UNIVERSITY OF EDUCATION, WINNEBA

THE EFFECT OF THINK-PAIR-SHARE ON STUDENTS PERFORMANCE ON CIRCLE THEOREM-PLANE GEOMETRY II: THE CASE OF ASANKRANGWA SENIOR HIGH SCHOOL

SIMON NII BORTEY APPIAH

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DECLARATION

STUDENT'S DECLARATION

I, Simon Nii Bortey Appiah, declare that this thesis, with the exception of quotations and references contained in the published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

SIGNATURE:……………………………………………...

DATE:……………………………………..……………...

SUPERVISOR'S DECLARATION

I, hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of thesis/dissertation/project as laid down by the University of Education, Winneba.

NAME OF SUPERVISOR: DR PETER AKAYUURE

SIGNATURE:…………………………………………

DATE:…………………………………………………

DEDICATION

To my Father, Mr. Kwame Appiah and my Mother, Comfort Appiah

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I thank the Almighty God for the gift of life in good health and financial stability which aided me to complete my work successfully.

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LIST OF ACRONYMS

GES	Ghana Education Service
IBE-UNESCO	International Bureau of Education-United Nations Educational, Scientist, and Cultural Organization
ICT	Information and Communication Technology
MMDA	Metropolitan, Municipal and District Assembly
MoE	Ministry of Education
RME	Realistic Mathematics Education
SEIP	Secondary Education Improvement Project
SHS	Senior High School
WAEC	West Africa Examination Council
WASSSCE	West African Senior Secondary School Certificate Examination

ABSTRACT

The aim of the study was to determine the effect of using think-pair-share teaching approach as an instructional tool in the teaching and learning of circle theorem-plane geometry II on the performance of Senior High School students. The study employed quasi experimental design, using non-equivalent control pre-test post-test design. Purposive sampling technique was used to select two intact classes for the study, one intact class was used as control group and the other intact class as the experimental group. The sample size consisted 24 participants in the control group and 24 participants in the experimental group. The experimental group was taught circle theorem-plane geometry II using think-pair-share teaching approach while the control group was taught circle theorem-plane geometry II using the traditional teaching approach. Pre-test and post-test were carried out concurrently on the groups using teacher-made achievement test. The achievement test was based on Ghana Education Service syllabus. Independent sample t-test and paired sample t-test were used to analyze scores of the teacher-made achievement test. The finding showed that there is a statistically significant positive effect for the participants who employed think-pair-share to learn circle theorem-plane geometry II. Thus, the participants taught with think-pair-share teaching approach performed better than their counterpart who did not use the think-pair-share approach to learn circle theorem-plane geometry II. Also, the effective use of think-pair-share teaching approach made teaching and learning of circle theorem-plane geometry II very enthralling, intriguing, thought-provoking and easy to understand. Therefore, the use of think-pair-share approach should be encouraged and employed by mathematics teachers and other teachers as it enhances students' performance.

CHAPTER 1

INTRODUCTION

1.0 Overview

This chapter contains the background to the study which establishes the context of the study and explains why the study is important. In the statement of the problem, the study gives concise description of the problem that the study seeks to address and identifies the current state. The purpose of the study that states the ultimate goal of the study and the overall direction or focus of the study is also presented. The significance of the study that elaborates how the study is beneficial to the development of students, teachers, Ministry of Education, the Ghana Education Service and science society in general is discussed. The delimitation which spells out the characteristics, the limit, and the scope that describes the boundaries of the study is described. The limitations of the study are the characteristics of the research design or methodology that impacted the interpretation of the findings from the study are stated. Finally the organization of the study that provides a map to guide readers reading and understanding of the study is presented

1.1 Background to the study

Geometry as one of the classical disciplines of mathematics permeates many fields of study such as ship navigation, architecture, telescope making, driving, photography, graphic designing, astronomy and astrology where concepts of geometry are applied. This is why geometry is suggested to be among the crucial branches of mathematics for national development if its concept is understood. A nations' growth and development and the quality of life of its people depend on the in-depth knowledge in mathematics and the concept of geometry. This is because Architects, Cartographers, Photogrammetrists, Drafters, Mechanical Engineers, Surveyors, Urban and Regional planner depends solidly on mathematics and geometry. For that matter, the progress and improvement of geometry concept are linked to the prosperity of the state. This tells why geometry is highly esteemed. For this reason, among the thirty (30) topical units in the core mathematics syllabus for SHS, ten (10) are geometry topics. Again, the Secondary Education Improvement Project (SEIP) module designed for SEIP beneficiary Senior High Schools (SHS) which is made up of 11 modules has six (6) of them are in Geometry. This shows how important Geometry is in the national mathematics curriculum and given priority to nations' development agenda. In Ghana, a student who fails the Core Mathematics paper that contains 17% of geometrical concepts at the Basic Education Certificate Examination (BECE) or at the West African Secondary School Certificate Examination (WASSCE) cannot progress to the next level of his/her education.

The current Core Mathematics curriculum is based on a student-centered approach. The aim of this student-centered curriculum is to assist students build their individual mathematical concepts by their understandings and intuitions and describe abstract and concrete structure of Mathematics by utilizing their understanding (Ministry of Education, 2012). The mathematics curriculum is essentially based on the principles of constructivism and mathematics teachers are to create a conducive environments that would enable students to actively explore different problem strategies in learning Mathematics (Nabie, Raheem, Agbemaka, & Sabtiwu, 2016). Literature attests that teachers who utilize innovative approaches of teaching geometry, such as the Think-pair-share interactive teaching approach are likely to build high confidence in interactive pedagogical strategies and focus their lessons to improve students' performance in Mathematics (Sitorus & Masrayati 2016). The interactive modes of teaching enable students' to formulate individual ideas and share these ideas with another student. Nabie (2013) reported in his book 'Understanding Primary Mathematics Methods for Teaching' that

> interactive approach of teaching mathematics changes the role of the teacher as an 'expert' to a teacher as a learner; from a reproductive thinker to an autonomous thinker; from reproducing knowledge to creating and discovering knowledge; from passive recipient to active decision maker; from convergent and rule abiding to divergent and stepping outside rules to create original ideas; from one right answer to multiple solutions; from the conception of mistakes as flaws to mistakes as learning devices; from external evaluation and direction to self-evaluation and self-direction; and from individualism and competition to collaboration. (p. 158-*159)*.

Also, interactive learning is a hands-on, real-world approach to education. According to Standford University School of Medicine (2012), interaction learning actively engages the students in wresting with the material. It reinvigorates the classroom for both students and faculty. Lectures are changed into discussions, and students and teachers become partners in the journey of knowledge acquisition (Carl, 2020). For instance, IBE-UNESCO (2008) admonishes the practice of involving learners in the educational process by encouraging them to bring their own experience and knowledge into the process, while contributing to defining or organizing their learning. Also, the Mathematics Association of Ghana (2001) had earlier recommended the significance of using interactive approach in teaching to enhance students' achievement and learning of mathematics.

In view of this, the Secondary Education Improvement Project (SEIP) made a remarkable step towards the use of interactive approach to teaching and learning geometry which is believed to be an effective instructional approach to enhancing Senior High School students' understanding of geometry (SEIP, 2015). SEIP is of the view that teaching and learning geometry through the interactive approach will enhance students' understanding of geometry.

Governments of both developed and developing countries have recognized, as a matter of urgency, the role of interactive teaching approach in redefining their democratic activities. This, for that reason, calls for the integration of interactive strategies into the teaching and learning all over the world. The importance of the integration of interactive teaching strategies into education is recognized as providing opportunities for developing skills that have the potency to change pedagogical practices and to reform curricula (MAG, 2001)

However, despite the positive impact of innovative approaches of teaching on student achievement and strong advocacy for using interactive strategies in the teaching and learning of geometry, classrooms in Ghana are still characterized by traditional method of teaching (Tay & Wonkyi, 2018). The traditional approach is characterized by lectures/oral exposition that is more teacher-centered rather than learner-centered.

Lim and Hwa (2007) indicated that teaching and learning Mathematics in schools is laden with traditional approach of teaching and textbook-oriented method where learners have a tendency to memorize mathematical formulae and laws without understanding the concepts. This teaching approach has resulted in general detest for Mathematics by students and poor performance in Mathematics in local (WASSCE) (Abreh, Owusu & Amedahe, 2018) and international examinations (Trends in

International Mathematics and Science Study, TIMSS) (Fredia-Kwarteng 2005; Appiah 2010). Thus, national and international reports show that Ghanaian students perform poorly in higher order thinking problems. The National Education Assessment (NEA) report shows that the mean score of Mathematics for P3 and P6 were 41.8% and 39.6% respectively (Ministry of Education, 2009). The 2003 Trends in International Mathematics Science Study (TIMSS) report by Anamuah-Mensah, Mereku & Asabere-Ameyaw (2004) indicates that Ghanaian students scored zero in advance and higher-level thinking in the content domains tested. Unfortunately, geometry was one of the topical areas in which candidates' performance was weak.

Johnston-Wilder and Mason (2005), blamed students' lack of interest and understanding of Geometry on teachers' poor teaching skills and lack of resources for presenting geometrical shapes to students. They argue that the ordinary primary tutor has an anxiety of the very word 'geometry.' This is why David, Tahta & Brookes (1979), are of the view that it is difficult to encourage any form of geometry to be taught at all in primary schools, and some books for primary teachers devote little time or space to it.

Pupils who proceed to the senior high school, therefore, have very weak foundation in Geometry. Perhaps, one of the reasons why so much time is spent on arithmetic than on Geometry in the primary school is that skills and techniques in arithmetic are very much more in evidence.

Furthermore, Battista (2007) argues that teaching geometry for students to learn meaningfully requires an understanding of how students construct their knowledge of various geometric topics. School geometry is commonly regarded as a key topic within which to teach mathematical argumentation and proofs and/or to develop students' deductive reasoning and creative thinking. Although, deductive reasoning and proof are central to making progress in Mathematics, it remains the case that students at the senior high school have great difficulty in constructing and understanding theorems in Geometry.

Gender difference is no exception to the factors contributing to students' difficulties or low achievement in Mathematics. Gender difference in Mathematics achievement and ability has remained a source of concern as researchers seek to address the under-representation of women at the highest levels of Mathematics (Asante, 2010). This difference could occur as a result of the methods a teacher uses in the Mathematics classroom.

With the dominance of traditional methods in Mathematics instruction in Ghana coupled with students' learning difficulty in geometry, one probable approach for enhancing instruction and student learning could be implementing realistic interactive instructional method such as the use of think-pair-share.

Think-pair-share is a collaborative and interactive teaching strategy first proposed by Frank Lyman of University of Maryland in 1981 (Kaddoura, 2013). Think-pair-share is a teaching and learning strategy where students work together to solve a problem or answer a question about an assigned reading. This strategy requires students to think individually about a topic or answer to a question and share ideas with classmates.

Of all the interactive teaching approaches, think-pair-share has been described as more effective over others since discussing a problem with a partner maximizes participation, focuses attention and engages students in comprehending the assigned material (Lom, 2012). The think-pair-share interactive teaching strategy has made it possible for students to think individually about certain concept of geometry. It has taught students to share their ideas on concept of geometry with their classmates and builds oral communication skills (Afthina, Mardiyana & Pramudya, 2017).

While it is promising to see that several previous studies have demonstrated positive effects of think-pair-share interactive instructional approach lessons on students' achievement, literature available indicates that many of these studies are not centered on think-pair-share interactive instructional approach in teaching and learning of geometry, particularly in Ghana. Therefore, it is against this background that the study is conducted to determine the extent to which the use of think-pair-share as an interactive instructional strategy will enhance students' understanding of geometry.

1.2 Statement of the Problem

Students of Asankrangwa Senior High School (SHS) over a decade have registered abysmal performance in core mathematics in the West African Senior Secondary School Certificate Examination (WASSSCE). The Ghana Education Service (GES) classified Asankrangwa SHS as one of the Secondary Education Improvement Project (SEIP) schools (SEIP, 2015). The main objective of SEIP is to increase SHS Science and Mathematics teachers' content and pedagogical knowledge to enable effective teaching and learning in the schools. During an interaction with the Director who designed the SEIP module, Prof. Mereku, he revealed that during the survey of SEIP schools', mathematics teachers mentioned that they needed another approach in teaching geometry since it dominates the topics in the WASSSCE and the SEIP module.

The under listed topics were selected from a survey conducted in the 125 SHS benefiting from SEIP six (6) modules were geometry related. Module 1 (Geometry); Module 2 (Mensuration); Module 4 (Bearings and Vectors); Module 5 (Trigonometry);

Module 7 (Circle Theorem-Plane Geometry II) and Module 9 (Construction). The Ghana Education Service is hopeful that if these topics are taught with the appropriate teaching methods, it will enhance students' performance in the West African Senior Secondary School Certificate Examination (WASSSCE) (SEIP, 2015.). Out of the eleven (11) modules, six (6) were geometry related topics. It therefore implies that the SEIP recognize the challenges mathematics teachers face in teaching geometry topics. This was evidenced in the Science and Mathematics training workshop for SEIP SHSs teachers held in Kumasi and Accra from 20th September to 3rd October, 2015. During the period, mathematics teachers who teach in SEIP schools were actually taught all the geometry related topics first. Facilitators used the cooperative teaching and learning strategy (think-pair-share) during the workshop and challenged all participants to move away from the traditional way of teaching geometry to employ modern and innovative methods of teaching to allow students to process new information and, through discussion and peer to peer interaction and assign meaning to what they learnt. Students' will again develop problem-solving skills, conjecturing, deductive reasoning, intuition, visualization, perspective, logical argument and proof students' problems.

1.3 Purpose of the study

The purpose of the study was to determine the effect of using think-pair-share interactions as an instructional tool in the teaching and learning of circle theorems under Plane Geometry II of Senior High School syllabus on students' performance.

1.4 Research objectives

The study seeks to:

1. Identify Asankrangwa SHS form 2 students' difficulties in solving problems in circle theorem-plane geometry II.

2. Determine the extent to which think-pair-share will address Asankrangwa SHS 2 students' difficulty in solving circle theorem-plane geometry II problems.
3. Ascertain the difference in performance of female and male students' taught circle theorem-plane geometry II using think-pair-share.

1.5 Research questions

The study is aimed at answering the following questions:

1. What are Asankrangwa Senior High School Form 2 students' difficulties in solving problems in circle theorem-plane geometry II?
2. To what extent will the use of think-pair-share address Asankrangwa Senior High School Form 2 students' difficulty in solving problems in circle theorem-plane geometry II?
3. Is there any significant difference in the performance of female and male students' taught circle theorem-plane geometry II using think-pair-share?

1.6 Hypothesis

To determine the effect of using think-pair-share or traditional method as instructional tools in the teaching of Circle Theorem-Plane Geometry II on the performance of female and male students, the hypothesis below is formulated.

Ho: there is no significant difference in the performance of female and male students' taught circle theorem-plane geometry II using think-pair-share.

1.7 Significance of the study

The findings of this study will be a resource material for policy makers, agencies of quality education, Secondary Education Improvement Project (SEIP), teachers and other stakeholders as to whether utilization of think-pair-share in the teaching of circle theorem-plane geometry II actually enhance students' conceptual understanding of the above topic. It will also generate relevant information that could inform curriculum developers on the ways to design the curriculum by integrating think-pair-share into the teaching and learning of Mathematics due to its impact in the study. Students would benefit from this study since they would be able to think critically and have problem solving skills and use it to achieve better understanding of mathematical concepts. It would benefit senior high school mathematics teachers by giving them insight on how think-pair-share interactive strategy can be used to enhance their teaching.

Again the findings of the study will provide information on how the use of think-pair-share as an interactive instructional strategy motivate and engage students to participate in the teaching and learning process since they see themselves as partners with the teachers to share ideas. The study will also serve as a source of information for scholars and researchers who want to embark on similar study. Finally, the outcomes and recommendations of this study will create the much needed awareness and attention among Mathematics teachers to enhance their teaching pedagogy and make Mathematics lively, loving and interesting to students at all level.

1.8 Delimitations of the study

One public senior high school in the Amenfi West Municipal and two intact classes in the Second-year were used for the study. The main rationale for using the Second-year students in the study is that the topic, circle theorem-plane geometry II is within the scope of the Second-Year Mathematics Curriculum. Again, the study was confined to properties of circle and theorems of circle. From a total of 125 Senior High Schools benefiting from Secondary Education Improvement Project nationwide, only 1 school was purposely selected for this study which clearly limited the scope of the current research. This implies that generalization cannot be extended beyond the school where the study took place, but schools with similar features. By this limitation, the study by no means claims to be conclusive. It would rather serve as an eye opener for future researchers on related studies

1.9 Limitations of the study

The study had limitations of all quantitative type research such as clarity of teacher-made achievement test designed by the researcher and respondent understanding of some terminologies. The limited time available in the midst of covid-19 pandemic collecting data may prove to be a challenge to conducting a detailed and thorough research. By these limitations, the study by no means claims to be conclusive. It would rather serve as an eye opener to study the use of think-pair-share to enhance reduce drastically students' difficulty in solving circle theorem-plane geometry II problems.

1.10 Organization of the Rest of the study

The rest of the study is organized as follows. Chapter two deals with the review of related literature on the theoretical framework of the study, interactive method of teaching geometry, think-pair-share strategy in teaching Geometry, effect of using think-pair-share in teaching and learning Mathematics, nature of think-pair-share, geometry, difficulty students face in solving application questions involving geometry and summary of the findings from the literature review. Chapter three is concerned with methodology for the study and focuses on research design, population, sample and sampling procedures and data analysis. Chapter four presents the results of the findings of the study. Chapter five is the final chapter of the study. It gives the summary of the study and draws conclusions on the key findings of the study. It outlines recommendations from the study and suggests areas for further research.

CHAPTER 2

REVIEW OF RELATED LITERATURE

2.0 Overview

The study is about the effect of Think-Pair-Share on students' performance on Circle-Theorem-Plane Geometry II. The researcher seek to identifying students difficulties in solving problems in circle theorem-plane geometry II. Again, the researcher seek to determining the extent to which think-pair-share will address students difficulty in solving circle theorem-plane geometry II problem and finally, exploring to ascertain the difference in performance of female and male students' taught circle theorem-plane geometry II using think-pair-share. The researcher intends to do know the effect of think-pair-share on students' performance on circle theorem – plane geometry II by reviewing literature under the following sub-headings.

i. Theoretical Framework of the study
ii. Effect of using think-pair-share strategy in teaching and learning mathematics
iii. Gender, mathematics and think-pair-share
iv. Think-pair-share strategy in teaching and learning
v. Traditional method of teaching and learning mathematics
vi. Students' difficulties in solving problems on circle theorem-plane geometry II
vii. Potential for using think-pair-share within Mathematics Education

2.1 Theoretical Framework

The theoretical framework is a collection of theories that support a research (Ofori & Dampson, 2011). Therefore, the theory supporting this study is social constructivist's theory. In social constructivist learning theory, the learner is part of a cooperative group assigned a task to read, interpret and translate their act under the guidance of the teacher (facilitator) to create a shared understanding of their assigned act and use that shared understanding as a basis for their construction of the modern-day learning (McMahon, 1997).

Unlike traditional teaching approaches where students learn by memorizing whatever teachers say, the social constructivist give students in their cooperative groups the opportunity to discuss and brainstorm to share ideas with teachers and other group to bring out new ideas on board for general class discussion and by these learners' ideas are recognized and improved through various instruction methods that keenly involve them. Also, social constructivism permits discussion and interactive discourse among students since they afford students the opportunity to use language as a demonstration of their independent thoughts. In this case, discussion and the interactive discourse elicit sustained responses from that encourage meaning-making through negotiating with the ideas of other students. This type of learning promotes retention and in-depth processing associated with the cognitive manipulation of information (Nystrand, 1996).

Again, while learning plane geometry, students are expected to be able to state and use the circle theorems, identify the tangent as perpendicular to the radius at the point of contact, verify that the angle between the tangent and the chord at the point of contact is equal to the angle in the alternate segment and finally, verify that tangents drawn from an external point to the same circle are equal, when measured from their point of contact. This knowledge of social constructivism help students construct knowledge through interaction with others with the guidance of the facilitator. The social constructivist profess large and small group discussion afford students opportunities to exercise self-regulation, self-determination, and a desire to persevere with tasks (Corden, 2001).

To further support the rationale for employing this theoretical framework in this study, Li & Lam (2005) remarks that think-pair-share supports social constructivist approach, lesson is student-centered, instructor-facilitated instructional strategy in which a small group of students is responsible for its own learning and the learning of all group members. Students interact with each other in the same group to acquire and practice the elements of a subject matter in order to solve a problem, complete a task or achieve a goal. This implies that think-pair-share permits social constructivist instructors of Mathematics to plan learning atmospheres that raise curiosity and stimulate experientially-based understanding attained by means of interaction and discussion in the classroom. It is against these above-mentioned reasons that this study seeks to employ the theory of social constructivism to determine the extent the use of think-pair-share will enhance reduce drastically students' difficulty in solving circle theorem-plane geometry II problems.

2.2 Effect of Using Think-pair-share Strategy in Teaching and Learning Mathematics

There are number of studies that have shown that using Think-pair-share is a better way of teaching and learning Mathematics.

Miratika, Asmin, Mulyono & Minarni (2018) conducted a study on the topic "The effect of cooperation learning of type think-pair-share based on Mandailing Culture to Mathematical problem solving ability of the students at MSS Ali Imron

Medan." They employed pre-test and post-test quasi experimental design with 32 students divided into experimental and control groups. They employed analysis of variance (ANOVA) to determine the statistical difference in students' mathematical problem solving abilities taught with cooperative model learning type think pair share oriented to Mandailing culture and students taught with ordinary learning (conventional approach) based on the IMA that students get. They found out from their study that think-pair-share enhanced the problem solving and mathematics learning outcomes of students. Think-pair-share as an interactive and cooperative teaching and learning strategy improve mathematical problem solving skills if and only if the steps in the learning process are fully implemented.

It is obvious from Miratika, Asmin, Mulyono & Minarni's (2018) study that when Think-pair-share is fully utilized in classroom teaching and learning process, it will enhance better teaching and learning. The ability of think-pair-share to increase students' mathematics problem solving ability is an indication that such an interactive and cooperative strategy can be used to enhance students' conceptual understanding of geometry.

Akanmu (2019) used 2X2X3 pretest, posttest, quasi experimental, non-equivalent and non-randomized control group design to survey "the effects of think-pair-share on senior school students' performance in mathematics in Ilorin, Nigeria." (p. 109). Among 118 SS 2 students divided into an experimental and a control group. The experimental group was taught using think-pair-share strategy while students in the control group received tuition through traditional teaching approach. There was a statistical significance difference in the performance of students taught using think-pair-share compared with their counterpart in control group and also determine a statistically significant difference in the knowledge retained by students taught set theory in mathematics using think-pair-share.

Akanmu (2019) observed in his work difference in the mean achievement scores of students taught with Think-pair-share strategy and that of students taught with traditional method. It revealed statistically significance difference in the performance of students taught set theory using think-pair-share strategy. It also found statistically significant difference in the knowledge retained by students taught set theory in mathematics using think-pair-share compared with their counterparts in the control group in favor of think-pair-share.

According to Akanmu (2019), the use of think-pair-share improved students' performance in mathematics and also it improved the retention ability of the students.

It is clear from the study of Akanmu (2019) that teaching and learning set theory with Think-pair-share strategy helped improved students' performance and their retention ability.

Choirul, Siti & Raden (2018) conducted an experimental research model using static group comparison design to examine the effect of Think-pair-share learning with contextual approach on 58 junior high school students' mathematics problem solving ability. Choirul, Siti & Raden (2018) used t-statistic test to test to determine the statistical difference that exists between the two groups. They observed that both groups were at different learning level. The experimental group was more successful than the control group- students' problem-solving ability in the experimental group was better than the control group. This shows that Think-pair-share learning with contextual approach has an effect on students' problem solving ability. It is reasonable to think that Think-pair-share learning with contextual approach can be used as a reference to conduct learning in mathematics class as well as to train students' problem solving ability.

In a study conducted by Yarisda (2019) on the topic "the use of cooperative learning models of Think-pair-share in mathematics learning." Yarisda employed the randomized control group only design model and Lilliefors test of normality with sample size of 71 students. It was realized that the mathematics learning outcomes of students using cooperative learning model of think-pair-share are better than the mathematics learning outcomes of students who use conventional learning in the XI science class MAN 2 Padang. According to Arends (2004), the Think-pair-share type of cooperative learning model is structured and in its implementation strongly emphasizes cooperation between students in solving problems independently (Think), pairing (Pair), then presentation in front of the class (Share). Think-pair-share provide an opportunity for students to explore the ability of cooperation with other people, both their desk mates and classmates, expressing opinion or responding other students' opinions.

It is clear with respect to the study of Yarisda (2019) that the use of cooperative learning models Think-pair-share in mathematics learning will almost always have positive outcome if implemented correctly than the conventional approaches.

A quasi-experimental control group pre-test post-test design study conducted by Bertha & Athanasius (2019) examined the effect of Think-pair-share cooperative learning model on grade 12 learners' performance in quadratic functions in Twashika Secondary School in Luanshya. Two classes were randomly assigned to experimental group and control group consisted of 42 students, 19 students in the experimental group and 23 in the control group. F-test, T-test and descriptive statistics were used to determine the statistical difference that existed between the groups. They observed that both groups were at different learning level. Results from analysis indicated by Bertha and Athanasius showed statistically significant difference existed between the post-test scores of both experimental and control groups.

As indicated by Bertha and Athanasius, the effectiveness of Think-pair-share cooperative learning model on learners' performance, it is very appropriate for mathematics teachers to employ the Think-pair-share model of cooperative learning in order to improve both learners' performance and attitudes towards quadratic functions.

A study conducted by Afthina, Mardiyana & Pramudya (2017), on the topic "think pair share using realistic mathematics education approach in geometry learning." The quasi experimental study was aimed to determine whether the impact of mathematics learning applying Think-pair-share using realistic mathematics education viewed from mathematical-logical intelligence in geometry learning.

The study findings of Afthina, Mardiyana & Pramudya (2017), showed mathematics achievement applying Think-pair-share using realistic mathematics education approach gives a better result than those applying direct learning model. Therefore, learning process which employs the full application of Think-pair-share learning strategy will give positive influence towards students to comprehend given materials.

2.3 Gender, Mathematics and Think-pair-share

Globally, researchers (Preckel et al., 2008; Else-Quest, Hyde & Linn, 2010; Frenzel, Goetz, Pekrun, and Helen Watt, 2010; Ganley & Lubienski, 2016) have launched studies in few settings to explore and examine factors that affect gender achievement in Mathematics. Some of the research on performance in mathematics has highlighted a traditional gender gap in favour of boys (Aunola, Leskinen, Lerkkanen & Nurmi, 2004; Githua & Mwangi, 2003). Other researchers (Lindberg, Hyde, Petersen,

& Linn, 2010) have concluded that the gender gaps in mathematics are insignificant. Despite this, Brown & Kanyongo (2010) showed that girls have obtained slightly better grades in mathematics over the last four decades than boys. These findings were also supported by the findings of Robinson & Lubienski (2013).

A multifarious number of such studies have identified from their studies different factors that has contributed gender gap between boys and girls. According to Eccles & Roeser (2011) findings support findings of other researchers who perceive that traditionally, girls' lower performance in mathematics is contributed by both internal and external contextual factors- for example lower perceived support for learning mathematics. Other researchers like Riegle-Crumb, Farkas, & Muller (2006), attributed girls' drop in performance to their mathematics feelings that their classrooms were unattractive, uncomfortable and hostile. Such results concerning mathematics are supported by general findings indicating that teacher and peer support are positively connected to academic attitudes, achievement, emotions, learning, motivation and self-efficacy (Danielsen, Wiium, Wilhelmsen, & Wold, 2010; Eccles & Roeser, 2011).

According to Gherasim, Butnaru, & Mairean (2013), gender effect variables as achievement goals, classroom environments and achievement in mathematics among young adolescents showing that girls obtained higher grades in mathematics than boys. Girls reported (a) higher classroom support, lower performance-avoidance goals (Shim, Ryan, & Anderson, 2008) and (b) more mastery of the learning materials (Pekrun, Elliot, & Maier, 2006). Another important aspect found by researchers was teaching practice, especially the behaviour of the teacher, such as (a) being responsive and helpful (Patrick, Ryan, & Kaplan, 2007; Puklek Levpuscek & Zupancic, 2009) (b) being supportive (Ahmed, Minnaert, van der Werf, & Kuyper, 2010). Yet another aspect, students' attitudes, was studied by Jones & Young (1995), who found that boys had more favourable attitudes towards mathematics and science than girls. Emotions towards mathematics were studied by Frenzel, Pekrun, & Goetz (2007) who found that girls experienced less enjoyment and pride than boys. Boys, on the other hand, experienced less anxiety and less hopelessness towards mathematics than girls. They also found that girls felt slightly more shame than boys (Frenzel et al., 2007).

In a study executed by Yaw (2013), in the Tamale Metropolis in Northern Ghana on the topic "Gender differential in academic performance in mathematics among senior high school final year students, it was revealed that male students significantly performed better than female students at the metropolis. The findings of Yaw are similar to the findings of a study conducted by Eric (2001), aimed at examining mathematics achievement of boys and girls in primary schools in the Central region of Ghana. In his study which consisted of 450 pupils from 5 randomly selected schools among classes 3, 4 and 6, findings indicated general poor performance by both sexes in each of the classes but identified significant difference in achievement in favour of boys in only class 6.

Helena, Eric & Daniel (2018) also support the assertion that boys perform better than girls in mathematics base on their investigation in the Brong-Ahafo region of Ghana which was aimed at examining gender differences in performance in mathematic among pre-service teachers. Their descriptive survey showed a significant difference in performance in favor of boys after analysis.

Ato & Adelaide (2015), reported similar findings "experimental group differ significantly on the post-test scores from the control group." The study which employed quasi-experimental design to examine gender differences and mathematics achievement of the Ghana National College science classes who were randomly selected for the study had 42 and 40 students in the experimental group and control group respectively. The independent sample t-test and paired sample t-test were used to find the mean differences between the groups.

Asante (2010), referring to Collins, Kenway & McLeod (2000), contends that schools build up typical resistances amongst male and female students through gendering of information and characterizing of specific subjects for male. Interestingly, female students are adapted in the general public to trust that mathematics is a male subject, and it is worthy for them to drop it. Again, Bassey, Joshua, & Asim (2009) contended that the nature of teaching and learning mathematics entrenches male predominance over the female.

In a study conducted by John & Benjamin (2015), on the topic "Gender differences in mathematics achievement and retention scores". Findings revealed that male and female students who were taught algebra using the problem-based learning did not significantly differ in achievement and retention scores thereby highlighting that male and female students are capable of competing and collaborating in mathematics. It means that performance is a function of orientation, not gender.

Similarly, Susana, Bibiana, Isabel, Iris & Antonio (2020), found no significant gender differences in academic performance in their study "gender differences in mathematics motivation: differential effects on performance in primary education." It was however revealed that the explanatory power of attitudes toward mathematics was clearly more significant in boys than in girls.

According to Akanmu (2019) study on the effect of Think-pair-share on senior high school performance in mathematics in Ilorin, Nigeria, no statistical significant difference in the performance of students taught set theory in mathematics using Think-pair-share based on scoring levels. This means that gender of a student does not affect students' performance.

Findings from the study executed by Bertha & Athanasius (2019) on the effect of think-pair-share cooperation learning model on learners' performance showed a significant difference between the performance of males and females.

From the literature, several issues may be related to the gender difference in Mathematics performance: classroom interactions, curricular materials, beliefs, social and cultural norms, teaching approach, students' attitudes, students' interest and self-esteem, tutors' gendered attitudes. These various issues might have inferences on the type of teaching and learning techniques used in Mathematics classroom that is suitable for both girls and boys. Also, there is the need to give both boys and girls equal chances and trials to learn Mathematics to bridge the gap between boys and girls. For these reasons, the study selected gender as a variable due to the current world trend and research emphasis on gender performance in Mathematics. Hence, the researcher deemed it necessary to use Think-pair-share teaching approach to determine gender difference in Mathematics performance of students of Asankrangwa senior high school.

2.4 Think-pair-share Strategy in Teaching and Learning

Currently, think-pair-share strategy has received much recognition by researchers than the other interactive teaching strategies. The benefits of think-pair-share to both teachers and learners has compelled colleges of education mathematics teachers especially those in Ghana to employ think-pair-share strategy as the mode of lesson delivery.

Think-pair-share is a collaborative and interactive teaching strategy first proposed by Frank Lyman of University of Maryland in 1981. Think-pair-share is a teaching and learning strategy where students work together to solve a problem or answer a question about an assigned reading. This strategy requires students to think individually about a topic or answer to a question and share ideas with classmates (Kaddoura, 2013).

The Think-pair-share strategy start with 'Think', the teacher provokes students' thinking with a question or problem. The student take few minutes, maybe 5 minutes to think about the question. The next stage is 'Pair', using desk mate or the immediate mate available, students' pair up to talk about the answer each came up with. They compare their mental or written notes and identify the answers they think are best, most convincing and most appropriate. After students talk in pairs for few moments, the teacher calls for pairs to 'Share' their thinking with the rest of the class. The teacher can do this by calling on each pair or take answers as they are called out. Often, the teacher will record these responses.

According to Lom (2012), in the first phase of think-pair-share, the instructor asks question or poses a problem or gives a task to the class. Learners are then given a set amount of time during which they are expected to quietly and independently think about or write their answers. During this time the students' role changes from a reproductive thinker to an autonomous thinker; from reproducing knowledge to creating and discovering knowledge (Nabie, 2013). At the 'think' stage every student becomes an active learner, a critical thinker and good self-debater and time conscious since each student will have to support their answers with reasons which are logical within the set working time. The question by the teacher which provoke students thinking should require engagement with higher order learning skills of evaluation, analysis or synthesis; the amount of time learners should be given should be related to the difficulty of the question being asked; students will require varying amounts of time and students must provide a justification for their answer whether correct or wrong and if there are any concerns about students staying engaged during the think time, the course instructor can ask students to write their answers ((Lom, 2012). During the second phase 'Pair' of the activity, students' pair up or split into small groups to discuss and compare their thoughts (Lom, 2012). During this time the student has moved from convergent and rule abiding to divergent and stepping outside rules to create original ideas (Nabie, 2013). At this time the student can be instructed to pick the best answer, generate as many possible responses as possible, or come to a consensus depending on the question asked. Groups are ideally heterogeneous, with a mix of learning abilities, communication styles, ethnicities and genders. The composition of the groups should be changed periodically, approximately every six weeks. As in the think phase attention should be paid to the amount of time given for discussion: too much time and students will become bored and get off task, too little time and they will become frustrated. Observing the groups during the discussion phase can help the instructor get a sense of the appropriate amount of time required for most groups to produce an answer to the question.

In the final phase of the think-pair-share activity students rejoin the large group and are asked to 'Share' their responses with the class as explored by (Lom, 2012).

The 'Share' phase allow students the opportunity to discuss their answers with a small group of peers, rehearse their answers, and get approval from their group members prior to being asked to share with the larger class. When students have the opportunity to discuss their answers, the student move away from one right answer to multiple solutions. The student will appreciate that most often there are multiple solution to a question thereby applying this skill in real life situation. At the sharing stage, students can self-evaluate and self-direct. Sharing can be done by cold calling, asking for volunteers, requesting diverse responses, going around the room, etc. the instructor can also ask the groups to write their responses and collect these at the end of the class.

In a study conducted by Anaduaka (2018), on the topic “Effect of using think-pair-share in teaching and learning mathematics,” was aimed at investigating the impact of think-pair-share on the mathematics achievement of attention-deficit of hyperactive students. Findings revealed that think-pair-share instruction employed on the experimental group was better on increasing mathematics achievement of attention-deficit of hyperactive students. He again indicated that the think-pair-share helped students to think on a given topic by enabling students to formulate individual ideas and share the ideas with other students. When students are actively engaged in the process of learning, they think and their thinking become focused when they discuss with a partner and more of the critical thinking is retained after a lesson if students have the opportunity to discuss and reflect on the given topic.

The findings of Bertha & Athanasius (2019), supported the findings of Anaduaka (2018). They found significant difference existed between the post-test score of the experimental group and the control group-the think-pair-share model of cooperative learning improved both learners’ performance and attitudes towards quadratic functions of students in the experimental group.

Ariana (2013), professed that the use of think-pair-share encourages students’ participation in discussing and promotes forming and critiquing arguments both in small and large groups. Rowe (1972), in his research confirmed that the think-pair-share technique is a combination of many beneficial classroom practices which increases the number of students participating in whole class discussion and increase discussion based on evidence. This confirmed the findings of Cooper and Robinson (2000) and

Rowe (1972) agreeing that using the think-pair-share learning approach in his classroom allowed him to increase the amount of students' participation in class discussion, increased the number of long explanations students gave, and increases their comfort when sharing their thoughts and ideas.

According to Lom (2012), Raba & Harzallah (2015), and Parker (2009), there are additional benefits on the use think-pair-share. When students have appropriate "think time", the quality of their responses improves; students are actively engaged in the thinking; thinking becomes more focused when it is discussed with a partner; more of the critical thinking is retained after a lesson if students have an opportunity to discuss and reflect on the topic; many students find it safer or easier to enter into a discussion with another classmate, rather than with a large group; no specific materials are needed for the strategy, so it can easily be incorporated into lessons. According to Phungsuk, Viriyavejakul & Ratanaolarn (2017), studies building on the ideas of others is an important skill for students to learn; it promotes increased involvement of students and students develop increased comfort and skill with oral presentation. Because all students have the opportunity to share their answer and thinking there is an increased opportunity for them to get feedback both from the course instructor and from their peers, the quality of student's responses are improved with the increased wait time and opportunity for discussion. The think-pair-share strategy is designed to differentiate instruction by giving students time and structure to think or a given worksheet, allowing them to formulate individual views of these ideas and share with their partners.

Yarisda (2019), also used the cooperative learning model, think-pair-share in mathematics learning and found out that the mathematics learning outcome of students using the cooperative learning model think-pair-share are better than the mathematics learning outcome of students who use conventional learning in the XI science class

MAN 2 Padang. Similarly, in a study conducted by Choirul, Siti & Raden (2018), on the effect of think-pair-share learning with contextual approach on junior high school students' mathematics problem solving ability, students' problem-solving ability in the experimental group was better than the control group. They emphasized that think-pair-share learning model offers a learning process to a more challenging activity which is started by involving students to think about a problem given by a teacher.

In a study done by Akanmu (2019), on the effects of think-pair-share on senior school students' performance in mathematics in Ilorin, Nigeria, the researcher found a statistically significant difference in the performance of students taught set theory using think-pair-share compared with their counterparts in the control group in favor of think-pair-share group and a statistically significant difference in the knowledge retained by students taught set theory in mathematics using think-pair-share compared with their counterparts in the control group in favor of think-pair-share group.

According to Afthina, Mardiyana, & Pramudya (2017), think-pair-share using realistic mathematics education approach in geometry learning, the result revealed that there was mathematics achievement on the group which the think-pair-share strategy model using RME approach than direct learning model. Students with high mathematical-logical intelligence achieve better mathematics achievement than students with average and low mathematical-logical intelligence; while students with average mathematical-logical intelligence achieve better mathematics achievement than students with low mathematical-logical intelligence; there is no interaction between learning model and level of students' mathematical logical intelligence in mathematics achievement. The impact of application of think-pair-share learning model using RME approach affect the increase of students' activeness in learning activity and students' comprehension toward geometry learning. It also increases awareness of students that mathematics application is used as well as beneficial in daily life aspects. In spite of external factors such as learning model and learning approach, it is important as well to consider internal factors, for example, mathematical-logical intelligence of students.

2.5 Traditional Method of Teaching and Learning Mathematics

In this technique, the teacher stands at the front of the classroom near the chalkboard or marker board and the students are sitting in straight columns looking with an open note book and pen ready to take note. In 60 minutes' math class, the teacher starts by recalling what was taught in the previous lesson and highlight helpful materials for the first 10 minutes, followed by a comprehensive presentation of the new topic for 40 minutes uninterrupted, the students are then asked to begin practicing the new content by answering and completing multiple problems for the last 10 minutes until class time is finished. This is a typically traditional instruction that involve no other activities in the class. This classroom environment is mainly controlled by studying with pen and paper (Pierce & Ball, 2009).

According to a study by Fletcher (2003) and Osafo-Affum (2001), the Ghanaian mathematics teacher has assume the role of a lecturer systematically communicating the structure of mathematics, regardless of the level at which mathematics is taught. Teaching of mathematics in the Ghanaian classroom is teacher-centered (Fredua-Kwarteng & Ahia, 2015). According to them, the typical traditional way of the teacher presenting comprehensively new topic uninterrupted on the chalkboard or the marker board and students listen to their teacher and copy note rather than asking questions for explanations. This makes the learning of mathematics not interactive and consequently, mathematics is learnt by memorizing facts, theorems or formulas instead of exploring for conceptual understanding of the topic.

According to Panina & Vavilova (2008), in the traditional method, the student acts as an object of educational activity where the student learn and reproduce the material that is transferred to them by the teacher or another source of knowledge.

In a study conducted by Strutchens, Harris & Martin (2001), findings show that students learn geometry by only memorizing geometric theorems and properties rather than by exploring and discovering the underlying theorems and properties. By these approach students do not grasp conceptual understanding of geometry. For instance, if students memorize geometry theorems without understanding the concept, they are unable to apply these theorems to solve related problems. The lack of conceptual understanding often discourages students leading to abysmal performance in questions involving geometry.

2.6 Students' difficulties in solving problems on circle theorem-plane geometry II

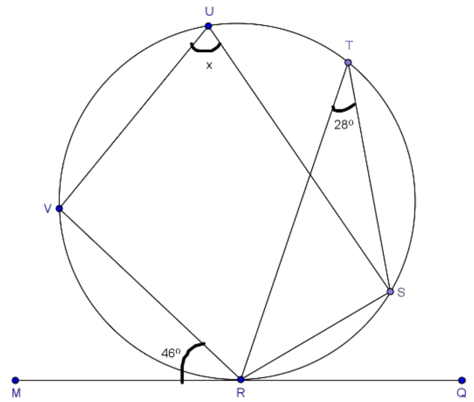
Several studies have shown students' difficulties in learning geometry (Mason, 2002; Udo Usoro, 2011 and Axsen, 2012). These difficulties were observed as a result of teaching methods, geometric language, visualization abilities, non-availability and obsolescence of instructional materials, poor reasoning skills, inadequate school curriculum and lack of proof by students.

According to Jones (2002), concerning issues in the teaching and learning of geometry, most students are faced with difficulties in the learning of geometry and therefore do not realize the importance and the beauty of it. According to Jones, teachers teach geometry by informing students of the properties associated with plane or solid shapes, requiring them to learn the properties and then to complete exercises which show that they have learned the facts. Such an approach can mean that little attempt is made to encourage students to make logical connections and explain their reasoning. It is therefore important that students have a good knowledge of geometrical facts if they are to develop their spatial thinking and geometrical intuition. For instance, some facts can be introduced informally, others develop deductively or found through exploration.

According to SEIP (2015), the requisite learning experience developed at JHS and SHS I geometry I for learning high school core mathematics geometry II that are considered taught (even though not) makes the teaching of plane geometry II very difficult. According to Jones (2002), in the planning approaches to teaching and learning geometry, it is important to ensure that the provision in the early years of secondary school encourages students to develop an enthusiasm for the subject by providing opportunities to investigate spatial ideas and solve real life problems and that there is a good understanding of the basic concepts and language of geometry in order to provide good foundation for future work and to enable students to consider geometrical problems and communicate ideas. This is the case that most students did had little or no opportunity to investigate spatial ideas and let alone solve real life problems.

The Chief Examiner for Core Mathematics reported candidates' difficulties in solving problems involving geometry such as cyclic quadrilateral (WAEC, 2016).

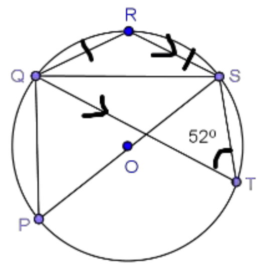
The figure below is the test item (question 3a) under the compulsory section of Core Mathematics WASSCE 2016.



In the diagram, $\angle RTS = 28^0, \angle VRM = 46^0, MQ$ is a tangent to the circle VRSTU at the point R. Find $\angle VUS$.

The candidate had difficulty in recognizing that for any cyclic quadrilateral, angles in the opposite segments are supplementary and the exterior angle is equal to the opposite interior angle. Again the candidate was expected to identify that the angle formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment.

Again, candidates had difficulty in solving test item 7b in the same year 2016 in Part II. The figure below is question 7b.

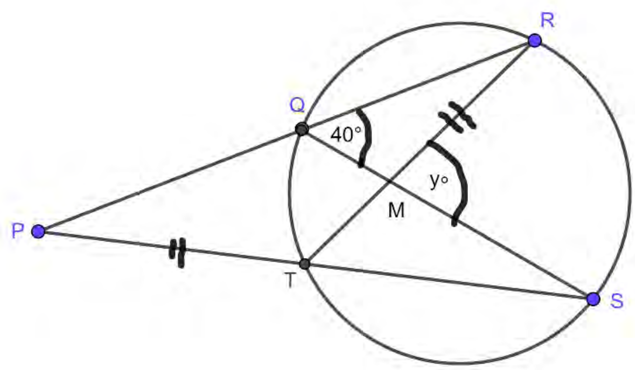


In the diagram, PQRST is a circle with centre O. If PS is a diameter, $RS // QT, |QR| = |RS| \; and \; \angle QTS = 52^0$, find:

(i) $\angle SQT$;
(ii) $\angle PQT$.

The Chief Examiner reported on candidates' difficulties in solving circle theorem questions involving cyclic quadrilateral because most candidates were unable to recognize the figure as cyclic quadrilateral. In this question, candidates were expected to recognize that angles in the opposite segments are supplementary. Again candidates were unable to deduce that angles subtended from a diameter and formed on the circumference is a right angle (angles in a semicircle is a right angle). In the same diagram, candidates have difficulty in recognizing that triangle QRS is isosceles and finally they had difficulty applying the angle property of trapezium.

The Chief Examiner for Core mathematics recorded some weaknesses of the candidates as difficulty in solving problems involving geometry such as, cyclic quadrilaterals, tangent and chord theorem (WAEC, 2017). Both test items 8a and 8b as shown below are circle theorem problems.



In the diagram, $\angle RQS = 40^0, |RT| = |PT| \; and \; \angle RMS = y$. Find the value of y.

In the question 8b, candidates were expected to illustrate the information in a diagram and find the values of certain angles but candidates had difficulty in illustrating the information in a diagram. Question 8b is shown below:

XY is a tangent to a circle LMN at the point M. XLN is a straight line, $\angle NXM = 34^0$ *and* $\angle NMY = 65^0$.

(i) Illustrate the information in a diagram.

(ii) Find the value of:

$(\alpha)\angle MLX$;
$(\beta)\angle LNM$.

The difficulties of students in solving problems involving circle theorem goes on and on even now.

By observation, the West African Senior School Certificate Examination (WASSCE) core mathematics for school candidates has more questions in circle theorem-plane geometry II than any other topic in the core mathematics syllabus. This observation was attested by Mr. Frederick Ofori, a facilitator for teachers' professional development seminar organized under the auspices of SEIP at Asankrangwa Senior High School. If the observation is true, then students' ability or inability to answer correctly circle theorem questions will influence students' grade in WASSCE Core Mathematics. According to the facilitator, several complains in the form of formal letters has been written to WAEC to set questions on all the topics in the core mathematics syllables and not to skew test items on a particular topic. The trend of the WASSCE core mathematics test items has still not changed and therefore there is the need to use every appropriate means to address students' difficulties in solving problems in circle theorem. The researcher believes that if the mathematics teacher would explain to students the importance of geometry in everyday life and its influence on the grade in the Core

Mathematics WASSSCE exams, the student will make every effort to understand provided the mathematics teacher will make the class very interactive by employing the use of think-pair-share in teaching circle theorem-plane geometry II.

2.7 Potential for Using Think-pair-share within Mathematics Education

According to Afthina and Pramudya (2018), think-pair-share can improve students' mathematical problem solving and mathematical communication skills. This is because think-pair-share encourages students' participation in the teaching and learning activities. The student is given time to think and solve a mathematical problem and discuss their solution with small group and large group. In doing this the student learn new skills in solving problems as they compare their solutions and the teacher also elaborate further for clearer understanding. Again, because students are made to participate in discussion and critiquing arguments students' mathematical communication is improved. According to Choirul, Siti and Raden (2018), think-pair-share learning with contextual approach on students' mathematics problem solving ability improved since learning begins with the stage of individual thinking to solve a problem, and then discussing individual results by pairing with desk mate to discuss individual outcome and come out with a common solution to present to the class. Several scholars agree that think-pair-share is the most appropriate cooperative or interactive strategy which will improve students' problem solving skills.

According Napitupulu & Surya (2017), think-pair-share is a cooperative learning model that is considered to arouse students' interest in mathematics and make students more active and socialize, encourage cooperation among students in learning the material, so that it can improve students learning outcomes.

Think-pair-share is expected to improve mathematical problem solving skills provided that the steps in the learning process are fully implemented (Miratika, Asmin, Mulyono & Ani, 2018). The 3 stages in think-pair-share is very important. According to studies of several researchers, the use of think-pair-share will enhance performance provided the steps are strictly adhered to.

2.8 Summary of Literature Review

From the review of related literature, it is evident that the use of think-pair-share in learning environment improves performance of students in mathematics, help teachers to change their classroom to investigative environment, improves critical thinking, it helps students' socialize and participate in discussion and enable students to comprehend difficult and abstract concepts in geometry. It is therefore recommended to use think-pair-share to enhance students' understanding of mathematics and improve learning. Base on the literature reviewed it was manifested that think-pair-share has not been extensively researched into especially in Ghana and in the field of mathematics.

CHAPTER 3

METHODOLOGY

3.0 Overview

This study was guided by the following research questions:

1. What difficulties do Asankrangwa Senior High School Form 2 students' have in solving problems in circle theorem-plane geometry II?
2. To what extent will the use of think-pair-share address reduce Asankrangwa Senior High School Form 2 students' difficulty in solving circle theorem-plane geometry II problems?
3. Is there any significant difference in the performance of female and male students' taught circle theorem-plane geometry II using think-pair-share?

This chapter provides a comprehensive description of the methodology used in the study for data to respond to the stated questions. It comprises the research design, population, sample and sample procedure, instruments, data collection and data analysis procedures. It also describes ethical considerations made in finding out the effect of using think-pair-share in teaching circle theorem – plane geometry II on the performance of students.

3.1 Research Design

Research design is the overall plan for collecting data in order to tackle the objectives of the study (Fraenkel & Wallen, 2000). The essence of research design is to support the researcher on the type of data to collect, how to collect, process and analyze them in order to answer the research questions or test the research hypothesis. This study was structured basically within the framework of quasi-experimental research design. Quasi-experimental research is a model that allow researchers to answer critical questions about the relationship between variables by determining whether there are significant differences between variables (Butin, 2010). Specifically, non-equivalent control group pre-test post-test design, a quasi-experimental research design, was employed because intact classes of unequal number of students were used and the respondents were not randomly selected and allocated to the groups (Creswell, 2008). The quasi-experimental research design involving pre-test and post-test was used to examine the effect of the use of think-pair-share as an instructional tool in teaching circle theorem-plane geometry II on the performance of students.

A non-equivalent control group pre-test post-test design, a quasi-experimental research design is the most important research design for investigating cause and effect relationships between two or more variables (Gall, Gall & Borg 2005). Quasi-experimental design is frequently used in educational research since it is often difficult and sometimes unethical to randomly assign students to settings. The strength of quasi-experimental research generally lies in their practicality, more feasible and generalizability (Gall, Gall & Borg 2005). However, two limitations are generally associated with quasi-experiments. First, quasi-experiments do not allow researchers to determine the order by which variables occur. Secondly, quasi-experiments do not rely on random assignment. Instead, subjects are assigned to groups based on non-random criteria (Lauren, 2021). Also, without proper randomization, statistical test can be meaningless (Shuttlework, 2008).

In the Ghanaian senior high school classroom settings, it was very difficult to use true experimental design for a study. This is because no school would allow a researcher to disorganize classes assign to students who are already in their various classes into different academic programs for the purpose of research. For this reason, random assignment of participants to groups was impossible. This is why the researcher could not employ other research designs because they involve random assignment of participants and therefore would make it inappropriate and unethical to use for this study.

Consequently, variables of this study were categorized into independent variables and dependent variables. In this study, there were two independent variables, which were the approaches used in teaching and learning circle theorem-plane geometry II, thus think-pair-share approach and the traditional approach. The dependent variable of this study was students' performance on circle theorem-plane geometry II achievement test while the possible covariate of this study was students' performance on readiness test for circle theorem-plane geometry II. These scores were analyzed to establish whether a significant difference exist between the control group and the experimental group or not.

In summary, the quasi-experimental design for this study was the non-equivalent 'pre and post' test with treatment. Two non-equivalent intact-classes of SHS form two students were used in the study. These comprises of think-pair-share learning group and control group.

The design of the study consists of three phases. These three phases are pre-test stage, treatment stage and post-test. The pre-test phase was carried simultaneously on both the control group and the treatment group before administering the treatment. The treatment stage is the second phase. This is where the experimental group was taught circle theorem-plane geometry II using think-pair-share while the control group was taught circle theorem-plane geometry II using traditional method of teaching (without using think-pair-share). The next phase was the post-test to both experimental group and control group after three weeks. After the participants went through the three phases, the test results were evaluated to determine whether think-pair-share has effect on students' achievement in circle theorem-plane geometry II or not.

3.2 Population

The target population for this study was all second year Senior High School 2 (SHS2) students of 2020/2021 academic year in the Amenfi West Municipality in the Western Region of Ghana. The accessible population consisted of a set of second year students of Asankrangwa Senior High School. The reason for choosing only Asankrangwa Senior High School and not involving Asankrangwa Senior High Technical School even though the two schools are all beneficiaries of SEIP, Asankrangwa SHS was operating under the double track system whereas Asankrangwa Senior High Technical School was just one track. Also, the reason for choosing Asankrangwa Senior High School was to ensure that all other factors that might affect the result of the study, except the performance in the teacher-made achievement test.

3.3 Sampling and Sampling Procedure

A purposive sampling technique was used to select two intact classes (Class A and Class B). Purposive sampling was used because of the special features of the two classes in facilitating the purpose of the study (Creswell, 2009). In purposive sampling the units of the sample are selected and not by random procedure, but they are carefully selected for the study because of their distinctive features and characteristics. In all, there were two General Art B intact classes in the gold track: 2General Art B 1 and 2General Art B 2. The two intact classes for the study was together as one class. They were divided into two equal halves to ensure covid-19 protocol; social distance in the classrooms. This means that the difference between the two classes is their class label and are thought by different teachers especially in core mathematics. These two classes were chosen to ensure fair comparison in terms of performance in the achievement test in the pre and post-tests. Therefore, the sample size consisted of 48 students of which 24 were in the control group (Class A) and 24 were in the experimental group. The control group was made up of 10 boys and 14 girls and the experimental group was made up of 10 boys and 14 girls.

The reason for selecting the intact class was that all the lessons were taught during the instructional hours. Also, the intact class was used for the study so that the contents treated would be beneficial to the entire class. Again, the entire class was used to avoid disruption when school is in session. The SHS form 2 classes were used because the topic treated in the study was among the SHS form 2 topics in the mathematics syllabus for SHS and the school would not allow the researcher to teach or reteach the topic in the other forms.

The reason for selecting the General Arts programme was that traditionally students offering this programme do not show much interest in the study of mathematics and that they might not be very much different in mathematics performance. After the initial mathematics achievement test (pre-test) which was administered, the outcome of the test disclosed that both the students in the control and experiment group were comparable in aptitudes before the treatment was administered.

3.4 Research Instruments

In view of the nature of research questions been surveyed, the achievement test was used in gathering the data for the study. Therefore, an achievement test designated Circle Theorem-Plane Geometry II Achievement Test guide was the main instrument for data collection.

3.4.1 Achievement Tests (Pre-test and Post-test)

The test items on the teacher-made achievement test were constructed based on the lesson taught and the learning objectives in the SHS core mathematics curriculum. The aim of this instrument was to provide a measurement of achievement. The teacher-made achievement test was preferred in this study to other types of tests due to the following reasons: it reflects instruction and curriculum; it is sensitive to students' ability and needs; it provide immediate feedback about students' progress; and finally, it can be made to reflect small changes in knowledge (O'Malley, 2010)

The pre-test consisted of 10 subjective type questions which were based on core mathematics syllabus objectives 2.10.2, 2.10.3, 2.10.4 and 2.10.5 (Ministry of Education, 2010). The questions covered all the 6 theorems treated under circle theorem-plane geometry II in the mathematics teaching syllabus. Participants' in each group were given 150 minutes to complete the test. The pre-test was done to determine the initial entry points and compare difference between experimental and control group before treatment. Also, post-test consisted of 10 subjective test items that are slightly different from questions in the pre-test, however the questions measured the same difficulties level of participants. See Appendix F for post-test and Appendix G for marking scheme for scoring the test. Post-test was used to measure participants' achievement after the treatment.

Generally, the researcher wanted to know participants' ability and their difficulties in answering the pre-test test items. In test item 1, participants were expected to recognize that angle formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment. This is the appropriate theorem participants are expected to use to get two linear equations involving two variables. Participants' were expected to solve them simultaneously to get the value of the valuables. In test item 2, participants were expected to recognize that an interior angle of a cyclic quadrilateral is equal to the exterior angle of the quadrilateral. Participants' were expected also to recognize that for any cyclic quadrilateral, the sum of the opposite interior angles are supplementary. Finally, they were expected to recognize that angles subtended from the same chord or arc formed on the circumference are equal. The researcher with test item 3 expected participants to recognize that angles in the same segment are equal. In test item 4 participants' were expected to recognize that angles on a straight line sum up to 180^0, participants' were expected to recognize that triangle BOC is isosceles, therefore the angles OBC and OCB are equal. Again, participants' were expected to recognize that angles subtended from a diameter formed on the circumference is 90^0. The researcher with test item 5 expected participants' to consider the smaller circle first and recognize that the angle subtended by an arc of a circle at the center is twice that of the angle at the circumference. Participants' were expected to consider the circle BPOQ and recognize that for cyclic quadrilateral, the sum of the opposite interior angles sum up to 180^0. In test item 6, participants were expected to recognize that that the length of OS and OT are equal to the radius, therefore angles ROS and ROT are equal. Participants' were expected to also recognize that angles RSO and RTO are equal to 90^0. Participants' were expected to recognize that the length of tangents to a circle from an external point are equal and finally, they were expected to recognize that the sum of the interior angles of a triangle is 180^0. In test item 7, participants were expected to recognize that the angle subtended by an arc of a circle at the center is twice that of the angle at the circumference. They were also expected to recognize that angle at a point is 360^0. In test item 8, participants' were expected to recognize that angles formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment and also identify that the sum of the interior angle of a triangle is 180^0. In test item 9, participants were expected to recognize that angles formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment. Participants' were again were expected to recognize that angles subtended from a diameter formed on the circumference is 90^0 and finally, participants' recognize that the sum of the interior angles of a triangle is 180^0. Finally, in test item 10, participants' were expected to recognize that angles subtended from a diameter formed on the circumference is 90^0, again participants' were expected to recognize that the sum of the interior angle of a triangle is 180^0 and finally recognize that for any cyclic quadrilateral, the sum of the opposite angles is 180^0

3.5 Treatments

The think-pair-share approach was applied to the experimental group whereas traditional approach of teaching was applied to the control group. These approaches are described in this section.

3.5.1 Think-pair-share approach to teaching circle theorem-plane geometry II

The think-pair-share approach is a collaborative learning strategy where students work together to solve a problem or answer a question (Anaduaka, 2018). Think-pair-share requires students to think individually about a topic or answer to a question, and share ideas with classmates. Students were taught circle theorem-plane geometry II by using think-pair-share which was designed hand in hand with the lesson plan by the researcher according to activities in senior high school students' mathematics curriculum. This implies that the treatment in the experimental group was affected by collaborative learning unlike the control group where all the lessons were taught using teacher-centered approach. Lesson plans for the treatment group were designed to ensure that classroom instruction reflect the aims and objectives of teaching Circle Theorem-Plane Geometry II as in the mathematics curriculum. The lesson plans indicated the lesson objectives, duration of the lesson, contents to be treated, teaching and learning activities, assessments and remarks. For instance, the theorem 'the angle subtended by an arc of a circle at the center is twice that of the angle at the circumference' can be illustrated with think-pair-share when the teacher guiding students to use a pair of compasses to draw a circle with a given radius and with O as the center and a point P is marked on the circumference. The teacher then guides students to use a ruler to draw two chords from point P to cut the circumference at points A and B respectively. After students are directed to join OA and OB, the radii, the facilitator will then ask the students to measure $\angle APB$ *and* $\angle AOB$ with a protractor. The teacher then ask students what they have noticed. At this point students are given few minutes (2 minutes) to examine the relationship between the two angles. They then discuss with their desk mate and agree on common grounds and then share with the class.

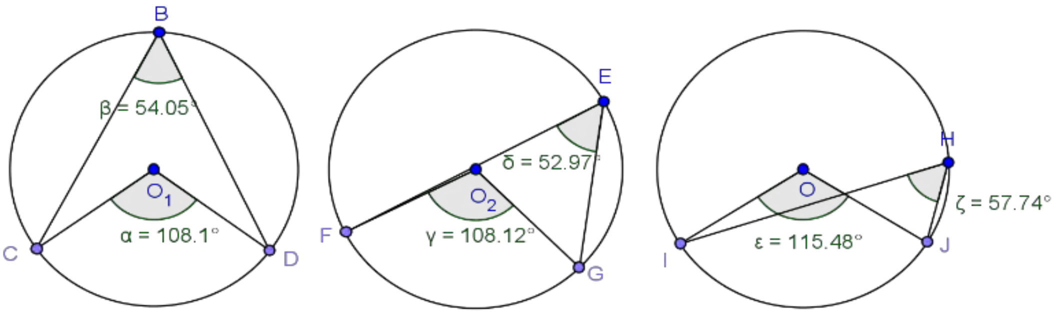


Figure 3.5.1: Diagrammatic Representation of Circle theorem

3.5.2 Control Design: Traditional Teaching Approach

The traditional teaching approach used in this study refers to the teaching approach where the teacher stands in front of the classroom near the chalkboard or the marker board and the students are sitting in straight columns looking with an open note book and pen ready to take note. In this approach also, the teacher starts by recalling what was done in the previous lesson followed by a comprehensive presentation of the new topic. Thus, the researcher wrote on the board and the students strictly followed the instruction the teacher gave, and active participation of the student was not encouraged. In this teaching approach, the geometry instruction was not in line with the social constructivism theory but mainly in lecture format and, therefore, instruction was teacher-centered. For instance, to illustrate the theorem "the angle subtended by an arc of a circle at the center is twice that of the angle at the circumference," the teacher would have to draw three diagrams as in figure 3.5.1 and measure $\angle APB$ *and* $\angle AOB$ with a protractor. The accuracy of the results obtained cannot be assured as it would depend on the reliability of the construction and measuring tools. The teacher does this all alone whiles students watch because the teacher has rendered them passive learners. Unlike the think-pair-share approach where students take instruction from the facilitator to think about the diagram that has been constructed and students' ability to differentiate it from other diagrams. Also, the students are made to pair and discuss with desk mate to know how their desk partners also see the diagram and finally desk mate share how they see the diagram with the class. After students have argued and agreed that on the diagram, the teacher instruct students to measure $\angle APB$ *and* $\angle AOB$ with a protractor, each student is asked to establish the relationship between the two angles and discussing their findings with desk mate and finally the desk mate presenting their common solution to the class.

Nevertheless, with regards to traditional method of exploring various concepts of Circle Theorem-Plane Geometry II, teaching can be stressful and time consuming. These routine activities do not need a large amount of concentration. However, the main differences between the two approaches of teaching and learning of Circle Theorem-Plane Geometry II are shown in the Table 3.1

Table 3.1 Differences in the two approaches of teaching circle theorem-plane geometry II

Think-Pair-Share Approach	Traditional Approach
i. Student-centered	i. Teacher-centered
ii. Students create their own version of knowledge by active participation in learning activities	ii. Students learn mainly from teachers' explanations
iii. Students were more responsible for their own learning critically thinking about a problem and discussing their findings with desk mate and finally with the class	iii. Students were engaged in repeated practice for mastery of skills
iv. Teachers provide students with problem solving situations to investigate in individually, then discuss with desk mate and finally present their solution to the class	iv. Teacher explains thoroughly the mathematical rules and procedures before giving students mathematical problem
v. Teachers focus more on conceptual understanding	v. Teachers focus more on procedural understanding
vi. Teachers engage students in situations that might bring about contradictions and then encourage students discussions	vi. Learning activities provided are focused on memorization of skills and procedures by doing repetitive practice

3.6 Validity and Reliability of Instruments

Validity is the extent to which results obtained from the analysis of the data actually represents the phenomena under study (Mugenda & Mugenda, 2003; William, 2014). The validation of the instruments was carried out to check or prove the accuracy of the instrument used during the pilot study. This checks the appropriateness of the data collection instrument, that is, achievement tests.

In order to ascertain content validity, the test was constructed based on the instructional objectives of the lesson taught and the specific objectives in senior high school mathematics curriculum. Also, comments and inputs were made about the content of the research instrument by researcher's supervisor and they were found to be acceptable. Again, to ensure content validity of the research instrument the researcher relied on the knowledge of other researchers (Heale & Twycross, 2015) and (Korb, 2012), who were familiar and well-versed with the construct being measured. These subject-matter experts were provided with the achievement test items for their input. Test items were also given to five (5) SHS mathematics teachers including the head of the mathematics department to cross-check and contribute to the content areas that were tested in this study in order to further ensure that the content that was chosen was within the approved domain of the study for the SHS students concerned.

Reliability is the ability of the instrument to give consistent results after a number of repeated trials (Kerlinger, 2003). It is the extent to which items in an instrument generate consistent responses over several trials with different respondents in the same setting or circumstances (Fraenkel &Wallen, 2003). A reliability test was conducted with the aim of testing the consistency of the research instruments in order that the research instrument would be improved by revising items. To determine the reliability of the instrument, a pilot study was conducted.

Piloting determines whether questions and directions are clear to respondents and whether they understand what is required of them. Piloting is done to determine the feasibility of using a particular research instrument in a major study. It provides an opportunity to try out the instructions for completion of the instrument, especially if it is being used for the first time. Piloting entails a trial administration of a newly-developed instrument in order to identify flaws and time requirements (Shilubane, 2010). The researcher piloted the instrument on a small sample of 34 Form two General Arts C students of Asankrangwa SHS. The piloting was done in Asankrangwa Senior

High School because they have the same characteristics as the sample for the study. The split-half method was used to check the reliability of the test instruments.

In the split-half method, the construction of a single test is required. The number of items are split into two parallel halves (even number of test items) or one half more than one (odd number of test items). Spearman-Brown formula was used to correlate participants' scores from each half to test the reliability of the test. The value of the reliability coefficient was .836 which indicates a high degree of reliability of the test instrument (Fraenkel & Wallen, 2003). The reliability coefficient of .836 means 83.6% of variability in scores is due to true score differences amongst students while the 16.4% is due to error in measurement. Taking sides with Kline (1999) and George and Mallery (2003), a reliability coefficient greater than .50 shows a homogeneous test. Thus, the whole test between odd and even items of the achievement test has very good reliability and therefore the achievement test used in the study is within the acceptable standard of the instrument being reliable.

According to Fraenkel and Wallen (2009), internal validity means the degree to which a researcher confidently concludes that observed differences on the dependent variable are directly related to the independent variables in a study, not to some other variables. Bhandari (2020), construe internal validity to be the extent to which you can be confident that a cause-and-effect relationship established in a study cannot be explained by other factors. In this study, potential peril to internal validity include testing (the pre-test influences the post-test), maturation (the outcomes of the study vary as a natural result of time), selection (groups are not comparable at the beginning of the study), history (an unrelated event influences the outcomes) and instrumentation (different measures are used in pre-test and post –test phases). In this study, two intact groups were purposely chosen for both the control and the experimental groups therefore could not become a threat to the study. The achievement tests were administered to all the participants in their respective classes, therefore, location threat was also reduced by satisfying similar conditions in all classes during the time of administering the instruments.

Testing could also threaten the validity of a study. In this study, testing did not threaten the study because various achievement tests such as pre-test and post-test were used. To reduce the threat, equal time was given to all the respondents in the two groups. There were two mathematics teachers, one (the researcher) taught the experimental group using the think-pair-share approach and the other mathematics teacher taught the control group using the traditional method of teaching. The two teachers were involved in the scoring of the test to ensure objectivity. Maturation, including emotional, psychological and physiological processes, within study subjects across time someway somehow affect the dependent variable (Martella, Nelson, & Marchand-Martella, 1999). Since the same number of treatment lesson periods were used for groups at the same duration, maturation did not affect internal validity of this study. For that reason, if there any maturation threat to the study, the groups would have been affected.

3.7 Data Collection Procedure

The researcher was given an introductory letter by the Head of Department of Mathematics in the University of Education, Winneba to the Headmaster of Asankrangwa SHS to be given the necessary support for the collection of data.

The Headmaster then informed the teachers and the students of the purpose of the study and the need for their participation and maximum cooperation for the success of the study. A suitable date was then fixed for the commencement of the study. Two weeks before commencement of the main study, the head of the mathematics department and other faculty members oriented students chosen for the study.

One week before the main study, pre-test was administered, marked and analyzed to determine the entry level of each group, readiness and difficulties students encountered in solving geometry problems. The main study took four weeks. Each week the researcher and the mathematics teacher designated for the other class met their respective students in both the control group and the experimental group four instructional periods, subjecting the experimental group to think-pair-share approach and the control group to the traditional approach. The groups were taken through the treatment. Lessons were designed on circle theorem-plane geometry II to teach students to discover the following:

i. The angles subtended by an arc of a circle at the center is twice that of the angle at the circumference
ii. The angle subtended from the diameter and formed on the circumference is right angle
iii. Angles in opposite segments are supplementary.
iv. The exterior angle is equal to the opposite interior angle.
v. Angles in opposite segments of any cyclic quadrilateral supplementary.
vi. The exterior angle is equal to the opposite interior angle.
vii. A diameter or radius is perpendicular to the tangent to the circle at the point of contact
viii. Angle formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment.
ix. The angle formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment.

x. The lengths of tangents to a circle from an external point are equal.

The two groups worked on the same mathematical tasks but the control group was taught with the traditional method while students in the experimental group were taught using the think-pair-share approach, Bailey (2008).

Lessons were taught using the traditional approach to students in the control group by following the approved curriculum of mathematics for senior high schools. The traditional teaching method was used as a dominant instructional approach to teaching and learning of circle theorem-plane geometry II although there were many activities in the curriculum that have not been prepared based on the students centered approach of teaching (Ministry of Education, 2012). Nevertheless these activities were not employed in the control group. Few activities about circle theorem-plane geometry II were presented to students by drawing on the marker board. For each 60 minutes instructional period with the control group, the researcher starts by recalling what was taught in the previous lesson and highlighting helpful materials in the form of discussion. The researcher gave definitions of concepts about circle theorem-plane geometry II by writing properties and theorems about circles and if need be, draw figures which are not drawn to scale on the marker board and students are asked to write them in their notebooks. The researcher then asked students to begin practicing the new content by answering and completing multiple problems for the last 10 minutes until instructional period is finished. Only few students were allowed to willingly explain their solutions to the class when the instruction period of that lesson has not ellapsed. Activities and exercises in students' textbooks were given as homework or assignment to students.

The experimental group on the other hand, were taught circle theorem-plane geometry II by using think-pair-share interactive teaching strategy with worksheets designed hand in hand with the lesson plan by the researcher according to activities in senior high school students' mathematics curriculum. This means that the treatment in the experimental group was affected by collaborative learning unlike the control group where all the lessons were taught using traditional approach. The worksheets supported students to use think-pair-share to explore and examine the properties and theorem of geometric figures such as triangles, quadrilaterals and theorems of circle according to the instruction given to them. In this group, students were given the opportunity to think to solve problem involving circle theorem-plane geometry II individually and independently. Students are encouraged to exchange information through the discussion of properties and theorem of geometry and finally, students of desk mates are encouraged to share information to the larger class. The researcher identifies students' difficulties and guide them by providing feedback to students' questions. During the teaching and learning period, students were given one or two assessment test item(s) in class to assess their short-term learning in each class lesson and were done for both control group and the experimental group. These class exercises were marked by the researcher. Although, the scores in the class exercises were not cumulated in the post test for data analysis, it helped them in the post test.

3.8 Ethical considerations

According to Resnik (2018), ethics in research means the discipline that study standards of conduct, such as philosophy, theology, law, psychology or sociology. It is an approach, strategy, procedure or perspective for deciding how to act and for analyzing complex problems and issues. Protection of participants and their views were assured by obtaining informed consent, protecting privacy and ensuring confidentiality.

The researcher in doing this, describe the study, the purpose and the possible benefits were mentioned to participants. Participants are permitted to freely withdraw or exit at any time if they deemed it fit. Works in the form of ideas, writings, drawing and other intellectual properties were duly acknowledged. In the case of unpublished document, permission was sought from the owners.

3.9 Data Analysis

Quantitative data was used to measure the scores of the participants in the achievement test. This was used to give a statistical report with correlations, comparisons of means and statistical significance of findings (Johnson & Christensen, 2008). After 150 minutes duration for the achievement test, the researcher took participants responses. A marking scheme was designed by the researcher to ensure reliability of scoring the subjective test. Participants' responses were shared among the researcher and the mathematics teacher who handled the control group. After marking, moderation of participants' assessment was done to ensure that marks and grades are as valid, reliable, and fair as possible for all the participants and all the markers (ALTC, 2012). Moderation of assessment checks that marking is consistent such that an assessment item would be awarded the same marks by any marker. The task of moderation is to minimize discrepancies among assessors before participants' receive their marks (Sadler, 2009). Descriptive statistics such as means, standard deviations, percentages, tables were employed by the researcher to describe the general performance of students in both the control group and the experiment group in the pre-test and post-test.

The independent sample t-test was used to compare the means between two unrelated groups on the same continuous, dependent variable. The independent sample t-test was used to compare the means of mathematics achievement scores between the control group and the experimental group in the pre-test scores and between the gender in the control group and the experimental group as well as between the genders in the control group before the treatment to know their entry level of the mathematics achievements. An independent sample t-test was used to answer research question 2.

Again, the paired sample t-test was used to test to find whether the performance of students within each group improved or not while the effect size (eta statistic square) was used to determine the magnitude of improvement in each group. According to Pallant (2001), the criterion for interpreting eta squared values are 0.0 = small effect, 0.06 = moderate effect and 0.14 = large effect. The purpose was to determine whether there were statistically significant difference between each student score in the pre-test and post-test.

Test of the Assumptions of t-tests

The paired sample t-test and the independent sample t-test are parametric tests and therefore there are some assumptions that need to be met before they are used to analyses any quantitative data. The data that were collected in this study warranted the use of paired sample t-test and the independent sample t-test. This is because the scores from the achievement tests were treated as having interval scale and continuous. Again, the distribution of the data was approximately normal (see Appendix H). Another assumption that was met before the t-tests were used, was homogeneity of variance as shown in Table 3.2.

Table 3.2: Homogeneity of variance test for pre-test and post-test

		F	Sig.
Pre-test scores	Equal variances assumed	.010	.923
Post-test scores	Equal variances assumed	.007	.935

The results in Table 3.2 show that the variances are equal since the p-values reported are all greater than the alpha value of .05. This is evidence that the homogeneity of variance assumption for operating independent sample t-test was met.

The slight progress in the students' performance could be the use of sequences of exercise the students were taken through. Even though in the control group, the lesson was mainly based on traditional method and no manipulation of Think-pair-share, the students were focused throughout the lesson.

CHAPTER 4

RESULTS AND DISCUSSION

4.0 Overview

This chapter presents and discusses the results of the study. Data was collected from students using pre-test and post-test. The data collected from the achievement tests were marked and scored. The data was used to gather quality information in a measured and systematic manner to ensure accuracy and facilitate data analysis. The data was used to support decisions or provide evidence. The results are presented in this chapter under four main subheadings;

1. What are Asankrangwa Senior High School Form 2 students' difficulties in solving problems in circle theorem-plane geometry II?
2. To what extent will the use of think-pair-share address Asankrangwa Senior High School Form 2 students' difficulty in solving problems in circle theorem-plane geometry II?
3. Is there any significant difference in the performance of female and male students' taught circle theorem-plane geometry II using think-pair-share?

4.1 Demographic Characteristics of Participants

Table 4.1 presents gender distribution of the participants in the study

Table 4.1: Gender distribution of the participants

Gender	Control Group		Experimental Group		Total	
	N	%	N	%	N	%
Male	10	20.83	10	20.83	20	41.67
Female	14	29.16	14	29.16	28	58.32
Total	24	49.99	24	49.99	48	100

Source: Field work, 2021

Table 4.1 shows that out of the 48 participants, 41.67% were males whiles 58.32% were females. This implies that majority of the respondents for the study were females.

4.2 Research Question 1: What are Asankrangwa Senior High School Form 2 students' difficulties in solving problems in circle theorem-plane geometry II?

To investigate Asankrangwa Senior High School 2 students' difficulties in solving problems in circle theorem-plane geometry II, participants were made to answer ten essay type questions test items on circle theorem-plane geometry II in the pre-test. To answer the research question, the researcher took time to scrutinize students' solutions item by item of all the ten test items of all the participants in the pre-test to find out the test items which were attempted by the participants, test items which were correctly responded and participants' difficulties from their solution.

Table 4.2 shows the distribution of attempted, correct responses and difficulties of the participants' in solving problem in circle theorem-plane geometry II.

Table 4.2: Distribution of attempted, correct responses and difficulties in circle theorem-plane geometry II pre-test test items (n=48)

Test items	Attempted		Correct response		Difficulties
	N	%	N	%	
1	48	100	15	31.25	Majority of participants were unable to recognize that angle formed by tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment.
2	48	100	4	8.33	Large number of participants were unable to recognize that the interior angle of a cyclic quadrilateral is equal to the exterior angle of the quadrilateral. Again, majority of participants were unable to recognize that angles subtended from the same chord or arc formed on the circumference are equal.
3	48	100	13	27.08	Most of the participants were unable to recognize that angles subtended by an arc of a circle at the center is twice the angle at the circumference.
4	43	89.5	4	8.33	Greater number of participants were unable to recognize that angles on a straight line sum up to 180^0. Majority of participants were unable to recognize that triangle BOC is isosceles and therefore unable to apply the angle properties of isosceles triangle. Again, most participants were unable to recognize that angles subtended from a diameter formed on the circumference is right angle. Finally, participants were unable to recall that the sum of the interior angle of a triangle is 180^0. Most of the participants were unable to recognize that angles subtended by an arc of a circle at the center is twice the angle at the circumference.
5	37	77.08	1	2.08	Majority of the participants were unable to recognize that angles in opposite segments of any cyclic quadrilateral is supplementary.
6	29	60.41	1	2.08	Most participants were unable to recognize that the lengths of tangents to a circle from an external points are equal. Again, majority of participants were unable to recall that the sum of the interior angle of a triangle is 180^0.

Table 4.2: Cont. Distribution of attempted, correct responses and difficulties in circle theorem-plane geometry II pre-test test items (n=48)

Test items	Attempted		Correct response		Difficulties
	N	%	N	%	
7	47	97.91	1	2.08	Most of the participants were unable to recognize that angles subtended by an arc of a circle at the center is twice the angle at the circumference. Again, most participants were unable to recognize that angle at a point is 360^0
8	21	43.75	1	2.08	Majority of participants were unable to recognize that angle formed by tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment. Finally, participants were unable to recall that the sum of the interior angle of a triangle is 180^0.
9	16	33.33	2	4.16	Majority of participants were unable to recognize that angle formed by tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment. Again, most participants were unable to recognize that angles subtended from a diameter formed on the circumference is right angle. Finally, participants were unable to recall that the sum of the interior angle of a triangle is 180^0.
10	9	18.75	0	0	Most participants were unable to recognize that angles subtended from a diameter formed on the circumference is right angle. Majority of the participants were unable to recall that the sum of the interior angle of a triangle is 180^0. Finally, majority of the participants were unable to recognize that angles in opposite segments of any cyclic quadrilateral is supplementary.

Table 4.2 indicates that out of the 48 respondents who took part in the pre-test, all the 48 participants attempted test item 1 representing 100% but only 15 participants representing 31.25% responded correctly. Similarly, all the 48 participants attempted test item 2 representing 100% but only 4 of the responded correctly representing 8.33%. For test item 3 also, all the 48 participants representing 100% attempted it but 13 of them representing 27.08% answered correctly. Test item 4 was attempted by 43 participants resenting 89.5% and only 4 representing 8.33% responded correctly. Test item 5 was attempted by 37 participants representing 77.08% but only 1 responded correctly representing 2.08%. Similarly, test item 6 was attempted by 29 participants representing 60.41% but only 1 was correctly responded to. Test item 7 was attempted by 47 participants representing 97.91% and only one representing 2.08% responded correctly. Again, 21 participants representing 43.75% attempted test item 8 and only one representing 2.08% responded correctly. Test item 9 was attempted by 16 participants representing 33.33% and 2 representing 4.16% answered correctly. Finally, test item 10 was attempted by 9 participants representing 18.75% and no one representing 0% answered correctly.

Table 4.3 shows the descriptive analysis using frequency and percentages of participants' performance in pre-test.

Table 4.3: Descriptive analysis (frequency & percentage) of participants' performance in pre-test (N=48)

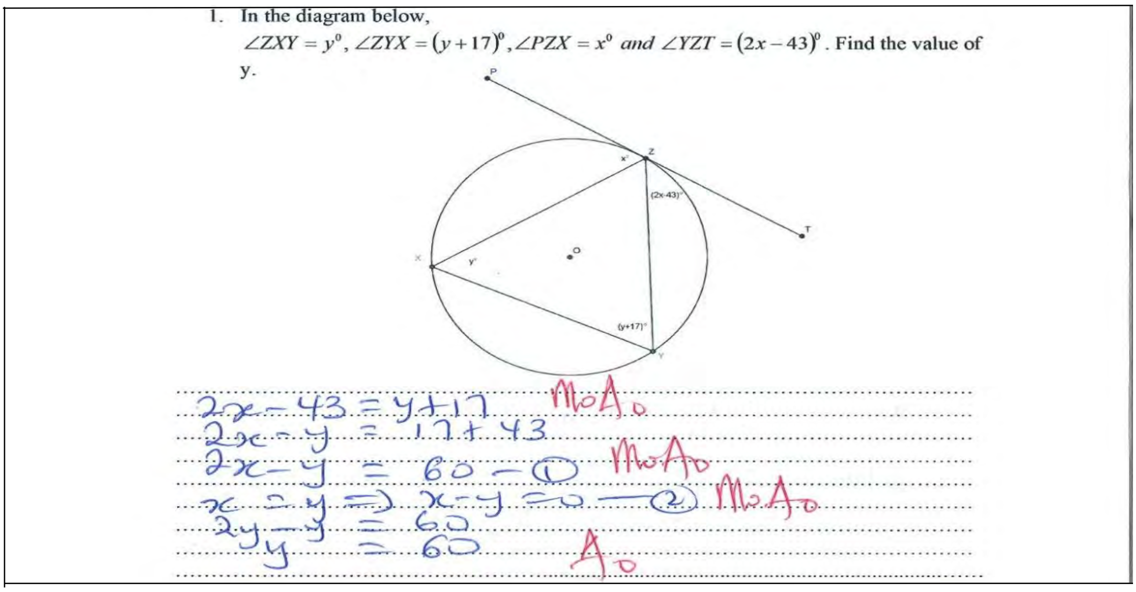
Test items	Frequency	Percentage (%)	Rating
1	15	31.25%	Below Average
2	4	8.33%	Below Average
3	13	26.08%	Below Average
4	4	833%	Below Average
5	1	2.08%	Below Average
6	1	2.08%	Below Average
7	1	2.08%	Below Average
8	1	2.08%	Below Average
9	2	4.16%	Below Average
10	0	0.00%	Below Average

The above shows the result of participants' performance on each test item answered correctly. The result revealed that 15 participants (31.25%) answered question 1 correctly. The remaining 33 participants (68.75%) were unable to answer question 1 correctly because the participants were unable to recognize that angle formed by tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment. The result again revealed 4 participants (8.33%) answered test item 2 correctly and the remaining 44 participants representing 91.67% were unable to answer it correctly because they had difficulty in recognizing that the interior angle of a cyclic quadrilateral is equal to the exterior angle of the quadrilateral and also had difficulty recognizing that angles subtended from the same chord or arc formed on the circumference are equal. Test item 3 was answered correctly by 13 participants (27.08%), the remaining 35 participants (72.92%) who were unable to answer the test item had difficulty recognizing that angles subtended by an arc of a circle at the center is twice the angle at the circumference. Test item 4 was attempted by 43 participants

(89.5%) and among which only 4 participants (8.33%) answered the test item correctly. The 5 participants (10.4%) who did not attempt the test item and the 43 participants (89.6%) who were unable to answer the test item had difficulty recognizing that angles on a straight line sum up to 180^0. Again, they were unable to recognize that triangle BOC is isosceles and therefore they were unable to apply the angle properties of isosceles triangle. They were also unable to recognize that angles subtended from a diameter formed on the circumference is right angle and finally participants were unable to recall that the sum of the interior angle of a triangle is 180^0. Test item 5 was attempted by 37 participants (77.08%), the 11 participant (22.91%) who did not attempt the test item and the 36 participants (81.81%) who answered wrongly had difficulty in recognizing that angles subtended by an arc of a circle at the center is twice the angle at the circumference and also participants were unable to recognize that angles in opposite segments of any cyclic quadrilateral is supplementary. Test item 6 was attempted by 29 participants (60.41%). The 19 participants (39.5%) who did not attempt it and the 29 participants (60.41%) who answered the test item wrongly had difficulty recognizing that the lengths of tangents to a circle from an external points are equal and again, majority of participants were unable to recall that the sum of the interior angle of a triangle is 180^0. Test item 7 was attempted by 47 participants (97.91%), the one participant (2.08%) who did not attempt the test item and the 46 participants (95.83%) who answered the test item wrongly had difficulty recognizing that angles subtended by an arc of a circle at the center is twice the angle at the circumference and finally participants were unable to recall that the sum of the interior angle of a triangle is 180^0. Test item 8 was attempted by 21 participants (43.75%). The 27 participants (56.25%) who did not attempt it and the 21 participants (43.75%) who answered test item 8 wrongly had difficulty in recognizing that angle at a point is 360^0 and difficulty in recognizing that angle formed by tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment. Test item 9 was attempted by 16 participants (33.33%). The 32 participants (66.66%) who did not attempt the test item and the 14 participants (29.16%) who answered wrongly had difficulty recognizing that angle formed by tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment. Again, the participants had difficulty recognizing that angles subtended from a diameter formed on the circumference is right angle and finally, participants had difficulty to recall that the sum of the interior angle of a triangle is 180^0. Test item 10 was attempted by 9 participants (18.75%). The 39 participants (81.25%) who did not attempt the test item and the 9 participants (18.75%) who answered wrongly had difficulty recognizing that angles subtended from a diameter formed on the circumference is right angle. They also had difficulty recalling that the sum of the interior angle of a triangle is 180^0 and finally had difficulty recognizing that angles in opposite segments of any cyclic quadrilateral is supplementary.

A sample of common error made by the participants on test item 1 is shown in Box 4.1.

Box 4.1: A sample of how a participant worked on test item 1

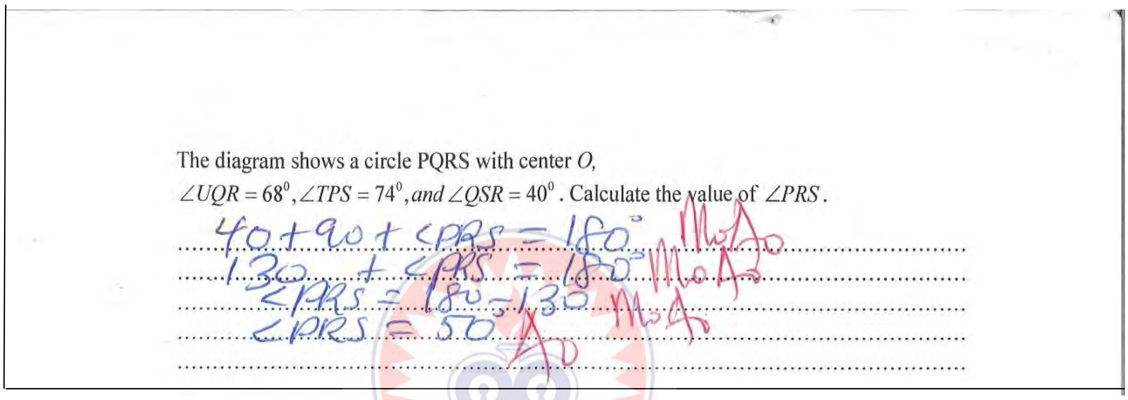

1. In the diagram below, $\angle ZXY = y^o$, $\angle ZYX = (y+17)^o$, $\angle PZX = x^o$ *and* $\angle YZT = (2x-43)^o$. Find the value of y.

$2x - 43 = y + 17$ MoAo
$2x - y = 17 + 43$
$2x - y = 60$ — (1) MoAo
$x = y \Rightarrow x - y = 0$ — (2) MoAo
$2y - y = 60$
$y = 60$ Ao

From Box 4.1, the participant had difficulty in recognizing that angle formed by tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment. The participant perceived that line PT and line XY are parallel and therefore angles TZY and XYZ are alternating.

A sample of common error made by the participants on test item 2 is shown in Box 4.2

Box 4.2: A sample of how a participant worked on test item 2


The diagram shows a circle PQRS with center O, $\angle UQR = 68^0, \angle TPS = 74^0, and\ \angle QSR = 40^0$. Calculate the value of $\angle PRS$.

$40 + 90 + \angle PRS = 180^\circ$

$130 + \angle PRS = 180^\circ$

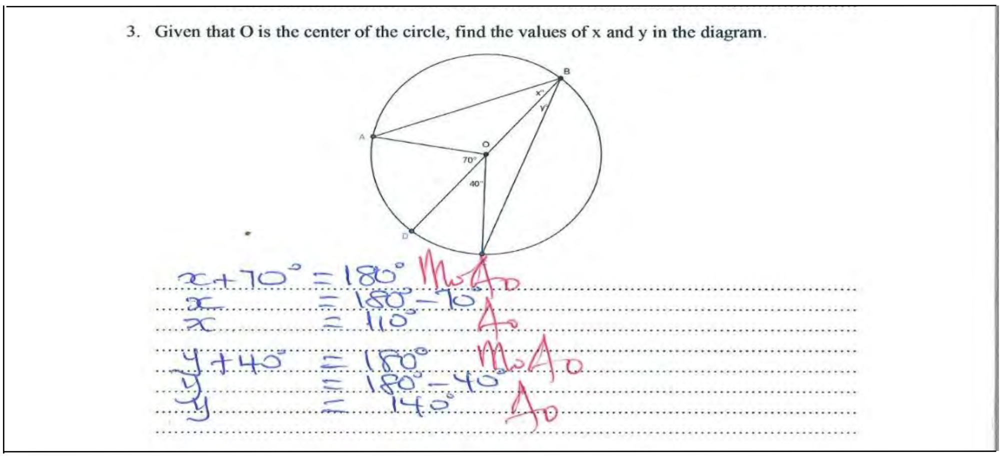
$\angle PRS = 180 - 130$

$\angle PRS = 50^\circ$

From Box 4.2, the participant had difficulty in recognizing that the interior angle of a cyclic quadrilateral is equal to the exterior angle of the quadrilateral and also had difficulty recognizing that angles subtended from the same chord or arc formed on the circumference are equal. The participant perceived that the quadrilateral is a kite or a rhombus and therefore the lines SQ and PR intersect at a right angle. See Appendix E for marking scheme for test item 2.

A sample of common error made by the participants on test item 3 is shown in Box 4.3

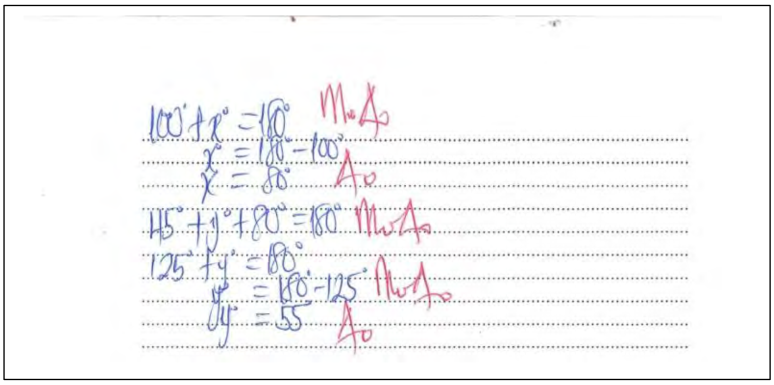
Box 4.3: A sample of how a participant worked on test item 3



From Box 4.3, the participant had difficulty in recognizing that the angle at the centre of a circle is twice the angle at the circumference subtended by the same arc or chord. The participant again had difficulty in recognizing that the triangles AOB and BOC are isosceles. The participant perceived that since BD is a straight line then $x + 70 = 180^0$ (i.e. angles on a straight line sum up to 180^0). See Appendix E for marking scheme for test item 3.

A sample of common error made by the participants on test item 4 is shown in Box 4.4

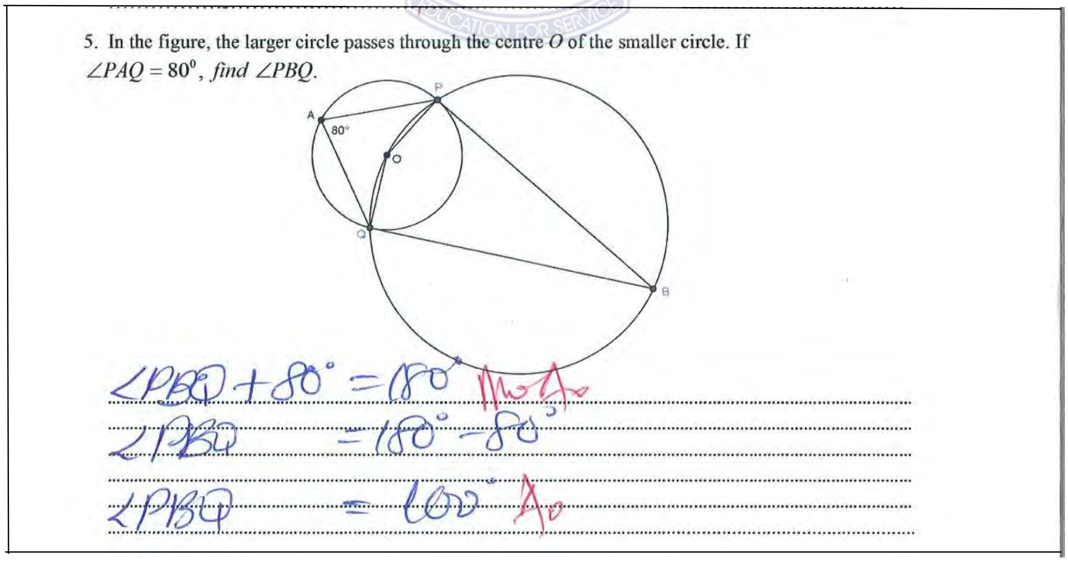
Box 4.4: A sample of how a participant worked on test item 4



The participant had difficulty in recognizing that $\angle DOB$ is a straight angle. Again, the participant had difficulty to recognize that the angles on a straight line sum up to 180^0. The participant had difficulty recognizing that triangle BOC is isosceles. The participant had difficulty to recognize that angles subtended from a diameter formed on a circumference is right angle (90^0) and finally, the participant had difficulty to recognize that the sum of the interior angle of a triangle is o 180^0. The participant perceived that the quadrilateral DFCO is cyclic which is not. Again, the participant perceived that $\angle BED$ is 80^0 which is why the participant stated $45^0 + y^0 + 80^0 = 180^0$. See Appendix E for marking scheme for test item 4.

A sample of common error made by the participants on test item 5 is shown in Box 4.5

Box 4.5: A sample of how a participant worked on test item 5

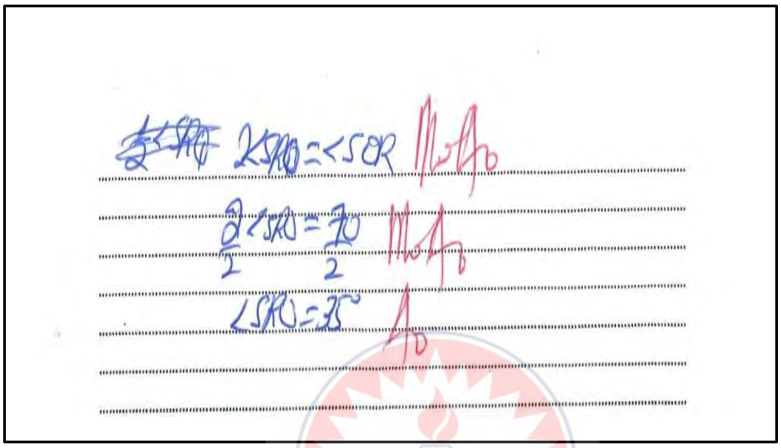


From Box 4.5, the participant had difficulty recognizing that the angle at the centre of a circle is twice the angle at the circumference subtended by the same arc or chord. Further, the participant had difficulty recognizing that BPOQ is a cyclic quadrilateral.

The participant perceived that APBQ is cyclic quadrilateral and therefore stated that $\angle PBQ + 80^0 = 180^0$. See Appendix E for marking scheme for test item 5.

A sample of common error made by the participants on test item 6 is shown in Box 4.6

Box 4.6: A sample of how a participant worked on test item 6

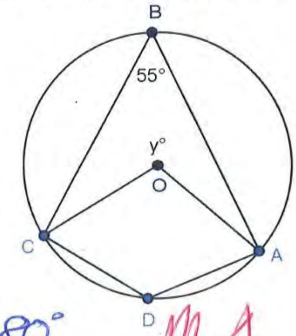


From Box 4.6, it was observed that the participant had difficulty in recognizing that tangents to a circle from an exterior point are equal and also could not recognize that the point of intersection of the radius and the tangent on the circumference is 90^0. Finally, the participant had difficulty recognizing that the sum of the interior angle of a triangle is 180^0. The error the participant committed was stating that $2\angle SRO = \angle SOR$ meanwhile no two angles were subtended from the same arc or chord formed at the center and at the circumference. See Appendix E for marking scheme for test item 6.

A sample of common error made by the participants on test item 7 is shown in Box 4.7.

Box 4.7: A sample of how a participant worked on test item 7

7. In the diagram, O is the centre of the circle ABCD. If $\angle ABC = 55^0$, find the value of y.



$55° + y° = 180°$ M1 A0

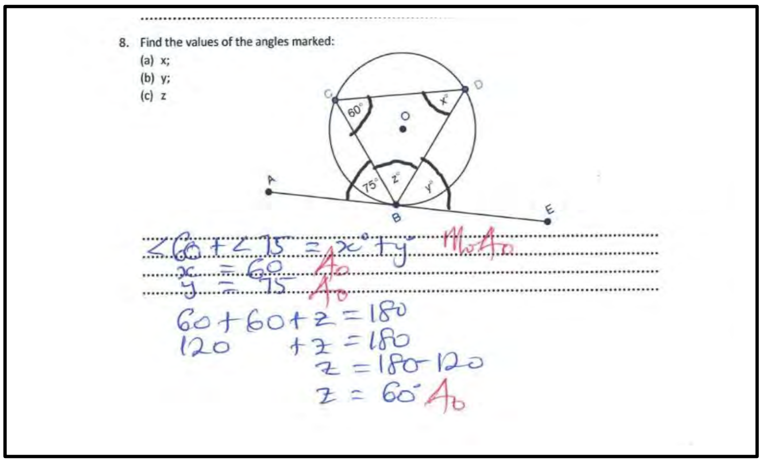
$y = 180° - 55°$

$y = 125°$ A0

From Box 4.7, the researcher observed that the participant had difficulty recognizing the angle at the centre of a circle is twice the angle at the circumference subtended by the same arc or chord. The participant had difficulty recognizing that the opposite angles in a cyclic quadrilateral are supplementary. The participant committed an error by saying that $55^0 + y^0 = 180^0$ instead of $55^0 + \angle CDA = 180^0$ (cyclic quadrilateral). See Appendix E for marking scheme for test item 7.

A sample of common error made by the participants on test item 8 is shown in Box 4.8

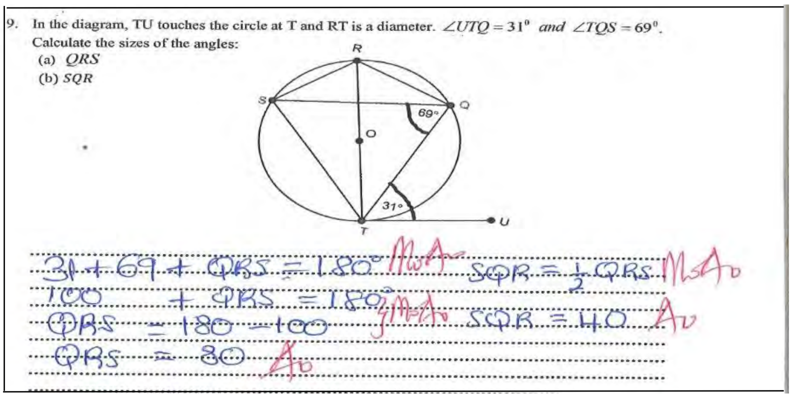
Box 4.8: A sample of how a participant worked on test item 8



From Box 4.8, the researcher observed that the participant had difficulty recognizing that the angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment. It was also observed that the participant had difficulty recognizing that the sum of the interior angle of a triangle is 180^0. The participant committed error by saying $\angle 60 + \angle 75 = x^0 + y^0$ and proceeding to conclude that x is 60^0 and y is 75^0. See Appendix E for marking scheme for test item 8.

A sample of common error made by the participants on test item 9 is shown in Box 4.9

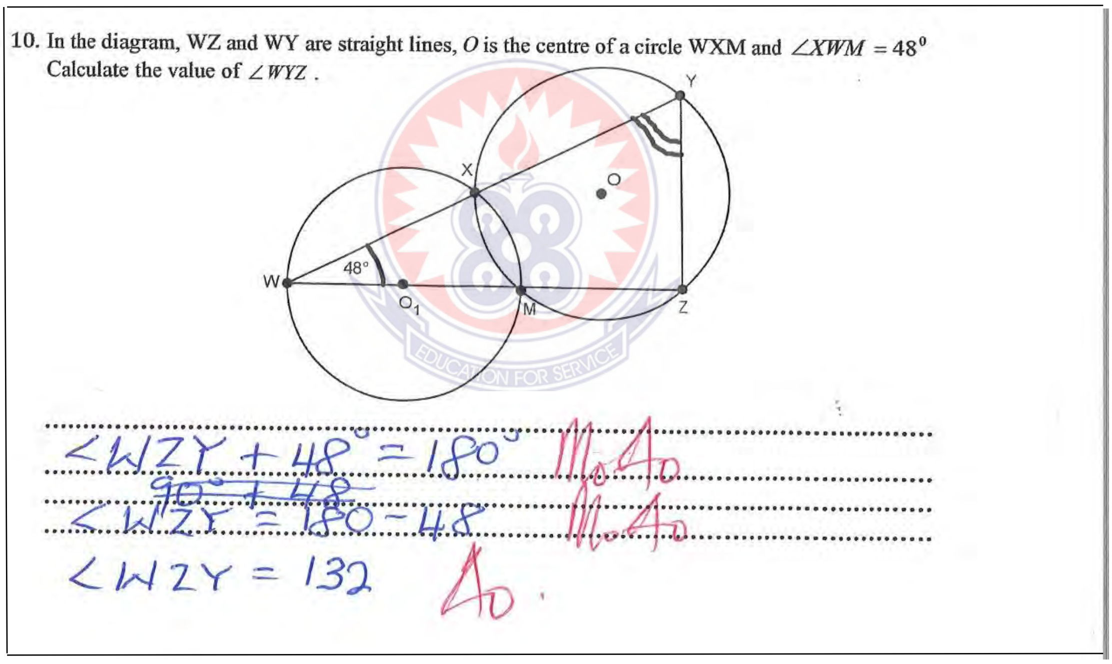
Box 4.9: A sample of how a participant worked on test item 9



From Box 4.9, the researcher observed that the participant had difficulty recognizing that the angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment. Again, the participant had difficulty recognizing that the angle subtended by the diameter on the circumference is 90^0. It was also observed that the participant had difficulty recognizing that the sum of the interior angle of a triangle is 180^0. The participant committed an error by saying that $31^0 + 69^0 + \angle QRS = 180^0$.

A sample of common error made by the participants on test item 10 is shown in Box 4.10

Box 4.10: A sample of how a participant worked on test item 10



From Box 4.10, it was observed that the participant had difficulty recognizing that the angle subtended by the diameter on the circumference is 90^0. Again, it was also observed that the participant had difficulty recognizing that the sum of the interior angle of a triangle is 180^0 and finally the participant had difficulty recognizing that the opposite angles in a cyclic quadrilateral are supplementary. It was observed that the participant perceived that $\angle WZY + 48^0 = 180^0$ which is an error..

4.3. Research Question 2: To what extent will the use of think-pair-share address Asankrangwa Senior High School Form 2 students' difficulty in solving problems in circle theorem-plane geometry II?

In answering the second research question, the results obtained in the pre-test and post-test were examined and compared for the two groups-experimental and control groups. The next two sections presents the descriptive statistics on the overall performance of the students before and after the experiment.

4.3.1 Performance of the Students Taught Circle Theorem-plane Geometry using the Think-Pair-Share Approach.

The results of analysis of the effect of think-pair-share teaching approach on students' performance is presented in Table 4.4

Table 4.4: Descriptive statistics of Students Taught Circle Theorem-Plane Geometry II using the Think-Pair-Share Approach (N=48)

Test	Minimum	Maximum	Mean	Std. Deviation
Pre-test	4	25	12.88	5.698
Post-test	7	31	18.92	6.782

Source: Field work, 2021

Table 4.4 compares the pre-test and the post-test results of participants in the experimental group. The results showed an improvement in participants understanding of circle theorem-plane geometry II in the post-test. The minimum score and maximum obtained by participants in the pre-test were 4 and 25 respectively out of 50. However, the minimum score in post-test was 7 whiles the maximum score was 31. The mean score of students in the pre-test was 12.88, while that of the post-test was 18.92, an increase of 6.04. This improvement in scores might be due to the use of think-pair-share approach in teaching circle theorem-plane geometry II. To ascertain whether or not the difference observed in the means are statistically different when taught with think-pair-share approach, a paired sample t-test was conducted to compare the pre-test and the post-test scores. Table 4.5 presents the results of the paired sample t-test on the pre-test and post-test performance of students taught with think-pair-share approach.

Table 4.5: Results of the paired sample t-test on the pre-test and post-test performance of participants taught Circle Theorem-Plane Geometry II using Think-Pair-Share approach (Experimental Group)

Test	Mean Difference	Std. Deviation	Std. Error Mean	*t*	*df*	Sig.	Eta square
Pre-test- Post-test	6.042	2.274	.464	13.015	23	< .001	.975

Source: Field work, 2021

A paired sample t-test was conducted to compare the pre-test and post-test scores of the participants taught with think-pair-share teaching approach. The paired sample t-test was examined to find out if the means core difference (M=6.042, SD=2.274) between the post-test and the pre-test of the experimental group was statistically significant. This was done to evaluate the effect of think-pair-share on participants' achievement in circle theorem. The results from Table 4.5 indicate a statistically significant increase in participants' achievement from pre-test to post-test, $t(23)=13.015$, $p< .001$. The eta squared statistics (.975) indicate large effect size. This means that 97.5% of the variance in the scores of the achievement tests (pre-test and post-test) of the experimental group was elucidated by the teaching approach (think-pair-share) for teaching circle theorem-plane geometry II. Again, the results imply that after the participants had gone through the intervention, their understanding in solving problems involving circle theorem-plane geometry II was enhanced significantly. This means that think-pair-share as an instructional tool had a positive impact on the participants' achievement in circle theorem-plane geometry II. Box plot was used to show the significant improvement of the experimental group from pre-test to post-test as shown in Figure 4.1.

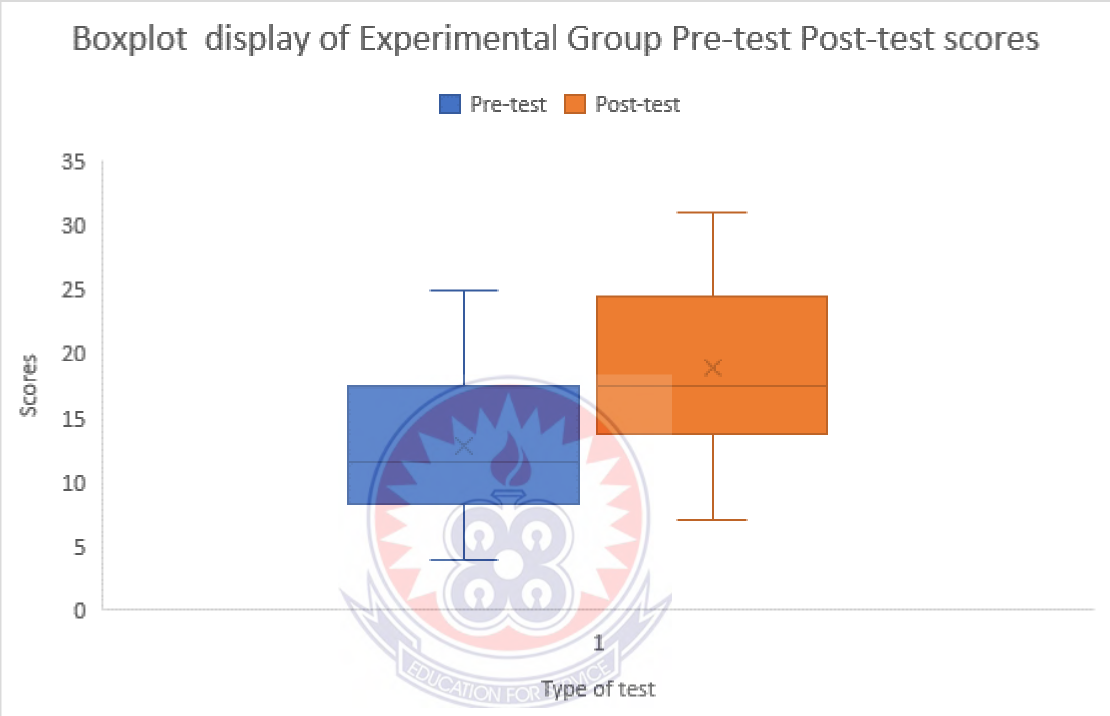


Figure 4.1: Box plot showing the difference in performance between the pre-test and post-test scores of participants Taught Circle Theorem-Plane Geometry II using Think-Pair-Share Approach

In this graphical presentation, Figure 4.1 displays the extent in which performance of participants has been enhanced. There has been significant increase in the achievement test scores of participants from pre-test to post-test.

4.4.2 Performance of Participants Taught Circle Theorem-Plane Geometry II using Traditional Teaching Approach

The result of the analysis of the traditional teaching approach on participants' performance is presented in Table 4.6.

Table 4.6: Descriptive statistics of participants Taught Circle Theorem-Plane Geometry II using Traditional Teaching Approach

Test	N	Minimum	Maximum	Mean	Std. Deviation
Pre-test	24	3	24	11.25	6.052
Post-test	24	8	32	18.50	6.711

Source: Field work, 2021

Table 4.6 shows comparison of the pre-test and post-test results of participants with the control group. The minimum score participants obtained in the pre-test was 3, while the maximum score was 24. However, in the post-test, the minimum score was 8, while the maximum score was 32. The mean score of participants in the pre-test was 11.25, while that of the post-test was 18.50, an increase of 7.25. This is an indication that in the post-test, most of the participants in the control group performance was enhanced. To determine whether or not the difference observed in the means are statistically different, a paired sample t-test was conducted to compare the pre-test and the post-test scores. Table 4.7 presents performance of students taught with Traditional learning approach.

Table 4.7: Paired Sample t-test of the Participants Taught Circle Theorem-Plane Geometry II using Traditional Teaching Approach

Test	Mean Difference	Std. Deviation	Std. Error Mean	*t*	*df*	Sig.	Eta Square
Pre-test-Post-test	7.250	3.365	.687	10.554	23	< .001	.846

Source: Field work, 2021

A paired sample t-test was examined to compare the pre-test and post-test scores for the students taught with traditional teaching approach (control group). The result as presented in table 4.7 reveals that the mean score difference between the post-test and pre-test of the control group was 7.250 with corresponding standard deviation of 3.365. The paired sample t-test was examined to find out if the mean score difference (M = 7.250, SD = 3.365) between the post-test and pre-test of the control group was statistically significant. This was done to assess the effect of traditional teaching approach on participants' achievement in circle theorem-plane geometry II. The results from Table 4.7 indicated a statistically significant increase in participants' achievement from pre-test to post-test, t (23) = 10.554, p = 0.001. The eta square statistics (.846) discovered that traditional teaching approach also has large effect on participants' performance in circle theorem-plane geometry II. Graphically, Figure 4.2 displays the extent to which participants' performance was enhanced. There was a significant improvement in the scores of the control group after the treatment. From the result, it can be seen that participants gained from the traditional teaching approach. This results is an indication that a well-structured traditional teaching approach can also improve participant's performance in learning circle theorem-plane geometry II.

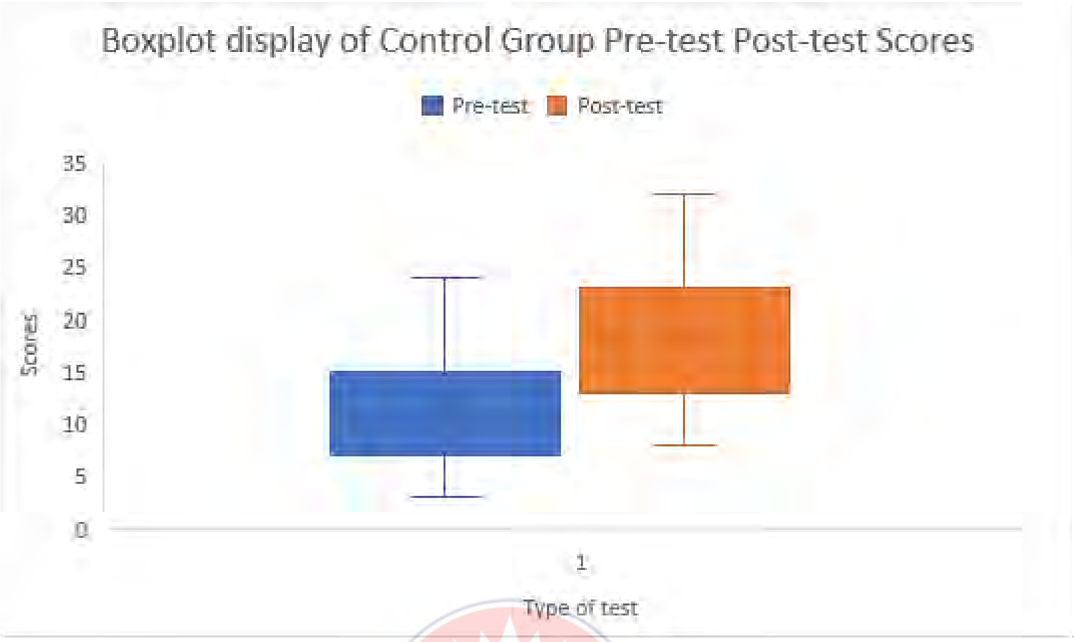


Figure 4.2. Box plot showing the difference in performance between the pre-test and post-test scores of participants Taught Circle Theorem-Plane Geometry II using Traditional Teaching Approach

The enhancement in participants' performance could be the use of sequence of exercises the participants were taken through. In this group, the lesson was mainly based on the traditional method and there was no manipulation of think-pair-share. Participants were focused throughout the lesson. This means that if the traditional teaching approach integrate with series of activities, performance can be enhanced to some extent.

4.4.3 Comparing performance of Participants Taught Circle Theorem-Plane Geometry II using Think-Pair-Share Approach and those Taught with Traditional Approach

The Independent sample t-test compare the means of the two independent groups (the control group and the experimental group) in order to determine whether there is statistical evidence that the associated population are significantly different. The independent sample t-test compares the effectiveness of the Think-Pair-Share approach and the Traditional approach on the performance of participants' achievement scores. The scores of the pre-test and post-test were compared using the independent sample t-test with $\alpha = 0.05$, to adjust for pre-test differences that existed between the control group and the experimental group.

Table 4.8: Independent Sample T-Test of the Performance of Participants Taught Circle Theorem-Plane Geometry II using Think-Pair-Share Approach and Traditional Approach (N=48)

	Group	N	Mean	Std. Deviation	df	t-value	Sig.	Eta Squared
Post-test	1	24	17.75	5.795	46	-.641	.386	.390
	2	24	18.92	6.782				

The statistics from Table 4.8 shows an independent samples t-test which was conducted to compare the effect of using think-pair-share teaching approach and the traditional teaching approach. There was a statistically significant difference in the scores for participants who were taught circle theorem-plane geometry using think-pair-share, (M=17.75, SD=5.795); t (46) = -.958; *p* = .923 and participants taught with the traditional approach, (M=18.92, SD=6.782); t (46) = -.641; *p* = .386. This shows that participants who were taught with the think-pair-share teaching approach performed better than their counterpart who were taught using the traditional approach. The eta squared statistics value of .390 indicates a very large effect size. That is, 39.0% of the variance in the dependent variable (achievement test scores) is expounded by the independent variable (teaching method). This implies that the extent of the difference between the mean score performance of participants taught with think-pair-share approach and the traditional approach is very large. The high scores of participants in the experimental group indicates that the use of think-pair-share in the teaching of circle theorem-plane geometry II enhanced performance of participants. This means that when learners are taught using think-pair-share as an instructional tool, their performance would be enhanced to a greater extent than learners taught using traditional approach in most Ghanaian classrooms. Finding from the results support Choirul, Siti & Raden (2018) who found out that there was a statistically significant difference in mathematics achievement between students in the experimental group taught with think-pair-share and their counterpart in the control group in terms of problem solving ability. The finding is again validated by Akanmu (2019) who indicated that the use of think-pair-share improved students' performance in mathematics.

In brief, the import of the finding from this study shows that the participants taught with think-pair-share approach performed better than those taught with the traditional approach, further, the participants taught with think-pair-share were able to apply the appropriate circle theorem to solve problems which require application. Therefore, having observed the success that think-pair-share as an instructional tool had in this study, it would be appropriate to use it more often in the teaching and learning of mathematics in Ghanaian classrooms, as this could be the solution to students' difficulty in geometry, especially circle theorem-plane geometry in the West African Secondary School Certificate Examinations.

4.5: Research Question 3: Is there any significant difference in the performance of female and male students' taught circle theorem-plane geometry II using think-pair-share?

In answering research question 3, the results obtained in the pre-test and post-test were examined and compared for the both the experimental group and the control group. The next two sections present the descriptive statistics on the overall performance of the male and female participants' before and after the treatment.

4.5.1 Gender differences in mean achievement on circle theorem-plane geometry II before the treatment

Table 4.9 shows the independent sample t-test for the experimental group pre-test scores showing differences in mean achievement in circle theorem-plane geometry II of female and male participants taught using think-pair-share.

Table 4.9: Independent sample t-test results for experimental group pre-test scores showing gender differences in mean achievement on circle theorem-plane geometry II

Gender	N	Mean	Std. Deviation	*df*	t-value	*p*-value
Female	14	12.50	5.004	22	-.374	.114
Male	10	13.40	6.802			

Source: Fieldwork, 2021

The result in Table 4.9 indicates there was no statistically significant difference between the mean scores of females (M = 12.50, SD = 5.004) and male (M = 13.40, SD = 6.802) in the experimental group thus [*t* (22) = -0.374, p = 0.114. The result in table 4.9 shows that the male participants achieved better than their female participants, the results indicates that the difference in the mean scores happened by chance rather than intention since there was no evidence to suggest that any significant differences existed between female and male participants in the experimental group. This results therefore suggest that the performance of both the female and male participants of the experimental group in the pre-test were almost the same before the treatment was administered.

The independent sample t-test results for the control group's pre-test scores showing differences in mean achievement on circle theorem-plane geometry II of female and male participants taught without think-pair-share are presented in Table 4.10.

Table 4.10: Independent sample t-test results for control group pre-test scores showing gender differences in mean achievement on circle theorem-plane geometry II

Gender	N	Mean	Std. Deviation	*Df*	t-value	*p*-value
Female	14	11.21	4.949	22	-.033	.067
Male	10	11.30	7.631			

Source: Fieldwork, 2021

The results in Table 4.10 show that there was no statistically significant difference between the mean scores of female (M = 11.21, SD = 4.949) and male (M = 11.30, SD = 7.631); *t* (22) = 0.033, p = 0.067). This result implies that performance of both female and male participants in the control group in terms of ability was similar before treatment was administered.

4.4.3 Gender difference in mean achievement on circle theorem-plane geometry II after treatment

With respect to differences in mean achievement on circle theorem-plane geometry II of female and male participants taught using, or without, think-pair-share after the post-test, an independent sample t-test was conducted to test the null hypothesis that "there is no significant difference in the performance of female and male students' taught circle theorem-plane geometry II using think-pair-share". The results are presented in Tables 4.11 and 4.12.

Table 4.11: Independent sample t-test results for experimental group's post-test scores showing gender differences in mean achievement on circle theorem-plane geometry II

Gender	N	Mean	Std. Deviation	*Df*	*t*-value	*p*-value
Female	14	18.64	6.767			
				22	.229	.742
Male	10	19.30	7.150			

Source: Fieldwork, 2021

The result from the independent sample t-test in Table 4.11 which was conducted to find out the difference in the mean scores of the females and males in the experimental group in the post-test.. The result shows that there was no statistically significant difference between the mean scores of female (M = 18.64, SD = 6.767) and male (M = 19.30, SD = 7.150); t (22) = 0.229, p = 0.742). Even though the result in Table 4.11 reveals that the male participants achieved slightly better in the post-test than their female counterparts, the result shows that the difference in the mean score was due to chance since there was no sufficiently enough evidence to conclude that significant differences existed between males and females participants in the experimental group. The results therefore suggest that both the female and male participants of the experimental group in the post-test performed at the same level to some extent. Further, the outcome of the analysis connotes that the effect of think-pair-share as an instructional tool on the achievement of female and male participants did not differ significantly, therefore, the use of think-pair-share as an instructional tool in teaching circle theorem-plane geometry II had no significant effect on gender difference in mathematics achievement.

4.5. Discussion of Findings

The abysmal performance of participants in the achievement tests revealed that Asankrangwa Senior High School Form 2 students' have difficulties in solving problems in Circle Theorem-Plane Geometry II. The findings revealed that participants were unable to state and use the circle theorem; participants were unable to identify the tangent as perpendicular to the radius at the point of contact; participants were unable to verify that the angle between the tangent and the chord at the point of contact is equal to the angle in the alternate segment and lastly, participants were unable to verify that tangents drawn from an external point to the same circle are equal, when measured from their point of contact. These difficulties were observed as a result of teaching approach. This confirms the findings of a study conducted by Jones (2002). The study revealed that teachers teach geometry by informing students of the properties associated with plane or solid shapes.

Findings from Research Question 2 revealed that participants taught Circle Theorem-Plane Geometry II using Think-Pair-Share approach to a large extent performed better than those taught using the traditional approach. The study of Afthina, Mardiyana and Pramudya (2017) on Think-Pair-Share using realistic mathematics education approach in the teaching and learning of geometry elucidated mathematics achievement. This finding is confirmed by the finding of a study conducted by Choirul, Siti and Raden (2018). The study revealed that Think-Pair-Share learning with contextual approach improved students' problem solving ability. Similarly, Miratika, Asmin, Mulyono and Minarni (2018) confirmed that Think-pair-Share approach enhanced the problem solving and mathematics learning outcomes of students. According to Akanmu (2019), the use of Think-Pair-Share approach improved students' performance in mathematics. Further, the findings of Yarisda (2019) revealed that mathematics learning outcomes of students who used cooperative learning model of Think-Pair-Share were better than the mathematics learning outcome of students who use conventional learning. Finally, in the findings of Bertha and Athanasius (2019), it was elucidated that the effectiveness of Think-Pair-Share cooperative learning model improves performance.

The study elucidated that gender gabs in mathematics achievement was not significant. According to the study of Lindberg, Hyde, Petersen, and Linn (2010), it was revealed that gender gabs in mathematics achievement was not significant. Similarly, in a study conducted by John and Benjamin (2015), on gender differences in mathematics achievement and retention scores, it was revealed that male and female students did not significantly differ in achievement and retention scores. Further, Akanmu (2019) also discovered that gender of student does not affect students' mathematics performance. Finally, the study conducted by Susana, Bibiana, Isabel, Iris and Antonio (2020) found no significant gender differences in academic performance of their study.

In summary, the findings from the study elucidate that think-pair-share as an instructional tool can improve students' mathematical problem solving in geometry especially Circle Theorem-Plane Geometry II.

CHAPTER 5

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.0 Overview

This chapter presents a summary of the findings, conclusion and the recommendations to the areas that call for further research.

5.1 Summary

The purpose of the study was to determine the effect of using think-pair-share as an instructional tool in the teaching and learning of circle theorem-plane geometry II on the performance of Senior High School 2 students in the Amenfi West Municipality. Again, the study examined the effect of teaching strategies (Think-pair-share approach and traditional approach) on the performance of students with respect to gender.

The study used quasi-experimental research design (nonequivalent control group design with pre-test and post-test). The researcher targeted all SHS 2 population of Asankrangwa SHS. Two intact classes were used for the study; twenty-four (24) General Art B1 students as control group and twenty-four (24) General Art B2 students as experimental group making a total of forty-eight (48) students and two (2) mathematics teachers, the researcher and the General Arts B1 mathematics teacher were used in the study. Purposive sampling technique was used to select the two intact classes for the study. The data collection was done by the researcher and the General Arts B1 mathematics teacher.

Teacher-made Mathematics Achievement Tests for both Pre-test and Post-test were the instrument used in the study. Both the Pre-test and Post-test items were based on the Senior High School Mathematics 2 Syllabus for Core Mathematics.

The Pre-test scores and the Post-test scores were analyzed using Statistical Package for Social Science (SPSS) software version 28. Percentages, frequencies, mean scores, and standard deviations were the descriptive statistics used to describe the general performance of the participants. The independent sample t-test was used to compare the mean scores of the pre-test achievement score between the control group and the experimental group before the treatment. It was to determine whether there is statistical evidence that the associated population are significantly different. Again, the independent sample t-test was used to determine whether there was significant difference between the control group and the experimental on post-test scores. Additionally, the independent sample t-test was used to determine whether there was significant difference between female and male students' performance with respect to the teaching approach. Paired sample t-test was used to examine whether significant difference exists in the pre-test post-test of the control group and the experimental group after the treatment. The key findings of the study are summarized below.

5.2. Major Findings

The purpose of the study was to determine the effect of using think-pair-share as an instructional tool in the teaching and learning of circle theorem-plane geometry II on the performance of Senior High School 2 students. The major findings from the study show that:

- Performance of students who were taught circle theorem-plane geometry II with think-pair-share approach to large extent was enhanced greatly than those who were taught using the traditional method of teaching and learning.

- The study revealed that if think-pair-share as an instructional tool is appropriately implemented in the teaching and learning of circle theorem-plane geometry II, performance of students will improve.
- The findings again indicate that the interactive nature of think-pair-share as an instructional tool would provide an opportunity for students to explore and cooperate with other students, both their classmates, expressing opinion and responding to other students' opinions to enhance performance of circle theorem-plane geometry II than those taught using the traditional approach as has been highlighted in the study.
- The findings also reveal that Think-pair-share as an instructional tool would arouse students interest in mathematics and make students more active and socialize, and actively participate in any mathematical discourse through critical thinking to enhance performance of circle theorem-plane geometry II more than students taught using the traditional approach as an instructional tool.

5.3 Conclusion

The study, therefore, concludes that think-pair-share is one of the solutions to poor performance in questions involving circle theorem-plane geometry II. This implies that if think-pair-share is used in the teaching and learning of circle theorem-plane geometry II in senior high schools in Ghana, it will enhance students' performance.

5.4 Recommendations

From the summary of the main findings of the study, certain recommendations are made for mathematics teachers, policy makers, school authorities and administrators and future researchers.

1. The Head Master of Asankrangwa Senior High School should organize in-service training for mathematics teachers to equip them with the necessary skills on the use of think-pair-share strategy as an instructional tool to enhance performance in the teaching and learning of mathematics and other subjects.
2. SEIP should encourage mathematics teachers to employ think-pair-share as an instructional tool since it has the potency to improve performance of both male and female students' towards mathematics if mathematics teachers appropriately use think-pair-share in teaching and learning mathematics. This is because the study has shown that the use of think-pair-share as an instructional tool improves the performance of both male and female.
3. Mathematics teachers should consider students' knowledge on types of triangles, their properties, properties of quadrilaterals and progressing to Plane Geometry 11. Most students' difficulties in solving problems involving circle theorem-plane geometry II is as a result of the students' inability to differentiate types of plane figures and their properties and relating them to geometry.
4. Ministry of Education and the Ghana Education Service should increase the number of workshops and in-service training for mathematics teachers on best strategies such as Think-pair-share to be used to teach circle theorem-plane geometry II and other geometry related topics.
5. Finally, mathematics is dynamic, therefore mathematics teachers' professional development is very necessary, and therefore the mathematics teacher should

subscribe to interactive forms of teaching and learning mathematics to make the mathematics lesson very interactive to involve every student.

5.5 Suggestions for further studies

Following the focus of this study, few areas are suggested for further studies. The study concentrated on examining the effect of think-pair-share in teaching only circle theorem-plane geometry II. A study can be undertaken on using think-pair-share to investigate other areas of geometry and any mathematics related topic. This will not only provide enough literature on the importance of using think-pair-share but also know that think-pair-share as an instructional tool is applicable in every level of learning provided it is implemented appropriately.

Further and detailed studies that will involve all 125 SEIP schools need to be done to establish the causes of consecutive poor trend in WASSSCE core mathematics and attempt to compare the usefulness of think-pair-share as an instructional tool with others to teach circle theorem-plane geometry II and geometry related topics and other topics.

REFERENCES

Abreh, M.K., Owusu, K.A., & Amedahe, F.K. (2018). *Trends in Performance of WASSCE Candidates in the Science and Mathematics in Ghana: Perceived Contributing Factors and the Way Forward.*

Afthina, H., Mardiyana, M., & Pramudya, I. (2017). *Think pair share using realistic mathematics education approach in geometry learning*.

Ahmed, S.A.A.S. (2006). The effect of using (Think-Pair-Share) strategy in the development of critical thinking in math and in the attitudes of life for the students of the preparatory stage, *Unpublished MA Thesi*s, Suez Canal University, College of Education, Port Said, Egypt.

Ahmed, W., Minnaert, A., van der Werf, G., & Kuyper, H. (2010). Percieved social support and early adolescents' achievement: The meditational roles of motivational beliefs & and emotions. *Journal of Youth & Adolescence*.

Ajai, J.T., & Imoko, I.I. (2015). Gender differences in mathematics achievement and retention scores: A case of problem-based learning method. *International Journal of Research in Education and Science (IJRES)*

Akanmu, M. A., (2019). *Effects of think-pair-share on senior school students' performance in mathematics in Ilorin,* Nigeria.

ALTC (2012). Moderation checklist. *http://resource.unisa.edu.au/course/view.php?id=285&topic=1*

Anaduaka, U.S., (2018). *Effect of think-pair-share cooperative strategy on mathematics achievement of attention-deficit hyperactive study.*

Anamuah-Mensah, J., Mereku, D. K., & Asabere-Ameyaw, A. (2004). Ghanaian junior secondary school students' achievement in mathematics and science*: Results from Ghana's participation in the 2003 trends in international mathematics and science study.* Accra: Ministry of Education, National TIMSS Centre.

Arends, R. I. (2004). *Learning to teach (6th Ed.)*. New York: McGraw-Hill.

Ariana, S. (2013). Finding the Effects of Think-Pair-Share on Student Confidence and Participation. *Hornors Projects 28.*

Asante, K.O. (2010). Sex Differences in Mathematics Performance among Senior High Students in Ghana. *African Journals Online.*

Ato, K. A., & Adelaide, K.O. (2015). Gender Differences and Mathematics Achievement of Senior High School Students: A Case of Ghana National College. *Journal of Education and Practice, 6*(33).

Aunola, K. E., Leskinen, E., Lerkkanen, M.-K., & Nurmi, J.-E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology.*

Bailey, R. A. (2008). *Design of Comparative Experiments*. Cambridge University Press. ISBN 978-0-521-68357-9.

Bassey, S.W., Joshua, M.T., & Asim, A.E. (2009). Gender Differences and Mathematics Achievement of Rural Senior Secondary Students in Cross River State, Nigeria. *African Journals Online, 10*

Battista, M. T. (2007). The Development of Geometric & Spatial Thinking. *In F. K. Lester (Ed.), Second H & book of Research on Mathematics Teaching* and *Learning. Pp. 843-908*. Charlotte, NC: Information Age.

Bertha, H & Athanasius, L. (2019). The Effect of Think-Pair-Share Cooperative Learning Model on Grade Twelve (12) Learners' Performance in Quadratic Functions: A Case Of Twashuka Secondary School in Luanshya. *International Journal of Research and Innovation in Social Science.*

Bhandari, P. (2020). *Understanding internal validity*.

Butin, D. W. (2010). *The education dissertation: A guide for practitioner scholars.* Thousand Oaks, CA: Corwin.

Carl, M.B. (2020). *How interactive learning will shape the classrooms of the future*.

Choirul, A., Siti, M.A., & Raden, S. (2018). The Effect of Think-Pair-Share Learning with Contextual Approach on Junior High School Students' Mathematics Problem Solving Ability. *Proceedings of Mathematics, Informatics, Science and Education International Conference.*

Collins, A., Brown, J.S. & Newman, S.E. (1989). Cognitive apprenticeship: Teaching the craft of reading, writing and mathematics. In L.B. Resnick (ed.). Knowing, learning and instruction: Essays in honor of Robert Glaser. Hillsdale, NJ: Erlbaum. *Originally circulated in ms, 1987*.

Collins, C., Kenway, J., & McLeod, J. (2000*). Factors Influencing the Educational Performance of Males and Females in School and their Initial Destinations after living school.* Commonwealth Department of Education, Training and Youth Affairs.

Cooper, J. L., & Robinson, P. (2002). *Getting started: Informal small-group strategies in large classes.* New Directions for Teaching & Learning.

Corden, R. (2001). Group discussion and the importance of a shared perspective: learning from collaborative research. *Qualitative Research, 1*(3), 347-369.

Creswell, J. W. (2008). *Educational research: Planning, conducting and evaluating quantitative and qualitative research (3rd ed.).* Upper Saddle River, NJ: Merrill.

Creswell, J. W. (2009). *Research Design: Qualitative, quantitative and mixed methods approach (3 ed.).* Thousand Oaks, CA: Sage.

Curriculum Research Development Division (2012). *Teaching syllabus for Core Mathematics (Senior High School).* Accra: Ministry of Education.

Danielsen, A. G., Wiium, N., Wilhelmsen, B., & Wold, B. (2010). Perceived support provided by teachers and classmates and students' self-reported academic initiative. *Journal of School Psychology*.

Eccles, J. S., & Roeser, R. W. (2011). Schools as developmental contexts during adolescence. *Journal of Research on Adolescence*.

Else-Quest, N. M., Hyde, J., & Linn, M. C. (2010). Cross-National Patterns of Gender Differences in Mathematics: A Meta-Analysis. *Psychological Bulletin, 136* (1), 103-127.

Eric, M.W. (2001). Gender Differences in Mathematics Achievement in Primary Schools in Ghana. *Mathematics Connection 2(1).*

Fletcher, J. (2003). Constructivism and mathematics education in Ghana. *Journal of the Mathematical Association of Ghana.*

Fraenkel, J. R., & Wallen, N. E. (2003). *How to design and evaluate research in education, (5th ed.).* New York: McGraw-Hill Publishing Co.

Fraenkel, J. R., & Wallen, N. E. (2009). *How to design and evaluate research in education (7th ed.).* New York: McGraw-Hill Publishing Co.

Fredricks, J. A., & Eccles, J. S. (2002). Children's competence and value beliefs from childhood through adolescence: Growth trajectories in two male-sex-typed domains. *Developmental Psychology, 38*(4), 519–533. https://doi.org/10.1037/0012-1649.38.4.519

Fredua-Kwarteng, Y., & Ahia, F. (2015, December 10). Confronting National Mathematics Phobia in Ghana' (Part 2). *GhanaHomePage. Retrieved from http://www.ghanaweb.com/GhanaHomePage/NewsArchives/artikel.php?ID=61432*

Frenzel, A. C., Pekrun, R., & Goetz, T. (2007). Girls and mathematics – A "hopeless" issue? A controlvalue approach to gender differences in emotions towards mathematics. *European Journal of Psychology of Education*.

Frenzel, A. C., Watt, H. M. G., Goetz, T., & Pekrun, R. (2010). Development of Mathematics Interest in German Adolescents: Influences of Gender, Family and School Context.

Gafoor, K. I. (2012). The effect of using (think, pair, share) strategy in Acquisition of mathematical concepts for Third stage students of Teachers Training institute. *Diyala Journal of Human Research.*

Gall, M. D., Gall, J. P., & Borg, W. R. (2005). *Educational research and introduction (7 ed.).* New York: Pearson Publication.

Ganley, C.M., & Lubienski, S.T. (2016). Mathematics confidence, interest, and performance: Examining gender patterns and reciprocal relations. *Learning and Individual Differences, 47.*

George, D., & Mallery, P. (2003). *SPSS for Windows step by step: A simple guide and reference, 11.0 update (4 ed.).* Boston: Allyn and Bacon.

Gherasim, L. R., Butnaru, S., & Mairean, C. (2013). Classroom environment, achievement goals & maths performance: gender differences. *Educational Studies.*

Githua, B. N., & Mwangi, J. G. (2003). Students' mathematics self-concept and motivation to learn mathematics: relationship and gender differences among Kenya's secondary-schools students in Nairobi and Rift Valley provinces. *International Journal of Educational Development.*

Heale, R., & Twycross, A. (2015). Validity and reliability in quantitative research. *Evidence-Based Nursing 18*(3), 66-67.

Helena, N. K. T., Eric, M.W., & Daniel, A. (2018). Gender Differences in Performance in Mathematics among Pre-Service Teachers in the Brong-Ahafo Region of Ghana. *International Journal of Education, Learning and Development, 6*(5), 38-45.

IBE-UNESCO (2008). "*The Right to Education for All: Global Perspectives in Inclusive Education*". Regional Preparatory Workshop on Inclusive Education-Gulf Arab States for the 48th Session of the ICE.

Johnson, B., & Christensen, L. (2008). *Educational research: Quantitative, qualitative, and mixed approaches.* Thousand Oaks, CA: Sage Publications.

Johnston-Wilder, S., & Mason, J. (2005). *Developing thinking in geometry*. London: Paul Chapman Publishing.

Jones, K. (2002). *Issues in the teaching and learning of geometry. In L. Haggarty (Ed.), Aspects of teaching secondary mathematics: Perspectives on practice.* London, UK: Routledge

Kerlinger, F. N. (2003). *Foundations of Behavioural Research*. Harcourt Brace Jovanovich.

Kline, P. (1999). *The handbook of psychological testing (2 ed.).* London: Routledge.

Lauren, T. (2021). *Quasi-experimental design: Definition, types and examples.*

Li, M. P., & Lam B. H. (2005). *Cooperative learning, Hong Kong:* Education of Hong Kong Institute.

Lim, C. S., & Hwa, T. Y. (2007). Promoting mathematical thinking in the Malaysian classroom: issues and challenges. *Centre for Research on International Cooperation in Educational Development (CRICED).* University of Tsukuba.

Lindberg, S. M., Hyde, J. S., Petersen, J. L., & Linn, M. C. (2010). New trends in gender and performance in mathematics: A meta-analysis. *Psychological Bulletin.*

Lom, B. (2012). Classroom Activities: Simple Strategies to Incorporate Student-Centered Activities within Undergraduate Science Lectures. *Journal of Undergraduate Neuroscience Education.*

Lubienski, S. T., Robinson, J. P., Crane, C. C., & Ganley, C. M. (2013). Girls' and boys' mathematics achievement, affect, and experience: Findings from ECLS-K. *Journal for Research in Mathematics Education*.

Lyman, F. (1981). *The responsive classroom discussion, in Anderson, A. S. (Ed.), Mainstreaming Digest*. College Park, MD: University of Maryland College of Education.

Lyman, F. (1981). Think-Pair-Share: An Ending Teaching Technique. *MAA-CIE Cooperative News.*

Martella, R. C., Nelson, J. R., & Marchand-Martella, N. E. (1999). *Research methods: Learning to become a critical research consumer.* Boston: Allyn and Bacon.

Mason, J. (2002). *Minding Your Qs and Rs: effective questioning and responding in the mathematics classroom, in L. Haggerty (Ed.) Aspects of Teaching Secondary Mathematics: perspectives on practice*. Routledge Falmer, London.

Mason, M. (2002). *The van Hiele levels of geometric understanding*. Retrieved from http:/jwilson.coe.uga.edu/EMAT8990/GEOMETRY, 2002.

Mathematics Association of Ghana (2001). *Mathematics Connection, Volume 2*.

McCarthy, J. P., & Anderson, L. (2000), "Active Learning Techniques versus Traditional Teaching Styles: Two Experiments from History and Political Science," *Innovative Higher Education*.

McMahon, M. (1997). *Social Constructivism and the World Wide Web-A Paradigm for Learning.* School of Multimedia and Learning Technologies. Edith Cowan University

Ministry of Education (2012). Teaching syllabus for mathematics (Senior High School). Accra: *Curriculum Research and Development Division (CRDD)*.

Ministry of Education. (2010). *National Syllabus for Senior High School Core mathematics.* Accra: Ministry of Education.

Miratika, ., Asmin, ., Mulyono, ., & Minarni, A. (2018). The Effect of Cooperative Learning of Type Think Pair Share Based on Mandailing Culture To Mathematical Problem Solving Ability Of The Students at Middle Secondary School Ali Imron Medan. *Advances in Social Sciences Research Journal,* 5(10). https://doi.org/10.14738/assrj.510.5239

Mugenda, O., & Mugenda, A. (2003). *Research Methods: Quantitative & Qualitative Approaches*. Nairobi, Kenya: Acts Press.

Nabie, M. J. (2013). *Understanding primary mathematics methods for teaching.* KAB Impression. Winneba – Ghana.

Nabie, M. J., Raheem, K., Agbemaka, J. B., & Sabtiwu, R. (2016). Multiple solutions approach (MSA): Conceptions and practices of primary school teachers in Ghana. *International Journal of Research in Education and Science (IJRES)*

Napitupulu, W.R., & Surya, E. (2017). The application of cooperative learning type tps (think pair share) in improving the ability of problem solving and mathematical learning results student. *Magister Program Mathematics Education, UNIMED*

National Council of Teachers of Mathematics (2003). *Principles and standards for school Mathematics.* Reston, V. A: NCTM.

Nystrand, M. (1996). *Opening dialogue: Understanding the dynamics of language and learning in the English classroom*. New York: Teachers College Press.

Ofori, R., & Dampson, D. G. (2011). *Research Methods and Statistics using SPSS*. Payless Publications Limited, Amakom, Kumasi Oglesby

O'Malley, P. (2010). *Students' evaluation: Steps for creating teacher-made test. Paper presented at Assessment Group Conference-School programme.* Maryland: Kennedy Krieger Institute.

Osafo-Affum, B. (2001). Mathematics crisis in our schools: Causes and remedies. *Mathematics Connection*.

Pallant, J. (2001). *SPSS survival manual: A step by step guide to data analysis using SPSS for windows (version 10 and 11).* Australia: Glasgow, Bell and Bam Ltd.

Panina, T.S., & Vavilova, L.N. 2008. *Modern methods of activation of training*. Moscow: Academy.

Parker, C. (2009). *Foundational Methods: Understanding Teaching and Learning*. Toronto: Pearson Education.

Patrick, H., Ryan, A. M., & Kaplan, A. (2007). Early adolescents' perceptions of the classroom social environment, motivational beliefs, and engagement. *Journal of Educational Psychology.*

Pekrun, R., Elliot, A. J., & Maier, M. A. (2006). Achievement goals and discrete achievement emotions: A theoretical model and prospective test. *Journal of Educational Psychology.*

Phungsuk, R., Viriyavejakul, I., & Ratanaolarn, T. (2017). Development of a problem-based learning model via a virtual learning environment. *Kasetsart Journal of Social Sciences, 38*(3)

Pierce, R., & Ball, L. (2009). Perceptions that may affect teachers' intention to use technology in secondary mathematics classes. *Educational Studies in Mathematics.*

Preckel, F., Götz, T., Pekrun, R., & Kleine, M. (2008). Gender Differences in Gifted and Average-Ability Students Comparing Girls' and Boys' Achievement, Self-Concept, Interest, & Motivation in Mathematics. *Gifted Child Quartey, 52*(2), 146-159.

Puklek L. M., & Zupancic, M. (2009). Achievement in math in early adolescence. The role of parental involvement, teachers' behaviour, and students' motivational beliefs about math. *Journal of Early Adolescence*.

Raba, A. A. A. M., & Harzallah, H. T. M. (2015). Effective Teaching from An-Najah National University M.A. Students' Perspectives. *Journal of Languages and Culture. htThink-Pair-Share://doi.org/10.5897/JLC2015.0325*.

Resnik, D. B. (2018). What is Ethics in Research and why is it important. *International Journal of Science and Research, 7(10)*

Riegle-Crumb, C., Farkas, G., & Muller, C. (2006). The Role of Gender and Friendship in Advanced Course Taking. *Sociology of Education, 79*(3), 206-228.

Rowe, M. (1972). *Wait-time and rewards as instructional variables: Their influence on language, logic, and fate control.* Columbia: New York City.

Sadler, D. R. (2009). Indeterminacy in the use of preset criteria for assessment and grading in higher education. *Assessment and Evaluation in Higher Education, 34*(2)

Secondary Education Improvement Project (2015). Module for Core Mathematics.

Shilubane, H. N. (2010). *University of Venda.* Retrieved May 4, 2015, from University of Venda web site: *http://uir.unisa.ac.za/bitstream/handle/10500/1450/04chapter3.pdf?sequence=5*

Shim, S. S., Ryan, A. M., & Anderson, C. J. (2008). Achievement goals and achievement during early adolescence: Examining time-varying predictor and outcome variables in growth-curve analysis. *Journal of Educational Psychology.*

Shuttlework, M. (2008). How to choose between different research methods: *Experiment Resource.* Retrieved February 23, 2016, from http://www.experiment-resources.com/different-research-methods.html

Sitorus, J., & Masrayati (2016). Students' creative thinking process stages: Implementation of realistic mathematics education. *Thinking Skill and Creativity.*

Slavin, R. (2007). *Educational research in an age of accountability*. Boston: Pearson Education.

Strutchens, M. E., Harris, K. A., & Martin, W. G. (2001). *Assessing geometric and measurement understanding using manipulatives mathematics teaching in the middle school.* Brunei: Brunei Works Press.

Susana, R., Bibiana, R., Isabel, P., Iris, E., & Antonio, V. (2020). Gender differences in mathematics motivation: Differential effects on performance in primary education. *Frontiers in Psychology, 10,* Article 3050. https://doi.org/10.3389/fpsyg.2019.03050

Tay, M. K., & Wonkyi, T.M (2018). Effect of using Geogebra on senior high school students' performance in circle theorems. *African Journal of Educational Studies in Mathematics and Sciences, 14.*

West African Examination Council, (2016). *Chief Examiners' Report.* Accra, Ghana: WAEC.

West African Examination Council, (2017). *Chief Examiners' Report.* Accra, Ghana: WAEC.

West African Examination Council, (2018). *Chief Examiners' Report.* Accra, Ghana: WAEC.

William, M. (2014). *Reliability and Validity of a Scale to Measure Prejudice toward Old Order Amish.*

Yarisda, N. (2019). The Use of Cooperative Learning Models Think Pair Share in Mathematics Learning. *Journal of Physics: Conference Series, 1387*.

Yaw, O. (2013). *Gender differential in academic performance in mathematics among Senior High School (SHS) students in the Tamale Metropolis in Northern Ghana.* Faculty of Mathematics Science: Navrongo Campus.

APPENDICES

Appendix A

INTRODUCTORY LETTER

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October 21, 2021

TO WHOM IT MAY CONCERN

LETTER OF INTRODUCTION

I write to introduce to you the bearer of this letter Mr. Simon Nii Bortey Appiah a postgraduate student in the University of Education. Winneba. He is reading for a Master of Philosophy degree in Mathematics Education and as part of the requirements of the programme, he is undertaking a research titled — *The Effect of Think-Pair-Share on Students Performance on Circle Theorem- Plane Geometry II: The Case of Asankrangwa Senior High School.*

He needs to gather information to be analysed for the said research and he has chosen to do so in your institution. I would be grateful if he is given the needed assistance to carry out this exercise. Thank you.

Yours faithfully

Prof. Michael J. Nabie
Head of Department

APPENDIX B

Lesson plan on Think-Pair-Share approach to teaching circle theorem-plane geometry II

(Experimental design)

Subject: Core Mathematics

Topic: Circle Theorem-Plane Geometry II

OBJECTIVES

By the end of the lesson the student will be able to:

i. Discover the relationship between the angle subtended at the centre and that at the circumference by an arc or a chord.
ii. Find the value of the angle subtended by a diameter at the circumference.
iii. Find the relationship between opposite angles of a cyclic quadrilateral.
iv. Identify the tangent as perpendicular to the radius at the point of contact.
v. Verify that the angle between the tangent and the chord at the point of contact is equal to the angle in the alternate segment.
vi. Verify that tangents drawn from an external point to the same circle are equal when measured from their point of contact.

Related Previous Knowledge

Students are able to:

a. Mention and identify all the various parts of a circle
b. Perform basic algebraic arithmetic
c. Identify various special triangles such as the right-angled triangle, equilateral triangle and isosceles triangle.
d. Mention properties of triangles and quadrilaterals

Teaching and Learning Materials

1. Mathematical set and non-programmable scientific calculator
2. Worksheets will be available for desk mate presentation, thus activities in the lesson will be designed alongside with the students worksheets

Advanced Preparation

The researcher gives brief history about TPS strategy and how it is applied in the learning of circle theorem-plane geometry II and other mathematics topics. The researcher will then introduce students to how the TPS cooperative learning strategy works in a geometry classroom. The students will then be made to follow the three instructional phases. The instructions will be based on Frank Lyman of University of Maryland proposed phases of instructions. The student continue working with TPS under the instructions and guidance of the researcher.

EXECUTION OF THE LESSONS

ACTIVITY ONE

Teacher assists students by organizing them in small group (2 desk mate per group) to find out that the angle subtended by an arc of a circle at the centre is twice that of the angle at the circumference.

1. Using a pair of compasses, the teacher guide the students to draw a circle with a suitable radius with center O on a blank worksheet and mark point P on the circumference.
2. Using a ruler, the teacher guide the students to draw two chords from P to cut the circumference at A and B respectively.
3. OA and OB, the radii are joined.
4. The students are instructed to measure $\angle APB \text{ and } \angle AOB$ with a protractor.
5. At this point the teacher give students 2 minutes to **think** how $\angle APB \text{ and } \angle AOB$ are related. After the two minutes, the teacher ask students to **pair** with their desk mate to **share** their agreed findings with the class.

EXPECTED RESULTS FROM DISCUSSIONS

a. When the instructions are carefully followed, the angles subtended by an arc of a circle at the centre is twice that of the angle at the circumference.

b. Conversely, the angle formed on the circumference is half the angle formed at the centre.

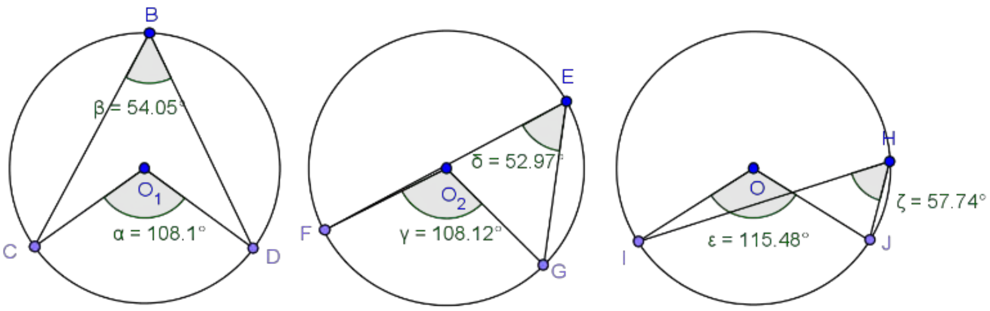


Figure 1: Diagram showing the angle at the centre and the circumference of the circles

Conclusion

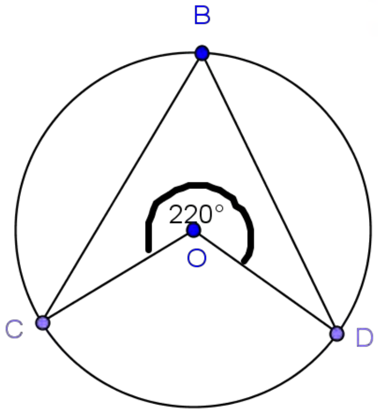
Lead the students to summarise the main ideas in the lesson

Assessment/Evaluation

Let students do class exercise.

QUESTION 1

Find the value of ∠CBD



EXPECTED ANSWER

To find the value of ∠CBD, note that

2(∠CBD)=∠COD

220°+∠COD (minor sector)=360° (angle at a point)

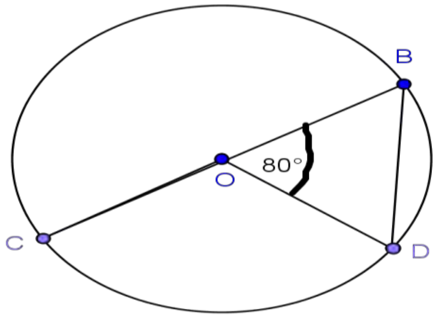
∠COD(minor sector)=140°

2(∠CBD)=∠COD

2(∠CBD)=140°

Therefore ∠CBD=70°

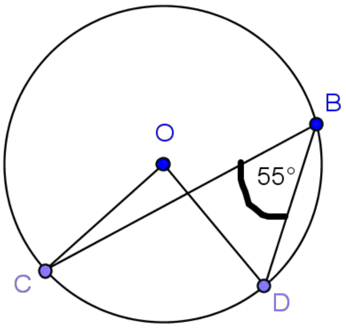
QUESTION 2
Find the value of ∠BDO



EXPECTED ANSWERS
Triangle OBD is isosceles,
Therefore OB=OD (radii)
∠OBD+∠BDO+∠DOB=180°
(sum of interior angles of a triangle)
∠OBD+∠BDO+80°=180°
∠OBD+∠BDO=100°
Since OB=OD, ∠OBD=ODB=50°

∠COB is a straight angle (180°)
∠COD+∠DOB=180°
∠COD+80°=180
∠COD=100
2(∠CBD)=∠COD
2(50°)=∠COD
∠CBD=∠BDO=50°

QUESTION 3
Find the value of ∠COD (minor sector)



To find the value of ∠COD (minor sector)
2(∠CBD)=∠COD
2(55°)=∠COB
110°=∠COB
Therefore the value of ∠COB is 110°

ACTIVITY TWO

Teacher guide students to find the value of the angle subtended by a diameter at the circumference.

1. Using a pair of compasses, the teacher guide students to draw a circle with a given radius on a worksheet.
2. Students are then instructed to draw a diameter AB.
3. Locate points P and Q on the circumference.
4. Students are guided to draw chords PA, PB, QA and QB.

5. Each student is asked to measure $\angle APB$ and AQB.
6. The teacher again asked students what they notice.
7. The teacher give students 2 minutes to **think** and critically examine the two angles $\angle APB$ and AQB. After the two minutes, the teacher ask students to **pair** with their desk mate to agree on common solution(s) and finally **share** their thoughts and findings with the class.

EXPECTED RESULTS FROM DISCUSSIONS

a. When the instructions are strictly adhered to, then any angle subtended from the diameter and formed on the circumference is always perpendicular, 90^0.

b. Also the angle in a semicircle is a right angle.

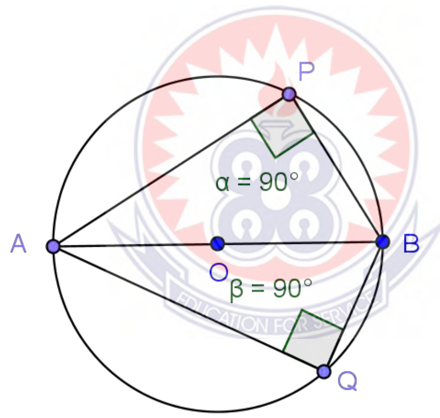


Figure 2: Diagram showing the angles formed on the circumference subtended by the diameter

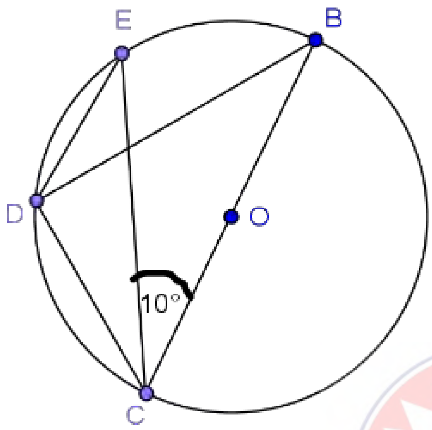
Conclusion

Lead the students to summarise the main ideas in the lesson

a. The angle subtended from the diameter and formed on the circumference is right angle

QUESTION 4

In the figure below, find the value of ∠BDC



EXPECTED ANSWER

BC is a diameter.

The ∠BDC formed on the circumference, was subtended from the diamer.

Therefore ∠BDC is 90°

Assessment/Evaluation

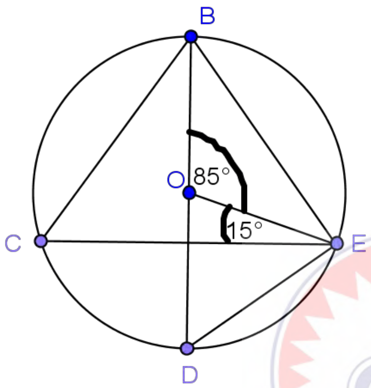
Let students do class exercise.

QUESTION 5

In the figure below find the value of:

(a) ∠CBD

(b) ∠CED



BD is a diameter

Triangle is isosceles, SO

∠OBE=∠BEO.

∠BOE+∠OBE+∠BEO=180°

But ∠BOE=85°

85°+∠OBE+∠BEO=180°

∠OBE+∠BEO=95°

∠OBE=∠BEO=47.5°

∠BEO+∠OEC+CED=90°

(∠BED=90°)

42°+15°+∠CED=90°

∠CED=90°-57°=33°

Therefore ∠CED is 33°

∠CED=∠CBD

(Both angles subtends from the chord or arc)

∠CBD is 33°

ACTIVITY THREE

Teacher guide students to find the relationship between opposite angles of a cyclic quadrilateral.

1. Students follow teacher's instruction to draw a circle with a given radius.
2. Students are allowed to take any four points A, B, C, and D on the circumference of the circle.
3. Students are made to join the points A, B, C, D to form a quadrilateral ABCD.
4. Students are asked to measure angles $\angle ADC, \angle DAB, \angle ABC, \angle BCD \text{ and } \angle ADE$.

   Calculate each sum.

   a. $\angle DAB + \angle BCD$

   b. $\angle ABC + \angle ADC$
5. The teacher then ask students the relation between $\angle ABC \text{ and } \angle ADE$?
6. Students are given 2 minutes to explore how $\angle ABC \text{ and } \angle ADE$ are related. After the thinking time, they joy their desk mate to put their ideas together and then present their solution to the whole class.

EXPECTED RESULTS FROM DISCUSSION

Students who followed each step with rapt attention will notice that any concyclic quadrilateral:

a. Angles in opposite segments are supplementary.
b. The exterior angle is equal to the opposite interior angle.

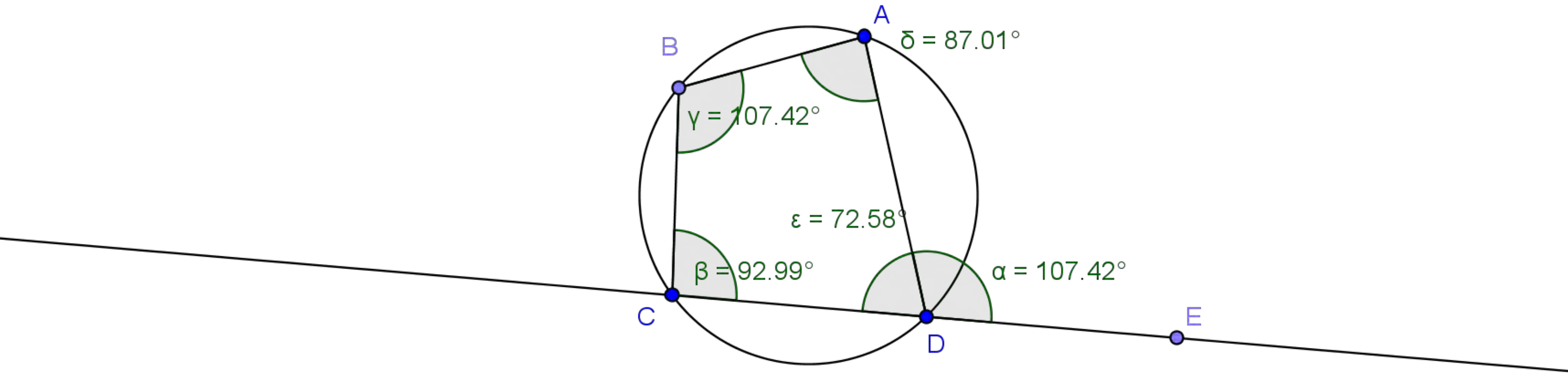


Figure 3: Diagram showing angles in opposite segment of cyclic quadrilateral

Conclusion

Lead the students to summarise the main ideas in the lesson

c. Angles in opposite segments of any cyclic quadrilateral supplementary.

d. The exterior angle is equal to the opposite interior angle.

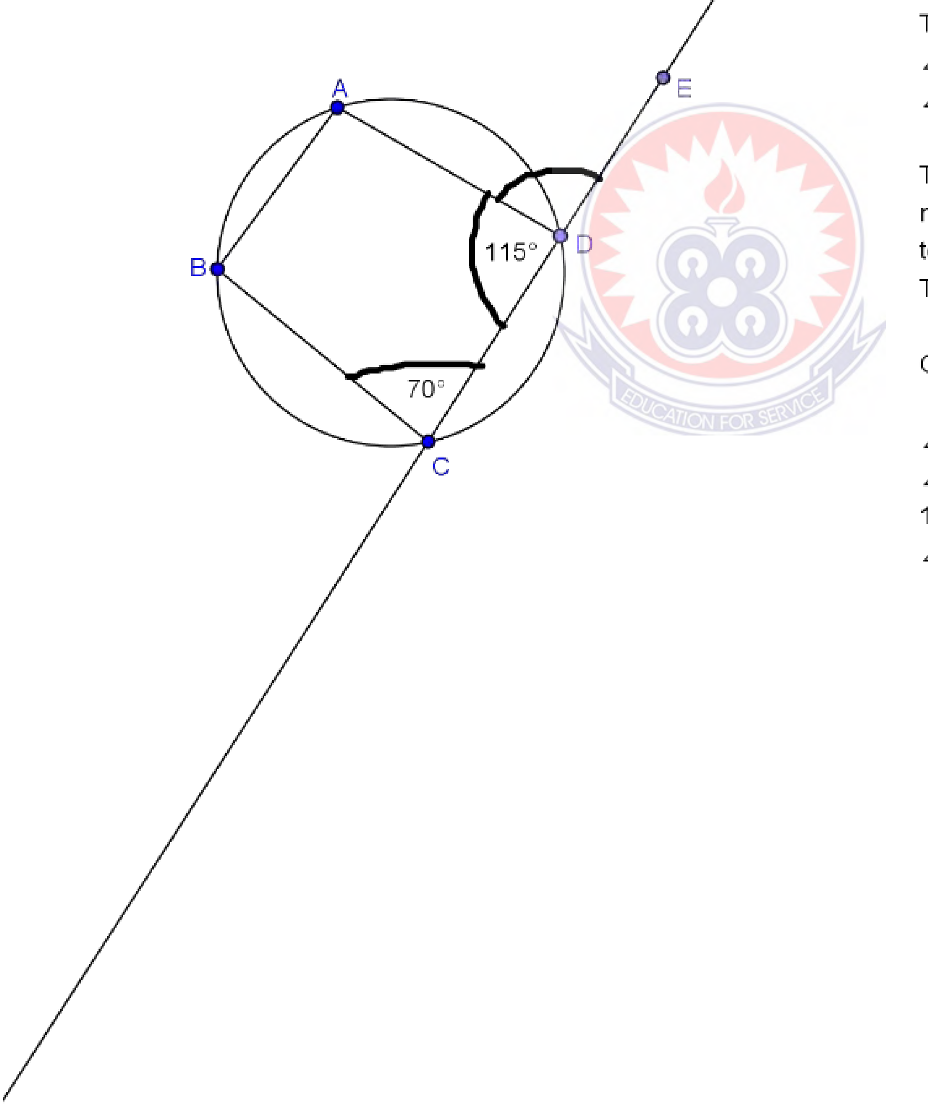
Assessment/Evaluation

QUESTION 6

In the diagram below, find the value of:

(a) ∠ADE

(b) ∠ABC



EXPECTED ANSWERS

For any cyclic quadrilateral,
the sum of the two opposite angles
are supplementary

Therefore ∠ABC+∠ADC=180°

∠ABC+115°=180°

∠ABC=65°

To find the value of ∠ADE,
note that the exterior angle is equal
to the opposite interior angle

Therefore ∠ABC=∠ADE=65°

OR

∠CDE=180° (straight angle)

∠ADC+∠ADE=180°

115°+∠ADE=180°

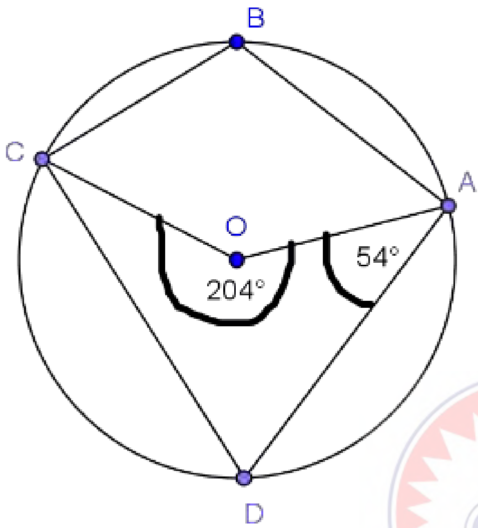
∠ADE=65°

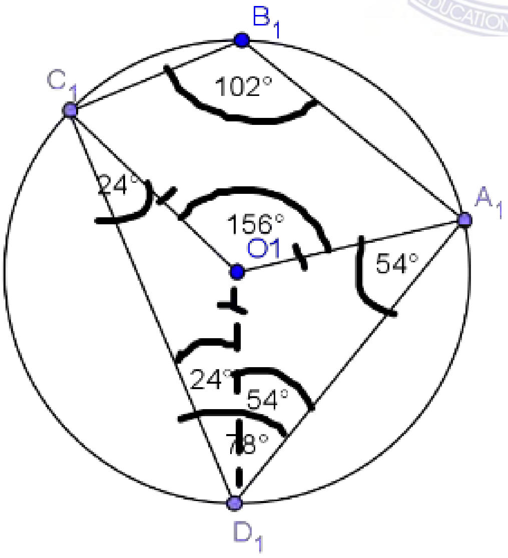
QUESTION 7

In the figure below, find the value of:

(a) ∠ABC

(b) ∠OCD





EXPECTED ANSWER

∠AOC=204° (major sector angle)

∠AOC=156° (minor sector angle)

Angle at a point O is 360°

2(∠ADC)= ∠AOC(minor sector angle)

2(∠ADC)=156°

∠ADC=78°

∠ADC+∠ABC=180°

(cyclic quadrilateral)

78°+∠ABC=180°

∠ABC=102°

(a) Therefore the value of ∠ABC is 102°

Triangle AOD is isosceles,

∠ODA= ∠OAD=54°

Similarly, Triangle OCD is isosceles

∠OCD= ∠ODC=24°

Therefore the value of ∠OCD is 24°

ACTIVITY FOUR

Teacher guide students to verify that the tangent is perpendicular to the radius at the point of contact

1. Students draw a circle with a given radius
2. Students are instructed to draw a tangent XTY to meet the circle at T.
3. Students then draw a diameter DOT.
4. Using protractor, students are instructed to measure $\angle DTX$ *and* $\angle DTY$

   Students are then asked to write what they noticed. After which they are allowed to discuss with their desk mate to examine what they noticed together and finally desk mate will share with the class.

EXPECTED RESULTS FROM DISCUSSION

Students who followed each step with rapt attention will notice that a diameter or radius is perpendicular to the tangent to the circle at the point of contact.

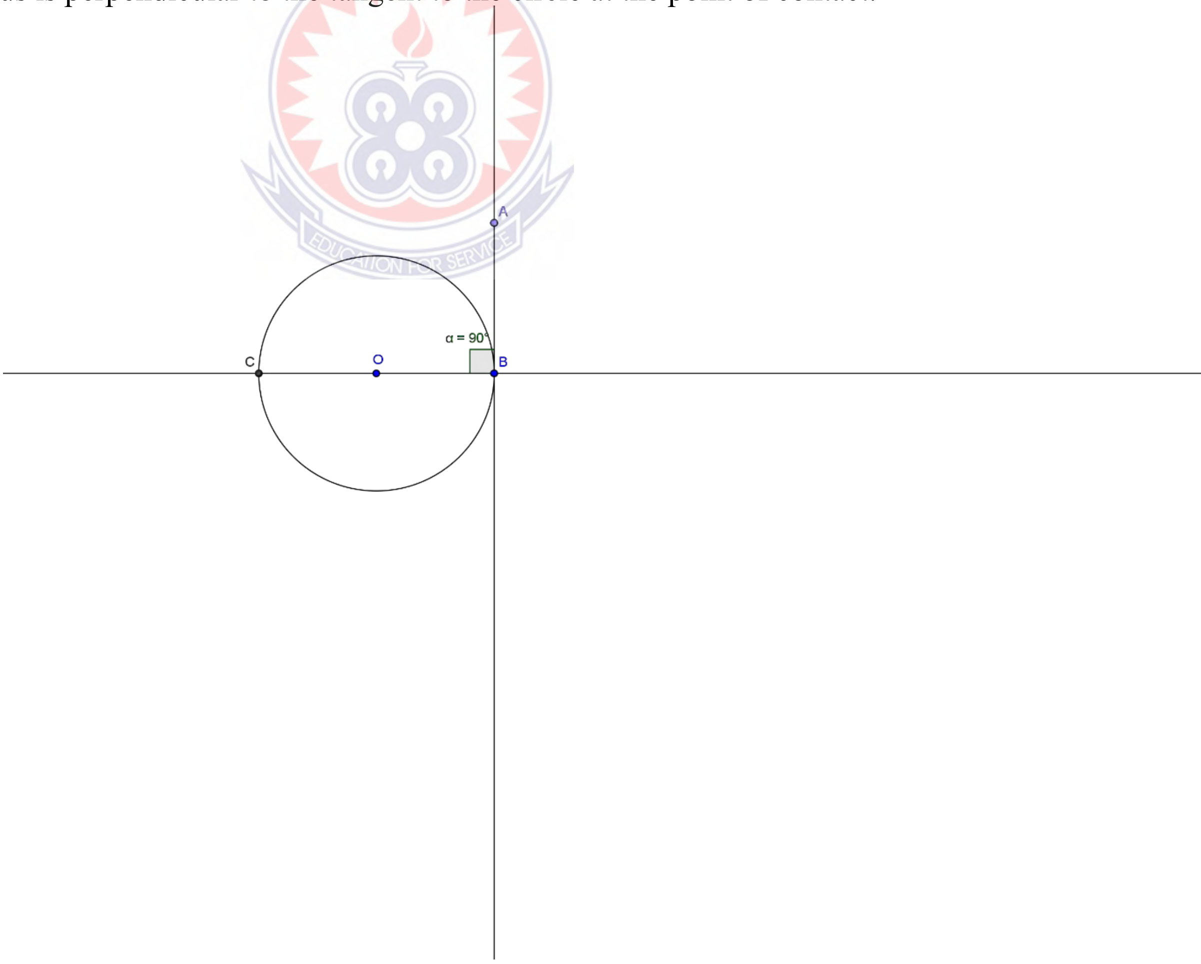


Figure 4: Diagram showing perpendicularity of tangent and radius of a circle.

Conclusion

Lead the students to summarise the main ideas in the lesson

A diameter or radius is perpendicular to the tangent to the circle at the point of contact

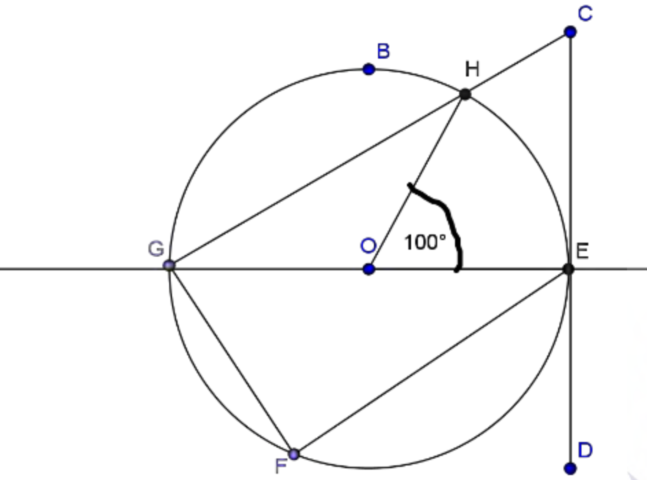
Assessment/Evaluation

In the figure below, find the value of:

(a) ∠GCE

(b) ∠GEF

(c) ∠DEF



EXPECTED ANSWER
∠GOE=180°
(angles on a straight line is 180°)
∠GOH+∠HOE=180°
∠GOH+100°=180°
∠GOH=80°
Triangle GOH is isosceles
Therefore ∠OGH=GHO=50°
∠GHO+∠OHC=180°
(angles on a straight line is 180°)
50°+∠OHC=180°
∠OHC=130°
∠OEC=90°
(A diameter or radius is perpendicular to the tangent to the circle at the point of contact)
Considering the quadrilateral EOHC,
The sum of the interior angle of any quadrilateral is 360°,therefore
∠EOH+∠OHC+∠HCE+∠CEO=360°
100°+130°+∠HCE+90°
∠HCE=40°
(a) Therefore is 40°

(b) ∠GHO=∠GEF
(angles subtended from the same chord or arc are congruent)
Therefore ∠GEF=50°

(c) ∠CED=180°
(straight line angle)
∠CEO+∠OEF+∠FED=180°
90°+50°+∠FED=180°
∠FED=180°-140°
∠FED=40°
Therefore ∠FED is 40°

ACTIVITY FIVE

Teacher assist students to verify the alternate angle theorem by drawing.

1. Students follow teacher's instruction to draw a circle with a given radius.
2. Students are allowed to take any four points A, B, C, and D on the circumference of the circle.
3. In the diagram, teacher instruct students to make AC a chord
4. Let C be the point where the tangent ECF meets the circle.
5. Teacher ask students to measure $\angle ACF$ and $\angle ABC$.
6. Also, teacher ask students to measure $\angle ACE$ and $\angle ADC$.
   a. The teacher give students 2 minutes to examine $\angle ACF$ and $\angle ABC$ and how they are related. After the thinking time, they joy their desk mate to put their ideas together and then present their solution to the whole class.
   b. The teacher give students another 2 minutes to investigate how $\angle ACE$ and $\angle ADC$ are related. After the thinking time, they joy their desk mate to put their ideas together and then present their solution to the whole class.

EXPECTED RESULTS FROM DISCUSSION

When the instructions are carefully followed, students will realise that

(a) Angle formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment.

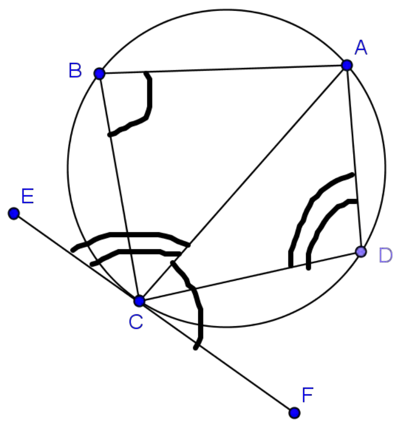


Figure 4: Diagram showing angle between tangent and a chord.

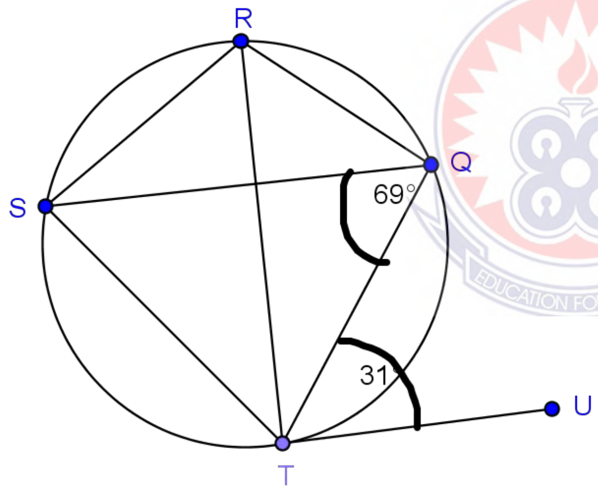
Conclusion

Lead the students to summarise the main ideas in the lesson

The angle formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment.

Assessment/Evaluation

QUESTION 8
In the diagram, TU touches the circle at T and RT is a diameter.
∠UTQ=31° and ∠TQS=69°
Calculate the size of:
(a) ∠QRS;
(b)∠SQR;
(c) ∠QTS



∠TRQ=∠TSQ=31°
(Angles in the alternate segment are equal)
∠SRT=69°
(Angles in the same segment are equal)

(a) ∠QRS=∠SRT+∠TRQ
∠QRS=69°+31°
∠QRS=100°
Therefore the size of ∠QRS is 100°

(b) ∠RQS=∠STR
(Angles in the same segment are equal)
∠RQS+69°=90°
(Angles in a semicircle equal 90°)
∠RQS=21°
Therefore ∠SQR is 21°

(c) ∠STR+∠RTQ=∠QTS
But, ∠RTQ+31°=90° (RT is a diameter)
∠RTQ=90°-31°
∠RTQ=59°
Therefore ∠QTS=21°+59°
Therefore ∠QTS=80°

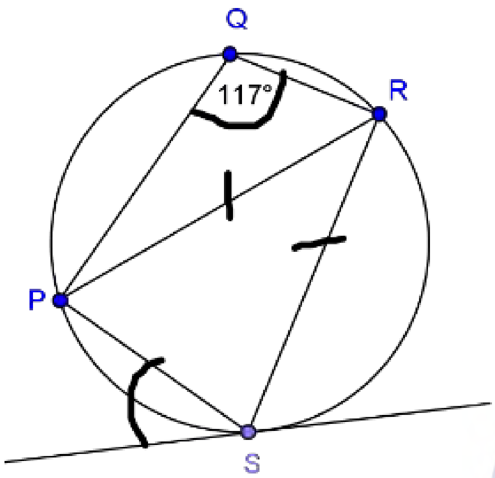
QUESTION 8

In the diagram,

TS is a tangent to the circle at S.

|PR|=|RS| and ∠PQR=117°.

Calculate ∠PST.



EXPECTED ANSWER

∠PQR+∠PSR=180°

(cyclic quadrilateral)

117°+∠PSR=180°

∠PSR=63°

Triangle PRS is isosceles

∠RPS=∠PSR=63°

(Sum of interior angle of a triangle is 180°)

∠RPS+∠PSR+∠SRP=180°

63°+63°+ ∠SRP=180°

∠SRP=180°-126°

∠SRP=54°

∠SRP=∠PST

(angle formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment)

Therefore ∠PST is 54°

ACTIVITY SIX

Teacher guide students to verify that two tangents drawn from an external point, T, to a circle at points A and B are equal in length.

1. Teacher instruct students to draw a circle of a given radius.
2. Teacher ask students to draw two radii to meet the two tangents from an external point P.
3. Label point of intersection A and B.
4. Draw a line from point P to the center of the circle O.
5. Teacher ask students to measure $|PA|\ and\ |PB|$
    a. Teacher ask students to examine the relation between $|PA|\ and\ |PB|$
    b. Teacher ask students to pair with their desk mate, compare and agree on common grounds.
    c. Teacher then ask desk mate to share their findings or results with the class.

EXPECTED RESULTS FROM DISCUSSION

When the instructions are carefully followed, students will realise that the lengths of tangents to a circle from an external point are equal.

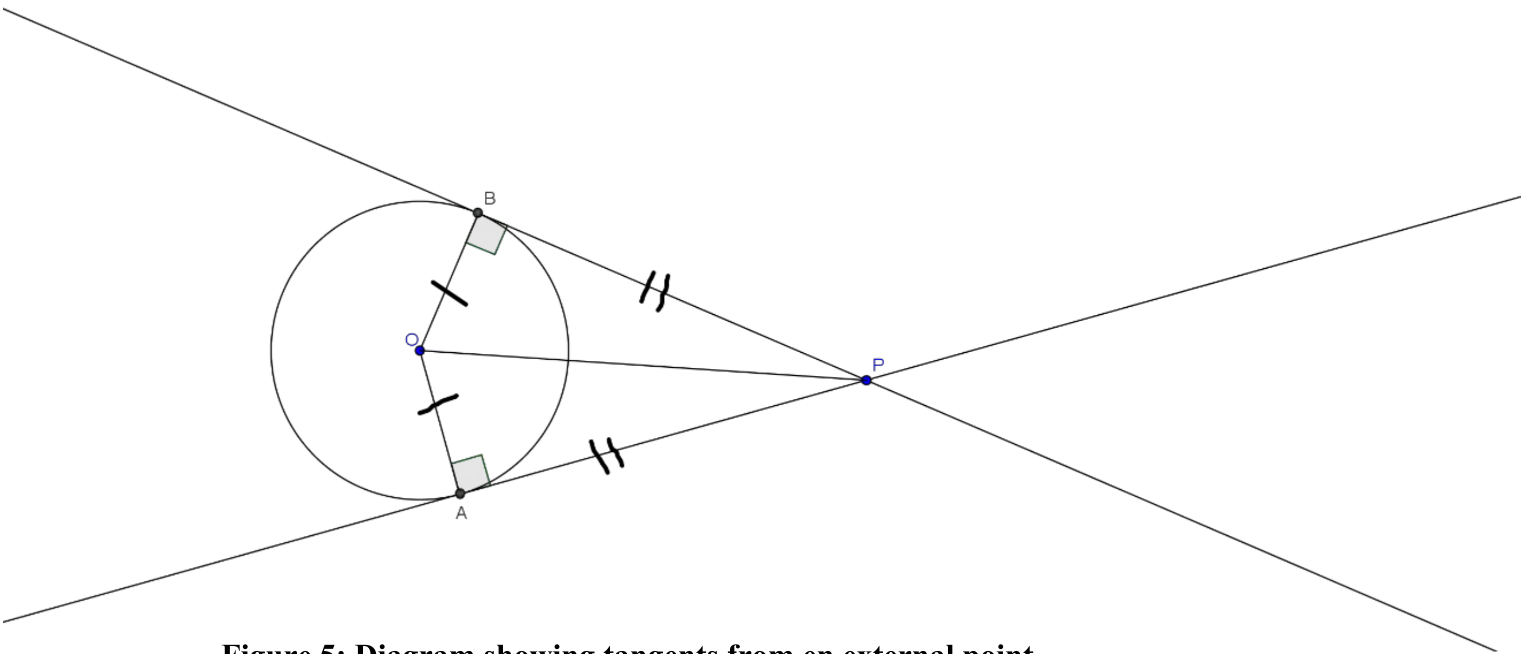


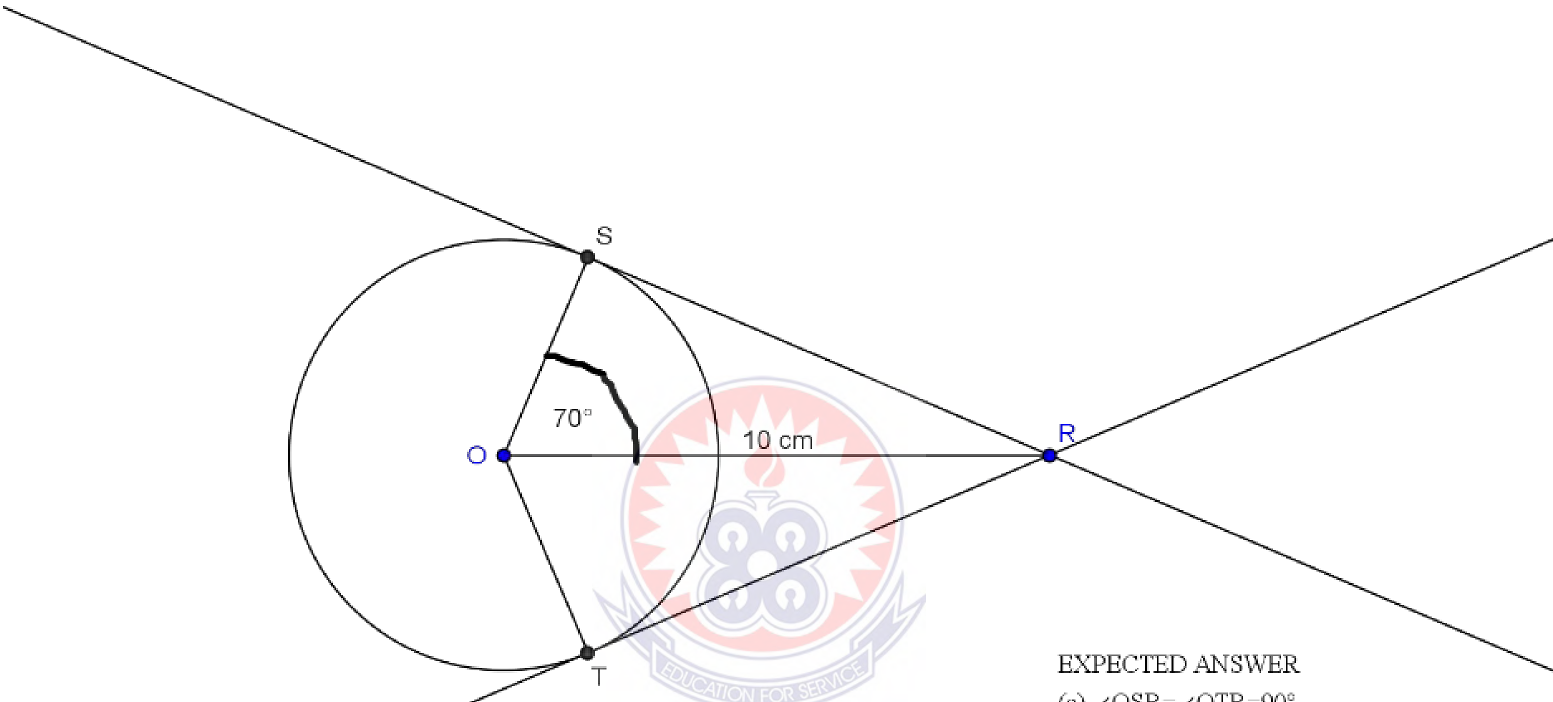
Figure 5: Diagram showing tangents from en external point.

Conclusion

Lead the students to summarise the main ideas in the lesson

The lengths of tangents to a circle from an external point are equal.

Assessment/Evaluation



QUESTION 9

In the diagram,

O is the centre of the circle and RS and RT are tangents to the circle from R.

$\angle SOR = 70°$ and $|OR| = 10$ cm.

(a) What is the size of $\angle ORT$?

(b) Find $|TR|$, correct to one decimal place.

EXPECTED ANSWER

(a) $\angle OSR = \angle OTR = 90°$

$(SR \perp OS)$

$\angle ORS = \angle ORT$

(Congruent angles, $\triangle OSR \equiv \triangle OTR$)

$70° + \angle ORS = 90°$

$\angle ORS = 20°$

Therefore size of $\angle ORS$ is 20°

(b) In $\triangle OTR$, $\cos 20 = |TR|/10$

$10 \cos 20 = |TR|$

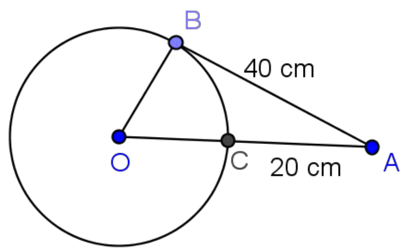
$|TR| = 9.3969$

Therefore $|TR|$ is 9.4 cm

QUESTION 10

In the diagram, O is the centre of the circle and OA is a straight line that cuts the circle at C. |AC|=20 cm and AB is a tangent to the circle at point B, |AB|=40 cm.

Find the radius of the circle.



EXPECTED ANSWER

$\angle ABO$ is a right angle. Let r represent the radius of the circle. Using Pythagoras theorem:

$$(AB)^2 + (BO)^2 = (AO)^2$$
$$40^2 + r^2 = (20 + r)^2$$
$$1600 + r^2 = 20^2 + 40r + r^2$$
$$1600 = 400 + 40r$$
$$1600 - 400 = 40r$$
$$1200 = 40r$$
$$30 = r.$$

The radius of the circle is 30 cm

APPENDIX C

Lesson plan on Traditional approach to teaching circle theorem-plane geometry II

(Control design)

Subject: Core Mathematics

Topic: Circle Theorem-Plane Geometry II

OBJECTIVES

By the end of the lesson the student will be able to:

i. Discover the relationship between the angle subtended at the centre and that at the circumference by an arc or a chord.
ii. Find the value of the angle subtended by a diameter at the circumference.
iii. Find the relationship between opposite angles of a cyclic quadrilateral.
iv. Identify the tangent as perpendicular to the radius at the point of contact.
v. Verify that the angle between the tangent and the chord at the point of contact is equal to the angle in the alternate segment.
vi. Verify that tangents drawn from an external point to the same circle are equal when measured from their point of contact.

Related Previous Knowledge

Students are able to:

a. Mention and identify all the various parts of a circle
b. Perform basic algebraic arithmetic
c. Identify various special triangles such as the right-angled triangle, equilateral triangle and isosceles triangle.
d. Mention properties of triangles and quadrilaterals

Teaching and Learning Materials

1. Mathematical set and non-programmable scientific calculator
2. Worksheets will be available for desk mate presentation, thus activities in the lesson will be designed alongside with the students worksheets

EXECUTION OF THE LESSONS

INTRODUCTION

Teacher introduces the lesson by reviewing relevant previous knowledge of students through questions and answers by asking students to mention different part of circle. The response by students compel the teacher to introduce the lesson.

Expected Answer

Arc, chord, circumference, diameter, sector, segment and radius

DEVELOPMENT STAGE OF THE LESSON

ACTIVITY ONE

Teacher guide the student through demonstration and illustration on the marker board to discover the relationship between the angle subtended at the centre and that at the circumference by an arc or a chord. Using a pair of compasses, the teacher follow the steps below to show the student that angles subtended by an arc of a circle at the centre is twice that of the angle at the circumference. Using a ruler, the teacher guide the students to draw two chords from P to cut the circumference at A and B respectively.

1. OA and OB, the radii are joined.
2. The students are instructed to measure $\angle APB \text{ and } \angle AOB$ with a protractor.
3. At this point the teacher ask the student his or her observation on $\angle APB \text{ and } \angle AOB$

EXPECTED RESPONSES

It can be deducted from the lesson that the angle subtended by an arc of a circle at the centre is twice that of the angle at the circumference. Conversely, the angle formed on the circumference is half the angle formed at the centre.

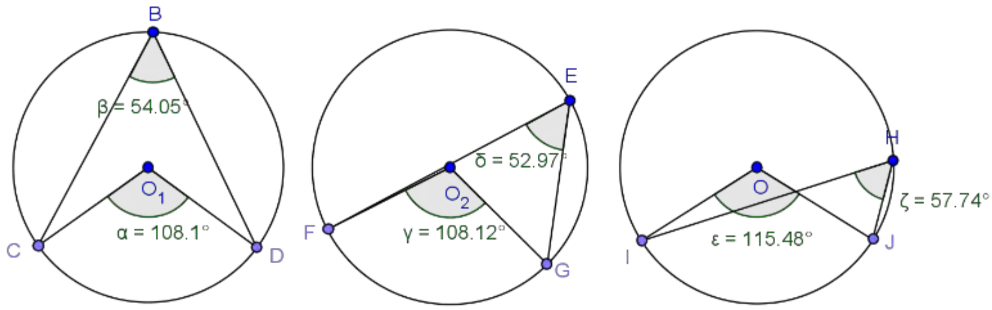


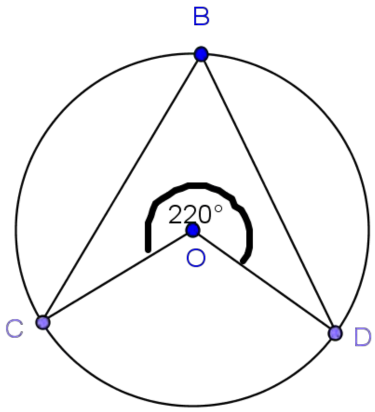
Figure 1: Diagram showing the angle at the centre and the circumference of the circles

Assessment/Evaluation

Let students do class exercise.

QUESTION 1

Find the value of ∠CBD



EXPECTED ANSWER

To find the value of ∠CBD, note that

2(∠CBD)=∠COD

220°+∠COD (minor sector)=360° (angle at a point)

∠COD(minor sector)=140°

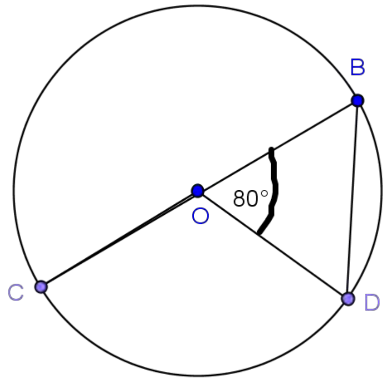
2(∠CBD)=∠COD

2(∠CBD)=140°

Therefore ∠CBD=70°

QUESTION 2

Find the value of ∠BDO



EXPECTED ANSWERS

Triangle OBD is isosceles,

Therefore OB=OD (radii)

∠OBD+∠BDO+∠DOB=180°

(sum of interior angles of a triangle)

∠OBD+∠BDO+80°=180°

∠OBD+∠BDO=100°

Since OB=OD, ∠OBD=ODB=50°

∠COB is a straight angle (180°)

∠COD+∠DOB=180°

∠COD+80°=180

∠COD=100

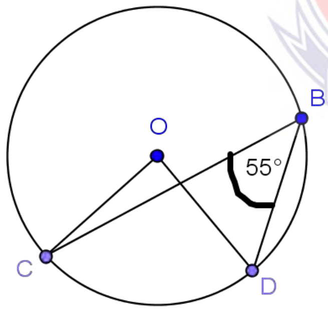
2(∠CBD)=∠COD

2(50°)=∠COD

∠CBD=∠BDO=50°

QUESTION 3

Find the value of ∠COD (minor sector)



To find the value of ∠COD (minor sector)

2(∠CBD)=∠COD

2(55°)=∠COB

110°=∠COB

Therefore the value of ∠COB is 110°

ACTIVITY TWO

Teacher guide students through demonstration and illustration on the marker board to find the value of the angle subtended by a diameter at the circumference. Using a pair of compasses, the teacher follow the steps below to show the student that any angle subtended from the diameter and formed on the circumference is always perpendicular, 90^0 and also the angle in a semicircle is a right angle.

1. Using a pair of compasses, the teacher guide students to draw a circle with a given radius on a worksheet.
2. Students are then instructed to draw a diameter AB.
3. Locate points P and Q on the circumference.
4. Students are guided to draw chords PA, PB, QA and QB.
5. The student is asked to measure $\angle APB \text{ and } AQB$.
6. The teacher again asked students what they notice.

EXPECTED RESPONSES

a. When the instructions are strictly adhered to, then any angle subtended from the diameter and formed on the circumference is always perpendicular, 90^0.

b. Also the angle in a semicircle is a right angle.

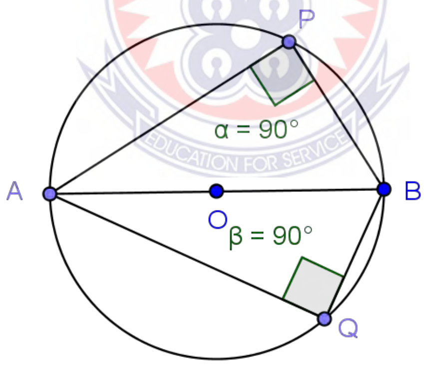


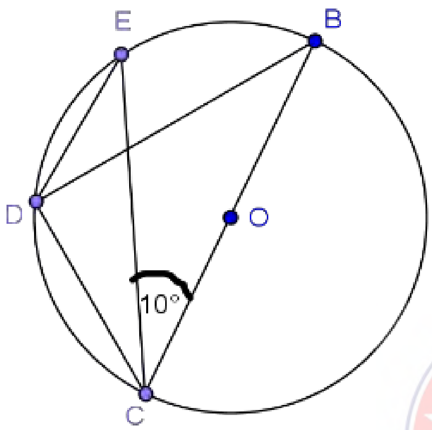
Figure 2: Diagram showing the angles formed on the circumference subtended by the diameter

Assessment/Evaluation

Let students do class exercise.

QUESTION 4

In the figure below, find the value of ∠BDC



EXPECTED ANSWER

BC is a diameter.

The ∠BDC formed on the circumference,
was subtended from the diamer.

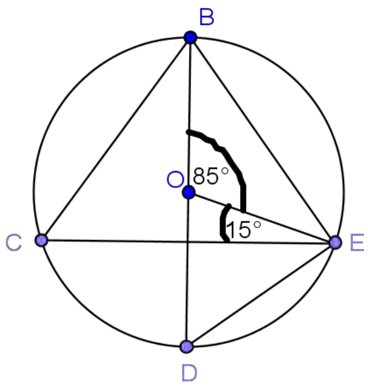
Therefore ∠BDC is 90°

QUESTION 5

In the figure below find the value of:

(a) ∠CBD

(b) ∠CED



BD is a diameter

Triangle is isosceles, SO

∠OBE=∠BEO.

∠BOE+∠OBE+∠BEO=180°

But ∠BOE=85°

85°+∠OBE+∠BEO=180°

∠OBE+∠BEO=95°

∠OBE=∠BEO=47.5°

∠BEO+∠OEC+CED=90°

(∠BED=90°)

42°+15°+∠CED=90°

∠CED=90°-57°=33°

Therefore ∠CED is 33°

∠CED=∠CBD

(Both angles subtends from the chord or arc)

∠CBD is 33°

ACTIVITY THREE

Teacher guide students through demonstration and illustration on the marker board to show the student the relationship between opposite angles of a cyclic quadrilateral. The teacher follow the steps below to show this theorem to the student.

1. Draw a circle with a given radius.
2. Label any four points A, B, C, and D on the circumference of the circle.
3. Students are made to join the points A, B, C, D to form a quadrilateral ABCD.
4. Students are asked to measure angles

   $\angle ADC, \angle DAB, \angle ABC, \angle BCD \text{ and } \angle ADE$.

   Calculate each sum.

   a. $\angle DAB + \angle BCD$

   b. $\angle ABC + \angle ADC$
5. The teacher then ask students the relation between $\angle ABC \text{ and } \angle ADE$?

EXPECTED RESPONSES

It can be deduced that any concyclic quadrilateral:

a. Angles in opposite segments are supplementary.
b. The exterior angle is equal to the opposite interior angle.

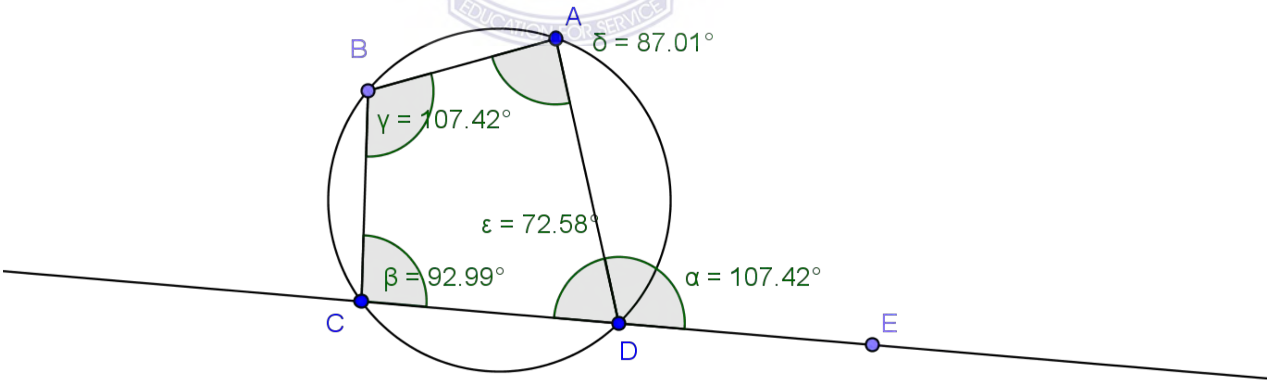


Figure 3: Diagram showing angles in opposite segment of cyclic quadrilatera

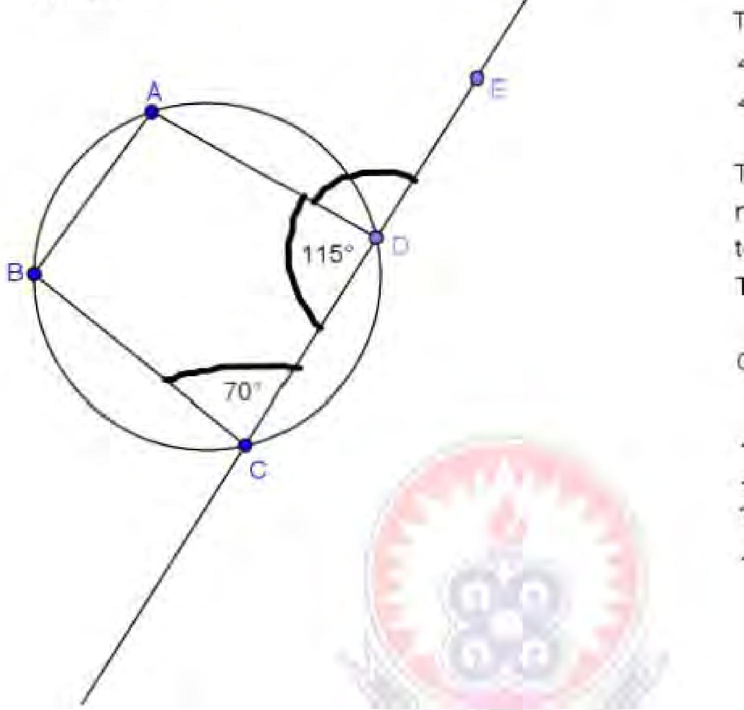
Assessment/Evaluation

QUESTION 6

In the diagram below, find the value of:

(a) ∠ADE

(b) ∠ABC



EXPECTED ANSWERS

For any cyclic quadrilateral, the sum of the two opposite angles are supplementary

Therefore ∠ABC+∠ADC=180°

∠ABC+115°=180°

∠ABC=65°

To find the value of ∠ADE, note that the exterior angle is equal to the opposite interior angle

Therefore ∠ABC=∠ADE=65°

OR

∠CDE=180° (straight angle)

∠ADC+∠ADE=180°

115°+∠ADE=180°

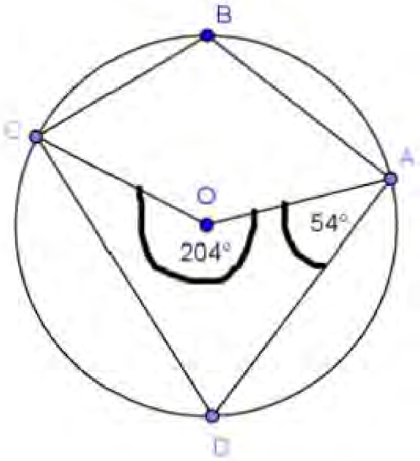
∠ADE=65°

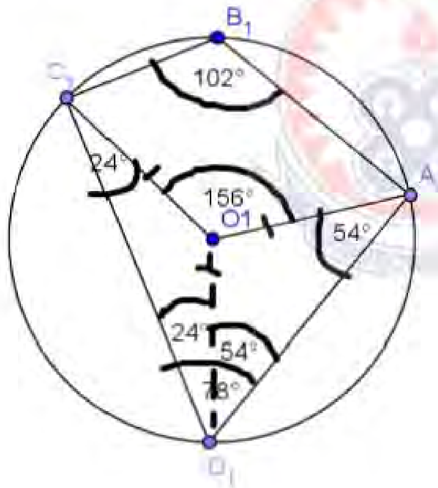
QUESTION 7

In the figure below, find the value of:

(a) ∠ABC

(b) ∠OCD





EXPECTED ANSWER

∠AOC=204° (major sector angle)

∠AOC=156° (minor sector angle)

Angle at a point O is 360°

2(∠ADC)=∠AOC(minor sector angle)

2(∠ADC)=156°

∠ADC=78°

∠ADC+∠ABC=180°

(cyclic quadrilateral)

78°+∠ABC=180°

∠ABC=102°

(a) Therefore the value of ∠ABC is 102°

Triangle AOD is isosceles,

∠ODA=∠OAD=54°

Similarly, Triangle OCD is isosceles

∠OCD=∠ODC=24°

Therefore the value of ∠OCD is 24°

ACTIVITY FOUR

Teacher guide students through demonstration and illustration on marker board to verify that the tangent is perpendicular to the radius at the point of contact

1. Students draw a circle with a given radius
2. Students are instructed to draw a tangent XTY to meet the circle at T.
3. Students then draw a diameter DOT.
4. Using protractor, students are instructed to measure $\angle DTX$ *and* $\angle DTY$

   Students are then asked to write what they noticed.

EXPECTED RESPONSES

It can be deduced that a diameter or radius is perpendicular to the tangent to the circle at the point of contact.

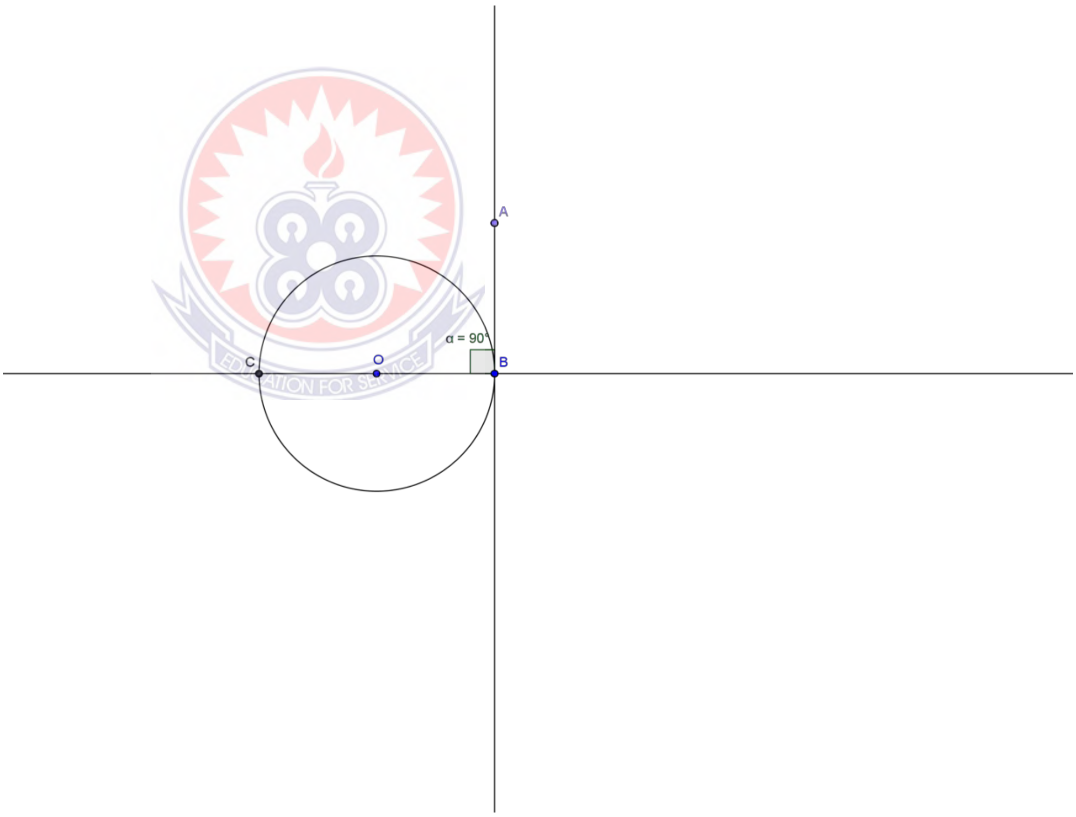
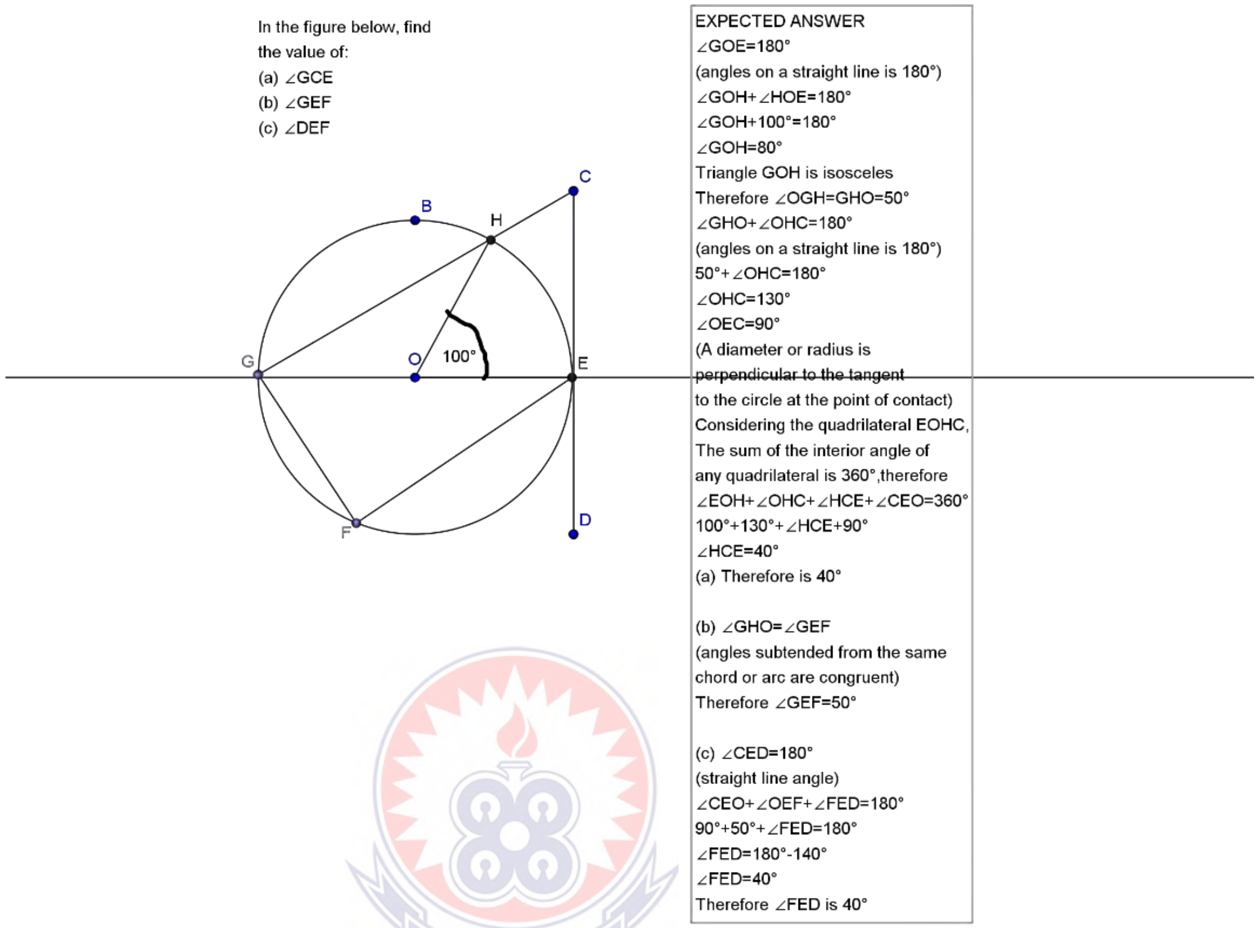


Figure 4: Diagram showing perpendicularity of tangent and radius of a circle.

Assessment/Evaluation



ACTIVITY FIVE

Teacher assist students through white marker board demonstration and illustration to verify the alternate angle theorem by drawing. The teacher went through the following steps to verify the theorem.

1. Students follow teacher's instruction to draw a circle with a given radius.
2. Students are allowed to take any four points A, B, C, and D on the circumference of the circle.
3. In the diagram, teacher instruct students to make AC a chord
4. Let C be the point where the tangent ECF meets the circle.
5. Teacher ask students to measure $\angle ACF$ *and* $\angle ABC$.
6. Also, teacher ask students to measure $\angle ACE$ *and* $\angle ADC$.

EXPECTED RESPONSES

It can be deduced that angle formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment.

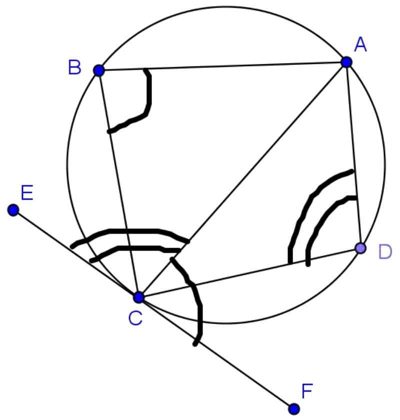


Figure 4: Diagram showing angle between tangent and a chord.

Assessment/Evaluation

QUESTION 8

In the diagram, TU touches the circle at T and RT is a diameter.

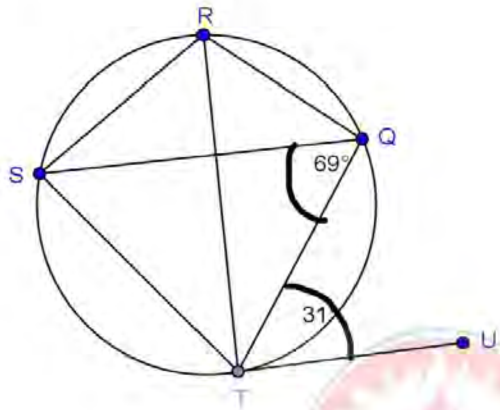
∠UTQ=31° and ∠TQS=69°

Calculate the size of:

(a) ∠QRS;

(b) ∠SQR;

(c) ∠QTS



∠TRQ=∠TSQ=31°

(Angles in the alternate segment are equal)

∠SRT=69°

(Angles in the same segment are equal)

(a) ∠QRS=∠SRT+∠TRQ

∠QRS=69°+31°

∠QRS=100°

Therefore the size of ∠QRS is 100°

(b) ∠RQS=∠STR

(Angles in the same segment are equal)

∠RQS+69°=90°

(Angles in a semicircle equal 90°)

∠RQS=21°

Therefore ∠SQR is 21°

(c) ∠STR+∠RTQ=∠QTS

But, ∠RTQ+31°=90° (RT is a diameter)

∠RTQ=90°-31°

∠RTQ=59°

Therefore ∠QTS=21°+59°

Therefore ∠QTS=80°

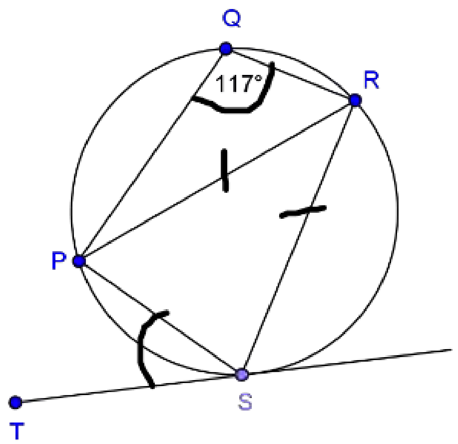
QUESTION 8

In the diagram,

TS is a tangent to the circle at S.

|PR|=|RS| and ∠PQR=117°.

Calculate ∠PST.



EXPECTED ANSWER

∠PQR+∠PSR=180°

(cyclic quadrilateral)

117°+∠PSR=180°

∠PSR=63°

Triangle PRS is isosceles

∠RPS=∠PSR=63°

(Sum of interior angle of a triangle is 180°)

∠RPS+∠PSR+∠SRP=180°

63°+63°+ ∠SRP=180°

∠SRP=180°-126°

∠SRP=54°

∠SRP=∠PST

(angle formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment)

Therefore ∠PST is 54°

ACTIVITY SIX

Teacher guide students through white board marker demonstration and illustration to verify that two tangents drawn from an external point, T, to a circle at points A and B are equal in length. The teacher went through the following steps to verify the theorem.

1. Teacher instruct students to draw a circle of a given radius.
2. Teacher ask students to draw two radii to meet the two tangents from an external point P.
3. Label point of intersection A and B.
4. Draw a line from point P to the center of the circle O.
5. Teacher ask students to measure $|PA|$ *and* $|PB|$

Teacher ask students to examine the relation between $|PA|$ *and* $|PB|$

EXPECTED RESPONSES

It can be deduced that the lengths of tangents to a circle from an external point are equal.

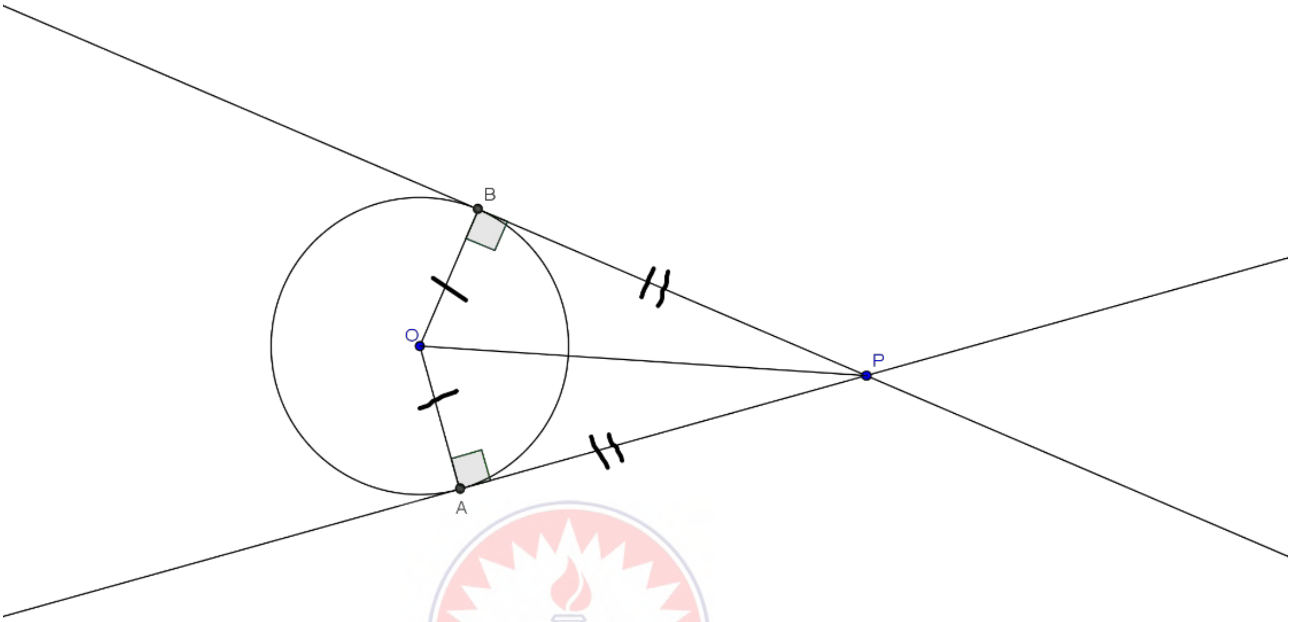
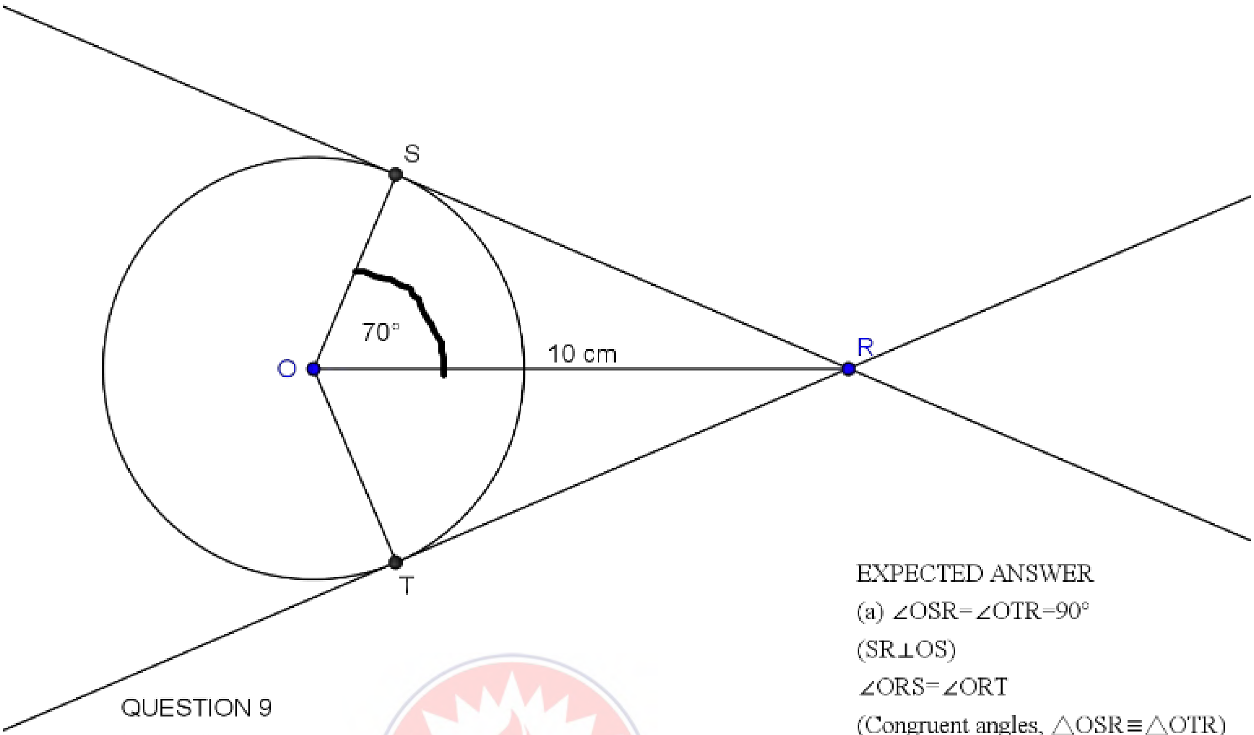


Figure 5: Diagram showing tangents from en external point.

Assessment/Evaluation



QUESTION 9

In the diagram,

O is the centre of the circle and RS and RT are tangents to the circle from R.

∠SOR=70° and |OR|=10 cm.

(a) What is the size of ∠ORT?

(b) Find |TR|, correct to one decimal place.

EXPECTED ANSWER

(a) ∠OSR=∠OTR=90°

(SR⊥OS)

∠ORS=∠ORT

(Congruent angles, △OSR≡△OTR)

70°+∠ORS=90°

∠ORS=20°

Therefore size of ∠ORS is 20°

(b) In △OTR, cos20=|TR|/10

10 cos 20= |TR|

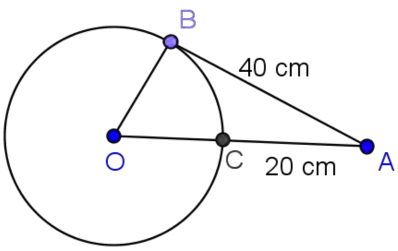
|TR|=9.3969

Therefore |TR| is 9.4 cm

QUESTION 10

In the diagram, O is the centre of the circle and OA is a straight line that cuts the circle at C. |AC|=20 cm and AB is a tangent to the circle at point B, |AB|=40 cm.

Find the radius of the circle.



EXPECTED ANSWER

$\angle ABO$ is a right angle. Let r represent the radius of the circle. Using Pythagoras theorem:

$$(AB)^2 + (BO)^2 = (AO)^2$$
$$40^2 + r^2 = (20 + r)^2$$
$$1600 + r^2 = 20^2 + 40r + r^2$$
$$1600 = 400 + 40r$$
$$1600 - 400 = 40r$$
$$1200 = 40r$$
$$30 = r.$$

The radius of the circle is 30 cm

APPENDIX D

ACHIEVEMENT TEST (Pre-test)

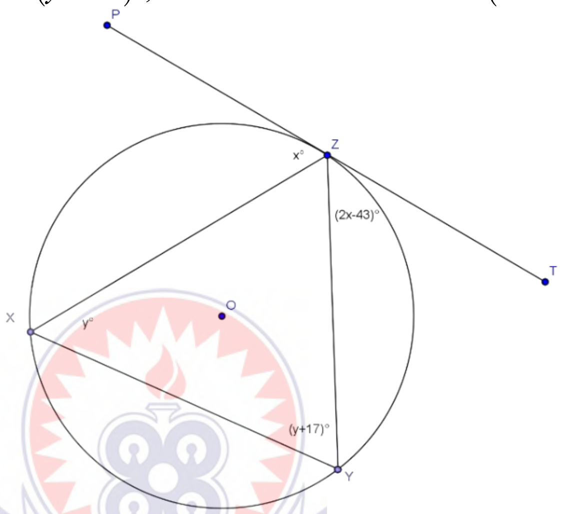
Name: ……………………………………………………… Duration: $2\frac{1}{2}$ *hours*

Gender:…………………… Age:………………

This paper consists of ten questions. Answer all questions and provide all necessary details of working and give your answers as accurately as data allow.

[50 marks]

1. In the diagram below, $\angle ZXY = y^0$, $\angle ZYX = (y+17)^0$, $\angle PZX = x^0$ *and* $\angle YZT = (2x-43)^0$. Find the value of y.



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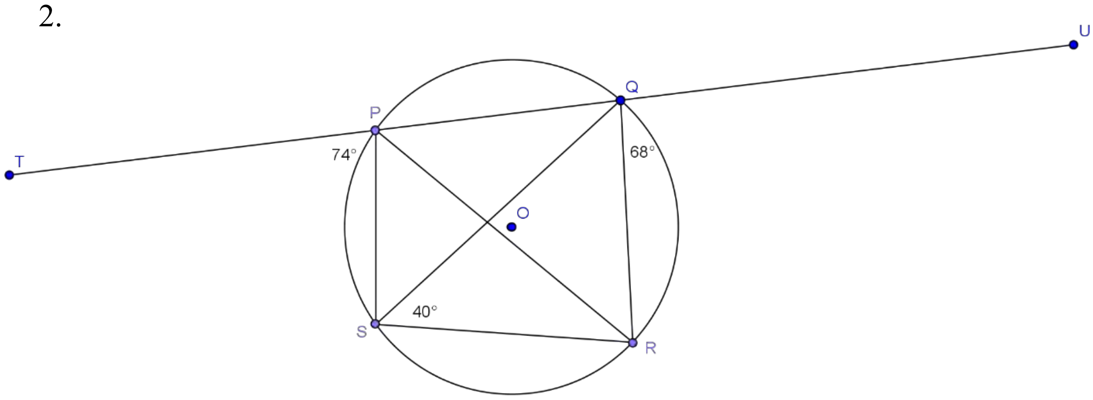

The diagram shows a circle PQRS with center *O*,
$\angle UQR = 68^0, \angle TPS = 74^0, and\ \angle QSR = 40^0$. Calculate the value of $\angle PRS$.

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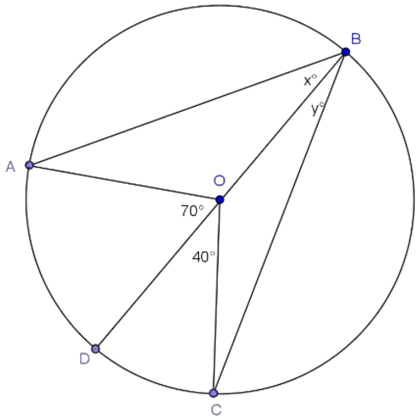
3. Given that O is the center of the circle, find the values of x and y in the diagram.



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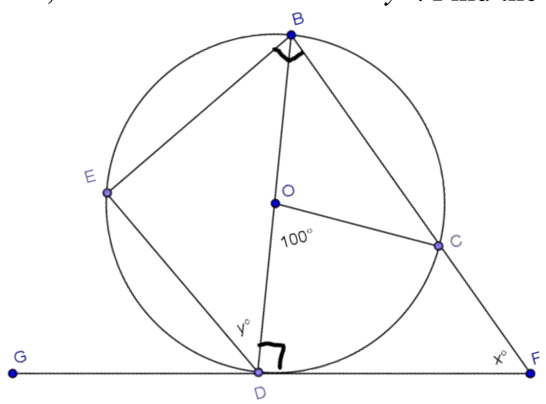
4. In the figure, O is the center of the circle, BD is a diameter, $\angle DOC = 100^0, \angle BFD = x^0$ *and* $\angle BDE = y^0$. Find the values of angles x and y.



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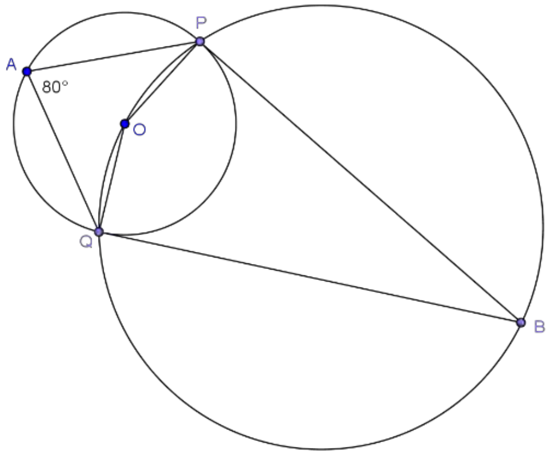
5. In the figure, the larger circle passes through the centre O of the smaller circle. If $\angle PAQ = 80^0$, *find* $\angle PBQ$.



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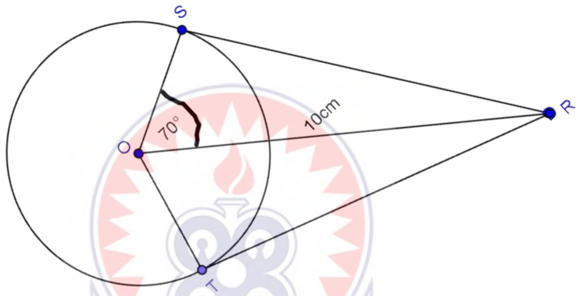
6. In the diagram, O is the centre of the circle and RS and RT are tangents to the circle from R. $\angle SOR = 70^0$ *and* $|OR| = 10cm.$ What is the size of $\angle ORT$?



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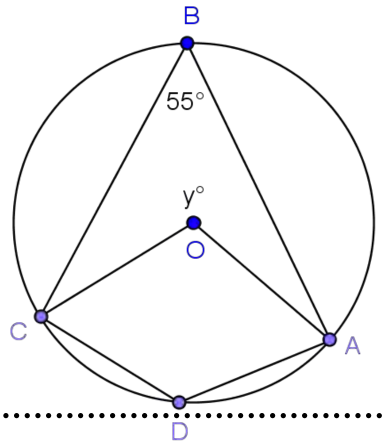
7. In the diagram, O is the centre of the circle ABCD. If $\angle ABC = 55^0$, find the value of y.



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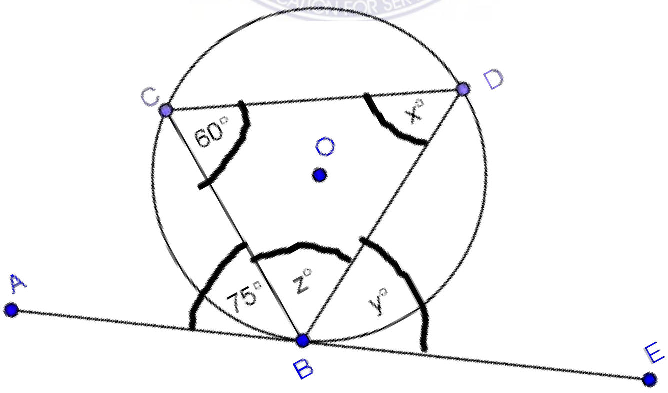
8. Find the values of the angles marked:
   (a) x;
   (b) y;
   (c) z



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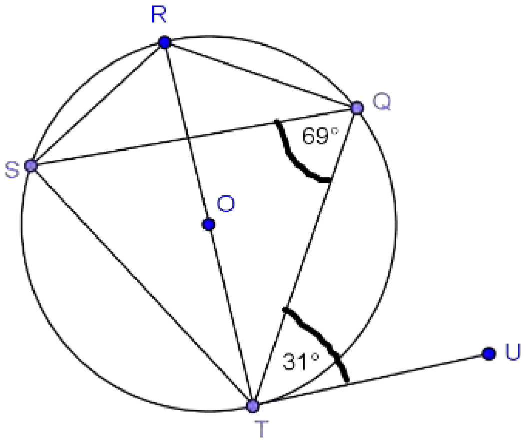
9. In the diagram, TU touches the circle at T and RT is a diameter.
$\angle UTQ = 31^0$ *and* $\angle TQS = 69^0$. Calculate the sizes of the angles:
(a) QRS
(b) SQR



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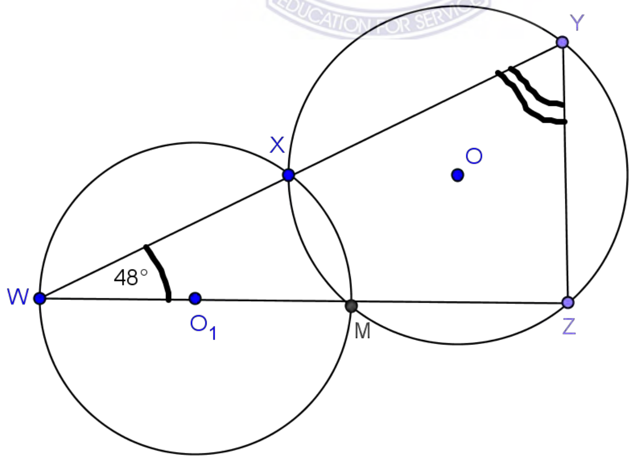
10. In the diagram, WZ and WY are straight lines, O is the centre of a circle WXM and $\angle XWM = 48^0$ Calculate the value of $\angle WYZ$.



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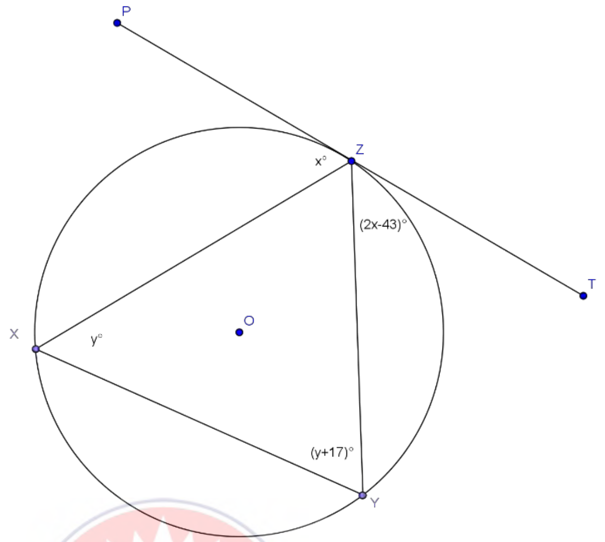
APPENDIX E

MARKING SCHEME FOR PRE-TEST

1. In the diagram below,
   $\angle ZXY = y^0$, $\angle ZYX = (y+17)^0$, $\angle PZX = x^0$ *and* $\angle YZT = (2x-43)^0$. Find the value of y.

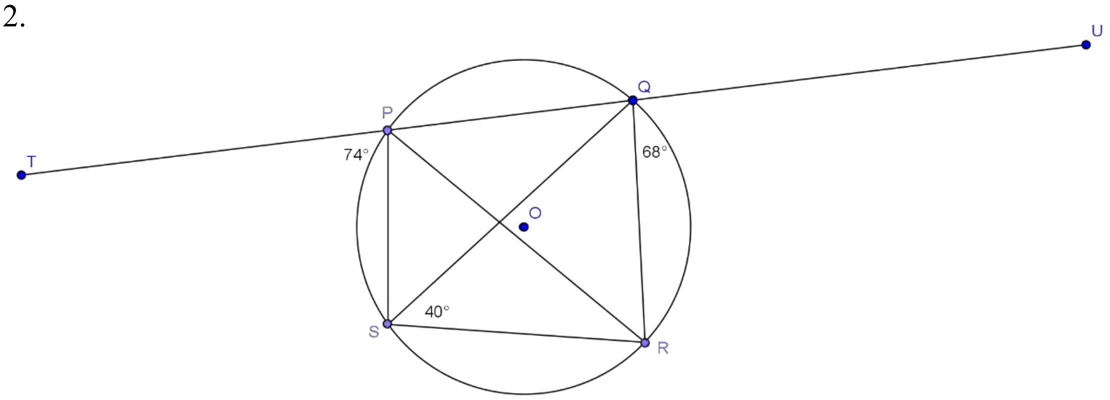


Solution Mark(5) Comment

Solution	Mark(5)
$x = y + 17$	$\frac{1}{2}M$
$y = 2x - 43$	$\frac{1}{2}M$
$x = 2x - 43 + 17$	$1M$
$-x = -26$	
$x = 26$	$1A$
$y = 2(26) - 43$	$1A$
$y = 52 - 43$	
$y = 9$	$1A$
Total marks	**(5)**

Comment: Angle formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment

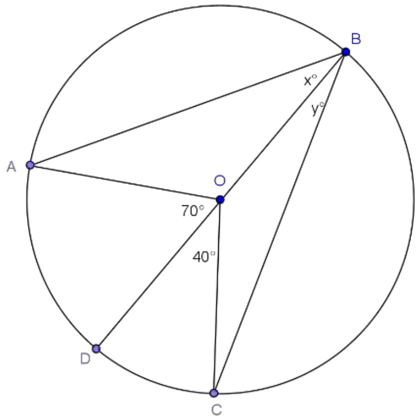
2.



The diagram shows a circle PQRS with center *O*,
$\angle UQR = 68^0, \angle TPS = 74^0, and\ \angle QSR = 40^0$. Calculate the value of $\angle PRS$.

Solution	Marks	Comments
$\angle RPQ = \angle RSQ = 40^0$	$\frac{1}{2}M$	Angles subtended from the same arc are equal
$\angle RSP = \angle RQU = 68^0$	$\frac{1}{2}M$	The exterior angle is equal to the opposite interior angle
$\angle RSQ + \angle QSP = \angle RSP$	$\frac{1}{2}M$	
$40^0 + \angle QSP = 68^0$	$\frac{1}{2}A$	
$\angle QSP = 28^0$	$\frac{1}{2}A$	
$\angle QRP = \angle QSP = 28^0$	$\frac{1}{2}M$	Angles subtended from the same arc are equal
$\angle QRS = \angle SPT = 74^0$	$\frac{1}{2}M$	The exterior angle is equal to the opposite interior angle
$\angle QRP + \angle PRS = \angle SPT$	$\frac{1}{2}M$	
$28 + \angle PRS = 74$	$\frac{1}{2}A$	
$\angle PRS = 46^0$	$\frac{1}{2}A$	
Total marks	**(5)**	

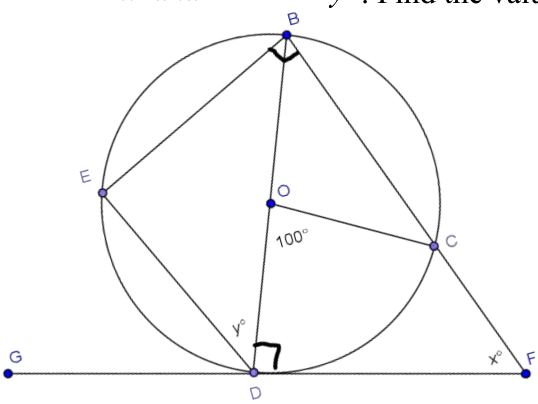
3. Given that O is the center of the circle, find the values of x and y in the diagram.



Solution Marks(5) Comment

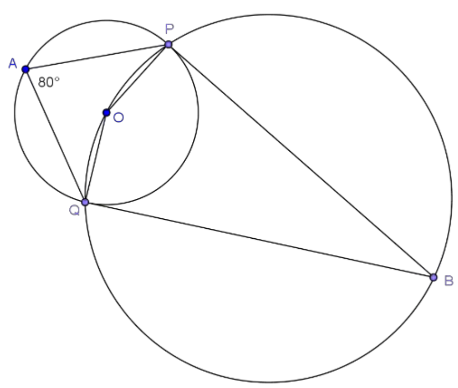
Solution	Marks(5)	Comment
ΔOAB *is isosceles*		
$\angle AOD + \angle AOB = 180^0$	1M	
$\angle AOB = 110^0$	1A	
$\angle ABO = \angle OAB = 35^0$	1A	
$\frac{1}{2}(110) = \angle ABC$	1M	
$55 = \angle ABC$		
$\angle ABO = 35^0$		
$\angle OBC = 20^0$	$\frac{1}{2}A$	
Therefore the values of x and y are 35^0 *and* 20^0 *respectively*	$\frac{1}{2}A$	
Total marks	**(5)**	

4. In the figure, O is the center of the circle, BD is a diameter, $\angle DOC = 100^0, \angle BFD = x^0$ *and* $\angle BDE = y^0$. Find the values of angles x and y.



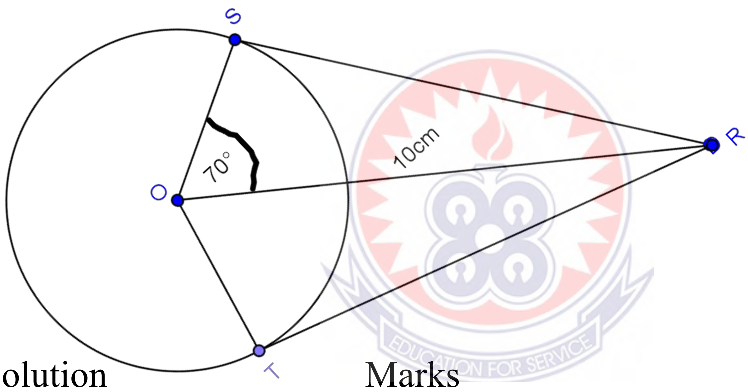
Solution	Marks	Comments
$\angle OBC = 50^0$	$\frac{1}{2}A$	
$2(\angle OBC) = \angle DOC$	$\frac{1}{2}M$	The angles subtended by an arc of a circle at the centre is twice that of the angle at the circumference
$\angle DBE = 40^0,\ \angle FBE = 90^0$	$\frac{1}{2}A$	angle subtended from the diameter and formed on the circumference is always perpendicular, .
$\angle COB = 80^0$	$\frac{1}{2}M$	Angles on a straight line sum up to 180^0
$\Delta OCB\ is\ isosceles,\ \angle OCB = 50^0$		
$y = \angle BDE = 50^0 = \angle OCB$	$1B$	Angles subtended from the same arc formed on the circumference are congruent
$\angle OCF = 130^0$	$\frac{1}{2}M$	Angles on a straight line add up 180^0
$\angle BDF + 100 + 130 + x = 360$	$\frac{1}{2}M$	Sum of interior angle of a quadrilateral is
$\angle BDE = 360 - 230 = 130$		
$x = 130^0$	$1A$	
Total marks	**(5)**	

5. In the figure, the larger circle passes through the centre O of the smaller circle. If $\angle PAQ = 80^0,\ find\ \angle PBQ.$



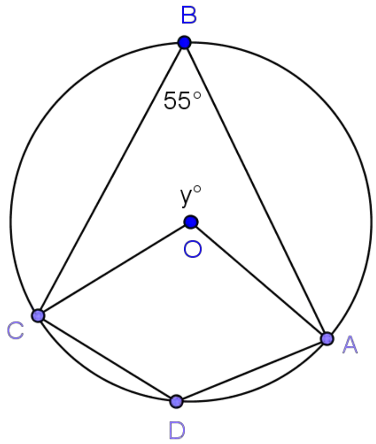
Solution	Marks	Comments
$\angle POQ = 160^0$	$1A$	
$2(\angle PAQ) = \angle POQ$	$1M$	The angles subtended by an arc of a circle at the centre is twice that of the angle at the circumference
$\angle PBQ + \angle POQ = 180^0$	$1M1A$	Angles in opposite segments of a cyclic quadrilateral are supplementary.
$\angle PBQ = 20^0$	$1A$	
Total marks	**(5)**	

6. In the diagram, O is the centre of the circle and RS and RT are tangents to the circle from R. $\angle SOR = 70^0$ *and* $|OR| = 10cm.$ What is the size of $\angle ORT$?



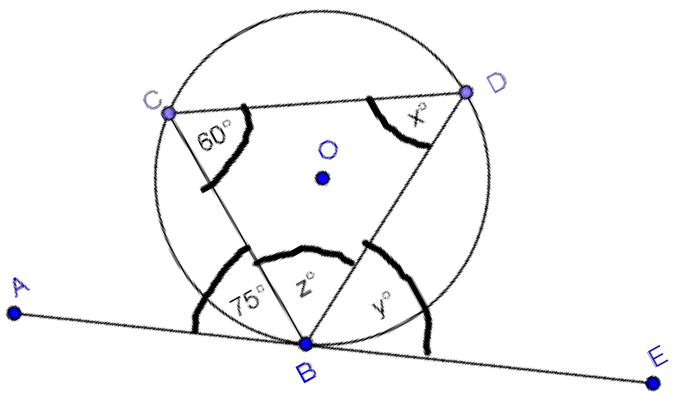
Solution	Marks	Comments
$\angle RSO = 90^0$	$1M$	The angle formed at the point of intersection of a tangent and a radius on the circumference is 90
$\lvert SR \rvert = \lvert RT \rvert$		
$\sin 70 = \dfrac{\lvert SR \rvert}{10}$	$1M1A$	
$\lvert SR \rvert = 10\sin 70 = 9.396$		
$\lvert TR \rvert = \lvert SR \rvert \approx 9.4cm$	$1A$	
$\cos(\angle ORT) = \dfrac{9.4}{10}$		
$(a)\ \angle ORT = \cos^{-1}\left(\dfrac{9.4}{10}\right) = 19.948 \approx 20^0$	$1A$	
Total marks	**(5marks)**	

7. In the diagram, O is the centre of the circle ABCD. If $\angle ABC = 55^0$, find the value of y.



Solution	Marks	Comments
$\angle COA(\min or\ \sec tor) = 2(55) = 110^0$	$1M1A$	The angles subtended by an arc of a circle at the centre is twice that of the angle at the circumference
$\angle COA(major\ \sec tor) = 250^0$	$1M1A$	Angle at a point is 360^0
$y + 110 = 360$		
$y = \angle COA(major\ \sec tor) = 360^0 - 110 = 250^0$	$1A$	
Total marks	**(5)**	

8. Find the values of the angles marked:
(d) x;
(e) y;
(f) z

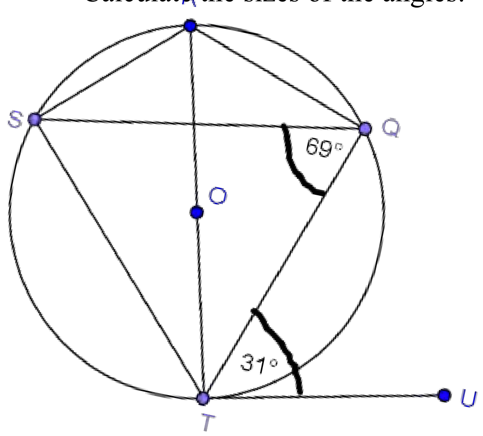


Solution	Marks	Comments
$x = 75$	$\frac{1}{2}A$	Angle formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment
$2 + x + 60 = 180$	$\frac{1}{2}M$	
$z + 75 + 60 = 180$	$\frac{1}{2}M$	Sum of interior angles of a triangle is 180.
$z = 180 - 135$		
$z = 45^0$	$1A$	
$y = 60^0$	$1A$	Angle formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment
$i.e\ 75 + z + y = 180$	$\frac{1}{2}M$	
$Therefore\ x = 75^0,\ y = 60^0\ and\ z = 45^0$	$1A$	
Total marks	**(5)**	

9. In the diagram, TU touches the circle at T and RT is a diameter. $\angle UTQ = 31^0\ and\ \angle TQS = 69^0$. Calculate the sizes of the angles:

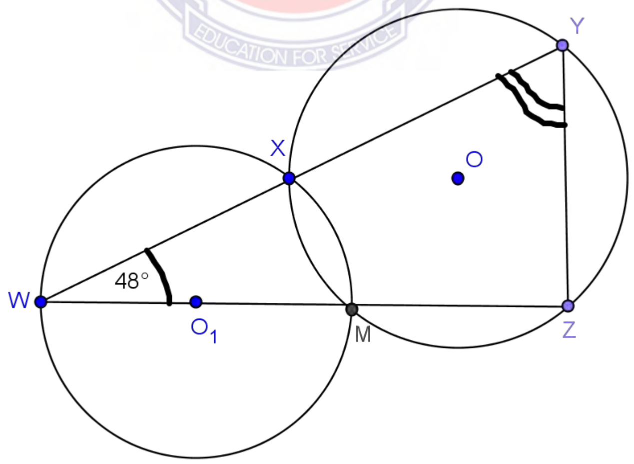
(c) QRS

(d) SQR



Solution	Marks	Comments
$\angle RQS = \angle SQR = 21^0$	$1B$	The angle subtended from the diameter and formed on the circumference is right angle
$\angle RQT = 90^0$		
$\angle TSQ = 31^0 = \angle STU$	$1M\ 1A$	Angle formed by a tangent to a circle and a chord through its point of contact is equal to the angle in the alternate segment
$\angle RSQ = 59^0$	$1A$	
$\angle RST = 90^0$		
$\angle STQ = 90^0 = \angle QTS$	$1A$	Sum of interior angles of a triangle is 180.
$\angle TRQ = 31^0$		
$\angle RTQ = 59^0$		
Total marks	**(5)**	

10. In the diagram, WZ and WY are straight lines, O is the centre of a circle WXM and $\angle XWM = 48^0$ Calculate the value of $\angle WYZ$.



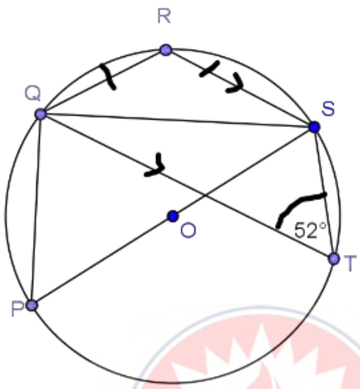
Solution	Marks	Comments
$\angle WXM = 90^0$	$\frac{1}{2}M$	The angle subtended from the diameter and formed on the circumference is right angle
$\angle WMX = 42^0$	$\frac{1}{2}B$	
$\angle YXM = 90^0$	$\frac{1}{2}B$	
$\angle MZY = 90^0$	$\frac{1}{2}M$	Angles in opposite segments of any cyclic quadrilateral supplementary
$\angle XMZ = 138^0$	$\frac{1}{2}M$	
$\angle WMX + \angle XMZ = 180^0$	$\frac{1}{2}M \frac{1}{2}A$	
$\angle XYZ = 42^0$	$\frac{1}{2}A$	
$\angle XYZ = \angle WYZ = 42^0$	$1A$	
Total marks	**(5)**	

APPENDIX F

Achievement Test (Post-Test)

[50 marks]

1. , PQRST is a circle with center O. If PS is a diameter, $RS // QT$, $|QR| = |RS|$ *and* $\angle QTS = 52^0$, find:
   (a) $\angle SQT$
   (b) $\angle PQT$



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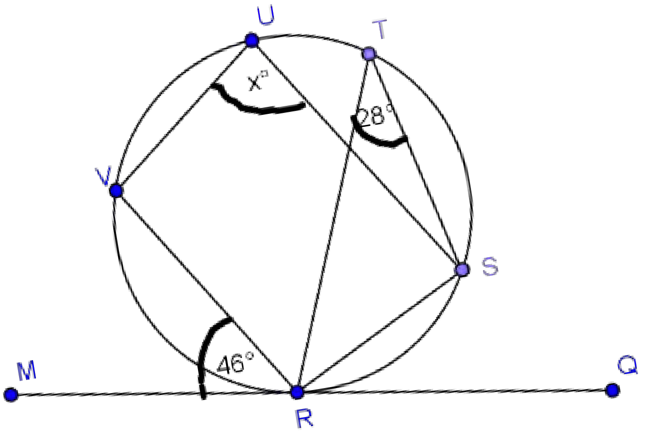
2. In the diagram, $\angle RTS = 28^0$, $\angle VRM = 46^0$, MQ is a tangent to the circle VRSTU at the point R. Find $\angle VUS$.



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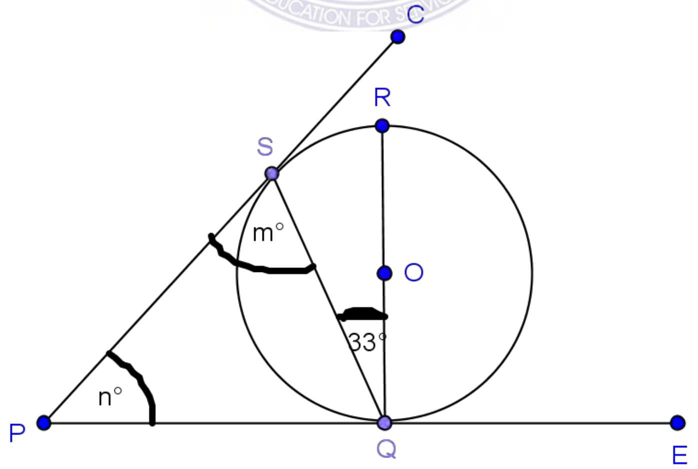
3. In the diagram, PQ and PS are tangents to the circle centre O. If $\angle PSQ = m,\ \angle SPQ = n\ and\ \angle SQR = 33^0$, find the value $(m + n)$.



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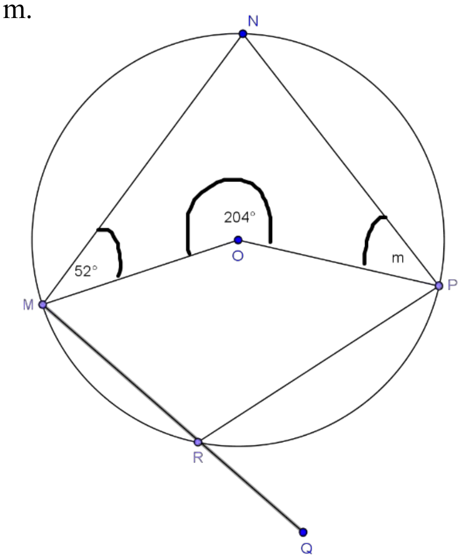
4. In the diagram is a circle MNPR with centre O. The reflex angle at O is $204^0, \angle NMO = 52^0$. Find the value of m.



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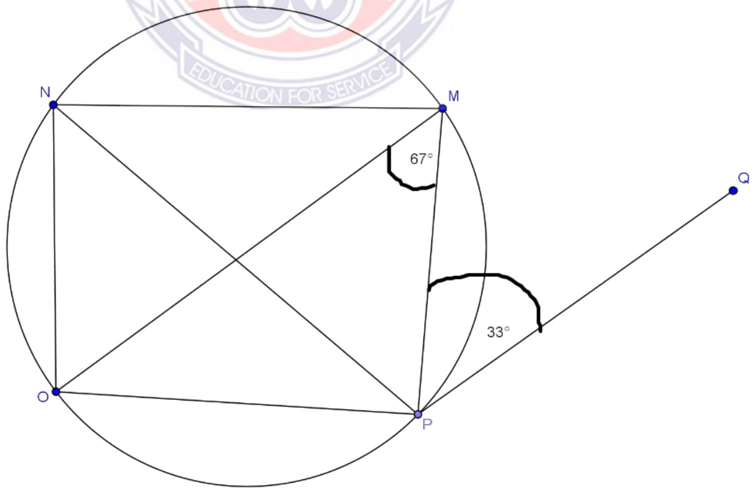
5. In the diagram PQ touches the circle MNOP at P and NP is a diameter. $\angle MPQ = 33^0$ and $\angle PMO = 67^0$. Find:

$(i) \angle MNO;$

$(ii) \angle MPO.$



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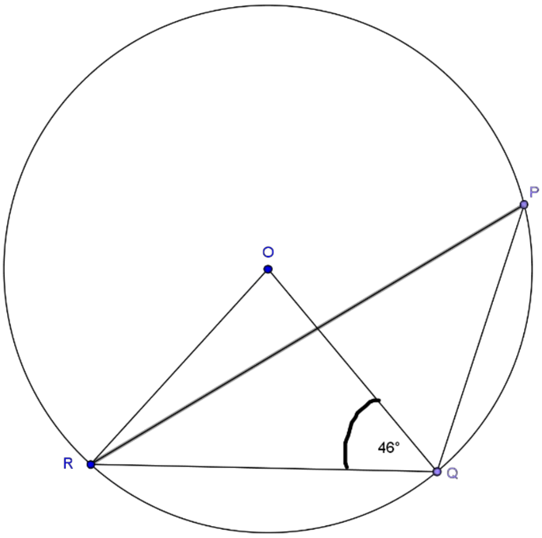
6. In the diagram, PQR is a circle with centre O. If $\angle RQO = 46^0$, find $\angle RPQ$.



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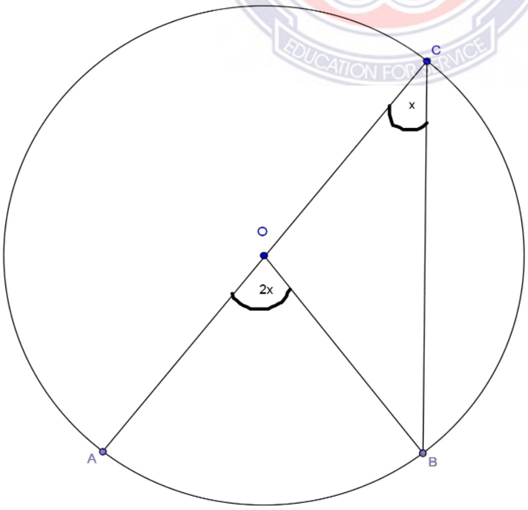
7. In the diagram, AC is a diameter, $\angle AOB = 2x$ and $\angle BCO = x$. Find the value of the angle $\angle BOC$



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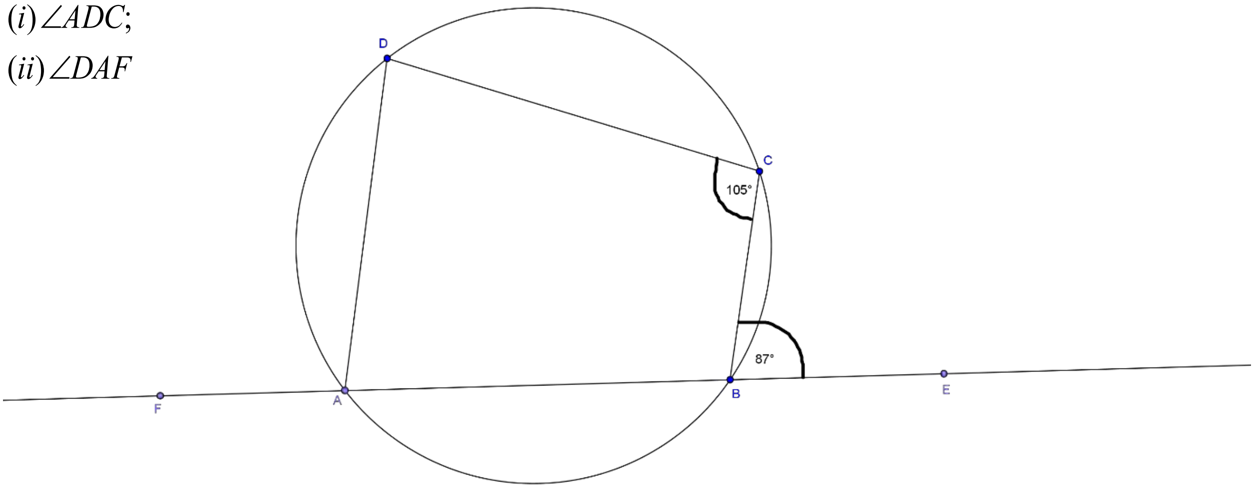
8. In the diagram $\angle BCD = 105^0$ *and* $\angle EBC = 87^0$. Find the values of:
   $(i)\angle ADC$;
   $(ii)\angle DAF$



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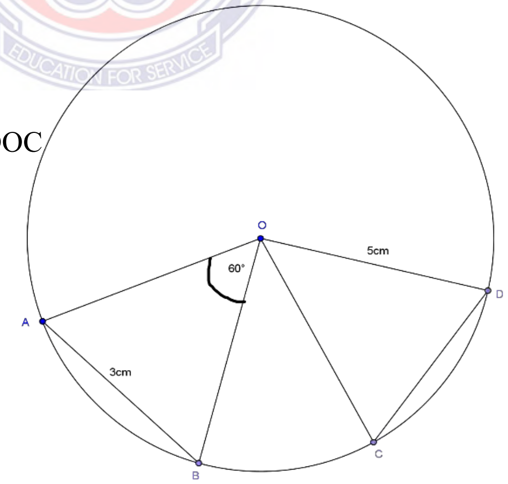
9. In the diagram O is the centre, $\angle AOB = 60^0$ and the length of $OD = 5cm$. Find the values of:
   (a) $\angle COD$;
   (b) The magnitude of CD;
   (c) The magnitude AO;
   (d) What type of triangle is DOC



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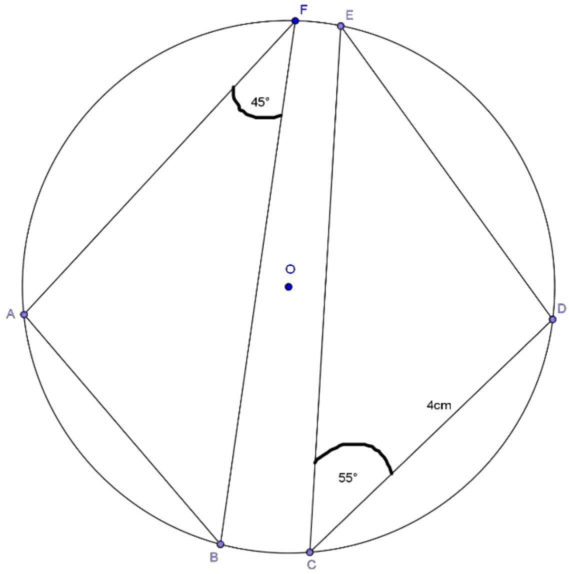
10. In the diagram, O is the centre of the circle, $\angle AFB = 45^0, \angle DCE = 55^0$ *and* $|CD| = 4cm$.

Find:

(a) $\angle DEC$;

(b) $|AB|$.



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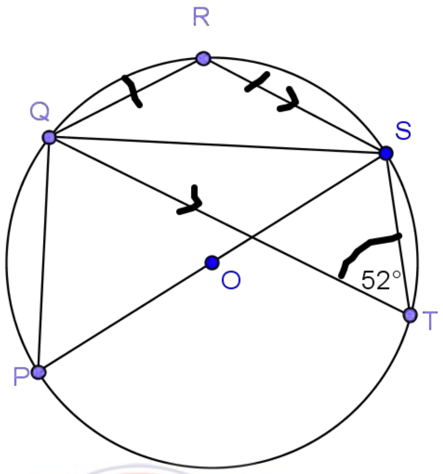
APPENDIX G

MARKING SCHEME FOR POST-TEST

1. In the diagram, PQRST is a circle with center O. If PS is a diameter, $RS // QT$, $|QR| = |RS|$ and $\angle QTS = 52^0$, find:

(c) $\angle SQT$

(d) $\angle PQT$



Solution	Marks (5)	Comment
		The sum of the opposite interior angle of a cyclic quadrilateral is 180^0
$\angle QRS + 52^0 = 180^0$	$M\frac{1}{2}\ A\frac{1}{2}$	
		Angles subtended from the same chord formed on the circumference are equal
		Angles subtended from the diameter formed on the circumference is 90^0

$\angle QRS = 128^0$

ΔQRS *is isosceless*

$\angle RQS = \angle RSQ = x$ $M\frac{1}{2}A\frac{1}{2}$

$128^0 + x + x = 180^0$

$2x = 180 - 128$

$2x = 52$

$x = 26^0$

$\angle RQS = \angle RSQ = 26^0$ $\frac{1}{2}M$

$\angle QPS = 52$

$\angle PQS = 90^0$

$\angle SRQ + \angle RQT = 180^0$

$128 + (26 + x) = 180$

$x = 26^0$

(a) $\angle SQT = 26^0$ $1A$

Co-interior angles sum up to 180^0

(b) $\angle RST + \angle STQ = 180^0$

Co-interior angles sum up to 180^0

but $\angle RST = \angle RSQ + \angle QST$

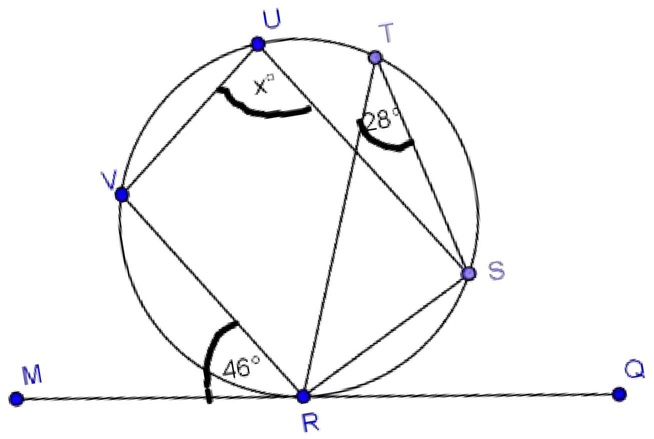
$\angle PTS = 90^0$ $\frac{1}{2}B$

Angles subtended from the diameter formed on the

$\angle PQT = \angle PST = 64^0$

$\angle PQT = 64^0$ $1A$

2. In the diagram, $\angle RTS = 28^0$, $\angle VRM = 46^0$, MQ is a tangent to the circle VRSTU at the point R. Find $\angle VUS$.



Solution | Marks (5) | Comment

$\angle MRV = \angle USR = 46^0$ — $B1$

$\angle UVR + \angle RSU = 180^0$

The sum of the opposite interior angle of a cyclic quadrilateral is 180^0

$\angle UVR + 46 = 180$

$\angle UVR = 180 - 46$

$\angle UVR = 134^0$ — $M\frac{1}{2}A\frac{1}{2}$ $1\frac{1}{2}$

$\angle RTS = \angle SRQ = 28$ — $B1$

The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment

$46 + \angle VRS + \angle SRQ = 180$

$46 + \angle VRS + 28 = 180$

$\angle VRS = 180 - 74$

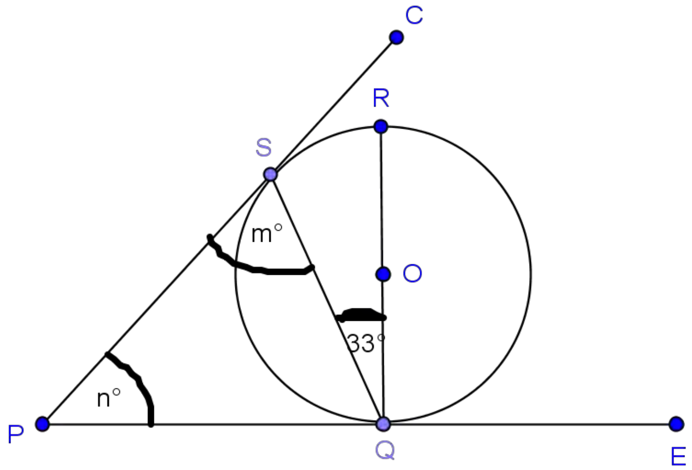
$\angle VRS = 106^0$ — $M\frac{1}{2}A\frac{1}{2}$ $\frac{1}{2}$

$\angle VRS + x = 180$

$x = 180 - 106$

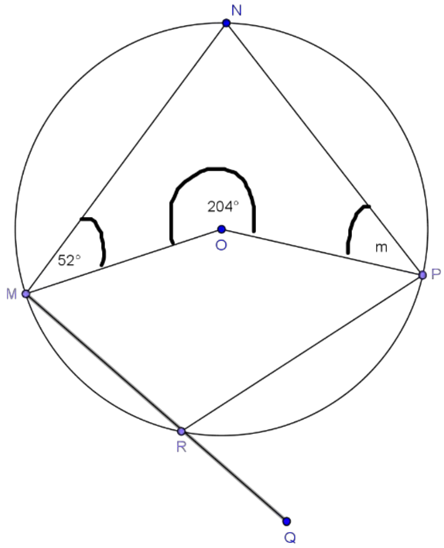
$x = 74^0$ — $A1$

3. In the diagram, PQ and PS are tangents to the circle centre O. If $\angle PSQ = m,\ \angle SPQ = n\ \ and\ \ \angle SQR = 33^0$, find the value $(m+n)$.



Solution	Marks (5)	Comment
$therefore\ \angle OSQ = \angle OQS = 33^0$	$B1$	ΔSOQ is isosceles
$\angle SOQ = 144^0$	$B1$	The sum of the interior angle of a triangle is 180^0
$\frac{1}{2}\angle SOQ = \angle QPS$		The quadrilateral PQOS is a kite
$\angle QPS = 57^0$	$M\frac{1}{2}A\frac{1}{2}$	
$\angle PQO = \angle PSO$		The sum of the interior angle of a quadrilateral is 360^0
$\angle PQR = 94.5^0 = \angle PSO$	$M\frac{1}{2}A\frac{1}{2}$	
$So\ \angle PQS = 61.5^0 = \angle PSQ$		
$m = 61.5^0$		
$n = 57^0$		
$Therefore\ m+n = 118.5^0$	$A1$	

4. In the diagram is a circle MNPR with centre O. The reflex angle at O is $204^0, \angle NMO = 52^0$. Find the value of m.

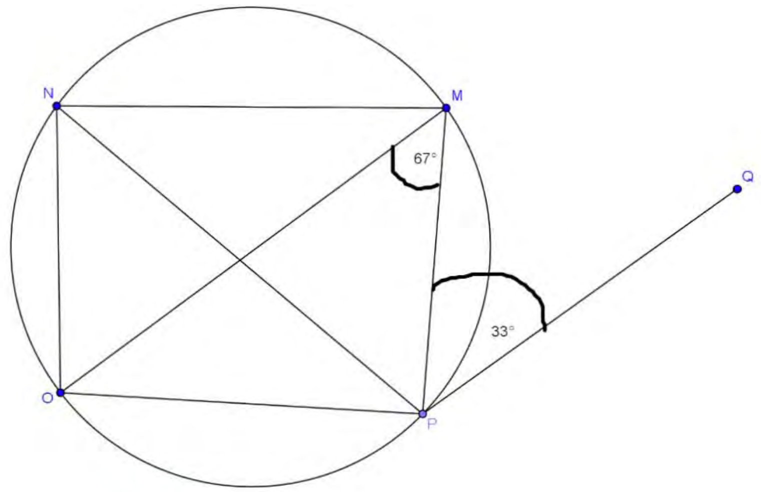


Solution	Marks (5)	Comment
$\angle MOP(\min \sec) = 156^0$	$M\frac{1}{2}A\frac{1}{2}$	Angle at a point is 360^0
$\angle MNP = \frac{1}{2}(204) = 102^0$	$B1$	The angle at the centre of a circle is twice the angle at the circumference subtended by the
$\angle MNP + \angle MRP = 180^0$		The sum of the opposite angles of a cyclic quadrilateral is 180^0
$102^0 + \angle MRP = 180^0$		
$\angle MRP = 78^0$	$M\frac{1}{2}A\frac{1}{2}$	
$\angle MNO = 52^0$		ΔMNO *is isosceless*
$\angle MON = 76^0$		The sum of the interior angle of a triangle is 180^0
$\angle NOP = 128^0$	$M\frac{1}{2}A\frac{1}{2}$	ΔONP *is isosceless*
$\angle OPN = \angle ONP = 26^0$	$1A$	

5. In the diagram PQ touches the circle MNOP at P and NP is a diameter. $\angle MPQ = 33^0$ and $\angle PMO = 67^0$. Find:

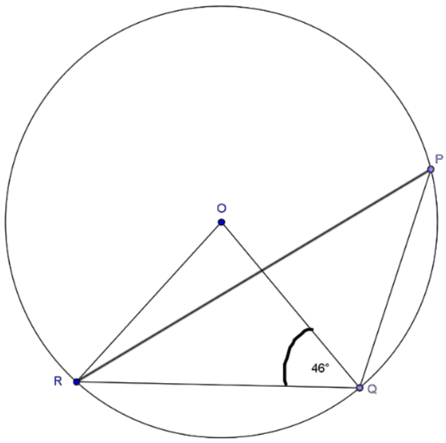
$(i)\angle MNO;$

$(ii)\angle MPO.$



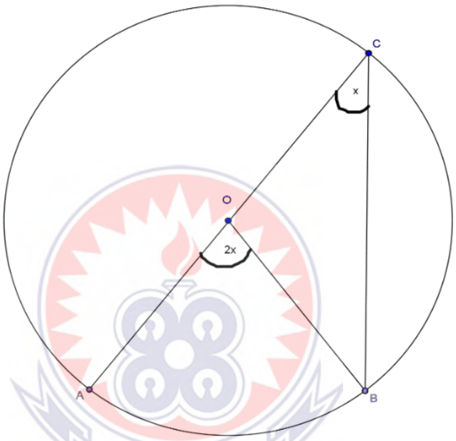
Solution	Marks (5)	Comment
$\angle NMP = 90^0$	$M\frac{1}{2}A\frac{1}{2}$	The angle subtended by the diameter on the circumference is
$\angle NMO = 23^0$		
$\angle MOP = \angle MPQ = 33^0$		The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment
$\angle NOM = 57^0$	$M\frac{1}{2}A\frac{1}{2}$	
$57^0 + 23^0 + \angle MNO = 180^0$	$M\frac{1}{2}A\frac{1}{2}$	Sum of the interior angle of a triangle is 180^0
$80^0 + \angle MNO = 180^0$		
$\angle MNO = 100^0$	$A1$	
$\angle NMO = \angle NPO = 23^0$		Angles subtended from the same chord and formed at the circumference are equal
$\angle NOM = \angle NPM = 57^0$		
$\angle MPO = \angle MPN + \angle NPO$		
$\angle MPO = 57^0 + 23^0$		
$\angle MPO = 80^0$	$A1$	

6. In the diagram, PQR is a circle with centre O. If $\angle RQO = 46^0$, find $\angle RPQ$.



Solution	Marks (5)	Comment
$\angle ROQ = \angle RQO = 46^0$	$M\frac{1}{2}A\frac{1}{2}$	ΔROQ is isosceless
$\angle ROQ + \angle ORQ + \angle RQO = 180^0$	$M\frac{1}{2}A\frac{1}{2}$	The sum of the interior angle of a triangle is 180^0
$\angle ROQ = 88^0$	$B1$	
$\angle RPQ = \frac{1}{2}\angle ROQ$	$M\frac{1}{2}A\frac{1}{2}$	
$\angle RPQ = \frac{1}{2}(88)$		
$\angle RPQ = 44^0$	$A1$	

7. In the diagram, AC is a diameter, $\angle AOB = 2x$ *and* $\angle BCO = x$. Find the value of the angle $\angle BOC$

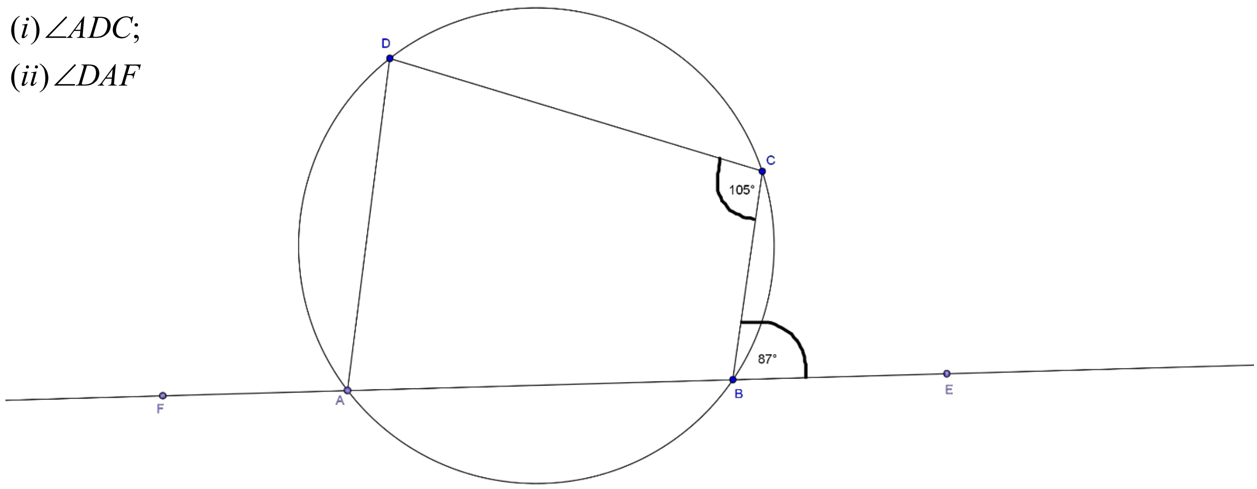


Solution	Marks (5)	Comment
$\angle OBC = \angle BCO = x$	$M1A1$	Triangle BOC is isosceless
$y + x + x = 180^0$	$M\frac{1}{2}A\frac{1}{2}$	Sum of the interior angle of a triangle is 180^0
$2x + y = 180^0$	$M\frac{1}{2}A\frac{1}{2}$	
$y = 180 - 2x$		
$\angle BOC = 180 - 2x$	$A1$	

8. In the diagram $\angle BCD = 105^0$ *and* $\angle EBC = 87^0$. Find the values of:

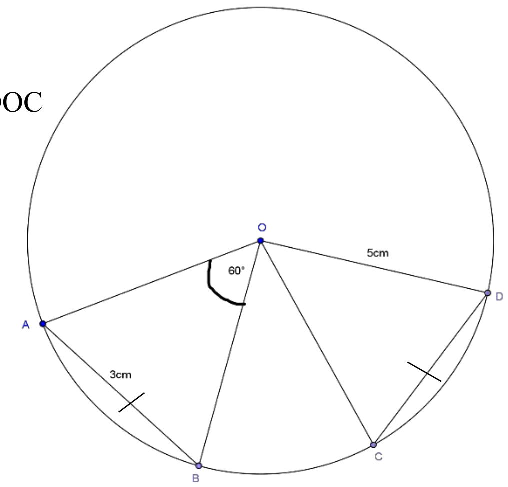
$(i) \angle ADC$;

$(ii) \angle DAF$



Solution	Marks (5)	Comment
$\angle DAB + \angle BCD = 180^0$	$M\frac{1}{4}A\frac{1}{4}$	The sum of two opposite angles in a cyclic quadrilateral is 180^0
$\angle DAB = 75^0$	$M\frac{1}{4}A\frac{1}{4}$	
$\angle ABC + \angle CBE = 180^0$	$M\frac{1}{4}A\frac{1}{4}$	Angles on a straight line sum up to 180^0
$\angle ABC + 87^0 = 180^0$		
$\angle ABC = 93^0$	$M\frac{1}{4}A\frac{1}{4}$	
$\angle ABC + \angle ADC = 180^0$	$M\frac{1}{4}A\frac{1}{4}$	The sum of two opposite angles in a cyclic quadrilateral is 180^0
$93^0 + \angle ADC = 180^0$		
$\angle ADC = 87^0$	$A1$	
$\angle DAF + \angle DAB = 180^0$	$M\frac{1}{4}A\frac{1}{4}$	Angles on a straight line sum up to 180^0
$\angle DAF + 75^0 = 180^0$		
$\angle DAF = 105^0$	$A1$	

9. In the diagram O is the centre, $\angle AOB = 60^0$ and the length of $OD = 5cm$. Find the values of:
   (e) $\angle COD$;
   (f) The magnitude of CD;
   (g) The magnitude AO;
   (h) What type of triangle is DOC



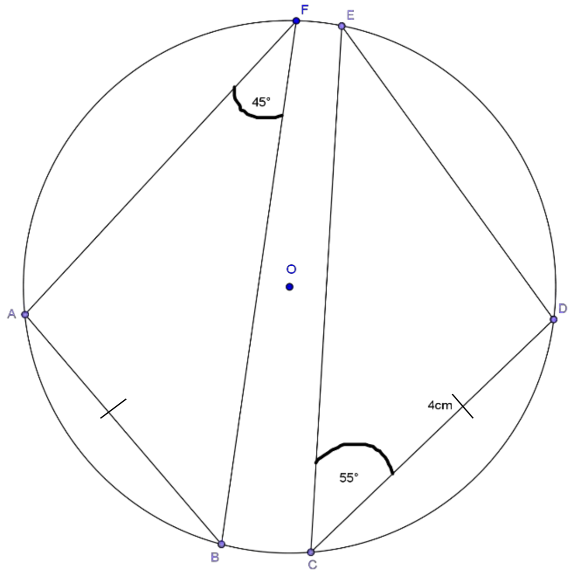
Solution	Marks (5)	Comments
$\angle COD = \angle AOB = 60^0$	$1B$	The angles subtended by equal chords at the centre of a circle are equal
$\lvert CD\rvert = \lvert AB\rvert = 3cm$	$1B$ $M\frac{1}{2}A\frac{1}{2}$	The angles subtended by equal chords at the centre of a circle are equal
$\lvert AO\rvert = \lvert DO\rvert = 5cm$	$1B$	The angles subtended by equal chords at the centre of a circle are equal
ΔDOC *is isosceless*	$1A$	

10. In the diagram, O is the centre of the circle, $\angle AFB = 45^0, \angle DCE = 55^0$ *and* $|CD| = 4cm$.

Find:

(c) $\angle DEC$;

(d) $|AB|$.



Solution	Marks (5)	Comment
$\angle DEC = \angle AFB = 45^0$	$M1A1$	The angles subtended by equal chords at the circumference of a circle are equal
	$M\frac{1}{2}A\frac{1}{2}$	
$\lvert AB\rvert = \lvert CD\rvert = 4cm$	$M1A1$	The angles subtended by equal chords at the circumference of a circle are equal

APPENDIX H
ASANKRANGWA SENIOR HIGH SCHOOL
SCHEME OF WORK FOR THE STUDY- CORE MATHEMATICS
2020/2021 ACADEMIC YEAR
SHS 2 SECOND SEMESTER (15TH NOVEMBER TO 6TH DECEMBER, 2021)

WK	DATE	ENDING	TOPIC	SUB-TOPIC/CONTENT	OBJECTIVE	REFERENCE
1	15-19/11/2021	19/11/2021	PLANE GEOMETRY II	The circle as a locus. Circle Theorems	Draw circles for given radii. State and use the circle theorems.	
2	22-26/11/2021	26/11/2021		Perpendicularity of tangent and radius of a circle.	Identify the tangent as perpendicular to the radius at the point of contact.	
3	29/11/-3/12/21	3/12/2021		Angles between tangent and a chord.	Verify that the angle between the tangent and the chord at the point of contact is equal to the angle in the alternate segment.	
4	6-10/12/2021	12/2021		Tangents from an external point.	Verify that tangents drawn from an external point to the same circle are equal when measured from their point of contact.	
5	13-17/12/2021	17/12/2021	Achievement test (Post-test)			

APPENDIX I

Time Table

	1 7:30-8:30	BREAKFAST 8:30-9:30	2 9:30-10:30	3 10:30-11:30	4 11:30-12:30	5 12:30-13:30	LAUNCH BREAK	6 14:30-15:30
MONDAY					2 ARTS B1			2 ARTS B2
TUESDAY	2 ARTS B1					2 ARTS B2		
WEDNESDAY					2 ARTS B1			
THURSDAY	2 ARTS B2					2 ARTS B1		
FRIDAY			2 ARTS B2					