UNIVERSITY OF EDUCATION, WINNEBA

JUNIOR HIGH SCHOOL PUPILS' CONCEPTION OF LINEAR INEQUALITIES IN ONE VARIABLE IN HAVE CIRCUIT OF THE AFADZATO SOUTH DISTRICT IN GHANA

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DECLARATION

Student's Declaration

I, Kenneth Kofikuma Puem, declare that this thesis, with the exception of quotation and reference in this publication are all my efforts and have all been duly identified and acknowledged for presentation.

Signature: ....

Date: ....

Supervisor's Declaration

We hereby declare that the preparation and presentation of this thesis were supervised in accordance with the guidelines for the supervision of project works laid down by the University of Education, Winneba.

Prof. Charles Kojo Assuah (Principal Supervisor)

Signature: ....

Date: ....

Mr. Clement Ali (Co-Supervisor)

Signature: ....

Date: ....

DEDICATION

To my family, especially, my grandfather, Mr. Albert Addo Kwame (W O I), my daughter Janessia Yayra Puem.

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I wish to first and foremost give thanks to the Almighty God for protecting and guiding me throughout this postgraduate programme to a successful completion of this thesis.

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TABLE OF CONTENTS

Contents **Page**

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

The main purpose of study was to investigate junior high school pupils' conception of linear inequalities in one variable in 'Have' Circuit of the Afadzato South District in the Volta Region of Ghana. Specifically, the study sought to examine pupils conceptions of linear inequalities, to identify the benefits pupils derived from understanding linear inequalities concepts in mathematics and to identify conceptual change in male and female pupils learning outcomes in linear inequalities in 'Have' Circuit of the Afadzato South District. Three research questions were raised to answer the research problem. The study adopted descriptive survey as a research design. Simple random sampling, stratified sampling techniques and Yamene's formula (1967) for sample size determination were used to select three hundred and ten (310) pupils from all nine (9) junior high schools in the Have Circuit. A questionnaire, interview guide and Algebra Diagnostic Test were the instruments used to collect both quantitative and qualitative data for the study. Analysis of data was carried out using descriptive statistics and t-test analysis. The findings of the study revealed that 125 (40.3%) of the number of pupils have developed the desired conceptions of linear inequalities which enabled them to perform well on the algebra diagnostic test. With regard to the benefits of understanding linear inequalities concepts in mathematics, 146 (47.1%) indicated that it enhanced their understanding of word problems that are associated with the inequality concept. Moreover, the findings revealed that the mean difference was statistically significant (Mean Deviation = 1.516, $p < 0.05$); Standard Error Difference of 1.606 (Mean Deviation = 1.516, $p < 0.05$; Standard Error Difference = 1.606). This statistical achievement mean the test actually had positive impact on the boys' and the girls' performance. Based on the findings of the study, it was recommended that stakeholders should encourage and motivate pupils to improve upon their study habits for better performance in mathematics at the basic level as this has the tendency to stimulate and motivate students to see mathematics as an easy subject since every concept in mathematics is related to another higher concept. Moreover, it was recommended to the authorities of basic education (headmasters and headmistresses, District and Municipal Chief Executives of Education and Ministry of Education) to provide logistics such as teaching and learning materials at the basic education level as far as the learning of mathematics is concerned to arouse the interest of the pupils in the learning of mathematics at that level. Furthermore, mathematics clinics, workshops and in-service training should be organised regularly for mathematics teachers at the basic level to update or upgrade their knowledge on efficient contemporary strategies and methodologies that should be used to teach mathematics at the basic level. In addition, effective supervision and encouragement of mathematics teachers at the basic level on how to teach effectively linear inequalities will help pupils at the basic level to develop interest and appreciate for the mathematics subject so that they can pursue it at a high level.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This chapter discusses the background to the study, statement of the problem, purpose of the study, objectives of the study, research questions, significance of the study, delimitation of the study, limitation of the study and organization of the study.

1.1 Background to the Study

The school as a social unit deal with the most exciting facet of human existence. It is essentially charged with the responsibility of providing wholesome experience and opportunities for social intervention, self-expression, and self-development and to shape attitudes, opinions and values of students. The basic school is known to be the foundation of all levels of education. Many countries have invested much time, financial and human resources in the development and study of mathematics at the basic level (Ansari, 2004).

Development in almost all areas of life is based on effective knowledge of science and mathematics. There simply cannot be any meaningful development in virtually any area of life without knowledge of science and mathematics. It is for this reason that the education systems of countries that are concerned about their development put great emphasis on the study of mathematics (CRDD, 2007).

Mathematics arises from the attempt to organise and explain the phenomena of our

environment and experience (Bell, 1993). It has been expressed thus: "Mathematics is ... an activity of organising fields of experience" (Freudenthal, 1973, p. 123); and

"Mathematics concerns the properties of the operations by which individual orders, organises and controls his environment" (Peel, 1971, p. 157). These descriptions are somewhat non-specific, though Peel is clearly referring to basic mental operations such as classifying, comparing, combining, and representing. A more specific characterisation of mathematics is given by Gattegno (1963):

> *To do mathematics is to adopt a particular attitude of mind in which what we term relationships per se are of interest. One can be considered a mathematician when one can isolate relationships from real and complex situations and later on when relationships can be used to create new situations in order to discover further relationships. Teaching mathematics means helping one's pupils to become aware of their relational thought, of the freedom of mind in its creation of relationship; it means encouraging them to develop a liking for such an attitude and to consider it as a human richness increasing the power of intellect in its dialogue with the universe"* (p.55).

Other authors have offered more descriptions which emphasise the logical aspects. Kaput and Roschelle (1998) regard "mathematics as a culturally shared study of patterns and languages that is applied and extended through systematic forms of reasoning and argument" (p. 155). Accordingly, it is both an object of understanding and a means of understanding. They reiterate that these patterns and languages are an essential way of understanding the worlds we experience – physical, social, and even mathematical. While our universe of experience can be comprehended and organised in many ways – through the arts, humanities, the physical and social sciences (Kaput & Rochelle, 1998) – important aspects include those that are subject to measure and quantification, that embody quantifiable change and variation, that the world we inhabit and construct, and that involve algorithms and more abstract structures (Kaput & Rochelle, 1998). In addition, "mathematics embodies languages for expressing, communicating, reasoning, computing, abstracting, generalising and formalising" (p. 155) – all extending the limited powers of the human mind. The authors conclude that mathematics embodies systematic forms of reasoning and argument to help establish the certainty, generality, and the reliability of our mathematical assertion.

Battista (1999), however, opines that “mathematics is a form of reasoning”. Thinking mathematically consists of thinking in a logical manner, formulating and testing conjectures, making sense of things, and forming and justifying judgments, inferences, and conclusions. Research has shown that mathematical behaviour is demonstrated by being able to recognise and describe patterns, construct physical and conceptual models of phenomena, create symbol systems to help in representing, manipulating, and reflecting on ideas, and inventing procedures to solve problems (Battista, 1999). Mathematics is also known as a language of science (Jaworski, 2006), an ideal tool for modelling scientific theories, deriving qualitative consequences from them and forecasting events. Mathematics is used in areas as diverse as space research, weather forecasting, geological exploration, and high finance. Mathematics is also a language (Jaworski, 2006) for everyday life, a central part of human communication, and a means of articulating patterns, relationships, rationality and aesthetic.

Meanwhile Bramall and White (2000) add that mathematics is a science related to life that has been used and valued by people ever since the emergence of civilisation or even before then, in many known and unknown ways. Ernest (2000) suggests that we understand our lives through the conceptual meshes of the clock, calendar, working timetables, travel planning and timetables, finance and currencies, insurances, pensions, tax, measurements of weight, length, area and volume, graphical and geometric representations, etc. To Skemp (1985) “mathematics is one of the most powerful and adaptable mental tools which the intelligence of man has constructed for his own use cooperatively over the centuries. Hence, its importance in today’s world of rapidly advancing science, technology, manufacturing, and commerce’’(p. 447). These tools include logical reasoning, problem solving skills, and the ability to think in abstract ways. Mathematics therefore, is important in everyday life and the subject transcends cultural boundaries where its importance is universally recognised.

For all these reasons, mathematics is a central subject in the school curriculum for students at all levels and lends itself as a tool and a way of thinking to many other subjects.

The main rationale for teaching mathematics in Ghana is focused to enable all Ghanaian young people to acquire the mathematical skills, insights, attitudes and values that they will need to be successful in their chosen careers and daily lives. It is vital to note that, the role mathematics plays can easily be seen in our everyday life such as the activities we undertake in our homes, at school, at work places, at market places and in almost every human endeavour. Mathematics as a logical body of knowledge can be used as a guide for arriving at result in a systematic way (Ansari, 2004).

Linear inequalities, an aspect of algebra in mathematics have traditionally been introduced to students when they have acquired the necessary arithmetic skills such as; insight, attitudes and values. Linear inequalities have usually been developed separately from arithmetic without taking advantage of their strong link. “Usually in arithmetic (mathematics) we apply operations to numbers and obtain results after each operation; but in linear inequalities, we usually do not start solving a problem using the given numbers, doing calculations with them, and obtaining a numeric result. In linear inequalities, students have to identify the unknowns, variables and relations among them, and express them symbolically in order to solve the problem” (Martinez, 2002, p. 8).

In its recent final report, the National Mathematics Advisory Panel (NMAP) (2008) outlined three fundamental elements in mathematics that students need to master in order to be fully prepared for exploring linear inequalities. These three critical elements are: (1) fluency with numbers, (2) fluency with fractions, and (3) particular aspects of geometry and measurement. Fluency, according to them means conceptual understanding and problem-solving skills as well as computational fluency. Linear inequalities have a vital role and is considered to be a gatekeeper in mathematics teaching and teaching (Knuth et al., 2005). It is defined as a generalized form of arithmetic that uses symbols, letters and signs for the purpose of generalization. As a result of use of symbols, letters and signs, a linear inequality is classified as an abstract subject (Ansari, 2004).

Students, generally, consider linear inequalities as one of the most difficult areas of mathematics. Consequently, many have misconceptions and difficulties in learning linear inequalities. Learning of mathematics starts when people begin to learn how to count. In the process, we use mathematics in our everyday lives without even realising it. In these situations, we learn the basic numerical concepts in order to communicate with and relate to others (Ansari, 2004).

Mathematics skills develop as we grow and become involved in more and more activities. For example, measuring flour while making cakes are some of the ways through which we make use of mathematical concepts in our everyday lives. What we need in these occasions is mostly common sense, a practical approach towards obtaining a solution and some prior experience. Everyday mathematics does not require the 'brains' of an academic mathematician (Ansari, 2004). Sometimes it is surprising how people even with barely any formal education can deal with calculations so quickly. Many children have well developed informal and intuitive mathematical competence before they start formal education (Clements & Sarama, 2004; Ginsburg, 2002; Ginsburg, Inoue, & Seo, 1999; Kilpatrick, Swafford, & Findell, 2001; Pepper & Hunting, 1998; Urbanska, 1993; Young-Loveridge, 1989).

Children engage in all kinds of everyday activities that involve mathematics (Anderson, 2000), and consequently develop a wide range of informal knowledge (Baroody & Wilkins, 1999; Perry & Dockett, 2004). From infancy to preschool, children develop a base of skills, concepts and understanding about numbers and mathematics. Perry and Dockett (2002) noted that much of this learning has been accomplished without the assistance of formal lessons and with the interest and excitement of the children intact. This is a result that teachers would do well to emulate in our children's school mathematics learning (p. 96). However, as children learn mathematics in a formal setting (the school curriculum), the sense they make of what they are presented with can differ from what the teachers might expect. The concepts can be counterintuitive and they do not understand the fundamental ideas or basic concepts covered in the mathematics class. (Clements & Sarama, 2004). Learning mathematics with understanding has increasingly received attention from mathematics educators and psychologists and has progressively been elevated to one of the most important goals of the mathematical education of all students, the realisation of this goal has long been problematic (Stylianides, & Stylianides, 2007).

For too many people, mathematics stop making sense somewhere along the way. Mathematics arises from the attempt to organise and explain the phenomena of our environment and experience (Bell, 1995). It has been expressed thus: "Mathematics is ... an activity of organising fields of experience" (Freudenthal, 1973, p. 123); and "Mathematics concerns the properties of the operations by which individual orders, organises and controls his environment" (Peel, 1971, p. 157). These descriptions are somewhat non-specific, though Peel is clearly referring to basic mental operations such as classifying, comparing, combining, and representing.

Mathematics is also known as a language of science (Jaworski, 2006), an ideal tool for modelling scientific theories, deriving qualitative consequences from them and forecasting events. Mathematics is used in areas as diverse as space research, weather forecasting, geological exploration, and finance. Mathematics is also a language for everyday life, a central part of human communication, and a means of articulating patterns, relationships, rationality and aesthetic (Jaworski, 2006).

Meanwhile, Bramall and White (2000) state that mathematics is a science related to life that has been used and valued by people ever since the emergence of civilisation or even before then, in many known and unknown ways. Ernest (2000) suggests that we understand our lives through the conceptual meshes of the clock, calendar, working timetables, travel planning and timetables, finance and currencies, insurances, pensions, tax, measurements of weight, length, area and volume, graphical and geometric representations, etc. To Skemp (1985) mathematics is one of the most powerful and adaptable mental tools which the intelligence of man has constructed for his own use cooperatively over the centuries.

More so, mathematics has long been thought of as a subject for only those with special talents. Bulletin, School Division (1989) stated that, until now students still have the perceptions that mathematics is the most difficult subject. This was, proven by Junior High School students' lack of confidence in their mathematical answers because they constantly seek confirmation for their answers from their teachers and parents. But across nations, this attitude is now changing. Mathematics is no longer for the few, but for all (Mcllrath & Huitt, 1995).

Lau, Sigh and Hwa (2009) emphasized that the mathematics skills required for the youth of today to function in the workplace are different from that for youth of yesterday. "Mathematics curriculum state that the strong mathematical competences developed at the basic and secondary levels are necessary requirements for effective study in mathematics, science, commerce, industry and vocations as well for those pupil(s) terminating their education at that level" (Curriculum Research and Development Division (CRDD), 2007, p.12). The history of mathematics education in Ghana dates back to the colonial era when castle schools were established. Under this school system, despite religious education being predominant, arithmetic was taught as a component of the school curriculum and this had enhanced their trade and commerce (Annabele-Addo, 1980).

Currently, mathematics is studied as a core and as an elective subject in Ghana. It is a compulsory subject to be studied by pupils at the basic level (primary and junior high schools) and senior high schools. The rationale behind this policy is to help the pupils to develop interest in the use of mathematics and the ability to conduct investigations using mathematical ideas. It is the acquisition of these qualities that mathematics education in Ghana aims to emphasis in the school system (CRDD, 2007). The subject is also studied as a core in all junior high schools and it is intended to build on the knowledge and competencies developed at the primary level. The student is expected at the J.H.S level to develop the required mathematical competence to be able to use his/her knowledge in solving real life problems. The current syllabus is based on the premise that all students can learn mathematics and that all need to learn mathematics (CRDD, 2007). One needs no further description to accept the subject as important and dear to the heart of curriculum designers in Ghana. Apart from the fact that mathematics is compulsory for all pre-university students, the subject is a hurdle to be cleared by all students who wish to enter into the university.

Moreover, every student is required to pass four core subjects in addition to two elective subjects to guarantee a Senior High School admission in Ghana (CRDD, 2007). Hence, the importance of mathematics in today's world of rapidly advancing science, technology, manufacturing, and commerce cannot be understated (CRDD, 2007). Mathematics therefore, is important in everyday life and the subject transcends cultural boundaries where its importance is universally recognised. For all these reasons, mathematics is a central subject in the school curriculum for students at all levels and lends itself as a tool and a way of thinking to many other subjects. Most of these everyday activities discussed above deal with linear inequalities.(Ansari, 2004)

1.2 Statement of the Problem

Inequality is a term used very often in mathematics contexts and real-life situations. If one opens a mathematics book in Junior High School, inequalities are often seen under algebra, explicitly used to express relationships between numbers or to write restrictions for quantities or implicitly embedded in the domain or behaviour of functions. In higher levels of mathematics, inequalities are everywhere. In analysis, for example, inequalities are a means of proof (Burn, 2005). In optimisation, inequalities off all sorts are the standard tools. In function theory, inequalities are a means of writing constraints and deriving domains. In school mathematics inequalities are studied mainly in algebra courses. Moreover, they are studied as a subsection of equations.

Many of the published studies present inequalities in parallel with equations (e.g., Kieran, 2004; Dreyfus & Hoch, 2004; Vaiyavutjamai & Clements, 2006a&b). There are a few studies where inequalities were connected to the study of functions (e.g., Boero, Bazzini, & Garuti, 2001; Garuti, Bazinni, & Boero,. 2001; Sackur, 2004). Presenting results from inequalities either in relation with equations or in connection with functions, most of the studies report on weaknesses of the pedagogy employed for teaching inequalities. The learning of inequalities encounters multiple obstacles and the students" work on inequalities exhibits misconceptions. According to West African Examination Council (WAEC) report on Basic Education Certificate Examination (BECE) 2017, the report also indicates that students do not answer questions in linear inequalities in algebra well (mathematics). According to the report, students who attempt questions in this area (linear inequalities) do not answer these questions properly. They end up missing up the signs, therefore, making the procedure incorrect and also the answers as well. This hitch has triggered the researcher to delve into the above issue to find out about students' conceptions of linear inequalities.

1.3 Purpose of the Study

The purpose of the study is to find out junior high school pupils' conceptions of linear inequalities in one variable in Have Circuit of the Afadzato South District in the Volta Region of Ghana.

1.4 Research Objectives

1. To examine pupils conceptions of linear inequalities in Have Circuit of the Afadzato South District.
2. To identify the benefits pupils derived from understanding linear inequalities concepts in mathematics in the Have Circuit of Afadzato South District.
3. To identify conceptual change in male and female pupil's learning outcomes in linear inequalities in Have Circuit of the Afadzato South District.

1.5 Research Questions

1. What conceptions of linear inequalities do pupils in Have Circuit of the Afadzato South District have?
2. What benefits do pupils derive from understanding linear inequalities concepts in mathematics in the Have Circuit of Afadzato South District?
3. What conceptual change in learning outcomes do male and female pupils have in linear inequalities in Have Circuit of the Afadzato South District?

1.6 Significance of the Study

First, the findings of this study would bring to light deficiencies existing in the learning of mathematics, especially in Algebra which would inform mathematics educators and policy makers on practical steps to take to improve the mathematics standards at the JHS level.

Second, the findings of this study would help inform the Ministry of Education to develop a comprehensive strategy to equip existing junior high schools with the necessary teaching and learning facilities that promote learning of mathematics at that level.

Third, the findings of this study would add to the already existing knowledge that policy makers and other educational stakeholders possess concerning linear inequality in the JHS mathematics.

1.7 Operational definition of terms

Linear Inequality

A linear inequality involves a linear expression in two variables by using any of the relational symbols such as, $<$, $>$, $\leq$, or $\geq$. A linear inequality divides a plane into two parts. If the boundary line is solid, then the linear inequality must be either, $\leq$ or $\geq$. If the boundary line is dotted, then the linear inequality must be either $>$, or $<$,

An inequality is a statement in which two expressions (algebraic or numeric) are connected with one of $<$, $\leq$, $>$, or $\geq$. A solution of an inequality is a number that, when substituted into the variable in the inequality, makes the statement of inequality true. To solve an inequality is to find all solutions of it. The set of all solutions is also called the solution set.

Concept/Conception

A concept is a notion or statement of an idea, expressing how something might be done or accomplished that may lead to an accepted procedure. The ability to form or understand mental concepts and abstractions is called conception.

Variable

Variable is a symbol used to represent an element of a set. In addition, numbers, variables are commonly used to represent vectors, matrices and functions. Making algebraic computations with variables as if they were explicit numbers allows one to solve a range of problems in a single computation. A typical example is quadratic formula, which allows one to solve every quadratic equation by simply substituting the numeric values of the coefficients of a given equation for the variables that represent them.

1.8 Delimitation of the Study

There are many public junior high schools in the Afadzato South. The study was limited to schools in the Have Circuits in the Afadzato South. This was limited to this circuit due to the focus of the study to find out students' conception of linear inequalities in one variable. This is also the circuit with the largest student population in the district, therefore the researcher decided to use this area since the researcher deals with a large population of respondent. This is a result of the circuit having most JHS schools in the district at Afadzato South. The researcher found it more expedient to use Afadzato South District for the study due to the fact that, it is the area in which the researcher works as a teacher.

The study was also conducted in the public schools only in the area. No private school was covered. The researcher used only public basic school because of the number of schools in the circuit as a whole. In addition, the study was restricted to linear inequalities in one variable. This is because it is one of the most typical areas in the JHS mathematics syllabus that particular aspect of it cannot be skipped.

1.9 Limitations of the Study

The limitation regarding the methodology of the study can be mentioned. Data was collected as written work alone in terms of responding to pre-set question within a time frame to them the respondent. When administering the surveys, time was sometimes an issue; some participants claimed that given more time, they would have answered time, questions well enough. For instance, there is no assumption of equal intervals between the categories, hence a rating of four indicates neither that is twice as powerful as two nor that it is twice as strongly felt. Also using a Likert scale, the researcher has no way of knowing if the respondents might have wished to add any other comments about the issues under investigations (Cohen, Manion & Morrison, 2004).

1.10 Organisation of the Study

This study contains five chapters. Chapter One is the background of the study, statement of the problem, research questions, the significance of the study and the limitations and delimitation of the study. Chapter Two describes the review of related literature relevant to the study. Chapter Three describes the research methodology and procedure used in it. It covered the research design, population of the study, sampling and sampling techniques, and instruments of the study, validity of the study, reliability of the study, data collection procedure and data analysis procedures.

Chapter Four presents the research findings, the analysis and discussion of the data collected. Chapter Five presents summary, conclusion and recommendation. The first section of this chapter presents a brief overview of the background, statement of the problem, purpose of the study, objectives of the study, research questions, significance of the study, delimitation, limitation and organisation of the study. The second section of this study looks at related literature of the study, the third section looks at the methodology of the study, section four speaks on result and discussion and section five looks at major findings of this study to each of the original research questions.

1.11 Summary

This chapter looks at the background to the study, the statement of the problem, purpose of the study, objectives of the study, research questions, and significance of the study, delimitation, limitation and organisation of the study. The study talks about the school as a social unit that deals with providing wholesome experience and opportunities for interventions. The study continue to look at problem statement, what exactly constituted the problem, the purpose of the study, the objectives that guided the study, the research questions that also guided the study was look at: what conceptions of linear inequalities do pupils have, what benefits do pupils derive from understanding linear inequalities concepts in mathematics, what conceptual change in learning outcomes do male and female pupils have in linear inequalities. The importance of the study was look at together with delimitation and limitation of the study and finally the organisation of the study.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

This chapter discusses the review of related literature relevant to the study. Thematic areas covered include the theoretical framework, conceptual change model, students' conceptions and nature of alternative conceptions, genesis of students' alternative conceptions and nature and significance of students' conceptions

In this review, literature will be provided to support the study under the following areas:

1. Conceptual change model
2. Theoretical Framework
3. Related literature in
    a) Students' Misconceptions of linear inequality Concepts.
    b) Misconceptions in relation to the use of literal symbols.
    (c) Misconceptions of translation
    (d) Students' Conceptions and Nature of Alternative Conceptions.
    (e) Genesis of Students' Alternative Conceptions.
    (f) Nature and Significance of Students' Conceptions.
4. Conceptual change model

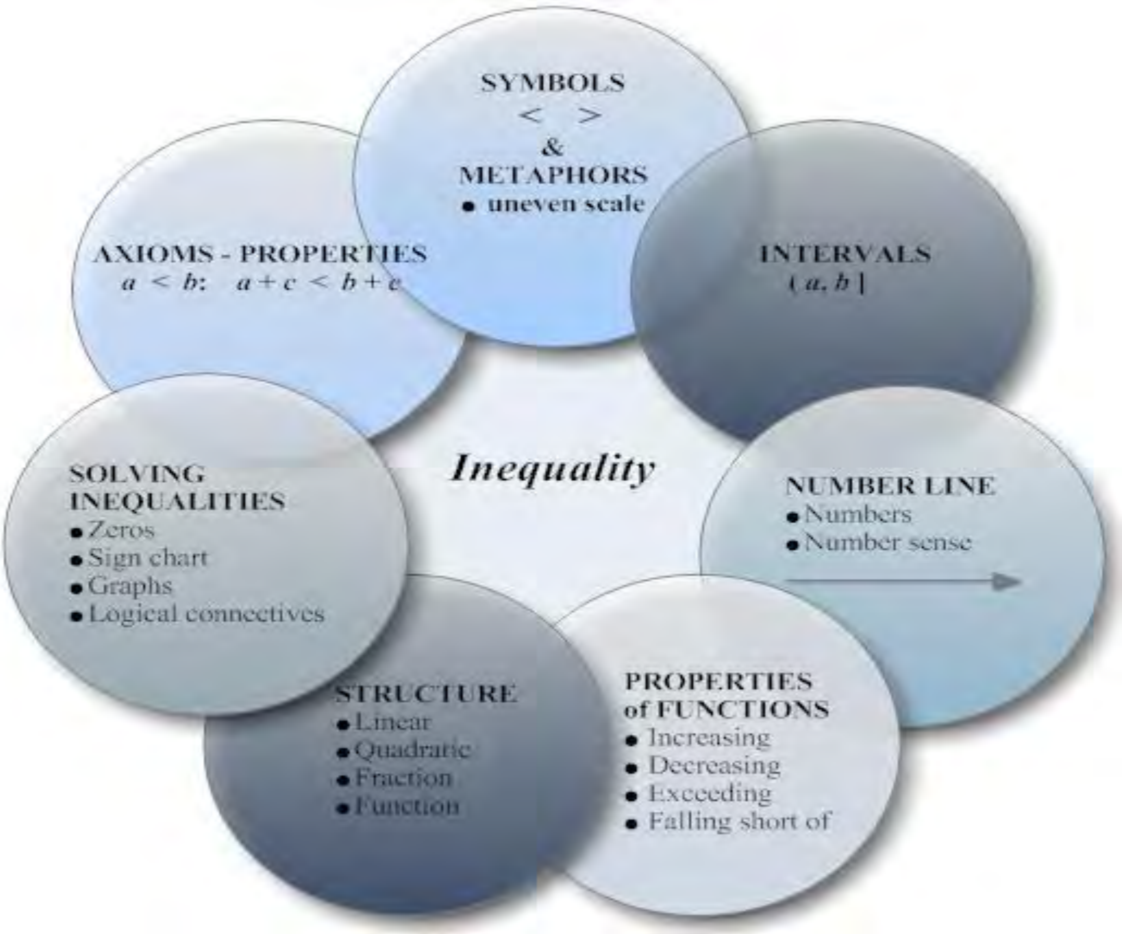

Figure 1: A concept Map of Inequality

Euclid recognized the conditions when one length was larger than another and expressed that in words such as "falls short" of or "is in excess of". However, no indication of inequalities for numbers, except in Archimedes" work in science, mathematics and engineering, is found in the ancient texts (Greer, 2004). Several inequalities, such as the inequality of the arithmetic-geometric means of two quantities or triangle's inequality, which were known to ancient mathematicians, are found in geometric contexts.

2.1 Theoretical Framework

It is a framework based on an existing theory in a field of inquiry that is related and/or reflects the hypothesis of a study. It is a blueprint that is often 'borrowed' by the researcher to build his/her own house or research inquiry. It serves as the foundation upon which a research is constructed. The theoretical framework guides the researcher so that he or she would not deviate from the confines of the accepted theories to make his/her final contribution scholarly and academic. The theoretical framework also guides the kind of data to be accrued for a particular study (Lester, 2005). It aids the researcher in finding an appropriate research approach, analytical tools and procedures for his/her research inquiry. It makes research findings more meaningful and generalizable (Akintoye, 2015).

2.1.1 The perspective of constructivism

The basic and most fundamental assumption of constructivism is that knowledge does not exist independently of the learner, it is constructed. Several philosophers and educators are associated with constructivism. Among the most prominent ones are Piaget (1970), Blumer (1969), Kuhn (1996), von Glasersfeld (1989), and Vygotsky (1978). The major philosophical and epistemological assumptions of constructivism are: (1) There is a real world that sets boundaries to what we can experience. However, reality is local and there are multiple realities. (2) The structure of the world is created in the mind through interaction with the world and is based on interpretation. Symbols are products of culture and they are used to construct reality. (3) The mind creates symbols by perceiving and interpreting the world. (4) Human thought is imaginative and develops out of perception, sensory experiences, and social interaction. (5) Meaning is a result of an interpretive process and it depends on the knowers' experiences and understanding (Cobb, 1994; Jonassen, 1992a; Philips, 1995).

Von Glasersfeld (1989), one of the radical constructivists, traces the origins of constructivism to the Neapolitan philosopher Giambattista Vico. Vico argued that one can only know what he constructed. God created the real world, so only God can know the real world. Man constructs reality so man can only know that he constructed. He argued that knowledge never represents the real world and any knowledge that is constructed does not correspond to the external reality. All we can know is the knowledge we construct and not the external real world constructed by God. There are several schools of thought within the constructivist paradigm (Cobb, 1994; Prawat & Floden, 1994). The two most prominent ones are personal constructivism and social or sociocultural constructivism.

Their major difference has to do with the locus of knowledge construction. For the personal constructivists' knowledge is constructed in the head of the learner while she is re-organizing her experiences and cognitive structures (Piaget, 1970; Von Glasersfeld, 1989). For the social constructivists, knowledge is constructed in communities of practice through social interaction (Kuhn, 1996; Lave & Wenger, 1991; Vygotsky, 1978).

Cobb (1994) argues that the two approaches cannot be separated because both complement each other. While discussing specifically mathematics education he argued that "mathematical learning should be viewed as both a process of active individual construction and a process of enculturation into the mathematical practices of wider society" (p. 13).

Unless the socially constructed knowledge is being processed in the individual's mind and related to her experiences, it will not be meaningful. The assumptions set forth by constructivist epistemology have several implications for the nature of learning and instruction that are antithetical to objectivist approaches. Since constructivists believe that there are multiple truths and realities, education should be encouraging multiple perspectives. Learners interpret their world and educators have to account for the meaning-perspectives of the learners and for their interpretations of the world. Constructivism does not reject the idea that a real world exists. But what it argues is that the world can never become known in one single way. The physical world sets certain boundaries within which multiple perspectives can be negotiated and constructed. For constructivists, learning is meaning-making. People create meaningful interpretations of their environment by taking action and reinterpreting the world (Blumer, 1969). Human choices and actions are a result of interpretation of the world.

This study of the development of understanding of the concept of variable occurred during classroom activities in a specific classroom that has elements that the researcher classifies within the realm of constructivism. Given the various ways in which the term constructivism has been used, the lack of a clear definition, and the obscurity of its implications for pedagogy, it is important, at this point, to highlight some agreed upon principles of constructivism. The study would also outline some different forms constructivism can take, possible characteristics of pedagogical practice, and the study falls into the realm of what is considered constructivism. The purpose of the study relates to interaction in a mathematics classroom and is based on the idea that individual students can create their own subjective knowledge within the social context of the classroom. The general term for this idea is constructivism (Ernest, 1992). Noddings (1990) outlined some general principles that are agreed upon by most constructivists. These are: (1) all knowledge is constructed, at least in part, through reflective abstraction, (2) cognitive structures are activated during the construction process, (3) these cognitive structures are under constant development, enhanced through purposeful activity, and (4) the acknowledgement of constructivism as a cognitive position leads to an adoption of methodological constructivism in research and teaching methods.

Constructivism has been highly debated in educational research especially in the context of mathematics, particularly with regards to the objectivity of knowledge. Even among those who call themselves constructivists, there is no consensus as to what it implies for learning. There are many versions of constructivism. Prawat (1996) describes six versions and Richardson (2003) suggests that there might even be up to 18 different versions. For the purposes of situating the research into the realm of constructivism, the researcher describes Prawat's (1996) six versions of constructivism, distinguish between the two main types of constructivism, radical and social, outline some characteristics of constructivist pedagogical practice, and then outline the view of constructivism for the study.

2.1.1.1 Different versions of constructivism

For the purposes of this study the researcher expands Prawat's (1996) notion that there are six versions of constructivism that fall into two main categories: modern and postmodern. The modern versions are schema (radical) constructivism and information processing theory that represents traditional epistemological stances towards constructivism and hold that knowledge is primarily the property of individuals. The postmodern versions are generally called social constructivism and refute the idea that knowledge is primarily the property of the individual. Social constructivism can be further delineated into sociocultural theory, symbolic interactionalism, social psychological construction, and Deweyan versions of constructivism. The critical difference between the various forms of constructivism seems to be the focus given to the individual or the collective. This distinction can be described in terms of the mind-world dilemma. Cobb (1994) explains the mind-world dilemma as a general conflict between radical constructivists, who view the mind as being located in the learner's head as an individual account of cognition and the social constructivists who view the mind as being located in the individual as part of the social action. The mind-world dilemma poses problems for pedagogy if either extreme is adopted.

2.1.1.2 Modern versions of constructivism

The modern versions of constructivism are schema (radical) constructivism and information-processing theory. They hold that knowledge is the sole property of the individual (Cobb, 1994). Schema (radical) constructivism has roots in the ideas of Piaget and more recently in ideas of Glasersfeld (1990). The focus is on the ways in which individuals construct their own knowledge through the assimilation and accommodation of new understandings into existing cognitive schemes. Piaget attributed most learning to cognitive maturity. Piaget's idea of reflective abstraction is one of importance in the discussion of constructivism. Cobb, Bouffi, Mclain, and Whitenack (1997) defined reflective abstraction, as Piaget did, as a process or episode that allows individual children to reorganize their mathematical activity. It may occur first as a whole class but must ultimately occur on an individual level for true conceptual development to occur. The interpretations, within radical constructivism, of Piaget's ideas emphasizing individual construction of knowledge, have been criticized for not giving enough emphasis on social influences on learning.

2.1.1.4 Information-processing constructivism

The second variety of modern constructivism is information-processing. Prawat (1996) described this variety as one where cognitive structures are built in the individual's head and the validity of these structures is judged according to their fit with structures present in the world. Assimilation of information into existing cognitive structures is more the focus in this variety than is accommodation in the radical variety. According to Prawat (1996), the postmodern versions of constructivism came about as solutions to three main challenges to modern versions. These are: (1) knowledge is primarily the property of the individual and there may be no objective reality, (2) science will solve the mind-world dilemma, once and for all, proving where the mind is located, and (3) knowledge must be the product of an inferential system based on facts and proof of conclusions. Social constructivism has its roots in the ideas of Vygotsky (1986) that individuals construct knowledge through social interactions.

Ernest (1992) stated that, at any time, mathematics is determined by a set of artifacts such as books or papers, a set of people, and a set of linguistic rules followed by these people. Therefore, social interactions and negotiation of meaning are believed to influence the construction of individual cognitive functions. Although teachers may wish to maintain classroom debate at an intellectual level, it is important to keep in mind that the negotiation of meaning in a classroom environment, as in society at large, is influenced by power relationships, and dominant opinions (Ernest, 1992; Richardson, 2003).

In a classroom, power relationships such as teacher-student and dominant opinions such as those of the teacher and of students whose peers deem them adept at mathematics, have an impact on interactions and can impact the views students express in discussions. Vygotsky (1986) called the disparity between what students can accomplish unassisted and what can be accomplished when assisted the zone of proximal development (ZPD). Learning activities, for success, must occur in this zone to result in conceptual change. Pedagogical problems can be seen with the type of constructivism that focuses too heavily on the social collective as Vygotsky has been criticized for doing so at the expense of the individual. Despite agreement on the challenges addressed by social constructivism, the four versions represent very different solutions. The four types of social constructivism, described below, are: sociocultural theory, symbolic interactionalism, social psychological construction, and Deweyan constructivism (Prawat, 1996).

2.1.1.6 Sociocultural theory of constructivism

Sociocultural theory rejects the notion that knowledge is the property of the individual and instead views knowledge as the property of the collective, as a social construct. Psychological tools are seen as mediators between the child and reality. Objects and events in the world exert an indirect effect on tools or artefacts. Through processes of shared meaning, cultural artefacts connect individuals to society and society to individuals. The individual constructs the social and at the same time is constructed by the social. Thus "sociocultural theory relies on socially constructed artefacts to define *individuals* within a culture" (Prawat, 1996, p. 221).

2.1.1.7 Symbolic interactionalism

Symbolic interactionalism is the version of social constructivism that takes into consideration both the individual and the collective in equal measure. Prawat (1996) stated that its strength is that it accounts for how a group of individuals build up collective knowledge but also how individuals within the group have their own unique thoughts about the collectively created meaning. Individuals do not automatically come up with the same personal meaning as another person in the group. Instead, individuals create their own meanings and act towards objects according to the meanings they have created for them. Artefacts are not seen as extensions of the individual but rather as part of the object world to which the individual reacts, thus "symbolic interactionalism relies upon socially constructed artefacts to define *objects and events* in the world" (Prawat, 1996, p. 221).

2.1.1.8 Social psychological constructivism

Social psychological constructivism holds that the mind is in language and that language as a result of communal relations is the carrier of truth. The view is that there is nothing outside language to which individuals may refer in order to validate the truthfulness of the language the community has chosen to use (Prawat, 1996). Prawat outlines some of the inherent weaknesses of this version of constructivism as it's leaving no room for reality in its construction and that it does not account for how individuals manage to operate simultaneously on the individual and collective levels, in order to create new knowledge and understanding. The final version of constructivism, according to Prawat (1996) is idea based (Deweyan) constructivism. He explains that this version attempts to solve the mind-world dilemma through its focus on ideas. Ideas can, under this view, offer a solution to the dilemma as they can move back and forth between the barriers that separate mind from world. Actions are also seen at the heart of ideas and knowing is seen as doing. Prawat (1996) explained that the Deweyan version of constructivism can be seen as a solution to the problem that social constructivism offers no way for students to carry on a dialogue with the real world of objects and events. Now that the six main versions, as seen by Prawat (1996) have been outlined, the researcher would outline some possible characteristics of constructivist pedagogy and explain the view of constructivism that guides this study.

2.1.1.10 Possible characteristics of constructivist pedagogy

Cobb (1988) notes that constructivism does not provide a definite pedagogical plan. He explains that while constructivism is attractive as a theory of learning, it causes great problems when one tries to apply it to instruction, because the assumptions of constructivism are in direct opposition to those of traditional transmission type teaching. He further explains that teaching results from varying degrees of imposition (transmission view) and negotiation (constructivist view) and that the prescription of instructional recommendations from theories that emphasize structure and meaning are problematic at best, because we cannot explain how students construct concepts that are more advanced and complex than before the instruction began. The lack of a pedagogical plan in making the transition from the belief in constructivism as a learning theory, to its use as an instructional theory leaves the teachers with the challenge of figuring it out for themselves.

Richardson (2003) outlines five characteristics of practice that have been seen in classrooms where teachers claim to use constructivism as a guiding theory. These are (1) attention to the individual and their personal backgrounds, (2) facilitation of group dialogue with the purpose of developing shared understandings, (3) provision of opportunities for students to justify and explain responses and possibly change or add to their existing beliefs and understandings by engaging in the planned tasks, (4) development of students' awareness of their own understandings and learning processes, and (5) planned and unplanned use of direct instruction, references to text or web sites. This last characteristic has been questioned because direct instruction is often seen as representing the transmission view of teaching and learning and is not usually associated with constructivist pedagogy (Richardson, 2003).

It can be seen however, as a practical modification as the theory of constructivism meets the reality of classroom teaching and learning. Teachers attempting to put constructivist theory into practice must strike a balance between teaching methods. Teachers have the ultimate responsibility for student learning and sometimes in the name of moving lessons along, direct questioning or instruction is used. Although the five characteristics can be viewed as elements of constructivist pedagogy, they are not specific practices. They are approaches to strive for and eventually become part of a teacher's practice. They represent one way of helping students learn; however, if we accept constructivism as a learning theory, then we must also accept that students can and would learn and create meaning from various sorts of instruction, many of which can be classified into the realm of constructivism so there is not only one way to teach according to constructivist principles (Richardson, 2003).

Constructivist pedagogy can be "thought of as the creation of classroom environments, activities, and methods that are grounded in a constructivist theory of learning, with goals that focus on individual students developing deep understandings in the subject matter of interest and habits of mind that aid in future learning" (Richardson, 2003, p. 1627). Given this general definition of constructivist pedagogy and the acknowledgment that students learn in different ways, teachers may find it useful to try to find a balance between what Sfard (1998) called the acquisition and participation metaphors of learning, especially as a starting point in moving towards constructivist pedagogy. The acquisition metaphor focuses on concept development and learning acquisition of the individual. The teacher plays a central role in the transmission of knowledge. Following the participation metaphor, knowing comes about through evolving bonds between individuals and others based on communication and discourse. The teacher plays the role of facilitator and questioner, versus dispenser of knowledge.

2.1.2 Students' misconceptions of linear inequality concepts

Alternative conceptions research has documented that student:

> 'enter instruction with conceptual configurations that are culturally embedded; are tied into the use of language; and connected to other concepts; have historical precursors; and are embedded in a cycle of expectation, prediction, and confirmation or rejection. For students… it appears that the course of learning is not a simple process of accretion, but involves progressive consideration of alternative perspectives and the resolution of anomalies (Confrey, 1990a, p. 32).

Recent studies focusing on errors and misconceptions in school mathematics are difficult to find (Barcellos, 2005), although some past studies were carried out to focus on students' errors caused by correctly using buggy algorithms or incorrectly selecting algorithms in elementary arithmetic (Brown & Burton, 1978; Brown & VanLehn, 1982) and in elementary algebra (Matz, 1982; Sleeman, 1982). In the efforts to explore the conceptual basis of students' procedural errors or buggy algorithms, existing research tended to attribute students' learning difficulties to their underdevelopment of logical thinking (e.g., Piaget, 1970), lack of understanding of mathematical principles underlying procedures (e.g., Resnick et. al., 1989), lack of proficiency or knowledge (Anderson, 2002; Haverty, 1999), or poor understanding of mathematical symbols (e.g., Booth, 1984; Fujii, 2003; Stacey & MacGregor, 1997; Usiskin, 1988).

In contrast, rather than characterising students' difficulties or misconceptions in terms of a deficiency model, McNeil and Alibali (2005) claimed that "earlier learning constrains later learning" (p. 8), that is, students' misconceptions may be caused by their previous learning experiences. Researchers agree that students enter classrooms with different conceptions due to different life experiences or prior learning. An important task for teachers is to identify students' preconceptions and misconceptions in order to help students learn mathematics effectively and efficiently. Ignoring students' misconceptions may have negative effects on students' new learning and may also result in the original misconceptions being reinforced. In the following sub-section, literature on some misconceptions and their genesis is reviewed briefly.

2.1.3 Misconceptions in Relation to the use of Literal Symbols

A linear inequality uses its own standardized set of signs, symbols and rules about how something can be written (Drijvers et al., 2011). Linear inequalities seem to have its own grammar and syntax and this makes it possible to formulate algebraic ideas unequivocally and compactly. In this symbolic language, "variables are simply signs or symbols that can be manipulated with well-established rules, and that do not refer to a specific, context-bound meaning" (Drijvers et al., 2011, p. 17).

There is a great deal of empirical evidence (Vosniadou, Vamvakoussi, & Skopeliti, 2008) showing that rational number reasoning (learning rational numbers) is very difficult for students at all levels of instruction and in particular when new information about rational numbers comes in contrast with prior natural number knowledge (Ni & Zhou, 2005) in their previous learning experiences. Students have difficulties interpreting and dealing with rational number notation, in particular when it comes to fractions (Gelman, 1991; Stafylidou & Vosniadou, 2004). They do not realise that it is possible for different symbols (e.g., decimals and fractions) to represent the same number and thus they treat different symbolic representations as if they were different numbers (Khoury & Zazkis, 1994; O'Connor, 2001; Vamvakoussi & Vosniadou, 2007).

These difficulties are exacerbated to the study of algebra in which students are introduced to the principled ways in which letters are used to represent numbers and numerical relationships – in expression of generality and as unknowns – and to the corresponding activities involved with these uses of letters … justifying, proving, predicting, and solving, resulting in a series of misconceptions with the use of notation – a tool to represent numbers and quantities with literal symbols but also to calculate with these symbols (Kieran, 2007).

With regard to students' possible misconceptions of the meaning of variable, often contexts call for multiple usages and interpretations of variables. Students must "switch from one interpretation to another in the course of solving a problem which makes it difficult for an observer and for the individual himself to disentangle the real meaning being used" (Kuchemann, 1981, p. 110). In conclusion, "the meaning of variable is variable; using the term differently in different contexts can make it hard for students to understand" (Schoenfeld & Arcavi, 1988, p. 425), hence, giving rise to many misconceptions that students have in understanding the different uses of literal symbols in different contexts.

Previous studies have demonstrated a series of misconceptions which students have in relation to the use of literal symbols in algebra. A common naïve conception about variables is that different letters have different values. Alternatively, when students think of literal symbols as numbers they usually believe that they stand for specific numbers only (Booth, 1984; Collis, 1975; Kuchemann, 1978, 1981; Knuth et al., 2008 Stacey & MacGregor, 1997). This misconception is illustrated by students' responses of "never" to the following question: "When is the following true – L + M + N = L + P + N always, never or sometime?" Kuchemann (1981) reported in the CSMS project that 51 percent of students answered "never" and Booth (1984) reported in SESM project that 14 out of 35, that is, about 41 percent of 13 – 15-year-old students responded likewise. Olivier (1989) reported that 74 percent of 13-year-old students also answered "never". Fujii (2003) have likewise found a high proportion of students in his sample doing the same. Kieran (1988) in her study found out that students do not understand that multiple occurrences of the same letter represent the same number. Even students who have been told and are quick to say that any letter can be used as an unknown may, nonetheless, believe that changing the unknown can change the solution to an equation.

One of the best known of the misconceptions is the "letter as object" misconception, described by Kuchemann (1981), in which the letter, rather than clearly being a placeholder for a number, is regarded as being an object. For example, students often view literal symbols as labels for objects, that is, they think that 'D' stands for David, '*h*' for height, or they believe that '*y*' - in the task "add 3 to 5y"- refers to anything with a '*y*' like a yacht. The term "fruit-salad algebra" (MacGregor & Stacey, 1997b) is sometimes used for this misconception, infamously presented in examples such as "a for apples and b for bananas, and so 3a + 2 is like 3 apples and 2 bananas, and since you can't add apples and bananas we just write it as 3a + 2." One difficulty is that 3a in algebra does not represent 3 apples, but three times an unknown number. The second difficulty here concerns the mathematical idea of closure: in saying we cannot add apples and bananas we contradict the fact that 3a + 2b is the sum.

According to Chick (2009), the letter as object misconception may be reinforced by formulas like A = L * B, where *A* = area. Another typical sign whose use in arithmetic is inconsistent with its meaning in algebra is the equal sign. Beginning students tend to see the equal sign as a procedural marking telling them "to do something", or as a symbol that separates a problem from its answer, rather than a symbol of equivalence (Behr, Erlwanger, & Nichols, 1976).

Equality is commonly misunderstood by beginning algebra students (Falkner, Levi, & Carpenter, 1999; Knuth et al., 2008). Several researchers have noted that such a limited view of the equal sign exists among some students in secondary schools (Herscovics & Linchevski, 1994) and also at college level (Bell, 1995).

Moreover, Steinberg, Sleeman, and Ktorza (1990) showed that eighth- and ninth-grade students have a weak understanding of equivalent equations. Even college calculus students have misconceptions about the true meaning of the equal sign. Carpenter, Levi, and Farnsworth (2000) posit correct interpretation of the equal sign is essential to the learning of algebra because algebraic reasoning is based on students' ability to fully understand equality and appropriately use the equal sign for expressing generalisations. For example, the ability to manipulate and solve equations requires students to understand that the two sides of the equation are equivalent expressions and that every equation can be replaced by an equivalent equation (Kieran, 1981). Consider the equation $2x - 3 = 11$. Some students see the expression on the left-side as a process and the expression on the right-side as the result (Linchevski & Herscovics, 1996; Knuth et al., 2008).

2.1.4 Misconceptions of translation

Drijvers et al. (2011) regard an important part of linear inequalities in algebraic activity is the translation of a problem or situation into linear inequalities. These researchers are of the opinion that this involves more than translation; it concerns building a structure that algebraically represents the problem variables and their mutual relationships in the situation which is essentially described as modelling. Among students' greatest difficulties is modeling equations from problem situations. Translating from verbal relational statements to symbolic equations or from English to "maths" causes students of all ages a great deal of confusion.

Lodholz (1990) observed that writing equation from word problem is often a skill taught in contrived situations or in isolation. The researcher attributes this to be one of the causes for another misconception related to direct translation of verbal statement in English. Lodholz reiterates that mechanical word problems that require students to write an expression that represents "5 more than 3 times a number," when taught apart from opportunities for application, can cause students difficulty when interpreting meaningful sentences later. Students may translate English sentences to algebraic expression, simply moving from left to right. For example, "Three less than a number" is interpreted by many students as "3 – x" since the words "less than" (which means to subtract) follow the 3. Teachers must be aware of these misconceptions and address them in instruction (Lodholz, 1990). Another incorrect direct transliteration of verbal statements into algebra, where $3a + 4b$ could be derived from 3 apples and 4 bananas (irreverently known as "fruit-salad algebra"), has been shown to be a frequent source of error (Kuchemann, 1981), and is of course encouraged by some algebra texts.

2.1.5 Students' conceptions and nature of alternative conceptions

Ausubel, in the preface to his book, Educational Psychology: A Cognitive View, says that "If I had to reduce all of educational psychology to one principle, I would say this: The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly" (Ausubel, 1968, p. vi). This widely shared principle in educational psychology stated by Ausubel (1968) posits that the most important factor to influence learning is the student's previous knowledge. The acceptance of this principle resulted in greater interest within the mathematics education research in exploring students' conceptions (Confrey, 1990). Since the time of Piaget, researchers have been keenly interested in how students view concepts of science and mathematics (Confrey, 1990).

Interest in student conceptions surged with the emergence of constructivism (Osborne, 1996; Solomon, 1994), which brought along with it "a language with new descriptive power" (Solomon. 1994, p. 6). This language, Solomon argues, transmuted the study of common student mistakes, previously of little appeal to anyone, to something exciting and of great interest. Essential to the constructivist view is the recognition that learning is viewed as a process of active construction which is shaped by students' prior knowledge and conceptions. Many researchers agree that the most significant things that students bring to class are their conceptions (Ausubel, 1968, 2000; Driver & Oldham, 1986).

This current cognitive structure of the students composing a set of abstract ideas, concepts and generalisation builds upon facts is the organisation of the student's present knowledge in a subject (Confrey, 1990). The students' previously stored knowledge or ideas play an important role when teaching new concepts (Dochy & Bouwens, 1990; Goss, 1999). During instruction students generate their own meaning based upon their experience and abilities (Nakhleh, 1992). For meaningful learning to occur, new knowledge must be related by the student to the relevant existing concepts in the students' cognitive structure (Ausubel, 1968, 2000). Teachers can be astonished to learn that despite their best efforts, students do not understand fundamental ideas or basic concepts covered in mathematics class. Some of the students give the right answers but these are only from correctly memorised algorithms. Students are often able to use algorithms to solve numerical problems without completely understanding the mathematical concepts (West & Fensham, 1974).

Furthermore, concept means in everyday life 'a term' or 'a word' (Bolton, 1972, p. 23; Nelson, 1985). Concepts are described both in older and newer sources in a similar way. For instance, it is said, that "concepts are perceived regularities or relationships within a group of objects or events and are designated by some sign or symbol" (Heinze-Fry & Novak, 1990, p. 461). Concepts can be considered as ideas, objects or events that help us understand the world around us (Eggen & Kauchak, 2004). Concepts that are embedded in mental structures are defined by Cohen, Manion and Morrison as follows:

> *Concepts enable us to impose some sort of meaning on the world; through them reality is given sense, order and coherence. They are the means by which we are able to come to terms with our experience.* (Cohen, Manion & Morrison, 2000, p. 13).

Each student constructs and reconstructs a wide range of complex, integrated, idiosyncratic, and epistemologically legitimate conceptions on an ongoing basis as s/he negotiates his/her classroom experience (Confrey, 1990). Therefore, students' real-world conceptions play a critical role in their view of the world (Novick & Nussbaum, 1982). So, what is a conception? The term 'conception' itself must be explained since it is widely used in science and mathematics education but with very different meanings (Kaldrimidou & Tzekaki, 2006). Conceptions are systems of explanation (White, 1994). Glynn and Duit (1995) viewed conceptions as learner's mental models of an object or an event. Duit and Treagust (1995) however define conceptions as "the individual's idiosyncratic mental representations", while concepts are "something firmly defined or widely accepted" (p. 47). Consequently, Kattmann, Duit, Gropengieer and Komorek (1996) define conceptions "as all cognitive constructs which students use in order to interpret their experience" (p. 182). These constructs are located on different epistemological levels of complexity, comprising for example concepts, intuitive rules, thinking forms, and local theories (Gropengie ver, 2001; Prediger, 2008). Osborne and Wittrock (1983) summarised student conceptions succinctly in their statement that "children develop ideas about their world, develop meanings for words used in science [mathematics], and develop strategies to obtain explanations for how and why things behave as they do" (p. 491). Children develop these ideas and beliefs about the natural world through their everyday experiences. These include sensual experiences, language experiences, cultural background, peer groups, mass media as well as formal instruction (Duit & Treagust, 1995).

Some of these ideas and beliefs such as those about "the failure to accept the possibility of dividing a smaller by a larger number, and the assumption that multiplication always makes bigger and division smaller" (Bell, 1986, p. 26) may be similar across cultures as children have very similar personal experience with phenomena. These categories of children's beliefs, theories, meanings, and explanations will form the basis of the use of the term student conceptions. Simply stated, "conceptions can be regarded as the learner's internal representations constructed from the external representations of entities constructed by other people such as teachers, textbook authors or software designers" (Treagust & Duit, 2008, p. 298).

Students' conceptions are critical to subsequent learning in formal lessons because there is interaction between the new knowledge that the students encounter in class and their existing knowledge. Research with infants has shown that the process of constructing naïve physics starts soon after birth. By the time children go to school they are deeply committed to ontology and causality that distinguishes physical from psychological objects and which forms the basis for the knowledge acquisition process (Carey, 1985;

Vosniadou, 1994; 2001). Naïve physics facilitates further learning when the new, to be acquired information is consistent with existing conceptual structures. In the learning of mathematics, students also develop a "naïve mathematics" (Vosniadou, 2001) on the basis of everyday experience, which appears to consist of certain core principles or presuppositions (such as the presupposition of discreteness in the number concept) that facilitate some kinds of mathematical learning but may inhibit others (Gelman, 2000).

Johnstone (2000) states that when a person tries to store material in long term memory and cannot find existing knowledge with which to link it, s/he may try to 'bend' the knowledge to fit somewhere, and this gives rise to erroneous ideas (Gilbert, Osborne & Fensham, 1982). When people place instances in sets, or concepts, different from those determined by the community of scientists, the term misconceptions (Nesher, 1987; Perkins & Simmons, 1988) is used. Other researchers use other terms for these constellations of beliefs (Confrey, 1990), including the following: alternative frameworks (Driver, 1981; Driver & Easley, 1978), student conceptions (Duit & Treagust, 1995), preconceptions (Ausubel, Novak, & Hanesian, 1978), alternative conceptions (Abimbola, 1988; Hewson, 1981), intuitive beliefs (McCloskey, 1983), children's science (Osborne & Freyberg, 1985), children's arithmetic (Ginsburg, 1977), mathematics of children (Steffe, 1988), naïve theories (Resnick, 1983), conceptual primitives (Clement, 1982) and private concepts (Sutton, 1980).

Mathematics education researchers have conceptualised these conceptions as a 'belief system' (Frank, 1985), as a 'network of beliefs' (Schoenfeld, 1983), as a 'mathematical world view' (Silver, 1982) and as 'conceptions of mathematics and mathematical learning' (Confrey, 1984). Although different authors may have good reasons for preferring particular terms in the context of particular studies, there is no generally agreed usage across the literature (Taber, 2009).

There is currently a large body of research literature that would be summarised in articles by Duit (2009). Tsamir and Bazzini (2001) presented the results of a study on student's reactions to traditional methods when facing non-standard learning tasks. The traditional method tested was a teacher demonstrating in small steps how to solve a certain type of problem and then students were given a lot of practice to master the procedure. The subjects of the study incorrectly solved linear inequalities or were unable to explain their work. The study suggests that the root of students‟ misconception in regard to inequalities is the way they were previously instructed in this topic. The traditional process of solving linear inequalities is very similar to solving linear equations, except for one detail: you reverse the inequality sign whenever you multiply or divide the inequality by a negative number. This common and handy way to introduce inequalities in an algebra course may be the root of many misconceptions students have when solving inequalities (Bazzini & Tsamir, 2001).

Teaching inequalities in a traditional manner, such as a teacher presenting a "sequence of topics, theorems and rules, which are demonstrated in suitable sets of examples," (p.61) followed by home assignments that are usually a "repetition of tasks similar to those experienced in class" (p.61), does not leave room for students‟ creative participation in the process of learning. Bazzini and Tsamir concluded that doing algebra should involve *understanding*, rather than "just formal manipulation" (p.67).

Similar conclusions were derived from an experimental study on linear inequalities in Thailand by Vaiyavutjamai and Clements (2006a). The study measured whether there was lasting improvement in the performance and understanding of inequalities. Their main finding is that inequalities are hard to master, especially when presented to students in a traditional teaching style. After taking an intensive course on inequalities, many students remained confused about the meaning of an inequality and about what the solutions to inequalities represented. Six months after the study, students in low- and medium-stream classes performed only slightly better than they had performed at the pre-teaching inequalities stage (Vaiyavutjamai & Clements, 2006a; b).

Vaiyavutjamai and Clements (2006a) based their study's framework on evidence that traditional teaching and assessment in school mathematics isolates skills into small compartments and thus fails to assist learners in making suitable, long-lasting cognitive connections. The results of their study were pretty pessimistic: all students, apart from the best, showed a "rules without reason" (p.20) approach to solving inequalities. These types of results were not singular: Linchevski and Sfard (1991), Tsamir and Almog (2001), and Tsamir and Bazzini (2001) presented similar results.

Abramovich (2006) used a spread sheet as a modelling tool for working with inequalities in the mathematical reduction context. The study draws from the Standards document (NCTM, 2008) for teaching and recommendations for teachers, which also includes the familiar approach of reducing a difficult problem to a more familiar one. Abramovich recognized that:

> Many problems in geometry can be reduced to the study of algebraic equations and inequalities; by the same token, many problems in algebra can be reduced to the study of their geometric representations. In calculus, the investigation of infinite processes, such as convergence of sequences and series, can be reduced to the study of inequalities between their finite components (p.527).

He claimed that even though inequalities are recognized as powerful tools in pure mathematics, only a few mathematics education studies – such as Boero and Bazzini's 2004 paper – have revealed this didactical aspect of inequalities. Abramovich's (2006) spread sheet approach to using inequalities in problem solving and the graphing calculator for solving inequalities (Abramovich & Ehrlich, 2007) are recommended by the authors to prospective teachers of secondary mathematics in preparation for teaching inequalities. Their recommendation followed a study that blames students‟ misconceptions in regards to inequalities on teachers‟ inadequate knowledge in this area (Abramovich & Ehrlich, 2007).

Published studies identified several *common errors* in students work when solving inequalities (Tsamir, Almog, & Tirosh, 1998; Linchevski & Sfard, 1991; Tsamir et al. 2004).

A comprehensive list of common errors associated with solving inequalities includes:

(1) Multiplying or dividing the two sides of an inequality by the same number without checking whether the number is positive, negative or zero.

(2) Solving linear equalities instead of equations

Inequalities, with their multiple semiotic registers of representation – algebraic, interval, functional, and graphical (Sackur, 2004) – present many ways in which students could make errors. “Equations were found to serve as a prototype in the algorithmic models of solving [linear] inequalities” (Tsamir & Bazzini, 2002, p.7).

However, as it can be seen from the above list, this approach to teaching linear inequalities seems to be the one that could generate most of the listed errors. In addition, linear inequalities are not the only type of inequalities affected by that pattern; rational inequalities introduced as special cases of equations are also affected by eliminating the denominator. Students very often eliminate the denominator when solving equations, without thinking that such an action might influence the sign of the entire algebraic expression. For the graphical approach to solving inequalities, the main problem is student's ability to correctly read the graph and convert what they see into a valid solution.

2.1.6 Nature and significance of students' conceptions

Students' existing ideas are often strongly held, resistant to traditional teaching and form coherent though mistaken conceptual structures (Driver & Easley, 1978). Rather than being momentary conjectures that are quickly discarded, misconceptions consistently appear before and after instruction (Smith, diSessa, & Roschelle, 1993).

Students may undergo instruction in a particular topic, do reasonably well in a test on a topic, and yet, do not change their original ideas pertaining to the topic even if these ideas are in conflict with the concepts they were taught (Fetherstonhaugh & Treagust, 1992). Duit and Treagust (1995) attribute this to students being satisfied with their own conceptions and therefore seeing little value in the new concepts. Therefore, it is difficult for students to change their thinking. Another reason the authors proposed was that students look at the new learning material "through the lenses of their pre-instructional conceptions" (p. 47) and may find it incomprehensible. Osborne and Wittrock (1983) state that students often misinterpret, modify or reject scientific viewpoints based upon the way they really think about how and why things behave, so it is not surprising that research shows that students may persist almost totally with their existing views (Treagust, Duit, & Fraser, 1996).

When the students' existing knowledge prevails, the scientific concepts are rejected or they may be misinterpretation of the concepts to fit or even support their existing knowledge. As Clement (1982a) has shown in elementary algebra where college students make the same reversal error in translating multiplicative reasoning relationships into equations (e.g., translating "there are four people ordering cheesecake for every five people ordering strudel" into "4C = 5S"), whether the initial relations were stated in sentences, pictures, or data tables. In domains of multiplication (Fischbein, Deri, Nello, & Marino, 1985), probability (Shaughnessy, 1977), and algebra (Clement, 1982a; Rosnick, 1981), misconceptions continue to appear even after the correct approach has been taught. Sometimes misconceptions coexist alongside the correct approach (Clement, 1982a). Such results are compatible with the conceptual theoretical framework, which predict difficulties in learning when the new knowledge to be acquired comes in conflict with what is already known (Vosniadou, 1994).

If the concepts are accepted, it may be that they are accepted as special cases, exceptions to the rule (Hashweh, 1986), or in isolation from the students' existing knowledge, only to be used in the classroom (de Posada, 1997; Osborne & Wittrock, 1985) and regurgitated during examinations. Additional years of study can result in students acquiring more technical language but still leave the alternative conceptions unchanged (de Posada, 1997). In the mathematics education research, there has been much evidence to show that prior knowledge about natural numbers stand in the way of understanding rational numbers. Students make use of their knowledge of whole numbers, to interpret new information about rational numbers (Moskal & Magone, 2000; Resnick, Nesher, Leonard, Magone, Omanson, & Peled, 1989; Vamvakoussi, & Vosniadou, 2004).

This gives rise to numerous misconceptions pertaining to both conceptual and operation aspects of numbers. For example, properties of natural numbers such as "the more digits a number has the bigger it is" are used in the case of decimals (Stafylidou & Vosniadou, 2004). And in the context of mathematical operations, the well-known misconception such as "multiplication always makes bigger" reflects the effects of prior (existing) knowledge about multiplication with natural numbers (Fischbein, Deri, Nello, & Marino, 1985).

These pre-concepts are tenacious and resistant to extinction (Ausubel, 1968, 2000); deep seated and resistant to change (Clement, 1987); and consistent across individuals, very strongly held and not easily changed by classroom instruction (McCloskey, 1983). Smith et al. (1993) opine that students can "doggedly hold onto mistaken ideas even after receiving instruction designed to dislodge them" (p. 121). This persistence does not necessary mean that instruction has failed completely.

However, it is vital to acknowledge that misconceptions because of their strength and flawed content can interfere with learning expert concepts. Research has demonstrated the persistence of student misconceptions and the tendency for regressing to these preconceptions even after instruction to dislodge them. Mestre (1989) reports "students who overcome a misconception after ordinary instruction often return to it only a short time later" (p. 2).

2.1.7 Genesis of students' alternative conceptions

Arguably the most general source of conceptions is limited experience or exposure limited examples - both in learning outside of school and during formal instruction (Greer, 2004). Alternative conceptions may arise when students are presented with concepts in too few contexts or when concepts presented are beyond their developmental level (Gabel, 1989).

It may be that, when first introduced to a new exposition of a scientific idea, students have not yet attained as high a level of abstract thinking as the instructors assume.

Perhaps instructors provide part of the concept or a more simplified concept in the belief this will lead students to better understanding, as sometimes happens when the concept is difficult or known to be troublesome. Such limited explanation, however, may prevent some students from crossing a cognitive threshold and entering through a door to a higher level of understanding (Liljedahl, 2005).

Other possible explanations could be both the pace at which the algebra concepts are covered and also the formal approach often used in its presentation. It seems that many teachers and textbook authors are unaware of the serious cognitive difficulties involved in the learning of algebra. As a result, many students do not have the time to construct a good intuitive basis for the ideas of algebra or to connect these with the pre-algebraic ideas that they have developed. They fail to construct meaning for the new symbolism and are reduced to performing meaningless operations on symbols they do not understand (Herscovics & Linchevski, 1994). Within the domain of algebra, Kieran (1992) contended that “one of the requirements for generating and adequately interpreting structural representations such as equations is a conception of the symmetric and transitive character of equality – sometimes referred to as the ‘left-right equivalence’ of the equal sign” (p. 398).

Yet, there is abundant literature that suggests that students do not view the equal sign as a symbol of equivalence (i.e., a symbol that denotes a relationship between two quantities), but rather as an announcement of the result or answer of an arithmetic operation (e.g., Falkner, Levi, & Carpenter, 1999; Molina & Ambrose, 2008). It has been suggested that this well documented (mis)conception might be due to students’ elementary school experiences (Carpenter, Franke, & Levi, 2003; Seo & Ginsburg, 2003), where the concept of equality and its symbolic instantiation are traditionally introduced during students' early elementary school years, with little instructional time explicitly spent on the concept in the later grades and the equal sign was nearly always presented in the operations equals answer context (e.g., 2 + 5 = 7).

Another source of confusion is the different meaning of common words in different subjects and in everyday use. This applies not only to words, but also symbols. Harrison and Treagust (1996) reported that students were confused between the nucleus of an atom and the nucleus of a cell, and that one student actually drew a cell for an atom. They also found students having alternative conceptions about electron clouds, and cautioned that teachers need to qualify the sense in which they transfer the attributes of the analog to the target. Many scientifically associated words are used differently in the vernacular (Gilbert & Watts, 1983), for example, energy has a cluster of 'life-world' associations that do not match its technical use (Solomon,1992). Likewise, Tall and Thomas (1991) indicate that, in the natural language 'and' and 'plus' have similar meanings. Thus, the symbol 'ab' is read as '*a* and *b*' and interpreted as 'a+b'. Schmidt has pointed out how linguistic cues in scientific terminology may lead to inappropriate inferences being drawn (Schmidt, 1991).

In the same way, verbal cues certain words such as 'more', 'times', reduction' have strong associations with particular operations, and these sometimes "displace the true meanings of the phrases in which they appear" (Bell, Swan & Taylor, 1981). For example, "the milkman brought 11 bottles of milk on Sunday. That was four more than he brought on Monday. How many bottles did he bring on Monday?" Here, the word 'more' was shown to greatly increase the likelihood of addition being chosen as the correct operation to perform (Anderson, Reder & Simon, 2000; Nesher & Teubal, 1975).

The problem raised by Fischbein (1987, p. 198) is that: A certain interpretation of a concept or operation may be initially very useful in the teaching process as a result of its intuitive qualities (concreteness, behavioural meaning, etc.). But as a result of the primacy effect that interpretation may become so rigidly attached to the respective concept that it may become impossible to discard it later on. The initial interpretation of a concept may become an obstacle which can hinder the passage to a higher-order interpretation - the more general and more abstract – of the same concept. For example, an early understanding of natural number and its properties supports children's understanding of notions such as potential infinity (Hartnett & Gelman, 1998), while at the same time it stands in the way of students' understanding of the properties and operations of rational numbers (Moskal & Magone, 2000; Yujing & Yong-Di, 2005).

There is a great deal of evidence showing that students at various levels of instruction make use of their knowledge of natural number to conceptualise rational numbers and make sense of decimal and fraction notation, often resulting in making systematic errors in ordering, operations, and notation of rational numbers. Many researchers attribute these difficulties to the constraints students' prior knowledge about natural numbers imposes on the development of rational number concept. McDermott (1988) suggests that some alternative conceptions may arise from failure to integrate knowledge from different topics and from concept interference that comprises "situations where the correct application of a conception by students is hindered by their misuse of another concept that they have learned" (p. 539). This occurs when students do not have an adequate conceptual framework to know which concept to apply in a situation. Concept interference may also be due to set effects (Hashweh, 1986) where certain knowledge or conceptions are brought to mind due to strong ties with certain features of a given situation through previous experience. For example, students may have as part of their concept image for subtraction that when things are subtracted the numbers become smaller. This could become a conflict when they are later introduced to negative numbers because when they are subtracted the numbers become larger (Gallardo, 1995, 2002; Vlassis, 2001). Additionally, student conceptions of the associative and commutative properties of numbers think that subtraction like addition, is commutative, that order does not affect the answer (Bell, Greer, Grimison, & Mangan, 1989; Brown, 1981), so 7 – 3 and 3 – 7 are the same, or rather they have the same answer.

According to Olivier (1984), the main contributory factor for seeing subtraction as commutative is probably that students have extensive experience of the commutatively of addition and multiplication when learning tables. They are over-generalising over operations. Booth (1984), Chalouh and Herscovics (1988) refer to the students' frame of reference or the context of a problem on the answer that the student chooses and also the uncertainty about what is required as the source of the misconception. For example, students may often hear teachers say that "you can't add unlike terms" but then in a test are instructed: "Add 4 onto 3n" (Kuchemann, 1981, p. 108). Since these are unlike terms, how can they be added? Similarly, when told "X is any number: "Write the number which is 3 more than x" (Bell, Costello & Kuchemann, 1983, p. 138), the large number of students who gave x an arbitrary value and added 3 to it may have thought that since x is "any number" that is what they are supposed to do.

Other wrong responses reflect students' assumption, developed from arithmetic, about what an acceptable "answer" should look like (e.g., that it should not contain an operation sign). Teachers need to be aware that they also can be the sources of alternative conceptions. When teachers have the same alternative conceptions as their students (Wandersee, Mintzes, & Novak, 1994), they think that there is nothing wrong with their students' conceptions. Consequently, teachers can unwittingly pass their own alternative conceptions to their students, and the way they teach, for instance, using imprecise terminology, can also cause confusion (Chang, 1999; Lin, Cheng, & Lawrenz, 2000). The incorrect direct transliteration of verbal statements into algebra, where 3a + 4b could be derived from 3 apples and 4 bananas (irreverently known as "fruit-salad algebra"), has been shown to be a frequent source of error (MacGregor & Stacey, 1997b; Kuchemman, 1981), and is encouraged by some algebra texts.

In research conducted by MacGregor and Stacey (1997a), they found that a new misinterpretation of algebraic letters may be caused by students misinterpreting teachers' explanations. Students sometimes misinterpret 'X' is any number. It appears that when teachers stress that "X without the coefficient is 1X", they may also be misunderstood, since some students interpret this to mean "X by itself is one". Another possible source of confusion is the fact that in the context of indices the power of X is 1 if no index is written (i.e., X=X 1). When interviewed, several Year 10 students explained, "By itself X is 1, it's because it hasn't got a number". Thus, teachers must be aware that they "cannot assume that what is taught is what is learned" (Driver & Scott, 1996, p. 106). Gray and Tall (1994) underlie the fact that the same notation may be viewed as signifying a process or an object, so that, for example, a teacher may offer a representation of the function y= 2+ 3 as an example of a linear function, but the learner may see it as an example of a procedure (for drawing a graph from an equation) (Bills, Dreyfus, Tsamir, Watson, & Zaslavsky, 2006). Given the number of students taught over a teaching career, the generation of alternative conceptions can be quite significant. Teachers should realize that textbooks also can contain errors and misleading or conflicting illustrations and statements which can give rise to alternative conceptions (Boo, 1998; de Posada, 1999); hence, textbook should not be regarded as infallible.

Teachers need to know their students' alternative conceptions in order to help them lower the status of these conceptions in favour of the accepted algebra concepts. Unfortunately, teachers are often unaware of their students' alternative conceptions (Treagust et. al., 1996), that is why Posner, Strike, Hewson and Gertzog (1982) maintain that teachers "should spend a substantial portion of their time diagnosing errors in thinking and identifying moves used by students to resist accommodation (p. 226). The above literature suggests that students' conceptions play an important role in their on-going understanding and learning in formal lessons. That some of these conceptions can be stable, widespread among students, can be strongly held, resistant to change, and can interfere with learning expert concepts causing students difficulties in their learning of certain algebra concepts. The purpose of the following section is to examine not only the inhibiting interference from these conceptions but also the other inhibiting interferences causing students difficulties in the learning of algebra. Relevant literature related to typical students' difficulties and misconceptions is reviewed and discussed in greater detail so as to have a better understanding of the problems faced by the students.

2.2 Summary

This chapter reviewed literature regarding students' conceptions and nature of alternative conceptions, students' difficulties and misconceptions of algebra concepts, students' attitudes towards mathematics, and diagnostic teaching methodology that incorporates cognitive conflict focusing on its correlation to student achievement and conceptual change. The critical role that students' existing knowledge (conceptions) plays in learning is central to the theory of conceptual change and is embedded within a broad scope of constructivism. The underlying theme in conceptual change literature is that it is difficult to assess. As a result, a conceptual change model in which learning is described as a process in which an individual changes his/her conceptions by capturing new conceptions, restructuring existing conceptions or exchanging existing conceptions for new conceptions. This change to the central conception occurs rationally as a result of perceived conflict with existing concepts in reference to perceived intelligibility, plausibility, and fruitfulness of the new concept.

Fostering academic achievement and conceptual change, as the research indicates, requires a good teaching methodology in an optimal learning environment. A review of the literature provided information about associations between students attitudes, achievement (outcomes) and instruction for using cognitive conflict to assess the effectiveness of diagnostic teaching methodology.

CHAPTER THREE

METHODOLOGY

3.0 Overview

This chapter describes the method and procedure used to carry out the study. This includes: research design, the population, the sampling technique and sample, the instrumentation, data collection procedure and data analysis.

3.1 Research Design

Research designs are set of guidelines and instructions that are followed in conducting research. The choice of research designs for a particular study is based on the purpose of the study (Cohen, Manion & Morrison, 2004). For this study, descriptive survey research design was selected for the design because this form of design allows for a large number of respondents (cohen et al, 2004). According to Gay (1992), the descriptive survey design is an attempt to collect data from members of the population in order to determine the current status of that population with respect to one or more variables.

Osuala (2001) indicated that descriptive survey research gives a picture of a situation or a population. It is basic for all types of research in assessing the situation as a pre-requisite for inferences and generalizations. It also helps or enables researcher to collect data on a large number of people. In this context, the study was intended for a very large population of students, hence the selection of the descriptive survey design.

Descriptive survey design is useful because it provides important information regarding the average member of a group. Specifically, by gathering data on a group of people, a researcher can describe the average member, or the average performance of a member, of the particular group being studied. Descriptive research design is highly regarded by policy makers in the social sciences where large populations are dealt with using questionnaires, which are widely used in educational research since data gathered by way of descriptive survey represents field conditions (Osuala, 2001).

3.2 Population of the Study

Cohen et al (2004) explain that a population is a group of elements or cases, whether individuals, objects or events, that conforms to specific criteria which can then be generated to the population. In this study, students from all junior high schools (JHS) in the Afadzato South District of Ghana constituted the target population of the study with a population of one thousand, three hundred and seventy one (1,371) students. These groups of students come from a town where almost all the facilities needed for students to learn can be found there. Most of these pupils are Christian whiles a few belong to other religious denominations.

3.3 Sampling Techniques and Sample

Burns and Grove (2001) described sampling criteria as the characteristics or attributes essential for membership in the target population. The researcher used simple random sampling and stratified sampling techniques and Yamane's formula (1967) for sample size determination in selecting three hundred and ten (310) pupils from all nine (9) junior high schools in the Afadzato Circuit for the study.

3.3.1 Yamane's Formula (1967)

The researcher adopted Yamane's formula (1967) to estimate the sample size from the total population of the nine (9) junior high schools in the study area which was systematically randomly selected for the study. The researcher adopted the Yamane's formula to determine the sample size of the study, for the fact that the formula is relatively easy to use as compared to other statistical approaches of determining sample size, as it makes use of a finite population or a population whose total number is known and the margin of error or significance level which forms the basis of sample size determination.

Moreover, the Yamane's formula is suitable to determine a sample size when a researcher is faced with selecting samples from two or more homogenous groups or clusters (Singh & Masuku, 2014). The formula is given by,

$n = \frac{N}{(1+N * e^2)}$,

where

n = sample size

N = The population size

e = Specified margin of error or significance level

Table 1: Sample Distribution of the Study

Schools	No. of Pupils	Percentage (%)
Have D/A "A" JHS	33	10.6
Have D/A "B" JHS	32	10.3
Agate D/A JHS	34	11.0
Have R/C JHS	32	10.3
Have E.P. JHS	40	12.9
Have Ando No. 1 Presby JHS	35	11.3
Have Alavanyo D/A JHS	34	11.0
Have Ando No. 2 JHS	36	11.6
Hadzidekope D/A JHS	34	11.0
Total	**310**	**100**

3.4 Instruments of the Study

A combination of instruments was used for data collection which includes; questionnaire which assesses students' understanding and achievement; and the Test for Mathematics-Related Attitudes (TOMRA), which measures students' attitudes towards mathematics (see Appendix E). These two survey instruments were administered to students in paper-and-pencil format.

3.5 Interview

The researcher used semi-structured interview (see Appendix D) with which respondents responded to preset open-ended questions derived from a performance test administered to pupils. The questions incorporated pupils' conceptions and challenges on the topic under study and how teachers can help to address them. This instrument provided a face to face interaction between the researcher and the respondents enabling the researcher to clarify conflicting issues. The research implored the help of an assistant who help in taking notes during the interview process.

3.6 Questionnaire

An algebra diagnostic test was considered by the researcher to be the most efficient means of identifying conceptions and measure learning gains in terms of time available. Due consideration was given to ensure that the test would be of valid use by additional researchers and that the algebra diagnostic test would achieve the three fold purpose intended:

This search resulted in the location of two instruments, Blessing's (2004) Algebraic Thinking Content Knowledge Test for Students and Perso's (1991) Diagnostic Test that would adhere to the guidelines previously established by the researcher. As a result, Perso's (1991) Diagnostic Test and Blessing's (2004) Algebraic Thinking Content

Knowledge Test for Students were adapted and given a name change, Algebra Diagnostic Test (see Appendix D).

3.7 Validity of the Study

To ensure validity of the instruments, experts, supervisors, mathematics teachers, and colleagues were consulted for suggestions. They helped to evaluate whether the items were relevant to the research questions and their suggestions helped to establish the items' face and content validity, before they were administered as opined by (Anderson & Morgan, 2008). Face and content validities were established by submitting the instrument to the researcher's supervisors for review. The instruments were given to an expert to ascertain how they meet face and content validity. The suggestions as given by the expert were used to effect the necessary changes to improve upon the instrument. The queries that came out of the item analyses were catered for. All these actions were taken to ensure that the instruments were capable of collecting quality and useful data for the study

3.8 Reliability of the Study

According to Amin (2005), reliability is the extent to which an instrument will produce consistent scores when the same groups of individuals are repeatedly measured under the same conditions.

Reliability, according to Cohen et al. (2003), means that scores obtained from an instrument are stable and consistent. The Cronbach reliability value computed was 0.88.

3.9 Ethical Considerations

Permission was obtained from the school head teacher before administering the questionnaire to the student and teachers and informed verbal consent was also obtained from the teachers. Participants were made aware that, their participation was voluntarily.

Participants were assured of their confidentiality of their responses. They were aware that, the information they provided was not going to be made public, and none of respondents' name, addresses, date of birth and any possible means by which their identity will be made public was requested. All references were duly acknowledged to avoid plagiarism

3.9.1 Permission

The researcher took an introductory letter from the Department of Basic Education, of University of Education, Winneba (see Appendix A) which enabled him to seek permission and approval from the head-teachers and mathematics teachers of the sampled schools for the study. The research on the first established a cordial relationship with the staffs of the selected schools in order to acquire the necessary assistance and support needed for the study.

3.9.2 Confidentiality

Confidentiality is one of the cherished elements in every human relation. The researcher in this regard assured the respondents of high degree of privacy both in written and in verbal form. In addition to the previous statement, names of respondents were not captured in any part of the study unless the consent of the respondent was obtained in a written form.

3.10 Data Collection Procedures

Prior to embarking on the data collection exercise, the researcher made preliminary contacts with the headteachers as well as mathematics teachers in the selected schools. The heads of the various schools organized their mathematics teachers for a meeting where the purpose of the research was explained to the respondents.

One week was used to collect the data for the study. On the day for the data collection, the researcher assigned a unique index numbers to all the pupils sampled for the study. Each index number was made up of letters and numbers. This was done to ensure that accurate data were anonymously gathered from the participants in the study and also to ensure the confidentiality of the pupils. Three days were used to collect qualitative data through a semi-structured interview process and classroom observations whereas two days were used to collect quantitative data through the use of questionnaire prepared by the researcher and validated by the researcher's supervisor (see Appendix B). The scores of pupils obtained from the questionnaire were used in the analysis.

3.11 Data Analysis Procedures

After the data were collected, cleaning of data were done to help the researcher to get rid of errors that may result from coding, recoding, missing information, influential cases or outliers.

Consequently, the data were carefully entered into Statistical Package for Social Sciences (SPSS) version 25, while ensuring that the data points matched with or corresponded to the required individual. In the SPSS, boys were coded or assigned a numerical value of 1 and girls were coded or assigned a numerical value of 2 throughout the analysis of the data collected.

Moreover, the study aims at finding out junior high school students' conceptions of the linear inequalities in one variable in 'Have' Circuit of the Afadzato South District in the Volta Region of Ghana, and in order to address the research questions associated with the study the following statistical procedures or tools were used to analyse the data which answered research questions: descriptive statistics and t-test. The qualitative data or categorical data realised from the interview and questionnaire were analysed descriptively with charts while the quantitative data from the questionnaire was analysed using the t-test. The independent t-test was used to determine whether a difference exists between the boys and the girls after the performance test was administered; this was done to determine whether linear inequalities concepts impact on boys and girls learning outcomes in Have Circuit of the Afadzato South District. The researcher paid attention to the assumptions underpinning the independent t-test analysis.

3.12 Summary

This study involved three research questions. The nature of the research questions mandated that various instruments be used in order to gain answers relevant to the questions being asked. The instrument that aided in data collection for the study was interview and questionnaires. Yamane's formula,$n = \frac{N}{(1+N*e^2)}$, was used to determine the sample size for the study. The researcher used three hundred and ten (310) respondents as the sample size for the study out of a total population of one thousand, three hundred and seventy one pupils (1,371). A detailed description of data collection and analysis procedures including quantitative and qualitative data was also provided.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Overview

This chapter presents the result and discussions of the analysis conducted. The demographic characteristics of participants are presented first, follows by result and discussion of each researched questions.

4.1 General and Demographic Information

The demographic information on the sample and the results are presented in this chapter on three main subheadings that reflected the following research questions.

1. What conceptions of linear inequalities do pupils in Have Circuit of the Afadzato South District have?
2. What benefits do people derive from understanding linear inequalities concepts in mathematics in the Have Circuit of Afadzato South District?
3. What conceptual change in learning outcomes do male and female pupils have in linear inequalities in 'Have' Circuit of the Afadzato South District?

Table 2 indicates the gender distribution of the junior high school pupils used in the study from Have Circuit of the Volta Region of Ghana.

Table 2: Gender Distribution of the Pupils as used in the Study from Have Circuit of the Volta Region of Ghana

Gender	No. of Pupils /Teachers	Percentage (%)
Pupils		
Boys	155	50
Girls	155	50
Total	**310**	**100**
Teachers		
Males	24	80
Females	6	20
Total	**30**	**100**

From Table 2, it could be seen that a total of three hundred and ten (310) participants or sample from the nine (9) junior high schools in the Afadzato Circuit took part in the study. The total number of boys who took part in the study was 155 representing 50% of the sample. That of the girls was also 155 (50%). From the Table 1, it was evident that equal number of boys and girls were sampled for the study, indicating that the researcher was not bias towards a particular group of people as demonstrated in the table. Nonetheless, 24 (80%) male mathematics educators in the Have Circuit participated in the study as against 6 (20%) female mathematics educators in the circuit. This revelation indicated how majority of the teachers perceive the teaching of mathematics has been dominated by male teachers in the Have Circuit.

Table 3 presents the age distribution of the pupils and teachers across the junior high schools in the area. The ages of the pupils have been grouped into three categories; those whose ages fall within 7-10 years, those whose ages are between 11-14 years and those who are above 15 years of age.

Table 3: Age Distribution of Pupils and Teachers in the Study

Age Range	No. of Pupils / Teachers	Percentage (%)
Pupils		
7-10	103	33.2
11-14	156	50.3
Above 15	51	16.5
Total	**310**	**100**
Teachers		
20-29	13	43.3
30-39	12	40.0
40-49	3	10.0
50-59	2	6.7
Total	**30**	**100**

From Table 3, the 11-14 age range represented the modal age group with 156 (50.3%). Closely followed by 11 -14 age group was the 7-10 age range with 103 representing 33.2%. Those who were above 15 years of age represented 51(16.5%). Moreover, 13 teachers representing 43.3% falls with 20-29 age bracket. In the same vein, teachers with 30-39 age range represents 12 (40%). Also, 3 teachers within 40-49 age range represented 10%. The least age range was 50-59 with 2 two teachers representing 6.7%.

1. Research Question 1: What conceptions of linear inequalities do pupils in 'Have Circuit' of the Afadzato South District have?

The first research question of the study sought to examine the students' conception of linear inequalities in the Have Circuit located in the Afadzato South District of the Volta Region of Ghana. The categorical data collected with the questionnaire (see Appendix B) and the interview guide (see Appendix D) designed for the pupils were used to respond to this question.

The response of the pupils from the questionnaire which answers the first research question has been presented in Table 4.

Table 4: Students' Conceptions of linear Inequalities District

Concepts of linear inequalities	**Mode**	**Percentage (%)**
Equality and inequality concepts are the same (125 pupils SD).	125	40.3
I can solve an inequality of $2x > 4$ (250 pupils SA)	250	80.6
I can solve this inequality $1-2x < 5$ (200 pupils SA).	200	64.5
I can solve $1-2x > 2(6-x)$ (100 pupils SA)	100	32.3
I can work out for the solution of this, $2x + 4 \geq 24$ (140 pupils SA).	140	45.2
I can work out for the solution of $m/3 – 3 \leq - 6$ (86 pupils A).	86	27.7

From Table 4, 125 pupils representing 40.3% strongly disagreed with the assertion that *equality and inequality concepts are the same*. In addition, 250 (80.6%) strongly agreed that they could apply the concepts of inequality to solve the inequality $2x > 4$. It could be seen that this percentage was high indicating that about two-thirds of the sample have developed the require concepts to be able to solve $2x > 4$. Meanwhile, 200 (64.5%), 100 (32.3%), 140 (45.2%) and 86 (27.7%) acknowledged that they could apply the concepts of inequality to solve $1-2x < 5$, $1-2x > 2(6-x)$, $2x + 4 \geq 24$ and $m/3 – 3 \leq - 6$ respectively.

The Table 5, below presents the responses of the mathematics teachers in the various schools sampled for the study in Have Circuit of Afadzato South District on the concepts of linear inequalities.

Table 5: Mathematics Teachers' Conception of linear Inequalities

Concepts of linear inequalities	Mode	Percentage (%)
Examining the students at the end of each topic and making necessary review to correct their mistake of students' (23 teachers SA).	23	76.7
Motivate students to learn mathematics on their own (19 teachers SA).	19	63.3
Encourage students to form small group to practice (23 teachers SA).	23	76.7
Involving students' in the teaching and learning process (18 teachers SA).	18	60
My students can solve m /3-3≤ -6 (23 teachers SA)	23	76.7

From the table, the views of the mathematics teachers on inequality concepts from the nine schools sampled for the study have been presented. It is refreshing to note that 23 (76.7%) teachers out of 30 acknowledged that their students could solve *m /3-3≤ -6* as presented in teachers' questionnaire. Indicating that the teachers had adequately taught their students inequality as a topic in the mathematics syllabus for the junior high school in the country. More so, 23 teachers representing 76.7% indicated that they do examine their pupils at the end of each topic and making necessary review and corrections to reinforce the understanding of the pupils in the inequality concept. Furthermore, 63.3% of the teachers corresponding to 19 teachers indicated that they did motivate their pupils to learn in order to grasp the inequality concept. Meanwhile, 23 (76.7%) of the teachers revealed that during the teaching of inequality concept, they encourage pupils to form small groups to practice the solving of questions related to inequality, while 18 (60%) teachers responded that they actually involved pupils in the teaching and learning process.

The above revelations indicated that majority of the teachers sampled actually tried to assist their pupils to grasp the concepts underpinning the learning of linear inequalities.

4.2 Pupils' Responses from the Interview Session

For the first question on the interview guide: *What sorts of images or examples come to mind when you consider the concept of inequality?*

In response to the above question on the interview guide (see Appendix C), 300 (96.8%) pupils indicated that the *"symbols of inequality such as* $\leq$, $\geq$, $<$ *and* $>$*"* come into mind when they consider the concept of inequality. Also, 150 pupils representing 48.4% indicated the fact that "*the symbol (s) of the inequality is changed or reversed when the inequality is multiplied or divided by a negative value*". The above responses of the pupils indicated that they have a deeper understanding of the integral concepts associated with inequality.

What are the ways in which inequalities and equations are the same and/or different?

Furthermore, with regard to second question on the interview guide shown in the italics above, 160 (51.6%) indicated that "*the procedure for solving an inequality problem is similar to the procedure for solving a linear equation of one variable*". Meanwhile, 306 (98.7%) pupils demonstrated that "linear *equations have the equal to sign (=) while inequalities may use any of the symbols, i.e.* $\leq$, $\geq$, $<$ *and* $>$*"*. More so, 200 (64.5%) pupils indicated that "*inequality sign is changed when it is multiplied with a negative value whereas in the case of linear equation, the sign (equal to, =) does not change when it is multiplied with a negative value*".

For the third interview guide question: *What does a solution of an inequality mean?*

In response to that question, 120 (38.7%) of pupils indicated that the solution of an inequality means "*a number which when substituted for the variable makes the inequality a true statement*".

In addition, for the fourth interview guide question: *Can you tell me an interesting fact you have learned/discovered lately about inequalities?* 250 (80.6%) indicated that "*adding or subtracting the same number on both sides of the inequality expression does not change the inequality*". Again, 303 (97.7%) demonstrated that the "multiplying an inequality with a negative value changes the inequality". For the interview guide question 5 and 6, majority of pupils demonstrated mastery of the inequality concept and provided the desired solution to the problem posted.

Research Question 2: What benefits do people derive from understanding linear inequalities concepts in mathematics in the 'Have Circuit' of Afadzato South District?

This second research question of the study sought to examine the benefits of the conception of linear inequalities in the Have Circuit located in the Afadzato South District of the Volta Region of Ghana. The categorical data collected with the questionnaire (see Appendix B) and the interview guide (see Appendix C) designed for the pupils were used to answer this question.

The Table 6 presents descriptively, the responses of pupils on the benefits of the learning of linear inequalities. The modal responses together with their corresponding percentages have been presented in Table 4.5 below.

Table 6: Benefits of Understanding Linear Inequality Concepts

Benefits of understanding linear inequality concepts	**Mode**	**Percentage (%)**
Learning of linear inequalities would form basis for the learning of other lessons (140 pupils A).	140	45.1
Learning of linear inequalities will make word problems associated it easier (146 pupils SA).	146	47.1
Learning of linear inequalities will arouse pupils' interest (150 pupils SA).	150	48.1
Learning of linear inequalities will enable pupils interpret mathematical statements with confidence (170 pupils SA).	170	54.8
Learning of linear inequalities will enable pupils to logically present solutions relating to it (160 pupils SA).	160	51.6
The learning of the linear inequalities will allay pupils' fears (127 pupils SA).	127	41.0
The learning of linear inequalities will address the misconceptions of it (250 pupils SA)	250	80.6

From Table 6, 140 pupils corresponding to 45.1% agreed that the learning of linear inequalities would form basis for the learning of other concepts in the mathematical syllabus as well as other subjects like integrated science. More so, 146 (47.1%) strongly agreed with the fact that the learning of inequalities will make or enhance their understanding of word problems that are associated with the inequality concept. Meanwhile, 150 pupils representing 48.1 strongly agreed with the notion that the learning of inequalities will arouse pupils' interest.

Moreover, 170 (54.8%) pupils strongly agreed that as a benefit to the studying of inequality, the concept will help them to interpret mathematical statements with confidence. In addition, 160 pupils representing 51.6% indicated that the learning of the inequality will enable them to logically present solutions relating to the concept. Again, 127 (41%) accepted that the learning of linear inequalities will allay their fears.

Nonetheless, 250 (80.6%) strongly agreed that will help address their misconception that they have always associated with the learning of the concept.

Research Question 3: What conceptual change in learning outcomes do male and female pupils have in linear inequalities in 'Have' Circuit of the Afadzato South District?

This third research question of the study sought to examine the impact of inequalities concepts on boys and girls in the Have Circuit located in the Afadzato South District of the Volta Region of Ghana. The quantitative data collected through pupils' scores after the administration of achievement test on linear inequalities was used to answer the question.

4.3 Descriptive Statistics of the Study

The Table 7 displays the descriptive statistics of the scores of the sample (pupils) after taking part in the achievement test of the study.

The descriptive statistics of the pupils' achievement test scores has been presented in

Table 7: Descriptive Statistics of Pupils' Achievement Test Scores

	Gender	N	Mean	Std. Deviation	Std. Error Mean
Score	Boys	155	51.06	14.225	1.143
s1	Girls	155	49.54	14.043	1.128

From the table, the boys recorded a mean and standard deviation of (M = 51.06, SD = 14.23) respectively with a standard error mean of (Std. Error Mean = 1.143) while the girls recorded a mean and standard deviation of (M = 49.54, SD = 14.04) with a standard error mean of (Std. Error Mean = 1.128). Moreover, comparing the two performances indicated that the boys performed slightly higher than the girls, even though their standard deviation and standard error mean were almost the same. The boys slightly outperformed the girls with a mean difference of 1.52.

Meanwhile, a t-test was conducted to investigate whether a significant change (impact) exists between the performance of boys and the girls after taking part in the achievement test on linear inequality.

Table 7, which has been split into Table 7a and Table 7b (due to lack of space) presents the analysis of the independent samples t-test on the pupils' achievement test scores.

Table 8a: Independent Samples T-Test of Pupils' Achievement Test Scores

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	**Sig.**	**t**	**Df**
Score s1	Equal variances assumed	.179	.672	.944	308
	Equal variances not assumed			.944	307.949

Table 8b: Independent Samples T-Test of Pupils' Achievement Test Scores

		t-test for Equality of Means		
		Sig. (2-tailed)	**Mean Difference**	**Std. Error Difference**
Scores 1	Equal variances assumed	.000	1.516	1.606
	Equal variances not assumed	.000	1.516	1.606

From Table 7, the Levene's Test for Equality of Variance was computed (F = .179, p > 0.05). This statistical test indicated that equal variances were assumed between boys and girls as far as the achievement test scores were concerned this was because the calculated significance level of .672 was greater than 0.05 (p = .672 > 0.05). However, had the significance level been lesser than 0.05, we would have accepted Equal Variances not Assumed and used calculated statistics along that row for the discussion.

More so, from Table 7b, t-test for Equality of Means was carried out, and it was revealed that there was a mean difference of 1.516 (Mean Deviation = 1.516) which was statistically significant with Standard Error Difference of 1.606 (Mean Deviation = 1.516, $p < 0.05$; Standard Error Difference = 1.606); indicating that the achievement test actually had positive impact on the boys' and the girls' performance.

4.4 Discussion of Results

The findings of the study were discussed with regard to the following research objectives that derived the research question:

1. What are the students' conceptions of linear inequalities in 'Have Circuit' of the Afadzato South District?
2. What are the benefits in understanding linear inequalities concepts in mathematics in the 'Have Circuit' in Afadzato South District?
3. What conceptual change learning outcomes do male and female pupils have in linear inequalities in Have Circuit of the Afadzato South District?

Findings and discussion

Research Question 1: What are the students' conceptions of linear inequalities in Have Circuit of the Afadzato South District?

The findings of the study revealed that with regard to the concept of linear inequalities, 125 pupils representing 40.3% strongly disagreed with the assertion that *equality and inequality concepts are the same*. In addition, 250 (80.6%) strongly agreed that they could apply the concepts of inequality to solve the inequality $2x > 4$. It could be seen that this percentage was high indicating that about two-thirds of the sample have developed the requite concepts to be able to solve $2x > 4$. Meanwhile, 200 (64.5%), 100 (32.3%), 140 (45.2%) and 86 (27.7%) acknowledged that they could apply the concepts of inequality to solve $1-2x < 5$, $1-2x > 2(6-x)$, $2x + 4 \geq 24$ and $m/3 - 3 \leq -6$ respectively. These revelations have shown that certain salient concepts of linear inequalities that are crucial to the understanding of the topic have been grasp by the pupils. These findings agree with Greer (2004) when he stated that the most general source of conceptions is limited experience or exposure limited examples - both in learning outside of school and during formal instruction (Greer, 2004). Alternative conceptions may arise when students are presented with concepts in too few contexts or when concepts presented are beyond their developmental level (Gabel, 1989).

Moreover, the findings of the study are in line with Liljedahl (2005) that when students are first introduced to a new exposition of a scientific idea, students have not yet attained as high a level of abstract thinking as the instructors assume. Perhaps instructors provide part of the concept or a more simplified concept in the belief this will lead students to better understanding, as sometimes happens when the concept is difficult or known to be troublesome. Such limited explanation, however, may prevent some students from crossing a cognitive threshold and entering through a door to a higher level of understanding.

Furthermore, it was seen that linear inequality uses its own standardized set of signs, symbols and rules about how something can be written (Drijvers et al., 2011). Linear inequalities seem to have its own grammar and syntax and this makes it possible to formulate algebraic ideas unequivocally and compactly. In this symbolic language, "variables are simply signs or symbols that can be manipulated with well-established rules, and that do not refer to a specific, context-bound meaning" (Drijvers et al., 2011, p. 17).

However, for meaningful learning to occur, the teacher (mathematics educator) has to guide the students to discover that new knowledge must be related to the relevant existing concepts in the students' cognitive structure (Ausubel, 2000). This prompted the researcher to seek the views of the mathematics teachers in the study area. It was revealed that 23 (76.7%) teachers out of 30 acknowledged that their students could solve $m/3-3 \leq -6$ as presented in teachers' questionnaire. Indicating that the teachers had adequately taught their students inequality as a topic in the mathematics syllabus for the junior high school in the country. More so, 23 teachers representing 76.7% indicated that they do examine their pupils at the end of each topic and making necessary review and corrections to reinforce the understanding of the pupils in the inequality concept. Furthermore, 63.3% of the teacher corresponding to 19 teachers indicated that they did motivate their pupils to learn in order to grasp the inequality concept.

Meanwhile, 23 (76.7%) of the teachers revealed that during the teaching of inequality concept, they encourage pupils to form small groups to practice the solving of questions related to inequality, while 18 (60%) teachers responded that they actually involved pupils in the teaching and learning process. The above revelations indicated that majority of the teachers sampled actually tried to assist their pupils to grasp the concepts underpinning the learning of linear inequalities. These findings agree with the fact that when teachers fail to accurately present the initial interpretation of a concept to students, it may become an obstacle which can hinder the passage to a higher-order interpretation - the more general and more abstract – of the same concept. For example, an early understanding of natural number and its properties supports children's understanding of notions such as potential infinity (Hartnett & Gelman, 1998), while at the same time it stands in the way of students' understanding of the properties and operations of rational numbers (Moskal & Magone, 2000; Yujing & Yong-Di, 2005; Johnstone, 2000; Bills, Dreyfus, Tsamir, Watson & Zaslavsky, 2006).

Research Question 2: What are the benefits in understanding linear inequalities concepts in mathematics in the Have Circuit in Afadzato South District?

The findings of the study indicated that 140 pupils corresponding to 45.1% agreed that the learning of linear inequalities would form basis for the learning of other concepts in the mathematical syllabus as well as other subjects like integrated science. More so, 146 (47.1%) strongly agreed with the fact that the learning of inequalities will make or enhance their understanding of word problems that are associated with the inequality concept. Meanwhile, 150 pupils representing 48.1 strongly agreed with the notion that the learning of inequalities will arouse pupils' interest.

Moreover, 170 (54.8%) pupils strongly agreed that as a benefit to the studying of inequality, the concept will help them to interpret mathematical statements with confidence. In addition, 160 pupils representing 51.6% indicated that the learning of the inequality will enable them to logically present solutions relating to the concept. Again, 127 (41%) accepted that the learning of linear inequalities will allay their fears. Nonetheless, 250 (80.6%) strongly agreed that will help address their misconception that they have always associated with the learning of the concept.

This revelation is in line with fact that the understanding of linear inequality on the part of students play an important role in their on-going understanding of higher order concepts in mathematics as well as in formal lessons. That some of these conceptions can be stable in their mental structures, widespread among students, can be strongly held, resistant to change, and cannot interfere with the learning of certain algebra concepts (Vosniadou, Vamvakoussi, & Skopeliti, 2008; Richardson, 2003; Anderson, 2002; Haverty, 1999; Booth, 1984; Collis, 1975; Kuchemann, 1978, 1981; Knuth et al., 2008 Stacey & MacGregor, 1997).

Research Question 3: How did the evidence of students' conceptual change in linear inequalities concepts impact on boys and girls learning outcomes in Have Circuit of the Afadzato South District?

The findings revealed that the boys marginally performed better than the girls in the administration of the achievement test as demonstrated in the analysis of the t-test. From table 7, the Levene's Test for Equality of Variance was computed ($F = .179$, $p > 0.05$). This statistical test indicated that equal variances were assumed between boys and girls as far as the achievement test scores were concerned this was because the calculated significance level of 0.672 was greater than 0.05 ($p = 0.672 > 0.05$). However, the significance level been lesser than 0.05, we would have accepted Equal Variances not Assumed and used calculated statistics along that row for the discussion.

More so, from Table 7, t-test for Equality of Means was carried out, and it was revealed that there was a mean difference of 1.516 (Mean Deviation = 1.516) which was statistically significant with Standard Error Difference of 1.606 (Mean Deviation = 1.516, $p < 0.05$; Standard Error Difference = 1.606); indicating that the achievement test actually had positive impact on the boys' and the girls' performance. This finding agrees with McNeil and Alibali (2005) who stated that earlier learning constrains later learning that is, students' misconceptions and conceptions may be caused by their previous learning experiences. If a mathematical concepts art explained to students devoid of ambiguities, when the students' achievement in such concepts are measured through achievement tests and examination, the students will always perform well.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.0 Overview

This chapter gives the summary of the study findings based on the study results. It also illustrates the conclusion of the study. It finally, makes recommendations when adopted by the relevant educational authorities, impact on the teaching and learning of linear inequalities.

5.1 Summary of the Findings

The main purpose of the study was to examine the conceptions of linear inequalities among the pupils of Have Circuit in the Afadzato South District, to identify the benefits in understanding linear inequalities concepts in mathematics in the Have Circuit in Afadzato South District and to investigate whether there is a significant difference in the performance of low achievers and to identify the evidence of students' conceptual change in linear inequalities concepts impact on boys and girls learning outcomes of students in Have Circuit of the Afadzato South District. The following research hypotheses were raised and answered in the study:

1. What are the students' conceptions of linear inequalities in Have Circuit of the Afadzato South District?
2. What are the benefits in understanding linear inequalities concepts in mathematics in the Have Circuit in Afadzato South District?
3. How did the evidence of students' conceptual change in linear inequalities concepts impact on boys and girls learning outcomes in Have Circuit of the Afadzato South District?

The findings of the study revealed that with regard to the concept of linear inequalities, 125 pupils representing 40.3% strongly disagreed with the assertion that *equality and inequality concepts are the same*. In addition, 250 (80.6%) strongly agreed that they could apply the concepts of inequality to solve the inequality $2x > 4$. It could be seen that this percentage was high indicating that about two-thirds of the sample have developed the require concepts to be able to solve $2x > 4$. Meanwhile, 200 (64.5%), 100 (32.3%), 140 (45.2%) and 86 (27.7%) acknowledged that they could apply the concepts of inequality to solve $1\text{-}2x < 5$, $1\text{-}2x > 2(6\text{-}x)$, $2x + 4 \geq 24$ and $m/3 - 3 \leq -6$ respectively.

More so, the findings of the study revealed that 23 (76.7%) teachers out of 30 acknowledged that their students could solve $m/3\text{-}3 \leq -6$ as presented in teachers' questionnaire. Indicating that the teachers had adequately taught their students inequality as a topic in the mathematics syllabus for the junior high school in the country. More so, 23 teachers representing 76.7% indicated that they do examine their pupils at the end of each topic and making necessary review and corrections to reinforce the understanding of the pupils in the inequality concept. Furthermore, 63.3% of the teacher corresponding to 19 teachers indicated that they did motivate their pupils to learn in order to grasp the inequality concept. Meanwhile, 23 (76.7%) of the teachers revealed that during the teaching of inequality concept, they encourage pupils to form small groups to practice the solving of questions related to inequality, while 18 (60%) teachers responded that they actually involved pupils in the teaching and learning process.

Meanwhile, 140 pupils corresponding to 45.1% agreed that the learning of linear inequalities would form basis for the learning of other concepts in the mathematical syllabus as well as other subjects like integrated science. More so, 146 (47.1%) strongly agreed with the fact that the learning of inequalities will make or enhance their understanding of word problems that are associated with the inequality concept. Meanwhile, 150 pupils representing 48.1 strongly agreed with the notion that the learning of inequalities will arouse pupils' interest.

Moreover, 170 (54.8%) pupils strongly agreed that as a benefit to the studying of inequality, the concept will help them to interpret mathematical statements with confidence. In addition, 160 pupils representing 51.6% indicated that the learning of the inequality will enable them to logically present solutions relating to the concept. Again, 127 (41%) accepted that the learning of linear inequalities will allay their fears. Nonetheless, 250 (80.6%) strongly agreed that will help address their misconception that they have always associated with the learning of the concept.

In addition, it was revealed that there was a mean difference of 1.516 (Mean Deviation = 1.516) which was statistically significant with Standard Error Difference of 1.606 (Mean Deviation = 1.516, $p < 0.05$; Standard Error Difference = 1.606); indicating that the achievement test actually had positive impact on the boys' and the girls' performance.

5.2 Conclusion of the Study

The study agrees based on all the findings in the study that irrespective of the school category or the pupils category, when linear inequalities concept (instruction) is properly planned focusing on all the theories underlying the teaching of a mathematics lessons, pupils would be able to excel in the mathematics subject in the external examination, thereby invoking the interest and the study of mathematics at high levels of their education.

5.3 Recommendations of the Study

It is mathematics clinics, workshops and in-service training should be organized regularly for mathematics teachers in the basic level to update or upgrade their knowledge on efficient contemporary strategies and methodologies that should be used to teach mathematics at the basic level. In addition, effective supervision and encouragement of mathematics teachers at the basic level on how to teach effectively linear inequalities will help pupils at the basic level develop interest and appreciation for the mathematics subject so that they can pursue it at a high level.

It is recommended that parents and mathematics teachers (educators) should encourage and motivate pupils to improve upon their study habits for better performance in mathematics at the basic level as this has the tendency to stimulate and motivate students to see mathematics as an easy subject since every concept in mathematics is related to another higher concept.

Moreover, it is recommended to the authorities of basic education (headmasters and headmistresses, District and Municipal Chief Executives of Education and Ministry of Education) to provide the necessary assistance at the basic education level as far as the learning of mathematics is concerned to arouse the interest of the students in the learning of mathematics at that level.

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APPENDICES

APPENDIX A

Letter of Introduction

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF EDUCATIONAL STUDIES
DEPARTMENT OF BASIC EDUCATION

Date: May [illegible]

The District Director
Afadzato South District Educ. Directorate
P. O. Box PMG
Ve-Golokuati
Afadzato South District, V/R

Dear Sir/ Madam,

LETTER OF INTRODUCTION

We forward to you, a letter from Mr. Kenneth Kofikuma Puem, a second year M.Phil student of the Department of Basic Education, University of Education, Winneba, with registration number 8170030012.

Mr. Kenneth Kofikuma Puem is to carry out a research on the Topic "*Junior High School Students' Conception of Linear Inequalities in One Variable in 'Have Circuit' of the Afadzato South District*"

We would be grateful if permission is granted him to carry out his studies in the District.

Thank you

Yours faithfully,

DEPT. OF BASIC EDUCATION
UNIVERSITY OF EDUCATION
WINNEBA, GHANA

MRS. SAKINA ACQUAH (PHD)
(Ag. Head of Department)

www.uew.edu

APPENDIX B

Questionnaire for Students

UNIVERSITY OF EDUCATION WINNEBA
FACULTY OF EDUCATIONAL STUDIES
DEPARTMENT OF BASIC EDUCATION

Thank you for taking time to complete this questionnaire. Please answer the question to the best of your knowledge. Your thoughtful and truthful responses will be greatly appreciated. Your individual name or identification number is not required and will not at any time be associated with your responses. Your responses will be kept completely confidential.

SECTION A

Please mark with (√)

1. Gender: Male [ ] Female [ ]

2. J.H.S form...

3. (a) Have you participated in any mathematics workshops, debate or quiz lately?
....

(b) If yes, what experience have you gained.....................................

Items	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
1.Equality and inequality concepts are the same					
2. I would prefer to do a mathematics problem on a topic than to read about it in a mathematics magazine.					
3. It is better to be told mathematics facts than to find them out from doing a mathematics problem.					

4. It is better to ask the teacher the answer than to find it out by doing a mathematics problem.					
5. I really enjoy going to mathematics lessons.					
6. Mathematics lessons are a waste of time.					
7. I would prefer to find out why something happens by solving a mathematics problem than by being told.					
8. The different ways of seeing inequalities refers to the **'conception of inequalities'**					
9. Doing mathematics problems is not as good as finding out from teachers.					
10. I would rather agree with other people than to do a mathematics problem to find out for myself.					
11. I would prefer to do my own mathematics problems than to find out information from a teacher.					
12. I would rather find out about things by asking an expert than by doing a mathematics problem.					
13. I would rather solve a problem by doing mathematics than be told the answer.					

14. I would prefer to do mathematics problems than read about them.					
15. Mathematics one of the most interesting school subjects.					
16. I can solve an inequality of $2x > 4$					
17. I can solve this inequality $1-2x < 5$					
18. I can solve $1-2x > 2(6-x)$					
19. I can work out for the solution of this, $2x + 4 \geq 24$					
20. I can work out for the solution of, $m/3 - 3 \leq -6$					

APPENDIX C

Questionnaire for Teacher Instructions

UNIVERSITY OF EDUCATION, WINNEBA

FACULTY OF EDUCATIONAL STUDIES

DEPARTMENT OF BASIC EDUCATION

Thank you for taking time to complete this questionnaire. Please answer the question to the best of your knowledge. Your thoughtful and truthful responses will be greatly appreciated. Your individual name or identification number is not required and will not at any time be associated with your responses. Your responses will be kept completely confidential.

SECTION A

Please mark with tick (√)

1. Gender [ ] male [ ] female [ ]

2. Years of teaching experience ……………

3. Years of teaching J.H.S. Mathematics. …………..

4. Are you professional teacher? Yes [ ] No [ ]

5. What is your highest academic qualification?

Certificate A [ ] Diploma [ ] Bachelor's Degree [ ] SSCE/SEC

Other, (please specify) ……………………………………………………

6a. Have you participated in any in-service or workshop on mathematics teaching?
Yes [ ] No[ ]

6b. If yes, state the number of times you have participated in such programme.

……………………………………………………………………………………
……………………………………………………………………………………
……………………………………………………………………………………
……………………………………………………………………………………

SECTION B

Mark with (√) to indicate your actual classroom practice.

Strongly Agree Agree Neutral Disagree Strongly Disagree

Items	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
7. Make connections between mathematics and other discipline					
8. Use additional mathematics textbook(s) as instructional tools					
9. Use marking scheme and other electronics devices to supplement your teaching					
10. Enjoy teaching JHS mathematics					
11. Take students' prior understanding into consideration when planning a lesson					
12. Cover all mathematics concepts in the syllabus					
13. Involve parents in mathematics education of their children					
14. Advice students' about job opportunities or prospect in mathematics					
15. Examining the students' at the end of each topic and making necessary review to correct their mistake of students'					
16. Conducting frequent test					
17. Motivate students' to learn mathematics on their own					
18. Encourage students' to form small group to practice					
19. Involving students' in the teaching and learning process.					
20. My students can solve m/3-3_<-6					

APPENDIX D

Interview Guide

UNIVERSITY OF EDUCATION WINNEBA

FACULTY OF EDUCATIONAL STUDIES

DEPARTMENT OF BASIC EDUCATION

Thank you for taking time to complete this questionnaire. Please answer the question to the best of your knowledge. Your thoughtful and truthful responses will be greatly appreciated. Your individual name or identification number is not required and will not at any time be associated with your responses. Your responses will be kept completely confidential.

1. What sorts of images or examples come to mind when you consider the concept of inequality?
2. What are the ways in which inequalities and equations are the same and/or different?
3. What does a solution of an inequality mean? Exemplify.
4. Can you tell me an interesting fact you have learned/discovered lately about inequalities?
5. You've got a text message from Ernest- your homework partner - that reads: "Missed the class on inequalities. I have to prepare for the quiz. Don't know how to start. Help me with the steps of solving a linear inequality." E-mail him back the steps for solving linear inequalities.
6. An hour later an e-mail from Ernest arrives: "I followed your steps and solved a whole bunch of inequalities. Thanks. Then I attempted this one: $1- 2x > 2(6 - x)$. I worked out the algebra and got this $1 - 2x >12 - 2x$ and then ended up with: $0 >11$. Here I got stuck. Please help." E-mail him back. The message should contain your feedback on Ernest's work as well as your input to Jamie's further understanding of inequalities.

APPENDIX E

Algebra Diagnostic Test

In each of the following linear equations, find the truth set and represent your results on a number line. All questions carry equal marks.

1. $\mathbf{2y + 3 < 4y - 3}$
2. $\mathbf{10 - 5x < 3 + 2x}$
3. $3\,(2x - 4) \leq 6 + 2(x + 5)$
4. $\frac{1}{3}(5x - 4) > x + \frac{11}{12}$
5. $1 + \frac{3}{8}(x - 2) \geq \frac{3}{4}(x - 2) + 4$
6. $\frac{2x-1}{4} - \frac{x-2}{3} > 1$
7. $\frac{5x-3}{6} + \frac{2x-4}{4} < 1$
8. $3\,(3x - 5) > 6$
9. $x < 4x + 9$
10. $\frac{2x-2}{4} - \frac{2x-1}{3} \leq 1$

APPENDIX F-1

Analysis of the Various Themes of the Questionnaire

EQUALITY AND INEQUALITY CONCEPTS ARE THE SAME

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	51	16.5
	Agree (A)	46	14.8
	Neutral (N)	19	6.1
	Disagree (D)	69	22.3
	Strongly Disagree (SD)	125	40.3
Total		310	100%

- Mode = 125

APPENDIX F-2

I WOULD PREFER TO DO A MATHEMATICS PROBLEM ON A TOPIC THAN TO READ ABOUT IT IN A MATHEMATICS MAGAZINE

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	134	43.2
	Agree (A)	66	21.3
	Neutral (N)	20	6.5
	Disagree (D)	60	19.4
	Strongly Disagree (SD)	30	9.7
Total		310	100%

- Mode = 134

APPENDIX F-3

IT IS BETTER TO BE TOLD MATHEMATICS FACTS THAN TO FIND THEM OUT FROM DOING A MATHEMATICS PROBLEM

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	73	23.5
	Agree (A)	66	21.3
	Neutral (N)	16	5.2
	Disagree (D)	100	32.3
	Strongly Disagree (SD)	55	17.7

Total		310	100%

- Mode = 100

APPENDIX F-4

IT IS BETTER TO ASK THE TEACHER THE ANSWER THAN TO FIND IT OUT BY DOING A MATHEMATICS PROBLEM

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	85	27.4
	Agree (A)	41	13.2
	Neutral (N)	25	8.1
	Disagree (D)	109	35.2
	Strongly Disagree (SD)	50	16.1
Total		310	100%

- Mode = 109

APPENDIX F-5

I REALLY ENJOY GOING TO MATHEMATICS LESSONS

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	95	30.6
	Agree (A)	140	45.1
	Neutral (N)	0	0.0
	Disagree (D)	40	12.9
	Strongly Disagree (SD)	35	11.4
Total		310	100%

- Mode = 140

APPENDIX F-6

MATHEMATICS LESSONS ARE A WASTE OF TIME

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	93	30.0
	Agree (A)	40	12.9
	Neutral (N)	0	0.0
	Disagree (D)	133	42.9
	Strongly Disagree (SD)	44	14.2

Total		310	100%

- Mode = 133

APPENDIX F-7

I WOULD PREFER TO FIND OUT WHY SOMETHING HAPPENS BY SOLVING A MATHEMATICS PROBLEM THAN BY BEING TOLD

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	146	47.1
	Agree (A)	45	14.5
	Neutral (N)	0	0.0
	Disagree (D)	65	30.0
	Strongly Disagree (SD)	54	17.4
Total		310	100%

- Mode = 146

APPENDIX F-8

THE DIFFERENT WAYS OF SEEING INEQUALITIES REFER TO THE 'CONCEPTION OF INEQUALITIES'

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	150	48.4
	Agree (A)	41	13.2
	Neutral (N)	0	0.0
	Disagree (D)	62	20.0

	Strongly Disagree (SD)	57	18.4
Total		310	100%

- Mode = 150

APPENDIX F-9

DOING MATHEMATICS PROBLEMS IS NOT AS GOOD AS FINDING OUT FROM TEACHERS

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	80	25.8
	Agree (A)	41	13.2
	Neutral (N)	50	16.1
	Disagree (D)	92	29.8
	Strongly Disagree (SD)	47	15.2
Total		310	100%

- Mode = 92

APPENDIX F-10

I WOULD RATHER AGREE WITH OTHER PEOPLE THAN TO DO A MATHEMATICS PROBLEM TO FIND OUT FOR MYSELF

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	75	24.2
	Agree (A)	46	14.8
	Neutral (N)	0	0.0
	Disagree (D)	132	42.6

	Strongly Disagree (SD)	57	18.4
Total		310	100%

- Mode = 132

APPENDIX F-11

I WOULD PREFER TO DO MY OWN MATHEMATICS PROBLEMS THAN TO FIND OUT INFORMATION FROM A TEACHER

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	102	32.9
	Agree (A)	88	28.4
	Neutral (N)	0	0.0
	Disagree (D)	63	20.3
	Strongly Disagree (SD)	57	18.4
Total		310	100%

- Mode = 102

APPENDIX F-12

I WOULD RATHER FIND OUT ABOUT THINGS BY ASKING AN EXPERT THAN BY DOING A MATHEMATICS PROBLEM

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	97	31.3

	Agree (A)	93	30.0
	Neutral (N)	10	3.2
	Disagree (D)	63	20.3
	Strongly Disagree (SD)	47	15.2
Total		310	100%

- Mode = 97

APPENDIX F-13

I WOULD RATHER SOLVE A PROBLEM BY DOING MATHEMATICS THAN BE TOLD THE ANSWER

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	170	54.8
	Agree (A)	42	13.5
	Neutral (N)	0	0.0
	Disagree (D)	43	13.9
	Strongly Disagree (SD)	55	17.8
Total		310	100%

- Mode = 170

APPENDIX F-14

I WOULD PREFER TO DO MATHEMATICS PROBLEMS THAN READ ABOUT THEM

		Frequency	Percentage (%)

Valid	Strongly Agree (SA)	160	51.6
	Agree (A)	42	13.5
	Neutral (N)	0	0.0
	Disagree (D)	53	17.1
	Strongly Disagree (SD)	55	17.7
Total		310	100%

- Mode = 160

APPENDIX F-15

MATHEMATICS IS ONE OF THE MOST INTERESTING SCHOOL SUBJECTS

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	127	41.0
	Agree (A)	61	19.7
	Neutral (N)	0	0.0
	Disagree (D)	59	19.0
	Strongly Disagree (SD)	63	20.3
Total		310	100%

- Mode = 127

APPENDIX F-16

I CAN SOLVE AN INEQUALITY OF 2X > 4

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	250	80.6
	Agree (A)	50	16.1
	Neutral (N)	0	0.0
	Disagree (D)	3	1.0
	Strongly Disagree (SD)	7	2.3
Total		310	100%

- Mode = 250

APPENDIX F-17

I CAN SOLVE THIS INEQUALITY 1-2X < 5

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	200	64.5
	Agree (A)	100	32.3
	Neutral (N)	3	1.0
	Disagree (D)	3	1.0
	Strongly Disagree (SD)	4	1.3
Total		310	100%

- Mode = 200

APPENDIX F-18

I CAN SOLVE 1-2X > 2(6-X)

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	100	32.3
	Agree (A)	91	29.4
	Neutral (N)	0	0.0
	Disagree (D)	92	29.7
	Strongly Disagree (SD)	27	8.7
Total		310	100%

- Mode = 100

APPENDIX F-19

I CAN WORK OUT FOR THE SOLUTION OF THIS, $2X + 4 \geq 24$

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	120	38.7
	Agree (A)	140	45.2
	Neutral (N)	0	0.0
	Disagree (D)	30	9.7
	Strongly Disagree (SD)	20	6.5
Total		310	100%

- Mode = 140

APPENDIX F-20

I CAN WORK OUT FOR THE SOLUTION OF $M/3 – 3 \leq - 6$

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	83	26.8
	Agree (A)	86	27.7
	Neutral (N)	0	0.0
	Disagree (D)	78	25.1
	Strongly Disagree (SD)	63	20.3
Total		310	100%

- Mode = 86

APPENDIX F- 21

MAKE CONNECTIONS BETWEEN MATHEMATICS AND OTHER DISCIPLINE

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	8	26.7
	Agree (A)	6	20.0
	Neutral (N)	3	10.0
	Disagree (D)	6	20.0
	Strongly Disagree (SD)	7	23.3
Total		30	100%

- Mode = 8

APPENDIX F-22

USE ADDITIONAL MATHEMATICS TEXTBOOK(S) AS INSTRUCTIONAL TOOLS

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	11	36.7
	Agree (A)	5	16.7
	Neutral (N)	0	0.0

	Disagree (D)	9	30.0
	Strongly Disagree (SD)	5	16.7
Total		30	100%

APPENDIX F-23

USE MARKING SCHEME AND OTHER ELECTRONICS DEVICES TO SUPPLEMENT YOUR TEACHING

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	14	46.7
	Agree (A)	6	20.0
	Neutral (N)	0	0.0
	Disagree (D)	9	30.0
	Strongly Disagree (SD)	1	3.3
Total		30	100%

- Mode = 14

APPENDIX F-24

ENJOY TEACHING JHS MATHEMATICS

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	21	70.0
	Agree (A)	5	16.7

	Neutral (N)	0	0.0
	Disagree (D)	3	10.0
	Strongly Disagree (SD)	1	3.3
Total		30	100%

- Mode = 21

APPENDIX F-25

TAKE STUDENTS' PRIOR UNDERSTANDING INTO CONSIDERATION WHEN PLANNING A LESSON

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	17	56.7
	Agree (A)	7	23.3
	Neutral (N)	1	3.3
	Disagree (D)	3	10.0
	Strongly Disagree (SD)	2	6.7
Total		30	100%

- Mode = 17

APPENDIX F-26

COVER ALL MATHEMATICS CONCEPTS IN THE SYLLABUS

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	11	36.7
	Agree (A)	9	30.0
	Neutral (N)	2	6.7
	Disagree (D)	5	16.7
	Strongly Disagree (SD)	3	10.0
Total		30	100%

- Mode = 11

APPENDIX F-27

INVOLVE PARENTS IN MATHEMATICS EDUCATION OF THEIR CHILDREN

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	21	70.0
	Agree (A)	3	10.0
	Neutral (N)	1	3.3
	Disagree (D)	4	13.3
	Strongly Disagree (SD)	1	3.3
Total		30	100%

- Mode = 21

APPENDIX F-28

ADVICE STUDENTS ABOUT JOB OPPORTUNITIES OR PROSPECT IN MATHEMATICS

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	22	73.3
	Agree (A)	2	6.7
	Neutral (N)	0	0.0
	Disagree (D)	4	13.3
	Strongly Disagree (SD)	2	6.7
Total		30	100%

- Mode = 22

APPENDIX F-29

EXAMINING THE STUDENTS AT THE END OF EACH TOPIC AND MAKING NECESSARY REVIEW TO CORRECT THEIR MISTAKE OF STUDENTS'

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	23	76.7
	Agree (A)	4	13.3
	Neutral (N)	0	0.0
	Disagree (D)	3	10.0
	Strongly Disagree (SD)	1	3.3
Total		30	100%

- Mode = 23

APPENDIX F-30

CONDUCTING FREQUENT TEST

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	22	73.3
	Agree (A)	4	13.3
	Neutral (N)	0	0.0
	Disagree (D)	2	6.7
	Strongly Disagree (SD)	2	6.7
Total		30	100%

- Mode = 22

APPENDIX F-31

MOTIVATE STUDENTS TO LEARN MATHEMATICS ON THEIR own

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	19	63.3
	Agree (A)	7	23.3
	Neutral (N)	0	0.0
	Disagree (D)	3	10.0
	Strongly Disagree (SD)	1	3.3
Total		30	100%

- Mode = 19

APPENDIX F-32

ENCOURAGE STUDENTS TO FORM SMALL GROUP TO PRACTICE

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	23	76.7
	Agree (A)	5	16.7
	Neutral (N)	0	0.0
	Disagree (D)	2	6.7
	Strongly Disagree (SD)	0	0.0
Total		30	100%

- Mode = 23

APPENDIX F-33

INVOLVING STUDENTS' IN THE TEACHING AND LEARNING PROCESS

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	18	60
	Agree (A)	7	23.3
	Neutral (N)	1	3.3
	Disagree (D)	3	10.0
	Strongly Disagree (SD)	1	3.3
Total		30	100%

- Mode = 18

APPENDIX F-34

MY STUDENTS CAN SOLVE M /3-3≤ -6

		Frequency	Percentage (%)
Valid	Strongly Agree (SA)	23	76.7
	Agree (A)	3	10.0
	Neutral (N)	0	0.0
	Disagree (D)	2	6.7
	Strongly Disagree (SD)	2	6.7
Total		30	100%

- Mode = 23