UNIVERSITY OF EDUCATION, WINNEBA

THE SPATIAL ABILITY EFFECT ON PRE-SERVICE TEACHERS' BASIC GEOMETRY CONTENT KNOWLEDGE IN RELATION TO VERBAL REASONING

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DECLARATION

Student's Declaration

I, Peter Akayuure, declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

Signature: ……………………..…………….

Date: …………………………..…………….

Supervisors' Declaration

This dissertation has been read and approved as meeting the requirements of the School of Graduate Studies, University of Education, Winneba.

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DEDICATION

This thesis is dedicated to my MUM and DAD,

MR AND MRS. AKAYUURE AKUYELE ABULE

for inculcating in me commitment to God, love for mankind and positive study attitudes.

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I am most thankful to GOD ALMIGHTY, for providing me with the aptitude, wellbeing and opportunity to complete this thesis. In addition, this thesis might not have been accomplished without the support and interest shown by many persons:

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LIST OF ABBREVIATIONS

Con:	Conditional Knowledge in Geometry
Dec:	Declarative Knowledge in Geometry
KTG:	Basic Geometry content knowledge
MGCFA:	Multi-group Confirmatory Factor Analysis
MSEM:	Multi-group Structural Equation Modeling
Pro:	Procedural Knowledge in Geometry
SA:	Spatial Ability
SEM:	Structural Equation Modeling
Sos:	Mental Rotation
Sp:	Spatial Perception
Svs:	Spatial Visualization
Sy:	Nonsense Syllogism
Vc:	Verbal Comprehension
VR:	Verbal Reasoning Ability

ABSTRACT

Research in cognitive psychology suggests that cognitive factors such as spatial abilities and verbal reasoning are important in developing geometric knowledge of students in elementary school. However, these cognitive abilities are less explored in teacher education. The study adopted cross-sectional survey design to investigate relationships among knowledge types in geometry, and how spatial ability and verbal reasoning relate to and account for knowledge in basic shape and space among pre-service teachers. A sample of 757 pre-service teachers from 12 public colleges of education in Ghana participated in the study. Cognitive tests on spatial ability, verbal reasoning and basic Geometry content knowledge were validated for the main data collection. The instrument was found to be reliable and achieved measurement invariant across gender and programmes offered by pre-service teachers. Descriptive statistics and structural equation modeling were employed to analyze and estimate model parameters to answer the research questions and hypotheses. Bootstrapping was conducted to determine significance of effect sizes in the relationship paths connecting spatial ability, verbal reasoning and basic Geometry content knowledge. The study found that pre-service teachers performed better in procedural and declarative tasks than in conditional tasks. Pre-service teachers also did better in 2-D tasks than in 3-D tasks. The declarative, conditional and procedural knowledge were found to be significantly interrelated in a way that defines pre-service teachers" content knowledge. From the structural equations, pre-service teachers" spatial ability significantly affected their basic Geometry content knowledge. Verbal reasoning was also found to intermediate the effect of spatial ability on basic Geometry content knowledge. Finally, the results of multigroup structural equation modeling show that the relationship among spatial ability, verbal reasoning and basic Geometry content knowledge was structurally invariant across programme of study but moderated by gender of pre-service teachers. Implications for the design of curriculum, teaching, learning and assessment tasks in basic geometry at colleges of education were discussed.

CHAPTER ONE

INTRODUCTION

1.1. Overview

In this chapter, Background of the Study, Statement of the Problem, Purpose of the Study, Research Objectives, Research Questions, Research Hypotheses, Significance of the Study and Delimitations have been discussed according to sections. In the background of the study section, concerns regarding the trends of students‟ knowledge deficiencies, historical lapses in geometry curriculum structure, instructional issues and performance problems have been discussed. The issues of spatial competencies and verbal reasoning influencing geometry learning in the contexts of teacher education have also been highlighted. In the statement of the problem section, the neglect of spatial ability development in pre-service teachers‟ geometry knowledge for teaching, the research gap and the likely consequences in the teaching of spatial geometry at the basic schools in Ghana were discussed. Based on the argument in literature that spatial factor and geometric knowledge were related, the purpose, objectives and hypotheses of the study were formulated, and a model constructed for structural equation modelling. The significance of the study in terms of pre-service teachers‟ content Basic Geometry content knowledge, the epistemology of geometry and spatial courses at colleges of education in Ghana were assumed pending analysis of data and findings of the study.

1.2. Background of the Study

Geometry is one of the domains of mathematics which developed from natural outgrowth of human exposure to cultural artifacts and spatial elements of the world. With its roots from the ancient Greece (Cofie, 2011; Hayden, 1981), Geometry was part of medieval subjects appreciated by early mathematicians as channel for improving algebraic abstraction and deductive reasoning. In modern times, Geometry content occupies a large proportion of school mathematics curriculum and serves the utilitarian role of advancing present-day computer technologies, engineering, architecture, physics, astronomy, art, chemistry, biology, geology and mathematics education (Gunhan, 2014; Arici & Aslan-Tutak, 2013; Golan, 2011; Mullis, Martin & Foy, 2008). Knowledge of geometry is a prerequisite for studying mathematics disciplines because it is linked to the growth of students‟ mathematical competencies and problem-solving abilities (Luneta, 2015). Universally, mathematics curriculum demands that students completing school geometry should be able to develop conceptual and procedural knowledge to analyze the characteristics and relationships of two-three dimensional shapes, describe spatial relationships, apply transformation and symmetric properties to analyze mathematical situations; and use visualization, spatial reasoning and geometric modelling to solve problems (Luneta, 2015). Developing this duality of knowledge has been debated as to how such mathematical knowledge develop.

In attending to these curriculum demands of geometry education, school geometry over decades has gone through series of transformations and neglect worldwide (Gogoe, 2009; Hayden, 1981). Hayden (1981) was concerned that Greece geometry was corroded by text and watered-down at the high schools when some aspects were removed because they were either irrelevant or difficult to "teach as well as to learn" (Luneta, 2015, p.1). During the New Math era in the 1960-70s, geometry was further downgraded by the then new math movement which acclaimed set theory as basis for learning mathematics. These transformations appeared to have led to geometric knowledge being ill-defined, uncoordinated or at extreme cases eliminated in curriculum documents or neglected by teachers in their classroom practices. Subsequently, there was a neglect of geometry in the colleges of education which created geometry knowledge gaps among teachers and learners across the generations between the 1960s and 2000s. In response to the global concerns that Euclidean geometry was limited to plane surface problems and hence *Euclid Must Go* (Hayden, 1981), Euclidean definitions and proofs in geometry were significantly relegated in Ghana. Geometry became a course of study only in 2005 after 2003 education review report indicated that teachers‟ mathematical knowledge for teaching was not meeting pupils‟ curriculum goal for learning mathematics. Contemporary geometers on the other hand argue for the inclusion of the earlier Euclidean and Non-Euclidean geometries which are relegated in school curriculum (Cofie, 2011). Despite these arguments, recent educators and researchers in many countries appear rather to be concerned with how teachers and students engage in developing geometry knowledge already ascribed in the school mathematics curriculum (Ministry of Education, 2012; Mullis, Martin & Foy, 2008).

Different proportions of geometry knowledge have been ascribed in mathematics curriculum from elementary through high school to college level. In the Ghanaian mathematics curriculum, Shape and Space, which was hitherto called elementary geometry, covers about 17% of the mathematics content areas at primary school and comprises definitions, properties, generalizations, procedures and their connections regarding points and lines, angles and triangles, quadrilaterals, prisms and pyramids (Ministry of Education, 2012). At the Ghanaian junior and senior high school, more than 30% of the six mathematics content areas is assigned to geometry topics such as plane geometry, mensuration, trigonometry and vectors and transformation in a plane. These content areas are intended to promote conceptual and procedural knowledge. In the primary school mathematics curriculum in particular, Shape and Space are emphasized in order to develop pupils‟ ability to "[o]rganize and use spatial relationships in two or three dimensions, particularly in solving problems" (Ministry of Education, 2007, p. iii) relating to the physical world and advance studies in science, technology, engineering and mathematics disciplines (STEM) (Akayuure, Asiedu-Addo & Alebna, 2016). Developing conceptual knowledge, procedural knowledge and spatial relationships which are at the heart of geometry curriculum however remains an unresolved problem in pedagogical literature. This makes the goal of geometry education still far-fetched.

Recent local and international studies have raised concerns regarding Ghanaian pupils‟ deficiencies in geometric knowledge and lack of spatial relations of shape and space concepts. It has particularly been observed that up to 60% of Ghanaian candidates at the junior and senior high levels lack conceptual understanding of basic shape and space, and procedural knowledge when tackling questions on circle theorems, mensuration and three-dimensional problems in pre-tertiary Mathematics (Asemani, Asiedu-Addo & Oppong, 2017; Acquah, 2011; Anamuah-Mensah, Mereku & Ghartey-Ampiah, 2008). Asemani, Asiedu-Addo and Oppong (2017) for instance, reported that 42.5% of the senior high school students in Ghana could not attain any of the five van Hieles Geometric thinking levels at all, with 33%, 22.4% and 0.5% only reaching visualization, abstraction and rigor levels of thinking in geometry.

These deficiencies were identified over a decade ago as some outgrowth of teachers‟ poor knowledge because most teachers were noted to have avoided the teaching of solids such as prisms and pyramids (Gogoe, 2009; Institute of Education, 1995). Teachers have often been blamed for failing to teach for conceptual understanding with some authorities questioning particularly the basic teachers‟ knowledge base and instructional decisions towards the effective teaching of shape and space concepts. These concerns were upheld in the 2003 education review report and led to the introduction of geometry course at the colleges of education to develop subject matter knowledge and expose pre-service teachers to the pedagogy on how to teach shape and space effectively at the basic level of education (Acquah, 2011).

However, semester by semester achievement reports pointed out the wide disconnect between pre-service teachers‟ knowledge of concepts, procedures and their relationships in geometry. Series of the chief examiners‟ reports, spanning three decades have continually portrayed that Ghanaian pre-service teachers apparently lack sound knowledge of basic space concepts (Institute of Education, 2014; 2007; 1995). The Institute of Education at the University of Cape Coast which is responsible for evaluating students‟ learning at over 46 public colleges of education in Ghana, lamented on how most pre-service teachers found it difficult to reason spatially, discriminate between pyramids and prisms, grasp the concept of order of rotational symmetry or demonstrate knowledge of hierarchical classification of quadrilaterals (Institute of Education, 2007). With these assessment reports becoming disconcerting to declaim, findings in recent empirical studies are rather corroborating previous ones. A recent study by Armah, Cofie and Okpoti (2017) confirmed findings by Gogoe (2009) that majority of pre-service teachers in Ghana struggle to visualize common shapes especially when drawn in unconventional positions. Can this visualization problem be contributor to the reported conceptual and procedural knowledge problems among pre-service teachers?

The recent findings have raised concerns about the quality of pre-service teachers" mathematical knowledge and the future of mathematics instruction in Ghana. Consequently, stakeholders have currently embarked on policies and projects aimed at transforming teacher education processes in Ghana. A project dubbed Transforming Teacher Education and Learning (T-TEL), which begun in 2014 under the sponsorship of UKAid with professional support from Cambridge Education, sought to address performance problems through innovative pedagogical approaches and curriculum revolutions ((Ministry of Education, Ghana, 2015). However, the question of which pedagogical approach could optimize the development of both conceptual and procedural knowledge remains unclear.

Aside pedagogies, pre-service teachers" personal factors which affect geometric performance have been suggested (Adolphus, 2011). Apart from affective factors such as attitudes and interest, cognitive psychologists have identified low spatial ability and poor verbal reasoning as significant variables affecting knowledge types existing in the long-term memory (Cevirgen, 2012; Edwards, Figueras, Mellanby & Langdon, 2011). Recent studies in Ghana (Armah, Cofie & Okpoti, 2017; Akayuure, et al, 2016; Gogoe, 2009) have already connected the pre-service teachers" poor geometric knowledge to low spatial ability. Studies establishing this causal relationship however remains unclear, inconclusive and quite rare. Till date, questions relating to students" lack of ability to comprehend and tackle questions on common shapes and spatial concepts in Ghana remain only suggestive (Institute of Education, 2014) and hence open to enquiry. This present study argues that the performance problems among pre-service teachers stem from the interplay between primitive cognitive abilities (spatial and verbal abilities) and different knowledge types in geometry. Investigating such an interplay is therefore crucial in understanding the causes of pre-service teachers‟ knowledge gaps in geometry for pedagogical implications.

Regardless of limited studies, cognitive science theories such as Gardner‟s (1983) multiple intelligence theory acknowledged how limited spatial ability affects the learning of various mathematics domains. Cevirgen (2012) observed that spatial ability is the basis for declarative, conditional and procedural knowledge construction in geometry. Spatial ability involves thoughts of visual stimuli and mental manipulation of two- and three-dimensional spaces. Such thoughts when generated, retained, retrieved and transformed, represent figural contexts or visual information which form foundation for formal geometric thought. This suggests that teachers‟ knowledge of figural contexts in geometry may be accounted for by their spatial ability.

Some specific studies argued that lower spatial thinking ability correlates positively with poor geometric reasoning and achievement at high school levels (Riastuti, Mardiyana & Pramudya, 2017; Cakmak, Isiksal, & Koc, 2014; Arici & Aslan-Tutak, 2013; Konyalioglu, Aksu & Penel, 2012). Arici and Aslan-Tutak (2013) identified that spatial visualization, perception and orientation affect students‟ geometric reasoning and problem solving. Konyalioglu, Aksu and Penel (2012) have also indicated that the difficulties that students face in understanding and solving geometric problems emanate from poor visuo-spatial skills. According to Riastuti, Mardiyana and Pramudya (2017), students with difficulties in analyzing, interpreting and understanding what they see and hear mostly fail in processing solutions to geometric problems. In all these studies, little is known about the constituents of geometric knowledge of the Ghanaian pre-service teachers and how spatial ability constrains the development of the knowledge types in geometry. Also, while the effect of spatial ability on geometric knowledge may have been investigated somehow (Riastuti, Mardiyana & Pramudya, 2017; Cakmak, Isiksal, & Koc, 2014; Arici & Aslan-Tutak, 2013; Cevirgen, 2012), studies on the effect of geometric knowledge on spatial ability are rare in literature and nonexistent in Ghana.

Learning constrains in the linguistic context of geometry have also been acknowledged in the development of spatial reasoning when considering conceptual and procedural knowledge in geometry (Institute of Education, 2007). Geometry by nature is the language of figural relationships, logics, axioms and proofs deduced from defined and undefined terms (Fijuta, Jones & Yamamoto, 2004). The word triangle for instance, is a defined term in geometry with spatial connotation which refers to a plane of tri-angles. However, some geometric terms hold certain everyday meanings and representations which differ from geometric considerations. For instance, a point is conventionally and spatially represented by a dot on a paper. However, conceptually, a point is never a dot in Euclidean Geometry since it has no length, no breadth and no size (Cofie, 2011). The conflict between daily meaning and geometric definition shows how logical reasoning in language described in literature as verbal reasoning, can be key to students‟ success in geometry. This conflict poses difficulties in developing logical proofs, in moving across the van Hieles Geometric thinking levels and in developing conceptual and procedural knowledge for solving geometric story problems (Mullis, Martin & Foy, 2008). These difficulties breed misconceptions that limit students‟ ability to grasp concepts meaningfully beyond noticing the geometric figures (Battista, 1990). Consequently, the development of conceptual and procedural knowledge is constrained (Cervigin, 2012). While these pieces of evidence have been portrayed in literature, the degree of relationship between verbal reasoning and geometry knowledge remains questionable in the context of teacher education.

1.3. Statement of the Problem

One of the aims of pre-service teachers" education in Ghana is to enable them develop sound knowledge and ability in mathematics and geometry to teach effectively upon completion (Institute of Education, 2014). Regrettably however, studies in Ghana between 2009 and 2017 have questioned the quality of pre-service teachers" knowledge for teaching elementary geometry (Asemani, Asiedu-Addo & Oppong, 2017; Gogoe, 2009). Currently, many Ghanaian pre-service teachers are apparently, struggling to completely reach the foundation levels of geometric reasoning which are visualization and abstraction (Armah, Cofie & Okpoti, 2017). Majority are reportedly having problems in interpreting the definition of a square, a rhombus and a rectangle based on symmetry property. This phenomenon is of great concern since the level of pre-service teachers" geometric knowledge significantly determines their future success in teaching elementary geometry and mathematics (Unal, Jakubowski & Corey, 2009). Pupils who would be taught by these pre-service teachers are likely to develop superfluous understanding of geometry and negative attitudes in disciplines relating to mathematics and geometry.

This study thus questions why pre-service teachers who have progressively learnt basic and higher-level geometries, from primary through junior high to senior high schools, should struggle to visualize and abstract properties of shape and space at college level. What causal factors relate to pre-service teachers" inability to deal with basic ideas like symmetry, congruence, order of rotation or surface area of a pyramid? Several factors may explain this problem. However, critical examination appears to point at limited reasoning with spatial concepts. This study is therefore focused on the theoretical outlook that pre-service teachers‟ spatial reasoning ability is a major influential factor in understanding geometry. Clements and Sarama (2011) emphasized the theoretical relationship between spatial ability and geometry knowledge. However, the nature of such relationship remains empirically debatable as some researchers view spatial ability as just a blind-spot (National Research Council, 2006). In Ghana, little is known about how spatial abilities relate to basic Geometry content knowledge or how much they account for variance in geometric knowledge of learners. Currently, no formal measurement procedure exists for measuring pre-service teachers‟ spatial ability and how they relate to geometric achievement.

Several studies in spatial ability and knowledge in geometry have argued contentiously about differences in gender, discipline-specifics and age groups of students (Baki, Kosa & Guven, 2011; Hegarty & Waller, 2005). A large proportion of these studies concentrated on students at the basic, middle and secondary levels. There has been some neglect of spatial reasoning issues in the definition, development and measurement of pre-service teachers‟ basic Geometry content knowledge. The few studies that exist elsewhere also failed to explain any direct and indirect effects or causal relationships among spatial factors and geometry knowledge types and how gender, specific discipline or age groupings differentiate such relationships. Because spatial ability is one cognitive aspect with gender gap favoring males (Baki, Kosa &Guven, 2011; Linn & Petersen, 1985), it may be one possible reason for female pre-service teachers‟ underperformance and inability to teach shape and space. Thus, the argument that learners‟ gender and academic discipline could moderate the effect of spatial ability on geometry performance (Bruce, Flynn & Moss, 2016; Baki, Kosa & Guven, 2011) remains an issue for enquiry in the contexts of teacher education.

Some studies have often raised the issue of likely interference or confounding effect of verbal reasoning on geometric knowledge and spatial abilities (Haciomeroglu, 2016; Fredua-Kwarteng & Ahia, 2014; Haj-Yahya & Hershkowitz, 2013). Haj-Yahya and Hershkowitz (2013) underscored that knowledge, visual perception and verbal-logical arguments interact to affect students‟ responses in geometry. Within the sub-Saharan Africa, verbal apprehension like spatial ability, is yet to be considered as antecedent or mediating factor affecting mathematical or geometric knowledge of pre-service teachers (Fredua-Kwarteng & Ahia, 2014). Nonetheless, teachers‟ verbal reasoning in terms of mathematical text comprehension, communication, logical deductive reasoning and processing of textual information, is fundamental for their success in geometric representations and teaching. With recent reports that Ghanaian pre-service teachers are having difficulties in reasoning in text and in comprehending questions in geometry (Akayuure, et al, 2016), investigating how verbal reasoning separately accounts for or mediates spatial ability effect on geometric knowledge for teaching is warranted.

1.4. Purpose of the Study

The purpose of the study was two-fold: first, to investigate how knowledge types in geometry relate to and account for each other and second, to determine how spatial ability directly or indirectly via verbal reasoning relate to and accounts for Basic Geometry content knowledge.

1.5. Objectives of the Study

In relation to the purpose of the study, specific research objectives were formulated to guide the study. These included to find out:

1. (a) how pre-service teachers perform in tests involving
   (i) spatial ability,
   (ii) verbal reasoning and
   (iii) basic geometry content knowledge;

   (b) how pre-service teachers" knowledge types in geometry affect each other.
2. if there is significant effect of spatial ability on basic Geometry content knowledge;
3. how verbal reasoning mediates the effect of spatial ability on the acquisition of basic Geometry content knowledge;
4. if the relationship among spatial ability, verbal reasoning and basic Geometry content knowledge differ by (i) gender and by (ii) programme of study.

1.6. Research Questions

In line with the objectives of the study, the following research questions were formulated to guide the study.

1. (a) How do pre-service teachers perform in tests involving spatial ability, verbal reasoning and knowledge types for teaching geometry?

   (b) How do knowledge types in geometry affect each other?
2. How do spatial ability and basic Geometry content knowledge relate to each other?

3. To what extent does verbal reasoning mediate the effect of spatial ability on basic Geometry content knowledge?
4. Do the relationships among spatial ability, verbal reasoning and knowledge in geometry differ by (i) gender and (ii) programme of study?

1.7. Research Hypothesis

The main null hypotheses formulated for the study were:

H01: There is no relationship among spatial ability, verbal reasoning and basic Geometry content knowledge of pre-service teachers.

H02: The structural model describing the relationship of spatial ability, verbal reasoning and basic Geometry content knowledge of pre-service teachers is invariant across gender and programme of study.

In addition to the above main research hypotheses, specific hypotheses were derived in Chapter Three.

1.8. Significance of the Study

Literature review on mathematics teacher knowledge revealed that investigations into cognitive factor models that influence domain-specific knowledge acquisition have not yet been exhaustive. Geometry is a unique domain in mathematics with specific cognitive factors. Thus, in developing pre-service teachers" geometric knowledge for teaching, there is need to understand how cognitive factors such as spatial ability and verbal reasoning directly or indirectly affects their ability to learn and teach concepts. Such knowledge can inform mathematics educators, curriculum developers, assessment developers and other stakeholders in the field of teacher education in addressing learning and performance issues.

Part of the motivation for this present study is grounded on the acknowledgement by Wai, Lubinski and Benbow (2009) that spatial competencies are key for the acquisition and application mathematical knowledge. After 70+ years when the report by Super and Bechrach identified the critical role that spatial ability plays in STEM education, relatively little application of spatial ability is found in curriculum and instruction in educational settings (Moss, Bruce, Caswell, Flynn & Hawes, 2016). Yet, for classroom instruction, Erkek, Işıksal and Çakıroğlu (2017) described teachers‟ spatial knowledge as important resource.

To effectively apply spatial components of geometry in the classroom, teachers themselves need to possess appropriate spatial abilities that will enable them teach shape and space geometry. Surprisingly, there has not been explicit effort to investigate or measure the spatial competency of pre-service teachers at the college of education in Ghana. In line with the current efforts by the government of Ghana, educators and stakeholders to transform teacher education and learning processes, the outcome of this study could significantly influence how pre-service teachers learn geometric concepts and solve problems related to spatial geometry. The outcome of the study could clarify or define a framework to guide geometry teaching and assessment of spatial competences of pre-service teachers to ascertain their ability to teach shape and space content as the basic schools.

There is currently a public outcry about the mathematical reasoning levels of students in Ghana. Özdemir and Göktepe-Yildiz (2014) showed that increases in students‟ spatial skills results in increases in their geometric reasoning skills. Since Geometry occupies a large proportion of mathematics curriculum and serves as a prerequisite for learning advanced mathematics (Šipuš & Cižmešija, 2012), developing pre-service teachers‟ geometric reasoning should have positive impacts on their mathematical thinking and achievement. Therefore, the outcome of the study was anticipated to have significant impact on pre-service teachers" mathematical knowledge for teaching and by extension improved performance among pupils in Ghana.

It has been proven that spatial thinking and verbal reasoning can be invoked through model-based instructions (Septia, Prahmana, Pebrianto, Wahyu, 2018; Akayuure, et al, 2016; Boakes, 2009). This present study could serve as motivation for tutors to explore new and creative ways of improving spatial-verbal reasoning skills of pre-service teachers at the colleges of education in Ghana. This would go a long way to influence basic school teachers" practices towards the development of fundamental aspects of mathematics learning such as spatial, verbal and geometric thinking skills among pupils. This has the potential of breaking the cycle of poor geometric knowledge and mathematics achievement problems currently reported at pre-tertiary levels of education in Ghana.

Research on spatial ability and geometric knowledge has been conducted elsewhere in recent times (Bruce, Flynn & Moss, 2016; Özdemir & Göktepe Yildiz, 2014; Sipus & Cizmesija, 2012; Baki, Kosa & Guven, 2011). However, a review of literature suggested that these studies did not examine jointly, the contribution of spatial thinking and verbal reasoning to geometry learning regarding pre-service teachers. The finding of the study would therefore fill the current knowledge gap on the direct and indirect effects of pre-service teachers" spatial ability and verbal reasoning on their learning of geometry.

Also, a review of literature suggested that most geometry studies have not considered explicitly various knowledge types such as declarative, conditional and procedural knowledge as theorized in Alexander and Judy (1988) and Smith and

Ragon (2005). Often, geometry knowledge is taken as a uni-dimensional construct (Cevirgen, 2012). Arguably, this does not pinpoint the source of performance problems to allow for appropriate prescription of interventions needed to address such learning problems. In this study, pre-service teachers" geometry knowledge is viewed in the lens of Alexander and Judy (1988). The interactions among these knowledge types and the role one plays towards the development of the other are also examined with respect to the structure of measurement errors and effect sizes. The outcome of such investigation would clarify the developmental precedence of geometric knowledge types and help in the choice of instructional tasks and construction of items for learning and assessment.

Finally, the Ghanaian mathematics curriculum requires that basic school teachers support pupils to develop both verbal and spatial thinking. One of the objectives states that pupils should be assisted (by teachers) to "[o]rganize and use spatial relationships . . . in solving problems" (Ministry of Education, 2012, iv). It is unlikely that a teacher can effectively provide her/his pupils with spatial learning opportunities if s/he has limited spatial abilities (Erkek, Işıksal & Çakıroğlu, 2017). Therefore, to be able to implement the above objective, the pre-service teachers who were being prepared to teach at the basic school should themselves be led to acquire such spatial experience and competencies. Spatial skills development is however currently absent at the colleges of education in Ghana. Thus, the outcome of this study could help curriculum developers, educators and teachers to consider incorporating spatial thinking and verbal reasoning activities in geometry and mathematics education. Such consideration might include the introduction of spatial-verbal reasoning courses into the basic teacher training and learning programmes at the colleges of education in Ghana. This might lead to the emphasis on spatial reasoning at the basic school level in lieu of the current curriculum and instructional practices.

1.9. Delimitation

The current trend of poor geometric achievement, which might largely be attributed to teacher factor and knowledge framework, seem to be affecting all levels of education in Ghana. To address this, it is believed that transforming pre-service teachers‟ knowledge for teaching can ensure positive change. Therefore, the study was limited to pre-service teachers for basic schools in Ghana.

Shape and space forms a key content area in the basic school mathematics curriculum in Ghana. This supposes that those who are trained as teachers should have full grasp of the content to be able to teach effectively. This study was a thoughtful spotlight on what knowledge pre-service teachers gained in shape and space in college geometry course for their teaching practicum at the basic schools in Ghana. Even though knowledge for teaching undergirded the study, the emphasis was on content knowledge for teaching shape and space. Other aspects of teacher knowledge comprising of pedagogical and curriculum knowledge were excluded to narrow the scope of investigation for better interpretation. Finally, the study was constrained to cognitive factors affect pre-service teachers‟ knowledge acquisition in shape and space. Thus, relationships among content knowledge types and between spatial ability and verbal reasoning were the focus for the study.

1.10. Organization of the Study

This thesis consists of five major chapters. Chapter One provides an overview of geometry education and spatial ability with emphasis on pre-service teachers at the colleges of education in Ghana. Chapter Two begins with a brief theoretical overview of relation between geometry and spatial ability from the Piaget"s theory of spatial development. This is followed by the description of theoretical constructs including the conceptualization of Basic Geometry content knowledge, spatial ability and verbal reasoning. The chapter then reviews existing literature on the relationship between geometric knowledge, spatial ability and verbal reasoning. The last section presents the conceptualization of the study and a summary of the chapter. Chapter Three provides an overview to the methodological framework used for the study. The chapter is devoted to detailed discussion of the structural equation modelling, data collection and analysis in the study. Chapter Four provides a detailed description and interpretation of the analysis of data. The chapter describes participants" performance on tasks eliciting spatial abilities, verbal reasoning and knowledge types for teaching shape and space. The chapter then evaluates SEM assumptions, measurement and structural models, and the effect of spatial ability on basic Geometry content knowledge mediated by verbal reasoning and moderated by gender. Finally, major findings are discussed. Chapter Five contains a summary and conclusions of the study. The chapter highlights the implications for the study as well as provides recommendations for teaching, learning, assessment and further research areas. Finally, some limitations are acknowledged.

1.11. Definitions of Terminologies

- Basic Geometry content knowledge: Knowledge on shape and space comprising angles, triangles, quadrilaterals, prisms and pyramid as enshrined in Ghanaian basic school mathematics curriculum
- Comprehension: The ability to understand textual information in context to geometry

- Conditional knowledge: Knowing "why", "when" and "where" to access geometric facts, definitions, properties, generalizations and employs a procedure.
- Declarative knowledge: Knowing facts, generalizations, theories, and truths about the world events. It is also referred to as knowing "what".
- Mental rotation: The cognitive ability to rotate objects in ones‟ own mind.
- Procedural knowledge: Knowing "how to" apply the rules and principles, recall steps to perform tasks correctly and confirm result in geometry.
- Spatial ability: The ability to visualize objects, perceive mental images and reasoning about spatial relations of objects.
- Spatial perception: The ability to determine spatial relationships with respect to the orientation of one‟s own body, in spite of distracting information.
- Spatial visualization: The ability to manipulate a visual image in two- and three-dimensional spaces.
- Syllogistic reasoning: The ability to thinking logically and draw valid inferences regarding some conclusions from given premises or logical statement.
- Verbal comprehension: The ability to reason and comprehend textual information.
- Verbal reasoning: The ability to analyze information and solve problems using language-based reasoning involving reading, listening, conversing, writing and thinking.

CHAPTER TWO
LITERATURE REVIEW

2.1. Overview

This chapter presents the theoretical background and reviews literature on the main variables under study. While several theoretical foundations exist in geometry studies, no specific framework exists in literature to define geometric content knowledge for teaching. Therefore, the relationship between the theory of knowledge types (Alexander & Judy, 1988), the theory of spatial ability (Linn & Petersen, 1985) and verbal reasoning (Gardner, 1983) underpinned the study. The relationship among these theories provided grounds for investigating content Basic Geometry content knowledge, how spatial ability relates to basic Geometry content knowledge and how verbal reasoning mediates such relationship among pre-service teachers in Ghanaian colleges of education.

2.2. Theoretical Foundation for the Study

This study conceptualized the relationship between spatial ability and geometric knowledge within the theory of spatial development (Piaget & Inhelder, 1967) reinforced by geometric models of the Fischbein (1993), Duval (1988) and van Hieles (1986). Piaget and Inhelder expounded a theory about the development of spatial sense which theorized that the representational space that develops first in learners is topological in nature as opposed to Euclidean. These topological properties are characterized by proximity, separation, order and continuity. Jean Piaget (1896–1980) was a Swiss psychologist whose work was significant during the 1940s when geometry was downgraded to a point that it lost its status in the field of mathematics. His earlier investigation established that children‟s conception of space was the foundation of geometric intuition. In separate landmark experiments, Piaget with his team members theorized that spatial or topological conceptions preceded geometric thought (George, 2017). The theory emphasizes that the development of geometric knowledge progresses through four spatial stages. These stages are enshrined in ones‟ perceptual, mental representational and conceptual spaces (Piaget & Inhelder, 1967). This theory supports the notion that physical manipulative activities promote not only perceptual abilities but more importantly imaginational, representational and conceptual space developments. The implication of Piaget‟s theory of spatial development led to manipulatives been recommended for instructional engagement at elementary school level. However, it is often argued that such activities are not necessary when engaging adults to learn college geometry. Nevertheless, research continue to show that spatial sense guarantees the understanding of topological, projective and Euclidean relationships such as similarity, parallelism and distance. Consequently, limited spatial sense affects the learning of geometry among adults as among children. Even though Piaget‟s theory of spatial development related to children between the ages of 2 to 12, the theory is applicable to adults learning geometry since according to van Hieles, geometric thinking progresses through levels rather than chronological age. Thus, it can be conceived that college level learners who do not achieve the foundational level of visualization could struggle to progress to level two in their geometric thinking regardless of their ages.

The importance of spatial reasoning in the learning of geometry is demonstrated right from the level one of the van Hieles‟ theory. Sipus and Cismesija (2012) applied the van Hieles theory in studying the spatial abilities of students of mathematics education. The theory distinguishes five hierarchical developmental levels of students‟ understanding of spatial ideas (Sipus & Cizmesija, 2012). These include visualization, analysis, informal deduction, formal deduction to rigor (George, 2017; Vojkuvkova, 2012). At the visualization level, the student recognizes spatial components such as the size and orientation of shapes, but specific properties may not be identified. At the analysis level, the students proceed to use vocabulary relating to geometric properties and extend their spatial notion of size and orientation to specific properties of a shape. At the informal deduction level, the student recognizes and reasons about spatial relationships between shapes. At the formal deduction level 4, the student understands deduction, postulates, theorems, and proofs. At the final level of rigor, the student understands and can work with axiomatic systems in geometry. It can therefore be admitted that the first three level of geometric conceptions depend largely on individuals‟ spatial sense. Pre-service teachers are hence supposed to operate at least to level three in order to teach shape and space effectively.

The relationship between spatial sense and geometric knowledge has also been postulated in Duval‟s theory of geometric thought (Duval, 1988). According to Duval, geometric learning involves three recursive cognitive processes of visualization, construction and reasoning. Like Piaget and the van Hieles, Duval recognizes linguistic systems, visual representations and even gestures as basis for geometric construction and reasoning. Duval has further theorized a reciprocal relationship between visualization and geometric construction as well as between geometric reasoning. Portraying the importance of spatial visualization, Duval emphasized that the meteorological decomposition of the whole shape into parts to reconstruct another figure ensure the discovery of geometrical properties.

Fischbein in his theory of figural concepts, acknowledges that without spatial images, geometry would not exist as a branch of mathematics (Fischbein, 1993). Fischbein therefore emphasizes imagery as building blocks for geometric knowledge construction. This supposes that if pre-service teachers have limited spatial imagery, they might not be able teach geometry effectively.

In all the theoretical frameworks reviewed, spatial ability assumes theoretical relation with geometry knowledge. However, there are also arguments that students can understand geometry without visualization. Haj-Yahya and Hershkowitz (2013) for example, established that students in grade 10 understood and explained inclusive quadrilateral relationships based on verbal definitions without any visuals. Therefore, in examining learning and performance problems in geometry among pre-service teachers, it might be inconceivable to neglect the causal effect of spatial factors. However, this theoretical relation has not been investigated in the context of pre-service teachers in Ghana. This study, therefore, hypothesized causal relation between spatial ability and geometric knowledge and investigated how pre-service teachers" spatial ability affects the development of their geometric knowledge for teaching.

2.3. Basic Geometry Content Knowledge

This study argued that effective instruction in geometry requires teachers with sound content knowledge of concepts and procedures as well as spatial abilities. Drawing from the theoretical foundation of knowledge for teaching propounded by Shulman"s (1986), this study considered content knowledge of geometric concepts and procedures as the central knowledge needed by pre-service teachers for effective teaching upon graduation. As a result, knowledge of pedagogy, curriculum and students are only utilized if the teacher possesses adequate content knowledge. Shulman defines content knowledge of the teacher as the knowledge of the structure and methods of discourse which encompasses knowledge of concepts, theories, ideas, proofs and evidences as well as practices and approaches to develop this knowledge (Fernandez, 2014). Such content knowledge is a core part of the knowledge base of teachers and when not properly developed and well-coordinated in the memory can affect teachers‟ classroom practices. Future or pre-service teachers should therefore be made to develop or acquire this type of content knowledge for teaching. Despite this, research into how these constituents of knowledge develop or are acquired by pre-service teachers is quite rare. Little is known about whether limited conceptual knowledge shapes procedural knowledge in geometry or vice versa. Understanding this might help in choosing suitable pedagogical interventions to address geometric performance problems and improve the knowledge base of pre-service teachers.

Cervigin (2012) argues that acquiring geometric content knowledge is often influenced by cognitive factors such as spatial ability. The cognitive perspective on learning is based on the assumptions that knowledge of concepts, procedures and conditions must be well-organized in an individual‟s long-term memory (Schneider & Stein, 2010). As human learning is multi-faceted, knowledge acquisition is dependent on the individual‟s cognition and the interaction effects of concepts, procedures and abilities stored in the declarative and procedural memories. Consequently, if these knowledge aspects are structured in adverse ways, it can affect the knowledge growth and the quality of performance of pre-service teachers.

In recent literature, very rare empirical work exists on the geometric knowledge of pre-service teachers from the spatial-verbal perspective. However, cognitive theorists such as Piaget and Gardner have theorized the critical role of spatial and verbal reasoning abilities in learning aspects of mathematics such as geometry (Cevirgen, 2012). These abilities make up part of the main cognitive abilities of human beings. In his psychological study, Carroll (1993) examined the mathematical, spatial and verbal abilities of students and reported of their shared commonality which theorizes their relationships. This study was framed within this relationship theory where the mathematical knowledge is narrowed to the field of geometry so that the main variables considered include geometric knowledge, spatial ability and verbal reasoning. The study focused on examining a relationship model in which spatial and verbal reasoning abilities are hypothesized to contribute directly or indirectly to an interconnected geometric knowledge of pre-service teachers. Furthermore, it is regarded that the quality of pre-service teachers‟ ability to teach geometry of shape and space depends on the interplay between these three aspects. In reviews of literature on teachers‟ geometric knowledge for teaching, there is currently void to this argument and as a sequel such knowledge remain undefined and less understood.

2.4. Knowledge Types in Geometry

Following Shulman‟s model of teacher knowledge, several related mathematical knowledge frameworks have been proposed and examined (Ball, Thames, & Phelps, 2008; Gogoe, 2009; van der Sandt & Nieuwoudt, 2003; Blanco, 2001). However, the framing of pre-service teachers‟ mathematical knowledge for teaching has since remained tentative and continuing. A critical analysis of various teacher knowledge frameworks in mathematics revealed that the frameworks have largely focused on describing teachers‟ knowledge for teaching in broad terms rather than on how pre-service teachers develop the content knowledge for teaching. No study has empirically investigated the developmental precedence of different facets of pre-service teachers‟ geometric knowledge with the aim to understand learning constraints and pedagogical interventions. For example, in developing pre-service teachers‟ Basic Geometry content knowledge, it remains unclear whether conceptual knowledge influences the development of procedural knowledge or the vice versa.

Nevertheless, these knowledge types are most prevalent in the development of various aspects of mathematics such as geometry and cannot be ignored.

To fill this gap, the present study adopted the developmental knowledge framework involving declarative (knowing that), conditional (knowing why) and procedural knowledge (knowing how to) to understand learning problems of pre-service teachers in geometry (Alexander & Judy, 1988). In the perspective of modern-day cognitive science, understanding the developmental precedence of declarative, procedural and conditional knowledge types is key to providing instructional interventions (Schneider & Stein, 2010). These fragments of knowledge can be investigated in isolated or interrelated, context-bound or context-general manner (Cevirgen, 2012; Schneider & Stein, 2010; Alexander & Judy, 1988). The present study assumed the interrelatedness of the declarative, conditional and procedural knowledge in the context of teaching shape and space.

2.4.1. Declarative Knowledge

Declarative knowledge is a type of conceptual knowledge (Schneider & Stein, 2010). It refers to what cognitive psychologists conventionally considered as store of facts and experiences in the long-term memory (ten Berge & van Hezewilk, 1999). A consensus in literature has been that declarative knowledge refers to “knowing-that” or “knowing what” (Yilmaz & Yalcin, 2012). In other words, it entails an open descriptive knowledge that something is the case. This form of knowledge is equated to explicit knowledge where learners are often not conscious of but can recount on demand. In delineating declarative knowledge, Schneider and Stein (2010) provides the following descriptors:

- knowledge of facts,
- knowledge of generalization,

- knowledge of theories and hypotheses,
- opinions, and
- beliefs and attitudes

In terms of geometry knowledge, the learner‟s recognition and recall of (i) geometric concepts such as triangle, polygon, etc. (ii) statement of facts/properties such as a cube has eight vertices and (iii) geometric generalizations such as $(n-2)180°$ for finding the sum of interior angle of a regular polygon of n sides, constitute declarative knowledge. To illustrate further, if a learner can classify triangles as three-sided figures, provide examples such as isosceles or scalene triangles, and discriminate triangles from quadrilaterals, then the learners is said to hold significant declarative knowledge of triangles. To elicit declarative knowledge from learners, Smith and Ragan (2005) suggested words like "what" and "which" as such knowledge relates to the remembering level of Bloom‟s Revised Learning Taxonomy. Furthermore, in assessing declarative knowledge in geometry, Cervigin (2012) highlighted four descriptors which may be elicited from learners:

- labelling and naming of geometric figures;
- statement of facts about geometric figures;
- listing of properties of figures; and
- organization of discourse involving geometric terms

Regarding the above descriptors, if the learner can label and name various forms of prisms, list or identify properties of prisms and distinctively describe prisms using appropriate terminologies, then such a learner demonstrates declarative knowledge.

Declaring one‟s knowledge requires recall, retrieve and use of prior experiences. This entails the individual‟s ability to a establish connection which ensures the integration of the new knowledge and prior knowledge as well as construction of the knowledge into well-ordered components that aid new knowledge transfer into the long-term memory. According to Anderson (2005), retrieval and use of declarative knowledge is consciously slow. This is because the learner needs to recall the specific facts in the process of thinking and replying accurately.

2.4.2. Procedural Knowledge

According to Yilmaz and Yalcin (2012), the set of "knowing how to" do or step-by-step instructions to take to accomplish certain tasks is described as procedural knowledge. According to Yilmaz and Yalcin (2012), knowing-how to involves knowledge of procedures or the possession of a skill, a trained capacity or a technique of routine competencies or critical skills. This knowledge is therefore related to doing or solving a problem. The descriptors of procedural knowledge include the learners‟ knowing how to:

- assemble knowledge of concepts, facts, theories,
- make rational predictions of rules and procedures,
- apply critical judgments of appropriate procedures to follow,
- arrive at conclusions for choice of procedures,
- decide on the best course of actions, and
- execute the actions successfully (Aydin, 2007)

In terms of geometry, this study views procedural knowledge as the knowledge of strategies, illustrations, sketches, algorithms and computational approaches needed to arrive at solutions to geometric problems. Aydin (2007) highlights four steps to be considered in applying procedural knowledge as follows:

- determine if a situation requires doing a cognitive task;
- recall the correct procedures;
- complete various stages in the procedure; and
- analyze the completed procedure (p.17)

In terms of geometric knowledge for teaching, one"s ability to provide instances of strategies and explanations required by a learner to arrive at the correct solution designates one"s procedural knowledge.

2.4.3. Conditional Knowledge

Conditional knowledge is the second component of conceptual knowledge which also captures procedural knowledge. Unlike the declarative knowledge which deals with knowing-that, learning relating to *knowing why, when and where* to access facts or employ a procedure is described as conditional knowledge (Alexander & Judy, 1988).

An individual who possesses conditional knowledge can determine the contexts and conditions under which other knowledge types are applicable in accomplishing tasks. Conditional knowledge thus entails relational rules which are networks of condition-to-action sequences (Yilmaz & Yalcin, 2012). In its explicit sense, Aydin (2007) expressed that conditional knowledge is characterized by if-then statements that suggest relations between two or more concepts in a domain. The learner anticipates the condition and subsequently recognizes the effect and action to be taken.

In geometry, conditional knowledge concerns propositions, principles, postulates, axioms, theorems and laws in geometry (Yilmaz & Yalcin, 2012). A typical example relates to the axiom of incidence in Euclidean geometry which states that *"[e]very plane contains at least three points"* (Cofie, 2011). Understanding this axiom requires knowing the relevant concepts of the situation of a plane, followed by knowledge of appropriate rules for obtaining a plane using points before finally concluding on whether in deed at least a set of three points can make a situational plane. Drawing from the above descriptions, a typical question requiring the use of conditional knowledge might be illustrated as follows:

"when does a rhombus become a square?"

In this example, the learner demonstrates conditional knowledge if s/he can state similar and dissimilar properties of both rhombus and square as well as the procedures needed to configure a rhombus into a square and vice versa. A conditional statement is then reached based on the properties and procedures.

As indicated by Aydin (2007), conditional knowledge is reached when the relevant declarative and procedural knowledge have been underlined. In this study, conditional knowledge is assessed based on the pre-service teachers" ability to provide justification, predictions and explanations to learning problems/misconceptions as well as representations of shapes and space concepts ascribed in the mathematics curricula for basic schools and colleges of education in Ghana.

To summarize, studies in geometry have usually treated geometry knowledge as a unidimensional construct with little account on its inherent knowledge compositions and configurations (Cevirgen, 2012; Gogoe, 2009). It can however be supposed that configuring geometry knowledge into separate components might help explain some underlying learning difficulties. In this study, knowledge of geometry is differentiated into three types - declarative, conditional and procedural - to investigate their relations and amount of variance accounted for in them by spatial and verbal factors. Star (2007) remarked that adopting conceptual-procedural framework can be constrained by the question of whether one is measuring knowledge type or knowledge quality. While knowledge type is limited to what is known, knowledge quality describes how well knowledge is understood in terms of its deep, superficial level or in between these two extremes. In this study, the three knowledge types are being referred to for two reasons. First, the study focused on quantify nearly what pre-service teachers might know after completing college geometry course. In line with this, conceptual knowledge (declarative and conditional) would refer to knowledge of concepts, principles and definitions including misconceptions, while procedural knowledge demarcates knowledge of procedures, including sequence of actions and algorithms used in problem solving (Star, 2007).

Second, studying knowledge types was deemed necessary as Cevirgen (2012) underscored the distinctive practical implications of differentiating and clarifying knowledge structure for teaching and learning geometry and other fields in mathematics. Analyzing the factor structure of geometry knowledge in term of declarative, conditional and procedural knowledge types would help highlight fundamental performance problems among pre-service teachers at the colleges of education in Ghana. For example, if pre-service teachers‟ conditional knowledge limits their declarative knowledge, appropriate strategy could be used to boost it instead of relying on generalized strategies of teaching.

2.5. The Theory of Spatial Ability

Spatial ability comprises a set of abilities such as perceiving, interpreting, understanding and appreciating the real world (Turgut & Yilmaz, 2012). Like Gardner (1983), Carroll (1993) described spatial ability as a higher order of fluid intelligence responsible for inferences and understanding of relationships between objects. Spatial skills are required for everyday dealings and for educational or professional successes. For everyday dealings, spatial sense is required for human beings to locate directions, read maps, perceive objects and understand different changes in environment. In mathematics, spatial manipulations, configurations and representations of situations are largely needed in geometric problem-solving situations (Akayuure, et al, 2016). In view of its critical role in national mathematics curricula, National Science Board (2010) echoed the need to emphasize spatial development as prerequisite for mathematical knowledge growth particularly at basic school level.

A review of existing literature revealed several related definitions for spatial ability. For the over 70 years of studies, spatial ability, researchers portray different lens about spatial conception. As a sequel, when referring to spatial conception terms such as visual-spatial ability, spatial skills, spatial sense, spatial reasoning, spatial knowledge are often used (Ministry of Education, 2012; National Science Board, 2010; Wai, Lubinski & Benbow, 2009). In this study, the term spatial ability is adopted as working lexicon while other terms are used interchangeably to maintain conceptual and contextual meaning in literature.

Just like its name, definitions of spatial ability have been largely similar but different in context. For example, earlier researchers such as Linn and Petersen (1985), Lohman (1994) described spatial ability with reference to mental processes of perceiving, storing, recalling, creating, editing and communicating spatial images (Turgut & Yilmaz, 2012). According to Linn and Petersen, spatial ability entails mental processes such as "representing, transforming, generating and recalling of symbolic nonlinguistic information" (p. 1482). These skills encompass understanding, manipulating, reorganizing or interpreting relationships visually as spatial ability.

After nearly a decade, Lohman (1994) reviewed Linn and Petersen"s definition and described spatial ability as "the ability to generate, retain, retrieve, and transform well-structured visual images" (p. 1000). Olkun (2003) also defined the spatial ability as the mental manipulation of objects and their parts in two dimensional and three-dimensional spaces. In his review of spatial literature, Other authors described spatial ability as the ability to recognize and mentally manipulate the spatial properties of objects and the spatial relations among objects (Prokysek & Stipek, 2016). In a bid to differentiate spatial notions from verbal notions, spatial skills comprise locating, orienting, decomposing, balancing, diagramming, symmetry, navigating, comparing, scaling and visualizing as spatial skills (Bruce, 2014).

In view of varied definitions, recent literature (Turgut & Yilmaz, 2012; Lubinski, 2010) acknowledges that the use of specific definitions depends on the researchers" interests and purpose. In this study therefore, the definition by Olkun (2003) is adopted as a working definition. Accordingly, spatial ability is operationalized for the study as an individual"s mental ability to perceive the visual world accurately and infer about the relationships between various entities. This definition is preferred in this study because it covers students" aptitude to apprehend and interpret images and mentally operate them through transformations, rotations or visual relationships (Seah, 2013; Taylor & Tenbrink, 2013). This definition also supports the factorability of spatial ability into three factors as proposed in this study. Above all, this kind of factorability of spatial ability embraces the entire scope of spatial contents enshrined in the Ghana basic school mathematics curriculum which pre-service teachers are trained to teach.

2.5.1. Factorial Structure of Spatial Ability

There are wide criticisms about the factorability of spatial ability. Since the mid-1940s when spatial factor structure became an area of study, its construct validity and underlying number of factors have not been clearly agreed upon (Erkek, Işıksal & Çakıroğlu, 2017; Hawes, Moss, Caswell, Naqvi & MacKinnon, 2017; Ramful, Lowrie & Logan, 2017). What remain consistent however, is fact that past and current factor analytic researches and meta-analyses have repetitively viewed spatial ability as multi-dimensional.

A review of core literature from the period of McGee (1979), Linn and Peterson (1985) through to van Niekerk (1995) till date, suggests that several models of sub factors have been developed and used to explain spatial ability. Further analysis also revealed that the models of spatial sub factors reflected attributes such as researchers‟ interests, levels of spatial experience, dimensional reasoning, spatial task demands among other conceptions (Turgut & Yilmaz, 2012).

Regarding the order of experience of space dimensions, spatial orientation which describes a three-dimensional experience of space was distinguished from spatial insight which describes a two-dimensional experience of space by van Niekerk (1995). According to her, an individual need to experience a three-dimensional cube before he/she may be able to describe it verbally, develop a mental image of it and finally make a two-dimensional drawing of the cube.

In terms of dimensional reasoning, McGee was the earliest to describe five sub factors of spatial skills as spatial visualization, spatial orientation, spatial perception, mental rotation and mental relations (David, 2012; Marchis, 2012). Tatre (1990) did an analysis of McGee‟s classification and found an overlap where certain activities of spatial relations linked with spatial orientation tasks. For him, spatial visualization can be factored into mental rotation and mental transformation.

With respect to spatial task demands, the theoretical paper by Lohman (1994) and the Meta-analysis conducted by Linn and Petersen (1985) reduced spatial factors from five to spatial perception, mental rotation and spatial visualization (Turgut, 2015; Uttal et al, 2013; David, 2012). According to these authors, spatial perception tasks are centered on an individual‟s recognition and understanding of spatial relationships with respect to their own body positions. Also, mental rotation tasks are a function of time and one‟s ability to accurately relate images mentally. These tasks involve the ability to imagine how objects appear when rotated in two- or three-dimensional space. Finally, spatial visualization tasks entail complex task operations requiring the use of multiple steps, multiple solution strategies of mental rotations or spatial perceptions and analytical strategy to resolve the spatial tasks. Thus, spatial visualization can be factored into mental rotation and mental transformation.

An analysis of past and present spatial literature shows that most research in spatial ability refer to spatial visualization, mental rotation and spatial perception sub-factors (George, 2017; Turgut, 2015; Turgut & Yilmaz, 2012). These authors argued that the definitions, descriptors and cognitive tests used to assess these sub factors have been widely embraced and readily available. In line with this, these three sub-factors of spatial ability are adopted for examination in this study.

2.5.2. Mental Rotation, Spatial Visualization and Spatial Perception

Mental rotation is one of the three spatial factors identified by Linn and Petersen (1985). According to Linn and Petersen (1985), mental rotation refers to a Gestalt-like process relating to the "rotation of a two- or three-dimensional figure rapidly and accurately" (p.1483). This factor measures ability to rotate figures in the mind with respect to time. In literature tasks involving mental rotation include Purdue spatial visualization test, Shepard and Metzler mental rotation test, card rotation test, cube comparison, progressive matrices and the Vandenberg tests (George, 2017; David, 2012; Nemeth, 2007). Research involving these mental rotation tasks has shown male superiority. The source of this superiority was traced to the fact that males tend to adopt holistic analogue which is less time consuming as opposed to step-by-step strategies adopted by females (George, 2017; Turgut, 2015).

The second sub factor of spatial ability identified by Linn and Petersen is the spatial perception. It involves spatial relationships "with respect to the orientation of the subject‟s own body, in spite of distracting information" (Linn & Petersen, 1985, p. 1482). Spatial perception is partially characterized by two cues. First is the reliance on gravitational/kinesthetic cues and second is the focus on dis-embedding or overcoming distracting cues. In literature, spatial perception is often used to explain spatial orientation. For example, Erkek, Işıksal and Çakıroğlu (2017) described spatial orientation as one‟s ability to perceive how one object located in space is structurally related to other objects. Bahr and de Garcia (2010) described spatial orientation as the ability to look at a fixed figure from several different points of view. In earlier studies, spatial orientation referred to the ability to imagine how a given object or set of objects would appear, from a spatial perspective, different from that in which the objects are shown. Both spatial perception and spatial orientation concern the individual‟s ability to:

- determine relationships between different spatial objects;
- recognize the identity of an object when it is seen from different angles, or when the object is moved;

- consider spatial relations where the body orientation of the observer is essential
- perceive spatial patterns and to compare them with each other;
- remain unconfused by the varying orientations in which a spatial object may be presented; and
- perceive spatial patterns or to maintain orientation with respect to objects in space.

Turgut and Yilmaz (2012) noted that spatial perception needs to be well-developed in students because it is an important part of science, technology, engineering and mathematics education. Literature reviews show that spatial perception tasks demonstrate marginal gender differences favoring males. Piaget‟s Water level task and Purdue spatial perception test, and Rod and Frame test are often used to measure spatial perception (Turgut, 2015).

As highlighted in the earlier section, spatial visualization is the third of Linn and Petersen‟s sub factors of spatial ability. According to Linn and Petersen, spatial visualization may involve the "processes required for spatial perception and mental rotations but are distinguished by the possibilities of multiple solution strategies" (p. 1484). Ekstrom, French, Harman and Dermen (1976) defined spatial visualization as the mental ability to manipulate a visual image. According to Lohman (1979), spatial visualization is the ability to comprehend imaginary movements in a 3-dimensional space or the ability to manipulate objects in imagination. It is also described by Tartre (1990) as the ability to predict specified transformations of geometric figures or imagine mental rotations of objects or their parts in 3-D space. One distinct attribute of spatial visualization is in the task demand. Tasks requiring flipping, sliding or complex mental rotation of one or more visualized objects, mental folding and rearrangement of pieces of an object to form the whole object are often those demanded in spatial visualization. Because spatial visualization requires keeping track of multistep procedures to finish the task, multi-task tests such as surface development, paper folding hidden figures, paper form board, differential aptitude test are often used to measure spatial visualization ability (Robichaux, 2003; Vandenberg & Kuse, 1978). Most of these tests have yielded no or little gender differences.

In most curriculum and instructional materials, visualization which encompasses imaginary activities involving two- and three-dimensional objects is commonly equated to spatial visualization (Ministry of Education, 2012; National Science Board, 2010; Wai, Lubinski & Benbow, 2009). However, spatial visualization can only be viewed as an aspect of visualization. McGee (1979) clarified this with four distinct attributes of spatial visualization which entails the skill or ability to:

- imagine the rotation of a depicted object, the folding and unfolding of a solid and the relative changes of position of objects in space;
- mentally visualize a configuration in which there is movement among its parts;
- comprehend imaginary movements in three dimensions and to manipulate objects in the imagination; and
- manipulate or transform the image of a spatial pattern into other arrangements in the mind.

Because its attributes embrace many other spatial components, spatial visualization forms the basis for scientific, technological, engineering and mathematical knowledge, skills and generalizations (Erkek, Işıksal & Çakıroğlu, 2017). Thus, spatial visualization is perhaps the basis for the development of shape and space geometry in the mathematics syllabus at the colleges of education.

The three sub factors are theoretically related in that they all concern mental processing of objects or imagery (Turgut, 2015). However, the main difference between these sub spatial factors centers on the spatial image. While spatial visualization deals with the sensory cognition of spatial relationships of the spatial image itself in verbal or graphical forms, mental rotation relates to the observer mentally rotating the image in time. Spatial perception however deals with recognizing or maintaining ones‟ orientation with respect to objects in space.

To be successful in geometry, it can be argued that students need to have good spatial reasoning abilities such as spatial visualization, mental rotation and spatial perception. This argument is consistent with current assessment frameworks of Programme for International Student Assessment and Trends in International Mathematics and Science Study where spatial competences of pupils are now tested (Mullis, Martin & Foy, 2008). While this is viewed as a paradigm shift towards holistic realization of the goal of learning mathematics (Flynn & Hawes, 2014), an attempt to promote teachers‟ knowledge and spatial competencies to undertake spatial tasks remains ill-informed. With the belief that teachers‟ subject matter knowledge in geometry greatly predicts students‟ learning (Shulman, 1986), the extent to which factors, such as spatial ability, relate to or affect the geometric knowledge of pre-service teachers is worth-investigating.

2.6. Theory of Verbal Reasoning

The argument in literature as to whether verbal reasoning is mediated by spatial thinking or vice versa remains unending. Both Polk and Newell (1995) and van

Hieles (1986) noted that verbal reasoning is the heart of abstractions and deduction since language structure influences students" progression from the visual, concrete structures to abstract structures. For example, in their discrete hierarchical categorization of geometrics thinking levels, the van Hieles, portrayed that object/spatial visualization (level 1) precedes analysis (level 2), informal deduction (level 3), formal deduction (level 4) and rigor (level 5) of geometric thinking (Armah, Cofie & Okpoti, 2017). Duval (1988) however assumes that visualization, reasoning and constructions are bilaterally related. These theories suggest that for any verbal abstraction and deductive reasoning to take place, visualization is necessary. However, Roux (2005) found that more proficient language learners did better on visual level than less proficient language learners suggesting that verbal reasoning precedes visual.

Verbal reasoning involves repeatedly re-encoding a given problem and reasoning using concepts framed from words (Mellanby & Langdon, 2010; Burton, Welsh, Kostin &van Essen, 2009; Polk & Newell, 1995). In other words, verbal reasoning concerns students" reasoning about vocabulary, syllogism, verbal analogies and comprehension of texts involving concepts. As described by Gardner (1983), verbal reasoning influences the learning of many disciplines. Often, people who obtain good scores in verbal reasoning tests are generally held to be quick in grasping written concepts and successful in solving problems using verbal information. They could as well construct sound logical decisions involving words and verbal information.

Geometric thought is usually expressed in words. Hence, students" verbal reasoning is supposed to predict the level of abstractions and deductions of geometric content. This therefore implies that pre-service teachers who acquire limited verbal reasoning ability in shape and space concepts during their learning at the colleges of education may lack the verbal ability to effectively deliver lessons on shape and space upon completion.

Verbal reasoning is an aspect of cognition in language proficiency (Burton, Welsh, Kostin &van Essen, 2009). For second language learners, as pertains in Ghana, verbal reasoning can be a major factor in the learning school subjects like science and mathematics. In terms of Geometry education, students‟ verbal reasoning could affect their logical reasoning, understanding of definitions, propositions, proofs and overall achievement (Gunhan, 2014). Verbal reasoning ability can influence how a student comprehend definitions of a given polygon or make inferences in proofs when communicating or solving problems geometrically. In teacher education, it may be argued that teachers with good verbal reasoning abilities could effectively communicate the precise concept image and concept definition in geometry to students. A pre-service teacher or a practicing teacher‟s poor verbal reasoning may therefore be a conduit for misrepresentations of geometric concepts and breeding grounds for transfer of misconceptions to students.

Comparatively, research suggests that measures of verbal quantitative reasoning appear to be better predictors of both real-world learning and academic achievement than figural reasoning tests (Lohman & Lakin, 2009). Despite this, there is an apparent lack of empirical work on how verbal factors impact on the learning of mathematics and geometry in teacher education. Based on this literature gap, an investigation into pre-service teachers‟ verbal reasoning ability and its direct or indirect effect on geometric knowledge types for teaching is justified.

2.7. Studies on Relationships between Constructs

One theoretical relationship between geometry and spatial ability is illustrated in hierarchical geometry thinking levels theorized by van Hieles. According to van Hieles, geometric thought develops through 5 level including visualization, analysis, informal deduction, deduction and rigor (Armah, Cofie & Okpoti, 2017). The theory considers spatial ability (i.e. visualization of shapes and space) as the foundation for complete linear hierarchical progression in geometric knowledge growth. Though not explicit in theoretical definitions in geometry, it can be viewed from the point of test performance that ones" ability to abstract geometric definitions, properties, axioms among other geometric statements depends substantially on verbal reasoning ability. Therefore, it might be hypothesized that the development of geometric knowledge may be impeded with limited spatial and verbal reasoning abilities. This study thus conceptualized a relationship where spatial ability and verbal reasoning are said to impact on geometric knowledge of pre-service teachers.

A number of empirical studies have examined the relationships between knowledge and spatial ability (Luneta, 2015; Gunhan, 2014; Uttal & Cohen, 2012). However, most of these studies have not considered how spatial ability relates to different components of knowledge especially among pre-service teachers. The following literature review demonstrates such limited knowledge gap.

2.8. Declarative, Conditional and Procedural Knowledge Relationships

According to long-term memory research, a distinction between procedural knowledge and declarative knowledge systems have been described with some authors considering these systems as parallel and analogous. While noting the functional distinction, ten Berge and van Hezewilk (1999) conceded that declarative knowledge is an outgrowth of procedural knowledge. They argue ontogenetically that learning and remembering procedures happens before facts are developed. Therefore, it may be irrational to suppose that knowledge of facts evolved before knowledge of procedural skills. On the other hand, other researchers have argued that declarative knowledge is necessary condition for procedural knowledge to exist (Cervigin, 2012; Ashby & Crossley, 2010; Haapalasa & Kadijevich, 2000).

Arguably, studies on the developmental relations between different types of knowledge have since yielded inconsistent and unclear results. In a recent study, Schneider, Rittle-Johnson and Star (2011) tested whether the predictive relations between conceptual and procedural knowledge were bidirectional using equation solving. Two measurement points each from two samples of middle school students who differed in prior knowledge were analyzed. The results showed that conceptual and procedural knowledge had stable bidirectional relations that were not moderated by prior knowledge but contributed independently to procedural flexibility.

According to Yilmaz and Yalcin (2012), the greatest part of human knowledge is in procedural and declarative systems. They further argued that procedural and declarative knowledge types are interrelated. Ashby and Crossley (2010) particularly clarify the neural correlates of switching between procedural and declarative categorization systems. Cevirgen (2012) demonstrated in a structural equation modelling the bilateral relationships between declarative, conditional and procedural knowledge types in geometry. His sample was 8th graders at senior high school in Turkey. While most studies appear to justify bilateral relations, none of these studies involved the subjects of pre-service teachers.

In a study to understand students" misconceptions, Luneta (2015) established that students" errors were related to their procedures for solving geometry questions and because “most of the learners were conceptually weak, their procedures were flawed too" (p. 9). This suggests that conceptual knowledge is pre-requisite for procedural knowledge. This is however in contrast with ten Berge and van Hezewilk‟s (1999) observation that remembering procedures happens before facts are developed.

Aydin (2007) reviewed literature on the relationships among declarative, conditional and procedural knowledge. His review portrayed that the learning of mathematics and geometry, involves linking declarative, conditional and procedural knowledge. According to him, it is during such linkages that students use their conditional knowledge to select appropriate procedures or rules, recollect declarative knowledge of such rules and consequently use their procedural knowledge of algorithms to solve problems. Aydin and Ubuz (2010) also examined the reciprocal relationships between declarative, conditional, and procedural knowledge of 297 tenth graders in secondary schools in Cankaya district of Ankara, Turkey. Their finding hinted that declarative knowledge has significant positive effect on both conditional and procedural knowledge. Also, procedural knowledge was found to have significant direct effect on conditional knowledge. Finally, declarative knowledge had positive indirect effect on conditional knowledge in the presence of procedural knowledge of the tenth graders. They further found substantial correlations among declarative, conditional and procedural knowledge. Their findings supported earlier studies (Rittle-Johnson & Koedinger, 2005). In line with these findings, Aydin (2007) argued that learning geometric concepts separately from procedures or vice versa yields no differential effect.

The argument as to whether concepts must be learnt first before procedures or procedures before concepts in mathematics has been historically documented (Aydin & Ubuz, 2010). These arguments have led to distinction of four theoretical views namely the generic view, the dynamic interaction view, the simultaneous activation view and the inactivation view (Lauritzen, 2012; Haapalasa & Kadijevich, 2000). The generic view states that procedural knowledge is a necessary and not sufficient condition for conceptual knowledge. In other words, procedures occur prior to the conceptual knowledge. The dynamic interaction view however contrasts this view by stating that conceptual knowledge is a necessary but not sufficient condition for procedural knowledge. To the dynamic interaction, conceptual knowledge makes it possible for construction of procedures. The simultaneous activation view however seeks for a balance by stating that procedural knowledge is necessary but also sufficient condition for conceptual knowledge. The final view about the relation between conceptual and procedural knowledge claims that procedural and conceptual knowledge are not related. This view which seeks for neutral perspective is described as the inactivation view. While debate on the developmental primacy continues in literature, it is clear from such arguments that at least these three knowledge types exist and hence provides opportunity for further enquiry into the nature of relationship between these knowledge types. The present study on how pre-service teachers" knowledge of concepts and knowledge procedures relate is therefore essential.

Most studies in mathematics have limited the knowledge types to conceptual and procedural (Lauritzen, 2012; Aydin & Ubuz, 2010; Haapalasa & Kadijevich, 2000). Literature review however confirmed that conceptual knowledge entails conditional knowledge and declarative knowledge (Aydin & Ubuz, 2010). As a result, to understand pre-service teachers" geometric knowledge, the relation between declarative, conditional and procedural knowledge in geometry were considered relevant. As future teachers, pre-service teachers need not only understand procedures but also have deep declarative and conditional knowledge to ensure strong conceptual knowledge. These types of knowledge would enable them to provide meaningful learning contexts and solution strategies to pupils they would teach in future. Understanding how these three knowledge types relate to each other might suggest pedagogical consequences for effective knowledge construction.

2.9. Spatial Ability and Geometric Knowledge Relationships

Critical analyses of research in geometry (Wahab, Bin Abdullah, Bin Abu, Mokhtar & Bt Atan, 2015; Gunhan, 2014; Uttal & Cohen, 2012; Hull, 2011; Lubinski, 2010) and mathematics curriculum documents (Ministry of Education, 2012; Guzel & Sener, 2009; National Council of Teachers of Mathematics, 2010), suggest that spatial ability influences geometry learning. Such influence may be mediated by students" verbal reasoning abilities. Very few studies have however examined the theoretical relationships between spatial abilities and various types of geometric knowledge (Cevirgen, 2012; Eryaman, 2009). In current literature, a void of studies on spatial ability in the context of pre-service teachers" knowledge-base in geometry is seen as rather astounding.

A synthesis of research in the field of spatial ability since 1957 revealed that much of the spatial studies have been in STEM (Sipus & Cizmesija, 2012; Turgut & Yilmaz, 2012). Most of these studies argued that spatial abilities play prominent role in learning mathematics and regarded spatial ability as significant factor in scientific and mathematical thinking. Hegarty and Waller (2005) assert that spatial ability is required for an individual"s construction and comprehension of abstract representations in mathematical problem solving. Several other studies (Taylor & Tenbrink, 2013; Atebe & Schäfer, 2011; Boakes, 2009) have documented that human thinking processes which stimulate understanding and logical reasoning for effective geometric problems solving depend largely on spatial visualization, mental rotation and spatial perception. Boakes (2009) particularly highlighted that lack of spatial sense creates learning deficiencies in science and mathematics disciplines.

Spatial sense is one"s perception of space which corresponds to some less prescribed ways of looking at two- and three-dimensional space through activities like paper-folding, transformations, tessellations, and projections (The Ontario Ministry of Education, 2014; Guzel & Sener, 2009). The basis of Geometry is the study of shape and space and so, there may be little argument regarding the theoretical association between spatial ability and geometry. However, empirical results are still inconclusive about the level or extent to which one influences the other. One argument in literature has been that spatial ability is a primitive ability and hence might not be needed in adult learning (Flynn & Hawes, 2014; Uttal & Cohen, 2012). Closely related to this argument is the view raised by some authorities that in adulthood, ones" spatial competency development remains stable and cannot be improved further (Boakes, 2009). While recent evidences are beginning to show contrary (Akayuure et al, 2016), these arguments appear to have resulted in the neglect of spatial competency issue in teacher education or college level. Regarding pre-service teachers, there are still limited studies and doubts regarding the effect size of spatial ability on geometry knowledge acquisition.

In terms of selecting learning and teaching activities in mathematics and geometry, Baki, Kosa and Guven (2011) identified visualization activity as a trigger tool. Indeed, earlier studies have shown positive relationship between various forms of spatial abilities and geometric knowledge. Battista (1990) investigated the strategies and abilities employed by pre-service elementary teachers in solving geometry problems. Their study revealed that visualization strategy which was used most frequently proved to be effective in students" performance even though analytic strategy also resulted in fair performance. In line with this, Kospentaris, Spyrou and Lappas (2011) considered visualization as an important factor that provokes the choice of strategy when students undertake geometry tasks. Haj-Yahya and Hershkowitz (2013) also found that the position of a shape, which relates to spatial orientation, affects its identification and classifications.

In a case study of four participants, Unal, Jakubowski and Corey (2009) looked at the differences in learning geometry among high, mid-range and low spatial ability pre-service teachers enrolled at a university in the southeast United States of America. The Purdue rotation object test (ROT) and Mayberry van Hieles protocol were employed before and after instruction to characterize their spatial ability and geometric thinking levels. Evidence showed that initial as well final spatial ability scores of the four pre-service teachers who participated in the study did affect changes in development of geometric understanding. While acknowledging the consequences of low spatial ability on pre-service teachers" instructional decisions and choice of geometric tasks for students, Unal, Jakubowski and Corey (2009) argued that more studies need to be carried out to further understand how important spatial ability is in the process of reasoning in geometry.

Guzel and Sener (2009) investigated low and high spatial ability students" potential creativity in geometry at Anatolian and State High Schools, in Marmara region of Turkey. Data were gathered through a Likert-like Purdue spatial ability instrument and three open-ended problems in geometry. The study showed that spatial ability improves students" understanding of symbols, shapes, tables, and figures. In their study, students with high spatial ability comprehended geometric drawings easily, commented correctly with visualized information, created contexts among different concepts easily and adopted ways of different thinking towards generalize complex concepts.

Pittalis and Christou (2010) did an analysis of the structure of grade 5 to 9 students‟ 3-dimensional geometry thinking with the aim of identifying different types of reasoning and their relations with spatial ability. After administering two cognitive tests, they identified four types of reasoning including representation of 3-dimensional objects, spatial structuring, conceptualization of mathematical properties and measurement. It was observed that geometry reasoning types and spatial abilities could be modeled as distinct constructs. The study then concluded that students‟ spatial ability labelled as spatial visualization, spatial orientation and spatial relations factors, was a strong predictive factor of four types of reasoning in 3-dimensioanl geometry thinking.

Özdemir and Göktepe-Yildiz (2014) investigated the reciprocal relationship between spatial ability and geometric knowledge among pre-service mathematics teachers in Turkey. Three pre-service mathematics teachers with low, average and high spatial skills were engaged in a clinical interview to analyze their spatial abilities in line with the Structural Observed Learning Organization (SOLO) taxonomy. The results revealed that the pre-service teacher at low spatial visualization skill corresponded to the lowest level of SOLO taxonomy. The responses of the two pre-service teachers at the middle and high levels of spatial visualization skills matched with multi-structural and relational levels respectively. The study concluded that as the spatial visualization skill increases, the level of the pre-service teachers „geometric reasoning progresses into higher levels of SOLO taxonomy. The study suggested that, to develop spatial ability, the use of dynamic geometry programs and abstract materials were necessary. While the findings were significant, Özdemir and Göktepe-

Yildiz"s (2014) research design involving clinical interviews and sample size of three pre-service teachers is arguably subjective, less reliable and limited in generalizations. As supposed in this present study, perhaps, establishing such relationships through structural equation modelling with large sample, which accounts for variance-covariance and measurement errors, could lead to much clearer and objective conclusion than the current finding by Özdemir and Göktepe-Yildiz (2014).

In line with previous studies (Pittalis & Christou, 2010; Boakes, 2009), Özdemir and Göktepe-Yildiz (2014) argued that space is the right field of study to develop students" spatial skills and therefore must be given prominence when teaching school geometry at all levels. Several national mathematics and geometry curriculum documents (Ministry of Education, Ghana, 2012; National Council of Teachers of Mathematics, 2010) have already highlighted the need to develop students" spatial reasoning. Regrettably, the teaching of space is yet to be effectively done in most mathematics and geometry classrooms in Ghana (Akayuure, et al, 2016). It may be quite likely that the very teachers who are obliged, by their professional responsibility, to promote the development of students" spatial ability are limited in their own spatial abilities. Such limited spatial abilities if they exist among practicing teachers, may be traced to lack of exposure to spatial concepts at the colleges of education or earlier school work. The recent concerns that a few proportions of Ghanaian pre-service teachers could visualize and abstract geometric shapes (Armah, Cofie & Okpoti, 2017; Asemani, Oppong & Asiedu-Addo, 2017), obviously conveys in the question of whether the trend of poor performance in geometry among pre-service teachers is the consequence of limited spatial knowledge or vice versa.

2.10. Spatial Ability and Verbal Reasoning

In line with spatial ability or bodily-kinesthetic intelligence, verbal reasoning is considered an important component in many influential intelligence models (Carroll, 1993; Gardner, 1983). In the viewpoint of Gardner (1983), verbal processing skill and spatial abilities complement each other in developing higher level thinking skills among students. For success in Science, Technology, Engineering and Mathematics disciplines, the National Science Board (2010) maintains that verbal and spatial skills should be discretely defined in the list of mathematics skills in national mathematics syllabi. In an empirical study, Battista (1990) found among high school students that spatial visualization and verbal-logical reasoning abilities were significant factors of geometry achievement and geometric problem solving. Haciomeroglu (2016) conducted a study on the relationship among object, spatial, and verbal cognitive styles and spatial ability, verbal-logical reasoning ability and exam scores. The study found that low and average object visualizers had significantly higher mean exam scores than high object visualizers.

According to Haj-Yahya and Hershkowitz (2013), students provided verbal definitions of inclusive quadrilateral relationships more accurately without the presence of visuals. These findings suggest the need to expose those undergoing teacher training should be exposed that verbal reasoning opportunities to allow for effective deductions of figural and linguistic contexts of geometry. This would prepare them effectively to teach shape and space concepts when they assume their professional roles in future.

2.11. The Mediating Role of Verbal Reasoning

In geometry learning, a learner"s verbal reasoning may affect his/her understanding of geometric definitions, properties, proofs and arguments (Duval, 1988). However, this relationship has not been fully studied in the context of pre-service teachers. Research establishing any inter-relationships and effect of pre-service teachers" spatial ability, mediated by verbal reasoning abilities, on their geometric knowledge is quite rare in literature and completely new in Ghanaian context.

Theoretical frameworks in geometry (Fischbein, 1993; Duval, 1988; van Hieles, 1986) largely underscore visual skills or spatial orientation as key to geometric knowledge construction. A critical examination of these theoretical frameworks reveals two issues. First, among the frameworks, the issue of verbal reasoning appears to have been perceived implicitly or perhaps ignored completely. Second, with regards to geometry instructions at colleges of education in Ghana, it seems none or little spatial contexts are often applied in teaching concepts or properties of shapes and space. This study argues that without the development of adequate spatial abilities to complement verbal reasoning or vice versa, pre-service teachers are likely to face learning difficulties in college geometry and in their future teaching of shape and space concepts. This argument though plausible, remains inconclusive in current literature. In other words, limited studies exist on the relationships and contributions of spatial and verbal reasoning abilities to pre-service teachers" geometric knowledge for teaching. This study therefore draws on a relationship theory to examine hypothesized relationships between the verbal reasoning and spatial ability with respect to geometry knowledge of pre-service teachers at the colleges of education in Ghana.

2.12. Gender Differences and Academic Programmes

In studies on spatial ability, gender has been studied as an important issue across science and non-science related disciplines. Nevertheless, a review of past and recent literature suggests that the existence or lack of gender difference regarding spatial ability remains highly debatable. Earlier meta-analytic studies examined the issue of gender difference in spatial abilities between 1974 and 1982 (Metz, et al, 2012; Hoffler, 2010). With respect to tests of spatial perception and mental rotation, they found that men generally performed better than women. However, in the spatial visualization tests, men and women performed equally. Another meta-analysis of studies that measured mental rotation abilities using mental rotation tests and the Purdue Spatial Visualization Test Revised was conducted by Voyer (2011). The analysis revealed that, when the length of time allowed for the tests was constrained, significant difference in gender scores existed favoring males.

Apart from meta-analytic studies, individual empirical studies have examined the gender factor in spatial ability development. While Battista (1990) reported in his cross-sectional study that high school males outperformed females in most situations requiring spatial visualization, Parsons (2004) found sex difference in Mental Cube Test but no such sex differences in Virtual Reality Spatial Rotation (VRSR) system. Parsons (2004) attributed the contrasting results to cognitive load in the use of the VRSR system.

The contrasting and debatable results drew more discipline specific studies into the subject of gender. Consequently, gender disparity in spatial ability is highlighted particularly among Science, Mathematics, Engineering and Technology domains. In many engineering education researches, gender disparity in spatial-skills performance favored males (Sorby, 2009; Nemeth, 2007). Nemeth (2007) investigated the engineering students" spatial ability development and concluded that the difference in male and female students" spatial ability performances favored males. In mathematics teacher education, Erkek, Işıksal and Çakıroglu (2017) conducted a study on pre-service teachers" spatial visualization ability and spatial anxiety levels with respect to their gender and programs of study. The study involved 1007 3rd and 4th year undergraduate pre-service teachers enrolled in elementary mathematics, science, and early childhood education programs from four universities in Ankara, Turkey. Data gathered from Spatial Visualization Test and the Spatial Anxiety Scale revealed a statistically significant gender differences between the spatial visualization ability and spatial anxiety scores of participants. Similarly, Turgut and Yilmaz (2012) investigated the relationship among 193 pre-service primary mathematics teachers" gender, academic success and spatial ability at Dokuz Eylul University. The study focused on spatial visualization and spatial orientation factors. They concluded that there was no significant gender difference in the spatial abilities.

Diverse arguments regarding the reasons for gender differences in spatial abilities have been noted in literature (Yang & Chen, 2010). While some authors attributed such differences to genetic factor and hormonal changes, other authors observed environmental factors like childhood play, test environment and educational experience. Genetically, spatial ability is a recessive feature carried by the X-chromosomes in females. In neuroscience studies, gender difference in spatial ability were related to hemisphere specialization where spatial processing is dedicated to the right cerebral hemisphere which is stronger in males (Langdon & Warrington, 2000). Males" ability to think holistically also appears more efficient in spatial task execution than females who apply double checking and analytic approaches. Males" dominance in spatially inclined tasks over females is found to emerge at adolescence and maintained in adulthood. Earlier study by Johnson and Meade (1987) with 1800 students between the ages of 6 and 18 years revealed that male advantage in spatial ability appeared at 10 and maintained at that magnitude through 18.

Aside these factors, the gender differences in spatial ability is often disputed based on its operationalization or some fundamental research flaws (Sorby 2009). It has been pointed out that the lack of unified definition of spatial abilities and its sub-factors leads to inconsistencies in data collected. The use of smaller sample size, different tests, factor loading problems and overgeneralization of findings have all been reported as likely reasons for the disagreements in gender differences regarding spatial ability. In factor analytic studies, spatial ability displayed different pattern of relationship in its factor loadings which tended to yield different magnitude of gender difference. Linn and Petersen (1985) found that not all spatial ability factors exposed gender differences. Particularly, the differences were larger in mental rotation but smaller in spatial perception.

Spatial visualization is found to be more related to mathematics performance in girls than boys. Studies have reported that spatial training can bridge the gender gap in performance on spatial visualization tests (Prokysek & Stipek, 2016; Boakes, 2009). It was also realized that spatial visualization improved for both male and females with proper instruction involving model building, working with 3-D and solving spatial visualization problems. In terms of spatial orientation, gender differences were found to relate to field dependence, field independence and Piagetian and maze tasks.

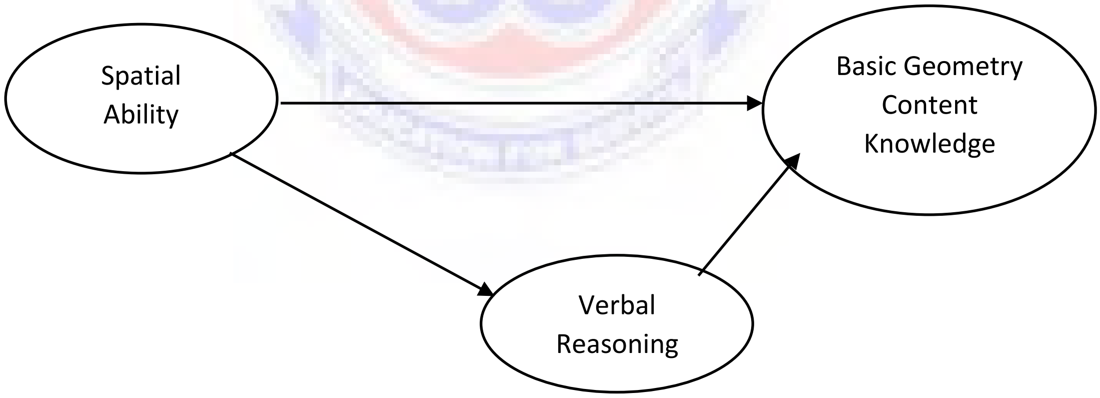
According to Metz, Donohue and Moore (2012), differences between males and females in spatial ability were widely established across discipline, but the magnitude and source of this difference remain open to debate. More studies would therefore help to further determine what and how teaching practices might bridge the gender gap in spatial ability. It might also help clarify why there are fewer numbers of females in STEM professions since Turgut and Yilmaz (2012) observed that female representation was due to their lag in spatial skills. Age differences of sample have often been raised with some studies noting that young students differ in their spatial ability from adults. Prokysek and Stipek (2016) found a dependency of mental rotation ability on the age of subject where most successful university students were within the age of 27 and 32 and the least successful group was between age of 42 and 52. Based on these disparities, the present study examined how spatial ability may influence geometry knowledge acquisition by gender in the context of teacher professional education.

2.13. Conceptualization of the Study

Pre-service teachers‟ geometric knowledge for teaching is a requirement for their future classroom practice. However, the issue of what constitute pre-service teachers‟ geometric knowledge for teaching shape and space and which factors affect its development have not been fully studied. Reviews of literature shows that the issue has only been investigated using the general framework of geometric knowledge such as those proposed by van Hieles, Duval and Fischbein. No known study looked at Ghanaian pre-service teachers‟ knowledge for teaching in terms of knowledge attributes in the long-term memory. This current review of literature did not reflect any study that looked at declarative, conditional and procedural knowledge in shape and space relative to pre-service teachers including how the development of these knowledge types are affected by cognitive dimensions such as spatial and verbal reasoning abilities. The only known empirical studies which examined geometric knowledge types using the declarative, conditional and procedural knowledge dimension were Aydin and Ubuz (2010); and Cevirgen (2012). Both studies were however conducted among secondary students in Turkey. Thus, findings might not be generalized to pre-service teachers in Ghana because of age, educational and contextual variations. Since pre-service teachers‟ in-depth declarative, procedural and conditional knowledge outline their ability to define concepts, use procedures and explain the relationships to their students, understanding these knowledge types is imperative.

For the study, pre-service teachers‟ geometric knowledge for teaching shape and space concepts at the basic schools was conceptualized as an interplay between basic Geometry content knowledge and spatial ability as mediated by verbal reasoning (See Figure 1). Based on previous findings which established varied covariances in knowledge types in geometry (Cevirgen, 2012; Aydin & Ubuz, 2010), this study hypothesized bilateral relationships between pre-service teachers‟ declarative, conditional and procedural knowledge types in geometry. The strength of such bilateral relationships determines the nature of geometric knowledge acquired by pre-service teachers (Star & Stylianides, 2013). Furthermore, it was hypothesized, based on literature and Piaget theory of spatial development, that there exist direct and indirect effects of spatial ability measured by spatial visualization, mental rotation and spatial perception on knowledge for teaching measured by declarative, conditional and procedural knowledge (Gunhan, 2014; Turgut & Yilmaz, 2012; Uttal & Cohen, 2012). Erkek, Işıksal and Çakıroğlu (2017) acknowledged that no teacher with limited spatial abilities can effectively provide pupils with the right spatial learning opportunities. Hence, establishing these effects would provide description of the relationship between pre-service teachers‟ spatial experience and geometric intuition in spatial concepts in basic school curriculum in Ghana.

The presence of verbal reasoning in spatial and geometry research is often highlighted, yet it is not often directly investigated (Cevirgen, 2012). In view of this, the intermediating effect of verbal reasoning between spatial ability and knowledge in geometry has been hypothesized in this present study. Finally, programmes and gender differences in performance in geometry and spatial ability tests have been acknowledged. However, till date, conclusions on such disparities have been inconsistent and disputed (Prokysek & Stipek, 2016). To understand how significant pre-service teachers‟ type of programme and gender affect the relationship between spatial ability, verbal reasoning and knowledge in geometry, this present study hypothesized that such differences existed. Figure 1 presents the conceptual framework for the study where hypothesized relationships have been illustrated in path diagram. The single headed lines pointing from the variable is the direction of effect to the variable affected. The three ovals in Figure 1 represent the latent variable of spatial ability (SA), verbal reasoning (VR) and basic geometry content knowledge



(KTG).

Figure 1 Conceptual Framework for the Study

2.14. Summary

In this chapter, distinction of knowledge types residing in the long-term memory were conceptualized in relation to pre-service teachers‟ content knowledge for teaching shape and space concepts at the basic school level. Three knowledge types were adopted as sub factors of pre-service teachers‟ Basic Geometry content knowledge. They consist of declarative, conditional and procedural knowledge types. Declarative knowledge refers to knowledge of geometric facts, properties and definitions which can be recalled from the long-term memory of pre-service teachers. Conditional knowledge describes the aspect of conceptual knowledge which deals with relationships between concepts and procedures in geometry and conditions requiring pre-service teachers to relate and apply such concepts and procedures. Procedural knowledge also refers to the knowledge of procedures required to execute a given geometric task such as process to find the area of a square.

From the theoretical viewpoint in which geometry entails both figural and linguistics elements, the theory of spatial ability and verbal reasoning were presented. After synthesizing past and current meta-analytic and factor analytic studies, three spatial factors (spatial visualization, mental rotation and spatial perception) were adopted for the study. Literature reviewed in this study suggests that the relations between spatial ability and basic Geometry content knowledge have not been studied on large scale. This present study drew on the argument in literature that spatial ability and geometric knowledge are two related construct which can be modeled as different constructs and investigated to find out how they relate. The mediation effect of verbal reasoning has often neglected in spatial and geometry research (National Science Board, 2010). Also, following several inconsistencies regarding the differential effects of discipline specific and gender in performance in geometry and spatial ability tests, further investigation from the angle of pre-service teachers in Ghana was imperative.

Finally, as a result of existing research deductions, implications and knowledge gap in literature in relation to limited knowledge in geometry in Ghana, this study investigated the significant relational paths using covariance-based structural equation modelling.

CHAPTER THREE

METHODOLOGY

3.1. Overview

This chapter presents a description of the research methodology adopted in collecting data for analyses and interpretations of results upon which conclusions were drawn. The key sections of the chapter include the design of the study, population and sample, data collection instruments, checks of validity and reliabilities as well as data collection and analysis procedures. The design of the study which is described in section one of the chapter considered the nature of knowledge and its enquiry method and the plan of flow of research process comprising structural equation modelling steps. The population and sample section identified the sampling frame, characteristics of the group under study as well as the distributional attributes of the sample representing the target population under study. The data collection section also identified the type and nature of the instruments employed to gather data while the validity and reliability section presented issues regarding dependability of the instruments using pilot study of a sample similar in characteristics to the sample for the main study. The last section covered the data analysis procedure and criteria for interpreting results of analysis to address the research questions and hypotheses.

3.2. Design of the Study

This study was structured within an empirical setting where data were essentially quantified for purpose of investigating the theoretical relational paths connecting spatial ability, verbal reasoning and Basic Geometry content knowledge. To investigate these relational paths, quantitative research method was employed in a seemly positivist epistemology to scientific enquiry. From such positivist epistemology, human knowledge and cognitive abilities can be measured. Accordingly, it was possible to quantify the relationship between the knower and what can be known, though not in the strictly objective manner (Cohen, Manion & Morrison, 2007). This suggests that data collected from participants were presumed to represent their knowledge and ability to solve problems. This approach was deemed suitable for conducting statistical experiments that tested the significances and magnitudes of effect of hypothesized relationships among spatial ability, verbal reasoning and geometric knowledge for teaching.

The study adopted a cross-sectional survey design to collect test scores of pre-service teachers at the colleges of education in Ghana and accordingly, examine relationships among spatial ability factor, verbal factor and Basic Geometry content knowledge. A cross-sectional study is one that produces a „snapshot" of a population at a particular point in time for data collection (Cohen, Manion & Morrison, 2007, p. 215). In this design, data were gathered at one point in time to described variables, examine their relationship structure and degree of common variations. Thus, the survey ensured that data collected from participating pre-service teachers were suitable for investigating relationships and co-variations between the variables of spatial ability, verbal reasoning and knowledge of geometry for teaching.

Despite the advantages of this design, there are some methodological limitations associated with the application cross-sectional studies. For example, Cohen, Manion and Morrison (2007) cautioned that taking a single sample at one moment in time may be ineffective for investigating trends or changes in geometric knowledge of pre-service teachers. Investigation of trends or changes are however beyond this study since the focus of investigation was to describe relationship between prevailing factors affecting geometric knowledge. This study could however provide basis for future trend studies.

3.2.1. The flow of the research design

The research process involved seven stages. Firstly, the theoretical and empirical justifications (Bruce, Flynn & Moss, 2016) that spatial ability and verbal reasoning relate to and account for geometric knowledge acquisition was hypothesized in the context of pre-service teachers. Secondly, based on the above, a hypothetical model was specified involving three factors of spatial ability, two factors of verbal reasoning and three-tier knowledge types identified in Alexander and Judy (1988). This was described as model construction and was particularly informed by the theoretical and empirical works reviewed in chapter two of this report. Thirdly, three tests were adopted and constituted for data collection. Fourthly, tests were administered to pre-service teachers at selected colleges of education in Ghana. Fifthly, the hypothesized model was identified and analysis in Analysis of Moment Structures (AMOS) software and estimates tested and evaluated. Sixthly, analysis of data was carried out in relation to the research questions and hypotheses. Finally, the results were interpreted and discussed in line with related theories and empirical literature on teachers‟ geometric knowledge acquisition. Figure 2 is the diagrammatic representation of sequence of processes of the study.

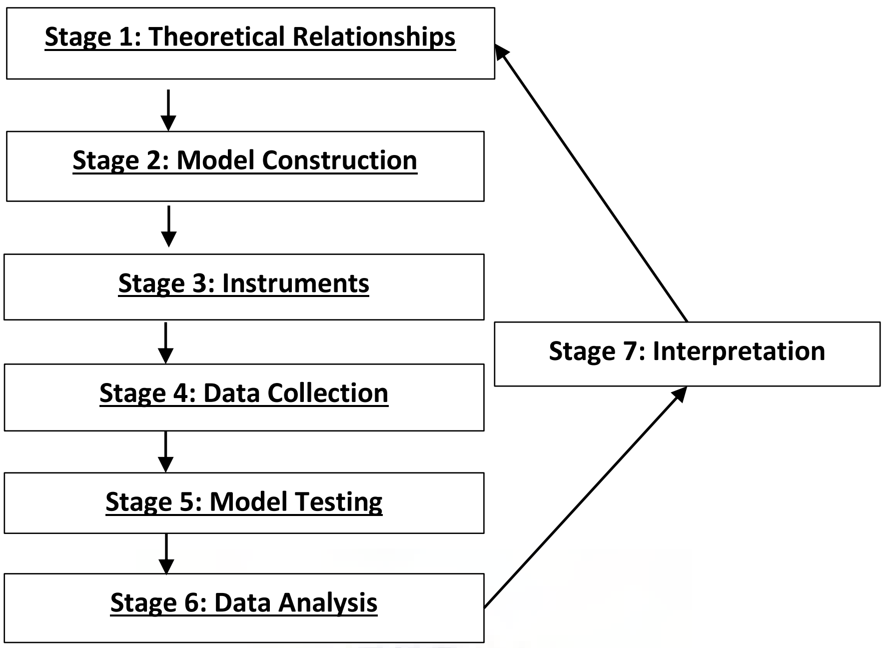


Figure 2 The Flow of the Research Design Process

3.2.2. The Structural Equation Modeling methodology

Structural equation modeling (SEM) was employed during the model construction, estimation, testing and evaluation. SEM comprises variation, co-variation, confirmatory factor analyses and regression of relationship patterns among variables (Kline, 2015). In this study, it included modeling of relationships among spatial ability, verbal reasoning, basic Geometry content knowledge and examining linearity, correlated independences, measurement errors, correlated error terms and latent variables measured by multiple indicators.

SEM was preferred because it allowed for statistically comprehensive and iterative testing of overall model fitness to data. This also provided room to holistically and inclusively address the main research questions and hypothesized patterns of relationships among the set of measured and latent variables (Kline, 2015). SEM ensured a straightforward examination of the patterns of covariance between variables and amount of common variance accounted for in the hypothesized model.

Some authors have cautioned about the use of SEM (Kilne, 2015; Nachtigall, Kroehne, Funke & Steyer, 2003). Kline (2015) stated that SEM analysis requires a large sample size to maintain the statistical power to find true relationships which might be practically difficult to attain in certain study designs. Bagozzi and Yi (2011) also pointed out two limitations regarding the use of SEM. They identified that SEM (1) is susceptible to many assumptions and (2) can lead to gullible interpretation when dealing with complex models. It must be recognized however that; these limitations are not related to only SEM. Even first-generation statistical techniques such as regression are also constrained by similar practical issues. Furthermore, issues of statistical assumptions can still be performed to ascertain any issues of violation. Besides, several estimation methods are currently available to address different characteristics exhibited by data (Arbuckle, 2013). Bagozzi and Yi (2011) therefore maintained that SEM is currently the only integrative function which researchers can be more precise in their specification of hypotheses and constructs. This study adopted SEM to allow for the running of statistical tests and simultaneous comparison of complex phenomena or models as required in this study (Kline, 2015).

In accordance with SEM literature (Kline, 2015), five steps were employed in the study. A diagrammatic representation of these steps is shown in Figure 3.

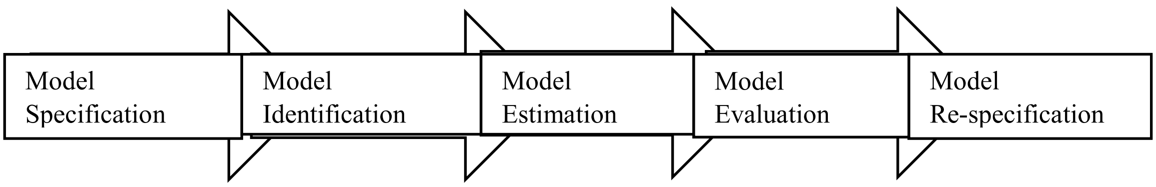


Figure 3 Flow of Structural Equation Modeling Steps

As shown in Figure 3, SEM steps comprise model specification, model identification, model estimation, model evaluation and model re-specification. Detailed descriptions of how each step was applied in this study follows.

Step 1: Model Specification

As suggested in Bagozzi and Yi (2011), the first step in SEM is to specify hypotheses and provide reasons for the assigning latent and observed variables in the study. This study hypothesized that basic Geometry content knowledge comprises declarative, conditional and procedural knowledge. These types of knowledge however have theoretical relationship and covariance well-acknowledged in literature. The theoretical foundation for the hypothesized relationship took into consideration the generics, dynamics interaction, the simultaneous activation and inactivation views of knowledge relationship (Lauritzen, 2012). While the generics view procedural knowledge to promote conceptual knowledge, the dynamic interaction reverses this view. However, this study assumed the simultaneous activation view, where both procedural and conceptual knowledge types occur simultaneously and co-vary in their development.

Additionally, since conceptual knowledge is made up of declarative and conditional knowledge, it was also considered that they would relate to procedural knowledge. Cevirgen (2012) stressed the reciprocal relationships between declarative, procedural and conditional knowledge. Aydin (2007) also indicated that an individual"s knowledge of concepts, facts, and generalizations can be derived from his/her knowledge of procedures and vice versa. The condition under which a procedure is being adopted is likely to co-vary with one"s declarative and procedural knowledge. Existing studies that determined these relationships reflected the context of secondary level students only (Cevirgen, 2012; Aydin, 2007). In the case of pre-service teachers" knowledge of geometry, no known study investigated how knowledge of concepts related to procedures or affected the choice of procedures for dealing with geometric tasks. One main related research question formulated for the study was: How do pre-service teachers‟ knowledge types in geometry affect each other? The research question was specified in the following null hypothesis:

H_{01}: There is no relationship among pre-service teachers' declarative, conditional and procedural knowledge.

In the model specification, spatial ability was hypothesized to have direct effect on pre-service teachers‟ knowledge in geometry. This was based on theory of Piaget‟s spatial development (Piaget & Inhelder, 1967) supported by geometric thinking frameworks including van Hieles (1986), Duval (1993) and Fischbien (1993). Research on the influence of spatial ability as a single construct on geometric knowledge has been well-documented. However, little is known about the nature and degree of such relationship when dealing with spatial sub factors and knowledge configurations in geometry among pre-service teachers in Ghana. Regarding this limited evidence, the present study questioned "how do spatial ability and basic Geometry content knowledge relate to each other?" Accordingly, the following hypothesis was formulated for testing:

Ho_2: There is no direct effect of spatial ability on Basic Geometry content knowledge.

From literature reviews, it remains uncertain about how verbal reasoning mediates the effect of spatial ability on pre-service teachers‟ geometry knowledge. However, the contribution of verbal reasoning in the learning of various mathematics discipline including geometry have been acknowledged (Lohman & Lakin, 2009). Most test items measuring geometric knowledge including the widely used van Hieles‟ geometric test, are based largely on verbal reasoning. This suggests that perhaps an individual‟s verbal reasoning might mediate how s/he applies his/her spatial ability to attend to geometry tasks. To examine this, Research Question 3 was raised as: "How does verbal reasoning mediate the effect of spatial ability on Basic Geometry content knowledge? In line with this research question, the following null hypotheses was specified:

H_{O3}: There is no verbal reasoning intermediating the effect of spatial ability on knowledge for teaching.

Lohman and Lakin (2009) argued that measures of verbal reasoning appear to be better predictors of both real-world learning and academic achievement than figural or spatial reasoning tests. As concerns are raised about the ability of pre-service teachers to reason and comprehend geometric problems coupled limited empirical evidence to clarity these concerns, it appeared imperative to investigate the effect of verbal reasoning on Basic Geometry content knowledge. To determine the effect of verbal reasoning on geometry knowledge of pre-service teachers, the following hypothesis was stated:

H_{04}: There is no effect of pre-service teachers' verbal reasoning on Basic Geometry content knowledge.

Studies on the moderating effect of gender and academic programme on spatial ability and geometric knowledge have produced conflicting results. While some studies found that gender differences affect students" spatial ability and achievement in geometry (Baki, Kosa & Guven, 2011), others argued that such differences could be explained by several factors including choice of cognitive tasks (Bruce, Flynn & Moss, 2016). Furthermore, students pursuing science related disciplines develop high spatial and geometric understanding than non-science related students. To test how these unique characteristics of pre-service teachers affect the relationship among spatial ability, verbal reasoning and knowledge in geometry, the following hypothesis was specified.

H_{05}: The relationship among spatial ability, verbal reasoning and knowledge in geometry is invariant across gender and programmes.

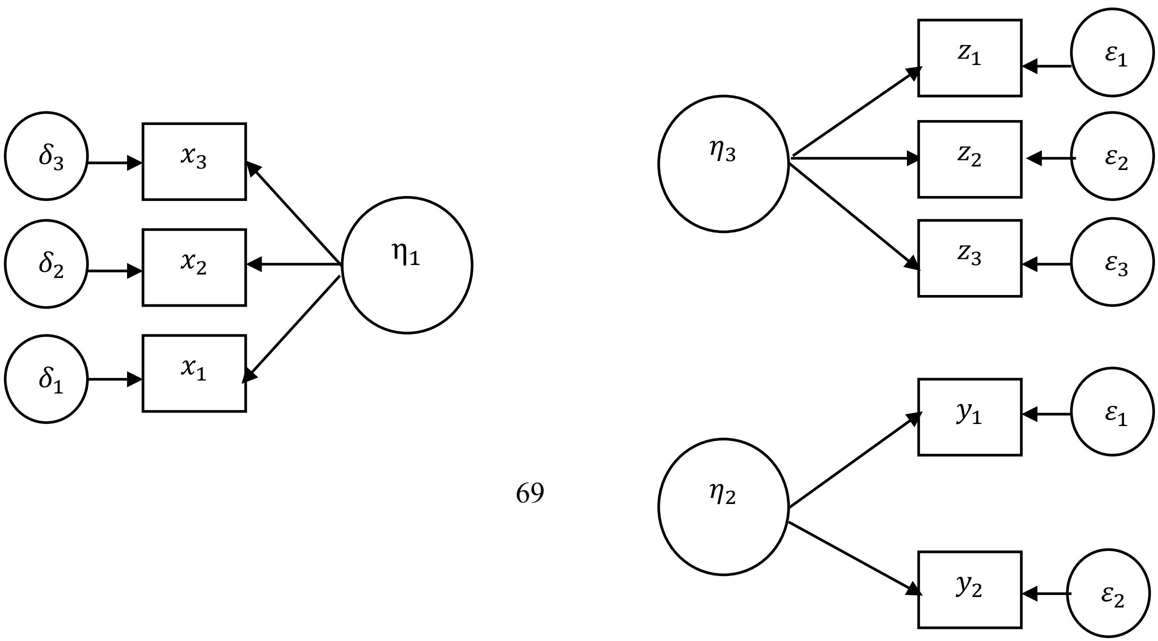
By linking these hypotheses stated above, the entire hypothesized model was specified in parts – the measurement part and structural part – pictorially in the path diagram in Figure 4 and Figure 5. The measurement model and the latent variable (structural) model formed the basis for developing a structural equation modeling in AMOS and for addressing the two main hypotheses of the study:

H01: There is no relationship among spatial ability, verbal reasoning and Basic Geometry content knowledge.

H02: The structural model describing the relationship of spatial ability, verbal reasoning and basic Geometry content knowledge is invariant across gender and programme of study.

3.2.3.1. The Measurement part of the model

As shown in Figure 4, the exogenous latent variable of spatial ability is represented by η_1. This latent variable is theoretically defined and estimated by three indicator items that comprise test scores of spatial perception (x_1), spatial



visualization (x_2) and mental rotation (x_3) (Linn & Petersen, 1985). On the other hand, the endogenous latent variables of verbal reasoning and basic Geometry content knowledge are represented by η_2 and η_3 respectively. The verbal reasoning was derived from test scores of comprehension (y_1) and syllogism (y_2). Finally, the indicator items z_1, z_2 and z_3 denoted test scores for declarative, conditional and procedural knowledge used to estimated Basic Geometry content knowledge.

Figure 4 Measurement Model with Three Latent Variables and Eight Indicator Items

The arrows connect the latent variables to their respective observed variables (indicator items). The direction of each arrow shows the causal direction and signifies that the indicator item has the effect on the latent variable. Consequently, the equations in the measurement model where $\lambda's$ are coefficients, are described as follows:

$$x_1 = \lambda_1\eta_1 + \xi_1 \ldots\ldots\ldots\ldots\ldots\ldots [1]$$
$$x_2 = \lambda_2\eta_1 + \xi_2 \ldots\ldots\ldots\ldots\ldots\ldots [2]$$
$$x_3 = \lambda_3\eta_1 + \xi_3 \ldots\ldots\ldots\ldots\ldots\ldots [3]$$
$$y_1 = \lambda_4\eta_2 + \varepsilon_1 \ldots\ldots\ldots\ldots\ldots\ldots [4]$$
$$y_2 = \lambda_5\eta_2 + \varepsilon_2 \ldots\ldots\ldots\ldots\ldots\ldots [5]$$
$$z_1 = \lambda_6\eta_3 + \epsilon_1 \ldots\ldots\ldots\ldots\ldots\ldots [6]$$
$$z_2 = \lambda_7\eta_3 + \epsilon_2 \ldots\ldots\ldots\ldots\ldots\ldots [7]$$
$$z_3 = \lambda_8\eta_3 + \epsilon_3 \ldots\ldots\ldots\ldots\ldots\ldots [8]$$

The first three equations [1-3] defined the exogenous latent variable of spatial ability. The next two equations [4-5] defined endogenous latent variable of verbal reasoning whiles the last three equations [6-8] defined the endogenous latent variable of Basic Geometry content knowledge. Therefore, generalizing these equations and transforming into matrix forms gives:

$$x = \Lambda_x\eta_1 + \xi$$
$$y = \Lambda_y\eta_2 + \varepsilon$$
$$z = \Lambda_z\eta_3 + \epsilon$$

In which case

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad y = \begin{bmatrix} y_2 \\ y_2 \end{bmatrix} z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}, \quad \Lambda_x = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}, \quad \Lambda_y = \begin{bmatrix} 0 & \lambda_4 \\ 0 & \lambda_5 \end{bmatrix},$$

$$\Lambda_z = \begin{bmatrix} 0 & 0 & \lambda_1 \\ 0 & 0 & \lambda_2 \\ 0 & 0 & \lambda_3 \end{bmatrix}, \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix}, \quad \xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} \text{ and } \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_1 \\ \epsilon_1 \end{bmatrix}$$

In line with SEM assumptions, the measurement errors denoted by $\xi's$, $\varepsilon's$ and $\epsilon's$ in the equations are assumed be to uncorrelated with each other or with $\eta's$ (Arbuckle, 2013; Lauritzen, 2012). To maximize measurement, the measurement errors are also assumed to result into zero expected value and satisfy homoscedasticity assumption (Kline, 2015). Equations [1-8] would be solved to assess the strength of factor loadings, construct validities, reliabilities and measurement errors as well as test measurement invariances across gender and programme.

3.2.3.2. Structural part of the model

The structural part of the model otherwise known as the latent variable model (Arbuckle, 2013) comprise the relationships among spatial ability (η_1), verbal reasoning (η_2) and basic Geometry content knowledge (η_3) (see Figure 5). The essence of the structural model was to examine significant path coefficients in the relationships as well as the extent of covariances or correlations exhibited by associated variables. Each of the structural path therefore represents specific hypothesis in the study. To explore the theoretical relationship between ability and knowledge to teach in the context of pre-service teachers, it was hypothesized that basic Geometry content knowledge depends on verbal reasoning ability. Then, spatial ability was assumed to have both a direct and an indirect effect on basic Geometry content knowledge in which the indirect effect passes through verbal reasoning. A path diagram reflecting the hypotheses about the relations between the latent variables is shown in Figure 5 whereby the direction of each arrow indicates the direction of effect of one latent variable on the other.

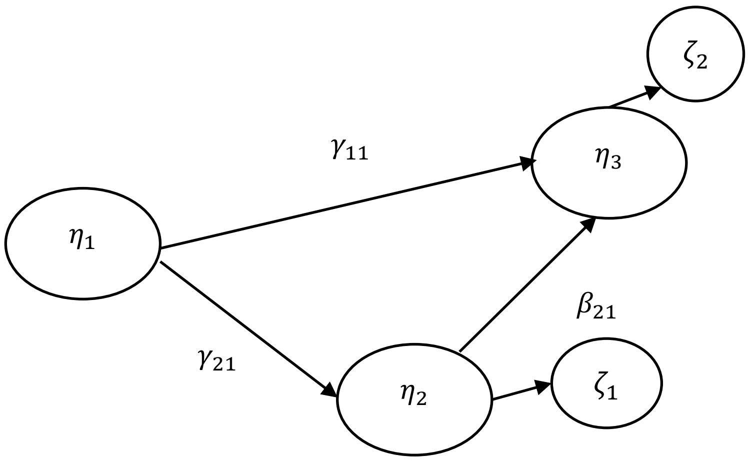


Figure 5 Hypothesized Latent Variable Model

From Figure 5, the following structural equations can be obtained:

$\eta_3 = \gamma_{11}\eta_1 + \zeta_1$……………..[11]
$\eta_3 = \beta_{21}\eta_2 + \gamma_{21}\eta_1 + \zeta_2$ ……………..[12]
Which can be generalized into the matrix $\Lambda = \mathrm{B}\eta + \Gamma\xi + \zeta$ ……………..[22]
And

$\mathrm{B} = \begin{bmatrix} 0 & 0 \\ \beta_{21} & 0 \end{bmatrix}$ $\Gamma = \begin{bmatrix} \gamma_{11} & 0 \\ \gamma_{21} & 0 \end{bmatrix}$, $\xi = \begin{bmatrix} \xi_1 \\ 0 \end{bmatrix}$, $\zeta = \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}$ where γ_{11}, γ_{21} and β_{21} are regression parameters and ζ_1 and ζ_1 are errors.

Eventually, the full model as shown in Figure 6 refers to a single model which links the measurement model and the structural model parts together.

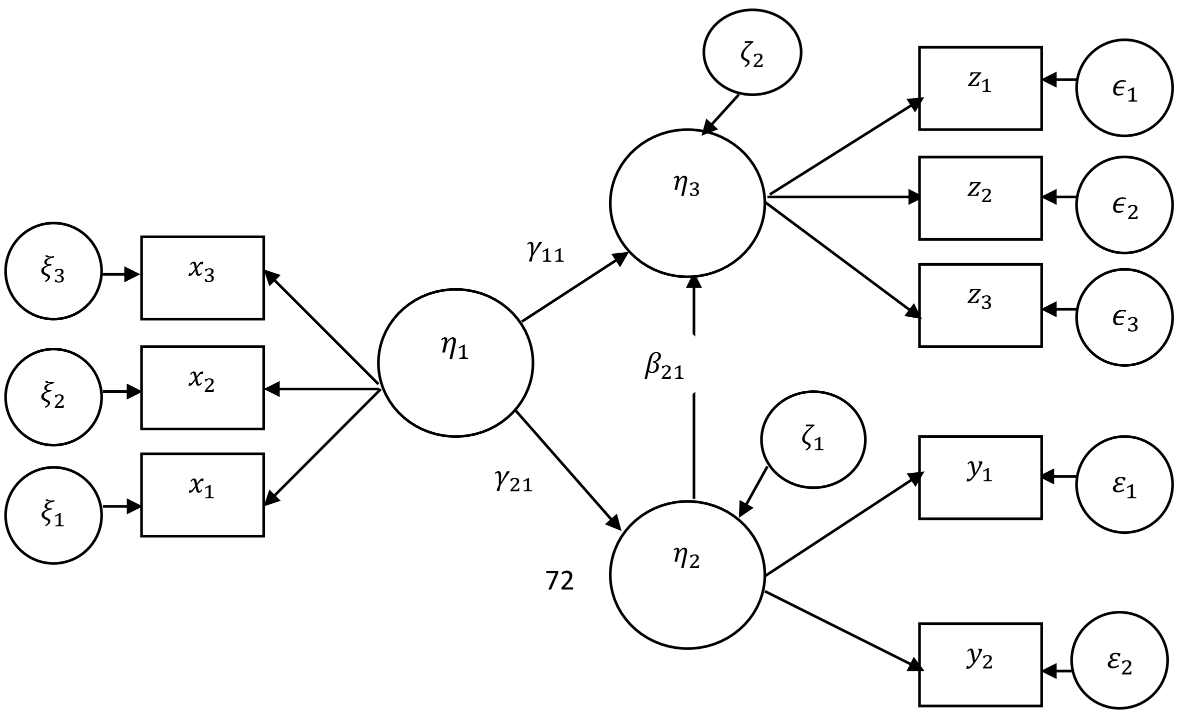


Figure 6 Full Model Linking Measurement Part and Structural Part of the Hypothesized Model

3.2.3.3. Addressing the research questions and accompanying hypotheses

The final part of the model specification was to examine and clarify whether the model could in deed address the intended research questions and hypotheses. It should be indicated that Research Question 1 was addressed using descriptive and observed correlation statistics. Therefore, the model largely focused on addressing Research Questions 2, 3 and 4 using structural equations.

In this final model, equation $\eta_3 = \gamma_{11}\eta_1 + \zeta_1$…………….[11] was solved to address Research Question 2: "How do spatial ability and basic Geometry content knowledge relate to each other? This entailed whether the parameter γ_{11} in equation [11] was significantly different from zero. Therefore, by referring to the structural model in Figure 5 and research hypothesis H_{02}, we have:

$$H_{02}: \gamma_{11} = 0$$

Furthermore, the equation $\eta_3 = \beta_{21}\eta_2 + \gamma_{21}\eta_1 + \zeta_2$ ……………..[12]would be solved to address Research Question 3: "To what extent does verbal reasoning mediate the effect of spatial ability on Basic Geometry content knowledge?". In that case, the parameters β_{21} and γ_{21} were estimated and tested separately using critical ratios to find out if they significantly differ from zero. The squared multiple correlation revealed the statistical significance of equation [12]. Thus, by referring to Figure 5 and the research hypothesis H_{03}, we tested:

$$H_{03A}: \gamma_{21} = 0$$

And

$$H_{03B}: \beta_{21} = 0$$

Finally, Research Question 4 was “Do the relationships between spatial ability, verbal reasoning and knowledge in geometry differ by (i) gender and by (ii) programme?”

The full model was maintained to answer these research questions by examining the structural invariance across gender and across programmes of study using multi-stage SEM (MSEM).

To be able to address these research questions and their corresponding hypotheses, the full model should first fit data well. If the model fit data well, the model fit indices would demonstrate a non-significant difference between the observed covariances or correlations derived from data and the estimated covariances/correlations matrix.

This would answer the main hypothesis H01: “There is no a structural relationship between spatial ability, verbal reasoning and Basic Geometry content knowledge” and allow for addressing all other research questions and hypotheses.

Step 2: Model Identification

One key issue in SEM was the question of whether the specified model is mathematically identified (Kline, 2015). This step was needed to check whether it was possible to apply SEM analysis to find unique estimates of the parameters in the set of equations of the hypothesized model. Model identification also provided information regarding whether at least a unique solution can be obtained for each parameter estimate in the model from the observed data (Byrne, 2010). Arbuckle (2013) stated three possibilities that models can be identified for SEM analysis. These include:

- unidentified model
- just-identified model or

- over-identified model

For unidentified model, the number of observations would be less than the number of model parameters. If the parameters to be estimated exceeded the item variances and co-variances, estimates would not be computed. This would require the model to be redefined or restructured. For just-identified model, exactly sufficient degrees of freedom should exist for estimating all free parameters in the unique set of path coefficients. According to Arbuckle (2013), it is often worthless evaluating model fit of just-identified models since it only defines trivial perfect fit. For model to be over-identified, the observations should necessarily be more than the model parameters. Over-identified models are usually the subject matter for investigation or evaluations since they provide multiple solutions.

To identify the model, we needed to calculate the degrees of freedom representing the difference between the number of distinct sample moments and number of parameters to be estimated. The formula $[q\ (q + 1)]/2$, where q represents the number of observed or measured variables, can be used to identify the model (Kline, 2015). For this study, the full model comprises eight ($q = 8$)observed variables. Using the above formula, there were 36 data-points and 19 distinct parameters to be estimated resulting in difference of 17 degrees of freedom. With complex models, arriving at the degrees of freedom manually could be cumbersome because of the large number of distinct sample moments and parameters in the model. A statistical software known as Analysis of Moment Structures (AMOS) was used to perform the identification in this study. AMOS was deemed preferable in model fitting process because it provided reasonable warnings about under-identification conditions. AMOS was also user-friendly and allowed the researcher to avoid complex syntax in LISREAL, R or similar applications. The researcher also found that AMOS output tables and graphical analyses were also easy to understand and interpret by those with less statistical background. Hence, AMOS was considered good software for me as a new SEM user. The analysis of the variable counts and computation of degrees of freedom of the hypothesized model is presented as follows;

Variable counts

Number of variables in the model:	1
Number of observed variables:	
Number of unobserved variables:	3
Number of exogenous variables:	1
Number of endogenous variables:	0

Computation of degrees of freedom

Number of distinct sample moments:	6
Number of distinct parameters estimated:	9
Degrees of freedom (36 - 19):	7

In the analysis, the difference between the number of distinct sample moments and the number of distinct parameters to be estimated was 17. Since $17 > 0$, the specified model was over-identified and qualified for parameter estimation.

Step 3: Model Estimation

To apply SEM, an adequate sample size was required, and the data would need to meet normal distributional assumptions. For example, Maximum Likelihood Estimation method in SEM requires multivariate normally distributed continuous variables. As a rule of thumb, Kline (2015) recommended that the sample size should be more than 25 times the number of parameters to be estimated, the minimum being a subject- parameter ratio of 10:1. For this study, 19 parameters would be estimated. Hence, the expected sample size would be 475. However, Kline (2015) recommended a lower bound of total sample size of at least 200. Using AMOS was a suitable way of solving the sets of structural equations specified in this study. It provided estimation of the model parameters in composite form. Several SEM parameters estimation techniques including Maximum Likelihood Method of Estimation (ML), Generalized Least Squares (GLS) and Unweighted Least Squares (ULS), Scaled free Least squares (SLS) and asymptotically distribution free (ADf) are incorporated in AMOS. The choice of one of these techniques for model estimation was based on the variable scale and distribution properties of the data. Among these techniques, ML has been widely used in several studies involving SEM. For estimating model parameters in this study, ML was preferred based on its advantages of producing asymptotically efficient, unbiased and reliable estimates with reasonably large size of sample (Hair, Black, Babin & Anderson, 2010).

Step 4: Model Evaluation

In this study, Chi-Square Test and four other absolute model fit indices were used to evaluate the model. The absolute indices were Goodness of Fit Index (GFI), Adjusted Goodness of Fit Index (AGFI), Root Mean Square Error of Approximation (RMSEA) and Root Mean Square Residual (RMR). In evaluating model involving SEM, these indices are often considered adequate for testing the model fit with data. However, further examination of the extent to which the model fits data were conducted using the baseline comparison indices including as Normed Fit index (NFI), Relative Fit Index (RFI), Incremental Fit Index (IFI), Tucker-Lewis Index (TLI) and Comparative Fit Index (CFI). Detail descriptions of each of the model fit indices are presented as follows:

The Chi-Square Test (χ^2)

The Chi-Square test was used to examine statistical significance of the difference between two structural equation models of the data in which one model was a nested subset of the other. According to Arbuckle (2013), χ^2compares the modified model with the independent model to examine difference between the sample variance-covariance matrix and the estimated variance-covariance matrix. The hypothesized model was accepted if there is no significant difference between the unconstrained original model and the nested constrained modified model. In other words, if no significance difference existed, then the model would not depart from the data collected.

Kline (2015) noted that for larger samples, the Chi-square would be predisposed to give significant values. Hence, Chi-Square test was not used exclusively to evaluate the model because it could yield unreliable index since the sample size was very large. Arbuckle (2013) suggested, the normed chi-square ($\chi 2/df$) which describes the ratio of the chi-square value and its degrees of freedom. If the normed chi-square ($\chi 2/df$) value is less than 3.0 or 5.0 the model is tenable.

Goodness of Fit Index (GFI)

Goodness of Fit Index (GFI) was based on the ratio of the sum of the squared differences between the observed matrix and the reproduced matrix to the observed variances in the study. GFI ranges from 0 to 1 and tends to be larger as the sample size increases. GFI values exceeding 0.9 indicate good fit to the data (Kline, 2015). Adjusted Goodness of Fit Index (AGFI) is a variant of GFI for degrees of freedom of a model relative to the number of variables. AGFI also ranges from 0 to 1. Values close to 0 reflect poor fit whiles values close to 1 signify good fit of model to data.

Root Mean Square Error of Approximation (RMSEA)

Root Mean Square Error of Approximation (RMSEA) describes the discrepancy per degree of freedom. Thus, RMSEA values less than or equal to 0.05 significant level indicated good model with values close to 1 revealing near perfect fit. However, in line with McGrew, Keith, Floyd and Taub (2008); and Hu and Bentler (1999), RMSEA value less than .06 was also accepted as good fit.

Baseline Comparisons Indices

Five baseline comparisons indices were also used to evaluate the hypothesized model fit to data. These indices included Normed Fit Index (NFI), Relative Fit Index (RFI), Incremental Fit Index (IFI), Tucker-Lewis Index (TLI) and Comparative Fit Index (CFI). The Tucker-Lewis Index also called NNFI compares a proposed model"s fit to a nested baseline or null model and assesses the degrees of freedom from the proposed model to the degrees of freedom of the null model. Arbuckle (2013) recommends the use of TLI because of its known robustness against variations in sample size. These indices according to Hooper, Coughlan and Mullen (2008) provided possibility for comparing the hypothesized model to the saturated and independence models for fit to data. The cut off values for these indices to conclude on the good of fit of a hypothesized model are presented in Table 1.

Table 1 Cut off Values (Criteria) of the Fit Indices used in the Model Evaluation

Type of Fit Indices	Fit Index	Cut off Criterion or Thresholds in SEM
Absolute fit Indices	Chi-Square (χ^2)	Non-significant is generally agreed by authorities (Hooper, Coughlan & Mullen, 2008)
	Relative/normed chi-square ($\chi 2/df$)	Recommendations range from as high as 5.0 (Hooper, Coughlan & Mullen, 2008) to as low as 2.0 (Tabachnick & Fidell, 2007).
	Goodness-of-Fit Index (GFI)	Traditional omnibus cut-off point of 0.90 but a higher cut-off of 0.95 is more appropriate ((Hooper, Coughlan & Mullen, 2008); (Kline, 2015)
	Adjusted Goodness-of-Fit Index (AGFI)	Generally accepted value is 0.90 or greater (Hooper, Coughlan & Mullen, 2008)
	Root Mean Square Error of Approximation (RMSEA)	RMSEA ≤ 0.05 A cut-off value close to .06 (Hu & Bentler, 1999) or upper limit of 0.07 (Kline, 2015) suggests $< .08$
	Root Mean Residual (RMR)	Obtaining values less than .05 or as high as 0.08 are deemed acceptable (Hu &Bentler, 1999) (Hooper, Coughlan & Mullen, 2008)
Incremental Fit Indices or Baseline comparisons Fit Indices	Normed Fit index (NFI)	Recommendations as low as 0.80 as a cutoff have been preferred however Bentler and Hu (1999) have suggested NNFI ≥ 0.95 as the threshold. (Hooper, Coughlan & Mullen, 2008)
	Relative Fit Index (RFI)	RFI ≥ 0.95 as the threshold (Hooper, Coughlan & Mullen, 2008)
	Incremental Fit Index (IFI)	IFI ≥ 0.95 as the threshold (Hooper, Coughlan & Mullen, 2008)
	Tucker-Lewis Index (TLI)	TLI or NNFI ≥ 0.95 as the threshold (Hooper, Coughlan & Mullen, 2008). TLI ≥ 0.90 is acceptable threshold
	Comparative Fit Index (CFI)	A cut-off criterion of CFI ≥ 0.90 was initially advanced however, recent studies have shown that a value greater than 0.90 is needed (Hooper, Coughlan & Mullen, 2008)

Step 5: Model Re-specification or Modification

In instances of misfit or poor fit of the specified model, re-specification was done to adjust those indices which failed to achieve the mandatory values. Usually, AMOS suggested certain modifications to the paths or error variances by reporting the amount of reduction in discrepancy and the margin of change in estimate when the covariance between errors are treated as a free parameter (Arbuckle, 2013). Hence, with the aid of AMOS, and depending on the effect on the model indices, such paths or error covariances were added or deleted during evaluation. Another option was to allow the error variances to co-vary. One other advantage of the use of AMOS was the fact that when the models were unidentified during testing, AMOS would either request for imposition of additional constraints or declare the model inadmissible. In such situations, additional constraints could be included to ensure identifiable model.

3.3. Population under Study

This study was framed within the basic school teacher training context in Ghana. In Ghana, the institutions which train pre-service teachers in Ghana are referred basically to as Colleges of Education. These colleges which offer 3-year diploma certificate in Basic Education are distributed in all ten regions in Ghana. Only senior high school graduates with requisite grades in core mathematics, core English and core science including at least two elective courses are often shortlisted and admitted into these colleges.

In this study, an entire population estimated was 16,045 second year pre-service teachers pursuing 3-year diploma in Basic Education at the 46 public colleges of education in Ghana. The target population however comprise 13, 604 from 39 colleges. Seven (7) colleges were not involved in the study because they appeared less-established having been recently absorbed by government as public colleges in 2016 and 2017. Of the 39 colleges considered, 15 colleges offer mathematics/science programmes and 24 offers general programmes. The 15 colleges are mandated to train specialist teachers in science, mathematics and technical skills for Junior High Schools and Primary schools in Ghana (Institute of Education, UCC, 2014). The general programmes colleges are also mandated to train generalist teachers for only primary schools in Ghana. Notwithstanding these core mandates, few of these colleges also offer other specialist diploma programmes such Early childhood, French and Hearing/Visually Impaired. After subtracting these other specialist diploma programmes, approximately 3,861 and 8,697 of the second-year pre-service teachers were in the science/mathematics and general programmes colleges respectively

(Institute of Education, UCC, 2017). The focus of the present study was therefore specifically on the science/mathematics specialists and the generalists in their second year. This choice was based on the nature of geometry course content as well as the fact that second year pre-service teachers were exposed to the full content of geometry required to teach at basic school by the end of second year when they prepare to undertake their practicum.

The course contents of the syllabus of the colleges of education in Ghana have been differentiated according to specialist and generalist pre-service teachers (Institute of Education, UCC, 2014). In terms of geometry, both the specialist and generalist pre-service teachers were exposed to ascribed geometry content among other mathematics courses. In the first year second semester of the generalist programmes, the pre-service teachers took FDC 122 Geometry and Trigonometry and in their second year second semester they were exposed to PFC 222 Methods of Teaching Basic Mathematics including shape and space concepts. For specialist pre-service teachers in science and mathematics, FDC 112M Algebra and Geometry course was taught in the first semester of first year and PFC 222M Methods of Teaching Basic School Mathematics, including shape and space, was taught during their second semester of second year. A comparative analysis of the course descriptions showed that there was more content depth in the specialist science programmes than in the general programmes. The content variations suggested the need to study the knowledge of geometry of pre-service teachers separately to clarify any differences in each programme. The categorization was therefore to ensure that the sample bore characteristics depicting the difference in geometry content taught at the colleges of education in Ghana.

3.3.1. Sampling and sample size

Having described the nature and distribution of the pre-service teachers considered for the study, the sample and sampling techniques adopted were considered. A two-stage sampling technique involving stratified sampling of colleges and simple random sampling were employed to select the colleges. This involves dividing the population into homogeneous groups (Cohen, Manion & Morrison, 2007). Thus, the number of public colleges of education in Ghana were stratified into those offering science or general programmes. There were 15 and 24 colleges in each category. A ratio of 15:24 was used in which the sample was determined by reducing the ratio to the lowest terms of 5:8. This was deemed as the lowest proportional sample representative of the programmes distribution at the colleges. Simple random sampling was employed in which all 15 science colleges were labelled in pieces of paper and kept in bin. Then, five (5) colleges were drawn one after the other without replacement to represent the science category. Similar approach was employed to select eight (8) colleges from the 24 to represent the general programmes category. This ended up with thirteen colleges; five (5) science colleges and eight (8) general programmes colleges. In all, 13 colleges of education were to be considered as initial sample. However, data were not obtained from one of the general programmes colleges because the colleges vacated as of the time the researcher got to that college.

Several attempts to visit the college to collect data before vacation period proved futile. It was not possible to access that year group again since they were scheduled to begin their practicum on the field the succeeding semester. The main study therefore relied on the data collected from the 12 remaining colleges which represented about 31% of the 39 colleges in Ghana. As a guiding principle in cross-sectional survey design, a minimum random sample of 20% of the population of second years was recommended (Robson, 2002). This 31% was therefore considered adequately representative and generalizable to the number of colleges under study. All second-year pre-service teachers of the selected colleges were supposed to provide data for the study. However, those who responded to the instruments were 953 as distributed in Table 2.

Table 2 Sample for the Study

/n	College of Education	eneral	(	cience	:
	College A, Upper East			9	
	College B, Upper West	2	6		
	College C, Northern Region	21	1		
	College D, Brong-Ahafo	1	7		
	College E, Ashanti			4	
	College F. Ashanti	29	1		
	College G Western Region			5	
	College H, Volta Region	7	2		
	College I, Volta Region			6	
0	College J, Volta Region	9	8		
1	College K, Central Region			7	
2	College L, Eastern Region	3	9		
	TOTAL	92	5	61	53

It must be noted that the educational attributes of students from the selected colleges of education were comparable to all other college students in Ghana in the sense that they:

- are admitted under same qualification from senior high school certificate examinations.
- use unified national syllabus developed by the Institute for Education, University of Cape Coast
- are taught by tutors who bear similar professional certificates in education
- write the same external examination to be able to progress from one year to another.

Thus, the sample could be described as having representative characteristics of the study population.

3.3.2. Sample size requirement

Further analysis to ensure adequacy of the sample size requirement for the study was conducted using the margin of error of 5%. According to Yemen‟s (1996) table of sample size requirement, a minimum sample of 357 was needed for the population of 3,861 science pre-service teachers while the population of 8,697 general programmes pre-service teachers required a minimum sample of 370. Since the sample of 361 science pre-service teachers and 592 general programme pre-service teachers exceeded 357 and 370 respectively, the samples were acceptable.

3.4. Data Collection Instruments

In this study, data were collected using tests. The tests comprise sections A, B and C on spatial ability, section D on verbal reasoning and section E on Basic

Geometry content knowledge. The spatial ability tests were used to measure three spatial factors - spatial visualization, spatial perception and mental rotation. The verbal reasoning test comprises nonsense syllogistic statements and comprehension tasks. Finally, the geometric knowledge test was structured to measure declarative, conditional and procedural knowledge on basic shape and space. Participants‟ gender and programme of study were included in the instrument to highlight participants‟ characteristics.

3.4.1. Spatial ability tests

In search for the type of spatial abilities tests, Study (2012) cited that over 50 different tests were found in literature. These tests include 2D and 3D mental rotations, speed visual explanation, flexibility and speed closure, long-term spatial location memory and measurement of space. Though, these tests differ in format and scope, many of them have similar levels of reliability and validity. In his overview of tests of cognitive spatial ability, Study (2012) declared that the choice of these tests depends on the type of spatial skill to be assessed within the specific field or discipline. In addition to the declaration by Study (2012), the study took into consideration those types of spatial ability tests which could minimize verbal ability interferences since verbal reasoning was also measured in the study. In view of this, the paper folding, card rotation and water level task were adapted. The paper folding and card rotation are part of factor-referenced cognitive tests developed by Ekstrom, French, Harman & Dermen (1976) and distributed for worldwide for spatial ability research. Yurt and Sünbül (2014) recently found Cronbach Alpha Reliability Coefficient of 0.75 (n=70) for the Paper Folding Test and 0.72 (n=70) for the Mental Rotation Test. These tests are explained in detail in the following sub sections.

Paper Folding Test

The paper folding test (i.e. Vz-2) which involved the punched holes was selected from the Educational Testing Services kit for factor-referenced cognitive tests revised version (Ekstrom, French, Harman & Dermen,1976, p. 286). The test is documented to be suitable for grades 9 to 16 in the United States of America. In Ghana, this grade levels range from Junior High School 3, Senior High School, Colleges of Education, to university and other tertiary levels of education. Second year pre-service teachers who constituted the sample in the study were comparable to grade 14 in terms of the educational structure in Ghana. The test was therefore deemed suitable for pre-service teachers considered in this study.

The paper folding test contains 10 items to be completed in 3 minutes and widely used to measure spatial visualization skill (Ekstrom, French, Harman & Dermen, 1976). This test involved diagrams of punched holes on a folded paper before opening it for mental examination (refer to Appendix A for the paper folding test items). In each test item, a paper was folded a given number of times in sequence before a hole was punched through it. The paper was then unfolded. The task of respondent was to choose from 5 images, which image depicted the right positioning of the holes if the paper was unfolded. The task is a mental exercise and hence respondents were not supposed to do any physical paper folding by themselves before choosing the right option. Tasks 1 involved one-step folding while tasks 2, 3, 4 and 5 involved two steps simple horizontal and vertical configurations. Tasks 6, 7, 8, 9 and 10 involved two to three-step mental configuration involving horizontal, vertical and diagonal foldings. Participants were required to keep the previous folding steps in mind while imagining the next folding arrangement. In the final step, ones‟ ability to relate the set of folding steps and visualize the positioning of the punched holes in an unfolded configuration was the focus.

The reliability estimates in a normal sample studies using this paper folding tasks ranged from .75 to .84 and hence indicated high internal consistency. In a recent study in Ghana among pre-service teachers, Akayuure et al (2016) reported a reliability of .77. This test was therefore selected based on its high internal consistency in studies of normally distributed sample.

Mental Rotation Test

Literature search revealed that two popular tests - card rotation and cube comparison tests - were often used in testing mental rotations. Vandenberg and Kuse (1978) reported that these tests showed respectable reliability coefficients of about .83 to .88. Burton and Fogarty (2003) however found that the Cronbach alpha coefficient for the Card Rotation Test of .96 was comparatively higher than .80 for the Cube Comparison Test. Blazhenkova and Kozhevnikov (2008) also reported an internal reliability of .88 for card rotation test using K-R 20. Thus, in this study, the card rotation test (S-1) was preferred for the measure of mental rotation based on its high consistency reported in literature (Olkun, 2003). The test can be seen in page 246 of Ekstrom, French, Harman and Dermen (1976). The test is suitable for grades 8 to 16. Each item of the card rotation test consisted of a given object displayed on the left column and eight similar objects on the right column representing options. Respondents were required to indicate in terms of rotation whether the objects on the right is the same as (S) or different from (D) the object at the left. The test contained 10 items which should be taken in 3 minutes. The first group of tasks 1, 2, 6, 8 and 9, involved objects made from straight lines. These tasks appeared quite easy to rotate mentally. However, the second group of tasks 3, 4, 5, 7 and 10, involved straight and curvilinear objects. These tasks required the respondent to mentally rotate the figures in clockwise or anticlockwise direction. These second tasks were assumed to be more demanding than the first group of tasks. Each respondent scores of mental rotation score was calculated as the number of items correctly identified. The highest score for a respondent was 80 (10 items of 8 pairs of options) (refer to Appendix A for the card rotation test items).

Water Level Task

Water level task (WLT) was adapted for the study. The test was originally developed by Piaget and Inhelder (1967) and became a widely used test for measuring spatial perception. The tasks measure the ability to perceive space in the Euclidean system. The respondent was supposed to draw a straight line to show the water surface level when a half-filled bottle of water was titled in a given orientation. There were eight bottles tilted in different degrees to the horizontal (refer to Appendix A for the water level task). Respondent was scored correct if the line s/he drew to represent the water level lie within an error of margin of 5 degrees of the horizontal.

For the water level test, task 1 was a bottle in its upright position, 90 degrees to the horizontal. In Task 2 and task 6, the bottle was tilted to form an angle of 30 degrees to the positive x-axis. In task 3 and 4, the bottle was 45 degrees to the negative and positive x-axis respectively. The bottle in task 5 was 60 degrees to the positive x-axis. In tasks 7 and 8, the bottle was 15 degrees to the negative x-axis and positive x-axis respectively. Finally, the view of the bottle in task 9 was 30 degrees to the negative x-axis. Reliability coefficients reported in previous studies ranged from .80 to .92. Even though this test was originally designed for 9 to 12-year old children, recent literature portray that they were suitable for measuring spatial knowledge of secondary and college students (George, 2017).

3.4.2. Verbal reasoning test

Blazhenkova and Kozhevnikov (2008) stated that while there are many validated visual-spatial scales, few comparable verbal scales appear to exist in literature. The items testing verbal reasoning also measured relatively homogeneous and bounded set of skills. They typically comprise word sequences, synonyms, antonyms, non-sense syllogisms, analogies, diagramming relationship, inferences and comprehension. In this study, only nonsense syllogism and comprehension were considered. This is because word sequence, synonyms and antonyms reflect more of verbal ability in English language and subject to prior knowledge than verbal reasoning (Langdon & Warrington, 1995; Polk & Newell, 1995). The emphasis in the present study was on ability to encode textual information logically without necessarily applying previous knowledge. Hence, nonsense syllogism and comprehension which relate to logical reasoning were considered appropriate.

Nonsense Syllogism

Syllogistic reasoning can be important in understanding geometric definitions, properties, axioms, proofs and inferences. Some of the test items used to measure the van Hieles‟ geometric thinking levels were typically based on syllogistic reasoning. In syllogistic reasoning, respondents were given only one model logical statements to provide valid conclusions. The researcher selected syllogisms for which the valid conclusions depended on only one model because according to Polk and Newell (1995), multiple models were difficult and unsuitable for students who have not studied logics. The level of difficulty of verbal reasoning test was also determined by the length of the statements, complexity of the text, simplicity of words and time constraints. Based on this, the research adapted the nonsense syllogisms from the Educational Testing Services kit for factor-referenced cognitive tests (refer to

Appendix B for the nonsense syllogisms). For the syllogistic statements, tasks 1, 2, 4 and 5, used the "all" logical reasoning while tasks 3 and 6 used the "some" logical argument. The last task 7 used the "no" logical argument.

Comprehension Passage and Statement

The comprehension tasks 1 and 2 were adopted from van Hieles" test items (refer to Appendix B). The first task is a short passage on an unconventional geometric property of intersecting and parallel lines whose definitions must be determined from the passage and not the usual geometric definitions. Participants were required to apply the definitions to identify intersecting and parallel lines or otherwise. Four questions were asked from the passage. The second comprehension task involved another unusual geometry invented by a mathematician which relied on the premise that "The sum of measures of the angles of a triangle is less than 180^0". Participants were required to choose from among four statements the one which is valid with respect to the premise.

3.4.3. Basic Geometry content knowledge test (KTG)

One other focus of the study was on knowledge of basic shape and space concepts needed to teach at the basic schools in Ghana. In the colleges of education curriculum, pre-service teachers need to possess this kind of specialized knowledge of geometry to be able to effectively teach the shape and space concepts in the Ghanaian basic mathematics curriculum (Institute of Education, 2014). Therefore, test items were designed to measure pre-service teachers" content knowledge of shape and space concepts and problems in the basic school mathematics syllabus (Ministry of Education, 2012). The content scope within which pre-service teachers should know to teach these concepts included relations basic angles, triangles, quadrilaterals, prisms, pyramids and their properties.

To inform the nature and structure of the test, test items from international geometric test, van Hieles test, Q-level test, geometric knowledge for teaching (Gogoe, 2009; Blanco, 2001), and end-of-semester geometry examinations for colleges in Ghana were critically analyzed. After extensive review, it appeared multiple choice items might promote guess work and conceal the actual performance of pre-service teachers on knowledge-based tasks. Furthermore, it was quite clear that allowing participants to provide short answers and solutions would effectively elicit their knowledge about shape and space. The present study focused on specialized knowledge for teaching Geometry which measured largely rather than pedagogical interpretation. This was deemed necessary since their content knowledge problems have been well-document (Armah et al, 2017). Therefore, instead of using tasks on students" representations, understanding, deduction of errors and misconception, and difficulties (Baumert, et al, 2010), the tasks concentrated on content knowledge about angles, triangles, quadrilaterals, prisms and pyramids. The items reflected the three knowledge types in this study (see Appendix C). One set of sub-tasks were designated to measure declarative knowledge of pre-service teachers. These items comprise identifications, definitions, recall of properties, representation and classification of shapes. The other two sets of items were designed to measure conditional and the procedural knowledge of shape and space.

For declarative knowledge, tasks 1 and 2 involved declarative knowledge on the formation of angles. Tasks 6a and b were on the concept of triangle and its properties. Tasks 9a, b, c and d concerned the concept of square and its properties as well as the lines of symmetry and rotational orders. Tasks 10a and b concerned recognizing triangular prism and its faces. Finally, 16a and b involved naming one unique property of a pyramid and the faces of rectangular pyramid.

For conditional knowledge, tasks 3 and 4 were open problems eliciting participants‟ reasoning regarding angles and number of angles formed when two straight lines cross each other. Task 7 involved the concept of angle measures in every isosceles triangle. Task 9f was on transformation of quadrilateral into another quadrilateral when two opposite sides were increased by 2 units. The concept of prism and pyramid were also included in the tasks. Tasks 12 was an opened question that elicited conceptual knowledge of a given prism when all its faces were squares and the base was pentagon. Tasks 15 and 18 concerned justifications of why a cylinder is or is not a prism and why a cone is or is not a pyramid. Task 19 required a justification of relationship between a cuboid and a cube. The last task 20 concerned the determination of a new figure configured when the vertices of a triangular prism are pulled together straight up to form a tip.

The tasks designed to measure procedural knowledge also involved angles, triangles, quadrilaterals, prisms and pyramids. Task 5 related to knowing how to sketch angles formed when two straight lines cross each other in the plane. This concept and its procedural knowledge are often applied in many problem-solving situations in geometry. Task 8 was however centered on finding the area of a right-angled triangle. For quadrilaterals, task 9e, 9g and 9h were meant to assess how to find the area of quadrilaterals such as the kite, sketch line symmetry and find area of a square transformed into rectangle. Tasks 11a and b also elicited knowledge on sketching the net and finding the area of rectangular prism, whiles tasks 13 and 14 concerned finding the volume of a milo tin and the surface area of a cube. Finally, task 17 was designed to measure the knowledge in finding the volume of a cone.

To ensure representation and content validity, an item specification table was created for declarative, conditional and procedural questions. Declarative knowledge questions focused on facts, names, and lists and involved the "what . . .", "how many . . ." and "name one . . ." types of tasks. Conditional knowledge tasks focused on understanding a network of condition-action sequences and predicting what happens "if" one of the variables in the sequence changes within the context of "if . . . then . . ." and "why . . ." relationship statements. Procedural knowledge tasks involved "how to" problems such as the sketch of figures, use of rules and formulae in solving problems. The distribution of the task items based on the three knowledge types is shown in Table 3 (refer to Appendix C for test items).

Table 3 Items Specification by Knowledge Types for Teaching Geometry

Content	Declarative	Conditional	Procedural
Angles	1 and 2 angle formation concepts	3 and 4: corresponding angles and number of the angles formed when two straight lines cross each other	5.Sketching the angles formed when two straight lines cross each other
Triangles	6a and b triangle and its properties	7.concept of same angle measures in every isosceles triangle	8. finding the area of right-angled triangle
Quadrilaterals	9a and b recognizing a square and its unique properties 9c and d line of symmetry and rotational order	9f. new shape when two opposite sides of a square is increased by 2units	9e. finding the area of kite 9g. sketching lines of symmetry 9h. finding the area of a transformed square(rectangle)
Prisms	10a and b recognizing a triangular prism and its faces	12. if all faces of a prism are squares and the base is pentagon, how will you call such a prism? 15.justifying why whether a cylinder is a prism 18.justifying whether a cone is a pyramid	11a and b: Sketching the net and finding the area of a rectangular prism
Pyramids	16a and b naming one unique property of a pyramid and faces of a rectangular pyramid	19. justifying when a cuboid is a cube 20.determining a new figure formed when the vertices of a triangular prism are pulled to a tip	13. finding the volume of a milo tin (cylindrical) 14. finding the surface area of a cube 17.Finding the volume of a cone

As shown in Table 3, the items reflected the entire scope of shape and space contents in the basic school mathematics syllabus as well as in the courses on Geometry and trigonometry, and Methods of teaching basic school mathematics. The items were also evenly specified such that the tasks for each knowledge type embraced a specific content area of shape and space.

3.5. Threat to Validity Issues

The internal and external validities of the study were considered. Fraenkel and Wallen (2006) explained internal validity of a study as observed differences on dependent variable that are not due to some other unintended variables. Wide differences in sample characteristics such as educational experience, gender, age gap, can often lead to threat to internal validity of a study. In this study, the pre-service teachers were assumed to have had similar prior educational experiences since they were all products of the senior high schools in Ghana. They were also second year students who as at the time of the study had completed both content and methodology courses on shape and space. As also shown in Table 4, number of males were more than the number of females and those offering science were less than those offering non-science programmes. These distributions reflected fairly the gender and programme representations of pre-service teachers at the colleges of education in Ghana. In terms of age, majority of the pre-service teachers were within the ages of 21 and 25 signifying that they were a class of young adults. A cross tabulation of pre-service teachers‟ gender by their age range and programme offered at senior high school is illustrated in detail in Table 4.

Table 4 Distribution of Pre-service Teachers' Gender by Age range and SHS Programmes (Pilot data)

Gender	Age Range	SHS Programme of Study					Total
		Science	Business	General arts	Visual arts	Others	
Male	20 and below	12	0	7	0	0	19
	21-25	74	41	47	23	63	248
	26-30	30	15	15	3	9	72
	31-40	3	0	0	0	0	3
	Total	**119**	**56**	**69**	**26**	**72**	**342**
Female	20 and below	4	5	8	3	0	21
	21-25	15	39	70	8	22	155
	26-30	2	2	6	0	6	16
	Total	**21**	**46**	**84**	**11**	**28**	**190**
Over all	20 and below	16	5	15	3	0	39
	21-25	89	80	117	31	85	402
	26-30	32	17	21	3	15	88
	31-40	3	0	0	0	0	3
	Total	**140**	**102**	**153**	**37**	**100**	**532**

The spatial ability tests were administered and supervised by the researcher within the classroom setting. This was done to ensure no location threat from collector characteristics and bias (Robson, 2002). The presence of the researcher largely ensured that respondents did not copy and were not aided to do the tasks. All participants were assumed to have therefore performed the tasks to the best of their abilities.

External validity refers to the extent to which the results of a study can be generalized (Fraenkel & Wallen, 2006). Three of the external validity issues were addressed. First was the population validity or representativeness. A random sampling technique was used to ensure that the study sample bore characteristics that were representative of the population of pre-service teachers in Ghana. Therefore, the generalizations of the findings of the study could be done on similar subjects at the colleges of education. Second was ecological validity. According to Fraenkel and Wallen (2006), ecological validity refers to what extent the results of a study can be extended to other setting or conditions. To ensure this, the spatial ability and verbal reasoning tests were adapted because of their use in many countries and educational settings (Ramful, Lowrie & Logan, 2017; Turgut & Yilmaz, 2012; Wai, Lubinski & Benbow, 2009).

To optimize this, the test items on geometric knowledge were carefully selected and constructed in line with the college-level and basic school mathematics syllabus on shape and space. Moreover, experts‟ reviews, pre-testing and pilot study were done to ensure the items measured the constructs intended and to eliminate items that seemed unclear, too difficult or too easy for respondents. This also helped to address the third external validity known as content validity.

3.6. Pre-Testing and Pilot Study

Pilot studies were important aspects of the study. They assist in providing useful information on the item structure, content/nature of the data, feasibility of data collection environment and accessibility. One paramount importance of a pilot study is to provide evidence of validity and reliability of the data collection instruments (Robson, 2002). While the general issue of improving design of research was examined, the pilot study here focused on the validity and reliability of the spatial, verbal reasoning and geometry tests. The spatial ability and verbal reasoning tests which were adapted from literature, needed to be validated in the Ghanaian context. A preliminary study to check feasibility, appropriateness and likely errors occurring during the main study was therefore imperative (Robson, 2002). Furthermore, since the geometry test items were constructed or derived from different study context, exploring construct validity and reliability were crucial to the success of the main study.

The pilot study was conducted using a random sample of pre-service teachers at two-science and two-general colleges of education. These pre-service teachers were used since they had characteristics like those used for the main study. To avoid any influence of performance of research participants during the main data collection, this sample were excluded from the main study. The sample for the pilot study comprises 532 second year pre-service teachers: 73.7% from the general colleges and 26.3% from science colleges. In terms of gender from general programmes colleges, 41.9% were males and 31.9% were females. Moreover, 22.4% of the 532 were males and 0.4% were females from science colleges. These proportions were representative of the normal population distributions in the Ghanaian colleges of education across gender and programme groupings.

Prior to piloting the tests, my supervisors examined the content and item structure as well as the scoring scheme of the spatial ability, verbal reasoning and geometry tests (See Appendix C). They found that the spatial and verbal reasoning tasks elicited the kind of reasoning they purported to measure and that the tests were useful, usable and applicable to the Ghanaian context. After critically scrutinizing the geometry test, they also established that the test content covered all the shape and space contents in the basic mathematics curriculum of which the pre-service teachers were exposed to at the colleges of education. The content coverage and item structure were also judged as appropriate for measuring declarative, conditional and procedural knowledge.

Two tutors from two of the colleges of education who taught "Methods of teaching mathematics course" were also requested to assess the content, items and ability of the participants to provide answers to the geometry test items. Both tutors were unanimous about the appropriateness of the content and the items structure. However, one tutor was doubtful about the ability of the students to answer the questions. According to him *"our students are used to chew, pour and forget. I doubt* *whether they can remember all these basic things"*. This tutor who judged the items as easy, did not suggest how the items could further be made less difficulty. Consequently, based on the overall remarks, the items were considered suitable for testing the knowledge-base of the pre-service teachers on basic shape and space concepts.

To administer the tests, clearance and formal approvals from the graduate committee of the Department of Mathematics Education, University of Education, Winneba and authorities of four selected colleges of education, were obtained. Consents from pre-service teachers participating in the pilot study were also sought before the administration of tests. One research assistant was recruited and trained to assist the researcher in the data collection. The researcher and the assistant visited each college at agreed times. All participating pre-service teachers assembled in their respective classrooms at agreed convenient times and took the tests. At each test centers, the preambles for the tests were explained including the allowable time for the tests. The three tests were administered to more than 532 pre-service teachers. However, 532 participants completed and returned their papers within the stipulated time.

Validity of Tests

Validity of a test refers to the ability of a test to serve its purpose adequately (Kline, 2015; Süß & Beauducel, 2015). For example, a test meant to measure students" knowledge of geometry must in deed contain items on geometry and such items must measure the required knowledge to a reasonable level of precision. Failure to attain precision of measure could render the instrument invalid. Checking validity issues of data collection instrument prior to the main data collection ensured some level of certainty regarding the suitability and adequacy of measurement indicators for each construct. It also helped to sieve out unwanted data to ensure items which did not measure the constructs well were deleted or reconstructed to reflect the construct properties. To perform the validity tests, the data of the pilot sample of 532 participants were keyed into SPSS, examined for missing data and cleaned up.

3.6.1. Construct validity of spatial ability (SA)

Though the paper folding, card rotation and water level tasks have been validated many times in studies since 1976, they were not known to have been carried out in large scale among pre-service teachers or even in Ghana. The only known study in Ghana which used these tests was a quasi-experimental study by Akayuure et al (2016). Dimension reduction through factor analysis using SPSS23 was carried out to validate the underlying spatial ability structure in the items (Kline, 2015). The analysis which included 28 observed variables employed principal component method. Descriptively, Kaiser-Meyer-Olkin (KMO) measure and Bartlett"s test of sphericity values for the items were examined for sampling adequacy and significant difference of values of items respectively. Table 5 presents the result of these tests.

Table 5 KMO and Bartlett's Test for Spatial Ability Construct

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.901
Bartlett's Test of Sphericity	Approx. Chi-Square	6092.948
	Df	378
	Sig.	.000

As shown in Table 5, the KMO measure revealed a value of 0.901. This value is greater than 0.5 (Field, 2013) or 0.60 (Tabachnick & Fidell, 2007) which suggested that the distribution of values was very sufficient to conduct a factor analysis. The Chi-Square value of 6092.948 from the Bartlett"s test of Sphericity as shown in Table 5 was also significant at .000 level. The statistical significance indicated that the data were adequate and appropriate for principal component analysis of the spatial factor structure.

With regards to the factor structure, three-factor solution was predicted in line with literature. Literature purported that paper folding, card rotation and water level tests measure three different spatial abilities (Vandenberg & Kuse, 1978; Piaget & Inhelder, 1967). Despite this, it was imperative to conduct exploratory analysis of the factor structure of the items because of variation in learning settings between Ghana and countries where the tests were validated. Hence, in the first analysis, a full factor solution based on eigenvalue of 1 or above was run. The result revealed that 64.6% of the sample variance was accounted for by the items. This yielded five-factor solution in which all card rotation items loaded on one-factor but the first three items of paper folding loaded on a separate factor. The water level tasks loaded on two factors with one factor loadings very high.

With reference to previous factor analytic studies, this result was not interpretable (Blazhenkova & Kozhevnikov, 2008). Therefore, to ensure interpretability of the factor solution, further analyses using the correlation matrix and scree plot were conducted and the presumed three factors were rotated in Varimax using Kaiser Normalization. The three factors were set since single factor of spatial ability have been disputed in literature (Cervigin, 2012). The output as presented in Table 6, limited the Eigen values to minimum of 2.442 and resulted in approximately 60.4% of the sample variance accounted for by the items. Consequently, further analysis of item to item loading was done. In the rotated component matrix, it was observed that no item loaded lower than ± 0.3 to be suppressed (Leech, Barett & Morgan, 2005) or ± 0.5 to be excluded in the factor structure (Tabachnick & Fidell, 2007). The final three-factor structure corresponded with Linn and Petersen‟s (1985) spatial ability classification. The eigenvalues, percentage and cumulative percentage of variances accounted for in each of the three spatial abilities are shown in Table 6.

Table 6 Percentages of Variance Explained in Spatial Ability

Factor	Initial Eigenvalues			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cum. %	Total	% of Variance	Cum. %
Spatial visualization	8.474	30.266	30.266	7.059	25.211	25.211
Mental rotation	3.715	16.884	47.150	3.791	17.780	42.991
Spatial perception	2.442	13.267	60.417	3.781	17.426	60.417

In the three-factor solution as shown in Table 6, all paper folding items loaded on spatial visualization, all card rotation items loaded on mental rotation and all water level tasks loaded on spatial perception in accordance with literature. Table 6 also shows that, 25.2%, 17.8% and 17.4% of the total variance were accounted for by the spatial visualization, mental rotation and spatial perception items. The total variance accounted for by all three factors was 60.4%. Since the variance captured by the items measuring the three spatial ability factors exceeded 50%, it can be concluded that the items cumulatively demonstrated the construct validity. As shown in Figure 7, the scree plot of component numbers against Eigen values confirmed the three-factor structure.

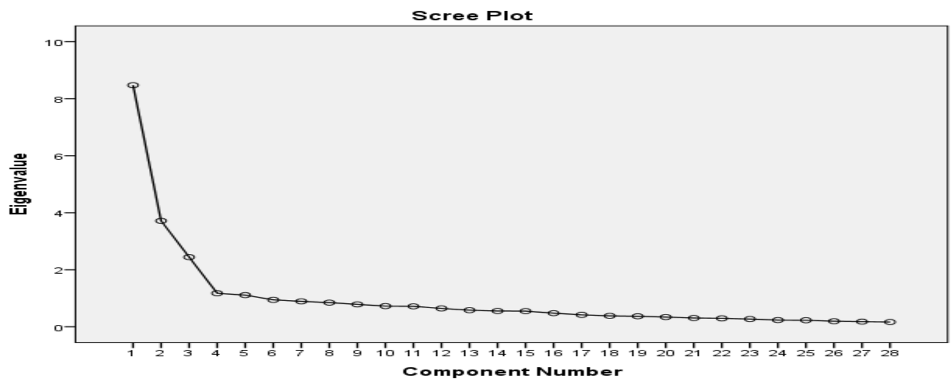


Figure 7 Scree Plot of Spatial Ability

From the scree plot in Figure 7, it could be observed that substantial amount of sample variance was captured by the items on spatial ability. The sample variance remained corresponded to Eigen values less than 1 and hence were not principally accounted for in spatial components. Table 41 in Appendix D displays the factor loadings for each item on their respective principal components.

As can be observed Table 1 in Appendix D, all 28 items loaded between .524 and .850. These values are above .300 and hence the items could be considered significant indicators for the measure of spatial ability.

3.6.2. Construct validity of knowledge types of Geometry (KTG)

An Exploratory Factor Analysis of 23 observed variables were run in SPSS 23 and AMOS respectively to examine the principal structure of the items of KTG. It was very important to investigate the factor structure because the items which were constructed by the researcher have not been tested for validity.

First, the Kaiser-Meyer-Olkin (KMO) measure was computed to check the adequacy of the distribution of KTG to proceed with factor analysis. From the range of 0 to 1, the rule of thumb for proceeding to conduct factor analysis is KMO value of 0.60 or above (Tabachnick & Fidell, 2007). Second, Bartlett‟s test of Sphericity measure was computed at 0.05 significance level to test for the appropriateness of data for factor analytic study. Table 7 shows the computed KMO and Bartlett‟s test of Sphericity values of KTG from SPSS version 23.

Table 7 KMO and Bartlett's Tests of KTG

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.864
Bartlett's Test of Sphericity	Approx. Chi-Square	7491.820
	Df	153
	Sig.	.000

As shown in Table 7, the KMO measure was 0.86. This value was greater than 0.60 and suggested that the distribution of data was adequate for conducting factor analysis. The Bartlett"s test of Sphericity was also significant at .000 indicating that principal component analysis method could be carried out with the KTG data.

The Exploratory Factor Analysis (EFA) using principal component method was then performed where one knowledge factor was first explored. This revealed 37.8% of the sample variance was accounted for by items. The factor loading included Eigen values of 6.81 which was far more than Kaiser"s suggested criterion of 1.0 (Kline, 2015). The data were further explored by a second factor analysis conducted with restriction of the number of factors to two and using Varimax rotation with Kaiser Normalization. The result revealed 57.7% of the sample variance was accounted for by the items. The items extracted were limited to Eigen values of 3.58 or higher. The final factor structure was then explored by referring to the hypothesized three factor structure of knowledge types. A full factor analysis was run then based on Eigen values of 1 or greater. The output revealed a three-factor solution from data with 67.9% of the sample variance accounted for by the items of KTG.

After examining the item loadings, the three principal components 1, 2 and 3 were held as declarative knowledge, conditional knowledge and procedural knowledge respectively as indicated in Table 8.

Table 8 Initial Eigenvalues and Rotation Sums of Squared Loadings of KTG

Factor	Initial Eigenvalues		Rotation Sums of Squared Loadings		
	Total	% of Variance	Total	% of Variance	Cumulative %
Declarative knowledge	6.81	37.84	5.22	29.01	29.01
Conditional knowledge	3.58	19.89	4.10	22.76	51.77
Procedural knowledge	1.83	10.19	2.91	16.15	67.92

Extraction Method: Principal Component Analysis.

As shown in Table 8, 29.0%, 22.8% and 16.2% of the total variance were accounted for by the declarative knowledge, conditional knowledge and procedural knowledge items. The total variance accounted for by all three factors was 67.9%. The rest of variance corresponded to the eigen values less than 1. Hence, such variance was principally unaccounted for in the knowledge components. The scree plot in Figure 8 shows a visual illustration of the component numbers against their respective Eigen values.

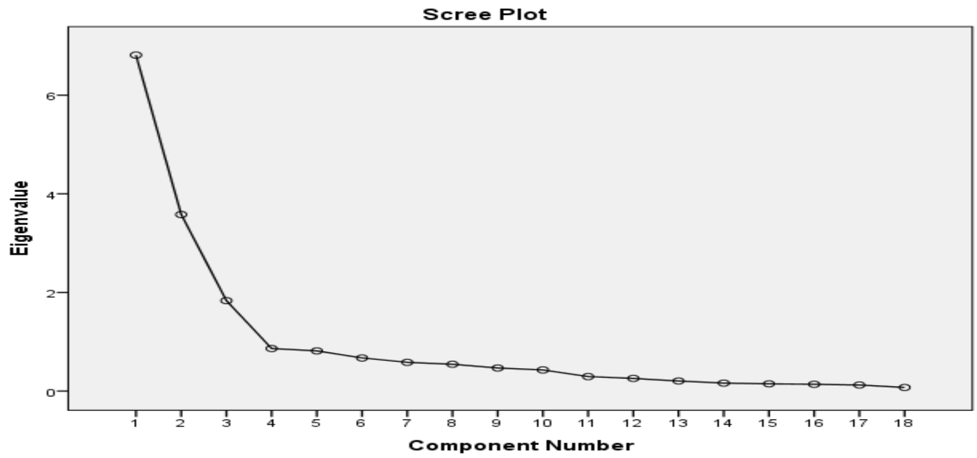


Figure 8 Scree Plot of Knowledge Type Factorization

From the Scree plot in Figure 8, a uni-dimensionality was rejected and the presumed two and three-factor model was interpretable. The three-factor solution was preferred for further Confirmatory Factor Analysis based on the high sample variance accounted for by the items and their alignment with the hypothesized three knowledge structure. Since the sample variance captured by the items measuring the three knowledge types exceeded 50%, it can be concluded that the items were highly validity.

Despite the large sample variance captured, five items did not load up to .3 or higher during the factor analysis and were deleted. They included tasks 9c and 9d on line of symmetry and rotational order, and pentagonal prism which loaded on declarative knowledge. Also, task 14 on surface area of a cube and configuration of triangular prism into a pyramid respectively which loaded on procedural knowledge. Last, tasks 12, 18 and 19 on a pentagonal prism, relationship between a cone and a pyramid; and relationship between a cube and a cuboid, which loaded on conditional knowledge. The loadings of items which were retained for further analysis are displayed in Table 42 in Appendix D.

Table 42 in Appendix D shows that all the items measuring basic Geometry content knowledge loaded between .498 and .996. These values were all higher than .300 and hence suggested that the items were significant indicators for the measure of the three knowledge types.

3.6.3. Construct validity of verbal reasoning test (VR)

An Exploratory Factor Analysis of 12 observed variables were run in SPSS 23 to examine the principal structure of the items constituting to the VR construct. The Kaiser-Meyer-Olkin (KMO) measure was computed to check the adequacy of the distribution of VR to proceed with factor analysis. From the range of 0 to 1, the rule of thumb for proceeding to conduct factor analysis is KMO value of 0.60 or above (Tabachnick & Fidell, 2007). Second, Bartlett"s test of Sphericity measure was computed at 0.05 significance level to test for the appropriateness of data for factor analytic study. Table 9 showed the computed KMO and Bartlett"s test of Sphericity values of VR from SPSS version 23.

Table 9 The KMO and Bartlett's Test for Verbal Reasoning Construct

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.877
Bartlett's Test of Sphericity	Approx. Chi-Square	11783.82
	Df	91

	Sig.	.000

As shown in Table 9, the KMO measure was 0.877. This value was greater than 0.60 and suggested that the distribution of the data was adequate for conducting factor analysis. The Bartlett‟s test of Sphericity was also significant at .000 indicating that principal component analysis method could be carried out with the VR data.

Series of EFA using principal component method was explored to determine the factor structure of items measuring verbal reasoning. The result of analysis yielded two factor structure with substantially large sample variance corresponding to Eigen values greater than 1. After examining the item loadings, the two principal components 1 and 2 were labelled as nonsense syllogism and verbal comprehension as indicated in Table 10.

Table 10 Percentages of Variance Explained in Verbal Reasoning

Component	Initial Eigenvalues			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cum %	Total	% of Variance	Cum %
Nonsense Syllogism	8.23	68.54	68.54	5.48	45.68	45.68
Verbal Comprehension	1.39	11.62	80.16	4.14	34.49	80.16

From Table 10, 45.7% and 34.5% of the total variance of 80.2% were accounted for by syllogism and comprehension items respectively. The Varimax rotation with Kaiser Normalization produced factor loadings for each item are shown in Table 10. While seven items loaded on syllogism, five items loaded on comprehension. All factor loadings above the threshold of .3 and were therefore retained for further analysis. The scree plot in Figure 9 shows a visual illustration of the component numbers of verbal reasoning items against their respective Eigen values.

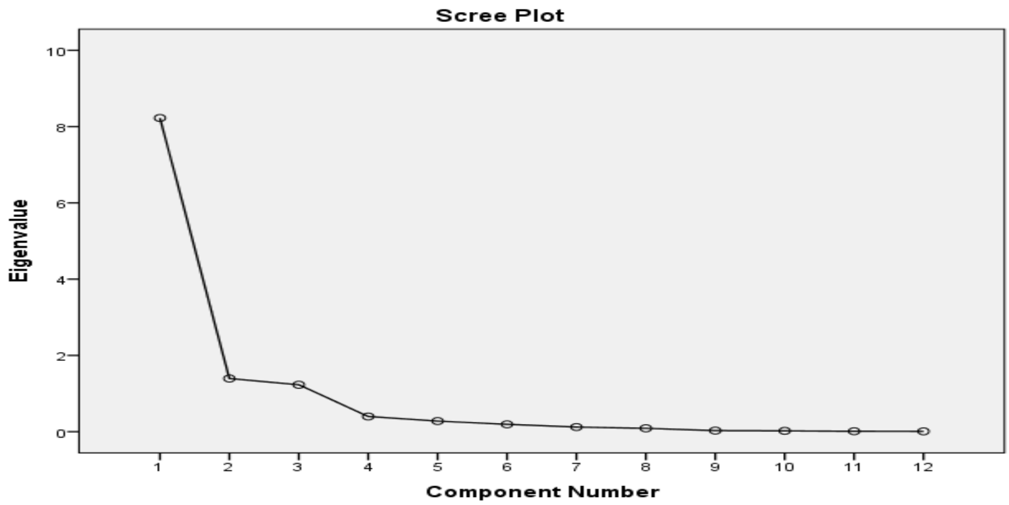


Figure 9 Scree Plot for Verbal Reasoning Ability

From the scree plot in Figure 9, it could be observed that substantial amount of sample variance was captured by the items on verbal reasoning. The remaining 19.8% of the sample variance corresponded to Eigen values less than 1 and hence were not principally accounted for in verbal reasoning components. Table 43 in Appendix D displays the factor loadings of each item on their respective principal components.

From Table 43 in Appendix D, all 12 items loaded between .379 and .927. These values are above .300 and hence the items could be considered valid indicators for the measure of verbal reasoning.

3.7. Reliability of Tests (pilot)

Research findings are usually accepted by stakeholders and applied in real life contexts when such findings are deemed to be reliable, consistent, dependable, replicable and trustworthy (Robson, 2002). At the heart of all these characteristics is the reliability of the instrument used in collecting data (Cho, 2016; Fraenkel & Wallen, 2006). Reliability of data collection instrument is described as the extent to which the instrument is consistent and can provide similar result when used in similar situations or among a similar sample frame. In this study, the essence of checking reliability of the three tests was to ensure internal consistency of the tests. The following sub sections described in detail the reliabilities of each of the tests used in the study.

3.7.1. Reliability of spatial ability tests (pilot)

To ascertain reliability of spatial ability tests, Cronbach Alpha reliability coefficients were calculated in SPSS 23 for the Paper Folding Test (PFT), Mental Rotation Test (MRT) and Water Level Tasks (WLT) using data from the pilot sample of 532. In the case of the 10 items of PFT, the reliability coefficient was found to be .794. For the 10 items of the MRT, the reliability coefficient was .954. The eight items of WLT also yielded the reliability coefficient of .780. For all test items, the computation of Type C interclasses correlation coefficients using consistency definition, where between-measure variance is assumed no interaction effect, yielded the values ranging from .745 to .950 for 95% confidence interval. The overall reliability coefficient was also computed taken together all 28 items from the PFT, MRT and WLT. A reliability coefficient of .92 was obtained. These reliability values were greater than the minimum rule of thumb of .70 (Cho, 2016; Kline, 2015) demonstrating that the spatial ability tests were internally consistent and hence reliable.

3.7.2. Reliability of basic Geometry content knowledge test (pilot data)

The Cronbach Alpha reliability coefficients were calculated for the KTG items using the pilot sample of 532. The analysis was performed separately for the items of the three KTG and for the overall items. In the case of the 5 items of DEC, the computed reliability coefficient was found to be .837. For the 5 items of the CON the computed reliability coefficient was .919. For the 8 items of the PRO the computed reliability coefficient was .913. The computation of Type C interclasses correlation coefficients using consistency definition also yielded average measure values of .794

and .954 at 95% confidence interval. The overall reliability coefficient was also computed taken together all 18 items. A reliability coefficient of .897 was obtained. These reliability values were greater than the minimum value of .70 (Kline, 2015). The result therefore signifies that the KTG items were largely consistent and hence reliable.

3.7.3. Reliability of verbal reasoning test (pilot)

The Cronbach Alpha reliability coefficient was computed for the verbal reasoning test. The reliability coefficient was found to be .941. The computation of Type C interclasses correlation coefficients using consistency definition yielded the same average measure values of .941. This measure was significant at .000. The reliability value was greater than the rule of thumb of .70 indicating that the verbal reasoning test was consistent and hence reliable.

3.8. Ethical Issues and Data Collection Procedure for Main Study Data

The data collection begun with the issues of research ethics and integrities. The University of Education Research Ethics Policy, where the PhD study took place, was under consideration by the Academics Planning Committee as at the time of data gathering. However, ethical approval and clearance for the collection of the data for the study was given by my supervisors and the Graduate Board of the Department of Mathematics Education, University of Education, Winneba. This was done following my successful presentation of a comprehensive proposal for the study.

To ensure integrity of the data collection process, consent letters were presented to heads of the colleges of education selected for the study (see Appendix F). The letters requested for permission and clearance to collect data. After permission was granted by the principals, one tutor was often assigned to consult other tutors to assist in organizing the test administrations. The tutors assisted the researcher to assemble the students for further interactions and agreement to participate in the study. Only second year students who agreed to take part in the study were requested to assemble for the test administration. To ensure access to data collection environment, various dates were agreed upon by the principal or vice-principals, tutors and students for the administration of the tests. The researcher then visited the selected colleges with one research assistant on scheduled dates. In four of the colleges, one mathematics tutor was assigned by principals to assist in organizing and distributing the test papers to participating students. In the rest of the eight colleges, two tutors assisted the researcher together with the research assistant to administer the tests.

The test administrations took place in the evenings after the official lessons on the colleges" timetables for teaching ended. This was agreed upon by the tutors, participants and the researcher in order not to disrupt regular lessons. In each classroom, the participating pre-service teachers were given numbers according to their sitting positions. This was needed to ensure the collation of the different sets of tests by respondents, without students using their own names. The instructions and purpose of the study were reiterated to respondents after the sets of tests were distributed to respondents. Because the spatial and verbal tests were to be answered strictly in 3 minutes in each case, respondents were requested to answer and return them within time before proceeding to answer the geometry items. The geometry tests were not time bound. However, most respondents completed all tests within 1 hour. Few respondents were found to have copied or shown less interest in answering the tests or submitted incomplete scripts. These scripts were excluded from the sample.

3.9. Main Data Analysis Procedure

Data collected were thoroughly explored using relevant data coding and screening techniques including descriptive statistics, treatment of missing data and examination of outliers and normality. This was necessary to obtain clear insight into the data characteristics (Robson, 2002).

3.9.1. Dealing with empty/incomplete test papers

A total of 953 test papers were collected from respondents. A thorough assessment were conducted in which all empty and incomplete test papers were removed before and after scoring was done by the researcher. In all, 196 representing 20.6% of the test papers were either returned empty or incomplete. Out of the 196 incomplete responses, 92 representing 46.9% were from geometry test, 40 representing 20.4% from verbal ability and 64 representing 32.7% from spatial tests. The rest of the test papers were then scored using the Scoring rubrics in Appendix C1. The scores of these papers constituted data for the study.

3.9.2. Scoring and coding variables

Relevant codes were diverse and defined in the variable view of SPSS version 23 file and prepared for data to be entered. For the spatial visualization, mental rotation and spatial perception tests, 1 score was assigned for correct answer and 0 for incorrect answer. However, for the spatial perception, because each item contains 8 sub items, the number of correct responses in each item was summed up as score for that item. Thus, the expected total score for the 10 items was 80. This was rescaled by dividing the total score by 8 to reflected same maximum scoring scale of 10. For the geometry questions, each question was scored on continuous scale from 0 for lack knowledge to 2 for full knowledge with 1 denoting partial knowledge.

For the bio data, nominal scale was used where gender was coded 1 for male and 2 for female, type of college was coded 1 for Science College and 2 for general programme college, SHS programmes offered were coded nominally from 1 to 5 and finally the age ranges were coded from 1 to 4. Table 11 displays these coding scheme.

Table 11 Scoring Codes for all Measured Variables

Variable	Defined	Code
Spatial visualization	Correct response	1
	Incorrect response	0
Mental rotation	Correct response	1
	Incorrect response	0
Spatial perception	Correct response	1
	Incorrect response	0
Syllogism	Correct response	1
	Incorrect response	0
Comprehension	Correct response	1
	Incorrect response	0
Basic Geometry content knowledge	Lack of knowledge	0
	Partial knowledge	1
	Absolute knowledge	2
Gender	Male	1
	Female	2
College type	Science college	1
	General college	2
SHS programmes	Science	1
	Business	2
	General arts	3
	Visual arts	4
	Others	5

3.9.3. Examination of data entry and missing data

The data were entered into prepared SPSS file and exploratory analyses were done in which missing data and errors during coding or entering of data were investigated and corrected. Arbuckle (2013) cautioned about the issue of incomplete or missing data with SEM analysis in AMOS regarding fit measures of hypothesized, saturated and independence models. According to Arbuckle (2013), missing or incomplete data require extensive computation and can lead to solutions not being reached or parameter estimates being lowered. Besides, model fit indices such as CFI of independence model and chi-square statistic of the hypothesized model cannot be computed in AMOS when the data file contains incomplete data (Arbuckle, 2013).

Therefore, to ensure high level of precision in the data entered and facilitate the use of AMOS for SEM, all entries were cross-checked case by case before descriptive statistics were run which confirmed no missing data or errors. The descriptive statistics including minimum, maximum, mean, skewness and kurtosis scores were extracted from the analysis and presented in tabular form in section 3.16.

3.9.4. Cronbach alpha reliability, congeneric reliability and average variance extracted

To ensure internal consistency or reliability of the latent variables, the average inter-item correlation was conducted using Cronbach alpha reliability, congeneric reliability and average variance extracted. Lee Cronbach developed the Cronbach Alpha formula in 1951 to measure how internally consistent a test is along the scale of 0 to 1 (Lauritzen, 2012). Internal consistency means the extent to which all the items in a test measure the same concept or construct or the inter-relatedness of the items within the test. The formula estimating reliability using Cronbach Alpha is defined by:

$$\alpha = \frac{N.\bar{c}}{\bar{v} + (N-1)}$$

Where: N =the number of items

$\bar{c}$ = average covariance between item-pairs

$\bar{v}$ =average variance

Table 16 displays the measure of Cronbach alpha and construct reliability coefficients obtained from SPSS and AMOS. The Cronbach alpha coefficients of .76, .52 and .69 were obtained for spatial ability, verbal reasoning and basic Geometry content knowledge respectively. Except verbal reasoning, the reliability coefficients show acceptable internal consistencies. Cronbach Alpha does not only measure test homogeneity but also the length of the test. Cronbach alpha proves to be sensitive to the number of indicator items and tends to give small coefficients even if the items were reliable. Therefore, the low coefficient for the verbal reasoning might be due to the fewer number of indictor items used.

To address the above concern, Cho (2016) argues that congeneric reliability should be used as alternative for calculating reliability coefficients. This study involved fewer number of indicator items for each factor. It was thus necessary to further calculate the reliability coefficients of the constructs using congeneric reliability which appears less sensitive to number of indicator items. The congeneric reliability (CR) is the reliability of a unidimensional congeneric measurement model. The CR is defined mathematically by:

$$\mathrm{CR} = \frac{\left(\sum \lambda_{\mathrm{yi}}\right)^2}{\left(\sum \lambda_{\mathrm{yi}}\right)^2 + \sum_{\mathrm{i}} \mathrm{var}(\varepsilon_i)}$$

Where k is the number of items in the measurement model, λ_i the factor loading of the ith item and ε the measurement error.

Table 16 displays the estimated values of the congeneric reliabilities of the three latent variables. The CR values were .79, .67 and .63 for spatial ability, verbal reasoning and basic Geometry content knowledge respectively. These values were within acceptable thresholds and showed that the respective indicator items of each latent variable in the measurement model reflected the same underlying latent variable.

The CR formula has been criticized for not been able to measure completely the amount of variance either accounted for by the construct or unexplained due to measurement error. The average variance extracted (AVE) is an alternative measure that attempts to capture much of the variance of the construct (Tabachnick & Fidell, 2007). The AVE is defined by the formula as follows:

$$AVE = \frac{\sum \lambda_{yi}{}^2}{\sum \lambda_{yi}{}^2 + \sum var(\varepsilon)}$$

To fully account for the variance of each of the constructs as opposed to the variance owing to measurements, the AVEs for the three constructs were estimated using SPSS. Table 12 shows the estimated values of average variance extracted in each of the latent variables in the model. In examining reliability with AVE, an estimated AVE>.50 suggested that the variance captured by the construct was larger than the variance due to measurement error. As shown in Table 12, the estimated AVEs for spatial ability, verbal reasoning and basic Geometry content knowledge were .73, .55 and .64 respectively. These AVEs indicated that the variances accounted for by each construct were adequate and hence the items used were deemed reliable measures.

Table 12 Estimates of Reliability of Constructs in the Measurement Model

Estimate	Spatial ability	Verbal reasoning	Basic Geometry content knowledge
Cronbach‟s Alpha	.76	.52	.69
Congeneric Reliability	.79	.67	.63
Average Variance Extracted (AVE)	.73	.56	.65

3.9.5. Checking SEM assumptions

Many authorities have argued that SEM can compensate for certain violations of multivariate assumptions and thus checking multivariate assumptions might be irrelevant (Kline, 2015; Hair et al, 2010; Hooper, Coughlan & Mullen, 2008)). Other authors have however maintained that violating key assumptions such as multivariate normality, positive definiteness and minimum sample size requirement might truncate solution to equations or influence the estimation of model parameters and make conclusions less meaningful or inadmissible (Byrne, 2012). In this study, while relaxing on strict attainment of statistical assumptions, it was deemed prudent to check outliers, multivariate normality, positive definiteness, multicollinearity and minimum sample size requirement for the use of SEM. These assumptions were examined as follows:

3.9.5.1.Outliers and assumption of normality

There are factors which account for extremely low or high scores in a data set usually described as outliers. These include errors in data recording or entry, social desirability and motivated mis-reporting, methodological and sampling errors (Byrne, 2012). A typical methodological issue in this study was time constraints in test administration which possibly affected ill-prepared participants leading to incomplete solutions. This had the tendency to result in bimodal, skewed asymptotic or flat distributions depending on the sampling characteristics. Outliers have serious effects on statistical analyses since they often increase error variance thereby reducing statistical power of tests. Outliers also can influence the overall data to violate assumptions of sphericity and multivariate normality and contribute to serious biases in interpretation and conclusion of analysis. In literature, the main ways of testing for bivariate and multivariate outliers in research data include the use of indices of influence, leverage and distance (Byrne, 2012). Another key aspect involved examining residuals in which any standardized residual which exceeds 3 is deemed to be an outlier.

In this study the outliers were examined using the Mahalanobis distance and the standard residuals matrix obtained from AMOS. Checks for outliers in the data using the boxplots of all variables investigated in the study were obtained in SPSS. Two outliers were detected from spatial visualization data (entry number 667 and 189). They were two males whose scores were as low as zero. There were also four male participants whose score of zero categorized them as outliers from the declarative knowledge data. Finally, there were three male and two female participants who scored zero in all test items on conditional knowledge. Even though, there were several factors associated with these outlier scores, it seemed plausible for test takers to score zero in test in a typical learning setting. Further examinations of these outliers also indicated that they might have little influence on the results of the analysis of the associated variables. Therefore, the outliers were assumed as occurring at random under normal test taking conditions and were not removed or transformed before conducting further analysis.

To use structural equation modeling, the assumption of normality was required. Most statistical analyses (e.g. SEM) are based on the central limit theorem where population parameters are estimated by sample statistics. Hence, violating normality assumption could cause the chi-square value of the model and the standardized errors to demonstrate biasness. This could distort model validity since conclusions from model evaluation are based on likelihood ratio from the chi-square distribution.

For univariate normally distributed variables, values of skewness and kurtosis were examined. Highly skewed data would display abnormal distributions and when analyzed, might not represent the true attributes of the population. Also, data with flat distributions could produce statistical estimates that might not be consistent and representative of the population parameters. To assume approximate normality, at least marginal evidences of skewness and kurtosis would be acceptable as it is usually practically difficulty to obtained absolute normal distribution (Byrne, 2012). For the observed variables in this study, estimates of skewness and kurtosis with corresponding critical ratio and significant values are displayed in Table 13.

Table 13 Assessment of Outliers and Multivariate Normality for Continuous Variables

Variable	Min	Max	Skew	c.r.	sign	Kurtosis	c.r.	Sign
Sy	.00	9.00	-.07	-.74	.389	-.96	-5.37	.002
Vc	.00	5.00	-.96	-10.72	.001	.15	.85	.358
Pro	.00	8.0	-.14	-1.58	.209	-.80	-4.52	.035
Con	.00	5.00	-.34	-3.80	.051	-.03	-.18	.668
Dec	.00	5.00	-.31	-3.44	.064	-.29	-1.62	.204
Sos	2.00	10.00	-.59	-6.68	.010	-.46	-2.60	.107
Svs	.00	10.00	-.34	-3.78	.052	-.67	-3.76	.053
Sp	1.00	8.00	-.54	-6.06	014	-.11	-.63	.426
Multivariate						17.02	18.51	.295

As shown in Table 13, the skewness values ranged from -.96 to -.07 with corresponding critical ratios ranged from -10.72 to -1.58. The critical ratios of the three variables (verbal comprehension, mental rotation and spatial perception) were statistically significant which meant that these variables did not show characteristics of normal distribution. However, since their absolute values of skewness did not exceed 2.0 or 3.0, they could be described as moderately normal (Byrne, 2012). The result also shows kurtosis values ranged from -.96 to -.03 with two variables, syllogism and procedural knowledge. This signals a high kurtosis. However, since no kurtosis values exceeded 7.0, the result shows no firm evidence of kurtosis. Hence, generally, data did not indicate serious violation of univariate normality.

Conformity to the assumption of multivariate normality of dependent variables were assessed in line with suggestion by Teo, Tsai and Yang (2013). SEM assumes that the endogenous variables are (1) continuous with approximately normally distributed residuals and (2) their joint distribution also having approximately joint multivariate normality (Kline, 2015). The first condition was partly satisfied since both endogenous variables (verbal reasoning and Basic Geometry content knowledge) were measured on continuous scales. For the multivariate normality, the Mardia‟s critical ratio of 18.51 produced p=.295. This critical ratio suggested a non-significant difference at .05 level cutoff.

3.9.5.2.Positive definiteness and variances

In conducting multivariate analysis such as SEM, it was required that the determinant of matrix of covariance or correlation must not be negative or zero. This principle is called the positive definiteness or semi-positive definiteness (Kline, 2015). If this principle is violated, the interpretation of the model might be inadmissible. To test for positive definiteness, the determinant of correlation matrix of all eight observed variables was computed using SPSS. The result revealed a positive determinant of .042. Since a non-zero determinant was achieved, positive definiteness was not violated. Despite this, positive definiteness was continually assessed by AMOS throughout the model fitting and modifications stages (Arbuckle, 2013). AMOS has in-built algorithm to detect and caution users when this assumption is violated.

Another key assumption in SEM analysis was that the variance of any of the variables could not be greater than ten times the variance of any other variable in the model (Arbuckle, 2013). For the sample of 757, the variances of 5.38, 2.66 and 2.44

were obtained for spatial visualization, mental rotation and spatial perception respectively. Similarly, the variances of 5.65 and 1.53 were obtained for syllogism and verbal reasoning respectively. Finally, 1.18, 1.10 and 4.20 were the variances for declarative, conditional and procedural knowledge respectively. Among the variables considered for the study, none of their variances was closed to ten times another variance. Hence, this assumption was not violated.

3.9.5.3.Multicollinearity

If the independent variables in a study were highly correlated, it might imply the variables were strong dependent on each other and could constitute a single construct. Performing model analysis with such variables might not yield useful result. In SEM, the independent or the predictor variables must not exhibit multicollinearity where variables are said to be high correlated. This was assessed using tolerance interval and variance inflated factor (VIF) (Kline, 2015) as well as using matrix of bivariate correlations among variables (Tabachnick & Fidell, 2007). The tolerance defined by 1 - R^2 where R^2 was calculated by regressing the independent variable of interest onto the remaining independent variables in the multiple regression analysis. The acceptable threshold of tolerance was .10. On the other hand, the VIF refers to the ratio of variance in a model with multiple terms, divided by the variance of a model with one term alone. The VIF is the reciprocal of the tolerance as shown mathematically as:

$$VIF_i = \frac{1}{1-{R_i}^2}$$

where ${R_i}^2$ are the coefficient of determination of each independent variable

An analysis of data in this study shows that, the tolerance for spatial visualization, mental rotation and spatial perceptions were .46, .29 and .46

respectively. The syllogism and the comprehension were also .47 and .77 respectively. Finally, the tolerance for declarative, conditional and procedural knowledge were .69, .69 and .56 respectively. No value was therefore smaller than .10.

A VIF value above 10 signified high correlation or evidence of severe multicollinearity (Lauritzen, 2012; Byrne, 2012). The VIF for the spatial visualization, mental rotation and spatial perceptions were 2.19, 3.46 and 2.17 respectively. Also, the VIF for syllogism and comprehension were 2.13 and 1.30 respectively. Similarly, the VIF of 1.46, 1.44 and 1.78 were obtained for declarative, conditional and procedural knowledge respectively. All the VIF values were less than 10 and hence suggested that the variables did not exhibit multicollinearity to unnecessarily inflate variances in the model.

3.9.5.4.Sample size adequacy

Another key requirement for SEM considered in this study was the adequacy of the sample size. In model analysis, the number of subjects from whom data were obtained could affect the statistical power and effect sizes of variables. In other words, the power of the statistical test relies on the sample size (Kline, 2015; Bagozzi & Yi, 2011). Usually, larger sample sizes are recommended to capture adequately the characteristics of the population under study and facilitate reasonably valid inferences. According to Kline (2015), the sample size greater than 200 is often required for SEM analysis. A-priori Sample Size for Structural Equation Models Calculator online to estimate an adequate sample size for the present study. The recommended standard moderate effect size of .30, desired statistical power of .80 and p-value of .05 were set. For the 3 latent constructs and 8 observed variables involved in this study, 256 participants were expected to attain minimum sample size adequacy. The sample taken for this study was 757. Therefore, it was concluded that the sample size was acceptable for model testing and evaluation. Issues relating to sampling biases were also addressed through the sampling processes described earlier in section 3.9.1 of this chapter.

3.10. Data Analysis

Descriptive statistics were employed to answer research question 1. Mean, standard deviation, minimum and maximum scores of participants were obtained and displayed in tables. These scores were used to describe participants" performance in tasks involving spatial ability, verbal reasoning and basic Geometry content knowledge. Participants" performance in verbal reasoning was further examined using proportions of participants obtaining correct answers. Also, to determine how well participants performed in the basic Geometry content knowledge, the scores were categorized into low, moderate and high and presented in tables for interpretations. Out of the maximum scores, participants who obtained scores from $0 - 3$ (i.e. the lower 33.3% of the distribution of performance) were categorized as low in performance. Those who obtained scores from $4 - 7$ (i.e. occupying middle 33.3% of the distribution) were categorized as moderate in performance while those who scored from $8 - 11$ (upper 33.3% of the distribution) were categorized as high in performance. Correlations between declarative, conditional and procedural knowledge were obtained and examined to answer research question 1 b and the corresponding hypothesis.

Research questions 2, 3 and 4 with their corresponding hypotheses were addressed using SEM. To use SEM, the main assumptions, validity and reliability analyses were first conducted in SPSS and in AMOS for each construct and corresponding factors using main data from 757 participants. Confirmatory factor analyses were conducted to examine the variables of spatial ability, verbal reasoning and geometry knowledge types respectively. The spatial, verbal and knowledge measurement models involving observed variables were built in AMOS and run to confirm construct validities.

A two-step structural equation modeling strategy via AMOS was employed in estimating and evaluating the parameters. This comprised analyzing the (i) measurement coefficients and (ii) structural coefficients (Kline, 2015). The goodness of fit indices of the overall model were reported using two categories of model fit indices. First, absolute goodness of fit indices comprising the Chi-Square test, Normed Chi-Square test, Goodness Fit Index (GFI), Root Mean Residual (RMR) and the Root Mean Square Error of Approximation (RMSEA) were evaluated for model fit to data. The Chi-square test and its Normed Chi-Square test were used to test departure or deviation of the data from the specified model (Hooper, Coughlan & Mullen, 2008). The GFI, RMR and RMSEA values were used to examine the extent to which the specified model was closed to data. Second, the baseline comparisons fit indices, comprising the Normed Fit Index (NFI), Relative Fit Index (RFI), Incremental Fit Index (IFI), Tucker Lewis Index (TLI) and Comparative Fit Index (CFI), were further evaluated to compare the model with the baseline or independent model. The use of both absolute and comparative indices was intended to allow for full judgement of the model fitness and to determine which theoretical notions are supported empirically (Kline, 2015). Multi-group confirmatory factor analysis and multigroup structural equation modeling (MSEM) were the main analytic procedures used to examine measurement invariance and structural invariance across gender and across programmes. Since the model was identified as saturated, path deletion and reverse arrow path directions were employed to improve the model indices and select a more parsimonious model. For path analysis, effect sizes were used to determine the association between the variables under investigation. An effect size is an indicator of the magnitude of association between two or among several variables (Kline, 2015). The effect size in SEM is equivalent to squared multiple correlation or its adjusted squared multiple correlation in multiple regression analysis. In relation to SEM, the squared multiple correlation depicted the amount of variance accounted for by the set of latent independent variables. It also determined the strength of relationships between the variables considered. The following criterion proposed by Cohen (1988), in standardized effect size values, were used as rule of thumb to judge the magnitude of effect of spatial ability and verbal reasoning on Basic Geometry content knowledge.

Effect Size	**Range of Values**
Small effect	below . 09
Intermediate effect	between .09 and .25
Large effect	. 25 or greater

The total effects of the latent independent variable were decomposed into direct and indirect effects. The output of effect sizes was presented in path diagrams and tables and interpreted under headings of relationships between variables. Table 14 highlights the types of statistical analysis used to answer specific research questions and related hypotheses.

Table 14 Research Questions (hypotheses) and Analytic Procedures

Research Question	Hypothesis	Analysis

1.	(a)How do pre-service teachers perform in tests involving: (i) knowledge types for teaching geometry? (ii)spatial ability? (iii) verbal reasoning? (b) How do knowledge types in geometry affect each other?	Ho_1: There is no relationship among pre-service teachers‟ declarative, conditional and procedural knowledge.	Descriptive and correlational analysis and MGCFA
2.	How do spatial ability and basic Geometry content knowledge relate to each other?	Ho_2: There is no direct effect of spatial ability on Basic Geometry content knowledge.	SEM-path analysis
3.	To what extent does verbal reasoning mediate the effect of spatial ability on Basic Geometry content knowledge?	Ho_3: There is no verbal reasoning intermediating the effect of spatial ability on Basic Geometry content knowledge. Ho_4: There is no effect of pre-service teachers‟ verbal reasoning on Basic Geometry content knowledge.	SEM- path analysis
4.	Do the relationships between spatial ability, verbal reasoning and knowledge in geometry differ by gender and programme?	Ho_5: The relation between spatial ability and knowledge in geometry mediated by verbal reasoning is invariant across gender and programmes.	MGCFA and MSEM – Invariance analysis

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1. Overview

This chapter presents the result of analysis of the main data of the study. The presentation began with descriptive statistics of participants‟ performance in the three latent constructs considered in the study. Following this was the assessment of conformity or likely violation of the prerequisite assumptions for SEM application. This was then followed by examination of the measurement model of spatial ability, verbal reasoning and basic Geometry content knowledge constructs under investigation. Evaluation of structural equations involved two approaches. The first comprises examinations of departure of data from the specified model tested for non-significance at .05 using Chi-square test (Kline, 2015). Other absolute fit indices were also evaluated. The second approach involved an examinations of baseline comparison model fit indices. The hypothesized model was tested for both measurement and structural invariance across gender and across programme of study by participants. To compare the nested covariance structure models, the standard procedure involving the likelihood ratio test of difference in fit was used to test the null hypothesis that the model fits identically in the population. Path analyses were conducted to examine relationships between spatial ability, verbal reasoning and basic Geometry content knowledge as well as direct, indirect and total effect sizes. The results are presented based on themes that reflect the research questions and hypotheses formulated for the study.

4.2. Gender and Programme of Study

A cross tabulation of distributions of gender and academic programme of study by the 757 participants is displayed in Table 15.

Table 15: Frequency Distribution of Participants' Gender and Programme of Study

Gender	Programme of Study		Total
	General	Science	
Male	308	187	495
Female	95	167	262
Total	403	354	757

As shown in Table 15, there were relatively more males than females who took part in the study. Similarly, the male participants in the general programmes category were considerably more than their female participants. However, the number of participants pursuing general programmes did not differ substantially from those pursuing science at the colleges. A critical observation of Table 15 suggests that the distribution of participants by gender and by programme of study mirrored the population distribution at the college of education (Institute of Education, 2017). The sample was therefore considered representative of population of second year pre-service teachers at the colleges of education in Ghana and the distributions were suitable for the purpose of the study.

4.3. Performances in Spatial Ability, Verbal Reasoning and Basic Geometry Content Knowledge

The first objective of the study was to examine the performances of participants in tasks involving spatial ability, verbal reasoning and basic Geometry content knowledge. The results of analysis are presented as follows:

4.3.1. Performance in spatial ability tasks

The spatial ability variables were measured with the observed variables of spatial visualization, mental rotation and spatial perception. Participants" scores for each of the three variables were summed up to represent their respective measures.

Thus, the highest possible score for spatial visualization tasks was 10. The possible scores for mental rotation ranged from zero to 10 while that of spatial perception ranged from zero to eight. Descriptive statistics of scores obtained by participants in the spatial ability tasks are displayed in Table 16.

Table 16 Descriptive Statistics of Participants' Scores in Spatial Ability Tasks

Spatial Ability	N	Mean (%)	St. Dev. (%)	Min (%)	Max (%)	Possible Max(%)
Spatial Visualization	757	5.95(59.5)	2.319(23.2)	0(.0)	10(100)	10(100)
Mental Rotation	757	7.25(72.5)	1.360(13.6)	2(20.0)	10(100)	10(100)
Spatial Perception	757	5.95(74.4)	1.561(19.5)	1(12.5)	6(75.0)	8(100)
Overall	757	19.14(68.4)	5.480(19.6)	0(.0)	26(92.9)	28(100)

***percent score in parenthesis**

For spatial visualization, the results as displayed in Table 16 indicate mean performance of 5.95 with the standard deviation of 2.319 scores. This standard deviation corresponds to 23.2% of the maximum score of 10. Even though the mean performance might be described as representative of average score considering the total of 10, the standard deviation appeared to indicate wide variations between individual scores away from the mean. This indicates that the spatial visualization tasks might have been difficult for participants. For mental rotation, the maximum score was 10 and the minimum score was two. The mean performance was 7.25 with standard deviation of 1.360. The standard deviation gives a small variation of 13.6% which indicates that the mental rotation tasks appeared somehow easy for participants to answer than the spatial visualization. Finally, for spatial perception, the mean performance was 5.95 with standard deviation of 1.561. Proportionally, this standard deviation represents 19.5% of the possible maximum value of eight. This variation is quite small signifying that the tasks used to measure spatial perceptions were not too easy and not too difficult. The overall mean performance of 19.14 with the standard deviation of 5.480, represents 68.4% of the possible maximum score of 28. With regards to research objective 1(a) (i), the entire result shows that participants demonstrated average performance in the three spatial ability tasks.

4.3.2. Performance in verbal reasoning tasks

As indicated in Chapter Three, the verbal reasoning was measured with seven statements on nonsense syllogism, and four tasks on a passage and one task on logical statement on verbal comprehension. Descriptive statistics of scores obtained by participants in the verbal reasoning tasks are displayed in Table 17.

Table 17 Descriptive Statistics of Participants' Scores in Verbal Reasoning Tasks

Spatial Ability	N	Mean (%)	St. Dev (%)	Min (%)	Max (%)	Possible Max(%)
Nonsense syllogism	757	5.26(46.6)	2.378(33.9)	1(14.3)	7(100)	7(100)
Verbal Comprehension	757	3.81(76.2)	1.235(17.6)	1(20.0)	5(100)	5(100)
Overall	757	9.93(82.6)	3.111(25.9)	1(.08)	12(100)	12(100)

***percent score in parenthesis**

As shown in Table 17, the mean performance on nonsense syllogism items was 5.26 with standard deviation of 2.378 reflecting 33.9% of the maximum score of 7. Also, the mean performance in verbal comprehension was 3.81 with standard deviation of 1.235 reflecting 17.6% of the maximum score of 5. The large variations show that the set of tasks includes questions with different degrees of difficulty. However, the overall mean of 9.93 with standard deviation of 3.111 represents as large as 82.6% of the maximum score of 12.

Further analysis using proportions shows that majority of the participants demonstrated substantial ability to logically encode textual information. For the nonsense syllogisms items, between 70% and 83% of the participants got correct answers thereby demonstrating good reasoning abilities with text. For the comprehension passage, substantial proportions (above 68%) of the participants demonstrated good comprehension in the first three questions. However, it was surprising that majority (81.8%) could not comprehend the fourth question. Perhaps, the question appeared ambiguous or difficult for participants. For the second set of comprehension tasks, more than 78% obtained correct answers demonstrating their ability to process new information. In terms of research objective 1a (ii), the overall results portrayed that approximately 83% of the participants could reason through and process written textual information.

4.3.3. Performance in basic Geometry content knowledge tasks

Participants" performance on the basic geometry tasks were categorized into declarative, conditional and procedural knowledge. Descriptive statistics of scores obtained by participants in tasks on basic Geometry content knowledge types are displayed in Table 18.

Table 18 Descriptive Statistics of Participants' Scores in Basic Geometry Content Knowledge Tasks

Knowledge Types	N	Mean (%)	St. Dev (%)	Mim (%)	Max (%)	Possible Max (%)
Declarative Knowledge	757	6.98(63.5)	2.103(19.1)	0(.0)	8 (72.7)	11(100)
Conditional Knowledge	757	4.84(44.0)	2.102(19.1)	0(.0)	10(90.0)	11(100)
Procedural Knowledge	757	9.52(50.1)	4.070(21.4)	0(.0)	16(84.2)	19(100)
Overall	757	21.35(52.1)	6.651(16.2)	0(.0)	34((82.9)	41(100)

From Table 18, the mean performance on declarative knowledge tasks was 6.98 with standard deviation of 2.103 reflecting 19.1% of the possible maximum score of 11. For conditional knowledge tasks, the mean performance was 4.84 with standard deviation of 2.102 which corresponds to 19.1% of the possible maximum score of 11. Finally, for procedural knowledge tasks, the mean performance was 9.52 with standard deviation of 4.070 which corresponds to 21.4% of the possible maximum score of 19. Participants" overall mean performance of 21.35 with standard deviation of 6.651 reflects 52.1% of the possible maximum score of 41. The entire results show that participants performed averagely well in tasks on basic Geometry content knowledge.

Further analysis was conducted to examine the percentage distributions of pre-service teachers" performance in declarative, conditional and procedural tasks in the five main content on shape and space. The scores were categorized into low, moderate and high performance as shown in Table 19, Table 20 and Table 21.

Table 19 Percentage of Participants Obtaining Correct Answers on Declarative Tasks

Declarative knowledge		Low	Moderate	High
Angles	Angle formation concept	12.0	87.6	.4
Triangles	Triangle and its properties	9.8	88.1	2.1
Quadrilaterals	Recognizing square and its unique properties	16.4	80.4	3.2
Prisms	Recognizing triangular prism and its faces	14.3	85.6	.1
Pyramids	Naming of one unique property of pyramid and faces of rectangular pyramid	11.2	88.6	.1
Average		**12.9**	**86.0**	**1.1**

Score ranges: Low = 0 – 3; *Moderate* = 4 – 7; *High* = 8 – 11

In this study, declarative knowledge refers to the ability to form angles, describe properties of a triangle, recognize a square and its unique properties, recognize triangular prism and its faces and name property of pyramid and faces of rectangular pyramid. The result in Table 19 shows that at least 86.0% of the participants demonstrated moderate declarative knowledge. Specifically, 87.6% demonstrated declarative knowledge on angle formation, 88.1% on triangular and its properties 80.4% on uniqueness of square in the family of quadrilaterals, 88.6% on angular prism and its properties as well as 85.6% on names and properties of pyramids. As shown in Table 19, only a few (less than 5%) of the participants demonstrated high declarative knowledge in all the five content areas. For example, regarding quadrilaterals, 96.8% of the participants could only figure out the class in which a square belongs to and mention its unique property. The overall results show that majority of participants exhibited moderate declarative knowledge regarding basic geometry.

Conditional knowledge refers to ability to explain why, when and where certain concepts and procedures related and could be applied to specific instances in geometry. Table 20 displays the percentages of participants‟ performance in each conditional task in basic geometry.

Table 20 Percentage of Participants Obtaining Correct Answers on Conditional Task

	Conditional knowledge	Low	Moderate	High
Angles	Corresponding angles and number of angles formed when two straight lines cross each other	12.3	66.7	21.0
Triangles	Concept of same angle measures in every isosceles triangle	12.7	64.1	23.2
Quadrilaterals	New shape when two opposite sides of square increase by 2 units	15.6	49.4	35.0
Prisms	Justifying why whether cylinder is a prism	54.0	45.7	.3
Pyramids	Determining new figure formed when triangular prism vertices are pulled to a tip	69.5	25.6	4.9
Average		**32.8**	**50.3**	**16.9**

Score ranges: Low = $0-3$*; Moderate =* $4-7$*; High =* $8-11$

Similar to the declarative knowledge, substantial proportions of participants attained moderate scores on conditional knowledge as shown in Table 20. However, compared to declarative knowledge, high proportions of the participants had low scores in conditional knowledge tasks. For example, less than 13% of the participants demonstrated low knowledge in angle measures, in isosceles triangle and in recognizing the number of angles formed when two straight lines cross each other. Also, while 23.2% had high scores on tasks measures and properties of isosceles triangle, up to 35% demonstrated high knowledge on quadrilaterals. However, as large as 54.0% of participants did not demonstrate high knowledge on relationship between cylinder and prism or transformation of prisms into pyramids. Overall, averagely 32.8%, 50.3% and 16.9% of participants demonstrated low, moderate and high conditional knowledge. These results portrayed that, not many participants had high scores on the tasks on conditional questions on angles, triangles, quadrilaterals, prisms and pyramids.

Procedural knowledge in this study was limited to knowing how to use procedures, sketch shapes and find areas and volumes of shapes. The result of proportions analysis of participants‟ performance is shown in Table 21.

Table 21 Percentage of Participants Obtaining Correct Answers on Procedural Tasks

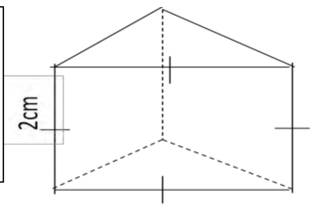
	Procedural Knowledge	Low	Moderate	High
Angles	Sketching angles formed when two straight lines cross each other	16.8	27.6	55.6
Triangles	Finding area of right-angled triangle	44.8	5.3	49.9
	Finding area of kite	12.9	34.9	52.2
	Sketching lines of symmetry	52.6	44.4	3.0
Quadrilaterals	Finding area of transformed square(rectangle)	77.3	19.7	3.0
Prisms	Sketching net and finding area of rectangular prism	35.4	50.3	14.3
	Finding volume of milo tin (cylindrical)	16.8	81.6	1.5
Pyramids	Finding volume of cone	54.0	32.9	13.1
Average		**38.8**	**37.1**	**24.1**

Score ranges: Low = 0 – 3*; Moderate* = 4 – 7*; High* = 8 – 11

Compared with declarative and conditional knowledge, the proportions of participants who had high scores in procedural tasks as shown in Table 21, were considerably high. Except procedures relating to lines of symmetry, area of transformed square and volume of cylinders, more than 50% of the participants attained high score in procedural tasks. For procedural tasks on angles, areas of right-angled triangle and kite, majority (50-56%) of participants demonstrated high procedural knowledge. When the area of square was transformed into rectangle, more than 50% of the participants were unable to solve the task completely. While few numbers (.3-13%) reached high procedural knowledge on prisms and pyramids, a large proportion (50-82%) demonstrated low to moderate procedural knowledge. Cumulatively, 38.8%, 37.1% and 24.1% of participants demonstrated low, moderate and high procedural knowledge respectively in angles, triangles, quadrilaterals, prisms and pyramids.

Regarding research objective1 (a) (iii), the result show that participants‟ knowledge on shape and space contents was general moderate. Despite this, majority demonstrated better knowledge in procedural knowledge and declarative knowledge than in conditional knowledge. Also, it appears the tasks relating to three-dimensional solid geometry (prism and pyramids) were more difficult to be performed by participants than tasks involving two-dimensions or plane geometry. For example, considering tasks 10 and 11 as shown in Box 1, majority of participants had difficulty in identifying the name of the prism or determining the faces in the given prism (declarative knowledge). More than 35% of participants also had difficulty in naming prism and determining its faces (declarative knowledge), in sketching correctly a rectangular prism (procedural knowledge) or explaining the relation between triangular prism and triangular pyramid (conditional knowledge). This means that participants had difficulties in dealing with tasks which required spatial reasoning across all three types of knowledge.

10. (a)What is the name of this figure?......................................
    (b) How many faces are there in the figure…………………………..



11. (a) Sketch the net of a rectangular prism.
    (b) What is its area if all the faces are equal

Box 1: Sample Task on Shape and Space

4.4. Knowledge Types Relationships

The second part of the first objective of the study was to examine the relationship between participants‟ declarative, conditional and procedural knowledge regarding basic geometry. The result of correlational analysis indicated in Table 22 shows positive correlations of .381, .460 and .439 between declarative and conditional knowledge, declarative and procedural knowledge, and procedural and conditional knowledge respectively. The analysis shows that all correlation coefficients were moderate but highly significant at .05 level. The results demonstrated that there exist positive and significant bilateral relationships in participants" knowledge types.

Table 22: Correlations between Knowledge Types in Geometry

	Declarative Knowledge	Conditional Knowledge	Procedural Knowledge
Declarative Knowledge	1		
Conditional knowledge	.381**	1	
Procedural Knowledge	.460***	.439**	1

***significant at .001*

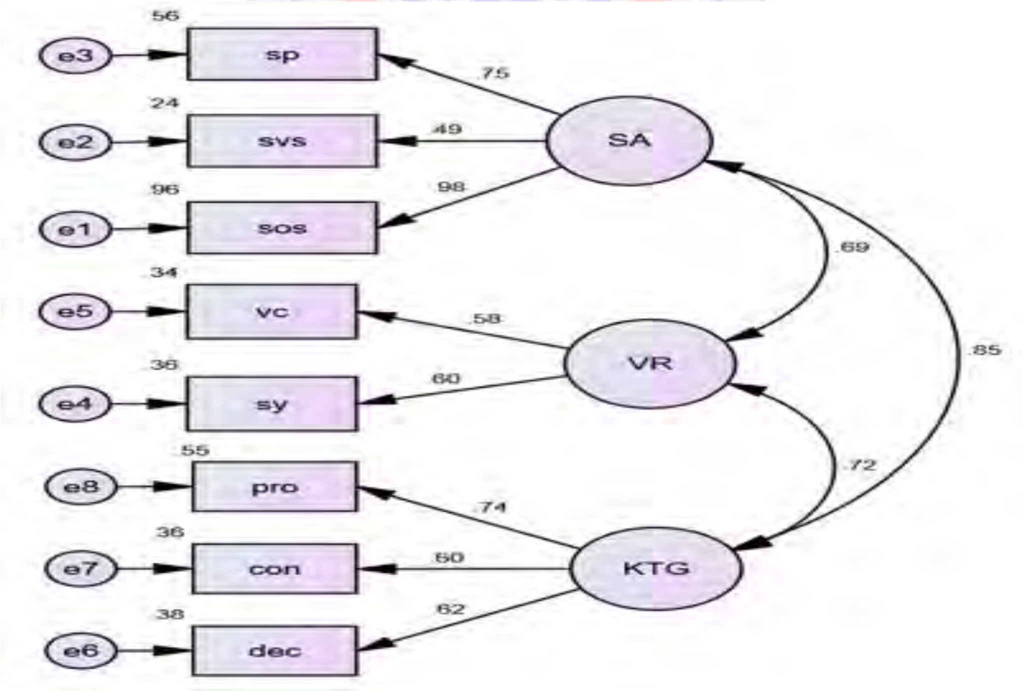
Further analysis using CFA was conducted in the next section 4.5.1 to further examine the how declarative, conditional and procedural knowledge respectively and cumulatively accounted for variance in participants" basic Geometry content knowledge.

4.5. Relationship among Spatial Ability, Verbal Reasoning and Basic Geometry Content Knowledge

The second and third objectives of the study were to investigate how spatial ability directly or indirectly via verbal reasoning relate to and accounts for Basic Geometry content knowledge. The investigation involved the use of SEM which linked both measurement and structural models.

4.5.1. Full measurement model

An investigation of the measurement part of the hypothesized model in this study was conducted to ensure that observed variables used for the model construction produced reasonable fit measures. This was done by examining the measures of reliability and acceptable magnitude of regression parameters describing the latent variables they purported to measure. Thus, each measurement part of the model was specifically evaluated to reveal construct validities and reliabilities. Since the constructs were derived from observed variables, estimates of construct validities explained how much observed variables have successfully measured what was intended to measure. On the other hand, examining reliability ensured the proportion of consistency of the measurement of each latent variable cumulatively by the observed variables. The examination begun with factor loadings analysis since the strength of measurement model largely depends on how each factor loads on its latent variable. Figure 10 displays the standardized factor loadings corresponding to the three latent variables: spatial ability (SA), verbal reasoning (VR) and basic Geometry content knowledge (KTG). The entire measurement estimates are displayed is Appendix D.



Model fit Indices :$\chi^2 = 38.251$, *df=17*, $\frac{\chi^2}{df} = 2.391$, *RMSEA=.043 and CFI=.991*

Figure 10 Standardized Factor Loadings in CFA Model (three factors)

4.5.1.1. *Factor loadings and correlations as reliabilities*

From the correlation matrix of a standardized solution, the factor loadings are standardized values ranging from -1 and 1. Thus, the factor loading described the reliability of items. An observation of the standardized solution in Figure 10 shows that mental rotation had the highest loading of .99 followed by spatial perception which loaded as high as .75 on spatial ability factor. The third and last indicator item of spatial ability was spatial visualization which loaded as low as .49 on the factor structure. Factor loadings of .50 and above are recommended and acceptable value for consistency of observed variable (Kline, 2015). In this study, spatial visualization which loaded below .50 might be described as a suspicious observed measure. However, since the loading of .49 is approximately .50, the item was deemed to have loaded reasonably. It must be noted that spatial visualization tasks appear rather more difficult since they require more complex configuration than the other two spatial factors (Cevirgen, 2012). While several reasons might be adduced, the multi-steps involved in performing spatial visualization tasks was, perhaps, one explanation for the low loading. The proportion of variances explained by each measure were estimated as shown in Figure 11.

For verbal reasoning, two indicator items which comprise comprehension and syllogism loaded .58 and .60 respectively on the factor structure. These loadings show that the effects of the two indicator items on the verbal reasoning factor were evenly distributed. Finally, for Basic Geometry content knowledge, the factor loadings were .74, .60 and .62 for procedural, conditional and declarative knowledge measures respectively. Even though, declarative knowledge had the highest effect on Basic Geometry content knowledge, the contributions of all three indicator items were substantially high and consistently distributed on the factor structure. Further examination of the error terms or the variances unaccounted for in each indicator items suggested that all factor loadings contributed satisfactorily in measuring each of the three latent variables considered for the study.

4.5.1.2. *Inter-item correlations*

The inter-item correlation coefficients also explained the strength of relationship between two items which loaded on one latent variable. Too low correlation between two items meant that the both items did not have similar characteristics required to measure same latent variable. Differently, too high correlation meant that the two items had almost the same characteristics and so only one could be retained as indicator of the underlying latent variable. Literature recommended item-item correlation coefficient of .30 and .85 as the floor and the ceiling values respectively (Byrne, 2012). For spatial ability, the correlation coefficients were .37 for spatial visualization (sv) and spatial perception (sp), .73 for mental rotation (sos) and spatial perception, and .48 for spatial visualization and mental rotation. These correlations portray reasonable relationships since none was lower than .30 to suggest the item did not indicate the same latent variable. None of these correlation coefficients was also higher than the recommended threshold of .85 to suggest the said two items were nearly the same. In term of the items loading on verbal reasoning, the correlation coefficient was .35 for syllogism (vr) and comprehension (vc) .35. For Basic Geometry content knowledge (KTG), the correlation coefficients were .37 for declarative (dec) and conditional (con) knowledge, .44 for conditional and procedural (pro) knowledge, and .46 for declarative and procedural knowledge.

As shown in Table 23 (see covariances in Appendix D), all inter-item correlations suggested acceptable consistencies and thus signified that the items sufficiently correlated with each other in measuring their respective latent variables. The correlation coefficients of items measuring spatial ability and those measuring verbal reasoning justified the expected relationship. Also, the correlation coefficients of items measuring spatial ability and basic Geometry content knowledge and the correlation coefficients of items measuring verbal reasoning and basic Geometry content knowledge were all moderate.

Table 23 Matrix of all Correlations between Constructs and Inter-item Correlations

	KTG	VR	SA	Pro	Con	Dec	Vc	Sy	Sp	Svs	sos
KTG	1.000										
VR	**.724**	1.000									
SA	**.849**	**.690**	1.000								
Pro	**.745**	.539	.632	1.000							
Con	**.597**	.432	.506	**.444**	1.000						
Dec	**.620**	.449	.526	**.461**	**.370**	1.000					
Vc	.424	**.585**	.404	.315	.253	.263	1.00 0				
Sy	.432	**.596**	.411	.321	.257	.267	**.349**	1.000			
Sp	.633	.514	**.745**	.471	.377	.392	.301	.307	1.000		
Svs	.420	.341	**.495**	.313	.250	.260	.200	.203	**.369**	1.000	
Sos	.830	.675	**.978**	.618	.495	.514	.395	.402	**.729**	**.484**	1.000

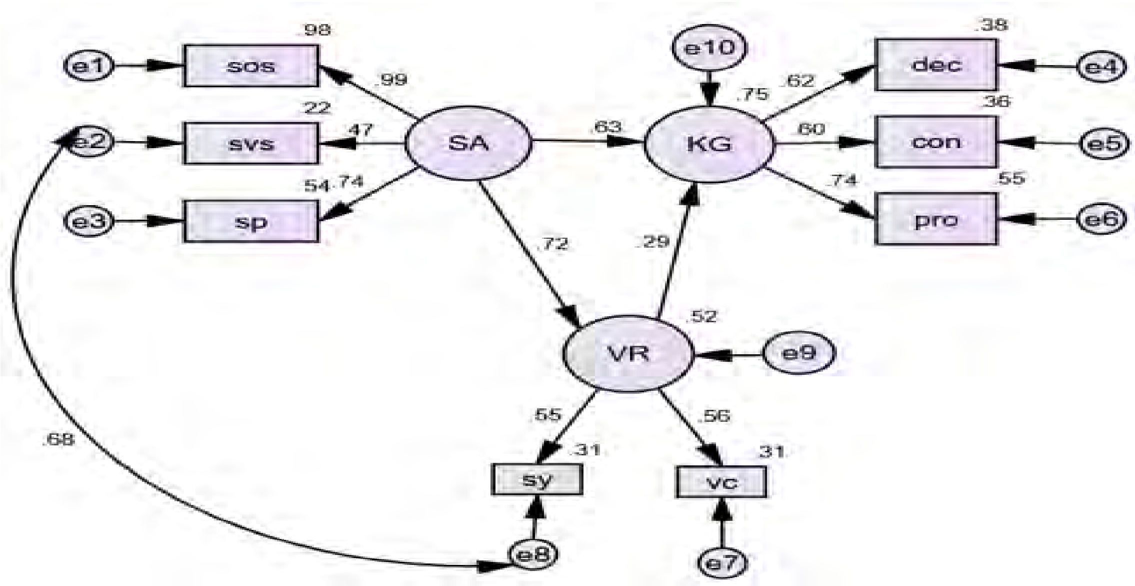
N/B: Figures in bold are correlations between constructs or inter-item correlations.

Apart from the inter-item correlations, the hypothesized model in this study assumed a reasonable degree of correlation between items that measure different constructs and tested the extent of correlation in the presence of measurement errors. The correlations between spatial ability and verbal reasoning was .69, between verbal reasoning and basic Geometry content knowledge was .72 and between spatial ability and basic Geometry content knowledge was .85. All these correlation coefficients show that the three constructs had significantly strong bilateral relationships.

4.5.2. The structural model

The structural model which was earlier specified and identified in Chapter three of this thesis was evaluated using multi-group structural equation modeling (MSEM). For convenience, it should be recalled that the theoretical relationship of declarative, conditional and procedural knowledge types was adopted to defined Basic Geometry content knowledge. A review of meta-analytic and factorial studies on spatial ability also led to the adoption of spatial visualization, mental rotation and spatial perception as indicators of spatial ability. Lastly, syllogism and comprehension were considered indicators of verbal reasoning.

During the model specification, it was hypothesized that spatial ability would have both direct and indirect effects on basic Geometry content knowledge with verbal reasoning mediating the effect. The output of analysis of the model in standardized estimated parameters (regression weights, factor loadings and the residual variances) is shown in Figure 11. The three latent variables are labelled SA for spatial ability, VR for verbal reasoning and KTG for Basic Geometry content knowledge. The goodness-of-fit of the model was evaluated using the Chi-square values (χ^2) with its degrees of freedom (df) and the p-values (sign), the relative Chi-square value (CMIN/DF), Root mean square error of approximation (RMSEA) and Residual Mean Residual (RMR) estimated by the maximum likelihood estimator method (MLE).



χ^2 = 38.251, DF = 16, P-value = .001; CMIN/DF= 2.391; RMSEA=.043; RMR=.070.

Figure 11 The AMOS Output of Initial Model Estimates by ML Estimation

As shown in the path diagram in Figure 11, the parameters estimated indicate that the model might not have achieved good fit to data as expected since the Chi-square value demonstrated significance. The p-value of .001 derived from the Chi-square value of 43.289 did not seem to support the prior hypothesis that the sample variance and the model-implied variance are equal. Arbuckle (2013) however, observed that chi-square test tends to be sensitive with large sample size and does not give good measure for evaluating model fit for large sample. Arbuckle suggested normed Chi-square test (CMIN/DF) as alternative when using sample size larger than 200. Other absolute indices such as, RMSEA, RMR together with the baseline comparison fit indices should also be examined. When the CMIN/DF is less than 3 or 5, the model is said to be good fit to data (Kline, 2015). Since sample size of 757 used in this study was deemed large, the normed chi-square value was further examined. The result from Figure 11 revealed CMIN/DF of 2.391 and portrayed that the model is a very good fit to data. The RMSEA and the RMR values of .043 and .070 were both less than the criterion values of .05 and .08 respectively. The GFI and AGFI of .987 and .971 were also higher than .95 and suggested that the model fits reasonably well with data. The baseline comparison indices were examined further to support the model fit. The NFI (.984), RFI (.972), IFI (.991), TLI (.984) and CFI (.991) were all higher than .95 and showed that model fits very well to data.

In the model, spatial ability was specified as an exogenous unobserved (independent latent) variable explaining two unobserved endogenous variables (verbal reasoning and Basic Geometry content knowledge). A closer look at the structural regression equations in Figure 11 revealed that most of the relationships supported the initial hypothesized relationships. Refer to Appendix D for full parameter estimates including standardized and unstandardized values and corresponding p-values. Thus, the theoretical equations were solved as follows:

Structural Equations

Variable			**Estimate**	**S.E.**	**C.R.**	**P**	R^2
VR	<---	SA	.306	.026	11.673	***	.516
KTG	<---	VR	.287	.081	3.530	***	.746
KTG	<---	SA	.263	.032	8.226	***	

As can be seen, the regression weight of spatial ability corresponded with critical ratio of 11.673 which was significant at .000. In the second equation, the regression weight of verbal reasoning also reflected a critical ratio of 3.530 which demonstrated statistical significance of .000. The squared multiple correlation of .516 demonstrated a reasonably effect size or moderate amount of variance in verbal reasoning accounted for by spatial ability. In the final equation, the regression weights yielded a critical ratio of 8.226 corresponding to statistical significance of .000. The squared multiple correlation of .746 demonstrated a large effect size or amount of variance in basic Geometry content knowledge accounted for by spatial ability and verbal reasoning. The total effect comprises the direct effect of spatial ability and the indirect effect along verbal reasoning. AMOS output value for the indirect effect size was .088 (standardized to .210). The standardized direct effects of spatial ability and verbal reasoning are shown in Figure 11. While the path from spatial ability to basic Geometry content knowledge had significantly large direct effect, the indirect effect of .210 accounted for by verbal reasoning may be described as moderate (Kline, 2015). This indirect effect provided some evidence prompting further model reexamination on the strength of the indirect path.

4.5.3. Model re-Specification and testing

To explore competing forms of the relationship among the three latent variables in the study, an alternative model involving only the indirect path from spatial ability to basic Geometry content knowledge mediated by verbal reasoning was assumed and tested. Figure 12 is a visual display of the alternative models.

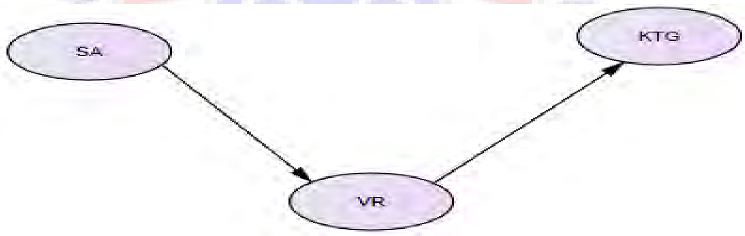


Figure 12 An Alternative Model of Spatial Ability on Basic Geometry content knowledge via Verbal Reasoning

While exploring the more preferred model that fits data through model re-specification, it was expected that central research questions particularly regarding the „*effect of spatial ability on basic Geometry content knowledge* and *the mediating role of verbal reasoning''* remained unaffected (Lauritzen, 2012). An analysis of both models showed that in deed these research questions which prompted the investigation were not altered with the re-specification. In the alternative model, a weaker indirect path through verbal reasoning would reinforce the original research hypothesis that spatial ability had significant magnitude of direct effect on Basic Geometry content knowledge.

In this re-specification, the alternative model was nested in the initial model where the number of parameters to be estimated differed by one degree of freedom. And, as the two models were nested with the same number of observed and latent variables, chi-square difference test was used to compare goodness of fit (Byrne, 2012; Muthen & Muthen, 2012). Thus, an investigation for a more parsimonious model between the two models was conducted using the chi-square difference test. Figure 13 (See Appendix D) displays the AMOS output of standardized estimates of the alternative model.

Figure 13 in Appendix D shows that the alternative model attained chi-square value of 64.41, df=17 and p-value=.000. Further model fit examination was done using the alternative CMIN/DF since the chi-square value was significant. The CMIN/DF of 3.79 was less than 5.0 and portrayed that the model fits the data well. This was not supported by the RMR of .094 which was greater than .08. However, the RMSEA of .061 was less than .08 and showed that the model produced satisfactorily reasonable fit to data. Furthermore, the GFI (.979) and AGFI (.955) were both higher than .95 signifying that the model had good fit to data. The baseline comparison indices including NFI (.975), RFI (.958), IFI (.981), TLI (.969) and CFI (.981) were all also higher than .95 suggesting the model fits substantially well to data. To determine which of the models was more parsimonious, the chi-square difference test was performed at .05 significance level. Table 24 displays $\Delta\chi^2$ value, the degree of freedom and two main model fit indices.

Table 24 Chi-square Difference Test for Initial and Competing Models

Model	Chi-square	Df	p-value	RMSEA	RMR

Competing model	64.41	17	.000	.061	.094
Initial (Original) model	38.25	16	.001	.043	.070
Difference	26.16	1	.0001	-	-

Using the change chi-square value of 10.64 and $df = 1$, the p-value of .000 was obtained from Microsoft Excel computation. This result as shown in Table 24 was significant at .05. According Byrne (2012), the alternative model should be supported if the Chi-square difference value shows non-significance after adjusting for change in degrees of freedom. In this case, the difference was significant and hence the alternative model fitted the data significantly worse than the initial model.

A critical comparative observation of other model indices and statistics shown in Table 25 confirmed that the initial model produced better fit to data than the alternative model. For example, the RMSEA=.043 and the RMR=.070 for the initial model were far smaller than those (RMSEA=.061 and RMR=.094) for the alternative model. According to Byrne (2010), to compare alternative model with same data, evaluation of the models with RMSEA, RMR, GFI and CFI should be complemented with Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC) and Expected Cross-Validation Index (ECVI). From Table 25, the result showed that AIC, CAIC and ECVI of the initial model were smaller than the alternative model signifying that the initial model was better fit to data. The initial model was hence the preferred choice for further analysis. To avoid ambiguity from now onwards, the initial model demonstrated more parsimony and would be referred to as the final model or the model in the succeeding analysis. Table 25 displays full model fit indices of both models.

Table 25 AMOS Output of Model Indices for Initial and Competing Models

Fit Indices	Initial model	Competing model	Thresholds
χ^2(df)	38.25 (16)	64.41 (17)	p>.05 (Non-sign.)
CMIM/DF	2.39	3.789	<3 or <5

RMSEA	.043	.061	<.05
RMR	.070	.094	<.08
GFI	.987	.979	>.950
AGFI	.971	.955	>.950
NFI	.984	.975	>.950
RFI	.972	.958	>.950
IFI	.991	.981	>.950
TFI	.984	.969	>.950
CFI	.991	.981	>.950
ECVI	.104	.135	The lesser the better
AIC	78.733	102.412	The lesser the better
CAIC	190.839	209.370	The lesser the better

Even though the model fits the data statistically well, there was the need to ascertain whether the three observed variables which appeared skewed and two observed variables which indicated flat kurtosis respectively influenced the model in any way. In attending to such situation, Arbuckle (2013) suggested that researchers should also estimate model parameters with alternative estimators and compare with the original estimator method. In line with this suggestion, other methods of parameter estimators recommended for non-normally distributed variables were compared with Maximum Likelihood (ML). They include the Unweighted Least squares (ULS), scaled -free least squares (SLS) and Asymptotically distribution-free (ADf). The estimated model fit indices from these methods are displayed in Table 26 displays.

Table 26 Estimates with Maximum likelihood (ML), Unweighted Least squares (ULS), scaled -free Least Squares (SLS) and Asymptotically Distribution-free (ADf)

Equations			ML	GLS	ULS	SLS	ADf
Structural Equations							
VR	<---	SA	.306	.311	.323	.303	.316
KTG	<---	VR	.287	.279	.160	.293	.227
KTG	<---	SA	.263	.269	.305	.258	.282
Measurement Equations							
Svs	<---	SA	.675	.688	.707	.705	.679
Sos	<---	SA	1.000	1.000	1.000	1.000	1.000
Dec	<---	KTG	1.000	1.000	1.000	1.000	1.000
Con	<---	KTG	.933	.935	.940	.941	.945
Pro	<---	KTG	2.256	2.259	2.252	2.253	2.278
Sp	<---	SA	.712	.716	.697	.690	.699

Equations			ML	GLS	ULS	SLS	ADf
Structural Equations							
Vc	<---	VR	1.000	1.000	1.000	1.000	1.000
Sy	<---	VR	1.887	1.915	1.822	1.847	1.940
Model fit indices							
Chi-square			38.251	53.134	72.236	14.615	34.952
p-value			.001	.004	-	-	.004
RMSEA			.043	.040	-	-	.040
RMR			.070	.076	.052	.060	.071

The result in Table 26 shows that none of the estimates significantly differed by the estimation methods. All the chi-square values revealed statistical significance due to perhaps the large sample size. However, the alternative CMIN/DF values were all less than 3.0 which suggest a good model fit to data for all the estimation methods. Comparative analysis of the factor loadings and regressions also suggests that the choice of one of the estimation methods could not yield substantially different measurement and structural equations. For example, the regression weights for KTG on SA were .263, .269, .305, .325 and .282 when using ML, GLS, ULS, SLS and ADf estimators respectively. The RMSEA of .043, .040 and .040 corresponding to ML, GLS and ADf estimation methods respectively did not also differ substantially from each other.

Critical observation of all estimates in Table 26 indicates that the ML generally produced more conservative values which further suggests that if the model fitted data well with ML then it would fit with the other methods as well. An examination of the correlation matrices in the five techniques used also revealed structural similarities rather than differences. The model estimates using ML method was therefore maintained for the further model in this study.

4.5.4. Evaluation of model fit

The first aspect of the model evaluation was an examination of the residual covariances. According to Arbuckle (2013), in large samples often greater than 200 participants, the standardized residual covariances should have a standard normal distribution. Also, the residual covariances should be less than two in absolute value if the model would be correct. From the analysis of the standardized residual covariances as shown in Table 27, it revealed that except between comprehension and conditional knowledge (2.430), all other standardized residual covariances were less than two in absolute values. This appears predictable since univariate normality test conducted earlier in this study portrayed that comprehension variable was not statistically normally distributed. Nonetheless, due to the robustness of SEM, these residual covariances did not seem to have much influence on parameter estimates.

Table 27 Standardized Residual Covariances

	sy	Vc	Sp	pro	Con	Dec	sos	Svs
Sy	.416							
Vc	1.161	.000						
Sp	-.090	-1.646	.000					
Pro	-.825	.410	.147	.000				
Con	1.116	**2.430**	-1.205	-.138	.000			
Dec	-.296	-.092	.360	-.006	.244	.000		
sos	.437	-.428	.013	.189	-.358	-.016	.000	
Svs	.551	1.644	.009	.655	.142	.183	.287	.218

The second aspect of the model evaluation involved an examination of the significance and non-significance of regression weights. Table 28 displays the AMOS output of unstandardized regression weights, standard errors and critical ratios (with corresponding significance values) and squared multiple correlations of each of the observed variables in the measurement models. All the critical ratios (which ranged from 9.594 to 23.609) were statistically significant at .05 level. This means that their contributions to the model were significant.

Table 28 Measurement Equations with R^2

Variables			Estimate	S.E.	C.R.	P	R^2
Sp	<---	SA	.712	.030	23.609	***	.738
Svs	<---	SA	.675	.049	13.702	***	.473
Sos	<---	SA	1.000				.991
Dec	<---	KG	1.000				.620
Con	<---	KG	.933	.070	13.239	***	.599
Pro	<---	KG	2.256	.147	15.382	***	.743
Vc	<---	VR	1.000				.558
Sy	<---	VR	1.887	.197	9.594	***	.552

The squared multiple correlation analysis (R^2) as shown in Table 28, was another important aspect of the model which was examined. These estimates described the amount of variance accounted for in each variable or the magnitude of effect one variable had on the other. For spatial ability, 73.8%, 47.3% and 99.1% of the variances of spatial perception, spatial visualization and mental rotation were captured. Also, 62.0%, 59.9% and 74.3% of the variances of declarative, conditional and procedural knowledge respectively were accounted for by Basic Geometry content knowledge. Finally, relatively low proportions of 55.8% for syllogism and 55.2% for comprehension were accounted for by verbal reasoning. According to Kline (2015), proportions higher than 25% could be described as substantially large and reasonable effect sizes of good model.

Having determined how spatial ability (SA), verbal reasoning (VR) related to and accounted for the variance in basic Geometry content knowledge (KTG), further analysis was done to test the Research Hypotheses H_{02}, H_{03} and H_{04} (see Table 18) using their structural equations as follows.

Structural equations

Variable Relationship			γ	S.E.	C.R.	P	R^2
VR	<---	SA	.306	.026	11.673	***	.516
KTG	<---	VR	.287	.081	3.530	***	.746
KTG	<---	SA	.263	.032	8.226	***	

Firstly, to accept the null hypothesis H_{02} to the solution of the theoretical equation $\eta_3 = \gamma_{11}\eta_1 + \zeta_1$……………..[11] (see chapter 3), $\gamma_{11} = 0$. However, as can be seen above, the regression weight of spatial ability ($\gamma_{11} = .263 \neq 0$) is different from zero and corresponded with critical ratio of 11.673 which was significant at .000. Thus, the alternative hypothesis is supported. This means that the effect of spatial ability on basic Geometry content knowledge is statistically significant.

Secondly, for the null hypothesis H_{03}, the regression weight of verbal reasoning ($\gamma_{21} = .306$) also reflected a critical ratio of 3.530 which demonstrated statistical significance of .000. The squared multiple correlation of .516 demonstrated a moderate amount of variance in verbal reasoning accounted for by spatial ability. The result means that spatial ability affects verbal reasoning significantly and accounts for 51.6% of its variance.

Finally, to accept the null hypothesis H_{04} regarding the solution to the theoretical equation $\eta_3 = \beta_{21}\eta_2 + \gamma_{21}\eta_1 + \zeta_2$ ……………..[12], $\beta_{21} = 0$ and $\gamma_{21} = 0$. However, the regression weights ($\beta_{21} = .287$ and $\gamma_{21} = .306 \neq 0$) differ from zeros and yielded statistical significance of .000. The result means that the effect of spatial ability on basic Geometry content knowledge is significantly mediated by verbal reasoning ability. Furthermore, .746 of the variances was accounted for by spatial ability and verbal reasoning in basic Geometry content knowledge.

4.6. Relationship Across Gender and Across Programmes

The fourth objective of the study was to determine whether relationship among spatial ability, verbal reasoning and basic Geometry content knowledge differ by (i) gender and (ii) programme of study. In other words, the study sought to determine if the hypothesized model fitted data reasonably and consistently across gender as well as across the programmes. This was tenable if the null hypothesis H_{05} is not rejected.

Most studies often assume that the measurements for observed variables are invariant and proceed to examine mean-difference between groups. However, simply assuming measurement invariance could result in inaccurate conclusions when this assumption is violated (Sass & Schmitt, 2013). Byrne (2010) particularly suggested that testing measurement invariance should be a prerequisite if researchers intend to test structural coefficients of a model across groups. In line with this suggestion and to be able to test the structural invariance, the model was specified for analysis of measurement invariance across groups. The essence was to cross-validate the model and allow for evaluation of differences in the relationships of the constructs across groups.

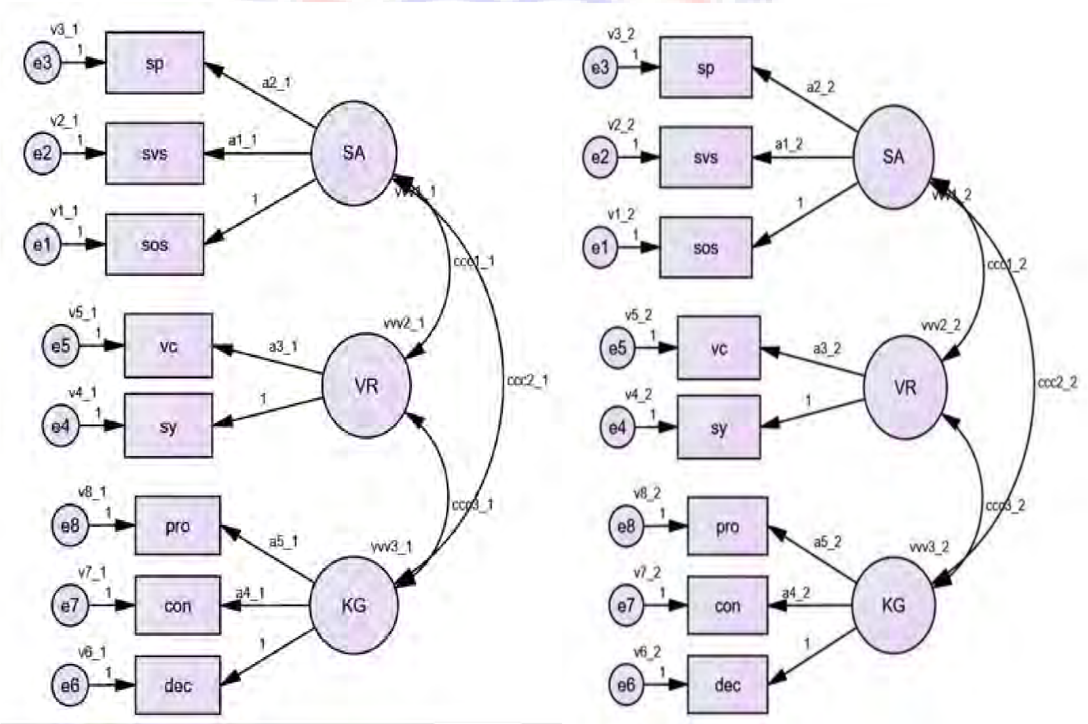
The measurement invariance testing was conducted by comparing the Confirmatory Factor Analysis (CFA) model of relationships between spatial ability, verbal reasoning and basic Geometry content knowledge based on gender and on programme of study by participants. In measurement invariance testing, the baseline model was estimated first and compared with separate estimations of the model for each group. For example, the baseline model was compared with the model for males and for females. The process is described as multi-group invariance analyses (Kline, 2015; Arbuckle, 2013). The essence of measurement invariance analysis was to test how the baseline model compared with observed structures of two or more variables. Various absolute and baseline comparison fit indices were analyzed to determine how well the prior model fits the data as well as how well the model fits in comparison with the saturated model or the independent model. Figure 13 shows AMOS output of the standardized estimates of baseline measurement model. These estimates were examined to provide evidence of the model goodness-of-fit and compare separate models between the two genders and later between the two programmes offered by participants.

4.6.1. Multi-Group invariance analysis

The invariance analysis takes place sequentially through a series of stages of adding or deleting constraints to the initial factor model. In literature, three main approaches exist for testing measurement invariance. These include item response theory, testing for cognitive equivalence and multiple group confirmatory factor analysis (MGCFA). In this study, the MGCFA was preferred because of its wide acceptance in literature (Byrne, 2010) and its applicability in the AMOS software (Arbuckle, 2013; Hair, Black, Babin & Anderson, 2010) employed in this study. Unlike other approaches that employ many separate procedures, the multi-stage MGCFA provided a well-designed way to examine several measurement issues through a single procedure.

To apply MGCFA in this study, the forward approach was employed using the AMOS software (Arbuckle, 2013). This approach involved sequentially imposing constraints and investigating the non-significance of χ^2 difference test ($\Delta\chi^2$) between two nested models of the unconstrained model which assumes no invariance and one constrained model. The first stage was the establishment of configural invariance which involved finding out if the same variables had statistically significant factor pattern for same constructs in the groups. The next stage involved checking if the measurement weights (factor loadings) were equal in different groups. If they were equal, the metric invariance was satisfied. A strong factorial invariance was then examined by testing if the intercept on observed variables could be set equal on different groups. The final stage was to test if the measurement residuals were equal across different groups. This equivalence condition when met would make the model attain a strict factorial invariance. A strict factorial invariance portrayed that the factor loadings, the covariances, the intercepts and the measurement residuals were statistically equivalent across groups. This would imply that indicator items were equally reliable across groups. If all items were invariant, groups could be compared using mean performances or regression coefficients of structural model (Byrne, 2010).

This study was focused on these comparisons across gender and programme of study. Hence, examining measurement invariance was necessary. A detailed specification of the stages of MGCFA for testing the measurement invariance across groups of participants in this study is outlined in Appendix E. Figure 13 is diagrammatic representation of MGCFA.



NB: $a's$ are factor loadings, $v's$ are measurement errors and $ccc's$ are correlations and $vvv's$ are variances. E.g. $a1\_1$ is factor loading in male model and $a1\_2$ is factor loading in female model.

Figure 13 Diagram for MGCFA Analysis with Specified Parameters for Male and Female

4.6.1.1. *Measurement invariance across gender*

Table 29 displays the results of goodness-of-fit indices (GFIs) for the unconstrained model in terms of baseline, as well as for male and female. The results show that Chi-square values of 38.25, 32.45 and 27.46 are statistically significant for the baseline, male and female groups respectively. However, alternative test, the normed chi-square values of 2.39, 2.03 and 1.72 were all less than the threshold of 3. These values suggest the model in each case fit the data well. The GFIs for the two groups were above the threshold of .95 with female model yielding better absolute goodness of fit indices than the male with the same model specification. On the other hand, the baseline comparison fit indices were relatively higher in the male group. The GFI of .984 and CFI of .989 were higher than threshold of .95 whiles the RMSEA of .052 is closed to .05. These goodness-of-fit indices confirmed that the unconstrained model was a very good fit to data.

When all parameters of the model were unconstrained across gender, the chi-square value of 51.15 with df=26, portrayed significant value p=.002 and CMIN/DF=1.80. The difference in GFIs (∆NFI, ∆IFI, ∆RFI and ∆TFI) were also less than the threshold of .01 (Cheung & Rensvold, 2002) Thus, the unconstrained model satisfied the configural invariant hence gender groups decomposed into the same number of factors, with the same items associated with each factor. Principally, configural invariance may fail when participants from different groups attach different meanings to same construct due to different conceptual frames of reference. Other likelihood problems relating to configural non-invariance include data collection problems or translational errors. The result in this study however shows that males and females attached similar conceptual frames of reference to the constructs considered in this study.

Table 29 Separate Goodness of Fit Indices for Male, Female and Unconstrained

Model	Baseline N=757	Male N=495	Female N=262	Unconstrained N=757
CMIN (DF)	38.25(26)	32.45(16)	27.46(16)	51.15(26)
P	.001	.009	.037	.002
CMIN/DF	2.391	2.028	1.716	1.796
RMSEA	.043	.046	.052	.038
RMR	.141	.148	.097	.078
GFI	.987	.984	.975	.981
AGFI	.971	.963	.944	.947
RFI	.972	.963	.944	.962
IFI	.991	.989	.986	.991
TLI	.984	.981	.976	.981
CFI	.991	.989	.986	.991

Since the assumption of configural invariance of unconstrained model was achieved, the factor loadings or measurement weights (model A) were constrained to be equivalent and tested for invariance. The result is displayed in Table 30 (refer to model A). The chi-square difference ($\Delta\,\chi^2$) test yielded a value of 4.661 with df=5 and a non-significant p-value of .459. The range of values from -.003 to .002 for ΔNFI, ΔIFI, ΔRFI an ΔTFI respectively were less than the criterion cut-off of .01 which supported the result of the $\Delta\,\chi^2$ value. The result shows that the changes in the model fit indices were quite minimal. Therefore, metric invariance was established. It was hence concluded that the spatial ability construct, for example, had the same operational meaning across groups. Similarly, both the verbal reasoning and basic Geometry content knowledge were also homogeneously operationalized by the male and female participants. According to literature, the metric invariance requirement could usually be difficult to satisfy. Thus, if the noninvariant items constitute only a small portion of the model, cross-group comparisons may not be affected to any meaningful degree (Byrne, 2010). The items responsible for the overall non-invariance could be located through series of testing and acknowledged, freed or removed entirely. The result in this study however achieved the metric invariance for the CFA model and hence portrayed that all factor loadings were assumed equivalent across gender. This means that the strength of the relationship between each indicator item and its underlying construct was the same across both genders.

The invariance of the structural covariances was examined next. Table 30 shows the result of comparison of model A which constrained factor loadings and model B which constrained both factor loadings and structural covariances. The ∆NFI, ∆IFI, ∆RFI and ∆TFIs (between -.002 and .003) appear very small and suggest no substantial differences in the model A and model B cross gender. The $\Delta \chi^2$s of 7.88, df=6 with p=.246 show that there was non-significant difference across gender. Thus, the structural covariances can be described as invariant across gender in this study.

The final stage examined whether measurement errors were invariant across gender. Thus, model B and its nested model C which included equivalence of measurement errors were compared. The result as shown in Table 34 indicates a $\Delta \chi^2$ of 54.54 df=12 with p=.000. The ∆NFI, ∆IFI, ∆RFI and ∆TFIs (between -.012 and .022) were all greater than .01. These indices suggest that there was significant difference between model B and C in which model B appeared more parsimonious. In other words, invariant residual variance was not achieved since model C appeared to fit data worse than model B. As suggested by Byrne (2012), when GFIs achieve high values, differences in the GFIs could replace the $\Delta \chi^2$ test as alternative method to test measurement invariance. The ∆GFIs of model of constrained residual variances revealed marginal differences. According to Cheung and Rensvold (2002), ∆GFIs are superior to $\Delta \chi^2$ when testing invariance because ∆GFIs are not often affected by sample size and model complexity. They further established that CFI particularly is a robust statistic for testing the between-group invariance of CFA models and hence

△CFI≤ 0.01 justifies invariance characteristics. In this study, it was observed that the △CFI (.991-.975=.016) for the residual invariance was closer to .01. Thus, the model could be accepted as having satisfied the residual invariance.

Table 30 AMOS Output of Model Fit Summary and Nested Model Comparisons (Unconstrained-Constrained)

Table 30a: Model Fit Summary

Model	CMIN	DF	P	CMIN/ DF	CFI	GFI	AGFI	RMSEA
U	51.148	26	.002	1.967	.991	.981	.947	.038
A	55.809	31	.004	1.800	.991	.979	.951	.035
B	63.697	37	.004	1.722	.991	.976	.954	.033
C	118.240	49	.000	2.413	.975	.960	.942	.046
D	78.707	46	.002	1.711	.988	.972	.957	.033

Table 30b: Nested Model Comparisons

Model	DF	CMIN	P	△NFI Delta-1	△IFI Delta-2	△RFI rho-1	△TLI rho2
Assuming model Unconstrained to be correct							
A	5	4.661	.459	.002	.002	-.003	-.003
B	11	12.549	.324	.004	.004	-.005	-.005
C	23	67.092	.000	.023	.024	.009	.009
D	20	27.559	.120	.010	.010	-.005	-.005
Assuming model Measurement weights to be correct:							
B	6	7.888	.246	.003	.003	-.002	-.002
C	18	62.431	.000	.022	.022	.012	.012
D	15	22.898	.086	.008	.008	-.002	-.002
Assuming model Structural covariances to be correct:							
C	12	54.543	.000	.019	.019	.013	.014
D	9	15.010	.091	.005	.005	.000	.000

U=Unconstrained, A= Measurement weights, B= Structural covariances, C= Measurement residuals, D=Partial measurement residuals (free v_3 and v_7)

While agreeing with Byrne (2010), the obvious question - *what accounted for slight non-invariance of residual variance across gender?"* still required answers. Therefore, to address this question, the source of non-invariance was investigated through a step-by step deletion and freeing of equality constraints on each of the measurement errors (Cheung & Rensvold, 2002). The step-by-step invariant analysis was conducted in AMOS leading to the detection of the indicator items which produced the residual non-invariance. The analysis resulted in model D where the errors for mental rotation (v1_1≠v1_2) and conditional knowledge (v7_1≠v7_2) were freed and the rest constrained to equality. As shown in Table 34, $\Delta\chi^2$=15.01, df=9 yielded p=.091 which suggests that Model D did not fit the data worse than model B. Changes in the baseline model comparison indices ranged from .000 to .005. Therefore, model D was said to have achieved partial residual invariance across gender. The influence of errors from mental rotation and conditional knowledge was not surprising since earlier analysis pointed out that these indicator items had slightly flat kurtosis. It could also be guessed that the non-invariance of error variance came from the nature of scale and the scoring scheme adopted here. The spatial ability items were summed from dichotomous responses which could easily correlate due to lack of spread in scale. Also, the items for conditional basic Geometry content knowledge were subjective and difficult and errors might accumulate randomly across gender.

The overall MGCFA, nonetheless, shows that the CFA model achieved measurement invariance except a suspicious lack of residual invariance resulting from two residual errors from mental rotation and conditional knowledge variables. The CFA model was therefore said to be cross-validated and hence provided basis for structural path invariance testing. This subsequently provided basis to answers research question 4 and its hypothesis regarding relationship among spatial ability, verbal reasoning and basic Geometry content knowledge across gender. The next section presents the analysis of the measurement invariance across programme offered by participants.

4.6.1.2. *Measurement invariance across programmes*

Apart from gender, participants in this study were characterized into those at colleges studying to become science-specialist or generalist teachers. Literature

(Duffy et al, 2015; Institute of Education, 2014; Uttal & Cohen, 2012) suggests that people studying science were more likely and better inclined to comprehend science, mathematics and geometric concepts than those pursuing general programmes such as general arts and business courses. This has been the rationale for separating pre-service teachers into science specialists who would teach relatively high mathematics and science concepts and the generalists who would teach elementary concepts on shape and space.

Therefore, in evaluating the model on geometry knowledge, it was important to examine whether the model exhibited invariances across the two programmes. Table 31 displays the result of goodness-of-fit indices (GFIs) for separate models for science and general programmes as well as for the unconstrained model in terms of baseline to prove factorial validity. The result in Table 31 shows Chi-square values of 9.83, df=13 with non-significant p=.708 for general programmes participants. The science model revealed a chi-square value of 34.18, df=13 with a significant p=.001. The normed chi-square values of .76 and 2.63 for the general and science programmes were all less than the threshold of 3. The RMSEA for the science and the general programmes were .068 and .000 respectively. Even though the RMSEA for the science was greater than .05, Byrne (2010) suggested that RMSEA value less than .08 is an acceptable fit index. Furthermore, as shown in Table 31, all the GFIs for the two groups were above the threshold of .95 except the AGFI (.939) and IFI (.939) which differ marginally from .95. Cumulatively, these values suggest that the model for each group fits the data well, with the general programmes model demonstrating better fit to data than the science.

Table 31 Model fit Indices for Programme Type and Unconstrained Models

Model	General=403	Science=354	Unconstrained=757

Model	General=403	Science=354	Unconstrained=757
CMIN (DF)	9.83(13)	34.18(13)	44.02(26)
P	.708	.001	.015
CMIN/DF	.756	2.629	1.693
RMSEA	.000	.068	.030
RMR	.089	.091	.090
GFI	.994	.978	.986
AGFI	.983	.939	.962
RFI	.982	.939	.960
IFI	1.00	.982	.994
TLI	1.00	.961	.987
CFI	1.00	.982	.993

Turning to unconstrained model, the chi-square value was 44.02 with df=26, p=.015 and CMIN/DF=1.69. Also, the baseline comparison fit indices were substantially high. The GFI of .986 and CFI of .993 were higher than threshold of .95 whiles the RMSEA of .030 was lower than the highest acceptable value of .05. These absolute fit indices established that the unconstrained model fits the data well. It was also observed that the difference in GFIs (∆NFI, ∆IFI, ∆RFI and ∆TFIs) between the unconstrained and the general; and unconstrained and science models seemed quite small and satisfied the requirement for factorial validity and invariance analysis across the two programmes. The model therefore satisfies the configural invariance for programme of study.

Having achieved configural invariance, the measurement invariance was examined. The factor loadings or measurement weights (model A) were constrained to be equivalent and the model tested for invariance across programmes. The result is displayed in Table 32. The chi-square difference ($\Delta\ \chi^2$) test shows a value of 1.02 with df = 5 and a non-significant p-value of .961. The ∆NFI, ∆IFI, ∆RFI and ∆TFI as shown in Table 32b were less than the criterion cut-off of .01. The result shows that changes in the model fit indices were negligible. Therefore, metric invariance was established signifying that the constrained model did not fit the data worse off than the unconstrained model. It must be noted that equality of measurement intercept invariance was combined with the metric invariance analysis since all measurement intercepts were already set to 1 for model identification purposes in AMOS software which was employed in this study.

An assessment of the full invariance of measurement residuals (MRI) across programmes revealed similar findings as depicted in earlier invariance analysis across gender. However, as shown in Table 32, when measurement residuals were constrained except that of mental rotation and conditional knowledge, an invariance across programmes was achieved. This result reflects a partial measurement residual invariance (MRI*) across programmes and hence satisfies the condition for further invariance analysis and comparisons of regression weights in the model.

By default, AMOS software has incorporated both factor variance and covariances into structural covariance in multi-group analysis. Therefore, the invariance of the structural covariances was examined under a more restrictive measurement invariance involving factor variance and structural covariances. Table 32 shows the result of comparisons.

Table 32 AMOS Output of Model Fit Summary and Nested Model Comparisons (Unconstrained-Constrained)

Table 32a: Model Fit Summary

Model	CMIN	DF	P	CMIN/DF	GFI	AGFI	CFI	RMSEA

Model	CMIN	DF	P	CMIN/DF	GFI	AGFI	CFI	RMSEA
Unconstrained	44.02	26	.015	1.69	.986	.962	.992	.030
MI	45.04	31	.049	1.45	.986	.968	.994	.024
SCOVI	59.83	37	.010	1.62	.981	.964	.990	.029
MRI	107.72	49	.000	2.20	.969	.955	.975	.040
MRI*	65.97	47	.035	1.40	.980	.969	.992	.023

Table 32b: Nested Model Comparisons for Gender Invaraint

Model	DF	CMIN	P	∆NFI	∆IFI	∆RFI	∆TLI
Assuming Model Unconstrained to be Correct:							
MI	5	1.02	.961	.000	.000	-.006	-.006
SCOVI	11	15.82	.148	.007	.007	-.002	-.002
MRI	23	63.70	.000	.026	.027	.012	.012
MRI*	21	21.96	.402	.009	.009	-.007	-.007
Assuming Model Measurement Weights to be Correct:							
SCOVI	6	14.80	.022	.006	.006	.004	.004
MRI	18	62.68	.000	.026	.026	.017	.018
MRI*	16	20.94	.181	.009	.009	-.001	-.001
Assuming Model Structural Covariances to be Correct:							
MRI	12	47.89	.000	.020	.020	.013	.014
MRI*	10	6.14	.803	.003	.003	-.005	-.005

MI= Measurement invariance, SCOVI= Structural covariances invariance, MRI= Measurement residuals invariance, MRI=Partial measurement residuals invariance (free v_3 and v_7)*

From Table 32b, the ∆NFI, ∆IFI, ∆RFI and ∆TFI (between -.00 and .003) appear to be very small and suggest no substantial differences in the model cross programmes. The small difference yielded $\Delta\,\chi^2$ of 15.82, df=11 with p=.148 which shows a non-significant difference across programmes. However, when compared with measurement invariance model, the chi-square value of 14.80 with df=6 resulted in the p=.022. Whereas the differences in the GFIs were deemed marginal, the significance of chi-square difference value suggested non-invariance across programmes. Thus, the structural covariances could not be described as invariant across programmes. The result suggests the likelihood of moderating effect of programme type. Further analysis was conducted to examine the sources of the non-invariance using full structural invariances test which incorporate the path analysis of the model. This analysis is shown under structural invariance across programmes.

4.6.2. Test for structural invariance

An important part of model evaluation involved the determination of whether the paths of relationships between latent variables were invariant across sample and population groupings such as gender and academic programmes. The determination of invariance explained any existing moderating effect of group variables on the relationships in the model and predicted whether the model could be applicable across groups. For participants in this study, differenc es in gender and in academic programmes offered at the colleges of education were significant group variables which could moderate relationships between the latent constructs. Therefore, the present study investigated whether the final model was structurally invariant across gender and across programmes. The analysis was conducted using the multi-group structural equation modeling (MSEM) framework. Basically, MSEM compared regression weights in the model across grouping to determine statistical equivalence.

4.6.2.1. *Structural invariance across gender*

In examining structural invariance, it was important to examine the structural covariances to give a clear idea of the equivalences or differences in covariances or correlations across gender. Since covariance invariance was achieved during the measurement invariance analysis, we proceed to examine its matrices across gender. Table 33 displays these covariances in standardized values (correlations) to facilitate invariant comparisons.

Table 33 Correlations of Estimates (Structural) of Measurement Weight for CFA

Female	SA	VR	KTG	Male
SA_F	1.000	.754	.810	SA_M
VR_F	.437	1.000	.651	VR_M
KTG_F	.786	.676	1.000	KTG_M

NB: The lower left matrix is Female covariance coefficients and the upper right matrix is Male covariance coefficients.

In Table 33, the low and upper arms of matrix represent the covariance matrix in standardized values, for females and males respectively. Except for correlation between spatial ability and verbal reasoning (.437) in the female model which seemed weak, all correlations were substantially high since they exceeded .5. The highest correlation was between spatial ability and basic Geometry content knowledge suggesting they relative dependency and causal effect. Considerable differences in correlations across gender was manifested between spatial ability and verbal reasoning. This was suggestive of moderating effect of gender and portrayed the need to analysis the structural invariance.

Table 34 and 35 display the result of the invariance analyses of the structural coefficients of the model. From the MSEM analysis in Table 34, the structural weights, the structural covariances and the structural residuals were constrained to equivalence across gender groups and compared with the unconstrained model for invariances. All three models which have been nested by forward MSEM demonstrated good absolute and baseline comparisons GFIs. Except the chi-square values which exhibited sensitivity to the large sample size in this study, the normed chi-square values were all lower than the highest threshold of 3.0. The RMSEAs were all also smaller than .05 and supported the GFI, AGFI and CFI which exceeded the threshold of .95. Since these models were nested, the chi-square difference test and changes in GFIs were used to examine any significant invariance of the models across gender.

Table 34 Model Fit Summary for Invariance across Gender

Model	CMIN	DF	P	CMIN/DF	GFI	AGFI	RMSEA	CFI
Unconstrained	59.93	32	.002	1.873	.981	.957	.034	.988
Structural weights	70.96	35	.000	2.027	.978	.954	.037	.985
Structural covariance	73.68	36	.000	2.047	.977	.954	.037	.984

Model	CMIN	DF	P	CMIN/DF	GFI	AGFI	RMSEA	CFI
Structural residuals	74.64	38	.000	1.964	.976	.955	.036	.984

The result as shown in Table 34 revealed a $\Delta\ \chi^2$s of 11.03, df=3 with p=.012. The ∆NFI, ∆IFI, ∆RFI and ∆TFIs (between .004 and .005) were all less than 01. The result suggests there was significant difference in the structural weights between male and female models. Also, when the unconstrained model was compared with the structural covariances model which nested the structural weights, the results revealed a $\Delta\ \chi^2$s of 13.75, df=4 with p=.008. The ∆NFI, ∆IFI, ∆RFI and ∆TFIs (between .004 and .006) were all less than 01. Finally, when the structural residuals model was nested into the structural covariances model, the $\Delta\ \chi^2$s of 14.71, df=6 yielded p=.023. The ∆NFI, ∆IFI, ∆RFI and ∆TFIs (between .002 and .006) were all less than 01 thus the model satisfied the structural invariance. Even though ∆GFIs were less than recommended threshold (Byrne, 2012), $\Delta\ \chi^2$ showed that the models which constrained the structural weights, the structural covariances and the structural residuals were significantly difference from the unconstrained model. This suggests that these nested models fitted the data worse off than the unconstrained model. The structural invariance was therefore not satisfied.

Table 35 Nested Model Comparisons: Assuming Model Unconstrained to be Correct:

Model	DF	CMIN	P	NFI	IFI	RFI	TLI
Structural weights	3	11.03	.012	.005	.005	.004	.004
Structural covariances	4	13.75	.008	.006	.006	.004	.004
Structural residuals	6	14.71	.023	.006	.006	.002	.002
Assuming model Structural weights to be correct:							
Structural covariances	1	2.72	.099	.001	.001	.000	.000
Structural residuals	3	3.68	.298	.002	.002	-.001	-.002
Assuming model Structural covariances to be correct:							
Structural residuals	2	.96	.619	.000	.000	-.002	-.002

The evidence of no structural invariance across gender meant that gender could constitute a moderator on the relationship paths between spatial ability, verbal reasoning and Basic Geometry content knowledge. Critical examination of the regression weights or structural coefficients was required to detect if the effect of the moderator on the path was plausible. In view of this, a pairwise parameter comparison using multi-group analysis was carried out to determine the path(s) of relationship contributing to the non-invariance. In relation to the model, the following hypotheses were derived and tested for evidence of gender moderating the paths of relationship in the model:

H_{m1}: Is path $SA \longrightarrow VR$ for males statistically different from path $SA \longrightarrow VR$ for females?

H_{m2}: Is path $VR \longrightarrow KTG$ for males statistically different from path $VR \longrightarrow KTG$ for females? And finally

H_{m3}: Is the path $SA \longrightarrow KTG$ for males statistically different from the path $SA \longrightarrow KTG$ for females?

The result of multi-group analysis was presented in critical ratios for differences between parameters. These critical ratios were therefore compared to the two-tailed standard z-score of 1.96 in absolute terms. For H_{m1}, the critical ratio for the difference between the coefficients for the male and female models was 1.31. This value was less than 1.96 signifying that the path from SA to VR for males‟ model was not statistically different from the path SA to VR for the females‟ model. Also, for SA to KTG, the critical ratio of .39 did not exceed the standard z-score of 1.96. Therefore, the path from SA to KTG for males was not statistically different from the same path in the case of the female model.

Finally, the critical ratio of −2.56 was obtained for the difference between the coefficients for the male and female models in Table 36. Since 2.56 is greater than the 1.96, the result shows that the path from VR to KTG for males was statistically different from the path from VR to KTG for the females. The overall pairwise parameter comparisons indicates that the source of the non-invariance across gender in the model came from the relationship between verbal reasoning (VR)and basic Geometry content knowledge (KTG).

Table 36 Pairwise Comparisons by Critical Ratios for Differences between Parameters

Paths	**b1_1**	**b2_1**	**b3_1**	b1_2	b2_2	b3_2
b1_1	.000					
b2_1	.239	.000				
b3_1	-.205	-.277	.000			
b1_2	**1.312**	.570	1.360	.000		
b2_2	-3.720	**-2.558**	-3.296	-4.070	.000	
b3_2	.243	-.085	**.386**	-.968	2.707	.000

b1_1= path $SA \longrightarrow VR$ for male; b1_2=path $SA \longrightarrow VR$ for female; b2_1=path$vr \longrightarrow KG$ for male; b2_2=path $VR \longrightarrow KG$ for female; b3_1=path $SA \longrightarrow KG$ for male and b3_2=path $SA \longrightarrow KG$ for female

The coefficients along the path from verbal reasoning to basic Geometry content knowledge was investigated to detect the actual change in the relationship across gender. The result shows that the coefficient for the female was larger than that for the male. The change of .289 (female=.421 and male=.132) between the standardized coefficients was large. In conclusion, the analysis suggests that the mediating role of verbal reasoning in the relationship between spatial ability and basic Geometry content knowledge was moderated by gender.

4.6.2.2. *Structural invariance across programmes*

The analysis of the structural invariance across programmes begun with comparison of the matrices of correlations for each of the two programmes of study by the participants. Table 37 displays these correlations in standardized values to facilitate comparisons along the same scale. The low part of the matrix shows the correlations for the general programmes while the upper part shows the correlations for science participants. From the Table 37, all correlations were substantially high. The highest correlation was between spatial ability and Basic Geometry content knowledge. Considerable differences in correlations across programmes did not seem to exist between the three constructs. This evidence supported earlier CFA analysis that the model satisfied structural covariances invariance across programmes.

Table 37 Correlations (Structural Covariances) of Measurement Weight for CFA by Programmes

	SA	VR	KTG	
SA_G	1.000	.672	.839	SA_S
VR_G	.672	1.000	.742	VR_S
KTG_G	.839	.743	1.000	KTG_S

SA_G = *spatial Ability of General programme;* VR_G -*Verbal Reasoning of General programme*- KTG_G = *Basic Content knowledge of general programme:*
SA_S = *Spatial Ability of science programme;* VR_S -*Verbal Reasoning of Science programme*- KTG_S = *Basic Content knowledge of Science programme*
NB: The lower left matrix is generalist covariance coefficients and the upper right matrix is science covariance coefficients.

However, since the structural coefficients represented the paths of effect from one latent variable on the other, it was considered useful to establish whether there was structural invariance across programmes. From the MSEM analysis, the structural weights, the structural covariances and the structural residuals were constrained to equivalence across programmes in nested models and compared with the unconstrained model for invariances. Table 38 displays the result of the analyses. All three models which have been nested by forward MSEM demonstrated good absolute and baseline comparisons GFIs.

Table 38 Model Fit Summary for Structural Invariance across Programmes

Model	CMIN	DF	P	CMIN/DF	GFI	AGFI	RMSEA	CFI
Unconstrained	70.97	32	.000	2.218	.977	.948	.040	.983
SI	71.82	35	.000	2.052	.977	.952	.037	.984

Model	CMIN	DF	P	CMIN/DF	GFI	AGFI	RMSEA	CFI
SCOVI	71.82	36	.000	1.995	.977	.953	.036	.985
SRI	76.21	38	.000	2.006	.975	.953	.036	.984

SI=Structural invariance; SCOVI=structural covariance invariance

From Table 38, except the chi-square values which exhibited sensitivity to the large sample size in this study, the normed chi-square values ranged from 1.20 to 2.22. These values were all lower than the ceiling threshold of 3.0 and hence signified that the three models fit well to the same data. The RMSEAs were all also smaller than .05 and supported the GFI, AGFI and CFI which were not too distant from the threshold of .95. Since these models were nested, the chi-square difference test and changes in GFIs were used to establish any statistical invariance of the models across programmes. The result is displayed in Table 39.

Table 39 Nested Model Comparisons for Programmes of Study

Model	DF	CMIN	P	∆NFI	∆IFI	∆RFI	∆TLI
1. Assuming model Unconstrained to be correct							
SI	3	.84	.839	.000	.000	-.004	-.004
SCOVI	4	.84	.933	.000	.000	-.005	-.005
SRI	6	5.24	.514	.002	.002	-.005	-.005
2. Assuming model SI to be correct:							
SCOVI	1	.00	.993	.000	.000	-.001	-.001
SRI	3	4.39	.222	.002	.002	-.001	-.001
3. Assuming model SCOVI to be correct:							
SRI	2	4.39	.111	.002	.002	.000	.000

SI=Structural invariance; SCOVI=structural covariance invariance

Table 39 revealed a $\Delta\chi^2$ of .84, df=3 with p=.839 when the structural weights were constrained, and nested model compared with the unconstrained model. The ΔNFI, ΔIFI, ΔRFI and ΔTFI were also all less than .01. Both the $\Delta\chi^2$ and $\Delta GFIs$ values shows that there was no significant difference between the models. In other words, the constrained model did not fit the data worse off than the unconstrained model. Further invariance analysis revealed that when the unconstrained model was compared with the constrained structural covariances model which was nested in the structural weights, the $\Delta\chi^2$ of .84, df=4 which yielded p=.933, did not fit the data worse off than the unconstrained model. Finally, when the constrained structural residuals model was nested into the structural covariances model, the $\Delta\chi^2$ of 4.39, df=2 produced p=.111. The ΔNFI, ΔIFI, ΔRFI and ΔTFIs were all less than .01.

The overall estimates show that the model satisfied full structural invariance across programmes. According to Byrne (2010), models which satisfy both measurement and structural invariance are consistent and can be applied across categories to test difference in means and regression weights under the given population. The model in this study achieved both measurement and structural invariance with respect to the two programmes classifications at the colleges of education in Ghana. The model could therefore be applied across programmes offered by the population of pre-service teachers at the colleges of education.

4.7. Effects Sizes in the Model

The structural equations derived from the three-latent variables model described the linear relationships between independent variables and the dependent variables. In the model, spatial ability represented the unobserved exogenous (independent) variable affecting unobserved endogenous variables of verbal reasoning and Basic Geometry content knowledge. Similarly, Verbal reasoning was specified as a mediating variable directly affecting Basic Geometry content knowledge. In the first equation labelled [11S], the result indicates that a unit score in spatial ability results in .593 scores in basic Geometry content knowledge with a margin of error of .069 scores. In the second equation [12A], a unit score in spatial ability results in .578 scores in verbal reasoning with an error of .458 scores. The squared multiple correlation ($R^2 = .525$) shows that spatial ability explained about 53% of the variance in verbal reasoning. In the third equation [12S], the result shows that a unit score in spatial ability together with another unit score in verbal reasoning yields .620 scores in basic Geometry content knowledge with an error of .167 units. The squared multiple correlation ($R^2 = .745$) suggests that both spatial ability and verbal reasoning significantly accounted for as high as 74.5% of the variance in Basic Geometry content knowledge.

Having confirmed the good fit of data to the latent variable model and obtained solutions to the structural equations, path analysis was used to examine the effect sizes of one latent variable on the other (Arbuckle, 2013). The path analysis looked at the statistical significance of both causal and correlational relationships of latent variables in the model. To determine statistically significant paths, Arbuckle (2013) suggested the use of bootstrapping method. Therefore, bootstrapping was performed in AMOS to obtain direct, indirect and total effects with corresponding standard errors and significant values. To optimize the result, 3000 bootstrap samples recommended by Arbuckle (2013), were performed under 95% bias-corrected confidence intervals. The estimates are displayed in Table 40.

Table 40 Overall Model Effects with Standard Errors and Significant Values (Sample =757)

	SA			VR		
Effect	Direct	Sd	Sign	Direct	Sd	Sign
VR	.718	.048	.001	-	-	-
KTG	.629	.097	.001	.293	.110	.005
Effect	Indirect	Sd	Sign	Indirect	Sd	Sign
VR	-	-	-	-	-	-
KTG	.210	.086	.004	-	-	-
Effect	Total	Sd	Sign	Total	Sd	Sign
VR	718	.048	.001	-	-	-
KTG	.840	.029	001	.293	.110	.018

The direct effects are the path coefficients which are interpreted similarly as coefficients in linear regression. As shown in Table 40, the positive direct effect of .63 with standard error of .097 was obtained for the path from spatial ability to Basic Geometry content knowledge. Also, the positive direct effect of spatial ability on verbal reasoning was .72 with standard error of .048. Similarly, the positive direct effect of verbal reasoning on knowledge on geometry was .29 with standard error of .110. All the three direct effects specified in the model were statistically significant at .05. When verbal ability was specified as a mediating factor, an indirect effect of .21 was found between spatial ability and Basic Geometry content knowledge. This indirect effect was positive and statistically significant at .004 as shown in Table 40. The indirect effect resulted in a total direct effect of spatial ability on basic Geometry content knowledge of .84. Since the total effect was positive and statistically significant, it can be concluded that the mediating role of verbal reasoning was important to the acquisition of Basic Geometry content knowledge.

Further analysis of the bootstrapping results revealed the indirect effects of spatial ability and verbal reasoning to each of the observed knowledge types. The output was examined to clarify how spatial ability affect each of the knowledge types in geometry. The analysis shows that the indirect effect of spatial ability on declarative (.577), conditional (.557) and procedural knowledge (.672) were statistically significant at .001 respectively. Similarly, the standardized indirect effects of verbal reasoning on declarative (.333), conditional (.325) and procedural knowledge (.393) were significant at .005 level.

4.8. Discussions

In line with the four main research questions stated in Chapter one, the findings of the study are discussed as follows:

In this study, pre-service teachers" basic Geometry content knowledge were identified as declarative, conditional and procedural knowledge. This is in line with the three-tier knowledge structure in Alexander and Judy (1988), and Schneider and Stein (2010). The three-factor knowledge type provided clearer picture of the contribution of each type of knowledge to performance problems in geometry for appropriate interventions.

In terms of declarative knowledge, it was found that more than 80% of the 757 pre-service teachers demonstrated moderate performance with about 13% demonstrating low declarative knowledge in angles, triangles, quadrilaterals, prisms and pyramids. Only a few of the pre-service teachers demonstrating high performance on all declarative tasks. This suggests a conceptual lapse and lack of in-depth knowledge of definitions, properties and representations of shape and space in elementary geometry. This result corroborates previous empirical research findings (Armah, Cofie & Okpoti, 2017; Gogoe, 2009) and college-wise assessment reports (Institute of Education, 2014; 2007; 1995) which indicated that majority of pre-service teachers in Ghana have conceptual knowledge problems regarding common shapes and space. The present study also reinforces conclusion by Haj-Yahya and

Hershkowitz (2013) that students have limited declarative knowledge in the form of superfluous definitions of geometric concepts and do not consider necessary and sufficient set of attributes of shapes.

In the literature, conditional knowledge defines the „why", „when" and „where" aspects of conceptual knowing in mathematics (Lauritzen, 2012) or geometry (Aydin & Uzbuz, 2007). Therefore, it tells how deep or superficial an individual conceptual knowledge is regarding geometry. The present study shows that only 15% of the pre-service teachers demonstrated high performance on conditional knowledge of shape and space with majority failing to fully justify or explain conditional statements regarding angles, triangles, quadrilateral, prism and pyramids. This suggests that majority of these pre-service teachers might struggle to explain relationships among basic shapes and spaces to pupils they would teach at the basic schools. In contrast, a substantial number of these pre-service teachers demonstrated high procedural knowledge in angles, triangles, quadrilaterals, prisms and pyramids. This supposes that many of these pre-service teachers understand how to effectively apply procedures, formulas or algorithms to solve problems relating to shape and space better than they do for declarative and conditional problems. This result is consistent with findings by Haapalaso and Kadijevich (2000); and Star and Stylianides (2013) that most students have gaps in their conceptual and procedural knowledge. According to these authorities, many students tend to be fluent in applying procedures without meaningful understanding of the related concepts. This knowledge gap is a conceptual problem which can be attributed to the conventional direct teaching approaches that emphasize more on solution with formulae rather than geometric concept definitions, concept images and figural relationships. The overall results therefore revealed that pre-service teachers seem to demonstrate better knowledge in procedural tasks than in conceptual (declarative and conditional) tasks.

Furthermore, the results show that declarative, conditional and procedural tasks relating to prisms and pyramids appeared more difficult to be performed by pre-service teachers than tasks on quadrilaterals, triangles and angles. Apparently in this study, majority of pre-service teachers struggled in identifying the name of the given prism (declarative knowledge), in determining the faces in the prism (conditional knowledge) or even in sketching rectangular prism (procedural knowledge) and deriving its area (conditional knowledge). Three-dimensional knowledge forms the basis for developing spatial competencies needed for advances in science, technology, engineering and mathematics (STEM) (Riastuti, Mardiyana & Pramudya, 2017; Cakmak, Isiksal, & Koc, 2014; Gunhan, 2014; Arici & Aslan-Tutak, 2013; National Science Board, 2010). As a result, if pre-service teachers have poor knowledge in three-dimensional shapes, the tendency is that they might not be able to develop 3-D competences in their pupils to undertake future STEM carriers. The present finding gives a new direction for researchers to refocus on 3-D competence development which according to Cevirgin (2012) have been neglected in educational studies.

Relationship between knowledge types provides educators, teachers and curriculum experts the understanding of the kind of learning opportunities to introduce to promote meaningful learning among learners. In this study, declarative, conditional and procedural basic Geometry content knowledge were found to have significant bilateral effect on each other. This result is similar to findings by Cervigin (2012); Aydin and Ubuz (2010). The finding also supports two theoretical views about the development of concepts and procedures in mathematics. First, the simultaneous activation theoretical view was supported implying that both conceptual knowledge (declarative and conditional) and procedural knowledge in geometry develop simultaneously in the same direction (Haapalaso & Kadijevich, 2000). Second, the dynamic interaction view was also supported suggesting that conceptual knowledge and procedural knowledge were dynamically related. This means that, declarative and conditional knowledge are necessary though not the only conditions for procedural knowledge. Differently, conceptual knowledge makes it likely for construction of procedures in geometry. Indeed, contrary to finding by Lauritzen (2012), it seems realistic to admit that an individual ability to apply procedures hinges largely, but not entirely, on the ability to identify, discriminate, generalize and abstract shapes and space concepts. This result adds more evidence to the long-standing counter arguments (Haapalasa & Kadijevich, 2000) regarding whether concepts must be taught before procedures or vice versa. The result demonstrated that both concepts and procedures contribute to each other and could be learnt relationally but the effect of concepts on procedures is elevated. Research into neural networks of human mind has already acknowledge that, declarative memory and procedural memory are distinct, yet analogous in the long-term memory (ten Berge & van Hezewilk, 1999). Therefore, pre-service teachers" knowing of facts, properties and definitions of shapes could provide link to how to discriminate them and address associated learning difficulty.

Another significant finding in this study involved the bilateral relationship between declarative and conditional knowledge in geometry. Little is known in literature about this theoretical relationship (Aydin & Uzbuz, 2007). However, finding in the study indicates that declarative and conditional knowledge exist distinctly even if they correlate when measured. This suggests that developing pre-service teachers" ability to identify and discriminate quadrilaterals for instance, is necessary for their ability to generalize the inclusive classes of quadrilaterals. Similarly, developing pre-service teachers‟ knowledge regarding inclusive classes of quadrilaterals simultaneously impacts on their knowledge regarding the identification and discrimination of parallelism and symmetrical properties of quadrilaterals. This finding suggests that the development of pre-service teachers‟ basic Geometry content knowledge must address the "what", the "why", the "where" and the "when" aspects of knowledge in an integrated rather that in isolated lessons.

In line with spatial literature (Linn & Petersen, 1985), the present study identified three spatial subfactors for measuring spatial ability namely spatial visualization, mental rotation and spatial perception. On tasks related to spatial visualization, the study found that pre-service teachers‟ performance was within average. The standard deviation of 23% further shows that pre-service teachers found the tasks at different degrees of difficulty. However, the standard deviations of 16% and 20% around the means of 5.95 and 7.25 shows that the mental rotation and spatial perceptions tasks were not too difficult for pre-service teachers. Both findings corroborate previous finding where spatial visualization tasks were known to be more complex and difficult to perform than mental rotation and spatial perception tasks (Linn & Petersen, 1985). Unlike mental rotation and spatial perception which are functions of accuracy to rotate images mentally and regulate spatial relations with respect to ones‟ body position respectively, spatial visualization entails complex task operations on multiple steps and multiple solution strategies. This explains why spatial visualization is not only described as a trigger tool for geometric reasoning (Baki, Kosa & Guven, 2011) but also an important factor in the choice of strategies for executing geometry tasks (Kospentaris, Spyrou & Lappas, 2011). This form of spatial ability is particularly required when dealing with real world figural representations as in architectural designs, molecular imagery, computer graphics and artworks. Despite the central role of spatial visualization in geometry, both spatial perception and mental rotation are also important since such abilities enable people to deal with reasoning regarding shapes and relative locations in spaces (Haj-Yahya & Hershkowitz, 2013). Such reasoning tends to facilitate knowledge on the inclusion properties and relationships between shapes (Haj-Yahya & Hershkowitz, 2013). Thus, spatial visualization, mental rotation and spatial perception are important spatial factors likely to affect the level of pre-service teachers‟ knowledge in geometry.

It is often theoretically acknowledged that individuals with well-developed spatial ability can deal with geometric aspects of their environments and shapes (George, 2017; Sipus & Cizmesija, 2012; Piaget & Inhelder, 1967). At the heart of the present study was the empirical investigation of the relation between spatial ability and basic Geometry content knowledge among pre-service teachers. The result of the analysis indicated positive correlation between spatial ability and Basic Geometry content knowledge. It was further found that spatial ability had significantly positive effect on Basic Geometry content knowledge. An effect size of .63 was found which demonstrated large magnitude of effect of spatial ability on Basic Geometry content knowledge. In terms of the three knowledge types, spatial ability was found to affect declarative knowledge, conditional knowledge and procedural knowledge separately. These findings suggest that pre-service teachers who had low score in spatial tasks also had low scores in each set of tasks involving the three types of Basic Geometry content knowledge. The finding tends to illustrate causal relationship between pre-service teachers‟ spatial ability and knowledge for teaching. Therefore, as reported in similar studies (Akayuure et al, 2016), spatial ability is a significant factor which can predict pre-service teachers‟ acquisition of Basic Geometry content knowledge.

At the nucleus of the study was the testing of hypothesized model to describe any relations among pre-service teachers verbal reasoning ability, spatial ability and Basic Geometry content knowledge. The aim was to examine whether the cognitive abilities (spatial and verbal reasoning abilities), which are often downplayed in adult learning, affect pre-service teachers" geometric knowledge as theoretically espoused (Carroll, 1993). The hypothesized relationships were supported as the model demonstrated good fit to data in absolute and comparative indices. Both spatial ability and verbal reasoning accounted for nearly 75% of the variance in basic Geometry content knowledge. The path analysis of structural coefficients in the model shows that the effect of spatial ability on knowledge for teaching was significantly mediated by verbal reasoning. This finding suggests that verbal reasoning is a significant mediator in determining how spatial ability affected Basic Geometry content knowledge. The fact that the path between verbal reasoning and basic Geometry content knowledge was statistically significant supports the observation by Gunhan (2014) that students" verbal reasoning affects their understanding of geometric definitions, propositions, proofs and performances.

Findings of the present study confirmed the theoretical notion that spatial ability, verbal reasoning and basic Geometry content knowledge are significantly related. However, there has been expanding literature regarding the differences in spatial ability based on gender and academic discipline. The MSEM analyses shows that while the model demonstrates structural invariance across programmes of study, the path between verbal reasoning and basic Geometry content knowledge for the males differs statistically from that for the females. This result portrays gender as moderator to the effect of verbal reasoning on basic Geometry content knowledge. The present result suggests that disparities in geometric performance of pre-service teachers can be attributed to disparities in verbal reasoning across gender. Studies relating to gender as a moderator are however rare in literature.

CHAPTER FIVE

SUMMARY AND CONCLUSIONS

5.1. Summary

In Ghana, the outcry about what affects geometry learning has often emanated from semester assessment reports. Few empirical studies have examined the problem from cognitive perspective and using statistical procedures involving large samples. Even in main stream literature, little attention had been paid to how spatial ability affects pre-service teachers‟ basic Geometry content knowledge using holistic analytic procedures like the structural equation modeling. Research in the past has only focused on the psychometric and differential attributes of spatial ability in relatively small samples of elementary and secondary students. No known study has so far addressed spatial ability in geometric knowledge for teaching in Ghana using large sample size.

To address pre-service teachers‟ limited knowledge in geometry, the study investigated the knowledge structure and effect of spatial ability on its acquisition. It has particularly been recognized that, pre-service teachers with underdeveloped spatial sense tend to struggle in understanding spatial concepts. Therefore, it is reasonably to hypothesize that the pre-service teachers‟ limited knowledge in geometry might be explained by their limited spatial competencies. In line with this, Unal, Jakubowski and Corey (2009) noted that the interaction between geometric knowledge and spatial ability is a fundamental field to investigate. The focus of this present study was to investigate geometric knowledge structure of pre-service teachers in terms of shape and space concepts, examine the effect of spatial ability on geometry knowledge for teaching where verbal reasoning mediates such effect, and finally investigate how relationships between spatial ability, verbal reasoning and basic Geometry content knowledge are moderated by gender and programmes of study at the colleges of education in Ghana. Accordingly, the study was driven by the research questions:

1. (a) How do pre-service teachers perform in tests involving:
    (i) spatial ability,
    (ii) verbal reasoning and
    (iii) basic geometry content knowledge;

   (b) How do pre-service teachers" knowledge types in geometry affect each other?
2. How do spatial ability and basic Geometry content knowledge relate to each other?
3. To what extent does verbal reasoning mediate the effect of spatial ability on Basic Geometry content knowledge?
4. Do the relationships between spatial ability, verbal reasoning and knowledge in geometry differ by gender and by programme of study?

In line with these research questions, structural equation modeling methodology was adopted to investigate how spatial ability, verbal reasoning and basic Geometry content knowledge relate to and account for each other. The aim was to clarify the rising concerns about pre-service teachers" limited geometric knowledge for teaching and how spatial factors might account for knowledge in geometry among pre-service teachers.

The study was cross-sectional descriptive survey design which employed multigroup CFA and multigroup SEM analytic techniques to examine not only relationships between latent variables, but also the measurement and structural invariances across gender and college programme.

5.2. Summary of Main Findings

The following were the main findings of the study.

- On how pre-service teachers perform in tests involving knowledge types for teaching geometry, the study found that majority of the participants did better in declarative and procedural tasks than conditional tasks. The overall performance was however average. It was also found that participants performed better on tasks in two-dimensional shapes than in tasks on three-dimensional shapes.
- Regarding spatial ability, the study found that participants" performance in all three spatial subfactors were average. Furthermore, participants did relatively better in tasks on mental rotation and spatial perception than in spatial visualization.
- With respective to verbal reasoning, the study found that more than 70% of the participants performed well in all tasks on nonsense syllogism and verbal comprehension.
- On relationships among the knowledge types, the study found that there are positive bilateral relationships between declarative, conditional and procedural Basic Geometry content knowledge.
- On how spatial ability and basic Geometry content knowledge relate, it was found that there is strong positive correlation between them. Furthermore, spatial ability has direct effect on the overall basic Geometry content knowledge. More so, spatial ability has positive effect on declarative, conditional and procedural basic Geometry content knowledge.

- Regarding whether verbal reasoning mediate the effect of spatial ability on basic Geometry content knowledge, the results showed a statistically moderate indirect effect. It was also found that spatial ability accounted for 51.6% of variance in verbal reasoning.
- For the total effect of spatial ability on basic Geometry content knowledge, the study found that both spatial ability and verbal reasoning accounted for nearly 75% of the variance in basic Geometry content knowledge.
- Regarding the role of gender and programmes offered by participants, the study found that the relationship among spatial ability, verbal reasoning and basic Geometry content knowledge is significantly moderated by gender but not by programme of study. Gender moderated only the relationship path between verbal reasoning and Basic Geometry content knowledge. The model of relationship among spatial ability, verbal reasoning and basic Geometry content knowledge is found to be invariant across programme but moderated by gender.

5.3. Conclusion

Two main conclusions are drawn in line with the purposes of the study. First, the study concludes that there is significant relationship among pre-service teachers" declarative, conditional and procedural Basic Geometry content knowledge. The study extends research on the relationship among constituents of pre-service teachers" knowledge for teaching mathematics. In the above relationship, all the three distinct content knowledge types exist in an interrelated way where the crux defines the knowledge that pre-service teachers need for effective teaching of geometry (Figure 14). The integral knowledge acquired because of the interrelatedness of declarative, conditional and procedural knowledge should be the focus for teacher content knowledge development in geometry. The relationship reinforces the pedagogical power of simultaneous activation view about knowledge development which states that conceptual (declarative and conditional) knowledge and procedural knowledge develop simultaneously.

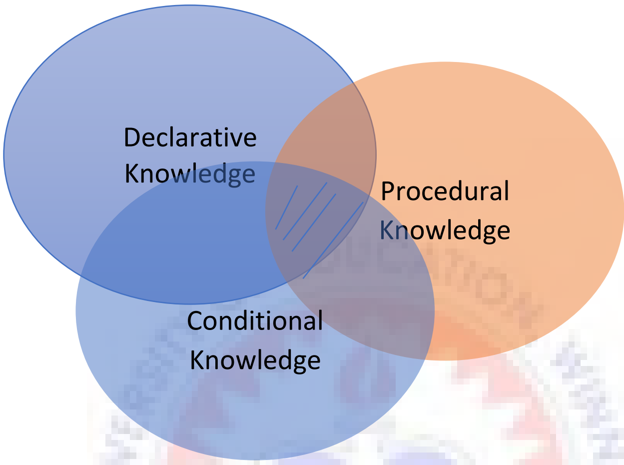


Figure 14 Basic Geometry Content Knowledge

Second, the study further concludes that there is a strong relationship between spatial ability and knowledge for teaching shape and space. If verbal reasoning intermediates the relationship, spatial ability tends to account for nearly three-fourths of the variance in pre-service teachers‟ knowledge for teaching shape and space. As depicted in the final model in Figure 15, this finding confirms Piaget‟s theory of spatial development in geometry and further extends knowledge on the relationship function between spatial ability and geometric knowledge to the intermediation variable of verbal reasoning. Along the mediation path, a significant finding of great didactical importance is that gender moderates the effect of spatial ability on Basic Geometry content knowledge.

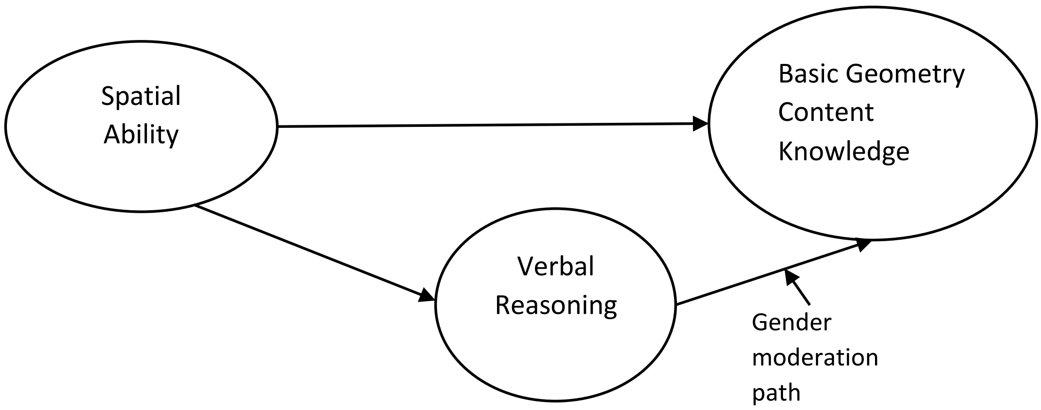


Figure 15 Final Model of Relationship among Spatial Ability, Verbal Reasoning and Basic Geometry content knowledge

The fact that about one-fourth of the variance in knowledge for teaching is not accounted for by of the two cognitive abilities in the model justifies the need to further augment Piaget‟s theory with factors affecting geometric knowledge acquisition. These factors may include learner‟s affective factors, teacher-factors and environmental factors.

5.4. Implications of the Study

There are three main implications resulting from findings of the study on relationships among spatial ability, verbal reasoning and Basic Geometry content knowledge.

5.4.1. Implication for teaching and learning

Shape and space content occupy more than 17% of Ghanaian basic school mathematics curriculum and accounts for pupils‟ future understanding and carrier choices in science, technology, engineering and mathematics (Ministry of Education, 2012). The content knowledge of teachers affects how they enact this curriculum demand and people who are trained to teach shape and space content ought to have mastery of this relevant content. However, the difficulties encountered by pre-service teachers‟ in dealing with shape and space tasks implies they are unlikely to effectively deliver lessons on shape and space. This has the predisposition to greatly influence pupils" understanding of STEM. STEM carriers are important in national development. Therefore, if pre-service teachers fail to promote STEM competences in their pupils, then the Ghana nation risks in scientific and technological advancement soon.

In current literature, emphasis is placed on teaching for relational understanding. In other words, learners should be taught to think and appreciate *the whats, the how tos* and *the whys* of knowing rather than imitate procedures. Developing declarative and conditional knowledge leads to relational thinking, which is the emphasis of contemporary teaching. However, in the study, a large proportion of pre-service teachers demonstrated better ability to perform procedural tasks than declarative and conditional tasks. This means the teaching practice at the colleges of education is probably not providing pre-service teachers with opportunities that would enable them understand concepts relationally. There is therefore the need to modify the current procedural teaching approaches by stressing on the declarative-conditional knowledge among pre-service teachers. As acknowledged in Gogoe (2009), if tutors focus their teaching practice on core conceptual competencies or declarative-conditional-procedural relations, pre-service teachers are likely to develop mastery of the basic concepts needed for teaching geometry.

It is surprising in this study to find that pre-service teachers whose daily lives reevolve around 3-D objects rather performed better in 2-D tasks than in 3-D tasks. This finding portrays the view that teachers tend to teach school geometry in a way that does not reflect learners" daily experience or spatial reasoning which forms the core epistemology of geometry. The finding further suggests that majority of these pre-service teachers are likely to dwell more on plane geometry and less on solid geometry in their lesson enactment at the basic schools. Three-D knowledge is the basis for most scientific discoveries since the days of Einstein, Kekule, Pythogoras and is currently dictating multimedia technology, telecommunications and the new trend of industrial revolutions. There is therefore the need to place more emphasis on developing pre-service teachers‟ 3-D knowledge through spatial training and experiential learning so that they would in turn impart such knowledge to the younger generation. Perhaps in-service teachers who are the products of such limited teaching practices should also be made to develop 3-D competencies through workshops on solid geometry. Improving pre-service and practicing teachers‟ 3-D competencies should be a sure response to the recent call to promote spatial curriculum in the basic schools.

Since spatial ability affects basic Geometry content knowledge significantly, teaching practices such as modeling, origami and dynamic Geometry Software which improve spatial competencies would enhance the three knowledge types in geometry. The improvement of geometric knowledge through spatial activities must also take into consideration the mediating effect of verbal reasoning in the form of logical reasoning in texts and comprehension.

In deciding on how to improve geometry content knowledge, caution must be taken to address gender disparities since male and female differ in their verbal reasoning effect on Basic Geometry content knowledge. Also, since the correlation between spatial ability and verbal reasoning differ across gender, it is important that tutors support their teaching process with visuospatial materials in order to bridge the performance gap emanating from spatial ability difference that favour males.

5.4.2. Implication for curriculum development

The study on pre-service teachers" basic Geometry content knowledge could be considered as an evaluation of the implemented geometry curriculum in the colleges of education. The finding that pre-service teachers demonstrated inadequate knowledge in shape and space therefore implies that they are not well-prepared to effectively teach shape and space in the basic school curriculum. The finding in this study also suggests that the current state of geometry content at colleges does not place emphasis on basic shape and space content. Previous study by Gogoe (2009) identified that the content of the geometry course at the colleges of education in Ghana, differed largely from the content of the basic school geometry. This seems to explain why pre-service teachers are limited in their knowledge for teaching shape and space at the basic schools.

Spatial skills are necessary for day-to-day activities and for scientific discoveries and innovation (Hawes et al, 2017; Newcombe, 2010). For instance, the discovery of the structure of DNA and theory of relativity by Einstein were borne out of spatial thinking. This explains the rationale for the recent global call to spatialize basic school mathematics curriculum as basis for emerging computer and information technological advancements (Gunhan, 2014; Janelle, Hegarty & Newcombe, 2014; National Council of Teachers of Mathematics, 2010; Mullis, Martin & Foy, 2008). It may be argued that providing pre-service teachers with adequate spatial competencies should be the starting point. Indeed, enacting a spatially-oriented curriculum at the colleges to promote spatial competences of pre-service teachers could facilitate implementation of spatial components at the basic school mathematics curriculum. By placing emphasis on spatial ability in college geometry curriculum, tutors might have ample time to emphasize on the basic 2- and 3-D geometry contents such as angles, triangles, quadrilaterals, prisms and pyramids. Another curriculum approach might be to design a full course or a unit under shape and space to expose pre-service teachers to learning and assessment tasks on spatial visualization, mental rotation and spatial perception.

5.4.3. Implication for assessment

The performance demonstrated by pre-service teachers in the declarative, conditional and procedural tasks on basic shape and space means that tutors at the colleges of education should not assume that pre-service teachers will exit college system with sound acclimation on shape and space. Such an assumption would be arbitrary and have dire consequence on their future teaching practices. A formal assessment procedure for assessing pre-service teachers" spatial competencies to teach shape and space is required. Since the instrument used in this study achieved measurement invariance across gender and programme of study, it can be seen as a model for evaluating spatial abilities, verbal reasoning and knowledge for teaching shape and space at the basic school.

5.5. Recommendations

Based on the implications highlighted in the previous section, the following recommendations have been enumerated with respect to the actors within the scope of the study.

- When assessing pre-service teachers, tutors should develop assessment task scale that specifies declarative, conditional and procedural knowledge in geometry. This approach can help tutors and other assessors detect which knowledge type is lacking in order to adopt appropriate intervention measures.
- Tutors should measure pre-service teachers" spatial visualization, mental rotation and spatial perception, especially, prior to internship of second year

students to ascertain whether they are spatially competent to teach shape and space content at basic schools in Ghana.

- Tutors need to teach shape and space content with real manipulatives. They need to particularly employ spatial training activities involving origami, dynamic geometric software (GeoGebra, geometer‟s sketchpad), physical manipulatives and models when teaching shape and space contents to promote pre-service teachers‟ spatial competencies.
- The gender disparity remains a significant issue in geometry. Tutors need to be conscious of this and adopt visuo spatial and verbal reasoning tasks that support differentiation, reduce moderating effect of gender and enhance female pre-service teachers‟ geometric knowledge.
- A course on epistemology of geometry and spatial ability should be mounted to expose pre-service teachers to the history, knowledge structure and curriculum goal for geometry. This will enable them understand and adopt teaching practices suitable for understanding geometry.

5.6. Future Research

The study explored pre-service teachers‟ basic Geometry content knowledge and how spatial ability and verbal reasoning relate to and accounted for such knowledge. The results not only provide justification to focus on their spatial ability as basis to develop declarative, conditional and procedural knowledge, but also act as a foundation studies for future research. Also, quite rare in empirical literature was finding where verbal reasoning mediated the effect of spatial ability on Basic Geometry content knowledge. This is a new, yet significant finding that needs follow up investigations. Thus, this study could be replicated in the colleges of education to find out if similar findings would be obtained. There is also the need to examine how to improve spatial abilities through quasi-experiments or qualitative approaches.

This study focused on participants" grouping variables such as gender and programme of study as moderating variables. Even if gender moderated the relationship between verbal reasoning and geometric knowledge, when gender was not differentiated the relationship proved statistically significant. To further validate the moderating role of gender, a replicative study is suggested.

5.7. Limitations of the Study

While a plethora of factors affecting geometry knowledge acquisition exist in literature, this study was limited to the cognitive dimensions. The study focused on spatial and verbal components affecting knowledge for teaching elementary geometry. The choice of these cognitive components was informed by theoretical views expounded by cognitive psychologists including Piaget, van Hieles, Duval, Fischbein, Gardener and Carroll. Therefore, caution should be taken when extending the interpretation and application of findings in this study to personal/affective dimensions such as attitude, interest, or environmental factors such as class size, instructional materials, among others.

The basic Geometry content knowledge was narrowed to the specialized content knowledge and did not include the entire content knowledge on and pedagogical knowledge for teaching geoemtry. This was deliberately considered to provide deeper clarity on how pre-service teachers understand shape and space concepts at declarative, conditional and procedural levels and may be viewed as a step to further enrich our understanding of teacher content knowledge within the framework of mathematical knowledge for teaching.

The data collection process was characterized by time and accuracy tests, and achievement test which was developed by the researcher. Limitations that come with the use of test cannot be ignored. For example, participants might have developed certain anxieties during the testing process which could have led to some measurement errors yielding slight skewness and kurtosis in some of the observed variables. However, as indicated by Kline (2015), SEM is a robust statistical procedure that accommodates such minor skewness and kurtosis as it was in this study. Thus, these limitations did not very much affect results and conclusions.

While the model can be applied across programmes, caution need to be taken when differentiating the model into gender since the relationship between verbal reasoning and geometric knowledge in the female model is weak. It is however be noted that, the model could be generalizable to specialist and generalist pre-service teachers at Ghanaian colleges of education.

5.8. Final Thought

It appears obvious that curriculum actors and mathematics tutors at the colleges of education need to return to the basis to emphasize on spatial ability of pre-service teachers. The incessant neglect of this ability in pre-service teachers" mathematics curriculum implementation would only continue to create conceptual gaps and poor understanding not only in shape and space content but in the entire mathematics course. It should also be noted that if pre-service teachers transport their limited spatial competency to classrooms, they may become breeding ground for misconceptions, misrepresentations and performance problems among pupils at the basic schools. This would in turn create learning problems among pupils as they transition from basic to the secondary and higher levels. This trend would consequently impede geometric and mathematical knowledge growth in the country and limit learners‟ potentials of going into STEM carrier choices. The answer regarding why pre-service teachers struggle to visualize and abstract basic shape and space concepts could be 75% explained by the interrelatedness of spatial ability, verbal reasoning and Basic Geometry content knowledge. Interventions and reforms aimed at addressing pre-service teachers‟ performance problems in geometry ought to consider these elements.

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APPENDICES

Appendix A

Spatial ability Tests

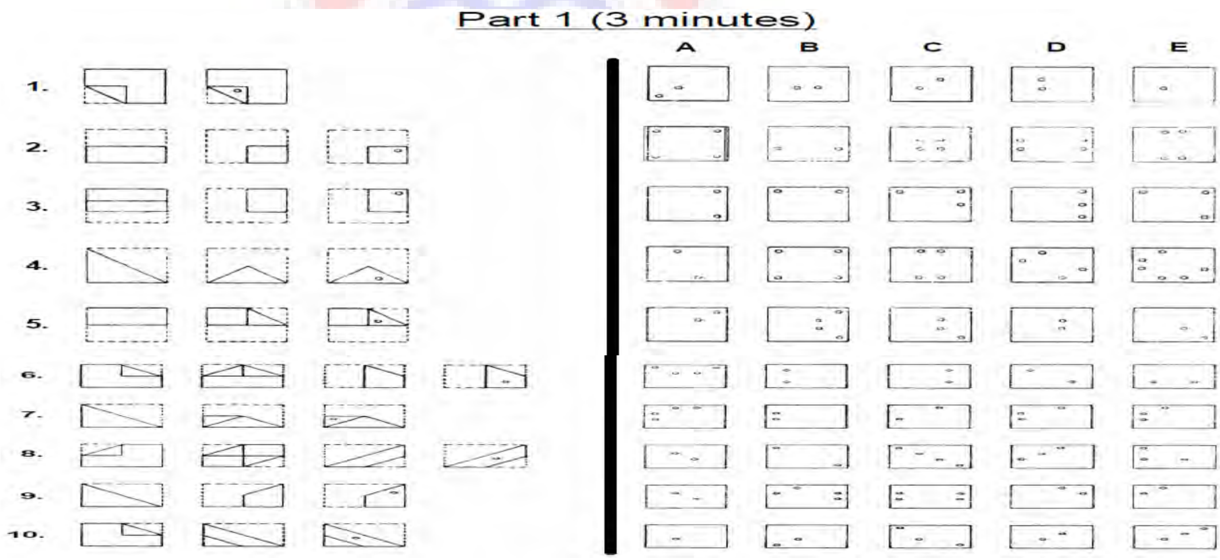
This test consists of four parts: paper folding test, card rotation test, water level task, verbal reasoning and basic Geometry content knowledgeitems. You are to answer all questions to the best of your ability. The test is for research purpose and will not be used for grading you at the college. Your responses shall remain confidential. Thank you.

Bio data

1. *Gender*: Male [ ] Female [ ]
2. *Age range:* 20 and below [ ] 21-25 [ ] 26-30 [ ] 31-40 [ ] 41 and above [ ]
3. *College Programme*: Science [ ] General [ ]
4. *SHS programme Offered*: Science [ ] Business [ ] General Arts [ ] Visual Arts [ ] Others:……………………………….

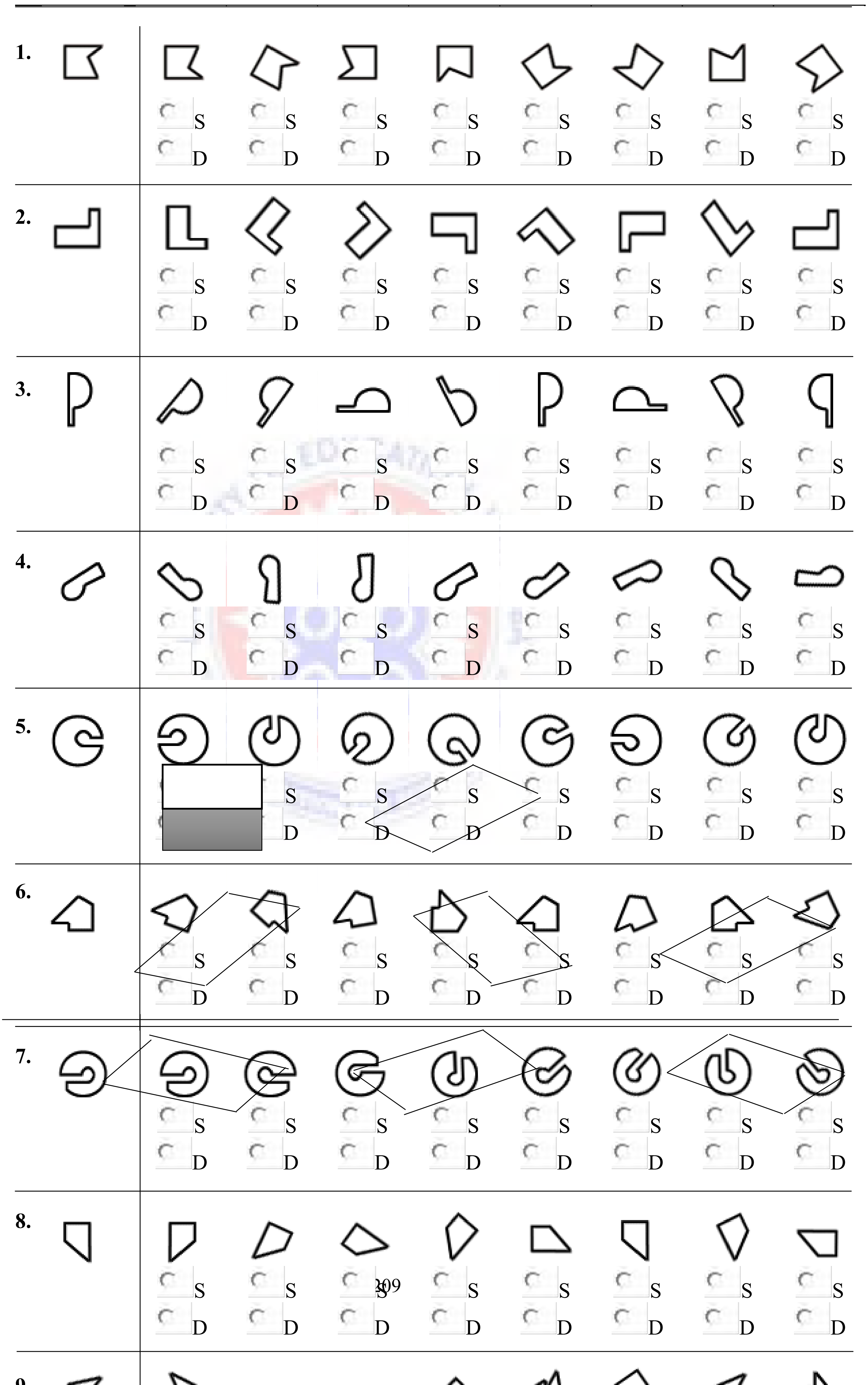
Paper Folding Test

At the left column, a square paper is folded a number of times and a hole is punched through. One of the five figures at the right of the vertical line shows where the holes are punched when the square paper is completely unfolded. You are to decide which one of these figures is correct location of the holes and tick on that figure. **Caution! This test is a mental test and so you should not fold any paper yourself in order to check the correct answers. Use your mind.**



Card Rotation Test

Indicate which figure is the SAME (S) or DIFFERENT (D) from the one on the left if the card is rotated without lifting it. Tick S if the same orientation or D if different orientation from the first card**. Please tick S or D below each of the 8 cards in each question**



Appendix B

Verbal reasoning

The following items are designed to assess your ability to understand text and think logically through the information provided. Some of the items may not make sense ordinarily but they should only be interpreted within the information provided. You are to answer as many questions as you can in 8 minutes. Select the letter which corresponds to the correct answer

1. **Syllogism:** The following are two pair of nonsense statements with a conclusion drawn from them. You are to determine whether the conclusion is good reasoning (G) or poor reasoning (P).

Nonsense Syllogism –SL-1

All trees are fish. All fish are horses. Therefore, all trees are horses.	G	P
All trees are fish. All fish are horses. Therefore, all horses are trees.	G	P
Some swimming pools are mountains. All mountains like cats. Therefore, all swimming pools like cats.	G	P
All swimming pools are mountains. All mountains like cats. Therefore, all swimming pools like cats.	G	P
All elephants can fly. All giants are elephants. Therefore, all giants can fly	G	P
Some carrots are sports cars. Some sports cars play the piano. Therefore, some carrots play the piano.	G	P
No solid shape has a flat face. A cuboid has 6 flat faces. Therefore, a cuboid is not a solid shape.	G	P

2. **Verbal comprehension 1: Use only the information provided here to answer the questions that follow:** In a certain nonsense F-geometry, there are exactly 4 points and six lines. Every line contains exactly two points. If the points are P, Q, R and S, then the lines are {P, Q}, {P, R}, {P, S}, {Q, R}, {Q, S} and {R, S}. Here are how the words intersect and parallel mean in F-geometry. The lines {P, Q} and {P, R} intersect at P because {P, Q} and {P, R} have P in common. The lines {P, Q} and {R, S} are parallel because they have no points in common. From this information.

   i. Which other lines intersect?
      A. {P, R} and {Q, S}
      B. {P, R} and {Q, S}
      C. {Q, R}and {R, S}
      D. {P, S} and {Q, R}

   ii. Which other lines are parallel?
      A. {P, R} and {Q, S}
      B. {P, R} and {Q, R}
      C. {Q, R}and {R, S}
      D. None of the above

   iii. Which lines are not parallel and not intersecting?
      A. {P, R} and {Q, S}
      B. {P, R} and {Q, S}
      C. {Q, R}and {R, S}
      D. None of the above

   iv. Which is correct?

A. {P, R} and {Q, S} intersect
B. {P, R} and {Q, S} are parallel
C. {Q, R}and {R, S} are parallel
D. None of the above

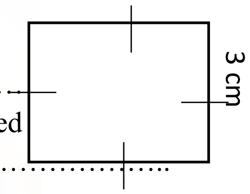
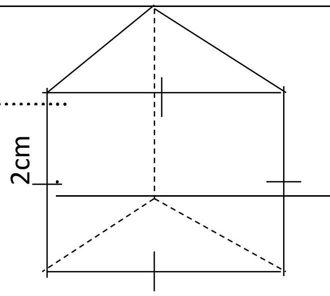
3. **Verbal comprehension 2**: **Use only the information provided here to answer the questions that follow**: There is a geometry invented by a mathematician in which the following is true:

   "G:The sum of measures of the angles of a triangle is less than 180^0.
   Which is correct?
   A. G made a mistake in measuring the angles of the triangle
   B. G made a mistake in logical reasoning
   C. G has a wrong idea of what is meant by "true"
   D. G started with different assumptions than those in the usual geometry

Appendix C

Basic Geometry content knowledge

The Purpose of this part is to assess pre-service teachers" knowledge on basic shape and space. You are to provide answer to the items by either ticking, writing down or sketching in the paper.

Knowledge of angles 1. All angles are formed by two or more lines True [ ] False [ ] 2. The angle measure of an acute angle is a subset of angle measure of a right-angle True [ ] False [ ] 3. Corresponding angles are equal no matter the figure. True [ ] False [ ] 4. How many angles are formed when two straight lines cross each other? . . . . . Explain:………………………………….. 5. Sketch to show the number of angles in the space on the right-hand side	Sketch work to (5) here
Knowledge of triangles  6. (a) Mention one type of triangle ………………………………. (b) Mention two main properties of the triangle you mentioned (i) …………………………………………………………………….. (ii) …………………………………………………………………….. 7. All isosceles triangles have the same angle measures True [ ]False [ ] 8. What is the area of a right-angled triangle whose height and base measure 2cm each? *(show work on right-hand side)*	Show work to (8) here
Knowledge of quadrilaterals 9. (a) Which group of plane figure does it belong to? ……………………………………………………. (b) Mention one unique property of this figure that make it different from all others…………………………………………………….................. (c) How many lines of symmetry has the figure? …………………………... (d) How many order of rotation has the figure? …………………………… (e) If the figure above is transformed into a kite, what is its area? …………………………………………………………………….. (f) If two opposite sides of this figure increase by 2 units, what new shape is formed? …………………………………………………………... (g) Sketch this new figure and show its lines of symmetry. (h) Find the area of the new figure you have sketched. *(show work below)*	Show work to (9g) here
Knowledge of prism 10. (a) What is the name of this figure?................................... (a) How many faces are there in the figure? …………………………………………………  	Sketch your answer to (11) here

11. (a) Sketch the net of a rectangular prism. (b) What is its area if all the faces are equal …………………………………………………………………………………….. 12. If all faces of a prism are squares and the base is pentagon, how will you call such a prism? ………………………………………………………… 13. Find the volume of a milo tin base area is $4cm^2$ and the height is 2 cm. ………………………………………………………………………………… 14. If the area of each face of a cube is $8cm^2$, find its total surface area? ………………………………………………………………………………… 15. Is a cylinder a prism? Yes [ ] No [ ] Why?…………………………………………………………………………. ……………………………………………………………………………... ……………………………………………………………………………...	
Knowledge of pyramid 16. (a) Name one property of a pryramid..………………………………………. (b) How many faces has a rectangular pyramid?.. 17. Find the volume of a cone with circular base of $6cm^2$ and height of 2cm. ……………………………………………………………………………............. 18. Is a cone a pyramid? Yes[ ] No[ ] Why?....................................... ……………………………………………………………………………... 19. When does a cuboid become a cube? ……………………………………………………………………………… 20. If all vertices at the base of a triangular prism are pulled together to form a tip, which new figure is formed? ……………………………………….;	Sketch to show (20) here

Appendix C1

<u>Scoring Scheme for Paper folding test</u>

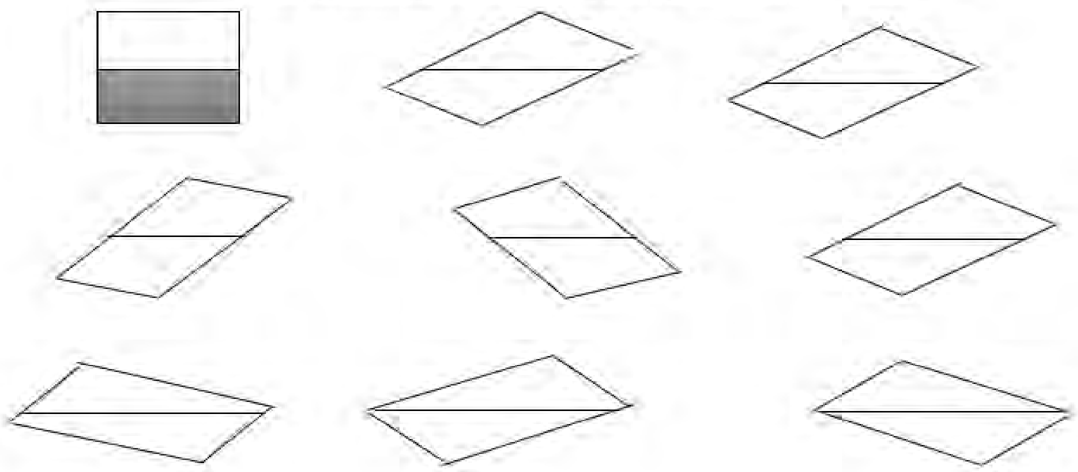
1. A
2. D
3. B
4. D
5. B
6. E

7. A
8. C
9. E
10. E

Scoring Scheme for Card Rotation test

1. D S S D D S D S
2. S S S D S S S S
3. S D D D S S S D
4. S S D S D D D S
5. D S D D S S D S
6. S D S S S S D D
7. S D S D D S S S
8. D D S S D S D D
9. D D S S D S S D
10. S D D S D D S S

Scoring Scheme for Water Level Tasks (Researcher's own sketch)



Scoring Scheme Verbal Reasoning

1. **Nonse syllogism**
   - i. G
   - ii. P
   - iii. G
   - iv. G
   - v. P
   - vi. G
   - vii. G

ii. **Verbal comprehension**

Verbal comprehension 1:

- i. C
- ii. A
- iii. D
- iv. B

Verbal comprehension 2: D

Scoring Scheme for Basic Geometry Content Knowledge Test
Knowledge of angles:

1. false [1 mark]
2. true [1 mark]

3. false [1 mark]
4. more than 4 angles [1 mark]
   Different ways exist so many angles [2 marks] give 1 mark for "*more than 4 angles*"
5. sketching B1B1B1 [3 marks]

Knowledge of triangles

6. (a) any type of triangle(equilateral) [1 mark]
   (b) All sides and all angles equal [2 marks] give 1 mark for each property
7. false [1 mark]
8. $\frac{1}{2} \times 2 \times 2 = 2cm^2$ M1M1A1 [3 marks]
9. (a) quadrilateral {squares) [1 mark]
   (b) It has equal sides and equal angles [1 mark]
   (c) 4 lines of symmetry [1 mark]
   (d) 4 order of rotation [1 mark]
   (e) $\frac{1}{2}(product\ of\ diagonals)$; same area=9cm^2 M1A1[2 marks]
   (f) Rectangle [1 mark]
   (g) Sketching B1B1B1 [3 marks]
   (h)$3 \times 5cm = 15cm^2$ M1M1A1 [3 marks]

Knowledge of prism

10. (a) triangular prism [1 mark]
    (b) 5 faces [1 mark]
11. (a) correct sketch [3 marks]
    (b) $6(L \times B)$ M1A1 [2 marks]
12. pentagonal prism [1 mark]
13. $4 \times 2 = 8cm^2$ M1M1A1 [3 marks]
14. $6 \times 8 = 48cm^2$ M1M1A1 [3 marks]
15. No [1 mark]
    It has curve at base [2 marks]

Knowledge of Pyramid

16. (a) It has apex [1 mark]
    (b) 5 faces [1 mark]
17. $6 \times 2 = 12cm^3$ M1M1A1 [3 marks]
18. (a) No [1 mark]
    (b) The base of a cone is a curve, but the base of a pyramid is a polygon [2 marks]
19. When all its sides are equal [2 marks]
20. Triangular prism [2 marks]

Appendix D

Appendix D1: Various Outputs of Construct Validity analysis

Table 41 Rotated Component Matrix for Spatial Ability Items

	Component		
	Spatial visualization	Mental rotation	Spatial perception
PF1			.518
PF2			.609
PF3			.541
PF4			.524
PF5			.623
PF6			.642
PF7			.657
PF8			.527

PF9			.586
PF10			.550
CR1	.792		
CR2	.792		
CR3	.786		
CR4	.804		
CR5	.820		
CR6	.850		
CR7	.820		
CR8	.829		
CR9	.843		
CR10	.826		
SP1		.554	
SP2		.769	
SP3		.673	
SP4		.794	
SP5		.757	
SP6		.653	
SP7		.649	
SP8		.574	

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

Table42 Factor Loadings of KTG Items

Item indicator	Declarative	Conditional	Procedural
CK(3&4)		.710	
CK(1 and 2)	.800		
CK(6a&b)	.922		
CK(9a&b)	.798		
CK(5)			.923
CK(8)			.953
CK(9e)			.886
CK(9g)			.649
CK(9h)			.792
CK(10a&b)	.590		
CK(7)		.498	
CK(1&b)			.504
CK(13)			.654
CK(16a&b)	.924		
CK(9f)		.996	

CK(15)	.750	
CK(17)		.679
CK(11a&b)	.753	

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

Table 43 Verbal Reasoning Factor Loadings by Components

Observed Variables	Component	
	Syllogism	Comprehension
VR1A	.865	
VR1B	.857	
VR1C	.847	
VR1D	.859	
VR1E	.629	
VR1F	.871	
VR1H	.858	
VR1I		.923
VR2i		.379
VR2ii		.828
VR2iii		.920
VR3		.928

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

Table 44 Measurement Model Outputs

Factor Loadings from CFA

			Stand. Estimate	Unstand. Estimate	S.E.	C.R.	P
sos	<---	SA	.978	1.000			
svs	<---	SA	.495	.090	.007	13.759	***
Sp	<---	SA	.745	.091	.004	24.668	***
Sy	<---	VR	.596	1.000			
vc	<---	VR	.585	.510	.053	9.687	***
dec	<---	KTG	.620	1.000			
con	<---	KTG	.597	.930	.071	13.192	***
pro	<---	KTG	.745	2.263	.147	15.391	***

Covariances from CFA

			Estimate	S.E.	C.R.	P	Label
SA	<-->	KTG	7.289	.568	12.828	***	par_6
SA	<-->	VR	12.457	1.240	10.042	***	par_7
VR	<-->	KTG	.691	.076	9.074	***	par_8

Correlations from CFA

Correlations			Estimate
SA	<-->	KTG	.849
SA	<-->	VR	.690

Correlations			Estimate
VR	<-->	KTG	.724

Variances from CFA

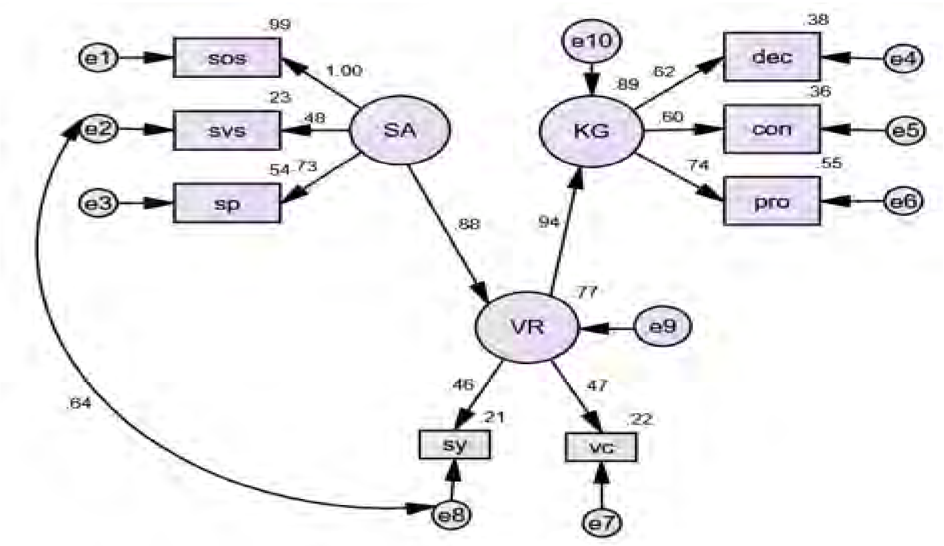
Variables	Estimate	S.E.	C.R.	P
SA	162.475	9.696	16.757	***
VR	2.006	.301	6.656	***
KTG	.454	.053	8.601	***
e1	7.395	4.239	1.745	.051
e2	4.057	.217	18.717	***
e3	1.081	.063	17.086	***
e4	3.640	.277	13.151	***
e5	1.002	.074	13.557	***
e6	.728	.044	16.722	***
e7	.710	.042	17.010	***
e8	1.869	.137	13.598	***

Squared Multiple Correlations from CFA

Variables	Estimate
pro	.554
con	.356
dec	.384
vc	.342
Sy	.355
Sp	.556
svs	.245
sos	.956

Table 45 Covariances Matrix for all variables

	KTG	VR	SA	Pro	Con	dec	vc	sy	sp	svs	sosscaled
KTG	.454										
VR	.691	2.006									
SA	.911	1.557	2.539								
pro	1.027	1.564	2.062	4.193							
con	.422	.643	.847	.955	1.103						
dec	.454	.691	.911	1.027	.422	1.182					
vc	.352	1.023	.794	.797	.327	.352	1.523				
sy	.691	2.006	1.557	1.564	.643	.691	1.023	5.647			
sp	.665	1.136	1.852	1.504	.618	.665	.579	1.136	2.432		
svs	.655	1.120	1.826	1.483	.610	.655	.571	1.120	1.332	5.370	
sosscaled	.911	1.557	2.539	2.062	.847	.911	.794	1.557	1.852	1.826	2.654



Chi-square value=64.41,df=17, p-value=.000,CIM/DF=3.789;RMSEA= .061; RMR=.094.

Figure 16 The AMOS Output of Alternative Model Estimates by ML

Appendix E: Procedure for Invariance Analysis

Stage 1: The first condition for the use of MGCFA was to ensure model consistency or best fit for each group. This involved preforming single-groups analyses using the model. The goodness of fit indices of the model to data were determined separately for each gender without specifying that any parameter estimates were the same across gender. The path diagram of separate models of male and female are shown in Figure 16.

Stage 2: This stage begun the actual multigroup testing. At this stage, configural invariance was assumed and measurement weights or factor loadings were constrained to equality with other parameters estimated freely. The factorial invariance would indicate conceptual agreement in the type and number of underlying constructs, and the items associated with each construct. In the model showing in

Figure 16, the constrained parameters (equivalence of factor loadings across groups) were defined mathematically as:

a1_1=a1_2
a2_1=a2_2
a3_1=a3_2
a4_1=a4_2
a5_1=a5_2

The result of analysis is displayed in Table 25 under the label model A.

Stage 3: This stage depended on the previous stage. When measurement weights or factorial invariance was correct, the equivalence of structural covariances across groups (for gender and for programme of study) was tested. Thus, the measurement weights, structural covariance and the factor variances were all set to equivalence across groups and tested for chi-square differences and changes in goodness-of-fit indices. In the model specification showing in Figure 16, the following mathematical equations were tested across groups:

a1_1=a1_2
a2_1=a2_2
a3_1=a3_2
a4_1=a4_2
a5_1=a5_2
ccc1_1=ccc1_2
ccc2_1=ccc2_2
ccc3_1=ccc3_2
vvv1_1=vvv1_2
vvv2_1=vvv2_2
vvv3_1=vvv3_2

The output of the AMOS analysis is shown in Table 25 under model B.

Stage 4: Residual variance is the portion of item variance not accounted for in the latent variable. The test for the equivalence between-group residual variance establishes whether the same degree of measurement errors occur among the latent constructs across groups. Residual non-invariance may suggest participants belonging to one group were inconsistent in their responses due to item scale, vocabulary or cultural influences. The test for measurement residual invariance involved constraining the structural covariances, measurement weights and testing whether measurement residuals including item intercepts, were equivalent across groups. With reference to Figure 16, the following mathematical equivalence were tested for invariance.

a1_1=a1_2
a2_1=a2_2
a3_1=a3_2
a4_1=a4_2
a5_1=a5_2
ccc1_1=ccc1_2
ccc2_1=ccc2_2
ccc3_1=ccc3_2
vvv1_1=vvv1_2
vvv2_1=vvv2_2
vvv3_1=vvv3_2
c1_1=c1_2
c2_1=c2_2
c3_1=c3_2
c4_1=c4_2
v1_1=v1_2
v2_1=v2_2
v3_1=v3_2
v4_1=v4_2
v5_1=v5_2
v6_1=v6_2
v7_1=v7_2
v8_1=v8_2

This described the full measurement model.

Appendix F

Appendix F1: Sample of Permission Letters

UNIVERSITY OF EDUCATION, WINNEBA

DEPARTMENT OF MATHEMATICS EDUCATION

P. O. Box 25, Winneba, Ghana. Tel: 233- 03323-20989, E-mail: maths @uew.edu.gh

14th April, 2017

The Principal,
…………………………………………………
…………………………………………………
…………………………………………………
…………………………………………………

Dear Sir/Madam,
My name is Peter Akayuure, a graduate student at the University of Education, Winneba pursuing PhD in Mathematics Education. As part of my study, I am investigating pre-service teachers‟ spatial ability and geometric knowledge for teaching shape and space at some selected colleges of education in Ghana.
The participants of the study are second-year pre-service teachers at the colleges of education offering science and general programmes. Considering the nature of the programme distributions, your college is specially selected for the study. I would therefore be very grateful if you could allow me to involve your students in the study.
If you agree, I would request for your tutors to assist me in arranging and administering my instrument to the students as may be deemed convenient.
Your consent and assistance will be highly appreciated.

Thank you.

Yours sincerely,
Peter Akayuure

Appendix F2: Sample of Permission Letters

E. P. COLLEGE OF EDUCATION, BIMBILLA

Our Ref: ....

Your Ref: ....

P. O. Box 16
Bimbilla, N/R

Office Tele: 037-2093548/ 037-2093606
037-2093605 / 037-2093608

Email: ebimbico@yahoo.com

Date: ....

23rd June, 2017

Peter Akayuure (PhD Candidate)

Department of Mathematics Education

University of Education, Winneba

Dear PhD Candidate,

RE: **PERMISSION TO CONDUCT RESEARCH**

With reference to your letter, dated 14th June, 2017. I hereby consent and grant you permission to meet with the participating students of my college for your data collection in respect of your PhD study on *preservice teachers' spatial ability and basic geometry content knowledge.*

Thank you.

for

A . Abu-Wemah

(Principal)

E.P College of Education, Bimbilla

PRINCIPAL
E.P COLLEGE OF EDUCATION
BIMBILLA