UNIVERSITY OF EDUCATION WINNEBA

TOPIC:

USING DIAGRAMS THAT WILL ENHANCE CONCEPTUAL UNDERSTANDING AND SKILLS DEVELOPMENT IN CIRCLE THEOREMS AMONG SHS 2 STUDENTS OF MANKESSIM SENIOR HIGH SCHOOL

ANTHONY BRUNO

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ANTHONY BRUNO

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A DISSERTATION SUBMITTED TO THE DEPARTMENT OF MATHEMATICS EDUCATION, FACULTY OF SCIENCE EDUCATION AND SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES, UNIVERSITY OF EDUCATION, WINNEBA, IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF MASTER OF EDUCATION IN MATHEMATICS EDUCATION

2013

DECLARATION

STUDENT'S DECLARATION

I hereby declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

SIGNATURE………………………

DATE………….............................

SUPERVISOR'S DECLARATION

I hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines on the supervision of thesis as laid down by the University of Education, Winneba.

NAME OF SUPERVISOR: PROF. SAMUEL ASIEDU-ADDO

SIGNATURE…………………………

DATE…………………………......

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DEDICATION

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TABLE OF CONTENTS

Page

CHAPTER 2

LITERATURE REVIEW

CHAPTER 3

LIST OF TABLES

ABSTRACT

The study investigated the impact and the importance of the use of circle diagrams in the application of circle theorems towards geometry instructions. This study sought to find out what type of instructional approach mathematics teachers use in delivering the concept of circle theorems in mathematics classrooms. Also to find out perceptions students hold towards the learning of circle theorems in geometry. The study also sought to look at the extent diagrams can be used to improve upon learning for application of the concepts of circle theorems in geometry.

The study was carried out in Mankessim Senior High School (SHS) in the Central region. The sample consisted of 63 form two students of the selected Senior High School. Business two (2) class was chosen for the study in the school. All the students in the Business class were used with the exception of two students who were absent during the start of the study. Questionnaires, Pre- test and Post- test were administered on mathematics teachers and students in the school. The data were then examined to obtain the attitudes students hold towards the use of circle diagrams in the application questions relating to circle theorems in geometry.

The findings from the research questions were analysed which indicated that majority of the students liked the use of diagrams to enable them understand the geometry concept, like in the case of circle theorems. Also, majority of the students felt satisfied when using the circle diagrams in learning the concepts of circle theorems in geometry. The findings from pre-test and the post-test results indicated that the understanding and application of circle theorems using diagrams have improved upon the level of conceptual knowledge of the students in a given mathematical area. The results indicated that students‟ knowledge and understanding and application have improved as a result of the use of circle diagrams.

CHAPTER 1

INTRODUCTION

BACKGROUND TO THE STUDY

Mathematics has always been a challenging academic subject in schools. It consists of numerous domains that continue to develop in a cumulative manner toward increasingly complex topics. Therefore it is not surprising to note that many students perceive mathematics as a boring and tedious subject they have to learn in school and that it requires them to memorize rules and applying them. If they do get correct answers for the exercises they did, it is often assumed that they have understood the mathematical concepts like in the case of circle theorems. However, this is not a true picture of having attained mathematical knowledge and competence. In fact, it is often much later that learning problems, especially in the area of mathematical comprehension, which begins to crop up revealing a serious lack of real understanding of fundamental mathematical concepts. When students encounter difficulties in mathematics learning, the seemingly common reaction to resolve the issue is to get them to practice more because most of us believe that practice makes perfect.

Mathematics teachers have to be encouraged to take time to reflect and ponder why there are students who continue to fail mathematics despite extra remedial lessons and provision of learning support for mathematics. In this way, mathematics teachers ought to be advised to identify and/or examine the errors these students commit, misconceive or respond in certain ways when working out sums or mathematical problems. From this observation or examination of error patterns, we can actually learn more and thus, become better equipped to manage the various learning difficulties students encounter in their mathematics learning. It has also been observed that the attitude toward mathematics learning can also impact student‟s performance. Academic failure can inhibit the student‟s desire to perform in mathematics as well as negatively impact his or her self-confidence regarding mathematics. Early failures in mathematics learning can result in anxiety about performance in mathematics learning and this can continue into high school, college and adulthood. To solve this problem among students, mathematics teachers can resort to the use of activities where students can actively be involved in the learning process. When diagrams are used to teach mathematical concepts that require students to visualize the concept taught will enhance conceptual learning. In the case of learning geometry concepts and for that matter the concepts of circle theorems, diagrams must be used to digest the concepts for understanding and application. It is important to note that in order to acquire mathematical concepts one should visualize the importance and use of diagrams, because mathematics is the field in which preconditions are crucial and so before the teaching process student backgrounds on the subject should be tested.

In fact diagrams are important in mathematics education. Plainly the movement to accord diagrams a substantial role in mathematics is crucial to a philosophy of real mathematics. They help to illustrate and are frequently used to help explain mathematical concepts. Students often create them with pencil and paper as an intuitive aid in visualizing relationships among variables, constants, and functions, and use them as a guide in writing the appropriate mathematics to solve the problem. However, such diagrams generally assist only in the initial formulation of the required mathematics, not in "debugging" or problem analysis. This can be a severe limitation, even for simple problems with a natural mapping to the temporal dimension or problems with complex spatial relationships. Mathematical sketching derives from the familiar pencil-and-paper process of drawing support diagrams which facilitate the formulation of mathematical expressions; however, with mathematical sketching, users can also leverage their physical intuition by watching their hand-drawn diagrams to animate in response to continuous or discrete parameter changes in their written formulas Thus, I have decided to focus on a particular subset of dynamic illustrations to explore the mathematical sketching paradigm. In this current situation the use of mathematical diagrams can create a dynamic illustration for concept understanding. In other words, mathematical diagrams when utilized properly in our mathematics lessons or classrooms can create a mathematics environment which will close the gap that students perceive of mathematics as abstract and difficult and begin to develop interest in learning mathematics. In addition to the sketches/diagrams which mathematics teachers use in their lessons, drawings of the actual diagrams showing the various parts of the true diagram should not be left out.

The drawings need to be transformed, but we also want to extract some geometrical properties from them to keep them close to their original representations. Thus, a delicate balance is needed between retaining the essence of the drawings and transforming them into something coincident with the mathematics. From the definition of mathematical sketching, drawing rectification is critical but must be somewhat transparent to the Mathematical sketching which is therefore a form of visualization, consisting, as it does, of a subset of the many visualization, algorithms, tools, and systems. Mathematical sketching takes data and transforms them into a representation that can provide insight into a particular phenomenon. In other words, rectification should involve little cognitive effort on the user‟s part.

Apparently, many students preparing for their West African Senior School Certificate Examination in Mathematics cannot boast of being confident in the fundamental concepts of Circle Theorems. It is the task of the mathematics teachers to help their students to understand and develop confidence in the application of these circle theorems. This task is not, however, quite easy. The task demands time, practice, devotion, and sound pedagogical principles. Personal experience as a teacher educator informs that the students need to acquire as much experience of independent work as possible as being fed. Although students need to construct personal understandings, if they are left alone without appropriate assistance, they may make no progress in their conceptual development. However if the teacher gives too much help, nothing is left to the students to think about the student begins to note down concepts learnt without understanding. When the student learns this way he or she easily forgets the concepts. In other words, the teacher should judiciously assist the student but not to give him/her everything. When assistance becomes too much the student tends to be a passive recipient, and it should not be too little also. In this way the student will have a reasonable share of the work to accomplish. In teaching circle theorems this way, the student will be investigating problems and be applying the concepts appropriately with little or no help from the teacher.

The students` inability to solve problems in circle theorems should not be a foregone conclusion that the topic is left out. The strategy used and students attitude towards the topic can cause harm. Apart from circle theorems perceived to be difficult by students, the students also lack the ability to direct thought processes along fruitful channels of applying the concepts to circle related problems.

In recent times, educationists advocate that teaching and learning of mathematics in the classroom should be more problem solving oriented and student centered. Students should be allowed to learn mathematics through the experiences that their teachers provide. Thus, students understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition towards mathematics are all shaped by the teaching they encounter in school. This means that the improvement of mathematics education for all students requires effective mathematics teaching in all classrooms.

In conclusion students` weakness in applying the concept of circle theorems to solve circle related problems is of great concern to every mathematics teacher. Mathematics teachers need to know the ideas with which students have difficulty in and find ways to help bridge the common misunderstandings. Skinner (1969) asserted that good teaching is the ability to arrange proper sequence of reinforcements for the students. Therefore I deem it a duty for mathematics teachers to involve students in mathematics lessons through a gradual representation of the concepts to be thought. Bruner (1966) also insisted that, the goal of teaching is to promote the general understanding of the structure of the subject. When the student understands the structure of a subject, he or she sees it as a way that permits many other things to be related to it meaningfully. Bruner (1966) further stated that, when learning is based on structure, it is more long lasting than when it is not based on structures. Thus it should be of great concern of every mathematics teacher to involve mathematics learners through activities using sketches and diagrams.

STATEMENT OF THE PROBLEM

Circle theorems pose great challenge to most students in Mankessim Senior High School in Mfantseman municipality in Ghana. The students encounter difficulties in applying the concept of the theorem to answer questions involving circle theorems. They lack some basic skills which include: finding unknown angles and formulating algebraic equations to solve for unknown angles and fail to apply the concepts of angle properties to solve problems of angles found in circles. It is also observed that questions that demand the application of the theorems but are not in diagrammatic form pose problems to the students. It was also clear among students that they can recall all the theorems with their numbers as stated in some mathematics books or pamphlets but cannot fruitfully apply them to solve circle related problems. They also lack the skill of applying the theorems which state that;

1) angle that subtends at the centre of a circle by an arc is double the angle that subtends by it on any point on the remaining part of the circle,

2) angle in a semicircle is a right angle,

3) angles in the same segment formed at the circumference of a circle are equal.

All these were observed when marking the end of term examinations after teaching the concepts of circle theorems among other topics taught within the term. About 95% of the students did not answer application questions on circle theorems. Although students have options to choose from but I was expecting more students to choose application questions on circle theorems. This motivated me to research into why students fail to choose and answer questions that demand the application of circle theorems.

Learning geometry and for that matter concepts of circle theorems may not be easy, and a large number of students fail to develop an adequate understanding of geometry concepts and geometry reasoning. The lack of understanding in learning geometry often causes discouragement among the students. When questions on circle theorems are made compulsory in their examinations, students come out of examination room complaining of the paper being difficult because they could not answer the questions related to the application of the theorems simply because they did not learn the theorems due to the perception they had about the concepts that they are difficult. A personal experience encountered in teaching mathematics over the past year in Mankessim Senior High School is that students do not answer questions on circle theorems when they are asked during mid-term test and end of term examinations. Those preparing for the WASSCE fail to answer questions on the application of the circle theorems during mock examinations. It is observed that students cannot translate questions on circle theorems into diagrams to pave way for easy recognition of the demand for the application of the circle theorems. Furthermore when more than one of the theorems is involved in a question, students are found wanting. In addition when the question set on the application of the circle theorems is not the same as the example worked in class they fail to apply their knowledge of the theorems. Other senior colleague teachers share the same experience with me when I complained to them at a departmental meeting in the school. Therefore this study is designed to explore appropriate diagrams to teach the circle theorems which will enhance conceptual understanding and skills development in circle theorems so that students will appreciate to learn and apply the theorems appropriately.

PURPOSE OF THE STUDY

This study aims at providing students the opportunities to explore mathematical ideas and to develop conceptual understanding of circle theorems. The study will help students use appropriate theorems with diagrams to provide a variety of manipulative skills to overcome their misconceptions about the circle theorems which they have perceived of as difficult to learn and to apply. It is also meant to assist students form appropriate equations for unknown angles in the diagrams to understand properly the circle theorems and apply them appropriately. The study also seeks to provide students the opportunity to link other geometric concepts learnt to the learning and understanding of the concepts of circle theorems but not to learn the circle theorems in isolation. This can be done through the effective use of activities and students involvement in mathematics lesson.

RESEARCH QUESTIONS

The aim of this study is to find out why students in Mankessim Senior High School have difficulty answering questions involving circle theorems and develop an intervention strategy as a way of helping students to remedy any difficulty and misconception they might have. Therefore, the study seeks to address the following questions:

1. What instructional techniques are used in teaching and learning the concept of circle theorems in mathematics classroom in Mankessim Senior High School?
2. What is difficult about the circle theorems that scare demotivate students from learning the circle theorems concepts?
3. What is the effect of diagrams on students` perceptions on questions involving circle theorems?
4. To what extent do diagrams enhance the teaching and learning of circle theorems?

SIGNIFICANT OF THE STUDY:

The study of circle theorems enables our students to use and apply them appropriately to answering questions associated with circles. Our students these days like the use of mathematical formulae in solving mathematical problems, thus they are able to memorize mathematical formulae and apply them correctly. For theorems the students have to study well the problems before applying the appropriate theorem to answering questions on circle theorems. The student who understands these theorems very well shows interests in the concept grasped and prove to others that he/she is the master of the concept. These students who have understood well the theorems sometimes help their friends through these theorems in class and thereby remove the fears students have in learning some mathematical concepts and theorems. Moreover students feel secured and confident applying the theorems.

In addition to the above the finding of the study would guide curriculum developers for planning and designing mathematics curriculum for pre-university education. The study would also provide information to the Colleges of Education and Teacher Education Universities in designing academic programmes. The findings of the study would provide relevant literature to other researchers who wish to conduct into the teaching of circle theorems. Further, the study would serve as base for organizing in-service training courses for teachers who teach mathematics at the Senior High Schools.

DELIMITATION:

This research focuses on the students in Mankessim Senior High School. The students to be involved in the study are students in Business 2. Some Mathematics teachers in the school will be involved in answering questionnaires for the research. Data on the academic performance of the students will be collected from the school`s academic head. Other information relevant to the study will also be taken from the school administration.

ORGANIZATION OF THE STUDY

This write up is made up of five chapters. Chapter one is introduction and is made up of background to the study, statement of the problem, purpose of the study, research questions, significance of the study and delimitations. Literature review takes chapter two. Chapter three is methodology and consists of research design, population and sample, research instruments, validity and reliability, data collection procedure, the intervention and scoring and data analysis procedure. Chapter four will deal with results and discussions of the result, while chapter five will consider the summary of findings, conclusion, recommendations and suggestions for further study.

CHAPTER 2

LITERATURE REVIEW

Mathematics is a living subject which seeks to understand patterns that permeate both the world around us and the mind within us. Although the language of mathematics is based on rules that must be learned, it is important for motivation that students move beyond rules to be able to express things in the language of mathematics. As teaching begins, students have opportunities to study mathematics as an exploratory, dynamic, evolving discipline rather than as a rigid, absolute, closed body of laws to be mesmerized. They should be encouraged to see mathematics as a science, not as a canon and reorganize that mathematics is really about patterns and not merely about numbers (National Research Council 1989 p 84). From this perspective, learning mathematics is "empowering." Mathematically powerful students are quantitatively literate. They are capable of interpreting the vast amount of quantitative data they encounter on daily basis and making balanced judgments on the basis of those interpretations. They use mathematics in practical ways, from simple applications such as using proportional reasoning for recipes or scale models, to complete budget projections, statistical analysis and computer modeling. They are flexible thinkers with a broad repertoire of techniques and prospective for dealings with novel problems and situations. They are analyzed, both in thinking issues through themselves and in examining the arguments put forth by others.

Understanding in mathematics education

Hiebert & Carpenter (1992) asserted that, one of the most widely accepted ideas in mathematics education is that students should understand mathematics. In other words when students understands a concept taught very well he/she stands a good chance of applying the concepts learnt to solve problems. Sierpinska (1994) starts her book on understanding in mathematics with the following questions: "how to teach so that students understand? What exactly don't they understand? What do they understand and how?" This is to say that to begin teaching concepts the teacher must take into consideration what the students already know and their level. Pirie & Kieren (1994) mention the interest towards teaching and learning mathematics with understanding, which is shown in recent curricular reforms in many countries. The use of the term „understanding'(or 'comprehension') is varied, depending on institutional contexts, although the dominant psychological approach emphasizes the mental facet of understanding. The book by Sierpinska (1994) represents an important step forward, when discerning between understanding acts and processes and when relating "good understanding" of a mathematical situation (concept, theory, problem) to the sequence of acts of overcoming obstacles specific to this situation. Through the historic-empirical approach it is possible to identify meaningful acts for understanding a concept. Nevertheless, we think that taking the notion of object as primitive and deriving meaning from understanding cause some difficulties in analyzing the processes of assessing students „understanding. From my point of view, a theory of conceptual understanding useful for mathematics education should not be limited to saying, for example, that understanding the concept of circle theorems is a person's mental experience assigning some object to the term"s circle theorems. A cultural entity, a very complex, not ostensive object is designated with the terms` circle theorems'. Therefore, in order to define what the concept of circle theorems is, we need to clarify a previous question: What object must the student assign to the term 'circle theorem' so that the teacher may say that he/she understands the object circles. His problem of understanding is, consequently, closely linked to how the nature of mathematical knowledge is conceived. Mathematical terms and expressions denote abstract entities whose nature and origin should be researched for elaborating a useful and effective theory for what it is to understand such objects. This research requires answering questions such as: What is the structure of the object to be understood? What forms or ways of understanding exist for each concept? What are the possible and desirable aspects or components of mathematical concepts for students to learn at a given time and under certain circumstances? How are these components developed? If, for example, we consider mathematical knowledge as internally represented information, understanding occurs when the representations achieved are connected by progressively more structured and cohesive networks (Hiebert & Carpenter, 1992). However, we consider it excessively reductionist to view mathematical activity as information processing. From my point of view, the theories of understanding derived from this conception do not adequately describe the teaching and learning processes of mathematics, especially the social and cultural aspects involved in these processes.

Mathematical problems

Mathematical concepts and notation provide students with familiar structures for organizing information in a problem; they also help students understand and think about the problem. When students have a strong understanding of mathematical concepts and notation, they are better and able to recognize the mathematics present in the problem, extend their understanding to new problems and explore various options when solving problems. Building from students" prior knowledge of mathematical concepts and notation is instrumental in developing problem-solving skills.

In solving mathematical problems teachers of mathematics are humbly advised to use activities that involve the students" engagement in a variety of cognitive actions including accessing and using previous knowledge and experience. To successfully overcome mathematical problems, students should coordinate previous experiences, knowledge, familiar representations and patterns of inference, to generate new representations and related patterns of inference that resolve the tension or ambiguity that prompted the original problem. What does it mean for students to organize previous knowledge and experiences to generate new knowledge? It is clear that if students are to be engaged in activities in order to overcome their problems teachers must give guidance to their students in the classroom. For effective teaching and learning of mathematics concepts the mathematics classroom environment must be attractive and student-centered with variety of activities guided by the teacher. Students fear learning mathematics concepts simply because the type of classroom environment that must be created for teaching and learning seem not attractive enough for students to participate in the learning process. Teachers of mathematics must encourage their students to learn mathematics concept by involving the students in activities that will enable students show interest in learning mathematics concepts. Students usually experience difficulties in solving problems in areas such as:

1) comprehension of the problem posed,

2) strategic knowledge, and

3) inability to translate the problem into a mathematical form.

Actually the level of difficulty of the problem depends on problem solvers` perceptions of whether they had suitably categorized the situation, interpreted the intended goal, identified the relevant resources and executed adequate operations to lead toward a solution. The difficulty here suggests that the problem solvers` characteristics are the most important determinants of problem difficulty. It has been noticed with great concern that geometry in general and circle theorems in particular which was once popular in our Senior High School curricula now has little place in school mathematics. This branch of mathematics is either left out completely or partially taught by mathematics teachers. This phenomenon makes it so difficult for students to develop interest and confidence of its value. In a typical classroom situation, the students asked, "where is the place of circle theorem in our daily lives which makes teachers worry us with circle theorems?" "Of what benefit is the learning of circle theorems to our daily lives?" Also several students hold the view that circle theorems is just too difficult to work with and wish that it is removed completely from the mathematics syllabus.

According to Thomaidis (1991) students‟ reactions to geometrical concepts is clearly expressed through various questions, such as:

a) Why do we have to prove things that are blatantly obvious?

b) What is the use of all these theorems and exercises?

c) Why must geometrical constructions be made using straight edge and compasses?

The first negative reactions towards theoretical geometry are usually followed by a passive if not indifferent attitude which certainly does not favour the materialization of the course‟s ambitions and aims. Of course, if students are to be led to learn an area of mathematics it is important to let them know what they are to achieve from learning it. The teacher must provide opportunities for students to see the relevance of the mathematical concepts they learn in their real life situations

Students‟ difficulties in answering questions on circle theorems

According to Strutchens, Harris & Martin (2001), students learn geometry particularly circle theorems, by memorizing the properties rather than by exploring and discovering the underlying properties. Geometry knowledge learned in this way is limited and superficial. Eventually these students find difficulty in applying that limited geometry knowledge in solving circle related problems. This lack of understanding often discourages the students, invariably leading to poor performance in questions involving circle theorems. A number of factors have been proposed to explain what makes learning of circle theorems very difficult for students.

First, the geometry language, which involves specific terminology, is unique and needs particular attention and understanding before it can be used meaningfully. Misuse of geometry terminology can lead to misconceptions of geometric knowledge (Lappan, Fey, Fitzgerald, Friel and Philips 1996). This is to say that the various parts of a circle make students worry about the distinction between some of them; for instance a chord and a diameter, circumference and arc. Furthermore the numbering of the theorems to make them unique as stated in some of the mathematics books makes the learning of the theorems too rigid which may lead to misconceptions.

Furthermore, circle theorem requires visualizing abilities but many students cannot easily visualize it. Due to their limited experiences in circle theorems, students may not have enough opportunities to develop and exercise their spatial thinking skills for effective learning of circle theorems. Another problem is that traditional approaches of learning circle theorems do not seem to help students achieve the intended learning outcomes in the curriculum. By using just textbooks and without activities in the classroom hamper optimal learning. In the Ghanaian context, this concern is shown in the low achievements of students in circle related problems in public examinations (The Chief Examiners Report, WAEC, 2003 - 2006). There is an urgent need to change the traditional mode of teaching circle theorems to one that will be more rewarding for both teachers and students. Specifically, learners must be given opportunities to personally investigate and discover the circle theorems to enable their understanding of the subject in depth and also in relation to the other fields of mathematics.

According to WAEC, chief examiner"s report (WAEC, 1995) on students" performance in SSSCE Mathematics (Core), most candidates avoided the questions on circle theorems and the few who attempted it; many were unable to solve the problem accurately because they were not able to write correct algebraic equation needed to solve the problem. This may be attributed to students" inadequate knowledge of circle theorems.

Candidates who attempted the circle theorems question hardly knew what to do. This shows that such topic in the syllabus is not receiving the needed attention that it deserved from both students and teachers. It is therefore advisable that students and teachers give some time to geometry. Furthermore students need to understand the theorems very well in order to apply them to solve problems relating to circle theorems. To solve this problem student need to know that:

a) Angles subtended at the centre O are two (2) times that it subtends at the circumference by the

same chord or arc.

b) Opposite angles in a rhombus are equal.

c) Opposite angles in a cyclic quadrilateral are supplementary.

Among the many strategies that have been suggested to improve efficacy in solving mathematics problems, using diagrams has been described as one of the most effective. Using diagrams was the most efficient among strategies that had been suggested as helpful. Mathematics teachers‟ frequent use of diagrams in class is another strong indicator of the value of diagrams to their users. Numerous studies on the use of diagrams have attempted to explain their contribution to efficiency. For example, Larkin & Simon (1987) argued that diagrammatic representation is computationally more efficient than sentential representation because it minimizes labels, gathers related information in one place, and facilitates easier recognition of the situation with the help of the visual system. Cheng, (2002) Koedinger & Terao, (2002); Stern & Aprea (2003), also share the same view. For example Stern & Aprea. (2003) found that actively constructing and using linear graphs as reasoning tools while learning economics can better facilitate a transfer effect across subject content compared to a condition of receptive diagram use (i.e., where learners receive the diagrams to use rather than constructing their own).

Students' perception about circle theorems

Craig‟s (2001) in her thesis considered calculus word problems and students perceptions of the difficulty of the problem. She looked at the variables of concrete versus abstract and the types of representation in the problem. The important variables for the perception of difficulty were familiarity of the problem, the context and whether there was a visual representation. Smith (1994) concluded in his studies that the conceptual difficulty of the mathematics was the important variable in the students‟

performance. In the present study I consider a series of tasks requiring differing conceptual skills and the students‟ perceptions of their difficulty. I examine the students‟ perceptions before and after completing the task to identify any changes that may have occurred. I look at students‟ performance on the end-of-term examinations to see whether learning has occurred and to find which variables may cause significant differences in performance.

Summary.

In teaching mathematics concepts, the teaching environment must be attractive enough to motivate learners learn the concept with little or no difficulty. The teacher must involve the learner in the teaching and learning processes. Teachers must not set rules in learning a concept, rather teachers must guide learners to establish their own rules to learn and understand the concept. In the case of learning the concept of circle theorems numbering of the theorems is not necessary as found in some mathematics pamphlets sold in Ghanaian markets. For instance in those pamphlets one can read things like, theorem 1 says this, theorem 2 says that and so on. The teaching must devoid of such practices which may pave way for some misconceptions to set in as mathematics being a difficult subject.

CHAPTER 3

METHODOLOGY

RESEARCH DESIGN

The design is basically action research. The researcher used diagrams which involved the students through varied activities among SHS students of Mankessim SHS to understanding the concepts of circle theorems and improve upon the application of the knowledge acquired to solving circle related problems. The researcher administered test on circle theorem to Business 2 students in the School and majority of them scored abysmally very low marks. It was therefore imperative on the researcher to provide remedial tutorials, using diagrams through series of activities as an appropriate intervention tool.

The researcher thoroughly took systematic steps by leading the students through the concept of circle theorems successfully by identifying the problems learners encounter when learning the concepts, researching into it, suggesting some actions to be taken, evaluating the results, and revisiting the same problem. The problem of the division between theory and practice was thus, catered for after successfully completing the process. This type of design gave the researcher many opportunities to revisit the previous state of the problem as a prelude to the subsequent state(s).

POPULATION

The population for the research was 110 SHS 2 students from the departments of Business and Visual Art of 2012/2013 academic year group of the school. Each class had averagely 50 students. This group of students studied circle theorems during the third term period of study in the second year. The choice of Mankessim Senior High School was based on the fact that the researcher was teaching in the school and the class chosen was due to the researcher`s familiarity with the students in those classes. The researcher has spent barely a year in the school teaching mathematics and has been friendly with both the teaching staff and the students and as such taken information from both the teachers and the students to carry out his research was not a problem to the researcher. The population comprises both males and females (boys and girls) averagely aged 17 years and can speak both Fante, Twi and English Language. Thus there was no communication barrier in information flow as the research was on-going.

SAMPLE

Circle theorem is a core mathematics topic in SHS 2 syllabus and all SHS 2 students in Ghana are supposed to learn these theorems in geometry. However, for the purpose of implementing the diagrammatic approach to overcome the problem of learning the circle theorems, Business 2A students in Mankessim Senior High School were purposively selected for this study. The choice of this class was based on the researcher‟s preliminary observation and diagnosis of the students‟ level of understanding of the circle theorem in the School. It was evident that the Business 2A students had difficulty in conceiving the concepts under Circle theorems. 63 students out of 65 in Business 2A class comprising 34 boys and 29 girls were used in the study. The remaining two (2) students in the class were absent from school on the first day the study began. The researcher therefore excluded them on their return to school. The researcher used purposive sampling because of the large size of the number of students in the classes he was handling in the school. The Business 2A class was sampled for the study because the class size was manageable to enable the researcher engage in thorough work/research and to come out with findings that will be applicable to all other SHS 2 students in the school.

RESEARCH INSTRUMENTS

The researcher used two instruments in collecting data for the study. These were questionnaires and tests. The tests included pre-test and post-test. The pre-test was used to diagnose students" difficulties in circle theorems and post-test to evaluate the effectiveness of the intervention. The questionnaires were administered to solicit teachers` and students" perceptions on the concepts of circle theorems. The same questionnaires were administered on both teachers and students.

QUESTIONNAIRES

Both teachers and students were made to respond to the questionnaire. Ten mathematics teachers and 63 students were involved in responding to the questionnaires. The questionnaire was made up of 13 items meant to solicit students" perception about the learning of circle theorems before using the diagrams. A time frame of 30 minutes was allowed for submission. The questionnaire was structured using a five-point Likert scales ranging from strongly disagree to strongly agree.

PRE AND POST TESTS

Two tests were used to collect data on students" achievements before and after implementing the activities which involved the use of diagrams. The first test was a pre-test which was structured to diagnose and determine students" level of achievements in circle theorems (see Appendix B). The second test was a post-test which was aimed at assessing the level of achievements of students after using of the activity method with diagrams (see Appendix C). Both tests had the same structure and scope but with different number of test items aimed at assessing students" level of achievements which is the main construct under investigation. The test items which intended to focus on application of knowledge were made up of 10 theory questions in the pre-test and 14 questions in the post-test all involving theorems in cyclic quadrilateral, angles subtended at the centre of a circle, angles in a semi-circle and angles on the circumference which subtend on the same chord in the same segment. Questions 1, 2, 3, 4, 5 and 6 of the test items in the pre-test were diagram free whilst questions 7-10 were in diagram forms. The diagram free questions intend to let students transform the questions into diagrams and apply the theorems to solve the questions in the test. Those that involve diagrams require students to apply their understanding of the concepts in solving the problems. These questions were carefully designed to conform to the scope of Circle Theorems in the SHS 2 syllabus and similar to those included in West African Senior School Certificate Examination.

Reliability

Specific printed questions with instructions were given to the students by the researcher to answer to find out the problems students encounter when applying the circle theorems in answering circle related questions. To ensure that all questions were clear to the students, the questions were given to the Head Of Mathematics Department in the school to vet and moderate the questions to suit students level of understanding. The test took duration of an hour which was seemed to be a good ample time for students to operate within. Hence, time limit was not a factor affecting the data. Supervision was normal under any normal examination environment. After the duration for the test the answer scripts were collected and marked. The test was conducted as a formal assessment. The results from the test were analyzed and discussed in class with students later.

Validity

The questions in the instrument were reviewed by one senior mathematics teacher in the school and the head of mathematics department in an attempt to make sure that the questions in the instruments are valid that is the test has content validity. Moreover, no explicit help was given to the students in order to minimize the "help factor" that can influence the validity of the data. The test questions were questions taken from mathematics textbooks for senior high schools.

DATA COLLECTION PROCEDURE

Data collection procedures started with the administration of the pre-test to the sample. The researcher together with two other mathematics teachers supervised the students during the 60 minutes test period. This was done in order to ensure that each student did an independent work. The Headmaster of the school was contacted for permission. Students" consents were sought before conducting the pre-test, putting in the intervention and administering the post-test. The self-structured questionnaire on students" perception was also administered to (63) students and ten (10) mathematics teachers a day after the pre-test was conducted. Both teachers and students answered and returned the questionnaires to the researcher within an average period of 30 minutes.

The pre-test was followed by a two week teaching process using sketches/diagrams of circles to effectively demonstrate the circle theorems one after the other to enhance the learning of the theorems. The scope of the topic on circle theorems as ascribed in the SHS 2 core mathematics syllabus was covered within the two weeks period. In all, there were three lessons of 80 minutes in each case.

At the end of the teaching period, a post-test was conducted under the supervision of the researcher and one mathematics teacher. After the stipulated time of 60 minutes the test scripts were collected. All the 63 students took part in answering the post-test items. The post-test answer scripts were marked and the scores were analyzed using SPSS. The scores of the pre-test and the post-test were compared to draw conclusions on students` performance in the application of the circle theorems in solving circle related questions.

THE INTERVENTION

The intervention was carried out after the pre-test and questionnaire were administered and students" difficulties were identified. The intervention lasted for two weeks. The intervention was mainly based on the scope of the topic as ascribed in the SHS core mathematics syllabus. In all there were three lessons of 80 minutes in each case. In each lesson at least two theorems of circle were taught, all through practical activities and based on diagrammatic representations and demonstrations. The activities were drawing, measuring and comparing of angles. Each of the three lessons was carried out as follows:

LESSON ONE

The duration of the lesson was 80 minutes. This lesson was aimed at assisting students to investigate state and apply the following theorems in solving problems related to circles. That is;

a) Equal chords subtend equal angles at the centre of a circle:

b) Equal chords subtends equal angles at the circumference of the circle

c) The angle which an arc or chord of a circle subtends at the centre of the circle is twice that angle which it subtends at any point on the remaining part of the circumference.

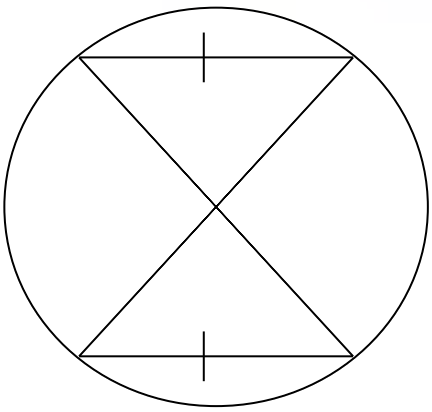
Activity 1

Students were assisted to carry out the following activities individually.

a) On a sheet of paper and with any convenient radius draw a circle and mark the centre O.

b) Draw two chords AB and CD such that AB = CD

c) Join the points A, B, C and D to the centre O as shown on the diagram below



1) Measure angle AOB and angle DOC.

2) Repeat the activity using different radii and vary the positions of A, B, C and D.

Students' Observation

The angles AOB and DOC are equal in all the activities. In any circle the angles formed by equal chords to the circle at the centre are always equal.

By this a theorem was propounded as:

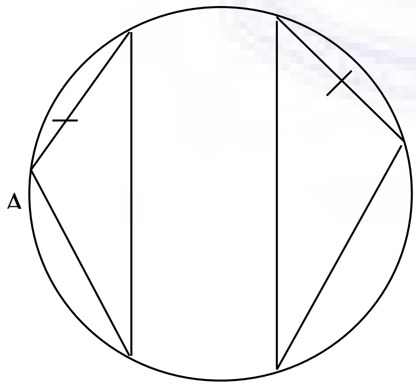
Angles formed by equal chords at the centre of a circle are equal.

Activity 2

1. On a sheet of paper and with any convenient radius draw a circle.

2. Locate on the circumference the points A, B, C, D, E and F such that AC = EF

3. Join the points ABC and also DEF separately to form triangles as shown below



1. Measure the angles ABC and EDF

2. Repeat the activity using different lengths such that AC = EF and vary the positions of the points.

Students' Observations

Students observed from the activities that angles ABC and EDF are equal. Based on the discovery by students through the activities, the students with the assistance of the researcher concluded by stating another theorem as;

Equal chords subtend equal angles at the circumference of the circle.

Activity 3

a) Draw on a sheet of paper, a circle centre at O with any convenient radius.

b). Mark on the circumference, the points P, Q and R as shown on the diagrams below

c) Measure the angles POR and PQR in each of the two diagrams.

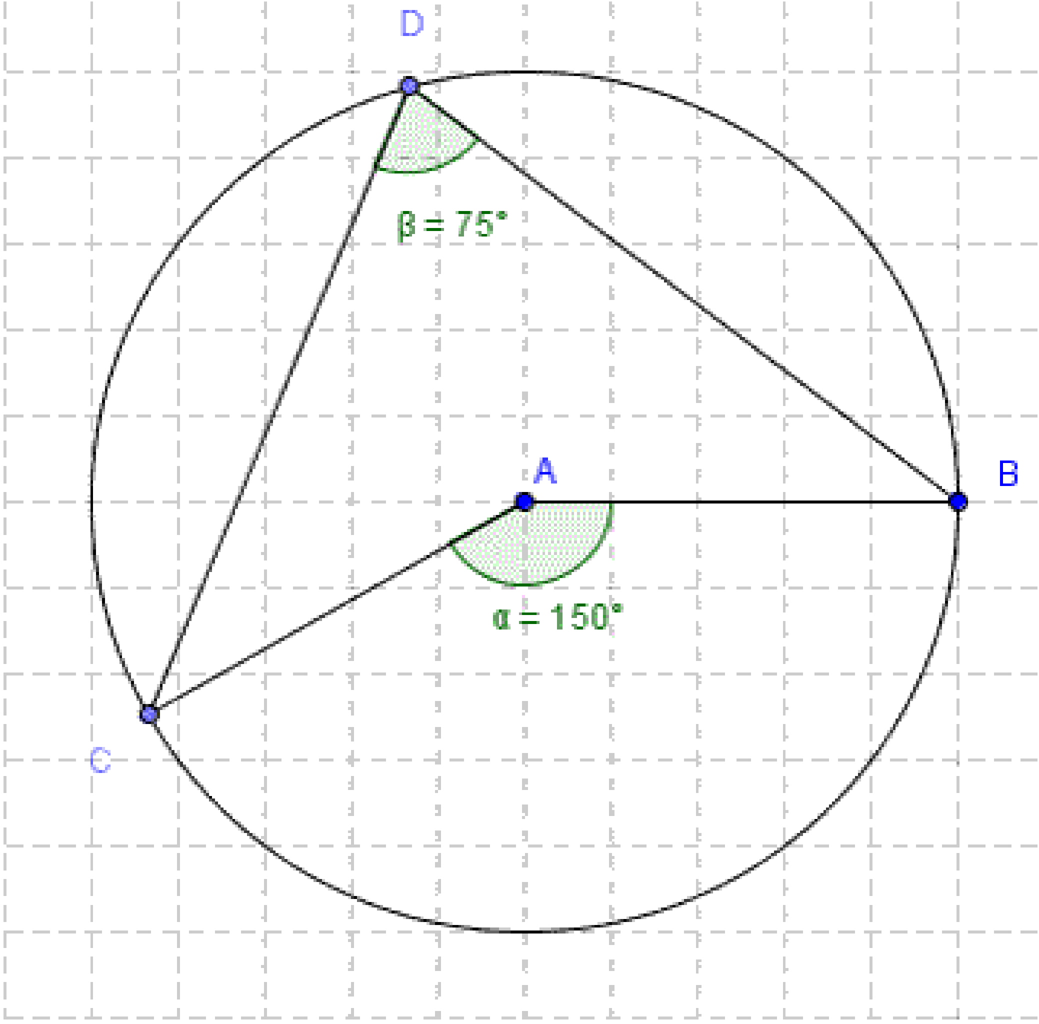
d) Repeat the activity but using different locations of P, Q and R on the circumference.

Students' Observations:

Students observed that in all the activities, angle POR is twice the size of the angle PQR. That is angle POR = 2 × angle PQR.

Example:

What is the relationship between the angles CDB and CAB in the diagram below.



Students' answers

a) Angle CAB is twice the angle CDB

b) Angle CDB is half the angle CAB

Students then state a theorem based on their discovery as:

The angle formed by an arc of any circle at the centre of the circle is twice the angle the same arc forms at the circumference.

LESSON TWO

The duration of the lesson was 80 minutes. The lesson was intended to assist students to investigate, discover, state and apply the following theorems in solving circle related problems.

a) An angle subtended by a diameter at the circumference of a circle is a right angle.

b) Angles in the same segment of a circle are equal.

c) Opposite angles in a cyclic quadrilateral are supplementary.

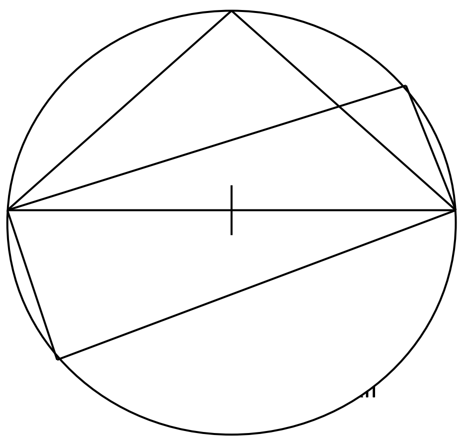
Activity 1

a) Draw on a sheet of paper and with any convenient radius, a circle centre O.

b) Draw the diameter AB of the circle.

c) Join the extreme points A and B of the diameter to meet at the points P, Q and R on the

circumference of the circle as shown on the diagram below.



Measure the angles APB, AQB and ARB.

Repeat the activity using different radii and different locations of A, B, P, Q and R on the circumference.

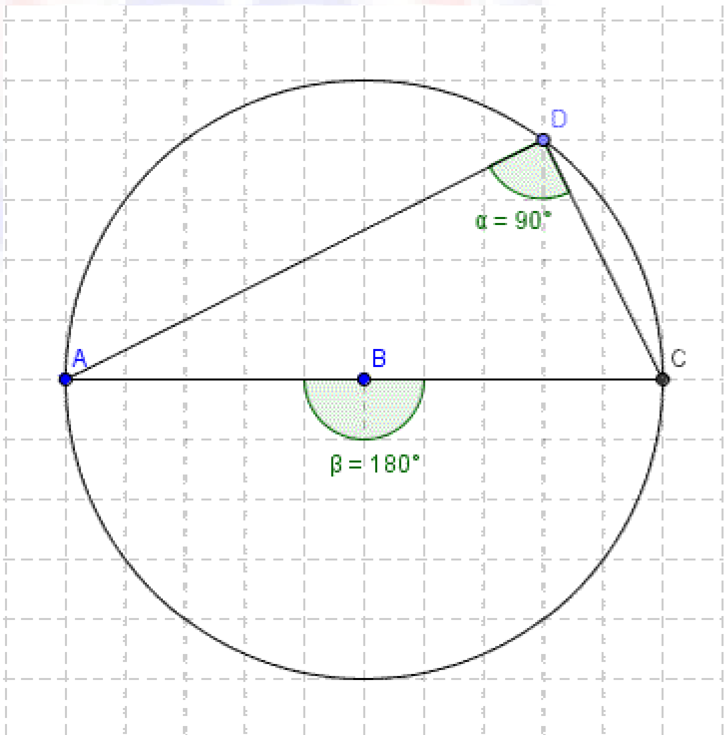
Students' Observations

Students observed that the angle APB = angle AQB = angle ARB = 90°.

Based on their investigations and conclusions, students stated the theorem as follows;

An angle subtended by a diameter at the circumference is a right angle.

Example below;



From the above example students measure angle ADC.

Students conclude that angle ADC=90^0.

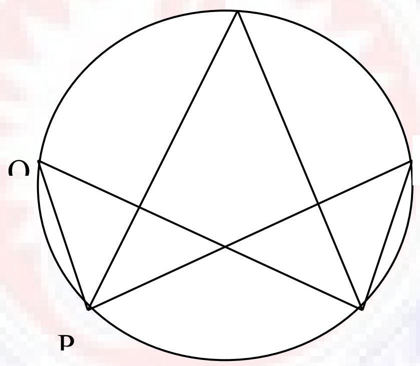
Activity 2

a) Draw a circle using any convenient radius.

b) Mark on the circumference, the points P, Q, R, S and T.

c) Join arc PT to Q, R and S so that they form angles at the circumference as shown on the

diagram below.



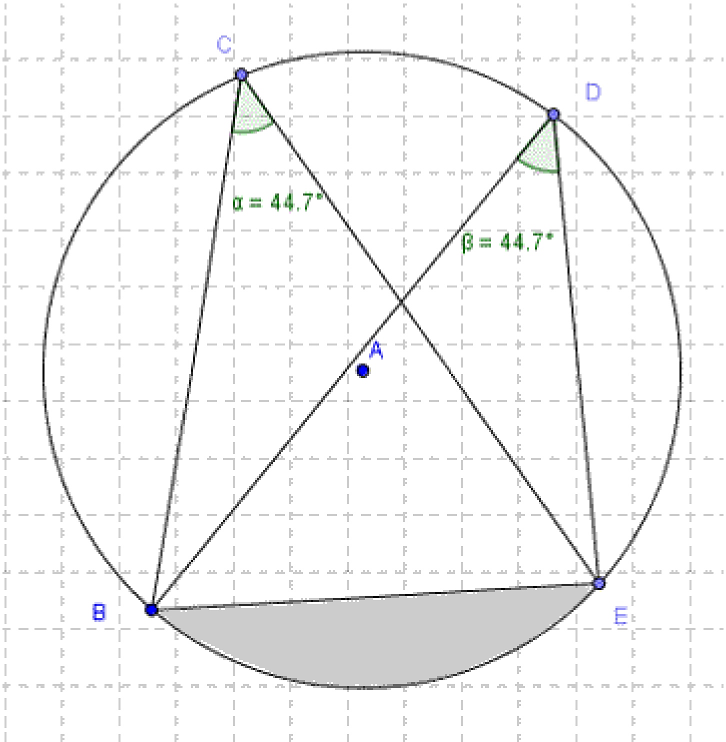
a) Using a protractor, measure the angles PQT, PRT and PST.

b) Repeat the activity using different radii and vary the positions of P, Q, R, S and T on the

circumference.

Students' Observations

Students noticed that the angles PQT = PRT = PST.



The students concluded based on their observations by stating that:

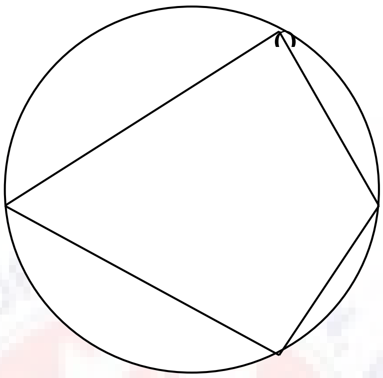
Angles formed by the same arc/chord in the same segment are equal.

Activity 3

a) Draw a circle using any convenient radius.

b) Mark four points P, Q, R and S on the circumference.

c) Join the points to obtain a cyclic quadrilateral as shown.



a). Measure all the angles in the cyclic quadrilateral.

b). Find the sums of the two pairs of opposite angles.

c). Repeat the activity using different radii and vary the positions of the points P, Q, R and S.

Students' Observations

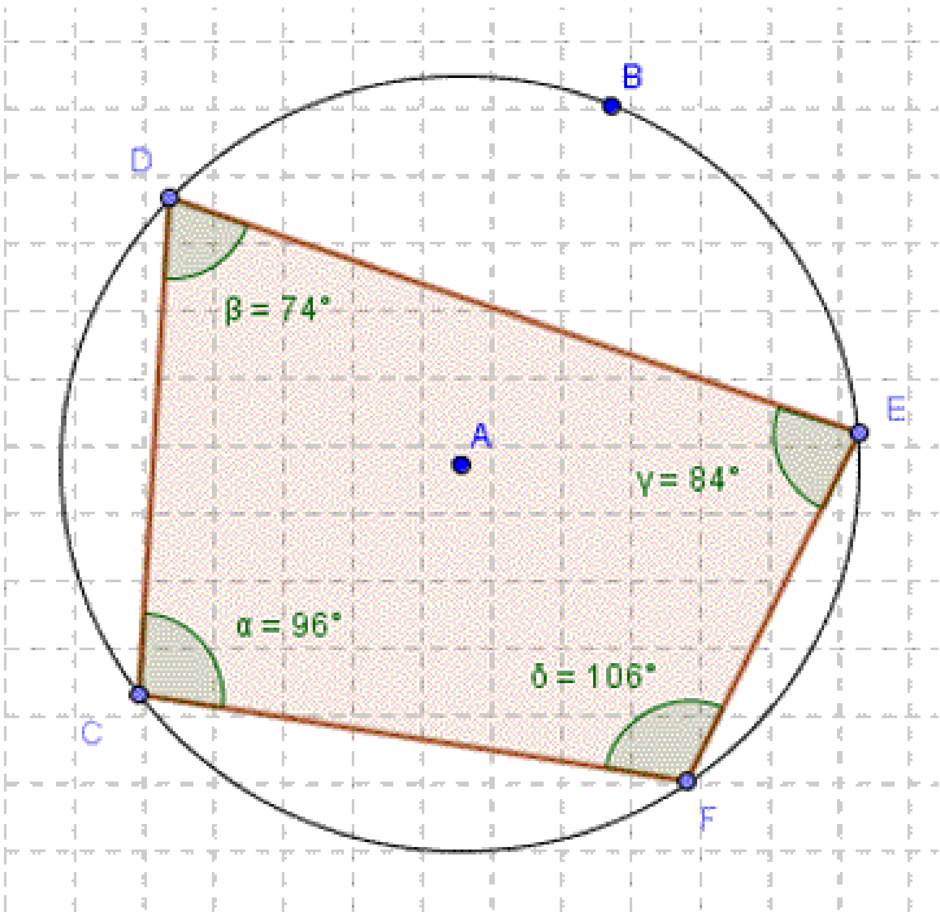
Students observed that the sum of opposite angles in any cyclic quadrilateral is equal to 180°. That is angle QRS + angle QPS = 180° and also angle PQR + angle PSR = 180°.

Students concluded by stating a theorem that;

Opposite angles in a cyclic quadrilateral are supplementary.

Example:

Measure the opposite angles in the diagram below.



Students answer:

Angles CDE and CFE are opposite and supplementary (i.e. $74^0 + 106^0 = 180^0$).

Also angles DEF and DCF are supplementary (ie $96^0 + 84^0 = 180^0$).

LESSON THREE

The duration of the lesson was 80 minutes. The lesson seeks to assist students to investigate the following theorems.

a) If one side of a cyclic quadrilateral is extended, the exterior angle formed is equal to the interior opposite angle.

b) The angle between a tangent to a circle and the chord through the point of contact are equal respectively to the angle in the alternate segments.

A tangent to a circle forms an angle of 90° to the radius of the circle.

Activity 1

a) Draw a circle with any convenient radius,

b) Locate the points P, Q. R and S on the circumference of the circle,

c) Join the points P, Q, R and S to form a cyclic quadrilateral,

d) Extend line PS to a point T outside the circle. Measure angles PQR and RST,

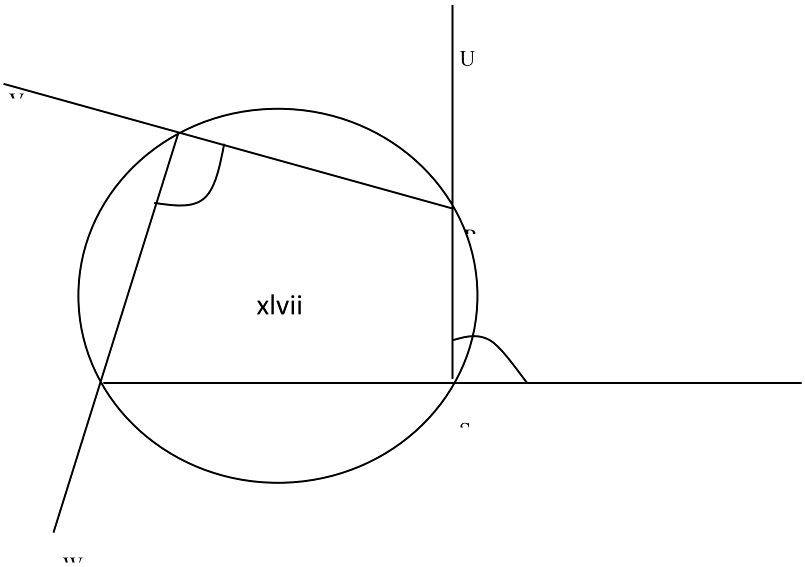
e) Extend line SR to a point U outside the circle. Measure angles QPS and QRU,

f) Extend line RQ to a point V outside the circle. Measure angles PSR and PQV,

g) Extend line QP to a point W outside the circle. Measure angles QRS and SPW,

h) Repeat the activity using different radii for the circle and vary the positions of the points P, Q, R, S and T.

See diagram below.



Students' Observations

The students noticed that, in all the activities; if one side of a cyclic quadrilateral is extended, the exterior angle formed is equal to the interior opposite angle. Therefore the students arrived at a theorem that;

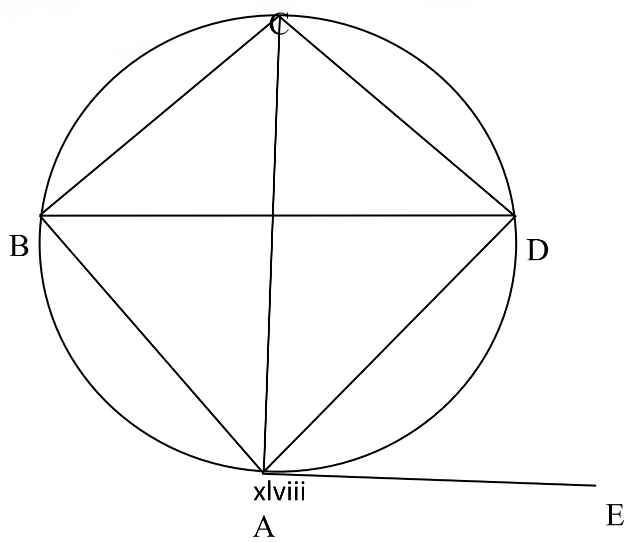
If one side of a cyclic quadrilateral is extended, the exterior angle formed is equal to the interior opposite angle.

Activity 2

a) Draw a circle using any convenient radius,

b) Locate the points A, B, C and D on the circumference of the circle,

c) Draw a tangent AE to the circle at A, Join; A to B, A to C, A to D, B to C, B to D and C to D, as shown on the diagram.



a) Measure and compare

i) angles DAE, ABD and ACD;

ii) angles CAE and ABC.

b) Repeat the activity by varying the positions of A, B, C, D, and E.

Students' Observations

Students noticed that angles DAE, ABD and ACD are all equal. They also observed that angles ABD and ACD are angles in the alternate segment of the angle formed by the tangent AE, and the chord AD, to the circle. Also, angles CAE and ABC are equal. Angle ABC is an angle in the alternate segment of angle CAE. Through the activity performed, the students observed and concluded that;

Angles in opposite segments are equal.

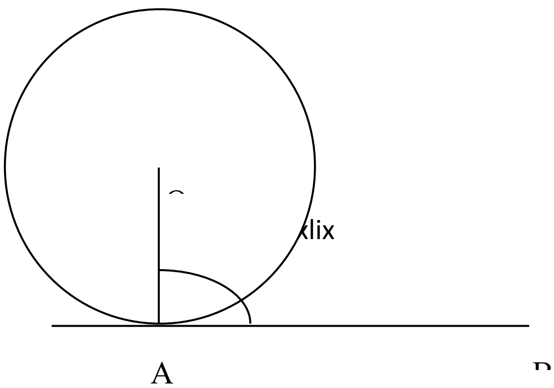
Activity 3

a) Draw a circle centre O using any convenient radius,

b) Locate a point A on the circumference of the circle,

c) Join O to A,

d) Draw a tangent AB to the circle through A as shown on the diagram below.



a) Measure the angle OAB,

b) Repeat the activity by varying the position of A.

Students' Observation

Students observed that in all the activities, angle OAB is 90°. This made the students arrived at a theorem that;

A tangent to a circle forms an angle of 90° to the radius of the circle at the point of contact. Looking at all the activities performed during the lessons, numbering of the theorems as some books and authors have proposed, perhaps have scared our students from learning the theorems. Learning the theorems alone without the numbers will not be a problem to the students, but when the theorems are attached to numbers then students are restricted to learning the theorems with their numbers. I proposed that the theorems must be learnt or taught without numbering. This is because if the theorems are attached with numbers such as theorem 1; theorem 2 and so on it will make the learning of the theorems difficult because as soon as one misses a given number to a particular theorem makes one found wanting. Teachers of mathematics must begin to realize and avoid teaching our students that theorem 1 says this or theorem 2 says that and so on. Theorems must be learnt exclusively without numbers as shown in the intervention or teaching process.

SCORING AND DATA ANALYSIS PROCEDURE

The test items on the pre-test were based on students‟ knowledge and understanding of circle theorems and the scores were taken as such. The scoring was based on correct presentation of facts on the application of the theorems. Students were to represent the information given in some of the questions without diagrams in diagrammatical form before forming the algebraic equations to solve them. It is infact difficult to solve such questions without transforming the questions into diagram. Rather, it appears easy to identify the theorems if any, in the diagram if it is drawn. For instance, if a student states an algebraic equation after drawing the diagram from the test items correctly, it shows the students‟ ability to „devise a plan‟. Such a student was awarded a mark for drawing and forming the algebraic equation correctly in order to solve for the unknown angle(s) requested to find otherwise loses a mark. Similarly, solving the equation devised and indicating the appropriate theorem reflects the student ability to „carry out the plan‟ and getting correct answer confirms it. The scores from the questionnaire were fed into SPSS for analysis. The result from the analysis was used to determine the kind of perceptions students have in learning the concepts of circle theorems.

Data on the scores of the pre-test and post-test were also fed into SPSS to compare means of the two tests to determine the relationship between their means and also to determine students‟ perception about the learning of theorems under circles. The analysis was used to determine the degree by which students have improved upon the understanding and application of the circle theorems.

Table 3.1 Number of teachers who ticked the various responses of choice.

Item No.	Perceptions about learning circle theorems	Responses SA	A	D	SD
1.	It is just one of the difficult topics.	4	4	2	0
2.	It is just one of the easiest topics.	1	8	1	0
3.	The concepts are easy to memorize.	6	4	0	0
4.	The concepts are difficult to memorize.	1	7	2	0
5.	It has disjointed concepts.	6	4	0	0
6.	The concepts are linked.	1	7	2	0
7.	Peer influence makes me not to like the topic.	1	1	3	5
8.	I have problems relating patterns.	5	5	0	0
9.	It has a lot of principles one must recall.	4	5	0	1
10.	It is easier to relate patterns.	3	6	1	0
11.	It has just some few principles one must recall.	4	4	2	0
12.	It is difficult to picture the algebra of a given question.	8	2	0	0
13	It is easy to formulate the algebra of a given question.	6	4	0	0

From the table out of the ten (10) teachers who responded the questionnaire eight (8) of them strongly agreed that students find it difficult picturing the algebra in a given question. This is an indication that students will not answer questions relating to circles most especially questions that demand the application of circle theorems in any examination and answer other questions which they feel easy. Most of the teachers either strongly agree or agree to most of the perceptions students have about the learning of the theorems. It is an indication that teachers have been observing students and have taken notice of that.

Table 3.2: Number of students who ticked the various responses of choice.

		Responses			
Item No.	**Perceptions about learning circle theorems**	**SA**	**A**	**D**	**SD**
1.	It is just one of the difficult topics.	42	16	3	2
2.	It is just one of the easiest topics.	6	14	1	42
3.	The concepts are easy to memorize.	46	17	8	5
4.	The concepts are difficult to memorize.	6	32	5	20
5.	It has disjointed concepts.	50	10	1	5
6.	The concepts are linked.	3	15	40	5
7.	Peer influence makes me not to like the topic.	44	11	7	1
8.	I have problems relating patterns.	41	17	2	3
9.	It has a lot of principles one must recall.	39	18	1	5
10.	It is easier to relate patterns.	3	14	38	8
11.	It has just some few principles one must recall.	6	5	31	21

12.	It is difficult to picture the algebra of a given question.	23	15	13	12
13	It is easy to formulate the algebra of a given question.	49	9	1	4

Table 3.2 shows the responses by 63 students. And from the responses the students have indicated that: a) it is difficult to formulate the algebra of a given question, b) a lot of principles are involved in the learning of the circle theorems, c) their peers have not been encouraging or motivating them to learn the concepts of the theorems and d) the concepts are disjointed. From their responses it is clear that students` perception about the concepts that they are difficult is true. In this regard students will show no interest in learning the concepts. They will either leave a class or pay no attention to the mathematics teacher anytime he/she makes any attempt to teach the concepts.

Table 3.3 Number of students and their solutions to the questions in the pre-test test items.

No of students	Questions without diagrams		Questions with diagrams	
	Correct solution	Incorrect solution	Correct solution	Incorrect solution
63	2	61	11	52

Table 3.4. The table shows the number of students and their solutions to the questions in the post-test test items.

No of students	**Questions without diagrams**		**Questions with diagrams**	
	Correct solution	Incorrect solution	Correct solution	Incorrect solution
63	38	25	50	13

CHAPTER 4

ANALYSIS OF DATA AND DISCUSSIONS.

The pre-test results identify that most of the students could not represent the information given in the test items without diagrams into diagrams. For this reason all of the students could not solve those questions correctly. They needed to picture the information from the diagram to enable them apply the appropriate theorems to solving the problems. The difficulties encountered by students were perhaps their inability to construct meaning from the problems. However the approach by students to solve the other questions which involved diagrams seemed better than those without diagrams. The students were able to identify some of the known theorems, formed algebraic equations to solve for the unknown angles in the circles given in the test item. In addition to this it was also identified that most of the students could not apply the concepts appropriately and correctly to solving the circle questions. These results were meant to guide the researcher in planning and implementing the circle diagrams to mediate the difficulties identified.

Table 4.1 presents descriptive statistics of the 63 SHS students‟ achievements in the pre-test in this study. Students‟ achievements were categorized into knowledge acquisition and its application (see Table 4.1).

4.1 Result from the Pre-test

No of students	**Questions with diagrams**		**Questions without diagrams**	
	Correct solution	Incorrect solution	Correct solution	Incorrect solution
63	11	52	2	61

From the table 4.1 out of the 63 students who took part in the pre-test 11 that is 17% of them got the solution of the questions with diagrams correct and only 2 of them representing 3% of the students got the solution of the questions without diagrams correct. This is an indication that

majority of the students cannot interpret questions on the application of circle theorems correctly.

Table 4.2 Result from the Post-test

No of students	Questions with diagrams		Questions without diagrams	
	Correct solution	Incorrect solution	Correct solution	Incorrect solution
63	56	7	50	13

Table 4.2 shows an improvement in the application of the theorems after the intervention. 79% of the students could solve questions on problems without diagrams and about 89% of them solving questions on application of circle theorems with diagrams correctly. This shows that correct use of diagrams on the interpretation and application of the theorems is necessary. Application as used in this study refers to students" ability to devise correct algebraic equations to solve for unknown angles using appropriate theorems in solving a given cyclical diagram.

Table 4.3: Percentage distribution of students' achievement in pre-test item on applications of theorems on circles

Aspects of applications of circle theorems by test items	Appropriate (%)	Inappropriate (%)	Total (%)
Pre-test item on circle with angles in the segments			
Formulation of algebraic equation	27.0	73.0	100.0
Students" strategy/approach	33.3	66.7	100.0
Students" answer provided	20.6	79.4	100.0
Pre-test item on circle diagram with a tangent to a circle centre O			
Formulation of algebraic equation	12.3	87.7	100.0

Students‟ strategy/approach	20.1	79.9	100.0
Students‟ answer provided	20.1	79.9	100.0
Pre-test item on concepts and theorem in cyclic quadrilateral			
Formulation of algebraic equation	12.6	87.4	100.0
Students‟ strategy/approach	11.1	89.9	100.0
Students‟ answer provided	9.1	91.9	100.0

With circle theorems, three aspects of the application were identified from the analysis of students‟ performance in pre-test items. In all the three pre-test items answered by the 63 students, between 73.0 - 87.7% of the students had difficulty setting up the correct algebraic equations to solve for the various unknown angles in the cyclical diagrams. This reflected the likelihood of limited knowledge on formulating equations and poor understanding of the various properties of geometrical circles. As also seen in Table 4.3, the large proportion of inappropriate solutions were due to the inappropriate formulation of the algebraic equations. It was also clear during the analysis of the solutions that students either misrelated the circle theorems to their equation or did not indicate a theorem to signify the mathematical actions taken. Few students therefore were able to obtain correct answers as shown in Table 4.1 above.

In general, the two causes of students‟ low achievements related to application included:

limited knowledge in formulating algebraic equations and Inadequate knowledge in the geometry of circle properties and theorems as a result of teaching method used.

These areas were emphasized and taken care of during the implementation of circle diagrams in the teaching periods.

Research question one.

What instructional techniques are used in teaching and learning the concept of circle theorems in mathematics classroom in Mankessim Senior High School?

It was observed during my period of teaching in the school that a lot of students in the school were not answering questions involving the application of the circle theorems during the end of term mathematics paper. Students who attempted answering questions on the applications of the theorems were quoting the theorems on the paper instead of applying them to answer the questions. When I enquired to know the root cause of why students were not answering questions on the application of circle theorems, some of the students I questioned answered that the concepts were difficult to learn. Moreover some of their mathematics teachers made them copy the concepts of the theorems as they write them on the chalkboard with no explanation. In addition students complained bitterly that their teacher never solved examples on the application of the theorems with them in class. The only example he solved with them was not well understood simply because the teacher did not take time to explain the steps he was taken to solve the problem.

On the part of some of the teachers consulted, the explanation given was that students failed to memorize the theorems as stated in the mathematics textbooks given to them. Furthermore students failed to apply the theorems as appropriate and applicable, they always looked up to the teacher to tell them what theorems to use in answering a particular circle related questions. They also mentioned that students failed to link the idea of properties of angles to solving problems relating to angles in the circles.

Based on the two sides listened to, the researcher realized that each side is shifting the blame on the other, that is either the students refused to learn the theorems because of the methods teachers used in teaching the concepts in the classroom for onward application or some of the teachers in the school lacks the skill to teach the concepts or the school does not support in providing the needed teaching and learning materials into the of mathematics in the school. In addition to above one can conclude that in teaching some of the concepts like in the case of circle theorems the teaching is devoid of activities, students are not involved in any activity thereby making the teaching unattractive for students to learn. Infact teaching circle theorems a lot of circle diagrams need to be drawn with parts clearly indicated and let students draw some and identify on their own some parts in them before introducing the circle theorems without necessarily assigning numbers to the theorems hence the intervention of using circle diagrams to teach circle theorems by the researcher.

Research question two.

What is difficult about the circle theorems that scare or demotivate students from learning the circle theorems concepts?

Another difficulty students encounter when learning the theorems is that the circle theorems have been associated with numbers for example Theorem 1; Theorem 2; Theorem 3 and so on. The numbering of the theorems makes the theorems unique, thus making students learn them as though the theorems were rules. Stating the theorems in one`s work without it correct number makes one`s solution wrong. More so different placement of chords in a circle other than placing it at the top or down in the circle makes students found wanting when questions are asked about them. This was observed when marking the pre-test test items. Most students did not see that major or minor segment can be place sideways in a circle. Emphasis should not be placed on our students to either see a chord at the top or bottom in a circle rather mathematics teachers should emphasize on the definition of a chord as a line drawn to meet the circumference at two ends in a circle. Lack of knowledge about these scored the students with low marks in the pre-test conducted.

Table 4.4 The distribution of students according to four main perceptions about circle theorem prior to the use of circle diagrams.

Perceptions of students about circle theorems	Frequency	Percentage
It has disjoined concepts	52	82.5%
It is difficult to picture the algebra of the topic	35	55.6%
It is often difficult to relate the patterns in the circle	57	90.5%
It has a lot of principles one must recall	57	90.5%

An analysis of the perceptions of students about circle theorems shows that majority of the students perceive circle theorems as involving disjoined concepts, difficult algebra, patterns and principles which must be remembered, comprehended and related to other concepts. Out of the 63 students in the class between 55.6% and 90.5% hold these perceptions as shown in Table 4.4

Research question three.

What is the effect of diagrams on students` perceptions on questions involving circle theorems?

Students" perceptions were recognized to have significant influence on their learning process. The main perceptions of the students prior to and after the application of the theorems using diagrams were also sought and compared in order to find out any changes in perceptions in learning circle theorems. Table 4.4 shows the differences in students" responses to four main perceptions of students about the learning of circle theorems.

Table 4.5: Comparison of students' perceptions before and after the use of circle diagrams

Perceptions	Chi-Square Value	df	Asymp. Sig. (2sided)
It has disjoined concepts	4.918	1	0.027*
It is difficult to picture the algebra of the topic	3.927	1	0.048*

It is often difficult to relate the patterns in the circle	3.896	1	0.050*
It has a lot of principles one must recall	0.015	1	0.902

*significant as 0.05 level

df is degree of freedom

The result indicates significant changes in three of the perceptions identified prior to the teaching intervention. Before the application of the theorems using diagrams it was found that about 83% - 91% of the students indicated that circle theorems have disjoined concepts and often with difficult algebra and patterns. The analysis of responses from students using 0.05 significance levels indicated significant differences in students‟ perceptions. There is however no significant difference between students perception that circle theorems contains principles which must be recalled in order to solve related problems. The result means that circle diagram has a positive effect on students‟ perceptions that the circle theorems contain disjoined concepts with difficult algebra and patterns.

Research question four.

To what extent do diagrams enhance the teaching and learning of circle theorems?

In this study, the level of students‟ achievements were one of the parameters used to determine the effectiveness of the diagrams in addressing students‟ inadequate knowledge in geometrical properties of the circle and limited knowledge in formulating correct algebraic equations from a given circle problem. Table 4.5 displays the mean scores of the 63 students who took both the pre-test and the post-test.

Table 4.6: Comparison of students' mean scores in pre and post tests with the application of the circle theorem using diagrams

Using circle diagrams	Mean scores		Difference
	Posttest	Post-test	
Understanding	5.133	7.013	2.080
Devise plan	1.000	2.683	1.683
Carry out the plan	0.619	2.730	2.111
Look back	0.810	2.730	1.920

The understanding and application of circle theorems using diagrams measures the level of conceptual knowledge of the students in a given mathematical area. In this study, comparing the pre-test with the post-test, the mean score of the post-test had increased to 7.013. The difference of 2 scores in students" performance between the pre-test and the post-test suggests that students" understanding seem to be enhanced after intervening with the use of circle diagrams.

The main difficulty of the students which resulted into the low performance in the pre-test was their inability to apply the concepts, principles and theorems of circle to solve circle related problems. The difference of 2 scores in students" performance between the pre-test and the post-test suggests that students" understanding seem to be enhanced after intervening with the circle diagrams.

A student" ability to devise a plan recognized from the solution when the student was able to formulate a correct algebraic equation appropriate for solving the circle problem. Out of the 3 maximum score, students average pre-test score increased from

1.000 to 2.683 with the difference of 1.683. This reflects students‟ improvement in formulating algebraic equations which was the root cause of their poor achievements in solving problems related to circles. Carrying out the plan relates to the ability of the students to proceed to associate the appropriate main principles or theorems to each step in the process of solving the problem. After the intervention process, students mean score increased from 0.619 to 2.730 suggesting an improvement in applying the theorems under circles to solve related problems. A students‟ ability to look back or evaluate his/her final answer(s) signifies the students‟ ability to connect his/her knowledge of the properties of the circle with the solution(s). The difference of 1.920 between the mean scores of the pre-test and the post-test indicates an improvement in their achievements. These results indicated that students‟ knowledge and understanding and application have improved as a result of the use of circle diagrams

DISCUSSION.

An analysis of the perceptions of students about circle theorems prior to the intervention shown that majority of the students perceived circle theorems as involving disjoined concepts, difficult algebra, patterns and principles which must be remembered, comprehended and related to other concepts. These perceptions by students were not different from that outline by Thomaidis (1991) about students‟ reactions to the learning of geometrical concepts. However, after using circle diagrams as an intervention, their perceptions were reorganized to have significant influence on their learning process. This was explicitly shown in the improvement of students‟

achievements in the post-test. In this study, the level of students" achievement was one of the parameters used to determine the effectiveness of the use of circle diagrams for the application of circle theorems in addressing students" inadequate knowledge in geometrical properties of the circle and limited knowledge in formulating correct algebraic equations from a given circle problem.

The main difficulty of the students which resulted into the low achievement in the pre-test was their inability to apply the concepts, principles and theorems of circle using the circle diagrams to solve related problems. The difficulties encountered by students were not different from that outlined by Strutchens, Harris, & Martin (2001) and Lappan, Fey, Fitzgerald, Friel & Philips (1996) when they proposed that students" difficulties were as a result of memorizing properties rather than exploring to discover properties and misuse of geometrical terminologies often leads to misconceptions of geometrical knowledge. The students" difficulties in the pre-test leading to their low achievements in circle theorems is a clear manifestation of the Chief Examiner"s Report (WAEC, 1995: 1997: 2005) about students" difficulties in answering circle theorem questions.

However, after applying the theorems using circle diagrams to solve circle related problems, students were able to devise a plan, recognized from the solution when the student was able to formulate a correct algebraic equation appropriate for solving the circle problem. These results indicated that students" knowledge and understanding and application have improved as a result of the use of circle diagrams to apply the concepts of circle theorems to circle related problems.

CHAPTER 5

SUMMARY OF FINDINGS, CONCLUSION AND RECOMMENDATIONS

SUMMARY OF FINDINGS

The study which was conducted in Mankessim Senior High School, using 63 Business 2 students was designed to find out students difficulties and perceptions in learning and applying the concepts of circle theorems and to use direct and appropriate intervention like the use of circle diagrams as a teaching and learning strategies to improve their achievements. Students` problem and difficulty in solving questions relating to the application of the circle theorems is when the question or the problem is framed in sentence form without the involvement of diagrams. In summary students‟ difficulties in applying the circle theorems were identified to include:

1) learning the theorems with accompanying numbers;

2) average knowledge and understanding of the theorems on circles.

Disproportionately, wide variations in the students‟ knowledge and understanding of the theorems on circles seem a difficulty in applying the various theorems in circles to solve related problems. Inability of students to apply their knowledge of circle properties to solve circle related problems. Difficulty setting up the correct algebraic equations to solve for the various unknown angles in the cyclical diagrams, mispresentations of circle theorems in formulating equations or did not indicate a theorem to signify the mathematical actions taken limited knowledge in formulating algebraic equations, inadequate knowledge in the geometry of circle properties and theorems.

Students‟ perceptions of circle theorems were identified to include:

It has disjoined concepts

It is difficult to picture the algebra of the topic

It is often difficult to relate the patterns in the circle

It has a lot of principles one must recall.

After the intervention, students can now use and apply the appropriate theorem to answer a circle related problems correctly.

Students‟ perception about circle theorems being difficult was minimized.

Students were using circle diagrams to answer circle related questions.

CONCLUTION.

It is important to note that in order to acquire mathematical concepts one should visualize the importance and use of diagrams, because mathematics is the field in which preconditions are crucial and so before the teaching process student backgrounds on the subject should be tested. Students should be allowed to learn mathematics through the experiences that their teachers provide. Thus, students understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition towards mathematics are all shaped by the teaching they encounter in school. This means that the improvement of mathematics education for all students requires effective mathematics teaching in all classrooms. Mathematics teachers need to know the ideas with which students have difficulty in and find ways to help bridge the common misunderstandings. Mathematics teachers must help students to solve problems but not to create problems for students to solve. Therefore I am of the view that in teaching practical oriented topics or concepts like in the case of circle theorems, care must be taken not to leave the learner out of the learning process. Looking at the intervention used to reduce the misconceptions students or learners had about the concepts of circle theorems, I believe strongly that if mathematics teachers use or apply the steps used by the researcher in the intervention, learners will understand the concepts learnt and apply them appropriately at all times. Setting of rules for our students or learners to learn a concept is not helpful rather it promotes root learning. Mathematics textbooks used in our Ghanaian classrooms must be thoroughly looked at before allowing them into our schools. Most of these textbooks look attractive but are not useful to solving the mathematics problems confronting us in Ghana. If mathematics teachers prepare adequately before going to teach a concepts in the classroom I believe the teacher will consider the type of textbooks, teaching and learning materials to use and the type of environment to create to enable the smooth teaching of a mathematics concepts. Mathematics teachers should come together to help students or learners reduce or possibly eradicate the phobia and misconceptions learners have in mathematics.

RECOMMENDATIONS.

The study explored students‟ difficulties in circle theorems and used circle diagrams strategies to improve students ‟achievements in circle theorems. Based on the difficulties identified and the improvement in students` achievements when the intervention of the use of circle diagrams in applying the circle theorems to circle related problems during the lessons, the researcher recommends the following:

The Ghana Education Services should organise in-service training workshop for mathematics teachers in the Senior High Schools on how to teach circle theorems using circle diagrams with activities.

Mathematics Teachers should use practical approaches to teach the properties of geometrical concepts.

The teaching of geometrical concepts in schools should reflect real life situations. For instance, the teaching of the concept of circle can be linked to pegging an animal in a grass field to graze. The Ministry of Education in collaboration with the Curriculum Research and Development Division of the Ghana Education Service should revise the mathematics syllabus to include and expand the scope and umbrella of geometry at pre-university level of education.

The teaching and development of concepts in circle theorems should be more activity oriented and must involve students in investigating, conjecturing and discussing.

SUGGESTIONS FOR FURTHER STUDIES

The researcher suggests the extension of this study to cover many schools. Further study into how teachers teach circle theorems in senior high schools is also recommended. In addition, the use of appropriate circle diagrams with activities to improve students‟ understanding and achievements in applying the circle theorems is recommended for further study. In conclusion the circle theorems must be taught and learnt without associating numbers to the theorems and last but not the least, parts of circle like the chord, segment, arc and others must be well defined for students or learners to visualize but not to draw for them on the boards.

REFERENCES

Ainsworth, S., & Th Loizou, A. (2003). The effects of self-explaining when learning with text or diagrams. Cognitive science, 27, 669-681.

Bruner, J. S. (1966). *Towards a Theory of Instruction.* Cambridge: Massachuse Belknap Press.

Cheng, P. C. H. (2002). Electrifying diagrams for learning: principles for complex representational systems. Cognitive Science, 26, 685e736.

Cheng, P. C. H. (2004). Why diagrams are (sometimes) six times easier than words: benefit beyond locational indexing. In A. Blackwell,

Craig,T. (2001), Factors affecting Perceptions of Difficulty in Calculus Word problems. Unpublished Master Thesis, University of Cape Town.

Herbert J., & Carpenter, T. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning189(pp.65-97). New York: Macmillan.

Koedinger, K. R., & Terao, A. (2002). A cognitive task analysis of using pictures to support pre- algebraic reasoning. In C. D. Schunn, & W. Gray (Eds.), Proceedings of the twenty-fourth annual conference of the Cognitive Science Society (pp. 542e547). Mahwah, NJ: Lawrence Erlbaum Associates.

Lappan , G., Fey, J., Fitzgerald, W. M., Friel, S., & Philips, E. D. (1996). *Shapes and design. Two dimensional geometry*. Palo Alto, CA: Seymour Publications

Larkin, J. H., & Simon, H. A. (1987). Why a diagram is (sometimes) worth ten thousand Words. Cognitive science, 11, 65-99.

Mayer, R. E. (2003). The promise of multimedia learning: using the same instructional Design methods across different media. Learning and instruction, 13, 125-139.

National Research Council. (1989) *Everybody counts: A report to the nation on the future of mathematics education.* Washington D.C: National Academy Press.

Pirie, S.E.B. & Kieren, T.E. (1994a). Growth in mathematical understanding: How can we characterize it and how can we represent it? Educational Studies in Mathematics, 26, 165-190-191

Pirie, S.E.B. & Kieren, T.E. (1994b). Beyond metaphor: Formalising in mathematical Understanding within constructivist environments. For the Learning of Mathematics, 14(1), 3943.

Sierpinska, A. (1994). Understanding in mathematics. Washington, DC: Falmer Press.

Skinner, B. F. (1969). *"Why Teachers fail'': Educational Psychology*. New York: Van Nostrand-Reinhold.

Smith, E. (1994). Exponential functions, rates of change, and the multiplicative unit. Educational Studies in Mathematics, 26, 135-64.

Stern, E., Aprea, C., & Ebner, H. G. (2003). Improving cross-content transfer in text processing by means of active graphical representation. Learning and Instruction, 13, 191e203.

Strutchens, Harris & Martin, (2001). *Assessing geometric and measurement understanding using manipulatives. Mathematics teacher in middle School, 6*(7), 402-405.

Thomaidis, Y. (1991). Historiocal Digression in Greek Geometry Lessons. *For the Learning of Mathematics. Special issues on history in mathematics education*, *11* (2), 37-43.

West African Examinations Council (1995). Senior *Secondary School Certificate Examination. Chief Examiner's Report.* Accra: West African Examinations Council.

West African Examinations Council (1997). Senior *Secondary School Certificate Examination. Chief Examiner's Report.* Accra: West African Examinations Council.

West African Examinations Council (2003). West *African Senior School Certificate Examination. Chief Examiner's Report.* Accra: West African Examinations Council.

West African Examinations Council (2004). *West African Senior School Certificate Examination. Chief Examiner's Report.* Accra: West African Examinations Council.

APPENDIX A

Questionnaire for Students.

This questionnaire which you are requested to kindly answer is aimed at soliciting your perceptions about learning circle theorems in the Senor High School. Your response will enable the researcher to develop an intervention strategy as a way of assisting you to improve upon your answering skills and achievements in problems related to circle theorems.

To ensure complete anonymity, please do not write your name on the questionnaire.

Tick your response to the statements about learning circle theorems.

Item No.	Perceptions about learning circle theorems	Responses			
		SA	A	D	SD
1.	It is just one of the difficult topics.				
2.	It is just one of the easiest topics.				
3.	The concepts are easy to memorize.				
4.	The concepts are difficult to memorize.				
5.	It has disjointed concepts.				
6.	The concepts are linked.				
7.	Peer influence makes me not to like the topic.				
8.	I have problems relating patterns.				
9.	It has a lot of principles one must recall.				
10.	It is easier to relate patterns.				
11.	It has just some few principles one must recall.				

12.	It is difficult to picture the algebra of a given question.				
13	It is easy to formulate the algebra of a given question.				

SA = Strongly agree, A = Agree, D = Disagree, SD = Strongly disagree

APPENDIX B

Pre-test for Students.

These questions which you are kindly requested to answer are aimed at finding out your level of difficulties in problems related to circle theorems. The results will enable the researcher to develop an intervention strategy as a way assisting you to improve upon your problem solving skills and achievements in problems related to circle theorems.

To ensure complete anonymity, please do not write your name on the question paper.

[Application of knowledge]

Answer all the questions. Show all necessary details of working together with the answer.

1. In a diagram TB is a tangent to the circle at B. |BD| is a diameter. Angle ABT= 29^0 and angle BAC = 59^0. Calculate:

i) angle ADC

ii) angle ABC

iii) angle CAD.

2. PR is a diameter of a circle PQR, angle PRQ = 58^0. Find angle RPQ.

3. In a diagram, A, B, C, and D are four points on a circle. |AB| = |AC|, angle BDC = 50^0 and angle ABD = 30^0. Calculate angle CAD.

4. In a circle with centre O, angle WXZ and angle WYZ are two angles formed at the circumference in the same segment and subtend on the chord WZ. If angle WOZ is 135^0 find:

angle WXZ

angle WYZ

5. In a circle with centre O, ABCD is a quadrilateral inscribed in the circle with BD and AC as diagonals of the quadrilateral. BT is a tangent at B. If angle ABT is 31^0 and angle BAC is 60^0. Find:

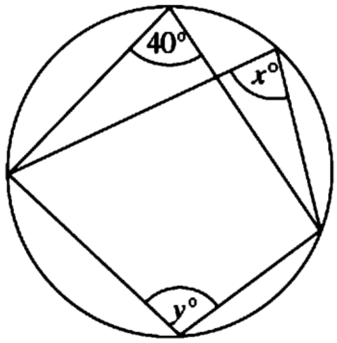
(i) angle ADC

(ii) angle ABC subtends the same chord in the same

angle CAD

6. In a circle with centre O, AB is a chord and angle AOB is 120^0. Find the angle ACB at the circumference which segment.

7. Use the diagram below to answer the questions under it.



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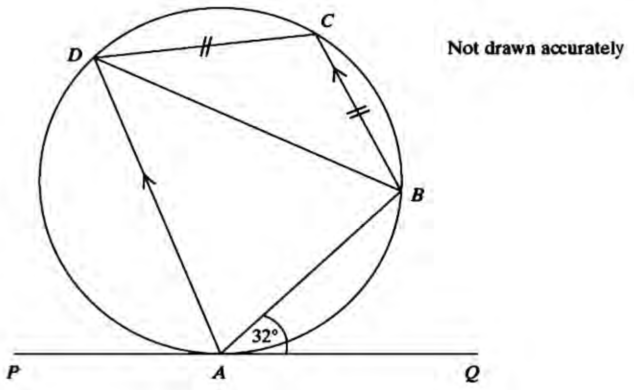
(i) Write down the value of x.

(ii) Calculate the value of y.

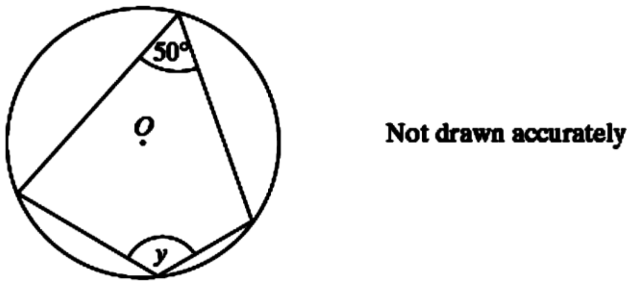
8. *ABCD* is a cyclic quadrilateral. *PAQ* is a tangent to the circle at *A*.

BC = *CD*. *AD* is parallel to *BC*. Angle *BAQ* = 32°.



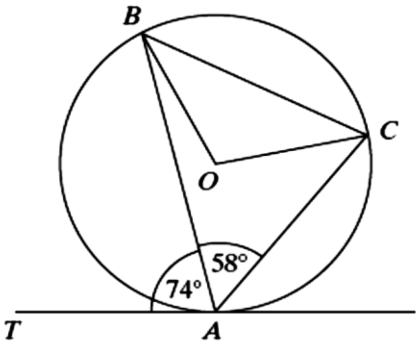
Find the size of angle *BAD*.

9. The diagram shows a different circle with centre *O*.



Work out the size of the angle marked *y*.

10. TA is a tangent to the circle at A. Angle BAC = 58° and angle BAT =74°.



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(i)Calculate angle *BOC*.

(ii) Calculate angle *OCA*.

APPENDIX C

Post-test for Students.

This paper which you are requested to kindly answer is aimed at finding out your level of difficulties and achievements in problems related to circle theorems after the intervention strategy.

To ensure complete anonymity, please do not write your name on the question paper.

Answer all the questions and show all necessary workings.

[Application of knowledge]

1. In a diagram O is the centre of the circle. The points A, B and C are on the circumference of a circle. Angles CAO and AOB are 32^0 and 140^0 respectively. Calculate

i) angle OBC

ii) angle COB.

2 in a diagram A , B, C and D are points on a circle. |AB| = |AC|, angle BDC = 50^0 angle ABD = 30^0. Calculate the value of angle CAD.

3. in a diagram: P, Q, R and S are points on a circle.|PQ| = |PS|, angle PQS = 40^0 and angle QSR = 25^0. Calculate the value of;

i) angle QPS

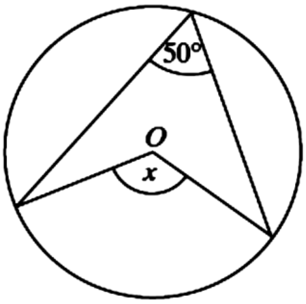
ii) angle QRS

iii)angle RQS

4) P, Q, R and S are four points on a circle with centre O, angle PQR = 100^0 and angle OPQ = 40^0. Find the value of angle ORQ.

5). The diagram shows a circle with centre *O*.

Work out the size of the angle marked *x*.



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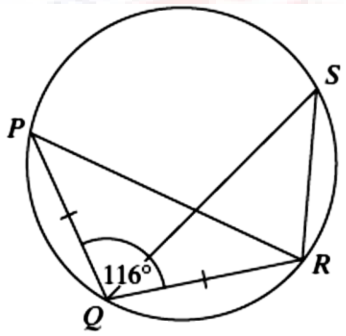
6). Points *P*, *Q*, *R* and *S* lie on a circle. *PQ* = *QR Angle PQR* = 116°



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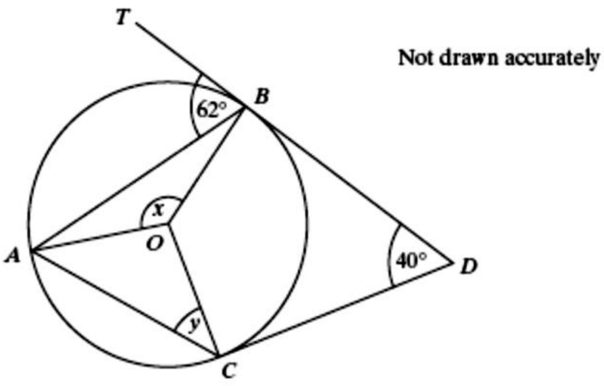
Find the value of angle *QSR*.

7). The tangent *DB* is extended to *T*.

The line *AO* is added to the diagram. Angle *TBA* = 62°

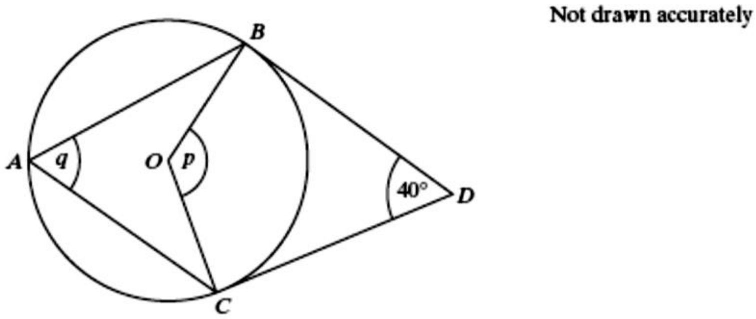


(i) Work out the value of x.

(ii) Work out the value of y.

8). A, B and C are points on the circumference of a circle with centre O.

BD and CD are tangents. Angle $BDC = 40°$



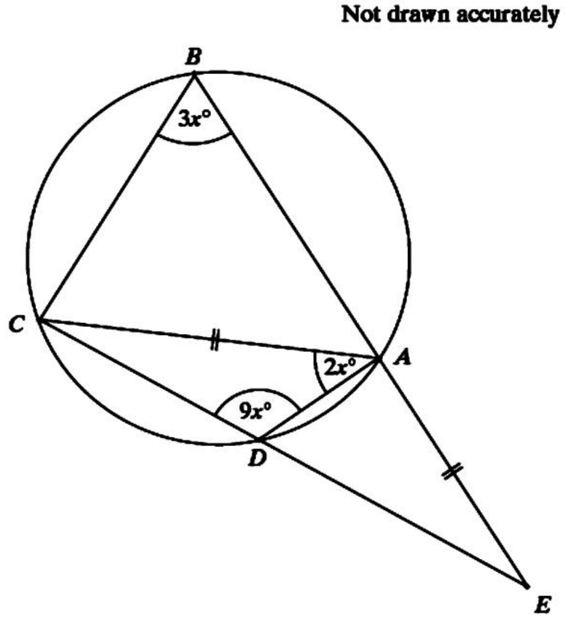
(i) Work out the value of p.

(ii) Hence write down the value of q.

9). The diagram shows a cyclic quadrilateral $ABCD$.

The straight lines BA and CD are extended and meet at E.

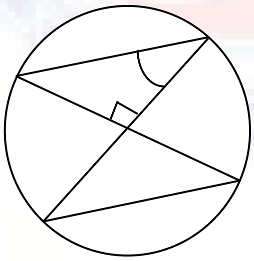
$EA = AC$ Angle $ABC = 3x°$ Angle $ADC = 9x°$ Angle $DAC = 2x°$



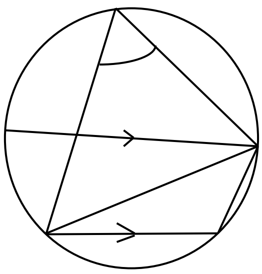
(i) Show that $x = 15$

(ii) Calculate the size of angle *EAD*.

10).In the diagram O is the centre of the circle. Find the value of angle RPS.



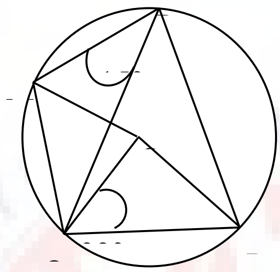
11). In the diagram, LM is a chord parallel to the diameter KN of the circle KLMNP. If the angle NPL = 60°, calculate the value of the angle MLN.



12). In the diagram, O is the centre of the circle. Angle OQR = 32° and angle MPQ = 15°. Calculate the values of

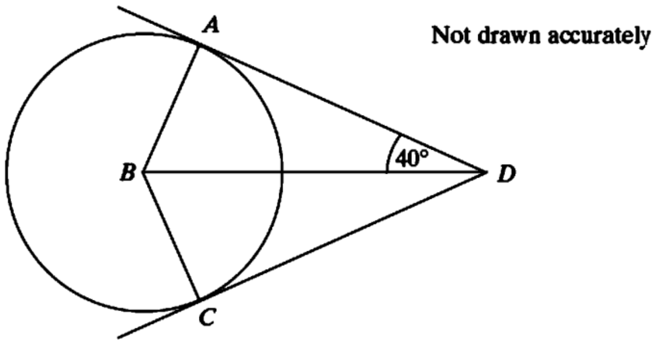
(i) Angle QPR

(ii) Angle PQO



13). *A* and *C* are points on the circumference of a circle centre *B*.

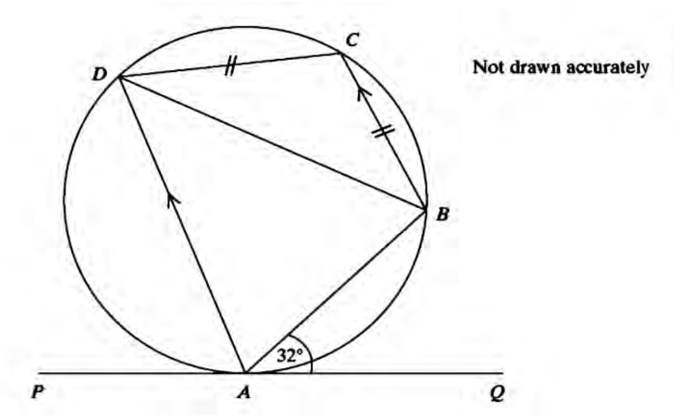
AD and *CD* are tangents. Angle *ADB* = 40°.



Find the value of angle *ABC*.

14). *ABCD* is a cyclic quadrilateral. *PAQ* is a tangent to the circle at *A*.

BC = *CD*. *AD* is parallel to *BC*. Angle *BAQ* = 32°.



Find the size of angle *BAD*.

Thank you very much for your help.