UNIVERSITY OF EDUCATION, WINNEBA

THE EFFECT OF INCORPORATING GEOGEBRA IN CYCLE MODEL OF TEACHING CALCULUS AT UNIVERSITY OF EDUCATION, SEKONDI STUDY CENTRE.

GIDEON COBBINAH

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A thesis in the Department of Mathematics Education, Faculty of Science, submitted to the School of Graduate Studies in partial fulfilment of the requirements for the award of the degree of Master of Philosophy (Mathematics) in the University of Education, Winneba

APRIL, 2023

DECLARATION

Student's Declaration

I, GIDEON COBBINAH, declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

Signature: ………………………….

Date: ……………………………..…

Supervisors' Declaration

We hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of thesis as laid down by the University of Education, Winneba.

Supervisor's Name: Dr. Peter Akayuure

Signature: ………………………….

Date: ……………………………..…

DEDICATION

This work is dedicated to my son PRAYER NANA BADU AMUZU COBBINAH.

ACKNOWLEDGEMENTS

It is always said that, he who cut a path knows not where it's crooked except the bystander. It is obvious the completion of this project was immensely contributed by number of people who I cannot mention their names all. However, through their many and diverse ways toward the completion of this project work, I wish to say that, I am very grateful for all of their assistance and guidance most importantly, Dr. Peter Akayuure, who serve as a personal mentor, lecturer and supervisor to this work, I humbly and sincerely appreciate his perseverance, patience and dedication. God abundantly bless you in all your endeavors.

In addition to the data obtained from University of Education Winneba, Sekondi study center, students totally volunteered to participate in this study, and I sincerely thank you all.

May God bless you all.

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ABSTRACT

With the rapid growth of technology in the 21st century, traditional teaching and learning methods are considered outdated and not suitable for the active learning processes of the constructivist learning approach. The adjustment of existing methods and the development of new ones to teach and learn calculus with the help of technology is needed. This study aimed to investigate the effect of the use of cycle model incorporating GeoGebra software on university students' learning of calculus in terms of proficiency in calculus and attitudes to using GeoGebra. A quasi-experiment with a pre-test post-test design and questionnaires was used. The study was conducted at a University of Education-Sekondi study center with 66 students. The data were collected over four weeks in semester two of the 2021/2022 academic year. The quantitative and qualitative data were analyzed using SPSS. The results indicated that incorporating GeoGebra in the cycle model of teaching calculus had a more positive effect on students' conceptual and procedural understanding when compared to students who were taught using a traditional teaching approach. Students in the GeoGebra group showed greater improvement in procedural understanding, with an effect size of d = 1.2 and a percentile gain of 49%; in conceptual understanding of differential calculus. However, the students in non-GeoGebra group showed only slight improvement with an effect size of d = 0.02 and a percentile gain of 2%. Students expressed positive attitudes and perceptions towards the use of GeoGebra for learning differential calculus. While the GeoGebra oriented learning approach to calculus has the potential to improve proficiency, it remains critically important that it should be designed (cycle model) and aimed to fill a specific gap. Based on the results, it is imperatives that students are being taught calculus by incorporating GeoGebra in cycle model.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This chapter provides an introduction to the research study. It also includes the statement of the problem, the purpose of the study and research questions which guide the study and further highlights the significance of the study.

1.1 Background of the Study

Society at large continues to be shaped by science and technology in the 21st century. In this regard, each country must reconsider its capacity to remain relevant in the competitive global arena. The present century has been marked by rapid technological developments; the learning environment has thus undergone irreversible changes, and individuals can learn whatever they choose, as long as they have access to technology that is paired with the skill to use it effectively. Technology has become the foundation of this modern industrial society. Technology-based instruction aims to stimulate students' active participation, purposeful learning and task-oriented activities. The integration of technological aids, specifically in the teaching of mathematics is a move away from teacher-centered instruction towards a learning-centered approach in which the student's conceptualization of subject matter takes center stage. Teaching and learning mathematics, the implementation of information and communications technology (ICT) in the classroom has been slower than expected. Some factors hampering the implementation of new educational technologies are mentioned by researchers (De Witte & Rogge, 2014; Agyei & Voogt, 2010).

Agyei and Voogt, (2010) point out that the slow implementation of educational technology may be the result of a lack of teacher professional development. De Witte and Rogge (2014) argue that the shortage of computers in schools restricts the use of technology. In the same vein, Safdar, Yousuf, Parveen and Behlol (2011) believe that the financial outlay and resources required by these technologies are responsible for their slow implementation of the integration of technology in education is intended to expedite and enhance the mastering of subject content. However, studies on the use of technology in the learning of mathematics have revealed different findings in terms of improvement (or not) in learning. For instance, Biagi and Loi (2013), Goodison (2002) found that the use of technology did not lead to any visible improvements in mathematics learning.

In contrast, several scholars have reported gaps in the use of technology in the teaching in the mathematics classroom (Curri, 2012; Miller & Glover, 2007; Novotná & Jančařík, 2018; Tay, Lim, Lim & Koh, 2012). However, another study revealed that one of the best methods of enhancing student achievements in various mathematical topics, for instance, calculus is the use of technology in the teaching and learning process (Eyyam & Yaratan, 2014). Calculus has a wide range of applications in disciplines such as economics, engineering, science, business, computer science and information systems (Mendezaba & Tindowen, 2018). As a branch of mathematics, the concepts embedded in calculus are abstract and complex (Gordon, 2004; Sahin, Zachariades et al., 2007). As such, students need higher-order thinking skills to cope with calculus. Sahin et al. (2015) argue that calculus is often the main reason for the failure of students at the undergraduate level because of the way these students have been trained. In their study, Bressoud, Ghedamsi, Curri (2012) found that students' difficulties with calculus emerged between secondary school and tertiary education.

Tall, Smith and Piez (2008) argue that “calculus can be taught more by using technology from all fields of mathematics”. In a series of research studies demonstrating the power of technology, Tall (1986, 1990, 2003, 2013) found that digital technology enhanced visualization skills, enable programming language and improved students’ understanding of the concepts of calculus. Tall (2019) points out that digital technology enhances the teaching and learning of calculus by allowing students to make fast and accurate numerical calculations, to manipulate symbols and to create dynamic figures that help them to visualize abstract concepts. Several researchers have demonstrated that most difficulties encountered by students in calculus arise from a poor understanding of function concepts (Breidenbach, Dubinsky, Hawks, & Nichols, 1992; Williams, 1991).

To explore these difficulties, educators have resorted in their teaching to instructional multimedia such as graphic software (Robutti, 2010; Lavicza, 2010), computer algebraic systems (Ozguiin-Koca, 2010; Mignotte, 1992; Durán, Pérez, & Varona, 2014) or a combination of both (Antohe, 2009; Dikovic, 2009; Hohenwarter & Jones, 2007; Hohenwarter & Lavicza, 2009). It is known that the concepts involved in calculus include limits and continuity of functions, derivatives, integration, and the fundamental theorem. A function may be constant or a function of a single, two, three or more variables containing dependent and independent variables. Students’ first encounter with the concept of functions is in the form of a univariate mathematical relationship where the value of a single independent variable (x) determines the value of a single dependent variable (y). In calculus, this concept is expanded to functions that can have multivariable inputs or outputs (vector-valued functions). The visualization and conceptualization of these functions allow the human mind to observe, infer, and predict change and growth (Tall, 2019). At a conference at the University of Agder, Tall (2019) reviewed the evolution and rapid growth of calculus over the past half-century, and the role of digital technology that has helped to make calculus meaningful in a wide range of applications (Tall, 2019, p. 2). Arango, Gaviria and Valencia (2015) and Nobre et al. (2016) concur that the use of technologies as an alternative and novel way of teaching and learning calculus may support students' understanding of the abstract and complex theoretical ideas that characterize this field of mathematics. On a practical level, interactive technology such as graphing calculators and mathematics software helps students to visualize change and growth through graphical representations (Moses, Wong, Bakar, & Mahmud, 2013; Arslan, Kutluca, & Özpınar, 2011; Liang & Sedig, 2010).

However, despite this, the interdependence of technology and education has in the past two decades attracted students to the sciences. Students' regard mathematics as "boring, a burden, scary" (Eng, Han, & Fah, 2011). Such attitudes may stem from students being forced to memorize formulae, algorithms, and steps to achieve good scores in tests and examinations. Calculus by its very nature demands step-by-step processes to understand the concepts, definitions and theorems (Matthews, Hoessler, Jonker, & Stockley, 2013). Students have difficulty relating algebraic ideas to graphically represented calculus notions (Tall & Vinner, 1981; Ubuz, 2007). Calculus teachers mostly make use of traditional methods in their teaching (Lasut, 2015). sComputational procedures take preference over the true understanding of calculus concepts (Lasut, 2015). Axtell (2006) is concerned that this sequential method of instruction does not help students to understand the basic concepts of calculus. As a result of the lack of true understanding, Studies by Fluck and Dowden (2013) and Nobre et al. (2016) found that many students did not know how to convert calculus concepts to applications in the physical world.

Effective teaching programmes in the 21st century are characterized by the integration of technology in education (Pierson, 2001). Gündüz and Odabasi's (2004) study revealed that we can no longer regard the integration of technologies in the learning environment of the classroom as an option; it is an obligation in the information age. This use of technology in the classroom requires thorough planning of how it is to be used to facilitate mathematical understanding (Zho, Pugh, Sheldon, & Byers, 2002). In this study, I investigated the effect of incorporating GeoGebra in teaching calculus at University of Education. GeoGebra was used in this study as it is open-source software, it is simple to use, and anybody can download the software free from the internet.

Although there are currently several technologies available to enhance the teaching and learning of mathematics, the choice of the most appropriate technological tool can be difficult (Ruthven, Hennessy, & Brindley, 2004). My decision was guided by the ease of use of GeoGebra and the fact that it is a multi-platform, dynamic mathematical software package designed for students at all levels of education and has a wide range of applications. The program contains both dynamic geometric software (DGS) and computer algebra systems (CAS) (Hewson, 2009; Hohenwarter, Hohenwarter, & Lavicza, 2008). The software can also be manipulated in various ways in the same window. GeoGebra was designed specifically for educational purposes and has been used in the field of mathematics teaching; it comprises creative and interactive visual application tools that help students to understand complex theoretical mathematical ideas. GeoGebra's display is composed of an algebra window (a window with 2D and 3D graphics), an input bar, an input environment spreadsheet, CAS, statistical abilities and calculus tools.

1.2 Statement of the Problem

In my seven years of experience in higher education in Ghana, I have observed that technology has not been used to support students' performance in calculus, either in or outside the classroom. Ghanaian university students regard calculus as difficult and conceptually challenging. At Sekondi study centre where I lecture, students don't do well in mathematics and more especially calculus and they tend to complain bitterly about their performance to me and I cannot fathom.

Mathematics is a compulsory subject in all sciences courses, which meets with resistance, particularly because there are no preparatory bridging courses for students (Semela, 2010). Semela (2010) identified several factors contributing to the challenges, namely students' weak mathematics background as a result of teachers' poor qualifications and a lack of job opportunities ousted the teaching profession, and inadequate pedagogical content knowledge. However, despite this, the interdependence of technology and education has in the past two decades attracted students to the sciences.

Students' regard mathematics as boring, a burden, scary, etc and such attitudes are as a result of students being forced to memorize formulae, algorithms, and steps to achieve good scores in tests and examinations. Calculus by its very nature demands step-by-step processes to understand the concepts, definitions and theorems. Calculus has a wide range of applications in disciplines such as economics, engineering, science, business, computer science and information system. As a branch of mathematics, the concepts embedded in calculus are abstract and complex and as such, students need higher-order thinking skills to cope with calculus. Sahin et al.(2015) argue that calculus is often the main reason for the failure of students at the undergraduate level because of the way these students have been trained. In their study, Bressoud, Ghedamsi, Curri (2012), Martinez-Luaces and Törner (2016) found that students' difficulties with calculus emerged between secondary school and tertiary education.

Tall, Smith and Piez (2008) argue that "calculus can be taught more by using technology from all fields of mathematics. Calculus teachers mostly make use of traditional methods in their teaching. Most of the time, the traditional method of teaching and learning can be seen as talk and chalk using the traditional paper-pencil approach, while scaffolding is being employed within lectures method-oriented classrooms.

Axtell (2006) is concerned that this sequential method of instruction does not help students to understand the basic concepts of calculus. As a result of the lack of true understanding, students do not know how to convert calculus concepts to applications in the physical world.

Many students fail to complete mathematics in degree courses. Fear of failure and lecturers' pedagogies and instructional methods have been cited as factors in the high attrition rate in mathematics (Bligh, 2000; Booth, 2001; Knight & Wood, 2005; Novak, Patterson, Gavrin & Christian, 1999).

Bligh (2000) argues that lectures are an ineffective teaching method that leads to a tendency among students to memories rather than to develop a conceptual understanding of mathematics. Scholars such as Handelsman et al. (2004), Hurd (1998), and Williams, Papierno, Makel and Ceci (2004) found that courses at the tertiary level focus more on memorization and less on conceptual understanding.

Glasson and Lalik (1993) proposed activities in the classroom that encourage active learning and student participation. Thalheimer (2003) supports this, arguing that learning occurs only when students are cognitively engaged in a process of questions and answers. In Ghana, little research has been done on integrating technology into mathematics teaching at either school or university level, especially in teaching with open access software like GeoGebra. Teaching in Ghana is still traditional, and teacher-centered. GeoGebra has produced a lot of success in some studies but the way it is being taught becomes a challenge. Thus, this study adopted a model known as the cycle model and investigated its effects on students' learning of calculus through GeoGebra.

1.3 Purpose of the study

The purpose of the study is to investigate the effect of incorporating GeoGebra in cycle model of teaching calculus at university of education, Sekondi study centre.

1.4 Objectives of the Study

The following are the objectives of the study;

a. To compare the level of proficiency in differential calculus of students in two groups: those taught using GeoGebra (experimental Group) and those taught using conventional lecturing (control Group).

b. To compare the level of proficiency in differential calculus pre- and post-test in experimental Group.

c. To investigate students' perception of the use of mathematical software (GeoGebra) in learning calculus concepts.

1.5 Research Questions

The following research questions is being posed in this study:

Specific research questions:

a) How does the level of proficiency in differential calculus compare in students taught using GeoGebra (experimental Group) and students taught through conventional lecturing (control Group)?
b) How does the level of proficiency in differential calculus compare within the experimental group pre- post incorporating the use of GeoGebra?
c) What are students' perceptions towards using mathematical software (GeoGebra) when learning calculus concepts?

1.6 Research Hypothesis

H_0: There is no effect on students' proficiency in calculus at University of Education using GeoGebra Mathematical software.

H_i: There is an effect on students' proficiency in calculus at University of Education using GeoGebra Mathematical software.

1.7 Significance of the Study

The study will be imperative to students in that it investigates the effects of incorporating GeoGebra in solving calculus. This study will be of immense benefit to other researchers who intend to know more on this study and can also be used by non-researchers to build more on their research work. This study contributes to knowledge and could serve as a guide for other study. I selected GeoGebra as an appropriate tool to teach and learn mathematics at the tertiary level. Research has shown that using technology when teaching students is important in increasing students' involvement in STEM. I also believe that using technology to teach mathematics may encourage links with other disciplines such as the social sciences; some scholars have investigated the use of GeoGebra in the teaching of the social sciences (Arini & Dewi, 2019). As GeoGebra is free software, there is no cost implication for parents or policy makers when students use the program. Furthermore, the use of this software is believed to have a positive impact on student's attitudes, beliefs and perceptions of calculus and provide an alternative approach to learning calculus concepts and solving related problems, whether in algebra or calculus. It is further hoped that teachers will use this study to enhance their students' understanding of the concepts of calculus and even devise interventions based on the one documented in this thesis. The study may also prove significant for students who enjoy learning mathematics in an e-learning (online) environment. In addition, the findings of the study may provide information on how students with different abilities communicate with their peers when engaging in activities in the classroom. Such information is crucial when planning lessons for large classes that include students are of varying abilities. It was hoped that the findings would reveal that the integration of technology is an aid to students learning of mathematics, particularly calculus. The study was also intended to help teachers to redefine their role as facilitators and guide in the learning process. As a lecturer myself, the findings of these and other scholars inspired me to do further research in this area: the purpose of the study was to investigate the effect of incorporating GeoGebra software in solving and learning calculus by students at University of Education.

1.8 Scope of the Study

The study is on the effect of incorporating GeoGebra mathematical software in solving and learning calculus at University of Education. The study was carried out at University of Education, Sekondi Study center in Takoradi metropolis. A fair balance of male and female respondents will be sampled.

1.9 Limitations of the Study

This study was not conducted without some limitations. One possible limitation was that the study included self-reported views. It is difficult to determine whether students answered the questions honestly, providing their genuine feelings towards the three scales of perceptions. Depending on social appeal, students may respond based not on what they feel, but on what they think is socially acceptable. The results obtained from the questionnaire may thus not reflect students' actual feelings. A second issue that might have affected the data quality in this study was the low level of computer ability of students in the experimental group; they might have failed to benefit fully from the approach, especially during the externalization stage of the cycle model. In addition, the smooth implementation of the intervention was affected by electrical outages and the absence of a well-organized mathematics laboratory. This situation affected the study, although I did my best to continue the experiment by changing my schedule. That was managed by arranging classes at the times when the university generator was functioning as a power supply for some purpose, such as to power the cafeteria or library.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

The purpose of this literature review is to provide an overview of the theories and research findings of studies conducted in the discipline of mathematics teaching to address the research questions. This review focused on literature concerned with the following topics:

- The use of technology in education in the 21st century
- Software used in the teaching and learning of mathematics
- GeoGebra software, its components and students' attitudes to using it
- Student's proficiencies in Mathematics education
- The challenges that mathematics poses for students
- Teachers' beliefs about using technology in the classroom.

The focus of the study was on the use of technology in the teaching and learning of mathematics at the university level. In this review, I used a wide range of cross disciplinary sources including books, journal articles, thesis and dissertations, and conference proceedings.

2.1 The use of technology in education in the 21st century

The 21st century has seen a technological revolution that has had a significant impact on education. The term technology can be defined in a variety of ways, depending on the field of its application; literally, it refers to the use of hardware, while metaphorically it can be applied to real-world problem solving (Heinich, Molenda, Russell, & Smaldino, 2002). Huang, Spector and Yang (2019) argue that there are two components to technology: hardware and software. Hardware comprises the tool that embodies the technology, material or physical object, while software comprises the information base underlying the tool. Some technologies may lack one or both components and may simply take a standard procedure or general-purpose algorithmic approach. While technology is not a replacement for human intelligence, it certainly reduces the uncertainty in cause-effect relationships involved in achieving the desired outcome. It is a systematic application of knowledge to solve problems (Huang et al., 2019). As such, it has many applications in education and is indispensable in learning and teaching (Pierce & Ball, 2009; ten Brummelhuis & Kuiper, 2008). Technology in education benefits not only researchers but also teachers, governments and funding agencies. Educational technology (EdTech) includes the use of hardware, software, digital content, data and information systems that support and enrich teaching and learning, improving education management and delivery.

In mathematics specifically, technology enables discovery and promotes the discovery method and experimentation. These advantages have encouraged the integration of technology in the mathematics education community and among policy makers (Lavicza, 2008). The extent to which technology is used in mathematics education differs from country to country, however, and even within countries its growth and use may vary greatly from place to place, for example from rural to urban areas. Reasons for this uneven adoption of EdTech include differences in policy initiation and infrastructure expansion. In Southeast Asia (Singapore and Malaysia, for example), the government has led the integration of technology (ICT) in the education system. In Malaysia, various types of technology or dynamic mathematical software such as Geometry's Sketch Pad, Autograph, and the Graphing Calculator have been integrated into secondary school mathematics (Bakar Ayub & Tarmizi, 2010). Although open software is still new to Malaysia, the internet is widely accessed, even by children. Tapscott (2009) found that children in technologically advanced countries naturally develop technological capabilities, are dependent on technology, regarding it as natural as breathing, and resist teaching that makes use of the old "telling" paradigm (Prensky, 2008). Because of this rise in the use of technology in education, educators need to integrate technology in their learning and teaching processes and use it as a tool to support the new teaching paradigm (Prensky, 2008). Scholars define the integration of technology or the use of technology in education in a variety of ways; there is no standard definition of the term (O'Dwyer, Russell, & Bebell, 2004). The integration of technology may be viewed either in terms of the use of computers in the teaching process or the presentation of teaching materials (O'Dwyer et al., 2004).

Dockstader (1999) defines the integration of technology in the classroom simply as a way of using computers effectively and efficiently in teaching and learning to enhance student learning. The present study argues that technology should form an integral part of the curriculum, not merely for the sake of integrating it, to support, learning. Most developed countries have adopted technology-based instruction to keep up with the ever-increasing demand for development and progress that characterizes the 21st century (Eyyam &Yaratan, 2014; Lasut, 2015).

Students who use technology can discover mathematical concepts, test their emerging mathematical understanding, both procedural and conceptual, experiment and visualize (Olive et al., 2010). In contrast, the use of GeoGebra affected learners' learning and positively affected the teacher's beliefs regarding teaching and learning even for those teachers found in high-poverty, rural settings where the availability of technological resources is limited (Mthethwa, Bayaga, Bossé & Williams, 2020). Lacey (2010) argues that a learner-centred classroom and the integration of technology (such as the use of a 3D printer) supports cognitive development, problem-solving, and active engagement by students in the learning process. Furthermore, modelling and simulation of a range of mental and natural processes become possible when using technology, and computer-based educational environments can provide context and support for meaningful problem-solving activities. This wider view of bridging the zone of proximal development is consistent with Vygotsky's emphasis on human-tool interaction. Despite this evidence, some studies have challenged the benefits-only view of technology integration in education (Mantiri, 2014).

Mantiri's study revealed some disadvantages and challenges associated with educational technology, including copyright issues, the dangers of dehumanized teaching and of breaching privacy and security. In this regard, Jaffee (1997) lists four valuable pedagogical principles and practices that should occur in the technologically integrated classroom, namely active learning, mediation, collaboration, and interactivity. Active learning involves students' interaction with the subject content in constructing knowledge. Mayer (2009) argues that learning is an enduring change in students' knowledge, attributable to their experience. Learning involves three simultaneous processes, namely acquisition, transformation and evaluation of activities (Bruner, 2006). In most descriptions of learning, the starting point of learning is the interaction of students with their environment (e.g., the scaffolding of students with the help of GeoGebra and the integration of new knowledge with existing knowledge. Jaffee (1997) emphasizes the imperative of active learning that students must do more than merely receive information. Barak, Lipson and Lerman (2006) found that the use of technology forces students to be engaged, motivated, and focused on activities in the classroom, activities in which they not only learn theoretical concepts but also practice hands-on programming. The advantages of technology in the mathematics classroom can be affected by students' confidence in using this technology. Galbraith and Haines (1998) found that students' confidence was a factor in the use of technology: those with low levels of confidence in using technology felt disadvantaged while those with high levels of confidence felt self-assured when using technology. To sum up, the use of technology in teaching and learning in the 21st century cannot be questioned, if it is properly implemented. Applying technology in the classroom requires a more active learning process (Barak et al., 2006; Jaffee, 1997); the engaged student in a classroom environment is a problem solver (Lacey, 2010); technology-oriented classrooms enhance students' learning (Dockstader, 1999; Nobre et al., 2016).

The most effective teaching method in the 21st century involves the effective integration of technology in the classroom (Pierson, 2001). Lastly, to integrate technology in the learning and teaching process, all partners in the education process need to understand the technology (e.g., government, teachers, students, parents, school leadership etc.). The implementation of technology in the classroom must be carefully planned (taking note of the available infrastructure), bearing in mind the criteria for technology implementation (Jaffee, 1997; Ruthven, 2009). It is not the technology itself that facilitates new knowledge and practice; it affords the development of tasks and processes that open new pathways to knowledge (Olive et al., 2010).

2.2 Types of Software in Mathematics Instruction

In this subsection, I discuss the usefulness of different types of free mathematics software applications, their application in learning and teaching mathematics and their value, particularly in teaching the topic of calculus. It is widely acknowledged that students benefit from the teaching of mathematics through technological means

(Dossey, McCrone, & Halvorsen, 2016; Heinich et al., 2002; Lavicza et al., 2019; NCTM, 2000; Pierce & Ball, 2009; ten Brummelhuis & Kuiper, 2008). Inayat and Hamid (2016) focus on the advances of technological tools such as Computer Algebra Systems (CAS) and Dynamic Geometry Systems (DGS), and the combination of the two packages in GeoGebra, in terms of their effectiveness in the teaching and learning of mathematics. They argue that such applications promote more effective learning in a student-centered and dynamic environment. They found that in mathematics, innovation in the teaching and learning process was shaped by modern digital technologies offered by web-based applications. A web image has been used to characterize this new way of teaching mathematics in the digital age.

Tall (2019), as discussed in conference proceedings mentioned above, together with several other researchers have shown that the use of computerized technology in Mathematics education has many advantages (Ayub et al., 2008; Ayub, 2008). Curriculum developers, educators and students all benefit from the advantages of educational technology, not least because students are attracted to this visually entertaining and interactive mode of learning. The introduction of technology in mathematics instruction elevates the level of motivation and affect displayed by students in science-related courses of study. Inayat and Hamid (2016) and Keong, Horani and Daniel (2005) found that technology-oriented mathematics education enhanced students' understanding of basic concepts. Interactive software can provide an immediate response to students' input, enables interaction and cooperation among students, improves skills, stimulates active participation and assists in the integration of theory and models (Inayat & Hamid, 2016). Two categories of mathematical software for educational purposes are prominent (Hohenwarter, Kreis & Lavicza, 2008; Inayat & Hamid, 2016): Computer algebra systems (CAS) software such as Derive, Mathematica, Maple and MuPAD; Dynamic geometry software (DGS) such as Geometer's Sketchpad, Cabri Geometry software. In each category, effective mathematics instruction tools can be found. Both CAS and DGS are essential for higher education, while DGS is also suitable for primary school since it features a mouse-driven user interface and is rich in visualization. Some software has been developed for specific applications and others have multipurpose applications; some are available free of cost and others must be purchased; some are area or country bound while others are globally available (Papp-Varga, 2008).

Papp-Varga (2008) observes that Graph is an open-source application suitable for teaching functions, which can be categorized under software with specific packages. Maple can be categorized as software with general packages as it works for almost all fields of mathematics. Some mathematics software such as GeoGebra, has been translated into several languages while others are restricted to one language. Most types of software can be installed on personal computers, notebooks, mobile cell phone devices and laptops. Some educational software packages for mathematics teaching and learning come at a cost in the market, and many students, teachers and schools cannot afford to buy them. Subsequently, free open-source software, readily available from the internet, is in high demand, especially in developing countries. Apart from interactive software applications, courseware and teaching materials are also available. Given the Ghanaian educational setting, the focus in this research study is on free open-source mathematical software suitable for teaching and learning calculus at the tertiary level. From the available applications in this category, namely GeoGebra, Wolfram Alpha and Desmos, I chose GeoGebra because it is user-friendly, time-saving, simple to use and easy to manipulate.

Any student can download the software onto his or her electronic device at no cost. This freeware is gaining popularity around the world for both educational and research purposes. Despite the obvious advantages of teaching technology integrated mathematics, many studies have revealed that the integration of technology into mathematics teaching has been slow when compared to the speed at which technology has evolved (Lavicza, 2010; Lavicza et al., 2019). Some teachers are fearful of integrating technology into their classrooms because their skills, knowledge and abilities may be overshadowed by those of their increasingly proficient 21st-century students (Lavicza et al., 2019). The digital age is accompanied by the imperative to conduct technology training for teachers; (Bekene, 2020) regards this as the first phase of the integration of technology into classrooms, as teacher professional development in the implementation of technology into teaching. Because of the development of GeoGebra, by 2001 the two types of mathematics software mentioned above had increased to three common types, known as DGS, CAS and a combination of the two. However, before 2001 DGS and CAS had not been linked in one program. GeoGebra mathematical software, developed in 2001, integrates the possibilities of both DGS and CAS in one program (Antohe, 2009; Dikovic, 2009; Hohenwarter & Jones, 2007). Kllogjeri and Shyti (2010) argue that GeoGebra software provides bidirectional representations, making it different from software developed previously. For example, GeoGebra makes it possible to write an algebraic equation in one window and the graph of the equation will be displayed in a graphic window.

2.3 GeoGebra Software in Teaching and Learning Mathematics

2.3.1 Characteristics of Geogebra

Among the multitude of mathematics software programs available in the global market, GeoGebra has gained exceptional popularity as a freely downloadable multi-stage dynamic mathematical software package. It was developed for educational purposes and its use spans all levels, from elementary to university level. It combines the functionality of CAS and DGS in one user friendly application (Hewson, 2009; Hohenwarter et al., 2008; Hohenwarter, Hohenwarter & Lavicza, 2009). Hewson (2009) points out that GeoGebra software is attractive both in terms of price and of the way it encourages collaboration in learning and teaching. For the end-user in the classroom and at home, there are no licensing concerns and after downloading, it can operate offline. Since its development by Markus Hohenwater in 2001, GeoGebra has built a user community in 190 countries and has been translated into 55 languages (Furner, 2020).

2.3.2 Components of Geogebra

The components of GeoGebra and their applications have been developed interestingly and appealingly. Akanmu, (2015) listed the elements of GeoGebra as, among others, menus, tools, views, input bar, tool bar, graphics window and algebra window.

2.3.3 Successful integration of GeoGebra in mathematics instruction

Notwithstanding the speed of technological developments in the 21st century, the majority of teachers do not find its integration in the classroom without difficulties (Ruthven et al., 2004). Teachers are aware that students need the motivation to tackle problems by themselves and to become involved in practical activities. These are the elements of constructivist theory, which is generally regarded as the most effective approach to mathematics teaching and learning. Based on a large body of research, educators are equally aware that the integration of technology in the classroom has the potential to give rise to motivation, interest and involvement. GeoGebra provides users with considerable opportunity to engage in true constructivist learning (Hohenwarter et al., 2008). The mere introduction of technology into the classroom does not necessarily affect the motivation, interest and involvement of students in mathematics, however. It requires training for educators to master the technology and its uses (Cuban, Kirkpatrick, & Peck, 2001; Ruthven & Hennessy, 2002). One of the indicators of successful integration of technology in the classroom occurs when students become cognitively involved in the mathematics they learn and do, using technological tools. Karadag and McDougall (2011) investigated the ability of GeoGebra software to provide activities that would involve students cognitively. They found that students created mathematical objects, that they were able to conceptualize ideas and to form relationships among these ideas. They were able to perceive mathematical objects in the physical environment and through social interaction and it became clear that technology was capable of introducing concepts to the working memory where they were systematically processed as integrated knowledge. From an educator's perspective, GeoGebra is a useful aid when creating mathematics tasks. These tasks include the preparation of teaching and learning materials such as test banks (which may reduce repetition of test items from year to year), module preparation, progress tests and summaries, in both technological and traditionally oriented situations.

In the technologically oriented classroom, the use of dynamic software such as GeoGebra accelerates these tasks (Jaffee, 1977; Ruthven, 2009), while allowing best teaching and learning practices to be maintained. GeoGebra can create precise figures that can be manipulated both in the classroom and at home. The literature reveals that GeoGebra creates an atmosphere conducive to the learning of mathematics, in the sense that it stimulates creative thinking and promotes a problem-solving orientation (Selvy, Ikhsan, Johar, & Saminan, 2020; Žilinskiene & Demirbilek, 2015; Zulnaidi & Zamri, 2017). The software is simple to use, which helps to reduce the teacher's role to that of a knowledgeable guide, while students take on an active role by doing tasks by themselves, only calling for help when they find activities difficult. Fahlberg-Stojanovska and Stojanovska (2009) found that learning mathematics through technology motivates students to engage in the process of searching for solutions at a higher level, not only in finding the solutions. A further advantage of GeoGebra mathematics software is that it helps students to learn calculus by simultaneously displaying the answer of a task in the algebra view window with its visual representation in the dynamic geometry view. This dual-mode of representation facilitates the making of connections and relationships, a prerequisite for high-level mental functioning. However, GeoGebra not only unlocks higher-level thinking; it also enables mathematical thinking at all developmental levels. This allows teachers to explore students' potential in mathematics and to unlock their skills (Aydin & Monaghan, 2011). GeoGebra mathematical software is a cloud-based service and like Office 365 it offers online data processing and self-actualization of certain actions (Semenikhina, Drushlyak, Bondarenko, Kondratiuk & Dehtiarova, 2019). It also has great potential in an e-learning environment (Albano & Iacono, 2018; Antohe, 2009; Dikovic, 2009; Gülseçen, Reis, Kabaca & Kartal, 2010). There is evidence (Zulnaidi, Oktavika, & Hidayat, 2019) that the use of technology in general, and GeoGebra in particular (Zengin, Furkan, & Kutluca, 2012) enhances students' achievement. Zengin et al. (2012) investigated the effect of GeoGebra software on students' achievement in trigonometry by using a control group that received constructivist instruction and an experimental group whose instruction included the use of GeoGebra software. They found a significant difference in achievement between the experimental group and the control group; those using GeoGebra achieved significantly better than those who were taught without GeoGebra. GeoGebra has several innovative functions that empower the user when tackling complex tasks. Diković (2009) lists some of the uses of GeoGebra as follows:

GeoGebra is a calculator of graph functions. GeoGebra enables the student to sketch the graph of a simple linear function not only in one variable, such as $(x)=3x+5$, but also in two variables, such as $f(x,y)=\frac{x^2+y^2}{x-y}$. In Figure2.2 these functions are displayed in a single GeoGebra window.

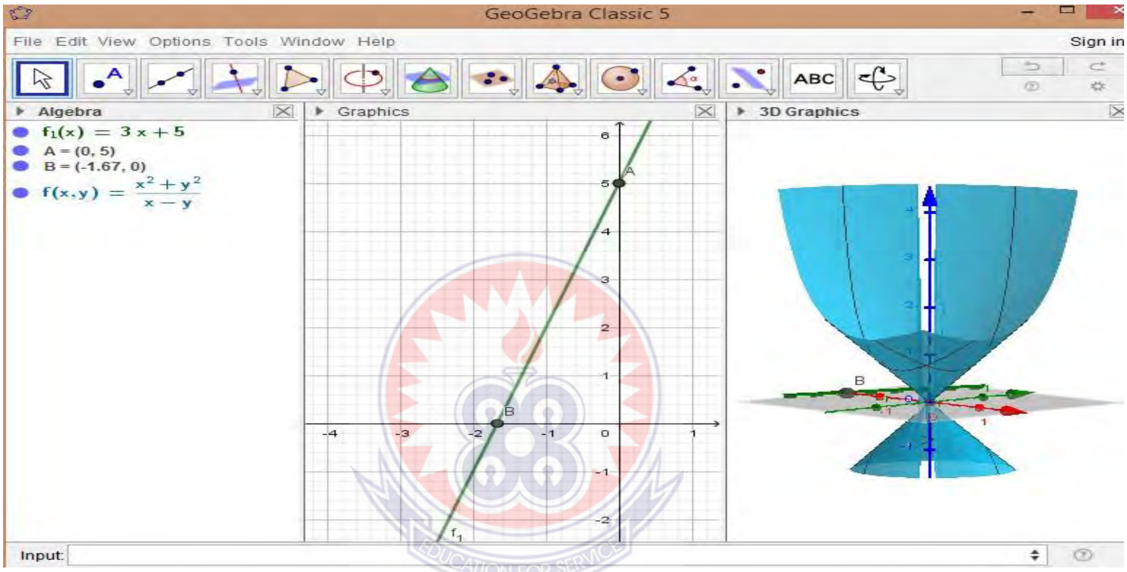


Figure 2.2: The graphs of functions in one and two variables in a GeoGebra window

GeoGebra can be used for investigation projects. Students can apply GeoGebra to their investigations since it allows experimentation with various representations of a mathematical idea, it is visually rich and it promotes a problem-solving disposition, also termed heuristics (Bruner, 2006). Because of these multiple representations, connections between and among various mathematical ideas can be made. In the case illustrated in Figure 2.2, the CAS window represents the two functions symbolically and the DGS windows represent the graphs of the two functions visually. Students can experiment with these multiple representations by adding, changing, and manipulating input elements at will.

GeoGebra enables original creations. Students can personalize a graphed function in the GeoGebra interface by changing the language and display elements such as font type, size and color, the coordinates of axes, the thickness of lines and line styles. GeoGebra simplifies the understanding of mathematical concepts. A complex and abstract idea, such as the real function f=xsin $\left(\frac{1}{x}\right): R/\{0\} \to R$ approaching zero, once mapped and visualized, may be easier to conceptualize, as illustrated in Figure 2.3 below.

When studying the properties of this function near zero, GeoGebra provides a visual conceptualization of the limits of the function of f at zero.

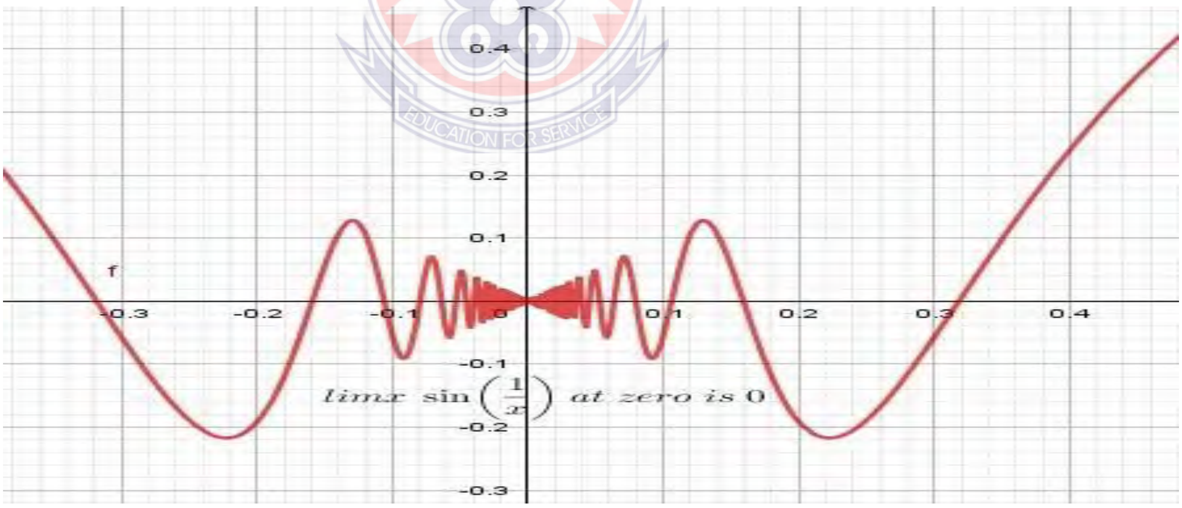


Figure 2.3: The graph of a real function approaching zero

GeoGebra enables cooperative learning and teaching. Mathematical problem solving can be approached cooperatively, in pairs, small groups, or by the whole class. It can be used in individual student presentations and in group presentations, as well as when teaching mathematical modelling. Within a pleasant, interactive and friendly environment, students tend to participate actively, cooperating and collaborating. All this enhances the understanding of the problem and the development of problem-solving strategies (Pasco & Roble, 2020). e. GeoGebra enables the generation of mathematical objects. The GeoGebra software allows students to create new graphs or edit existing ones. It allows the user to easily publish a worksheet as a Web page and in so doing, make online e-learning possible in a virtual classroom. Apart from uploading the activities of students together with their sketched figures onto the GeoGebra platform, students can access activities created by their teacher by simply clicking on the link and using the password to enter the virtual classroom. Diković (2009) found that GeoGebra encouraged not only the students but also their teachers to use the software in their classrooms, whether they were conventional or virtual.

2.4 Students' Proficiency in Conceptual and Procedural Knowledge in Mathematics Education

2.4.1. History of mathematics education and mathematical proficiencies

Learning theories influenced the evolution of mathematics education. These theories can be classified under the two umbrella terms, behaviorism and constructivism. The method of transmission of mathematics knowledge differs in these theories of learning. Behaviorism combines explicit teaching and direct instruction as a method of knowledge transmission, sometimes known as a traditional teaching method, while in constructivism knowledge is constructed when it was imposed or integrated into existing knowledge. This is known as the active teaching method (Hechter, 2020). The procedural-formalist paradigm and the cognitive-cultural paradigm are two paradigms in the history of mathematics education; the procedural-formalist paradigm is built on behaviorist foundations [transmission of knowledge] and cognitive-cultural paradigm is built on the foundation of socio-constructivism [promotes the active role of the student and improvement of conceptual understanding through reflection and shared experiences] (Ellis & Berry III, 2005). Vygotsky believed that knowledge was made within the process of communication and interaction with others and that scaffolding would lead to the storage of information in the mind and used by the students in the environment (base of the developed cycle model (Vygotsky, 1978). Mathematics is considered to be a difficult subject (Kinnari, 2010). The reviewed literature argues that the transmission of mathematical knowledge in the classroom may be facilitated by the use of h technology, as in this study with the use of GeoGebra together with traditional/conventional methods that are "embedded in culture, human experience and social interaction''(Hechter, 2020). Within the classroom, mathematics tasks activities commonly take place while social interaction occurs. Tasks are the basis of students' learning in the classroom (Stein & Smith, 2011). Tasks that need students to recall step in a monotonous manner lead to one type of student thinking [multiple choice question types]; tasks that require students to think theoretically and that encourage students to make connections lead students to different ways of thinking [working out problem questions]. Students who have difficulty linking the statistical words or calculations with their graphical, tabular or other representations may improve with the help of technology (Ocal, 2017). In general, in mathematics the recognition of students' starting level (proficiency) by using a pre-test of differential calculus containing the tasks of two types of knowledge (conceptual and procedural) can lay the foundation for successful learning situations in the environment (Kinnari, 2010). The word proficiency in this study refers to students' fluency in both types of knowledge (conceptual and procedural understanding) that can be discretely measured, quantified, and stratified using the tasks of differential calculus before and after intervention (Ellis & Berry III, 2005; Kilpatrick, 2001). The term mathematical proficiency has been referred to as mathematical literacy by Kilpatrick (2001) who posits five strands of mathematical proficiency. These are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. This study concentrated on the two types of knowledge known as conceptual understanding, and procedural understanding.

Conceptual understanding: students' grasp of mathematical concepts, operations, and relations, that is knowledge that students understand.

Procedural understanding: students' skills that they use to follow mathematical procedures, and whether they use them flexibly, accurately, efficiently and appropriately.

2.4.2 Developing tasks for mathematics proficiency

If mathematics is dealt with in the classroom as a priori knowledge, based on objective reasoning alone, without taking the experiences of students with mathematics or the meaning they make of what they have learned, this can be taken into account by pre and post-test which allows teachers to identify student's mathematics achievement by discretely measured, quantified, and stratified the delivered pre and post-test (Ellis & Berry III, 2005). Hence, the activities given to the students may be developed depending on the concepts of the two types of understanding – conceptual and procedural understanding. Conceptual understanding refers to an integrated and functional grasp of mathematical ideas that allows students to reconnect with the designed tasks (Kilpatrick, Swafford, & Findell, 2001). Proficiency in representational activities demands conceptual understanding of the mathematical concepts involved (definition of limits, derivatives, etc.), the operations (addition, subtraction, division and multiplication) and the relations (the combination of concepts such as the relation between natural exponentials and logarithms [$e^{\ln x}=x$]). It also requires strategic competence to formulate and represent that information. Hence, the conceptual tasks require the ability to recall or connect to previous knowledge. In calculus, we know that $\lim_{X\to 0}\left(\frac{\sin X}{X}\right) = 1$,but if the task for students is given to evaluate $\lim_{X\to 0}\left(\frac{\sin X}{5X}\right)$ students need to connect the previous knowledge of basic limits to the given rule for the building of students' prior knowledge (Sumartini & Maryati, 2021). Once students have conceptualized the rule they can simply recall answers. Sumartini and Maryati (2021) suggest two measurements of conceptual understanding, implicit and explicit measures. These measurements of conceptual understanding are implicit measures and relate to evaluations where one makes definitive choices, ranks quality, and compares numbers; explicit measures, on the other hand relate to definitions and explanations. The factors that hinder the recalling or reconnecting of students to previous knowledge to new knowledge occur in the classroom and these conditions should be identified by the teachers (Stein & Smith, 2011). Factors that are associated with making connections include:• Scaffolding of student activities. • Students' own exploration. • Teachers or capable students modelling high-level performance. • Teachers providing activities [questioning, comments, and feedback]. • Tasks developed based on students' prior knowledge. • Teachers making frequent connections in conceptual tasks. • Sufficient time for exploration.

2.4.3 Procedural understanding

The procedure is the knowledge that shows the order or sequence of actions for comprehensive learning of all the components (Zulnaidi & Zamri, 2017). They elaborated procedural understanding by examples of questions asking students to solve the function equation of $(x) = x^2 + 1$; to determine the formula of inverse function f^-

$^{1}(x)$ and to graph the functions. According to the question, students are required to find the formula of an inverse function. In this case, students need to recall the ways how to find inverse functions such as:

Step one: Letting $y = (x) = x^2 + 1$

Step two: Interchange the variables x with y that is $x = y^2+1$

Step three: Solve for y variables that is $y = \pm\sqrt{x} — 1$

Step four: Set $y = f^{-1}(x)$

Here we understand that to arrive at the required formula students need to know these steps or procedures. Thus, in this study procedural tests are tests that require step by step activities to arrive at the answers. Procedural understanding is the knowledge of procedures, when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently (Kilpatrick et al., 2001). These two types of knowledge were also discussed in the framework of this study in general, even if procedural or conceptual tasks/tests are presented for the students' things to be considered in measuring student's cognitive level should be considered.

2.5 Students' Perceptions of Geogebra in Learning Mathematics

I report on the literature about the perceptions of students about using GeoGebra, particularly when learning differential calculus (DC). We all have our own unique set of perceptions, attitudes, and behaviors (Aiken, 2002). Attitudes and perceptions are closely related (Pickens, 2005) and in this study, I use the terms interchangeably. The process by which people interpret and organize sensations to produce a meaningful experience of the world is known as perception (Pickens, 2005). Many scholars believe that students' attitudes to mathematics are formed by ''social forces'' (Singh, Granville, & Dika, 2002). One of these forces may be technology, including software such as

GeoGebra. If students are not sufficiently prepared to use technology in learning, this may affect the impact of technology-integrated instruction and students' perceptions of their ability to solve complex mathematical problems (Moos & Azevedo, 2009). Singh et al. (2002) argue that negative attitudes of parents and teachers affect students' attitudes towards their own abilities and interests; however, programs such as GeoGebra mathematical software may restore their motivation and improve their perceptions of their own achievements (Doğan & Içel, 2011). Shadaan (2013) investigated students' perceptions of GeoGebra in learning circle geometry and found the integration of GeoGebra in their teaching and learning produced a significant improvement in their level of thinking, creativity, critical reasoning and logical assumptions. It is the role of the teacher to determine the learning outcomes of students when using technology in teaching and learning mathematics (Smith, 2002). Leder, Pehkonen and Törner (2002) emphasize that students' beliefs about the social setting within which they learn have a decisive influence on problem-solving behavior, particularly on the affective aspects of learning (such as emotional reactions to class activities). If students experience positive emotions when using mathematical tools/software, studies have revealed that this has a decisive effect on their studies. Arbain and Shukor's (2015) study, found that students in the experimental group that had been taught using GeoGebra not only had positive perceptions towards learning mathematics but also performed better than students in the control group who had been taught by traditional methods.

2.6 Teaching and Learning Differential Calculus

2.6.1 Brief overview of studies on differential calculus (DC)

Various conceptualizations of calculus exist, as follows: Calculus is a branch of mathematics that deals with quantities approaching other quantities (Charles-Ogan &

Ibibo, 2018). Calculus is a branch of mathematics that deals with how a change in one variable is related to changes in other variables (Nobre et al., 2016). Tall (2009) describes a calculus course of study as the desire to quantify and express: How things change (the function concept), the rate at which things change (the derivative of functions), how they accumulate (the integral of functions); and the relationship between the two (the fundamental theorem of calculus and the solution of differential equations). Standard terminology in calculus includes the terms limits, derivatives and integrations of functions, while the main terms in differential calculus are limits and derivatives of functions. The big ideas in calculus are limits, derivatives, integrals and fundamental theorem, while the idea of series also features in the generalization of calculus, mathematical analysis (Tall, 2019). Arango et al. (2015) argue that traditionally explaining differential calculus can be dry and off-putting for students; they believe that the use of technology may render explanations more fruitful. As technology continues to develop at an astonishing pace, teaching and learning calculus becomes more possible and accessible. Technology has migrated from large mainframes to portable desktop computers, calculators, laptops and notebooks, while manual input of data, arithmetic and the subsequent creation of graphs have been replaced by automated calculations and graphs. This makes technology available anywhere, anytime. A study conducted in Brazil by Nobre et al. (2016) found that calculus (and the way it was taught) was the primary cause of failure among college and university students. Traditionally, students experience calculus as difficult, hard to understand and daunting; innovative methods and approaches are needed to make teaching and learning of calculus more effective (Charles-Ogan & Ibibo, 2018; Lasut, 2015). As early as the end of the last century, Rochowicz (1996) identified calculus as the subject that prevents many students from completing courses in science and engineering. According to his research, the calculus curriculum was outdated (even then) and needed to be revised to align with a technologically oriented educational curriculum. The rapid growth of technology in the 21st century is ongoing, and studies on the effect of combining technology and calculus instruction have also increased. Tall et al. (2008) identify the dynamic nature of both technology and calculus as the reason for this increased interest in such research. Recognizing the importance of calculus as the backbone of many science courses, other scholars (Durán et al., 2014; Lavicza, 2010; Mignotte, 1992; Ozguiin-Koca, 2010; Robutti, 2010) have argued that technology has the potential to simplify complex calculus concepts and is gaining ground as a research interest. The potential of technology in education to promote constructivist instruction is particularly appealing. Huang et al. (2019) list the characteristics of constructivist learning as follows: Instruction is student-centered, Learners actively construct internal psychological representations, Learning comprises the reorganization and reconstruction of old knowledge and the meaningful construction of new knowledge, Learning is not only individualized, but involves language centered social interaction, communication, and cooperation, Learning must be situationally embedded to support meaningful learning and the construction of meaning requires appropriate resources.

2.6.2 Learning differential calculus using GeoGebra software

The use of technology in teaching calculus stimulates student participation and motivation by relating subject content and concepts to visualization and experimentation (Nobre et al., 2016). GeoGebra mathematical software provides significant opportunities for meaningful learning and concept formation in calculus, geometry and algebra at various levels (Tatar, 2013). Ocal (2017) investigated the effect of GeoGebra on applications of derivatives in two calculus classrooms (experimental and control) involving 55 students. Students' conceptual understanding and scores both improved; however, there was no significant difference between the procedural knowledge of the experimental group and the control group. The National Research Council (2001) argues that conceptual and procedural knowledge of mathematics are interrelated components, with the first (conceptual understanding) taking the central position. Procedural fluency can be affected by basic instructional routines and by following steps, algorithms and methods or strategies of calculation and the application of formulae and rules. In the GeoGebra software-based mathematics classroom, the main task of the teacher is to guide students' work, as the software enables students to explore and discover mathematics concepts by themselves (Preiner, 2008). This idea is consistent with Vygotsky's classical cognitive constructivist theory. Preiner (2008) found that the simple way in which developers of GeoGebra designed the user interface of the software aligns with the characteristics of cognitive constructivism, particularly its visualizing and explorative capabilities, its contribution to multimedia environments for learning and the minimization of cognitive load in learning. Multimedia environments offer new ways of learning and teaching compared to traditional environments (Preiner, 2008). Akanmu (2015) agrees that technology, well-integrated into mathematics education, enhances students' achievements, "irrespective of gender" (Akanmu, 2015). In summary, studies on the integration of GeoGebra in differential calculus have found positive effects on student performance (Akanmu, 2015; Nobre et al., 2016; Ocal, 2017; Preiner, 2008; Tatar, 2013).

2.6.3 Beliefs about learning and teaching mathematics through technology

Several qualitative studies of teachers' and students' beliefs about mathematics learning with the use of technology, particularly GeoGebra, have been conducted. Teachers' beliefs about technology-oriented mathematics classrooms Teachers' perceptions of effective teaching and their cultural beliefs may influence their instructional practices; these beliefs mustn't widen the gap between theory and practice (Purnomo, Suryadi, & Darwis, 2016). There is no uniform definition of the term teacher beliefs in the literature. Ertmer (2006) defines beliefs as suppositions, commitments or ideologies. Variations in teachers' cultural belief systems influence how they view their students, what mathematics should be learned, and how this should be taught (Tirosh & Graeber, 2003). Galbraith and Haines (1998) view beliefs as a way of imitating a certain set of concepts, while attitudes are an emotional reaction to an object, to beliefs about an object, or behavior towards the object such as technology. They view emotion as heated or agitated arousal created by some stimulus. In their review of articles, they found that understanding students' attitudes and beliefs about learning is a crucial step in understanding how the mathematics learning environment is affected by the introduction of computers and other technology to the classroom. Ernest (1989) identified three main components of teachers' mathematical beliefs: the conception of the nature of mathematics as the basis of the philosophy of mathematics; the structures of mathematics teaching; and the process of learning mathematics. The conception of the nature of mathematics is fundamental as it has an impact on the structure of mathematics teaching and the process of learning mathematics (Speer, 2005; Thompson, 1992). Ernest (1989) reasons that the restructuring of teaching cannot take place unless teachers' beliefs about mathematics, its teaching structure and its learning process change. In general, teachers' beliefs are regarded as critical to the restructuring of mathematics education (Cooney & Shealy, 1997; Leder et al., 2002; Thompson, 1992). In particular, teachers' beliefs towards technology in the classroom, their beliefs about the potential of their students and teaching mathematics have a decisive impact on the success or failure of the implementation of technology (Windschitl & Sahl, 2002). Teachers who believe in the potential of instructional technology are catalysts for the transformation of teaching mathematics with technology. Teachers believe that the integration of GeoGebra in their classrooms is time-saving when preparing worksheets, tests, lecture notes and board work. Prepared work can be stored on a web page or the GeoGebra software; teachers can simply change the variables of the object to create a new set of instructional materials. Interactive lectures can also be created using GeoGebra, and can be uploaded on the internet (Hohenwarter et al., 2008). Zakaria and Lee (2012) found that teachers were positive perceptions about the use of GeoGebra as far as its features, tools and commands were concerned. In a quantitative survey, these researchers concluded that technology can be used as an alternative method in mathematics instruction. Tatar (2013) used a mixed-methods approach to investigate the effect of technology, in particular GeoGebra, on teacher perceptions and arrived at the same positive conclusion. Although educational technology is undeniably beneficial and positively perceived by teachers, Pierce and Ball (2009) found that a lack of time, skills and confidence may hinder its implementation in the classroom. They suggest ways for smooth implementation to overcome such barriers.

2.6.4 Students' beliefs about technology-oriented mathematics learning

Leder et al. (2002) explain that students' beliefs about mathematics are "implicitly or explicitly held subjective conceptions" that they believe to be true and "that influence their mathematical learning and problem solving''. Thompson (1992) states that although the term belief is not clearly defined, it is assumed that the reader knows what is meant in context. In this study, I use the term concerning students' and teachers' perceptions about the use of technology when learning calculus. Students' attitudes towards mathematics can be affected by technology. Akanmu (2015) found that Nigerian students' attitudes towards mathematics could be linked in a significant way to their knowledge of GeoGebra. Factors that could influence students' attitudes towards the use of GeoGebra include their attitude towards learning mathematics and their knowledge of the technology they will be using to master mathematical concepts (Anthony & Walshaw, 2007; Kele & Sharma, 2014). Anthony and Walshaw (2007) regard students' attitudes towards technology as a central concern when evaluating the impact of technologies on mathematics learning. Kele and Sharma (2014) found both negative and positive mathematical beliefs among the students in their study and concluded that teachers needed to develop or use new instructional approaches in mathematics instruction to encourage a positive disposition towards mathematics in all students. Mwei, Wando and Too (2012) noted that the majority of students developed constructivist learning strategies when exposed to computer assisted instruction (CAI). Han and Carpenter (2014) define beliefs about mathematics as the cognitive component of attitude, while feelings (emotions) about mathematics comprise the affective component of attitude (Akinsola & Olowojaiye, 2008). Behavioral responses are the observable elements of attitude that students display when dealing with mathematics (Ingram, 2015). Cognitive and affective components of attitude interact with each other and are both important in learning mathematics (Di Martino & Zan, 2007). Student responses to mathematics instruction, i.e., their mathematical behavior, are the overt expression of the cognitive and affective elements of attitude (Akinsola & Olowojaiye, 2008). Unsuccessful behavioral attitudes such as negative feelings manifest when students are not confident about mathematics (Di Martino & Zan, 2007). As soon as students observe the importance and value of mathematics in real life, however, they start to engage in learning, gaining confidence and becoming connected (Attard, 2012). Students' beliefs about mathematics influence their achievements, and the cognitive, emotive and behavioral aspects of attitude are intertwined. This holds also for students' cognition, affect and behavior as far as mathematical software is concerned. Guiding and scaffolding the effective use of the latest technology in mathematics learning helps students to solve mathematical problems with greater ease (Oldknow, Taylor, & Tetlow, 2010). This holds for the complex mathematical topic of calculus (Ayub et al., 2010), as reflected in the improved performance of students who learned calculus through the aid of technology. Two programs, Mastering Calculus Computer Courseware (MACCC) and SAGE software were investigated and their effect on the learning of calculus; however no significant difference in student performance was detected (Ayub, Sembok, & Luan, 2008). Hew and Brush (2007) list some barriers that affect the teaching and learning of mathematics through technology, including a lack of resources, negative attitudes and beliefs, institutional restrictions, the complexity of the subject and variations in culture, knowledge and skills. Complex mathematical tasks such as visualizing a 3D graph may be difficult for the teacher to demonstrate manually; this is simplified by using technology. Bos (2007) found a better understanding of such concepts among students who used technology than among those not using technology. It may be that such improvements result in altered beliefs about mathematics.

2.7 Theoretical Framework of the Study

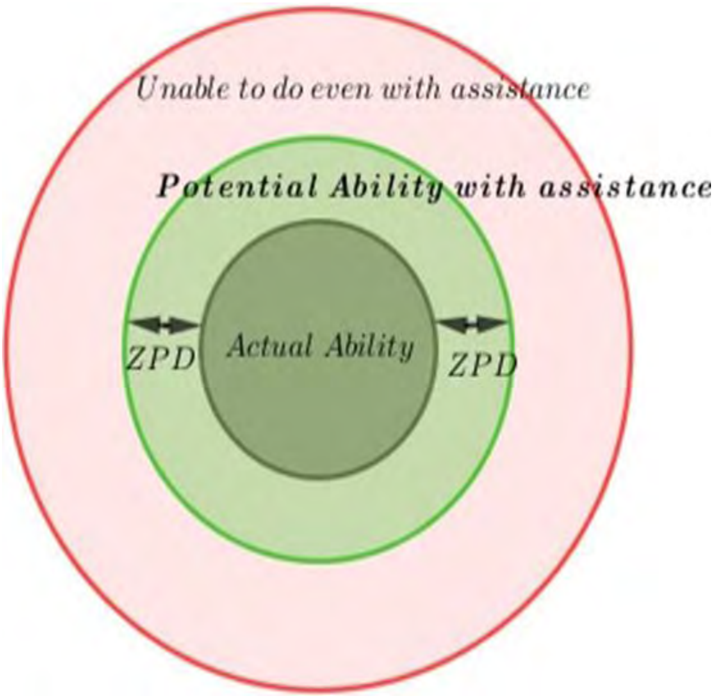
The theoretical framework of this study is grounded in Vygotsky's theory of learning and extends to the Zone of Proximal Development. The study outlines how communication between students and technology, between students and their peers (among students), and students and teachers (more knowledgeable adults) affect the learning of Mathematics, particularly calculus, with GeoGebra software. Using this theoretical framework, the researcher developed a model to observe how using GeoGebra/Technology in differential calculus relates to Vygotsky's ideas of the Zone of Proximal Development (ZPD) and scaffolding patterns (Wood, Bruner, & Ross, 1976).

2.8 Overview of Vygotsky's Theory of Learning

Social interaction plays a significant role in student learning both at school and in the wider environment. In this learning landscape, learning occurs both at school from the teacher or instructor and other people in the environment (world), that is from the human to the world of the object/technology (Lantolf & Appel, 1994). Learning is a social activity in which the engagement of students in learning takes place through their use of active cognitive and metacognitive knowledge and strategies (Leder, Pehkonen, & Torner, 2002). Metacognition encompasses student's self-regulation, self-determination, self-planning, and self-checking once they have received guidance (Daniels, 2001). In this space, learning can occur in social and cultural scenarios. The culture of a society provides students with the knowledge, as discussed in the literature on mathematics culture, and this affects student achievement. To this end, the sociocultural environment or milieu is the central idea in Vygotsky's theory of learning. His theory of cognitive development is based on a child's ability to learn things socially with the tools at hand (hands, hammers, computers) and to learn the culturally-based signs (language, writing, Mathematics). Morcom (2014) discusses learning, emotion, and motivation, all of which are central and interconnected processes in Vygotsky's sociocultural theory. According to this theory, students' and teachers' perceptions of the learning and teaching culture of Mathematics can be considered overtly or covertly. The famous scholar Wertsch (1985) views Vygotsky's theory as having three core themes. These are a) reliance on genetics, that is, developmental methods; b) the claim that higher mental functions of the individual have their origin in social processes; and c) the claim that mental processes can be understood only as mediated by signs and tools. In my study, Wertsch's (1985) third theme in particular led me to focus on Vygotsky's theory. Minick's (1987, as cited in Daniel, 2001) study of Wertsch's ideas on Vygotsky's theory revealed that Vygotsky's thinking moved from a focus on instrumental acts in 1925 to the analytic unit of psychological systems in 1930, and the modification of the descriptive principle from 1933 to 1937, with an emphasis on interactions and actions in individual participation. As cited in Daniels (2001), Vygotsky argues that humans master themselves when actions come from the external symbolic, cultural system, rather than by being conquered by and in them. From my understanding of the literature and Vygotsky's beliefs, learning takes place when an individual receives assistance from educated persons. From an educational point of view, Vygotsky argues that psychology reveals that the human mind is developed through the interactions of subjects with the world as well as the quality of the relationship between students, subject matter and tools (technology) in the classroom (Vygotsky, 1978). In this light, I used students as the subject and GeoGebra Mathematical software as the object to observe the interaction between these phenomena on students learning differential calculus. Modern technology or digital technology enhances human abilities to learn, especially in the subject of calculus, and emphasizes the interaction of technology and humans that enables them to increase their capacity to process expressions numerically, manipulate symbolically, create new theories and represent ideas visually (Tall, 2013). In general, according to Vygotsky, the method reveals the human mind's potential for future development to address the challenges of this century. Vygotskian theory developed from the Piagetian theory (Piaget, 1959). Since I followed Vygotsky's theory in this study, I do not discuss Piaget's theory in detail, however. In Vygotsky's view, teachers' interaction with students is the most important factor of the learning process (Vygotsky 1984), whereas in Piaget's view the teacher's interaction with students is a secondary factor. The reason I follow Vygotsky rather than Piaget is that I believe that the teacher must interact with students in the first phase as the students are new to the tools GeoGebra, as social interaction is key to cognitive development, as emphasized by Vygotsky. Lantolf and Appel (1994) regard development as a mediated mental activity. However, in Vygotsky's theory, students and teachers are all active mediators in the process of students' development, that is in teaching and learning. In the teaching and learning process, teachers' intervention in students' learning is essential. Social constructivism emphasizes the quality of importance of teacher-student communication in learning (Gallimore & Tharp, 1988). Social constructivism posits those ideas are constructed by the interaction between teachers and other students; in contrast, cognitive constructivism holds that ideas are created individually. In social constructivist theory, the learning process is cooperative, and knowledge is created not only through the interaction of teacher and student in the environment (psychologically), but also by students themselves (intra psychologically) (Churcher, Downs, & Tewksbury, 2014). Lastly, effective learning occurs in a particular place, according to Vygotsky's theory. A place at which 'good learning' occurs is the Zone of Proximal Development (Vygotsky,1978). In response to this notion, another scholar Doolittle (1995) found that not only learning but also cognitive development of students occurs in the zone. Vygotsky (1978) argues that students' intellectual and problem-solving abilities fall into three categories: 1) those that are performed independently (lower level), 2) performed with assistance (higher level), and 3) cannot be performed even with assistance. Those that cannot be performed even with assistance are those found to be beyond the ZPD (outside the concentric circles). This study considers students' intellectual and problem-solving abilities in activities that can be performed with assistance. Assistance, in this case, refers to providing hints and directions, rephrasing questions, modelling, asking the student to restate what has been discussed, or asking what he/she understands or has learned, or demonstrating the task or a portion of it (Bodrova & Leong, 2007; Jones, Rua, & Carter, 1998). All these activities can occur in the classroom in effective teaching and learning processes. In this study, the researcher guided students who were learning calculus with the tools in GeoGebra software. In general, studies have shown that teachers and students are mediators in the mathematics classroom; adding technology as a tool can lead to the development of a better understanding of mathematical concepts in interactions between individuals (peers), and between students and teachers (MKO). Daniels (2001) found that every meaning in a child's cultural development occurs at the social level (interpersonal) and the individual level (intrapersonal); this supports the arguments for the importance of the zone of proximal development.

2.9 Zone of Proximal Development and Learning Mathematics by using Technology

Vygotsky (1896-1934), a Russian psychologist, first introduced the term zone of proximal development (ZPD) in the 1930s. He defined this as the difference between what a student can do without assistance and what he or she can do with assistance (Vygotsky, 1978), and depends greatly on the" more knowledgeable other'' (MKO). The MKO is defined as an essential component of the learning process and is a teacher or lecturer who has more knowledge than his or her students (Vygotsky,1978). The zone of proximal (potential) development (ZPD) is the gap between what a student can do independently and what he or she could potentially do with support (guidance) and assistance (Daniels, 2001). Vygotsky (1984) divided ZPD into two categories of intellect: actual intellect, the distance between the actual developmental level as indicated by independent problem-solving ability, and potential intellect, indicated by problem-solving ability with adult guidance or in collaboration with more capable peers. In the present study, the more capable peers were the teachers. Students in the ZPD zone can be successful with instructional guidance (Blake & Pope, 2008). Literature reveals that the ZPD represents a maturation process. If students are nurtured properly, they will grow. Doolittle (1995) and Warford (2011) argue that social interaction in the learning process is at the heart of ZPD. Bodrova and Leong (2007) argue that the ZPD is not a static region but rather an active region of learning in which students develop experiences through participation. In this sensitive region, students learn cultural skills. This study proposed that one of these cultural skills could be mathematical skills. The ZPD is important for the learning and teaching of mathematics because it determines the scope of work to be covered. Gallimore and Tharp (1990) claim that the ZPD has four stages. In their study they identified these as stage 1: in this stage modifying for transfer, assistance and task performance is applied; stage 2: this is the stage where performance is monitored by self or assisted by self although the learner has not yet automatized the activity; stage 3: this is where the performance is automatized, fossilized and developed; and stage 4: this is the point at which the automation of performance leads to recursion through the ZPD.



2.10 Scaffolding in Teaching Mathematics by GeoGebra

The literature review revealed some agreement on the notion of a socio-cultural theory of the mind. In this respect, ZPD is based on Vygotsky's theory, at the heart of which is the notion of scaffolding or guidance. The term scaffolding was first introduced in the context of teaching and learning by Wood et al. (1976), who define scaffolding as a form of adult assistance that helps learners achieve a goal that they would not be able to do on their own. Doing difficult tasks, setting appropriate goals, and guiding students in the classroom are tasks of the teacher in the ZPD. Scaffolding may take several forms, including "increasing engagement, providing alternate learning strategies, resolving learning bottlenecks, and (paradoxically) taking away support to allow students to master the material" (Lee, 2014). Technology can scaffold student learning before, during and after class to provide appropriate assistance to students. Pea (2004) believes that there are two primary axes to support the processes of learning in the classroom. The first axis depends on students' needs and the resources that enable them to do more than they would do alone. This axis is social and involves interactive responses. Ruthven (2009) also used resources as classroom practice. The second axis comprises technology and the design of artefacts (Simon, 1996) and focuses on problem-solving. Theory building and design in education can encompass scaffolding (Quintana et al., 2004); technology supports learning and teaching and has become increasingly important in pedagogical design. For example, to demonstrate GeoGebra in teaching calculus in the classroom, I prepared a lesson plan that was compatible with this software. This led to a dramatic shift from a lecturer centered lesson plan to a technology-oriented lesson plan. This change in pedagogical design may be necessary if this software is to be applied to all chapters of a calculus textbook. Several scholars Quintana (2004) have found that software tools support students by simplifying their learning, and this, in turn, encourages their engagement in learning. Technology can support multiple methods of studying the same material and can provide visual scaffolds that help students to understand complex concepts. Providing direction to their study and showing students how to do activities can be regarded as scaffolding. Such scaffolding can gradually be withdrawn over some time. Scaffolding should be seen as temporary assisted learning in certain activities that leads to independence; the result will be that students may become self-governing and problem-solvers in their own activities (Lajoie, 2005). If the task is accomplished, then the scaffolding is slowly withdrawn. This dynamic system is recognized by both teacher and student. One of the best technologies to use in the teaching and learning of calculus is the GeoGebra Mathematical software as the software is dynamic. This program provides scaffolding by guiding and assisting students in their learning activities in the classroom. The dynamic system comprises three ideas that are important in defining the system. These are contingency, fading and responsibility. In my study I used the notion of contingency, which involves modifying and customizing the teaching lesson plan according to students' abilities, taking into account the students' calculus syllabus which was designed according to the Ghanaian higher education programmed. Pea (2004) defines the notion of scaffolding by listing questions to ask, such as What, Why and How, when determining which individuals require scaffolding. In the present study' What and Why questions were used to identify students who needed assistance and How questions were used to determine the type of scaffolding, such as guiding, focusing and modelling of activities. Instructional scaffolding is a mechanism for observing the process by which a student is helped to achieve his or her potential learning in education by a potential teacher (Stone, 1998). Vygotsky believes that with appropriate assistance in the ZPD, students will be able to move from the present zone of proximal development to the actual developmental level in the future (Vygotsky, 1978). In this case, assisting students by using a given technology within a given period and then stopping the guidance coincides with Vygotsky's ideas; students should master calculus by the use of GeoGebra, with some guidance. Social interactions play an important role in learning and teaching, for both students and teachers. The social and participatory landscape of teaching and learning in education can be explained by scaffolding and this term is used as a metaphor for educators and researchers in the ZPD (Daniels, 2001). Through social interaction, students learn from each other, as well as from adults, in this case teachers. This is illustrated in the ZPD in Figure 2.2. Students learn first through interactions with their peers and then on their own by internalization, finally reaching deep understanding (Fogarty, 1999). In this view, learning mathematics/calculus through the use of GeoGebra software fits the theory that I followed: I used the technology to teach the students to investigate the effects this had on students' achievement and understanding. In this study, Vygotsky's perspective was the most appropriate theory to use to interpret the data. Instrumental mediation allows the researcher to analyze the advantages of technologies in education (Elizondo-Rami & Hemandez-Solis, 2016). One of the characteristics of humans being is the building of tools, such as GeoGebra software. In principle, this amplifies an intentional activity, whether physical or cognitive. Wertsch (1985) defines Vygotsky's tools in two ways, techniques and psychological aspects. The tools are mediators or ladders of human activity in an environment for building the concepts of intended activities. Kozulin (2003) argues that both human mediation and symbolic mediation, in which the first enhances the learner's performance (in my case, learning mathematics with GeoGebra), and the second describes changes that occur in a learner's performance (in my case the result of the post-test after students had studied calculus with the aid of GeoGebra). In

Vygotsky's conceptualization, the term 'mediator' is defined as the ladder between an environmental stimulus and an individual response to this stimulus. In this study, this is referred to as the environment and the individual area of the Hypothesized cycle model of teaching mathematics using GeoGebra, as illustrated in Figure 2.6 below. This figure shows the relationship between an environmental stimulus and individual response to the stimulus as well as the ladder between environment stimulus (GeoGebra) and students learning differential calculus.

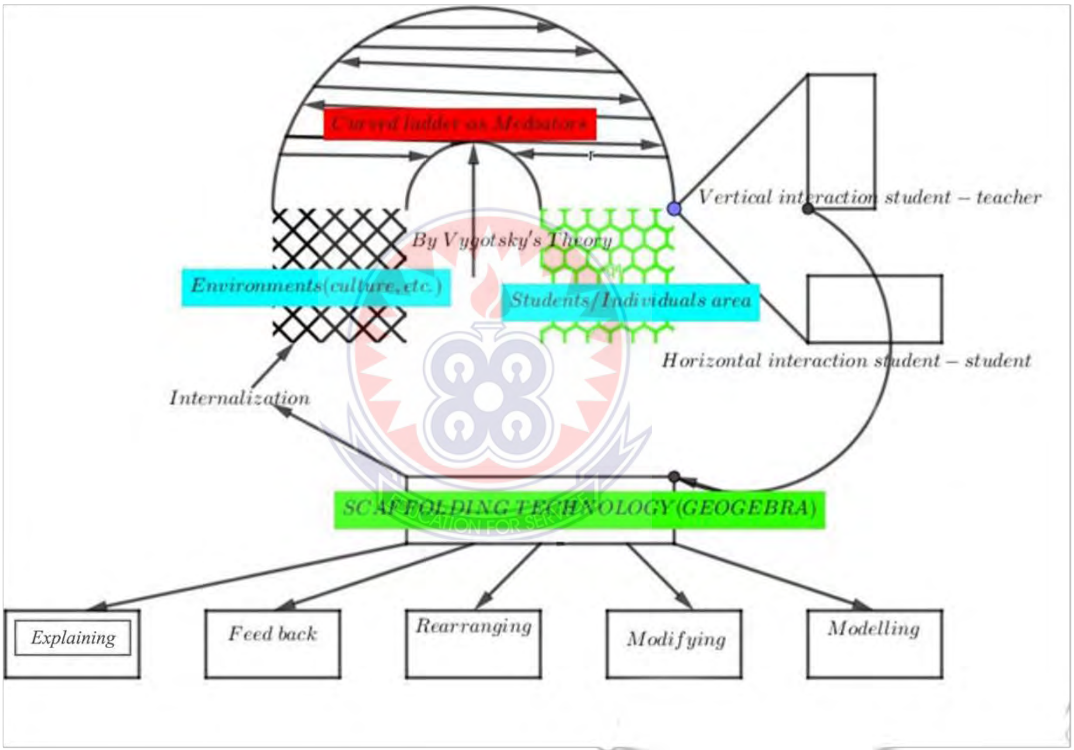


Figure 2.6: The interaction of environment, teachers, and students with technology (IEST). Hypothesized cycle model of teaching mathematics by GeoGebra

The model was developed from Figures 2.1 and 2.2 and the theoretical framework of the study. Figure 2.3 indicates five interaction treatments. These are students-environment, teacher content and tools, student-student, student-teacher, tools, and content, and educated individual environment (internalized activities). Within these environments is a variety of objects that may be human (students) or human created tools (the content of the lesson and technology). Indeed, culture is naturally part of an environment. Students are actors in the learning process and interact with the environment. In Vygotskian theory, this interaction is known as socio-cultural interaction. When human-created objects such as GeoGebra are introduced to the interaction process, as it is indicated in Figure 2.3, interaction may be between student-teacher or between student-student. In this model, student-teacher interaction is guided (scaffolded) by the technology of GeoGebra. Those students who are guided by more knowledgeable ones (MKO), most of whom will be teachers, will internalize the concepts (self-reflection) in the environment. When the self-reflection that occurs during internalization is reflected in the environment, for example in the reproduction of culture, externalization will immediately take place in, for example, the creation of new artefacts made possible by its transformation like that of human growth and understanding of required activities (Vygotsky, 1978b; Vygotsky, Leont'ev, & Luria, 1999). If a well-designed model and activities are implemented in classroom teaching and learning then externalization will be optimum (Vygotsky et al., 1999). I guided students in such environments to become new teachers, and if these newly qualified teachers apply these activities themselves, the process becomes cyclical. In this study, this cyclical process is referred to as the hypothesized cycle model, as illustrated in Figure 2.3. A learning cycle is not a teaching method; rather, it is a process of teaching and learning (Marek, Gerber, & Cavallo, 1999). In summary, studies have shown that in Zone of Proximal Development learning can be scaffolded equally by using interactive teaching methods or by using technology. In Vygotsky's theory of learning, students learn a given course (such as calculus) first by interacting with more a knowledgeable person, in this case, the teacher. Gradually, the student internalizes the knowledge and engages in activities independently by using tools in the given environment. As students' abilities grow, scaffolding is progressively decreased, an important aspect in ZPD. In other words, where students interact with each other, cooperative learning takes place, and the assistance of the individual occurs during these activities. For a long time, researchers have tried to understand the steps one takes when solving a problem to comprehend how the mind works and how best to educate the next generation (Singer & Moscovici, 2008). Taking this idea as a starting point, this study posits nine steps in the implementation of the Hypothesized Cyclical model in classroom learning and teaching: 1) Identification of area (environment) (laboratory class). 2) Identification of individual areas (teacher professional development and student ability, perception). As the researcher was a teacher the MKO occurs, thus teaching and learning can take place. The pre-test was used and to identify students' abilities. 3) State objectives of teaching a lesson with GeoGebra (review literature). 4) Design teaching materials (lesson plan that is compatible with GeoGebra). 5) Implementation of a lesson plan in the classroom (start scaffolding student-student, teacher student interaction). 6) Get feedback from students (responses). This could be in the form of a post-test and interview. 7) Evaluation of whether the method had achieved what was intended. Comparison of abilities before and after. 8) Internalization and externalization. See stage 3 in Figure 2.2. 9) Apply in the environment as in Step1.

Figure 2.7: Steps in implementation of Hypothesized Cycle model

I included the framework of the study as shown in figure 2.7 that used to give direction for my study (Akanbi, Amiri & Fazeldehkordi, 2015). The study employed the GeoGebra mathematical software to investigate the effect of it on students learning differential calculus either by self-exploration or social interaction (vertical and horizontal interaction) that they got because of scaffolding in the zone of proximal development by using Vygotsky’s theory.

The student development in the zone was investigated both in terms of two types of knowledge known as conceptual and procedural understanding. Conceptual understanding of students is increased with collaborative learning which is the central idea of Vygotsky’s theory in education (Hwang, Wu, & Kuo, 2013). The world of conceptual understanding is one of the three mental worlds of mathematics that build on human perceptions and actions by developing mental images (Tall, 2013a). Procedural knowledge is defined as “mental actions or manipulations, including rules, strategies, and algorithms, for completing a task”. Conceptual knowledge is defined as "knowledge about facts, [generalizations], and principles’ (Baroody, Feil, & Johnson, 2007). The difference between conceptual and practical or procedural knowledge is expressed as by Ivic (1991, as cited in Haapasalo & Kadijevich, 2000): Piaget made a distinction between 'practical knowledge' (savoir-faire) and 'conceptual knowledge', whereas Vygotsky dealt with three levels of knowledge: 'manifest content' (facts, data, and the like), 'instrumental knowledge' (methods, skills, procedures, etc.), and 'structural knowledge' (knowledge structures with underlying modes of thinking).

2.11 Teaching mathematics in the Zone of Proximal Development and Cooperative Learning in Classroom by GeoGebra

2.11.1 Teaching mathematics in ZPD

As discussed above, good learning takes place in the ZPD. Tharp (1993) defines the term teaching as assisting the performance through the ZPD and argues that teaching takes place when assistance is offered at points in the ZPD where performance requires assistance. In this study, the definition of teaching is redefined as the assistance of the performance of students by using GeoGebra in the classroom. Tharp (1993) identifies seven means of assisting performance and facilitating learning in the ZPD, as listed below: Modelling: Providing behavior for imitation. Modelling assists the learner by providing information and a remembered image that can serve as a performance standard. Feedback: The process of providing information on performance. Feedback is essential to improving performance because it allows the performance to be compared to the standard and thus encourages self-correction. Ensuring feedback is the commonest and single most effective form of self-assistance. Contingency management: Application of the principles of reinforcement and punishment of undesirable behavior. Instructing: Requesting specific action. This assists by selecting the correct response and by providing clarity, information and enhancing decision-making. It is most useful when the learner can perform some segments of the task but cannot yet analyze the entire performance or make judgements about what elements to choose. Questioning: A request for a verbal response that assists by producing a mental operation that the learner cannot or would not produce alone. This interaction assists further by giving the assistor information about the learner's developing understanding.

Cognitive structuring: Explanations. Cognitive structuring assists by providing explanatory and belief structures that organize and justify new learning and perceptions and allow the creation of new or modified schemata. Task structuring: Chunking, segregating, sequencing or otherwise structuring a task into or from components. It assists learners by modifying the task itself so that the units presented to the students fit into the ZPD when the entire unstructured task is beyond that zone. In this study, teaching mathematics through ZPD by using GeoGebra Mathematical software was applied to the experimental group. In the GeoGebra oriented classroom all seven identified means of assisting performance and facilitating learning mentioned by Tharp (1993) were implemented in the developed model.

2.11.2 Cooperative learning in ZPD

This study made use of learning activities with guidance from the teacher and discussion between peers and teachers, and as well as between peers themselves. These activities take up a large percentage of the teaching and learning process in the mathematics classroom, and with the aid of GeoGebra Mathematical software, this leads to cooperative learning. In this type of learning, students engage in activities both as a group and as an individual, with the help of the teacher. Cooperative learning is a form of small group teaching and learning in which students work actively in a social setting (Doolittle, 1995). Doolittle (1995) argues that social interaction between teachers and students forms the heart of ZPD; in a social context the ZPD must be regarded as the immersion of students in cooperative activities in a specific social environment.

2.12 Teaching Methods in Vygotsky's Theory and Hypothesized Cycle Model

As Vygotsky died before he had fully articulated his ideas there is no clear methodology for the teaching and learning process in the classroom in his theory. Furthermore, this idea itself needs investigation as there is no clarity on this method in his theory. Fani and Ghaemi (2011) contend that Vygotsky did not discuss any specific methodology for the use of ZPD in teacher education. In their paper, they discuss some factors that hinder the teacher's implementation of ZPD in the classroom. These factors include peers, mentors, contextual constraints, mediators' artefacts, and technology. They regard technology as an important factor when planning activities in the ZPD; one example of technology is GeoGebra, and they point out that technology has proved to be a reliable source of electronic scaffolding and thus a positive change in teacher's professional development (Fani & Ghaemi, 2011). However, in my opinion, there is an implication of teaching methodology in Vygotsky's theory. For example, Palincsar and Brown (1984, as cited Daniels, 2001) use the term 'reciprocal teaching method' to cover a combination of modelling, coaching, scaffolding and fading. Scaffolding is the central idea of Vygotsky's theory and in this dynamic system, the learning and teaching process consists of the four ideas that are important in defining the system. Fading is one is these concepts. Therefore, this theory has indirect references to teaching methods or methodology. For Daniels (2001), the reciprocal teaching approach involves summarizing, generating questions, clarifying and predicting the topic in the classroom. In general, in Vygotsky's theory, the teacher can use a reciprocal teaching method (modelling, coaching, scaffolding, and fading) by integrating technology such as GeoGebra in the classroom to teach mathematics. This is the argument of this study as scaffolding is part of both Vygotsky's theory and the Hypothesized Cycle model developed in this study. The main aim of this study is to give special considerations in integrating technological pedagogical and content knowledge (TPACK) in teaching students' differential calculus with GeoGebra, a dynamic multi-purpose mathematics software. According to Beken e (Bekene, 2020), GeoGebra oriented lesson is a way of implementing some developed steps or designed teaching-learning (lesson plan) in the classroom. "The designed teaching-learning scenario allows students and teachers to focus on specific mathematics learning and teaching and to make sense of the mathematics with foreseeable results for the full range of students in the classroom" (Bekene, 2020). On implementation stages of the Hypothesized Cycle model of this study, the teaching material used consists of the topics on differential calculus which can be considered as a GeoGebra oriented lesson plan for the experimental group and traditional oriented lesson plan for control groups. It is accepted that planning helps the teachers to organize and systematize the learning and teaching process. Therefore, planning is important for the teaching of students in control manner in the classroom and preparing detailed lesson plans is important, especially for beginner teachers who newly experience explicit instruction, modelling, guided practice, and scaffolding and proficient teachers were found to start their lesson plans with instructional activities included within the developed lesson plan(Allahverdi & Gelzheiser, 2021). The important components of lesson design (lesson plan design tool) sometimes known as task solutions help the communication between the students and teachers around contents (differential calculus), technology/GeoGebra, and pedagogy/developed cycle model during the teaching process

CHAPTER THREE

RESEARCH METHODOLOGY

3.0 Overview

This section will discuss the research process under the following headings: Research Design, Population, Sample and Sampling Procedure, Research Instruments and Data Analysis to be used to investigate the effect of the use of GeoGebra mathematical software on university students' learning of calculus.

3.1 Research Paradigm

Considering these questions and the differences between research paradigms and how each related to the objective of this study as discussed below, the researcher chose the pragmatic research paradigm for this study. The pragmatic paradigm is based on the researcher's plan to use a methodology that fits the problem to be investigated by the researcher (Teddlie & Tashakkori, 2009). In this case, the literature review revealed that quasi experimental method was appropriate when following the pragmatic research paradigm as it represents a compromise between the positivist and constructivist paradigms (Maarouf, 2019). Thus, a quasi-experimental method research approach was chosen for this study.

Traditionally, there are three common research paradigms: positivist, interpretivism, and critical theory. For example, the interpretive/constructivist paradigm tries understand and interpret what the subject is thinking about the concept (Kivunja & Kuyini, 2017). All these paradigms; positivist, interpretivism, and critical theory contain opposing ideas that have led to a "paradigm war" (Galvez, Heiberger, & Mcfarland, 2020, Maarouf, 2019,) in terms of the three philosophical dimensions of ontology, epistemology and methodology. As a result, the compromising paradigm known as the pragmatic paradigm has emerged (Teddlie & Tashakkori, 2009). Understanding the most significant differences between the research paradigms and how they approach (ontology, epistemology, and methodology) these three philosophical dimensions helped the researcher to choose the best research paradigm for this study. It is thus to discuss these dimensions. Guba and Lincoln (1994) argue that the philosophical dimensions present three fundamental, interconnected questions: The ontological question asks, "What are the form and the nature of reality?" Does "objective" reality exist "independent of the researcher"? The objective of this study is to investigate the effect of incorporating GeoGebra in teaching and learning calculus. By asking this question the researcher hoped to establish a reality somewhere between positivist (quantitative) and constructivist (qualitative) ways of knowing to examine the data in the study from both world views for triangulation purposes. The epistemological question "What is the nature of the relationships between the knower and what can be known/participant?" is concerned with the acceptable knowledge in the study field (Saunders, Lewis, & Thornhill, 2009). Morgan (2007) defines epistemology as the nature of knowledge and the relationship between researcher and participants in the study. Drawing on ontology to establish a reality between quantitative and qualitative ideology, the researcher's task was to scaffold students in their learning of calculus with the help of GeoGebra mathematical software, using the hypothesized cycle model of teaching mathematics by GeoGebra. Thus, there is a relationship between the researcher and participants in terms of knowledge. In the epistemological philosophical dimension, reality is represented by objects that are considered to be real, such as computers, trucks and machines (Saunders et al., 2009). Investigating the views of students on the use of GeoGebra before and after intervention was the task of the researcher in this study. Methodological questions include, how can the inquirer go about finding out whatever he or she beliefs can be known? The nature of the research question addressed in this study demanded the use of an explanatory methodology, which consisted of the investigation of the cause-and-effect relationships between the variables of the study such as teaching differential calculus with the help of GeoGebra (independent variables) and students' achievements and understanding (dependent variables) in the experimental group, and teaching calculus using conventional methods(independent variables) with their achievement(dependent variables) and hence, the study investigated the relationship between achievements and students' views on using GeoGebra which answered the methodological questions appeared in the study. Pragmatism holds that truth is what works at the time; it is not based on a dualism between reality independent of the mind and within the mind (Creswell & Poth, 2018). In keeping with this world view, this study used multiple methods or perspectives to validate quantitative and qualitative instruments by considering information obtained from the reviewed literature (students' perceptions of GeoGebra) because statistics cannot manipulate perceptions obtained from interviews, and to explain quantitative results for better contextualization in the intervention. Creswell (2013) believes that mixed research methods are suitable for research problems and questions in a study. Before the researcher administers instruments, he or she needs to explain the statistical results by talking to people; the researcher must determine whether the quantitative and qualitative results match. Having established the research method to be used in the study in any research, the next question is how to collect data (ways of obtaining data in terms of time available). In Quasi-experimental, data can be collected sequentially or concurrently to achieve the best understanding of the research problems (Creswell, 2014). The method allows the researcher to apply two types of research questions (to collect qualitative and quantitative methods), two types of sampling procedures (probability and purposive), two types of data (numerical and textual), two types of data analysis (statistical and thematic) and two types of conclusions (objective and subjective) (Tashakkori & Creswell, 2007).

3.2 Research Design

The general approach chosen for this study is a quasi-experimental design. A quasi-experimental study takes place in a real life setting as opposed to only a laboratory setting. Quasi-experiment is an empirical study used to estimate the causal impact of an intervention on its target population. A quasi-experimental study on the other hand is a type of evaluation which aims to determine whether a program or intervention has the intended effect on a study's participants (NCTI, 2011). Quasi-experimental studies lack one or more of the following key components of a true experiment which includes (1) pre-posttest design, (2) a treatment group and a control group, and (3) random assignment of study participants. Quasi-experimental studies are often an impact evaluation that assigns members to the treatment group and control group by a method other than random assignment. The quasi-experimental design that will be chosen for this study will be Pretest-Posttest non-equivalent group strategy. The purpose of this strategy is to use qualitative data and results to assist in explaining and assigning reasons for quantitative findings. Morgan, (1998) suggested that the mixed method design is appropriate to use when testing elements of an emergent theory resulting from the qualitative phase and that it could also be used to generalize qualitative findings to different samples. Golafshani (2003) described that, qualitative research uses a naturalistic approach that seeks to understand phenomena in context-specific settings such as real world setting in which the researcher does not attempt to manipulate the phenomenon of interest but only try to unveil the ultimate truth. This research design provides a quickly efficient and accurate means of assessing information about a population of interest.

A quasi-experimental pre-and post-test and a control group design were adopted for this study. Quasi-experimental research uses non-randomized assignments of the group of the study that are categorized into experimental and control groups (Shadish & Luellen, 2005).

3.3 Population, Sample and Sampling Techniques

It was imperative to choose University of Education students studying in the Department of Mathematics because I being a part time lecturer at the university, the problem of the study was raised there and also data collection will be easy. One group of undergraduate students of mathematics made up the participants of the study. The numbers of these students depend on the capacity of the department and the researcher used a purposeful sampling method to select an experimental and control group for the study. In total, 30 and 36 students learning mathematics were included in experimental and control groups.

Purposive sampling is a sampling technique in which researcher relies on his or her own judgment when choosing members of population to participate in the study.

Purposive sampling is a non-probability sampling method and it occurs when "elements selected for the sample are chosen by the judgment of the researcher. Researchers often believe that they can obtain a representative sample by using a sound judgment, which will result in saving time and money".

Alternatively, purposive sampling method may prove to be effective when only limited numbers of people can serve as primary data sources due to the nature of research design and aims and objectives. Purposive sampling is one of the most cost-effective and time-effective sampling methods available. Purposive sampling may be the only appropriate method available if there are only limited number of primary data sources who can contribute to the study. This sampling technique can be effective in exploring anthropological situations where the discovery of meaning can benefit from an intuitive approach.

3.4 Research Instrument

The instruments in this study were: 1) test (comprised of a multiple-choice test and word problem), prepared by the researcher and validated by my supervisor. This was referred to as the differential calculus achievement test (DCAT); 2) an interview; and 3) a questionnaire comprising closed-ended items. The reason the researcher used both multiple-choice questions and a word problem in the test was that multiple-choice questions are no longer regarded as a tool for providing a suitable response (Sharma, 2021; Shute & Rahimi, 2017) because they do not allow a sufficiently accurate assessment of students' knowledge and skills (Whittington & Hunt, 1999). A word problem, on the other hand, allows the teacher to assess what knowledge and skills the student have and which s/he does not (Morgan, 2007). This is closely related to investigating the student's competence in a certain domain. So, the combination of the two types of questions enabled the researcher to judge the competency of the students in the subject of differential calculus during and before the study.

3.5 Validity of Instrument

Validity refers to the accuracy of the inferences or interpretations the researcher makes from the test scores. The questionnaire and interview were checked by experts to ensure their reliability and validity. The validity and of questionnaires were discussed.

The instrument of this study was subjected to face validation. Face validation tests the appropriateness of the questionnaire items. This is because face validation is often used to indicate whether an instrument on the face of it appears to measures what it contains. Face validations therefore aims at determining the extent to which the questionnaire is relevant to the objectives of the study. In subjecting the instrument for face validation, copies of the initial draft of the questionnaire were validated by supervisor. The supervisor discussed the nature of the research and it standard expected, the selection of the research topic to be covered, the planning and timing of the successive stages of the research topic, literature and sources, research methods and instrumental techniques. The supervisor then discussed every chapter and made the necessary corrections throughout the end of the project.

Some of the factors affecting the internal validity, such as maturation, effects of history, selection, and design contamination of the study, were considered by the researcher while the study was underway. The difference between the pre-and post-test of differential calculus might be the result of the psychological maturation of the participants rather than differences in the independent variable. Also, differences between experimental and control groups might result from one group changing at a different pace than another (selection-maturation interaction). This is the invisible factor of internal validity of the study as the duration of the study, the age level, and education level of the study participants are somewhat the same of the cycle model (Creswell & Poth, 2018).

Events during a study might affect one group but not another, leading to differences between groups that are not solely the result of the independent variables. In nonexperimental history, this might refer to events happening (to a group of individuals) beyond the event that the researcher is studying (Creswell & Poth, 2018).

Certain attributes of one group are different from another before the study starts, coinciding with the stages of the cycle model. Hence, differences after treatment are not solely attributable to the independent variable, and thus the researcher selected the participants of the study with the same education level (Teddlie & Tashakkori, 2009). This factor was managed by selecting the study participants randomly into experimental and control groups so that the characteristics had the probability of being equally distributed among groups of the study (Creswell & Creswell, 2018).

As the study was employed within one university with the participants of the same education level, this factor of internal validity was enabled to be controlled by the researcher. However, the researcher protected students from using the instructional material (GeoGebra oriented lesson plan) that prepared them for teaching purposes to have at their home. The material is used only within the classroom environment.

3.6 Reliability of Instrument

In psychological and educational testing, reliability is the stability of test scores, that is the scores must be similar on every occasion. To assure reliability in this study, this researcher used the test-retest reliability method. The reliability of questionnaires were discussed. The closed-ended questionnaire was adopted and rearranged according to the context of the study following research by Bu, Mumba, Henson, and Wright (2013), and the researcher computed its reliability by using a five-point Likert scale starting from strongly agree = 1 to strongly disagree = 5, with the scales between 1 and 5 coded as Agree = 4, Neutral = 3 and Disagree = 2. As the closed-ended questionnaire was intended for students, the questionnaire was distributed for students and the reliability of the questionnaire obtained from students who participated in the pilot study was computed and Cronbach's alpha was found to be 0.917 for students, implying that the questionnaires were reliable. The questionnaire comprised 14 closed-ended items and five interview questions to investigate the perceptions of students on the use of GeoGebra as an instructional tool. If the value of Cronbach's alpha of an item is equal to or greater than 0.5, then the item is considered acceptable, implying that it is reasonably reliable (Salvucci, Walter, Conley, Fink, & Saba, 1997; Taber, 2018).

3.7 Data Collection Procedure

As part of the research, there was the need to issue a permission letter to the Coordinator of Sekondi study center of Codel of University of Education-Winneba to officially allow me to collect my data amidst the fact that am a part time lecturer at the study center.

A pre-test was administered to the experimental and control group before intervention. In the case of the experimental group in this study, the pre-test helped to categorize students as low or high achievers. The pre-test and the post-test were the same for the two groups in terms of content and the number of questions. The instructional materials for the two groups were also the same.

One section of the experimental group, was conducted in a computer laboratory over two weeks, while the control group was taught at the same time in an ordinary classroom. The laboratory classroom was arranged so that each student had a computer. The intervention was planned as a set of eight 50-minute lessons, in total about 400 minutes or approximately 7 hours using the dynamic mathematics software GeoGebra (DMSG) as well as conventional methods. The instructional material or lesson plan for the experimental group was designed to be delivered using computer-assisted teaching, in this case, GeoGebra software. The same instructional material was used to teach the control group but using traditional or conventional teaching methods in which the teacher must use the lecturer method, using talk and talk through paper-pen approaches.

Where there is no difference between the pre-test scores of two groups, the researcher uses a T-test or an ANOVA. This ensures that the results are real and helps the researcher to manage the initial group difference statistically.

Closed-ended questionnaires were administered to the experimental group after the intervention and students were interviewed. In short, the quantitative data was collected using two instruments, a closed-ended questionnaire and tests, and qualitative data was collected using a focused group interview. The focused group interview was chosen for this study because of the number of the individual chosen by the researcher. There were about five participants of individuals who participated in the interview thus simple to control the data. An interview is a specialized form of communication between people for a specific purpose associated with the research question of the study and whereas a focus group interview is a qualitative technique for data collection by discussions of the participants of the study on a given issue or topic (Dilshad & Latif, 2013).

An interview was used in determining students' perceptions of learning calculus through technology, in this case, GeoGebra, and used a closed-ended questionnaire and tests to investigate the effect on students' learning of differential calculus of GeoGebra software, and the extent to which this software enhanced students' learning of calculus, in terms of both achievement and understanding of differential calculus. I used the quantitative methods first (posed pre and post-test to students) and gave a greater emphasis in addressing the study's purpose, and the qualitative methods (perception of students after intervention employed) followed to help me explain the quantitative results of the study and in this method, qualitative data is enhanced me for an understanding of some aspect of the experiment.

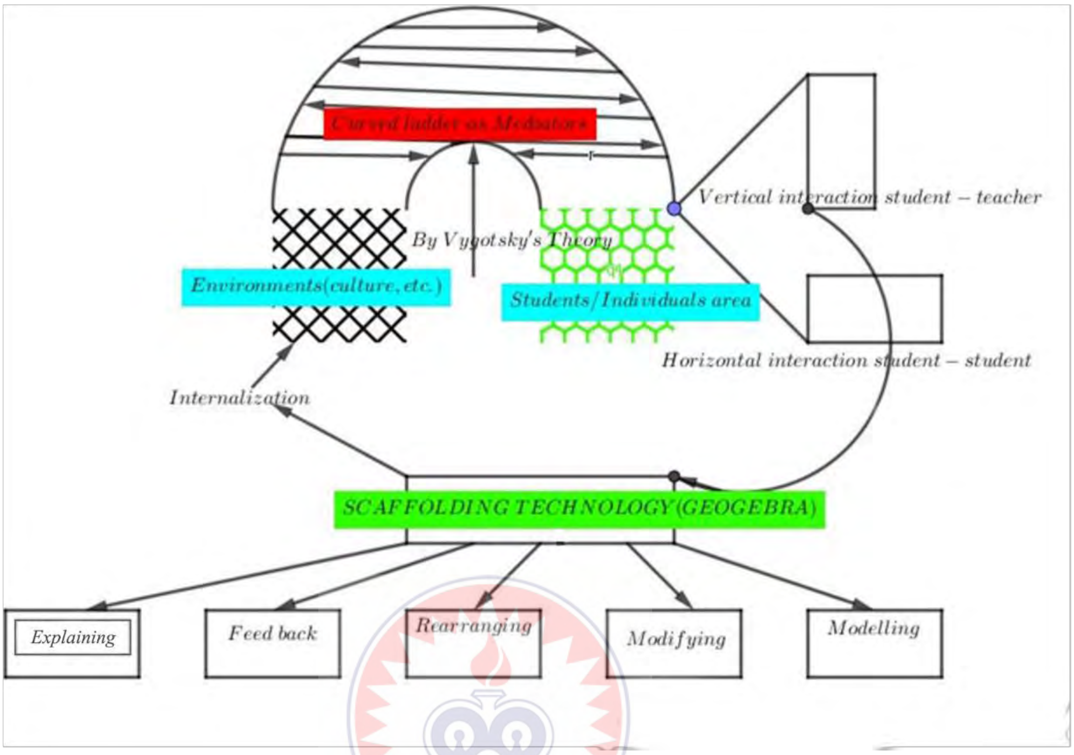


Figure 3.1: The interaction of environment, teachers, and students with technology (IEST). Hypothesized cycle model of teaching mathematics by GeoGebra

3.7.1 Stages in the hypothesized cycle model

Stage 1: I first identified the environment.

In my seven years of experience in higher education in Ghana, I have observed that technology has not been used to support students' performance in calculus, either in or outside the classroom. Ghanaian university students regard calculus as difficult and conceptually challenging. In Ghana, little research has been done on integrating technology into mathematics teaching at either school or university level, especially in teaching with open access software like GeoGebra. Teaching in Ghana is still traditional, and teacher-centered. Thus, this study developed a model known as the cycle model and investigated its effects of students' learning of calculus through GeoGebra.

This section provides information on the area (environment) in which the study took place, in this case, a laboratory classroom at the University of Education Sekondi Study Centre. Vygotsky's ideas are reflected in the community of practice thinking that addresses the need for continuous professional development and lifelong learning in the environment (Heinze & Procter, 2006). The environment can be viewed from two standpoints: the biological perspective (phylogenesis and fatal development) and the psychological perspective. The 'environment' or 'real world' can be articulated and described only in terms of viable intangible structures by observers (Glasersfeld, 1996). Within the school environment, teaching and learning activities occur, using a variety of reinforcements, such as praise, rewards, and grades. As this study depends on Vygotsky's theory, I based my view of the environment on psychological perspectives to initiate articulation of things in the environment, indicated on the left side of the figure above. I will search for a laboratory before starting the main study. I felt that it was important to determine the study area before commencing with the intended intervention, which is the base of the cycle model.

Below are pictures of student using GeoGebra software in solving differential calculus tests;

Stage 2: Identification of individual ability within this environment

The next stage of the cycle model is the identification of individual ability. This section considers teacher professional development and student ability. It was important to establish students' abilities before the intervention. I did this by administering a pre-test on proficiency in differential calculus developed by a researcher. Students' ability or proficiency will be analyzed by testing two types of understanding: conceptual and procedural understanding. According to the ASSURE model (analysis, state objectives, select instructional materials, utilize materials, require students, and evaluate), this step is regarded as the first step in the analysis, that is identifying students' characteristics on entering the programmes (Baran, 2010). The ASSURE model does not take the environment/workplace setting in which the programmes would be employed into account.

Stage 3: Feedback stages of cycle model.

After the implementation of the teaching and learning of topics in differential calculus with the help of GeoGebra mathematical software in the experimental group and with the aid of traditional methods in the control group, students' feedback on the activities in the classroom setting will be discussed. In this step of the cycle model, I will administer the post-test to both groups. I will also give the experimental group a questionnaire designed to elicit students' perceptions of the use of GeoGebra in the learning process. During the interventions, the activities suggested by Tharp (1993) will be used in the classroom setting. These activities will include scaffolded feedback on how to find the solution to the problem provided by the teachers. If students are given the correct answer immediately after making a mistake, the correct information will be better remembered. Finn and Metcalfe (2010) argue that "scaffolded feedback" builds on retrieval practice by providing "incremental hints" until students are able to find the correct answer themselves. The next step in the cycle model will be to investigate the mean gain or loss in students' proficiency or understanding after the intervention.

3.8 Data Analysis Procedure

The gathered data through the administration of pre-test and post-test was coded, tabulated and analyzed using SPSS statistical software according to the research question. Data collected from the control and experimental groups before, during and after the intervention was analyzed using the statistical software SPSS. Depending on the nature of the research questions and the data collected, different statistical techniques were employed. In the case of the quantitative data, the researcher used either a T-test or an ANOVA. The researcher observed the data from the two approaches separately, analyzed and interpreted it. Data were not merged as the study used an explanatory sequential design. Cohen, Manion, and Morrison (2018) argue that the use of different research methods is important for a better understanding of the issues of the study. Answers may be found using either of the approaches and the limitations of one method may be balanced out by the advantages of the other (Creswell, 2009).

In their study, Cohen, Manion, and Morrison (2018) found that the use of data collected using mixed methods enables researchers to generalize to the wider population. The quantitative data in the study were analyzed using T-tests and ANOVAs.

For the research question one which is; How does the level of proficiency in differential calculus compare in students taught using GeoGebra (experimental Group 1) and students taught through conventional lecturing (control Group 2)? the difference in means of the two groups, experimental and control groups, will be established using both pre-and post-test results.

For the research question two which is; How does the level of proficiency in differential calculus compare within the experimental group (Group 1) pre-and post the intervention incorporating the use of GeoGebra? Comparing in experimental group students' achievement in pre-test and post-test. Comparing the mean scores of the experimental and control groups on the post-test.

For research question three which is; What are students' attitudes and perceptions about learning calculus using GeoGebra software? The questionnaire and interview were used to determine the attitudes and perceptions of students about the use of GeoGebra and the existence of this and other software at the university.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Overview

The purpose of this study is to determine the effect on students' learning of calculus by being taught through GeoGebra Mathematical software at the university level. To achieve this purpose, instructional materials were designed, and the instruments for the study, which included a questionnaire featuring Likert scales and differential calculus achievement tests of both conceptual and procedural understanding, were implemented. The dependent variable was a differential achievement, which was measured using a pre-test and a post-test. A mixed-methods approach was followed to achieve the goals of the study. The research questions addressed in this study were: (1) How does the level of proficiency in differential calculus compare in students taught using GeoGebra (experimental Group 1) and students taught through conventional lecturing methods (control Group 2)? (2) How does the level of proficiency in differential calculus compare in the experimental group pre-and-post the intervention? (3) What are students' experiences and perceptions of using mathematical software (GeoGebra) in learning calculus concepts? In addressing these research questions, I used a developed cycle model that was the theoretical framework of the study.

4.1 Demographies of Participants

The first section of the questionnaire used in the study was used to obtain demographic information such as the gender and age of the participants in the experimental and control groups. Figure 4.1 shows the information for the experimental and control groups of students.

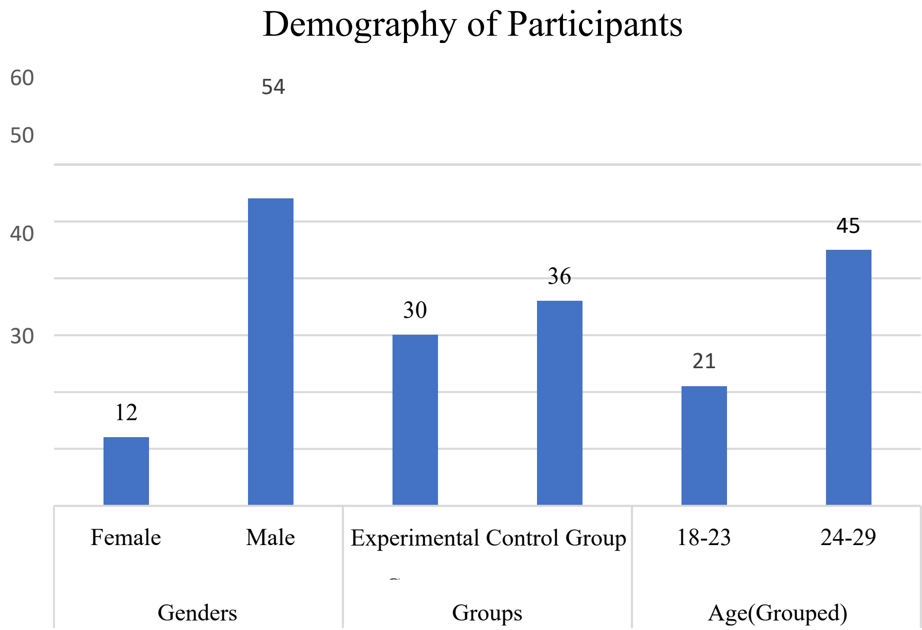


Figure 4.1: Participants' demographic information

4.2 Analysis of Group Differences in Pre-Test of Differential Calculus Achievement

In this section, students' scores on differential calculus achievement tests (DCAT) were obtained. The test comprised 20 items, 10 items on procedural knowledge and 10 on conceptual knowledge, developed by the researcher and administered at the beginning of the study. This pre-test was used to investigate the initial differences (if any) between the two groups in the study in terms of their performance in a differential calculus achievement test (DCAT) to address the two research questions in the study: (1) How does the level of proficiency in differential calculus in students teach using GeoGebra (experimental) and those taught through conventional lecturing (control) compare? (2) How does the level of proficiency in differential calculus in the experimental group compare pre-and-post the intervention incorporating the use of GeoGebra? Scores obtained from the pre-test were analyzed by applying an independent samples T-test, which compares the means of the two groups as shown in

Table 4.1 below. This showed that the pre-test was normally distributed in both groups in the study as the significance level in both tests was greater than 0.05.

Table4.1:Test normality of pre-test

		Kolmogorov-Smirnov			Shapiro-Wilk		
	Group	**Statistic**	**Df**	**Sig.**	**Statistic**	**df**	**Sig.**
Pre-test	Experimental	.119	30	.200	.946	30	.133
	Control	.145	36	.055	.957	36	.172

Table 4.2: Over all descriptive statistics of the two groups' proficiency in differential calculus before the intervention

Groups	Differential Calculus achievement (Before Intervention)			
	N	**Mean**	**SD**	**Std.Error**
Experimental	30	27.00	9.965	1.81944
Control	36	26.67	10.823	1.80388
Total	**66**	**26.82**	**10.364**	**1.27572**

Table 4.2 shows a mean difference of 0.33333 between Group1 (M=27.000) and Group 2 (M=26.6667). This indicates that the two groups were very similar as the difference was not significant at 0.05 (p=0.898>0.05) (see Table 4.3). Students in the two groups had similar academic backgrounds, with each group consisting of both high and low achievers. The uniformity in the results of the two groups was a good starting point for me to be able to deduce whether the effect of the treatment after the intervention had occurred. Hence, if the experimental group scored higher than the control group on the post-test, the researcher could assume that the differences had occurred because of the treatment in the study, by controlling other confounding variables. In this regard, I tried to control all the possible confounding variables such as time allocation for a lesson, the effect of the teacher (this was controlled by using the researcher as the teacher for both groups), and topics covered (this was controlled by focusing on the curriculum.

The one-way ANOVA is summarized in Table 4.3 below. This provided further analysis of the two groups and within the groups (experimental and control).

Table 4.3: Overall one: Way analysis of variance summary table comparing groups' achievement in differential calculus before treatment

	Differential Calculus Achievement Test (Before Intervention)				
Sum of Squares		**Df**	**Mean Square**	**F**	**Sig.**
Between Groups	1.818	1	1.82	.017	.898
Within Groups	6980.000	64	109.06		
Total	**6981.818**	**65**			

The results in Table 4.3 show that there was a statistically non-significant difference in pre-test differential calculus achievement ($F(1, 64) = 0.017$, $p = 0.898 > 0.05$). The dependent variable in this study was students' proficiency in differential calculus and this may have been influenced by the other variables (groups). Hence, the study investigated the conceptual and procedural understanding of both groups before treatment as a starting point, as tabulated in Table 4.4.

Table 4.4: Overall descriptive statistics of achievement in differential calculus of the two groups (Conceptual and Procedural understanding) before treatment

Student's proficiency within Groups			
Groups		Pre-test Conceptual	Pre-test Procedural
Experimental	Mean	16.3	10.7
	N	30	30
	Std. Deviation	6.557	5.833
Control	Mean	13.89	12.78
	N	36	36
	Std. Deviation	7.281	5.909
Total	**Mean**	**15.00**	**11.82**
	N	**66**	**66**
	Std. Deviation	**7.016**	**5.925**

Table 4.5: Students' proficiency by gender before intervention

		Students' proficiency by*Gender	
Gender		**Pre-test Procedural**	**Pre-test Conceptual**
Female	Mean	12.92	12.08
	N	12	12
	Std. Deviation	7.217	6.894
Male	Mean	15.46	11.76
	N	54	54
	Std. Deviation	6.955	5.759
Total	Mean	15.00	11.82
	N	66	66
	Std. Deviation	**7.016**	**5.925**

Table 4.4 shows that the mean scores of experimental groups 1 on both pre-procedural and pre-conceptual understanding of DCAT were M=10.6667 and M=16.3333 respectively with a mean difference of 5.6666. This indicates that students in this group had better conceptual understanding than procedural understanding before the intervention. The mean for the control group 2 was M=12.7778 and M=13.8889 for pre-conceptual and pre-procedural understanding respectively, with a mean difference of 1.1111, indicating that some students in the control group had the same level of procedural and conceptual understanding of differential calculus before the intervention. Table 4.5 shows that both male and female students had a better conceptual understanding of differential calculus than procedural understanding before the intervention. An ANOVA was calculated to determine whether if there was any significant difference between the mean scores of the groups in terms of two types of knowledge. The one-way ANOVA is summarised in Table 4.6 below.

Table 4.6: Over all one way analysis of variance summary table comparing groups' proficiency in differential calculus before treatment

Understanding		Sum of Squares	Df	Mean Square	F	Sig.
Pre-	Test Between Groups	97.778	1	97.78	2.017	.160
Conceptual	Within Groups	3102.222	64	48.47		
	Total	3200.000	65			
Pre-test	Between Groups	72.929	1	72.93	2.113	.151
Procedural	Within Groups	2208.889	64	34.51		
	Total	**2281.818**	**65**			

Table 4.6 indicates that there were statistically non-significant differences in both conceptual and procedural understanding of differential calculus before the treatment, with the values F (1, 64) =2.017, p=0.160>0.05 and F (1, 64) =2.113, p=0.151>0.05 respectively. Next, I was interested in investigating students' abilities within each group in terms of the two types of knowledge involved in understanding differential calculus.

4.3 Analysis of Students' Ability within Groups

When dividing students into two groups within the groups, I considered their pre-test score to investigate the GeoGebra treatment effects on diverse achievers. These were divided into two groups, higher achievers and lower achievers, using the pre-test score median of each group. Next, I categorized students into nested groups (below the median of 27.5 (low ability), 16 in number, and above-median of 27.5 as high ability (14 in number) for the experimental group. Of these students, only two of the female students were categorized as high achievers and none were higher achievers in procedural proficiency or conceptual proficiency. However, the sum of the two (procedural proficiency and conceptual proficiency) or one proceed the other (procedural proficiency proceed conceptual proficiency and vice versa) resulted in their categorization as high achievers (Finn & Metcalfe, 2010; National Research Council, 2001; Rittle-Johnson & Alibali, 1999). Twelve male students were higher achievers but only one male student was a high achiever in procedural proficiency; the others were becoming high achievers, as reflected in the sum of the scores on the two types of proficiency before intervention. Of the 36 students in the control group, 17 were included in the high achiever category as their scores were higher than the median of 25; 19 students were low achievers as their scores fell below the median of 25. Of these students, only three female students were high achievers, and none were high achievers in procedural proficiency or conceptual proficiency; the sum of their scores on the two types of proficiency allowed them to be categorized as high achievers (see Table 4.7). Fourteen males' students and three female students were high achievers in the procedural understanding of calculus; 12male students and two female students were high achievers in procedural understanding.

Table 4.7: Descriptive statistics of students' proficiency by gender before treatment

Proficiency Student Ability	Genders	Groups	Mean	SD	N
Pre-conceptual. Low ability	Female	Experimental	6.7	2.9	3
		Control	10.0	4.1	4
		Total	8.6	3.8	7
	Male	Experimental	13.5	4.7	13
		Control	9.7	5.8	15
		Total	11.4	5.6	28
	Total	Experimental	12.2	5.2	16
		Control	9.7	5.4	19
		Total	10.9	5.4	35
High ability	Female	Experimental	17.5	3.5	2
		Control	20.0	8.7	3
		Total	19.0	6.5	5
	Male	Experimental	21.7	4.4	12
		Control	18.2	6.1	14
		Total	19.8	5.6	26
	Total	Experimental	21.1	4.5	14
		Control	18.5	6.3	17
		Total	19.7	5.6	31
Total	Female	Experimental	11.0	6.5	5
		Control	14.3	7.9	7
		Total	12.9	7.2	12
	Male	Experimental	17.4	6.1	25
		Control	13.8	7.3	29
		Total	15.5	6.95	54
	Total	Experimental	16.3	6.6	30
		Control	13.9	7.3	36
		Total	15.0	7.0	66
Pre-conceptual. Low ability	Female	Experimental	5.0	5.0	3
		Control	11.3	4.8	4
		Total	8.6	5.6	7
	Male	Experimental	7.7	4.8	13
		Control	9.3	4.95	15
		Total	8.6	4.9	28
	Total	Experimental	7.2	4.8	16
		Control	9.7	4.9	19
		Total	8.6	4.9	35

High ability	Female	Experimental	15.0	7.1	2
		Control	18.3	5.8	3
		Total	17.0	5.7	5
	Male	Experimental	14.6	3.96	12
		Control	15.7	5.1	14
		Total	15.2	4.6	26
	Total	Experimental	14.6	4.1	14
		Control	16.2	5.2	17
		Total	15.5	4.7	31
Total	Female	Experimental	9.0	7.4	5
		Control	14.3	6.1	7
		Total	12.1	6.9	12
	Male	Experimental	11.0	5.6	25
		Control	12.4	5.9	29
		Total	11.8	5.8	54
	Total	Experimental	10.7	5.8	30
		Control	12.7778	5.9	36
		Total	11.8	5.9	66

N.B pre-conc = pre-test conceptual on pre-test, pre-pro = pre-test-procedural on pre-test

4.4 The difference between students' proficiency and students' ability

On admission to both groups of the study, students' ability was the same before intervention they had on differential calculus. Although students' ability before being introduced to differential calculus was very similar (see Table 4.2), there were some differences in their proficiency (see Table 4.6).

Table 4.8: Overall one: Way analysis of variance summary: Students' proficiency in differential calculus compared to their ability before treatment

Variables		Sum of Squares	df	Mean Square	F	Sig.
Pre-test conceptual	Between Groups	1278.940	1	1278.940	42.608	.000
	Within Groups	1921.060	64	30.017		
	Total	3200.000	65			
Pre-test procedural	Between Groups	785.505	1	785.505	33.597	.000
	Within Groups	1496.313	64	23.380		
	Total	**2281.818**	**65**			

Table 4.8 shows whether in terms of their ability, experimental and control group students' procedural and conceptual understanding of differential calculus differed before the treatment. The table shows that there were statistically significant differences in both conceptual and procedural understanding of differential calculus by student ability before the treatment with the values F (1,64) = 42.6, p < 0.5 and F (1,64) = 33.6, p < 0.5. To determine the extent of the difference between the two groups in terms of the two proficiencies, I used effect size (ES). For the ANOVA test, the effect size can be calculated by the formula:

$$Eta\ squared = \frac{Sum\ of the\ squares\ between\ groups}{Total\ sum\ of squared}$$

(Cohen et al., 2018). According to the formula, the effect size of the pre-conceptual understanding of the experimental and the control group was computed as:

$$Eta\ squared = \frac{Sum\ of the\ squares\ between\ groups}{Total\ sum\ of squared}$$

$$= \frac{1278.9}{3200.0}$$

$$=0.4$$

Eta squared = 0.4 indicates a small effect size; this, in turn, implies that there is a small difference between the two groups (experiment and control) in terms of pre-test conceptual understanding in terms of achievement (Cohen et al., 2018).

The effect size of the pre-test procedural understanding of the experimental and control group was computed as:

$$Eta\ squared = \frac{Sum\ of\ the\ squares\ between\ groups}{Total\ sum\ of\ squared}$$

$$= \frac{785.5}{2281.8}$$

$$=0.34$$

This indicates that pre-test procedural understanding of students had a small effect size, implying that there were small statistically significant differences in the two groups in pre-test procedural in differential calculus.

4.5 Analysis of Group Differences in Post-Test of Differential Calculus

After the intervention had been completed, the post-test was administered to both the experimental and the control group. The research questions of the study (1) How does the level of proficiency in differential calculus compare in students taught using GeoGebra (experimental Group 1) and students taught through conventional lecturing (control Group2)? and (2) How does the level of proficiency in differential calculus compare within the experimental group (Group 1) pre-and post-intervention incorporating the use of GeoGebra? To address these questions, a post-test was administered to both groups. The recorded post-test scores achieved after the intervention were analyzed and are reflected in Table 4.9.

Table 4.9: Over all descriptive statistics for two groups on differential calculus achievement after the treatment

Comparison of Pre-test Scores and Post-test Scores of Groups			
Interventions		Pre-test Scores	**Post-test Scores**
Experimental(N=30)	Mean	27.00	**41.17**
	Std. Deviation	9.965	**13.814**
Control(N=36)	Mean	26.67	**31.11**
	Std. Deviation	10.823	**11.409**
Total	Mean	26.82	**35.69**
	Std. Deviation	**10.364**	**13.442**

Table 4.9 shows that the mean score of the experimental Group 1 in the post-test was M= 41.1667 and that of the control Group 2 was M = 31.1111; the mean difference between the two groups was 10.05556, indicating that the scores of the two groups were significantly different at 0.05 ($p = 0.002 < 0.05$) after the intervention (see Table 4.11).

To determine which gender was responsible for the difference, I computed the overall descriptive statistics for the analysis of gender, as tabulated in Table 4.11.

Table 4.10: Pre-test scores and post-test scores by gender

Pre-test Scores and Post-test Scores by Gender			
Interventions		Pre-test Scores	Post-test Scores
Female	Mean	25.00	31.25
	Std.Deviation	11.078	9.324
Male	Mean	27.22	36.67
	Std.Deviation	10.264	14.075
Total	Mean	26.82	35.68
	Std.Deviation	**10.364**	**13.442**

Table 4.10 shows that both male and female students had benefited from the intervention. Next, I investigated which students' proficiency was causing the differences. For this, an ANOVA was calculated to investigate the difference in students' achievement in both types of knowledge in the post-test of differential calculus. These results are tabulated in Table 4.11.

Table 4.11: Over all one-way analysis of variance summary table comparing groups on differential calculus achievement after the treatment

Comparing groups on differential calculus understanding						
Statistic		**Sum of Squares**	**Df**	**Mean Square**	**F**	**Sig.**
Pre-test Scores	Between Groups	1.818	1	1.818	.017	.898
	Within Groups	6980.000	64	109.063		
	Total	6981.818	65			
Post-test Scores	Between Groups	1654.596	1	1654.596	10.495	.002
Within Groups		10089.722	64	157.652		
Total		**11744.318**	**65**			

The results in Table 4.11 show that there was a statistically significant difference in students' achievement in differential calculus post the intervention (F (1, 64) = 10.495, p= 0.002<0.05). There was a statistically significant difference in students' achievement in the pre-test of differential calculus (F (1,64) = 0.17, p = 0.898>0.05) with effect size (ES) d = 1. Thus, it could be argued that the improvement was the result of the treatment. Students' results on the test of conceptual and procedural understanding of differential calculus were analyzed.

Research Question1: Table 4.12 shows the level of proficiency in terms of students' conceptual and procedural understanding in differential calculus achievement when taught using GeoGebra (experimental Group 1) and when taught through conventional lecturing (control Group 2).

Table 4.12: Descriptive analysis of student proficiency in conceptual and procedural understanding

Pre-test conceptual-procedural, Post-test conceptual and post-test procedural with Groups					
Groups		**Pre-test Conceptual**	**Pre-test Procedural**	**Post-test Conceptual**	**Post-test Procedural**
Experimental	Mean	16.33	10.67	16.83	24.33
	Std. Deviation	6.557	5.833	8.146	7.512
Control	Mean	13.89	12.78	20.00	11.25
	Std. Deviation	7.281	5.909	9.562	8.399
Total	**Mean**	**15.00**	**11.82**	**18.56**	**17.20**
	Std. Deviation	**7.016**	**5.925**	**9.020**	**10.308**

Table 4.12 shows that students' conceptual and procedural understanding of the differential calculus material in GeoGebra software-assisted learning had improved whereas students' conceptual and procedural understanding of the differential calculus material in the control group indicated improvement only in terms of conceptual understanding. In the case of procedural understanding, nothing had changed, or understanding had diminished slightly. Further statistical tests were required on the post

test data, firstly a normality test on the results obtained from the post-test of differential calculus proficiency. The reason for carrying out further analysis was to determine whether the data were normally distributed or not, enabling me to choose the types of tests I used (parametric such as a t-test or non-parametric such as a Mann Whitney test) (Elliott & Woodward, 2007). Thus, the descriptive analysis of the normality test of the posttest data was computed and is tabulated in Table 4.13.

Table 4.13: Descriptive analysis of normality test of post-test data

Tests of Normality of post-test							
		Kolmogorov-Smirnov			**Shapiro-Wilk**		
	Groups	**Statistic**	**Df**	**Sig.**	**Statistic**	**df**	**Sig.**
Post-test	Experimental	.139	30	.144	.924	30	.033
Scores	Control	.128	36	.145	.962	36	.253

Kolmogorov-Smirnov and Shapiro-Wilk tests are designed to determine whether the observed data fit the shape of a normal curve (bell curve) closely. If a test does not reject normality, this suggests that a parametric procedure that assumes normality (e.g. a t-test) can be safely used (Elliott & Woodward, 2007). However, the results in Table 4.13 indicate that the data were normally distributed for the value of $p = 0.144$ and $p = 0.145$ for the experimental and control group in learning differential calculus respectively and were greater than 0.05 in the Kolmogorov-Smirnov test. In contrast, the Shapiro-Wilk test in Table 4.13 indicates that the data in the experimental group were not normally distributed as the p-value was less than 0.05; however, the data for the control group were normally distributed. Thus, further investigation using another test was required. Table 4.14 reflects the post-test data normality test for both types of understanding to determine for which types of proficiency the data were not normally distributed.

Table 4.14: Descriptive analysis of normality test of post-test proficiency data

Tests of Normality Post-test							
Groups		**Kolmogorov-Smirnov**			**Shapiro-Wilk**		
		Statistic	**Df**	**Sig.**	**Statistic**	**df**	**Sig.**
Conceptual	Experimental	.199	30	.004	.909	30	.014
	Control	.144	36	.057	.970	36	.434
Procedural	Experimental	.251	30	.000	.887	30	.004
	Control	.145	36	.055	.923	36	.016

Table 4.14 shows that data from the post-tests of both types of knowledge in the experimental group were not normally distributed; on the other hand, in the control group, these data were normally distributed. Table 4.10 above summarizes the descriptive statistics of the post-test of conceptual understanding scores for the experimental group (n = 30) and the control group (n= 36), M = 16.8333 (SD = 8.14559) and M = 20 (SD =9.56183) respectively. Descriptive statistics of the scores on the post-test of procedural understanding of differential calculus for the experimental group (n = 30) and control group (n = 36) were reported as M = 41.1667 (SD = 13.81424) and M = 11.2500 (SD = 8.39855) respectively. The skewness for participants in the two groups in terms of the scores on the post-test of conceptual understanding was computed as .329 and -.078 respectively, whereas for scores on the post-test of procedural understanding this was reported as .329 and .232, respectively. The kurtosis for participants in the experimental group and the control group in terms of the post-test of conceptual understanding was - 1.281 and -.547, whereas for scores on the post-test of procedural understanding this was reported as -1.189 and -.653, respectively. Their scores in terms of post-intervention procedural understanding were slightly positively skewed, which indicated that most participants tended to score lower than

the mean score. The result of negative kurtosis meant that their test score distributions for both types of understanding were flatter than the normal distribution, indicating that test scores were spread out rather than grouped. As no data were normally distributed in this study, the non-parametric test used the MannWhitney U test (Elliott & Woodward, 2007). The results of the Mann Whitney test on students' achievement on differential calculus are reported in Table 4.15.

Table 4.15: Mann Whitney U test on students' scores in differential calculus

Time	Groups	N	Effect size(r)	Mean Rank	Median	Sum Ranks	Of Z-value	U	P
Pre-test	Experimental	30	.19	37.42	15	1122.50	-1.551	422.5	.121
conceptual	Control	36		30.24	15	1088.50			
	Total	66							
Pre-test	Experimental	30	.71	30.03	10	901.00	-1.385	436	.166
procedural	Control	36		36.39	15	1310.00			
	Total	66							
Post-test	Experimental	30	.72	29.93	17.5	898.00	-1.397	433	.163
Conceptual	Control	36		36.47	20	1313.00			
	Total	66							
Post-test	Experimental	30	.83	47.10	20	1413.00	-6.729	132	.000
Procedural	Control	36		22.17	10	798.00			
	Total	**66**							

The Mann-Whitney U test showed that procedural proficiency/understanding was statistically significant in both experimental (Md = 20, n = 30) and control groups (Md =10, n = 36) after the intervention (U = 132, z = -6.729, p<0.05); students' scores in the post-test of conceptual understanding of differential calculus in the experimental group (Md=17.5,n=30) and control group (Md=20,n=36) did not show any visible significant difference between the two (U = 433, z = -1.397, p = 0.163>0.05); pre-intervention procedural understanding of differential calculus of students in the experimental group (Md = 10, n = 30) and of those in the control group (Md = 15, n = 36), (U = 436, z = -

1.385, $r = .71$, $p = .166>0.05$); conceptual understanding of differential calculus of students in the experimental group (Md = 15, n = 30) and students in the control group (Md = 15, n = 36), ($U = 422.5$, $z = -1.551$, $r = .19$, $p = 0.121>0.05$) post intervention also showed no visible significant difference. However, using computed effect size(ES), which can be calculated as $r=z/\sqrt{N}$, where N is the total number of participants and z is the z- value computed by SPSS, the groups had small to moderate differences in terms of pre-test conceptual, pre-test procedural and post-test conceptual understanding of differential calculus in both groups (Rice & Harris, 2005). In addition, observation of both types of knowledge in each group revealed that in the experimental group, both differential calculus proficiency (conceptual) (median=15 to median=17.5) and procedural (median= 10 to median = 20) had increased. In contrast, students' procedural understanding proficiency had diminished in the control group (median = 15 to median = 10), whereas the conceptual understanding of differential calculus increased (median=15 to median=20).

Therefore, for Research Question 1 that asks How does the level of proficiency in differential calculus compare in students taught using GeoGebra (experimental Group 1) and students taught through conventional lecturing (control Group 2)? the study found that students who learned differential calculus with the help of GeoGebra scored highly statistically differently, with students improving by 46% (see Table 4.17), and more students made greater progress in procedural understanding (see Table 4.15).

Research Question 2 asks how the level of proficiency in differential calculus compares within the experimental group (Group 1) pre and post the intervention incorporating the use of GeoGebra. The results from students' proficiency variables measured before and after the interventions were used to examine their progression from pre-test to post-test.

In this case, the subject was measured twice (before and after the intervention), giving a pair of observations. Thus, the progression of each group from pre-test to post-test on proficiency variables (conceptual and procedural) was analyzed by using the paired sample t-test, as all assumptions were met for all variables by the Levene test for equality of variances. The results are reflected in Table 4.16.

Table 4.16: Differences in student proficiency in experimental group

Student	Paired Differences of Experimental group							
Proficiency	**Mean gain**	**SD**	**Std. Error Mean**	**95%CI**				
				Lower	**Upper**	**t**	**Df**	**P**
Pre-test-Post-test	-14.2	12.3	2.25	-18.77	-9.57	-6.3	29	.000*
Pre-C-Post-C	-.5	7.92	1.45	-3.46	2.46	-.35	29	.732
Pre-P-Post-P	-30.5	13.5	2.5	-35.5	-25.5	12.4	29	.000*

Pre-C: Pre-test conceptual*Significant at 0.05

Post-C: post-test procedural

Pre-P: Pre-test procedural

Post-P: Post-test procedural

The results of a paired samples t-test, (see Table 4.16) indicate the mean gain in students' proficiency in the two types of knowledge between pre-test and post-test, and in particular the mean gain of conceptual and procedural understanding of DC before and after an intervention. The p-value for the comparison of pre-test and post-test conceptual understanding of differential calculus was p = 0,0.732 and 0, respectively. Students in the experimental group improved significantly in terms of procedural understanding (t (29) = -9.36, p<0.05, d = -30.5/13.5 = -2.35) but did not show a visible improvement in terms of conceptual understanding of DC when being taught using GeoGebra (t (29) = -35, p>0.05, d = -.5/7.92 = -0.06). In general, students in the experimental group improved their proficiency significantly (t (29) = -6.3, p<0.05, d =

-14.2/12.3 = -1.2). To determine the extent to which the improvement of students occurred after the intervention, I used Cohen's d effect size standard; this is the numerical method of interpreting the strength of a reported correlation, avoiding simply 'binarizing' matters. It states the effect size of 0.2 for small, 0.5 for medium and 0.8 and above for large (Cohen et al., 2018; Lakens, 2013; Mills & Gay, 2019). Table 4.17 indicates the interpretation of effect size computed in Table 4.16.

Table 4.17: Computed effect size of pre-test and post-test

Observations	Computed effect size	Percentile gain	Interpretation
Pre-test-post-test	-1.7	46%	Improvement is high as the value is greater than Cohen's d standard 0.8.
Pre-C-Post-C	-0.06	2%	Improvement is low as the value is smaller than Cohen's d standard 0.2.
Pre-P-Post-P	**-2.35**	**49%**	Improvement is high as the value is greater than Cohen's d standard 0.8.

The negative value indicates the direction of means and as is indicated in Table 4.16, negative values occurred as the means within post-intervention were subtracted from pre intervention on each observation. In other words, scores were lowered by the effect of the program used in the study.

4.6 Evaluation Stage of the Cycle Model

If the average post-test score is higher than the average pre-test score, it makes sense to conclude that the treatment might be responsible for the improvement. The difference between the control group's pre-test and post-test composite violence scores was -4.4444 (26.6667 – 31.1111) while the post-test difference between the experimental and control group was -10.1 (31.1111- 41.1667). The intervention, therefore, boosted the pre-post increase in the aggression score by 44% (-4.444/-10.1).

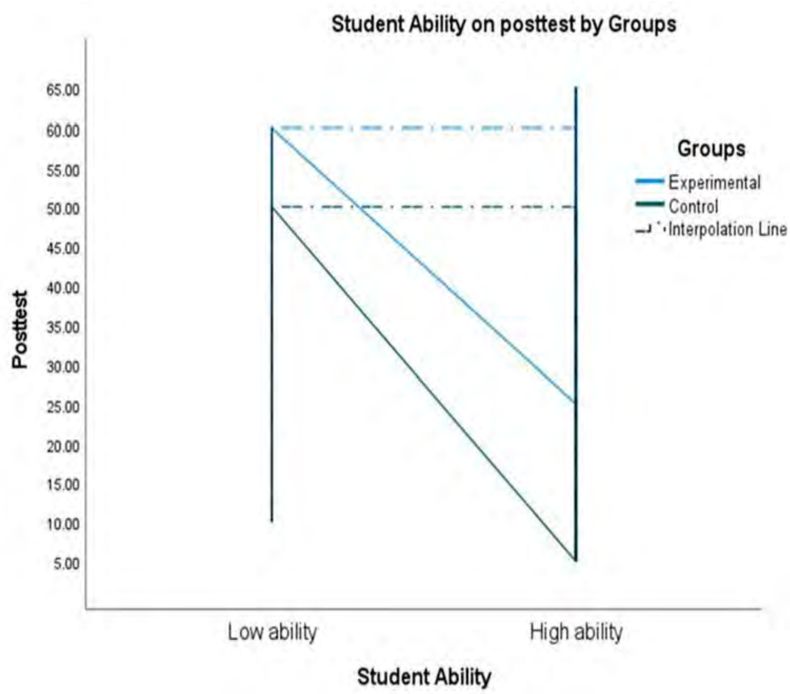


Figure 4.2: Student ability on posttest by group

Figure 4.2 shows that both high ability and low ability students were advantaged by the treatment, but students in the experimental group scored higher than the control group.

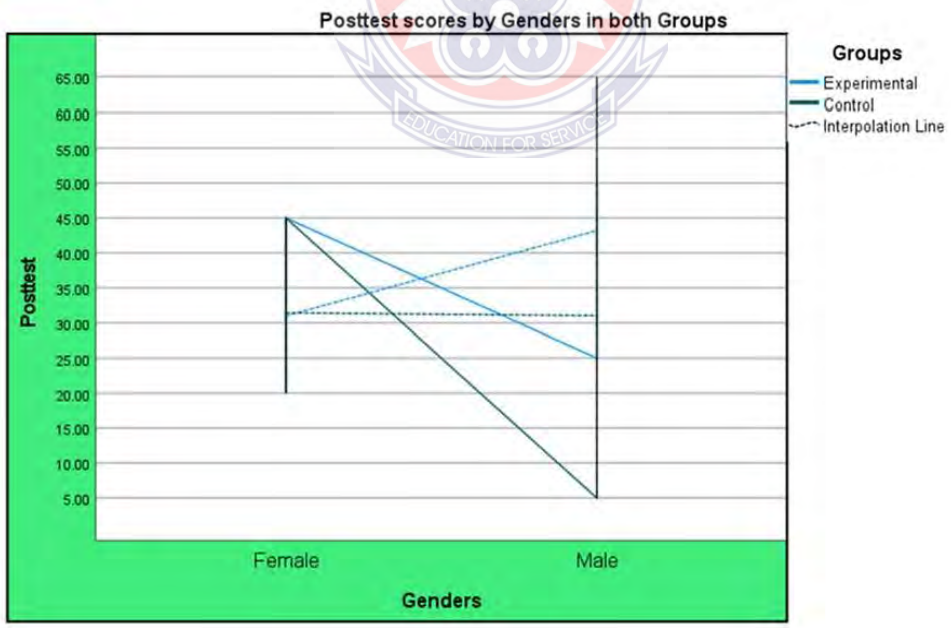


Figure 4.3: Gender difference in scores on post-test in both groups

Figure 4.3 indicates that both female and male students in the experimental group scored higher than students in the control group. These findings are in line with a study that found that female students learning a given course with the help of GeoGebra achieved scores that were superior to those of a control group taught by traditional methods. They also showed greater survival of learning impact, defined by learning output retained in memory as indicated in scores on a post-test (Alabdulaziz, Aldossary, Alyahya & Althubiti, 2021).

One of the aims of this study was to explore how student participants perceived learning with the aid of GeoGebra after the intervention had been implemented. To this end, in addition to the interview, a questionnaire was distributed to the experimental group after the intervention. The validity of the interview and the reliability and validity of the questionnaire was discussed in chapter three. Questionnaire items were grouped according to three core themes (perception towards the existence of the technology in the environment (the first step of the cycle model), perception towards scaffolding (the vertical and horizontal interaction step of the cycle model), and their preference for using technology/GeoGebra (individual perspectives step of the cycle model). This was called the three-perception scale. Items such as 4.5 are grouped under 'perceptions towards technology', items 1, 2, 3, 7, 8, 9, 10, 11 and 14 are categorized as 'perceptions 'towards technology in learning, and items such as 6, 12, 13 and 14 are categorized as 'perception towards scaffolding' during the intervention. The results of the analysis of responses are provided in Table 4.18.

Table 4.18: Percentages and means of perceptions scales

Items	**Scales**					
Preferences in the classroom	SD	DA	N	A	SA	Mean
1: At the beginning, I did not like GeoGebra	11(35.5)	7(22.6)	5(16.5)	6(19.4)	1(3.2)	2.3(3.7*)
2: I like GeoGebra because It is dynamic and free for everyone.	6(19.4)	4(12.9)	2(6.5)	7(22.6)	11(35.5)	3.433
3: Right now, I'm more open to learning using GeoGebra.	12(38.7)	?2.6)	3(9.7)	5(16.1)	3(9.7)	3.6
7: I think working with GeoGebra is frustrating.	6(19.4)	?9.0)	1(3.2)	5(16.1)	9(29.0)	.0667(2 933*)
8: I am comfortable with GeoGebra in learning calculus.	2(6.5)	-	2(6.5)	12(38.7)	14(45.2)	4.2
9: I do not want to use GeoGebra for my future study.	3(41.9)	7(22.6)	9.7)	2(6.5)	5(16.5)	2.3 (3.7*)
10:GeoGebra makes calculus more difficult for me.	(38.7)	7(22.6)	3(9.7)	5(16.1)	3(9.7)	2.3333 (3.6667*)
12: The instructional material in learning calculus through GeoGebra is well organized.	2(6.5)	2(6.6)	1(3.2)	11(35.5)	14(45.2)	4.1000
15: I achieved better marks after I learned calculus through GeoGebra software.	3(9.7)	1(3.2)	2(6.5)	9(29.0)	15(48.4)	4.0667
Overall	**2.16**	**1.42**	**0.72**	**2**	**2.424**	**3.7**
Existence of software	Scales					
4: There is mathematical software for learning calculus in secondary school.	10(32.3)	2(6.5)	2(6.5)	9(29.0)	7(22.6)	3.033
5: There is no mathematical Software for learning calculus.	3(9.7)	7(22.6)	3(9.7)	0(32.3)	7(22.6)	3.3667 (2 633*)
Overall	**0.42**	**0.291**	**0.162**	**0.613**	**0.452**	**2.8**
Scaffolding in the classroom	Scales					
6: I need a lot of help when doing new things by using technology like GeoGebra.	3(9.7)	3(9.7)	1(9.7)	9(29.0)	4(45.2)	3.9333
12: I get enough time to do the activity on my own in the Laboratory classroom.	8(25.8)	(22.6)	4(12.9)	(6.5)	(29.0)	2.900
13: I depended on others to do the activity while the program was running in the classroom.	7(22.6)	8(25.8)	2(12.9)	4(12.9)	(29.0)	3.000
14: I achieved better marks after I learned through GeoGebra Mathematical software.	3(9.7)	1(3.2)	2(6.5)	(29.0)	5(48.4)	4.0667
Overall	**0.678**	**0.613**	**0.42**	**0.774**	**0.152**	**3.5**

Note*indicates the reversed mean in positive statements.

Students' perceptions were elicited by a questionnaire consisting of 14 items (nine items for perceptions towards GeoGebra, two items on the existence of the technology and four items on scaffolding by GeoGebra). The questionnaire was distributed to the experimental group only to determine their perceptions based on their experience of using the GeoGebra software. The results of the analysis of the responses to the questionnaire reflect students' perceptions towards GeoGebra for teaching in the classroom (with an overall mean of M = 3.7) and perceptions of scaffolding activities (an overall mean of M = 3.5) in the classroom. These were positive whereas perceptions towards the existence of technology for the mathematics classroom were negative (with an overall mean of M = 2.8). It appeared that students were not familiar with the technology for teaching and learning calculus before the intervention. These students had never used GeoGebra before. This may be why they enjoyed using GeoGebra software for learning as it is a dynamic mathematical software (M = 3.7). The study found that the items in the questionnaire that had the highest mean were those which showed that students were comfortable using GeoGebra for learning calculus (M = 4.2), indicating that the software increased students' motivation, confidence, and achievement. The lowest mean was item 2.9, responses to which revealed that students did not think that working with GeoGebra was frustrating. Studies have found that technology in the classroom improves not only student performance and achievement but also student motivation (Harris, Al-Bataineh, & Al-Bataineh, 2016). GeoGebra software can increase students' interest, confidence, and motivation in learning calculus. These findings correspond to those of a study by Arbain and Shukor (2015). The three-perception scale was developed by condensing the items in each category/theme; negative statements were reversed and recoded into positive statements (Sadeghiyeh et al., 2019).

Table 4.19: Mean of perception scale

Perception scales	Gender	Mean	Std Deviation	N
Preference	Male(M=3.7111)	3.71	.548	30
	Female(M=3.7111)			
Existence	Male(M=2.82)	2.83	.834	30
	Female(M=2.9)			
Scaffolding	Male(M=3.5)	3.48	1.028	30
	Female(M=3.35)			

Table 4.19 shows that there was no difference in means according to gender in the three perception scales measuring perceptions of the use of technology/GeoGebra in classroom learning and teaching of differential calculus.

4.7 Internalization and Externalization Stages of Cycle Model

These findings suggest that students in the experimental group gained more advantage from the intervention than the control groups gained from traditional teaching. It was anticipated that students in the experimental group would internalize the GeoGebra mathematical software and externalize their knowledge in the environment with their mentors or students after they had completed their studies at university. Vygotsky's concept of internalization is a model of learning alienated activities; interconnected dialogic processes (scaffolding) (i.e. decontextualized) in which the individual uses sociocultural practices (teaching and learning of differential calculus with the aid of GeoGebra Mathematical software) through engagement with these interconnections (activities designed by Tharp (1993))(Smith, Dockrell, & Tomlinson, 1997).In Vygotsky's theory, externalization occurs when learning and teaching process outcomes in sociocultural practices are fossilized in terms of the cognitive proficiency (e.g., conceptual and procedural) of human adults. Behavior-based proficiency in competencies such as how to approach a task, how the subject's meta strategic understanding has evolved in the course of engagement with the task, and successful search procedures in the form of self-produced state-based feedback, may well constitute a separate layer of competence with a powerful potential role in the growing interaction between subject and environment (Smith et al., 1997).

In general, technological (GeoGebra) aids within the cycle model were provided to the students to increase students' motivation toward learning differential calculus, increasing students' opportunities to operate with mathematical representations of both conceptual and procedural knowledge, making learning more meaningful and enjoyable in the progression of ZPD by a scaffolder (teacher) (please see Figure 3.4), maximizing visualizations of the learned topic (differential calculus) by the software and maintaining the students' attention on the lesson to make them ready for applying the situations in the environment stages of the cycle model.

4.8 Apply in the Environment Stage of the Cycle Model

Those students who had internalized the activities were expected to externalize the activities again in the school environment, which is known as communities of practice. This stage was similar to step1 of the cycle model, but participants were now familiar with the environment and familiar with the activities they had engaged in during the treatment.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.0 Overview

This chapter concludes the thesis by providing a summary of the major findings and recommendations. The chapter opens with a brief overview of the research design, followed by a summary of the empirical findings. Finally, further reflections and implications of the study with recommendations are provided.

5.1 Summary and discussion of major findings

In this section, a summary of major findings of the study is organized according to the three research objectives, followed by the discussion of the findings, in Student proficiency, and perception scales.

5.1.1 Student proficiency

A paired samples t-test presented in Table 4.13 indicates the mean gain in students' proficiency in the two types of knowledge between pre-test and post-test and in particular the mean gain of conceptual and procedural understanding of DC before and after an intervention. The p-value for the comparison of pre-test and post-test conceptual understanding of differential calculus and pre-test and post-test procedural understanding were p = 0,0.732 and 0, respectively. Students in the experimental group improved significantly in terms of procedural understanding (t (29) 9.36, $p<0.05$, d = -13/7.6 =-1.7) but did not show a visible improvement in terms of conceptual understanding of DC (t (29) = -35, $p>0.05$, d = -.5/7.92 =-0.06). In general, students in the experimental group improved their proficiency significantly (t (29) = -6.3, $p<0.05$, d = -14.2/12.3 =-1.2). To determine the extent to which the improvement of students had occurred after the intervention, I used Cohen's d effect size standard. The combination of the two mathematical proficiencies of students in the understanding of differential calculus in the experimental group showed great improvement, with an effect size of $d = 1.7$ and with a percentile gain of 46%. Students in the experimental group showed great improvement in procedural understanding, with an effect size of $d = 1.2$ and a percentile gain of 49%; in conceptual understanding of differential calculus; however, the students showed only slight improvement with an effect size of $d = 0.02$ and a percentile gain of 2%. These findings indicate that using GeoGebra for teaching DC helped students to improve their procedural understanding more than their conceptual understanding, which is in contrast to the findings of Ocal (2017).

The analysis of post-test data using the Mann-Whitney U test indicated that procedural proficiency/understanding was statistically significantly different in the two groups ($U = 132$, $z = -6.729$, $p<0.05$), whereas student's proficiency in procedural understanding of differential calculus after the intervention ($U = 433, z = -1.397$, $p = 0.163 > 0.05$), and procedural understanding of differential calculus before the intervention (U = ($U = 422.5$, $z = -1.551$, $p = 0.121 > 0.05$) showed no visible significant difference (436, $z = -1.385$, $p = .166 > 0.05$) between the groups. Using computed effect size (ES), the groups showed small to moderate differences in terms of pre-intervention conceptual, pre-intervention procedural, and post-intervention conceptual understanding of differential calculus, indicating that there was a relationship between the two (Rice & Harris, 2005). In addition, when observing both types of knowledge in each group, the findings revealed that in the experimental group, students' differential calculus proficiency (conceptual: median = 15 to median = 17.5, and procedural: median = 10 to median = 20) had increased as had students' overall scores (Diković, 2009). In the experimental group, procedural understanding of differential calculus had increased more than conceptual understanding as GeoGebra enables students' visualization. The transformation of procedural to conceptual understanding requires an integral gradual reconstruction of students' perceptions towards the use of GeoGebra, even though the students expressed positive perceptions towards the use of GeoGebra during the study (Attorps, Björk, & Radic, 2011). Therefore, the findings indicated that instruction with GeoGebra had a positive effect on students' scores in both conceptual and procedural understanding of differential calculus, contrary to the findings of Ocal (2017), who reported that GeoGebra did not affect procedural understanding. However, procedural understanding can be considered as the mediator between conceptual understanding and student achievement (Zulnaidi & Zamri, 2017). In contrast, proficiency in procedural understanding was slightly diminished in the control group (median = 15 to median = 10), whereas proficiency in conceptual understanding of differential calculus was increased (median = 15 to median = 20). The findings by Handelsman et al. (2004), Hurd (1998) and Williams, Papierno, Makel, and Ceci (2004) revealed that at the college level, courses focused more on memorization and less on conceptual understanding and computational/procedural understanding of the material.

Finally, this study revealed that students in the experimental group were more advantaged than those in the control group in terms of both types of proficiency and had also developed positive attitudes towards the use of GeoGebra in the classroom when used with the developed cycle model in constructivism approaches. These findings are in keeping this those of several earlier studies on overall student achievement (Akanmu, 2015; Alkhateeb & Al-Duwairi, 2019; Arbain & Shukor, 2015; Doğan & Içel, 2011; Hutkemri, 2014; Jelatu, 2018; Nobre et al., 2016; Ocal, 2017; Preiner, 2008; Rohaeti & Bernard, 2018; Saha et al., 2010; Tatar, 2013; Thambi & Eu, 2013; Zulnaidi & Zamri, 2017).

5.1.2 Perception scale

The third research question in the study, what are students' experiences and perceptions towards using mathematical software (GeoGebra) in learning calculus concepts? was addressed by the questionnaire and interview. The items in the questionnaire and the questions asked in the interview were grouped according to three perception scales. These were the preference scale, the scaffolding scale and the existence scale (see Table 4.9). Findings from these scales revealed that students had developed positive perceptions towards using the software GeoGebra in the classroom in terms of the preference scale, and towards the scaffolding activities included in the model during the intervention. Students were neutral on whether technology was integrated into elementary and secondary school mathematics teaching and learning, suggesting that they were neutral about the existence of technology or of using technology, particularly GeoGebra, at the school level for learning calculus (Bretscher, 2014). These findings were consolidated in the interviews conducted with five students. In general, the existence of technology, a preference for technology, and scaffolding affected students' perception of the use of technology in the classroom, in line with the findings by Nikolopoulou and Gialamas, (2013), Thambi and Eu (2013) and Željka and Trupčević (2017).

5.2 Recommendations

With the current rapid technological advancement, good quality education cannot be achieved without the integration of technology. That is why the Ministry of Education has planned to implement a Ghanaian educational road map (Teferra et al., 2018). To this end, this road map (2019–2030) integrates technology such as Math Lab, Latex and Mathematica as one course named Mathematical Software for the Mathematics Department. However, not all these technologies are freely accessible from the internet.

GeoGebra Mathematical software is an open and freely available access software, however. This study thus recommends that the government integrates GeoGebra mathematical software in teaching differential calculus at the tertiary level. As the findings showed that the study was successful in improving both conceptual and procedural understanding of differential calculus, it is therefore recommended that both mathematics teachers and students be encouraged to use computer-based multimedia instruction. GeoGebra can be regarded as a multimedia tool to provide equal opportunities for students of different abilities (Anyanwu, Ezenwa, & Gambari, 2014). There are several models of learning being practiced by various universities abroad that work for all contexts of learning, such as the ASSURE and the ADDIE model. But in the cycle model used in this study, the duration and type of activities in the classroom depend on the context/environment and the reasons for learning by technology (see Chapter 2, the review of literature), the nature of the students, and the availability of technology and laboratories. These elements were considered in this study and evaluated, and it was decided that the cycle model using GeoGebra was most suitable for implementation in the intervention for the teaching and learning of differential calculus, following Vygotsky's theory of constructivism. The study was based on Vygotsky's ideas and the cycle model that was developed posits nine steps. This nine-step cycle model of learning differential calculus by GeoGebra benefited students. This study has shown the potential of a GeoGebra oriented classroom and the cycle model to benefit a developing country such as Ghana: the software is freely downloadable and can be installed on any computer or smartphone and it can be used offline. Developing countries, including my country Ghana, could thus use this nine-step cyclical model of implementation of GeoGebra in their own context as educational software technology is still out of reach for many developing countries. This is of course not the complete story and acquiring and using up-to-date technology has associated costs. The lack of internet access, especially in schools is also a constraint (Bekene, 2020; Mainali & Key, 2012). It is thus recommended that the GeoGebra program is included in mathematics curricula at all stages of education (Alabdulaziz et al., 2021). The study strongly recommended to the Ghanaian Government that the cycle model using technology, more specifically GeoGebra, was the best teaching process for all students at any educational level.

In summary, as the integration of technology in mathematics education cannot replace the teacher, teachers and students need to be equipped with both content knowledge (differential calculus), skills to effectively apply the given technology (for instance GeoGebra), and pedagogy (interactive teaching methods) to facilitate the teaching and learning processes (cycle model) for students' achievement (Koehler & Mishra, 2009).

5.3 Conclusion

This study shed light on the use and effect of GeoGebra in teaching and learning differential calculus in the Ghanaian context. Learners in the 21st century need technological support in the learning process because of the advancements made in technology for teaching and learning. A GeoGebra-oriented classroom uses one of these technologies that can be implemented in the classroom. Generally, the findings from this study were supported by previous studies discussed in Chapter 4. It developed a new cycle model for the implementation of the technology of GeoGebra in the classroom according to nine steps. Based on the discussion and the findings of the study, the following conclusions can be made. This study aimed to investigate the effect of GeoGebra software on students' learning differential calculus in terms of two psychologies of knowledge, that is conceptual and procedural understanding. It also investigated students' perceptions towards the use of GeoGebra. The GeoGebra classroom-oriented approach had a more positive effect on the conceptual and procedural understanding of students in learning differential calculus than the traditional teaching approach had on students in the control group. The gap in the zone of proximal development was reduced by using technology/GeoGebra and students were assisted in becoming self-learners after being scaffolded in the internalization stage of the cycle model. In the GeoGebra oriented classroom, students benefited more in terms of procedural understanding than conceptual understanding, while in the control group the reverse result was reported. The improvement in achievement/scores of students can be attributed to the vast learning opportunity they gained from the GeoGebra classroom-oriented approach. One of the advantages came from the interactivity and supplementary materials. What students found important and attractive during the intervention was scaffolding when explaining the concepts, modelling, rearranging of fixed differential calculus questions on topics discussed in the classroom, immediate feedback, discussion forums, and supplementary materials, both online and offline, such as reference books and collections of previous worksheets. Thus, the role of the teacher lay in identifying both environment and student ability, designing, guiding, helping, assisting, facilitating, giving feedback, evaluating, and motivating students to use their learning in the classroom and environment after they had developed their understanding (internalization) for externalization. In this regard, Vygotskian theory holds that cognitive development can be described as a process of internalizing culturally transmitted knowledge (that can be held by scaffolded) in the cycle model, in which the exposure to cultural models (cyclical model) stimulate a gradual internal process of knowledge growth (in both conceptual and procedural understanding) in students learning differential calculus with the help of GeoGebra (Nezhnov, Kardanova, Vasilyeva, & Ludlow, 2014; Vygotsky, 1978). The perceptions of students were found to be positive towards the GeoGebra classroom-oriented approach, as respondents agreed that scaffolding activities offered learning opportunities that were better than those in traditional classrooms. Perception is a part of the process of using technology (Bruce & Hogan, 1998). The study found that 74% of students were satisfied with the preferences of the GeoGebra lesson-oriented course offered in the study while 70% were also interested in scaffolding activities and seeing Tharp's (1993) activities included in the developed model during interventions.

Student respondents felt that the GeoGebra classroom-oriented approach was an interactive, engaging, convenient, and more resourceful approach to logical thinking and discovery. In addition, GeoGebra's classroom-oriented approach allowed students to become familiar with computers and to build some essential skills for their studies. The developed cycle model was evaluated and brought positive changes to students' learning of differential calculus, in terms of both perception and scores. These findings suggest that the cycle model that emanated from the study for learning and teaching could improve students' procedural and conceptual understanding. The study satisfied the principles of the fourth educational revolution which are that the teaching and learning process should be reshaped (Ally & Wark, 2020) and consistent with Common Core State Standards that do not recommend traditional teaching and learning approaches (Alabdulaziz et al., 2021). This study thus produced the cycle model for teaching and learning differential calculus using technology (Koehler & Mishra, 2009).

In summary, I strongly believe that the use of GeoGebra had a positive impact on visualization through self-exploration and social interaction when learning differential calculus, in terms of both scores and perceptions of students (Semenikhina et al., 2019).

Technology/GeoGebra provides an environment of communication and interaction between students and students, and teachers and students during the process of scaffolding (learning and education) that leads to effective teaching and learning landscapes (Ayub et al., 2008; Ayub et al., 2010; ten Brummelhuis & Kuiper, 2008). This creates positive perceptions among students towards the technology as an educational method and towards the subjects students study, which supports the findings of Alabdulaziz et al.(2021). Achieving conceptual and procedural understanding through combining different concepts can be significantly enhanced by using the digital tools of GeoGebra Mathematical software at the tertiary level, supporting the 21st century generation in the learning environment by employing the developed cycle model, which follows the concept of Koehler and Mishra (2009).

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APPENDICES

APPENDIX A

Introductory Letter

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF SCIENCE EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION
P. O. Box 25, Winneba, Ghana
math@uew.edu.gh
+233 (020) 2041076

September 07, 2022

Dear Sir/Madam,

LETTER OF INTRODUCTION: GIDEON COBBINAH (200008027)

I write to introduce to you the bearer of this letter, Mr. Gideon Cobbinah, a postgraduate student in the University of Education, Winneba. He is reading for a Master of Philosophy degree in Mathematics Education and as part of the requirements of the programme, he is undertaking a research titled –***The Use And Effect of Geogebra Software In Differential Calculus At University of Education, Winneba: An Investigation.*** He needs to gather information to be analysed for the said research and he has chosen to do so in your institution. I would be grateful if he is given the needed assistance to carry out this exercise. Thank you.

Yours faithfully,

Dr. Ali Mohammed

Graduate Coordinator

DEPARTMENT OF MATHEMATICS EDUCATION
UNIVERSITY OF EDUCATION
WINNEBA

www.uew.edu.gh

APPENDIX B

Consent Form for Students

Section A

Title of the questionnaire: An investigation of the effect of GeoGebra mathematical software on students' learning of mathematics

Dear Respondent

My name is Gideon Cobbinah and I am currently an Mphil student at the University of Education, Winneba (UEW), doing my thesis in the Department of Mathematics Education. This questionnaire forms part of my masters of philosophy research for the degree Mphil Mathematics Education at the University of Education, Winneba entitled: An investigation of the effect of GeoGebra mathematical software (GMS) on students' learning of mathematics (at University of Education, Winneba. You have been selected by a purposive sampling strategy. I invite you to take part in this survey. Permission to undertake this survey has been granted by the Ministry of Education (MOE) on behalf of University of Education, Winneba, and the Ethics Committee of the Department of Mathematics Education, UEW. If you have any research-related enquiries, they can be addressed directly to me or my supervisor. My contact details are phone: 0540788466 or e-mail: gideoncobbinah711@yahoo.com and my supervisor can be reached at 0245309437, e-mail: pakayuure@gmail.com Department of Mathematics Education, Faculty of Social Science, UEW. By completing the questionnaire, you imply that you have agreed to participate in this research study. Please return the completed questionnaire to the department secretary before the date indicated on the questionnaire. I would like to express my gratitude for your time and cooperation, beforehand, in completing this questionnaire. This study is purely for academic purposes. Your sincere, honest and timely responses are vital to the success of this study. There is no "right" or "wrong" answer here; rather, what is required is your opinions.

Section B

GIVING INFORMED CONSENT

This section indicates that you are giving your informed consent to participate in the research:

I confirm that I have read this consent requesting my consent and understand the information provided and do agree to participate in this study. I do understand that my participation is voluntary and I hereby add my signature below as I am over 18 years of age.

Participant's signature ________________________Date________________________

Section C

Demographic Information

Please, tick in the appropriate boxes

Age 18-23 [ ]   24-29 [ ]   30-35 [ ]   36-41 [ ]

Sex: Male [ ]   Female [ ]

Educational level: Undergraduate [ ]   Postgraduate [ ]

APPENDIX C

Student Questionnaire

The items use a five-point Likert scale ranging from strongly disagree (1) to strongly agree

(5). Note that 1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, 5 = strongly

agree.

Questions (Items)	1	2	3	4	5
1. At first, I did not like GeoGebra.					
2. I like GeoGebra because it is dynamic mathematical software and free for everyone.					
3. Right now, I’m more open to investigations using GeoGebra.					
4. There is mathematical software for learning calculus in secondary school.					
5. There is mathematical software for learning calculus, but I did not know how to manipulate the software at my institution.					
6. I need a lot of help when doing new things when using technology like GeoGebra.					
7. I think working with GeoGebra is frustrating.					
8. I am comfortable with GeoGebra when learning calculus.					
9. I do not want to use GeoGebra in my future studies.					
10. GeoGebra makes calculus more difficult for me.					
11. The instructional material for learning calculus through GeoGebra is well organized.					
12. I get enough time to do the activity on my own in the laboratory classroom.					
13. I depend on others to do the activity while the programmes is running in the laboratory classroom.					
14. I achieved higher marks after I learned calculus through GeoGebra software.					

Thank you in advance.

Interviews with students

1. Can you tell me what you gained and what you lost when you learned calculus through GeoGebra?

2. Do you think that learning calculus through GeoGebra software is useful for students?

3. Do you want to share this software with your friends?

4. Why do you think that not all subjects integrate software in their teaching and learning?

5. Is there any mathematical software you know of that you could use to study your other subjects? If so, tell me about it; if no, what is the reason for this, do you think?

CONSENT FORM FOR TEST QUESTIONS

Dear Students

I am an Mphil student at the university of education, Winneba (UEW) doing my thesis at the Faculty of Social Sciences in the Department of Mathematics Education. I am conducting a research study on an investigation of the effect of students' learning mathematics through GeoGebra software at University of Education, Winneba. Therefore, I request your assistance by inviting you to participate in the study by answering the questions below. The insights gained from these Pre-test Questions will provide helpful information, clarify mathematics student-teachers beliefs and help me to accomplish my research. The results would help to improve and develop mathematics teaching and learning at universities. The completion of these pre-test questions will take about 60 minutes. Your participation is voluntary, and you are free to discontinue at any time. As a participant, you have the right to ask for clarification and refuse to answer any questions. All information you provide will be kept strictly confidential and the researcher and the researcher's supervisors are the only ones who will be able to access this information.

Your name not be used or associated with the study. There are no risks to you or your privacy if you decide to participate in my study. But if you choose not to participate that is fine. However, your participation and your opinions are important in helping me to obtain answers to my research questions. I would appreciate your taking the time. If you are willing to participate in the research study, please put your signature here

……………………………………………………………………………………

QUESTION TYPES

Instructions

Answer all the questions carefully and neatly.

These are multiple-choice questions and questions requiring short answers.

The time allowed to complete these questions is 60 minutes.

Groups (Experimental Group or Control Group)

Name: ____

ID: ____

Part1: Choose the best answer and encircle it.

1. What is the value of $\lim_{n \to 0} \left(\frac{\sin 5x}{2x}\right)$?

   a. $\frac{5}{2}$ c. 1

   b. $\frac{2}{5}$ d. $\frac{1}{2}$

2. What is the value of $\lim_{x \to 0} \frac{\sqrt[3]{x+1} - 1}{x}$?

   a. 1/6 d. $\frac{1}{3}$

   b. 3 e. Does not exist

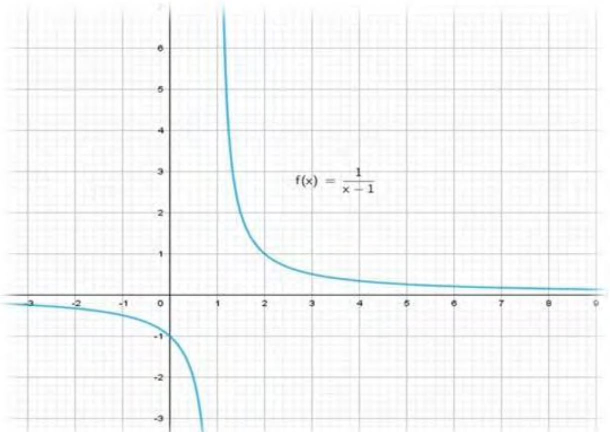
   c. 6

3. Find the values of $a \& b$ such that the diving board function

$$f(x) = \begin{cases} 2, x \leq -1 \\ ax + b < x \leq 3 \\ -2, x \geq 3 \end{cases}$$

   a. $b = 1,\ a = -1$
   b. $a = 1,\ b = -1$
   c. $a = 2,\ b = -2$

4. Assuming that the graph of the function $f(x) = \frac{1}{x-1}$ is given by



Which of the following is not true about this graph.

a. $f(x)$ is continuous in its domain

b. The vertical asymptote the function is line x=1

c. x - $axis$ is the horizontal asymptote of the function.

d. The value of $\lim_{x \to 1} f(x) = \infty$

5. Let (x) = $e^{\ln(x^2)}$ be given function. Which of the following is the derivative of (x)?

a. $f'(x)$=2x

b. $f'(x)$= $e^{\ln(x^2)}$

c. $f'(x)$=2

d. None of the above

6. Equation of tangent line to the curve $f(x) = x^2 + 2$ that passes through the point (0,2) is:

a. y = 2x+2

b. y = - 2x+2

c. y = 2

d. y = - 2

7. Let the composition function $h(x)$=$f(g(x))$ be given as the differentiable function of x. Which of the following is true about $h(x)$?

a. $\frac{d}{dx}\, h(x) = \frac{d}{dx}\, f\left(g(x) + \frac{f}{dx}\, g'(x)\right)$

b. $h'(x) = f'\big(g(x)\big) + g'(x)$

c. $h'(x) = f'\big(g(x)\big) \times g'(x)$

8. The derivative of $g(x)$=$cos(cos$–1(√x2+ 1)) is

a. $g'(x) = \sqrt{x^2 + 1}$

b. $g'(x) = \frac{2x}{\sqrt{x^2+1}}$

c. $g'(x) = \frac{x}{\sqrt{x^2+1}}$

9. Which of the following is true about the critical point(s) ''c'' of the function
$p(x) = \frac{x^3}{3} + x^2 + x + 1$

a) $c = \pm 1$ is the only critical point.

b) $c = 1$ is the only critical point

c) $c = -1$ is the only critical point

10. Let M_1 be the slope of the function $y = 5^x$ at the point $x = 0$ and let M_2 be the slope of the function $y = \log_5 x$ at $= 1$. Then

a) $M_1 = ln(5)\, M_2$

b) $M_1 = M_2$

c) $M_1 = -\, M_2$

d) $M_1 M_2 = 1$

e) $M_2 = ln(5)\, M_1$

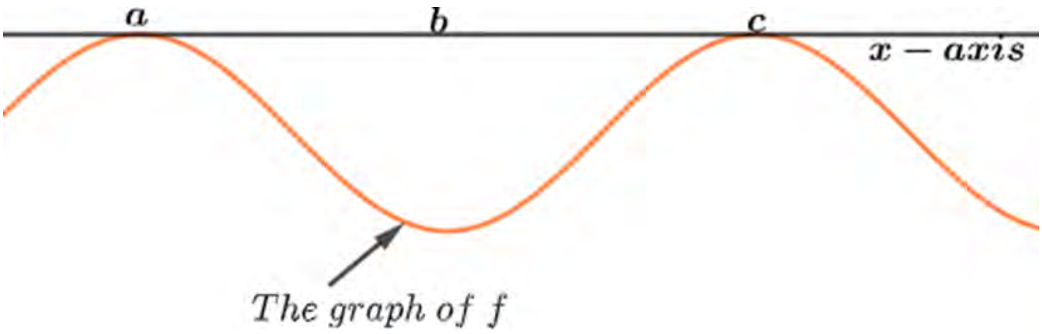
11. By using the power rule of derivatives, you that the derivative of $x^{1/3} = \frac{1}{3}x^{-2/3}$

for every $x \neq 0$. Then, $\lim_{x\to 8} \frac{\left(\frac{1}{8}\right)^{1/3} - 1}{x-8} = \frac{1}{2a}$, where $a =$ ______________

a) 4 c) 6

b) 8 d) 12

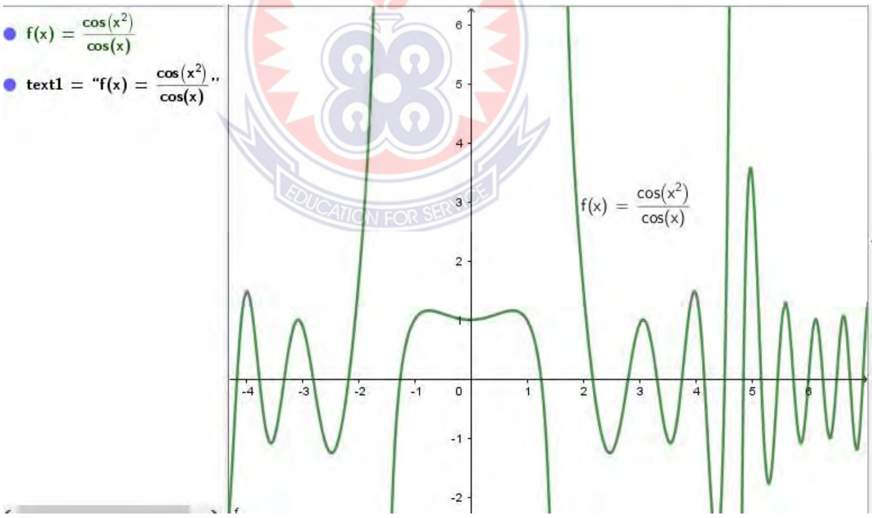
12. Suppose that the graph of the function f is drawn as in the following figure.



Which of the following is not true?

a. The function f is concave upward on the interval $[a, b]$
b. The function attains the minimum value at point b
c. The maximum values of the function occurs at the point a and c
d. The function has no inflection point.

13. The first derivative of the function $f(x) = \dfrac{\cos(x^2)}{\cos(x)}$ which is indicated in the following is:



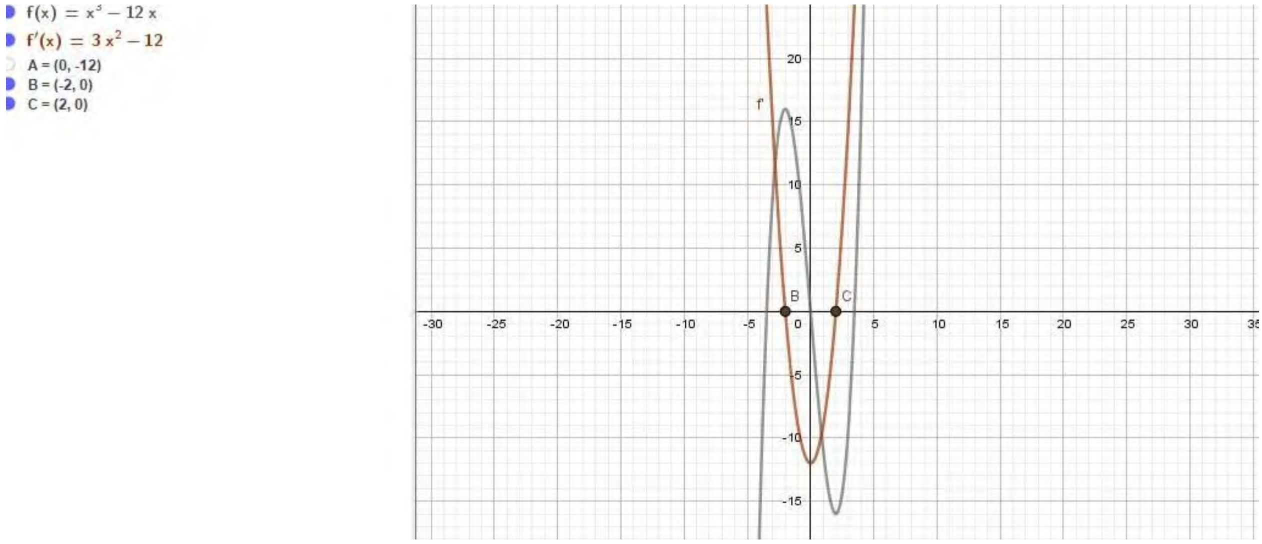
a. $\dfrac{\sin(x)\cos(x^2) - 2x\sin(x^2)\cos(x)}{\cos^2(x)}$

c. $\dfrac{\sin(x)\cos(x^2) + 2x\sin(x^2)\cos(x)}{\cos^2(x)}$

b. $\dfrac{\sin(x)\cos(x^2) - 2x\sin(x^2)\cos(x)}{\cos(x^2)}$

d. $\dfrac{\sin(x)\cos(x^2) - 2x\sin(x^2)\cos(x)}{\cos(x^2)}$

14. By using the following graph of the function $f(x) = x^3 - 12x$ determine



which of the following is true?

a. The turning point of the derivatives of the function $f(x)$ points A

b. Between point B and C of the function $f(x)$ is increasing

c. From point B to negative infinity the function $f(x)$ is decreasing

d. -2 is the only critical point of the function $f(x)$

15. Let $r(t)$ stand for the position of a particle at the time t. Which of the following is false?

a. $r'(t)$ represents the velocity of a particle at time t

b. $r''(t)$ represents the acceleration of a particle at time t.

c. $r'^{(t)}$ represents the length of a particle at time t.

16. What is the first derivative of the function $f(x) = \frac{x}{x^2+1}$ at = 0 ?

a. 0 c. 2

b. 1 d. 3

17. $\frac{dy}{dx}$ of $x^2 + \cos(xy^2) = xy$ is:

a. $\frac{dy}{dx} = \frac{2x - y - y^2\sin(xy^2)}{2xysin(xy^2) + x}$

b. $\frac{dy}{dx} = \frac{2x - y + y^2\sin(xy^2)}{2xysin(xy^2) + x}$

c. $\frac{dy}{dx} = \frac{2x - y^2\sin(xy^2)}{2xysin(xy^2) + x}$

18. Use the fact that $\lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x = e$. Then $\lim_{x\to\infty}\left(1 + \frac{1}{2x}\right)^{2x+1}$ is:

a. e^2

b. e^{-4}

c. e

d. e^{-1}
e. None of the above

Part 2: Work out the problems.

Show all necessary steps in finding the required answers and write your final answer carefully.

19. Find the equation of a tangent line to the function $xy = 1$ at $x = 1$ and sketch the graph of $xy = 1$.

20. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, x = 0 \end{cases}$ $is\ f(x)\ continous\ at\ x = 0?\ justify$