UNIVERSITY OF EDUCATION, WINNEBA

ERRORS AND MISCONCEPTIONS IN SOLVING PROBLEMS IN LINEAR INEQUALITIES AMONG PRESERVICE TEACHERS AT KOMENDA COLLEGE OF EDUCATION, GHANA

SAMUEL KOJO BINEY

MASTER OF PHILOSOPHY

2022

UNIVERSITY OF EDUCATION, WINNEBA

ERRORS AND MISCONCEPTIONS IN SOLVING PROBLEMS IN LINEAR INEQUALITIES AMONG PRESERVICE TEACHERS AT KOMENDA COLLEGE OF EDUCATION, GHANA

SAMUEL KOJO BINEY
(200028565)

A thesis in the Department of Basic Education,
Faculty of Educational Studies Education, submitted to the School of Graduate Studies in partial fulfillment of the requirements for the award of the degree of
Master of Philosophy
(Basic Education)
in the University of Education, Winneba

MARCH, 2022

DECLARATION

I, Samuel Kojo Biney hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Signature ……..………………

Date: ……………..…………….

Supervisor Declaration

We hereby declare that the preparation and presentation of this theses were supervised in accordance with the guidelines on supervision of theses laid down by the University of Education, Winneba

Name: Dr. Clement Ali (Principal Supervisor)

Signature ……..………………

Date: ……………..…………….

Name: Mr Nixon Saba Adzifome (Co-Supervisor)

Signature ……..………………

Date: ……………..…………….

DEDICATION

To my mum, Miss Joana Esi Twieku, and Mr and Mrs Aidoo

ACKNOWLEDGEMENTS

I express my sincere thanks to my supervisors, Dr Clement Ali and Mr Nixon Saba Adzifome for their hard work in supervision, guidance and ensuring that the very best was brought out of me.

I also express my gratitude to my family for supporting me in prayers. I also wish to thank my colleagues who gave me the morale in the process of putting this piece together.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

This research examined errors and misconceptions in solving problems in linear inequalities among preservice teachers at Komenda College of Education, Ghana. Four research objectives were formulated for the study. The study adopted the sequential explanatory design using the mixed-method approach for data collection. The simple random sampling technique was used to sample 285 preservice teachers from the Komenda College of Education to respond to the achievement test out of which fifteen (15) were conveniently sampled for the interviews. The instruments used for the study were an achievement test and a semi-structured interview guide. Frequencies and percentages were used as the main analytical tool for the quantitative phase of the study. One hundred and seventy-seven (177) responses were received from the achievement test out of the 285 sampled for the study. The quantitative data were analysed using Statistical Product and Service Solutions (SPSS version 26.0) using descriptive statistics specifically, the frequencies and percentages and the qualitative data were analysed thematically. It was found out that rules mixed up error, surface understanding, inability to assimilate concepts, carelessness, and poor understanding were the errors whiles discrete and separate operations, fact tests, overspecialized learning process on addition and/or subtraction, misapplication of commutative property using addition and subtraction and value of the digits, instead of place value were the misconceptions revealed in preservice teachers' solutions to linear inequality problems. The major source of these errors was teachers' teaching methods. Additionally, students perceived inequality as an amalgam of images or symbols encountered in a mathematics setting. Finally, teacher educators should vary their mode of teaching linear inequalities to preservice teachers, since their method of teaching influences students' mode of thinking and addressing problems in mathematics.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This chapter deals with the background to the study, statement of the problem, purpose of the study, objectives of the study, research questions, hypotheses, significance of findings, delimitation of the study, and organization of the study.

1.1 Background to the Study

Mathematics is increasingly recognised as one of the key subjects for access to different career paths and further education. This is because mathematics facilitates the mind and sets the foundation for pure science and social sciences (Gravemeijer, et al. 2017). However, mathematics is one subject that emphasises idea learning; hence, numerous misconceptions can be noticed in mathematics. According to Ersoy (2006), it is nearly hard to define any notion in mathematics without referring to numerous other concepts, as mathematics curricula have a spiral structure.

Mathematical inequalities are critical concepts that precondition for various topics, including algebra, trigonometry, and analytical geometry. A mathematical inequality is a statement constructed from expressions involving one or more of the symbols ($<$, $>$, $\leq$, $\geq$) or used to compare two numbers. Inequality resolution entails determining the value(s) of variables that maintain the correct order of the interaction. Linear inequalities, an aspect of algebra in mathematics, have traditionally been introduced to students when they have acquired the necessary arithmetic skills such as insight, attitudes and values. In linear inequalities, students have to identify the unknown variables and relations among them, and express them symbolically to solve the problem (Martinez, 2002). Thus, inequality plays a vital role in fundamental arithmetic concepts, serving as a critical entry point for various mathematical topics, including equations and various types of functions (Ralph & Scholtes, 1997; Salas & Obeysekera, 1982). Inequalities also play a vital part in the conceptual development of equality and equations, as inequalities have traditionally been viewed as complementary to students' grasp of equality (Tsamir & Almog, 2001). A solution to an inequality is a number that makes the inequality assertion true when replaced with the inequality variable. To resolve an inequality, one must first discover all of its solutions. The solution set is another term for the collection of all solutions.

Misconceptions have been identified as one of the most significant impediments to mathematics learning. Hansen (2006) define errors and misconceptions as mistakes learners make when solving problems that may be caused by carelessness, misinterpretation of symbols or text; a lack of relevant experience or knowledge related to that mathematical topic, learning objective, or concept; a lack of awareness or inability to check the given answer or the result of misconceptions. Hammer (1996) defined misconceptions as fundamentally different or incorrect views from those of experts on a particular subject or sector. In mathematics, misconceptions are imprecise or inaccurate understandings of mathematical structures (Champagne & Klopfer, 1983; Brown & Clement, 1989; Confrey, 1990).

Errors in mathematics can be factual, procedural, or conceptual, and may occur for a number of reasons. Identification of students' specific errors is especially important for students with learning disabilities and low performing students (Fuchs, Fuchs, & Hamlett, 1994; Salvia & Ysseldyke, 2004). Students' lack of knowledge could be a major reason why they cannot solve certain problems consistently (Hudson & Miller, 2006). According to Ginsburg (2000), factual errors are mistakes students make when they cannot recall a fact required to solve a problem or if they have not mastered basic facts. Procedural and factual errors are generally not due to inherent misunderstandings and are easier to identify than conceptual errors. Conceptual errors may look like procedural errors, but they occur because the student does not fully understand a specific math concept, such as place value (Ginsburg, 2000). To determine if an error is conceptual, teachers should check by asking the student to represent the problem with concrete objects or show and explain the steps used to solve the problem (Hudson & Miller, 2006).

Preservice teachers need to possess conceptual knowledge in their teaching of mathematics (Mereku, 2000), but many preservice teachers lack the conceptual understanding required to teach when they graduate from their studies (Prendergast & O'Donoghue, 2014). Despite the importance of teacher content knowledge being highlighted in numerous reports, studies have shown that preservice teachers possess a fragmented understanding of critical concepts such as algebra, trigonometry and statistics (Walsh, 2015; Walsh et al, 2017; Fitzmaurice et al, 2021).

Many students consider linear inequalities a problematic area of mathematics because they have misconceptions and difficulties in learning. According to Swan (2001), a misconception is not wrong thinking but a concept in the embryo or local generalisation that the individual has made. Therefore, it may be a natural stage of development and that ―although we can and should avoid activities and examples that might encourage them, misconceptions cannot simply be avoided" (p. 98). Therefore, it is essential to have strategies for remedying as well as for avoiding misconceptions. Makonye (2012) opined that those misconceptions are the underlying wrong beliefs and principles in one's mind that causes a series of errors. According to The National Council of Teachers of Mathematics (2000), students in the ninth through twelfth grades are required to explain inequalities using mathematical symbols and to comprehend their meaning through the interpretation of inequalities' solutions. Numerous researches (Eg. Almog & Ilany, 2012; Tsamir & Bazzini, 2004; Vaiyavutjamai & Clements, 2006) have discovered that many middle and high school pupils have misunderstandings and challenges that impede their ability to solve and comprehend equations effectively.

Therefore, unless teachers have a mastery of inequalities, they cannot teach students to grasp the meaning of equality even if they are capable of solving inequality questions. Teachers‘ shallow understanding of inequalities may have risen due to many factors, but the misconceptions they have about it are the dominant ones found so far (Akhtar & Stienle, 2013). Akhtar and Stienle (2013) further postulated that misconceptions are one of the main reasons for students‘ poor performance in mathematics. If teachers‘ misconceptions and errors are not diagnosed and at the same time corrected, then this would create a major problem for the students they teach (Baki & Çakıroğlu, 2010).

Vaiyavutjamai and Clements (2006) opined that a common inequality misconception that preservice teachers commonly possess the most is they regard inequalities as equations. Another common misconception preservice teachers possess is the interpretation of solution sets (Blanco & Garrote, 2007; Halmaghi, 2011; Kroll, 1986; Vaiyavutjamai & Clements, 2006). It was upon this bedrock that this study sought to investigate errors and misconceptions in solving linear inequalities among preservice teachers at Komenda College of Education, Ghana.

1.2 Statement of the Problem

Mathematics educators are encouraged to cultivate conceptual understanding in their classrooms rather than an exclusive focus on mastery of procedural skills (Ojose, 2015). In other words, determining and eliminating students‘ errors and misconceptions help teachers understand students' backgrounds and perceptions of an academic subject and shape their instructional methods (Murphy & Alexander, 2004). A common inequality misconception is that preservice teachers commonly regard inequalities as equations (Samo, 2009; Vaiyavutjamai & Clements, 2006). Another common misconception preservice teachers possess is the interpretation of solution sets (Blanco & Garrote, 2007; Halmaghi, 2011).

According to the WAEC chief examiner's report, students do not answer questions well in linear inequalities (BECE, 2017-2020). The reports have enumerated many errors and misconceptions in inequalities. However, Samo (2009) argued that many misconceptions in algebra experienced by students are seen to be a direct result of how it is taught by teachers and that such misconceptions stem from limitations in teacher content knowledge. These misconceptions were a result of a trickling down effect from teachers to their students, thus making it transitional. Thus, as educators, we need to know the possible reasons that lie behind these misconceptions and take precautions to provide more efficient learning environments (Ojose, 2015). In other words, determining and eliminating students' errors and misconceptions help teachers understand students' background and perceptions of an academic subject and shape their instructional methods (Murphy & Alexander, 2004)

Although there have been many studies conducted similar to the one under investigation; there is limited research that focuses directly on errors and misconceptions in linear inequalities among preservice teachers. For example, the study of El-Shara and Al-Abed (2010) aimed to diagnose errors that occurred in solving inequalities among mathematics majors at the University of Jordan. Also, cross-institutional mixed methods study; difficulties in learning inequalities in students of the first year of pre-university education in Spain was investigated by

Blanco and Garrot (2007). A study Conducted by Bicer et al. (2014) aimed to determine whether preservice teachers‘ have common difficulties and misconceptions about linear and quadratic inequalities showed that not only did the first-year preservice teachers possess difficulties and misconceptions with linear and quadratic inequalities, but also second, third-and fourth-year preservice teachers.

The College of Education in the educational system in Ghana is a crucial one because it is at this level that potential teachers are nurtured to become future teachers to teach in various basic schools in the country. The quality of any educational system lies in the effectiveness of the teachers.

If teachers who are to deliver the mathematics curriculum suffer from such deficiencies, then we have every cause to worry because it raises a lot of questions. Researchers have been conducting studies on determining and finding ways to eliminate the misconceptions in mathematics for many years (Türkdoğan, Güler, Bülbül & Danişman, 2015). Therefore, this study is thought to fill such a gap and be beneficial for teachers at all grade levels.

1.3 Purpose of the Study

The purpose of the study is to find the common errors and misconceptions preservice teachers of Komenda College of Education encounter in linear inequalities.

1.4 Research Objectives

The objectives of the study were:

1. To find out the errors revealed in preservice teachers‘ solution to linear inequality problems;
2. To find out the misconceptions revealed in preservice teachers‘ solution to linear inequality problems;

3. To identify the likely sources of the errors and misconceptions revealed in preservice teachers‘ solutions to linear inequality problems;
4. To identify the alternative conceptions preservice teachers have that are attributable to their errors and misconceptions in solving linear inequalities; and

1.5 Research Questions

The study was guided by the following research questions:

1. What errors are revealed in preservice teachers‘ solutions to linear inequality problems?
2. What misconceptions are revealed in preservice teachers‘ solutions to linear inequality problems?
3. What are the likely sources of the errors and misconceptions revealed in preservice teachers‘ solutions to linear inequality problems?
4. What alternative conceptions do preservice teachers have that are attributable to their errors and misconceptions in solving linear inequalities?

1.6 Significance of the Study

This was classified under practice, theory and policy. For practice, the study results would draw the attention of mathematics tutors in Komenda College of Education to the errors and misconceptions, its causes and effects on teaching linear inequalities among preservice teachers. With theory, it will help in theorizing possible errors and misconceptions in solving linear inequalities in the municipality and the country as a whole. Also on policy, findings from the study would reveal some of the deficiencies existing in the learning of mathematics especially linear inequalities and this would help inform mathematics educators and policy makers on what has to be done to improve the mathematics standard at the Colleges of Education in Ghana.

The findings from the research would inform educational stakeholders to develop strategies to equip existing schools with the necessary teaching and learning facilities that would promote the teaching and learning of linear inequalities. The study would also help prospective mathematics teachers to have an in-depth understanding of linear inequalities especially, in the areas of conceptual knowledge which would help them in their work as teachers after completing school. Moreover, the study would serve as reference material for other researchers.

1.7 Delimitations of the Study

The study was delimited to errors and misconceptions in linear inequalities among preservice teachers at Komenda College of Education, Ghana.

1.8 Limitation of the Study

This study was conducted to examine the errors and misconceptions preservice teachers commit when solving linear inequalities problems. The study focused on only the level 400 preservice teachers. Moreover, time constraints also limited the research activity. The time appointed for the data collection did not favour the research activity because the researcher was permitted to gather the data right after the preservice teachers were done with their examination, which affected the data collection. Preservice teachers were not willing to participate in the study especially the interview as a result of the COVID-19 pandemic and also with the intention of them not being mathematics major students.

1.9 Organisation of the Study

The study covers five chapters. Chapter One presents the introduction which is discussed under the following subthemes: Background to the study, statement of the problem, purpose to the study and research objectives. Moreover, it discusses the research questions, significance of the study, delimitations of the study, and organisation of the study. Chapter Two deals with the literature review, the empirical and conceptual reviews. Chapter Three deals with the research methodology adopted for the study. It discusses the research paradigm, research approach, research design, population, sample and sampling techniques and data collection instruments. It further presents the trustworthiness of the interviews, data collection procedures, data analysis procedures and ethical considerations. Chapter Four presents the results/findings, the analysis and discussion of the data collected. Chapter Five presents the summary, conclusions and recommendations based on the findings and conclusions of the study.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

This chapter highlights the literature review. It highlights significant theories and provides accounts from many sources related to the theoretical and empirical issues raised. The chapter focused on the various conceptualisations that reflect the objectives of the study. Empirical review from various articles was reviewed. Critical areas in the study included the meaning of errors and misconception of linear inequalities among pre-service teachers.

2.1 Theoretical Framework

The constructivist learning strategy was used in this study. It considered the contributions made by Jean Piaget, Jerome Bruner, Lev Vigotsky and Joseph Novak. The constructivist learning approach postulates that students construct their understanding of the world around them by reflecting on their own experiences. As a result, they make sense of their experiences using their mental conceptions. Taber (2009) asserted that learning refers to a search for meaning. As a result, the student must take an active role in the quest for new information and meaning, which should be based on issues that demand personal interpretation. Students may, however, adopt incorrect assumptions during the construction of meaning and comprehension, leading to misconceptions. The social aspects of learning form a crucial part of the constructivist view of learning. It means that people also learn from other individuals and not in isolation from others. According to Stears and Gopal (2010), learning occurs in the context of activities and social interaction, which are influenced by day-to-day cultural circumstances. According to social constructivist theorists, knowledge is both a socially constructed entity and a socially mediated activity. According to social constructivism, the student brings a wealth of prior information to the learning context. According to Vygotsky and Cole (1978), social interaction plays an essential role in the development of cognition, and every human child develops in the context of a culture. Therefore, human cognitive development is affected more or less by the individual's culture, including family environments and socio-cultural experiences. To a significant extent, this supports the assertion made by Tekkaya (2002) that some students' misconceptions result from ordinary experiences and observations. These preconceived beliefs, according to Tekkaya (2002), are vernacular misconceptions. Vygotsky and Cole (1978) assert that culture seems to make two kinds of contributions to children's intellectual development.

Initially, students acquire much of the content of their thinking from culture itself, and, secondly, they acquire the processes and means of their thinking from it. In short, according to Vygotsky and Cole (1978), culture teaches children both what to think and how to think. In this way, children are very likely to model their behaviour on the observed behaviour of their parents or role models. Learning is then viewed as dependent on social interaction. Hence, if the people they copy and learn from hold certain misconceptions on a particular subject, they are more likely to adopt those unfounded beliefs. One of the important components of learning that Vygotsky and Cole (1978) emphasised was that a child learns more effectively with the assistance of an adult and is limited to a certain space which he called the zone of proximal development (ZPD). Vygotsky and Cole (1978) postulated that the learner uses a word or concept label for communication purposes before that child has a fully developed understanding of that word. Hence a wrong utilisation of the word results in the wrong attainment of concepts. Therefore, the social aspects of learning are significant in this study because some misconceptions originate from everyday language (Tekkaya, 2002). According to Stears and Gopal (2010), constructivist approaches to learning support services and programmes are designed to build on students' understanding, concerns, and prior knowledge. These scholars (Bruner, 1990 & Snelbecker, 1974) state that socio-cultural knowledge arises from children's ethnic backgrounds, socioeconomic conditions, environment, and life circumstances. These circumstances may influence the perceptions and knowledge of the students such that they feel isolated and harbour many misconceptions, as they cannot link school mathematics and their socio-cultural knowledge.

On the other hand, constructivists like Jerome Bruner, in the theory of discovery learning, state that students participate in making many decisions about what, how, and when something is to be learned and even play a major role in making such decisions (Bruner, 1990). According to Snelbecker (1974), the student expects to explore examples to explore the principles or concepts to be learned, not to be presented with content by the teacher. Bruner's theory of discovery learning encompasses two key features, which are analytical thinking and intuition. Intuition relates to a gut feeling that leads to a problem without proof (Bruner, 1990). It should be noted that when students use discovery processes make inferences and speculations about scientific phenomena, they may also develop misconceptions. It is because the meaning is distinctive or unique to that particular person. Most constructivists recognise that students' learning is influenced by their context, beliefs, and attitudes; consequently, the current study investigates errors and misconceptions of pre-service teachers in solving problems in linear inequalities.

Furthermore, it is widely acknowledged that pre-service teachers are not ―blank slates" when entering the classroom. They arrive at school with preconceived notions about various things, including how they perceive and interpret the world around them. All students have diverse experiences prior to attending school and each has their own unique knowledge because of it (Churchill et al., 2013). Sometimes these views may be rather strange, even elaborate regardless of their content.

The idea of learners adding to their prior understandings is explained by the constructivist view which recognises that a person‘s learning is influenced by their prior experiences and understandings, thereby people do not simply gain knowledge but ‗construct' on or add to their former knowledge (Churchill et al., 2013). It emphasizes the significance of investigating pre-service teachers' errors and misconceptions, especially in solving problems in linear inequalities.

2.2 Conceptual Framework

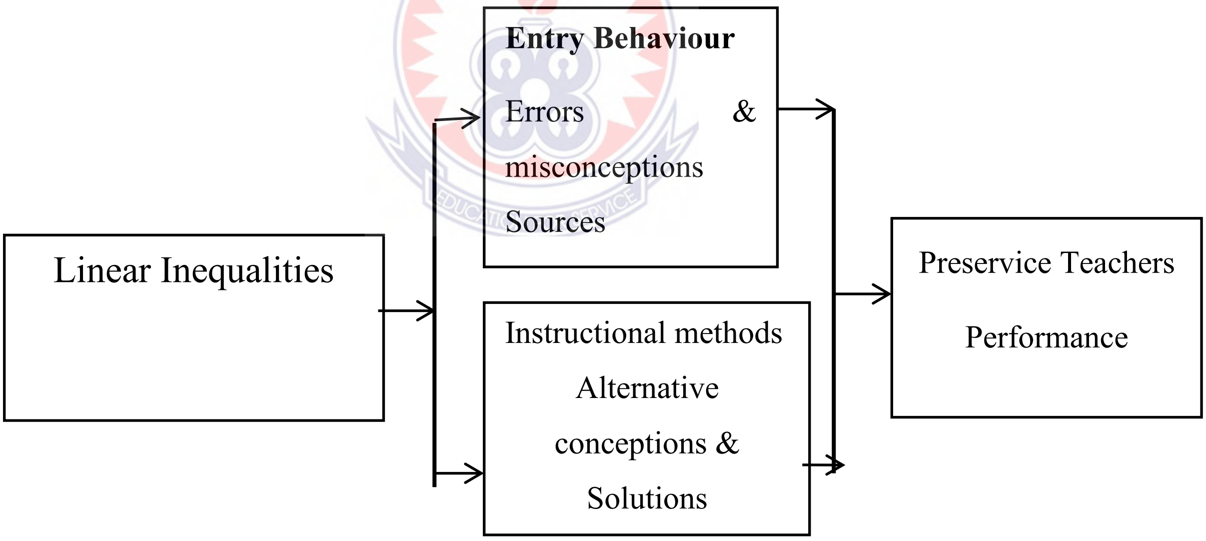

Fig. 1: Preservice Teachers' Errors and Misconceptions in Linear Inequalities

The model suggests that in learning linear ineqality, the entry behaviour of preservice teachers include errors and misconceptions and their sources. The teacher's teaching and learning materials used, methods and possible interventions play a critical role in correcting these errors and misconceptions. This was thought to have a direct impact on the conceptual understanding of preservice teachers which will result in high-quality learning, with the type of learning received by learners depending on the teaching methods. The type of learning that preservice teachers receive will influence their performance in linear inequalities.

2.2.1 Conceptual Review

This presents a conceptual literature review on errors and misconceptions, inequalities, instructional models related to the understanding of inequalities, overcoming student's inequality errors and misconceptions, the natal of alternative conceptions, the strategies of solving linear inequalities, preservice teachers conceptual and procedural knowledge, empirical studies, common errors and misconceptions in solving linear inequality problems and the conceptual framework on student errors and misconceptions in linear inequalities.

2.2.2 Concept of misconception

The belief of an incorrect fact does not constitute a misconception. Incorrect facts can be erased easily by communicating pertinent information. However, a misconception includes a deep framework of conceptual thinking that has been perpetuated through many years. To demonstrate intellectual respect for the learner who holds those views, some researchers prefer to refer to misconceptions as alternative frameworks or alternative conceptions (Bahar, 2003). Naive conceptions are another term used to label misconceptions. Naive conceptions highlight the formative aspect of learning and suggest that the conceptions result from youngsters attempting to explain their surroundings (Ridgeway & Dunston, 2000). Although various definitions of misconceptions are put forward by different researchers, Bahar (2003) summarizes misconceptions as corresponding to concepts with peculiar interpretations and meanings in students' explanations that are not scientifically accurate; that is, nature does not bear out as observable what a person may think. When students attempt to make sense of a situation or phenomenon in their surroundings, they may make mistakes. These beliefs are frequently formed at a young age.

Ojose (2015) defined misconceptions as misunderstandings resulting from incorrect interpretations, while Hannawa (2015) pointed out that misconceptions constitute wrong clarifications that most students accept as correct and those misconceptions are often used to illustrate beliefs that are incompatible with scientific explanations (Tippets, 2014). Thus, misconceptions can be described as the differences of opinion between students and experts, where students' understanding produces systematic errors (Winarso, 2017). According to Junus, (2018), mistakes and misconceptions occur when students make inappropriate generalizations about an idea. Nurul et al. (2019) further explained that students' failure to master the basic mathematics concepts would increase their probability of using the wrong strategies, resulting in errors and misconceptions in solving mathematical problems (Nurul et al., 2020). Misconceptions obstruct understanding of new concepts and information for various reasons, including a lack of prior knowledge about the concept, the learner's inability to connect what already exists with what is new, or the learner's misinterpretation of new concepts to match previous knowledge. The learner then sticks to his previous perceptions (Won et al., 2019).

Misconceptions emerge at the early stages of learning, whether in the classroom or as students interact socially with the world outside (Siyepu, 2015). Mathematical knowledge is cumulative, meaning that it is built on earlier mathematical knowledge and study (Siyepu et al., 2015). Brodie (2014) argued that most misconceptions stem from incorrect previous knowledge.

Students' misconceptions are created by students' misunderstandings, as they believe they are correct. It emphasises the significance of distinguishing between errors and misunderstandings when learning mathematics. Therefore, the most appropriate definitions of misconception and errors for this study is; misconceptions as misunderstandings of ideas or concepts while errors reflect incorrect applications, concepts, theories, or formulas. The terminology used to describe misconceptions and the different explanations available can be perplexing and intimidating. Rather, the focus should be on identifying the sources of misconceptions and devising measures to eradicate them. The more significant aim is to identify and change misconceptions. When a misconception is not corrected, it can have several harmful effects. If a misunderstanding is not corrected, it can be passed down across generations, resulting in a perpetually illiterate population (Chin & Chia, 2004). Studies by Eryilmaz (2002) mentioned that teachers need to identify and address common mathematical misconceptions among students to ensure the meaningfulness and effectiveness of mathematics lessons. Ocal (2017) suggested that teachers correct the misunderstandings of basic mathematical concepts before introducing a new concept. In this process, to prevent students from making the same mistakes, students need to be aware of the causes of the misconceptions and how to deal with them.

2.3 Errors

A primary premise in distinguishing between an error and a misconception is that errors are immediately detectable in learners' work, such as written text or speech, whereas misconceptions are frequently disguised from casual observation. Several studies have been done to determine the nature and causes of mathematical errors made by students (e.g., Davis 1984; Godden, Mbekwa & Julie, 2013; Hodes 1998; Olivier 1989; Luneta & Makonye, 2010; Mason, 2006; Matz 1980; Muzangwa & Chifamba, 2012; Orton 1983; Seng & Chen, 2010). These authors believe that learners make errors due to existing conceptual gaps or misconceptions embedded in their conceptual schemes. If errors and misconceptions were to be put on a continuum, one would have non-systematic errors on one end and the more serious systematic errors deeply rooted in misconceptions on the opposite end (Makonye, 2012).

2.3.1 Classification of errors

The majority of these classifications can be divided into two categories: non-systematic and systematic errors.

2.3.2 Non-systematic errors

Non-systematic errors are sometimes referred to as slips, and they might occur due to carelessness, misunderstanding material, or unintentionally forgetting something. Because there are no fundamental and flawed conceptual structures linked with such errors, preservice teachers will usually repair them on their own. Even though these errors were little blunders, they should not be overlooked because they can demoralise students and limit their performance if they are repeated, becoming a severe inhibitor to learning as mathematics builds on itself (Schnepper & McCoy, 2013)

2.3.3 Systematic errors

According to Star (2005), systematic errors are regularly defined as erroneous procedures, algorithms, or rules. Students' procedural knowledge, conceptual knowledge, or linkages between these two forms of information may be the source of systematic errors (Xiaobao & Ning, 2006). Making logically faulty deductions, applying an incorrect version of a concept or theorem and having the correct answer to the wrong question, or making a basic skill error (Schnepper & McCoy, 2013).

2.4 Errors and Misconception in Solving Linear Inequalities

Misconceptions and errors can beqw due to several reasons. Cultural views, interactions with others, particularly family members, and observations of others are some of the reasons (Chin & Chia, 2004). When teachers' scientific understanding is lacking, and their trust in the content is insufficient, they might also be guilty of creating misconceptions (Jarvis et al, 2005). It is common for misconceptions to occur in all divisions of scientific study. Misconceptions in mathematics include the problem of solving linear inequalities, algebra, and fractions. According to Li (2008), students have misconceptions regarding inequalities, which are a tough step up from equations for many students. Although the process for solving inequalities appears to be identical to that for solving equations, there are a few distinctions that students frequently ignore. Teachers can assist students in recognising these discrepancies and using multiple solution methods to solve inequalities. Many students disregard the importance of following the order of operations principles and solving the expression from left to right.

Furthermore, many students are unaware that parentheses can signify multiplication and grouping Gardella, (2008). Wang and Lin (2005) also discussed the ways students employ linear inequality problems. Students will attempt to express what they already know about solving equations with inequalities; nevertheless, they will fail to relate prior knowledge to the new concepts. When solving inequalities, students frequently use equation-solving procedures. This misunderstanding is reasonable because equations and inequalities "see" to be the same thing. McNeil and

Alibali (2005) argued that previous learning experiences might cause students' misconceptions.

Ciltas and Tatar (2011) stated the difficulties and misconceptions the students experienced in the subject of inequalities as follows;

1. Students deal with inequalities in the same way they treat equations.
2. Students forget the reverse of the inequality sign when dividing/multiplying both sides of the inequality by a negative number.
3. General errors when conducting algebraic operations include forgetting to distribute the number behind the parenthesis or neglecting the presence of the parenthesis.
4. Students had difficulty distinguishing between an equation and an inequality and comprehending that the solution to an inequality is an interval rather than a number.

Students apply previous knowledge to a new topic before having sufficient data in hand (Ashlock & Wright, 2001). When exploring new mathematical topics, including inequalities, this over-generalization can be the principal cause of students‘ mistakes (Vamvakoussi & Vosniadou, 2004). When students over-generalize, they may not realize that the new topic requires either partially or different mathematical processes. For instance, knowing how to solve equations can help solve inequalities; however, students may make mistakes when solving inequalities if they apply the same solution processes with equations (Almog & Ilany, 2012). In this regard, Almog and Ilany (2012) found that many students overgeneralize one specific equality rule ($a \times b = 0$ requires that at least either $a = 0 \; or \; b = 0$); however, the inequality $a \times b = 0$ requires that both a, and b, have the same sign. In this scenario, many students did not realize that the multiplication of two negative numbers yields a positive number or the multiplication of two negative numbers should yield an answer greater than zero. For example, Tsamir and Bazzini (2001) demonstrated that many high school students' responses $(y-2)(y+9) > 0 \; is \; (y-2) > 0 \; and \; (y+9) > 0$, but they simply forget the case that $(y-2) < 0 \; and \; (y+2) < 0$ also making the inequality true.

Another common and persistent misconception is expressing inequalities as equalities (Blanco & Garrote, 2007; Halmaghi, 2011; Kroll, 1986; Tsamir & Almog, 2001; Vaiyavutjamai & Clements, 2006). Because many students think that inequalities and equalities require the same mathematical solution process, they treat problems involving inequalities in the same manner as equations and assume the questions require similar processes (Blanco & Garrote, 2007; Vaiyavutjamai & Clements, 2006). Prestege and Perks (2008) conducted a study with prospective teachers about their understanding of inequalities and results showed that once students treat inequalities as equations and solve them, they put the sign back. For instance, solving the inequality: -65x 3 > 0 in the same way as solving equation: -65x 3 = 0. Then, they arrive at the conclusion that x 3 = 0, and then x= 0. When they put the sign back, many students find x > 0 to solve the inequality. However, students may forget that multiplying and dividing by a negative number changes the direction of the inequality, and their solution in actuality needed to be x < 0 (Almog & Ilany, 2012)

2.5 Interpretations of Inequalities

When students find the solution to inequalities, they do not always understand the meaning of their solutions (Becarra, Sisrisaengtaksin, & Walker, 1999 cited in Critchley et al., 2018). Vaiyavutjamai and Clements (2006) noted that students who treat inequalities as equations might find the correct answers; however, they cannot check whether or not they are arriving at the correct results. Tsamir and Bazzini (2004) found that students commonly believed ―solutions of inequalities must be inequalities". Additionally, Vaiyavutjamai and Clements (2006) noted that some students think only one value makes an inequality true, and they think solutions to inequalities cannot be an interval or infinite set. Due to these two misinterpretations, students have difficulty interpreting the results of inequalities. Overspecializing is another mathematical misconception in which students inappropriately restrict one special feature into other cases (Ashlock & Wright, 2001). Tsamir and Bazzini (2004) asked 148 Israeli high-school students about their understanding of inequalities, concluding that many students assume that the results of inequalities need to be inequalities.

However, solutions to inequalities can range from a single value to all numbers (Almog & Ilany, 2012). For example, when x is an integer, and$3 < x < 5$, only one value (4) satisfies this statement. It shows the result of inequality as a single number. However, when x is a real number, and $3 < x < 5$, x can be infinitely many real numbers between 3 and 5. However, Tsamir and Bazzini (2004) found that many high school students thought only one value makes the inequality true even if their solution set was infinite. Vaiyavutjamai and Clements (2006) studied 31 secondary school students' understanding of linear equations and inequalities and found that even if some students found the correct solution to inequalities, they tended to write a single value into the answer sheet. For example, for $6x \geq 6$ and x is an integer, even students who find $x > 1$ decide only ―1" answer this inequality. After the test, Vaiyavutjamai and Clements (2006) conducted interviews with students to obtain insight into their solving process. Students' responses during the interview were aligned with the test results, demonstrating that students believe only one value makes an inequality true.

Blanco and Garrote (2007) concluded there were two primary reasons why many students experience difficulty solving inequalities: a) a lack of arithmetic skills or knowledge, and b) the absence of semantic and symbolic meanings of inequalities. Other major types of difficulties include excluded values, the choice of logical connections, dividing or multiplying non-positive factors, and connections between the signs of given products and the signs of their factors (Tsamir, & Bazzini, 2004).

2.6 Instructional Models Related to Understanding of Inequalities

Particular teaching methods can help students understand their difficulties with inequalities. Tsamir and Almog (2001) found that students‘ understanding of inequality concepts is related to teacher instruction. In a study of instructional models and students‘ understanding of inequalities, three different teaching methods for inequalities, including algebraic manipulations, drawing a graph, and using the number line, were employed. Students had the highest number of incorrect solutions when they used algebraic solutions. However, drawing a graph usually yielded a correct solution (Tsamir & Almog, 2001). Before engaging in formal solutions with inequalities, students should have experience working with graphs and tables of values to make their learning more comprehensive. The function-based approach might have been useful since it enables students to develop their problem-solving strategies and visual thinking (Verikios & Farmaki, 2010). Using appropriate technologies like computer software and graphic calculators was beneficial because it helped students avoid misconceptions by developing their visual thinking (Abramovich & Ehrlich, 2007). Furthermore, calculator technology can help students develop their visual thinking and provide them with the opportunity to interpret inequality results easily and efficiently (Tsamir & Almog, 2001).

2.7 Overcoming Students' Inequalities Misconceptions and Difficulties

First, students should be encouraged to find examples of inequalities in their daily lives to make their learning more meaningful. For example, because inequalities are based on two concepts: boundary and direction, a basketball court might be given as an example that includes both boundary and direction concepts (Tent, 2000). To make students‘ learning more mathematical rather than pure memorization of algorithms, teachers should provide connections to real-life situations and other mathematical topics, which students are already familiar with, and arrange some classroom activities or games before introducing mathematical signs (Perks & Prestage, 2008).

Some additional suggestions from Tsamir and Bazzini (2004) for effective teaching and meaningful learning about inequalities are: a) be familiar with students‘ intuitive beliefs, b) create discussions about the differences and similarities between equality and inequality, c) encourage verbal analysis when students work with inequalities, d) encourage students to solve questions or problems about inequalities in more than one way to check the accuracy of results, and e) emphasis should be placed on the importance of ―zero". Blanco and Garrote (2007) also revealed in their findings that students; a) do not introduce the concept of inequality rapidly, b) make sure that symbols have semantic values, c) establish differences between the concepts of equality and inequality, d) use everyday life language, geometric language, and algebraic language in instruction, and e) use different methods to enrich students learning.

Students and educators alike easily get used to this type of learning, and misconceptions will not get fixed using this method (Novak, 2002). It is necessary to implement student-centred, constructivist experiences for meaningful learning to occur. Student-centred activities are also known as active learning. Students are involved in the process and must think for themselves (Burrowes, 2003). A constructivist learning activity is an experience that builds on old concepts while adding new ones. The catch is that when altering misconceptions, the old concepts also have to be changed. Processes that actual mathematician use, like inquiry and problem-solving, are considered student-centred and based on constructivist thought (Udovic et al., 2002). Other strategies can also include project-based learning with problem-solving strategies, guiding questions, and student.

2.8 Conceptions and Alternative Conceptions

According to the constructivist viewpoint, learning is considered an active construction process shaped by students‘ prior knowledge and conceptions. Many researchers agree that the most significant things that students bring to class are their conceptions (Taber, 2009). During instruction, learners construct their meaning depending on their prior knowledge and capacities (Nakhleh, 1992). The student must connect new knowledge to relevant current concepts in the learner‘s cognitive framework for meaningful learning (Ausubel et al., 1968). Teachers can be astonished to learn that students do not understand fundamental ideas or basic concepts covered in mathematics class despite their best efforts. Some of the students give the right answers, but these are only from correctly memorised algorithms. Students can often use algorithms to solve numerical problems without completely understanding the mathematical concepts (Agra et al., 2019).

Concept means a term or a word (Ben-Hur, 2006). Concepts are described both in older and newer sources in a similar way. For instance, it is said that concepts are perceived regularities or relationships within a group of objects or events and are designated by some sign or symbol (Heinze-Fry & Novak, 1990). Concepts can be considered ideas, objects, or events that help us understand the world around us (Eggen & Kauchak, 2004). Concepts enable us to impose some meaning on the world; through them, the reality is given sense, order, and coherence. They are how we can come to terms with our experience. (Cohen et al., 2010). Each student continuously develops and reconstructs a diverse array of sophisticated, integrated, distinctive, and epistemologically legitimate ideas as he or she negotiates his or her classroom experience (Confrey, 1990). Thus, learners' real-world conceptions are crucial to their worldview (Nussbaum & Novick, 1982). The term ‗conception' itself must be explained since it is widely used in science and mathematics education but with very different meanings (Sakonidis et al., 2006). According to Glynn and Duit (1995), Conceptions are learners' mental models of an item or an event. Conceptions, according to Duit and Treagust (1995), are the individual's mental representations, whereas concepts are ―something well defined or widely recognised" (p. 47). As a result, Puem (2019) define conceptions as ―all cognitive constructs used by students to explain their experiences. Concepts, intuitive principles, thinking patterns, and local theories, for example, are positioned on distinct epistemological degrees of complexity (Kim, et al., 2013 & Prediger, 2008).

Osborne and Wittrock (1983) summarised student conceptions succinctly in their statement that learners develop ideas about their world, develop meanings for words used in mathematics, and develop strategies to obtain explanations for how and why things behave as they do. Learners develop ideas and beliefs about the natural world through their everyday experiences. These include sensorial and linguistic experiences, cultural background, peer groups, mass media, and formal training (Treagust & Mann, 1998). Some of these ideas and beliefs, the failure to accept the possibility of dividing a smaller number by a larger number, and the assumption that multiplication always makes bigger and division smaller (Bell, 1986). Simply stated, conceptions can be regarded as the learner‗s internal representations constructed from the external representations of entities constructed by other people such as teachers, textbook authors or software designers (Treagust & Duit, 2008).

Students‘ conceptions are crucial to later learning in formal classes because interaction arises between the new knowledge that the students receive in college and their current knowledge in the teaching field. By the time student teachers enter college, they have developed a strong commitment to an ontology and causality that separates physical from psychological objects and serves as the foundation for knowledge acquisition (Carey & Dinh 1985; Vosniadou, 1994; 2001). When newly acquired knowledge is consistent with existing conceptual structures, naïve mathematics facilitates further learning. Mathematics education researchers have conceptualised these conceptions as a ‗belief system' (Frank, 1985), as a ‗network of beliefs' (Schoenfeld, 1983), as a ‗mathematical world view' (Silver, 1987), and as ‗conceptions of mathematics and mathematical learning' (Confrey, 1984). Although different authors may have good reasons for preferring particular terms in the context of particular studies, there is no generally agreed usage across the literature (Taber, 2009).

2.9 The Natal of Alternative Conceptions

Arguably the most general source of conceptions is limited experience or exposure to limited examples - both in learning outside of school and formal instruction (Greer, 2008). Alternative conceptions may arise when students are presented with concepts in too few contexts or beyond their developmental level (Gabel, 1989). It may be that, when first introduced to a new exposition of a scientific idea, students have not yet attained as high a level of abstract thinking as the instructors assume. Perhaps instructors provide part of the concept or a more simplified concept in the belief that this will lead students to better understanding, as sometimes happens when the concept is difficult or troublesome. However, such limited explanation may prevent some students from crossing a cognitive threshold and entering through a door to a higher level of understanding (Liljedahl, 2005). Other possible explanations could be both the pace at which the algebra concepts are covered and the formal approach often used in its presentation. Many teachers and textbook authors are unaware of the serious cognitive difficulties involved in the learning of algebra. As a result, many students do not have the time to construct a good intuitive basis for inequality ideas or connect them with the pre-inequality ideas they have developed. They fail to construct meaning for the new symbolism and are reduced to performing meaningless operations on symbols they do not understand (Herscovics & Linchevski, 1994). Within the domain of math, Kieran (1992) contended that one of the prerequisites for effectively generating and interpreting structural representations such as equations is an understanding of the symmetric and transitive nature of equality – sometimes referred to as the equal sign's 'left-right equivalence.

Nevertheless, there is substantial literature that implies pupils do not understand the equal sign as a symbol of equivalence (i.e., a symbol that shows a relationship between two quantities), but rather as an announcement of the outcome or response of an arithmetic operation (e.g., Falkner et al., 1999; Molina & Ambrose, 2008). It has been suggested that misconception might be due to students‘ elementary school experiences (Carpenter et al., 2003; Seo & Ginsburg, 2003). The concept of equality and its symbolic instantiation are traditionally introduced during students‘ early elementary school years, with little instructional time explicitly spent on the concept in the later grades, and the equal sign was nearly always present in the operations equals answer context (e.g., $2 + 5 = 7$).

Another source of confusion is the different meanings of common words in different subjects and everyday use. It applies not only to words but also to symbols. Many scientifically associated words are used differently in the vernacular (Gilbert & Watts, 1983); for example, energy has a cluster of ‗life-world associations that do not match its technical use (Solomon, 1994). Likewise, Tall and Thomas (1991) indicate that in the natural language, ‗and‘ and ‗plus‘ have similar meanings. Thus, the symbol ‗ab‘ is read as ‗a and b‘ and interpreted as ―$a + b$.‖ There is much evidence that students at various levels of education use their natural number knowledge to conceptualise rational numbers and make sense of decimal and fraction notation, leading to systematic errors in rational number ordering, operations, and notation. Many researchers (e.g, Gelman, 2000; Greer, 2004 & Van Hoof et al., 2017) attribute these difficulties to the constraint students‘ prior knowledge about natural numbers imposes on rational number concepts.

2.10 Nature and Significance of Conceptions

Students' existing ideas are often strongly held, resistant to traditional teaching, and form coherent thought mistaken conceptual structures (Driver & Easley, 1978). Rather than being momentary conjectures that are quickly discarded, misconceptions consistently appear before and after instruction (Smith, et al., 1994). Students may undergo instruction in a particular topic, do reasonably well in a test on a topic, yet do not change their original ideas about the topic even if they conflict with the taught concepts (Fetherstonhaugh & Treagust, 1992). Duit and Treagust (1995) attribute this to students being satisfied with their conceptions and seeing little value in the new concepts. Therefore, it is difficult for students to change their thinking. Osborne and Wittrock (1983) state that students often misinterpret, modify or reject scientific viewpoints based upon the way they think about how and why things behave, so it is not surprising that research shows that students may persist almost totally with their existing views (Treagust et al., 1996). When the students' existing knowledge prevails, the scientific concepts are rejected or misinterpreted to fit or even support their existing knowledge.

As Clement (1982) demonstrated in elementary algebra, college students make the same reversal error when translating multiplicative reasoning relationships into equations (e.g., when translating "four people order cheesecake for every five people order strudel" into "$4C = 5S$"), regardless of whether the initial relations were stated in sentences, pictures, or data tables. In domains of multiplication (Fischbein et al., 1985), probability (Shaughnessy, 1977), and algebra (Clement, 1982a; Rosnick, 1981), misconceptions continue to appear even after the correct approach has been taught. Sometimes misconceptions coexist alongside the correct approach (Clement, 1982). Such results are compatible with the conceptual, theoretical framework, which predict difficulties in learning when the new knowledge to be acquired comes in conflict with what is already known (Vosniadou, 1994). If the concepts are accepted, it may be that they are accepted as special cases, exceptions to the rule (Hashweh, 1986), or in isolation from the students' existing knowledge, only to be used in the classroom (de Posada, 1997; Osborne & Wittrock, 1985) and regurgitated during examinations. Additional years of study can result in students acquiring more technical language but still leave the alternative conceptions unchanged (de Posada, 1997). Students use their whole numbers to interpret new information about rational numbers (Moskal & Magone, 2000; Resnick, Nesher, Leonard et al., 1989; Vamvakoussi, & Vosniadou, 2004) which gives rise to numerous misconceptions about both conceptual and operation aspects of numbers. For example, properties of natural numbers such as "the more digits a number has, the bigger it is" are used in the case of decimals (Stafylidou & Vosniadou, 2004).

Moreover, in the context of mathematical operations, the well-known misconception, such as "multiplication always makes bigger," reflects the effects of prior (existing) knowledge about multiplication with natural numbers (Fischbein et al., 1985). These preconceptions are tenacious and resistant to extinction (Ausubel et al., 1968); deep-seated and resistant to change (Clement, 1987). Smith et al. (1994) opine that a student can doggedly hold onto mistaken ideas even after receiving instruction designed to dislodge them. This persistence does not necessarily mean that instruction has failed. However, it is vital to acknowledge that misconceptions because of their strength and flawed content can interfere with learning new concepts. Mestre (1989) demonstrated the persistence of student misconceptions and the tendency for regressing to these preconceptions even after instruction to dislodge them when students who overcome a misconception after ordinary instruction often returns to it only a short time later.

2.11 The strategies of Solving Linear Inequalities

There is not much debate on the strategies to use when solving linear inequalities in secondary or tertiary maths. The traditional method of solving equations is used, except that division or multiplication by a negative number results in inequality with the sign reversed. For example, solving $-4x + 4 \leq 8$results in

$-4x \leq 4$, which gives the simple-solution-discernable form $x \geq -2$. In interval form, the solution is $(-2 + \infty)$. In research on inequalities studies, three approaches for solving quadratic, polynomial, or rational inequalities have been identified and discussed: the graphic, the sign-chart (and the enhanced sign-chart), and the logical connectives technique.

Inequalities can also be solved by multiplying the inequality by the square of the least common denominator and then solving the resulting polynomial inequality with the sign-chart. A sign-chart used to solve inequalities consists of finding the intervals where the evaluated expression in the composition of the inequality is either greater than or less than zero. The intervals are bounded by all the zeros of the associated equations (from numerator and denominator in rational inequalities) aligned on a number line. The graphic method usually consists of creating a function associated with the inequality, graphing the function, comparing they with the $x - axis$ (or another y in some cases), reading the x values for the appropriate y, and giving the solution (Sackur, 2004).

2.12 Conceptual and Procedural Knowledge

Conceptual knowledge is knowledge about the facts, concepts or principles upon which something is based. Herbert and Lefevre (1986) define conceptual knowledge as knowledge that is rich in relationship, that can be thought of as an interconnected web of knowledge, a network in which linking relationships are as prominent as the discrete pieces of information. Such knowledge is described as interconnected through relationships at various levels of abstraction (Confrey, 1990) which plays a more important role in the learning of mathematics than a procedural one. Learners need to have conceptual understanding, as in the absence of which they will ineffectively indulge in problem-solving and follow wrong procedures to solve them (Confrey, 1990).

Zemelman et al, (2005) state that ‗without a true understanding of the underlying concepts, the serious problem [guaranteed] with learning other concepts. Focusing on understanding mathematical ideas makes students ‗far more likely to study mathematics voluntarily and acquire other skills as they are needed.' Teachers should know their learners' mathematical thinking to structure their teaching of new ideas to work with or correct those ways of thinking, thereby preventing learners from making errors (Sorensen et al, 2003). The way learners think about a concept depends on the cognitive structure learners have developed previously (Battista, 2001). Battista (ibid) also indicates that if learners cannot develop concepts by themselves, they will have a narrow understanding of those specific concepts and will not engage in problem-solving. Learners who do not have background knowledge in mathematics display numerous errors in solving mathematical problems, resulting in most learners grappling with quadratic equations by completing a square. Conceptual knowledge works hand in hand with procedural knowledge. Procedural knowledge should be taught in mathematics to reinforce understanding of mathematical concepts. Procedural knowledge is the ability of learners to use the relevant procedures in solving mathematical problems by following the rules, methods and procedures in different representations (Kanyaliglue et al., 2003 cited in Alhassan & Agyei, 2020; Herbert & Lefevre, 1986).

Procedural knowledge is a particular type of knowledge that learners display in solving problems and adhering to certain instructions when completing different tasks (Herbert & Lefevre, 1986). Luneta (2008) asserts that procedural knowledge is specific to a particular task, and this implies that some procedures are not appropriate to solve certain mathematical problems. Learners may grasp relevant procedures but fail to use them correctly in solving a mathematical problem (Siegler, 2009). Learners who lack this understanding frequently use wrong procedures, which generates systematic patterns of errors in solving problems. Accordingly, teachers should not focus only on factual errors but also on basic errors, especially when learners make the same procedural errors (Garnett, 1992; National Research Council, 2002; Stein & Smith, 1998).

Riccomini (2005) explains the instructional focus on facts as being problematic to teachers teaching parts of concepts or parts of procedural steps because teachers are trained to teach mathematics in terms of general concepts. Therefore, this helps in addressing the problem of learners solving quadratic equations by completing a square. Riccomini also states that the teachers‘ ability to recognise error patterns can be improved and that it might be possible to plan instructions that can help alleviate problematic patterns in this concept. Measures to this effect might be pre-service programmes, professional development opportunities in mathematics, refining curriculum materials, and continued research in mathematics for learners with disabilities.

There is a relationship between procedural knowledge and conceptual knowledge. The correlation is shown when a learner can execute the procedures correctly, which displays a good grasp of conceptual knowledge (Ziegler, Trninic & Kapur, 2021). If a learner has both procedural and conceptual knowledge, s/he can solve more complex problems of the same concept. Learners with conceptual understanding produce a substantial gain in both kinds of knowledge, but those with procedural understanding produce substantial gains in procedural knowledge but less in conceptual knowledge, which will ultimately impede learners‘ growth in mathematics. Nesher (1986) supports the view that if a learner can only be shown procedures of solving a particular problem without understanding the concept, it is very unlikely that such a learner would be in a position to solve more complex problems independently. If problems are difficult to solve, then learned procedures might not help and need a learner to have conceptual knowledge to solve them (Nesher, 1986). Conceptual knowledge also provides and constraints procedures to solve mathematical problems (Confrey, 1990).

2.13 Pre-service teachers' conceptual and procedural knowledge of linear inequalities

To be successful as a mathematics teacher, students must have a solid understanding of mathematical concepts and techniques. A solid comprehension of mathematical topics and methods instils confidence in the mathematics instructor in the classroom. When this knowledge is integrated with pedagogical knowledge, the teacher's competency is enhanced, and he is better able to address the student's learning issues and misconceptions. As a result, successful teaching necessitates the acquisition of conceptual and practical knowledge. Zakaria, Yaakob, Maat and Adnan (2010) pointed out that knowing mathematics has to do with understanding certain concepts and procedures.

Furthermore, there has to be a link between conceptual and procedural knowledge. It is imperative, as competence in mathematics is a combination of both concepts and procedures. Zakaria and Zaini (2009) examined the conceptual and procedural knowledge of algebra trainee teachers. The prospective instructors' conceptual and procedural expertise was found to be above average in the study. Engelbrecht, Harding, and Potgieter (2005) investigated undergraduate students' performance and confidence in procedural and conceptual mathematics and found that conceptual knowledge performed better than procedural knowledge. When pre-service mathematics instructors were tested to see how well they knew the subject they would teach, it was discovered that the majority of them could not substantiate their responses with conceptual understanding. Only approximately a quarter of the prospective instructors successfully clarified their answers using conceptual knowledge (Bryan, 1999). Stump (1996) surveyed secondary maths instructors to test their understanding of slope principles. The study evaluated the conceptual understanding of pre-service and in-service teachers, and the findings revealed that pre-service teachers had trouble distinguishing between linear and nonlinear equations and were unable to answer questions on the rate of change. Similarly, Faulkenberry (2003) researched secondary mathematics pre-service teachers' conceptual knowledge of algebra and found that preservice teachers gave explanations based on procedural knowledge. The research also found that experienced teachers' explanations were based on both conceptual and procedural knowledge.

2.14 Empirical Studies

This section of the theies presents the findings of various authors, scholars on pre-service teachers' errors and misconceptions in solving problems in inequalities. It also presents the sources of these errors and misconceptions in the teaching and learning of linear inequalities.

2.15 Common errors and misconceptions in solving inequalities

Bicer et al. (2014) opined that not only do middle and high school students hold misconceptions and have difficulties with inequalities, but college students also possess some of these same misconceptions, thus demonstrating difficulties in solving and interpreting inequalities. Bicer et al. (2014) conducted a survey in Texas in the

United States of America on pre-service teachers' understanding of linear and quadratic inequalities to determine whether they possess common errors and misconceptions with inequalities. Their research focused on mathematics pre-service teachers' understanding of linear and quadratic inequalities to determine whether they possess common misconceptions and difficulties with inequalities. Findings from their study indicated that many pre-service teachers have misconceptions and difficulties. Some of the misconceptions the authors described were pre-service teachers' lack of understanding of what inequalities' questions ask to find, having trouble with representing inequalities' solutions, and arithmetic errors that cause them to misunderstand inequalities.

In a similar context, Shalash (2019) also conducted a study using fifty (50) students in a general mathematics classroom at Al-Quds Open University in Jordan. The descriptive analysis method was employed in the study. The results showed some students have some misconceptions and misunderstandings in solving inequalities and operations on problem-solving with fractional linear inequalities. Their misconception and misunderstanding were identified when asked to carry out elementary algebraic operations and solve fractional linear inequalities. Moreover, Vaiyavutjamai and Clements (2006) conducted a study using 231 ninth-grade students revealed that the students often treated inequalities as equalities and demonstrated confusion of the meaning of solutions to inequalities.

Additionally, Serhan and Almeqdadi (2015) also surveyed Preservice teachers' linear Inequalities Solving Strategies and Errors at Emirates College for Advanced Education, United Arab Emirates. Fifty-one (51) pre-service teachers enrolled in an introductory mathematics course participated in this study. Data were collected from a test that consisted of four linear inequalities in one variable. The results of the study indicated that many pre-service teachers made few errors when solving linear inequalities. These include errors in representing the solution as interval or on the number line, multiplying/dividing by a negative number, and arithmetic errors. El-Shara and Al-Abed (2010) reserached to identify errors made by mathematics majors at the University of Jordan when solving inequalities. A single test was devised and administered to 188 male and female mathematics majors who had completed Calculus 101. The study's findings revealed certain typical blunders, such as mistaking an inequality for an equation, solving inequalities using commutative multiplication, and altering the direction of inequality when multiplying by a negative number. There were also some other calculating and thoughtless errors detected. The most prevalent errors varied from 5.7 per cent when multiplying by a negative number to 22.5 per cent when multiplying by a positive number. According to the study, faculty members should emphasise inequities for new students, offer exams to categorize them, and design appropriate treatment programmes.

Ureyen, Mahir, and Çetin (2006) conducted a study in the turkey universities to identify the mistakes pre-service teachers commit when undertaking inequalities. Their study aims to assess students' performances and to discover the errors made by students enrolled in a Calculus course when solving inequalities. A test was administered to undergraduates who had taken a calculus course at a Turkish university. The findings indicated that students struggle to solve inequities. The most often detected error was multiplying both sides of an inequality by an expression that included a variable without regard for the expression's sign.

Moreover, in Denbel's (2013) study on Students' Difficulties of Solving Inequalities in Calculus, the study tries to analyse the students' difficulties and explore the errors done by the students when finding solution sets for inequalities. For these purposes, a test was given to the college of natural and computational science students who took calculus I or applied mathematics I course in Dilla University, Ethiopia. The results showed that the students are not successful in solving inequalities. The most observed mistake was to multiply both sides of an inequality by an expression that includes a variable without paying attention to the sign of this expression. Moreover, a significant number of procedural and technical errors are made by the students. Tsamir and Almog (2001) also surveyed Students' strategies and difficulties: the case of algebraic inequalities using university and senior high school students as the Respondents. The study aimed at students' thinking about linear, quadratic, rational, and square-root inequalities. Findings show that using graphic representations of parabolas when solving rational and quadratic inequalities usually yielded correct solutions. Difficulties arose when students failed to reject the excluded values or chose inappropriate, logical connectives. The most prevalent source of difficulties was inappropriate analogies between equations and inequalities. The article concludes with some suggested educational implications.

Çıltaş and Tatar (2011) through a qualitative analysis method diagnosed learning difficulties related to the equation and inequality that contain terms with absolute value in Turkey. Their main aim was to diagnose learning difficulties in equations and inequalities that contain absolute value terms and make recommendations for teachers. A total of 170 ninth-grade students from four different high schools made up the study's sample. The research's data comprises a knowledge test with ten open-ended questions and interviews with students. Findings from the study indicated that the students had difficulty forming a correct solution set because they acted as if there was no absolute value while determining the solution set for the equation and inequality test. They were unable to internalise the concept of absolute value fully.

Naseer (2015), in the quest to help students unlearn the misconceptions and re-learn the correct conceptions, analysed students' errors and misconceptions in pre-university mathematics courses in England, Villa. The analysis aimed to bring awareness of some of the errors students make and the misconceptions they have concerning mathematical concepts and suggest how the formation of these errors and misconceptions can be remediated. The analysis revealed that misconceptions observed with inequality questions were very similar to misconceptions observed with equations. Furthermore, it was clear that students were trying to apply some ―rules" without having a fundamental understanding of how or why the rule works.

2.16 Summary of the Literature

The Learning theories formed the basis of the theoretical review. The social constructivist learning approach, discovery learning approach and many others were adopted for the study. The discovery learning theory postulated that when students use discovery processes to make inferences and speculations about scientific phenomena, they may develop misconceptions. It is because the meaning is distinctive or unique to that particular person. Moreover, various conceptualisations were discussed under the review. Empirical findings from the literature revealed that preservice teachers' errors and misconceptions about linear inequalities are due to their lack of conceptual and procedural knowledge in mathematical concepts. The literature identified that prospective teachers forget the reverse of the inequality sign when dividing/multiplying both sides of the inequality by a negative number. It was also noticed that many of the prospective teachers think that inequalities and equalities require the same mathematical solution process; they treat problems involving inequalities in the same manner as equations and assume the questions require similar processes.

CHAPTER THREE

METHODOLOGY

3.0 Overview

This chapter presents the systematic process of investigation used for the study. The chapter, specifically, explores the philosophical underpinnings and the decision of research methods and cycles used for the study. It also looks at how members were chosen and how the research instruments were developed and utilised for information gathering without neglecting ethical issues. This research aimed to investigate the errors and misconceptions in solving problems in linear inequalities among preservice teachers.

3.1 Research Paradigm

According to Saunders et al., (2009), a research paradigm is defined as the nature and growth of knowledge. This means that paradigms establish the basic principles and procedures by which the researcher determines what to examine, how to study it, which theory and technique to adopt, and how to interpret the study's results. Theoretical paradigms is critical in qualitative research, as qualitative research is conducted and/or evaluated using distinct assumptions from quantitative research (Krauss, 2005). As a result, researchers might adopt a variety of research ideologies, including positivism, interpretivism, and realism. To the researcher, these paradigms function as tailored goggles that modify perceptions of a phenomenon and influence how the observed phenomenon is made sense of (Bhattacherjee, 2012). The researcher's cognition is shaped by the paradigm (Bogdan & Biklen, 1982). Thus, while choosing an appropriate paradigm is a personal decision for each researcher (Panagiotopoulos et al., 2018), Kinash (2006) asserts that paradigms are contextualized beliefs held by social groups, and thus a researcher's choice of a paradigm may also be a product of the mind-set of the social group to which the researcher belongs at the time.

Positivism presupposes that reality exists apart from humans. It operates independently of our senses and is regulated by unchangeable laws. As Hutchinson (1988) puts it, "Positivists regard the world as 'out there' and explorable in a more or less static condition" (cited in Gall, Gall & Borg, 2006, p. 14). The objective is to quantify, control, forecast, build laws, and assign causality (Cohen, Manion, & Morrison, 2007).

Interpretivism believes that reality is a social creation that is subject to the individual's perceptions and interpretations (Walsham, 2006). As a result, the most effective method for ascertaining reality or comprehending a phenomenon is to see it in context. Numerous realities are built by individuals who encounter an interesting phenomenon. To the interpretive, attempting to establish truth in an external, objective sense is fruitless, as each researcher is unique, and so their personal opinions are certain to affect their research (Mack, 2010; Sobh & Perry, 2006; Walsham, 1995). The application of interpretive approaches by information systems researchers is increasing (Currie & Swanson, 2009). The interpretive paradigm is used in social research to assist the researcher in comprehending and explaining these contextualized realities as experienced and comprehended by the actors, of which the researcher is a member.

Critical theory's fundamental principle is to critique and resolve socioeconomic inequities that arise as a result of the race, social class, culture, religion, gender, and sexual orientation (Fay, 1987). According to critical theory, enslavement, alienation, and other types of dominance in society are the result of conception. Thus, the social reality is historical in nature and is formed and repeated by humans (Myers, 1997). While critical theorists think ideological subjects may consciously act to improve their socioeconomic circumstances, they also understand that their capacity to change is limited by many types of social, cultural, and political dominance. By exposing the restrictions of the status quo and empowering its subjects, this paradigm seeks to emancipate its subjects from ideologically frozen conceptions (Comstock, 1982), to abolish the roots of these social inequities.

The philosophy underpinning this study is based on the beliefs of the pragmatists. The pragmatists are of the view that the world should not be seen as a single unit but a multiplicity of units and hence, the use of multiple approaches to collect and analyse data (Biesta, 2010; Hall, 2013; Johnson, & Onwuegbuzie, 2004; Morgan, 2007; Pearce, 2012; Tashakkori, & Teddlie, 2010). The founders of Pragmatism all believe that "ideas are not out there waiting to be discovered, but are tools – like forks and knives and microchips– that people devise to cope with the world in which they find themselves" (Snarey, & Olson, 2003, p. 92). Pragmatists believe that ideas are social constructs delivered not by people but rather by gatherings of people and that human carriers and the environment entirely influence ideas that are generated. This occurs as a result of their use of multiple sources of information (Cohen, Manion, & Morrison, 2007).

These authors assume that their success relies on their immutability and adaptability because concepts are transient responses to real situations (Snarey & Olson, 2003). For this study, a particular phenomenon should not be investigated using only one source of data, using mixed methods. Again, in my opinion, the use of multiple data sources (achievement tests administration and interview conduct) helped understood the errors and misconceptions in solving problems in linear inequalities among preservice teachers.

3.2 Research Methodology

Rovai et al. (2013) state that a technique serves as a model for data collection, measurement, and analysis. Myers (1997) classifies research methods into two basic methods: qualitative and quantitative. Baiden et al., (2006), on the other hand, stated that research strategies fall into three main categories: quantitative, qualitative, and hybrid research. The choice between these three major categories is determined by several factors, including the study's objective, the research questions, and the type and simplicity with which the necessary information may be obtained (Naoum, 2012). The researcher's choice of research method is influenced by his or her underlying epistemological assumptions and does influence the data collection strategies used.

By quantifying the variance in a phenomenon, quantitative research can ascertain the magnitude of an issue or the presence of a relationship between its various facets (Boateng et al., 2016). Quantitative methods are used to identify and quantify features to develop statistical models that may be used to test hypotheses and explain data. It is related to the collection of numerical data and the generation of data to explain cause and effect relationships (Bhattacherjee, 2012). Its objective is to substantiate theories concerning phenomena. Quantitative research is increasingly being applied in the social sciences, for instance, through survey methods, laboratory experiments, formal methods, and numerical descriptive methods (Kuhn, 1970). In quantitative research, the instruments used to elicit and classify replies to questions are stricter and employ more structured methods.

Researchers in the social sciences (e.g., Merriam 2009 & Myers, 1997) created qualitative methods of inquiry to study social and cultural phenomena. Qualitative research aims to comprehend and explain how individuals construct their reality through their experiences and judgments (Merriam, 2009, p. 13). Qualitative research, in general, is predicated on a relativistic and interpretive ontology (methodological choice) that asserts that there is no objective reality but rather different realities formed by the people who encounter a phenomenon. Inductive reasoning is used to construct theories in qualitative research, and data sources include observation and participant observation (fieldwork), interviews and questionnaires, documents and texts, and the researcher's impressions and reactions (Myers, 1997).

In a mixed-methods study, qualitative and quantitative data are combined in one study. This approach helps researchers to thoroughly comprehend and explain complex events (Halcomb & Hickman, 2015). Johnson and Onwuegbuzie (2004) state that when researchers employ mixed methods, they gain an understanding of the advantages and disadvantages of qualitative and quantitative approaches, which enables them to capitalize on the researcher's strengths and neutralize the researcher's risks. The data collected is both numerical and verbal, and it is analysed using both qualitative and quantitative approaches.

This current study employed both the quantitative and qualitative methods based on the paradigm underpinning this work. Pragmatism is the philosophical debate that forces the merging of qualitative and quantitative research methods into a single sample. So placed, the confidence in doing what is best to obtain the intended outcome is Pragmatism. As an overarching research philosophy, Pragmatism allows researchers to choose amongst various research styles, as research questions that are answered eventually determine the approaches are better adapted (Morgan, 2007). While utilising qualitative analysis, some research problems are better answered, while others use quantitative approaches. The pragmatic philosophy underlying this analysis required suitable qualitative and quantitative methods to implement each particular objective systematically.

3.3 Research Design

In seeking to understand the errors and misconceptions in solving problems in linear inequalities among preservice teachers, the present study adopted the sequential explanatory mixed methods design for comparison to ensure cross-validation, corroboration, expansion, complementarity and triangulation (Creswell, 2021). The sequential explanatory mixed methods design as employed in this study consisted of two distinct phases. The quantitative (numeric) data was gathered during the first phase. The objective of the quantitative stage was to recognise the potential predictive power of designated items converged to explain a particular variable on the errors and misconceptions in solving problems in linear inequalities among preservice teachers and allow for the selection of participants for the interview.

During the second phase, a qualitative methodology was used to gather data from individual semi-structured interviews, which help brighten important analysis and teaching nexus. The rationale for adopting this approach to this current study was that the quantitative information and results provided an overall image of the interplay between research and teaching. In contrast, the qualitative information and their examination was refined, constantly explain those statistical outcomes by investigating members' perspectives in more profundity, making room for thick description regarding the errors and misconceptions in solving problems in linear inequalities among preservice teachers.

It is worthy of note that there are two variations of sequential explanatory mixed methods design: these discrepancies were due to the relation between qualitative approaches and the previous quantitative findings. If researchers focus on using quantitative knowledge to screen and involve subjects in a more comprehensive qualitative analysis, the sample selection model is being used. In my study, the selection of the participants for the interview was based on some critical findings from the quantitative results. So, there was the need to probe further to investigate the actual activities they engage in. The model of follow-up interpretations is being used in the quantitative stage to clarify and describe group variations or statistical associations. To better understand these discrepancies, this can be achieved by selecting sample subjects who fall into the corresponding groups and using qualitative approaches

The design requires that data analysis is done in a sequential manner, where the qualitative data from the interview were used to explain the quantitative data. In terms of priority in this research, since most of the research questions were susceptible to both inferential and descriptive statistics that fall under the quantitative domain's jurisdiction, a greater emphasis was placed on the quantitative analysis phase, yielding a quantitative-dominant mixed analysis (QUAN-QUAL) (Creswell, 2013).

The justification for applying the mixed method in this study is for purposes of development and expansion. It enables me to interpret the quantitative data, supported by the qualitative results, to enhance, expand, illustrate, or clarify findings derived from the quantitative strand regarding the errors and misconceptions in solving linear inequalities among preservice teachers. Also, to achieve the development purposes, the quantitative data were collected first, and the findings from the quantitative analysis informed the data collected and analysed during the qualitative phase (second phase) of this study. For purposes of expansion, since it has already been established in the gap analysis of the problem statement that most of the previous studies adopted either the quantitative or qualitative approach separately. In this current study, quantitative and qualitative analyses were utilised to increase the examination's scope and core interest (Creswell, 2013).

One major justification for the choice of the mixed method for this study is that new bits of knowledge and discoveries from one technique may improve the other strategy (Creswell, 2013). This pattern made the discussion very interesting because of the different perspectives due to the heavy reliance on multiple methods. In simple terms, utilisation of these two techniques allows meanings and findings to be elaborated, enhanced, clarified, confirmed, illustrated or linked (Saunders & Lewis 2012). The narrative approach is utilised for comprehensive explanations of observations from a statistical model and the other way around, which makes it possible for this research to generalise the effects on the population. Findings from the narrative approach were corroborated with those of the quantitative survey and assisted in making inferences and transfers to different settings with comparative states of this investigation (Creswell & Miller, 2000).

Data from the numerical quantitative phase complemented the results from the qualitative approach and vice versa which allowed for orderly checks on the approval or consistency of discoveries between the quantitative and qualitative phases of the examination. Data from the interviews supported investigations from the achievement test. Evidence from the findings of the two data sources was then captured and corroborated to give meaningful insights into the errors and misconceptions of preservice teachers in solving problems in linear inequalities (Yin, 2015). Describing the errors and misconceptions using the statistical figures on the preservice teachers alone may not be enough to reflect the reality. However, the use of follow-up explanation of the quantitative results with the qualitative results ensured an extensive clarification into the dynamics of the errors and misconceptions of preservice teachers and gave a thick description and rigorous insights into preservice teachers problem-solving in linear inequalities.

3.4 Population

Polit and Beck (2010) stated that population is the comprehensive assortment of phenomena or elements the researcher had an interest in and that these elements have similar characteristics. Thibaut (2020) differentiates between two types of population, the target population and accessible population. The target population is the total group of subjects to which a researcher would like to generalize the results of a study and accessible population is the group of subjects that is accessible to the researcher for a study from which the study sample can be drawn (Thibaut, 2020). The target population for this study comprised all 2000 preservice teachers in Komenda College of Education. The accessible population were all 1090 level 400 preservice teachers.

3.5 Sample and Sampling Procedures

A sample is a set of elements taken from a more significant population. It is, usually, a smaller group the researcher studies. Sampling refers to the process of choosing part of the population to represent the whole population (Amedahe & Asamoah-Gyimah, 2016). The study's sample frame was the Komenda College of Education in the Central Region of Ghana. According to Breakwell, Harmond, Fife-Shaw and Smith (2006), when a population is extremely large or infinite, it makes it impossible or too costly to study. Thus, to ensure a more detailed study of the element involved, a sample size of 285 preservice teachers was used for the study. The researcher chose this sample size provided by Krejcie and Morgan (1970), as cited in

Sarantakos's (2012) table for sample size determination, indicating that for a population of 1090, a sample size of 285 should be adequate. According to Krejcie and Morgan (1970), the table does not require any form of calculations in its usage. The table applies to any definite population.

Table 3.1

Table for Determining Sample Size of a Known Population

N	S	N	S	N	S	N	S	N	S
10	10	100	80	280	162	800	260	2800	338
15	14	110	86	290	165	850	265	3000	341
20	19	120	92	300	169	900	269	3500	346
25	24	130	97	320	175	950	274	4000	351
30	28	140	103	340	181	1000	278	4500	354
35	32	150	108	360	186	1100	285	5000	357
40	36	160	113	380	191	1200	291	6000	361
45	40	170	118	400	196	1300	297	7000	364
50	44	180	123	420	201	1400	302	8000	367
55	48	190	127	440	205	1500	306	9000	368
60	52	200	132	460	210	1600	310	10000	370
65	56	210	136	480	214	1700	313	15000	375
70	59	220	140	500	217	1800	317	20000	377
75	63	230	144	550	226	1900	320	30000	379
80	66	240	148	600	234	2000	322	40000	380
85	70	250	152	650	242	2200	327	50000	381
90	73	260	155	700	248	2400	331	75000	382
95	76	270	159	750	254	2600	335	1000000	384

Note: N is Population Size; S is Sample Size *Source: Krejcie & Morgan, 1970*

Fig 2: Krejcie and Morgan (1970) Table.

The simple random sampling technique was adopted to select the 285 students whereas the convenience sampling technique was used to sample fifteen (15) students from the 285 to take part in the interviews although all the students had an equal chance of being included in the study.

3.6 Data Collection Instruments

The instruments used for the data collection for this study are achievement tests (paper and pencil tests) and semi-structured interview guide.

3.6.1 Achievement tests

The main instrument used by the researcher to gather the quantitative data for the study was a paper and pencil test (Appendix A). The paper and pencil test included questions on linear inequalities that the respondents were supposed to solve. After the responses were collected, the researcher marked the test and at the same time paid critical attention to the errors involved in solving linear inequalities. The pencil and paper test used for the data collection was classified into two sections, namely A and B. Section A covered the demographic details of Respondents, including age, sex, and programme of study. Section B consisted of 10 questions on linear inequalities where the Respondents were supposed to provide the solution to the linear inequality problems.

3.6.2 Interview guide

The second stage of the data collection made use of detailed semi-structured face-to-face interview guide (Appendix B) to collect the qualitative data needed to respond to the research questions. Mugenda and Mugenda (1999) asserted that the interview guide provides participants with the opportunity to express themselves freely without limitation. This enabled a deeper understanding of the phenomenon being studied. The discoveries of the quantitative process were the reasons for the content of the interview questions. The questions on the interview guide focused on the issue of preservice teachers‘ conception, level of misconception, and sources of errors and misconceptions.

3.7 Validity of the Instruments

Punch (2003) argues that validity is determined by respondents' ability to honestly and thoughtfully respond to questions, which he claims is dependent not just on Respondents' disposition and mental state, but also on their ability to answer the instrument's questions. Lodico et al. (2010) state that validity is concerned with whether a test measures what it is meant to measure. Mugenda and Mugenda (2002) defined validity as the degree to which a test measures what it is intended to measure. To ensure the face and content validity of instruments, experts' opinions were sought from the supervisor, lecturers, and peers. Consultations with the supervisor, other lecturers, and peers helped to identify errors and offer the opportunity to modify and improve the instruments.

3.8 Reliability of the Instruments

Reliability is the degree to which scores on a test are consistent or stable over time (Lankshear & Knobel, 2004). It is to say that an instrument is regarded as reliable if it produces similar results on occasions when administered to the same Respondents. It also means reliability is how results are consistent over time and an accurate representation of the total population under study.

3.9 Pilot Test

To find out the errors and misconceptions of preservice teachers at Komenda College of Education, the achievement test was pilot tested in Komenda College of Education of the Central Region. According to Connelly (2007) as cited in (2017), extant literature has suggested that a pilot study sample should be 10% of the sample projected for the larger parent study. Given Connelly's assertion, the researcher used eighteen (28) teachers who represented 10% of the sample projected for the study (285 preservice teachers). The sample of eighteen (28) preservice teachers were conveniently sampled for the pilot-test. The researcher used this sampling technique after taking into consideration time and other resources at his disposal.

The researcher used the Spearman Brown Coefficient using Statistical Product for Services Solution (SPSS) version 26 to calculate the reliability. The scores of the respondents were keyed into the SPSS 26 and the resultant coefficient calculated was 0.84 which is greater than 0.70 and therefore deemed reliable. As prescribed by Creswell (2021), the general convention is to strive for reliability values of 0.7 or higher.

3.10 Trustworthiness

The aim of trustworthiness in a qualitative inquiry is to support the argument that the inquiry‘s findings are worth paying attention to (Polit & Beck, 2012). To bring about trustworthiness, the researcher incorporated five aspects of trustworthiness into the study: credibility, dependability, confirmability, authenticity, and generalizability.

3.10.1 Credibility

The significance of credibility stresses on multiple accounts of social reality is evident in the trustworthiness criterion of credibility. Shufutinsky (2020) asserted that, if there can be several possible accounts of an aspect of social reality, it is the credibility of the account that a researcher arrives at, that is going to determine its acceptability to others. Establishing the credibility of findings entails ensuring that research is carried out according to the canons of good practice and submitting research findings to the members of the social world (Amin et al., 2020). In other words, this criterion of trustworthiness examines if readers of the research believe what the authors are reporting. To ascertain credibility, the researcher used member checking. In the process of member checking, the researcher returns data, analytic categories, data interpretations, and/or even conclusions to the study‘ participants.

The argument is that by giving participants the opportunity to review research work, a researcher can claim that the work adequately presents their own and multiple realities. The use of member checking to determine the accuracy of the qualitative findings was done by taking the final report or specific descriptions back to participants and determining whether these participants felt that they were accurate. This did not mean taking back the raw transcripts to check for accuracy; instead, the researcher took back parts of the polished or semi-polished product, such as the significant findings, the cultural description, and so forth. It also provided the opportunity for a participant to recall additional points/ideas, correct errors, and provide context.

3.10.2 Dependability

As a parallel to reliability in quantitative research is the concept of dependability. Dependability indicates that the researcher's approach is consistent across different researchers and projects (Lemon & Hayes, 2020). To ascertain qualitative reliability (dependability), the researcher used a detailed, thick description to convey the findings. According to Little and Green (2021), this description may transport readers to the setting and give the discussion an element of shared experiences. When qualitative researchers provide detailed descriptions of the setting, for example, or offer many perspectives about a theme, the results become more realistic and richer. This entails ensuring that complete records are kept of all phases of the research process— problem formulation, selection of research participants, fieldwork notes, transcripts data analysis decisions, and so on, in an accessible manner. The researcher kept detailed records of the observation process to allow comparison. This procedure added to the dependability of the findings.

3.10.3 Transferability

Transferability describes the degree to which research findings will be applicable to other fields and contexts (Connelly, 2016). Researchers who are concerned about transferability should question whether their results will hold in another setting or group of participants. According to (Kyngäs et al., 2020), it is important to note that transferability is not the same as generalization in quantitative research because transferability is concerned with how readers will extend the results to their own situations, whereas generalization covers the extension of results from a sample to a broader population. Transferability, is therefore, affected by every stage of research, including the choice of research context and topic. During the research planning phase, a researcher should consider transferability by clearly describing the sampling techniques, potential inclusion criteria, and participants' main characteristic so that other researcher can assess whether the results drawn from this sample are applicable to other contexts (Kyngäs et al., 2020). To achieve this, the researcher gave a transparent report of the research process and results that is critical to achieving sufficient transferability. Every researcher is responsible for providing enough information about their study so that the audience can evaluate whether the findings are applicable to other contexts (Kyngäs et al., 2020).

3.11 Data Collection Procedures

Since the study adopted the sequential explanatory mixed methods, quantitative data were first collected using the achievement test after which the qualitative data was collected through interviews using the semi-structured interview guide. Data collection spanned from July to August 2021. To gain access to the college of education selected for the study, an introductory letter from the Department of Basic Education, University of Education, Winneba was presented to the Komenda

College of Education to seek permission to administer the test and conduct the interview as well. The purpose and intent of the research were indicated in the letter. After the collection of the administered test, the respondents who agreed to take part in the interview were prompted to be ready for the interview. An interview was carried out with 15 participants to obtain detailed information of preservice teachers' errors and misconceptions in solving linear inequalities. Green & Thorogood (2004) maintain that the experience of most qualitative researchers conducting an interview-based study with a fairly specific research question is that little new information is generated after interviewing 20 people or so belonging to one analytically relevant participant ‗category' (pp. 102–104). Lincoln & Guba (1985) proposed that sample size determination be guided by the criterion of informational redundancy, that is, sampling can be terminated when no new information is elicited by sampling more units

The interview was conducted in a suitable and relaxed position, depending on the consensus between all sides, for the observer and participants, and meeting times were scheduled. Therefore, the preservice teachers were interviewed in their respective classes. The interview lasted for 10 to 15 minutes.

3.12 Data Processing and Analysis

The data collected from the field were processed and analysed using Statistical Product for Service Solutions (SPSS version 26.0) and Microsoft Excel. The data comprised the main data for the objectives. The demographic data of the Respondents were analysed using frequency counts and percentages and represented in charts, graphs, and tables. The analysis of the specific research questions and objectives are highlighted below.

Research Question One was analysed using frequencies and percentages. The frequency and percentages were derived during the marking of the achievement test. During the marking of the achievement test, the researcher categorised the papers into each of the preconceived errors discussed in the literature review. Each error that the respondents committed while trying to solve the inequality questions had their frequencies and percentages tabulated.

Research Question Two was analysed using frequencies and percentages. The frequency and percentages were derived during the marking of the achievement test. During the marking of the achievement test, the researcher categorised the papers into each of the preconceived misconceptions discussed in the literature review. Each misconception that the respondents committed while trying to solve the inequality questions had their frequencies and percentages tabulated.

Research Question Three was analysed using thematic analysis. The responses from the interviews were transcribed by playing the recorded audios over and over until the exact words of the respondents were captured.

Research Question Four was analysed using frequencies, percentages and thematic analysis. Just as research question one was analysed, the researcher in this case categorised the respondents' solutions to the linear inequality questions in the achievement test into the alternative errors and misconceptions. Frequency counts and percentages were derived for the analysis. Furthermore, the interviews that were conducted to provide an in-depth response to this research question were also analysed.

3.13 Ethical Considerations

Ethical clearance was sought from the Institutional Review Board (IRB) of the University of Education, Winneba. As part of the process leading to the data collection, issues about informed consent, access and acknowledgement in the research setting, protection, obscurity, and classification arrangements were submitted to the IRB of the University of Education, Winneba for clearance to enable me to go ahead with the actual data collection. A covering letter was attached to the instrument to furnish the participants with the vital data needed to respond to the items. To ensure that no participants felt coerced, they were given a chance to indicate their willingness to participate in the research. Further, like Saunders, Lewis, and Thornhill (2007) suggest, the participants were pre-informed that the research report would be published and accessed in the public domain. However, the identity of each participant would never be revealed; hence, no risk in taking part in the study. Lastly, voluntary participation was assured in this study, and the data collected from the participants were treated with the utmost confidentiality and anonymity to help protect Respondents' identities. Also, a member check was done to ensure that the true record of the qualitative data was captured and analysed.

However, it was clarified to the respondents that their support in the study was deliberate (Neuman, 2017), and thus, they were encouraged to provide accurate and honest information if they were willing to participate. I explained to the participants, they reserved the privilege to pull out from the study anytime (Creswell, 2021), but this right ended after their instrument had been submitted. It was because of the difficulty of tracing back their test to be taken out of the analysis. Respondents were made mindful that the investigation was liberated from any psychological or physical maltreatment (Neuman, 2007). All COVID-19 protocols were also adhered to during the data collection.

3.14 Chapter Summary

This chapter presented the approaches and methods adopted for the study. It entailed every aspect performed in achieving the objectives of the study. It also highlighted the various instrumentations used for the study. Various procedures used in analysing the data obtained were discussed apparently.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Overview

The purpose of the study was to examine the errors and misconceptions in solving problems in linear inequalities among preservice teachers. The study employed the sequential explanatory mixed method. This chapter highlighted the results and discussions from the study. The chapter presented the results according to the purpose of the study. The chapter consist of two major sections. Section One present results on the demographic characteristics of the participants for the study. The second section consist of the responses obtained from the respondents according to the purpose of the study.

4.1 Demographic Characteristics of the Respondents

This section presented the background information of the participants. It included their age, gender, and programme of study. Even though 285 achievement tests were distributed, 177, representing 62.1% return rate were received. This was as a result of the timing of data collection, which was done immediately after preservice teachers had already written an end of semester examination and also preservice teachers were not willing to participate in the study with the intention of them not being mathematics major students. Hence, the analysis was conducted based on 177 achievement tests results and 15 interview responses. The analysis of the demographic data is presented using frequencies and percentages as shown in Table 1.

Table 1: Demographic Characteristics of the Respondents

Variable	Frequency (n)	Percentage (%)
Sex		
Male	83	46.9
Female	94	53.1
Total	**177**	**100**
Age		
18-22 years	94	53.0
23-27 years	65	37.0
28-32 years	10	6.0
33-36 years	8	4.0
Total	**177**	**100**
Programme of study		
Early Childhood	25	14.1
Home Economics	44	24.9
Visual Arts	13	7.3
Maths and Science	39	22.0
Maths and ICT	30	16.9
Primary Education	16	9.0
Agric Science	10	5.6
Total	**177**	**100**

Source: Biney (2021), Field data

Table 1 showed the summary of the respondents‘ demographic characteristics. The results indicated from Table 1 showed that one hundred and seventy-seven (177) Respondents were sampled for the study. Undoubtedly, 86(46.9%) of the Respondents were males whereas the remaining 94(53.1%) were their female counterparts. Hence, majority of the participants were female. The results could have a significant influence on the misconceptions about linear inequalities at the college level because, at the time of the data collection, the female respondents participated in the study than the males.

The ages of the participants were predetermined. Out of the 177 participants, it was revealed that (Frequency (n)=8, 4%) of the respondents were aged between 33-36 years. More importantly, (n=10, 6%) of them were also between the ages of 28-32 years. Additionally, (n=65, 37%) of them were also between 23-27 years, whereas the remaining (n=94, 53%) were also within 18-22 years. The results from the study can be concluded that the majority of the Respondents were within 18-22 years. These results show that most of the participants are young adults.

Out of the 177 participants, it was indicated that (n=25, 14.1%) of the students' programme of study was early childhood development. 44(24.9%) of the participants were Home Economics students whereas 13(7.3%) were visual Arts students. More importantly, Maths and science, and Maths and ICT were also some of the programmes offered by (n=39, 22.0%), and (16.9%) of the participants respectively. Additionally, it was indicated that (n=16, 9.0%) of the participants' study programme was primary education. Furthermore, the study indicated that 10(5.6%) of the participant's study programme was agriculture. The study concluded that the majority of the participants were Home Economics students which were followed by mathematics and science education students. It can also be concluded that students' programme of study could significantly affect their errors and misconceptions about the teaching and learning of linear inequalities.

4.2 Research Question One

What errors are revealed in preservice teachers' solutions to linear inequality problems?

The purpose of this research question was to explore the common errors preservice teachers commit in solving linear inequality problems. To do this, an achievement test was designed with questions involving linear inequalities. The test was given to the Respondents and they were allowed time to complete it. After the test papers were taken, the researcher marked and the common errors identified in the respondents‘ solutions were represented in Table 3. The analysis was done using frequencies and percentages to determine the number of respondents who have committed any of the errors in solving linear inequalities.

Table 2: Errors Committed

Students' errors	Type of errors	F (%)
Students failed to apply the basic arithmetic rules	Rules Mixed up	140(79.0)
Students failed to use the addition, subtraction operations inappropriately	Surface understanding	130(73.5)
Student inability to assimilate word problems into equations or inequalities	Inability to assimilate concepts	130(73.5)
Students incorrectly assumed that negative signs denote simply subtraction and do not change equations	Carelessness	125(70.6)
Students‘ inability to present the final answer on a number line	Poor knowledge	103(58.2)

Source: Researcher construct, 2021

From the results in Table 1, it was revealed that (n=140, representing 79.0%) of the preservice teachers were using the linear inequality operations incorrectly. In a test given to the preservice teachers, it was realised that most of the preservice teachers had difficulties with the usage of the linear inequalities‘ rules. Some apply the rule inappropriately when adding or subtracting from the inequality.

Figure 3 shows a sample of the rules mixed up error committed by one of the respondents during the test. Majority of the preservice teachers committed this error as they did not understand how and where to apply the rules. Although some were able to group like terms and also simplified the equations, they failed to apply the subsequent rules to make the inequality correct or valid. This could be attributed to the consequence of teaching rules before students are given the chance to understand the concept.

4.3 Research Question Two

What misconceptions are revealed in preservice teachers' solutions to linear inequality problems?

The purpose of this research question was to explore the common misconceptions do preservice teachers commit in solving linear inequality problems. To do this, an achievement test was designed with questions involving linear inequalities. The test was given to the Respondents and they were allowed time to complete it. After the test papers were taken, the researcher marked and the common misconceptions identified in the respondents' solutions were represented in Table 4. The analysis was done using frequencies and percentages to determine the number of respondents who have committed any of the misconceptions in solving linear inequalities.

Table 3: Misconceptions Committed

Students' misconception	Frequency	(%)
Expressing inequalities as equations	128	72.3
Representing inequalities on a number	134	75.7
Incorrect common denominator	114	64.4
Oversimplification	81	45.7
Only one value makes an inequality true	92	51.9

Expressing inequalities as equations

Many preservice teachers think that inequalities and equalities require the same mathematical solution process, they treat problems involving inequalities in exactly the same manner as equations, and assume the questions require similar processes (Blanco & Garrote, 2007; Vaiyavutjamai & Clements, 2006). In solving the inequality: -7 > $6t$ + 17 in the same way as solving equation -7 > $6t$ + 17, preservice teachers arrived at the conclusion that $t = -4$. When they put the sign back, preservice teachers simply find $x > -4$ as the solution to the inequality. However, preservice teachers may simply forget the rule that multiplying and dividing by a negative number changes the direction of the inequality, and their solution in actuality needed to be $t < -4$ (Kroll, 1986). Figure 8 shows a sample of this misconception from one of the respondents. From the results in Table 3, it was revealed that (n=128, representing 72.3.0%) of the preservice teachers had misconceptions in solving inequalities as equations.

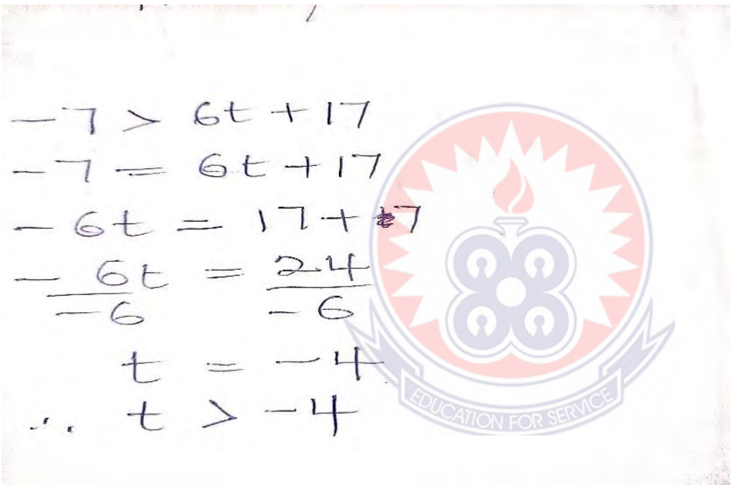

Figure 8: Inequalities as an equation

Representing inequalities on a number

After the achievement test, it was realised that (134 representing 74.5%) of the preservice teachers had a misconception representing inequalities on a number line. In this regard, when preservice teachers found an inequality solution as x>1, some participants shaded the opposite side or direction. This shows that pre-service teachers have limited geometrical understanding or they may not know how to read the inequality symbols. Some pre-service teachers lack efficient semantic value of mathematical terms such as ―greater than" or ―greater than equal to". Rubenstein and

Thompson (2001) suggested that some mathematical words need to be emphasized by teachers so that students grasp both the semantic and symbolic meaning. Tent (2000) shared her mathematics class activities about inequalities, and proposed that pre-service teachers should read inequalities in more than one way to increase their semantic and symbolic meaning about inequalities. Figure 9 shows a sample of this misunderstanding by a preservice teacher.

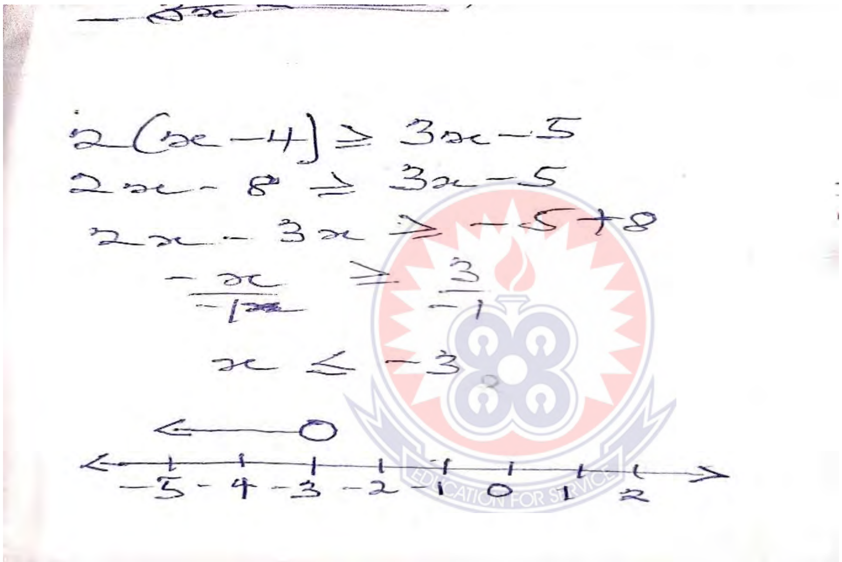

Figure 9: Representing inequalities on a number

Incorrect common denominator

Two different misconception types were detected in this group. They were incorrect calculation of the common denominator for two numbers or two letters. When calculating the common denominator of two numbers, some preservice teachers incorrectly chose the smaller number as the common denominator. This left the rest of the procedure incorrect. On the other hand, when the fractions were algebraic, preservice teachers considered the sum of their denominators as the common denominator instead of taking their product. Figure 10 shows a sample of this misconception committed by a preservice teacher in the test.

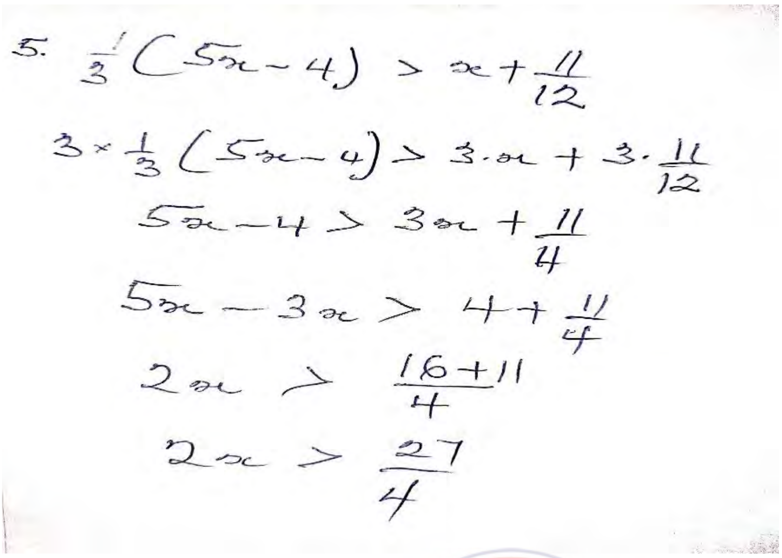

Figure 10: Incorrect common denominator

4.4 Research Question Three

What are the likely sources of the errors and misconceptions revealed in preservice teachers' solutions to linear inequality problems?

To determine the likely sources of the errors and misconceptions revealed in preservice teachers' solutions to linear inequality problems, the researcher interviewed fifteen (15) preservice teachers. The interview was conducted using an interview guide. Questions on the interview guide were geared towards soliciting information from the interviewees on why they committed the earlier identified errors in solving linear inequalities. This was to enable the researcher to identify the sources of these errors that were committed by the respondents. On the general question to the interviewees on why they committed the errors and what led to their misconceptions, the following theme was generated:

Sources of the Errors and Misconceptions Revealed to Linear Inequality Problems

Interviewee 1: My mathematics teacher uses the number line method as the teaching preference for the addition and subtraction of linear inequalities.

Interviewee 10: When it comes to the multiplication of linear inequalities, the teacher would ask us to memorize the multiplication table and the rules of multiplication and division.

Interviewee 3: Because we have many topics to complete before the semester is over when our teachers ask us to memorise something, we just memorise them without seeking further explanation. The teacher will later give us lots of exercises and we use what we have memorised to answer the questions.

Interviewee 7: Since we are many in the classroom, the teachers find it difficult to explain one thing over and over, especially to those who do not understand.

Interviewee 15: Teachers rush us through the linear inequalities because they said it is an easy topic even though not all students can understand something within a few hours. Weak students need more time to understand and at the same time, the teachers cannot wait for them.

Interviewee 9: Some teachers themselves find it difficult to explain the concept to us using real-life examples. So, they only give us formulas to memorise so that we can answer questions for them. Since we do not understand these things, we also find it difficult to explain to our friends why the things are how they are.

Interviewee 6: The textbooks that we use do not provide much insight on the linear inequality topics. Hence, I just memorise the formulas and use them to answer the questions.

From the interview, it was noted that in general, teachers who teach the preservice teachers devote little time to complete the topic of linear inequalities. All the interviewees agreed that their teachers used direct instruction and classroom lecture style in explaining the concept of linear inequalities. Additionally, the teachers would just provide their students with the rules and procedures to solve the linear inequality problems. This is because they need to finish all the topics in the curriculum within a certain time frame. Thus, it is impossible to merely focus on one topic and neglect the other topics. When this happens, students lack the understanding of the concept since they have not been thought with any examples to make the teaching real to them. For teachers, this is the reason to explain their inability to focus more on only one topic and to not finish the other topics. Therefore, they prefer to use any method of teaching that can reduce the time.

Furthermore, one of the reasons preservice teachers failed to grasp the concept of linear inequalities and they ended up committing errors and misconceptions is that there are too many students in one classroom with different abilities. Hence, a classroom may have students with strong cognitive abilities and students with weak cognitive abilities. Therefore, teachers have to spend more time to cater for the differing abilities of the students. This becomes difficult at a point since teachers have to move from one topic to the other because they are working within a time frame. This also justifies why the teachers rush the students through the topics without proper explanation of the concepts. The findings of this study confirm the findings of Khalid and Embong (2019) who found that teachers' teaching methods, teachers rushing to complete the extensive syllabus, and consequently, students resorted to memorizing rules because of surface understanding were the major sources of errors and misconceptions in understanding mathematical concepts.

4.5 Research Question Four

What alternative conceptions do preservice teachers have that are attributable to their errors and misconceptions in solving linear inequalities?

The purpose of this research question was to identify the alternative conceptions and conceptual bases to the errors and misconceptions preservice teachers' commit in solving linear inequalities. Thus, from the achievement test conducted, the alternative errors and misconceptions were presented in Table 3, showing the frequencies and percentages of the Respondents who have exhibited any form of alternative errors and misconceptions. Data obtained were analysed using the frequencies and percentages.

Table 4: Alternative Conceptions attributed to the Errors and Misconceptions of Preservice Teachers

Statements	Frequency (n)	Percentage (%)
Changing the direction of inequality when dividing by a negative number	92	49.1
Inequality as a strange relative of an equation	107	60.5
Inequality as an algebraic process	92	55.3
Inequality as an instrument for comparing known or imaginary quantities or a tool for expressing restrictions	90	50.8

Source: Biney (2021) field data.

of others and showed that most of the preservice teachers do not understand the idea of reversing the sign after dividing by a negative number. This was also noted by Basturk, (2009) in his study when he said, some students do not change the direction of an inequality sign when dividing or multiplying by a negative number.

Furthermore, in an interview with the preservice teachers, they were asked ―what sort of images comes to mind when they hear inequalities". The following theme was generated from the interview:

Pre- Service Teachers' Alternative Conceptions attributed to the Errors and Misconceptions

Interviewee 6: The sort of images and examples that come to mind are equations and graphs that are formed from inequality concepts.

Interviewee 2: Frustration and confusion and that x or y must be less than or equal to a number.

Interviewee 11: Mathematical inequality is when 2 numbers or variables do not match up as a final answer. You may have equations linking the same system but the product of the equation, rather the solutions do not equal to each other as they are supposed to. For example, the given equation is $A = B$ but the solution for A is 5 and the solution B is 6, therefore, coming to an inequality as $A \leq B$.

Interviewee 12: Number lines, triangles, confusion.

Interviewee 14: Lines on a graph, or line segments on a graph.

Again, groping for symbols, images, or words to describe the concept of inequality is visible in all of the above quotes. The first iteration of Task Conception 1 is associated with ―fumbling in the dark mansion" of mathematics and incoherently trying to describe the object one stumbles upon. Graphs with vague or no connection to inequalities are students' responses. Graphs of intervals used for working with fractions are provided as examples of inequalities.

Conception 2: Inequality as a strange relative of an equation

The title of this conception explicitly captures the referential aspect of inequalities that induced it. In some responses of the preservice teachers, it was identified that the majority (n=107, 60.5%) of the erroneous solution of an inequality is due to the student following the pattern of solving equations. In the interview, the researcher did not have to guess the logic behind the mistake because the subjects explicitly stated: ―treat the inequality as it would be an equation." The action here consists of the algebraic manipulation of inequality following the properties of transforming an equation into an equivalent one. The definition of inequality as an ―equation with unequal components" is the main metaphor for Conception 3. With this concept image in mind, students often replace the inequality symbol with the equal symbol and solve the equation, which often results in an erroneous solution. What is also interesting about this misconception is that it is not derived solely from looking at students' work and coding as in other groups of papers; it comes directly as students' declaration, their concept definition of inequality. It was documented those familiar procedures are performed on symbols that do not have natural conceptual embodiments (Tall, 2004). Here, the inequality is not encapsulated yet and the process of solving it is carried out in a routinized way based on the procedures known from equations; the familiar look of inequality invited not only the application of the procedure from equations but a complete substitution of the new symbol with the symbol which was more familiar.

The responses directing the ―what comes to mind when talking about linear inequalities" it was realised that the majority of the preservice teachers defined an inequality as an ―equation with unequal sides."

Interviewee 3**:** Solving linear inequalities is very similar to solving linear equations. The only difference is that the equal sign is substituted for an [in]equality sign. Both concepts are very much alike.

Interviewee 10: Two things that could be equal, but are usually either more than or less than.

Interviewee 8: An ―equation" that does not necessarily provide a solution showing one answer.

Interviewee 5: An image of a scale balancing, making sure that both sides are equal.

The interviewee correctly solves the inequality. However, with this conception, the majority of interviewees, not only had the pattern of solving equations in their mind when working on inequalities, but they replaced the inequality symbol with the equal sign. If they were unlucky and had a negative coefficient for x at the end of the solving process, they got the wrong solution for failing to change the sign of the inequality when dividing by a negative number.

Conception 2: Inequality as an algebraic process

From the results of the test and the interview, it was revealed that most (n=92, representing 55.3%) of the preservice teachers were considering inequality as an algebraic process. Here, inequality is seen as a process to be done, with rules to be followed. The rules are usually nicely stated on the right side of the work on the proposed item. Sometimes, ―the rules without reason" (Skemp, 1976) are transparent in little details, such as the last two lines in the transcript from students: the student

Interviewee 5: It is when something is compared to another. Then that one thing is either greater than the other one, smaller, greater than or equal and smaller than or equal. Example 0 > 0 the larger circle is greater in size than the smaller circle.

Interviewee 12: When I think of inequality, I think of a scale. There are different weights on both sides and their relationship to other changes when the weight on one of the sides of the scale changes. They can be equal. There can also be an infinite number of weights to use on the scale.

Interviewee 11: The concept of inequality brings to mind images like unbalanced scale (where one side is heavier/lighter than the other). Another image is of a power play in hockey where one side has fewer players than the other for a set period.

The conception of ―inequality as a comparison" is close to the formal definition of inequality. The first part of the quote from Respondent 7 looks more like Conception 0; however, the examples that accompany the definition use inequality symbols to compare given numbers which can be classified as Conception 1. Respondent 5 shows a real-life embodiment of inequality, a hockey game where the unequal forces are emphasized and whose effect is seen and emotionally lived. Also, the scale of Respondent 7 is dynamic, the relationship between sides changes when one side is modified. This idea can accommodate the axioms of inequalities. Moreover, the possibility that the scale could be in equilibrium and still represents an inequality shows an understanding of the mechanism of an inequality producing a single number as a solution.

4.6 Chapter Summary

This chapter presented the results and discussion of the study. The analysis of the results was presented in accordance with the objectives of the study. The discussions of the study were also presented based on the objectives of the study. The results were finally discussed based on both the results from the quantitative and the qualitative phases.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Overview

This chapter presents the summary of the study and reports on major findings. It highlights the conclusion of the study and its implications for practice. The implications were based on the major findings in the study. It further outlines some recommendations and suggestions for further research.

5.1 Summary of the Study

The purpose of this research was to examine the errors and misconceptions in linear inequalities among preservice teachers at Komenda College of Education. Specifically, four research objectives were formulated for the study that sought to explore errors and misconceptions revealed in preservice teachers‘ solutions to linear inequality problems; identify the likely sources of the errors and misconceptions revealed in preservice teachers‘ solution to linear inequality problems; identify the alternative conceptions preservice teachers have that are attributable to their errors and misconceptions in solving linear inequalities, and the possible solutions to the preservice teachers‘ errors and misconceptions attributed with linear inequality problems in Komenda College of Education.

In achieving the objectives of the study, the research adopted a mixed-method approach where the sequential explanatory method was used for the study. The study adopted the achievement test and interview as the instruments for the study. The simple random sampling method was used to sample 285 students who were Respondents to the achievement test. Out of the 285 Respondents, fifteen (15) of them were conveniently sampled to take part in the interviews. The researcher distributed

285 achievement tests, a returning rate of 177 representing 62.1% were completely answered and returned. The analytical tool used in analysing the data obtained from the field was the Statistical Product for Service Solutions (SPSS version 26.0) using descriptive statistics specifically the frequencies and percentages. The analysis of the data was presented per the objectives of the research. The final results obtained from the quantitative and the qualitative data were presented as the results and discussions from the study.

5.2 Summary of the Key Findings

Concerning research question one, the study found out that preservice teachers do not apply the basic rules in solving numerical problems without using the mathematical concepts. These results indicate that students do not have the procedural fluency and skills in carrying out the linear inequalities concepts. They perform the linear inequality operations the way they understand in arriving at a solution. The result of the study also found that most of the preservice teachers encounter difficulties in combining, integrating, or using information either given in the task or given as a result of calculation in solving linear inequality problems, they also find it a challenge or problem in manipulating symbols when solving linear inequalities. Additionally, the study found that students cannot formulate an equation or an inequality from the given word problem. They rather present the solution from the way they understand without applying any rules in solving the problem. Moreover, the study found that most of the students' errors and misconceptions about linear inequalities arise from confusion between the solution of the equation and inequation.

In research question two, the study found that teachers teaching methods, limited time, memorisation, among others were the sources of the errors and misconceptions revealed in preservice teachers‘ solutions to linear inequality problems.

Concerning research question three, the study found that students‘ alternative conceptions were a result of carelessness when using the various signs in linear inequalities, thus limiting them to present their solution on a number line and present their final solution on interval notation. Additionally, the study also discovered that preservice teachers have difficulty forming a correct solution set and could not fully understand the concept as a result of poor knowledge and procedural knowledge to operate. The study's findings also revealed that preservice teachers are perplexed by the relationship between the signs of given products and the signs of their elements because multiplying the inequality by a negative value does not affect the directions of the inequality.

Concerning research four, the study found that various approaches such as the function-based approach might have been helpful since they enable students to develop problem-solving strategies and visual thinking. The study also indicated that teacher educators must use conceptual problem representation by constructing linear inequalities from word problems to help students develop the arithmetic or algebraic schemata. Calculational operators must be used in this schematic problem model because both the derivation and solution of an equation may involve numerous stages. Students must apply their understanding of algebraic structure and syntax to generate the inequalities to create this schematic problem representation.

5.3 Conclusions

Based on the findings of the study, the following conclusions were drawn:

1. The study concluded that preservice teachers perform linear inequalities operations the way they understand solving due to lack of procedural skills,

and also encounter difficulties in combining, integrating, or using information either given in the task or given as a result of calculation in solving linear inequality problems.

2. Due to the nature of the concept of linear inequalities, teachers find it difficult to relate it to real-life situations. This makes the students not understand the concept in-depth. Furthermore, the teachers resort to teaching methods that subject students to rote learning and memorising.
3. The study concluded that preservice teachers cannot formulate an inequality from the given word problem, and the signs of given products and the signs of their factors seem to be confusing to the preservice.
4. The study concluded that preservice teachers are not able to present their solution on a number line, and are not able to present their final solution on interval notation due to their lack of arithmetic operations on linear inequalities.

5.4 Recommendations

The recommendations in this thesis may be valuable for educators, administrators, parents, and other stakeholders in facilitating the teaching of linear inequalities by ensuring that the difficulties preservice teachers encounter is addressed. The following suggestions were made in light of the observations and conclusions:

1. Teachers should take their time to explain to students how to apply basic arithmetic rules like multiplication with negative numbers, addition, and subtraction in linear inequalities.
2. The study recommends that teacher educators should use appropriate instructional materials that reflect the preservice teacher‘s conceptual

understanding in teaching linear inequalities to enable them to have a deeper understanding and formation of linear inequalities task.

3. The study recommends that teacher educators should teach preservice teachers the right procedures to change the direction of linear inequalities so they do not end up mixing them up. Also, the educators should thoroughly explain the concepts of linear inequalities to the preservice teachers so that their misconceptions will be corrected.
4. Teacher educators should make connections and build on previous concepts to construct new knowledge in teaching preservice teacher‘s linear inequalities.

5.5 Areas for Further Research

Further research needs to be conducted to investigate teacher educators' perceived challenges of teaching linear inequalities since this current study could not factor that in the analysis of the study.

REFERENCES

Abramovich, S., & Ehrlich, A. (2007). Computer as a medium for overcoming misconceptions in solving inequalities. *Journal of Computers in Mathematics and Science Teaching*, *26*(3), 181-196.

Agra, G., Formiga, N, Oliveira, P, Costa, L, Fernandes, M., & Nóbrega, M. (2019). Analysis of the concept of Meaningful Learning in light of the Ausubel's Theory. *Revista brasileira de enfermagem*, 8(*72)*, 248-255.

Akhtar, Z., & Steinle, V. (2013). Probing Students' Numerical Misconceptions in School Algebra. *Mathematics Education Research Group of Australasia*.

Alhassan, M. N., & Agyei, D. D. (2020). Examining Colleges of Education Mathematics Tutors' Conceptions in Teaching Completing the Square in Ghana. *International Journal of Scientific and Research Publications (IJSRP)*, *10*(3), 253-261.

Almog, N., & Ilany, B. S. (2012). Absolute value inequalities: High school students' solutions and misconceptions. *Educational Studies in Mathematics*, *81*(3), 347-364.

Amedahe, F. K., & Asamoah-Gyimah, E. (2016). Introduction to measurement and evaluation. UCC Press.

Amin, M. E. K., Nørgaard, L. S., Cavaco, A. M., Witry, M. J., Hillman, L., Cernasev, A., & Desselle, S. P. (2020). Establishing trustworthiness and authenticity in qualitative pharmacy research. *Research in Social and Administrative Pharmacy, 16*(10), 1472-1482.

Ashlock, T., & Wright, S. (2001). *Students at the Learning Edge: Advanced Technological Education Programs at Community Colleges*. Community College Press, One Dupont Circle NW, Ste. 410, Washington, DC 20036.

Ausubel, D. P., Novak, J. D., & Hanesian, H. (1968). *Educational psychology: A cognitive view* (Vol. 6). New York: holt, rinehart and Winston.

Bagni, G. T. (2005). Mathematics education and historical references: Guido Grandi's infinite series. *Normat-Nordisk Matematisk Tidsskrift*, *53*(4), 173-185.

Bahar, M. (2003). Misconceptions in biology education and conceptual change strategies. *Educational Sciences: Theory & Practice*, *3*(1), 55-64.

Baiden, F., Hodgson, A., & Binka, F. N. (2006). Demographic surveillance sites and emerging challenges in international health. *Bulletin of the World Health Organization*, *84*, 163-163.

Baki, A., & Çakıroğlu, Ü. (2010). Learning objects in high school mathematics classrooms: Implementation and evaluation. *Computers & Education*, *55*(4), 1459-1469.

Basias, N., & Pollalis, Y. (2018). Quantitative and qualitative research in business & technology: Justifying a suitable research methodology. *Review of Integrative Business and Economics Research*, *7*, 91-105.

Basturk, R. (2009). Assessing in-service teachers' instructional beliefs about student-centered education: A Turkish perspective. *Teaching and Teacher Education*, *25*(2), 350-356.

Battista, M. T. (2001). A research-based perspective on teaching school geometry. In *Subject-specific instructional methods and activities*. Emerald Group Publishing Limited.

Ben-Hur, M. (2006). *Concept-rich mathematics instruction: Building a strong foundation for reasoning and problem-solving*. ASCD.

Bhattacherjee, A. (2012). *Social science research: Principles, methods, and practices* (2nd ed.). CreateSpace Independent Publishing Platform.

Bicer, A., Capraro, R. M., & Capraro, M. M. (2014). Pre-service teachers' linear and quadratic inequalities understandings. *International Journal for Mathematics Teaching & Learning*.

Biesta, G. (2010). Pragmatism and the philosophical foundations of mixed methods research. *SAGE Handbook of Mixed Methods in Social and Behavioural Research*, *2*, 95-118.

Blanco, L. J., & Garrote, M. (2007). Difficulties in learning inequalities in students of the first year of pre-university education in Spain. *Eurasia Journal of Mathematics, Science and Technology Education*, *3*(3), 221-229.

Boateng, M. A., Danso-Appiah, A., Turkson, B. K., & Tersbøl, B. P. (2016). Integrating biomedical and herbal medicine in Ghana–experiences from the Kumasi South Hospital: a qualitative study. *BMC complementary and alternative medicine*, *16*(1), 1-8.

Breakwell, G. M., & Hammond, S., Fife-Shaw, & Smith, J. A. (Eds.) (2006). *Research methods in psychology*.

Brodie, K. (2014). Learning about learner errors in professional learning communities. *Educational Studies in Mathematics*, *85*(2), 221-239.

Brown, D. E., & Clement, J. (1989). Overcoming misconceptions via analogical reasoning: Abstract transfer versus explanatory model construction. *Instructional Science, 18*(4), 237-261.

Bruner, J. (1990). *Acts of meaning*. Harvard University Press.

Bryan, T. J. (1999). The conceptual knowledge of preservice secondary mathematics teachers: How well do they know the subject matter they will teach. *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal*, *1*, 1-12.

Burrowes, P. A. (2003). A student-centered approach to teaching general biology that really works: Lord's constructivist model put to a test. *The American Biology Teacher*, *65*(7), 491-502.

Carey, G. F., & Dinh, H. T. (1985). Grading functions and mesh redistribution. *SIAM Journal on Numerical Analysis*, *22*(5), 1028-1040.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically*. Portsmouth, NH: Heinemann.

Çekmez, E. (2021). Investigation of the effect of computer-supported instruction on students' achievement on optimization problems. *International Journal of Mathematical Education in Science and Technology*, *6*, 1-17.

Champagne, A. B., & Klopfer, L. E. (1983). *Naïve knowledge and science learning*. New York: Prentice Hall.

Chin, C., & Chia, L. G. (2004). Problem-based learning: Using students' questions to drive knowledge construction. *Science Education*, *88*(5), 707-727.

Churchill, D., King, M., Fox, B. (2013). Learning design for science education in the 21st century. *British Journal of Education Technology.*

Çıltaş, A., & Tatar, E. (2011). Diagnosing learning difficulties related to the equation and inequality that contain terms with absolute value. *International Online Journal of Educational Sciences*, *3*(2).

Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education*, *13*(1), 16-30.

Clements, D. H. (1987). Longitudinal study of the effects of Logo programming on cognitive abilities and achievement. *Journal of Educational Computing Research*, *3*(1), 73-94.

Cohen, L., Manion, L., & Morrison, K. (2007). Observation. *Research Methods in Education*, *6*, 396-412.

Cohen, L., Manion, L., Morrison, K., & Wyse, D. (2010). *A guide to teaching practice*. Routledge.

Comstock, D. E. (1982). Power in organizations: Toward a critical theory. *Pacific Sociological Review*, *25*(2), 139-162.

Confrey, J. (1990). Chapter 1: A review of the research on student conceptions in mathematics, science, and programming. *Review of Research in Education, 16*(1), 3-56

Creswell, J. W. (2013). *Steps in conducting a scholarly mixed methods study*. New York: Sage Publicati*ons.*

Creswell, J. W. (2021). *A concise introduction to mixed methods research*. New York: Sage Publications.

Creswell, J. W., & Miller, D. L. (2000). Determining validity in qualitative inquiry. *Theory into Practice*, *39*(3), 124-130.

Critchley, W. R., Pellet-Many, C., Ringham-Terry, B., Harrison, M. A., Zachary, I. C., & Ponnambalam, S. (2018). Receptor tyrosine kinase ubiquitination and de-ubiquitination in signal transduction and receptor trafficking. *Cells*, *7*(3), 22.

Currie, W. L., & Swanson, E. B. (2009). *Special issue on institutional theory in information systems research: contextualizing the IT artefact*.

Davis, R. B. (1984). *Learning mathematics: The cognitive science approach to mathematics education*. Greenwood Publishing Group.

De Posada, J. M. (1997). Conceptions of high school students concerning the internal structure of metals and their electric conduction: structure and evolution. *Science Education*, *81*(4), 445-467.

Denbel, D. G. (2013). Students difficulties of solving inequalities in calculus. *American Academic Scientific Research Journal for Engineering, Technology, and Sciences*, *5*(1), 57-62.

Driver, R., & Easley, J. (1978). Pupils and paradigms: A review of literature related to concept development in adolescent science students. *Review of Research in Education*, *16*(1), 3-56.

Duit, R. & Treagust, D. F. (1995). Students' conceptions and constructivist teaching approaches. In b. J. Fraser & H. J. Walberg (Eds.), *Improving science education* (pp. 46-69). Chicago: National Society for the Study of Education.

Eggen, P., & Kauchak, D. (2004). *Educational psychology: Windows on classrooms*. Ohio: Pearson Merrill Prentice Hall.

El-Shara, I., & Al-Abed, A. (2010). Error analysis of solving inequalities among mathematics majors at the University of Jordan. *Jordan Journal of Educational Sciences*, *6*(2), 93-108.

Engelbrecht, J., Harding, A., & Potgieter, M. (2005). Undergraduate students‘ performance and confidence in procedural and conceptual mathematics. *International Journal of Mathematical Education in Science and Technology*, *36*(7), 701-712.

Ersoy, Y. (2006). Innovations in mathematics curricula of elementary schools-I: Objective, content and acquisition. *Elementary Education Online*, *5*(1).

Eryilmaz, A. (2002). Effects of conceptual assignments and conceptual change discussions on students' misconceptions and achievement regarding force and motion. *Journal of Research in Science Teaching*, *39*(10), 1001-1015.

Falkner, K., Levi, L., & Carpenter, T. (1999). Early Childhood Corner: Children's Understanding of Equality: A Foundation for Algebra. *Teaching Children Mathematics*, *6*(4), 232-236.

Faulkenberry, E. E. D. (2003). *Secondary mathematics preservice teachers' conceptions of rational numbers*. Oklahoma State University.

Fay, B. (1987). *Critical social science: Liberation and its limits (Cornell paperbacks)* (F Second Printing ed.). Cornell University Press.

Fetherstonhaugh, T., & Treagust, D. F. (1992). Students' understanding of light and its properties: Teaching to engender conceptual change. *Science Education, 76*(6), 653-672.

Fink, A. (1995). *How to analyze survey data* (Vol. 8). Sage.

Fink, A. M. (2000). An essay on the history of inequalities. *Journal of Mathematical Analysis and Applications*, *249*(1), 118-134.

Fischbein, E., Deri, M., Nello, M. S., & Marino, M. S. (1985). The role of implicit models in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education*, *16*(1), 3-17.

Fitzmaurice, O., Walsh, R., & Burke, K. (2021). The ‗Mathematics Problem‘ and preservice post primary mathematics teachers–analysing 17 years of diagnostic test data. *International Journal of Mathematical Education in Science and Technology*, *52*(2), 259-281.

Frank, M. L. (1985). *Mathematical beliefs and problem solving (systems, anxiety, motivation, gifted)*. Purdue University.

Fuchs, L. S., Fuchs, D., & Hamlett, C. L. (1994). Strengthening the connection between assessment and instructional planning with expert systems. Exceptional Children, 61(2), 138-146.

Gabel, D. L. (1989). Let us go back to nature study. *Journal of Chemical Education*, *66*(9), 727.

Gall, M., Gall, J., & Borg, W. (2006). *Educational research: An introduction* (8th ed.). London: Pearson.

Gardella, F. J. (2008). *Introducing difficult mathematics topics in the elementary classroom: A teacher's guide to initial lessons*. London: Routledge.

Garnett, K. (1992). Developing fluency with basic number facts: Intervention for students with learning disabilities. *Learning Disabilities Research & Practice*, *7*(4), 210-216.

Garnett, P. J., & Treagust, D. F. (1992). Conceptual difficulties experienced by senior high school students of electrochemistry: Electrochemical (galvanic) and electrolytic cells. *Journal of Research in Science Teaching*, *29*(10), 1079-1099.

Gelman, S. A. (2000). Beliefs about the origins of human psychological traits. *Developmental Psychology,* 36(5), 663-678.

Green, J. & Thorogood, N. (2004). Qualitative methods for health research. London: Sage.

Greer, A. (2004). *A complete GCSE mathematics.* Stanley Thornes; 3rd edition.

Gilbert, J. K., & Watts, D. M. (1983). Concepts, misconceptions and alternative conceptions: Changing perspectives in science education. *International Journal of Mathematical Education in Science and Technology*, *52*(2), 259-281.

Gillespie, W. H. (1955). *The Gold Coast Police: 1844–1938*. Accra: The Government Printer.

Ginsburg, S. (2000). *The learning process,* Cambridge: University Press.

Glynn, S. M., & Duit, R. (Eds.). (1995). *Learning science in the schools: Research reforming practice*. London: Routledge.

Godden, H., Mbekwa, M., & Julie, C. (2013). An analysis of errors and misconceptions in the 2010 grade 12 mathematics examination: A focus on quadratic equations and inequalities. In *proceedings of the 19th Annual Congress of the Association for Mathematics Education of South Africa*, *1*, 1, 70-79.

Gow, J. (2010). *A short history of Greek mathematics*. Cambridge: Cambridge University Press.

Gravemeijer, K., Stephan, M., Julie, C., Lin, F. L., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future? *International Journal of Science and Mathematics Education*, *15*(1), 105-123.

Greer, R. D. (2008). The ontogenetic selection of verbal capabilities: Contributions of Skinner's verbal behaviour theory to a more comprehensive understanding of language. *International Journal of Psychology and Psychological Therapy*, *8*(3), 363-386.

Halcomb, E. J., & Hickman, L. (2015). *Mixed methods research*. London: Routledge.

Hall, B. C. (2013). *Quantum theory for mathematicians* (Vol. 267, pp. 279-301). New York: Springer.

Halmaghi, E. (2011). *Undergraduate students' conceptions of inequalities* (Doctoral dissertation, Education: Faculty of Education).

Hammer, D. (1996). More than misconceptions: Multiple perspectives on student knowledge and reasoning, and an appropriate role for education research. *American Journal of Physics*, *64*(10), 1316-1325.

Hannawa, A. F. (2015). 27. Miscommunication and error. In *Communication competence* (pp. 683-710). De Gruyter Mouton.

Hansen, R. S. (2006). Benefits and problems with student teams: Suggestions for improving team projects. *Journal of Education for Business*, *82*(1), 11-19.

Hardy, G. H., Littlewood, J. E., & Polya, G. (1934). *Inequalities*. Univ. Press, Cambridge.

Hashweh, M. Z. (1986). Toward an explanation of conceptual change. *European Journal of Science Education*, *8*(3), 229-249.

Heinze-Fry, J. A., & Novak, J. D. (1990). Concept mapping brings long-term movement toward meaningful learning. *Science Education*, *8*(3), 363-386.

Herbert, J., & Lefevre, P. (1986). *Conceptual and procedural knowledge: The case of mathematics.* Hillsdale: Lawrance Erlbaum Associates.

Herscovics, N., & Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational Studies in Mathematics*, *27*(1), 59-78.

Hinchey, P. H. (2008). *Action research primer* (Vol. 24). Peter Lang.

Hodes, M. (1998). Explorations in teaching inspiration: A seminar for students beginning dissertations. *The Journal of American History*, *84*(4), 1439-1446.

Hudson, P. & Miller, S. (2006). Designing and Implementing Mathematics Instruction for Students with Diverse Learning Need. Boston: Allyn & Bacon.

Hutchinson, J. N. (1988). General report: morphological and geotechnical parameters of landslides in relation to geology and hydrogeology. *International Symposium on Landslides, 5*, 3-35.

Institute of Education. (2019). *Inequalities in students' choice of STEM Subjects: An exploration of intersectional relationships*. CLS Working Paper 2015/6. Institute of Education-London.

International Program for Development Evaluation Training (IPDET). (2007). *Data collection methods*. IPDET Handbook.

Jarvis, T., McKeon, F., & Taylor, N. (2005). Promoting conceptual change in pre-service primary teachers through intensive small group problem-solving activities. *Canadian Journal of Science, Mathematics and Technology Education*, *5*(1), 21-39.

Johnson, R. B., & Onwuegbuzie, A. J. (2004). Mixed methods research: A research paradigm whose time has come. *Educational Researcher*, *33*(7), 14-26.

Junus, K. (2018). Assessing students‘ mathematical misconceptions through concept maps and online discussion transcripts: Inner product spaces. In *The 26th International Conference on Computers in Education (ICCE 2018)* (pp. 772-777).

Khalid, M., & Embong, Z. (2019). Sources and possible causes of errors and misconceptions in operations of integers. *International Electronic Journal of Mathematics Education*, *15*(2), em0568.

Kieran, C. (1992). *The learning and teaching of school Algebra*. London: Routledge.

Kim, Y., Jeong, J. H., & Lim, S. M. (2013). Multidimensional scaling analysis of the proximity of photosynthesis concepts in Korean students. *Journal of the Korean Association for Science Education*, *33*(3), 650-663.

Kinash, S. (2006). *Paradigms, methodology & methods* (pp.1-7). Australia: Bond University.

Krauss, S. E. (2005). Research paradigms and meaning making: A primer. *The Qualitative Report*, *10*(4), 758-770.

Krejcie, R. V., & Morgan, D. W. (1970). Determining sample size for research activities. *Educational and Psychological Measurement*, *30*(3), 607-610.

Kroll, R. (1986). *Metacognitive analysis of the difficulties caused by intervening factors in the solution of inequalities.* (Published Doctoral dissertation), Georgia State University).

Kuhn, T. S. (1970). Logic of discovery or psychology of research. In I.Lakatos & A. Musgrave, eds. *Criticism and the Growth of Knowledge.* Cambridge: Cambridge University Press. UK, 1–23.

Kyngäs, H., Mikkonen, K., & Kääriäinen, M. (2020). *The Application of Content Analysis in Nursing Science Research*. Springer: Nature Switzerland AG.

Lankshear, C., & Knobel, M. (2004). *A handbook for teacher research*. McGraw-Hill Education (UK).

Lemon, L. L., & Hayes, J. (2020). Enhancing trustworthiness of qualitative findings: Using Leximancer for qualitative data analysis triangulation. *The Qualitative Report*, *25*(3), 604-614.

Liljedahl, P. G. (2005). Mathematical discovery and affect: The effect of AHA! experiences on undergraduate mathematics students. *International Journal of Mathematical Education in Science and Technology*, *36*(2-3), 219-234.

Lincoln, Y. S., & Guba, E. G. (1985). Naturalistic inquiry. London: Sage.

Little, D., & Green, D. A. (2021). Credibility in educational development: trustworthiness, expertise, and identification. *Higher Education Research & Development*, 1-16

Lodholz, R. (1990). The transition from arithmetic to Algebra. In E. L. Edwards (Ed), Algebra for Everyone (Session 1, pp. 24-33). Reston, VA: *The National Council of Teachers of Mathematics.*

Lodico, M. G., Spaulding, D. T., & Voegtle, K. H. (2010). *Methods in educational research: From theory to practice* (Vol. 28). John Wiley & Sons.

Luneta, K. (2008). Error discourse in fundamental physics and mathematics: Perspectives of students‘ misconceptions. *ICET International Year Book*, *11*, 385-400.

Luneta, K., & Makonye, P. J. (2010). Learner errors and misconceptions in elementary analysis: A case study of a grade 12 class in South Africa. *Acta Didactica Napocensia*, *3*(3), 35-46.

Mack, L. (2010). *The philosophical underpinnings of educational research*. Sage.

Makonye, J. P. (2012). *Learner mathematical errors in introductory differential calculus tasks: A study of misconceptions in the senior school certificate examinations*. University of Johannesburg (South Africa).

Martinez, E. M. (2002). Learning mathematics at school.... and later on. *Down Syndrome News and Update*, *2*(1), 19-23.

Marton, F., & Pong, W. Y. (2005). On the unit of description in phenomenography. *Higher Education Research & Development*, *24*(4), 335-348.

Mason, J. (2006). Mixing methods in a qualitatively driven way. *Qualitative Research*, *6*(1), 9-25.

Matz, M. (1980). Towards a computational theory of algebraic competence. *The Journal of Mathematical Behavior School Science and Mathematics*, *108*(1), 4-8.

McNeil, N. M., & Alibali, M. W. (2005). Knowledge change as a function of mathematics experience: All contexts are not created equal. *Journal of Cognition and Development*, *6*(2), 285-306.

Mereku, K. (2000). School mathematics in Ghana: 1960–2000. *Mathematics Connection*, *1*(1), 18-24.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation* (3rd ed.). San Francisco, CA: Jossey-Bass.

Mestre, J. (1989). *Hispanic and Anglo students' misconceptions in mathematics*. ERIC Digest.

Molina, M., & Ambrose, R. (2008). From an operational to a relational conception of the equal sign. Thirds graders' developing algebraic thinking. *Focus on Learning Problems in Mathematics*, *30*(1), 61-80.

Morgan, D. L. (2007). Paradigms lost and pragmatism regained: Methodological implications of combining qualitative and quantitative methods. *Journal of Mixed Methods Research*, *1*(1), 48-76.

Moskal, B. M., & Magone, M. E. (2000). Making sense of what students know: Examining the referents, relationships and modes students displayed in response to a decimal task. *Educational Studies in Mathematics*, *43*(3), 313-335.

Mugenda, O. M., & Mugenda, A. G. (1999). *Research methods: quantitative and qualitative approaches*. Acts press.

Mugenda, O., & Mugenda, A. (2002). *Research methods* (2nd ed.). London: Sage Publication.

Murphy, P. K., & Alexander, P. A. (2004). Persuasion as a dynamic, multidimensional process: An investigation of individual and intraindividual differences. *American Educational Research Journal*, *41*(2), 337-363.

Muzangwa, J., & Chifamba, P. (2012). Analysis of errors and misconceptions in the learning of calculus by undergraduate students. *Acta Didactica Napocensia*, *5*(2), 1-10.

Myers, M. D. (1997). Critical ethnography in information systems. In *Information systems and qualitative research* (pp. 276-300). Springer, Boston, MA.

Nakhleh, M. B. (1992). Why some students don't learn chemistry: Chemical misconceptions. *Journal of Chemical Education*, *69*(3), 191.

Naoum, S. G. (2012). *Dissertation research and writing for construction students*. London: Routledge.

Naseer, M. S. (2015). Analysis of students‘ errors and misconceptions in pre-university mathematics courses. In *Proceedings: First International Conference on Teaching & Learning* (pp. 34-39).

National Research Council. (2002). *Scientific research in education*. National Academies Press.

Nesher, P. (1986). Are mathematical understanding and algorithmic performance related?. *For the Learning of Mathematics*, *6*(3), 2-9.

Novak, J. D. (2002). Meaningful learning: The essential factor for conceptual change in limited or inappropriate propositional hierarchies leading to empowerment of learners. *Science Education*, *86*(4), 548-571.

Nurul, A. İ. N. İ., Juniati, D., & Siswono, T. Y. E. (2020). Exploring the combinatorial reasoning of high school students with reflective and impulsive cognitive style in solving problems. *Journal for the Education of Gifted Young Scientists*, *8*(3), 1113-1124.

Nussbaum, J., & Novick, S. (1982). Alternative frameworks, conceptual conflict and accommodation: Toward a principled teaching strategy. *Instructional science*, 183-200.

Ocal, M. F. (2017). The effect of algebra on students' conceptual and procedural knowledge: The case of applications of derivative. *Higher Education Studies*, *7*(2), 67-78.

Ojose, B. (2015). Students' misconceptions in mathematics: analysis of remedies and what research says.

Olivier, A. (1989). Handling pupils‘ misconceptions. In *Pythagoras*.

Orton, A. (1983). Students' understanding of integration. *Educational Studies in Mathematics*, *14*(1), 1-18.

Osborne, R. J., & Wittrock, M. C. (1983). Learning science: A generative process. *Science Education*, *67*(4), 489-508.

Osborne, R., & Wittrock, M. (1985). The generative learning model and its implications for science education. *Studies in Science Education, 12*(1), 59-87.

Panagiotopoulos, G., Zogopoulos, C., & Karanikola, Z. (2018). The learning organization according to senge: Recording and validation of the park research tool in primary education schools in the prefecture of ilia. *Clobal Journal of Human Resource Management*, *6*(5), 1-19.

Parish, C. R., & Ludwig, H. J. (1994). Language, intellectual structures and common mathematical errors: a call for research. *School Science and Mathematics*, *94*(5), 235-239.

Patton, M. Q. (2002). Two decades of developments in qualitative inquiry: A personal, experiential perspective. *Qualitative Social Work*, *1*(3), 261-283.

Pearce, L. D. (2012). Mixed methods inquiry in sociology. *American Behavioral Scientist*, *56*(6), 829-848.

Perks, P., & Prestage, S. (2008). Tools for learning about teaching and learning. In *International Handbook of Mathematics Teacher Education: Volume 4* (pp. 265-280). Brill Sense.

Polit, D. F., & Beck, C. T. (2010). Generalization in quantitative and qualitative research: Myths and strategies. *International journal of nursing studies*, *47*(11), 1451-1458.

Polit, D. F., & Beck, C. T. (2012). *Nursing research*: *Principles and methods*.

Lippincott Williams & Wilkins.

Prediger, S. (2008). The relevance of didactic categories for analysing obstacles in conceptual change: Revisiting the case of multiplication of fractions. *Learning and Instruction*, *18*(1), 3-17.

Prendergast, M., & O'Donoghue, J. (2014). Unravelling the myth of effective teaching in mathematics. *Irish Journal of Academic Practice*, *3*(1), 1-22.

Puem, K. K. (2019). *Junior High School pupils' conception of linear inequalities in one variable in Have circuit of the Afadzato South District in Ghana* (Doctoral dissertation, University of Education, Winneba).

Punch, K. F. (2003). *Survey research: The basics*. London: Sage.

Radford, L. (2003). Gestures, speech, and the sprouting of signs: A semiotic-cultural approach to students' types of generalization. *Mathematical thinking and learning*, *5*(1), 37-70.

Ralph, D., & Scholtes, S. (1997). Sensitivity analysis of composite piecewise smooth equations. *Mathematical Programming*, *76*(3), 593-612.

Resnick, L. B., Nesher, P., Leonard, F., Magone, M., Omanson, S., & Peled, I. (1989). Conceptual bases of arithmetic errors: The case of decimal fractions. *Journal for research in mathematics education*, *20*(1), 8-27.

Riccomini, P. J. (2005). Identification and remediation of systematic error patterns in subtraction. *Learning Disability Quarterly*, *28*(3), 233-242.

Ridgeway, G., & Dunston, V. P. J. (2000). Content determines process: Seeing the familiar in new ways. *Reading & Writing Quarterly*, *16*(2), 119-138.

Rovai, A. P., Baker, J. D., & Ponton, M. K. (2013). *Social science research design and statistics: A practitioner's guide to research methods and IBM SPSS*. Watertree Press LLC.

Sackur, C. (2004). Problems related to the use of graphs in solving inequalities. In *Pme Conference, 1*(28), 1-15

Sakonidis, H., Kaldrimidou, M., & Tzekaki, M. (2006). Expert and novice primary teachers' intervening in students' mathematical activity: An insight. *International Group for the Psychology of Mathematics Education*, *323*.

Salas, J. D., & Obeysekera, J. T. B. (1982). ARMA model identification of hydrologic time series. *Water Resources Research*, *18*(4), 1011-1021.

Salvia, J., & Ysseldyke, J. E. (2004). Assessment (9th ed.). Boston: Houghton Mifflin Company.

Samo, M. A. (2009). Students' perceptions about the symbols, letters and signs in algebra and how do these affect their learning of Algebra: A case study in a government girls secondary school Karachi. *International Journal for Mathematics Teaching and Learning*, *5*(12), 13-18

Sarantakos, S. (2012). *Social research*. Macmillan International Higher Education.

Saunders, M. N., & Lewis, P. (2012). *Doing research in business & management: An essential guide to planning your project*. University of Surrey.

Saunders, M., Lewis, P., & Thornhill, A. D. R. I. A. N. (2007). Research methods. *Business Students 4th edition Pearson Education Limited, England*.

Saunders, M. N. K., Thornhill, A., & Lewis, P. (2009). *Research methods for business students* (5th ed.). London: Prentice Hall.

Schnepper, L. C., & McCoy, L. P. (2013). Analysis of misconceptions in high school mathematics. *Networks: An Online Journal for Teacher Research*, *15*(1), 625-625.

Schoenfeld, A. H. (1983). Beyond the purely cognitive: Belief systems, social cognitions, and metacognitions as driving forces in intellectual performance. *Cognitive Science*, *7*(4), 329-363.

Seng, J. L., & Chen, T. C. (2010). An analytic approach to select data mining for business decision. *Expert Systems with Applications*, *37*(12), 8042-8057.

Seo, K. H., & Ginsburg, H. P. (2003). You've got to carefully read the math sentence: Classroom context and children's interpretations of the equals sign. *Journal of Education and Practice*, *7*(26), 99-106.

Serhan, D., & Almeqdadi, F. (2015). Pre-service teachers‘ linear inequalities solving strategies and errors. *Global Journal of Mathematics, 6*(1), 1-15.

Sfard, A. (1995). The development of algebra: Confronting historical and psychological perspectives. *The Journal of Mathematical Behavior*, *14*(1), 15-39.

Shalash, B. (2019). Students' misconceptions and mistakes in general mathematics course among Al-Quds Open University students. *Higher Education Studies*, *7*(2), 67-78.

Shaughnessy, J. M. (1977). Misconceptions of probability: An experiment with a small-group, activity-based, model building approach to introductory probability at the college level. *Educational Studies in Mathematics*, *8*(3), 295-316.

Shufutinsky, A. (2020). Employing use of self for transparency, rigor, trustworthiness, and credibility in qualitative organizational research methods. *OD Practitioner*, *52*(1), 50-58.

Siegler, R. (2009). Implications of cognitive science research for mathematics education. *Colección Digital Eudoxus*, *4*(8), 89-96.

Silver, E. A. (1987). Theory and research for mathematics problem-solving. *Cognitive Science and Mathematics Education*, *33*.

Siyepu, S. W. (2015). Analysis of errors in derivatives of trigonometric functions. *International Journal of STEM Education*, *2*(1), 1-16.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, *77*(1), 20-26.

Smith III, J. P., DiSessa, A. A., & Roschelle, J. (1994). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *The Journal of the Learning Sciences*, *3*(2), 115-163.

Snarey, J., & Olson, P. (2003). Pragmatism's founding brothers. *Journal of Moral Education*, *32*(1), 91-95.

Snelbecker, G. E. (1974). *Learning theory, instructional theory, and psychoeducational design*. McGraw-Hill.

Sobh, R., & Perry, C. (2006). Research design and data analysis in realism research. *European Journal of Marketing*, *4*(2), 74-81.

Solomon, J. (1994). *The rise and fall of constructivism*. London: Sage.

Sørensen, J. G., Kristensen, T. N., & Loeschcke, V. (2003). The evolutionary and ecological role of heat shock proteins. *Ecology Letters*, *6*(11), 1025-1037.

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction*, *14*(5), 503-518.

Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education*, *36*(5), 404-411.

Stears, M., & Gopal, N. (2010). Exploring alternative assessment strategies in science classrooms. *South African Journal of Education*, *30*(4), 50-65.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School*, *3*(4), 268-275.

Stump, S. L. (1996). *Secondary mathematics teachers' knowledge of the concept of slope*. Illinois State University.

Swan, K. (2001). Virtual interaction: Design factors affecting student satisfaction and perceived learning in asynchronous online courses. *Distance Education*, *22*(2), 306-331.

Taber, K. S. (Ed.). (2009). *Progressing science education: Constructing the scientific research programme into the contingent nature of learning science*. Dordrecht: Springer Netherlands.

Tall, D. (2004). Reflections on research and teaching of equations and inequalities. In *PME Conference*, *28*, 1, 1.

Tall, D., & Thomas, M. (1991). Encouraging versatile thinking in algebra using the computer. *Educational Studies in Mathematics*, *22*(2), 125-147.

Tashakkori, A., & Teddlie, C. (2010). *Sage handbook of mixed methods in social and behavioral research*. Sage publications.

Tekkaya, C. (2002). Misconceptions as barrier to understanding biology. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, *23*(23).

Tent, M. W. (2000). Teacher to teacher: Inequalities: Part of every child's life. *Mathematics Teaching in the Middle School*, *5*(5), 292-295.

Tippets, B. S. D. (2014). *A parallel tree implementation for long range interactions*. University of California, Santa Barbara.

Treagust, D. F., & Duit, R. (2008). Conceptual change: A discussion of theoretical, methodological and practical challenges for science education. *Cultural Studies of Science Education*, *3*, 297-328.

Treagust, D. F., & Mann, M. (1998). A pencil and paper instrument to diagnose students‘ conceptions of breathing, gas exchange and respiration. *Australian Science Teachers Journal*, *44*(2), 55-59.

Treagust, D. F., Duit, R., & Fraser, B. J. (1996). Overview: Research on students‘ preinstructional conceptions–the driving force for improving teaching and learning in science and mathematics. *Improving Teaching and Learning in Science and Mathematics*, 1-14.

Tsamir, P., & Almog, N. (2001). Students' strategies and difficulties: The case of algebraic inequalities. *International Journal of Mathematical Education in Science and Technology*, *32*(4), 513-524.

Tsamir, P., & Bazzini, L. (2004). Consistencies and inconsistencies in students' solutions to algebraic ‗single-value‘inequalities. *International Journal of Mathematical Education in Science and Technology*, *35*(6), 793-812.

Türkdoğan, A., Güler, M., Bülbül, B. Ö., & Danişman, Ş. (2015). Studies about misconceptions in mathematics education in Turkey: A thematic review. *Mersin University Journal of the Faculty of Education*, *11*(2), 215-236.

Udovic, D., Morris, D., Dickman, A., Postlethwait, J., & Wetherwax, P. (2002). Workshop biology: demonstrating the effectiveness of active learning in an introductory biology course. *Bioscience*, *52*(3), 272-281.

Ureyen, M., Mahir, N., & Çetin, N. (2006). The mistakes made by the students taking a calculus course in solving inequalities. *International Journal for Mathematics Teaching and Learning*, *326*, 331.

Van Hoof J., Van Dooren W., & Verschaffel L. (2017). Number sense in the transition from natural to rational numbers. *British Journal of Educational Psychology* / Volume 87, Issue 1, 43-56

Vaiyavutjamai, P., & Clements, M. A. (2006). Effects of classroom instruction on student performance on, and understanding of, linear equations and linear inequalities. *Mathematical Thinking and Learning*, *8*(2), 113-147.

Vamvakoussi, X., & Vosniadou, S. (2004). Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction*, *14*(5), 453-467.

Verikios, P., & Farmaki, V. (2010). From equation to inequality using a function-based approach. *International Journal of Mathematical Education in Science and Technology*, *41*(4), 515-530.

Vlassis, J. (2004). Making sense of the minus sign or becoming flexible in ‗negativity'. *Learning and Instruction*, *14*(5), 469-484.

Vosniadou, S. (1994). Capturing and modeling the process of conceptual change. *Learning and Instruction*, *4*(1), 45-69.

In *Research in science education-past, present, and future* (pp. 177-188). Springer, Dordrecht.

Vygotsky, L. S., & Cole, M. (1978). *Mind in society: Development of higher psychological processes*. Harvard university press.

Walsh, M. N. (2015). Bounded rational points on curves. *International Mathematics Research Notices*, *2015*(14), 5644-5658.

Walsh, R., Fitzmaurice, O., & O'Donoghue, J. (2017). What Subject Matter Knowledge do second-level teachers need to know to teach trigonometry? An exploration and case study. *Irish Educational Studies*, *36*(3), 273-306.

Walsham, G. (1995). Interpretive case studies in IS research: nature and method. *European Journal of Information Systems*, *4*(2), 74-81.

Walsham, G. (2006). Doing interpretive research. *European Journal of Information Systems*, *15*(3), 320-330.

Wang, J., & Lin, E. (2005). Comparative studies on US and Chinese mathematics learning and the implications for standards-based mathematics teaching reform. *Educational Researcher*, 34(5), 3-13.

Winarso, W. (2017). A case study of misconceptions students in the learning of mathematics: The concept limit function in high school. *Jurnal Riset Pendidikan Matematika*, 4(6), 13-15.

Won, N., Liu, K., & Bukko, D. (2019). Developing instructional skills: Perspectives of feedback in student teaching. *Networks: An Online Journal for Teacher Research*, *21*(2), 8.

Wonu, N., & Zalmon, I. G. (2017). Diagnosis and remediation of senior secondary students' common learning difficulties in mathematics from chief examiners' report. *European Journal of Research and Reflection in Educational Sciences 5 (1), 24*, *31*.

Xiaobao, Y., & Ning, Z. (2007). Effects of the number of lanes on highway capacity. In *2007 International conference on management science and engineering, (pp. 351-356).* IEEE

Yin, R. K. (2015). *Qualitative research from start to finish*. Guilford publications.

Zakaria, E., Yaakob, M. J., Maat, S. M., & Adnan, M. (2010). Conceptual knowledge and mathematics achievement of matriculation students. *Procedia-Social and Behavioral Sciences*, *9*, 1020-1024.

Zakaria, E., & Zaini, N. (2009). Conceptual and procedural knowledge of rational numbers in trainee teachers. *European Journal of Social Sciences*, *9*(2), 202-217.

Zemelman S., Daniels H., & Hyde A. A. (2005). *Best practice: today's standards for teaching and learning in America's schools*. 3rd ed. Portsmouth, N.H., Heinemann.

Ziegler, E., Trninic, D., & Kapur, M. (2021). Micro productive failure and the acquisition of algebraic procedural knowledge. *Instructional Science*, *49*(3), 313-336.

APPENDIX A

ACHIEVEMENT TEST ON LINEAR INEQUALITY

UNIVERSITY OF EDUCATION WINNEBA

FACULTY OF EDUCATIONAL STUDIES

DEPARTMENT OF BASIC EDUCATION

Dear respondent,

I am a student of University of Education Winneba, pursuing an MPHIL Mathematics Education Programme at the Department of Basic Education, Ghana. The purpose of my study seeks to investigate —**The errors and misconceptions in solving problems in linear inequalities among preservice teachers"** which is a chosen area of study in partial fulfilment for the award of a Master of Philosophy in Basic Education at the University of Education, Winneba. The information you are to provide is purely for an academic exercise and will be treated with utmost confidentiality, anonymity, and privacy. There are no 'right' or 'wrong' responses.

SECTION A: DEMOGRAPHIC CHARACTERISTICS OF THE RESPONDENTS

1. Gender Male [ ] Female [ ]
2. Age 18-22yrs [ ] 23-27yrs [ ] 28-32yrs [ ] 33-36yrs [ ]
3. Programme/ Course…………………………….

SECTION B

Find the truth set and represent your results on a number line. All questions carry equal marks.

1. $3(4x - 1) \leq 15x + 12$
2. $2(x - 4) \geq 3x - 5$
3. $-4 - 3x \leq 20$
4. $-7 > 6t + 17$
5. $\frac{1}{3}(5x - 4) > x + \frac{11}{12}$
6. The least number in a set of real numbers is 24 and the greatest is 30. Find the range of the inequality.
7. A rental car company offers two options. Option 1 is GH¢100 per week plus 10 pesewas for each mile. Option 2 is GH¢125 per week plus 5 pesewas for each mile. How many miles per week would a person need to drive to make Option 2 more economical than Option 1?
8. When 5 is added to four times a certain number, the result is not more than twice that number added to 19. What is the number?
9. Calculate the range of values of y, which satisfies the inequality: $y - 4 < 2x + 5$.
10. Identify the solution set for $\frac{(x-1)}{3} + 4 < \frac{(x-5)}{5} - 2$

APPENDIX B

INTERVIEW GUIDE

Please provide your accurate responses to the following questions

1. What has influenced your solutions to the achievement test questions?
2. How do your teachers teach Linear Inequalities?
3. Do your teachers give you practical examples to demonstrate the importance of Linear Inequalities?
4. What sought of images come to your mind when you hear about Linear Inequalities?
5. From what you have learned, how will you explain Linear Inequalities?