured questionnaire which provided answers to the research questions using the conceptual model. The structured questionnaire was mainly associated with quantitative research, which was concerned with numbers (Norland-Tilburg, 1990) to investigate the degree of constructivism in mathematics lessons through CA, PUFM, IC and CGI instructional strategies at the colleges of education.

Participants for the study were third year teacher-trainees of the 2019/2020 academic year and the last batch of the Diploma programme who took some mathematics courses in the first and second years in the Colleges. In each of the college, a tutor was assigned to assist the researcher. With a convenience sampling technique, primary data were collected from the respondents through the use of self-administered questionnaire which were based on the research questions and the conceptual framework to measure the degree of constructivism that is being employed during mathematics lessons in the colleges of education. The participants responded to the questionnaire with no interference by the researcher even though he was present at the time of the exercise. With this non-interference, the items were not read out to the participants. Respondents to the questionnaire were those who were present at the time of administering the questionnaire when they were being taken through an orientation programme on off-campus teaching practice. The respondents studied mathematics in the first and second years and therefore had enough knowledge to respond to the questionnaire. A day each was dedicated for the three colleges to collect the data. Most tutors were present in the halls when the teacher-trainees responded to the questionnaire and this motivated the participants as they took the exercise seriously. Before the start of the exercise, the researcher and the tutors took time to explain to the participants the rationale behind the data collection. This action also motivated the respondents to fully participate in the exercise. The reliable and content valid questionnaire with the reduced variables was administered to the third year teacher-trainees of the three colleges of education; namely, Akatsi, Peki and St. Francis Colleges of Education which formed the sample for the study.

3.13 Ethical Considerations

Permission was sought from the various Principals of the Colleges of Education to conduct the study. When the statement of consent was reviewed, participants (teacher-trainees) were introduced to the objectives of the study and their consent was sought before they took part in the study. Consent forms to the participants therefore explained the reason for the research and were assured of the confidentiality of their responses. They were however, advised of their right to terminate the response process at any time.

3.14 Data Analysis

Data analysis requires a rigorous scientific approach which depends on knowledge of statistics, mathematics, measurement, logic, theories, experience, intuition and other variables that affect a situational context (Hair, Hult, Ringle & Sarstedt, 2014). Survey items were used to gather qualitative and quantitative data from third-year teacher-trainees of the 2019/2020 academic year. Statistical and computational analyses were performed to observe some occurrences which answered the research questions. The data collected in section A were qualitative but were quantified for the analysis. In section B, the data collected were quantitative and were analyzed using the confirmatory factor analysis, binomial test, descriptive statistics and the Partial Least Squares of the Structural Equation Modelling (PLS-SEM) as the research assumed a positivist epistemology.

3.14.1 Factor Analysis

Factor analysis is a statistical data reduction and analysis technique that strives to unearth underlying factors and explain correlation among multiple outcomes that results in a reduction of items into fewer numbers of dimensions. It attempts to discover the unexplained factors that influence the co-variation among multiple observations. These factors represent underlying concepts that cannot be adequately measured by a single variable. Thus it is used to simplify data, such as reducing the number of variables in regression models. For example, various measures of political attitudes may be influenced by one or more underlying factors. This statistical technique identified factors which were measured by a number of observed variables with their respective responses using the extraction and rotation methods (Field, 2005). The use of factor analysis depended on sample size (Field, 2005; Costello & Osborne, 2005; Tabachnik & Fidel, 2001) when the variables of the instrument were measured on an interval scale (Field, 2009) such as the discrete likert-scale (Ratray & Jones, 2007). The confirmatory factor analysis (CFA) is a special form of factor analysis, most commonly used to test whether the measurement of a construct are consistent with a researcher's understanding of the construct or factor. McCallum et al., (1999) indicated that when communalities after factor extraction are above 0.5, sample size between 100 and 200 is acceptable for factor analysis. However, if communalities after factor extraction are below 0.5, a sample size of about 500 is required for factor analysis.

In this study, sample size of 221 was considered. In effect, a KMO value of below 0.5 requires that more samples are to be collected. Also, a good sample size can be detected by KMO measure of sampling adequacy whose value must be above Kaiser's (1974) recommendation of 0.5 (Field, 2009). KMO represents the ratio of the squared correlation between variables to the squared partial correlation between variables (Field, 2009). Thus, KMO statistic varies between 0 and 1 such that when it is zero, the sum of partial correlation is large relative to the sum of correlations, hence factor analysis is inappropriate. However, a value close to 1 indicates that patterns of correlations are relatively compact; therefore factor analysis yields distinct and reliable factors. According to Hucheson & Sofroniou (1999), KMO value of between 0.5 and 0.7 is mediocre, between 0.7 and 0.8 is good, between 0.8 and 0.9 is great and above 0.9 is superb.

The next condition for factor analysis is the normality of the dataset which is hindered by the discrete nature of likert-scale scores which is not supposed to generalize the results beyond the sample (Field, 2009) and it is also not able to conduct a maximum likelihood factor analysis (Costello & Osborne, 2005). However, a p-value lower than 0.05 becomes a proof of normality when discrete scores are used hence making the normality of the dataset to be guaranteed and generalizable to the population (Field, 2009). One test to determine high or low correlation between variables is Bartlett's Test of Sphericity which compares the correlation matrix to the identity matrix with a determinant greater than 0.00001. Bartlett's Test of Sphericity is the test for null hypothesis that the correlation matrix has an identity matrix that ensures that the data is normally distributed. Essentially, it checks to see if there is a certain redundancy between the observed variables that can be summarized with a few numbers of factors. In this instance, the null hypothesis of the test is that the variables are not correlated and therefore rejected (Snedecor & George, 1989). In fact, there are relationships between two variables if the correlation coefficients of variables are not zero. So, for Bartlett's test to be significant, the p-values must be less than 0.05 indicating that the R-Matrix is an identity matrix. Bartlett's test therefore indicates the largeness of correlations between items telling us whether the correlation matrix is sufficiently different from an identity matrix. For instance, if Bartlett's Test of Sphericity has Chi-square value of approximately 3015.282 with a degree of freedom as 378 and significance level of 0.000, then ($X^2(378)=2989.77$, $p<0.001$) indicates that correlations with the R-matrix are sufficiently different from zero or identity matrix to warrant factor analysis. Kaiser's criterion is accurate when variables are less than 30 with communalities greater than 0.7 after factors are extracted. In addition, factor extraction takes place when sample size exceeds 250 with average communalities greater than 0.6 (Kaiser, 1974) such that the higher the communalities, the higher the number of factors extracted. However, Field (2005) reviewed many suggestions to indicate that about 300 cases are adequate for factor analysis with communalities of above 0.5.

To screen a dataset for factor analysis, we first look at the inter-correlation between observed variables such that the items extracted measure the same constructs. There is one set of R-matrices with two rows; the first row contains the Pearson Correlation Coefficients between all pairs of items while the second row contained one-tailed significance of these coefficients. A variable whose correlation coefficient is close to 0.8 is deleted and any variable whose significance value is greater than 0.05 is also deleted due to multicollinearity or singularity because these items do not correlate. In other words, factor analysis becomes problematic if the correlation coefficient of variables in the matrix are $r>0.8$ or $r<-0.8$ (highly correlated) or $0.3<r<0.3$ (lowly correlated).Any determinant of an R-matrix which is less than 0.00001is also checked for multicollinearity which is a problem in factor analysis (Field, 2005). However, multicollinearity does not matter when researchers use Principal Component Analysis. Extraction of factors is based on Kaiser's criterion of retaining variables when the eigenvalues are greater than 1 portraying the unidimensionality of the variables (Ravand & Baghaei, 2016). Before extraction, SPSS identify linear components that are equal to the number of variables within the data set. These variables are depicted by the eigenvalues which indicate how much a variable is explained. After extraction, SPSS displays the number of factors that have been extracted with eigenvalues greater than 1. In this instance, the percentage variable explanation by the first factor assumes a higher value than all other factors. The factors that explain the highest proportion of variance of the variable share are expected to represent the underlying constructs. Thus, if a factor explains lots of variance in a dataset, variables correlate highly with that factor or load highly on that factor. To improve the interpretability of extracted factors, the rotation technique is used to maximize the factor loading of each variable on each of the factors. The rotated solutions give the factor loadings for each variable in the dataset, which are used to interpret the meaning of the factors (Field, 2005). The rotation method gets factors that are different from each other, and helps to interpret them by putting correlated variable primarily under each of the factors. When a researcher decided that the extracted factors must be related, oblique rotation is used but when the extracted factors are to be independent, varimax rotations is adopted. In this study, the varimax rotation was used (Field, 2009; Tabachnik & Fidell, 2001; Reitvield &Van Hout, 1993).

3.14.2 Partial Least Square-Structural Equation Modelling (PLS-SEM)

Developing the statistical underpinnings of PLS-SEM, Wold (1975, 1982, 1985) and Lohmoeller (1989) aim at maximizing constructs to explain the variables by adopting an ordinary least squares estimation method (Ravand & Baghaei, 2016). SEM was considered as a second multivariate analytical technique that permits answering sets of interrelated research questions in a single, systematic and comprehensive manner that simultaneously analyze multiple variables to present measurements that are associated to the constructs. This involves the application of statistical methods such as factor analysis and regression (Hair, Hult, Ringle & Sarstedt, 2014). Combining factor analysis, regression analysis and advanced statistical analysis techniques like SEM, enable the researcher to simultaneously examine the relationships among observed and latent variables as well as between latent variables while taking measurement errors into account (Ravand & Baghaei, 2016). Partial Least Squares (PLS) is an approach to Structural Equation Models (SEM) that allows researchers to analyze relationships simultaneously using path models to visually display the hypotheses and variable relationships that are under examination (Hair, Hult, Ringle & Sarstedt, 2014), and ultimately estimating complex cause-effect relationship with latent variables. PLS-SEM as a variance based approach works efficiently with small data size and makes use of complex models with no assumptions about normal distributions (Cassel, Hackyl & Westlund, 1999). Hair, Ringle & Sarstedt (2011) also suggested that PLS-SEM is used when the research goal is about theory development and extension which involves the prediction of key constructs, when the models are structurally complex. PLS-SEM has a user-friendly software package, SmartPLS which requires little technical knowledge about the method as compared to other software packages (Ringle et al., 2015; Ringle et al., 2005).

a. Model Evaluation

Model evaluation is a two-step process in which reflective measurement and structural models are appraised. The dominant statistical tool in the context of confirmatory research is the test for overall model fit, where the number of correlations among observed variables exceeds the number of model parameters to be estimated. In a typical empirical research, these two types of research can be theoretically distinguished when they are combined such that the reflective measurement model is considered as confirmatory research and structural model as explanatory research (Beniteza, Henselerc, Castillob & Schuberthc, 2019). Employing the most recently proposed standards, this study considered PLSSEM as a causal confirmatory and explanatory research for data analysis. In the confirmatory research, the researcher aimed at understanding the causal relationships between theoretical concepts to confirm an assumed theory in order to obtain empirical evidence for the operational definition of the latent variables (Beniteza, Henselerc, Castillob & Schuberthc, 2019). Accordingly, this is done by imposing testable limits on the indicators and fixing path coefficients to a certain value, where it is assumed that the correlation between two indicators is the result of an underlying latent. Explanatory research also aims at understanding the causal relationship among the theoretical concepts but there are emphases on explaining specific phenomena which are treated as dependent variables in the structural model with the primary focus on the coefficient of determination (R^2) and the significance of path coefficient estimates.

The reflective measurement model in this research was evaluated to the extent that indicators were unidimensional; thus the latent variables explained the variations in the indicators through convergent validity (Ravand & Baghaei, 2016).

The convergent validity was the average variance extracted (AVE) from a set of indicators which explained more than one half of their variance (Fornell and Larcker, 1981). AVE which was the mean of the communalities of the indicators and associated with a given construct explained at least half of the variance of its observed variables. Thus, an AVE of 0.5 or higher was regarded as acceptable. The unidimensionalilty of the study was also assessed by examining the Cronbach's Alpha and composite reliability statistics whose indices were equal to 0.7 or higher and whose principal component analysis of each construct had the first eigenvalues greater than 1 while the subsequent ones were lesser than 1. In other words, the measurement model evaluation established convergent validity which was measured by checking how much of the indicators' variance a given construct had a factor loading of 0.7 or higher (Nunnally & Bernstein,1994). Consequently, observed variables with an outer or factor loading of 0.7 or greater were believed to be greatly acceptable (Hair, Sarstedt, Ringle & Mena, 2012), while the outer loading with a value less than 0.7 were discarded (Chin, 1998). Another most important reliability measure for PLS is ρA (Dijkstra & Henseler, 2015b); which was the only consistent reliability measure for PLS construct scores. In particular, Cronbach's Alpha typically underestimated the true reliability and therefore is regarded as the lower boundary to the reliability with composite reliability as the upper boundary (Sijtsma, 2009). Composite reliability is a measure of internal consistency in scale items, much like Cronbach's Alpha (Netemeyer, 2003). It can be thought of as being equal to the total amount of true score variance relative to the total scale score variance (Brunner & Süß, 2005).

A second quality criterion of path model analysis has to do with establishing theoretical differences in concepts between any pair of exogenous latent constructs. This difference in relationship raises the issue of discriminant validity (Henseler, Ringle & Sarstedt, 2015). This is an indication that the manifest variables that define any construct are distinct from variables in other constructs of the path model (Sarstedt, Ringle, Smith, Reams & Hair, 2014). The standard therefore, suggests that a construct should not show the same variance as any other construct which has high AVE value. Three criteria, the Fornell and Larcker criterion, cross-loadings (Fornell & Larcker, 1981) and the HTMT (Henseler et al., 2015) are considered to show discriminant validity. The Fornell-Larcker criterion says that a factor's AVE should be higher than its squared correlations with all other factors in the model. Thus, the quality of the reflective model which is shown by the square root of the AVE of each construct in the diagonal matrix must be higher than the related correlation in the corresponding rows and columns of the matrix (Fornell & Larcker, 1981). However, according to Ringle, Sarstedt and Straub (2012), the Fornell–Larcker criterion is ineffective in assessing the quality of the model because it relies on consistent factor loading estimates (Henseler et al., 2014). To solve this problem, Heterotrait-Monotrait (HTMT) ratio is developed to assess discriminant validity in the case of variance based estimators (Henseler, Ringle & Sarstedt, 2015) in two ways: (1) by comparing the variance-based estimates to a threshold value of 0.90 if constructs are conceptually very similar or 0.85 if the constructs are conceptually more distinct (Gefen, Rigdon & Straub, 2011) and (2) by constructing a confidence interval to examine whether HTMT is significantly smaller than 1 or below 0.85 or 0.90 (Ringle, Sarstedt & Straub, 2012). It is therefore concluded that HTMT is a reliable tool for assessing discriminant validity, whereas the Fornell–Larcker criterion has limitations that do not justify its reputation for rigor and its widespread use in empirical research (Beniteza, Henselerc, Castillob & Schuberthc, 2019). More precisely, the HTMT which is an upper boundary estimate for factor correlations clearly discriminate between two factors, with its value significantly smaller than one. The third discriminant validity test for a path model is the cross-loadings which are assessed to ensure that no indicator is incorrectly assigned to a wrong factor. Thus, the cross loadings of a PLS path model is evaluated when each measurement item correlates weakly with all other constructs except for the one to which it is theoretically associated (Gefen & Straub, 2005). These statistics are first proposed by Barclay et al. (1995) and Chin (1998) that each indicator loading should be greater than all of its subsequent cross-loadings. Bootstrapping as a non-parametric inferential technique is applied in order to obtain inference statistics for all model parameters with the assumption that the sample distributions convey information about the population distribution. Consequently, bootstrapping is the process of drawing a large number of sub-samples, with replacement of the original sample, and then estimating the model parameters for each sub-sample (Henseler, Hubona & School, 2015). In this instance, the path coefficients are evaluated for significance if the research is to be generalized to a population. The 4,999 bootstrap samples are sufficiently close to infinity for usual situations, and amenable to computation time which allows for a unanimous determination of empirical confidence intervals of 2.5% at the lower level and 97.5% at the upper level. In other words, to conduct the same test using a 95% confidence interval, the lower and upper limits of the confidence interval are calculated. If the value 0 (zero) does not fall within this interval (i.e., $0 \notin CI$) the null hypothesis is accepted, otherwise if $0 \in CI$, the null hypothesis is rejected (Kock, 2016). Hypothesis testing in inference statistics and in the context of PLS-SEM is usually conducted through the calculation of one- or two-tailed p values for each path coefficient to include empirical bootstrap confidence intervals (Kock, 2015b). Path coefficient is essentially a standardized regression coefficient, which is assessed with regards to their sign and absolute value. It is interpreted as the change in the dependent variable, if the independent variable is increased by one when all other independent variables remain constant (Ringle & Sarstedt, 2017). The greater the β value, the more substantial the effect on the endogenous latent construct. However, the β value had to be verified for its significance through the T-statistics test if the data are bootstrapped using 5000 sub-samples with no sign changes (Chin, 1998). When a hypothesis test is conducted and the path coefficient, β is greater than zero (β>0) at 5% significance level with one-tailed p value less than 0.05 ($p \leq 0.05$), the null hypothesis is accepted, otherwise it is rejected. Generally speaking, this quantity could be interpreted as the probability that β belongs to a distribution with mean of zero and standard deviation of σ. The T-ratio test can then be seen as a variation of this test, where the T-statistic is calculated as β/σ, and used instead of the corresponding p value for comparison against a threshold of 1.64 or 1.96 (Kock, 2016). The necessary condition in path modeling is to assess the "goodness" of the inner structural models such that the outer measurement model has demonstrated acceptable levels of reliability and validity. That is, there must be a sound measurement model before one can begin to assess the "goodness" of the inner structural model or to rely on the magnitude, direction, and/or statistical strength of the structural model's estimated parameters. Goodness-of-Fit (GOF) is therefore applied as an index for the complete model fit to verify that the model sufficiently explains the empirical data (Tenenhaus, Esposito, Chatelin & Lauro, 2005). The GOF values lie between 0 and 1, where values of 0.10 (small), 0.25 (medium), and 0.36 (large) indicate the global acceptance validation of the path model (Henseler, Hubona & Ray, 2016). The GOF is calculated by using the geometric mean values of the average communality (AVE values) and average R^2 as calculated by equation (1) (Tenenhaus, Esposito, Chatelin & Lauro, 2005).

$$GOF = (\text{Average } R^2 * \text{Average communality})^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The bootstrap-based tests of overall model fit indicate that the data are coherent with factor models, representing a confirmatory factor analysis. If the model does not fit the data, then the data contains more information than what the model conveys. Conducting a test of model fit is to help answer the question of how substantial the discrepancy between the model-implied and the empirical correlation matrix is. The approximate model fit criterion for PLS path modeling is the standardized root mean square residual (SRMR) which is the square root of the sum of the squared differences between the model-implied and the empirical correlation matrix (Hu & Bentler, 1998, 1999). A value of 0 for SRMR would indicate a perfect fit and generally, an SRMR value less than 0.05 with a cut-off value of 0.08 indicates an acceptable fit (Byrne, 2008).

b. Collinearity Statistics or Variance Inflation Factor (VIF)

Collinearity is simply a term used to describe two or more predictors in a regression model which are highly correlated. The VIF measures how much the variance of an estimated regression coefficient increases if the predictors are correlated. It detects multicollinearity in regression analysis when there's correlation between predictors (i.e. independent variables) in a model where its presence can adversely affect the regression results. In other words, VIF estimates how much variance of a regression coefficient is inflated due to multicollinearity in the model. It is calculated by taking a predictor variable and regress it against every other predictor in the model, giving it R-squared values. The numerical value for VIF which ranges from 1 upwards, tells about what percentage the variance is inflated for each coefficient. For example, a VIF of 1.9 implied that the variance of a particular coefficient is 90% bigger than what is expected if there was no multicollinearity. As a rule of thumb, a variance inflation factor of 1 implied that predictors are not correlated; between 1 and 5 implied that predictors are moderately correlated whilst VIF values greater than 5 implied high correlation. In general, a VIF above 10 indicates high correlation and a cause for concern where some authors suggest a more conservative level of 2.5 or above (Dodge, 2008; Everitt & Skrondal, 2010).

A PLS path model has favourable convergence properties (Henseler, 2010), however, as soon as the path models involve common factors, there is the possibility of Heywood cases (Krijnen et al., 1998); meaning one or more variances implied by the model would be negative. The occurrence of Heywood cases may be caused by an atypical or too-small sample, or the common factor structure may not hold for a particular set of indicators. If the researcher's aim is predictive then the assessment should focus on blindfolding (Tenenhaus et al., 2005) and the model's performance of holding out samples. Despite strong pleas for the use of confidence intervals, reporting p-values still seems to be more common (Cohen, 1994).

When an independent construct is deleted from the path model, it changes the value of the coefficient of determination (R^2) of the endogenous construct and defines whether the deleted latent exogenous construct has a significant influence on the value of R^2, of the latent endogenous construct. The f^2 is the degree of impact an exogenous latent construct has on an endogenous latent construct when the latter was deleted. If the effect, f^2 is 0.35, 0.15 and 0.02 then it said to be strong, moderate and weak respectively (Cohen, 1988). The coefficient of determination, R^2 therefore measures the overall effect size and the variance explained in the endogenous construct of the structural model indicating the model's predictive accuracy. According to Henseler, Ringle & Sinkovics, (2009) and Hair, Ringle, & Sarstedt, (2013), R^2 values of 0.75, 0.5 and 0.26 indicate that the results of the effect size are considered substantial, moderate, and weak respectively**.** With a value greater than zero, the predictive relevance of a model (Q^2) is used to measure the quality of PLS path model, when dataset is blindfolded (Tenenhaus, Esposito, Chatelin & Lauro, 2005). This is to find the cross-validated redundancy value in predicting the relevance of the endogenous latent construct.

Indirect effects and their inference statistics are important for mediation analysis (Zhao et al., 2010), and total effects for factor analysis (Albers, 2010). Direct effects, as the name implies, deal with the direct impact of one construct on another when the path is not mediated or transmitted through a third construct. Indirect effects can be defined as the impact of one construct on another, which is mediated or transmitted by a third construct. The total effect is the effect of an independent variable on a dependent variable, whereas a mediator is a variable that accounts for the effect of an independent variable on a dependent variable (Baron & Kenny, 1986; Hayes, 2009; Preacher et al., 2007).

CHAPTER 4

RESULTS/FINDINGS AND DISCUSSIONS

4.0 Overview

Chapter 4 presents the results of data analysis as well as discussion of key findings of the research. Results from the analysis were interpreted and discussed with respect to literature, and responses from the participants and the researcher's analytical background. The research questions of the study were therefore presented as follows:

1. What is the mathematical background of teacher-trainees in the colleges of education?
2. Which instructional strategies do college tutors mostly use in teaching mathematics?
3. What is the effect of teacher professional practice on tutors' instructional strategies?
4. What relationships exist between the instructional strategies and constructivism?
5. How do other constructs and constructivism affect teacher-trainees' performance in mathematics?

4.1 Research Question 1

What was the mathematical background of teacher-trainees in the colleges of education?

Table 4.1 shows the sample size and their percentage contribution from each of the three colleges of education. These were the numbers who responded and returned the questionnaire. In all, there were twenty-one (21) open-ended items in section A and 104 closed-ended items in section B.

Table 4. 1: Gender of Sample from the Colleges of Education

Colleges of Education	2018/2019 Population					
	Female		Male		Total	
	N	%	N	%	N	%
Akatsi	76	35.2	140	64.8	216	33.7
Peki	82	45.8	97	54.2	179	27.9
St. Francis	80	32.5	166	67.5	246	38.4
Total	**238**	**37.1**	**403**	**62.9**	**641**	**100.0**

The sample consisted of two hundred and sixteen (216) teacher-trainees from Akatsi College of Education, contributing 33.7% to the study sample, of which 35.2% were females and 64.8% were males. One hundred and seventy-nine (179) teacher-trainees came from Peki College of Education, contributing 27.9% to the sample and out of this, 45.8% were females and 54.2% were males. St. Francis College of Education, Hohoe was also made up of two hundred and forty-six (246) teacher-trainees, contributing 38.4% to the sample with 32.5% being females and 67.5% being males. On the whole, a convenience sample of 641 teacher-trainees took part in the study, making 37.1% females and 62.9% males.

The researcher looked at the age pattern of the respondents in the colleges of education as shown in Table 4.2.

Table 4. 2: Age of Respondents

Age	No. of Respondents	
	N	%
<20	34	5.9
20-24	392	61.2
25-29	166	25.9
30-34	45	7.0
Total	**641**	**100.0**

As indicated in Table 4.2, respondents numbering four hundred and twenty-six (426) who were under 25 years constituted 67.1% of the sample and two hundred and eleven (211) who were 25 years and above constituted 32.9% of the sample. It is evident from literature that more young students enter colleges because they turn to outperform the older ones (Matta, Ribas, & Sampalo, 2016). The research considered the representation of the respondents from the regions in Ghana where participants hailed from and also had their basic and secondary education.

Table 4. 3: Regional Representations of Respondents

Region	Regions where Respondents hailed from		Regions in which Respondents attended School			
			JHS		SHS	
	N	%	N	%	N	%
Volta	559	87.2	505	78.8	543	84.7
Others	65	10.1	125	19.5	88	13.8
No Response	17	2.7	11	1.7	10	1.5
Total	**641**	**100.0**	**641**	**100.0**	**641**	**100.0**

Table 4.3 shows that the majority (87.2%) of teacher-trainees for the study hailed from the Volta Region and the rest (10.1%) came from five (5) other regions which include Greater Accra, Ashanti, Eastern, Central and Brong-Ahafo (The study considered 10 regions before the year 2019, where the regions increased to 16). From the table, 78.8% and 84.7% of the respondents went to junior and senior high schools respectively in the Volta Region, revealing that the majority of teacher-trainees in the colleges of education in the Volta Region came from and went to basic and secondary schools in the Volta Region. This revelation justified the mathematics pass rates of 60.1% and 21.6% at the basic and secondary levels with 8th and 7th positions respectively on the national league table. These figures were lower than the national average pass rate of 72.6 % for BECE and 30.1 % for WASSCE for a period of five years (WAEC, 2019).

The researcher was interested in the distribution of programmes offered by the respondents at the SHS level as indicated in Table 4.4. This is because studies have shown that students who read sciences have a more positive attitude towards and understanding of mathematics than those in the humanities (Karjanto, 2017).

Table 4. 4: Programme offered at SHS

Programme	**Respondents**	
	No.	**%**
General Arts	287	44.8
Business	175	27.3
Sciences	130	20.3
Home Economics & Visual Art	46	7.1
Total	**641**	**100.0**

One most important criterion for offering science and technical programmes is a good understanding of mathematical concepts (Akinoso, 2011). From Table 4.4, only 20.3% of the respondents studied sciences at the SHS level with 44.8% and 27.3% studying Arts and Business programmes respectively and as little as 7.1% read Home Economics and Visual Art. The sciences consisted of those who studied general science, agricultural science and the technical programmes.

To form an opinion about the calibre of participants who went to the colleges, the researcher wanted to know if respondents got direct admissions to the colleges as delayed admission to college negatively affects students' performance (Burke, 2020).

Table 4. 5: Year completed SHS

Year of completion	**Respondents**		**No. of years of Staying at Home**
	N	**%**	
2004 & 2007	8	1.2	13 & 10
2009-2012	43	6.7	5-8
2013-2016	504	78.6	1-4
2017	86	13.5	0
Total	**641**	**100.0**	

From the responses as indicated in Table 4.5, only 86 (13.5%) of respondents had direct admissions into the colleges of education for the 2017/2018 academic year. This meant that the rest of the respondents stayed in the house for some number of years before getting admission into the colleges. In a particular case, as much as 78.6% of the respondents spent between one and four years at home before getting admission into the colleges. The reasons adduced to this trend are shown in Table 4.6.

Table 4. 6: Reasons for not going to College in 2017/2018 Academic Year

Reasons	N	%
Failure in all core subjects	102	15.9
Failure in Mathematics	25	3.9
Failure in English Language	68	10.6
Failure in Integrated Science	65	10.1
Failure in elective subjects	167	26.1
Financial Constraints	134	20.9
Others	25	3.9
No response	55	8.6
Total	**641**	**100.0**

The high percentage of those who didn't get direct admission have varying reasons some of which include poor passes in mathematics which had a percentage of 3.9 as indicated in table 4.6. The researcher was interested in this as it could help determine respondents' previous knowledge in mathematics and their readiness to become professional mathematics teachers.

Pass grades in mathematics was very crucial to this study as it was the core of the research. Having made the choice to become a teacher, the researcher wanted to know about the respondents' mathematics grades at both junior and senior high schools levels, since it is mandatory for all primary teachers to teach mathematics.

Table 4. 7: Pass Grades in Mathematics

Grades	Pass Grades in Mathematics			
	JHS (BECE)		SHS (WASSCE)	
	N	%	N	%
1-3	423	66.0	245	38.2
4-6	183	28.5	353	55.1
7-9	18	2.8	26	4.1
No Response	17	2.7	17	2.7
Total	**641**	**100.0**	**641**	**100.0**

The respondents' mathematics pass rate as seen in Table 4.7 indicates that 66.0% and 38.2 % passed mathematics with grades 1-3 at the Basic Education Certificate Education (BECE) and West African Secondary School Certificate Examinations (WASSCE) respectively. Comparing these figures revealed that teacher-trainees' performance at the BECE was better than that at the WASSCE as this may be due to poor understanding of mathematical concepts at the higher level. This is so because students' performance in lower secondary education is related to motivation received from knowledgeable adults to learn at school (Wijsman, Warrens & Westenberg, 2015). The indication therefore is that students at higher secondary education do not have this privileged of being motivated. From the table, 28.5% and 55.1% of the respondents had grades 4-6 at the BECE and WASSCE respectively, where grade 6 is the minimum qualification at WASSCE for admission into the colleges of education (WAEC, 2019).

Interest in teaching mathematics at the primary level is not a choice for teacher-trainees who are classroom teachers at the primary level according to the Ghana Education Service. So, the question was asked if participants were actually interested in teaching mathematics.

Table 4. 8: Do you like to teach Mathematics at the primary level?

Response	Respondents	
	N	(%)
Yes	467	72.9
No	159	24.8
No response	15	2.3
Total	**641**	**100.0**

From Table 4.8, those who did not want to teach mathematics were 159 forming 24.8%. A total of 467 respondents, representing 72.9% were interested in teaching mathematics at the primary level. This is a confirmation of a research conducted by Norton (2017) on primary school teachers' confidence in their mathematical content knowledge and confidence to teach specific primary mathematics concepts.

Table 4.9 shows the reasons why teacher trainees wish to teach mathematics at the primary level.

Table 4. 9: Reasons of wanting to teach Mathematics

Reasons	**N**	**%**
To promote personal and critical thinking skills	148	31.6
To help pupils' conceptual understanding	96	20.6
Have love, passion and interest for the subject	192	41.1
Motivation received from teachers and parents	15	3.2

No response	16	3.4
Total	**467**	100.0

From Table 4.9, 31.6% of the respondents who agreed to teach mathematics stated that the subject promotes personal and critical thinking skills, 20.6% said they want to help the pupils to have an in-depth mathematical conceptualization, 41.1% indicated their love and passion for and interest in the subject, and only 3.2% said it was due to the motivation received from their teachers and parents (Ahia & Fredua-Kwateng, 2004). The reasons adduced to interest in teaching mathematics by the respondents actually support the readiness of respondents' interest in learning the subject.

However, one hundred and seventy-four (174) respondents which constitute 27.1% of the sample said they would not like to teach mathematics due to the following reasons depicted in Table 4.10.

Table 4. 10: Reasons for not interested in teaching mathematics

Reasons	**N**	**%**
Subject is too difficult	43	24.7
Bad mathematical foundation	34	19.5
Don't like the subject	46	26.4
Other reasons	28	16.2
No response	23	13.2
Total	174	100.0

Table 4.10 indicates that 24.7% of the 174 respondents said the subject is too difficult to understand (Ernest, 1991) let alone teaching it, 19.5% did not have good mathematical foundation to enable them teach the subject, 26.4% indicated that they

just don't like the subject and 16.2% had other reasons which include inadequate resources (Azmidar et al., 2017).

One other reason that may encourage participants to teach the subject is their understanding of mathematical concepts that is seen from their performances in the colleges of education. So, Table 4.11 shows how participants performed in some mathematics courses at the colleges in their first and second years. Grades A to D, are passes with grade A, being the best, but grade E is a failure.

Table 4. 11: Mathematics Grades at the Colleges of Education

Grades	Respondents							
	1st Year				2nd Year			
	1st Semester		2nd Semester		1st Semester		2nd Semester	
	N	%	N	%	N	%	N	%
A	207	32.3	158	24.6	206	32.1	266	41.5
B	236	36.8	253	39.5	233	36.3	185	28.9
C	152	23.7	149	23.2	133	20.7	111	17.3
D	26	4.0	35	5.5	44	6.9	31	4.8
E	0.0	0.0	3	0.5	0.0	0.0	0.0	0.0
No response	20	3.2	43	6.7	25	3.9	48	7.5
Total	**641**	**100.0**	**641**	**100.0**	**641**	**100.0**	**641**	**100.0**

Table 4.11 shows that not more than 41.5% of the respondents passed mathematics courses with grade A for the two years in the colleges. This highest pass rate of 41.5% occurred in the second semester of the second year. This performance may be due to the fact that respondents improved on their understanding of mathematics concepts as they moved up. Furthermore, except for the second semester

of the second year which recorded a pass rate of 28.9% for grade B, all other semesters recorded a pass rate of not more than 39.5%. With respect to grade C, respondents' pass rates for the two years were between 17.3% and 24% and only 0.5% failed one course. In all, the average passes of 96.8% and 92.8% of the respondents were for the first and second semesters respectively in the first year. In the second year, 96.0% and 92.5% of the respondents passed mathematics courses in the first and second semesters respectively.

The teaching and learning of mathematics is crucial to every teacher and learner at all levels especially at the colleges of education. To find out how teacher-trainees received mathematics instructions from their teachers at the basic and secondary levels, the researcher asked the respondents to rate their mathematics teachers. Responses to this item are shown in Table 4.12.

Table 4. 12: Ratings of Mathematics Teachers

Rating Scale	Respondents			
	JHS		SHS	
	N	%	N	%
1-3	57	8.9	67	10.4
4-6	204	31.9	237	37.0
7-10	369	57.5	317	49.5
No response	11	1.7	20	3.1
Total	**641**	**100.0**	**641**	**100.0**

The ratings range from 1 being the lowest to 10 being the highest. At both levels, 57.5% and 49.5% of the respondents rated their teachers highly at points 7-10 at the JHS and SHS levels respectively. The table also indicates that the respondents with percentages of 31.9 and 37.0 rated their mathematics teachers on the scale of 4-6 points at the basic and senior high school levels respectively. Ratings of 1-3 which was the lowest attracted 8.9% and 10.4% for the junior and senior high schools teachers respectively. The ratings therefore indicate that teachers of mathematics at the basic and secondary levels delivered mathematics lessons well which inured to the benefit of the respondents.

Knowledge about respondents' choice of profession could also reveal if they are ready to become professional teachers which may consequently enhance academic instruction with particular reference to mathematics. Table 4.13 indicates participants' first choice of profession.

Table 4. 13: First Choice of Profession

Profession	Respondents	
	N	%
Teaching	224	34.9
Medicine	140	21.9
Accounting	66	10.3
Engineering & Security	104	16.2
Others	91	14.2
No response	16	2.5
Total	**641**	**100.0**

As the desire for the teaching profession is one of the factors that changes the educational landscape (Adegoke, 2003), the research sought to find out the first choice of respondents' profession. When the question was put only 34.9% of the respondents said teaching was their first choice profession whilst the remaining 65.1% mentioned other professions such as Medicine (21.9%), Accounting (10.3%), Engineering and Security (16.2%) and other professions (14.2%). The implication is that most of these teacher-trainees may be using the teaching profession as a stepping board to venture into their preferred professions. This means that teachers may therefore be in the classroom without their hearts and minds which may affect teaching and learning. Meanwhile, the teacher with strong disciplined knowledge and sound disposition towards teaching is the most important variable affecting student performance (Hattie, 2009).

Out of those whose first choice of profession was teaching, the researcher wished to know what informed their decision for the choice. Table 4.14 indicate the reasons assigned to the choice of their teaching profession.

Table 4. 14: Reasons for first Choice of Teaching Profession

Reasons	**N**	**%**
To impart Knowledge	116	51.8
Passion & Love	68	30.4
Motivation	19	8.5
Others	21	9.4
Total	**224**	**100.0**

For the two hundred and twenty-four (224) respondents who chose teaching as their first profession, 51.8% said they wish to impart academic and professional knowledge to the next generation, 30.4% said it is an act of love and passion for the profession, whilst 8.5% are into the teaching profession due to the motivation they received from their teachers and parents. These reasons are sufficient for good desire for the teaching profession (Balyer & Ozcan, 2014; Kyriacou & Coulthard, 2000; Thomson, Turner & Nietfeld, 2012; Yuce et al, 2013 cited in Lundstrom, Manderstedt & Palo, 2018).

For clarity and especially for all those whose first choice of profession was not teaching and yet desired to become professional teachers, the researcher was interested in knowing why they were in the colleges of education. Their responses were shown in Table 4.15.

Table 4. 15: Motivation of Respondents to become a teacher

	No. of Respondents	
Reasons	**N**	**%**
Love/ Interest	328	51.2
Mentor	94	14.7
Financial Constraints	120	18.7
No Response	99	15.4
Total	**641**	**100.0**

From the table, three hundred and twenty-eight (328) respondents forming 51.2% mentioned that they love and have interest in the teaching profession even though their interest was not to become professional teachers, 14.7% said they were influenced by their mentors such as teachers and parents, and 18.7% indicated that it was due to financial constraints. It must be put on record that the governments at a point in time gave teacher-trainees allowances to motivate and cushion them for their preparation into the teaching profession. This intervention may be best suited for those whose reason has to do with financial constraints.

One of the main purposes of this research was to investigate the instructional strategies that college tutors used in teaching mathematics especially about the use of constructivism as a teaching model. So, the question was asked as to whether respondents had any idea about constructivism as a teaching model. Table 4.16 shows the distribution of their responses.

Table 4. 16: Knowledge about Constructivism

College of Education	Respondents who had an idea			No Idea	Total
	Correct	Wrong	Total		
Akatsi	22	43	65	151	216
Peki	17	94	111	68	179
St. Francis	60	67	127	119	246
Total	**99**	**204**	**303**	**338**	**641**
%	**32.7**	**67.3**	**47.3**	**52.7**	**100.0**

Table 4.16 illustrates that a total of 303 respondents representing 47.3% of the study sample claimed they understood what is meant by constructivism when the question was asked. However, only 99 (32.7%) of 303 respondents could explain constructivism correctly with as much as 204 (67.3%) getting the understanding wrong. It was therefore indicative from the table that more than half of the respondents 338, which form 52.7% of the respondents have no idea about constructivism as a teaching model. These responses are not consistent with a research conducted by Ramsook & Thomas (2016) in Trinidad and Tobago where 96.2% of teacher-trainees revealed that they understand the principles of constructivism which subsequently influenced their personal philosophy of teaching and learning.

The findings for the effect of age on mathematics performance and other variables are shown in Tables 4.17 – 4.25. These results were confirmed by a research conducted by Owolabi & Etukiren (2014) which indicate that college students below the ages of 25 years have the highest mean score in an examination, as compared to the mean score of those whose ages are above 25 years.

Table 4. 17: Respondents' Performance in WASSCE Mathematics

Grades	Ages			
	<25 years		25-35 years,	
	N	%	N	%
A1	19	4.4	7	3.3
B2	55	12.9	25	11.7
B3	84	19.6	29	13.6
C4	80	18.7	35	16.4
C5	33	7.7	17	8.0
C6	75	17.5	41	19.2
D7-F9	16	3.7	9	4.2
No Response	66	15.4	50	23.5
Total	**428**	**100.0**	**213**	**100.0**

At the sitting of core mathematics examination at WASSCE, respondents whose ages were below 25 years had higher pass rates of A1, B2, B3 and C4 than respondents whose ages were 25 years and above as shown in Table 4.17. Contrary to most existing evidence, this finding conforms to the study by Pellizzari & Billari (2012), who analyzed the academic performance of university undergraduate students and conclude that the youngest students between the ages of 20 and 25 within a cohort perform better than their oldest peers, especially in mathematics.

Considering the performance of respondents in the colleges of education, 66.8% (428) of respondents were below 25 years and 33.2% (213) were 25 years and above as seen in Table 4.18.

Table 4. 18: 1st Year 1st Semester

Grades	**Ages**			
	<25		**25-35**	
	N	**%**	**N**	**%**
A	147	34.3	58	27.2
B+	86	20.1	37	17.4
B	75	17.5	36	16.9
C+	49	11.4	31	14.6
C	37	8.6	35	16.4
D+	8	1.9	3	1.4
D	10	2.3	5	2.3
E		0.0		0.0
No Response	16	3.7	8	3.8
Total	**428**	**100.0**	**213**	**100.0**

Examination in the first semester of the first year as in Table 4.18 revealed that 34.3% of respondents with ages below 25 years had grade A as compared to 27.2% of respondents whose ages were 25 years and above. More respondents at ages below 25 years had grade B+ (20.1%) and B (17.5%) when compared with 17.4% and 16.9% of respondents for grades B+ and B with ages 25 years and above.

Table 4. 19: 1st Year 2nd Semester

Grades	Ages			
	<25		25-35	
	N	%	N	%
A	116	27.1	41	19.2
B+	100	23.4	41	19.2
B	76	17.8	35	16.4
C+	52	12.1	32	15.0
C	34	7.9	30	14.1
D+	11	2.6	8	3.8
D	8	1.9	7	3.3
E	2	0.5	1	0.5
No Response	29	6.8	18	8.5
Total	**428**	**100.0**	**213**	**100.0**

In the second semester of the first year as shown in Table 4.19, 27.1%, 23.4% and 17.8% of respondents who were less than 25 years passed mathematics with grade A, B+ and B respectively whilst 19.2%, 19.2% and 16.4% of respondents who were 25 years and above passed with grade A, B+ and B respectively.

Table 4. 20: 2nd Year 1st Semester Results

Grades	Ages			
	<25		25-35	
	N	%	N	%
A	149	34.8	55	25.8
B+	69	16.1	59	27.7
B	70	16.4	25	11.7
C+	48	11.2	29	13.6
C	39	9.1	19	8.9
D+	19	4.4	9	4.2
D	10	2.3	8	3.8
E		0.0		0.0
No Response	24	5.6	9	4.2
Total	**428**	**100.0**	**213**	**100.0**

In the case of the first semester of the second year as in Table 4.20, 34.8%, 16.1% and 16.4% of the respondents who were below 25 years had grades A, B+ and B respectively as compared to 25.8%, 27.7% and 11.7% of respondents whose ages were 25 years and above for grades A, B+ and B respectively. On the contrary, only respondents whose ages were 25 years and above and had grade B+ were more than those with ages below 25 years.

Table 4. 21: 2nd Year 2nd Semester Results

Grades	Ages			
	<25		25-35	
	N	%	N	%
A	199	46.5	66	31.0
B+	70	16.4	41	19.2
B	44	10.3	28	13.1
C+	35	8.2	24	11.3
C	29	6.8	22	10.3
D+	8	1.9	2	0.9
D	13	3.0	9	4.2
E	0	0.0	0	0.0
No Response	30	7.0	21	9.9
Total	**428**	**100.0**	**213**	**100.0**

In Table 4.21, as much as 46.5% of respondents below the age of 25 years had grade A and 31.0% of respondents who were 25 years and above also had grade A. However, when 19.2% of respondents whose ages were 25 years and above had B+, 16.4% of the respondents whose ages were below 25 years also had B+. In a like manner, when 13.1% of the respondents with ages of 25 and above had grade B, 10.3% of respondents with ages below 25 years also had grade B. On the whole, respondents with ages below 25 years performed better than respondents whose ages were 25 years and above which confirmed the study conducted by Owolabi & Etukiren (2014).

Table 4. 22: Relationship between age and year of completion of SHS

Grades	Ages <25		25-34		Total for all the years	
	N	%	N	%	N	%
2004	0	0.0	2	0.9	2	0.3
2007	1	0.2	5	2.3	6	0.9
2009	3	0.7	5	2.3	8	1.2
2010	0	0.0	2	0.9	2	0.3
2011	0	0.0	7	3.3	7	1.1
2012	8	1.9	17	8.0	25	3.9
2013	43	10.0	44	20.7	87	13.6
2014	38	8.9	30	14.1	68	10.6
2015	129	30.1	39	18.3	168	26.2
2016	137	32.0	42	19.7	179	27.9
2017	**69**	16.1	**20**	9.4	**89**	13.9
Total	**428**	**100.0**	**213**	**100.0**	**641**	**100.0**

From Table 4.22, only 16.1% of respondents who were below 25 years and completed SHS in 2017 got direct admission to the colleges of education in the 2017/2018 academic year and as low as 9.4% of respondents of those whose ages were 25 years and above got direct admission into college in the same academic year. As little as 2.8% of respondents who were below 25 years and completed SHS between 2004 and 2012 got admission into college in 2017/2018 academic year whilst 17.7% of respondents who were 25 years and above and completed SHS between 2004 and 2012 got admission in the 2017/2018 academic year. The indication is that majority of older students delayed in going to school because young students always outperform the older ones (Owolabi & Etukiren, 2014).

On the whole, only 13.9% of all the respondents who completed SHS in 2017 got direct admission into the colleges in 2017/2018 academic year as 86.1% of the respondents stayed in the house for some number of years before being admitted into the colleges (Table 4.22). This trend has dire consequences on the calibre of teacher-trainees that are produced from the colleges as those who got direct admission into the colleges performed better than those who stayed in the house for some number of years before being admitted (Jacob & Ryan, 2018).

Table 4. 23: Age vrs Programme of Respondents at SHS

		Ages			
	Programme	**<25 years**		**25-35 years**	
		N	**%**	**N**	**%**
	Agriculture Science	20	4.7	16	7.5
Sciences	Technical	6	1.4	4	1.9
	General Science	66	15.4	17	8.0
	Sub Total	**92**	**21.5**	**37**	**17.4**
	Business	108	25.2	67	31.5
Humanities	General Arts	202	47.2	84	39.4
	Visual Art	14	3.3	11	5.2
	Home Economics	12	2.8	12	5.6
	Sub Total	**335**	**78.5**	**174**	**81.9**
Total		**430**	**100.0**	**211**	**100.0**

Table 4.23 showed that 21.5% of respondents below the age of 25 years offered science programmes whilst the majority of 78.5% offered the humanities. With respect to respondents whose ages were 25 years and above, 17.4% offered the

sciences and 81.9% offered the humanities. According to Pellizzari (2011), young students perform cognitively better than their older peers.

Table 4. 24: Age versus desire to teach mathematics by Respondents

Responses	Ages			
	<25 years		25-34 years	
	N	%	N	%
Yes	309	71.7	154	73.3
No	107	24.8	52	24.8
No Response	15	3.5	4	1.9
Total	**431**	**100.0**	**210**	**100.0**

Even though participants below the ages of 25 years performed better in mathematics during their school years than those who were 25 years and above, the latter's desire to teach mathematics after school was higher with a response rate of 73.3% than the former with response rate of 71.7% as in Table 4.24.

Table 4. 25: Programme at College of Education

Colleges of Education	General		Technical		Maths/Science		Total	
	N	%	N	%	N	%	N	%
Akatsi	119	24.6	52	100.0	45	42.5	216	33.7
Peki	179	37.1	-	-	-	-	179	27.9
St. Francis	185	38.3	-	-	61	57.5	246	38.4
Total	**483**	**100.0**	**52**	**100.0**	**106**	**100.0**	**641**	**100.0**
%		**75.4**		**8.1**		**16.5**		**100.0**

From Table 4.25, a total of 483 respondents offered General programme with 24.6% coming from Akatsi, 37.1% coming from Peki and 38.3% coming from St. Francis Colleges of Education. It is only Akatsi that offered Technical programme with 52 participants whilst a total of 106 respondents from Akatsi and St. Francis offered Maths/Science programme in which they contributed 42.5% and 57.5% respectively to the sample. On the whole, 75.4% of the respondents offered General programme, 8.1% offered Technical programme and 16.5% offered Maths/Science even though more than 70% of respondents wished to teach mathematics at the basic school.

Table 4. 26: Respondents' Programme offered at SHS versus Mathematics Grades

	Programmes			
	Sciences		Humanities	
Grades	N	%	N	%
A1	8	5.8	26	5.2
B2	17	12.2	70	13.9
B3	34	24.5	90	17.9
C4	33	23.7	114	22.7
C5	13	9.4	44	8.8
C6	16	11.5	117	23.3
D7-F9	2	1.4	13	2.6
No Response	16	11.5	28	5.6
Total	**139**	**100.0**	**502**	**100.0**

The sciences are made up of General Science, Agricultural Science and Technical whilst the humanities comprised of Business, General Arts, Visual Art and Home Economics. As students pursuing science programmes are expected to understand mathematics concepts easily, so is their good performance in mathematics (Karjanto, 2017). In addition, science students offer elective mathematics which supports their understanding of core mathematics better and faster than their counterparts who do not offer elective mathematics. From Table 4.26, 5.8% of respondents who offered science programmes at the senior high schools had grade A1. In addition, respondents who offered science programmes passed mathematics with grades B3 (24.5%), C4 (23.7%) and C5 (9.4%) as compared to humanities respondents of B3 (17.9%), C4 (22.7%) and C5 (8.8%) except for grades B2 (13.9%) and C6 (23.3%) in favour of humanities as in Table 4.26.

Table 4. 27: Relationship between programme of study and year of completion at SHS

Programme	Years							
	2004 & 2007		2009-2012		2013-2016		2017	
	N	%	N	%	N	%	N	%
Sciences	0	0.0	11	33.3	99	20.5	20	23.8
Humanities	10	100	22	66.7	384	79.5	64	76.2
Total	**10**	**100.0**	**33**	**100.0**	**483**	**100.0**	**84**	**100.0**

Only 10 respondents who offered humanities completed SHS in 2004 and 2007. Thirty-three respondents completed SHS between 2009 and 2012 with 33.3% being science graduates and 66.7% humanities graduates. Four hundred and eighty-three respondents completed SHS between 2013 and 2016 with 20.5% being science graduates and 79.5% being humanities graduates. Finally, out of a total of eighty-four respondents who graduated from SHS in 2017, 23.8% offered sciences and 76.2% offered humanities. Findings show that respondents had adequate knowledge in mathematics but most of them did not have any idea about constructivism. However, in responding to research question 1, the narration revealed that teacher-trainees in the colleges of education had adequate understanding of concepts hence good mathematics background.

4.2 Research Question 2

Which instructional strategies do college tutors mostly use in teaching mathematics?

Generally, three statistical tests were conducted to respond to research question 2. These were the binomial test with a benchmark value of 0.05, the descriptive statistic and composite score analyses of the mean likert-scale response for each instructional strategy. The binomial test was used because the experiment has been partitioned into two possible outcomes (i.e. success/failure or agree/disagree) with a probability of 0.5 for each outcome. Therefore, the responses were re-categorized into a scale of 1, 2 and 3 as disagree and a scale of 4 and 5 as agree (Kubiszyn & Borich, 1996). Bootstrapping of the descriptive statistics was conducted to indicate whether the results were statistically significant or not. The composite scores were used as a confirmatory test to the descriptive statistic. All these indices were calculated using SPSS version 2.0.

4.2.1 Predictor Variables to Constructivism

Cognitive Activation (CA)

CA is the ability of students to espouse their own learning theory to solve problems through tutors' support and encouragement.

Table 4. 28: Cognitive Activation

s/n	Items	SRN	Binomial Test Proportion <= 3	Binomial Test Proportion > 3	MLSR	Bootstrap Confidence Interval (%) 2.5	Bootstrap Confidence Interval (%) 97.5	S.D	Bootstrap Confidence Interval (%) 2.5	Bootstrap Confidence Interval (%) 97.5	SE
1	CA1	627	0.24	0.76	3.89	3.79	3.97	1.087	1.011	1.157	0.043
2	CA2	618	0.33	0.67	3.66	3.56	3.73	1.047	0.983	1.114	0.042
3	CA3	625	0.26	0.74	3.86	3.76	3.94	1.082	1.023	1.159	0.043
4	CA5	624	0.31	0.69	3.74	3.68	3.86	1.107	1.010	1.145	0.044
5	CA7	623	0.28	0.72	3.86	3.79	3.96	1.053	0.964	1.098	0.042
6	CA8	617	0.35	0.65	3.65	3.59	3.77	1.111	1.014	1.141	0.045
7	CA12	619	0.31	0.69	3.80	3.72	3.88	0.999	0.917	1.049	0.040
8	CA15	622	0.27	0.73	3.83	3.74	3.91	1.068	0.988	1.130	0.043
9	CA17	625	0.24	0.76	3.98	3.89	4.06	0.979	0.917	1.050	0.039
10	CA19	619	0.28	0.72	3.81	3.71	3.89	1.063	1.005	1.133	0.043
11	CA20	619	0.28	0.72	3.83	3.77	3.94	1.076	0.980	1.114	0.043
12	CA21	619	0.31	0.69	3.74	3.65	3.82	1.065	0.996	1.136	0.043
13	CA22	619	0.27	0.73	3.91	3.79	4.06	1.543	0.954	2.345	0.062
14	CA23	618	0.31	0.69	3.80	3.69	3.87	1.062	0.996	1.126	0.043
Mean		**621**	**0.29**	**0.69**	**3.811**	**3.724**	**3.904**	**1.096**	**0.983**	**1.207**	**0.044**
Composite Score Test					**3.810**						

MLSR- Mean Likert-Scale Response

SD - Standard Deviation

SE - Standard Error

SRN- Sample Response No.

On the average, 69% of the mean respondents of 621 agreed to the 14 items that college tutors used Cognitive Activation strategy to deliver mathematics lessons as indicated by the binomial test in Table 4.28. Supporting this claim by the respondents with a mean standard error of 0.044 and mean standard deviation of 1.096 which were significant at 95% confidence interval, the mean likert-scale response was 3.811. this response was collaborated by the composite score of 3.810. By these statistics, it was accepted among the teacher-trainees that college tutors used Cognitive Activation strategy to teach mathematics.

Instructional Coherence (IC)

IC is the teachers' capacity to ensure smooth flow of mathematics teaching and learning on the basis of connecting the old knowledge to the new. It is described as the interconnectivity of mathematical concepts in a lesson (Hiebert et al., 2003) and also explained as a link between structured content and classroom activities (Wang & Murphy, 2004).

Table 4. 29: Instructional Coherence

s/n	Items	SRN	Binomial Test Proportion		Descriptive Statistics Test Bootstrap						
					MLSR	Confidence Interval (%)		S.D	Confidence Interval (%)		SE
			<= 3	> 3		2.5	97.5		2.5	97.5	
1	IC61	623	0.30	0.70	3.74	3.67	3.83	1.014	0.936	1.073	0.041
2	IC64	622	0.25	0.75	3.89	3.81	3.96	0.940	0.883	1.015	0.038
3	IC65	616	0.24	0.76	3.88	3.80	3.95	0.942	0.870	1.010	0.038
4	IC67	617	0.30	0.70	3.78	3.69	3.85	1.017	0.959	1.088	0.041
5	IC68	618	0.28	0.72	3.84	3.76	3.93	1.030	0.974	1.104	0.041
6	IC73	620	0.23	0.77	3.98	3.90	4.05	0.940	0.878	1.017	0.038
7	IC74	614	0.23	0.77	3.97	3.88	4.03	0.966	0.892	1.027	0.039
8	IC75	619	0.26	0.74	3.89	3.81	3.97	0.974	0.918	1.049	0.039
9	IC80	617	0.23	0.77	3.90	3.83	3.98	0.955	0.863	0.998	0.038
10	IC81	619	0.25	0.75	3.90	3.83	3.97	0.899	0.830	0.951	0.036
11	IC82	619	0.26	0.74	3.87	3.80	3.96	0.991	0.911	1.054	0.040
12	IC83	618	0.24	0.76	3.94	3.87	4.03	1.000	0.913	1.053	0.040
13	IC84	624	0.25	0.75	3.89	3.81	3.97	1.047	0.973	1.120	0.042
Mean		**619**	**0.255**	**0.745**	**3.811**	**3.805**	**3.960**	**0.978**	**0.908**	**1.043**	**0.039**
Composite Score Test					**3.822**						

Table 4.29 represents the mean respondents of 619 who answered the 13 items that explained instructional coherence. Using the binomial test, 74.5% of respondents stated that tutors often use this strategy to teach mathematics. The mean likert-scale response of 3.811 to the items which was close to the composite score of 3.822 had a mean standard deviation of 0.978 and standard error (closeness of sample means of the population) of 0.039 at 5% significant level as indicated by the descriptive

statistics. These weights therefore confirmed tutors' use of Instructional Coherence strategy as a teaching model in mathematics class to in the colleges of education.

Profound Understanding of Fundamental Mathematics (PUFM)

PUFM is the ability of teachers to teach basic mathematics concepts from first principles to enhance students' understanding. Crucial to this strategy is the ability of learners to learn mathematics by knowing the 'why' and the 'how' during the problem-solving stage.

Table 4. 30: Profound Understanding of Fundamental Mathematics

s/n	Items	SRN	Binomial Test Proportion		Descriptive Statistics Test						
						Bootstrap					
					MLSR	Confidence Interval (%)		S.D	Confidence Interval (%)		SE
			<= 3	> 3		2.5	97.5		2.5	97.5	
1	PUFM33	620	0.29	0.71	3.85	3.79	3.96	1.005	0.940	1.066	0.040
2	PUFM34	617	0.29	0.71	3.87	3.81	3.98	0.999	0.929	1.054	0.040
3	PUFM35	618	0.28	0.72	3.83	3.75	3.92	1.041	0.973	1.109	0.042
4	PUFM36	616	0.31	0.69	3.80	3.72	3.91	1.102	1.034	1.166	0.044
5	PUFM39	614	0.26	0.74	3.90	3.82	3.98	0.983	0.906	1.044	0.040
6	PUFM40	618	0.32	0.68	3.76	3.67	3.84	1.011	0.956	1.083	0.041
7	PUFM42	619	0.28	0.72	3.87	3.80	3.96	1.001	0.934	1.068	0.040
8	PUFM43	617	0.35	0.65	3.76	3.68	3.85	1.026	0.963	1.084	0.041
9	PUFM45	618	0.32	0.68	3.76	3.69	3.86	1.039	0.967	1.093	0.042
10	PUFM46	616	0.30	0.70	3.86	3.79	3.95	0.965	0.885	1.011	0.039
11	PUFM47	617	0.28	0.72	3.88	3.81	3.97	0.966	0.903	1.033	0.039
12	PUFM48	613	0.26	0.74	3.94	3.86	4.02	1.030	0.961	1.097	0.042
13	PUFM49	621	0.27	0.73	3.87	3.77	3.94	1.036	0.981	1.113	0.042
14	PUFM50	616	0.28	0.72	3.86	3.79	3.95	0.993	0.931	1.071	0.040
15	PUFM59	618	0.24	0.76	3.95	3.86	4.02	0.993	0.918	1.065	0.040
	Mean	**617**	**0.289**	**0.711**	**3.851**	**3.774**	**3.941**	**1.013**	**0.945**	**1.077**	**0.041**
Composite Score Test					**3.849**						

Using the binomial test, a mean value of 71.1% of the mean respondents of 617 agreed that mathematics tutors in the colleges of education use Profound Understanding of Fundamental Mathematics (PUFM) strategy to teach as illustrated in Table 4.30. By the descriptive statistics, respondents supported this claim of the mean likert-scale response whose value of 3.851 lied between 3.774 and 3.941 confidence interval of 95%. This value was close to the composite score of 3.849. These statistics according to the respondents pointed out that tutors in the colleges of education teach mathematics using PUFM strategy which was measured by 15 items.

Cognitive Guided Instructions (CGI)

Balancing of knowledge acquired culturally with that obtained cognitively through scientific methods and classroom activities is referred to as Cognitive Guided Instruction**.** It is that instruction that encourages students to bring unscientific solutions to augment organized ones.

Table 4. 31: Cognitive Guided Instructions

s/n	Items	SRN	Binomial Test: Proportion <= 3	Binomial Test: Proportion > 3	Descriptive Statistics Test: MLSR	Confidence Interval (%) 2.5	Confidence Interval (%) 97.5	S.D	Bootstrap Confidence Interval (%) 2.5	Bootstrap Confidence Interval (%) 97.5	SE
1	CGI86	625	0.38	0.62	3.60	3.52	3.70	1.107	1.046	1.177	0.044
2	CGI88	620	0.26	0.74	3.84	3.76	3.91	0.959	0.890	1.025	0.039
3	CGI91	613	0.34	0.66	3.67	3.60	3.77	1.037	0.957	1.084	0.042
4	CGI93	616	0.28	0.72	3.81	3.74	3.89	0.967	0.893	1.019	0.039
5	CGI94	621	0.30	0.70	3.74	3.67	3.83	1.030	0.940	1.076	0.041
6	CGI95	620	0.29	0.73	3.79	3.72	3.89	1.061	0.987	1.124	0.043
7	CGI104	618	0.33	0.67	3.70	3.62	3.78	1.000	0.937	1.061	0.040
8	CG106	616	0.27	0.73	3.87	3.80	3.96	0.985	0.921	1.052	0.040
9	CG107	618	0.29	0.71	3.82	3.74	3.89	0.960	0.889	1.018	0.039
10	CGI108	610	0.33	0.67	3.74	3.66	3.82	1.032	0.966	1.095	0.042
Mean		**617**	**0.30**	**0.70**	**3.758**	**3.683**	**3.844**	**1.014**	**0.943**	**1.073**	**0.041**
Composite Score Test					**3.749**						

Responding to 10 items as in Table 4.31, 70% of the mean respondents of 617 settled on the fact that tutors in the colleges of education use Cognitive Guided Instruction strategy to teach mathematics at a significance level of 0.05, according to the binomial test. The mean likert-scale response of 3.758 with a mean standard deviation of 1.014 was significant at a confidence level of 95% with a standard error value of 0.041. The composite score of 3.749 which was close to the mean likert-scale response was an indication that teacher-trainees agreed that tutors in the colleges of education use Cognitive Guided Instruction strategy in mathematics lessons.

Summary of Results for Predictor Variables to Constructivism

Table 4. 32: Summary of Results of Predictor Variables to Constructivism

Instructional Strategies	Descriptive Statistics	Composite Score	Binomial Test (%)	P Value≤0.05	Result
CA	3.811	3.810	69.0	0.0	Significant
PUFM	3.851	**3.849**	71.1	0.0	Significant
IC	**3.851**	3.822	**74.5**	0.0	Significant
CGI	3.758	3.749	70.0	0.0	Significant
Mean	**3.818**	**3.808**	**71.150**	**0.0**	**Significant**

In conclusion, Table 4.32 stated that all instructional strategies were used by tutors of colleges of education to teach mathematics such that Instructional Coherence is the most used instructional strategy which had the highest binomial test percentage result of 74.5, highest descriptive statistics response of 3.851 but a composite score of 3.822 as compared to the composite score of 3.851 for PUFM.

4.2.2 Predictor Variables to Teacher Quality

In order to ensure accurate use of the instructional strategies, the researcher from literature considered internal and external motivation of teacher-trainees and relevant previous knowledge acquired by both the tutor and the teacher-trainees as factors that affect teacher quality in mathematics lessons. The three tests of binomial, descriptive statistics and composite score were conducted to either confirm the assertion or otherwise.

SDT_ Internal Motivation

Internal motivation takes place when an individual acts in a manner that satisfy his/her interest and in line with his/her own values and internal regulation (Chirkov, Vansteenkiste Tao, & Lynch, 2007) which enable him/her to learn a thing liberally.

Table 4. 33: SDT_ Internal Motivation

s/n	Items	SRN	Binomial Test Proportion <= 3	> 3	Descriptive Statistics Test MLSR	Bootstrap Confidence Interval (%) 2.5	97.5	S.D	Confidence Interval (%) 2.5	97.5	SE
1	SDTIM29	619	0.25	0.75	3.95	3.91	4.08	1.088	0.978	1.122	0.044
2	SDTIM30	617	0.28	0.72	3.88	3.81	3.97	1.028	0.948	1.084	0.041
3	SDTIM32	616	0.26	0.74	3.91	3.84	4.01	1.045	0.957	1.095	0.042
4	SDTIM38	574	0.25	0.75	4.01	3.94	4.13	1.155	1.008	1.328	0.048
5	SDTIM39	616	0.25	0.75	3.89	3.79	3.96	1.048	0.966	1.112	0.042
6	SDTIM40	616	0.22	0.78	4.00	3.93	4.09	0.953	0.865	1.008	0.038
7	SDTIM41	612	0.22	0.78	3.96	3.90	4.05	0.949	0.861	1.004	0.038
8	STDIM42	615	0.20	0.80	4.06	3.98	4.15	0.968	0.900	1.050	0.039
9	SDTIM44	618	0.23	0.77	3.96	3.90	4.06	0.999	0.908	1.050	0.040
10	SDTIM46	619	0.26	0.74	3.91	3.83	3.98	0.936	0.868	0.990	0.038
Mean		**612**	**0.242**	**0.758**	**3.953**	**3.883**	**4.048**	**1.017**	**0.926**	**1.084**	**0.041**
Composite Score Test					**3.950**						

The mean sample number of 612 of teacher-trainees in their response to the 10 items to measure internal motivation indicated that they are internally and personally motivated to learn mathematics hence the average percentage of 75.8 confirming the claim when the binomial test was conducted. In a further test, and at a 95% confidence interval, the descriptive statistics indicated participants' likert-scale response of 3.953 to the items at a 1.017 of standard deviation and standard error of 0.041. Finally, the composite score of 3.950 fully agreed that teacher-trainees were internally motivated to learn mathematics as seen in Table 4.33.

SDT_ External Motivation

External or extrinsic motivation is when students engage in learning with support from other people or factors for rewards such as promotion to the next level, and approval from teachers, parents and peers (Mueller, Yankelewitz & Maher, 2012).

Table 4. 34: SDT_ External Motivation

s/n	Items	SRN	Binomial Test Proportion		Descriptive Statistics Test					Bootstrap		
					MLSR	Confidence Interval (%)		S.D	Confidence Interval (%)		SE
			<= 3	> 3		2.5	97.5		2.5	97.5	
1	SDTEM50	617	0.28	0.72	3.82	3.73	3.91	1.057	0.994	1.130	0.043
2	SDTEM53	619	0.27	0.73	3.87	3.80	3.95	0.949	0.886	1.007	0.038
3	SDTEM57	619	0.25	0.75	3.94	3.84	4.01	1.029	0.965	1.101	0.041
4	SDTEM60	615	0.25	0.75	3.85	3.75	3.91	1.013	0.937	1.078	0.041
5	SDTEM66	606	0.27	0.73	3.86	3.79	3.95	1.045	0.969	1.103	0.042
6	SDTEM69	612	0.25	0.75	3.86	3.81	3.96	0.978	0.892	1.035	0.040
7	SDTEM88	616	0.24	0.76	3.90	3.83	3.99	0.991	0.928	1.062	0.040
Mean		**615**	**0.259**	**0.741**	**3.871**	**3.793**	**3.954**	**1.009**	**0.939**	**1.074**	**0.041**
Composite Score Test					**3.867**						

External motivation of teacher-trainees as a factor that affects teacher quality was significant when the binomial test was conducted on seven (7) items with a mean agreement of 74.1% from 615 sampled mean respondents. The mean likert-scale response of 3.871 was significant at a standard deviation and standard error of 1.009 and 0.041 respectively. The composite score of 3.867 which adequately supported the significance of the results indicated that teacher-trainees agreed that they are externally motivated to learn mathematics as shown in Table 4.34.

Relevant Previous Knowledge (RPK)

Relevant previous knowledge is the knowledge the learner and the teacher already have before meeting new information during a lesson. It is needed to facilitate understanding of current mathematics concepts.

Table 4. 35: Relevant Previous Knowledge

s/n	Items	SRN	Binomial Test		Descriptive Statistics Test							
									Bootstrap			
			Proportion		MLSR	Confidence Interval (%)		SD	Confidence Interval (%)		SE
			<= 3	> 3		2.5	97.5		2.5	97.5	
1	RPK4	628	0.23	0.77	4.02	3.96	4.13	1.046	0.952	1.094	0.042
2	RPK7	620	0.32	0.68	3.71	3.64	3.82	1.074	1.000	1.126	0.043
3	RPK8	623	0.23	0.77	4.00	3.94	4.10	1.025	0.934	1.082	0.041
4	RPK9	616	0.26	0.74	3.86	3.80	3.95	1.008	0.910	1.052	0.041
5	RPK10	619	0.27	0.73	3.87	3.82	3.97	0.933	0.851	0.976	0.037
6	RPK11	621	0.20	0.80	4.07	4.01	4.16	0.979	0.899	1.041	0.039
7	RPK13	624	0.30	0.70	3.79	3.73	3.90	1.043	0.943	1.079	0.042
8	RPK15	621	0.27	0.73	3.85	3.79	3.95	1.047	0.944	1.094	0.042
9	RPK16	628	0.24	0.76	3.93	3.88	4.03	0.989	0.899	1.035	0.039
10	RPK17	628	0.23	0.77	3.98	3.91	4.09	1.091	0.988	1.138	0.044
Mean		**623**	**0.255**	**0.745**	**3.908**	**3.848**	**4.010**	**1.024**	**0.932**	**1.072**	**0.041**
Composite Score					**3.906**						

On the average, 74.5% of the respondents in the study agreed that tutors and teacher-trainees use relevant previous knowledge in mathematics lessons as shown by the binomial test in Table 4.35. Also, the mean likert-scale response of 3.908 to the 10 items at a mean standard deviation of 1.024 and standard error of 0.04 and close to the composite score of 3.906 from 623 mean respondents is an indication that the tutors and teacher-trainees use RPK to understand new mathematics concepts.

Summary of Results for Predictor Variables to Teacher Quality

Table 4. 36: Summary of Results for Predictor Variables to Teacher Quality

Foundation to Instructional Strategies	Descriptive Statistics	Composite Score	Binomial Test (%)	P Value≤0.05	Result
SDT_IM	**3.953**	**3.950**	**75.8**	0.0	Significant
SDT_EM	3.871	3.867	74.1	0.0	Significant
RPK	3.908	3.906	74.5	0.0	Significant
Mean	**3.911**	**3.908**	**74.800**	**0.0**	**Significant**

According to Table 4.36, teacher-trainees' concluded that internal and external motivation and relevant previous knowledge largely predicts teacher quality in mathematics lessons with mean values of 74.8% in respect of the binomial test with a p value less than 0.05. The mean likert-scale response and composite score were 3.911 and 3.908 respectively. However, internal motivation of teacher-trainees subtly played the most significant role for predicting teacher quality in mathematics lessons with binomial test score of 75.8%, mean likert-scale response of 3.953 and composite score of 3.950. The implication is that when teacher-trainees are internally motivated they understand mathematical concepts with ease which consequently predicts teacher quality.

4.2.3 Results for Outer Variables

Measurement of Constructivism

Constructivist theory of learning emphasizes that knowledge is a product of one's cognitive act by building on previous knowledge that allows one to move to new knowledge (Lerman, 1996). Consequently, constructivism is an educational instruction that comprises numerous and diverse instructional strategies that help students to construct their own understanding of mathematical concepts.

Table 4. 37: Constructivism

			Binomial Test		Descriptive Statistics						
					Bootstrap						
s/n	**Items**	**SRN**	**Proportion**		**MLSR**	**Confidence Interval (%)**		**S.D**	**Confidence Interval (%)**		**SE**
			<= 3	**> 3**		**2.5**	**97.5**		**2.5**	**97.5**	
1	CONST4	625	0.24	0.76	3.93	3.89	4.08	0.981	0.906	1.122	0.039
2	CONST6	621	0.22	0.78	4.02	3.91	4.20	1.794	0.947	2.822	0.072
3	CONST7	622	0.20	0.80	4.03	3.96	4.12	0.948	0.880	1.025	0.038
4	CONST8	617	0.26	0.74	3.92	3.83	3.99	0.995	0.930	1.063	0.040
5	CONST9	622	0.22	0.78	3.97	3.90	4.07	1.091	0.999	1.148	0.044
6	CONST15	621	0.23	0.77	3.90	3.83	3.99	1.022	0.949	1.092	0.041
7	CONST16	624	0.20	0.80	4.01	3.93	4.08	0.976	0.909	1.051	0.039
8	CONST17	623	0.22	0.78	3.99	3.90	4.06	0.984	0.920	1.063	0.039
9	CONST18	622	0.18	0.88	4.17	4.03	4.38	2.255	0.904	3.738	0.090
10	CONST19	618	0.27	0.73	3.88	3.73	4.11	2.321	1.019	3.789	0.093
11	CONST20	619	0.25	0.75	3.86	3.79	3.95	1.026	0.956	1.090	0.041
Mean		**621**	**0.226**	**0.774**	**3.971**	**3.879**	**4.094**	**1.308**	**0.938**	**1.728**	**0.052**
Composite Score					**3.973**						

With a mean response sample of 621, Table 4.37 shows 11 items which explained constructivism. With the binomial test at 5% significance level, an average of 77.4% of the respondents stated that constructivism is a learning model in mathematics at the colleges of education. The mean likert-scale response indicated that the value of 3.971 which was between 3.879 and 4.094 at a 95% confidence internal was significant with a mean standard deviation of 1.308. A high value composite score of 3.973 which was close to the mean likert-scale response was an indication that tutors adopted constructivism in the teaching and learning of mathematics at the colleges of education. The value of 0.052 shows how close the sub-sample means were in the population.

Measurement of Teacher Quality

Teacher quality according to the conceptual model impacts on the instructional strategies of CA, PUFM, IC, and CGI which subsequently demonstrate the success of constructivism. Hattie (2009) describes quality teachers as those who challenge their pupils with problems in different contexts and ask them to apply what they have learned to new contexts in and out of the classroom. Therefore, teacher quality is the teacher's expertise to deliver quality instructions for effective learning by teacher-trainees.

Table 4. 38: Teacher Quality

s/n	Items	SRN	Binomial Test Proportion		Descriptive Statistics Test						
									Bootstrap		
					MLSR	Confidence Interval (%)		S.D	Confidence Interval (%)		SE
			<= 3	> 3		2.5	97.5		2.5	97.5	
1	TQ2	617	0.27	0.73	3.90	3.83	4.01	1.039	0.949	1.086	0.042
2	TQ3	616	0.23	0.77	3.96	3.91	4.07	0.989	0.890	1.030	0.040
3	TQ5	609	0.28	0.72	3.95	3.80	4.24	2.515	0.962	3.955	0.102
4	TQ6	616	0.35	0.65	3.80	3.66	3.83	1.892	0.972	1.098	0.076
5	TQ7	617	0.28	0.72	3.82	3.77	3.94	1.031	0.925	1.067	0.042
6	TQ8	616	0.27	0.73	3.85	3.81	3.97	0.97	0.882	1.017	0.039
7	TQ9	608	0.25	0.75	3.90	3.85	4.01	0.983	0.897	1.033	0.040
8	TQ12	589	0.26	0.74	3.88	3.79	3.95	0.996	0.927	1.063	0.041
9	TQ14	583	0.20	0.80	4.07	3.99	4.14	0.911	0.849	.985	0.038
10	TQ15	591	0.21	0.79	4.02	3.95	4.11	0.958	0.879	1.013	0.039
11	TQ16	590	0.22	0.78	3.96	3.88	4.04	0.983	0.903	1.044	0.040
12	TQ22	586	0.22	0.78	3.99	3.93	4.08	0.935	0.864	0.996	0.039
13	TQ35	590	0.29	0.71	3.8	3.74	3.92	1.075	0.995	1.139	0.044
14	TQ36	589	0.24	0.76	3.93	3.86	4.01	0.957	0.878	1.005	0.039
Means		**601**	**0.255**	**0.745**	**3.916**	**3.841**	**4.023**	**1.160**	**0.912**	**1.252**	**0.047**
Composite Score					**3.904**						

From Table 4.38, an average of 601 teacher-trainees responded to 14 items to measure teacher quality. Using the binomial test, 74.5% of the respondents stated that their tutors demonstrated a level of quality teaching during mathematics lessons. According to the descriptive statistics, the mean likert-scale response of 3.916 which is close to the composite score of 3.904 had a mean standard deviation of 1.160 at

95% confidence interval. In addition, the closeness of the sub-sample means of the population was depicted by a standard error value of 0.047.

Summary of Results for Outer Variables

Table 4. 39: Summary of Results for Outer Variables

Outer Variables	Descriptive Statistics	Composite Score	Binomial Test (%)	P≤0.05	Result
Teacher Quality	3.916	3.904	0.745	0.0	Significant
Constructivism	3.971	3.973	0.774	0.0	Significant

According to Table 4.39, teacher-trainees concluded that tutors in the colleges of education exhibited quality teaching skills as the binomial test revealed that 74.5% of the respondents agreed to the items, with a mean likert-scale response of 3.916 and a composite score of 3.904. With respect to constructivism, 77.4% of teacher-trainees agreed that constructivism as a teaching model is used in mathematics classes. Accordingly, the mean likert-scale response and the composite scores confirmed the fact that constructivism principle is adopted in mathematics classes with the composite scores of 3.971 and 3.973 respectively. This conclusion reflects the workshop organized by the Government of Ghana in collaboration with TTEL of the UK on teaching skills for tutors in all the 45 colleges of education.

4.3 Evaluation of Partial Least Squares-Structural Equation Modelling (PLS-SEM)

The first step in evaluating PLS-SEM results involves examining the measurement models with different relevant criteria for the reflective constructs. If all the required criteria of the measurement models are met, then there is the need to assess the structural model (Hair et al., 2017a). Similar to most statistical methods, PLS-SEM has rules of thumb that serve as guides to evaluating path model results (Chin, 2010; Götz et al., 2010; Henseler et al., 2009; Chin, 1998; Tenenhaus et al., 2005; Roldán & Sánchez-Franco, 2012; Hair et al., 2017a). Rules of thumb by convention are broad guidelines that suggest how to interpret PLS-SEM results in various disciplines. As an example, reliability for exploratory research should be a minimum of 0.60, while reliability for research that depends on established measures should be 0.70 or higher. The final step to interpret PLS-SEM results is by checking the robustness of the path model and the stability of results which depend on the research context and aim of the analysis with the available data (Hair, Risher, Sarstedt & Ringle, 2019).

4.3.1 Evaluation of Reflective Measurement Model

a. Factor Loadings

The first step in assessing reflective measurement model is to examine the indicator or outer loadings if they are equal or above the recommended loadings of 0.70 because the study depended on established measures. If the loadings meet this recommendation, then the constructs explain more than 50 per cent of the indicators' variance, thus providing acceptable item reliability (Götz, Liehr-Gobbers & Krafft, 2010; Hair, Sarstedt, Ringle & Mena, 2012) such that indicator loadings less than 0.7 were rejected (Chin, 1998) in this current study as shown in Table 4.40.

Table 4. 40: Outer (Factor) Loadings

Variables	CA	CGI	CONST	IC	PUFM	RPK	SDT-EM	SDT-IM	TQ	TTP
CA17	0.782									
CA19	0.796									
CA20	0.811									
CA21	0.793									
CA22	0.856									
CA23	0.852									
CGI104		0.732								
CGI106		0.806								
CGI107		0.776								
CGI108		0.755								
CGI86		0.716								
CGI88		0.779								
CGI91		0.776								
CGI93		0.826								
CGI94		0.832								
CGI95		0.825								
CONST15			0.766							
CONST16			0.,792							
CONST17			0.808							
CONST18			0.807							
CONST19			0.758							
CONST20			0.757							
CONST4			0.763							
CONST7			0.809							
CONST8			0.777							
CONST9			0.742							
Gr1stYr1stSem										0.776
Gr1stYr2ndSem										0.864
Gr2ndYr1stSem										0.794
Gr2ndYr2ndSe										0.885

m							
IC61	0.751						
IC64	0.814						
IC65	0.770						
IC67	0.765						
IC68	0.77						
IC73	0.797						
IC74	0.770						
IC75	0.791						
IC80	0.815						
IC81	0.799						
IC82	0.785						
IC83	0.823						
IC84	0.768						
PUFM33		0.735					
PUFM34		0.781					
PUFM35		0.792					
PUFM36		0.802					
PUFM39		0.764					
PUFM40		0.748					
PUFM42		0.825					
PUFM43		0.748					
PUFM45		0.771					
PUFM46		0.743					
PUFM47		0.799					
PUFM48		0.802					
PUFM49		0,801					
PUFM50		0.814					
PUFM59		0.701					
RPK11			0.765				
RPK13			0.787				
RPK15			0.817				
RPK16			0.826				

RPK17	0.796			
SDT_EM50		0.790		
SDT_EM53		0.782		
SDT_EM57		0.804		
SDT_EM60		0.812		
SDT_EM66		0.744		
SDT_EM69		0.793		
SDT_EM88		0.783		
SDT_IM29			0.775	
SDT_IM30			0.772	
SDT_IM32			0.772	
SDT_IM39			0.813	
SDT_IM40			0.839	
SDT_IM41			0.833	
SDT_IM42			0.841	
SDT_IM44			0.830	
SDT_IM46			0.813	
TQ12				0.782
TQ14				0.771
TQ15				0.780
TQ16				0.777
TQ2				0.769
TQ22				0.798
TQ3				0.791
TQ35				0.752
TQ36				0.784
TQ5				0.737
TQ7				0.765
TQ8				0.803
TQ9				0.763

In this analysis, the 104 items that were administered to the study sample reduced to 92 because items whose outer loadings were less than 0.7 in the constructs were deleted from the path model (Chin, 1998).

b. Convergent Validity

The next step in the reflective measurement model in this research was to evaluate the latent variables that explained the variations in the indicators (Ravand & Baghaei, 2016) which were examined through reliability and the validity of the constructs using convergent validity test. The convergent validity test indicated the values of Cronbach's Alpha, rho Alpha, and Composite Reliability whose statistics in this research were higher than the acceptable threshold of 0.7. In addition to the convergent validity, the average variance extracted (AVE) which explained more than one half of the variance in the construct as depicted in Table 4.41 (Fornell & Larcker, 1981) was also evaluated. In this step, the internal consistency of the items in the questionnaire were assessed using Jöreskog's (1971) composite reliability and Cronbach's Alpha reliability with high reliability values of 0.7 and above (Diamantopoulos et al., 2012; Drolet & Morrison, 2001). A high factor loadings of 0.7 or higher as in Table 4.40 and a high average variance extracted (AVE) of 0.5 or higher as in Table 4.41 were the pointers for convergent validity. AVE which was the mean of the communalities of the indicators associated with given constructs explained at least half of the variance of its observed variables as indicated in Table 4.41 (Hair et al., 2009 cited in Kazar, 2014).

Cronbach's Alpha reliability had less precision because the items were unweighted and produced lower values than composite reliability. In contrast, composite reliability items were weighted based on the individual construct's indicator loadings and therefore had a higher reliability function than Cronbach's Alpha. While Cronbach's Alpha may be conservative, the composite reliability may be liberal such that constructs' true reliability was typically viewed between these two extremes. As a solution to this, Dijkstra & Henseler (2015) proposed rho Alpha as an alternative measure of reliability to the exact constructs' indicators as true value which usually lies between Cronbach's Alpha and the composite reliability. Thus, the Cronbach's Alpha and composite reliability coefficients were the lower and upper limits respectively for the reliability test, with the rho Alpha lying in between them (Sijtsma, 2009). Consequently, rho Alpha may represent a good compromise if one assumes that the path model is correct. The outer measurement model determined the reliability of the constructs through the internal consistency and validity of the observed and unobserved variables. (Ho, 2013). While the internal consistency evaluations depended on the observed variables of the constructs, the convergent and discriminant validity tests assessed the strength of the constructs (Hair, Sarstedt, Ringle & Mena, 2012).

Table 4. 41: Convergent Validity

Constructs	Mean Factor Loadings	Cronbach's Alpha	Rho Alpha	Composite Reliability	AVE
CA	0.815	0.899	0.901	0.922	0.665
CGI	0.782	0.930	0.931	0.941	0.614
CONST	0.778	0.928	0.928	0.939	0.606
Grading	0.776	0.854	0.897	0.899	0.691
IC	0.787	0.949	0.949	0.955	0.619
PUFM	**0.775**	**0.953**	**0.953**	**0.958**	**0.602**
RPK	**0.798**	**0.858**	**0.859**	**0.898**	**0.638**
SDT_EM	0.787	0.898	0.900	0.919	0.620
SDT_IM	0.810	0.934	0.935	0.945	0.657
TQ	0.775	0.945	0.948	0.951	0.601

The findings as in Table 4.41 revealed that the Cronbach's Alpha values of the constructs which were greater than the recommended threshold of 0.70, were between 0.854 for teacher trainees' grades and 0.953 for profound understanding of fundamental mathematics (Hair et al., 2010; Nunnally, 1978) indicating that the items for the studied constructs in the measurement scale were internally consistent. The composite reliability ranged from 0.898 for relevant previous knowledge to 0.958 for profound understanding of fundamental mathematics, exceeding the threshold of 0.70 (Hair et al., 2010; Nunnally & Bernstein, 1994). In addition to confirming the reliability of the items, the values of rho Alpha for each construct which lied between the Cronbach's Alpha and the composite reliability had minimum and maximum values of 0.859 for relevant previous knowledge and 0.953 for profound understanding of fundamental mathematics respectively (Ringle, Wende & Becker, 2015). The rho Alpha, Cronbach's Alpha and Composite Reliability showed that the measurement scale used in the study was reasonably reliable as all the latent construct values exceeded the minimum threshold of 0.7 (Fornell & Larcker, 1981). Based on these results, all items demonstrated the reliability of the measurement scale. Furthermore in the study model, convergent validity was tested using Average Variance Extracted (AVE) for all the measured constructs with a minimum value of 0.506 and a maximum value of 0.687 which were above 0.5 thresholds (Fornell & Larcker, 1981). These values affirmed the validity of the latent variables of the model (Hair et al., 2014). Consequently, the reliability and validity of the constructs of the study measurement model were confirmed (Hair, Ringle & Sarstedt, 2011; Barclay, Thompson, dan Higgins, 1995).

In order to ensure the significance of the reliability and validity of the coefficients, the researcher conducted a bootstrapping with 5000 subsamples to indicate that the p values of the constructs were all less than 0.05 (Appendix H). The bootstrapping results therefore validated the sampled mean values which were very close to the original for each construct and lied between the lower and upper limits of the confidence interval (Ringle, Wende & Becker, 2015). The bootstrapping results for the convergent validity as found in Appendix H indicates that the p values of the constructs as in Rho Alpha, Chronbach's Alpha, Composite Reliability and Average Variance Extracted (AVE) were all less than 0.05.

c. Discriminant Validity of Constructs

The next step in the model evaluation process was to determine the discriminant validity of the latent constructs. This was to establish that the manifest variables for any construct are distinct from items in other constructs in the path model. To assess the discriminant validity, there was the need to establish Fornell and Larcker criterion, HTMT estimates and cross-loading values. The discriminant validity of the constructs was checked by using either the Fornell-Larcker criterion, the Heterotrait-Monotrait (HTMT) Ratio or cross loadings values.

i. Fornell-Larcker Criterion

The Fornell-Larcker criterion says that a factor's AVE should be higher than its squared correlations with all other factors in the model. Thus, the quality of the reflective model which is shown by the square root of the AVE of each construct in the diagonal matrix must be higher than the related correlation in the corresponding rows and columns of the matrix (Fornell & Larcker, 1981) as indicated in Table 4.42.

Table 4. 42: Fornell-Larcker Criterion

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDEM	SDIM	TQ	TTP
CA	0.815									
CGI	0.668	0.783								
CONST	0.503	0.597	0.778							
IC	0.736	**0.799**	0.619	0.787						
PUFM	**0.798**	0.740	0.566	**0.829**	0.776					
RPK	0.612	0.552	0.452	0.620	0.652	0.799				
SDEM	0.479	0.602	0.703	0.584	0.554	0.452	0.787			
SDIM	0.471	0.553	0.772	0.582	0.536	0.441	0.767	0.810		
TQ	0.540	0.636	0.483	0.617	0.595	0.439	0.606	0.502	0.775	
TTP	0.189	0.111	0.186	0.178	0.184	0.129	0.103	0.163	0.184	0.831

However, according to Ringle, Sarstedt & Straub (2012), the Fornell–Larcker criterion is ineffective in assessing the quality of the model because it relies on consistent factor loading estimates (Henseler et al., 2014). So, according to Table 4.42, the diagonal value of 0.776 for PUFM was less than the values of 0.829 for IC and 0.798 for CA in their corresponding rows. Also, the diagonal value of 0.787 for IC is lower than the value of 0.829 for PUFM in its corresponding column and lower than the value of 0.799 for CGI in its corresponding row. The discriminant validity among the constructs according to Fornell and Larcker (1981) criterion did not suggest the quality of the reflective model since the square root of AVE of the constructs (IC, CGI, PUFM and CA) in the matrix diagonal are not higher than the related correlation in corresponding rows and columns indicating no discriminality. This findings therefore supported Ringle, Sarstedt & Straub (2012) analysis which state that the Fornell–Larcker criterion is ineffective in assessing the quality of path models if it is not consistent with the factor loading estimates (Henseler et al., 2014).

ii. Heterotrait-Monotrait Ratio (HTMT)

The Heterotrait-Monotrait Ratio (HTMT) was therefore developed to solve the problem with Fornell–Larcker criterion (Henseler, Ringle & Sarstedt, 2015). In this instance, the comparative process of the variance-based estimates of constructs which are less than 0.85 indicates that the constructs are conceptually distinct but values higher than 0.85 and less than 0.90 indicates that the constructs are conceptually similar (Gefen, Rigdon & Straub, 2011; Ringle, Sarstedt & Straub, 2012).

Table 4. 43: Heterotrait-Monotrait Ratio (HTMT)

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDEM	SDIM	TQ	TTP
CA										
CGI	0.728									
CONST	0.549	0.642								
IC	0.795	0.849	0.657							
PUFM	**0.861**	0.784	0.599	**0.870**						
RPK	0.695	0.615	0.505	0.685	0.719					
SDT-EM	0.532	0.657	0.769	0.631	0.598	0.515				
SDT-IM	0.513	0.593	0.828	0.617	0.567	0.493	0.840			
TQ	0.569	0.663	0.504	0.637	0.614	0.482	0.639	0.523		
TTP	0.203	0.114	0.197	0.187	0.190	0.137	0.109	0.173	0.206	

Table 4.43 did fulfilled the HTMT criteria for assessing discriminant validity as the values for the correlations between PUFM and CA and between PUFM and IC were more than the acceptable value of 0.85 but lower than 0.9, indicating that manifest variables in these latent construct were distinct. Therefore, HTMT is a reliable tool to assess discriminant validity (Beniteza, Henselerc, Castillob & Schuberthc, 2019) in this current study.

iii. Cross-loadings

Another reliable test for discriminant validity for the reflective measurement models for this study was performed by evaluating all cross-loadings of the construct. This third discriminant validity test for the path model is the most effective since the cross-loadings are evaluated such that each measurement item correlates weakly with all other constructs except for the one to which it is theoretically associated (Gefen & Straub, 2005). As a rule of thumb, indicators of reflective measurement models should have the highest loading on their own underlying latent construct as compared to other constructs involved in the structural model (J. F. Hair et al., 2017) as shown from Table 4.44 to Table 4.53.

Table 4. 44: Cognitive Activation (CA)

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDEM	SDIM	TQ	TTP
CA17	**0.782**	0.577	0.415	0.610	0.655	0.536	0.412	0.390	0.443	0.175
CA19	**0.796**	0.522	0.389	0.582	0.621	0.484	0.379	0.362	0.421	0.141
CA20	**0.811**	0.499	0.404	0.581	0.593	0.450	0.371	0.389	0.420	0.145
CA21	**0.793**	0.520	0.401	0.559	0.616	0.443	0.358	0.366	0.381	0.135
CA22	**0.856**	0.590	0.435	0.643	0.698	0.535	0.422	0.413	0.499	0.178
CA23	**0.852**	0.554	0.416	0.619	0.712	0.539	0.398	0.383	0.469	0.146

In Table 4.44, the cross loadings of cognitive activation had all its values higher than all other constructs, hence its items are distinct from the items that defined other constructs. Discriminality was therefore established for cognitive activation.

Table 4. 45: Cognitive Guided Instruction (CGI*)*

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDEM	SDIM	TQ	TTP
CGI104	0.453	**0.732**	0.434	0.543	0.504	0.364	0.444	0.413	0.451	0.054
CGI106	0.526	**0.806**	0.474	0.603	0.577	0.458	0.476	0.440	0.491	0.088
CGI107	0.500	**0.776**	0.450	0.588	0.538	0.404	0.448	0.422	0.464	0.033
CGI108	0.471	**0.755**	0.451	0.599	0.518	0.377	0.439	0.408	0.460	0.043
CGI86	0.500	**0.716**	0.439	0.615	0.566	0.395	0.423	0.399	0.468	0.092
CGI88	0.574	**0.779**	0.469	0.667	0.612	0.447	0.479	0.445	0.506	0.073
CGI91	0.505	**0.776**	0.460	0.615	0.581	0.423	0.456	0.428	0.500	0.074
CGI93	0.559	**0.826**	0.495	0.651	0.602	0.495	0.495	0.477	0.516	0.132
CGI94	0.552	**0.832**	0.528	0.665	0.639	0.460	0.540	0.475	0.556	0.122
CGI95	0.574	**0.825**	0.470	0.694	0.639	0.483	0.506	0.420	0.551	0.139

From Table 4.45, cognitive guided instruction had all its cross loadings values higher than all other constructs in the corresponding rows, hence its items are distinct from the items of other constructs. With this finding, discriminality for cognitive guided instruction was established.

Table 4. 46: **Constructivism (CONST)**

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDEM	SDIM	TQ	TTP
CONST15	0.356	0.440	**0.766**	0.443	0.415	0.316	0.520	0.552	0.353	0.112
CONST16	0.336	0.431	**0.792**	0.444	0.407	0.320	0.516	0.572	0.369	0.117
CONST17	0.391	0.468	**0.808**	0.472	0.424	0.331	0.512	0.553	0.347	0.148
CONST18	0.368	0.459	**0.807**	0.486	0.440	0.367	0.518	0.614	0.373	0.187
CONST19	0.408	0.486	**0.758**	0.527	0.463	0.362	0.640	0.693	0.419	0.146
CONST20	0.386	0.473	**0.757**	0.485	0.437	0.344	0.614	0.659	0.424	0.145
CONST4	0.394	0.469	**0.763**	0.467	0.447	0.342	0.526	0.582	0.350	0.104
CONST7	0.448	0.498	**0.809**	0.523	0.478	0.372	0.577	0.605	0.397	0.115
CONST8	0.425	0.488	**0.777**	0.482	0.446	0.401	0.528	0.591	0.379	0.177
CONST9	0.393	0.428	**0.742**	0.471	0.434	0.356	0.506	0.572	0.341	0.190

Table 4.46 indicates that constructivism had all its cross loadings values higher than all other constructs, hence its items are distinct from the items of other constructs. With this revelation, constructivism has been discriminally established.

Table 4. 47: Instructional Coherence (IC)

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDEM	SDIM	TQ	TTP
IC61	0.612	0.618	0.479	**0.751**	0.684	0.535	0.460	0.477	0.486	0.067
IC64	0.577	0.623	0.517	**0.814**	0.698	0.556	0.503	0.513	0.519	0.184
IC65	0.599	0.637	0.502	**0.770**	0.668	0.535	0.467	0.443	0.487	0.158
IC67	0.562	0.632	0.497	**0.765**	0.632	0.510	0.446	0.459	0.439	0.118
IC68	0.598	0.622	0.512	**0.777**	0.666	0.540	0.455	0.460	0.470	0.164
IC73	0.616	0.627	0.539	**0.797**	0.681	0.497	0.475	0.474	0.523	0.196
IC74	0.568	0.618	0.465	**0.770**	0.631	0.457	0.462	0.470	0.480	0.148
IC75	0.585	0.636	0.489	**0.791**	0.642	0.475	0.452	0.455	0.481	0.112
IC80	0.541	0.639	0.460	**0.815**	0.641	0.464	0.471	0.458	0.470	0.127
IC81	0.548	0.635	0.470	**0.799**	0.609	0.438	0.427	0.415	0.490	0.161
IC82	0.539	0.592	0.429	**0.785**	0.609	0.412	0.412	0.397	0.464	0.123
IC83	0.599	0.640	0.464	**0.823**	0.650	0.488	0.459	0.463	0.479	0.126
IC84	0.570	0.645	0.492	**0.768**	0.653	0.422	0.474	0.461	0.509	0.130

Instructional Coherence as can be seen from Table 4.47 had its entire cross loadings values higher than all other constructs in the Table. This is an indication that the items that defined instructional coherence are different from the items for other constructs in the model. It is therefore imperative to conclude that discriminality was established for IC.

Table 4. 48: Teacher-Trainees' Performance (TTP)

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDEM	SDIM	TQ	TTP
G1Y1S	0.117	0.048	0.098	0.097	0.081	0.039	0.033	0.086	0.105	**0.776**
G1Y2S	0.145	0.093	0.180	0.150	0.149	0.097	0.111	0.156	0.144	**0.864**
G2Y1S	0.112	0.070	0.105	0.113	0.126	0.066	0.050	0.103	0.157	**0.794**
G2Y2S	0.221	0.133	0.204	0.202	0.218	0.184	0.120	0.173	0.189	**0.885**

The grades for teacher-trainees as seen from Table 4.48 had its entire cross loadings values higher than the cross loadings of other constructs. The items for grades were therefore very distinct from items of other constructs in the model hence discriminality was established for teacher-trainees' performance.

Table 4. 49: Profound Understanding of Fundamental Mathematics (PUFM)

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDEM	SDIM	TQ	TTP
PUFM33	0.641	0.534	0.412	0.594	**0.735**	0.484	0.391	0.384	0.402	0.101
PUFM34	0.673	0.549	0.463	0.625	**0.781**	0.524	0.404	0.465	0.459	0.190
PUFM35	0.688	0.582	0.456	0.626	**0.792**	0.568	0.439	0.404	0.496	0.203
PUFM36	0.684	0.555	0.442	0.636	**0.802**	0.558	0.429	0.420	0.460	0.124
PUFM39	0.626	0.531	0.389	0.605	**0.764**	0.514	0.357	0.387	0.425	0.140
PUFM40	0.591	0.562	0.380	0.604	**0.748**	0.485	0.396	0.373	0.421	0.116
PUFM42	0.626	0.627	0.457	0.671	**0.825**	0.510	0.461	0.403	0.481	0.114
PUFM43	0.576	0.604	0.416	0.610	**0.748**	0.435	0.378	0.393	0.457	0.112
PUFM45	0.581	0.563	0.397	0.630	**0.771**	0.473	0.393	0.365	0.415	0.071
PUFM46	0.569	0.528	0.386	0.599	**0.743**	0.467	0.412	0.397	0.418	0.173
PUFM47	0.608	0.582	0.438	0.691	**0.799**	0.500	0.465	0.441	0.476	0.146
PUFM48	0.578	0.567	0.468	0.674	**0.802**	0.493	0.482	0.451	0.483	0.163
PUFM49	0.613	0.609	0.489	0.682	**0.801**	0.486	0.469	0.449	0.521	0.146
PUFM50	0.618	0.607	0.472	0.684	**0.814**	0.529	0.482	0.465	0.495	0.162
PUFM59	0.605	0.594	0.493	0.694	**0.701**	0.543	0.473	0.425	0.494	0.168

The cross loadings for Profound Understanding of Fundamental Mathematics as indicated in Table 4.49 has its entire cross loadings value for each item higher than that of other constructs. Thus, the items were distinct from items of other constructs hence discriminality was established for Profound Understanding of Fundamental Mathematics.

Table 4. 50: Relevant Previous Knowledge (RPK)

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDEM	SDIM	TQ	TTP
RPK11	0.502	0.416	0.342	0.475	0.496	**0.765**	0.363	0.365	0.343	0.083
RPK13	0.457	0.392	0.345	0.462	0.488	**0.787**	0.354	0.345	0.323	0.050
RPK15	0.507	0.450	0.364	0.554	0.574	**0.817**	0.371	0.375	0.364	0.191
RPK16	0.519	0.494	0.418	0.532	0.555	**0.826**	0.379	0.372	0.363	0.102
RPK17	0.457	0.447	0.335	0.449	0.485	**0.796**	0.337	0.302	0.355	0.082

With respect to Table 4.50, relevant previous knowledge also had its cross loadings values higher than the cross loadings of other constructs in the model. Discriminality has therefore been established for relevant previous knowledge.

Table 4. 51: SDT_ External Motivation

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDEM	SDIM	TQ	TTP
SDEM50	0.391	0.473	0.562	0.457	0.453	0.374	**0.790**	0.660	0.447	0.120
SDEM53	0.410	0.493	0.578	0.454	0.440	0.327	**0.782**	0.672	0.432	0.064
SDEM57	0.350	0.468	0.542	0.442	0.418	0.366	**0.804**	0.617	0.468	0.056
SDEM60	0.394	0.532	0.608	0.529	0.495	0.390	**0.812**	0.641	0.475	0.064
SDEM66	0.352	0.404	0.467	0.415	0.405	0.326	**0.744**	0.503	0.469	0.091
SDEM69	0.338	0.454	0.589	0.441	0.413	0.387	**0.793**	0.605	0.471	0.071
SDEM88	0.402	0.491	0.534	0.474	0.431	0.322	**0.783**	0.543	0.557	0.098

The cross loadings for external motivation as indicated in Table 4.51 has its entire cross loadings values higher than that of other constructs; an indication that its items were all different from the items of other constructs, hence the establishment of discriminality for external motivation.

Table 4. 52: SDT_ Internal Motivation

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDEM	SDIM	TQ	TTP
SDIM29	0.364	0.446	0.598	0.459	0.412	0.367	0.597	**0.775**	0.397	0.138
SDIM30	0.347	0.404	0.624	0.437	0.375	0.328	0.611	**0.772**	0.386	0.091
SDIM32	0.374	0.438	0.602	0.472	0.430	0.379	0.593	**0.772**	0.404	0.169
SDIM39	0.397	0.474	0.630	0.502	0.466	0.387	0.630	**0.813**	0.407	0.130
SDIM40	0.398	0.432	0.634	0.485	0.466	0.381	0.637	**0.839**	0.386	0.110
SDIM41	0.386	0.462	0.646	0.464	0.422	0.357	0.634	**0.833**	0.419	0.087
SDIM42	0.403	0.458	0.634	0.486	0.465	0.357	0.627	**0.841**	0.433	0.171
SDIM44	0.372	0.460	0.648	0.440	0.442	0.339	0.615	**0.830**	0.396	0.148
SDM46	0.389	0.454	0.616	0.498	0.429	0.323	0.644	**0.813**	0.426	0.141

Internal Motivation cross loadings were all higher than the cross loadings for other constructs as indicated in Table 4.52. The indication was that its items were all not similar to the items of other constructs, hence of discriminality was established for internal motivation.

Table 4. 53: Teacher Quality

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDEM	SDIM	TQ	TTP
TQ12	0.333	0.424	0.342	0.405	0.390	0.313	0.395	0.339	**0.782**	0.199
TQ14	0.338	0.426	0.360	0.447	0.434	0.329	0.388	0.373	**0.771**	0.209
TQ15	0.347	0.443	0.314	0.436	0.421	0.348	0.388	0.315	**0.780**	0.184
TQ16	0.329	0.422	0.337	0.425	0.419	0.355	0.365	0.337	**0.777**	0.204
TQ2	0.535	0.564	0.454	0.566	0.557	0.370	0.554	0.457	**0.769**	0.129
TQ22	0.362	0.434	0.320	0.423	0.408	0.343	0.401	0.357	**0.798**	0.159
TQ3	0.534	0.561	0.465	0.582	0.561	0.400	0.587	0.478	**0.791**	0.185
TQ35	0.312	0.386	0.274	0.367	0.356	0.288	0.375	0.301	**0.752**	0.197
TQ36	0.362	0.395	0.317	0.414	0.417	0.324	0.415	0.350	**0.784**	0.222
TQ5	0.482	0.542	0.399	0.489	0.484	0.343	0.533	0.398	**0.737**	0.064
TQ7	0.457	0.549	0.379	0.492	0.464	0.303	0.534	0.419	**0.765**	0.047
TQ8	0.470	0.582	0.400	0.552	0.512	0.348	0.517	0.409	**0.803**	0.077
TQ9	0.445	0.555	0.414	0.500	0.468	0.323	0.521	0.434	**0.763**	0.044

From Table 4.53, Teacher Quality had its entire cross loadings values higher than all other constructs in the corresponding rows, hence its items are distinct from the items of other constructs. With this finding, discriminality for Teacher Quality was established. The findings in respect of the cross-loadings of the constructs confirmed the discriminant validity of the measurement model and suggested that the proposed conceptual model for the study was acceptable with the confirmation of high reliability and convergence with discriminant validity of the constructs.

4.3.2 Evaluation of the Structural Model `

Since the measurement model assessment is satisfactory, the next step in evaluating PLS-SEM results is to assess the structural model to test its robustness. Relationships between constructs in a structural model are derived from estimating a series of regression equations. Therefore to assess structural relationships, collinearity is first examined to make sure that relationships between constructs do not bias the regression results.

One of these processes was carried out by calculating the variance inflation factor (VIF) values. VIF is simply a term used to describe a situation where two or more predictors in a regression model are highly correlated and is referred to as collinearity. It measures how much the variance of an estimated regression coefficient increase, if the predictors are correlated. It detects multicollinearity in regression analysis when there's correlation between predictors (i.e. independent variables) in a model where its presence can adversely affect the regression results. In other words, VIF estimates how much variance of a regression coefficient is inflated due to multicollinearity in the model. It is calculated by taking a predictor variable and regress it against every other predictor in the model, giving it R-squared values. The numerical value for VIF which ranges from 1 upwards, tells about what percentage the variance is inflated for each coefficient. For example, a VIF of 1.9 implied that the variance of a particular coefficient is 90% bigger than what is expected if there was no multicollinearity. A rule of thumb for interpreting the variance inflation factor states that a value of 1ndcates that predictors of constructs are not correlated; between 1 and 5 implied that predictors are moderately correlated and values greater than 5 implied a high correlation. In general, a VIF above 10 indicates high correlation and a cause for concern where some authors suggest a more conservative level of 2.5 or above (Dodge, 2008; Everitt & Skrondal, 2010) and mostly occurring at lower values of 3-5 (Mason & Perreault, 1991; Becker et al., 2015).

Table 4. 54: VIF of Predictor Constructs

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDEM	SDIM	TQ	TTP
CA			2.932							1.922
CGI			3.169							2.498
CONST										1.635
IC	3.453	3.453	4.367							
PUFM	3.315	3.315	**4.384**	1.549						
RPK									1.292	
SDEM									2.523	
SDIM									2.493	
TQ	1.674	1.674	1.811	1.549	1.000					1.782
TTP										

From Table 4.54, the highest VIF value of 4.384 which was lower than the threshold of 5.0 depicted no collinearity between predictor constructs in the structural model.

Having satisfied that there is no collinearity in the predictor constructs, the standard assessment criteria for structural models were considered. These were the coefficient of determination, (R^2) (Sarstedt et al., 2014), the statistical significance and relevance of the path coefficients, (β) (Memon & Rahman, 2014), the effect size, (f^2) and the measurement of the predictive relevance, (Q^2) of the path model through blindfolding that is based on cross-validated redundancy (Shmueli et al., 2016). To ensure the significance of these values, bootstrapping was conducted in order to obtain the significance of these indicators of the models. In addition, the researcher assessed the overall model fit of the structural model using the standardized root mean square residual (SRMR) factor (Henseler et al., 2016) and the Goodness-of-Fit (GOF) index (Shahid, Zhu, Ahmed, Zaigham & Muhammad, 2018) to test the strength of the model in responding to the hypotheses.

a. Coefficient of determination, (R^2)

The coefficient of determination, R^2 of the latent variables of the endogenous construct assessed the structural model (Sarstedt et al., 2014). Thus, the R^2 criterion measured the variance to explain each endogenous constructs in order to predict the strength of the structural model (Chin, 1998); thus the measured R^2 is the model's explanatory power (Shmueli & Koppius, 2011) also referred to as in-sample predictive power (Rigdon, 2012). The R^2 as a function of number of predictor constructs indicates that the greater the number of predictor constructs, the higher the R^2. Ranging from 0 to 1, with higher values indicating a greater explanatory power, the rule of thumb is that an R^2 value of 0.75, 0.50 and 0.25 are considered substantial, moderate and weak respectively (Henseler, Ringle & Sinkovics, 2009; Hair, Ringle & Sarstedt, 2013), however, acceptable R^2 values are based on the context of the discipline under study. For example, when predicting stock returns, an R^2 value as low as 0.10 is considered satisfactory (Raithel et al., 2012).

Table 4. 55: Coefficient of Determination (R^2)

Constructs	R square	R square Adjusted
CA	0.657	0.655
CGI	0.682	0.681
CONST	0.421	0.416
IC	0.710	0.710
PUFM	0.354	0.353
TQ	0.402	0.400
TTP	0.062	0.056

Referring to Table 4.55, the coefficient of determination (R^2) of 0.421 for constructivism was explained by five (5) independent constructs of TQ, CA, PUFM, IC and CGI. Consequently, the R^2 value of 0.062 for TTP which was due to four latent constructs of CGI, CONST, CA and TQ in the model means that only **6.2%** of the variance explained teacher-trainees' performance (TTP). Even though the R^2 was weak for teacher-trainees' performance in mathematics, all the R^2 values of the endogenous construct in this study were significant as indicated in Appendix J and Figure 4.1.

a. Path coefficients (β)

Endogenous variables' path coefficients (β) assessed the quality of the structural model (Memon & Rahman, 2014). The path coefficients (β) are the expected variation in the dependent construct for a unit variation in the independent construct(s). The β values of every path in the hypothesized model were computed such that the greater the β value of a dependent construct, the more substantial the effect on the endogenous latent constructs. However, the β value had to be verified for its significance through the T-statistics test in Appendix I and Figure 4.2.

Table 4. 56: Path Coefficients (β)

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDT-EM	SDT-IM	TQ	TTP
CA			0.015							0.152
CGI			0.226							-0.161
CONST										0.140
IC	0.214	0.512	0.302							
PUFM	0.585	0.194	0.077	0.715						
RPK									0.202	
SDT-EM									0.482	
SDT-IM									0.044	
TQ	0.060	0.204	0.100	0.191	0.595					0.137
TTP										

From Table 4.56, the path coefficients of 0.015 from CA to CONST, of 0.077 from PUFM to CONST, of 0.060 from TQ to CA and of 0.044 from SDIM to TQ were not significant because the p values for these paths were 0.782, 0.226, 0.074 and 0.484 respectively (Appendix I). The rest of the paths were therefore significant because their p values were less than 0.05. To this end, bootstrapping procedure of 5000 subsamples was carried out to evaluate the significance of the research questions (Chin, 1998) as in Appendix I.

b. Effect Size, f^2

Assessing how the removal of predictor constructs affects the endogenous constructs' R^2 value in evaluating the quality of the path model. This metric referred to as effect size, f^2, is the degree of impact that each exogenous latent construct had on the endogenous latent construct when the exogenous construct is deleted from the path model. A change in the value of the coefficient of determination (R^2) defined the significance of the influence the removed latent exogenous construct on the value of R^2 of the latent endogenous construct. More specifically, the rank order of the predictor constructs' relevance in explaining an endogenous construct in the structural model is often the same as comparing the size of the path coefficients and the effect sizes, f^2. In such situations, the f^2 should only be reported (Nitzl et al., 2016). As a rule of thumb, R^2 values higher than 0.02, 0.15 and 0.35 when an endogenous path was removed depicted small, medium and large effect sizes respectively (Cohen, 1988) as indicated in Table 4.57.

Table 4. 57: Effect Size (f^2)

Constructs	Values	Decision
CA -> CONST	0.000	Weak
CGI -> CONST	0.028	Moderate
IC -> CONST	0.036	Moderate
PUFM -> CONST	0.002	Weak
TQ -> CONST	0.010	Weak
CA -> TTP	0.013	Weak
CGI -> TTP	0.011	Weak
CONST -> TTP	0.013	Weak
TQ -> TTP	0.011	Weak
TQ -> CA	0.006	Weak
PUFM -> CA	0.301	Moderate
IC -> CA	0.039	Moderate
IC -> CGI	0.239	Moderate
PUFM -> CGI	0.036	Moderate
TQ -> CGI	0.079	Moderate
PUFM -> IC	1.141	**Strong**
TQ -> IC	0.081	Moderate
RPK -> TQ	0.053	Moderate
SD-EM -> TQ	0.154	Moderate
SD-IM -> TQ	0.001	Weak
TQ -> PUFM	0.549	**Strong**

As shown in the Table 4.57, if the paths of CA, CGI, IC, PUFM, and TQ to CONST were deleted from the model, their respective effects on R^2 of CONST is 0.0 (weak), 0.028 (moderate), 0.036 (moderate), 0.002 (weak) and 0.010 (weak) respectively, hence minimal effect sizes though significant (Appendix K). Considering the deletion of the path models of CA, CGI, CONST and TQ from TTP, the effect sizes would be 0.013, 0.011, 0.013 and 0.011 respectively, all indicating weak effect sizes. Effect sizes of CA, when TQ, PUFM, and IC paths were deleted had values of 0.006 (weak), 0.301 (moderate) and 0.039 (moderate). Should the paths of IC, PUFM and TQ leading to CGI were deleted, moderates effect sizes would have been realised with values of 0.239, 0.036 and 0.079 respectively. PUFM and TQ had model paths leading to IC and should these paths be deleted from IC, f^2 values would be 1.141 (strong) and 0.081 (moderate) respectively. Suppose model paths of RPK, SDEM and SDIM to TQ were deleted, the effect sizes, f^2 would have values of 0.053 (moderate), 0.154 (moderate) and 0.001 (weak) respectively. Finally, a strong effect size of 0.549 is realised when the TQ path leading to PUFM is deleted. Therefore, the removal of any predictor construct from the path model affects the corresponding endogenous constructs by changing the value of R^2. So, in conclusion all paths leading to CONST were very important in the model. To this end, bootstrapping procedure of 5000 subsamples was carried out to evaluate the significance of the research questions as shown in Appendix K and Figure 4.3.

c. Path Model's Predictive Relevance, Q^2

Another means of assessing PLS path model's quality is to determine the predictive accuracy, Q^2 (Geisser, 1974; Stone, 1974) by using the blindfolding procedure (Tenenhaus, Esposito, Chatelin & Lauro, 2005). This procedure removes single points in the data matrix, assigns the removed points with the mean and estimates the model parameters (Rigdon, 2014; Sarstedt et al., 2014). The measurement of the predictive relevance, (Q^2) of the path model through blindfolding is based on cross-validated redundancy of the constructs (Shmueli et al., 2016). As such, the Q^2 combines aspects of out-of-sample prediction power and in-sample explanatory power (Shmueli et al., 2016; Sarstedt et al., 2017). Small differences between the predicted and the original values translate into a higher Q^2 value, thereby indicating a higher predictive accuracy. As a guideline, Q^2 values should be greater than zero for a particular endogenous latent constructs with the recommendation that the conceptual model can predict the endogenous latent constructs. Consequently, Q^2 values higher than 0.0, 0.25 and 0.50 depict small, medium and large predictive relevance of the PLS-path model.

Table 4. 58: Predictive Relevance, Q^2 of Path Model Quality

Constructs	SSO	SSE	Q^2 (=1-SSE/SSO)
CA	3846	2185.64	0.432
CGI	6410	3763.142	0.413
CONST	6410	4825.321	0.247
IC	8333	4725.197	0.433
PUFM	9615	7596.513	0.210
RPK	3205	3205	
SDT-EM	4487	4487	
SDT-IM	5769	5769	
TQ	8333	6432,436	0.228
TTP	2564	2467,602	0.038

Table 4.58 shows that the Q^2 value for the endogenous construct of TTP in this study model was equal to 0.038 depicting small predictive relevance. This value was higher than the threshold limit of 0.0, and therefore supported the quality of the path model. In addition, the Q^2 value of CONST of 0.247 and PUFM of 0.210 were greater than 0.0 indicating a small predictive accuracy of the structural model whilst IC, CA and CGI have medium predictive relevance of 0.433, 0.432 and 0.413 respectively.

Having positive contribution from RPK and SDEM, TQ had a small predictive relevance of 0.228 in the conceptualised structure model as indicated in Figure 4.4.

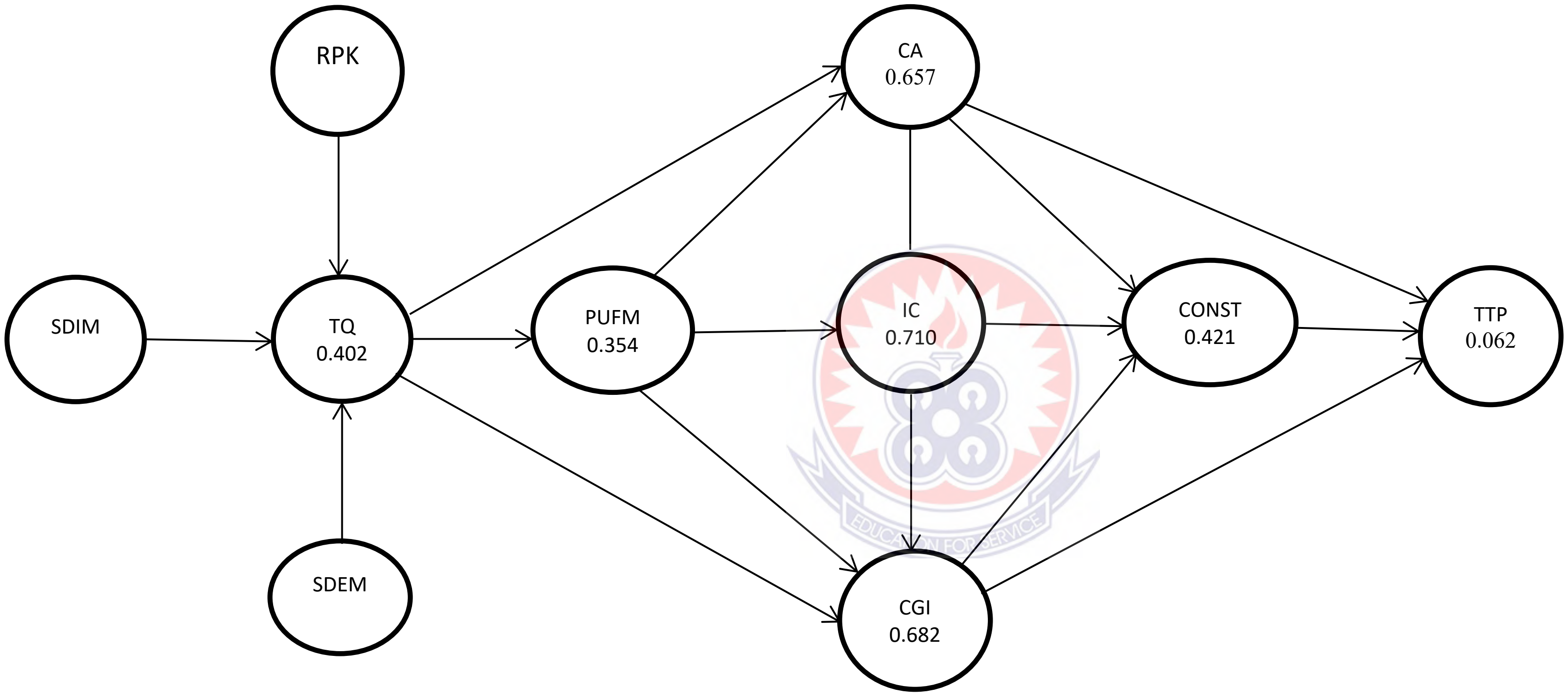

Fig. 4. 1 Coefficient of Determination, R2: 0.75≤ R2≤1.0- substantial; 0.5≤ R2<0.75 - moderate; 0.25≤ R2<0.5- weak

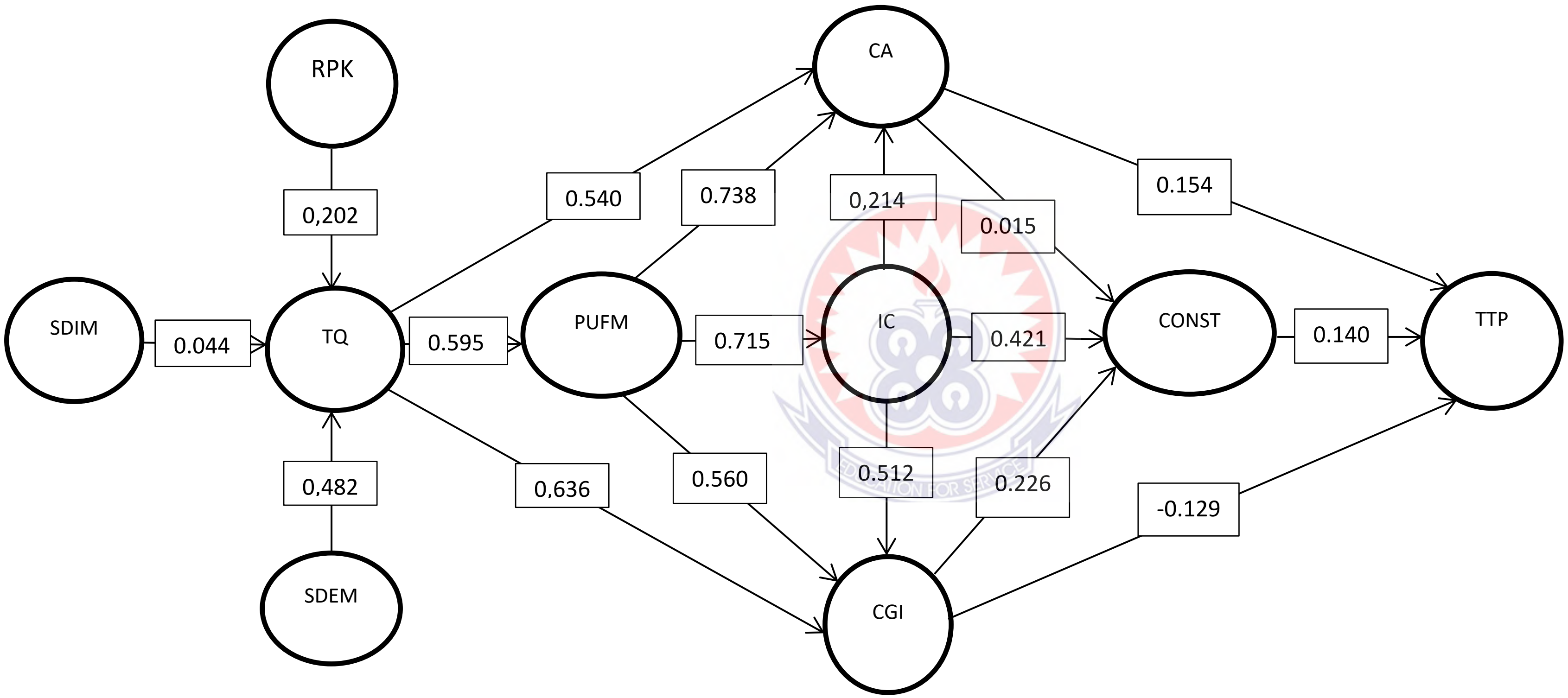

Fig. 4. 2 β: Non Significant Paths using T-Stats: CA -> CONST; PUFM -> CONST; TQ -> CA; SD-IM -> TQ

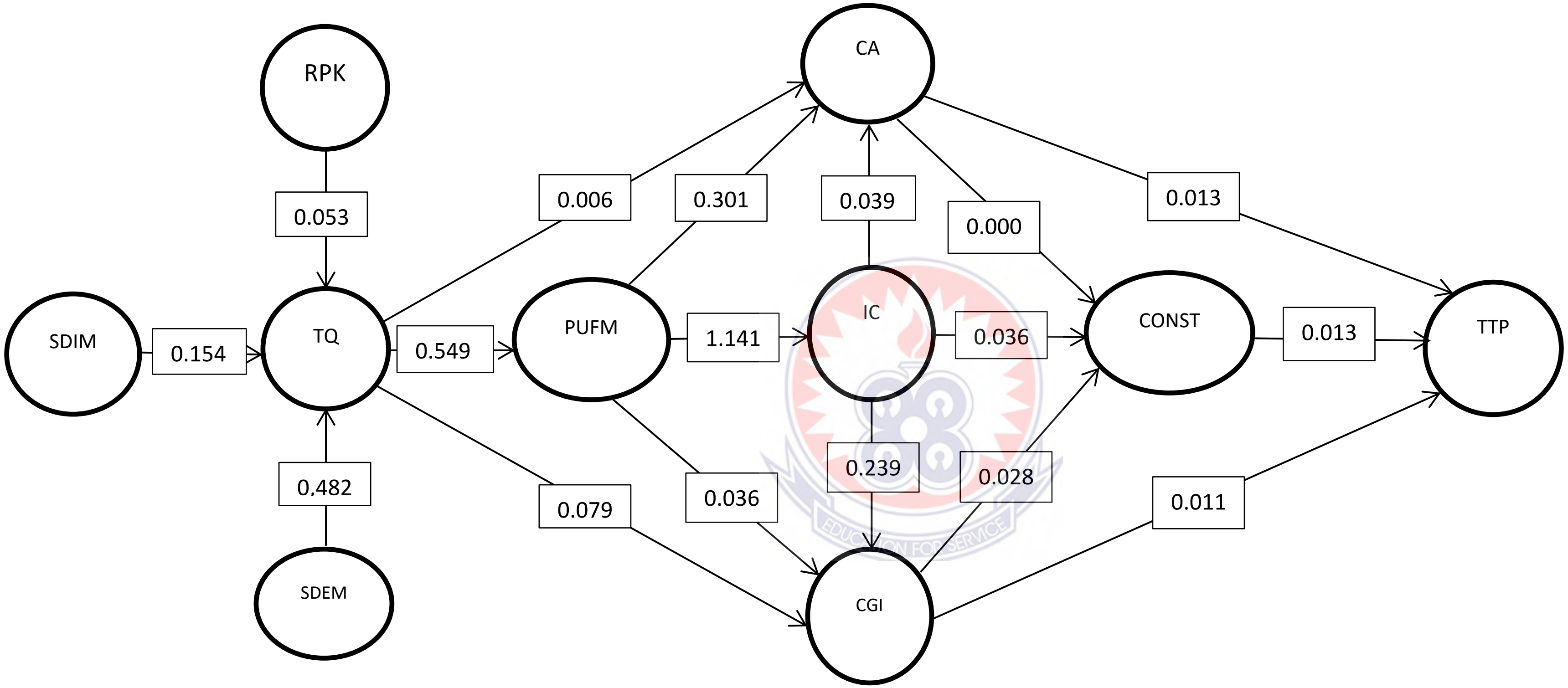

Fig. 4. 3 Effect Size (f2): f2≤0.02- Small; 0.02<f2<0.35-medium; f2≥0.35-large

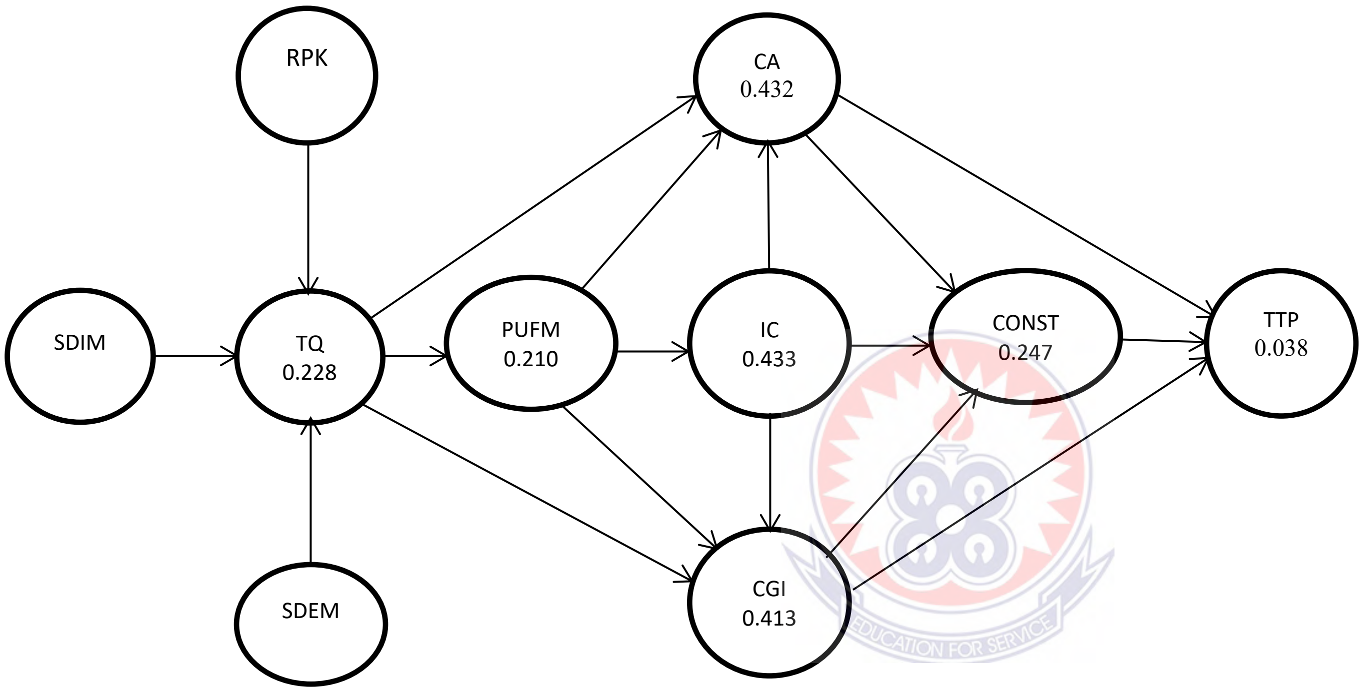

Fig. 4. 4 Predictive Relevance, Q2: 0.0<Q2<0.25-small; 0.25<Q2<0.5-medium; Q2>0.50-large

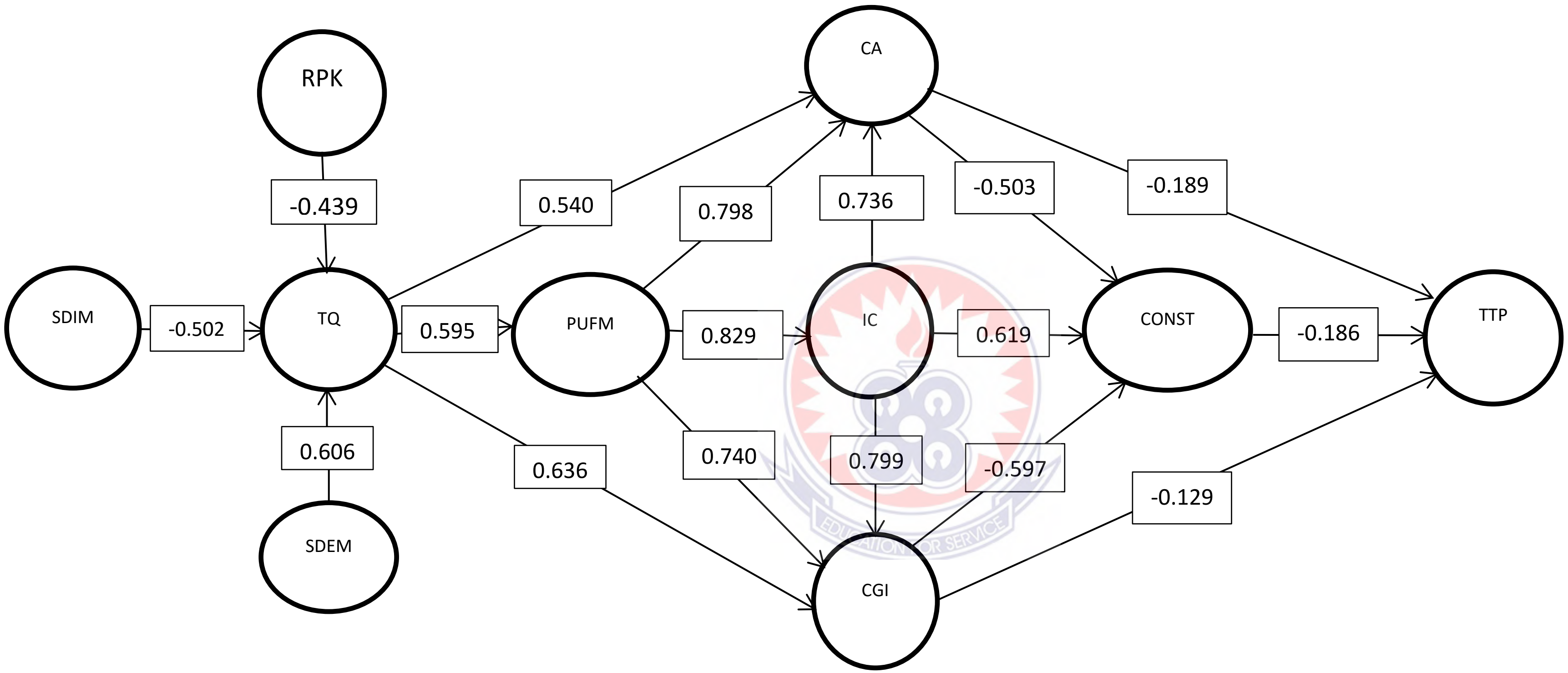

Fig. 4. 5 Correlation Coefficient, r

d. Model Fit of Structural Model

The overall model fit of the structural model was assessed using the standardized root mean square residual (SRMR) factor (Henseler et al., 2016). The SRMR, a measure of estimating model fit is an index of the average of standardized residuals between the observed and the hypothesized covariance matrices (Chen, 2007). An SRMR value equal to or less than 0.08 (=<0.08) indicate a good fit of the path model (Hu & Bentler, 1998), with a lower SRMR giving a better fit.

Table 4. 59: Model fit Summary

	Saturated Model	Estimated Model
SRMR	0.049	0.109
d_ULS	10.479	50.685
d_G	3.525	3.829
Chi-Square	12143.833	12802.224
NFI	0.762	0.749

Table 4.59 shows that the SRMR was 0.049, revealing that the conceptual model of the study had a good fit, with the measurement of the Chi-Square and NFI equalled to 12143.833 and 0.762 respectively. The SRMR figure was confirmed in Appendix L, where the value 0.049 lied between the confidence interval.

4.4 Research Question 3

What is the effect of teacher professional practice on tutors' instructional strategies?

With respect to the effect of independent (exogenous) constructs on Teacher Quality (TQ), the findings in Table 4.56 confirmed that RPK significantly influenced

TQ (β = 0.202, T =3.914, p<0.000) and external motivation (SD_EM) also positively affected TQ with significant values of (β = 0.482, T = 7.383, p<0.000). However, the influence of internal motivation (SD_IM) on TQ was insignificant (β = 0.044, T = 0.699, p<0.484) because teacher-trainees are required to take responsibility of motivating themselves in mathematics class (Appendix I and Figure 4.2). However, internal motivation has no effect on teacher quality as this has to do with the individual teacher-trainee even though external motivation is a recipe for internal motivation. This finding follows the assertion that external motivation triggers the internal motivation which has a positive effect on students, especially when they have low levels of intrinsic motivation (Brophy, 2004; Cameron, 2001; Lepper, Corpus & Lyengar, 2005). Therefore RPK and SD_EM had positive and significant effects on TQ resulting in a coefficient of determination (R^2) of 0.402 as indicated in Table 4.55 and figure 4.1 and confirmed by the p values and T statistics in Appendix J. It is therefore instructive that tutors must endeavour to revise current lessons in conjunction with previous lessons that are relevant to enable teacher-trainees connect mathematical concepts for teacher-trainees' understanding. In this regard, mathematics tutors are to encourage teacher-trainees to also revise previous topics or concepts that are related to lessons that will be taught. This underscored the need for tutors to make syllabuses or lesson plans available to teacher-trainees to enable them revise relevant previous knowledge and connect same to subsequent lessons. This strategy enhances the smooth flow of lessons as tutors will not be assuming that teacher-trainees are knowledgeable in the fundamentals that lead to understanding the current topic but have concrete idea of what teacher-trainees know about the basics of current lessons. Students who are highly motivated take studies seriously, accept challenges, participate in classroom activities and consider teachers' recommendations and as a result have high academic achievement (Pajares & Schunk, 2001; Wolters & Rosenthal, 2000) that enables them to display their real potential (Eggen & Kauchak, 1997). Therefore, external motivation which affects teacher quality emphasised the need for tutors to pay particular attention to motivating teacher- trainees and to ensure that they are encouraged to participate in lessons (de Leeuw, Hox & Dillman, 2008; Lehmann, 1986; Pokay & Blumenfeld, 1990).

For the effect of teacher quality (TQ) on instructional strategies, Table 4.56 showed that the effect of TQ on CA was not significant (β = 0.060, T = 1.785, p<0.074), however, the findings provided empirical support that TQ positively influence CGI with significant values of (β = 0.204, T = 5.166, p<0.000). Explanation to TQ not affecting CA stems from the fact that teacher-trainees are responsible for their cognitive reasoning when it comes to cognitive activation such that they think outside the box to evolve numerous ways of solving mathematics problems. These actions from the learners are possible when teachers challenge their students (Hattie, 2009). This is to say that students must not always be told to do a thing, but tutors must ask questions that will enable teacher-trainees to think critically and bring out innovative ideas in a mathematics class (Khalid, 2009). Again, the research findings revealed that TQ influenced IC such that the path coefficient of 0.191 was significant (β = 0.191, T = 4.453, p<0.000). This is evident with the fact that for any lesson to be consistent, teachers must be at the centre. Wong (2007) upheld that the tutors in the college of education needs to be the lead person in the mathematics classroom and not the teacher-trainees because it is the tutors who is supposed to design the learning activities, which is fundamental to understanding a mathematical concept. Consequently, the teacher is expected to lead the class and its learning activities

(Tsang et al., 2014). Additionally, TQ has positive effect on PUFM with significant values of (β = 0.595, T = 14.662, p<0.000). Teachers' understandings of mathematics afford them a broader and more varied range of strategies to represent and explain mathematics concepts (Ma, 1999). But an insufficient understanding of mathematical content limits the capacity of teachers to explain and represent mathematical contents to students in a way that make sense; a shortfall that cannot be balanced by pedagogical skills (Baumert et. al, 2010). Likewise, effect of TQ on CONST and TTP were positive and significant with values of (β = 0.100, T = 2.325, p<0.020) and β = 0.137, T = 2.261, p<0.024) respectively (Appendix I and Figure 4.2). These findings are due to the fact that for CGI, IC and PUFM to be effective as instructional strategies, tutors must play significant roles to cause the teacher-trainees to invoke the non-structural knowledge into the scientific knowledge, ensure consistent and smooth teaching and learning process and guarantee a successful use of basic mathematics principles in every lesson respectively for clear understanding of concepts. Except for the insignificant relationship between TQ and CA, there were significant relationships between TQ and other instructional strategies and constructs. The decision therefore is that teacher quality is very relevant to constructivism when it comes to teaching mathematics.

4.5 Research Question 4

What relationships exist among the instructional strategies and between the constructs and constructivism?

For the relationships among the instructional strategies, Table 4.56 indicated that IC and PUFM have positive effect on CA with significant values as (β = 0.214, T= 2.809, p< 0.005) and (β = 0.585, T= 8.170, p< 0.000) respectively as clearly seen in Appendix I. These paths consequently resulted in a coefficient of determination (R^2) of 0.657 for CA as seen in Table 4.55 and confirmed in Appendix J. Furthermore, IC, PUFM and TQ had positive effects on CGI because of the significant values of (β = 0.512, T= 8.606, p< 0.000), (β = 0.194, T= 3.574, p< 0.000) and (β = 0.204, T= 5.166, p< 0.000) respectively as shown in Appendix I, resulting in an R^2 value of 0.682 for CGI as seen in Table 4.55. A positive relationship also existed between TQ and PUFM with significant values of (β = 0.595, T= 14.662, p< 0.000) in Appendix I with an R^2 value of 0.354 in Table 4.55. There was also a positive relationship between PUFM and IC with significant value of (β = 0.715, T= 16.655, p< 0.000) and a relationship between TQ and IC with significant values of (β = 0.191; T = 4.453; p < 0.000) in Appendix I. The positive relationships between TQ and IC and that of PUFM and IC resulted in an R^2 value of 0.710 for IC as depicted in Figure 4.1, Table 4.55 and Appendix J. A tutor's knowledge in fundamental principles in mathematics lessons and his ability to teach consistently and sequentially affords the teacher-trainee to understand mathematics concepts thereby enabling the teacher-trainee to think extensively and outside the box to solve non-routine mathematics problems. CGI as a mathematics instructional strategy emphasise the incorporation of cultural values in mathematics lessons. The tutor who is at the centre of this instruction can only succeed when lesson are well-organized in order to teach from basic principles for the understanding of the teacher-trainees. From the foregoing, there were interrelationships among the instructional strategies which enhanced teacher-trainees' understanding of mathematical concepts.

Table 4. 60: Path Coefficients (β)

Constructs	CA	CGI	CONST	IC	PUFM	RPK	SDT-EM	SDT-IM	TQ	TTP
CA			0.015							0.152
CGI			0.226							-0.161
CONST										0.140
IC	0.214	0.512	0.302							
PUFM	0.585	0.194	0.077	0.715						
RPK									0.202	
SDT-EM									0.482	
SDT-IM									0.044	
TQ	0.060	0.204	0.100	0.191	0.595					0.137
TTP										

Table 4. 61: Coefficient of Determination (R^2)

Constructs	R square	R square Adjusted
CA	0.657	0.655
CGI	0.682	0.681
CONST	0.421	0.416
IC	0.710	0.710
PUFM	0.354	0.353
TQ	0.402	0.400
TTP	0.062	0.056

Results of the effect of instructional strategies on constructivism (CONST) shows that there are no positive effect of CA and PUFM on CONST because the values of (β = 0.015, T = 0.782, p<0.277) and (β = 0.077, T = 1.210, p<0.226) were respectively insignificant as shown in Appendix I. However, IC and CGI had positive effect on CONST with significant values of (β = 0.302, T = 3.096, p<0.002) and (β

= 0.226, T = 3.492, p<0.000) respectively with IC having the greater effect on CONST as indicated in Appendix I and Figure 4.2 such that the R^2 value for constructivism was 0.421 as in Table 4.55. This result stresses the fact that constructivist theory of learning describes how teaching consists of related activities, showing which aspects of the current subject-matter are linked to previous ones which are sometimes culturally related and understood by the teacher-trainees in order for them to construct their own mathematical concept (Pitkaniemi & Hakkkinen, 2012). To achieve the use of constructivism in mathematics lessons, teachers must take cognisance about the fact that lessons must consistently flow in a structured manner with the blend of culture of scientific reasoning and everyday understanding of concepts. This result coincided with research question 2 which indicated that IC was the most instructional strategies tutors in the colleges of education used to teach mathematics.

With a significant level of 0.0, other constructs such as RPK, SD_EM, SD_IM and TQ positively affected constructivism in this study as in Appendix N. Subsequently, these conclusions pointed to the fact that tutors and teacher-trainees recognized that they have roles to play when it comes to the construction of one's own mathematical knowledge.

4.6 Research Question 5

How do other constructs and constructivism affect teacher-trainees' performance in mathematics?

Considering the effect of constructs on teacher-trainees' performance (TTP), the findings in Table 4.56 confirmed that CA significantly influenced TTP because (β = 0.152, T = 2.796, p<0.005) values were significant (Appendix I and Figure 4.2).

This result was very weighty because for teacher-trainees to perform well in mathematics, they must be able to espouse their own theory to solve problems and to consequently construct their own understanding of concepts through tutors' support and encouragement. Also, the influence of TQ on TTP was positive and significant with the values (β = 0.137, T= 2.261, p< 0.024) because mathematics tutors must be effective in pedagogy and content for teacher-trainees to understand mathematics concepts with ease hence enabling them to perform well. However, the research findings indicated that CGI negatively influenced TTP with insignificant values of (β= -0.161, T = 2.491, p<0.013. In total, CA and TQ paths cumulatively resulted in a coefficient of determination, R^2 of 0.062 for TTP. From the results, CA had the strongest and positive effect on TTP. Conversely, TTP has significant effect on all the constructs according to Appendix N.

With regards to the effect of constructivism (CONST) on teacher-trainees' performance (TTP), the result indicated that there was a positive relationship between CONST and TTP with significant values of (β = 0.140; T = 2.546; p<0.011). For teacher-trainees to understand mathematical concepts, tutors must at all times use the constructivist theory of learning. The latent variable correlation as illustrated in Table 4.60 indicates that TTP, TQ, SDT_IM, SDT_EM, RPK, PUFM and IC had positive effects on constructivism. In this case, the tutor with strong disciplined knowledge and sound disposition towards teaching is the most important variable affecting teacher-trainee performance in mathematics (Hattie, 2009). Among multiple factors within the colleges of education, tutor effectiveness in adopting constructivism through the use of PUFM and IC is extremely important to the teacher-trainees' understanding of mathematical concepts. This is because teachers do matter most when it comes to school improvement and student learning (Stronge, 2010). In addition, tutors' external motivation on the lives of the teacher-trainees inure to activating their internal motivation.

Table 4. 62: Latent Variable Correlation

Path Model	Original Sample (O)	Sample Mean (M)	SD	T Statistics (|O/STDEV|)	P Values
TTP -> CONST	0,186	0,189	0,045	4.142	0.000
TQ -> CONST	0,483	0,484	0,049	9.767	0.000
SDT-IM -> CONST	0,772	0,771	0,029	26.632	0.000
SDT-EM -> CONST	0,703	0,702	0,039	18.246	0.000
RPK -> CONST	0,452	0,451	0,056	8.151	0.000
PUFM -> CONST	0,566	0,565	0,049	11.632	0.000
IC -> CONST	0,619	0,617	0,047	13.240	0.000

The results of the study from Table 4.62 therefore revealed that CGI, IC and TQ had positive and significant effects on CONST (R^2=0.421; p=0.000). By extension, CA, CGI, CONST and TQ had significant effects on TTP (R^2 = 0.062, p = 0.000). The predictive relevance, Q2 for TTP and CONST were 0.038 and 0.247 respectively, with the model fit (SRMR) value of 0.049. The final SEM results revealed that the CGI had the highest path coefficient (β =-0.161) as the major construct affecting TTP and IC as the highest path model value of β =0.302 affecting CONST. Therefore, tutors should pay more attention to CGI to improve TTP and pay attention to IC for successful CONST. Constructivism learning model therefore encompasses a lot of strategies for successes in mathematics lessons. So mathematics teachers must be conversant with most of the teaching strategies for successful mathematics lessons.

4.7 Summary

This study practically revealed that IC and PUFM as instructional strategies affected CONST whilst IC, CGI, and CA also as instructional strategies affected TTP using the PLS-SEM technique which is an effective technique for developing, analysing and validating complex models (Hair, Risher, Sarstedt & Ringle, 2019). Other constructs such as CONST and TQ influenced TTP. In addition, TQ, RPK, SDT_IM, and SDT_EM affected CONST. Generally, effective instructional strategies and methods results in good academic achievement rate which are seen in teacher competency in education and appropriate motivation and concentration in learning (Saritas & Akdemir, 2009). In this case, the teacher with strong disciplined knowledge and sound disposition towards teaching is one of the most important variables affecting student performance in mathematics (Hattie, 2009). Consequently, mathematics tutors and teacher-trainees in the colleges of education should focus on CGI and CA strategies respectively to improve TTP. It has also been shown that a detailed understanding of the curriculum scope, lesson planning and implementation, monitoring and evaluation substantially assists in the effective understanding of mathematical concepts to improve TTP. Accordingly, lack of experienced mathematics tutors and poor classroom management may lead to unsuccessful TTP in the colleges of education. These results were uniformed with regards to previous research in Fobih, Akyeampong and Koomson (1999) assert that significant part of problems confronting students' low academic performance has to do with teacher quality in terms of instructional skills and professional commitment. It is therefore recommended that tutors pay special attention to instructional strategies such as CGI to achieve an enhanced TTP because the results of this study recommended that CGI had significant and positive influence on TTP and that its quality could be enhanced by proper tutors' supervision and monitoring of the classroom activities as recommended by Remesh (2013) who indicated that the emerging changes in mathematics curricula calls for mathematics teachers with special training skills to monitor their own performance for continued and efficient performance of students at any stage. Tutors of mathematics must consider relevant previous knowledge in order to teach effectively and ensure that they motivate teacher-trainees adequately.

CA having effect on TTP imply that teacher-trainees must put in personal efforts to understand mathematics concepts since much does not depend on the tutors but on the teacher-trainees such that they must put in the effort to understand mathematical concepts by taking advantage of the available learning environment (Seidel & Shavelson, 2007). Additionally, for effective mathematics lessons, TQ is a factor in CGI, IC and CONST such that tutors must pay adequate attention in their efforts to use these instructional strategies effectively to enhance mathematics understanding. TQ depends on how well tutors consider relevant previous knowledge as part of constructing mathematics lessons and how they motivate the teacher-trainees before, during and after lessons. The values of 0.247 and 0.038 in the conceptual model predict the relevance of the path model as adequate to measure CONST and TTP respectively. In general, the Q^2 values for all the endogenous constructs were greater than zero, hence indicating predictive accuracy of the structural model.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Overview

This chapter of the study provided summary of major findings and their implications to teaching and learning of mathematics and the conclusions based on the insights gained from the findings. Recommendations for implementing teaching and learning strategies of mathematics in in the colleges of education and the possible areas for further research were outlined.

5.1 Summary of the Study

The study investigated tutors' use of instructional strategies that support constructivism in the teaching of mathematics in the colleges of education and its effect on teacher-trainees' performance. The problem for the study was at the instance of poor performance in mathematics at the basic and secondary levels and basic teachers' poor knowledge about constructivist theory especially in the Volta Region. The study therefore adopted the constructivist theory which is espoused by Piaget, Ausubel and Vygotsky and operationalized by Cognitive Activation (CA), Profound Understanding of Fundamental Mathematics (PUFM), Instructional Coherence (IC) and Cognitive Guided Instruction (CGI). Constructivism emphasizes that knowledge is a product of one's cognitive act by building on previous knowledge that allows one to move to new knowledge (Lerman, 1996). Its approach to learning holds the view that learners actively construct their own knowledge which is backed by the experiences they acquired (Elliott et al., 2000). Consequently, the focus of the study on the use of constructivism as the theoretical framework is to help teacher-trainees develop the ability to construct their own understanding of mathematical concepts so as to improve their performance both as learners and teachers. Subsequently, this performance is to ensure that teachers effectively teach pupils of first cycle schools and for the pupils to understand fundamental mathematics concepts through the construction of their own mathematical knowledge in order to build strong mathematical foundation. The study therefore examined the role of the tutor in establishing these instructional strategies in the classroom setting during mathematics lessons.

The investigation was conducted through the use of the researcher's own validated instrument which was designed with a 5-point likert scale and piloted on two colleges of education. These pilots were E.P. College of Education, Amedzofe and Holy Spirit College of Education, Ho respectively. The administration of the questionnaire to these pilot colleges was to ensure its reliability using the Cronbach's Alpha statistic and Cohen's Kappa interrater techniques for internal and external consistencies of the items respectively (McHugh, 2015). The data were reduced using the principal component analysis method of factor analysis to ensure the reliability and validation of the instrument. The validated instrument was later administered to the study sample whose data were analyzed using descriptive statistics, binomial test, composite score and partial least square structural equation model (PLS-SEM). In all, six hundred and forty-one (641) third-year teacher-trainees from Akatsi College of Education, Peki College of Education and St Francis College of Education, Hohoe in the Volta Region were considered as sample for the study. The sample consisted of two hundred and sixteen (216) respondents from Akatsi, contributing 33.7%; one hundred and seventy-nine (179) respondents from Peki, contributing 27.9%; and two hundred and forty-six (246) respondents from St. Francis-Hohoe, contributing 38.4%.

The reliability of the questionnaire for the study was very high with a Cronbach's Alpha value of 0.930 for the pilot study with an agreement ratio of 0.980; while the external consistency of the items was 0.933 using a Cohen's Kappa statistics. When the responses to the items by the two colleges were combined, the Cronbach's Alpha value was 0.931. Validity of the questionnaire expressed the degree to which the items measured the constructs that they sought to measure using the face validity, construct validity and content validity. All these procedures saw the reduction of the items in the questionnaire from 255 to 104 for all the constructs without losing their operational definitions and meanings. In conducting the construct validity, the principal component analysis method of factor analysis was employed. The data were normally distributed because the results of the kurtosis and skewness values lied between -1 and +1 respectively. This consequently satisfied the criteria of performing factor analysis, hence the sample adequacy, factor loadings, determinant of the matrices and the cumulative rotation sums of squared loadings (CRSSL) were determined using SPSS version 20. The research finally revealed that the items of the questionnaire were content valid to measure the constructs without missing their relevant meanings.

The conceptual understanding of mathematics as demonstrated by the teacher-trainees in the study indicated that they have some level of understanding mathematical concepts. However, they failed to demonstrate the understanding of constructivism. This may be due to the fact that tutors did not explain to the teacher-trainees that constructivism model was being used to teach, though the analysis of the data pointed to the fact that constructivism model was adopted in mathematics lessons in the colleges of education.

5.2 Findings of the study

The background of teacher-trainees was investigated to indicate their preparedness in learning mathematics and teaching same to the pupils in the first cycle schools in Ghana. From the study there were more male teacher-trainees (62.9%) than their female counterparts (37.1%) in the colleges of education. It was discovered that 87.2% of teacher-trainees hailed from the Volta Region where the colleges were located such that 78.8% and 84.7% attended junior and senior high schools respectively in the Volta Region. Findings from the research revealed that teacher-trainees had adequate background in mathematics such that as much as 94.5% and 93.3% passed mathematics with grades 1-6 to gain admission into the senior high schools and colleges of education respectively. The teachers of mathematics at the junior and senior high schools were highly rated for mathematics lessons delivery with 57.5% and 49.5% of the teachers being rated at points 7-10 (with 10 as the highest point) at the basic and secondary levels respectively. So, on the average 94.8% and 94.0% of the respondents passed mathematics courses in the first year and second years respectively in colleges of education though this pass rate did not resonate well with teacher-trainees' knowledge on constructivism which is believed to enhance mathematics performance (Jaworski, 1991, 1994). The major findings from the research showed that 52.7% of respondents had no idea about constructivism. Nevertheless, out of the 47.3% that claimed to have an idea about the teaching model, only 32.7% got the understanding of the constructivism concept right. This contradicts the findings obtained when Ramsook and Thomas (2016) conducted similar research on teacher-trainees in Trinidad and Tobago. Even though only 34.9% of the teacher-trainees indicated teaching profession as their first choice, as much as 72.9% said they would want to teach mathematics.

As much as 67.1% of respondents were below the age of 25 years. These categories of respondents performed better in mathematics than those who were 25 years and above (Owolabi & Etukiren, 2014). In addition, 24.6% of the respondents who offered science programmes at the senior high schools performed better in mathematics than those who offered the humanities (Karjanto, 2017). On the whole, only 13.9% of all the respondents who completed SHS in 2017 got direct admission into the colleges in the 2017/2018 academic year.

The proportion of variance in the TTP dependent variable that was explained by the independent variables of CA, CGI, IC and CONST was 0.062 whilst the proportion of variance in the CONST dependent variable which was 0.421 as explained by CA, CGI and IC. Stop From the analysis, it is clear that IC, PUFM, and CGI supported CONST in mathematics lessons. Thus the path coefficient of 0.302 indicated that IC had a direct effect on constructivism with a correlation coefficient of 0.619 between the two constructs. Also PUFM had a direct effect on CONST with a path coefficient of 0.077 and a correlation coefficient of 0.566. With respect to CGI, when CGI changed by one standard deviation, CONST changed by 0.226 standard deviation. The implication is that constructivist theory was seen in mathematics classes when tutors applied IC, PUFM, and CGI instructional strategies. In addition, external and internal motivation of self-determination theory and teacher quality (TQ) played significant role in constructivism.

According to the descriptive statistics, the mean likert-scale response and the composite scores confirmed the fact that constructivist theory was adopted in mathematics lessons with the scores of 3.971 and 3.973 respectively. In addition, 77.4% of the respondents with respect to the binomial test agreed that tutors in the colleges of education use constructivism during mathematics lessons. The findings from the analysis confirmed that RPK and external motivation (SD_EM) significantly influenced TQ. However, the influence of internal motivation (SD_IM) on TQ was insignificant because teacher-trainees were required to take responsibility of motivating themselves in learning mathematics. TQ positively influenced CGI, PUFM and IC instructional strategies. In addition, TQ has significant effect on CONST and TTP. Thus teacher quality was an important factor in teacher-trainees' performance in mathematics. The analysis indicated that IC and PUFM had positive effect on CA. Furthermore, IC, PUFM and TQ positively influenced CGI instructional strategy. Also, there were positive relationships that existed between TQ and PUFM, between PUFM and IC and between TQ and IC. The results of the study show that IC and CGI instructional strategies had positive effect on CONST with IC having the greater effect. The findings confirmed that CA and TQ significantly influenced TTP with CA having the stronger and positive effect. With regards to the effect of constructivism (CONST) on teacher-trainees' performance (TTP), the result indicated that there was a positive relationship between CONST and TTP.

5.3 Conclusion

The study revealed that even though tutors in the colleges of education use constructivism to teach mathematics, teacher-trainees were not aware of this teaching model as only 32.7% could correctly explain constructivism out of 303 who claimed to understand the teaching model. It was therefore indicative from the results that 52.7% of the respondents have no idea about constructivism as a teaching model.

These responses were not consistent with a research conducted by Ramsook & Thomas (2016) in Trinidad and Tobago where 96.2% of teacher-trainees revealed that they understand the principles of constructivism which subsequently influenced their personal philosophy of teaching and learning. Therefore, for the effective use of constructivism as a teaching model, tutors must concentrate on PUFM and IC as instructional strategies in mathematics class. Meanwhile, teacher quality supported constructivism instruction such that acquisition of relevant previous knowledge by the tutor and the teacher-trainee, motivation provided to the teacher-trainees by the tutor and teacher-trainees' internal motivation are equally important. The implication therefore was that there are correlations between constructivism and PUFM, IC, RPK, SDT_EM, SDT_IM and TQ. However, cognitive activation as an instructional strategy did not support constructivism; instead cognitive activation is a product of constructivism. The quality of mathematics tutors depended on RPK and their ability to motivate the teacher-trainees as the teacher-trainees also put in optimal efforts to motivate themselves.

If teacher-trainees appreciate mathematics and if the objective of helping the Ghanaian pupil to understand mathematical concepts using constructivism as spelt out in the Ghanaian primary mathematics curriculum is to be achieved, then based on the results of this study, measures must be put in place to help teacher-trainees to develop conceptual understanding of mathematics using cognitive activation, instructional coherence, profound understanding of fundamental mathematics and cognitive guided instructions which serve as foundation for constructivism.

5.4 Recommendation

From the findings and conclusions, it is recommended that

- The Colleges of Education mathematics curriculum must include the concept of constructivism as a course so that it can be applied to other subjects as well. By this, teacher-trainees will be taught the properties of this instructional model before they embark on their on- and off-campus teaching practices.
- During teaching practices, teacher mentors and supervisors must ensure that every teacher-trainee demonstrate adequate knowledge about constructivism and its applications during mathematics lessons.
- Mathematics tutors in must undergo continuous professional development that will expose them to constructivist theory and other instructional strategies such as cognitive activation, instructional coherence, profound understanding of fundamental mathematics and cognitive guided instructions.
- Teacher-trainees must be made to evaluate mathematics tutors n the colleges of education.
- Tutors must explicitly make teacher-trainees aware of the constructivist theory during mathematics lessons using CA, PUFM, IC and CGI instructional strategies (Rumsey, 2002).
- Teacher-trainees must be trained to become computationally literate as research has revealed that understanding of mathematical concepts and their appropriate application to life, relate to the adopted effective instructional strategies (Kahneman, Slovic & Tversky, 1982).
- Teacher-trainees should be made to understand and interpret mathematical concepts and procedures that were encountered in everyday life if the appropriate instructional strategies are used.

- College of education must institute a programme to specially take teacher-trainees whose interests are in mathematics to be taken through rigorous mathematics concepts as much as 72.9% of them wished to teach mathematics.

5.5 Limitation of the study

One identified limitation of the study was that teacher-trainees in responding to the questionnaire were really confused about which particular tutor they are assessing. This came to light when two students walked to me and said, they were taught by more than one tutor, so in responding to the questionnaire they were not clear in their minds as to who they were evaluating. Secondly, the selection of the sample was purposive which consequently has an effect on the generalizability of the study. Thirdly, the study could have been investigated using the best performing region in mathematics rather than a poor performing region as a sample. Fourthly, binomial analysis was used.

5.6 Areas for further research

Based on the limitation outlined, this study becomes a useful baseline study for future research in the Ghanaian colleges of education in

- Conducting similar research using the best performing region in mathematics as sample rather than the poor performing region.
- Replicating the study in other regions to ascertain the fact that SHS graduates from the regions attend colleges of education in the same region.

- Repeating the study to find out if mathematics tutors in the colleges of education mostly use instructional coherence in delivering mathematics lessons.
- Investigating the instructional strategies that a particular mathematics tutors in the colleges of education use.
- Exploring SHS mathematics teachers' conceptual understanding on the concept of constructivism.
- Multinomial analytical technique will be used to analyze the data.

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APPENDICES

Appendix A

This information is being collected for research purposes and for statistical analysis. It is purely an academic work. Your response will not be used for any legal or enforcement purpose. Rather the data you will provide in this survey is secret and confidential and shall not be divulged to a third party unless by your consent. I therefore pledge to protect the confidentiality of your data. However, its findings may be used to improve the teaching skills of mathematics teachers in the country. This is because the study will examine the consequent problems that are related to the teaching and learning of mathematics at the basic and senior high schools and to find adequate solutions for them, which in effect will support all student-teacher graduates. Accurate responses to the items will proffer exact solutions for the benefit the researcher, the respondents and the nation as a whole. Finally, responding to this questionnaire has no resultant effect, positive or negative on your final grades neither has it got anything to do with your tutor's promotion nor otherwise; instead it will inform policy makers and implementers to restrategize the methodology of teaching mathematics in all institutions of learning especially at the colleges of education. We therefore entreat you to recollect exactly what happened in your mathematics classes with respect to each statement in the questionnaire. Please be as fair as possible. Thank you.

SECTION A

Students' Bio-data

For each statement below, choose the response that best describes you.

1. Gender: Male ☐ ☐Female

2. Age: ☐ <20 ☐20-24 ☐25-29 ☐30-34 ☐35-39 ☐39<

3. What is the name of your District Assembly?……………………………………………

4. What grade did you obtain in mathematics at BECE?

1st attempt…………… 2nd Attempt…………… 3rd Attempt………………

5. On a scale of 1-10, where 1 is the lowest and 10 is the highest, how will you rate the teachers who taught you mathematics at JHS?....................................

6. In which TOWN did you attend JHS? ....

7. Which PROGRAMME did you offer at SHS?..

8. In which YEAR did you complete SHS?...

9. In which TOWN did you attend SHS?...

10. What grade did you obtain in mathematics at WASSCE?
☐1st attempt…………… ☐2nd Attempt……….. ☐3rd Attempt………………

11. On a scale of 1-10, where 1 is the lowest and 10 is the highest, how will you rate the teachers who taught you mathematics at SHS?......................

12. Which is your first choice of profession?...

13. What is the reason for your first choice?..

14. What motivated you to be trained as a professional teacher?.......................................

15. Will you enjoy teaching mathematics after school? ☐Yes ☐No

16. If yes, why?...

17. If No, why?...

18. State the grades you obtained in mathematics courses at the end of the 1st and 2nd years in the college of education

1st year ☐1. ………………… ☐2………………

2nd year ☐1…………………… ☐2…………………

19. Which programme are you offering at the college of education?................................

20. Which school and town will you do your teaching practice?.......................................

21. What do you understand by constructivist theory of learning?.....................................

SECTION B

In this section, I would want you to make reference to your mathematics tutor(s) in the college of education **ONLY**. Please tick the column that closely reflects your opinion about each item using the five point likert scale as shown below. Thank you.

1. Strongly Disagree 2. Disagree 3. Not Sure 4. Agree 5. Strongly Agree

	RELEVANT PREVIOUS KNOWLEDGE	**1**	**2**	**3**	**4**	**5**
1.	My tutor referred to what was taught yesterday to be the basis for teaching new topic.					
2.	My teacher used my previously disjointed knowledge to assist me to understand a particular mathematics topic.					
3.	My tutor used my fragmented knowledge in mathematics as a foundation for subsequent lessons.					
4.	My tutor reminded me to understand basic elementary mathematics principles which recur throughout mathematics learning.					
5.	My tutor related mathematics topics to other concepts for me to identify the connections in building new knowledge.					
6.	My tutor used my existing knowledge to accelerate the teaching process.					
7.	My tutor used my fundamental knowledge as basis for planning subsequent lessons.					
8.	My tutor gave me challenging tasks in which I applied my previous knowledge to solve.					
9.	My tutor provided me with challenging tasks using my existing experiences.					
10.	My tutor used my existing knowledge and ideas to explore new mathematics concepts.					
11.	My tutor linked new information to old ones that I already have.					

12.	My tutor made clear links among the past, present and future mathematics contents.					
13.	My tutor has a concept of connectivity to teach the subject dynamically.					
14.	My tutor demonstrated how teaching consists of related sequences.					
15.	My tutor established that current subject-matter is linked to previous ones.					
16.	My tutor showed me the interconnection among mathematics concepts to deepen my mathematical understanding.					
17.	My tutor related mathematical concepts to everyday life.					

s/n	**COGNITIVE ACTIVATION (CA)**					
	Teaching Strategies	**1**	**2**	**3**	**4**	**5**
1.	My tutor approached mathematics teaching in variety of ways.					
2.	My tutor appreciated different aspects of espousing mathematical ideas.					
3.	My tutor's teaching skills enabled me to solve mathematics problems by various methods.					
4.	My tutor gave me mathematical problems with no close or obvious method for solution.					
5.	I was encouraged by my tutor to reflect on problems that require thinking for extended time in order to solve the problems.					
6.	My tutor usually asked me to use my own procedures to solve mathematics problems.					
7.	My tutor always wanted me to explain how I solve mathematics problems.					
8.	My tutor made me to always explain why I chose particular					

	methods to solve problems.					
9.	My mathematics tutor encouraged a culture of exploratory talk in the classroom.					
10.	I suggested possible solutions to solving mathematics problems.					
11.	My tutor encouraged non-routine problem solving techniques.					
12.	My tutor empowered me to use discussions to discover problem solving techniques.					
13.	My tutor engaged me in higher-level thinking to solve mathematics problems.					
14.	I was exposed to think more deeply in order to find solutions to problems.					
15.	I was allowed to predict possible ideas to solving mathematical problems.					
16.	Using the available resources, I personally solved mathematical problems.					
17.	My tutor developed effective instructional processes to enable me understand mathematics.					
18.	My tutor solicited constructive feedback from me during mathematics lessons.					
19.	My tutor adopted positive approaches to correcting my errors and misconceptions.					
20.	I was made to focus on how to get answers to problems than simply focusing on the answer itself.					
21.	My tutor considered what I know and not my ignorance to teach mathematics.					
22.	Good organization of learning activities by my tutor gave me a high level clarity in the learning objectives.					

23.	My tutor represented the subject in varied ways to respond to my needs.					
24.	My tutor emphasized interactive discourse in mathematics class which help me to excel.					
25.	My tutor discouraged seatwork mathematics class.					
26.	My tutor asked higher order questions that enabled me to think outside the box.					
27.	My tutor always applied learning theories in mathematics class to enhance my understanding of mathematics.					
28.	My tutor used challenging tasks in the teaching process.					
	PROFOUND UNDERSTANDING OF FUNDAMENTAL MATHEMATICS (PUFM)					
	Teaching Strategies	**1**	**2**	**3**	**4**	**5**
29.	My tutor appeared in class and gave definitions of mathematical concepts.					
30.	I simply chew mathematical formulas or algorithm given to me by my tutor.					
31.	My tutor explained the logic and philosophy behind formulae.					
32.	I seriously rehearse mathematics formulas and recall them during exams.					
33.	My tutor had the capacity to explain mathematics contents to me.					
34.	My tutor is energetically clear in his mathematical knowledge and thoughts.					
35.	My tutor represented mathematics content in a way that made me to understand.					

36.	My tutor taught mathematics from basic or first principles.					
37.	My tutor balanced mathematics content with real life problems.					
38.	My tutor once upon a time confirmed the basic principles underlying place value.					
39.	My tutor demonstrated the basic principles that underlie basic mathematics operations.					
40.	My tutor established the basic principles underlying patterns and functions.					
41.	My tutor once upon a time demonstrated the basic principles underlying geometry.					
42.	My tutor explained to me the 'how' of solving mathematics problems.					
43.	My tutor clarified the 'why' of solving mathematics problems to me.					
44.	My tutor demonstrated a powerful personal mathematics experience.					
45.	My tutor explained the breadth, depth, and flexibility of any mathematics topic.					
46.	My tutor's mathematical understanding afforded him a more varied ways to represent mathematics concepts to me.					
47.	My tutor had broader approaches to explain mathematics concepts.					
48.	I observed that my tutor felt confident and comfortable during mathematics lessons.					
49.	My tutor did not only calculate correctly but also explain to me what it takes to get correct answers to problems.					
50.	My tutor had the ability to carry out tasks of deep mathematics teaching.					

51.	My tutor always asked recall questions.					
52.	My tutor made me to develop procedures to solving mathematics problems.					
53.	My tutor gave me confidence to understand solving non-routine problems.					
54.	My tutor was not limited to topic that was to be taught at a particular level only.					
55.	My tutor went outside a current topic to help me understand a topic that is being taught.					
56.	My tutor exposed me to basic principles of mathematics concepts.					
57.	My tutor used diagrams, symbols and teaching aids to represent mathematics concepts.					
58.	Using fundamental principles my tutor helped me solve real life problems.					
59.	The teaching model my tutor adopted helped me to understand mathematics concept.					
60.	My tutor told me about the advantages and disadvantages of using alternative methods to solve mathematics problems.					
s/n	**INSTRUCTIONAL COHERENCE (IC)**					
	Teaching Strategies	**1**	**2**	**3**	**4**	**5**
61.	My tutor used all necessary links to enable me understand mathematics concepts.					
62.	My tutor's learning theory and theory-in-use are consistent during mathematics lessons.					

63.	My tutor's set of beliefs and ideas about the nature of mathematics helped me to understand the subject.					
64.	The contents of a lesson reflect the stated objectives.					
65.	My tutor portrayed the subject as a collection of dynamic and continuous knowledge.					
66.	My tutor encouraged me regularly to learn mathematics.					
67.	My tutor consistently communicated with me using mathematical language.					
68.	My tutor's combination of the curriculum and his positive perception about mathematics improve my understanding of the subject.					
69.	During mathematics lessons my tutor did not digress; i.e. changing topics unnecessarily.					
70.	My tutor always did what he said during mathematics lessons; that is he practiced what he always said.					
71.	My tutor always kept to his promises in mathematics classes.					
72.	My tutor asked questions based on what he taught in a lesson.					
73.	My tutor's instructional practices informed me about how I am expected to teach mathematics to pupils.					
74.	Tests and examinations always reflected the objectives of lessons.					
75.	I noticed a smooth flow in all deliveries of mathematics concept by my tutor.					
76.	All tutors in the college shared similar teaching and learning strategies and goals.					
77.	My tutor organized learning using available relevant logistics to achieve mathematical goals.					
78.	My tutor always allowed my mathematical ideas to fit into what					

	was being taught.					
79.	My tutor emphasized underlying structures to knowledge connections.					
80.	My tutor demonstrated practical support that linked mathematics teaching to learning.					
81.	My tutor exhibited continuity in contents of the learning process.					
82.	My tutor always used activities that focused on challenging my mathematical thinking.					
83.	My tutor was constant in managing mathematics classes well all the time.					
84.	My tutor made connections between the mathematical theories and practices.					
	COGNITIVE GUIDED INSTRUCTION (CGI)					
	Teaching Strategies	**1**	**2**	**3**	**4**	**5**
85.	My tutor anchored the development of my mathematical thinking to his instructional practices and beliefs.					
86.	My tutor fused my cultural mind-set into mathematics concepts.					
87.	My tutor went through various stages to help me understand mathematics concepts.					
88.	My tutor understood that solving problems depended on my developmental stages.					
89.	My tutor agreed that he did not have the right answers to all problems all the time.					
90.	My tutor adopted a learner-centred method of teaching.					

91.	My tutor presented lessons with lots of activities which were intertwined with discussions.					
92.	My tutor acknowledged that we were on equal grounds to think critically and solve mathematics problems together.					
93.	My tutor made use of practical inputs during mathematics lessons.					
94.	My tutor's content knowledge and pedagogical skills in conjunction with my knowledge allowed him to design interesting lesson plans in mathematics.					
95.	My tutor's incorporation of his own ideas in presenting lessons made me to enjoy learning mathematics.					
96.	My tutor accepted that my inability to understand mathematics concept was not my fault.					
97.	My tutor's preparations and experiences contributed to my achievement in mathematics.					
98.	There is adequate cooperation and group effort that encouraged me to learn mathematics.					
99.	My tutor has never decided who the best student in class was or who was not.					
100.	My tutor considered my contribution to group discussions for completion.					
101.	My tutor allowed me to share my strategies, thoughts, enthusiasm, eagerness, satisfaction and success with my class mates.					
102.	My tutor possessed deep cultural knowledge about mathematics.					
103.	My tutor used word problems to teach mathematics contents.					
104.	My tutor's mathematics lessons were always related to the community or environment.					
105.	My tutor's mathematics concepts were socially-constructed.					

106.	My tutor's instructions were integral to my everyday life.					
107.	My tutor combined cultural environment and mathematical content for instructions.					
108.	My tutor took me through thoughtful mathematics reflection for cultural compatibility.					
109.	My tutor delivered mathematics instruction in a culturally responsive manner.					
110.	My tutor had knowledge about how I perceive mathematics.					
s/n	**SELF DETERMINATION THEORY (SDT)**					
	Student Learning	**1**	**2**	**3**	**4**	**5**
1.	Motivation is one of my basic psychological needs towards the construction of new mathematics ideas.					
2.	I am determined to learn mathematics and solve problems.					
3.	I spend time on mathematics tasks.					
4.	I maintain cognitive mathematical integrity because I engage in meaningful thinking.					
5.	I always learn mathematics on my own.					
6.	Learning mathematics on my own helped me to understand the subject very well.					
7.	Learning mathematics with my peers enhance my understanding of the subject.					
8.	I asked questions in mathematics classes.					
9.	I answered questions in mathematics classes.					
10.	I did well in mathematics when I was left alone.					
11.	My curiosity supported me to learn mathematics.					

Personal Effect		**1**	**2**	**3**	**4**	**5**
12.	Mathematics learning posed a lot of challenges to me.					
13.	I see mathematics as a path that helps me go a step forward in life to meet my desired goals.					
14.	I am one of those who shy away from mathematics.					
15.	I give up my intentions and desires about mathematics because of protracted failures.					
16.	I have interest and joy in learning mathematics.					
17.	I learn mathematics for internal satisfaction.					
18.	Learning mathematics is in line with my own values.					
19.	I have self-satisfaction whenever I learn mathematics.					
20.	I am bored with mathematics anytime I meet complex problems.					
21.	I have pleasure and satisfaction in mathematics when I am engaged in meaningful tasks.					
22.	In the face of disappointments, I still intensify my mathematics achievement.					
23.	My desperation made me to fail mathematics examinations.					
24.	I understand that my poor performance in mathematics limits choices for my success in life.					
25.	I have good perceptions about mathematics.					
Internal Support in Learning Mathematics		**1**	**2**	**3**	**4**	**5**
26.	I have control over solving mathematical problems.					
27.	My mathematical achievement is due to the motivation I have from within.					
28.	My belief systems have positive bearing on my ability to					

	understand mathematics.					
29.	My attitude towards mathematics is positive.					
30.	I believed that understanding mathematics is not only connected to being intelligent and talented.					
31.	Internal satisfaction made me to enjoy mathematics.					
32.	Internal regulations acting in the context of external connection helped me in mathematics lessons.					
33.	Learning mathematics is in line with my own values and internal regulation.					
34.	I have self-satisfaction whenever I learn mathematics.					
35.	I generate anxiety towards mathematics.					
	External Support in Learning Mathematics	**1**	**2**	**3**	**4**	**5**
36.	My tutor did everything possible including classroom activities to get me learn mathematics.					
37.	My tutor gave me enough confidence using variety of challenging strategies that made me to understand mathematics.					
38.	My tutor had time to explain mathematics concepts to me even outside the classroom.					
39.	I was always given the necessary support to learn mathematics.					
40.	My mathematics learning is sustained because I accepted the available social environment.					
41.	My self-assurance to LEARN mathematics was due to the support from social environment.					
42.	I have the emotional support from people to learning mathematics.					
43.	The environment in which I find myself favour me to learn mathematics.					

44.	I am motivated to learn mathematics because there is positive learner-tutor relationship.					
45.	I am inspired to learn mathematics because of the positive learner-learner relationship.					
46.	My tutor created an atmosphere of assurance for me to learn mathematics.					
47.	My tutor's mathematics activities gave me confidence for desired learning outcome.					
48.	I am always encouraged by my tutor to solve mathematics problems on my own.					
49.	My tutor regulated me through external forces to learn mathematics.					
50.	I learn mathematics because of the reward system.					
51.	I take mathematics studies seriously because I accepted challenges.					
52.	I participated in mathematics activities because my tutor empowered me.					
53.	I always considered my tutor's recommendations as a source of encouragement to understand mathematics.					
54.	My internal drive to learn mathematics comes as a result of an external push.					
55.	My tutor provided challenging class and home exercises which pushed me to learn mathematics.					
56.	Classroom activities adopted by my tutor influenced my mathematics understanding.					
57.	I have a high degree of success in mathematics because I have been appropriately supported.					
58.	I believe that challenging tasks are source of motivation in					

	mathematics learning.					
59.	Combinations of effective instructions and positive affection from my tutor inspired me to excel in mathematics.					
60.	I performed well in mathematics because I did not want to be punished or embarrassed.					
61.	The support I have from others made me to learn mathematics.					
62.	My parents supported me to learn mathematics.					
63.	The care I received from my tutor gave me a sense of belonging to the mathematics fraternity.					
64.	I focused on praise any time I did well in mathematics.					
65.	Owning mathematical ideas gave me a means of sustenance in mathematics learning.					
66.	Encouragement coming from my knowledgeable peers inspired me to learn mathematics.					
67.	My tutor built confidence in me as a prospective mathematics teacher.					
68.	My tutor's responses to my misconceptions encouraged me to learn the subject well.					
69.	My tutor's questions in mathematics class stimulated me to learn the subject very soundly.					
70.	I performed well in mathematics when there was a reward.					
71.	My mathematics learning is sustained because social environments were favourable.					
72.	My tutor created environments that empowered me to get the most from his instructional experiences.					
73.	My tutor motivated me during the instructional processes to solve mathematics problems.					

74.	My tutor's mathematical content knowledge and pedagogical skills are closely related to the learning environment.					
75.	A favourable environment was created for me to learn from my mistakes.					
76.	Effective classroom climate and management was made available for my mathematics achievements.					
77.	My tutor used supportive learning environments to enhance my understanding of mathematical concepts.					
78.	My tutor exposed me to activities in and outside classroom to understand mathematics.					

	CONSTRUCTIVISM	**1**	**2**	**3**	**4**	**5**
1.	As a student-teacher, I was instrumental in constructing my own mathematical ideas.					
2.	I had the opportunity to identify what I needed to learn.					
3.	I identified how mathematics concepts are applied in different situations.					
4.	I think critically to enhance the understanding of mathematical concepts.					
5.	I have the ability to make connections among mathematical concepts and procedures.					
6.	I summarized lessons to indicate my understanding of mathematical concepts.					
7.	I possess a set of broad learning techniques to solve mathematics problems.					
8.	I always select the necessary and appropriate resources to solve mathematics problems.					

9.	My acquisition of knowledge and skills help me to accomplish mathematics tasks.					
10.	I make personal choices to perform mathematical activities convincingly without pressure.					
11.	I have high academic achievement because I display my mathematical potential.					
12.	Solving simple mathematical problems encourage me to learn the subject.					
13.	I construct my own ideas to understand mathematics.					

	TEACHER QUALITY (TQ)	**1**	**2**	**3**	**4**	**5**
1.	My mathematics teacher has strong knowledge about the subject-matter.					
2.	My tutor has a good understanding of my mathematical knowledge.					
3.	My tutor has a sound outlook towards the teaching of mathematics.					
4.	My tutor knows how to teach what he understood.					
5.	My tutor knew about my interests so he taught effectively.					
6.	Knowing my background informed my tutor about how to teach.					
7.	My tutor's content-knowledge in mathematics was connected to my learning strategies.					
8.	My tutor is knowledgeable about my developmental stage in learning mathematics.					
9.	My tutor prepared me to work in groups during mathematics assignments.					
10.	My tutor's attitudes about mathematics positively influenced my level of understanding.					
11.	My mathematics learning was influenced by how well my tutor					

	explained concepts to me.					
12.	My tutor's mathematical beliefs have mostly influenced my learning strategies.					
13.	The confidence my tutor built in me increased my mathematical competences.					
14.	My mathematics tutor understood what he was about to teach.					
15.	My tutor asked stimulating questions in order to excite me to learn mathematics.					
16.	My tutor encouraged me to analyze mathematics problems.					
17.	My tutor's attitude has positive influence my understanding of mathematics topics.					
18.	My tutor communicated mathematics content well because of his level of education.					
19.	My tutor's content-knowledge directly determined my mathematical achievement.					
20.	I performed well in mathematics because tutor guided me with passion.					
21.	My tutor's preparations before lessons supported me to understand mathematics lessons.					
22.	My tutor's used effective pedagogical skills to teach mathematics to my benefit.					
23.	My tutor's interactions with me coupled with my prior knowledge enhanced my competency level.					
24.	My mathematics achievement is attached to the academic level of my tutor.					
25.	My tutor adopted his professionalism to teaching mathematics.					
26.	My tutor's ability to link concepts helped me to build strong					

	connections in acquiring mathematical knowledge.					
27.	My tutor created networks of knowledge for me when delivering mathematics lessons.					
28.	My tutor's understanding of mathematical concepts has great effects on me to learn.					
29.	My tutor has sufficient practical experiences in teaching mathematics.					
30.	My tutor really understood my developmental level of thinking as far as understanding mathematics is concerned.					
31.	My tutor built confidence in me as prospective mathematics teacher.					
32.	The smooth flow of mathematics lessons indicated that my tutor prepared well before coming to class.					
33.	My tutor did not rely solely on textbooks for mathematics instructions.					
34.	My tutor did not continue to teach when he knew that I was confused in mathematics lessons.					
35.	My tutor did not continue to teach anytime he was confused in a lesson.					
36.	My tutor's competences in mathematics assured me of my profession as a teacher.					
37.	My tutor's competence is anchored on his mathematical representations due to his personal outlook.					

Appendix B

Public Colleges of Education, Volta Region

COLLEGE OF EDUCATION	2015/2016			2016/2017			2017/2018			2018/2019		
	M	F	T	M	F	T	M	F	T	M	F	T
Akatsi College of Education	847	443	1290	830	470	1300	892	417	1309	867	396	1263
Dambai College of Education	481	220	701	461	213	674	552	245	797	545	219	764
E.P. College of Education, Amedzofe	402	197	599	390	196	586	407	205	612	399	200	599
Jasikan College of Education	603	510	1113	563	506	1069	634	547	1181	639	550	1189
Peki College of Education	544	302	846	534	337	871	542	352	894	507	367	874
ST. Francis College of Education, Hohoe	884	339	1223	783	351	1134	882	358	1240	803	371	1174
St. Teresa's College of Education, Hohoe	0	808	808	-	854	854	0	916	916	0	775	775
Total	**3761**	**2819**	**6580**	**3561**	**2927**	**6488**	**3909**	**3040**	**6949**	**3760**	**2878**	**6638**

Source: NCTE, 2019

M - Male F – Female T- Total

Appendix C

GHANA'S DISTRICT LEAGUE TABLE 2017
Strengthening Social Accountability for National Development
The District League Table is a simple tool that ranks the level of development in each of Ghana's 216 Districts using six indicators across six sectors. The District League Table aims to increase social accountability – where citizens and service providers demand of, and account to each other - for improving development across Ghana. It also helps to highlight inequities in development and where more support is required.
YOU CAN LOCATE YOUR DISTRICT'S RANK HERE
The Districts marked in red are ranking very low in the District League Table, those in green are ranking high.
The District League Table can be used:
The most improved District this year is Lambussie Karni in the Upper West region
unicef

Appendix D

BECE MATHEMATICS RESULTS

Year	Regions										
	BA	**W**	**A**	**GA**	**C**	**E**	**N**	**V**	**UE**	**UW**	**NA**
2013	87.43	88.4	85.42	82.55	70.63	70.7	69.39	56.51	59.15	62.23	**73.24**
2014	89.36	85.41	84.45	81.96	73.36	69.73	69.02	67.25	61.11	54.09	**73.57**
2015	82.87	86.1	82.97	80.57	77.52	68.91	63.13	62.62	57.94	52.91	**71.55**
2016	86.61	87.92	84.19	82.84	74.35	67.98	67.02	58.78	52.74	52.51	**71.49**
2017	89.11	86.51	84.85	79.73	74.36	67.37	70.08	59.72	55.67	50.59	**71.80**
Average	**87.08**	**86.87**	**84.38**	**81.53**	**74.04**	**68.94**	**67.73**	**60.98**	**57.32**	**54.46**	72.33
Position	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**7th**	**8th**	**9th**	**10th**	

Appendix E

WASSCE MATHEMATICS RESULTS

Year	Regions										NA
	BA	A	W	E	C	GA	V	UW	UE	N	
2013	52.34	43.62	40.3	40.03	36.49	28.77	27.31	24.19	28.63	14.98	**33.67**
2014	59.58	42.04	38.65	28.52	23.6	31.58	18.69	15.88	13.4	11.26	**28.32**
2015	36.72	34.65	30.03	20.74	21.57	25.19	12.63	9.03	8.72	7.73	**20.70**
2016	48.78	39.84	40.47	35.9	35.06	32.22	23.35	21.32	15.39	12.26	**30.46**
2017	76.25	50.21	49.67	42.52	43.92	35.15	26.01	19.51	17.72	11.82	**37.28**
Average	**54.73**	**42.07**	**39.82**	**33.54**	**32.13**	**30.58**	**21.60**	**17.99**	**16.77**	**11.61**	**30.08**
Position	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**7th**	**8th**	**9th**	**10th**	

Legend

Regions

A – Ashanti

BA – Brong-Ahafo

C – Central

E – Eastern

GA – Greater Accra

N – Northern

UE – Upper East

UW – Upper West

V – Volta

W – Western

NA – National Average

Appendix F

Nation's Senior High School Categorization

Total Number of Senior High Schools in Ghana	1633
Total Number of Category D Schools in Ghana	583
Percentage of Category D Schools in Ghana	35.7

Appendix F_1: Distribution of Senior High Schools in Ghana

Position	Regions	No. of Schools	%
1st	Ashanti	322	19.7
2nd	Greater Accra	229	14.0
3rd	Eastern	216	13.2
4th	Brong Ahafo	191	11.7
5th	Central	174	10.7
6th	**Volta**	**148**	**9.1**
7th	Western	132	8.1
8th	Northern	116	7.1
9th	Upper East	59	3.6
10th	Upper West	46	2.8
Total		**1633**	**100**

Appendix F_2: Distribution of Category D Schools in the Regions

Position	Regions	No. of Schools	%
1st	Ashanti	122	20.9
2nd	**Volta**	**87**	**14.9**
3rd	Eastern	84	14.4
4th	Brong Ahafo	74	12.7
5th	Central	59	10.1
6th	Western	52	8.9
7th	Greater Accra	45	7.7
8th	Northern	42	7.2
9th	Upper East	11	1.9
10th	Upper West	7	1.2
Total		**583**	**100.0**

Appendix F_3: Percentage distribution of category D Schools with respect to total schools in each Region

Position	**Regions**	**No. of All Schools**	**No. of D Schools**	**%**
1st	**Volta**	**148**	**87**	**58.8**
2nd	Western	132	52	39.4
3rd	Eastern	216	84	38.9
4th	Brong Ahafo	191	74	38.7
5th	Ashanti	322	122	37.9
6th	Northern	116	42	36.2
7th	Central	174	59	33.9
8th	Greater Accra	229	45	19.7
9th	Upper East	59	11	18.6
10th	Upper West	46	7	15.2
Total/Average		**1633**	**583**	**35.7**

Appendix F_4: Highest Category of schools in each Region

Position	Regions	Category	Number of Category Schools	% of Cat. D Schools
1st	**Volta**	**D**	**87**	**58.8**
2nd	Upper West	C	20	43.5
3rd	Greater Accra	F	95	41.5
4th	Western	D	52	39.4
5th	Upper East	C	23	39.0
6th	Eastern	D	84	38.9
7th	Brong Ahafo	D	74	38.7
8th	Ashanti	D	122	37.9
9th	Northern	D	42	36.2
10th	Central	D	59	33.9

Appendix F_5: Summary of School Categorization

Ashanti Region

Category	A	B	C	**D**	E	F	G	Total	**National %**
No.	6	58	59	**122**	3	64	10	322	
%	1.9	18.0	18.3	**37.9**	0.9	19.9	3.1	100	**19.7**

Central Region

Category	A	B	C	**D**	E	F	G	Total	**National %**
No.	6	21	40	**59**	3	36	9	174	

%	3.4	12.1	23.0	**33.9**	1.7	20.7	5.2	100	**10.7**

Eastern Region

Category	A	B	C	**D**	E	F	G	Total	**National %**
No.	9	35	45	**84**	8	21	14	216	
%	4.2	16.2	20.8	**38.9**	3.7	9.7	6.5	100	**13.2**

Brong Ahafo Region

Category	A	B	C	**D**	E	F	G	Total	**National %**
No.	8	47	20	**74**	3	27	12	191	
%	4.2	24.6	10.5	**38.7**	1.6	14.1	6.3	100	**11.7**

Greater Accra Region

Category	A	B	C	**D**	E	F	G	Total	**National %**
No.	7	14	23	**45**	6	95	39	229	
%	3.1	6.1	10.0	**19.7**	2.6	41.5	17.0	100	**14.0**

Northern Region

Category	A	B	C	**D**	E	F	G	Total	**National %**
No.	4	2	39	**42**	6	19	4	116	
%	3.4	1.7	33.6	**36.2**	5.2	16.4	3.4	100	7.1

Upper East Region

Category	A	B	C	**D**	E	F	G	Total	**National %**
No.	4	2	23	**11**	3	11	5	59	
%	6.8	3.4	39.0	**18.6**	5.1	18.6	8.5	100	**3.6**

Upper West Region

Category	A	B	C	D	E	F	G	Total	**National %**
No.	3	5	20	**7**	3	1	7	46	
%	6.5	10.9	43.5	**15.2**	6.5	2.2	15.2	100	**2.8**

Volta Region

Category	A	B	C	**D**	E	F	G	Total	**National %**
No.	3	16	11	**87**	10	14	7	148	
%	2	10.8	7.4	**58.8**	6.8	9.5	4.7	100	**9.1**

	Western Region								
Category	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**Total**	**National %**
No.	5	19	36	**52**	2	9	9	132	
%	3.8	14.4	27.3	**39.4**	1.5	6.8	6.8	100	**8.1**

Appendix G: Content Validation Index

Appendix G_1: Cognitive Activation

N/S/N	O/S/N	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6	SR	AR (I-CVI)	UA (S-CVI)
42	5	1	1	1	1	1	1	6	1.00	1
43	1	1	1	1	0	1	1	5	0.83	0
44	22	1	1	1	0	0	1	4	0,67	0
45	21	1	1	1	0	1	1	5	0.83	0
46	2	1	1	1	1	1	1	6	1.00	1
47	3	1	1	1	1	1	1	6	1.00	1
48	12	1	1	1	1	1	1	6	1.00	1
49	23	1	1	1	1	1	1	6	1.00	1
50	17	1	1	1	1	1	1	6	1.00	1
51	8	1	1	1	1	1	1	6	1.00	1
52	7	1	1	1	1	1	1	6	1.00	1
53	15	1	0	1	1	0	0	3	0.50	0
54	19	1	1	1	0	0	0	3	0.50	0
55	20	1	1	1	1	1	1	6	1.00	1
Averages		1.00	0.93	1.00	0.71	0.79	0.86	5.29	0.88	0.64

Appendix G_2: Profound Understanding of Fundamental Mathematics

N/S/N	O/S/N	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6	SR	AR (I-CVI)	UA (S-CVI)
56	34	1	1	1	0	1	0	4	0.67	0
57	35	1	1	1	1	1	1	6	1.00	1
58	50	1	1	1	1	1	1	6	1.00	1
59	47	1	1	1	1	1	1	6	1.00	1
60	33	1	1	1	1	0	0	4	0.67	0
61	46	1	1	1	1	1	1	6	1.00	1
62	49	1	1	1	0	1	1	5	0.83	0
63	59	1	1	1	1	1	1	6	1.00	1
64	36	1	1	1	1	1	1	6	1.00	1
65	42	1	1	1	1	1	1	6	1.00	1
66	39	1	1	1	1	1	1	6	1.00	1
67	40	1	1	1	1	1	1	6	1.00	1
68	43	1	1	1	1	1	1	6	1.00	1
69	45	1	1	1	1	1	1	6	1.00	1
70	48	1	1	1	0	0	0	3	0.50	0
Averages		1.00	1.00	1.00	0.80	0.87	0.80	5.47	0.91	0.73

Appendix G3: Instructional Coherence

N/S/N	O/S/N	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6	SR	AR (I-CVI)	UA (S-CVI)
71	82	1	1	1	0	1	1	5	0.83	0
72	84	1	1	1	1	1	1	6	1.00	1
73	83	1	1	1	1	1	1	6	1.00	1
74	75	1	1	1	1	1	1	6	1.00	1
75	74	1	0	1	1	1	1	5	0.83	0
76	80	1	1	1	1	1	1	6	1.00	1
77	81	1	1	1	1	1	1	6	1.00	1
78	65	1	1	1	1	1	1	6	1.00	1
79	64	1	1	1	1	1	1	6	1.00	1
80	61	1	1	1	1	1	1	6	1.00	1
81	67	1	1	0	0	1	1	4	0.67	0
82	68	1	1	1	0	1	1	5	0.83	0
83	73	1	1	1	1	1	0	5	0.83	0
Averages		1.00	0.92	0.92	0.77	1.00	0.80	5.54	0.92	0.62

Appendix G4: Cognitive Guided Instruction

N/S/N	O/S/N	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6	SR	AR (I-CVI)	UA (S-CVI)
84	94	1	1	1	0	0	0	3	0.75	0
85	95	1	1	1	1	1	1	6	0.92	0
86	88	1	1	1	1	0	1	5	0.83	0
87	86	1	0	1	1	1	1	5	0.92	0
88	93	1	1	1	1	1	1	6	1.00	1
89	91	1	1	1	1	1	1	6	1.00	1
90	106	1	1	1	1	1	1	6	1.00	1

91	107	1	1	1	1	1	1	6	0.92	0
92	104	1	0	1	1	1	1	5	0.83	0
93	108	1	0	1	1	1	1	5	0.86	0
Averages		1.00	0,70	1.00	0.90	0.80	0.90	5.30	0.90	0.30

Appendix G5: Constructivism

N/S/N	O/S/N	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6	SR	AR (I-CVI)	UA (S-CVI)
94	16	1	1	1	1	1	1	6	1.00	1
95	15	1	0	1	1	1	1	5	0.83	0
96	19	1	1	1	1	1	1	6	1.00	1
97	17	1	1	1	1	1	1	6	1.00	1
98	20	1	1	1	1	1	1	6	1.00	1
99	18	1	1	1	1	1	0	5	0.83	0
100	8	1	1	1	0	1	1	5	0.83	0
101	4	1	1	1	1	1	1	6	1.00	1
102	7	1	1	1	1	1	1	6	1.00	1
103	6	1	1	1	1	1	1	6	1.00	1
104	9	1	1	1	0	1	1	5	0.83	0
Averages		1.00	0.91	1.00	0.82	1.00	0.91	5.64	0.94	0.64

Appendix G6: Relevant Previous Knowledge

N/S/N	O/S/N	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6	SR	AR (I-CVI)	UA (S-CVI)
1	16	1	1	1	1	1	1	6	1.00	1
2	17	1	1	0	1	1	1	5	0.83	0
3	13	1	1	1	1	1	1	6	1.00	1
4	4	1	1	1	1	1	0	5	0.83	0
5	15	1	1	1	1	0	1	5	0.83	0
6	11	1	1	1	1	1	1	6	1.00	1
7	8	1	1	1	0	1	1	5	0.83	0
8	10	1	1	1	1	1	1	6	1.00	1
9	9	1	1	1	1	1	1	6	1.00	1
10	7	1	1	1	1	1	1	6	1.00	1
Averages		1.00	1.00	0.90	0.90	0.90	0.90	5.60	0.93	0.60

Appendix G_7: Internal Motivation

N/S/N	O/S/N	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6	SR	AR (I-CVI)	UA (S-CVI)
11	41	1	1	1	1	0	1	5	0.83	0
12	40	1	1	1	1	1	1	6	1.00	1
13	46	1	1	1	0	1	1	5	0.83	0
14	39	1	1	1	1	1	1	6	1.00	1
15	44	1	1	1	0	1	1	5	0.83	0
16	42	1	1	1	1	1	1	6	1.00	1
17	38	1	1	1	1	0	0	4	0.67	0
18	29	1	1	1	0	1	0	4	0.67	0
19	30	1	1	1	1	0	1	5	0.83	0
20	32	1	1	1	1	1	1	6	1.00	1
Averages		1.00	1.00	1.00	0.70	0.70	0.80	5.20	0.87	0.40

Appendix G_8: External Motivation

N/S/N	O/S/N	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6	SR	AR (I-CVI)	UA (S-CVI)
21	50	1	1	1	1	1	1	6	1.00	1
22	57	1	1	1	1	1	1	6	1.00	1
23	53	1	1	1	1	1	1	6	1.00	1
24	66	1	1	1	1	1	1	6	1.00	1
25	60	1	1	1	1	1	1	6	1.00	1
26	88	1	1	1	0	1	1	5	0.83	0
27	69	1	1	1	1	1	0	5	0.83	0
Averages		1.00	1.00	1.00	0,86	1.00	0.86	5.71	0.95	0.71

Appendix G9: Teacher Quality

N/S/N	O/S/N	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6	SR	AR (I-CVI)	UA (S-CVI)
28	15	1	1	1	1	1	1	6	1.00	1.00
29	14	1	1	1	1	0	0	4	0.67	0.00
30	22	1	1	1	1	1	0	5	0.83	0.00
31	16	1	1	1	0	1	1	5	0.83	0.00
32	36	1	1	1	0	1	1	5	0.83	0.00
33	35	1	0	1	1	1	0	4	0.67	0.00
34	3	1	1	1	1	1	1	6	1.00	1.00
35	7	1	1	1	1	1	1	6	1.00	1.00
36	8	1	1	1	1	1	1	6	1.00	1.00
37	9	1	1	1	1	1	1	6	1.00	1.00
38	2	1	1	1	1	1	1	6	1.00	1.00
39	12	1	1	1	1	0	1	5	0.83	0.00
40	5	1	0	1	0	1	1	4	0.67	0.00
41	6	1	0	1	1	0	1	4	0.67	0.00
Averages		1.00	0.79	1.00	0.79	0.79	0.79	5.14	0.86	0.43

SR = Sum Response AR = Average Response UA = Universal Agreement

I-CVI = Individual Content Validity Index S-CVI = Sum Content Validity Index

N/S/N = New Serial Number O/S/N = Old Serial Number

Appendix H: Convergent Validity (Bootstrapping- Mean, STDEV, T-Values, P-Values)

Appendix H_1: Rho Alpha

Constructs	Original Sample (O)	Sample Mean (M)	Standard Deviation (STDEV)	T Statistics (|O/STDEV|)	P Values
CA	0.901	0.901	0.010	86.589	0.000
CGI	0.931	0.931	0.007	127.535	0.000
CONST	0.928	0928	0.009	109.031	0.000
IC	0.949	0.948	0.006	152.242	0.000
PUFM	0.953	0.953	0.005	186.506	0.000
RPK	0.859	0.861	0.015	57.019	0.000
SDT-EM	0.900	0.900	0.012	77.803	0.000
SDT-IM	0.935	0.935	0.008	122.021	0.000
TQ	0.948	0.948	0.006	167.186	0.000
TTP	0.897	0.903	0.044	20.235	0.000

Appendix H_2: Chronbach's Alpha

Constructs	Original Sample (O)	Sample Mean (M)	Standard Deviation (STDEV)	T Statistics (|O/STDEV|)	P Values
CA	0.899	0.898	0.011	82.041	0.000
CGI	0.930	0.929	0.008	119.718	0.000
CONST	0.928	0.927	0.009	106.013	0.000
IC	0.949	0.948	0.006	150.731	0.000
PUFM	0.953	0.952	0.005	179.569	0.000
RPK	0.858	0.857	0.016	54.645	0.000
SDT-EM	0.898	0.896	0.012	74.036	0.000
SDT-IM	0.934	0.934	0.008	118.451	0.000
TQ	0.945	0.945	0.006	164.155	0.000
TTP	0.854	0.852	0.017	49.136	0.000

Appendix H3: Composite Reliability

Constructs	Original Sample (O)	Sample Mean (M)	Standard Deviation (STDEV)	T Statistics (|O/STDEV|)	P Values
CA	0.922	0.922	0.008	119.379	0.000
CGI	0.941	0.940	0.006	152.854	0.000
CONST	0.939	0.938	0.007	135.546	0.000
IC	0.955	0.954	0.005	181.215	0.000
PUFM	0.958	0.957	0.005	212.350	0.000
RPK	0.898	0.897	0.010	88.624	0.000
SDT-EM	0.919	0.918	0.009	104.431	0.000
SDT-IM	0.945	0.945	0.006	151.774	0..000
TQ	0.951	0.951	0.005	192.120	0..000
TTP	0.899	0.898	0.012	74.542	0.000

Appendix H4: Average Variance Extracted (AVE)

Constructs	Original Sample (O)	Sample Mean (M)	Standard Deviation (SD)	T Statistics (|O/SD|)	P Values
CA	0.665	0.664	0.024	27.991	0.000
CGI	0.614	0.612	0.026	23.893	0.000
CONST	0.606	0.604	0.028	21.447	0.000
IC	0.619	0.617	0.028	21.969	0.000
PUFM	0.602	0.600	0.026	23.060	0.000
RPK	0.638	0.637	0.025	25.332	0.000
SDT-EM	0.620	0.618	0.027	22.562	0.000
SDT-IM	0.657	0.655	0.027	24.760	0.000
TQ	0.601	0.599	0.025	23.823	0.000
TTP	0.691	0.689	0.027	25.434	0.000

Appendix I: Path Coefficients (T Statistics & P Values)

Path Model	Original Sample (O)	Sample Mean (M)	Standard Deviation (SD)	T Statistics (|O/SD|)	P Values	Decision
CA -> CONST	0.015	0.019	0.054	0.277	**0.782**	**Not Supported**
IC -> CONST	0.302	0.293	0.097	3.096	0.002	Supported
CGI -> CONST	0.226	0.226	0.065	3.492	0.000	Supported
PUFM -> CONST	0.077	0.080	0.063	1.210	**0.226**	**Not Supported**
TQ -> CONST	0.100	0.102	0.043	2.325	0.020	Supported
CGI -> TTP	-0.161	-0.166	0.065	2.491	0.013	Supported
CONST -> TTP	0.140	0.144	0.055	2.546	0.011	Supported
CA -> TTP	0.152	0.150	0.054	2.796	0.005	Supported
TQ -> TTP	0.137	0.141	0.060	2.261	0.024	Supported
IC -> CA	0.214	0.213	0.076	2.809	0.005	Supported
PUFM -> CA	0.585	0.585	0.072	8.170	0.000	Supported
TQ -> CA	0.060	0.060	0.034	1.785	**0.074**	**Not Supported**
PUFM -> IC	0.715	0.715	0.043	16.655	0.000	Supported
TQ -> IC	0.191	0.190	0.043	4.453	0.000	Supported
RPK -> TQ	0.202	0.205	0.052	3.914	0.000	Supported
SD-EM -> TQ	0.482	0.476	0.065	7.387	0.000	Supported
SD-IM -> TQ	0.044	0.048	0.062	0.699	**0.484**	**Not Supported**
IC -> CGI	0.512	0.510	0.060	8.606	0.000	Supported
TQ -> CGI	0.204	0.205	0.040	5.166	0.000	Supported
PUFM -> CGI	0.194	0.195	0.054	3.574	0.000	Supported
TQ -> PUFM	0.595	0.596	0.041	14.662	0.000	Supported

Appendix J: Bootstrapping of R^2

Constructs	Original Sample (O)	Sample Mean (M)	Standard Deviation (SD)	T Statistics (|O/SD|)	P Values	Decision
CA	0.657	0.658	0.040	16.478	0.000	Supported
CGI	0.682	0.683	0.037	18.279	0.000	Supported
CONST	0.421	0.427	0.058	7.269	0.000	Supported
IC	0.710	0.711	0.040	17.873	0.000	Supported
PUFM	0.354	0.357	0.048	7.360	0.000	Supported
TQ	0.402	0.409	0.048	8.450	0.000	Supported
TTP	0.062	0.071	0.022	2.826	0.005	Supported

Appendix K: Bootstrapping of f^2

Confidence Intervals

Path Model	Original Sample (O)	Sample Mean (M)	2.5%	97.5%
CA -> CONST	0.000	0.002	0.000	0.010
CA -> TTP	0.013	0.014	0.001	0.037
CGI -> CONST	0.028	0.030	0.006	0.070
CGI -> TTP	0.011	0.013	0.001	0.036
CONST -> TTP	0.013	0.015	0.,001	0.042
IC -> CA	0.039	0.044	0.003	0.121
IC -> CGI	0.239	0.243	0.123	0.401
IC -> CONST	0.036	0.040	0.004	0.112
PUFM -> CA	0.301	0.306	0.167	0.,485
PUFM -> CGI	0.036	0.038	0.009	0.083
PUFM -> CONST	0.002	0.004	0.000	0.017
PUFM -> IC	1.141	1.169	0.746	1.734
RPK -> TQ	0.053	0.058	0.015	0.122
SDT-EM -> TQ	0.154	0.157	0.074	0.274
SDT-IM -> TQ	0.001	0.004	0.000	0.020
TQ -> CA	0.006	0.008	0.000	0.027
TQ -> CGI	0.079	0.082	0.033	0.151
TQ -> CONST	0,010	0.012	0.000	0.033
TQ -> IC	0.081	0.084	0.030	0.166
TQ -> PUFM	0.549	0.564	0.358	0.825
TQ -> TTP	0.011	0.014	0.000	0.040

Appendix L: Model Fit (SRMR)

	Original Sample (O)	Sample Mean (M)	95%	99%
Saturated Model	**0.049**	0.028	0.,031	0.032
Estimated Model	**0.109**	0.033	0.039	0.042

Appendices M

Appendix M_1: Correlation Matrix for Cognitive Activation (CA)

		CA1	CA2	CA3	CA5	CA7	CA8	CA12	CA15	CA17	CA19	CA20	CA21	CA22	CA23
Correlation	CA1	1.000	.585	.446	.362	.420	.300	.342	.381	.407	.355	.330	.320	.285	.423
	CA2	.585	1.000	.532	.370	.378	.359	.383	.403	.423	.288	.227	.348	.243	.379
	CA3	.446	.532	1.000	.459	.325	.279	.334	.261	.421	.222	.201	.232	.244	.340
	CA5	.362	.370	.459	1.000	.355	.333	.256	.246	.298	.208	.171	.211	.239	.298
	CA7	.420	.378	.325	.355	1.000	.587	.331	.422	.297	.287	.273	.284	.242	.378
	CA8	.300	.359	.279	.333	.587	1.000	.393	.365	.288	.255	.207	.324	.218	.349
	CA12	.342	.383	.334	.256	.331	.393	1.000	.463	.404	.405	.403	.398	.246	.458
	CA15	.381	.403	.261	.246	.422	.365	.463	1.000	.434	.371	.360	.375	.268	.523
	CA17	.407	.423	.421	.298	.297	.288	.404	.434	1.000	.405	.379	.453	.346	.501
	CA19	.355	.288	.222	.208	.287	.255	.405	.371	.405	1.000	.533	.335	.293	.526
	CA20	.330	.227	.201	.171	.273	.207	.403	.360	.379	.533	1.000	.460	.322	.522
	CA21	.320	.348	.232	.211	.284	.324	.398	.375	.453	.335	.460	1.000	.350	.507
	CA22	.285	.243	.244	.239	.242	.218	.246	.268	.346	.293	.322	.350	1.000	.407
	CA23	.423	.379	.340	.298	.378	.349	.458	.523	.501	.526	.522	.507	.407	1.000
Sig. (1-tailed)	CA1		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	CA2	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	CA3	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	CA5	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	CA7	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000
	CA8	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000
	CA12	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000
	CA15	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000
	CA17	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000
	CA19	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000
	CA20	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000
	CA21	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000
	CA22	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000
	CA23	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	

a. Determinant = .006

Appendix M$_2$: Correlation Matrix for Instructional Coherence (IC)

		IC61	IC64	IC65	IC67	IC68	IC73	IC74	IC75	IC80	IC81	IC82	IC83	IC84
Correlation	IC61	1.000	.522	.446	.410	.495	.415	.451	.446	.408	.420	.383	.449	.419
	IC64	.522	1.000	.559	.470	.501	.478	.465	.459	.448	.468	.469	.504	.463
	IC65	.446	.559	1.000	.588	.474	.511	.372	.346	.452	.383	.400	.418	.376
	IC67	.410	.470	.588	1.000	.617	.456	.357	.420	.422	.356	.391	.406	.354
	IC68	.495	.501	.474	.617	1.000	.484	.428	.416	.465	.447	.382	.477	.386
	IC73	.415	.478	.511	.456	.484	1.000	.556	.451	.453	.397	.384	.418	.449
	IC74	.451	.465	.372	.357	.428	.556	1.000	.620	.408	.492	.375	.436	.440
	IC75	.446	.459	.346	.420	.416	.451	.620	1.000	.463	.493	.422	.524	.433
	IC80	.408	.448	.452	.422	.465	.453	.408	.463	1.000	.612	.646	.655	.477
	IC81	.420	.468	.383	.356	.447	.397	.492	.493	.612	1.000	.636	.602	.461
	IC82	.383	.469	.400	.391	.382	.384	.375	.422	.646	.636	1.000	.624	.505
	IC83	.449	.504	.418	.406	.477	.418	.436	.524	.655	.602	.624	1.000	.594
	IC84	.419	.463	.376	.354	.386	.449	.440	.433	.477	.461	.505	.594	1.000
Sig. (1-tailed)	IC61		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	IC64	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	IC65	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	IC67	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000
	IC68	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000
	IC73	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000
	IC74	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000
	IC75	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000
	IC80	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000
	IC81	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000
	IC82	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000
	IC83	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000
	IC84	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	

a. Determinant = .001

Appendix M3: Correlation Matrix for Profound Understanding of Fundamental Mathematics (PUFM)

		PU33	PU34	PU35	PU36	PU39	PU40	PU42	PU43	PU45	PU46	PU47	PU48	PU49	PU50	PU59
Correlation	PU33	1.000	.636	.596	.557	.424	.302	.400	.358	.470	.409	.421	.510	.411	.430	.355
	PU34	.636	1.000	.649	.603	.493	.378	.474	.405	.382	.458	.441	.567	.444	.509	.484
	PU35	.596	.649	1.000	.650	.485	.403	.444	.430	.481	.441	.446	.560	.508	.478	.514
	PU36	.557	.603	.650	1.000	.551	.508	.528	.461	.554	.435	.464	.532	.510	.509	.524
	PU39	.424	.493	.485	.551	1.000	.612	.510	.461	.504	.484	.504	.537	.493	.497	.407
	PU40	.302	.378	.403	.508	.612	1.000	.582	.480	.516	.391	.490	.502	.459	.502	.439
	PU42	.400	.474	.444	.528	.510	.582	1.000	.633	.551	.468	.457	.537	.512	.608	.498
	PU43	.358	.405	.430	.461	.461	.480	.633	1.000	.612	.463	.440	.418	.444	.488	.425
	PU45	.470	.382	.481	.554	.504	.516	.551	.612	1.000	.527	.572	.491	.460	.494	.423
	PU46	.409	.458	.441	.435	.484	.391	.468	.463	.527	1.000	.552	.483	.432	.523	.433
	PU47	.421	.441	.446	.464	.504	.490	.457	.440	.572	.552	1.000	.640	.550	.520	.445
	PU48	.510	.567	.560	.532	.537	.502	.537	.418	.491	.483	.640	1.000	.660	.636	.521
	PU49	.411	.444	.508	.510	.493	.459	.512	.444	.460	.432	.550	.660	1.000	.640	.431
	PU50	.430	.509	.478	.509	.497	.502	.608	.488	.494	.523	.520	.636	.640	1.000	.469
	PU59	.355	.484	.514	.524	.407	.439	.498	.425	.423	.433	.445	.521	.431	.469	1.000
Sig. (1-tailed)	PU33		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	PU34	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	PU35	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	PU36	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	PU39	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	PU40	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000
	PU42	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000
	PU43	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000
	PU45	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000
	PU46	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000
	PU47	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000
	PU48	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000
	PU49	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000
	PU50	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000
	PU59	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	

a. Determinant = .000

Appendix M_4: Correlation Matrix for Cognitive Guided Instructional (CGI)

		CGI86	CGI88	CGI91	CGI93	CGI94	CGI95	CGI104	CGI106	CGI107	CGI108
Correlation	CGI86	1.000	.535	.450	.411	.490	.457	.301	.346	.364	.384
	CGI88	.535	1.000	.474	.505	.521	.519	.300	.420	.398	.433
	CGI91	.450	.474	1.000	.656	.555	.523	.396	.478	.414	.397
	CGI93	.411	.505	.656	1.000	.608	.642	.395	.472	.424	.412
	CGI94	.490	.521	.555	.608	1.000	.645	.427	.473	.440	.451
	CGI95	.457	.519	.523	.642	.645	1.000	.303	.433	.411	.407
	CGI104	.301	.300	.396	.395	.427	.303	1.000	.580	.528	.501
	CGI106	.346	.420	.478	.472	.473	.433	.580	1.000	.576	.569
	CGI107	.364	.398	.414	.424	.440	.411	.528	.576	1.000	.580
	CGI108	.384	.433	.397	.412	.451	.407	.501	.569	.580	1.000
Sig. (1-tailed)	CGII86		.000	.000	.000	.000	.000	.000	.000	.000	.000
	CGI88	.000		.000	.000	.000	.000	.000	.000	.000	.000
	CGI91	.000	.000		.000	.000	.000	.000	.000	.000	.000
	CGI93	.000	.000	.000		.000	.000	.000	.000	.000	.000
	CGI94	.000	.000	.000	.000		.000	.000	.000	.000	.000
	CGI95	.000	.000	.000	.000	.000		.000	.000	.000	.000
	CGI104	.000	.000	.000	.000	.000	.000		.000	.000	.000
	CGI106	.000	.000	.000	.000	.000	.000	.000		.000	.000
	CGI107	.000	.000	.000	.000	.000	.000	.000	.000		.000
	CGI108	.000	.000	.000	.000	.000	.000	.000	.000	.000	

a. Determinant = .008

Appendix M5: Correlation Matrix for Relevant Previous Knowledge (RPK)

		RPK4	RPK7	RPK8	RPK9	RPK10	RPK11	RPK13	RPK15	RPK16	RPK17
Correlation	RPK4	1.000	.377	.394	.313	.311	.506	.401	.388	.406	.417
	RPK7	.377	1.000	.336	.329	.254	.392	.366	.395	.338	.217
	RPK8	.394	.336	1.000	.588	.434	.422	.365	.380	.378	.334
	RPK9	.313	.329	.588	1.000	.495	.396	.324	.274	.322	.246
	RPK10	.311	.254	.434	.495	1.000	.374	.393	.230	.333	.286
	RPK11	.506	.392	.422	.396	.374	1.000	.531	.434	.465	.468
	RPK13	.401	.366	.365	.324	.393	.531	1.000	.537	.498	.434
	RPK15	.388	.395	.380	.274	.230	.434	.537	1.000	.557	.425
	RPK16	.406	.338	.378	.322	.333	.465	.498	.557	1.000	.538
	RPK17	.417	.217	.334	.246	.286	.468	.434	.425	.538	1.000
Sig. (1-tailed)	RPK4		.000	.000	.000	.000	.000	.000	.000	.000	.000
	RPK7	.000		.000	.000	.000	.000	.000	.000	.000	.000
	RPK8	.000	.000		.000	.000	.000	.000	.000	.000	.000
	RPK9	.000	.000	.000		.000	.000	.000	.000	.000	.000
	RPK10	.000	.000	.000	.000		.000	.000	.000	.000	.000
	RPK11	.000	.000	.000	.000	.000		.000	.000	.000	.000
	RPK13	.000	.000	.000	.000	.000	.000		.000	.000	.000
	RPK15	.000	.000	.000	.000	.000	.000	.000		.000	.000
	RPK16	.000	.000	.000	.000	.000	.000	.000	.000		.000
	RPK17	.000	.000	.000	.000	.000	.000	.000	.000	.000	

a. Determinant = .028

Appendix M_6: Correlation Matrix for Student Internal Motivation (SDT_IM)

		IM29	IM30	IM32	IM38	IM39	IM40	IM41	IM42	IM44	IM46
Correlation	IM29	1.000	.496	.476	.450	.387	.427	.354	.423	.410	.366
	IM30	.496	1.000	.474	.348	.353	.344	.358	.382	.362	.403
	IM32	.476	.474	1.000	.406	.402	.460	.377	.462	.459	.407
	IM38	.450	.348	.406	1.000	.468	.429	.425	.443	.452	.366
	IM39	.387	.353	.402	.468	1.000	.607	.605	.553	.517	.420
	IM40	.427	.344	.460	.429	.607	1.000	.584	.614	.524	.525
	IM41	.354	.358	.377	.425	.605	.584	1.000	.553	.508	.525
	IM42	.423	.382	.462	.443	.553	.614	.553	1.000	.570	.470
	IM44	.410	.362	.459	.452	.517	.524	.508	.570	1.000	.517
	IM46	.366	.403	.407	.366	.420	.525	.525	.470	.517	1.000
Sig. (1-tailed)	IM29		.000	.000	.000	.000	.000	.000	.000	.000	.000
	IM30	.000		.000	.000	.000	.000	.000	.000	.000	.000
	IM32	.000	.000		.000	.000	.000	.000	.000	.000	.000
	IM38	.000	.000	.000		.000	.000	.000	.000	.000	.000
	IM39	.000	.000	.000	.000		.000	.000	.000	.000	.000
	IM40	.000	.000	.000	.000	.000		.000	.000	.000	.000
	IM41	.000	.000	.000	.000	.000	.000		.000	.000	.000
	IM42	.000	.000	.000	.000	.000	.000	.000		.000	.000
	IM44	.000	.000	.000	.000	.000	.000	.000	.000		.000
	IM46	.000	.000	.000	.000	.000	.000	.000	.000	.000	

a. Determinant = .013

Appendix M7 :Correlation Matrix for Student External Motivation (SDT_EM)

		EM50	EM53	EM57	EM60	EM66	EM69	EM88
Correlation	EM50	1.000	.406	.469	.526	.407	.348	.363
	EM53	.406	1.000	.495	.451	.311	.286	.353
	EM57	.469	.495	1.000	.469	.375	.371	.338
	EM60	.526	.451	.469	1.000	.445	.443	.339
	EM66	.407	.311	.375	.445	1.000	.500	.371
	EM69	.348	.286	.371	.443	.500	1.000	.383
	EM88	.363	.353	.338	.339	.371	.383	1.000
Sig. (1-tailed)	EM50		.000	.000	.000	.000	.000	.000
	EM53	.000		.000	.000	.000	.000	.000
	EM57	.000	.000		.000	.000	.000	.000
	EM60	.000	.000	.000		.000	.000	.000
	EM66	.000	.000	.000	.000		.000	.000
	EM69	.000	.000	.000	.000	.000		.000
	EM88	.000	.000	.000	.000	.000	.000	

a. Determinant = .130

Appendix M_8: Correlation Matrix for Constructivism (CONST)

		CON4	CON6	CON7	CON8	CON9	CON15	CON16	CON17	CON18	CON19	CON20
Correlation	CON4	1.000	.294	.502	.483	.413	.444	.379	.387	.209	.249	.437
	CON6	.294	1.000	.281	.283	.276	.168	.197	.172	.094	.100	.183
	CON7	.502	.281	1.000	.571	.514	.395	.361	.397	.188	.239	.433
	CON8	.483	.283	.571	1.000	.548	.393	.397	.383	.221	.199	.392
	CON9	.413	.276	.514	.548	1.000	.278	.281	.310	.190	.225	.354
	CON15	.444	.168	.395	.393	.278	1.000	.599	.574	.253	.217	.361
	CON16	.379	.197	.361	.397	.281	.599	1.000	.636	.259	.170	.417
	CON17	.387	.172	.397	.383	.310	.574	.636	1.000	.298	.251	.514
	CON18	.209	.094	.188	.221	.190	.253	.259	.298	1.000	.107	.246
	CON19	.249	.100	.239	.199	.225	.217	.170	.251	.107	1.000	.320
	CON20	.437	.183	.433	.392	.354	.361	.417	.514	.246	.320	1.000
Sig. (1-tailed)	CON4		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	CON6	.000		.000	.000	.000	.000	.000	.000	.010	.007	.000
	CON7	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000
	CON8	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000
	CON9	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000
	CON15	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000
	CON16	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000
	CON17	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000
	CON18	.000	.010	.000	.000	.000	.000	.000	.000		.004	.000
	CON19	.000	.007	.000	.000	.000	.000	.000	.000	.004		.000
	CON20	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	

a. Determinant = .029

- CON = CONST

Appendix M9: Correlation Matrix for Teacher Quality (TQ)

		TQ2	TQ3	TQ5	TQ6	TQ7	TQ8	TQ9	TQ12	TQ14	TQ15	TQ16	TQ22	TQ35	TQ36
Correlation	TQ2	1.000	.675	.196	.513	.502	.494	.395	.307	.327	.311	.341	.352	.362	.359
	TQ3	.675	1.000	.210	.450	.524	.496	.441	.362	.360	.314	.324	.372	.320	.354
	TQ5	.196	.210	1.000	.287	.238	.224	.252	.187	.044	.148	.128	.154	.092	.145
	TQ6	.513	.450	.287	1.000	.673	.669	.497	.395	.289	.317	.322	.374	.379	.353
	TQ7	.502	.524	.238	.673	1.000	.676	.532	.337	.269	.311	.260	.440	.334	.340
	TQ8	.494	.496	.224	.669	.676	1.000	.572	.467	.401	.333	.386	.442	.354	.365
	TQ9	.395	.441	.252	.497	.532	.572	1.000	.451	.334	.336	.363	.441	.359	.406
	TQ12	.307	.362	.187	.395	.337	.467	.451	1.000	.496	.420	.451	.435	.389	.434
	TQ14	.327	.360	.044	.289	.269	.401	.334	.496	1.000	.574	.663	.478	.322	.405
	TQ15	.311	.314	.148	.317	.311	.333	.336	.420	.574	1.000	.667	.529	.386	.438
	TQ16	.341	.324	.128	.322	.260	.386	.363	.451	.663	.667	1.000	.518	.384	.411
	TQ22	.352	.372	.154	.374	.440	.442	.441	.435	.478	.529	.518	1.000	.379	.440
	TQ35	.362	.320	.092	.379	.334	.354	.359	.389	.322	.386	.384	.379	1.000	.524
	TQ36	.359	.354	.145	.353	.340	.365	.406	.434	.405	.438	.411	.440	.524	1.000
Sig. (1-tailed)	TQ2		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	TQ3	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	TQ5	.000	.000		.000	.000	.000	.000	.000	.150	.000	.001	.000	.015	.000
	TQ6	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	TQ7	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000
	TQ8	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000
	TQ9	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000
	TQ12	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000
	TQ14	.000	.000	.150	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000
	TQ15	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000
	TQ16	.000	.000	.001	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000
	TQ22	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000
	TQ35	.000	.000	.015	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000
	TQ36	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	

a. Determinant = .001

Appendix N: Latent Variable Correlations

Mean, STDEV, T-Values, P-Values

Path Model	Original Sample (O)	Sample Mean (M)	SD (STDEV)	T Statistics (|O/STDEV|)	P Values
CGI -> CA	0.668	0.666	0.037	17.867	0.000
CONST -> CA	0.503	0.502	0.051	9.792	0.000
CONST -> CGI	0.597	0.596	0.047	12.828	0.000
IC -> CA	0.736	0.734	0.037	19.624	0.000
IC -> CGI	0,799	0.798	0.027	29.435	0.000
IC -> CONST	0.619	0.617	0.047	13.240	0.000
PUFM -> CA	0.798	0.797	0.025	31.815	0.000
PUFM -> CGI	0.740	0.739	0.032	23.355	0.000
PUFM -> CONST	0.566	0.565	0.049	11.632	0.000
PUFM -> IC	0.829	0.828	0.027	31.248	0.000
RPK -> CA	0.612	0.612	0.043	14.329	0.000
RPK -> CGI	0.552	0.552	0.046	11.960	0.000
RPK -> CONST	0.452	0.451	0.056	8.151	0.000
RPK -> IC	0.620	0.620	0.047	13.326	0.,000
RPK -> PUFM	0.652	0.651	0.042	15.654	0.000
SDT-EM -> CA	0.479	0.478	0.052	9.207	0.000
SDT-EM -> CGI	0.602	0.601	0.048	12.562	0.000
SDT-EM -> CONST	0.703	0,702	0.039	18.246	0.000
SDT-EM -> IC	0.584	0.582	0.048	12.122	0.000
SDT-EM -> PUFM	0.554	0.554	0.050	11.162	0.000
SDT-EM -> RPK	0.452	0.451	0.054	8.389	0.000
SDT-IM -> CA	0.471	0.469	0.051	9.228	0.000
SDT-IM -> CGI	0.553	0.551	0.051	10.873	0.000
SDT-IM -> CONST	0.772	0,771	0.029	26.632	0.000
SDT-IM -> IC	0.582	0.580	0,047	12.474	0.000
SDT-IM -> PUFM	0.536	0.535	0,049	10.917	0.000
SDT-IM -> RPK	0.441	0.439	0,053	8.367	0.000
SDT-IM -> SDT-EM	0.767	0.765	0,031	24.438	0.000
TQ -> CA	0.540	0.540	0,046	11.825	0.000
TQ -> CGI	0.636	0.635	0,042	15.246	0.000
TQ -> CONST	0.483	0.484	0.049	9.767	0.000
TQ -> IC	0.617	0.616	0.044	13.921	0.000
TQ -> PUFM	0.595	0.596	0.041	14.662	0.000
TQ -> RPK	0.439	0.440	0.049	8.901	0.000

TQ -> SDT-EM	0.606	0.606	0.043	14.257	0.000
TQ -> SDT-IM	0.502	0.502	0.051	9.909	0.000
TTP -> CA	0.189	0.190	0.047	3.985	0.000
TTP -> CGI	0.111	0.111	0.049	2.270	0.023
TTP -> CONST	0.186	0.189	0.045	4.142	0.000
TTP -> IC	0.178	0.178	0.051	3.516	0.000
TTP -> PUFM	0.184	0.184	0.047	3.881	0.000
TTP -> RPK	0.129	0.131	0.044	2.969	0.003
TTP -> SDT-EM	0.103	0.104	0.045	2.291	0.022
TTP -> SDT-IM	0.163	0.165	0.042	3.861	0.000
TTP -> TQ	0.184	0.186	0.045	4.072	0.000

Appendices O: Total Variance Explained

Appendix O_1: Cognitive Activation

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %
1	5.643	40.307	40.307	5.643	40.307	40.307	3.488	24.917	24.917
2	1.400	9.997	50.304	1.400	9.997	50.304	2.588	18.484	43.401
3	1.013	7.239	57.544	1.013	7.239	57.544	1.980	14.143	57.544
4	.819	5.848	63.392						
5	.729	5.206	68.597						
6	.677	4.833	73.431						
7	.614	4.388	77.818						
8	.571	4.079	81.897						
9	.540	3.855	85.752						
10	.474	3.384	89.136						
11	.439	3.137	92.273						
12	.394	2.814	95.087						
13	.361	2.581	97.668						
14	.327	2.332	100.000						

Extraction Method: Principal Component Analysis.

Cum. – Cumulative

Appendix O_2: Instructional Coherence

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %
1	6.574	50.573	50.573	6.574	50.573	50.573	3.898	29.985	29.985
2	1.137	8.748	59.320	1.137	8.748	59.320	3.814	29.336	59.320
3	.887	6.822	66.142						
4	.639	4.915	71.057						
5	.625	4.807	75.864						
6	.557	4.283	80.147						
7	.498	3.832	83.980						
8	.441	3.396	87.376						
9	.396	3.047	90.423						
10	.365	2.806	93.229						
11	.314	2.413	95.643						
12	.295	2.270	97.913						
13	.271	2.087	100.000						

Extraction Method: Principal Component Analysis.

Appendix O3: Profound Understanding of Fundamental Mathematics

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	7.927	52.847	52.847	7.927	52.847	52.847	5.173	34.487	34.487
2	1.103	7.356	60.204	1.103	7.356	60.204	3.858	25.717	60.204
3	.807	5.381	65.584						
4	.718	4.788	70.373						
5	.658	4.388	74.760						
6	.611	4.075	78.836						
7	.535	3.566	82.402						
8	.441	2.943	85.345						
9	.401	2.674	88.019						
10	.346	2.308	90.326						
11	.329	2.192	92.518						
12	.310	2.065	94.583						
13	.295	1.969	96.552						
14	.265	1.768	98.320						
15	.252	1.680	100.000						

Extraction Method: Principal Component Analysis.

Appendix O_4: Cognitive Guided Instruction

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %
1	5.218	52.181	52.181	5.218	52.181	52.181	3.570	35.701	35.701
2	1.141	11.407	63.588	1.141	11.407	63.588	2.789	27.888	63.588
3	.723	7.226	70.814						
4	.545	5.449	76.263						
5	.486	4.861	81.124						
6	.450	4.501	85.626						
7	.415	4.150	89.776						
8	.389	3.887	93.663						
9	.345	3.449	97.113						
10	.289	2.887	100.000						

Extraction Method: Principal Component Analysis.

Appendix O_5: Relevant Previous Knowledge

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %
1	4.545	45.447	45.447	4.545	45.447	45.447	3.343	33.428	33.428
2	1.140	11.400	56.847	1.140	11.400	56.847	2.342	23.419	56.847
3	.812	8.118	64.965						
4	.686	6.864	71.829						
5	.630	6.297	78.127						
6	.527	5.266	83.393						
7	.491	4.911	88.305						
8	.421	4.211	92.516						
9	.380	3.801	96.317						
10	.368	3.683	100.000						

Extraction Method: Principal Component Analysis.

Appendix O_6: SDT_ Internal Motivation

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %
1	5.116	51.162	51.162	5.116	51.162	51.162
2	.971	9.713	60.874			
3	.683	6.832	67.707			
4	.568	5.684	73.390			
5	.566	5.664	79.054			
6	.495	4.948	84.002			
7	.472	4.719	88.721			
8	.422	4.216	92.937			
9	.381	3.814	96.751			
10	.325	3.249	100.000			

Extraction Method: Principal Component Analysis.

Appendix O_7: SDT_ External Motivation

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %
1	3.423	48.902	48.902	3.423	48.902	48.902
2	.856	12.233	61.136			
3	.697	9.954	71.090			
4	.590	8.423	79.513			
5	.506	7.235	86.748			
6	.503	7.190	93.938			
7	.424	6.062	100.000			

Extraction Method: Principal Component Analysis.

Appendix O8: Constructivism

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %
1	4.446	40.416	40.416	4.446	40.416	40.416	2.887	26.249	26.249
2	1.202	10.931	51.347	1.202	10.931	51.347	2.761	25.099	51.347
3	.937	8.517	59.864						
4	.848	7.707	67.572						
5	.791	7.189	74.761						
6	.607	5.520	80.281						
7	.581	5.279	85.560						
8	.457	4.159	89.719						
9	.420	3.821	93.539						
10	.370	3.368	96.907						
11	.340	3.093	100.000						

Extraction Method: Principal Component Analysis.

Appendix O9: Teacher Quality

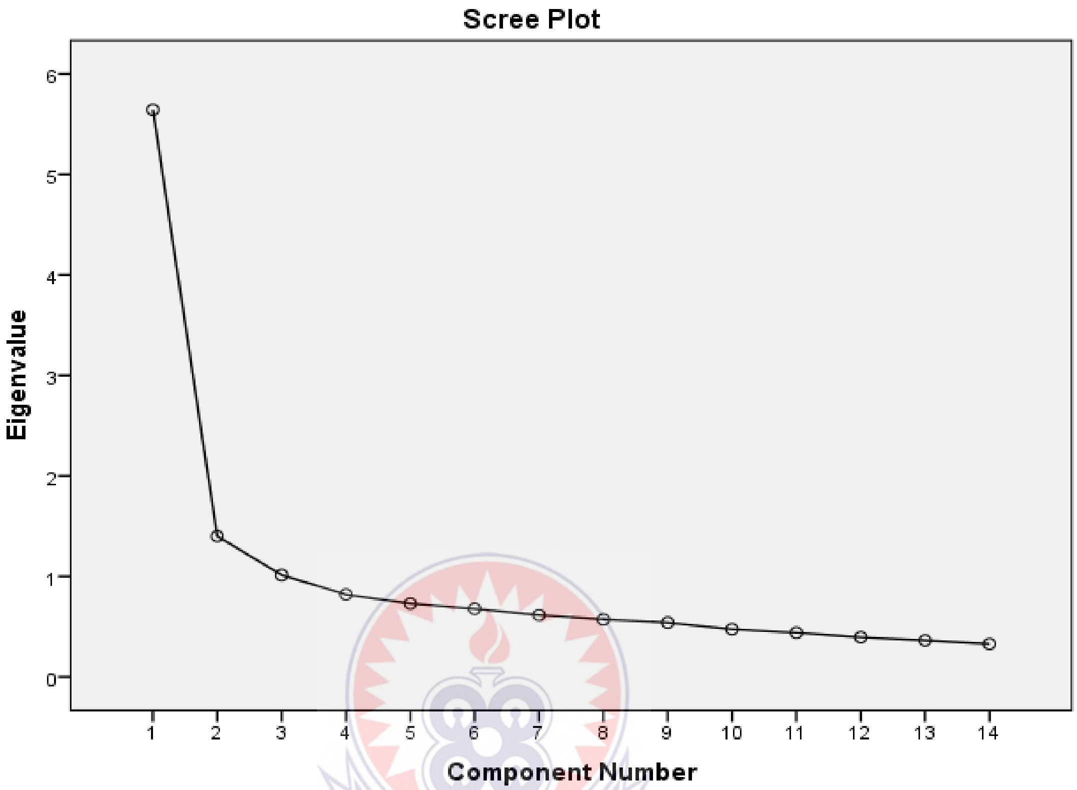
Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %
1	6.137	43.836	43.836	6.137	43.836	43.836	3.920	28.003	28.003
2	1.605	11.465	55.301	1.605	11.465	55.301	3.822	27.297	55.301
3	.921	6.579	61.879						
4	.848	6.058	67.938						
5	.790	5.639	73.577						
6	.632	4.515	78.092						
7	.563	4.021	82.113						
8	.473	3.379	85.492						
9	.460	3.287	88.779						
10	.386	2.759	91.538						
11	.352	2.517	94.055						
12	.295	2.107	96.162						
13	.277	1.981	98.144						
14	.260	1.856	100.000						

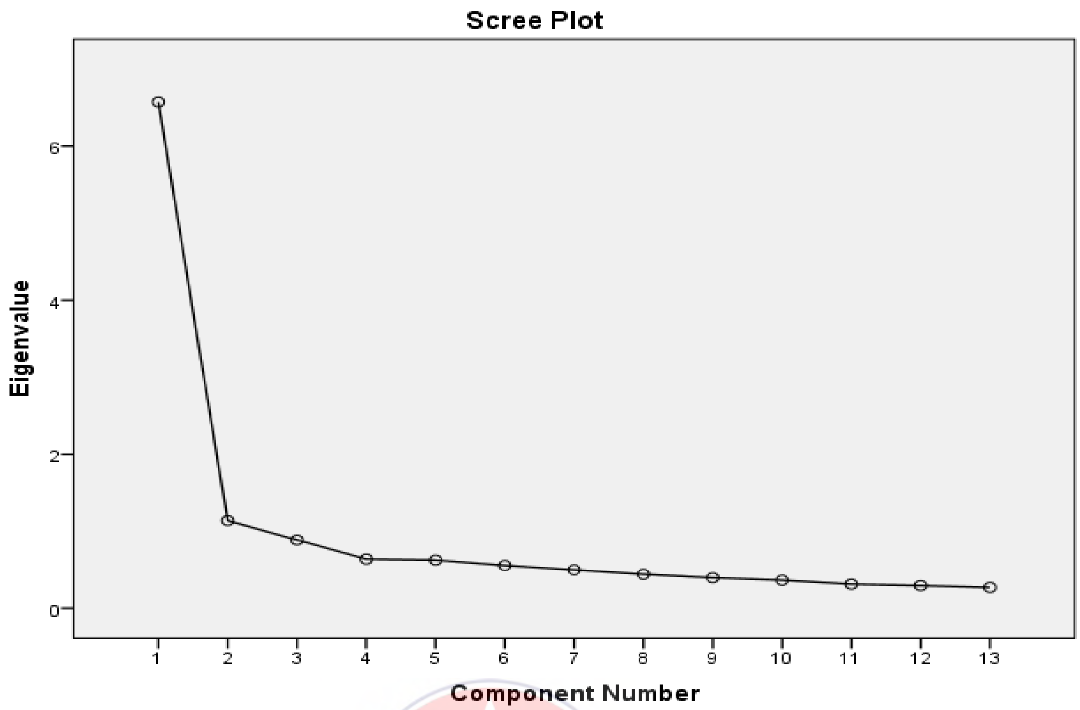
Extraction Method: Principal Component Analysis.

Appendices P: Scree Plots

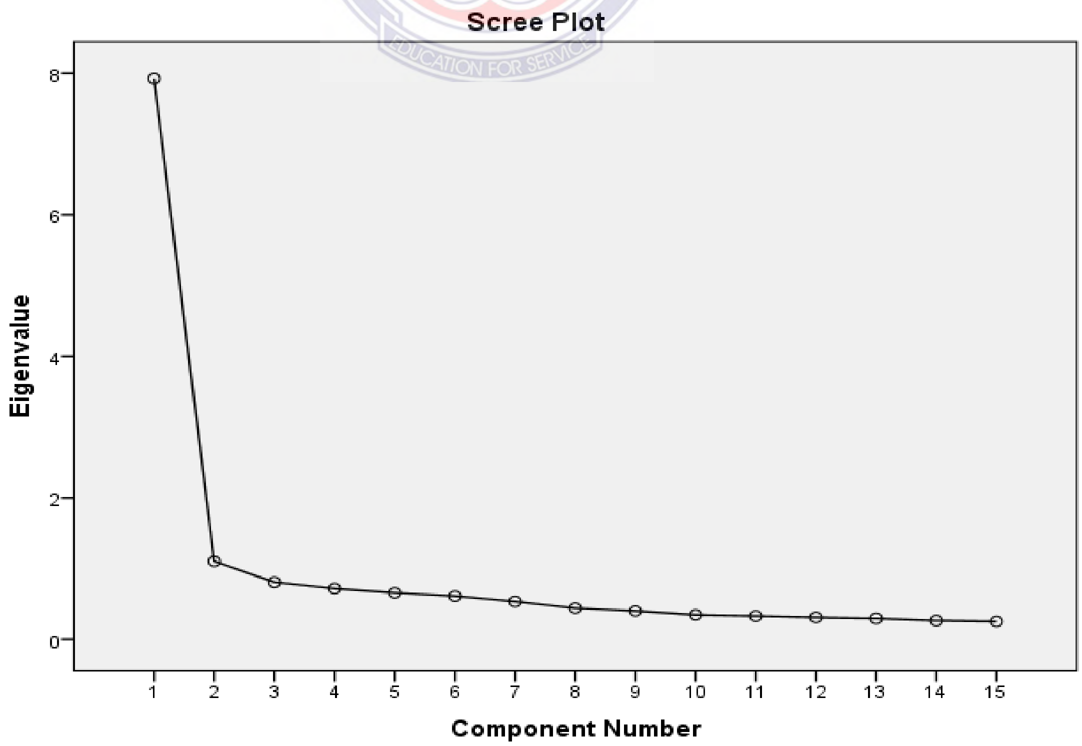
Appendix P_1: Cognitive Activation



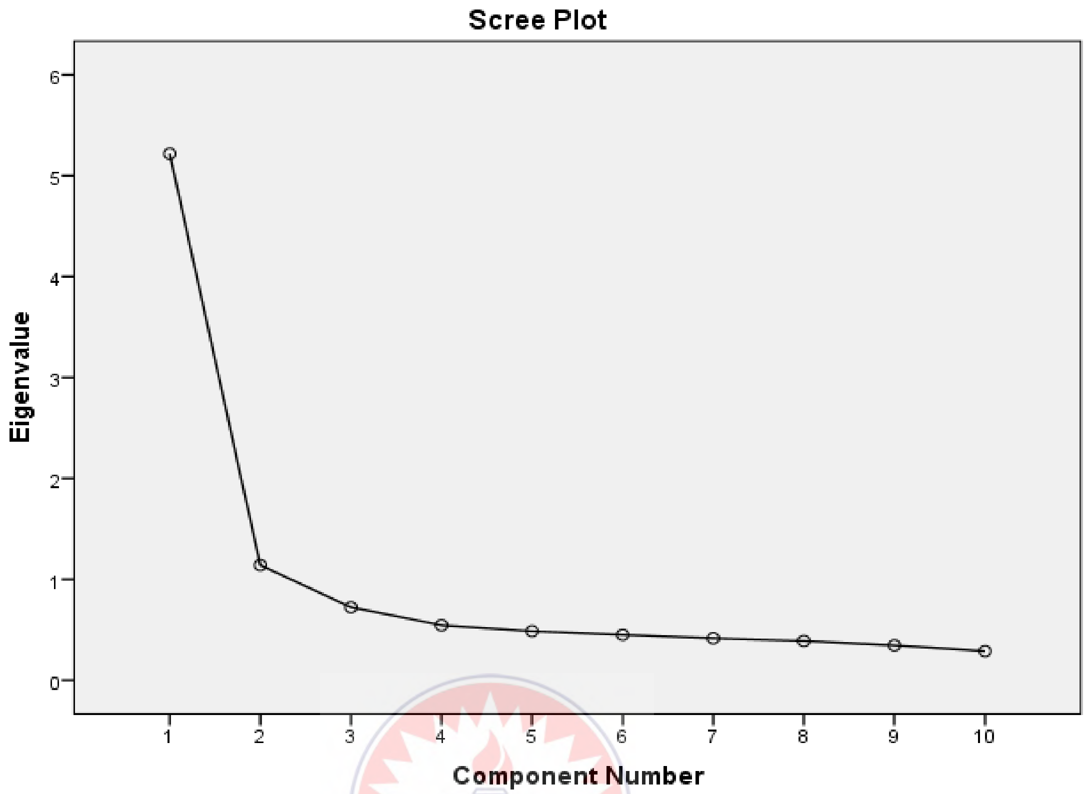
Appendix P2: Instructional Co



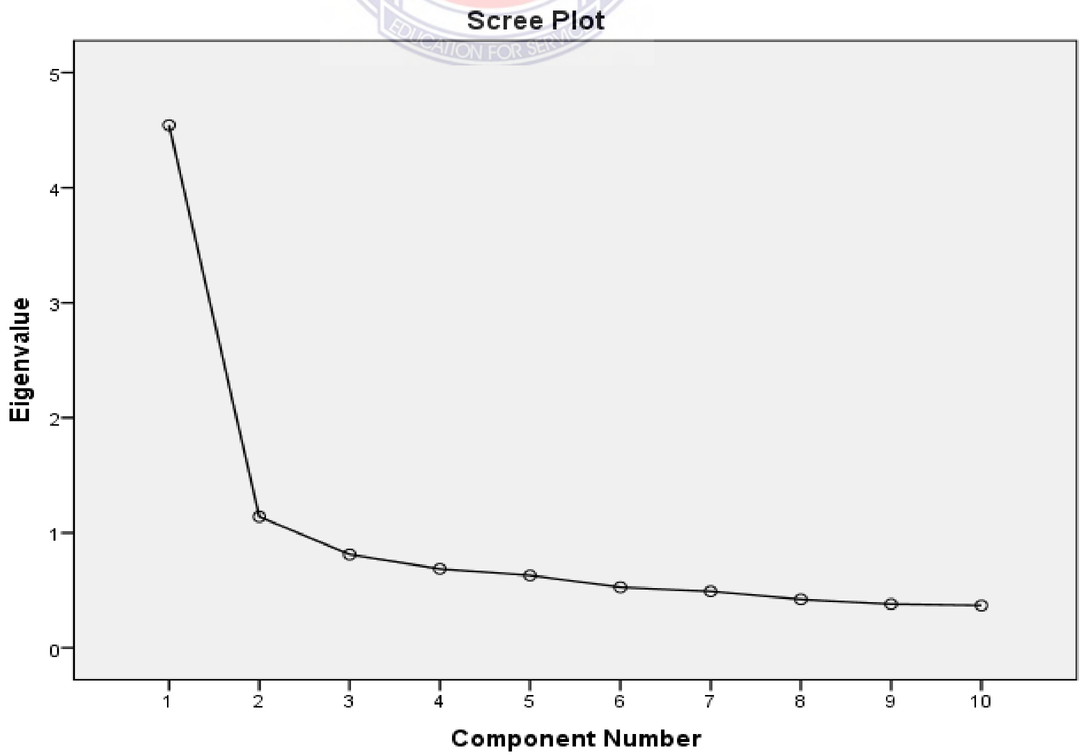
Appendix P3: Profound Understanding of Fundamental Mathematics



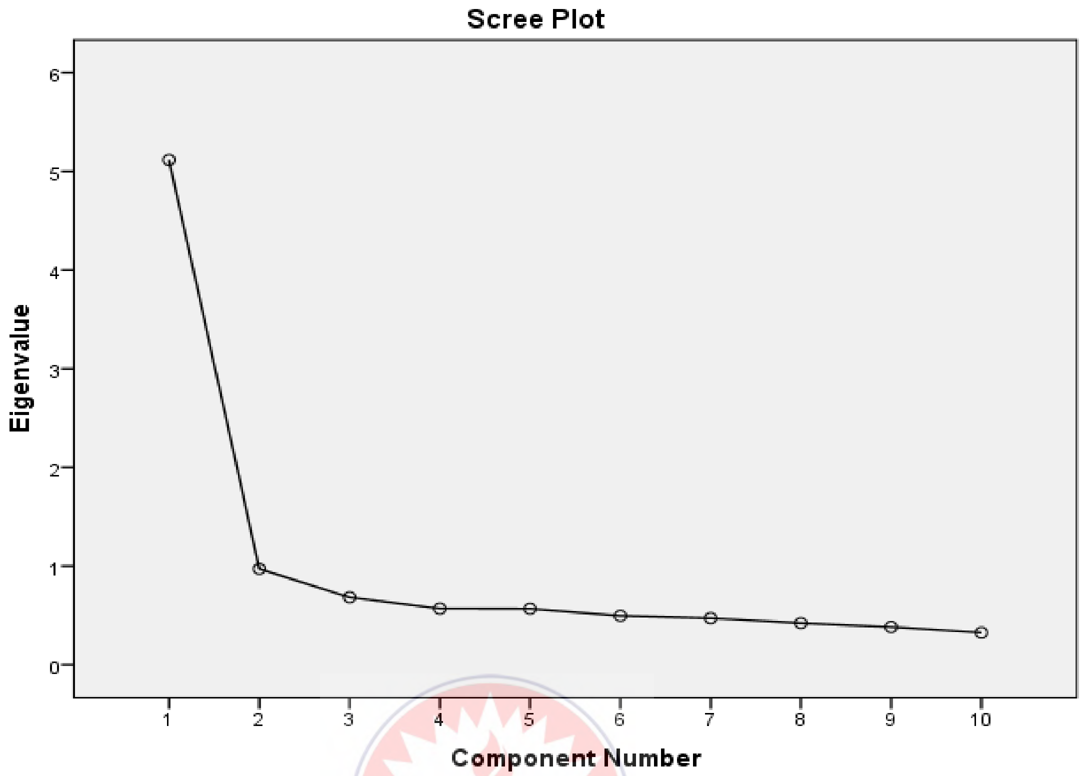
Appendix P4: Cognitive Guided Instruction



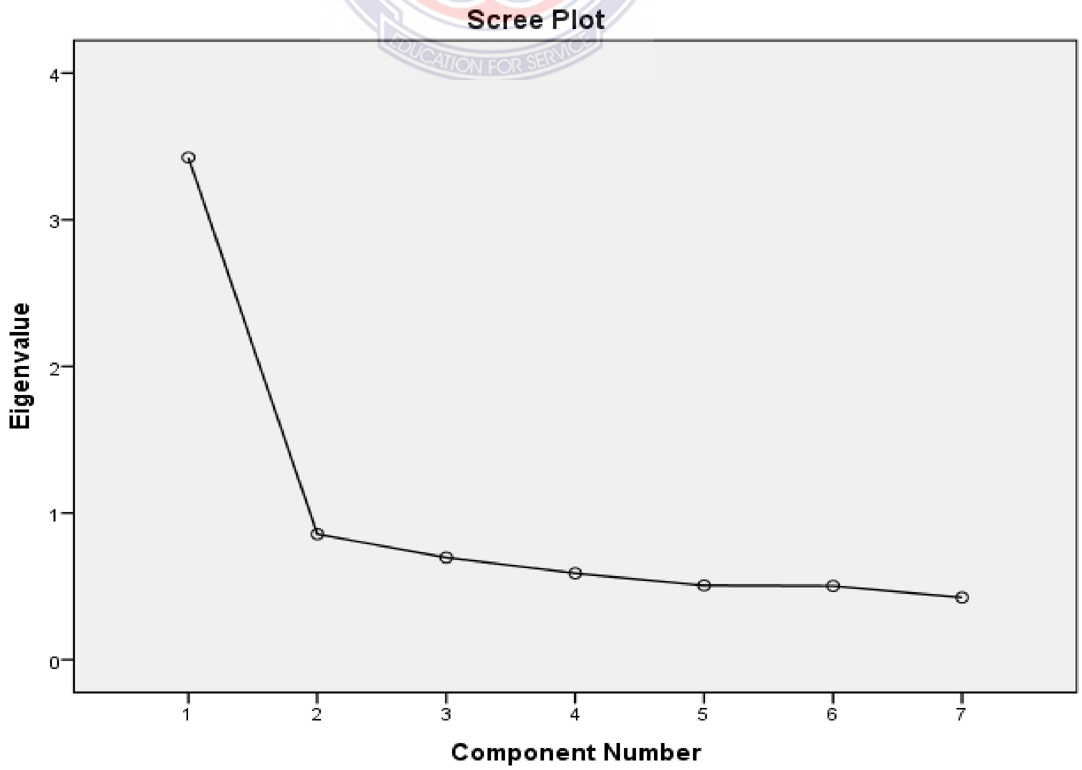
Appendix P5: Relevant Previous Knowledge



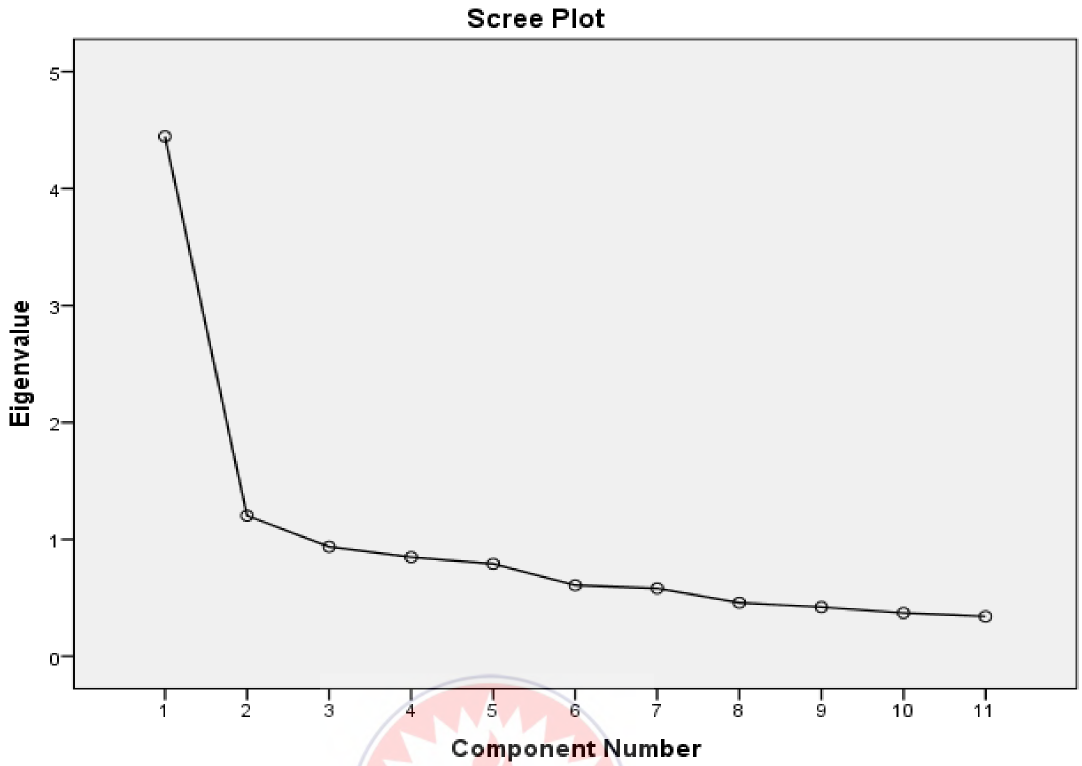
Appendix P$_6$: SDT_ Internal Motivation



Appendix P$_7$: SDT_ External Motivation



Appendix P_8: Constructivism



Appendix P_9: Teacher Quality