

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF SCIENCE EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION

**PRE-SERVICE TEACHERS' USE OF PROBLEM SOLVING SKILLS IN DEALING
WITH SIMULTANEOUS EQUATIONS INVOLVING WORD PROBLEMS**



BY

KUTTEN BOATENG JOHN

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A MASTERS' THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS
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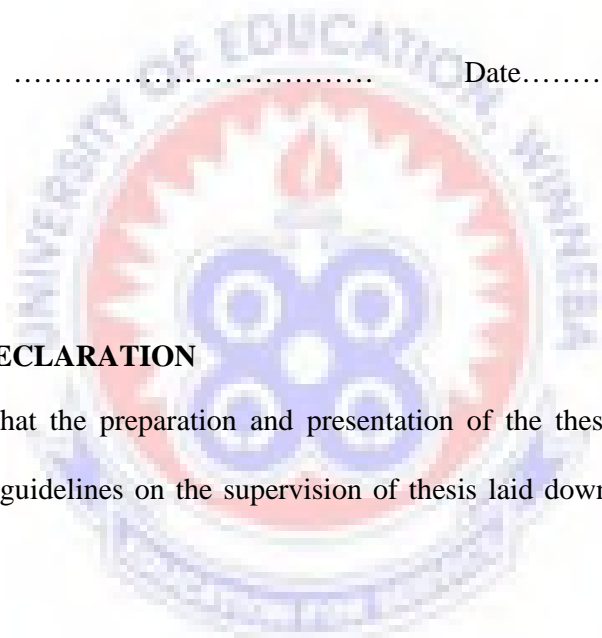
DECLARATION

CANDIDATE'S DECLARATION

I hereby declare that this thesis is the result of my own original research that no part of it has been presented for another degree in this University or elsewhere.

Candidate's Name Kутten Boateng John

Signature Date.....



SUPERVISOR'S DECLARATION

We hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on the supervision of thesis laid down by the University of Education, Winneba.

Supervisor's Name Prof. Kofi Damian Mereku

Signature Date

DEDICATION

This special work is dedicated to my wife, Evelyn Kuten Boateng and my four children, Ata Kutin-Boateng Sr., Ata Kutin-Boateng Jr, Sarfo Kutin-Boateng and Onyina Kutin-Boateng



#

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Every cloud, it is said has a silver lining and when the sun rays eventually penetrate the horizon, living things leap for joy. It is in a similar situation that I record with great joy the completion of this project.

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ABSTRACT

This study investigated how the use of problem solving techniques could improve pre-service teachers' performance in word problems of simultaneous equations as well as interest in the subject. The second-year pre-service teachers at the Wesley College of Education, Kumasi, were involved in the study. The study employed the survey design and data were collected using pre-test, post-test and questionnaire/interview. The data collected were analysed using descriptive statistics. The results indicated that the performance of the pre-service teachers' on word problems of simultaneous equations had tremendously improved after going through the problem-solving skills. It was observed that the pre-service teachers after going through the research project activities grew in confidence in solving word problems involving simultaneous equations. It was therefore recommended among others that the mathematics curriculum for pre-service training should be reviewed to ensure the development of and use of problem solving techniques in the teaching and learning of mathematics.

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CHAPTER 1

INTRODUCTION

1.0 Overview

This chapter looks at the background to the study which identifies what prompted the researcher to embark on this study. It is then followed by the statement of the problem, purpose of the study and significance of the study. Under this, it looks at the possible beneficiaries to the findings that might come out of this research.

1.1 Background to the study

The essence of mathematics is not to make simple things complicated but to make complicated things simple. Mathematics is both an art and a science and defining it is as difficult as trying to define music. One can listen to music, study music history and possibly even compose music and only then do one realizes what music is. One can study what great mathematicians have done and look at mathematical history and structure. “Mathematics is the means of sharpening the individuals mind, shaping his reasoning ability and developing his personality, hence its immense contribution (Asiedu-Addo & Yidana, 2000).

In Ghana, there are thirty-eight (38) Colleges of Education and they have been basically categorized into two (2); those that specialize in the teaching and training of science and mathematics teachers and those that offer general courses. Wesley College is one of the

Colleges of Education that have been selected to train science and mathematics teachers. The course of study in mathematics in the Colleges of Education is similar to the elective mathematics that is pursued in the Senior High Schools for which simultaneous linear equation is no exception. During the first semester, the researcher treated the topic simultaneous equation with questions like $3x + 4y = 6$, and $x - 3y = -11$. Even though they had come to read mathematics, their performance was not encouraging and this manifested more in the topic simultaneous equation – word problem.

The question “*The sum of the ages of a father and a son is 52 years. Eight years ago, the father was eight times as old as his son. How old is the father now?*” Of the two classes with a total of sixty (60); Science 2A and 2B, only four were able to work out the answer. During the mid-semester, a set of questions were given out which included this; “*A mother is three times as old as is the daughter. Six years ago, she was five times as old. How old is the daughter now?*” Here only two (2) were able to work it out. In the course of the semester, there were a series of mathematics competitions organized either by the school or in conjunction with a Ghanaian studying in the United Kingdom, for the two science classes A and B. Questions set on word problem could not be answered by all the contestants.

In January 2008 for instance, candidates who sat for either the first year end-of first semester or the re-sit examinations were heard complaining about their inability to tackle the word problem on question number ten (10) section B which says “one-half of number is added to one-third of the same number and the result is 4, find the number”. It is for this reason that the researcher has been moved to find out why pre-service mathematics teachers find it difficult in working out word problems. The observation is that these pre-service teachers lack the problem solving skills to work word problems.

1.2 Statement of the problem

The fact remains that every teacher trainee before gaining admission to pursue a three year diploma in basic education goes through a three year pre-tertiary education in the senior high school level. In these schools, those who offer science as a course have in their course outline or their course of study, *elective mathematics*. Under this subject, they study various topics which also include simultaneous equations – word problem. However, when they gain admission and enter the Colleges of Education, these trainee teachers are always found wanting whenever they are confronted with a question on simultaneous equations that deals with word problems. It is as if they have not studied the topic before or they have lost interest and confidence with respect to that topic.

As a result of this the researcher will want to find out how the use of problem solving skills can improve the learning of word problem of simultaneous equation especially at Wesley College of Education.

1.3 Purpose of the study and research questions

The reasons for undertaking this study was to make pre-service teachers state the sequence of steps involved in solving word problems of simultaneous equations. It was also to make pre-service teachers develop a plan to implement the solution to word problems of simultaneous equation. To achieve these, the following research questions were formulated to guide the study:

1. How has problem solving techniques improved pre-service teachers' performance in word problems of simultaneous equations?

2. How does the use of problem solving techniques increase students' interest in learning mathematics?

1.4 Significance of the study

One of the benefits of this research is that it will serve as a guide to mathematics teachers in Wesley College of Education in particular and other Colleges of Education as a whole to become aware of the impact of problem solving skills as an alternative method of teaching word problem of simultaneous equation.

In addition to this it will also enable pre-service teachers to come to the realization that questions on word problems under simultaneous equation are just any other form of simultaneous equation questions.

1.5 Limitation and delimitations

Two of the potential weaknesses related to this research were that;

1. During the course of the interview teachers felt reluctant to come out with the rightful information which affected the final analysis of the research work.
2. It was difficult to find a teacher who was willing to allow his/her class to be used for the study.

With respect to this study, it was restricted to two science classes; 2A and 2B in Wesley College of Education, as well the mathematics teachers found in the college. The research was an action research; therefore these were the classes where the problem was identified hence, reason for their selection and not any other classes.

1.6 Organization of the study

The remaining part of this research report was organized into four chapters. Review of literature in chapter two, reviews problem solving skills and how they can help solve word problems of simultaneous equations. Chapter three will look at the methodology for the study which consists of the data collection procedure, sample population, and data analysis procedure. Chapter Four, will discuss the findings, and Chapter Five summarizes the whole research and concludes with recommendations.



CHAPTER 2

LITERATURE REVIEW

2.0 Overview

This chapter deals with literature review and an essential feature of the chapter relates to problem solving skills and how they can help solve word problems of simultaneous equations. The review is broken down into the following sub-headings: the conceptual framework, problem solving and its characteristics, the phases and teachers' in teaching problem solving.

2.1 Conceptual framework

"When we think of mathematics books, we think of non-fiction, even though mathematics itself is predominantly fiction" (Pappas, 1999). Some of us may feel uncomfortable with the notion that mathematics is fiction, but the concepts and procedures of mathematics are all constructions of our minds, products of our attempts to understand our worlds, real and imaginary. Some mathematical ideas have obvious practical applications in our everyday lives, while other ideas seem very abstract; with little apparent connection to life as most of us experience it. All mathematical ideas, though, take shape through our attempts to communicate, and therefore find their way into our literature. Having an inherent sense of number (Dehaene, 1997), we express mathematical ideas in stories, essays, poems, books, and other forms of literature that convey life experiences, real or imagined. One way of connecting school mathematics to everyday life, then, is to draw attention to the mathematics embedded in the literature of

everyday life, to reveal the mathematics inherent in human thinking and communication about life experiences.

Many students approach mathematics with the attitude that “I can do the equations, but I’m just not a ‘word problems’ person.” No offence, but that’s like saying “I’m pretty good at handling a tennis racket, as long as there’s no ball involved.” The only point of handling the tennis racket is to hit the ball. The only point of mathematics equations is to solve problems. In that category, try this sentence instead: “I’ve never been good at word problems. There must be something about them I don’t understand, so I’ll try to learn it.” The number of research studies conducted in mathematics education over the past three decades has increased dramatically (Kilpatrick, 1992). Research findings indicate that certain teaching strategies and methods are worth careful consideration as teachers strive to improve their mathematics teaching practices. Among the methods worth considering is problem solving skills.

Students can learn both concepts and skills by solving problems. Research suggests that students who develop conceptual understanding early perform best on procedural knowledge later. Students with good conceptual understanding are able to perform successfully on near-transfer tasks and to develop procedures and skills they have not been taught. Students with low levels of conceptual understanding need more practice in order to acquire procedural knowledge.

2.1.1 What is Problem Solving?

Many writers like Evan and Lapping (1994) and Lester (1994) have attempted to clarify what is meant by a problem-solving approach to teaching mathematics. The focus is on teaching mathematical topics through problem-solving contexts and enquiry-oriented

environments which are characterised by the teacher helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying (Lester, F.K.J., Masingila, J.O., Mau, S.T., Lambdin, D.V., dos Santos, V.M. and Raymond, A.M. (1994). p.154). Specific characteristics of a problem-solving approach include:

- Interactions between students/students and teacher/students (Van Zoest et al., 1994).
- Mathematical dialogue and consensus between students (Van Zoest et al., 1994).
- Teachers providing just enough information to establish background/intent of the problem, and students clarifying, interpreting, and attempting to construct one or more solution processes (Cobb, P., Wood, T. and Yackel, E. (1991).
- Teachers accepting right/wrong answers in a non-evaluative way (Cobb et al., 1991).
- Teachers guiding, coaching, asking insightful questions and sharing in the process of solving problems (Lester et al., 1994).
- Teachers knowing when it is appropriate to intervene, and when to step back and let the pupils make their own way (Lester et al., 1994).
- A further characteristic is that a problem-solving approach can be used to encourage students to make generalisations about rules and concepts, a process which is central to mathematics (Evan and Lapping, 1994).

Also mathematical problem solving is a complex cognitive activity involving a number of processes and strategies. Problem solving has two stages: problem representation and problem execution. Successful problem solving is not possible without first representing the problem appropriately. Appropriate problem representation indicates that the problem solver has understood the problem and serves to guide the student toward the solution plan. Students who have difficulty representing mathematics problems will have difficulty solving them.

2.1.2 Characteristics of a problem

There are a number of variable attributes of problems. Problems vary in knowledge needed to solve them, the form they appear in, and the processes needed to solve them. The problems themselves also vary considerably, from simple addition problems in elementary school to complex social-cultural and political problems like those encountered in the Middle East. Intellectually, problems vary in at least four ways: structuredness, complexity, dynamicity, and domain specificity or abstractness.

Structuredness

Problems within domains and between domains vary in terms of how well structured they are. Jonassen (1997) described problems on a continuum from well-structured to ill structure. The most common problems that students solve in schools, universities, and training venues are well-structured problems. Like the story problems found at the end of textbook chapters or on examinations, well-structured problems require the application of a limited and known number of concepts, rules, and principles being studied within a restricted domain. They have a well-defined initial state, a known goal state or solution, and a constrained set of logical operators (a known procedure for solving). Well

structured problems also present all elements of the problem to the learners, and they have knowable, comprehensible solutions. Ill-structured problems, at the other end of the continuum, are the kinds of problems that are more often encountered in everyday and professional practice. Also known as wicked problems, these problems do not necessarily conform to the content domains being studied, so their solutions are neither predictable nor convergent. Ill-structured problems are also interdisciplinary, that is, they cannot be solved by applying concepts and principles from a single domain. For example, solutions to problems such as local pollution may require the application of concepts and principles from mathematics, science, political science, sociology, economics, and psychology. Ill-structured problems often possess aspects that are unknown (Wood, 1983), and they possess multiple solutions or solution methods or often no solutions at all (Kitchner, 1983). Frequently, multiple criteria are required for evaluating solutions to ill-structured problems, and sometimes the criteria are not known at all. Ill-structured problems often require learners to make judgments and express personal opinions or beliefs about the problem. For a long time, psychologists believed that “in general, the processes used to solve ill-structured problems are the same as those used to solve well-structured problems” (Simon, 1978, p. 287). However, more recent research in everyday problem solving in different contexts makes clear distinctions between thinking required to solve well-structured problems and everyday problems. Dunkle, Schraw, and Bendixen (1995) concluded that performance in solving well-defined problems is independent of performance on ill-defined tasks, with ill-defined problems engaging a different set of epistemic beliefs. Hong, Jonassen, and McGee (2003) showed that solving ill-structured problems in a simulation called on different skills than well-structured problems did, including the use of metacognition and argumentation. Other studies have shown differences in required processing for well-

structured and ill-structured problems. For example, communication patterns among problem solvers differed while teams solved well-structured versus ill-structured problems (Jonassen and Kwon, 2001). Groups that solved ill-structured problems produced more extensive arguments in support of their solutions when solving ill-structured problems because of the importance of generating and supporting alternative solutions (Cho & Jonassen, 2002). Although the need for more research comparing well-structured and ill-structured problems is obvious, it seems reasonable to predict that different intellectual skills are required to solve well-structured than ill-structured problems, and therefore the ways that we teach people to solve well-structured problems cannot be used effectively to teach people to solve ill-structured problems. Probably some very ill-structured problems cannot be taught at all. They must be experienced and dealt with using general intelligence and world knowledge.

Complexity

Problems vary in terms of their complexity. Problem complexity is determined by the number of issues, functions, or variables involved in the problem; the degree of connectivity among those variables; the type of functional relationships among those properties; and the stability among the properties of the problem over time (Funke, 1991). Simple problems, like textbook problems, are composed of few variables, while ill-structured problems may include many factors or variables that may interact in unpredictable ways. For example, international political problems are complex and unpredictable. Complexity is also concerned with how many, how clearly, and how reliably components are represented in the problem. We know that problem difficulty is related to problem complexity (English, 1998). The idea of problem complexity seems to be intuitively recognizable by even untrained learners (Suedfield, de Vries, Bluck, &

Wallbaum, 1996). The primary reason is that complex problems involve more cognitive operations than simpler ones do (Kluwe, 1995). Balancing multiple variables during problem structuring and solution generation places a heavy cognitive burden on problem solvers. Complexity and structuredness overlap. Ill-structured problems tend to be more complex, especially those emerging from everyday practice. Most well-structured problems tend to be less complex; however, some well-structured problems can be extremely complex and ill-structured problems can be fairly simple. For example, video games can be very complex well-structured problems, while selecting what to wear from our closet for different occasions is a simple ill-structured problem.

Dynamcity

Problems vary in their stability or dynamicity. More complex problems tend to be dynamic; that is, the task environment and its factors change over time. When the conditions of a problem change, the solver must continuously adapt his or her understanding of the problem while searching for new solutions, because the old solutions may no longer be viable. For example, investing in the stock market is often difficult because market conditions (for example, demand, interest rates, or confidence) tend to change, often dramatically, over short periods of time. Static problems are those where the factors are stable over time. Ill-structured problems tend to be more dynamic, and well-structured problems tend to be fairly stable.

Domain (Context) Specificity/Abstractness

Most contemporary research and theory in problem solving, claims that problem-solving skills are domain and context specific. That is, problem-solving activities are situated, embedded, and therefore dependent on the nature of the context or domain knowledge.

Mathematicians solve problems differently from engineers, who solve problems differently from political scientists, and so on. Problems in one organizational context are solved differently than they are in another context. Problems within a domain rely on cognitive operations that are specific to that domain (Mayer, 1992; Smith, 1991; Sternberg and Frisch, 1991). For example, students in the probabilistic sciences of psychology and medicine perform better on statistical, methodological, and conditional reasoning problems than do students in law and chemistry, who do not learn such forms of reasoning (Lehman, Lempert, and Nisbett, 1988). The cognitive operations required to solving problems within a domain or contexts are learned through the development of pragmatic reasoning rather than results from solving that kind of problem. Individuals in different domains or contexts develop reasoning skills through solving ill-structured problems that are situated in those different domains or contexts and require forms of logic that are specific to that domain or context. In sum, problems within a domain or context vary in terms of their structuredness, complexity, and dynamicity, but all problems vary also along another dimension between domains or contexts. Which affects problems more, context or problem type is not known.

2.1.3 Phases of problem solving

There are basically four phases of problem-solving according to Polya (1956) are:

- a) Understanding the Problem
 - i. You have to understand the problem.
 - ii. What is the unknown? What are the data? What is the condition?

- iii. Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?
- iv. Draw a figure. Introduce suitable notation.
- v. Separate the various parts of the condition. Can you write them down?

b. Devising a Plan

- vi. Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a plan of the solution.
- vii. Have you seen it before? Or have you seen the same problem in a slightly different form?
- viii. Do you know a related problem? Do you know a theorem that could be useful?
- ix. Look at the unknown! And try to think of a familiar problem having the same or a similar unknown.
- x. Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?
- xi. Could you restate the problem? Could you restate it still differently? Go back to definitions.

If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or data, or both if necessary, so that the new unknown and the new data are nearer to each other?

Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

b) Carrying Out the Plan

- i. Carry out your plan.
- ii. Carrying out your plan of the solution, check each step. Can you see clearly that the step is correct? Can you prove that it is correct?

c) Looking Back

- i. Examine the solution obtained.
- ii. Can you check the result? Can you check the argument?
- iii. Can you derive the solution differently? Can you see it at a glance?
- iv. Can you use the result, or the method, for some other problem?

2.2 Teachers role in teaching problem solving

Problem solving is the "heart of mathematics". Successful problem solving requires knowledge of mathematical content, knowledge of problem solving strategies, effective

self-mentoring, and a productive disposition to pose and solve problems. Teaching problem solving requires even more of teachers, since they must be able to foster content knowledge and positive attitudes in their students. A significant portion of a teacher's responsibility consists of planning problems that will give students the opportunity to learn important content through their explorations of the problems, and to learn and practise a wide range of problem solving approaches and strategies. The teacher must be courageous, for even a well planned lesson can veer off into uncharted territory. Students may make novel suggestions as they try to solve problems; they may make observations that give rise to new conjectures or explorations; they may suggest generalizations whose validity may be unknown to the teacher. Teachers must exercise judgement in deciding what responses to pursue; they must recognize the potential for both productive learning and improved attitudes when students generate new ideas. Teachers must also recognize that not all responses lead to fruitful discussions, and that time constraints do not allow them to pursue every interesting idea. It is the teacher's job to make the tough calls. To create an environment in which students reflect on their work as they engage it, the teacher must also be reflective. In fact, teaching is itself a problem solving activity. Effective teachers of problem solving must themselves have the knowledge and dispositions of effective problem solvers.

2.3 Solving word problems in mathematics

Many students sometimes have difficulty solving mathematics word problems of simultaneous equations because they often cannot decide what to do to solve the problem. Most textbooks are not very helpful when it comes to teaching students how to solve mathematics problems of simultaneous equations. They typically provide a four-step formula: (a) read the problem, (b) decide what to do, (3) compute, and (4) check

your answer. Understanding the problem is at the core of “reading” the problem. To understand the problem, students need to be able to represent the problem, which provides the basis for deciding what to do to solve the problem. From early on, most students acquire the skills and strategies needed to “read the problem” and “decide what to do” to solve it. Many students however, do not easily acquire these skills and strategies. Therefore, they need explicit instruction in mathematical problem solving skills and strategies to solve word problems of simultaneous equations and in their daily lives.

In teaching students how to solve mathematical word problems, they are at their developmental stage, expected to be able to solve problems like this one:

‘The Art Club is having a cookie sale. Each box of cookies costs Gh¢2.00. The first day Jennifer sold 6 boxes, Carlos sold 7, and Alex sold 3. How much did the Art Club make the first day of cookie sales?’

The following frequently asked questions provide the framework for each brief.

What is mathematical problem solving?

How do good problem solvers solve mathematics problems?

Why is it so difficult to teach students to be good mathematics problem solvers?

What is the content of mathematics problem solving instruction?

What are effective instructional procedures for teaching mathematics problem solving?

One of the most powerful problem representation strategies is visualization. Students should be able to use visualization effectively to represent mathematical problems. Many students do not develop the ability to use visual representation automatically during

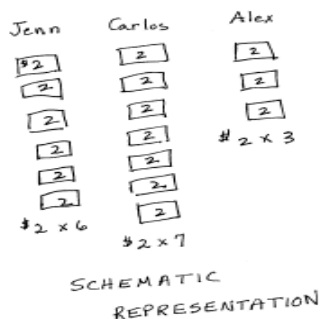
mathematics word problem solving. These students need explicit instruction in how to use visualization to represent problems.

Teaching mathematical word problem solving of simultaneous equation is a challenge for many teachers, many of whom rely almost exclusively on mathematics textbooks to guide instruction. Most mathematics textbooks simply instruct students to draw a picture or make a diagram using the information in the problem. Students may be incapable of developing an appropriate representation of the problem for a variety of reasons. First, they are generally operating at a fairly concrete level. Second, they are poor at visual representation. As a result, symbolic representation may not be possible without explicit instruction that incorporates manipulative and other materials that will help students move from a concrete to a more symbolic, schematic level. In other words, teachers must provide systematic, progressive, and scaffold instruction that considers the students' cognitive strengths and weaknesses.

Students who have difficulty solving mathematics word problems usually draw a picture of the problem without considering the relationships among the problem components and, as a result, still do not understand the problem and therefore cannot make a plan to solve it. So, it is not simply a matter of “drawing a picture or making a diagram;” rather, it is the type of picture or diagram that is important. Effective visual representations, whether with manipulative, with paper and pencil, or in one's imagination, show the relationships among the problem parts. These are called schematic representations (van Garderen & Montague, 2003).

Poor solvers tend

problem to make



immature representations that are more pictorial than schematic in nature. The illustration below

shows the difference between a pictorial and a schematic representation of the mathematical problem presented at the beginning of the brief. Other cognitive processes and strategies needed for successful mathematical word problem solving include paraphrasing the problem, which is a comprehension strategy, hypothesizing or setting a goal and making a plan to solve the problem, estimating or predicting the outcome, computing or doing the arithmetic, and checking to make sure the plan was appropriate and the answer is correct (Montague, 2003; Montague, Warger, & Morgan, 2000). Mathematical word problem solving also requires self-regulation strategies. Students are notoriously poor self-regulators. During this developmental period, it is imperative that they be explicitly taught how to self-instruct (tell themselves what to do), self-question (ask themselves questions), and self-monitor (check themselves as they solve the problem).

2.4 What good problem solvers do

Good problem solvers use a variety of processes and strategies as they read and represent the problem before they make a plan to solve it (Montague, 2003). First, they READ the problem for understanding. As they read, they use comprehension strategies to translate

the linguistic and numerical information in the problem into mathematical notations. For example, good problem solvers may read the problem more than once and may reread parts of the problem as they progress and think through the problem. They use self-regulation strategies by asking themselves if they understood the problem.

They PARAPHRASE the problem by putting it into their own words (Montague, 2003). They identify the important information and may even underline parts of the problem. Good problem solvers ask themselves what the question is and what they are looking for. VISUALIZING or drawing a picture or diagram means developing a schematic representation of the problem so that the picture or image reflects the relationships among all the important problem parts. Using both verbal translation and visual representation, good problem solvers are not only guided toward understanding the problem, but are also guided toward developing a plan to solve the problem. This is the point at which students decide what to do to solve the problem. They have represented the problem and they are now ready to develop a solution path.

They HYPOTHESIZE by thinking about logical solutions and the types of operations and number of steps needed to solve the problem (Montague, 2003). They may write the operation symbols as they decide on the most appropriate solution path and the algorithms they need to carry out the plan. They ask themselves if the plan makes sense given the information they have. Good problems solvers usually ESTIMATE or predict the answer using mental calculations or may even quickly use paper and pencil as they round the numbers up and down to get a “ballpark” idea. They are now ready to COMPUTE. So they tell themselves to do the arithmetic and then compare their answer with their estimate. They also ask themselves if the answer makes sense and if they have

used all the necessary symbols and labels such as dollar signs and decimals. Finally, they CHECK to make sure they used the correct procedures and that their answer is correct.

2.5 Why is it so difficult to teach students to solve mathematics word problems?

Students who are poor mathematical problem solvers, do not process problem information effectively or efficiently. They lack or do not apply the resources needed to complete this complex cognitive activity. Generally, these students also lack metacognitive or self-regulation strategies that help successful students understand, analyze, solve, and evaluate problems.

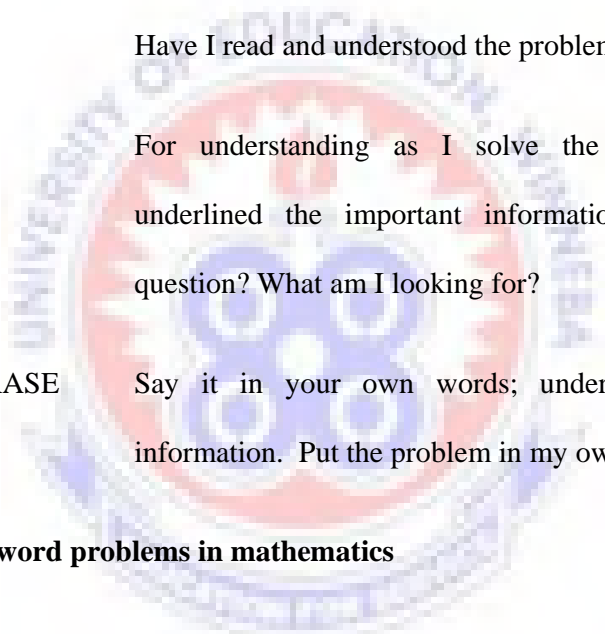
To help these students become good problem solvers, teachers must understand and teach the cognitive processes and metacognitive strategies that good problem solvers use. This is the CONTENT of mathematics problem solving instruction (Swanson, 1999). Teachers must also use instructional PROCEDURES that are research-based and that have been proven effective. These procedures are the basis of COGNITIVE STRATEGY INSTRUCTION, which has been demonstrated to be one of the most powerful interventions for students (Swanson, 1999).

2.6 What is the content of mathematics word problem-solving instruction?

The previous sections described the content of mathematics problem-solving instruction as the cognitive processes and metacognitive strategies that good problem solvers use to solve mathematical word problems. Students learn how to use these processes and strategies not only effectively, but efficiently as well. The following chart (chart 1) lists the processes and their accompanying self-regulation strategies that facilitate application of the processes (Montague, 2003). For students, teaching needs to be systematic. At the outset, reading, paraphrasing, and visualizing should be emphasized. In the initial stages,

manipulative should be used to develop the representation. Based on this pictorial representation of three-dimensional objects, a schematic representation using paper and pencil is formed. Eventually the representation is transformed into a symbolic mathematical representation using mathematical notation.

Mathematics word problem-solving processes require reading for understanding which according to Montague (2003) involve the following strategies:

- 
- SAY: Read the problem. If I don't understand, read it again.
- ASK: Have I read and understood the problem?
- CHECK: For understanding as I solve the problem. Have I underlined the important information? What is the question? What am I looking for?
- PARAPHRASE Say it in your own words; underline the important information. Put the problem in my own words.

2.7 Benefit of word problems in mathematics

Linking mathematics instruction to literature has become increasingly popular in recent years for a variety of reasons. Some suggest that the literature connection motivates students (Usnick & McCarthy, 1998), provokes interest (Welchman-Tischler, 1992), helps students connect mathematical ideas to their personal experiences (Murphy, 2000), accommodates children with different learning styles (Murphy, 2000), promotes critical thinking (Murphy, 2000), or provides a context for using mathematics to solve problems (Jacobs & Rak, 1997; Melser & Leitze, 1999). Hebert and Furner (1997) introduced the idea of "bibliotherapy" to help students see mathematics as a tool for making life easier.

Smith (1999) described the use of literature in designing lessons that place mathematical ideas in a cultural context. Despite the many suggestions and reasons for incorporating literature into mathematics instruction, however, relatively few formal studies of the benefits of literature-based mathematics have been reported.

Hong (1996) did find that students exposed to story-related mathematics exhibited a greater preference and aptitude for mathematics activities than did those of a comparison group. Whitin and Whitin (2000) explored the ways in which fourth-grade students use story, metaphor, and language to develop mathematical thinking skills and strategies, and their book offers ideas for using literature to inspire mathematical investigations and to teach mathematical concepts. Another research group (Karp, Brown, Allen, & Allen, 1998) examined the use of role models in literature to promote conceptual understanding and passion for mathematics among girls. In each of these studies, the value of literature-based mathematics instruction seems to be affirmed, but in what ways can literature be incorporated into mathematics instruction? Problem solving means many things to many people. For some, it includes an attitude or predisposition toward inquiry as well as the actual processes by which individuals attempt to gain knowledge. Usually, when teachers discuss problem solving on the part of students, they anticipate students will become involved with the thinking operations of analysis, synthesis, and evaluation (considered as higher-level thinking skills).

2.8 What are effective instructional procedures for teaching mathematics word problem?

Explicit Instruction, the basis of cognitive strategy instruction, incorporates research-based practices and instructional procedures such as cueing, modelling, verbal rehearsal,

and feedback. The lessons are highly organized and structured. Appropriate cues and prompts are built in as students learn and practise the cognitive and metacognitive processes and strategies. Each student is provided with immediate, corrective, and positive feedback on performance. Over learning, mastery, and automaticity are the goals of instruction. Explicit instruction allows students to be active participants as they learn and practise mathematics word problem solving processes and strategies. This approach emphasizes interaction among students and teachers.

2.9. Strategies in teaching mathematical word problem

Swanson (1999) identified the following eight components of effective strategy instruction. They are described as they would be used in teaching mathematical word problems.

1. Sequencing and Segmenting

Sequencing and segmenting means breaking the task into component subparts, providing short activities, and synthesizing the parts into a whole. For example, each cognitive process/self-regulation strategy routine is taught consecutively, beginning with reading the problem as a necessary first step for solving the problem. Students are taught to read the problem and then ask themselves if they understood it. They are then taught to go back and reread it or read parts until they decide they understand it. At this point, students have mastered a sequence of two important processes for solving mathematical problems.

2. *Drill-Repetition and Practice-Review*

This component includes daily tests to measure skill mastery, sequenced review, repeated practice, distributed review and practice, using the same or similar practice problems, and providing on-going and positive feedback. For example, the paraphrasing routine is taught and then students practice on their own or with peers.

“PARAPHRASE (your own words)

Say: Underline the important information. Put the problem in my own words.

Ask: Have I underlined the important information? What is the question?

What am I looking for?

CHECK: That the information goes with the question.”(Montague, 2003)

After the teacher models the routine and guides the students as they go through the routine, they are provided with practice until the routine becomes automatic. As they learn how to paraphrase mathematics word problems, they can evaluate themselves using a checklist and plot their improvement on a graph.

3. *Directed Questioning and Responses*

Cognitive strategy instruction uses a guided discussion technique to promote active teaching and learning. Students are engaged from the very beginning through an initial discussion of the importance of mathematical problem solving. With the teacher, they set individual performance goals and make a commitment to becoming a better problem solver. Teachers ask both “process-related” and “content-related” questions. Students are

directed by the teacher to ask questions. Students are also taught when and how to ask for help. (Montague, 2003).

4. Control Difficulty or Processing Demands of the Task

Arrange tasks from easy to difficult. The teacher provides simplified demonstrations, necessary assistance, appropriate cues and prompts, and guided discussion.

5. Technology

Technology extends beyond calculators and computers to include structured text and flow charts. It also includes structured curricula, scripted lessons, and video demonstrations. Students who are learning to be better mathematics problem solvers should be taught how to use calculators to facilitate computation.

6. Group Instruction

Students who have mathematics problem-solving difficulties should be taught in small groups (5-8 students), maximizing interaction between teachers and students. Interaction between teachers and students and among peers is the cornerstone of cognitive strategy instruction. Cognitive strategy instruction is intensive and time-limited.

7. Supplements to teacher and peer involvement

Students are given cue cards to study for homework as they memorize and learn the various routines in the comprehensive strategy. These routines can be added to a cue card ring as they accumulate additional routines. That is, students begin with learning the “reading” routine. When they have mastered how to read a mathematics word problem, they advance to the “paraphrasing” routine and add that cue card to their process/strategy

ring. This is used as a homework exercise and during class as a support when students are solving word problems independently. Students are expected to return to the general education math class and use what they have learned about solving mathematics problems. General education teachers must be made aware of the instruction that students are receiving and must supplement and support this instruction in the general education mathematics classes. To do this, it is essential that general and special education teachers communicate regularly about the students and the instruction, and coordinate what is being taught in the general education class. Continuity across general education is essential for student success. General education teachers must reinforce what students have learned to ensure that they apply this knowledge appropriately and also maintain acquired skills and strategies. (Montague, 2003)

8. *Strategy Cues*

Students are given reminders and prompts such as individual Student Cue Cards to carry with them for home and class use, Master Class Charts on the classroom walls, think-aloud protocols, and discussion about the benefits of using strategies.

2.10 Conclusion

A systematic, research-based mathematics problem-solving programme makes mathematical word problem easy to teach. Students are provided with the processes and strategies that make mathematics word problem easy to learn, and they become successful and efficient problem solvers. They also gain a better attitude toward problem solving when they are successful, and they develop the confidence to persevere. Real life mathematics situations, creates a challenge for students, and they begin to understand why they need to be good problem solvers. Cognitive strategy instruction in mathematical problem solving gives students the resources to solve authentic, complex

mathematical problems they encounter in everyday life. Teachers who are knowledgeable about the research underlying effective instruction will be able to justify the instructional time spent on small group instruction in mathematics word problem. They will also be able to explain how the supplemental instruction complements and builds on the mathematics curriculum.



CHAPTER 3

METHODOLOGY

3.0 Introduction

The purpose of this study was to improve students' performance in simultaneous equations involving word problems using problem solving skills. The chapter described the method employed for the study. It dealt with the research design which encompasses the design strategy, sample, data-collection procedure and analysis strategy. The section concluded with how the collected data was analyzed.

3.1 Research Design

In the study, the action research design was used. Action research is a research which is conducted with the purpose of solving classroom or local school problems through the application of the scientific methods. It is concerned with finding immediate solutions for local problems encountered since it involves local problem, it is conducted in the local environment (Awanta and Asiedu-Addo, 2008). It is based on the assumptions that teachers and students work best on problems they have identified for themselves, become more effective when encouraged to examine and assess their own work and then consider ways of working differently, work collaboratively with colleagues who help them in their professional development.

The study used both quantitative and qualitative research methods. The pre-test and post-tests were analyzed quantitatively. That is, the scores obtained by students in the pre-test and the post-test were organized into frequency distribution tables while the interview provided qualitative data. The means and standard deviations were calculated and used to

test hypothesis as to whether there was significant improvement in the performance of pupils in the post-test or not.

3.2 Population and sample

One hundred and forty five students of form 2A and 2B of Wesley College were selected to participate in the research. The sample for the study was made up of 44.5% females and 48.6%) males. This was to ensure that the study was not gender biased.

A cover letter, or letter of introduction (Appendix A), and consent for participation form, were distributed to all students. The cover letter explained the overall purpose of the study and the parameters of each child's involvement. The consent for participation form outlined that participation in the study was fully voluntary and could be ceased without repercussion at any time. The research is an action research, and it is a way where by the researcher could investigate and improve upon his way of teaching in his College, so the choice of the College is purposeful sampling. The sampling strategy used in this study is purposeful sampling because it was here that the teacher observed that the class had problem solving word problem. A total of 148 students of form two A and B were chosen for the study.

3.3 Data collection procedure

Three research instruments namely, interview, pre-test and post- test were used to gather information about students poor performance in algebraic concepts. Prior to the administering of the lesson, all the students were given a structured interview task to complete (Appendix A). This test was used to determine their prior knowledge of Algebra in general, before any instructions were given. The students completed the activity worksheets shown in Appendix B.

After the three-week lesson, (Appendix C), students were given a post-test (Appendix D), which contained similar concepts but different questions than the pre-test to determine the amount of knowledge students have gained from the lesson. The students were given a written test (Appendix D) to document their attitudes about the lesson. The students were not interviewed individually for this part, mainly due to time constraint and because there was also video documentation of the classes. Approximately two months after the lesson, the students were given another post-test (Appendix E). This test was exactly the same as the pre-test. The students would presumably not remember the exact answers to the pre-test, but would hopefully remember some of the concepts. For this reason, the pre-test was recycled instead of creating a third test.

3.4 Intervention design and implementation

This sub-section of the study describes the measures that were put in place to curb the weaknesses of the students under study. It showed how the intervention was used to bring the situation under control. The action plan was divided into three sub-sections; pre-intervention, implementation and post implementation stage. After analyzing the information gathered based on pre-school teachers' performance on word problem through the interview, the pre-test, and the post -test, it became evident that the poor performance of the students in the Algebraic concepts had been caused by the following factors:

- i. Students' inability to translate word problem into mathematical language.
- ii. Poor English background.
- iii. General perception about word problem.
- iv. Lack of interest and motivation in the topic.

Looking at the above factors, three of these factors, students' inability to translate word problem into mathematical language, general perception about word problem and lack of interest and motivation in the topic were relevant and therefore became the focus of this study.

Data was collected on the students with regards to their poor performance in Algebra through interview, pre-test and post-test. Based on the data gathered, these factors were put in place to solve the problem:

- Students were given a lot of activities to solve involving demonstration, discussion, and problem solving on Algebra using the discovery approach of learning.
- Extra tuition was given to students after classes' hours to keep them up and doing.
- Career guidance and counselling services were organized for students to keep them well informed of the prospects in the study of Algebra and Mathematics in general.

The pre-intervention stage was implemented as follows:

In order to sustain and arouse the interest of students in word problem, discovery approach was used to teach the course. The discovery approach involves demonstration, problem solving, discussion and the total embodiment of the Activity method of teaching. In these methods then, I involved students with a lot of activities on the topics to arouse and sustain their interest in class. A lot of examples were given to students. As part of the implementation design, students who were teacher-trainees were also

educated to use scientific calculators to support the learning where applicable. Personal calculators, Mathematical instruments, work book and other logistics to speed up the implementation design were adopted.

Hardworking, regular and punctual students were awarded. Challenging questions were designed to keep students up and doing in order not to relax which could lead to the failure of the objective of the study. This plan was used to inculcate in students the spirit of self-motivation in learning Algebra and for that matter their capability to translate word problem into mathematical language. Individual differences were taken into consideration in this study. Students were also given course outline and guidelines regarding the day to day activity in class work and discussion.

Student-centred approach to learning was the hallmark of students' success in this study. Based on the fact that students come from different socio-economic backgrounds and learning environments, I used mixed ability grouping based on interest, competence, gender, attitudes and social economic background to carry out the activity. Finally, as part of the intervention design, the mathematics department organized a forum to educate students on some prospects that await students who offer mathematics to the high levels. Students were made partakers of the Mathematics Association of Ghana (MAG) conference held at Kwame Nkrumah University of Science and Technology (KNUST), Kumasi. This was done in order to motivate students to take Mathematics learning as an integral part of their lives.

Two weeks after the intervention design, follow-up steps were taken to find out the effectiveness of the intervention design using the post-test technique. It was realized that students' performance in word problem had been improved tremendously. This was

made manifest during the mid-semester exams and the use of other assessment tools and the end of the Diploma in Basic Education Examinations from the Institute of Education, University of Cape Coast. During the just ended mid-semester exams, out of the 148 students who were introduced to the intervention, 16 scored A, 50 scored B⁺ 61 students scored B and the remaining students scored C⁺ and C respectively which was an excellent performance. Based on the above facts and figures produced from the just ended exams, the Head of Department, the Vice Principal Academic and other Mathematics tutors in the College all testified that students' performance in Mathematics has tremendously improved. The various activities offered to students are shown in appendix A, B and D.

Strategy Session Attendance

The strategy practice sessions averaged 100%, while the average attendance rates, by experimental group, was 83%. Attendance was typically not the result of absenteeism, but was due to the students teachers needing their presence in a required activity held during the practice sessions.

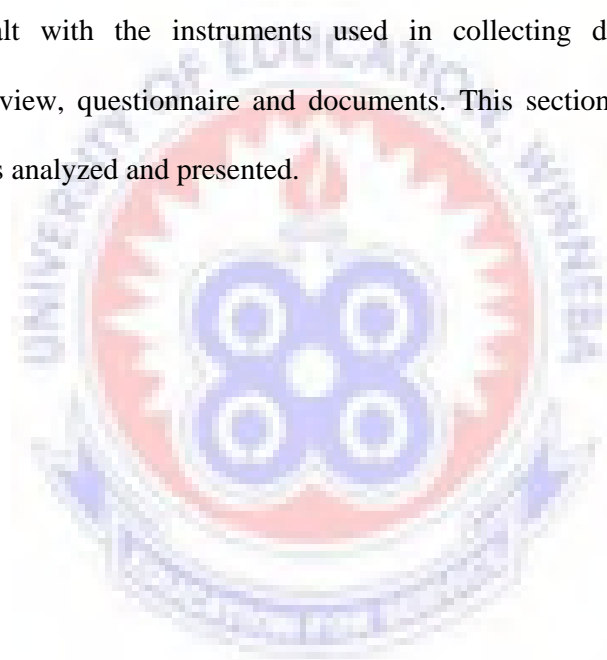
3.5 Data Analysis:

Pre-test measures were gathered from the two groups (2A and 2B) prior to the implementation of the strategy instruction intervention and post-test measures were collected from both groups following completion of the strategy instruction. The data collected were analysed using descriptive statistics. That is, the multiple data points collected at the pre- and post-testing measures for the dependent measures and items correct on the word problem tests, were plotted and described frequency tables, means and charts. The tables and figures were used in the analysis for easy comparison. It was

assumed that a treatment effect would result in positive changes which turned out to be true. Visual analysis of the figures indicated both groups had almost the same characteristics, and find word problems difficult.

3.6 Summary

The chapter has looked at the method employed for the study. It dealt with the research design which encompasses the design strategy, the data-collection and fieldwork strategy, and analysis strategy. It also included the population and sampling strategy. The chapter also dealt with the instruments used in collecting data which included observation, interview, questionnaire and documents. This section concluded how the collected data was analyzed and presented.



CHAPTER 4

RESULTS AND DISCUSSION

4.0 Introduction

This chapter deals with the analysis and interpretation of the data collected during the research. The analysis and findings were organized according to the following themes based on the research questions for the study:

- how the use of problem solving skills improved pre-service teachers' performance in the learning of word problems of simultaneous equations
- changes in students' perception about word problem using problem solving skills before intervention.

4.1 How techniques improved pre-service teachers' performance in word problems involving simultaneous equations

The major aim of the study was to find out if the use of problem solving techniques can improve pre-service teachers' performance in word problems involving simultaneous equations. The ages of the pre-service teachers' involved in the study ranged between 21 and 34 years. The age distribution of the participants of the study indicates the majority (66%) are still young or not more that 25 years old (Table 4.1).

Table 4.1 Distribution of participants by ages

Age	18-20	21-24	25-29	30 and above
No of students	5	93	27	23

As indicated in Chapter 3, the participants were put into two groups the experimental group and control group for the study. To find out whether or not the intervention will entails the use of problem solving techniques can improve pre-service teachers' performance in word problems involving simultaneous equations, the two groups were given a pre- and post test before and after the intervention. The scores of the tests were gathered from the experimental and control groups' prior to and after the implementation of the intervention are presented in Table 4.2 and 4.3 and Figures 1 and 2.

Table 4. 2 Scores obtained in the pre-test by the control and experimental groups

Scores	Control Group 2B			Experimental Group 2A		
	Number	%	Cum (%) ¹	Number	%	Cum (%) ¹
1 to 10	23	31	31	15	20	20
11 to 20	34	46	77	49	66	86
21 to 30	17	23	100	10	14	100
31 to 40	0	0	100	0	0	100
41 to 50	0	0	100	0	0	100
Total	74	100		74	100	

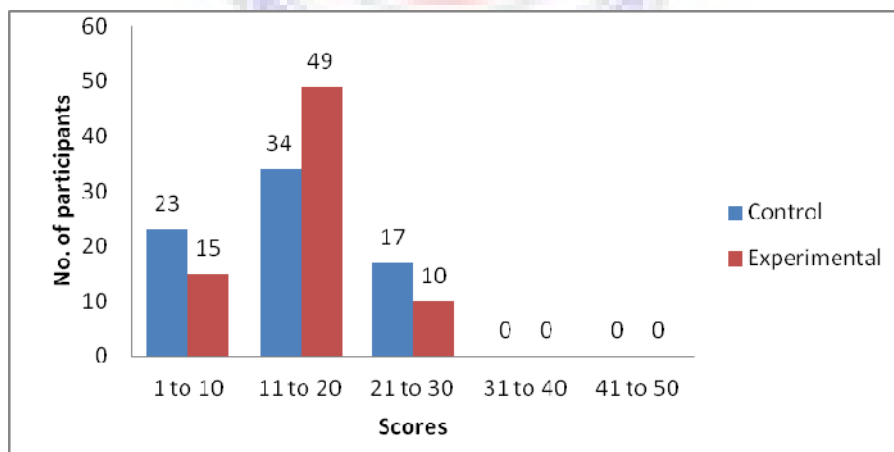


Figure 1 Scores obtained in the pre- test by the Control and Experimental Groups

Table 4.3 Scores obtained in the post-test by the control and experimental groups

Scores	Control Group 2B			Experimental Group 2A		
	Number	%	Cum (%) ¹	Number	%	Cum (%) ¹
1 to 10	18	31	31	0	0	0
11 to 20	13	46	77	0	0	0
21 to 30	31	23	100	8	11	11
31 to 40	9	0	100	23	32	43
41 to 50	3	0	100	43	57	100
Total	74	100		74	100	

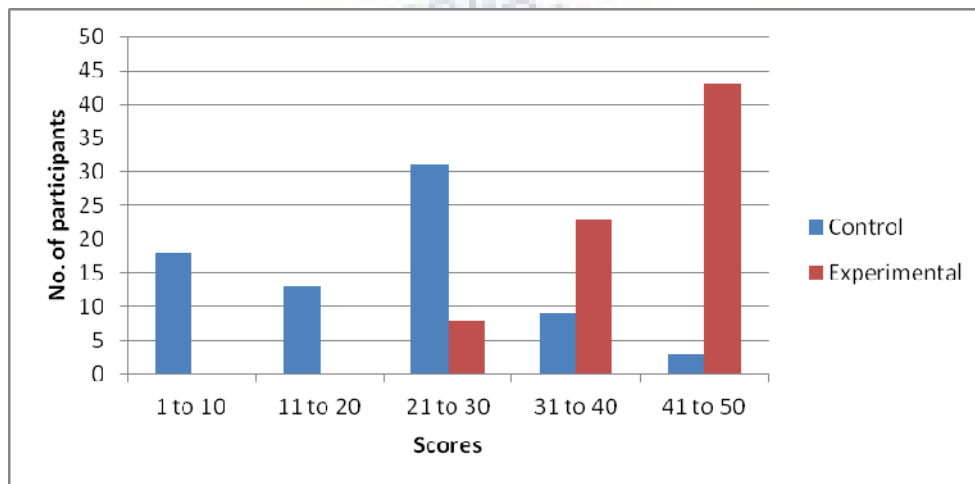


Figure 2 Scores obtained in the post-test by the control and experimental groups

As can be seen in Table 4.2, in the pre-test the lowest score was 10 for the control group with as many as 23 participants. The minimum score was also 10 for the experimental group with as many as 15 participants. In the pre-test nobody had above the score of 30. The highest score in the post-test for the experimental group was 50 with as high as 42 participants while the lowest score for the same group was 30 with only 8 participants (see Table 4.3). The highest score for the control group was 42 but with only 3

participants. Nobody in the experimental group after the post-test had below 20. The mean score for both two groups in the pre-test were 14.69 for the control group and 14.84 for the experimental group. The standard deviation for the pre-test was 7.31 for the control group and 5.73 for the experimental group. In the post-test the standard deviation for the control group was 11.04 while that of the experimental group was 6.82.

Analysis of the scores shows that the results of the pre-test for both 2A and 2B were almost the same, showing that both groups did not have any previous knowledge the problem solving skills. However, significant differences between the two groups were apparent in the scores after the intervention strategy. From the scores it is evident that students' performance increased significantly after the intervention strategy. The scores of the problem solving ability were analyzed to determine if the differences that existed on the ability and achievement measures after the intervention strategy were significant.

According to the scores, both the experimental and control groups may be considered equivalent before intervention, however, significant differences were evident between the experimental and control groups as seen in the post-test scores. Thus, the intervention strategy may be considered a possible confounding variable influencing math word problem solving effort.

4.2. Changes in perceptions about word problems before and after intervention

The second aim of the study was to find out if the use of problem solving techniques can change pre-service teachers' perceptions about word problems. Results of the key changes in the students' perceptions about word problems in interviews before, after and during the intervention are presented in Table 4.4.

Table 4. 4 Students’ perceptions about word problems in interviews before, after and during the intervention

	Perceptions before intervention	Perceptions after intervention
Experimental Group	1. Demonstrated poor knowledge of word problem solving	<ol style="list-style-type: none"> 1. Demonstrated deeper knowledge of mathematical word problem 2. Made richer description of their knowledge 3. Yielded strong knowledge of math word problem solving strategies
Control Group	1. Demonstrated poor knowledge of word problem solving	1. No significant change in the students’ perception about word problem

Considering the table above, students’ mathematics ability and mathematics problem solving ability in terms of knowledge, use, and control of mathematical problem solving strategies were assessed by the steps students followed in answering of the mathematical word problem. The experimental group showed significant change in their perception of mathematics word problem-solving after intervention looking at the format students followed in solving the questions.

The experimental group demonstrated deeper knowledge of mathematical word problem solving, following intervention. The experimental group’s post-test responses to the questions included richer descriptions of their knowledge of problem solving strategies. For example, a student initially described the following strategies she used in mathematics word problem-solving after the pre-test as, I read the problem, get the numbers, bring them down and try to solve it. This description was classified as little according to the guidelines outlined by Montague (1996). After receiving strategy instruction, the same student earned a much more rating as he listed all of the

mathematical problem solving strategies explicitly taught in the treatment, Read, paraphrase, visualize, hypothesize, estimate, compute, and check. To be expected, 9 out of the 10 (89%) experimental students responded with a listing of the strategies taught during the strategy instruction intervention.

Conversely, no significant change occurred in the control students' attitude toward mathematics word problem-solving, unlike the experimental group students, the control group did not show evidence of significant change across the overall knowledge of problem solving strategies. For example, a student reflected on her mathematics word problem-solving strategies at pretesting, I take it one by one, one step at a time, one problem at a time. At post-testing, she commented, I don't really have a strategy. I just attack and do it! Importantly, the control students did not have mathematical problem-solving strategy.

Student in the experimental group yielded strong knowledge of mathematics word problem-solving strategies in their post-testing response to the questionnaires; the responses were as follows; I read the problem and look at it closely. Then I make a picture in my mind. I solve it, and then check it. The response indicated awareness of reading the problem for understanding, problem representation through visualization, and the use of checking.

At post-testing, given the same probes, (e.g., what questions do you ask yourself while you are reading mathematics word problems? What questions do you ask yourself when you finish reading math word problems?) The student stated that he asks himself, what is the question saying? Did I double-check it to make sure I understood? His response

reflected good use of self-regulatory practices critical to self-assessing comprehension of mathematics word problems.

In sum, after participation in treatment, important and significant gains were evidenced in the experimental students' knowledge, use, and control of mathematics word problem-solving strategies, such that their awareness of these domains did not approximate that of the control group. Unlike the control group, the experimental group experienced significant growth in their attitude toward solving math word problems and in their comprehensive knowledge of overall mathematics word problem solving strategies.

Students perception about teaching word problem using problem solving skills after intervention 75% of the Pre-service teachers had heard about problem solving in mathematics in the college of education, but had never been taught using problem solving skill. The common teaching method students are used to the lecture method but after being introduced to problem solving skill, the students found this method more interesting and made lessons more practical and understandable. The relevance of these significant gains may perhaps best be reflected in the student's actual improvement in math word problem solving, as indicated.

4.3 Discussion of results

Analysis of the post-test performance of the treatment group suggested that the student s performance was new to better than what existed previously. These findings replicated those of Montague, Applegate, and Marquand (1993). While a significant improvement was demonstrated in the mathematical word problem solving of experimental students, a comparison group of control students did not outperformed the experimental students on the post-test measures.

As expected, in the present research the experimental students differed significantly from the control group students in their knowledge, use, and control of math word problem-solving strategies at post-test, indicating an improvement in problem representation and problem solution strategies.

Again, the significance of these findings is relevant in consideration of the experimental student's actual application of their knowledge, use and control of problem representation and solution strategies. With improvement gained on the post-test measures of word problem-solving, a correlation in the acquisition and application of strategy knowledge, use and control appears evident.

For mathematical knowledge to be useful, students must comprehend how procedures can function as tools for solving relevant problems. Yet, instruction for students often persists to focus on routines and memory instead of meanings and processes, or the relation between what is learned and the real world (Cawley & Parmar, 1992). In this research students enhanced their problem-solving performance, yet perhaps more importantly, they gained knowledge in critical metacognitive problem-solving activities. As Montague and Applegate (1993) outlined, without explicit instruction in cognitive and metacognitive strategies necessary for solving mathematical word problems, it is doubtful that students will learn to apply acquired mathematical skills and knowledge. This application of acquired skills and knowledge was reflected in the students' responses to inquiries about their knowledge, use and control of critical problem-solving strategies. Meaningful growth occurred across these domains evidencing significant

promise for parsimonious instructional practices in mathematical problem-solving. Teachers could introduce problem solving skills as an alternative method of teaching.

Woodward and Montague (2002) declared that although some special education researchers have investigated interventions consistent with mathematics reform, others continue to focus on traditional topics in mathematics. Clearly, exciting new paths have been forged as a result of the present research. Application of problem solving instruction to the classroom, and the resulting impact on learning, appears to be one of the most significant and pressing areas of future research.

Finally, future research on the maintenance and generalizability of cognitive strategy instruction in problem solving remains a venue for significant growth in the field of mathematics. Research indicates that metacognitive information about strategies plays a particularly critical role in the generalization and maintenance of strategies for students (Pressley & Woloshyn, 1995), yet conclusive longitudinal studies remain scarce in the literature. Therefore the present research adds depth to existing research (Montague, Applegate, & Marquard, 1993) through the use of a problem solving assessment instrument at pre- and post-testing. With this exploratory step, new insights were gained into the problem solving, and affective changes evidenced as a result of participation in strategy instruction.

CHAPTER FIVE

SUMMARY OF FINDINGS AND RECOMMENDATIONS

5.0 Introduction

This chapter presents a summary of the findings and highlights the significance of the study. It further outlines some of the recommendations and avenues for further research studies. As a novice researcher, I gained valuable experience in the process of conducting this research. Therefore, this chapter ends with a reflection of the experience I have gained from conducting this piece of research.

5.1 Summary of Study

The purpose of this study was to investigate the effects of problem solving strategy instruction on the mathematical problem-solving performance of students. The study as intended to make pre-service teachers develop plan to implement the solution of word problems involving simultaneous equations. The following research questions were formulated to guide the study:

1. How has problem solving techniques improved pre-service teachers' performance in word problems of simultaneous equations?
2. How does the use of problem solving techniques increase students' interest in learning mathematics?

The second-year pre-service teachers at the Wesley College of Education, Kumasi, were involved in the study. The study employed the survey design and data were collected using pre-test, post-test and questionnaire/interview. The data collected were analysed using descriptive statistics.

5.2 Findings

The results of the investigation show that the instruction strategy was efficacious in improving the mathematical word problem solving abilities of students. The effectiveness of the problem solving strategy was demonstrated through significant improvement in the mathematical word problem solving performance of the experimental students. That is, significant growth resulted on post-test measures of maths word problem-solving following treatment. This growth in performance did not approximate that of control students as the experimental students continued to outperform the control group students.

Results indicated that, unlike the control group, the experimental group of students experienced significant growth in their attitude toward solving mathematics word problems and in their comprehensive knowledge of overall math word problem solving strategies as a result of participation in problem solving strategy instruction. Additionally, after participation in treatment, important and significant gains were evidenced in the students' knowledge, use, and control of mathematics word problem-solving strategies. This finding, seems consistent; control group students generally appear to be lacking in strategy knowledge compared with their more proficient peers. Based upon their equivalent general knowledge of mathematics word problem-solving strategies at the intervention strategy, it is perhaps not surprising to find that, at post-testing, the experimental students exceeded their control group peers in this domain. Importantly, however, this finding lends significant support to the efficacy of the problem solving strategy instruction in bolstering the general strategy knowledge of students. The question of application of this knowledge may best be reflected in the improved performance of the students on the word problem-solving measures.

5.3 Limitations of the study

A significant limitation of the present study is the small sample size. The result might differ from a study which uses larger population especially pre-service teachers from all colleges of education in Ghana. Students selected for participation in the experimental group were not randomly selected from the population of students in the college nor were they randomly assigned to the experimental group, thus limiting the generalizability of the present results to only those students who participated in the study. Secondly, this study may not lend itself to replication since the situation/context might change as a result of the findings of this study. This may not be a fair representation to make generalizations but the aim of this study is not to draw general conclusions. This study therefore refers to the issue as pertaining to Wesley College of education, but any investigator wishing to venture into such area of research will find the findings useful as a starting point.

5.4 Implications for the policy makers

A problem solving method of teaching should be advocated for, so as to cultivate abilities to think scientifically and critically and to develop practical skills and problem solving skills among the students. More guidance should be given to teachers so that they would have more ideas of how to carry out an investigation teaching methodology and the expected level of attainment at college level. Obviously more support, in-service training and resources should be provided. Improvements in teachers' motivation to teach mathematics should be of major concern.

5.5 Conclusions and recommendations

The basis of the preceding research was predicated on the expectation that an instructional intervention would provide the strategic knowledge students require to

function successfully in mathematics classes. This expectation was met, students gained in their knowledge, use, and control of effective problem-solving strategies. Further, their actual mathematics word problem-solving performance increased significantly following strategy instruction. The implications of these findings lend strong evidence to the efficacy of strategy instruction in the inclusive classroom.



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APPENDICES

Appendix A Questionnaire for students before intervention

The University of Education, Winneba, Post Office Box 25, Winneba, C/R

Dear Students,

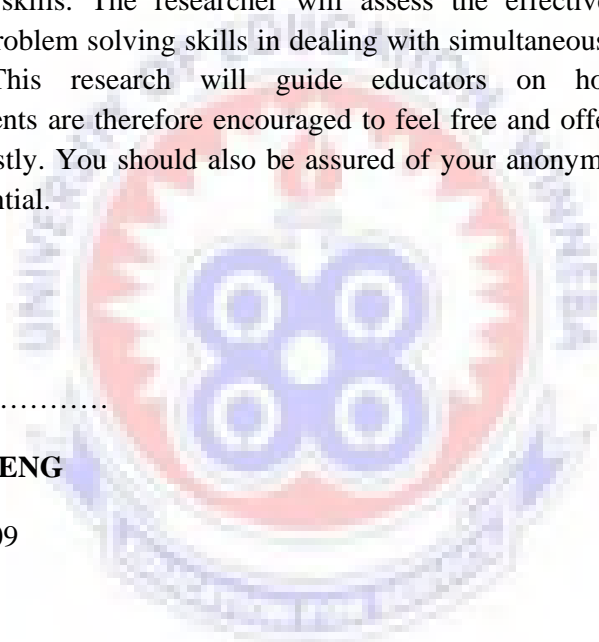
The purpose of this study is to collect and collate information on improving pre-service teachers' performance on simultaneous equations involving word problems using problem solving skills. The researcher will assess the effectiveness of pre-service teachers' use of problem solving skills in dealing with simultaneous equations involving word problems. This research will guide educators on how to teach their students. Respondents are therefore encouraged to feel free and offer accurate responses to questions honestly. You should also be assured of your anonymity since this will be treated as confidential.

Yours Sincerely,

.....

KUTTEN BOATENG

6th November, 2009



Background Information of Students

Please Tick (✓) As Appropriate

1. Sex: Male [] Female []
2. Age: 17-19 Years [] 20-25 [] 26 and above []

Perception about Teaching Word Problem Using Problem Solving Skills before Intervention

3. How do you read math word problems?
4. How many times do you read math word problems?
5. As you read, how do you help yourself understand the problem?
6. If you do not understand something about the problem, what do you do?.....
7. What questions do you ask yourself while you are reading math word problems?.....
8. What questions do you ask yourself when you finish reading math word problems?.....
9. How do you help yourself remember what the problem says?.....
10. Do you put what you read into your own words?.....
11. When you put the problem into your own words, how do you know what you said is correct?.....
12. Do you ever make a drawing of a problem or see a picture of the problem in your own mind?.....

13. Probes: What kind of picture? How often do you use drawings or pictures? When do you make?.....

14. How do you make a plan to solve a math word problem?.....



Appendix B Questionnaire for students after intervention

The University of Education, Winneba, Post Office Box 25, Winneba, C/R

Dear Students,

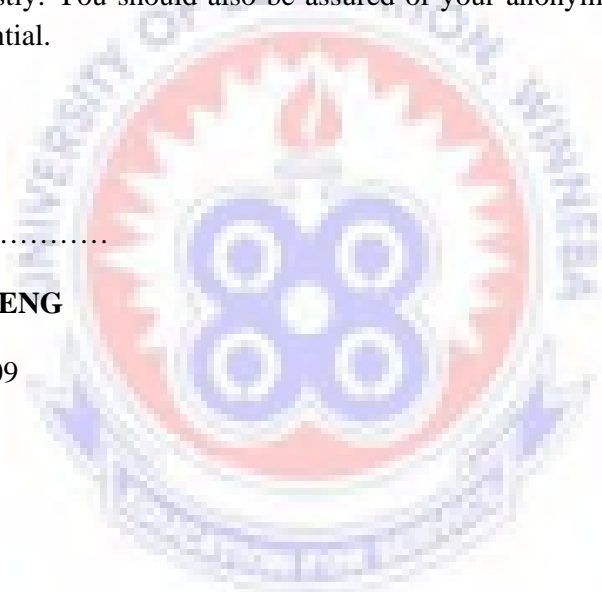
The purpose of this study is to collect and collate information on improving pre-service teachers' performance on simultaneous equations involving word problems using problem solving skills. The researcher will assess the effectiveness of pre-service teachers' use of problem solving skills in dealing with simultaneous equations involving word problems. This research will guide educators on how to teach their students. Respondents are therefore encouraged to feel free and offer accurate responses to questions honestly. You should also be assured of your anonymity since this will be treated as confidential.

Yours Sincerely,

.....

KUTTEN BOATENG

6th November, 2009



Perception about Teaching Word Problem Using Problem Solving Skills after Intervention

1. Have you heard of problem solving in mathematics before? Yes []
No []
2. Where did you first hear of it? JHS [] SHS [] College of Education []
3. Have you been taught using problem solving skills? Yes [] No []
4. Do you find any difficulty with word problem in mathematics? Yes [] No []
5. What teaching method does your teacher uses?
 - a. Group [] Demonstration [] Discovery [] Lecture []
6. Do you enjoy the methods used by your teachers in teaching word problems?
 - a. Yes [] No []
7. Can you say the teaching method used is the cause of your understanding or not understanding word problems? Yes [] No []
8. Has the problem solving skills improved your attitude/performance in word problem? Yes [] No []
9. Do you find word problem interesting when problem solving skill is used?
Yes [] No []
10. Do you understand word problems in mathematics when problem solving skill is used? Yes [] No []
11. Would you always encourage the continuous use of problem solving skill in teaching word problems in mathematics? Yes [] No []
12. If no, give reasons.....

Appendix C Pre-test

1. A woman is three times as old as her daughter. In 10 years' time the woman will be two times as old as her daughter. Find the age of the woman and the daughter.
2. The sum of the ages of a father and a son is 52 years. Eight years ago, the father was eight times as old as his son. How old is the father now?
3. A mother is three times as old as her daughter. Six ago, she was five times as old. How old is the daughter now?
4. A number of two digits is increased by 54 when the digits are reversed. The sum of the digits is 12. Find the number.
4. Aba is four years older than Asare and Asetu is twice as old as Asare. The sum of their ages is 96 years. Find Aba's age.
6. The ages of a father and a son are $10x$ years and x years respectively. In 32 years' time, the ratio of their ages will be 2:1. Find the sum of their present ages.
7. The cost of a packet of sugar is x cedis and a tin of milk is y cedis. If 3 packets of sugar and 4 tins of milk cost GH¢635.00 and 4 packets of sugar and 3 tins of milk cost GH¢695.00. Write down two equations connecting x and y hence find x and y .
8. A number of two digits is such that twice the ten digit is greater than the unit digit. When the digits are reversed the number is increased by 9. What is the number?
9. A bill of GH¢355 was paid using GH¢5 and GH¢20. If 35 notes were used altogether, how many of each were used?
10. Yaw is 5 years older than John and Mark is twice as old as Yaw. The sum of their ages is 103 years. Find each of their ages.

Appendix D Lesson plan

TOPIC: Simultaneous Equations with 2 unknowns

Sub-Topic: Simultaneous Equations – Word Problems

Objective: By the end of the lesson, students will be able to use problem solving skills to:

- (i) Translate word problems into mechanical sums.
- (ii) Solve word problems of simultaneous equations involving two digit numbers.

- R.P.K:
- (i) Students can identify two digit numbers like 45 as having 4 tens and 5 ones.
 - (ii) Students can tell their ages now together with their ages years back and years hence.
 - (iii) Students can work mechanical sums of simultaneous either by elimination or substitution methods.

Ref Book: Ordinary Level Mathematics by Harwood Clark, page 113.

Duration: 1 hour.

Teacher/Learner Activities:

- (i) Students to state their ages.
- (ii) Ask Students to state their ages in n year's time and m years ago.
- (iii) State the relationship between their ages and their parents.

Son: 10 years \rightarrow Father 40 years – 4 times

Daughter: 21 years \rightarrow Mother 63 years – 3 times

Tell the place value of 2 digit numbers

E.g. 36= 3 tens and 6 ones

A number $ab= 10a + b$

When the digits of 42 are reversed, it becomes 24, decreasing by $42-24=18$.

When ab is reversed, it becomes ba .

Core Point: A number of 2 digits is in the tens and ones.

E.g. 56= 5 tens + 6 ones

$ab= 10a + b$

When digits of a 2 digit number is reversed, it either increases or decreases.

E.g. 26 when it becomes 62 increases by $62-26=36$.

92 when it becomes 29 decreases by $92-29=63$.

A number of two digits is increased by 54 when the digits are reversed. The sum of the two digits is 12. Find the number.

Solution: Let the number be ab .

When reversed becomes ba . $ba-ab=54$

$$10b+a-(10a+b) =54$$

$$9b-9a=54$$

$$b-a=6$$

$$\underline{b+a=12}$$

$$2b=18$$

$$b=9$$

$$9-a=6 \rightarrow a=3$$

The number is 39.

Check: $93-39=54$.

Evaluation Exercise:

- (1) A mother is three times as old as his daughter. Six years ago, she was five times as old. How old is the daughter now?
- (2) A father is three times as old as his son. In 12 years' time, he will be twice as old. How old is the father now?
- (3) A number of two digit digits is such that twice the ten digit is 6 greater than the unit digit. When the digits are reversed, the number is increased by 9. What is the number?



Appendix E Post-test

1. A man is three times as old as his son. In 10 years' time the man will be two times as old as his son. Find the age of the father and the son.
2. The sum of the ages of a father and a son is 52 years. Eight years ago, the father was eight times as old as his son. How old is the father now?
3. A mother is three times as old as her daughter. Six ago, she was five times as old.
How old is the daughter now?
4. A number of two digits is increased by 54 when the digits are reversed. The sum of the digits is 12. Find the number.
5. Aba is four years older than Asare and Asetu is twice as old as Asare. The sum of their ages is 96 years. Find Aba's age.
6. The ages of a father and a son are $10x$ years and x years respectively. In 32 years' time, the ratio of their ages will be 2:1. Find the sum of their present ages.
7. The cost of a packet of sugar is x cedis and a tin of milk is y cedis. If 3 packets of sugar and 4 tins of milk cost GH¢635.00 and 4 packets of sugar and 3 tins of milk cost GH¢695.00. Write down two equations connecting x and y hence find x and y .
8. A number of two digits is such that twice the ten digit is greater than the unit digit.
When the digits are reversed the number is increased by 9. What is the number?
9. A bill of GH¢355 was paid using GH¢5 and GH¢20. If 35 notes were used altogether, how many of each were used?
10. Esi is 5 years older than Amina and Akwele is twice as old as Esi. The sum of their ages is 103 years. Find each girl's age.

Appendix F Solution to test items

1. Let the ages of father and son be $x::y$

$$x = 3y$$

In 10 years, time the ages will be

$$x + 10 : y + 10$$

Relationship in 10 years' time

The father's age is two times that of the son

$$x + 10 = 2(y + 10)$$

$$x + 10 = 2y + 20$$

$$x - 2y = 10 \text{ ----- (1)}$$

$$x - 3y = 0 \text{ ----- (2)}$$

$$(1) - (2) \quad y = 10$$

$$x = 3(10)$$

Father's age = 30yrs

Son's age = 10yrs.

2. Let the age of father be x and that of the son be y

$$x + y = 52 \text{ ----- (1)}$$

eight years ago the ages will be

$$x - 8 \text{ and } y - 8$$

$$x - 8 = 8(y - 8)$$

$$x - 8 = 8y - 64$$

$$x - 8y = -56 \text{ ----- (2)}$$

$$x + y = 52 \text{ ----- (1)}$$

$$(2) - (1) \quad -9y = -108$$

$$y = 12$$

$$x + 12 = 52$$

$$x = 40$$

Father : 40 yrs

3. Let the mother's age be x

Let the daughter's age be y

$$x = 3y \quad x - 3y = 0 \text{ -----(1)}$$

6 years ago, the mother's age was $x - 6$

6 years ago the daughter's age was $y - 6$

$$x - 6 = 5(y - 6)$$

$$x - 6 = 5y - 30$$

$$x - 5y = -24 \text{ -----(2)}$$

$$x - 3y = 0 \text{ -----(1)}$$

$$(2) - (1) \quad -2y = -24$$

$$y = 12$$

Daughter's age = 12yrs

Mother's age = 36yrs

4. Let the number be xy

When reversed, it becomes yx

$$yx - xy = 54$$

$$10y + x - (10x + y) = 54$$

$$10y + x - 10x - y = 54$$

$$9y - 9x = 54$$

$$y - x = 6 \text{ -----(1)}$$

Sum of digits $y + x = 12$ -----(2)

(1) + (2) $2y = 18$

$y = 9$

$9 - x = 6$

$9 - 6 = x$ $x = 3$

The number is 39

5. Let Asare's age be x

Aba's age will be $x + 4$

Asetu's age will be $2x$

Sum of their ages is 96

$x + x + 4 + 2x = 96$

$4x + 4 = 96$

$4x = 92$

$x = 23$

Aba's age : $x + 4$

$23 + 4 = \underline{27 \text{ yrs}}$

6. Father : Son

$10x$: x

In 32 years time:

$10x + 32 : x + 32 = 2 : 1$

$10x + 32 = 2(x + 32)$

$10x + 32 = 2x + 64$

$8x = 32$

$x = 4$

Father's age = 40yrs

Son's age = 4yrs

Sum of their ages : $40 + 4$

$$= 44$$

7. Let the cost of a packet of sugar be x

Let the cost of a tin of milk be y

Cost of 3 packets of sugar = $3x$

Cost of 4 tins of milk = $4y$

$$3x + 4y = 635 \text{ ----- (1)}$$

Cost of 4 packets of sugar = $4x$

Cost of 3 tins of milk = $3y$

$$4x + 3y = 695 \text{ ----- (2)}$$

$$(1) \times 4 \quad 12x + 16y = 2540 \text{ ----- (3)}$$

$$(2) \times 3 \quad 12x + 9y = 2085 \text{ ----- (4)}$$

$$(3) - (4) \quad 7y = 455$$

$$y = 65$$

Cost of a packet of sugar = 125

$$x = 125$$

$$y = 65$$

8. Let the number be ab

The ten digit is a and unit digit b

$$2a - b = 6$$

When the digits are reversed it becomes ba

$$ba - ab = 9$$

$$10b + a - (10a + b) = 9$$

$$10b - b + a - 10a = 9$$

$$9b - 9a = 9$$

$$b - a = 1 \text{-----} (1)$$

$$-b + 2a = 6 \text{-----} (2)$$

$$(1) + (2) \quad a = 7$$

$$b - 7 = 1$$

$$b = 8$$

The number is 78

9. Let the number of GH¢5 notes be x

Let the number of GH¢20 notes be y

$$x + y = 35 \text{-----} (1)$$

sum of the notes = GH¢355

$$5x + 20y = 355 \text{-----} (2)$$

$$(1) \times 5 \quad 5x + 5y = 175 \text{-----} (3)$$

$$(2) - (3) \quad 15y = 180$$

$$y = 12$$

$$x + 12 = 35$$

$$x = 23$$

No. of GH¢5 notes = 23

No. of GH¢20 notes = 12

10. Esi : Amina : Akwele

Let Amina's age be x

Esi will be $(x + 5)$ years

Akwele's age will be $2(x + 5)$

Sum of their ages: $x + 5 + x + 2(x + 5) = 103$

$$2x + 5 + 2x + 10 = 103$$

$$4x + 15 = 103$$

$$4x = 103 - 15$$

$$4x = 88$$

$$x = 22$$

Amina's age = 22

Esi's age = 27

Akwele's age = 5

