UNIVERSITY OF EDUCATION, WINNEBA

EFFECT OF USING THE GRAPHICAL SOLUTION TECHNIQUES ON SENIOR HIGH SCHOOL STUDENTS' PERFORMANCE IN SOLVING LINEAR INEQUALITIES

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DECLARATION

STUDENTS' DECLARATION

I, **ADOM THERESA** declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

SIGNATURE: ………………….

DATE: ………………………………

SUPERVISOR'S DECLARATION

I hereby declare that the preparation and presentation of this work were supervised in accordance with the guidelines for supervision of Dissertation as laid down by the University of Education, Winneba.

NAME OF SUPERVISOR: PROF. D. K. MEREKU

SIGNATURE: ……………………………………………

DATE: …………………………………………………..

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ADOM THERESA

Department of Mathematics Education

March 2020

DEDICATION

To my uncle, mother, siblings and my husband

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ABSTRACT

The study investigated the effect of graphical solution technique on students' performance in solving lineal inequalities, and their motivation to use the method in learning mathematics. The study used the mixed method design where samples of 65 senior high school students were through convenient and purposive sampling proceudres. An innovative lesson design using the graphical solution technique was used to teach an experimental group 34 students while 31 students were also taught using the traditional methods. Achievement tests (pre-test and post-test), questionnaire and interview guide were the main instruments used to collect both quantitative and qualitative data in the quasi-experimental strategy of inquiry. The results indicated that about 77% of the students were not aware of the uses of graphical solution technique in solving linear inequalities. The results from the pre- and post- tests also indicated that the use of graphical solution technique as a tool in learning linear inequalities had a positive effect on students' performance in solving linear inequalities. The means scores between the control and experimental groups in the post-test was not statistically significant, i.e., $p > 0.05$. The interview conducted on experimental group also revealed that students really motivated to learn with the graphical solution technique. Students' were also motivated to learn on their own due to the interest they had from learning with the method. Recommendations were made based on the need for stakeholders in education such as the Ministry of education, Ghana Education service, parents and teacher education divisions to develop and implement programs to equip new teachers with the requisite knowledge and skills in graphical solution to teaching inequalities and mathematics in general.

CHAPTER ONE

INTRODUCTION

1.0 Overview

The chapter presents the background to the study, problem statement, and purpose of the study, research questions, limitations and delimitations of the study, significance of the study and operations definitions of terms.

1.1 Background to the Study

Several definitions of Mathematics have been given by educationalist. Moursund (2006), defined mathematics as an inherently social activity in which a community of trained practitioners (Mathematical Scientists) engage in science of patterns based on observation and experimentation, to determine the nature of regularities in systems. In the view of Barrows (1994), Mathematics is the study of patterns and relationships; a science and a way of thinking, an art, characterized by order and internal consistency, a language of carefully defined terms and symbols; and a unique powerful set of tools to understand and change the world. These tools include logical reasoning, problem-solving skills, and the ability to think in abstract.

Around the world, Mathematics is largely being used as a critical filter for students seeking admissions to second cycle and tertiary institutions as well as professional institutions such as colleges of education, nursing colleges and polytechnics (Adetunde, 2007). Furthermore, if we look at the educational system in Ghana right from kindergarten, the learning of mathematics is one of the basic tools impressed upon. This shows that mathematics forms the foundation of any solid educational system. According to Nabie (2006), the fundamental objective of mathematics education is to enable children understand, reason and communicate mathematically and solve problems in their everyday life. It is clear that countries in the world which have taken the culture of mathematics and science seriously are leading, whereas those, in which their culture has paid little or no role, find themselves lagging and their very survival threatened (Sherrod, Dwyer & Narayan, 2009). The West Africa Examinations Council (WAEC), in their annual reports list mathematics as one of the subjects in which students usually perform poorly. Their report stated that, poor understanding of the questions concepts and procedures in mathematics were the weaknesses of students in their final examinations (WAEC, 2014).

The use of graphical techniques provides students with step by step procedures in learning a lot of mathematical concepts especially linear inequalities. It is considered an important mathematical topic and a prerequisite for many subjects such as algebra, trigonometry and analytic geometry. Bicer and Capararo (2014) stated that, linear inequalities occupy an important place in the basic mathematics concepts and being an important entry point for a lot of mathematical topics. Inequality is a mathematical sentence built from expressions using one or more of the symbols -, ≥, ≤ >, < to compare two quantities that may be equal. Inequality solving means finding the value(s) of variables that make the relationship correct order. The sense of inequality refers to whether the inequality symbol is the greater than symbol (>) or the less than symbol (<) (Frempong, 2012)

The solution set derived through solving linear inequalities "is the set of real numbers each of which when substituted for the variable makes the inequality true" (Frempong, 2012). In other words, finding all value of the variable for which the inequality is true or that satisfy the inequality (Larson, Hostetler & Edwards, 2008). Frempong, (2012) pointed out that, solving linear inequalities are "similar to the techniques for solving linear equations, except that when an inequality is divided or multiplied by a negative number, the sense of the inequality must be reversed". Therefore, the solution of equation $(4 - 2x = 0)$ is the value that takes the variable (x) and makes the expression $(4 - 2x)$ is equal to zero, while solution of the inequality $(4 - 2x < 0)$ is all the values of (x) that make the expression a negative value. Solving inequalities is considered to be an important topic in studying properties and applications on functions, which require students to be aware and to understand methods of finding the solution different types for each inequality (linear – non-linear and fractional).

The inequalities have two parts complement to each other, that do not complete the student knowledge in one part perfectly, but supplementing them. For example, Izsak (2008),asserted, "linear inequalities is difficult for students because the representations are abstract and required operations, especially those relating quantities in word-problem situations, conflict with operations students have learned to use through years of modeling with arithmetic". Students make a lot of errors in solving questions on linear inequalities as reported by the chief examiner's report (WAEC, 2014). For example, when students were given the question $4 + 3/4\ (x + 2) \leq\ 3/8\ x + 1$, few of them cleared the fractions by multiplying through by the appropriate least common multiple (LCM), grouped the like terms, and finally solved the inequality.

Many candidates however messed up with the clearing of the fractions. They cleared only the fractions and did not multiply the LCM by the whole numbers in the inequality; they obtained

$4 + 6(x + 2) \leq 3x + 1$ instead of $32 + 6\ (x + 2) \leq 3x + 8$ and this resulted into a wrong solution.

A lot of factors contributed to the inability of students understanding some concepts in mathematics. These include environmental factors, student and teacher related factors.

Without a doubt, the most important of them is teacher related. Ottevanger et etal (2007), attributed such failures to teacher's conceptual understanding on the topic; lack of important teaching competencies and lack of student's motivation in the subject. In view of these difficulties, research suggests that solutions to the problem of student inability to be successful in linear inequalities are many and frequently interconnected (Norton & Irvin, 2007). After studying quite a few research conclusions, Norton & Irvin (2007) pointed out some solutions which include making explicit algebraic thinking inherent in arithmetic in children's earlier learning.

Several pieces of research have shown that students' errors in linear inequalities can be ascribed to fundamental differences between arithmetic and linear inequalities. For instance, if students want to adopt an algebraic way of reasoning, they have to break away from certain arithmetical conventions and need to learn to deal with algebraic symbolism. Students in general perceive mathematics to be a very difficult subject. It is therefore, incumbent on teachers to motivate and sustain student's interest in it. These difficulties may be corrected by the use of instructional strategies, the methodology etc. (Ottevanger, 2007).

The mathematical application embedded on graphical solution techniques provides students with step by step ways of solving most mathematical problems. It fosters independent learning as well. That is, fostering independent studies. It helps the student to understand the solving process and others who have problems with their home works. It is helpful to high school, college students and teachers.

The use of graphical solution techniques in teaching and learning has proven to be useful and motivating to students. Graphical solution technique is method that helps students to solve problems and understand it well. A graph is a planned drawing, consisting of lines and relating numbers to one another. With the use of color and a little imagination you can quickly whip up a professional looking graph in no time at all. With technology at your fingertips you can make use of the computer. It is simply a more linear way of organizing that information. Graphical solution techniques are very beneficial in our modern society in so many ways.

Graph theory is an important branch of combinatorial mathematics. The theory originated from the Konigsberg bridge problem, and the mathematician Euler used the theory to address this problem. After hundreds of years of development, the graph theory has been used to solve the problems of the shortest path, network flow, dynamic planning, etc. It has been widely used in engineering fields, such as the analysis of drainage pipe network system, the optimal island distribution of smart grid, the train operation plan, and the tourism route optimization (Bondy and Murty, 1976).This improves students understanding on graph work and interpretation of them as well, since graphing and interpretation of graph are some of the major subject students like as stated in the chief examiner's report (WAEC, 2014).That is, nature of graphs can be explored easily, students will be able to tell the nature of a linear graph as a constant is added or subtracted. This method will help students to explore and learn at their own time. This enables students to work accurately with paper and pencil, and students are able to tell whether they are right or not. Students can also learn a wide range of concepts within a very short time. Graph work is seen to be strength to students when it comes to its interpretation. Larson, R., Hostetler, R. P., & Edwards, B. (2008) stated that method will give students opportunity to study the nature of graphs and through this, they can form mental pictures of various graphs and how they behave as variable or constant changes. The researcher believes that introducing students to the graphing method would improved students to understanding in graphical solution techniques. Students can also explore more with other mathematical method that can help them to understand mathematics easy without anyone teaching them.

1.2 Statement of the Problem

Algebra is a powerful problem-solving tool (Nickson, 2000). One of the major content areas covered to promote the gaining of mathematical knowledge and skills in school mathematics is algebra. At the Junior High School, algebra covers topics such as algebraic expressions, linear equations, linear inequalities, relations, mapping and others (Ministry of Education, 2007). According to the mathematics curriculum, these concepts are to help students establish the relationship between numbers and their usage in real life. In the domain of mathematics, algebra focuses on generalization and interpretation of patterns and relationships. Due to its usefulness to the development of the child, teachers need to teach students how they can understand these concepts better. Students in general have difficulties in algebra as well as linear inequalities; due to this, most of them regard mathematics to be difficult. Students therefore make a lot of errors when faced with linear inequalities problems. As a result of these errors, students still carry these difficulties throughout their academic ladder, making them unsuccessful in their program and job market as well. It is therefore incumbent on our students to find better ways of conceptualizing these concepts to improve their understanding on mathematics.

Students' understanding of algebra is central to their ability to solve mathematical problem of linear inequalities. According to Nickson (2000), difficulties that pupils have in linear Inequalities are centered on:

- The meaning of letters
- The shift to a set of conventions different from those used in arithmetic
- The recognition and use of algebraic structure.

Students in Senior High Schools (SHS) in Ghana are required to take the curriculum in mathematics syllabus seriously. According to the WASSCE core mathematics Chief examiner's report WAEC, (2011, 2013) students have difficult in arithmetic and illustration on the number line, also on how to reverse of the inequality sign. Form the researcher teaching experience, students' have difficulties in rearranging and manipulating of values and variables, and making connection to inequality vocabularies like 'inequality', 'greater than', 'less than', 'at least' and 'not more than' when solving linear inequalities.

Basically there are many different methods to help students to understand and solve linear inequalities with ease, among which graphical solution technique is part of the strategies used in improving student performance as revealed in the study by Almog & Ilany, (2012), Bazzini & Tsamir, 2004). The graphical solution technique is the solution attained from the intersecting point of the lines. Larson, R., Hostetler, R. P., & Edwards, B. (2008) Stated that graphical analysis is the method of investigation which performs graphing by taking the input from data tables. Graphical analysis is used to calculate statistics, integrals, tangents, and interpolations. It can be done by creating the graphs, histograms and data tables. It can perform automatic curve fits. It also adjusts the curves (or graphs) for the required parameters. It is possible to compare different parameters and the comparison of various parameters can understand easily in graphical analysis.

1.3 Objectives of the Study

The main objective of the study was assessed the effect of the use of graphical solution techniques on students' ability to solve linear inequalities and their attitude to the method (graphical techniques). Other specific objectives study would be:

- Examine students' awareness of the use of graphical solution techniques in solving linear inequalities.
- Ascertain the effect of the graphical solution techniques on students' performance in linear inequalities.
- Determine the various ways of graphical solution techniques and improve students' ability to solve linear inequalities.

1.4 Research Questions

The following research question will form the focus of study

- To what extent are students aware of the use of graphical solution techniques (GST) in solving linear inequalities?
- What does effect of the graphical solution techniques on students' performance in linear inequalities?
- In what ways does graphical solution technique(GST) improve students' ability to solve linear inequalities

To address the second research question, it is hypothesized that;

H_o: There is no significant difference in the performance in solving linear inequalities between students taught using the tradition method and student taught using the graphical solution techniques in solving linear inequalities.

1.5 Delimitation of the Study

The study was based on effect of graphical solution techniques in solving linear inequalities in mathematics at Kintampo Senior High School. The study was conducted on form two students in the Kintampo Senior High School in the Brong-Ahafo region of

Ghana. The school was chosen because of its proximity to the researcher and that the findings of the study cannot be generalized in other schools and institutions.

1.6 Limitations of the Study

The limitations of this research are considered in this section. The first is that the instrument was developed by the researcher, so the interpretation of the results should be done with caution. Secondly, the sample of sixty five (65) students (male and female) from Kintampo Senior High School who had completed studying equations and inequalities are not highly representative of students studying core mathematics in the district.

1.7 Significance of the study

This study is significant because it would enlighten students and teachers on how to design graphical solution techniques to improve the mathematical ability of students in solving both linear equations and inequalities.

The study would provide students with a transformative tool that would enable them visualize and manipulate mathematics objects freely. This would then lead to better understanding of graphical solution techniques. The study had helped teachers to design innovative mathematics lessons to improve proficiency in solving both linear equations and inequalities. The study would serve as a source of reference to curriculum planners when reviewing the mathematics curriculum in future.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

The rationale of this chapter was to reviewed and discussed related literature on how graphical solution techniques (step by step solver) can improve student understanding of linear inequalities. For the purpose of the study, the literature was reviewed on the following themes:

- Theoretical framework (constructivism)
- The history and concept of algebra
- The history and concept of linear inequalities
- History of the inequality symbols
- Usefulness of inequalities in mathematics
- Difficulties and misconception in learning linear inequalities
- Strategies
- Engagement and motivating students in using graphical method for solving problems in mathematics
- Impact of graphical method on student performance in mathematics

2.1 Theoretical framework

The emergence of graph as learning has coincided with a growing awareness and identification of alternative theories of learning that hold the great sway today based on constructivist principles. This would enable participants to apply theoretical concepts into practice, providing a more varied learning experience. In addition, constructivist approaches to learning increased by providing more hands - on activities for students, placing them at the center of the learning process, encouraging them to actively construct their own knowledge and share it with their peers (Bazzini &Tsamir, 2004). These principles hypothesize that learning is achieved by the active construction of knowledge supported by various perspectives within a meaningful context. This study was based on the constructivist point of view, the researcher believes that since graph has the capacity to promote and encourage the transformation of education from every teacher directed enterprise to one which supports more student-centered models, students can come out with more ideas the teacher might not even know. Graph by their very nature are tools that encourage and support independent learning. Students' using it for learning purposes become immersed in the process of learning.

The graphical solution technique is designed in such a way that when students are introduced to, they can explore and learn on their own at their own leisure time without any assistant from the teacher. Students can verify their answers via the method in doing their homework. Students can also explore different method for learning mathematics on their own and this will make students responsible for learning. Gardiner (2004) argues extensively that, even if the ability to use mathematics to solve daily life problems is a main goal for school mathematics, this cannot be seen as an alternative to basic knowledge and skills in mathematics. It may rather emphasize the pupils' need to be able to orient them in the world of pure mathematics as a necessary prerequisite to solving real-world problems.

Inequalities have traditionally been associated with symbolic expressions and manipulations of symbols. However, there has been an increasing interest in the use of graphical representations over the last 20 years or so. This is reflected in research reports of Bardini el etal, (2004) and Marcus el etal, (2007). The NCTM standards also recommend that, upper secondary school students should be able to "represent and analyze relationships using tables, verbal rules, equations and graphs". The standard is evident in Rule of four-model of multiple representations as presented by Huntley et al. (2007). In line with Duval (2006), the "tags" treatment and conversion have been added to the figure.

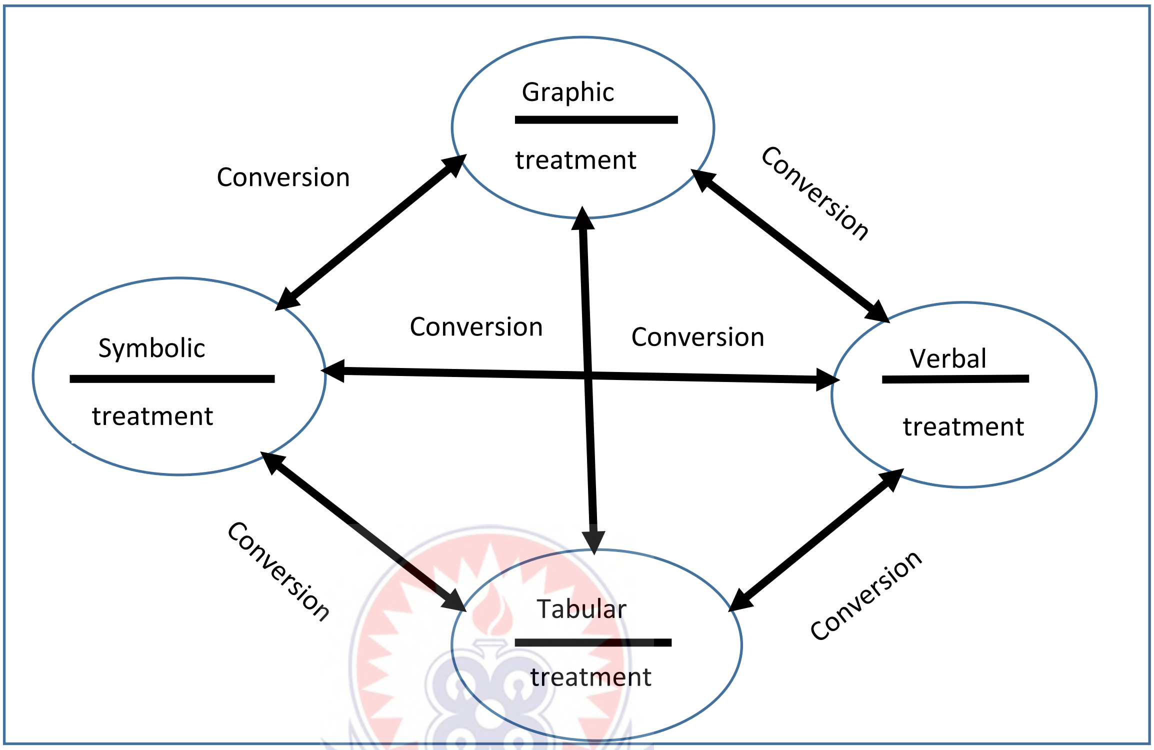

Figure 2: The rule of four- model of multiple representations

This paper mainly focusing on symbolic and graphic representations, and have tried to emphasize this part of the diagram by using shading and thicker lines.

Treatments are transformations of representations that stay within the same register, for example solving an inequality with purely symbolic manipulations. Conversions, on the other hand, are transformations where one changes a register without changing the objects being denoted, for example passing from the symbolic formulation of an inequality to a graphical representation of the same inequality. This is, according to Duval (2006), much more cognitively demanding than treatment. His view is supported by Sackur (2004), who noted that, in order to solve a standard inequality (symbolic formulation) graphically one must do the following work: Inequality on symbolic form

$(f(x)>g(x))$ → transform to functional form ($y=\cdots$and $y=\cdots$) → draw the graphs → compare the graphs → write down the truth set (for instance on the form $a<x<b$).

2.2 History and Concept of Algebra

Algebra is a branch of mathematics that plays vital role of developing and flourishing technology. It is one of the main areas of pure mathematics that uses mathematical statements such as term, equations, or expressions to relate relationships between objects that change over time.

The word "algebra" is a shortened misspelled transliteration of an Arabic title al-jebr w'al-muqabalah by the Persian mathematician known as al-Khwarizmi. The al-jebr part means reunion of broken parts and the second part, Al-muqabalah translates as "to place in front of, to balance, to oppose, to set equal''. Together they describe symbol manipulations common in algebra: combining like terms, moving a term to the other side of an equation etc.

Algebra is the branch of mathematics connecting the study of the rules of operations and relations and the constructions of concepts arising from them including terms, polynomials, equations and algebraic structures. Together with geometry, analysis, topology, combinatory and number theory, algebra is one of the main branches of pure mathematics.

Kieran (2006) stated that Algebra today plays the role that reading and writing did in the industrial age, if one does not have, one cannot understand much of sciences, statistics, Business or today's technology. Most students fail in WASSCE because their knowledge of algebra is weak.

The concept of algebra comes in so many forms. Many historically developed algebraic concepts can be observed in many of the current secondary school algebra curricula throughout the world. Tall D. (2001) gave and explained four fundamental conceptions of algebra. The first conception considers algebra as generalized arithmetic. In this conception, a variable is considered as a pattern-generalize. The key instructions for students in this conception are "translate and generalize". For example, the arithmetic expressions such as and could be generalized to give properties such as.

There is a close link between the cognitive processes involved in the learning of school algebra and the historical development of algebra as a symbol system (Kieran et al., 2006).

Historically, algebra has been transformed into many other forms of mathematics such as analytic geometry and calculus because of the power of algebra as generalized arithmetic (Egodawatte 2011).

The second conception suggests that algebra is a study of procedures for solving certain kinds of problems. In this conception, I have to find a generalization for a particular question and solve it for the unknown. For example, if I consider the problem "When 3 is added to 5 times a certain number, the sum is 43. Find the number." The problem translated into the algebraic language will be an equation of the form "$5x + 3 = 43$" with a solution of $x = 5$. Therefore, in this conception, variables are either unknowns or constants. The key instruction here is "simplify and solve" (Egodawatte, 2011).

In the third conception, algebra is used to be the study of a relationship between quantities.

Finally, the fourth conception views algebra as the study of structures. Under this notion, the variable is little more than an arbitrary symbol. The variable will become an arbitrary object in a structure related to certain properties. This is the view of variable found in abstract algebra. For example, when factorizing the problem, the conception of a variable represented in here is not the same as any previously discussed notions. The variable neither acts as an unknown nor an argument.

Egodawatte (2009) concluded in his studies "IS ALGEBRA REALLY DIFFICULT FOR ALL STUDENTS?" that, teachers should have a deeper knowledge of mathematical content as well as insights into student thinking. Teachers must identify the fundamental ideas that need to be taught and must understand the difficulties and misunderstandings that are likely to occur.

He added that problems that can be solved using algebra have different structures and processes. Using the same method to teach every student may not always work. Some diagnostic teaching is sometimes necessary. When students are given opportunities to verbalize their mental processes, it would mostly facilitate their transfer from arithmetic to algebra. However, this transition is neither automatic nor smooth. Sometimes, the structural differences between the two systems will interfere with a smooth transition process.

2.3 History and Concept of Linear Inequality

When teaching, learning or understanding of a concept encounters problems, there is a tradition in mathematics education research to turn the search for the answer to the problem toward the history of the concept (Burn, 2005).

Inequalities were first encountered in Classic Geometry, where it expressed factual relationships between quantities. The Hindu and Chinese mathematicians may have also had knowledge of those inequalities (Fink, 2000). In the big picture of the history of mathematics, Antique inequalities were captured and proved either in longhand expressions or in drawings.

Modern era is represented by the development of Algebra. There is no reference to any new inequality for almost two millennia, from Antiquity up to the 17th Century. However, the rise of Algebra and the adoption of mathematical symbols allowed inequalities to become more easily noticed in the big picture of mathematics. With the rise of the theory of functions, inequalities seemed to have gained greater relevancy. Mathematicians began working on proving the famous Antique inequalities.

Inequalities have been developed inside and through interaction between different branches of mathematics (Kjeldsen, 2002), like the theory of functions, linear algebra, mechanics, calculus, statistics and probability, to name only a few. Although, there are plenty of inequalities that were produced and used over the centuries, the big production of inequalities started with the appearance of the Journal of the London Mathematics Society.

Moreover, the first history of inequalities book was written by Hardy et al. in 1934 when edited the book Inequalities (Fink, 2000). Davis and Hersh (1998) argued that the production of mathematics has a rate of two hundred thousand theorems per year. A library search shows 95 papers per year in one of them. It may not be too misleading to assume that more than 200 papers on inequalities are written per year, published in the last 10 years in the two inequalities journals. On top of that, there is the five-volume anthology of inequalities edited Fink,(2000).

It seems that since Hardy, the development of inequalities has been remarkable. Thus, the history of inequalities continues and the production of inequalities is ongoing. The picture of mathematics is immense. "By multiplying the number of journals by the number of yearly issues, by the number of papers per issue and the average number of theorems per paper, their estimate came to nearly two hundred thousand theorems a year"(Davis & Hersh, 1998). This calculation takes into account only the new mathematics produced per year. Thus, taking into account all mathematics – the entire history of mathematics – makes the picture quite immense.

The goal of this paper has been to capture and convey some snippets of information regarding inequalities, extracted from the grand picture of the history of mathematics. This task, although not seemingly ambitious, was not free of surprises. At first, the lens used was not pointed at the region of the picture where inequalities were expected to be. In other words, in many of the famous History of Mathematics books (Katz, 2009), inequalities were not found in the index of topics. They were eventually found, however, disguised in unexpected forms. Those forms were pictures, verbal descriptions, or transcribed proofs, such as the sum of series, under Archimedes and Geometry in Katz (2009). Once the 'eye' trained enough to notice them in different eras, inequalities were seen in abundance.

However, someone could legitimately claim that, even though there is no visible epistemological obstacle related to inequalities, the fact that it took almost two millennia for inequalities to become a discipline is itself a signal that learners might have conceptual or psychological difficulties when dealing with them (Halmaghi, 2011). Inequalities were at first tools, and when the circumstances became favorable, they flourished into a discipline. Embedded in Geometry, they migrated to Algebra to get the power of symbols from there, and then they settled for good into the Theory of functions where they were enriched with new structures and philosophy. Embedded in functions, they grew omnipresent in many mathematical areas, from calculus to algebra, to statistics, to numerical analysis, to game theory. Paraphrasing Burn, I conclude this section with a historical account of the Inequality "encapsulates methods of proofs which originated in classical Greek mathematics, developed significantly during the 17th century and reached their modern form with [Hardy et al 1934]". Burn (2005) argues that not only periods of hardship, but also the actual developmental steps of a concept, can inform didactics.

My search into the history of inequalities revealed no epistemological obstacles. However, the study brought out that it is recorded and documented that inequalities are not easy concepts to manipulate. Even Hardy, the man who can be called the father of inequalities, confessed that there are plenty of inequalities which are hard to prove and that he had any amount of practice during the last few years, and we have found quite a number of which there seems to be no really easy proof. It has been our unvarying experience that the real crux, the real difficulty of idea, is encountered at the very beginning (Halmaghi, 2011). Thus, the answer to the investigation into the difficulty of understanding inequalities may not reside in the history of the concept, as expected. However, the answer could be deciphered from the history of inequalities in an unpredicted way.

It could be the case that mathematicians are not presenting inequality to the public with the same pomp and circumstance as when presenting equation results: Here lies the key to the relationship between equality and inequality in mathematics, between its poetry and its prose. Mathematics is founded on inequality, the commonest thing in the world. But the kind that mathematicians most pride themselves on, finished mathematics, mathematics for show to the public, is presented as much as possible in equality form (Tent, M. W., 2000). Inequalities are the back bones of many concepts and mathematical areas; therefore it is worth the effort of doing more research for clarifying what makes them hard to process.

2.4 The History of the Inequality Symbol(s)

It may be hard to believe, but for two millennia up to the sixteenth century, mathematicians got by without a symbol for equality. They had symbols for numbers and operators – just not one for equality (Lakoff and Núñez, 2000). If we can imagine volumes of mathematics developed throughout the centuries without the use of the equal sign, why would it be difficult to think of inequalities being employed or produced without the use of any special symbol? This section attempts an answer to the question: When in the history of inequalities were a symbol for inequality coined, how was the symbol used and how did the symbol influence the evolution of the inequality concept?

Several inequality symbols are used in mathematics. The most universally accepted in mathematics literature are: $<$ for less than, $>$ for greater than, $\leq$ for less than or equal to, $\geq$ for greater than or equal to, and $\neq$ for not equal to. In the previous section, it was also pointed out that, at some point in the history, mathematicians using inequalities in their work adopted the symbols suggested by Oughtred, and which are: greater than and less than. It was only in the 17th Century that either one of the two types of symbols for inequality came into being.

Tent (2000) remarked that inequality, one of the deepest-lying of the basic notions was the last to be symbolized. "At the divide between dearth and proliferation [of inequalities] stand Harriot"s inequality signs". In An Introduction to the History of Mathematics, Seltman and Goulding, R. (2007), documents that the symbols $<$ and $>$ were first introduced in mathematics-related texts by Thomas Harriot. Harriot was a mathematician who worked for Sir Walter Raleigh as the cartographer of Virginia, North Carolina today.

It is said that Harriot got inspired by the symbol on the arm of a Native American in coining the symbols for inequalities (Katz, 2009). The account states that Harriot decomposed the Native symbol into the two well-known symbols < and >. Seltman and Goulding, R. (2007). Argues that the origin of the symbols is less mystical than that. She argues that the inequality symbols are modifications of the equal sign, a symbol which was coined by Recorded as two horizontal, parallel and equal lines, to represent that what is on one side of the sign is exactly the same as what is on the left side of it. Katz (2009) indicates that, when producing the inequality signs, Harriot "took the equality in Recorder's sign to reside not in the two lengths, but in the unvarying distance between the two parallels". According to Seltman and Goulding, R. (2007), Harriot modified the distance between the two lines of the equal sign, to show that the biggest quantity lies on the side of the biggest distance between the lines.

Harriot used < to represent that the first quantity is less than the second quantity and > to represent that the first quantity is greater than the second quantity. "The symbol for „greater than‟ is > so that a > b will signify that a is greater than b. The symbol for „less than‟ is < so that a < b will signify that a is less than b" (Seltman & Goulding, 2007). Harriot was familiar with the symbolical reasoning introduced by Viete and, moreover, he transformed Viete‟s algebra into a modern form (Katz, 2009).

Harriot simplified Viete‟s notations to the point at which even a novice in the history of mathematics could understand his formulae, whereas one needs an index of notations to understand Viete‟s work. Tent(2000) considers Harriot to be the founder of "the English School of Algebraists". Harriot first used the symbol of inequality to transcribe the well-known inequalities of the means and then, he used the inequalities in his work to solve equations.

The inequality here imposes conditions on the coefficients of the given equation, which in turn helps prove that the cubic equation has only one solution. The sample of Harriot‟s work shown above may lead to a simple conclusion – which once they were coined and it was shown how they work, the inequality symbols became well established and were easily adopted. However, history shows that the mathematics community did not adopt Harriot‟s symbols immediately, possibly because Harriot did not publish his work or perhaps because at the same time, in 1631, Oughtred suggested for greater than and for less than. Halmaghi (2011) mention that ought red's inequality symbols were hard to remember, prompting many variations of the symbols to be circulated in the literature. Oughtred himself used for < and for > in some parts of his work. Many other derivations of Oughtred's symbols, as well as personal notations or improvised typewriting signs, were used to signal inequalities in the 17th and 18th centuries (Halmaghi, 2011). These new symbols were used to "represent inequalities on the continent" (Seltman & Goulding, 2007). More precisely, the < symbol is used to represent quantities that are different, the first one being less than the second one. The ≤ symbol incorporates the equality as well; it allows the first magnitude to be equal to the second one. It is well known that long before the appearance of symbolic algebra, people wrote all arguments in longhand. There were no symbols to represent the unknowns and there were no symbols to represent the relationship between unknowns as well. That was before Diophantus, during the „Rhetorical algebra‟ stage (Halmaghi, 2011). Writing mathematical statements in plain language is by no means incorrect. However, it may take several pages to describe a statement in plain language, while expressing the same statement in mathematical symbols could even take a single line. It is amazing how much the Greek mathematicians could accomplish by using rhetorical means of expressing inequalities and geometrical embodiments. Symbolic algebra produced the tools for a new embodiment of ideas and for inequalities a representation that is more abstract and specific. Moreover, the use of symbols allows for more work to be performed in a shorter time. Thus, the inequality symbol allowed for compression and aesthetic presentation of many old inequalities and spurred the development of a concept from a mere peculiarity. Radford (2006) argues that algebraic symbolism is "a metaphoric machine itself encompassed by a new general abstract form of representation and by the Renaissance technological concept of efficiency". Efficiency helped algebra prosper, while Harriot‟s inequality signs stimulated the proliferation of inequalities (Seltman & Goulding, 2007).

2.5. Usefulness of Inequalities in Algebra

Algebra is commonly referred to as the "gatekeeper subject." This is especially true in middle school years, when the challenges of adolescence can begin to rise, and when falling off track with school performance can greatly affect future success.

A report released by the National Center for Education Statistics Norway notes that the earlier a student proceeds effectively through Algebra. The more opportunities he or she has for reaching higher level mathematics courses in high school and completion of higher-level mathematics courses are related to higher likelihood of entering a 4-year college or university. When you begin to consider that your child's collegiate success is intertwined with their future professional success, then the true importance of Algebra begins to appear.

Algebra is more than just another class to pass. The skills learned in Algebra I, from understanding variables to development of abstract reasoning skills, will echo through your child's life and into their career.

Studies continue to emerge that prove Algebra is the key to success in the 21st century. With the fields of technology, engineering, mathematics, medicine, and education ever growing, your child's future can truly benefit from having a solid foundation on fundamental Algebraic concepts.

With Algebra, your children can open their future to endless possibilities. From our Sylvan Math Edge to our math mastery programs, your child will learn the skills they need to see soar with new found confidence and ability.

Algebra is the single most unsuccessful course in high school, the most failed course in community college, and, along with English language for nonnative speakers, the single biggest academic reason that community colleges have a high dropout rate. Although 60 percent of students enrolled at community colleges must take at least one course in math, about eighty percent of students never fulfill the requirement. They leave without graduating. They probably go through life thinking of them as the equivalent of "Big Dumb Gary."

Algebra is foundational to recognized mathematics, but it is not necessary for many important and useful forms of mathematical literacy. Statistics and data analysis, for example, often come easily to many who cannot understand abstract algebraic thinking. Over the years, I've worked with many wonderful professional computer programmers who hated algebra, some of whom dropped out of college or even high school because of it.

Inequalities are used all the time in the world around us—we just have to know where to look. Figuring out how to understand the language of inequalities is an important step toward learning how to solve them in everyday contexts.

You are challenged with mathematical inequalities almost every day, but you may not notice them because they are so familiar. Think about the following situations: speed limits on the highway, minimum payments on credit card bills, number of text messages you can send each month from your cell phone, and the amount of time it will take to get from home to school. All of these can be represented as mathematical inequalities. And, in fact, you use mathematical thinking as you consider these situations on a day-to-day basis.

Inequalities can be used to model a number of real-life situations. When converting such word problems into inequalities, begin by identifying how the quantities relate to each other, and then pick the inequality symbol that is appropriate for that situation. When solving these problems, remember that the solution will be a range of possibilities—inequalities do not have a single answer, as equations do. Absolute value inequalities can be used to model situations where margin of error is a concern.

When reading the prefaces of many books devoted to the theory of inequalities, I found one thing repeatedly stated: Inequalities are used in all branches of mathematics. But seriously, how important are they? Having finished a standard freshman course in calculus, I have hardly ever used even the most well-known inequalities like the Cauchy-Schwarz inequality. I know that this is due to the fact that I have not yet researched into the field of more advanced mathematics, so I would like to know just how important they are. While these inequalities are usually concerned with a finite set of numbers, I guess they must be generalized to fit into subjects like analysis. Can you provide some examples to illustrate how inequalities are used in more advanced mathematics?

Inequalities are extremely useful in mathematics, especially when we deal with quantities that we do not know exactly what they equate too. For example, let pn be the n-th prime number. We have no nice formula for pn. However, we do know that pn $\leq$ 2n. Often, one can solve a mathematical problem, by estimating an answer, rather than writing down exactly what it is. This is one way inequalities are very useful.

2.6 Difficulties and misconception in learning linear inequalities

Mathematical inequalities are one of the crucial mathematical topics requiring student understanding of various other mathematical topics such as algebra, trigonometry, and analytical geometry. Inequalities also play a critical role in evolving conceptual understanding of equality and equations because inequalities have been considered complementary to students' equality understanding (Tsamir & Almog, 2001).

Recent research has shown that students' understanding of equality and inequality concepts are related to each other by noting equality concepts should be aligned with comparison contexts since learning informal equality "the same as" with informal inequalities of "greater than" and "less than" helped students' understanding of relational concepts (NCTM, 2000). According to The National Council of Teachers of Mathematics (2000), ninth through twelfth grade students are predictable to be able to describe inequalities by using mathematical symbols and understand the meaning by understanding the solutions of inequalities.

However, various researchers (Almog & Ilany, 2012; Vaiyavutjamai & Clements, 2006; Tsamir & Bazzini, 2004) have found that many middle and high school students have misunderstandings and difficulties that cause them to misunderstand inequalities, and this impairs their ability to accurately solve and interpret equations. These misunderstandings and difficulties related to students' mathematical inequality understanding will be discussed later in the paper. More recently, Ellortan and Clements (2011) investigated elementary linear and quadratic equations and the inequality knowledge of 328 U.S. pre-service teachers, noting that many of the concepts these teachers held about inequalities were improper due to same reasons why students possess, even though they assumed otherwise.

Blanco and Garrote (2007) conducted a study with first year college students in Spain, and their findings showed that not only many middle and high school students possess misconceptions and difficulties about inequalities, but first year college students also possess these misconceptions in solving and interpreting inequalities and equations. Therefore, unless students have a mastery of inequalities, they cannot grasp the meaning of equality even if they are capable of solving equality questions. Although there is much research about students' understandings of the concepts surrounding equality, not enough attention has been sited on students' understanding of inequality concepts thus far (Tsamir & Almog, 2001; Verikios & Farmaki, 2010).

A review of many studies about students' understanding of linear and quadratic inequalities, two inequalities misconceptions students possess have found common. First, students regard inequalities as equations. In this regard, Vaiyavutjamai and Clements (2006) lead a study with 231 ninth-grade students. They found that the number of students preserved inequalities questions as equations. Second most common misconception students possess is the interpretation of solution sets (Blanco & Garrote, 2007; Halmaghi, 2011; Vaiyavutjamai & Clements, 2006).

For instance, Blanco and Garrote (2007) studied 91 students in their first year of college after getting instruction about inequalities. They found that students had difficulty understanding which principles made inequalities true and which did not. These students either thought that only one value made an inequality true or an inequality's result necessary to be an inequality. Tsamir and Bazzini (2004) administered a questionnaire to 148 high school students in a high-level algebra course, and 21 of them were interviewed regarding their answers. Interviews revealed that students supposed inequalities' solutions cannot be equations, but must also be an inequality.

2.7 Generalization of Equations to Inequalities

Students relate previous knowledge to a new topic before they have satisfactory data in hand (Ashlock, 2001). When exploring new mathematical topics including inequalities, this over-generalization can be the principal source of students' mistakes (Geoden, 2000). When students over-generalize, they may not realize that the new topic requires either partially or totally different mathematical processes. For instance, knowing how to solve equations can help in solving inequalities; however, students may make mistakes when solving inequalities if they relate the same solution processes with equations (Almog & Ilany, 2012).

In this regard, Tsamir and Bazzini (2001) found that many students overgeneralize one specific equality rule (a x b = 0 requires that at least either a = 0 or b = 0); however, the inequality a x b > 0 requires that both a and b have the same sign. In this scenario, many students did not realize that the multiplication of two negative numbers yields a positive number or the multiplication of two negative numbers should yield an answer greater than zero. For example, Tsamir and Bazzini (2004) demonstrated that many high school students' responses to (y - 2)(y + 9) > 0 is (y - 2) > 0 and (y + 9) > 0, but they simply forget the case that (y - 2) < 0 and (y + 9) < 0 also making the inequality true.

Another common and persistent misconception is expressing inequalities as equalities (Blanco & Garrote, 2007; Halmaghi, 2011; Vaiyavutjamai & Clements, 2006). Because many students think that inequalities and equalities require the same mathematical solution process, they treat problems involving inequalities in exactly the same manner as equations, and assume the questions require similar processes (Blanco & Garrote, 2007; Vaiyavutjamai & Clements, 2006). Prestege and Perks (2005) conducted a study with prospective teachers about their understanding of inequalities and showed that once students treat inequalities as equations and solve the equations, they simply put the sign back. For instance, solving the inequality: -65 x 3 > 0 in the same way as solving equation: -65 x 3 = 0. Then, they arrive at the conclusion that x3= 0, and then x= 0. When they put the sign back, many students simply find x > 0 as the solution to the inequality. However, students may simply forget the rule that multiplying and dividing by a negative number changes the direction of the inequality, and their solution in actuality needed to be x < 0.

Vaiyavutjamai and Clements (2006) posited that students who treat inequalities as equations may find the correct answers; however, they are not able to check whether or not they are arriving at the correct results. Tsamir and Bazzini (2004) found that students commonly believed "solutions of inequalities must be inequalities". Additionally, Vaiyavutjamai and Clements (2006) lamented that some students think only one value makes an inequality true, and they think solutions to inequalities cannot be an interval or infinite sets. Due to these two misinterpretations, students have difficulty interpreting the results of inequalities.

Overspecializing is another mathematical misconception, in which students inappropriately restrict one special feature into other cases (Ashlock, 2001). Tsamir and Bazzini (2004) asked 148 Israeli high-school students about their understanding of inequalities, concluding that many students assume that the results of inequalities need to be inequalities. However, solutions to inequalities can range from a single value to all of the numbers (Almog & Ilany, 2012). For example, when x is an integer, and 3 < x < 5, only one value (4) satisfies this statement. This shows the result of an inequality as a single number. However, when x is a real number, and 3 < x < 5, x can be infinitely many real numbers between 3 and 5. However, Tsamir and Bazzini (2004) found that many high school students thought only one value makes inequality true even if their solution set was infinite.

Vaiyavutjamai and Clements (2006) studied 31 secondary school students' understanding of linear equations and inequalities and found that even if some students found the correct solution to inequalities, they tended to write a single value into the answer sheet. For example, for $6x \geq 6$ and x is an integer, even students who find that $x \geq 1$ decide only "1" is the answer to this inequality. After the test, Vaiyavutjamai and Clements (2006) conducted interviews with students to obtain insight into their process of the solving the equation. Students' responses during the interview were aligned with the test results, demonstrating that students believe only one value makes an inequality true.

Major difficulties, Blanco and Garrote (2007) concluded there were two primary reasons why many students experience difficulty solving inequalities: a) a lack of arithmetic skills or knowledge, and b) the absence of semantic and symbolic meanings of inequalities. Other major types of difficulties include: excluded values, the choice of logical connections, dividing or multiplying non-positive factors, and connections between the signs of given products and the signs of their factors (Tsamir et al., 2004).

Instructional models related to understanding of inequalities particularly teaching methods can help students understand their own difficulties with inequalities. Tsamir and Almog (2001) found that students' understanding of inequality concepts is related to teacher instruction. In a study of instructional models and students' understanding of inequalities, three different teaching methods for inequalities, including algebraic manipulations, drawing a graph, and using the number line were employed. Students had the highest number of incorrect solutions when they used algebraic solutions. However, drawing a graph usually yielded a correct solution (Tsamir & Almog, 2001). Before engaging in formal solution with inequalities, students should have experience working with graphs and tables of values in order to make their learning more comprehensive. The function-based approach might have been useful since it enables students to develop their problem-solving strategies and visual thinking (Verikios & Farmaki, 2010).

Using appropriate technologies like computer software and graphic calculators were found to be beneficial because it helped students avoid misconceptions by developing their visual thinking (Abbromovich & Ehrlich, 2007). Furthermore, the use of calculator technology can help students develop their visual thinking and provide them with the opportunity to interpret inequalities' results easily and efficiently (Tsamir & Almog, 2001).

Suggestions from previous research to overcome students' Inequalities misconceptions and difficulties research have suggested different types of strategies in order to overcome these students' misconceptions and difficulties about inequalities. First, students should be encouraged to find examples of inequalities in their daily lives to make their learning more meaningful. For example, because inequalities are based on two concepts: boundary and direction, a basketball court might be given as an example that includes both boundary and direction concepts (Tent, 2000).

Another research study revealed that most students use their memory rather than mathematics when trying to solve inequalities. In order to make students' learning more mathematical rather than pure memorization of algorithms, teachers should provide connections to real-life situations and other mathematical topics, which students are already familiar with, and arrange some classroom activities or games before introducing mathematical signs (e.g., <, >) (Prestage & Perks, 2005).

Some additional suggestions from Tsamir and Bazzini (2004) for effective teaching and meaningful learning about inequalities are listed below: a) Be familiar with students' intuitive beliefs, b) Create discussions about the differences and similarities between equality and inequality, c) Encourage verbal analysis when students work with inequalities, d) Encourage students solve questions or problems about inequalities in more than one way to check the accuracy of results, and e) Emphasis should be placed on the importance of "zero". Other suggestions were provided by Blanco and Garrote (2007). According to their findings, student should; a) do not introduce the concept of inequality rapidly, b) make sure that symbols have semantic values, c) Clearly establish differences between the concepts of equality and inequality, d) use everyday life language, geometric language, and algebraic language in instruction, and e) use different methods to enrich students learning.

2.8. Engagement and Motivating Students in Using Graphical Method for Solving Problems in Mathematics

Improving students' problem-solving abilities is a major, if not the major, goal of student mathematics (National Council of Teachers of Mathematics, 2000). To address this goal, the researcher who is a university mathematics educator and senior high school teachers developed a math/science research project. It describes our unique approach to mathematical problem solving derived from research on reading and writing pedagogy, specifically, research indicating that students who use graphic methods to establish their ideas increase their comprehension and communication skills (Goeden, 2002)

Many teachers and students use graphic method to enhance the writing process in all subject areas, including mathematics. Graphic method helps students organize and then clarify their thoughts, infer solutions to problems, and communicate their thinking strategies. We designed a classroom action research project to study a problem-solving instructional approach in which students used graphic method. Our goal was to improve student achievement in three areas of our state's math assessment in open-response problems: mathematics knowledge, strategic knowledge, and mathematical explanation. We discuss graphic organizers and their potential benefits for both students and teachers, we describe the specific graphic organizer adaptations we created for mathematical problem solving, and we discuss some of our research results of using the two dimension graphic methods.

2.9 Benefits of Using Graphic Organizers in Mathematics Learning

A graphic organizer is an instructional tool student can use to organize and structure information and concepts and to promote thinking about relationships between concepts. Furthermore, the spatial arrangement of a graphic organizer allows the student, and the teacher, to identify missing information or absent connections in one's strategic thinking (Ellis, 2004). Senior high teachers already use many different types of graphic methods in the writing process. All share the common trait of depicting the process of thinking into a pictorial or graphic format. This helps students reduce and organize information, concepts, and relationships.

When a student completes a graphic method, he or she does not have to process as much specific, semantic information to understand the information or problem (Ellis, 2004). Graphic organizers allow, and often require, the student to sort information and classify it as essential or non-essential; structure information and concepts; identify relationships between concepts; and organize communication about an issue or problem.

Initial thinking is not a linear activity, especially in mathematical problem solving. Yet, the result of problem solving the written solution often looks like a linear, step-by-step procedure. Good problem solvers brainstorm different thoughts and ideas when first presented with a problem, and these may or may not be useful. Problem solvers can use a graphic organizer to record random information but not process it. A student can later reflect upon usefulness of the information and ideas. If the information and ideas help the student make relationships between concepts, then they are essential. A graphic organizer allows a student to quickly organize, analyze, and synthesize one's knowledge, concepts, relationships, strategy, and communication. It also gives every student a starting point for the problem-solving process.

So how does the use of the four corners and a diamond graphic organizer differ from the traditional Polya's four-step mathematical problem-solving hierarchy? In terms of objectives, it does not. Obviously, the four corners and a diamond graphic organizer is designed to help students understand the problem, devise a plan, carry out the plan, and look back. However, by having the non-linear layout of the graphic organizer, the student is not expected to do these "steps" in a hierarchical, procedural order that some students misapply. It is the implementation process, how students form their response that is the important aspect of the four corners and a diamond graphic organizer (Zollman, 2006).

The pictorial orientation allows students to record their ideas in whatever order they occur. If students first think of the unit for their final answer, then this is recorded in the fifth, bottom-right area. This idea (the unit), then, is not needed in the short-term memory because a reminder is recorded. If students first think of a possible procedure for their answer, this is recorded in the third, upper-right area. The four corners and a diamond graphic organizer allow, and even encourage, students to use their problem-solving strategies in a non-hierarchical order. A student can work in one area of the organizer and later work a different area. It also shows that completing a problem-solving response has several different, but related, aspects. Students do not begin writing a response until some information or ideas are in all five areas.

The graphic organizer especially encourages students to begin working on a problem before they have an identified solution method. As in the graphic method, the students then organize and edit their thoughts by text their solution in the traditional linear response. The steps for the open response write-up are as follows: (1) state the problem; (2) list the given information; (3) explain methods for solving the problem; (4) identify mathematical work procedures; and (5) specify the final answer and conclusions.

The graphic portion of the organizer allows all students to fill in parts of the solution process. It encourages all students to persevere to "muck around" working on a problem. Further, teachers quickly can identify where students are confused when solving a problem by simply examining the graphic organizer.

The teacher should model proper use of graphic organizer and have students work in groups when introducing this tool. Working in groups allows students to see that many problems can be worked in more than one way and that different people start in different places when solving a problem. In their small-group discussions, students identify relationships between the areas in the graphic organizer and among the various solutions.

Graphic organizers can benefit students when they take standardized state mathematics assessments, specifically open-response problem-solving items. The graphic organizer helps each type of student produce a more complete response in each of the three categories and, thus, receive a higher score.

2.10 Impact of Graphical Method on Student Performance in Mathematics

Senior high school teachers decided to use the open-response mathematics questions as the focus of their research on the effects of using graphic method. Teachers administered pre- and post-tests with their students to see if using the graphic method impacted their performance (Zollman, 2006).

All teachers reported dramatic improvements in students' mathematics scores on open-response items after implementing the graphic method. The percentage of students (N=186) who scored at the "meets" or "exceeds" levels on each of the open-response item categories on the pre-test was 4% for math knowledge, 19% for strategic knowledge, and 8% for explanation. After instructing students to use the graphic organizer in mathematical problem solving, the percentage of students scoring "meets" or "exceeds" on the post-test improved to 75% for math knowledge, 68% for strategic knowledge, and 68% for explanation (Zollman, 2006). Each teacher self-collected and self-scored these data using the state's scoring rubric. Overall scores increased from a 27% average on the pre-test to a 70% average on the post-test.

Data collected, analyzed, and triangulated from three sources—the teachers, the action research pre- and post-test data, and the students' work suggests that the use of the graphic method in mathematical problem solving may significantly help students coordinate their mathematical ideas, methods, thinking, and writing. The graphic organizer helped students coordinate various parts of mathematical problem solving: (a) What is the question? (b) What information is known? (c) What strategies might be used? (d) Which operations, procedures, or algorithms of the strategy need to be shown? (e) What explanations and reflections are needed to communicate the method(s) of solution? (Zollman, 2006).

Graphic method helps students communicate their thinking when they solve problems. The teachers found the use of graphic organizers in mathematical problem solving to be very efficient and effective for all levels of students. The teachers saw that their lower-ability students, who normally would not have attempted problems, had now written partial solutions. The organizer appeared to help average-ability students organize thinking strategies and help high-ability students improve their problem-solving communication skills (Zollman, 2006b). Students now had an efficient and familiar method for writing and communicating their thinking in a logical argument.

We hoped the students in our action research study would improve their problem solving with an instructional intervention from pre-test to post-test; however, no single instructional method directly affects learning (Duval, 2000). Rather, instruction is one of many factors that may influence learning. Others include the curriculum, the student, the class, and the teacher. Nevertheless, the teachers who conducted the action research described in this article believed the graphic method was associated with many of the positive outcomes in their students' problem-solving ability (Zollman, 2006).

The crucial factor in the effectiveness of any instructional method is how it is implemented. If four corners and a diamond graphic method is used as a linear, systematic procedure to teach problem solving, it will succeed sporadically. In fact, any direct teaching about problem solving is likely to have intermittent success. Giving students a chart of Polya's four steps in problem solving or a graphic method sheet may help students learn the steps of problem solving. However, students may remain uncertain about where to start a problem, confused by essential versus non-essential information, or unaware how to communicate important steps and reflections in their solutions. We found that graphic methods aid students in all three of these areas.

Allowing students to first use their own thinking–and then reflect, revise, and re-organize their knowledge, strategies, and communication helps them improve their problem-solving abilities. Initially, teaching about problem solving as a hierarchy of procedural steps is neither efficient nor effective. Our results confirm other studies that found teaching via problem solving is the key instructional process (Lester, 2007).

In culmination, effective reading and writing strategies like graphic organizers may have crossover effects in mathematics for students of all ability levels. We found that the graphic method, when properly used, was an extremely useful instructional method in the senior high school mathematics classroom. Our instructional approach helped students construct content knowledge and strategic knowledge and, we contend, it also improved their mathematical communication skills. In addition, graphic method allowed teachers to quickly identify the weaknesses and strengths of students' problem solving abilities. As teachers seek to expand and improve students' mathematical knowledge to help them solve problems, they may find that good teaching in reading and writing is good teaching in mathematics.

CHAPTER THREE

METHODOLOGY

3.0 Overview

The chapter provided a detailed description of the methodology employed in the study which includes the participants of the study, sampling and sampling procedures, research design, research instrument, data collection procedure and theme the data analysis.

3.1 Research Design

Trochim (2000) defined design-based research as "a systematic but flexible methodology aimed to improve educational practices through iterative analysis, design, development, and implementation, based on collaboration among researchers and practitioners in real-world settings, and leading to contextually-sensitive design principles and theories" (pp. 6-7). Design Based Research (DBR) studies use the term intervention to denote the object, activity, or process that is designed as a possible solution to address the identified problem (Wang & Hannafin, 2005).

A mixed method design (both quantitative and qualitative data) was used, which employed quasi-experimental design as a method to carry out the study. The rationale for this strategy was to use qualitative data and results to assist in explaining and assigning meaning and reasons for quantitative findings (triangulation of results).

According to Creswell (2003) quasi-experimental design is to test and determine whether a treatment/program/invention has the intended effect on a selected population. There are two common quasi-experimental designs: in the first one, data is collected from one selected population, no comparable group, which Creswell (2003) defined this type of research as "one group pre-test/post-test design" and Schuett (2006) identified as "before-and-after designs". He added that the main feature of this type of design on a comparison group was selected to compare with the treatment group. In the second design, the data collected from pre –test and post-test with both a treatment and the comparison group (i.e. control group). Both Creswell (2003) and Schuett (2006) named as “nonequivalent control group designs.” The questionnaire used to select the treatment group from the comparison group which aimed at finding out the number of students who are or not aware of the graphical solution techniques, and whether students make use of it in learning. Closed-ended and open-ended questionnaire on students’ performance and use of the graphical solution techniques in learning mathematics was given to students.

The questionnaire was chosen because it takes less time to administer it and also ensures the secrecy of respondents (Fraenkel &Wallen, 2000; Muijs, 2004). After which a quasi-experimental design employed for the study were the pre –test, and post-test with both a treatment and the comparison group (i.e. control group) designs.

In the experimental study, the quantitative aspects consist of an achievement test and closed-ended questionnaire whiles the open-ended questionnaire and interviews was the qualitative part.

The quantitative part was made of descriptive and inferential statistics to investigate how graphical solution techniques improved students’ performance in linear inequalities.

3.2 Population and sample

According to Best and Kahn (2007), a population viewed as a group of individuals who have one or more characteristics in common that are of interest to the researcher. The population was all the individuals of a particular type or a more restricted part of that group. Therefore, the population for the study was student in the Kintampo Senior High School, Brong-Ahafo Region. The sample for the study was SHS two (2) students in the

Kintampo Senior High School. The SHS two (2) are about two hundred and fifty students (250) students, consisting of one hundred and fifty (150) males and twenty (100) females.

The SHS two (2) were chosen because they are familiar with a lot of further mathematics and again they were available for the period of the study. That is after the intervention the researcher needs to monitor students' progress from time to time. SHS three (3) were preparing for their WAEC examinations and therefore were busy. Again, SHS one (1) are new in the system and are not familiar with a lot of further mathematics and will automatically not be aware graphical approach. Therefore the SHS two (2) were chosen as the sample for the study due to the above reasons.

In addition, Convenience sampling was to the select the SHS two (2) and random sampling was used to select students from the pure science class (A and B) to answer the questionnaire. Convenience sampling is a non-probability sampling technique where subjects are selected because of their convenient accessibility and proximity to their searcher (Castillo, 2009). Hence convenient sampling was employed in selecting the population. After the administration of the questionnaires; students who are aware were selected to from the two classes (A and B) as the experimental group and rest as the control group. The experimental group was randomly selected due to its special characteristics to the study. According to Creswell (2003), Random sampling refers to a variety of selection techniques in which sample members are selected by chance, but with a known probability of selection.

Again, the used of graphical solution techniques in learning instigates student-centered learning which is of much concern to mathematics educators (Mereku, 2004). The reason for selecting linear inequalities as a topic was to determine its effect on students' using graphical solution techniques is that students' difficulties in linear inequalities have been a thing of old. Students generally have a problem with linear inequalities right from Junior High School to Senior High School (WAEC's Chief Examiner's Report for JHS and SHS Mathematics, 2013-2015). To become a good student, the student should have indebt knowledge on linear inequalities since a lot of abstract mathematical concepts require linear inequalities as a baseline to understand.

3.3 Research Procedure

The main objective of the study was to investigate whether the use of graphical solution techniques in solving linear inequalities can aid students to understand linear inequalities better. To achieve this, students were taken through an innovative teaching approach different from what they know already (i.e. the traditional approach). The innovative teaching approach involved the use of graphical solution techniques which the researcher integrated into the teaching of some selected topics in linear inequalities. Lesson notes (see Appendix G and H) were designed covering areas like the use of graphical solution techniques in solving linear inequalities, a Double Inequalities and Absolute Value Inequalities on one variable and others.

The aim of the innovative teaching method was not only to improve understanding and performance but also to guide the student through how the graphical solution techniques can used to solve and also to instigate student-centered learning. The students were taken through a learning process to prepare them to use the graphical solution techniques.

The students were then given some questions (see appendix B) to try first using paper and pen before they were then taken through how the graphical solution techniques is used to arrive at the answer. Some students were also interview using an interview guide (see appendix F) within the lesson and after the lesson to find how the innovative teaching method will helps them to understand the concepts better.

3.4 Research Instrument

Questionnaire, interview and achievement test (pretest and posttest) were chosen as the instrument to collect data to answer the questions set for this study.

3.4.1 Achievement Tests (Pre-Test and Post-Test)

The aim is to use the pretest to find out student knowledge, understanding, difficulties, and idea on a particular concept before a treatment is given. On the other hand, a posttest was administered after a treatment to ascertain whether the treatment was effective in bringing out a better outcome as compared with the scores in the pretest. For the purpose of the study, the researcher chooses the achievement test to aid her to compare scores in both pretest and posttest of the control and experimental group respectively. . (See Appendices B and D for sample test).

3.4.2 Questionnaire

A questionnaire was one of the instruments used for data collection and in helped to answer research question one. The questionnaire was chosen because it took less time to administer them and also ensured the anonymity of respondents (Fraenkel & Wallen, 2000; Muijs, 2004). Questionnaire enabled the researcher to collect potential information about students' performance of the graphical solution techniques for solving linear inequalities and the extent to which students have use the graphical solution techniques in solving linear inequalities at Kintampo Senior High School. The questionnaires consisted of both closed and open-ended items. The open-ended questions enabled the researcher to probe a little deeper and explore student's attitude and feeling about the graphical solution techniques they would be use and how it would helped them in their studies.

One set of questionnaires would be developed and was given to student at the beginning of the study. The questionnaire served as a baseline study to investigate the extent to which students are aware and use the graphical solution techniques in solving linear inequalities. (See Appendix A)

3.4.3 Interview Guide

An unstructured interview guide would be conducted after the treatment to ascertain students' views, interest and their motivation to use the graphical solution techniques. Again, the interview also aids at finding students observations about using graphical solution techniques in solving linear inequalities. The interview also would help in understanding the "why's" behind students' performance on the posttest after the treatment and how they felt after the intervention. That is, making meaning of quantitative results (triangulation of results). (See Appendix F)

3.5 Reliability and validity of instrument

Reliability is the extent to which the result of an instrument yields the same results when measured on similar subjects at different times under the same condition. According to William (2006), reliability refers to consistency or 'dependability' of the measurement or the extent to which an instrument measures the same way each time it is used under the same condition with the same subjects. Validity, on the other hand, refers to the extent to which instruments measure what is intended to measure or how truthful the research results are (Joppe, 2000). To check for the validity of the instrument, the researcher was assisted by her supervisors in designing the questionnaires as well as the achievement test for content and construct as well as face validity. The approved questionnaire and achievement test by my supervisor was pilot-tested to establish not only its reliability but also to identify faulty items and ensure that the instrument is clearly understood by respondents.

3.6 Data collection procedure

Permission was sought from the Head of Department from the chosen school that is KINSS, Mathematics Department. After the approval, permission would sort from the algebra teachers in charge of the SHS two (2) students for a date to collect the data. The researcher was used fifth weeks in collecting data. In the first week, a closed-ended questionnaire with few open-ended and closed-ended questionnaires were given to SHS two (2) students' (pure science class). The aim was to find out the number of students who has little knowledge on graph and vice versa. It also aims to know the number of students who were aware of the graphical solution techniques in solving linear inequalities, and whether students make use of some of the graphical solution techniques when solving.

After the administration of the questionnaire, students who was aware of the technique were picked as the experimental group. This was to ensure that students selected for the experimental group have knowledge on graphical solution techniques. The students in the control group were taught by the tradition method they already know. Students were then taught linear inequalities using the graphical solution techniques. Students were brief on how these graphical solution techniques can improved their understanding in linear inequalities in general. The second week, the first lesson plan was taught, that is using graphical solution techniques in solving linear inequalities on one variables. Students were introduced to the graphical solution techniques, and how it works. Students were given some question to try using the graphical solution techniques, in each of the try works. Students were made to work out their solution using graph paper, pencil and ruler. This was to ensure that students understand the solving process.

Another lesson plan was taught in third week on using graphical solution techniques solving linear inequalities on double inequalities variables and their graph on till the fifth weeks. Posttest was given after the graphical solution techniques intervention. In the intervention, students were tests with graphical solution techniques on some selected topics in linear inequalities (linear inequalities on one variable and others). Students were taking through some linear inequalities on one variable graph with the help of the graphical solution techniques.

After implementing the design, the experimental group and the control group were given the same set of questions to try. The results of the two tests were analyzed to investigate the effect of graphical solution techniques design on students understanding of linear inequalities. The purpose of the interview was to find out their views or perceptions of their motivation about the use of graphical solution techniques, the advantages and disadvantages of using the graphical solution techniques in solving linear inequalities in general was also explored. Five (5) students were interviewed to triangulate the results in the pretest and posttest.

3.7 Piloting of the Survey Instrument

It is easy to overlook mistakes and ambiguities in question layout and construction when designing a questionnaire. Moreover, it is possible to design a questionnaire that is reliable because the responses are consistent, but may be invalid because it fails to measure the concept it intends to measure (Awanta and Asiedu-Addo, 2008). In view of this, the instrument was pilot tested. A pilot test of a questionnaire is a procedure in which a researcher makes changes in an instrument based on feedback from a small number of individuals who complete and evaluate the instrument (Creswell, 2003). Similar SHS 2 students at the Jema Senior High School were used for the pilot study. A total of Fifty (65) SHS 2 students from Jema Senior High School were used in the pilot study. The feedback of the respondents helped to improve the quality of the survey items in terms of content coverage, content validity, and reliability.

The questionnaire yielded a reliability coefficient of 0.678. The questionnaire was highly reliable since the reliability coefficients were above 0.5. The reliability statistics of the questionnaire is presented in Appendix D.

3.8 Data analysis

In order to analyze the effect of graphical solution technique on student understanding on linear inequalities, data collected was organized under the key variables. The independent variables being the treatment were the graphical solution technique and traditional teaching approach. The dependent variables being the 'students 'understanding of linear inequalities (i.e. students' gain scores in the test) and students' responses on the awareness and use of mathematical method in learning (i.e. students' ratings of the questionnaire).

The Statistical Package for Social Science (SPSS) software was used for the data analysis because it is user-friendly, does most of the data analysis one needs for quantitative analysis and is the most common statistical data analysis package used in educational research (Muijs, 2004). The responses from the test and questionnaire items were coded and captured for analysis using SPSS.

The data entries were done by the researcher in order to check the accuracy of the data. Data was cleaned before running any analysis. Cleaning the data helped the researcher to get rid of errors that could result from coding, recording, missing information, influential cases or outliers. The data was analyzed and presented largely using narrative, descriptive statistics (i.e. frequency distribution, percentages, charts, mean, median, and mode) and inferential statistics (i.e. independent sample t-test, paired sample t-test). The independent sample t-test was used because participants in each group are not related in any way that is the two groups are independent of each other. This independent sample t-test was used to answer research questions (2) which determine whether or not there is a statistically significant difference in the means of the control group and the experimental groups. The paired sample t-test was used to determine whether the means of each students pretest and posttest within the same group were significant. The independent sample t-test was used in analyzing the test and the questionnaire because it met the entire assumptions independent sample test. See Table 3.3 and Table 3.4 showing the homogeneity of variance for the test and the questionnaire.

In statistics, Levene's test is an inferential statistic used to assess the equality of variances for a variable calculated for two or more groups. Some common statistical procedures assume that variances of the populations from which different samples are drawn are equal. Levene's test assesses this assumption. It tests the null hypothesis that the population variances are equal (called homogeneity of variance or homoscedasticity). If the resulting P-value of Levene's test is less than some significance level (0.05), the obtained differences in sample variances are unlikely to have occurred based on random sampling from a population with equal variances. Thus, the null hypothesis of equal variances is rejected and it is concluded that there is a difference between the variances in the population (Levene, 1960).

Table 3. 1 Levene's Test of Equality of Variances for the Pre-test and Post-Test scores

		Levene's Test for Equality of Variances	
		F	Sig.
Equal variances assumed	Pre-Test	0.008	0.927
	Post-Test	0.865	0.356

From Table 3.5indicate that population variances are equal since the p-value is greater than the significance level (.356 > .05 and .927>.05), therefore we conclude that the null hypothesis is accepted and that equal variances are assumed.

Before the independent sample t-test was used, the researcher made sure that all the assumptions needed for independent samples t-test were met. The assumption is that, the test (dependent) variable is normally distributed within each of the two populations. 'The researcher used Q-Q plot to test the null hypothesis that "there is no significant departure from normality". To ascertain whether the dependent variables (scores of tests) were normally distributed for each combination of the levels of the independent variables, the researcher used Q-Q plots to test for the normality was employed. The researcher used this test for the normality of the scores (pre and post-tests) and the results are presented in Figures3.1 and 3.2.

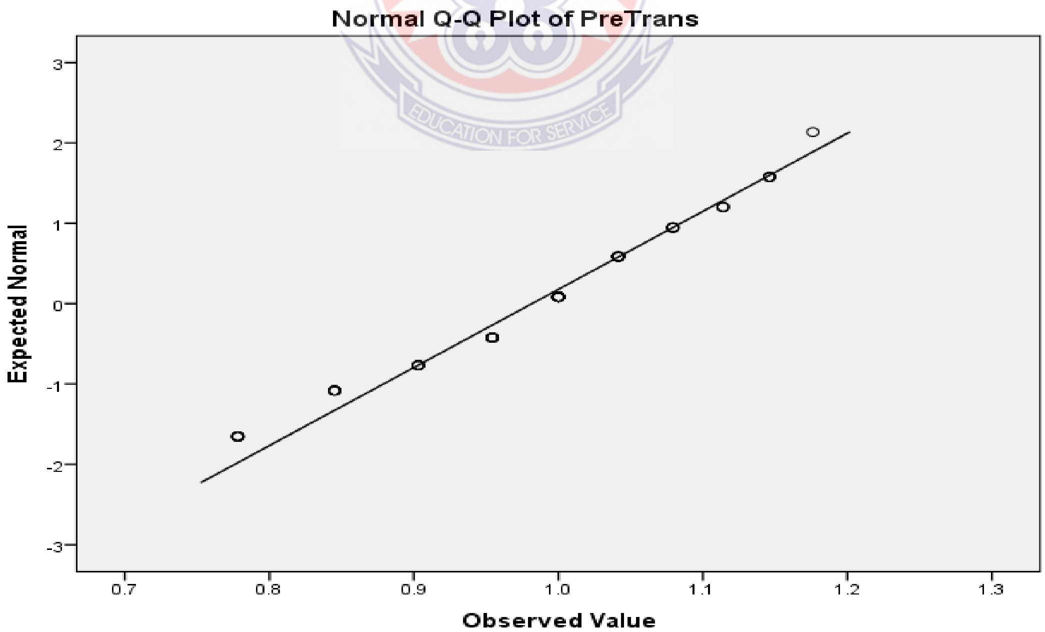

Figure 3. 1 Normal Q-Q plot of Scores for Pre-Test

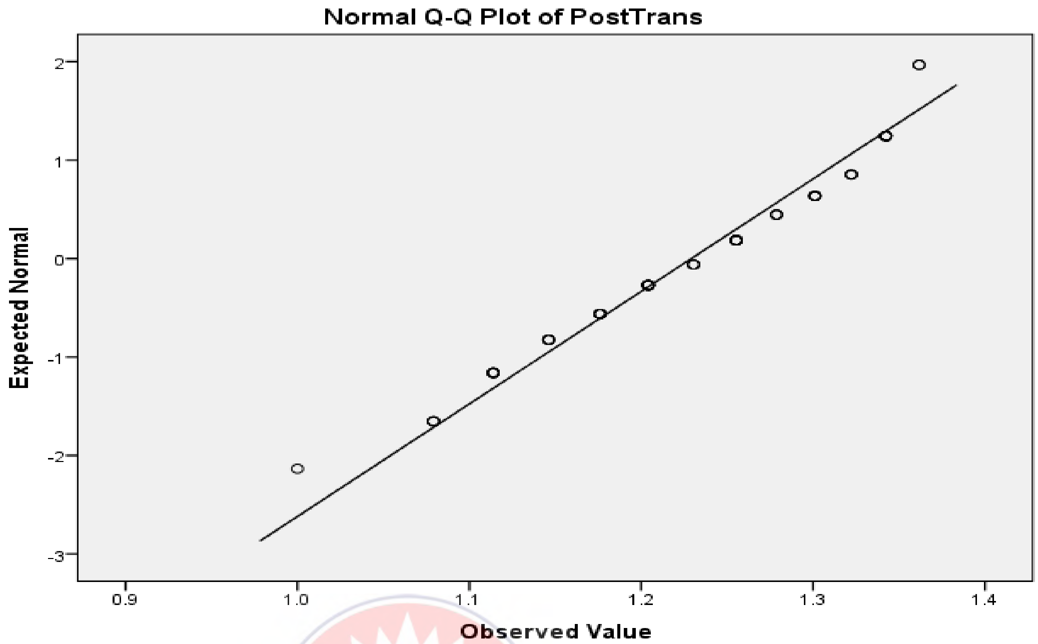

Figure 3. 2 Normal Q-Q plot of Scores for Post Test

Figure 3.1 and 3.2 presents the results of normal Q-Q plot of scores of tests. It can be observed from above figure that the dependent variable (scores of the pre- and post-tests) were normally distributed for each of the categories in the independent variables. That is, very little departure from normality for the scores variable since most of the values are not far away from the straight line. Thus, the normality assumption was not violated.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.0 Overview

This chapter focuses on the results of the analyses of the data and discussion of the major findings. The data were organized and presented using tables, figures, descriptive and inferential statistics. The result is presented under the following themes which broadly represent the research questions:

1. Demographic characteristics of participants the extent to which students are aware of and used graphical solution technique;
2. The effect of graphical solution technique on student performance in solving linear inequalities;
3. Student motivation to use graphical solution approach and their views about the lesson; and
4. Discussion of major findings

4.1 Demographic characteristics of participants

A total of sixty-five (65) SHS 2 students (i.e. 43 males and 22 females) participated in the study. Table 4.1 shows the gender distribution of participants in the study.

Table 4.1 Distribution of students' use of graphical solution technique

	Number of students	Percent
Male	43	66.2
Female	22	33.8
Total	**65**	**100**

From Table 4.1, it can be seen that out of the 65 respondent 43 students (66.2%) were males and the remaining 22 students (33.8%) were females. This means that females are underrepresented among the SHS 2 students. Fewer women are normally found in the science discipline, in the classes, out of over 200 students offering mathematics, only 90 are females. No wonder only 22 females were seen in the two groups who responded to the questionnaire.

Table 4.2 shows the age distribution of students that were involved in the study. This was to find the age range that used for the test.

Table 4.2 Descriptive statistics of the age range of students'

	Number of students	Percent
12-15	4	6.2
16-20	53	81.5
Above 20	8	12.3
Total	**65**	**100**

From Table 4.2, it can be seen that 4 students representing 6.2% were between the ages of 12- 15years, 53 students representing 81.5% were between the ages of 16-20 years and only 8 students were between the ages of above 20 years. It was clear that majority of the respondents (81.5%) students fall within the ages of 16-20. This means that majority of students are between the ages (16-20). This indicates that most of the respondents are coming directly from the junior high school per the curriculum of Ghana.

4.2 Extent to which student are aware of, and use, the various mathematical methods in solving linear inequalities

The participants of this research were senior high school (two) from kintampo senior high school, who had learned linear inequalities in SHS one (1) as prescribed by the curriculum of the of the GES program in the senior high level. Students' are supposed to have good knowledge of linear inequalities to help them in future. The study was based on the assumption that when students are made aware of the various methods use in solving linear inequalities, they are more likely in using them in learning mathematics topics. A set of questions (see Appendix A) was given to 65 students' to indicate whether or not they are aware of and use the various methods in solving linear inequalities. Table 4.2 shows the participants' responses on the meaning of linear inequalities.

Table 4.3 Descriptive Statistics of students' definition to linear inequalities

	Number of students	Percent
Correct definition	46	70.8
Wrong definition	19	29.2
Total	65	100

From Table 4.3 It is obvious that greater percentage (70.8%) out of the respondent knew the meaning of linear inequalities. Only few respondents (29.2%) did not know the meaning of linear inequalities. This indicates that student majority of the respondents are aware of linear inequalities. Tables 4.4 and 4.5 shows the distribution of the various methods and their awareness of those methods used in solving linear inequalities.

Table 4.4 Distribution of the type of methods used in solving linear inequalities

	Number of students	Percent
Algebraic solution only	35	53.8
Graphical solution only	12	18.5
Cimmino's solution only	1	1.5
Algebraic and graphical solution only	1	1.5
Algebraic and cimmino's solution only	12	18.5
Algebraic, graphical and cimmino's solution only	4	6.2
Total	**65**	**100**

Table 4.4 shows that 35 respondents representing 53.8% of the total respondents use algebraic solution to solve linear inequalities, while18.5% use either graphical or both algebraic and cimmino's, also cimmino's or algebraic and graphical solution use 1.5%and the remaining 6.2% say they use all the methods.

Table 4.5 Distribution of student aware of graphical solution technique

	Number of student	Percent
Yes	35	53.8
No	30	46.2
Total	**65**	**100**

From Table 4.5 it is obvious that greater percentage (53.8%) out of the respondent are aware of the graphical solution technique while few respondent (46 2%) are not aware. This indicates that student used graphical solution in learning and the solving of mathematics topics. Tables 4.6 will determine whether the student use graphical solution in learning and solving linear inequalities.

Table 4.6 Distribution of student using graphical solution technique in solving linear inequalities

	Number of students	Percent
Yes	22	33.8
No	43	66.2
Total	**65**	**100**

From Table 4.6 it is obvious that greater percentage (66.2%) out of the respondent are not using the graphical solution technique while few respondent (46 2%) are using. This indicates that student do not used graphical solution in learning and the solving of linear inequalities. Then the researcher wanted student to use this method to learn and solve linear inequalities.

Table 4.7 Distribution of student using Algebraic solution technique in solving linear inequalities

	Number of students	Percent
Yes	42	64.6
No	23	35.4
Total	**65**	**100**

From Table 4.7, it is obvious that greater percentage (64.6%) out of the respondent is using the algebraic solution technique while few respondents (35.4%) are not using. This indicates that majority of the students used algebraic solution in learning and the solving of linear inequalities.

Table 4.8 Distribution of students' responses on their awareness of mathematical methods and ability to list any three mathematical methods for solving linear inequalities

	Listing of methods used in solving linear inequalities						Total	
	Correct	Percent	Wrong comment	Percent	No response	Percent	Number	Percent
Yes, learn linear inequalities using methods	13	20	2	3	0	0	15	23
No, do not learn linear inequalities using methods	0	0	14	22	36	55	50	77
Total	13	20	16	25	36	55	65	100

From the responses in Table 4.8, it was gathered that only 23% of the students were aware of and listed the correct mathematical methods in learning and solving linear inequalities. This indicated that the majority 77% (50 out of the 65 valid responses) were not aware of this graphical method. In Table 4.8, it can be observed that only 13(20 %) of the students who said they were aware were able to list the methods correctly. It was also observed from the open-ended question that most of the respondents who indicated that they were aware was rather listing substitution, elimination and different methods rather than algebraic method which is the old method.

From the analyses above, it is clear that most of students were not aware of various methods in solving linear inequalities. The few who indicated they were aware of the various methods did not state they used them often in learning linear inequalities. In addition, data from interview support the results discussed above. Five (5) students were interviewed after the administration of the questionnaire. One student said," am not even aware that graphical solution can also use in solving linear inequalities". He further explained that if I was aware then mathematics would have been easy for me. Another student also said: Can one really learn with all this method? Can this method solve and explain the solving process? Wow, then this will be very helpful to mathematics students. These responses imply that students were not aware of and do not make use of this method in learning linear inequalities.

To conclude the baseline studies on whether students were aware of and use this method in learning, it was clear and obvious that students were not aware of this method in learning linear inequalities. Also, the few who were aware couldn't list about three examples of any of the methods. Some were able to list "two", others "one" and some were not even able to mention at all.

In addition to this, from the open-ended questions, some were also mentioning mathematics topics, meaning students are familiar and use some methods in learning mathematics which was very good as far as the learning of mathematics is concern. A Greater percentage (77) of students indicated not being aware and have not used the method. Almost all the students said they find it difficult using the old method in learning. Students were not aware of and use the graphical solution technique for learning linear inequalities to a very large extent of solving linear inequalities (76.9%). Results from the interview revealed that majority of the students were not aware of and do not make use of this method in learning linear inequalities.

4.3 The effect of graphical solution technique on student performance in solving linear inequalities?

To ascertain whether there was a significant difference between the means of the control and the experimental group the scores of both groups in the pre and post tests were compared. Table 4.9 presents descriptive statistics for the control group and the experimental group respectively.

Table 4.9 Descriptive statistics of the control and experimental group in the pretest and the post-test.

Test	Group	N	Minimum	Maximum	Mean	Std. Deviation
Pre-Test	Control	31	11	20	17.23	1.820
	Experimental	34	0	14	7.26	3.212
Post-Test	Control	31	14	20	16.45	2.501
	Experimental	34	14	20	17.35	1.668

From Table 4.9 above, it can be observed that the control group in the pre-test had a minimum score of 11 and a maximum of 20 with a mean of 17.23 (SD = 1.820) whereas in the experimental group had a minimum of 0 and a maximum of 14 with a mean of 7.26 (SD = 3.212).

Also after the treatment, the control group in the post-test had a minimum score of 14 and a maximum of 20 with a mean of 16.45 and a standard deviation of 2.501 whereas in the experimental group had a minimum of 14 and a maximum of 20 with a mean of 17.35 and a standard deviation of 1.668.

Therefore, to check whether the difference was statistically significant, an independent sample t-test was used at 0.05 level of significance to test the hypothesis that there is no statistically significant difference between the mean scores of the control group and the experimental group after the treatment (the use of the graphical solution technique) in teaching linear inequalities. The result of the test is shown in Table 4.10.

Table 4.10 Results of Independent sample t-test of students' means in the control and the experimental group in the pre and post-test

Test	Group	Mean	Std. Deviation	t-value	P-value
Pre-Test	Control	17.23	1.820	14.912	0.000
	Experimental	7.26	3.212		
Post-test	Control	16.45	2.501	1.538	0.129
	Experimental	17.35	1.668		

Table 4.10 presents the results of the independent-samples t-test performed on the pre-test scores of the 65 randomly selected students of the two independent groups (i.e. control and experimental groups). As can be seen in Table 4.10, comparison of the mean scores suggested that the control group performed better (mean = 17.23, SD=1.820) than those in the experimental group (mean = 7.26, SD=3.212). To test whether the difference in mean scores between the control and experimental groups was statistically significant, an independent-samples t-test was performed. The results of this test (Table 4.10) revealed that there was statistically significant difference ($t=14.912$; $p < 0.05$.) in the mean scores between the control group and experimental group. Therefore, the hypothesis that there is statistically significant difference between the mean scores of the control group and the experimental group before the treatment was retained. These results suggest that both the control group and the experimental group were almost at the same level of conceptual understanding in inequalities before the start of the intervention.

Similarly, in the post-test from Table 4.10, the means suggested that the experimental group performed better (mean = 17.35) than those in the control group (mean = 16.45). Also, to test whether the difference in mean scores between the control and experimental groups was statistically significant, an independent-samples t-test was again performed.

The results from Table 4.10 revealed that there was no statistically significant difference in mean scores between the experimental group (M = 17.35, SD=1.668) and control group (M = 17.23, SD = 1.820) at t=1.538; p > 0.05. Therefore, the null hypothesis that there is no statistically significant difference between the mean scores of the control group and the experimental group after the treatment was rejected in favor of the experimental group. These result also suggest that there has been a great improvement in the achievement of students in the experimental group than those in the control group.

In order to check whether there was a significant difference within each of the group's pre-test and post-test means. A paired sample t-test was conducted to test the null hypothesis that there was no statistically significant difference in their mean scores for the pre-test and the post-test. Table 4.11 shows the results of the Paired Samples t-test for Control and Treatment Groups' Pre and Post Tests.

Table 4.11 Results of Paired Samples t-test for Control and Treatment Groups' Pre and Post Test

Group	Test	N	Mean	SD	Paired Mean Differences	T	Df	Sig. (2-tailed)
Control	Post-Test	31	16.45	2.501	-0.774	14.912	30	0.153
	Pre-Test	31	17.23	1.820				
Experimental	Post-Test	34	17.35	1.668	10.881	1.538	33	0.000
	Pre-Test	34	7.26	3.212				

As can be seen in Table 4.11, paired samples t-test was conducted on the pre- and post-test scores in both control and experimental groups. The results of the paired samples t-test of the student in the control group indicated that there was no statistical significant difference in their mean scores for the pre-test (M =17.23, SD = 1.820) and the post-test (M = 16.45, SD = 2.501) at t=14.912; df = 30; $p > 0.05$. Also, in the experimental group, there was also a statistically significant difference in their mean scores for the pre-test (M = 9.26, SD = 3.212) and post-test (M = 17.35, SD = 1.668) at t= 1.538; df = 33; $p < 0.05$.

However, the paired mean difference for the control and experimental groups were found to be -0.77 and 10.88 respectively. The results from the tests showed that the paired mean difference of the experimental group is almost two times that of the control group. This difference indicates that the teaching and learning with the graphical solution technique is more effective than the traditional method. Also, a calculation of Cohen's d effect size revealed that the traditional teaching method has a medium effect of students' achievement while the graphical solution technique has a large effect on students' achievement. The effect sizes were found to be 2.05 and 3.45 for the control (traditional) group and the treatment group respectively.

In conclusion, results from the descriptive and inferential statistics revealed that the graphical solution technique really helped students (experimental group) to understand linear inequalities better than the control group.

4.4 What are Student motivation to use graphical solution technique and their views about the lesson?

In order to find out students' motivation of the use of graphical solution approach in learning linear inequalities, (10) students in the experimental group were interviewed using an interview guide (See Appendix F). The interview results revealed that the students found the lessons interesting and easy to understand. A greater number (90%)

of them were also motivated to use the approach. When students were asked their general impressions about the graphical solution approach used in the lesson, this is what some said:

Am highly impressed, what an approach! I now understand a lot of linear inequalities concepts better since the graphical approach solves the question in a systematic way (student A).

I am happy to know this, herr…..!!! Graph can also use to solve inequalities questions (Student B).

I had problem with how to change the sign when solving inequalities, but now I can use the approach to learn it and even the approach can interpret the solution for you. I am now motivated to learn mathematics now since one method can be used to solved so many topics(Student C).

The use of the approach helps to verify answers when learning, meaning one can check whether he is correct or wrong with approach (Student D).

Responses from student A, B, C, and D indicate that students had a lot of difficulties in how learning linear inequalities. The graphical approach in learning linear inequalities and other concepts have really enlightened the students on the various in learning mathematics.

On the issue of the student enjoyment, interest and engagement in learning with the innovative learning approach, all the students interviewed responded positively. The students indicated that they can now study on their own at their leisure. The researcher also observed that students' participation in the class was high, as they were engaged in sharing ideas among themselves. Some of the responses made in this regard are:

I understand these concepts better since the approach solves the question in a systematic way. Wow…Mathematics learning is now enjoyable. (Student A)

Seriously, I was motivated to learn since he was introduced to such approach that can solve, gives the steps and some functions as well. (Student B)

The use of the approach helps to verify answers when learning, meaning one can check whether he is correct or wrong with graphical approach. Using approach step-by-step arouses interest to learn more and saves time, in the sense that a lot of questions can be solved within a very short time. (Student C).

Furthermore, from observation, the students exhibited high levels of eagerness and attention due to the systematic nature of the Approach in solving questions. It was again observed that independent learning was exhibited. Students were seriously doing their own work after they had been introduced to the method.

With regard to their perceived motivation to use the Approach at their leisure and for teaching, the students said the introduction to the approach had enlightened them on the benefits of using the approach to study mathematics in general. They added that previously they were familiar with topics but not linear inequalities and it had really enlightened them in learning mathematics in general. They said that there was a lot of restrictions in using the algebra approach in learning mathematics which made it difficult to do with. Again, they said, with the approach, one can study at his or her own time and pace. Some of the students have these to say:

I learned that mathematical approach can be used to study mathematics. Previously I didn't know. I also learnt that the approaches don't just solve the problem but give steps as well (Student C).

I never knew there exist an approach that can solve and interpret the graphical process, like interpreting, the solution and many other things (Student F).

It was clear from the interview that all the participants exposed to the teaching graphical solution found the approach lesson impacting on what they knew already and also confirmed they never knew of any approach that could solve, interpret and graph problems in mathematics. The participants also indicated that they will be willing to teach their students such a wonderfully innovative way of learning mathematics. Their reason was that when students are told and taught how these approaches can be used, they can explore with another topics when they go home. Below are some of the responses:

> *Yes, because the approach makes mathematics easy to study and understand. The only thing is to introduce them to it and how to use it (Student B).*
>
> *I will only if they have the graph book, because if it has been introduced, I wouldn't have found mathematics that difficult (Student E).*
>
> *"I will even tell my friend about it, even if they don't have. This will make them to explore with the approach when they leave school (Student F).*

In addition to the questions on motivation, students were asked to mention challenges they faced in using the approach. The students indicated distraction, and laziness in using the approach as the major challenges. Below are some of the responses given by the students?

> *...the lesson was interesting but keying into the table was a little difficult. You need to explore more before thing becomes easy, however, I was happy when my answers came out. But with more examples I was able to overcome it (Student A).*

Sometimes the process given by the approach is difficult to understand, you need to sit and analyze the process before you can get the concept (Student D).

"Madam, the approach is excellent but it can't solve every problem under algebra. You need to be careful the type of question to used (Student E)

"My difficulty was with the various areas you need to explore before using the approach. but with constant practice this can be corrected. You have to explore the approach environment, how to key in and so on" (Student F).

In addition to the challenges, the students also gave the following as disadvantages of using the approach: the use of the approach makes students lazy". He explained that since the approach solves and gives the various procedures for a question, solving the question using paper and pencil becomes a problem for lazy students.

Reviewing critically the responses of the students, it was seen that students were really happy about the approach lesson and were ready to use the approach; however, they encountered some challenges. These challenges must be taken into consideration when planning a lesson on the mathematical approach and also make sure more problems are given to students to try out to overcome some of these challenges. In conclusion, it is obvious that using mathematical approach in learning really motivates students to learn. This was obvious from students' response that the use of the graphical approach helped them to learn on their own and everywhere (independent learning). This means the innovative learning approach was really helpful and the students are very likely to use it in teaching their students' in future.

4.5 Discussion of major findings

This research was aimed at assessing the impact of graphical solution technique has on students achievement in Inequalities and their motivation to use method in teaching and learning mathematics. The researcher conducted a baseline study to find out the extent to which students are aware and use the method in learning mathematics.

The results of the questionnaire indicated that majority (78.1%) of the students were not aware of that the method can also be used in learning inequalities. From the responses on the closed-ended questions in the questionnaire, a lot of students (92.2%) know the method but do not know that it can also be used for solving linear inequalities. Only 21.9% of the total number of students was aware as well as uses of other methods in learning. In addition, those who said they were aware, the majority of them could not list any example of methods correctly. Those who were able to list also were listing mathematics topics like simultaneous equation, relation and mapping, and etc. Furthermore, to know whether students use methods as part of their learning process, few students (40.6%) indicated using methods. This means students learn with other method but certainly not using the graphical solution technique. These results show that majority of the students are not aware of the various method in learning mathematics, and few learn with some of the methods. It is clear that students are not aware and do not use the various mathematical methods in learning linear inequalities to a very large extent.

Their study employed survey as a design to obtain a deeper insight into the nature of students' use of graphical solution technique as well as their attitude towards the educational on mathematics. Other researchers also did some finding on it also consistent with a study conducted by (Elena Halmaghi, 2011; Larson, Hostetler & Edwards, 2008;

Norton & Irvin, 2007) they concluded that linear inequalities is always difficult and brought different methods for solving it.

The effect of graphical solution technique lesson on students' performance in linear inequalities was also assessed. Results on the post-test were analyzed. It was discovered that the teaching technique (i.e. graphical solution technique design) led to an improvement in the performance of student achievement in linear inequalities. That is, the experimental group had a mean (M =7.26, SD = 3.212) which was highly statistically significant different at 5% level of significance from the control group (M = 17.23, SD = 1.820) at $p<0.05$. The conventional method used on the control group was not also significant (M = 16.45, SD = 2.501) at $p > 0.05$ but was lower than that of the experimental group. The result of the findings is consistent with that of Morell et. al. (2001) whose study also revealed that technology improves learning by taking teamwork to function actively in the classroom. Not only was the new method but also participants functioned as learners in the sense that they used the mathematical methods anytime and anywhere, in informal settings, in the course of their everyday activities. Students are motivated and improve their self-esteem after successfully using new method in learning.

Students' responses on their views and motivation about the graphical solution technique lesson were also sorted. Responses from the interview indicated that students felt happy and motivated to use the mathematical method in learning linear inequalities. These findings were consistent with a study conducted by Norton and Irvin (2007) suggested some solutions which include: making explicit algebraic thinking inherent in arithmetic in children's earlier learning (Lins & Kaput, 2004; Warren & Cooper, 2006), explicit teaching of nuances and processes of linear inequalities in an algebraic and symbolic setting (Kirshner & Awtry, 2004; Stacey & Chick, 2004) were considered to be interesting and enjoyable. They also discovered other motivational factors attributed to teachers use of mathematics method, which was: making the lesson more interesting, increasing pupils' motivation, improving the presentation of materials, making the teaching more enjoyable, improving the content of the lesson, and making the lesson more fun for the pupils. The finding is also in line with a study conducted by Tsamir and Almog (2001), which concluded that students were motivated and their self-esteem after successfully using the new method was improved. However, the interview conducted shows that students really love to learn with the new method and others as well. Students' were also motivated to learn on their own due to the quick feedback they get from learning with the method. Students centered learning was also observed.

Again, from some of the responses from the interview, it was concluded that in using such methods, care must to taken on the part of the teacher. Lessons should be well structured and fully guided in integrating such method in the mathematics classroom to avoid distraction. Learners and teachers, as well as the parents, should be taught a lesson on a function to fully utilize the environment. Also, teachers should encourage students to use the method in verifying their answer that is, students should try a problem out with calculate before using the method.

In conclusion, it can be argued from the results that students were not aware of the graphical solution technique in solving linear inequalities. Their difficulties were due to the fact that they find it difficult to use some of the methods. This may also result from lack of training on how to use this method in learning some mathematical concepts. Also during the intervention, the researcher observed that some students felt motivated and were happy in the course of the lesson. It was also observed that some of the students in the control group couldn't benefit from the conventional lesson due to the lack of motivation, interest and commitments level and the experimental group was so happy in class because they had never been taught with graphical solution technique.

CHAPTER FIVE

SUMMARY, CONCLUSION, AND RECOMMENDATIONS

5.0 Overview

This chapter presents a summary of the findings, conclusion and the recommendations to the areas that call for further research.

5.1 Summary

This research was aimed at assessing the impact of the use of graphical solution technique on students achievement in inequalities and their motivation to use graphs in teaching mathematics. A sample of 65 student was used in the study. The researcher used two achievements tests (pre- and post- tests), questionnaires and interview guide to collect data. The tests focused on the students' achievements before and after the treatment, while the questionnaire focused on student awareness and use of the various methods in learning mathematics. Also, the interview elicited information on students' impression, enjoyment and challenges in the use of the graphical step by step method.

The study used both quantitative and qualitative methodologies which employed a quasi-experiment as a strategy of inquiry. The study was preceded by a baseline study to check the extent at which student were aware of and use the method in learning mathematics (questionnaire was used). Convenient and purposive sampling were used to select the experimental group. After that student in the experimental group were taught with the innovative teaching method. A post-test was given to both the control group and the experimental group after the treatment. The data was analyzed and presented largely using narrative, descriptive statistics (i.e. frequency distribution, percentages, charts, mean, median, and mode) and inferential statistics (i.e. independent sample t-test). Independent sample t-test was used because participants in each group were independent of each other. The groups of participants are independent of one another. The constructivist approach to teaching was used as the theoretical framework to explain its effect on students.

5.2 Major Findings

The findings of the study are summarized and presented under the three sub-headings in line with the research questions.

5.2.1 Research Question One: To what extent are student aware of and use the graphical solution in solving inequalities?

The studies revealed that majority (78.1%) of the student were not aware of the various methods in learning mathematics. Also, the few who were aware couldn't list the three methods for solving inequalities. Some were able to list "two", others "one" and some were not even able to mention at all. In addition to this, from the open-ended questions, some were also mentioning difference for solving difference topics, meaning students were familiar and used some method in learning mathematics which was very good so far as the learning of mathematics is concern. A greater percentage of student indicated not being familiar with the various methods.

5.2.2 Research Question Two: What is the effect of graphical solution technique on student in solving and learning inequalities problems?

The study was find that there was a statistically significantly different between the groups. (Control and the experimental) with mean and standard deviation of ($M = 16.45$, $SD = 2.501$) and ($M=17.35$, $SD = 1.668$), $p < .05$. The independent sample t-test (Table 4.2.3) revealed that there was a significant difference in the achievement of students in the Posttest at $p= 0.000<0.05$ which indicate that there has been a remarkable improvement in the achievement of students in the experimental group. The study revealed that the use of graphical solution technique can positively influence students' learning mathematics including inequalities. Students' were able to solve a wide range of inequalities problems through the graphical solution technique. The student showed a high sense of independence in learning after they were introduced to the graphical solution technique.

5.2.3 Research Question Three: What are student motivations of use about the use of graphical solution technique?

Qualitative results derived from the interview showed that students were fully impressed and motivated to use the graphical solution technique in learning inequalities. Students were also indicated teaching their friends. This shows the extent at which the graphical solution technique helped students to understand inequalities better. However, it was revealed those students were highly engaged in lesson "the use of graphical solution technique for teaching and learning inequalities". The interview indicated that student have a positive perception of the innovative teaching approach and usefulness of graphical solution technique and easy use in teaching and learning of inequalities. This indicates that student generally view graph usage as a tool in learning mathematics an effective way of solving questions.

5.3 Conclusion

The main purpose of the study was to assess the impact of graphical solution technique (step by step solver) on student performance in inequalities in kintampo SHS . To assess the impact of graphical solution technique on students understanding of inequalities, a baseline study was first conducted using a questionnaire to ascertain the extent at which student were aware of and uses the various methods in learning mathematics. It was revealed from the baseline studies that majority (78.1) of students were not aware of the some methods in learning mathematics. The few of them who said they were aware couldn't list the three examples of methods (e.g. algebraic methods, graphical method, etc.) as indicated in the questionnaire. Some were also mentioning methods like elimination, substitution, among others. The study also revealed that about 90% of student use their leisure time learning difference etc. Even though student indicated learning with their graph but certainly not learning with the mathematical method. Also, a greater number of the student indicated that they find it very difficult in using a lot of the methods. After the baseline studies, it was revealed that students are not aware of some of methods in learning mathematics and also need training on how some of these methods are used in learning mathematics. It was also observed from the pre-test that students made a lot of mistakes in solving inequalities problems. A greater number of them had difficulty in drawing graphs of some quadratics function as well.

The study revealed that the use of graphical solution technique can positively influence students' learning mathematics including inequalities. As a result, the mean performance of the experimental groups was statistically significant from that of the control group after the experiment (See Table 4.2.2). Students' can solve a wide range of inequalities problems through the graphical solution technique. The students showed a high sense of independence in learning after they were introduced to graphical solution. Students were able to work in pairs, compare their answers and again expressed their feeling and motivation in using the graph methods. It was observed that students were happy and motivated using the Method when they realize how they can verify their answers as soon as possible. Also, after introducing students to the graph method, students were able to exploredand discovered for themselves other mathematical methods for learning a lot of concept in mathematics. Among the methods student discovered was math just to mention few. All these methods are used which students have access to in our world today. Students' centered learning was achieved as well.

However, it was revealed those students were highly engaged in lesson “the use of graphical solution in teaching and learning inequalities”. The interview indicated that students have a positive perception of the innovative teaching approaches and usefulness of graphical solution technique and easy use in teaching and learning of inequalities. This indicates that student generally view graph usage as a tool as an effective way of solving mathematics.

It is important for educators to find new ways of using graphical solution techniqe to enhance teaching and thereby to improve learning outcome. As the era of now days gradually shifts to solving of questions, there is the need for educators to find ways and means how these changes can bring about learning to benefit our students today. From the study, some of the students were aware of some methods for learning mathematics, which was very good so far as learning of mathematics is concern. However, a lot of the students were not aware of some of the methods in learning mathematics which is freely in solving inequalities. It was then clear that when students are made aware and taught how these methods can be used and how they can benefit from it in their studies, students will channel their time in learning mathematical concepts rather than just wasting their time on difference methods. There is also the need for our training colleges and Universities mandated to train teachers to include contents on how these methods should be used and also how to design some of the method in learning mathematics. However, when selecting the best mathematical application, several factors need to be considered to make sure content is blended with pedagogy to bring about positive learning outcome. The student should also be directed and supervised during learning in the classroom so that they don’t trespass doing a different thing rather than learning.

Finally, the interview revealed that student really loved to learn on their own. They said they can now learn at their own pace without any supervision. Students also outlined some advantages and disadvantages using the method. Teachers and students should take into consideration when learning with the graphical solution technique.

5.4 Recommendations

The results recorded from the research raise a number of issues of importance and interest to students, mathematics teachers and educational authorities. It is recommended that:

Teacher Training programs of the Universities and Colleges of Education should be updated so as to equip new teachers with the requisite knowledge and skills in some methods including graphical solution to teach mathematics, inequalities to be precise.

The teaching and learning mathematics in schools should practically advance from the mere traditional method to difference applications tools including graphical solution in mathematical methods.

It is suggested that the study should be replicated in many more schools (tertiary institution) to obtain the general picture of how graphical solution technique improve students' understanding of mathematics, not inequalities alone.

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APPENDICES

APPENDIX A: QUESTIONNAIRE FOR STUDENTS

UNIVERSITY OF EDUCATION, WINNEBA

DEPARTMENT OF MATHEMATICS EDUCATION

Dear students,

I am an M.Phil. Student offering mathematics education and I am writing my thesis on the use of graphical solution techniques in teaching and learning of mathematics. Please help by taking a few minutes of your time to answer this questionnaire about your personal experience with your graphical solution techniques in solving linear inequalities. The method will give students opportunity to study the nature of graphs and through this, they can form mental pictures of various graphs and how they behave as variable or constant changes. Thanks for your anticipated cooperation.

Instructions: Please tick

1. Gender

………..Male ………Female

2. Age?

…….12 – 15 ……..16 – 20 ……..above 20

3. Do you know linear inequalities?

……Yes ……No

If your answer to question 3 is yes, please continue with the rest of the questions.

4. What is the meaning of linear inequalities?

Ans.…………………………………………………………………………………………

……………………………………………………………………………………..

5. What method do you normally use for solving linear inequalities? (please you can tick more than one)

Algebraic Solution ………..

Graphical Solution …….

Cimmino's Solution Method …...

Others……….

Please specify here: ……………………………………………………………

6. Do you learn using graphical solution?

……Yes …….No

7. Are you aware of the various graphical solutions for learning and teaching topics in mathematics?

……….Yes (if yes, please specify). ……….No

8. If yes Please list three of the topics (specify here)………………………………………………

9. Do you learn graphical solution with any of topics you have listed above?

……Yes (if yes, please answer question 10

……..No (if no, please leave question 10) ……..sometimes

10. Please list the method you used in learning linear inequalities

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The question will only take not more than 5 minutes. We kindly appreciate your time and effort in answering the questions. Please answer the following questions by ticking the relevant box.

ITEM	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
I use Algebraic Solution often at my leisure time.					
I normally use graphical solutions in solving questions in some topics.					
Am familiar with the graphical methods in learning mathematics					
I use most of the mathematical method solving questions.					
I hardly use some of the methods in learning mathematics					
I find it difficult to use most of the mathematical methods					
I don’t use the methods in learning mathematics at all.					

Graphical solutions have helped me in understanding most of the topics in mathematics.					

Thank you for your time.

APPENDIX B: PRE-TEST ON PERFORMANCE IN SOLVING ALGEBRAIC PROBLEMS FOR BOTH GROUPS.

UNIVERSITY OF EDUCATION, WINNEBA

FACULTY OF SCIENCE EDUCATION

DEPARTMENT OF MATHEMATICS EDUCATION

Instructions: Answer all Questions

Read each question carefully and answer it, showing all necessary procedure and workings

1. Solve this inequality $7x - 7 < -2x + 2$

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2. Simplify the inequality $4(1 + x) \geq 2(3x - 5)$

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3. Simplify this inequality $5x + 3 > (4 - x) + 7x$

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4. Solve these rational inequalities $2 - x > 2x + 4 > x$

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5. Solve this double inequality $\frac{1}{3}(x + 2) > \frac{1}{4}(x-1)$

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APPENDIX C: MARKING SCHEME FOR PRE-TEST.

1. Simplify $7x - 7 < -2x + 2$

Solution **marks (4)** **comments**

$7x - 7 < -2x + 2$ group like terms

$7x + 2x < 2 + 7$ M1 add the common terms

$9x < 9$ M1 divide both side by the coefficient of X to find X.

$x < 1$ M2

2. Simplify $4(1 + x) \geq 2(3x - 5)$

Solution **marks (4)** **comments**

$4(1 + x) \geq 2(3x - 5)$ open the bracket

$4 + 4x \geq 6x - 10$ M1 group like terms

$4 + 10 \geq 6x - 4x$ M1 add the common terms

$14 \geq 2x$ M1 divide coefficient of X by both side

$x \leq 7$ M1 the change will change

3. Simplify the inequality $5x + 3 > (4 - x) + 7x$

Solution **marks (4)** **comments**

$5x + 3 > (4 - x) + 7x$

$5x + 3 > 4 - x + 7x$ M$\frac{1}{2}$ open the bracket

$5x + x - 7x > 4 - 3$ M$\frac{1}{2}$ group like terms

$-x > 1$ M1 divide both side by -1

$x < -1$ M2

4. Simplify $2 - x > 2x + 4 > x$

Solution **marks (4)** **comments**

$2 - x > 2x + 4 > x$ let 4 cross the >on both side

$2 - 4 - \mathrm{x} > 2\mathrm{x} > \mathrm{x} - 4$ M1 simplify the equation

$-2 - x > 2x > x - 4$ M$\frac{1}{2}$ group like terms

$-2 > 2x + x - x > -4$ M$\frac{1}{2}$ simplify the equation

$-2 > 2x > -4$ M1 divide both side by 2

$-1 > x > -2$ M1

5. Simplify $\frac{1}{3}(x + 2) > \frac{1}{4}(x - 1)$

Solution **marks (4)** **comments**

$\frac{1}{3}(x + 2) > \frac{1}{4}(x - 1)$ find LCM and multiply by both side

$12 \times \frac{1}{3}(x + 2) > 12 \times \frac{1}{4}(x - 1)$ M $\frac{1}{2}$ cancel to remove the fraction

$4(x + 2) > 3(x - 1)$ M$\frac{1}{2}$ remove the bracket on both side

$4x + 8 > 3x - 3$ M1 group like terms

$4x - 3x > -3 - 8$ simplify

$x > -11$ M2

APPENDIX D: POST-TEST ON PERFORMANCE IN SOLVING ALGEBRAIC PROBLEMS FOR BOTH GROUPS

UNIVERSITY OF EDUCATION, WINNEBA

FACULTY OF SCIENCE EDUCATION

DEPARTMENT OF MATHEMATICS EDUCATION

Instructions: Answer all Questions

Read each question carefully and answer it, showing all necessary procedure and workings

1. Solve this inequality $5 - 2x \geq 1$

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2. Simplify this inequality $3(x + 2) \leq 2(2x - 1)$

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3. Solve this inequality $3 + \frac{2}{7}x > x - 2$

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4. Solve this double inequality $-2 < \frac{1}{5}(2x - 4) \leq 2$

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APPENDIX E: MARKING SCHEME FOR THE POST-TEST

Solution

Let y_1 and y_2 be the new equation that is $y_1 = 5 - 2x$ and $y_2 = 1$.

Create a table for the linear equation and an interval for the x-axis

$y_1 = 5 - 2x$

x	-4	-3	-2	-1	0	1	2	3	4	5	6
y	13	11	9	7	5	3	1	-1	-3	-5	-7

From the graph

y_1 and y_2 meet at $x = 2$, put the answer into the main question to see whether the sign will change or not.

when $x = 3$

$5 - 2(3) \geq 1$

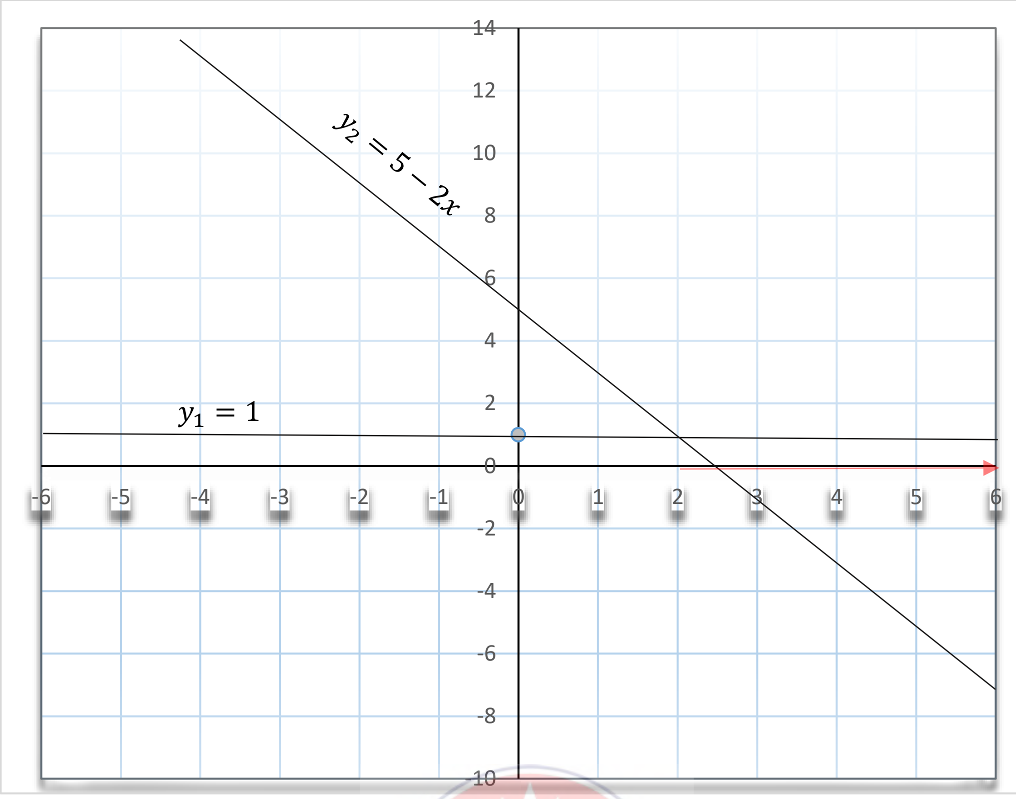
$-1 \geq 1$

when $x = 1$

$5 - 2(1) \geq 1$

$3 \geq 1$

Therefore the sign will not change that is $x \geq 2$.



1. Solve the inequality $3(x + 2) \leq 2(2x - 1)$

Solution

Let y_1 and y_2 be the new equation that is $y_1 = 3(x + 2)$and $y_2 = 2(2x - 1)$.

Create a table for both linear equation and an interval for the x-axis.

$y_1 = 3(x + 2)$

x	-10	-8	-6	-4	-2	0	2	4	6	8	10
y	-24	-18	-12	-6	0	6	12	18	24	30	36

$y_2 = 2(2x - 1)$.

X	-10	-8	-6	-4	-2	0	2	4	6	8	10
Y	-42	-34	-26	-18	-10	-2	5	14	22	30	38

From the graph

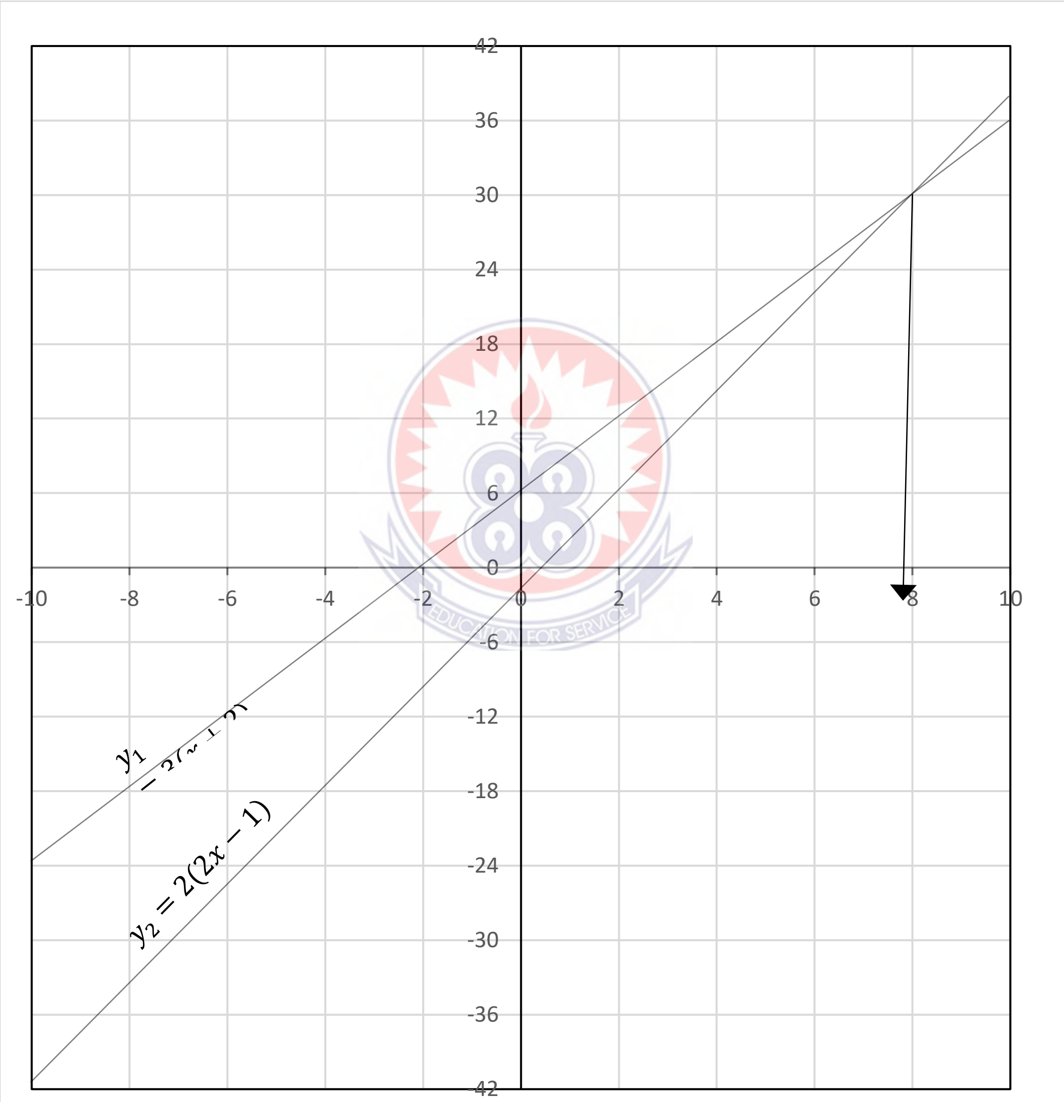
y_1 and y_2 meet at $x = 8$, then put the answer into the question to verify whether the sign will change or not. That is we will take the number less or greater the answer substitute into the question to verify the sign

When $x = 9$ when $x = 7$

$3(9 + 2) \leq 2((2 \times 9) - 1)$ $3(7 + 2) \leq 2((2 \times 7) - 1 \times)$

$33 \leq 34$ $27 \leq 26$

Therefore the sign will not change,$x \leq 8$.



2. **Solve the inequality $3 + \frac{2x}{7} > x - 2$**

Solution

Let y_1 and y_2 be the new equation that is $y_1 = 3 + \frac{2x}{7}$ and $y_2 = x - 2$.

Create a table for both the linear equations and an interval for the x-axis.

$y_1 = 3 + \frac{2x}{7}$

X	-8	-6	-4	-2	0	2	4	6	8
Y	0.7	1.3	1.9	2.4	3	3.6	4.1	4.7	5.3

$y_2 = x - 2$

X	-8	-6	-4	-2	0	2	4	6	8
Y	-10	-8	-6	-4	-2	0	2	4	6

Form the graph,

y_1 and y_2 meet at $x = 7$, then put the answer into the question to verify whether the sign will change or not. That is we will take the number less or greater the answer substitute into the question to verify the sign

When $x = 8$

$3 + \frac{2(8)}{7} > 8 - 2$

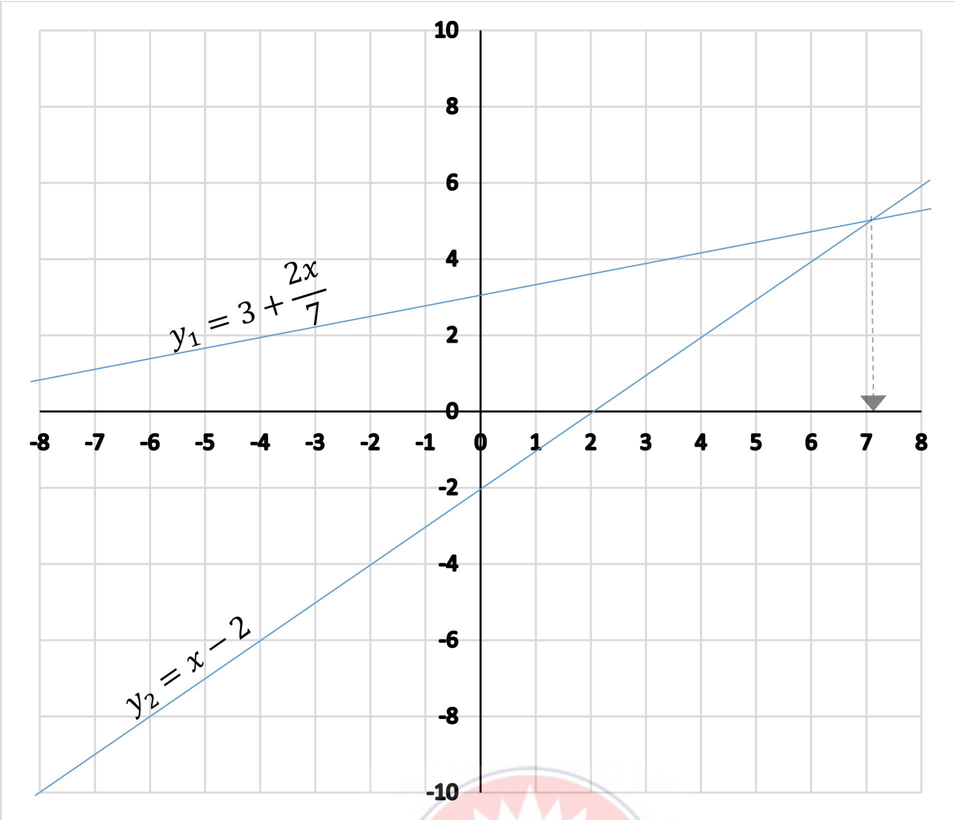
$5.3 > 6$

when $x = 6$

$3 + \frac{2(6)}{7} > 6 - 2$

$4.7 > 4$

Therefore the sign will chang, $x < 7$.



3. **Solve the double inequality $-2 < \frac{1}{5}(2x - 4) < 2$**

Solution

Let y_1, y_2, and y_3 be the new equation that is $y_1 = -2, y_2 = \frac{1}{5}(2x - 4)$ and $y_3 = 2$.

Because we have only one linear equation, let create a table for that (y_2) and an interval for all the equations.

$$y_2 = \frac{1}{5}(2x - 4)$$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
Y	-2.8	-2.4	-2	-1.6	-1.2	-0.8	-0.4	0	0.4	0.8	1.2	1.6	2	2.4

From the graph,

y_1 ,y_2 and y_3 at $x = -3$ *or* 7, then put the answer into the question to verify whether the sign will change or not. That is we will take the number less or greater the answer, substitute into the question to verify the sign

When $x = -4$

$-2 < \frac{1}{5}(2(-4) - 4) < 2$

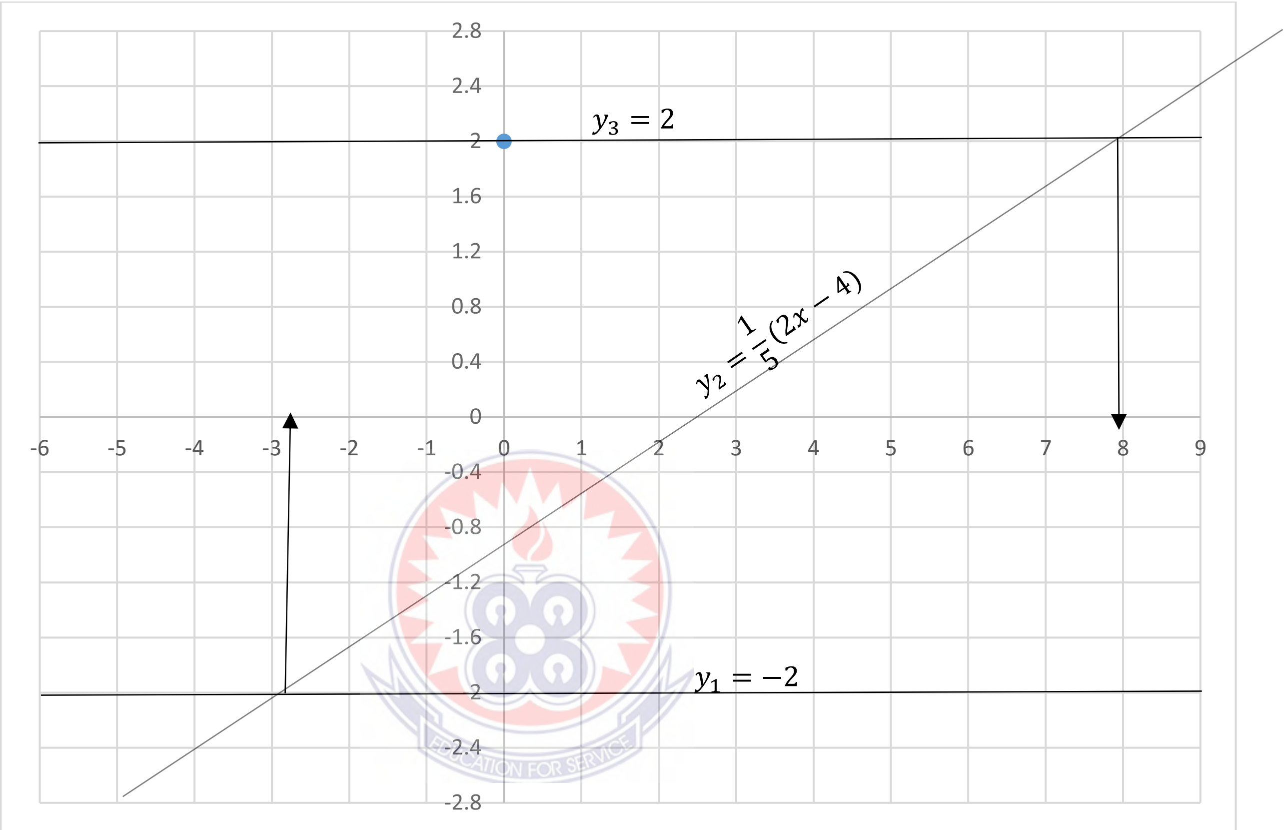
$-2 < -2.4 < 2$

when $x = 8$

$-2 < \frac{1}{5}(2(8) - 4) < 2$

$-2 < 2.4 < 2$

Therefore $-3 < x < 72$



APPENDIX F: INTERVIEW GUIDE

UNIVERSITY OF EDUCATION, WINNEBA

FACULTY OF SCIENCE EDUCATION

DEPARTMENT OF MATHEMATICS EDUCATION

1. What are your impressions about the graphical solutions approach in solving linear inequalities used in your class?
2. Did you enjoy learning linear inequalities with the graphical solutions approach?

   If **YES** why do you say you enjoyed?

   If **NO** why do you say you did not enjoy?
3. Can you tell or remember one thing you learned for the first time as a result of the use of the graphical solutions approach in your class? Or is there something you learned/understood or were able to do because of the graphical solutions approach? Explain.
4. What challenges did you face during the class when being taught using the Graphical solutions approach?

5. What are some of the disadvantages of using graphical method especially the graphical solutions approach used in the lesson?

6. Would you ever learn linear inequalities using graphical solutions approach?

If **YES** why would you learn with it?

APPENDIX G: LESSON NOTES USED FOR THE EXPERIMENTAL STUDY (FIRST WEEK)

GRAPHICAL SOLUTION TECHNIQUES FOR LESSON IN LINEAR INEQUALITIES

Lesson plan (first week)

Use of graphical solution techniques

Day 1

Duration: 2hours

Topic: linear inequalities

Sub-topic: linear inequalities

Objective: By the end of the lesson, the students will be able to

Use graphical solution technique to solve

- basic linear inequalities with the letter on one side
- basic inequalities with the letter on both side
- basic inequalities containing brackets

Relevant previous knowledge:

- students can divide numbers

INTRODUCTION

- Review students RPK on solving some question on some inequalities.

  Examples.

  1. Solve this using algebraic method $3x + 2 \geq 8$
  2. Solve this using algebraic method $5x + 7 > 3x - 9$.

DEVELOPMENT

TEACHER/LEARNER ACTIVITIES

Activity 1

Guide student to solve $3x + 2 \geq 8$ using the algebraic steps.

ACTIVITY 2

Lunch the method

Guide students through the following steps

Step 1 Rewrite the inequality as a comparison of two functions.

$y_1 < y_2$, $y_1 > y_2$, $y_1 \leq y_2$ and $y_1 \geq y_2$

Step 2 Graph the two functions on a single set of axes.

Step 3 Draw a vertical line through the point of intersection of the two graphs. Use a dotted line if equality is not included (< or>). Use a solid line if equality is included (≤ or≥).

Step 4 Mark the x values that make the inequality a true statement.

Step 5 Write the solutions in set notation

Then use graphic solution to solve the same question that is $3x + 2 \geq 8$.

CORE POINTS

Solution

$3x + 2 \geq 11$

$3x \geq 8 - 2$ (Group like term)

$\frac{3x}{3} \geq \frac{6}{3}$ (Divide each side by 3)

Therefore $x \geq 2$

Step 1

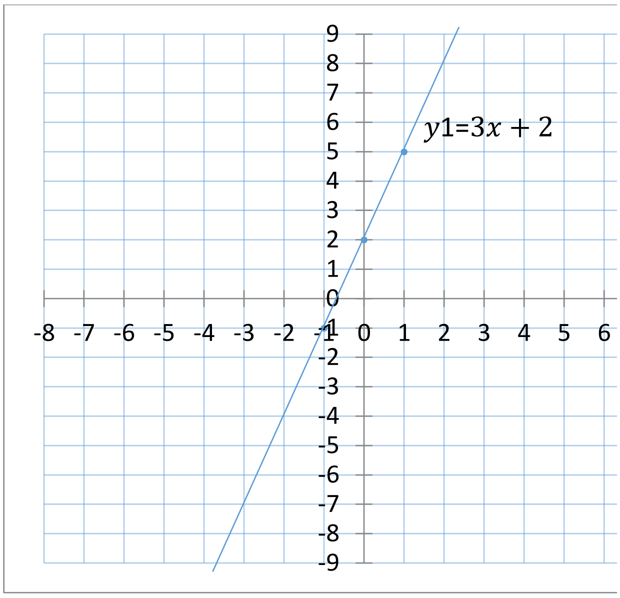
Write the inequality in two functions. That is

$y_1 = 3x + 2$ And $y_2 = 8$.

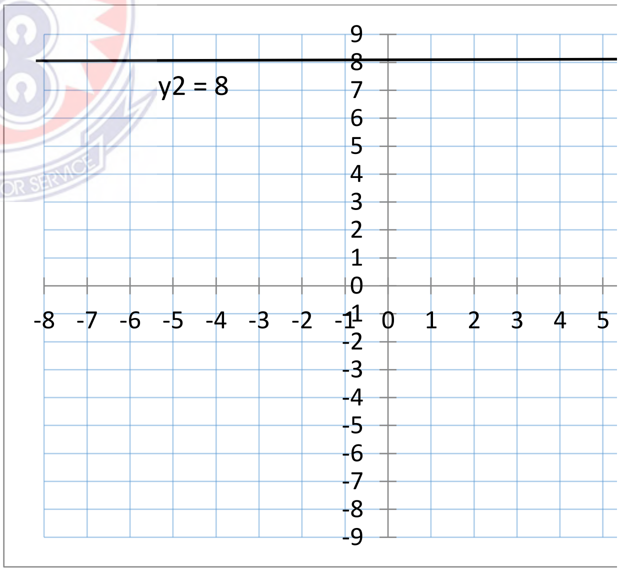
Frist $y_1 = 3x + 2$

Take some negative and positive number as the coordinates for x-axes and use it to find the coordinates of y-axes.

x	-1	0	1
y	-1	2	5



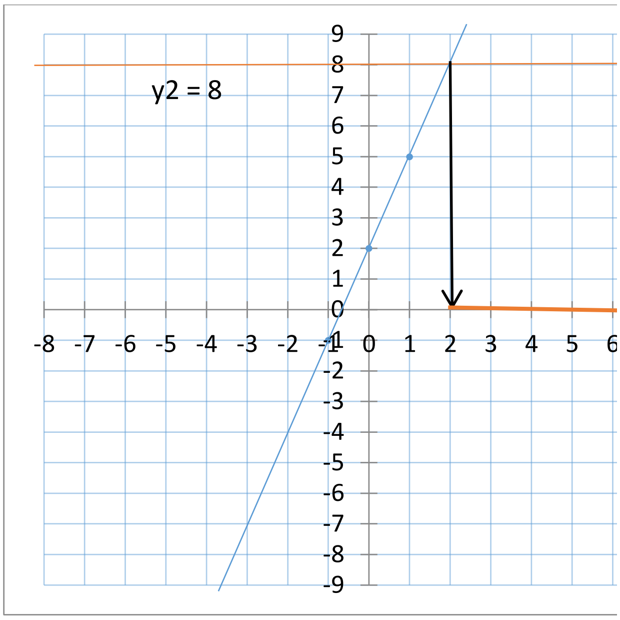
The second Graph is $y_2 = 8$ on the graph sheet



Now joint the two equations

$3x + 2 \geq 8$. You can see that the graphs appear to intersect at the point (2, 8).Use the *intersect* feature of the graphing utility to confirm this. The graph of y_1 lies above the graph ofy_2

to the left of their point of intersection, which implies that $y_1 \geq y_2$ for $x \geq 2$.



Guide students solve the next example $2x - 1 < x - 4$ using graphical solution technique.

Let rewrite the equation in two equations. That is $y_1 = 2x - 1$ and the $y_2 = x - 4$.

Frist Take some negative and positive number as the coordinates for x-axes and use it to find the coordinates of y-axes uses it to draw tables.

$y_1 = 2x - 1$

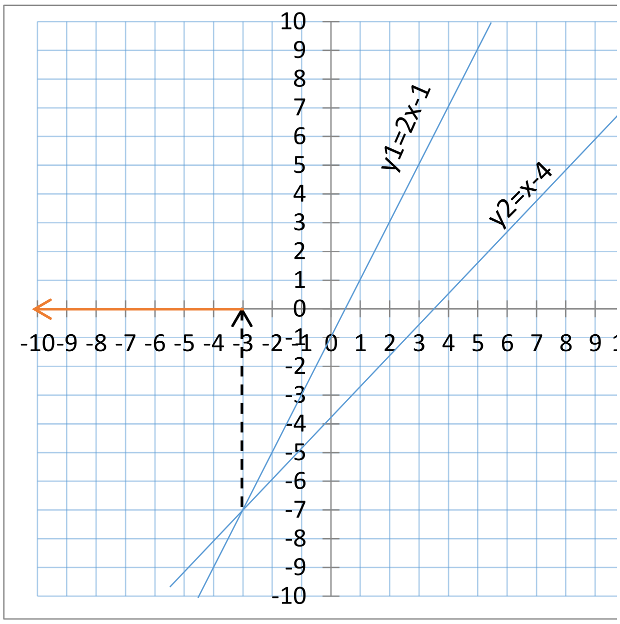
X	-1	0	1
Y	-3	-1	1

$y_2 = x - 4$

X	-1	0	1
Y	-5	-4	-3

Let's students analyze the steps for understanding of the question.

Now jointing the two equations $2x - 1 < x - 4$. You can see that the graphs appear to intersect at the point (-3, -7).Use the *intersect* feature of the graphing utility to confirm this. The graph of y_1 lies above the graph of y_2 to the left of their point of intersection, which implies that $y_1 < y_2$ for $x < -3$.



SEATWORK (in pairs)

Teacher asks students to explain their answers to the whole class after solving the problem.

Ask student to solve difference question using the graphical solution technique.

1. $4(x + 2) < 5x - 1$
2. $2x \geq 5x + 6$

Guide them also to solve using the graphical solution technique

Go round to check and guide students as they work.

CLOSURE

Homework:

Students are given homework to go and solve.

Solve the following graphically

1. $2(x-5) \geq 2x-1$
2. $5x+3 > 2(4-x)+7x$
3. $5x+1 > 10-x$
4. $7x-1 \leq 15x-15$

APPENDIX H: LESSON PLAN (SECOND WEEK)

Reference: Aki Ola series and SHS syllabus,

Duration: 2hours

Topic: linear inequalities

Sub-topic: Solving Inequalities involving fractions and double inequalities

Objective: By the end of the lesson, the students will be able to:

- Solving Inequalities involving fraction with graphical solution technique.
- Use the graphical solution technique in solving double equalities.

Relevant previous knowledge:

- Solve basic linear inequalities questions
- Students can use graphical solution technique for solving linear inequalities.

Teaching and Learning Mathematics: Graph sheets, pencil, easer and rule.

DEVELOPMENT

TEACHER/LEARNER ACTIVITIES	CORE POINT
Activity1	Solution
Teacher discusses the precious lesson with the students. Students are taking through another lesson again. Teacher explains the topic to student and solves a question using	1. $\frac{1}{3}(x+2) > \frac{1}{4}(x-1)$ Rewrite the inequalities in two form of equation. that is $y_1 = \frac{1}{3}(x+2)$ and $y_2 = \frac{1}{4}(x-1)$ than graph the two equations on the same graph sheet to meet at a point. Now take coordinates for x-axes and use it to find the y-axes for the two equations. $$y_1 = \frac{1}{3}(x+2)$$

the graphical solution technique.

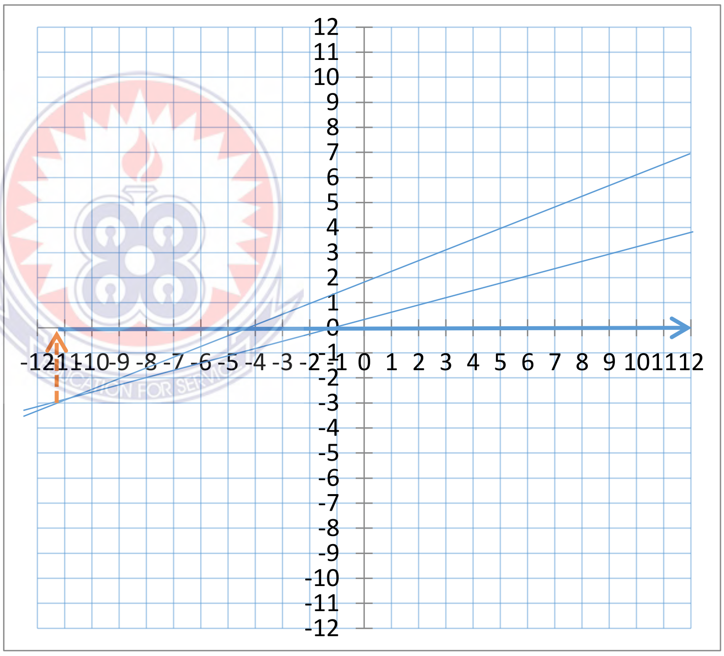
1. $\frac{1}{3}(x+2) > \frac{1}{4}(x-1)$

x	-5	0	1
y	-1	0.67	1

$$y_2 = \frac{1}{4}(x-1)$$

x	-3	0	5
y	-1	-0.25	1

Now jointing the two equations $\frac{1}{3}(x+2) > \frac{1}{4}(x-1)$. You can see that the graphs appear to intersect at the point (-11, -3).Use the *intersect* feature of the graphing utility to confirm this. The graph of y_1 lies above the graph of y_2 to the left of their point of intersection, which implies that $y_1 > y_2$ for $x > -11$.



2. $-3 \leq 6x - 1 < 3$

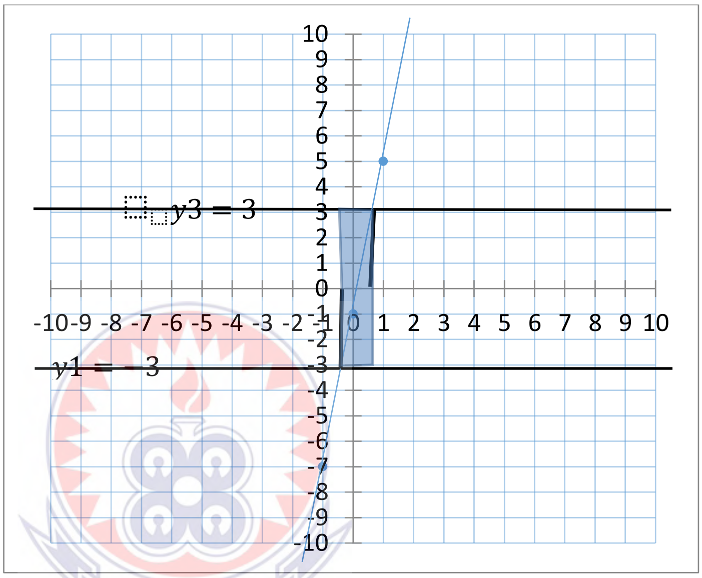
Solution

1. $-3 \leq 6x - 1 < 3$

Rewrite the inequalities in three form of equation. That is $y_1 = -3$, $y_2 = 6x - 1$ and $y_3 = 3$ then graphs the two equations on the same graph sheet to meet at a point. Now take coordinates for x-axes and use it to find the y-axes for the two equations.

Teacher help student to solve some questions using the graphical solution technique. Thus solve the following graphically

5. $-8 \leq 1 - 3(x - 2) < 13$
6. $-4 < 2x - 3 < 4$
7. $3 + \frac{2}{7}x > x - 2$



The graphs appear to intersect at the points $(\frac{-1}{3}, 3)$ *and* $(\frac{2}{3}, 3)$. Use the *intersect* feature of the graphing utility to confirm this. The graph **y₁** of lies above the graph of**y₂** to the right of the graph of **$(\frac{-1}{3}, 3)$** lies below the graph of y_3 to the left of **$(\frac{2}{3}, 3)$** This implies that $y_1 \leq y_2 < y_3$ when $\frac{-1}{3} \leq x < \frac{2}{3}$.

CLOSURE

Homework:

Students are given homework to go and solve.

CLOSURE

APPENDIX I: STRATEGIES FOR SOLVING INEQUALITIES (METHOD)

Many methods have been identified for solving linear inequalities, these methods includes Algebraic method, Cimmino's method and graphical method

Algebraic Method

The algebra is a study of procedures for solving certain kinds of problems. The problem translated into the algebraic language will be an equation of the form $5x + 10 > 40$ with a solution of $x>6$. Inequalities that have the same solution are called equivalent. There are properties of inequalities as well as there were properties of equality. All the properties below are also true for inequalities involving $\geq$ and $\leq$. The addition property of inequality says that adding the same number to each side of the inequality produces an equivalent inequality

$$\text{If } x > y, \text{ then } x + z > y + z$$

$$\text{If } x < y, \text{ then } x + z < y + z$$

The subtraction property of inequality tells us that subtracting the same number from both sides of an inequality gives an equivalent inequality.

$$\text{If } x > y, \text{ then } x - z > y - z$$

$$\text{If } x < y, \text{ then } x - z < y - z$$

The multiplication property of inequality tells us that multiplication on both sides of an inequality with a positive number produces an equivalent inequality.

$$\text{If } x > y \text{ and } z > 0, \text{ then } x\,z > yz$$

$$\text{If } x < y \text{ and } z > 0, \text{ then } xz < yz$$

Multiplication in each side of an inequality with a negative number on the other hand does not produce an equivalent inequality unless we also reverse the direction of the inequality symbol

$$\text{If } x > y \text{ and } z < 0, \text{ then } xz < yz$$

$$\text{If } x < y\text{and } z < 0, \text{ then } xz > yz$$

Division of both sides of an inequality with a positive number produces an equivalent inequality.

$$\text{If } x > y \text{ and } z>0, \text{ then } xz > yz,$$

$$\text{If } x < y \text{ and } z>0, \text{ then } xz < yz$$

And division on both sides of an inequality with a negative number produces an equivalent inequality if the inequality symbol is reversed.

$$\text{If } x > y \text{ and } z < 0, \text{ then } xz < yz,$$

$$\text{If } x < y \text{ and } z < 0, \text{ then } xz > yz$$

To solve a multi-step inequality you do as you did when solving multi-step equations. Take one thing at the time preferably beginning by isolating the variable from the constants. When solving multi-step inequalities it is important to not forget to reverse the inequality sign when multiplying or dividing with negative numbers.

To this point in this chapter we've concentrated on solving equations. It is now time to switch gears a little and start thinking about solving inequalities. Before we get into solving inequalities we should go over a couple of the basics first.

At this stage of your mathematical career it is assumed that you know that $a < b$ means that a is some number that is strictly less that b. It is also assumed that you know that $a \geq b$ means that a is some number that is either strictly bigger than b or is exactly equal to b. Likewise, it is assumed that you know how to deal with the remaining two inequalities $>$ (greater than) and $\leq$ (less than or equal to).

What we want to discuss is some notational issues and some subtleties that sometimes get students when they really start working with inequalities.

First, remember that when we say that a is less than b we mean that a is to the left of b on a number line. So, $-1000 < 0$ is a true inequality.

Next, don't forget how to correctly interpret ≤ and ≥ both of the following are true inequalities $4 \leq 4 - 6 \leq 4$. In the first case 4 is equal to 4 and so it is "less than or equal" to 4. In the second case -6 is strictly less than 4 and so it is "less than or equal" to 4. The most common mistake is to decide that the first inequality is not a true inequality. Also be careful to not take this interpretation and translate it to < and/or >. For instance, $4 < 4$ is not a true inequality since 4 is equal to 4 and not strictly less than 4.

Finally, we will be seeing many double inequalities throughout this section and later sections so we can't forget about those. The following is a double inequality $-9 < 5 \leq 6$ In a double inequality we are saying that both inequalities must be simultaneously true. In this case 5 is definitely greater than -9 and at the same time is less than or equal to 6. Therefore, this double inequality is a true inequality. On the other hand, $10 \leq 5 < 20$ is not a true inequality. While it is true that 5 is less than 20 (so the second inequality is true) it is not true that 5 is greater than or equal to 10 (so the first inequality is not true). If even one of the inequalities in a double inequality is not true then the whole inequality is not true. This point is more important than you might realize at this point.

In a later section we will run across situations where many students try to combine two inequalities into a double inequality that simply can't be combined, so be careful. The next topic that we need to discuss is the idea of interval notation. Interval notation is some very nice shorthand for inequalities and will be used extensively in the next few sections of this chapter. The best way to define interval notation is the following table. There are three columns to the table. Each row contains an inequality, a graph representing the inequality and finally the interval notation for the given inequality. Remember that a bracket, “[” or “]”, means that we include the endpoint while a parenthesis, “(” or “)”, means we don’t include the endpoint. Now, with the first four inequalities in the table the interval notation is really nothing more than the graph without the number line on it. With the final four inequalities the interval notation is almost the graph, except we need to add in an appropriate infinity to make sure we get the correct portion of the number line.

Also note that infinities NEVER get a bracket. They only get a parenthesis. We need to give one final note on interval notation before moving on to solving inequalities. Always remember that when we are writing down an interval notation for an inequality that the number on the left must be the smaller of the two. It’s now time to start thinking about solving linear inequalities. We will use the following set of facts in our solving of inequalities. Note that the facts are given for <. We can however, write down an equivalent set of facts for the remaining three inequalities. If a < b then a + c < b + c and a – c < b – c for any number c. In other words, we can add or subtract a number to both sides of the inequality and we don’t change the inequality itself. If a < b and c > 0 then ac < bc and ac < bc. So, provided c is a positive number we can multiply or divide both sides of an inequality by the number without changing the inequality. If a < b and c < 0 then ac > bc and ac > bc. In this case, unlike the previous fact, if c is negative we need to flip the direction of the inequality when we multiply or divide both sides by the inequality by c

Cimmino’s Solution Method

The problem of solving systems of linear inequalities arises in numerous fields. As examples, we can mention linear programming, image reconstruction from projections, image processing in magnetic resonance imaging, intensity-modulated radiation therapy

(IMRT). At the present time, a lot of methods for solving systems of linear inequalities are known, among which we can mark out a class of self-correcting iteration methods that allow efficient parallelization.

In this field, pioneer works are papers, in which the Agmon–Motzkin–Schoenberg relaxation method for solving systems of linear inequalities was proposed. The relaxation method belongs to the class of projection methods, which use the operation of orthogonal projection onto a hyperplane in Euclidean space. One of the first iterative algorithms of projection type was the Cimmino algorithm, intended for solving systems of linear equations and inequalities. Cimmino algorithm had a great influence on the development of computational mathematics.

A considerable number of papers have been devoted to the generalizations and extensions of the Cimmino algorithm. In many cases, systems of linear inequalities arising in the solution of practical problems can involve up to tens of millions of inequalities and up to hundreds of millions of variables. In this case, the issue of increasing scalable parallel algorithms for solving large-scale systems of linear inequalities on multiprocessor systems with dispersed memory becomes very urgent. When one creates parallel algorithms for large multiprocessor systems, it is important at an early stage of the algorithm design (before coding) to attain an analytical approximation of its scalability. For this purpose, one can use various models of parallel computation.

Nowadays, a large number of different parallel computation models are known. Each of these models generated a large family of parallel computation models, which extend and generalize the parent model. The problem of developing new parallel computation models is still important today. The reason is that it is impossible to create a parallel computation model, which is good in all respects. To create a good parallel computation model, the designer must restrict the set of target multiprocessor architectures and class of algorithms.

The parallel computation model BSF (Bulk Synchronous Farm) intended for cluster computing systems and iterative algorithms was proposed. The BSF model makes it possible to predict the upper scalability bound of an iterative algorithm with great accuracy before coding. The purpose of this article is to investigate the scalability of the Cimmino algorithm for solving large-scale systems of linear inequalities on multiprocessor systems with distributed memory by using the BSF parallel computation model. The rest of the article is organized as follows. Section 1 gives a formal description of the Cimmino algorithm.

In Section 2, the representation of the Cimmino algorithm in the form of operations on lists using higher-order functions Map and Reduce defined in the Bird–Meertens formalism is constructed. Section 3 is dedicated to an analytical investigation of the scalability of the Cimmino algorithm on lists using the BSF model cost metrics; the equations for estimating the speedup and parallel efficiency are given; the upper bound of the algorithm scalability depending on the problem size is calculated. In Section 4, a description of the implementation of the Cimmino algorithm on lists in C++ language using the BSF algorithmic skeleton and the MPI parallel programming library is presented; a comparison of the results obtained analytically and experimentally is given. In conclusion, the obtained results are summarized and directions for further research are outlined.

Euclidean inner product of a and x in Rn, bi ∈ R. To avoid triviality, we assume m > 2. We also assume that the system (1) is consistent. It is necessary to find a solution of the system of linear inequalities (1). To solve this problem, it is convenient to use a geometric language. Thus, we look upon x = (x1,...,xn) as a point in n-dimensional Euclidean space

Rn, and each inequality li (x) 6 0 as a half-space Pi. Therefore, the set of solutions of system (1) is the convex polytope M = m Ti =1 Pi. Each equation li(x) = 0 defines a hyper plane Hi:

The scalability and parallel efficiency of the iterative Cimmino algorithm used to solve large-scale linear inequality systems on multi-processor systems with distributed memory were investigated. To do this, we used the BSF (Bulk Synchronous Farm) parallel computation model based on the "master-slave" paradigm. The BSF-implementation of the Cimmino algorithm in the form of operations on lists using higher-order functions Map and Reduce is described. A scalability upper bound of the BSF-implementation of the Cimmino algorithm is obtained. This estimation tells us the following. If space dimension n is greater than or equal to the number m of inequalities, then the upper bound of the scalability of the Cimmino algorithm on lists increases in proportion to the dimension of the problem n. So, we may conclude that the Cimmino algorithm on lists is scalable well.

Graphical Method

Graphs are a common method to visually illustrate relationships in the data. The purpose of a graph is to present data that are too numerous or complicated to be described adequately in the text and in less space. When we teach our students about graphing inequalities we are preparing them to solve and graph an inequality on a coordinate plane. For example, when we teach students about an open versus closed circle, we are preparing them for a solid versus dashed line on a graph. When we teach shading to the left or right of this circle, we are preparing them for shading on either side of a line. In either graph we should be stressing that the circle and the line both act as boundaries in our solution. And a large part of the student's solution should be defining whether or not that boundary is part of the solution or not.

Finally, in this lesson when we show the student the "Point" test, we are preparing them for the same point test we would use in a linear inequality. *Note that using the Point Test, you want to pick a point other than your given point, i.e. if x > 3then we will not choose 3 as a point to test. In the examples we use 0, which is always good to use, unless your solution is n > 0.

When working with linear equations involving one whose highest degree (or order) is one, you are looking for the one value of that will make the true. But if you consider an as x + 2 < 7, then values of x can be 0, 1, 2, 3, any negative number, or any in between. In other words, there are many solutions for this inequality. Fortunately, solving inequalities involves the same strategies as solving a one equation. So even though there are an infinite number of answers to an inequality, you do not have to work any harder to find the answer.

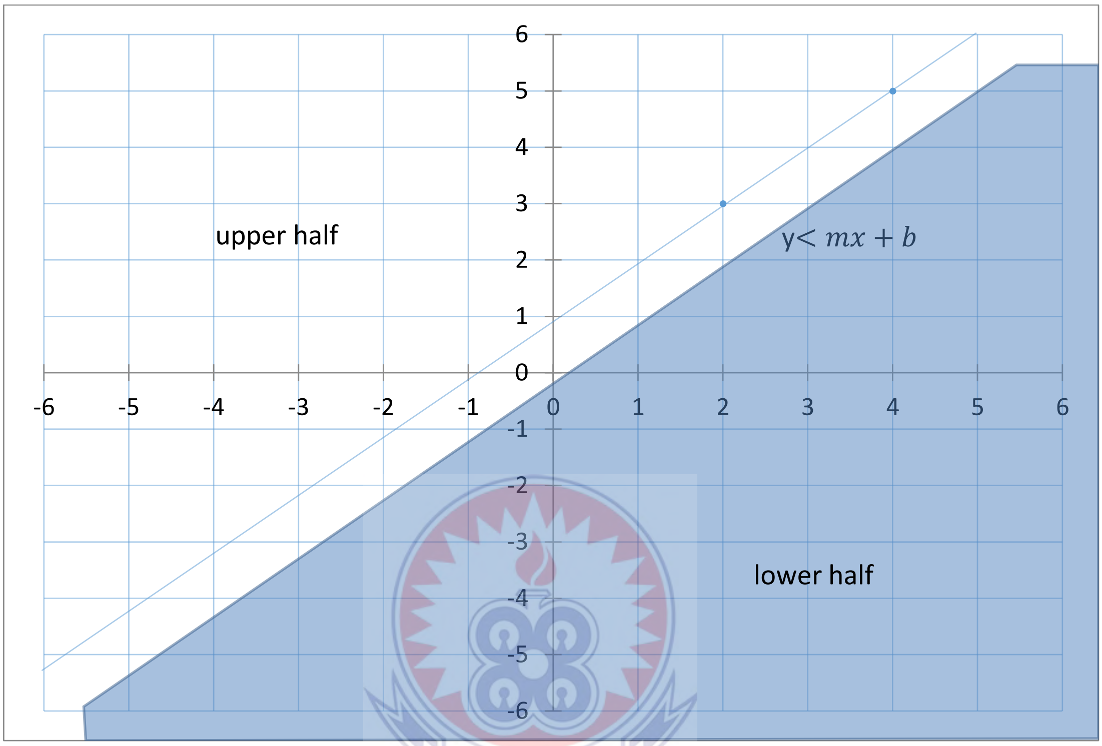
However, there is one major difference that you must keep in mind when working with any inequality. If you multiply or divide by a negative number, you must change the direction of the sign. Let's go back and look at x + 2 < 7. If this were an equation, you would only need to subtract 2 from both sides to have x by itself.

$$\begin{array}{rcl} x+2 & < & 7 \\ -2 & & -2 \\ \hline x & < & 5 \end{array}$$

Keep in mind that the new rule for inequalities only applies to multiplying or dividing by a negative number. You can still add or subtract without having to worry about the sign of the inequality.

But what would happen if you had 2(x + 1) +10 ≥ 12 ? Before solving, let's think about some values of x that will make this true. If you let x = -5 or -6 or any other value that is less than -5, then the equation will be true.

A linear inequality in two variables takes the form y > mx + b or y < mx + b. Linear are closely related to graphs of straight lines; recall that a straight line has the equation y = mx + b. When we graph a line in the coordinate plane, we can see that it divides the plane in half.



The solution to a linear inequality includes all the points in one half of the plane. We can tell which half by looking at the inequality sign:

> The solution set is the half plane above the line.

≥ The solution set is the half plane above the line and also all the points on the line.

< The solution set is the half plane below the line.

≤ The solution set is the half plane below the line and also all the points on the line.

For a strict inequality, we draw a dashed line to show that the points in the line are not part of the solution. For an inequality that includes the equals sign, we draw a solid line to show that the points on the line are part of the solution.

In the last few sections we graphed in one variable on the number line. We can also graph inequalities in one variable on the coordinate plane. We just need to remember that when we graph an equation of the type $x = a$ we get a vertical line, and when we graph an equation of the type $y = b$ we get a horizontal line.

Solving inequalities

1. Graph the inequality $x > 4$ on the coordinate plane.

First let's remember what the solution to $x > 4$ looks like on the number line.

The solution to this inequality is the set of all real numbers x that are bigger than 4, not including 4. The solution is represented by a line.

In two dimensions, the solution still consists of all the points to the right of $x = 4$, but for all possible y−values as well. This solution is represented by the half plane to the right of $x=4$. (You can think of it as being like the solution graphed on the number line, only stretched out vertically.)

The line $x = 4$ is dashed because the equals sign is not included in the inequality, meaning that points on the line are not included in the solution.

2. Graph the inequality $|y| < 5$

The absolute value inequality $|y| < 5$ can be re-written as $-5 < y < 5$. This is a compound inequality which can be expressed as $y > -5$ and $y < 5$

In other words, the solution is all the coordinate points for which the value of y is larger than -5 and smaller than 5. The solution is represented by the plane between the horizontal lines $y = -5$ and $y = 5$.