UNIVERSITY OF EDUCATION, WINNEBA

EFFECT OF PROBLEM-BASED LEARNING ON COLLEGE OF EDUCATION STUDENTS' ACHIEVEMENT IN, AND ATTITUDE TOWARD PROBABILITY

BUKARI, HAMIDU IBRAHIM

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of the requirements for the award of the degree of Master of Philosophy (Mathematics Education) in the University of Education, Winneba

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DECLARATION

Student's Declaration

I, Hamidu Ibrahim Bukari, hereby declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

Signature: …………………..………………

Date: ………………………..……………

Supervisor's Declaration

I hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of thesis as laid down by the University of Education, Winneba.

Name of Supervisor: Prof. S.K. Asiedu-Addo

Signature: …………………..………………

Date: ………………………..……………

DEDICATION

To my beloved and cherished wife Mrs. Sherifa Bukari and my children Bukari Abibata, Bukari Alhassan and Bukari Fuseina.

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A countless number of personalities have influenced my academic life in various positive ways. Indeed, I may not be able to mention all these mentors; I would like to first of all thank my supervisor Prof. Dr. S. K. Asiedu-Addo for his valuable comments, kind support, endless patience, continuous guidance and constructive criticism throughout this thesis write-up. Although, he had various responsibilities, he showed great patience to me, once again I am most grateful. May God richly bless you.

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ABSTRACT

The purpose of the study was to explore the effect of Problem-Based Learning Approach on College of Education students' achievement in, and attitude toward Probability. The research design for the study was Quasi-experimental. Convenience and simple random sampling techniques were employed to obtain a sample of 100 students for the study. Tests, questionnaire and semi-structured interviews were the instruments employed for data gathering. Probability achievement pre-test was administered to both experimental and control groups. An intervention was carried out and after that, probability achievement post-test was also administered to both groups. Independent samples t-test, ANCOVA and paired samples t-test were used in analysing data. The results indicated that there was a statistically significance difference between students Post-test scores of both Control group exposed to Traditional method and Experimental group exposed to Problem-Based Learning approach. Problem-Based Learning approach developed students critical thinking, good problem solvers and self-directed learners which would lead to life-long memory of probability concepts. It was recommended that Problem-Based Learning should be included in the Mathematics Curriculum at the Colleges of Education in Ghana and tutors should be given Workshops and Seminars on the use of Problem-Based Learning strategy in their Mathematics lessons.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This chapter presents the general background of the study, statement of the problem, purpose of the study, objectives of the study, research questions, significance of the study, delimitations and limitations; and the organization of the study.

1.1 Background to the Study

Specific attitude like patience, confidence and willingness help in problem solving in terms of Mathematics achievement. However, students do not like mathematics in Ghana and other parts of the world due to how they view or perceive mathematics. This perception has created a bad public image by students, describing mathematics as difficult, cold, abstract, theoretical and not an interesting subject (Arthur, Asiedu-Addo, & Assuah, 2017).

Mathematics plays a key role in shaping individuals with various aspect of private, social and civil life (Anthony & Walshaw, 2009). Mathematics is very important for students' achievement in every educational field, for producing informed citizens, success in careers and in personal fulfilment. It is also significant in many fields such as the Natural Sciences, Medicine, Engineering and Social Sciences. Hence mathematics has immersed contribution to the scientific development of a nation and intellectual growth and development of every person in a society.

One of the key roles mathematics plays in everyday life is problem solving according to Nabie, Akayuure and Seidu, (2013); Atteh, et al., (2014) and Karatas and Baki, (2013). Problem solving has become a primary goal of all mathematics instruction and an integral part of mathematical activity. One of the primary goals of mathematics education in Ghana had been to develop students' ability to solve everyday life problems. Mathematics tuition, importantly involves teaching the subject matter in real life situation of problem solving. The main aim of the mathematics curriculum at every level of Ghanaian education is to help develop everyday problem-solving skills of students. It is therefore, the role of the teacher to help students organize, interpret and present mathematical information accurately in written, graphical and diagrammatical forms and to analyse problems, select suitable strategies and apply appropriate techniques to obtain its solution (Ministry of Education, 2010). Tutors are therefore, required to integrate problem solving activities in every lesson to develop students' competencies, skills and reasoning. Problem solving in mathematics is a complex process which calls for a problem solver, engaged in a mathematical task to organize and deal with specifics to general pieces of knowledge.

Even though, problem solving is the heart of the mathematics curriculum, Ghanaian students at the Colleges of Education still face difficulties in dealing with probability problems. The Chief Examiner's Report, UCC (2017, 2018) on Statistics and Probability consistently indicated students' difficulty in understanding probability concepts, its application and solving of questions. The report pointed out students who answer probability problems performed poorly because they lacked basic Probability concepts that inhibits their problem-solving ability. This poor performance may be due to teacher-centred and traditional method approach employed in the teaching and learning discourse in the Ghanaian classroom (Mereku, 2010).

Mereku (2010) argued the teaching in the Ghanaian mathematics classroom was teacher-centred. The students only sit down and listen passively in the learning process which makes them lack mathematical analysis and logical reasoning. This showed that most teachers in Ghana especially at the College setting also adopt the lecture method or traditional method as their mode of mathematical instruction.

To overcome students' challenges, difficulties, poor academic achievement and poor attitudes toward probability problems, there is the need to employ an instructional approach that would help teaching and learning to be student- centred approach. Thus, problem-based learning is one of the potential instructional approaches which allow students to construct their own understanding of concepts. This study examined if teaching using problem-based learning on College of Education students could improve their achievement in probability problems. Problem-based learning is a constructivist learning approach that exposes students to mathematical problem where students solve problem based on mathematical concepts. Problem-based learning is also a classroom strategy that organises mathematics instructions around problem solving activities and give students' opportunities to think critically, present their own creative ideas and communicate with each other mathematically (Hmelo-Silver, 2004). The problem-based learning mathematics classroom focuses on problem-solving and conceptual understanding rather than on conceptual drill.

In problem-based learning, students are positioned as self-directed problem solver through collaborative activities to encourage them to find the problem and plan completion. The students are taken through collaborated activities and guided to familiarized skills that serves to reflect the findings through inquiry about the effectiveness of their way of thinking in solving the problem (Reviandari, Yaya, Utari, & Jozua, 2015).

Problem-based learning (PBL) is a constructivist pedagogical approach to learning in which students work in smaller groups to find solutions to complex problems (Bukari, 2019). Theoretically, Problem-based learning is based on "Constructivism" (Turan & Demirel, 2011) and its approach to instructional design is based on "Problem solution" and "Conceptual learning" (Espinoza & Sanchez, 2014). Constructivism is a theory about knowing and learning that knowledge cannot be directly transmitted but must be actively constructed by learners (Chowdhury, 2016). It is based on the belief that knowledge is not a thing that can be simply given by the teacher inside the classroom to the students but rather knowledge is constructed by learners through an active mental process of development; learners are the builders and creators of meaning and knowledge.

Probability was seen around 1660 by Pascal, Leibnitz as a game of chance, legal decision and annuities (Uspensky, 1937). He added that probability theory is a branch of applied mathematics with the effects of chance. In mathematics, the term "Probability refers to the study of randomness and uncertainty" (Asiedu-Addo, 2000). Probability is part of our everyday vocabulary, a complex concept with many dimensions in that we use words relating to probability such as probable, never, impossible, certainly, fairly, not likely; as undeniable facts of life (Agyapong, 2014; Mutodi & Ngirande, 2014). The study of Probability has been an integral part of mathematics curriculum designed for schools, right from basic level to tertiary level of education in Ghana. The concept of probability is applied in many fields of our lives such as trade, industries, banking and teaching. Experts in engineering, economics, business and medical fields require the concept of Probability to make informed decision in their fields of work. The theory of probability is applied in the areas of investments, insurance, weather forecasting and so on. Concepts of probability are very important tools in decision making, statistical analysis and many research areas.

Early research on probability understanding by Psychologists with College students focused on descriptive social setting within which questions relative but not necessarily numerical, likelihood could be placed (Tversky & Kahnman, 1983; Graham, 2005). For this reason, it was imperative to acknowledge sample spaces relative to social intuition and classroom practices.

Currently, researchers carried out studies on effect of problem-based learning (PBL) students' attitudes and achievements using Quasi-experimental control design and the results indicated the effectiveness of PBL for experimental groups compared to traditional (Control) groups (Han et al., 2016; Uygun & Tertemiz, 2014; Heyvaert et al., 2017; Iji, Emiakwu & Utubaku, 2015).

1.2 Statement of the Problem

The main problem of College of Education students was the conceptual understanding of probability and its application especially the use of outcome trees and tables. This was manifested in their classwork, quizzes and end of semester examinations. The researcher did content analysis of students' mid-semester examination and end of semester examination scripts and realised that most students have problems in answering probability questions. Most of the students got negative values and values greater than one, which was unacceptable. Agyapong, (2014) reported students inability to solve basic concepts of probability at the Senior High School level, as a result of the effect in understanding probability concepts at higher level of education in Ghana.The

Chief Examiner's Report for Statistics and Probability course on Colleges of Education reported that candidates had difficulty with the application of probability concepts. Students also had difficulty in identifying all the possible terms in probabilities as a result they provided one term instead of all three sets of terms as in the case of the question below (Chief Examiner Report, UCC., 2017).

> *Candidates were given "the probability that three women win elections in their respective electoral areas are 1/3, 3/5 and ¾ respectively. What is the probability that only one of them wins her electoral area?" candidates were not able to write all three terms to find the probability that only one of the three wins; that is;* $\left(\frac{1}{3}\times\frac{2}{5}\times\frac{1}{4}\right)+\left(\frac{3}{5}\times\frac{2}{3}\times\frac{1}{4}\right)+\left(\frac{3}{4}\times\frac{2}{3}\times\frac{2}{5}\right)$. *Most of them wrote only one out of the three probabilities.*

The Chief Examiner's Report, UCC (2018) also reported on Statistics and Probability course that, candidates could not use tree diagrams to answer probability problems, hence poor application of probability concepts.

> *Candidates were given "a bag contains seven discs, two of which are red and five green. Two discs are removed one after the other without replacement. Draw a tree diagram to illustrate the probability and what is the probability that they are of different colours?" Most of the candidates were only able to draw two branches out of six branches and were getting results greater than one which was unpardonable and unacceptable in the axioms of probability.*

Few of the candidates who understood the questions came out with relevant approach and solved them. The results showed that these difficulties in solving probability problems were common practice across all the Colleges of Education in Ghana. Students weaknesses in dealing with basic concepts involving decimals, fractions and percentages were identified as the main threat to the understanding of probability concepts (Tso, 2012). According to O'Connell (2010), College students have different types of errors in probability problems and outlined them as: text comprehension errors, conceptual errors, procedural errors and computational errors. These characteristics of error of probability are also experienced by College of Education students in Ghana (Chief Examiner' Report, UCC., 2018).

Attitudes determine student's ability and willingness to learn a course or work on a variety of assigned tasks available. Students perceived probability as abstract in nature. This makes students to develop a negative attitude towards probability concepts. Many of them dislike or show negative attitude to probability due to the influence of the attitude of their teachers who taught them at the Senior High School level. Borovnik (2012) noted that, the abstract and formal nature of probability often cause students to develop a negative attitude toward probability concepts

Asante and Mereku (2012) also pointed out that, mathematics pedagogy courses should be made practical for pre-service teachers and that they should be given enough opportunities to practice in order to teach effectively at the basic level of Ghanaian education. Hence, the need for an effective practical approach problem-solving skill such as problem-based learning approach. It was in this view that, the study focused on the effect of problem-based learning on College of Education students' achievement in, and attitude toward probability.

1.3 Purpose of the Study

The purpose of this study was to investigate the effect of problem-based learning approach on College of Education students' achievement in, and attitude toward probability.

1.4 Objectives of the Study

The specific objectives of the study were to:

1. identify the difficulties College of Education students encounter in solving probability problems.
2. investigate the effect of problem-based learning approach on College of Education students' achievement in probability problems.
3. determine the attitude of College of Education students towards probability problems.

1.5 Research Questions

The research sought answers to the following questions:

1. What difficulties do College of Education Students have in solving probability problems?
2. What is the effect of problem-based learning on College of Education students' achievement in probability?
3. What are the attitudes of College of Education students toward probability problems?

1.6 Significance of the Study

The study was important for it contributed to teaching and learning of probability at the Colleges of Education. The Problem-based learning allowed students to make decisions on their own. The effectiveness of problem-based learning helped students develop problem solving skills, self-directed learning and increase in their performance at the College of Education level of Ghanaian education. The study served as a resource for curriculum developers, policy makers and tutors to help improved Colleges of Education students' problem-solving ability in Ghana. The study also served as a baseline document for other researchers at the Colleges of Education in Ghana and the entire globe. Finally, it added new knowledge to mathematics education.

1.7 Delimitations of the study

The research was narrowed to only two Colleges of Education in the Northern Region, Tamale. It was also delimited to only second year students at the Colleges of Education.

1.8 Limitations of the Study

It was assumed that students responded to the test items honestly and impartially reflected their views. It was also assumed that all tests were administered under the same standard conditions and students were able to understand the test items correctly. The study was limited to second year general programme students at the Colleges of Education in Northern Region, Tamale.

Besides, the presence of the course tutor may have influenced students' mood and attitude. One major limitation was the difficult nature in organising students for probability sessions since they were not taking it for that semester exams.

1.9 Organization of the Study

The study outlined the effect of problem-based learning on Colleges of Education students' achievement in, and attitude toward probability. The study was also organised into five Chapters. Chapter one gave detailed about the background to the study and the statement of the problem. Other areas in this chapter include research questions, the purpose of the study and the significance of the study. The objectives of the study, the organisation of the study and limitations were all part of chapter one. Chapter two addressed review of related literature which involved the identification, location and analysis of documents containing information related to the research problem that had been written by scholars, mathematics educators and experts in the field of study.

Chapter two further pointed out research strategies and the specific procedures and instruments used in related research problem. Chapter three focused on the methodology of the study. Areas covered include the research design, the identification of the population, the sample and sampling procedure, instrumentation and data collection procedures. Chapter four focused on data analysis and discussion of results. It also outlined the results of both quantitative and interview of the data collected for the study. Chapter five consisted of the summary of the study. The areas covered include the summary of results based on the analysis of the data collected, the conclusions drawn from the results and recommendations made.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

This chapter dealt with the theoretical framework of the research and what other researchers have written on probability. It also had review of literature on various form of approach to the teaching and learning of probability. The areas covered in the literature review were: theoretical framework, theoretical inferences, concept of problem-based learning, attitude toward Probability and achievement, implementation of problem-based learning in mathematics education, evidence of empirical studies, concept of probability and difficulties and errors College students' make in solving probability problems.

2.1 Theoretical Framework

The theoretical framework for the research was based on George Polya's problem solving technique. Polya (1945) in his book "How to solve it", identified four basic steps of problem solving which had become the framework often recommended for teaching and assessing problem solving skills.

The four steps were:

1. understanding the problem;
2. devising a plan to solve the problem;
3. implementing the plan; and
4. reflecting on the problem.

It was obvious that students moved backwards and forwards between and across the steps in a problem-solving process. Students may be obliged to consider auxiliary problems if immediate connection cannot be found for a given situation. Students have to follow the steps with questions on each step as shown below.

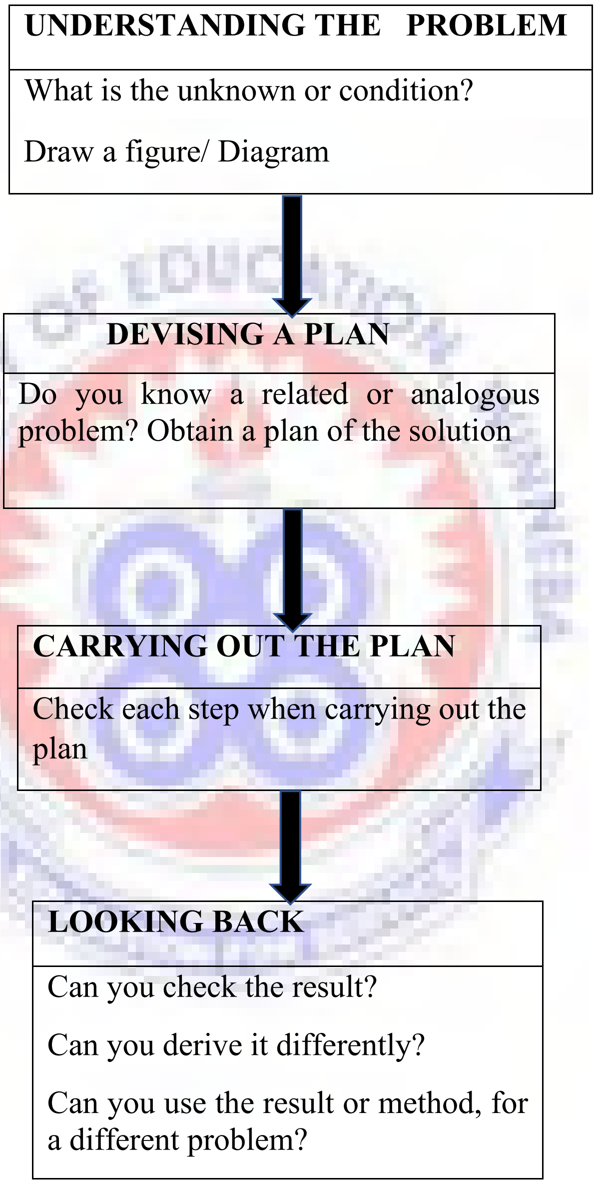


Figure 1: Stages of Polya Heuristics

The questions would guide the students to solve the problem. In view of (Maiorana, 1985 as cited by Atteh, Appoh-Andam & Obeng-Denteh, 2017) stated that, the teacher in the classroom does most of the talking, questioning and thinking, placing the students in a passive role rather than an active role. They argued that, this form of learning does not encourage critical thinking and problem solving on the part of students. Teachers should provide many opportunities for students to engage in problem solving where critical thinking takes place to provide the greatest benefit to students (Atteh, Appoh-Andam & Obeng-Denteh, 2017).

According to Atteh, Appoh-Andam and Obeng-Denteh (2017) most Mathematics teachers are with the belief that developing problem solving techniques in their students is of primary importance; however, few have an idea exactly what it is, how it should be taught and how should be assessed. They suggested four 4-step guideline that can be implemented in any Mathematics classroom or training setting to help students acquire problem-solving skills, which include:

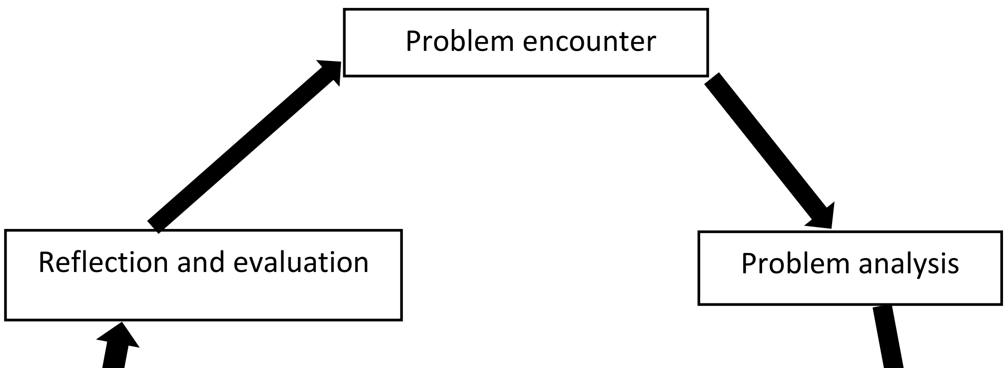
i. holistic understanding of the problem.
ii. identify method(s) for solution.
iii. apply the method(s) for solution
iv. check the accuracy of the solution.

The theoretical framework was further based on (Othman, Berhannuddin, & Abdullah, 2013) problem-based learning cycle model in probability problem-solving in the use of problem-based learning approach which conveyed information that helped students solve problems using tree diagrams, Venn diagrams and tables rather than the use of lecture-discussion approach such as formulas and calculations. The use of diagrams is generally acknowledged to be an integral part of mathematical knowledge (Zahner & Cortex, 2012). In the area of Probability certain types of schematic diagrams are conventionally used to represent important concepts, that is Venn diagrams for compound events, outcome tree diagrams for sequential experiments and students show diagrams in solving problems (Zahner & Cortex, 2012; Russell, 2000). Vygotsky (1978) also pointed out that when students are challenged by working with diagrams, they become useful within their boundary of competence and the zone of proximal development.

In general, what makes a diagram useful to a novice problem solver is both the appropriateness to the problem and need. The problem at hand must be difficult or complex enough that the diagram is needed and has a chance of having facilitating effect (Zahner & Cortex, 2012). They proposed a model of probability problem-solving by combining Reusser's (1996) and Casey's (1978) models. This model propounds that students make efforts to solve a problem by making ways to understand the problem text and then build a mental model of the problem. Problem solvers (students) then attempt to cast the problem in mathematical terms and possibly relate the current problem to familiar mathematical formulas or previously encountered problem. After that they proceed to develop a plan for solving the problem. Finally, they execute the chosen strategy, a final approach that sometimes does or does not occur is to check the solution for plausibility. The stages in Zahner and Cortex (2012) model include: text comprehensive, mathematical problem representation, strategy formulation and selection, and finally, execution of the strategy.

Othman, Berhannuddin and Abdullah (2013) carried out a study on, 5 ladders of active learning: An innovation learning steps in problem-based learning process. In fact, they defined problem-based learning as an educational approach at which learning is student-oriented and driven by a problem and students collaborate in groups to learn more about the problem by making findings, communicating to each other, apply knowledge and skills and enjoying the fruits of active learning. Problem-based learning has proven to be a successful educational strategy in many different fields of study all over the world and it was used as a strategy for development in the globalised higher education (Graaf & Kolmos, 2009; as cited in Othman, Berhannuddin & Abdullah, 2013). This is because of its popularity, problem-based learning has been accepted as one of the most powerful student-centred learning approaches that enable many institutions to make a significant change in teaching and learning process. Hence the theoretical framework of this study was further based on the problem-based learning cycle of Othman, Berhannuddin and Abdullah, (2013) which has six stages.

This model was adapted because of the time frame for the whole period of implementation; the student learning time and space, including classroom, outside classroom, individual learning, group learning; assisted learning and self-directed learning. The topic, scope and number of problems given to students demanded for this cycle. The reflection activity was included in the cycle because the activity forms the most crucial stage of problem-based learning cycle which help students achieve optimal learning outcomes (Hung , 2006). This model was very important for the study since problems were addressed using the stages to outline various concepts of probability using outcome tree diagrams and tables. Is a model that create room for students to explore in many real-life probability problems by going strictly to the stages and applying cycle appropriately. Below are the stages of the problem-based learning.



Source: Othman et al., (2013)

Figure 2: Problem-Based Learning Cycle

2.2 Theoretical Inferences

Vygotsky (1978) postulated that learning is transferred and corresponds to the learner who is in-charged of constructing meaning and the teacher's role is to guide and encourage on active participation of learners in order to promote the control of their own learning and creating possibilities of several interactions with the aim of sharing meaning in the classroom and fostering social nature of learning. Vygotsky (1978) theory of learning is acquired through social interaction and collaborative work based on personal and prior knowledge. The main concept is the social interaction which promoted acquisition of learning within the limit of development within a social and cultural context. Basically, the role of the teacher is to create situations for social interaction within classroom whereby students could learn with their peers in a collaborative and interactive way which promoted independent learning of concepts and procedures (Vygotsky, 1978).

Constructivism makes references to a set of theoretical elaborations, conceptions, interactions and practices having certain agreement with each other (Espinoza & Sanchez, 2014). The constructivist theory posits that knowledge, no matter its nature is elaborated by the learner through actions one takes to reality (Castillo, 2008). The constructivist theory assumes that people possesses varied knowledge schemes, which do not express general knowledge of reality but rather a partial knowledge configured according to the context in which they develop and live. This position not only allows to notice the difficulties that students usually have to learn but also provide a guide to develop more efficient teaching and learning strategies, using a teaching approach where the learner is the central protagonist considering their interests and skills to learn (Castillo, 2008).

Constructivist paradigm proposed sequences of teaching where the learner is put before experiences that enables him/her builds knowledge for understandable memory through relations in which the learner can find enough balance between the logic of his/her own structure and cognitive development. The teaching and learning of mathematics and also in the field of sociology, epistemology and psychology of learning are integrated roles of constructivist model. Hence, constructivist proposals have become the axis of a fundamental transformation of the teaching of mathematics.

Today, educational approaches that aimed analytical thinking, cause-effect relationships and individual who produce logical solutions can be applied to real life. In parallel with modernity and global change, different teaching methods and learning strategies have been tried in order to find solutions to problems encountered in mathematics education in Ghana (Atteh, et al., 2014; Atteh, Andam, & Obeng-Denteh, 2017). One of these approaches is problem solving which is the ability to analyse and determine a solution to a problem put before someone and the problem-based learning (PBL) which is one of the best constructivists learning environments (Hmelo-Silver, 2004). Problem-based learning approach is where learners develop solutions to a problem from everyday life by using the new information they have gained in the research process with the information available in cooperative environment (Duran, Ozdemir, & Kaplan, 2015). Problem-based learning is an approach that enables active participation in the process of knowledge and skills, allows learners to structure and evaluate information (Duch, Groh, & Allen, 2001). It encourages learners to work in small groups, providing learners with the opportunity to benefit from different situations (Massa, 2008). In PBL approach, the challenging problems from daily life to the learning environment lead to discussion of the learners to the cause of the problem and the solution to the problem. Finally, Problem-based learning is an approach that also aims in providing lifelong learning skills to learners in the process of problem solving through studies and experiences in different fields of education (Barrows, 2002).

2.3 Concept of Problem-Based Learning

Problem-based learning is a student-driven approach that give students opportunity to do research, blend theory and practice in the application of knowledge and skills to find practical solution to a defined problem (Savery, 2006). It is considered to be one of the most appropriate solution that increase students' learning motivation and to develop practical skills. Johari, Hasniza and Mokhtar (2013) stated that problem-based learning was first introduced in the world of education in 1960's by Prof. Dr. Howard Barrows at Mcmaster University in Canada.

Currently, PBL approach has been one of the most popular curriculum innovations in education. It is because, the approach helps students have clearer mind, been flexible with diversified ways of thinking and is considered as a paradigm of multidisciplinary studies. Problem-based learning is the combination of specific concepts in classroom context for increasing students thinking skills and problem-solving ability.

In addition, the challenge of globalisation today requires students to master problem-solving skills and also have positive attitudes and values besides good conceptual knowledge of mathematics (Shahbodin & Zareema, 2013). Azer (2009) views PBL as students-centred instructional strategy where they can solve problems collaboratively and reflect on their experiences. Similarly, Barrows (2002) explains PBL as an approach that requires students to gain effective skills for problem-solving in educational field by acquisition of knowledge through learning in team work in different subjects or disciplines.

Moreover, (Kazemi & Ghoraishi, 2012) indicated that in PBL students acquire knowledge in the context of meaningful problems and taking responsibility for their own learning. PBL is regarded as an active learning approach which plays a significant role in increasing students' level of interest and achievement which leads to lifelong memory and experience to students (Padmavathy & Mareesh, 2013). Among the varieties of PBL definitions is that students actively construct their own knowledge of mathematics. PBL's application in the educational field helps students understand and interpret knowledge deeply, to construct knowledge to achieved intrinsic motivation and act purposefully, think rationally with their colleagues (Dooley, 1997; Duch, Groh & Allen, 2001; Hmelo-Silver, 2004; Fatade, Mogari, & Arigbabu, 2013). Savery (2006) added that, although PBL did not originate from a philosophical background, its students centred characteristics make it well-aligned with theories of constructivism.

Students work in small group (Veneranda, 2014) and they learn collaboratively, communicatively and cooperatively. In PBL the main focus lies in problem-solving skills including different strategies, techniques and methods.

Besides, concept of PBL provides a clear scaffolding and guidance to facilitate student learning. In PBL, students learn content knowledge, strategies and self-directed learning skills through collaborative and communicative solving problems, reflecting on their experiences and engaging in self-directed inquiry processes (Hmelo-Silver, Duncan, & Chinn, 2007). Problem-based learning often uses text-based resources for both the problem and self-directed learning. In PBL environment, scaffolding is an extensive domain of research (Reiser, 2004) and researchers have developed theory-driven and empirically based design guidelines for incorporating effective scaffolding strategies to support learning (Hmelo-Silver, 2006) scaffolded problem-based environments provide learners with opportunities to engage in complex tasks that was otherwise beyond their current abilities. The tutor uses scaffolding to make sure that complex tasks are accessible, manageable and within student's zone of proximal development (Vygotsky, 1978).

The important feature of scaffolding was that it supported students' learning on how to do the task and why the task should be done that way (Hmelo-Silver, 2006). Scaffolding also guides learners through difficult task, shows important aspects of students work in order to force them to engage with key disciplinary frameworks and strategies (Reiser, 2004). Tutors play vital role in scaffolding and are mindful and showed productive engagement with the problem, tools and peers. Tutors guide students in the learning process, making them to think deeply and model the kinds of questions that students need to ask themselves, thus forming a cognitive apprenticeship (Hmelo-Silver & Barrows, 2006). Problem-based learning has positive effects on students in developing their problem-solving skills, critical and creative thinking skills, cooperative, collaborative and communicative skills (Abdullah, Tarmizi, & Abu, 2010; Koray, et al., 2008).

However, most of these studies were cross sectional in nature that leads to heated debates on its effectiveness (Abdul Kadir et al., 2016). Anecdotal evidence in forms of inconsistent and contradicting findings also indicates the need to further investigate the effectiveness of PBL on problem-solving skills.

Today, the use of PBL continues and its trend encourages researchers to undertake studies for determining the effectiveness of this contemporary approach. This is because PBL has increased recognition and its importance felt in the affective domain in many independent studies being investigated. However, different conclusions have been reached from studies. In some studies researchers argue that PBL has positive effects on attitudes in disciplines or courses (Abdullah, Tarmizi & Abu, 2010; Han, et al., 2016) whereas some identified that PBL has no effects or negatives effects on attitudes (Uygun & Tertemiz, 2014). Others argue that there are greater achievements in courses taking through by PBL treatment compared to traditional approach (Ajai, Imoko, & O'kwu, 2013; Iji, Emiakwu, & Utubaku, 2015). Researchers postulated that adoption of PBL help tutors to know when and how to apply scaffolding during instruction (Fatade, Mogari & Arigbagu, 2013; Chng, Yew, & Schmidt, 2011; De Grave, et al., 1999).

In contrast to the current traditional method of probability instruction in the classroom environments, a problem-based learning environment provides students with opportunities to develop their abilities to adapt and change methods or strategies to fit new situations (Abdullah, Tarmizi & Abu, 2010). PBL instructional strategy provides the needed impetus to review and re-align the existing curriculum of Colleges of Education to fit into the reform of Bachelor of Education Degree programme. Though, efforts have been made on the performance of students for some years, there is however little or no research carried out on the effectiveness of problem-based learning in probability problems at Colleges of Education in Ghana.

Table 1: Characteristics of Traditional Learning and Problem-Based Learning

	Traditional Learning	Problem-based learning
Role of tutor	Lecturer	Facilitator or Guide
Curriculum	Subjects	Problems
Audience disposition	Passive	Active
Organization	Large classes	Small groups
Approach	Tutor-directed	Self-directed

Source: Adapted from Barrows (1996) cited in (Grigg & Lewis, 2013).

2.4 Attitude toward Probability and Achievement

Attitude is one of the major factors that affects learning processes and had not attracted enough attention from stakeholders of education. It is therefore, important to consider learners contributions to their learning outcomes as a result of their belief and views about the subject matter, whether they like it or not and whether the see value in it. Attitudes are more or less positive emotions, beliefs, values and behaviours and hence affect individual way of thinking, acting and behaving which has a lot of implication to teaching (Mensah, Okyere, & Kuranchie, 2013). Attitudes though not directly observable are inferred from observable responses and behaviours which reflect a pattern of beliefs and emotions.

Most research studies supported the fact that student's success in mathematics depends upon the learners' attitude towards the subject as this determines their ability and willingness to learn, choice of action and response to challenges (Langat, 2015; Ayob & Yasin, 2017). It can therefore be inferred that negative disposition induces tendencies of fear, anxiety and stress where one resorts to other non-productive practices which finally prevents them from experiencing the richness of mathematics especially in probability and many approaches that could be used to develop competencies in the course probability.

In addition, researchers on attitudes point to the fact that attitude plays a crucial role in learning and achievement in Mathematics (Arhin, 2015; Mensah, Okyere &Kuranchie, 2013; Ayob & Yasin, 2017), hence determines the student's success in the course/ subject. Attitudes therefore, are psychological orientations developed as a result of one's experiences which influences a person's view of situations, objects people add how to respond to them either positively/negatively or favourably/unfavourably (Mensah, Okyere & Kuranchie, 2013).

In general, the conceptions students hold about probability determine how to approach probability tasks leading them into either productive or non-productive orientations. In many cases, students have been found to approach probability as procedural and formula-oriented. This prevents them from experiencing the richness of probability and the many approaches that could be used to develop competencies in probability.

According to Johnson and Johnson (1994), when students work in groups, they exhibit characteristics such as being supportive, encouraging, helpful and assisting each other excel academically. Students probability achievement can be enhanced if students are actively involved in discussion with their colleagues in group work, see advantage in solving complex task, explaining solutions to their colleagues as well as listening attentive to and trying to make sense of someone else's explanations (Pale, 2016;

Bayaga & Wadesango, 2014). Tutors who are effective with mathematical discourse and skilful in establishing good rapport with their students through representations, conventions and meanings of probability concepts would enhance positive attitude toward probability problems. It is deemed necessary that; tutors' approach should be directed at developing students' conceptual knowledge of probability rather than their memory skills and the tutors should also shape ideas and make connections between ideas in a more coherent and meaningful manner for probability achievement by students. When students are motivated intrinsically, they exhibit a number of pedagogically desirable behaviours when engaged in complex task, monitoring their own comprehension through elaborative process and adapting learning strategies boost the attitude to probability and hence its achievement.

Research centring on students' attitude towards mathematics study and its performance has increasingly received attention. Studies have shown that factors such as motivation and attitude have impacted on student's achievement (Cote & Levine, 2000; Duran, Ozdemir & Kaplan, 2015). The attitude of students could be influenced by the attitude of the tutor and his mode of delivery a lesson in the teaching and learning process. Studies carried on, have shown that tutors' methods of mathematics teaching and their personality greatly accounted for the students' positive or negative attitude towards mathematics and that without interest and personal effort in learning mathematics by students affects their performance in the subject (Pale, 2016). Students are therefore to interact and work together in groups to achieve the set objectives regardless of whether the grouping is heterogenous or homogenous, they are obliged to work together in order to exchange ideas, information and strategies among themselves with the tutor. Students gain better knowledge, interaction, achievement and social skills in a group setting than when not in a group setting (Johnson & Johnson, 1994). According to Pale (2016), fundamental assumption of the developmental perspective on student learning was that, interaction among students around appropriate tasks increases their mastering of critical concepts.

From the developmental perspective, the effects of problem-based learning on student probability achievement was largely due to the use of collaborative, communicative and cooperative tasks (Pale, 2016). In this view, students' opportunity to discuss, to argue, to present and hear one another's viewpoints is the critical element of student learning with respect to student probability achievement. Maio and Haddock (2010) posits that, students' attitude towards certain objects or subjects are dominated by behavioural component, cognitive component and affective component. They stated that behavioural component with regard to studying, past experience with a school-based learning can influence a student's current attitude towards studying. For instance, poor mathematics performance in Senior High School could lead to a negative attitude towards mathematics at the Colleges of Education. With the cognitive component, the emphasis is the decision for chosen a certain school and the study programme. Future students would weigh the advantages and disadvantages of certain institutions and their attitude would more or less be determined by these cognitive aspects, for example, information from the media or ranking tables in terms of performance. Finally, the affective component is based on societal values such as religion, moral, political views and so on. If the first study encountered in mathematics is experienced as joyful and fun, students are more likely to develop a positive affective attitude towards mathematics as opposed to a first impression that is characterised by an anxious atmosphere of scaring new students with enormous workload or difficulty they are faced with mathematics. This means that these components are intertwined and not independent of each other, all point in the same direction, that is, positive affective reaction are combined with positive cognitive reactions.

Studies also revealed that affective domain such as attitude towards mathematics and student's achievement are both clearly interrelated (Bayage & Wadesango, 2014; Mensah, Okyere & Kuranchie, 2013). In the context of mathematics education, researchers pointed out that, students' achievement is not only bound by cognitive domain but is also influenced by students' attitudes (Bayaga & Wadesango, 2014; Mensah, Okyere & Kuranchie, 2013). Basically, students with positive views of mathematics especially probability learning are believed to indirectly develop positive attitudes toward mathematics and thus, might lead them to obtain higher achievements. On the contrary, students with negative views towards mathematical probability which in turns could affect their learning and achievement (Bayaga & Wadesango, 2014; Mensah, Okyere & Kuranchie, 2013).

In general, the key definition of attitude in Social Psychology is the one that was proposed by Allport (1935) decade ago: "a mental and neutral state of readiness organized through experience, exerting a directive or dynamic influence upon individual's response to all objects and situations with which it is related" as cited by (Ayob & Yasin, 2017). Aiken (1970) concisely defined attitudes toward mathematics as individual's tendency to response positively or negatively to an object (that is, situation or concept). Nevertheless, there were some disagreements among researchers on these simple and comprehensive definitions of attitudes towards mathematics.

Modern researchers believed that the attitudes toward mathematics has been expanded by components such as affective (that is, emotions and beliefs), cognitive and behaviours (Martino & Zan, 2010; Maio & Haddock, 2010). Attitudes toward mathematics affective component shows emotions and feeling towards mathematical topics. Emotion simply refers to the feeling of liking or interest in mathematics, or the feeling of disliking or disinterest in mathematics. Mathematical Psychologists have also identified three types of learning that shape attitude based on affective components, namely classical conditioning, operant conditioning and observational conditioning (Lineros & Hinojosa, 2012; Mensah, Okyere & Kuranchie, 2013).

Classical conditioning refers to learning that takes place through repeated experiences with the environment and with stimuli (Lineros & Hinojosa, 2012; Mensah, Okyere & Kuranchie, 2013). In teaching and learning mathematics context, students being exposed to various forms of stimuli such as concepts, exercises, questioning and others are believed to gradually develop attitudes toward mathematics.

While operant conditioning learning is based on the belief that an individual has a tendency to repeat behaviours that produce positive effects and does not reiterate behaviours that cause negative effects (Lineros & Hinojosa, 2012; Mensah, Okyere & Kuranchie, 2013). For example, when students are rewarded after completing mathematics tasks, they would see such tasks as fun and eventually motivate them to do more of such similar tasks. On the contrary, when students are punished for not completing mathematics tasks, they might relate such tasks with unpleasant feelings and hence demotivate them to do more of such tasks.

Observational conditioning on the other hand, refers to an individual using their assessments towards other people’s actions and their consequences, in order to drive their thoughts, feelings and behaviours (Mensah, Okyere & Kuranchie, 2013). Observational conditioning reflects that an individual learns something indirectly and learning could happen at anytime and anywhere. In the context of teaching and learning mathematical probability, students are believed to form attitudes towards mathematical probability through observations of various tutors' actions in the classroom such as the way tutors solve mathematical probability problems and so on. The cognitive component of attitudes towards mathematical probability also indicates students' views of the importance of mathematical probability in their daily lives, either at the present time or in the future.

Ayob and Yasin (2017) identified opportunity to learn factors related to classroom teaching and learning processes as content coverage (teaching materials, task orientations, task based on daily life situation), teaching practice (instructional method) and teaching quality (classroom management, classroom organization, learning environment). If these factors are carefully put in place and become workable then students' attitude towards mathematical probability would be achieved. Bayaga and Wadesango (2014) stated that Mathematics academic achievement may be seen in a situation whereby a student views herself/himself as strong or weak in mathematics and this seem the most important factor in achievement. The influence and behaviour of an important personality like an educationist could contribute strongly to a student changing his/her attitude in agreement with that important educationist (Ismail & Anwang, 2009). Below was the Conceptual framework for the study.

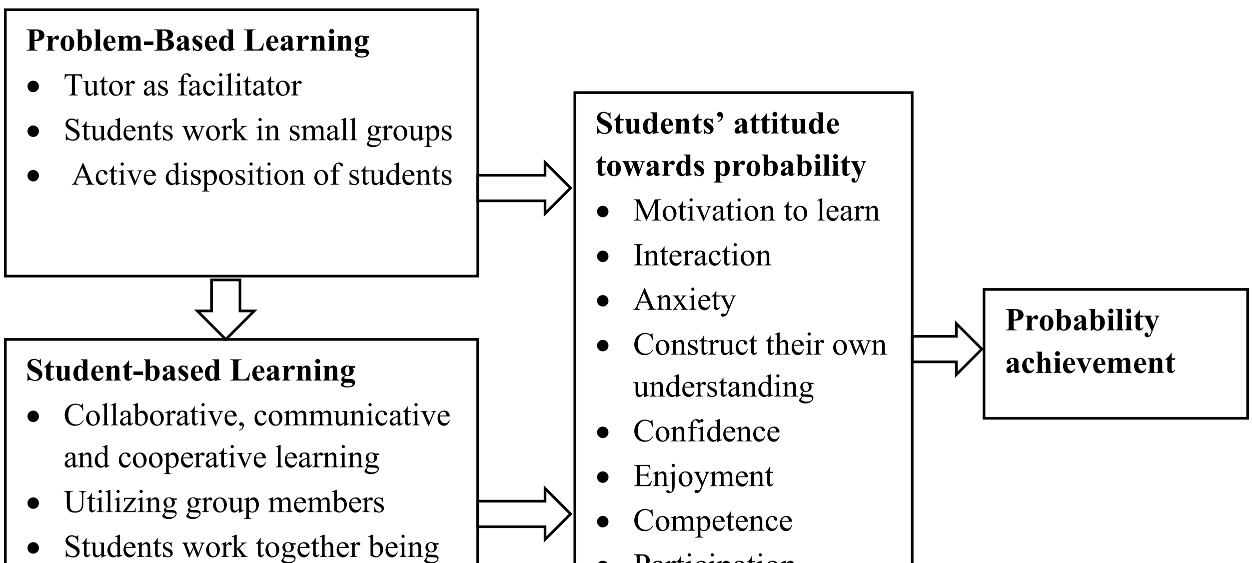


Figure 3: Conceptual Framework of Problem-Based Learning

2.5 Implementation of Problem-Based Learning in Mathematics Education

Research suggest that problem-based learning has proven as a successful educational method for development in a globalised higher institution and can prepare students to be flexible thinkers who can work productively with peers to solve problems, as well as communicate to each other and provide many essential skills to let students enjoy the fruits of active learning (Hmelo-Silver, 2004; Othman, Berhannuddin & Abdullah, 2013). The implementation of problem-based learning would be successful if problem-based learning students are based on problem-driven approach, self-directed learning and activities related to small group learning. The implementation of problem-based learning in mathematics education should be able to meet instructional needs and constraints for the level of self-directed learning by students. Problem-based learning should be based on factors that include various disciplines, learning goals, cognitive readiness and self-directed learning skills of students. Problem-based learning models are generated and practiced in mathematics educational setting (Savery, 2006). The development of problem-based learning model made problem-based learning flexible and robust pedagogical method for unique instructional needs in mathematics context.

Problem-based learning in mathematics education can be achieved if the implementation of problem-based learning in teaching and learning processes is based on tutor pedagogical content knowledge through effective scaffolding, problem-solving reasoning ability and self-directed learning by students. Mathematics educators should consider the following factors in the implementation of problem-based learning: problem-based learning instructional needs fulfilment, students' characteristics, problem-based learning model used in students practice and studies the rationale for selecting a particular problem-based learning model, the learning outcome generated and the type of assessment used (Hung, 2011).

Currently, the focus of problem-based learning assessment formats has shifted to not only assessment on student's performance from multiple choice questions but also learning processes through group assessment tasks and real-life easy type questions. Traditional teaching and learning situations have been and still dominant instructional methods for basic level, senior high school level and colleges of education setting in Ghana. Therefore, research have pointed out that, it was not realistic to expect students, most of who have studied under traditional curriculum for many years to shift their study habits and mindset within a short period of time (Hung, 2006). This shift in study habits from traditional approach to problem-based learning pedagogical approach require an external behavioural change and an internal psychological mindset change and this fundamental shift could be a long and difficult process if problem-based learning is not well implemented in mathematics education. Though, some habits are difficult to change if there are no immediate benefits from the change. Students old study habits could be difficult to change even when the instructional strategy has changed.

One major way to help problem-based learning students make an efficient and effective transition from traditional learning mindset to a problem-based learning mindset is to explicitly teach them the philosophy of problem-based learning. Knowing why, that is the philosophy of problem-based learning and rationale for application of problem-based learning as the instructional method may help students develop positive attitude toward the instructional method. This positive attitude would promote more active participation in the learning process and as a result would increase effective learning outcomes and a change of mindset. Effective teaching the problem-based learning philosophy and process can be seen as a short-term goal for developing a healthy mindset for studying under problem-based learning instruction, hence a successful implementation in mathematics education.

Besides, appropriate scaffolding is necessary for developing students' abilities and habits of the mind in assuming the role and tasks required in problem-based learning environments or learning situations. To provide appropriate scaffolding for problem-based learning students in order to implement in mathematics education (Simons & Ertmer, 2005) suggested that igniting students' interest and reducing the task to a level deemed achievable by the students could alleviate their difficulty in initiating the inquiry process; engaging students in the scientific self-directed learning process could help them see their misconceptions and providing prompts and modelling could promote students' reflective thinking about their solutions.

Another critical component is helping students to develop the mindset of taking responsibility for their own learning. However, in problem-based learning environments, the instructional goal is for the students to learn how to solve problems independently by conducting a scientifically good research and reasoning process.

Students' must identify what the problem is and what needs to be research and studied and then devised a solution (Simons & Ertmer, 2005). Thus, the criteria for evaluating problem-based learning students should be whether the students can articulate the critical elements of the problem, their process for solving it, and the solution proposed and defend their proposed solution and the rationale, rather than whether they match predetermined answers. When this achieved then, the implementation of problem-based learning in mathematics education can be additional knowledge for mathematics educators in the implementation of problem-based learning approach in the teaching and learning process.

Moreover, the successful implementation of problem-based learning apparently depends on many factors that will influence the effectiveness of problem-based learning strategy. In the context of this study, the factors that might influence the effectiveness of problem-based learning implementation are: Problem-based learning lesson plan or design, material development such as quality problem formulation, student problem-based learning presentation skills and tutor facilitation skills. This means that students can do well in Mathematics if problem-based learning is well implemented.

2.6 Related Empirical Studies

Espinoza and Sanchez (2014) carried out a study on problem-based learning to teach and learn statistics and probability. They employed quasi-experimental design where two groups are compared in academic performance, motivation and learning strategies. The results indicated a significant statistically change and positive evaluation in favour of the problem-based learning approach to traditional method of transmitting of learning.

Duran, Ozdemir and Kaplan (2015) carried out a research on the use of a problem-based learning approach: A case study of probability sample. The purpose of this study was to reveal the variation in the perceptions of middle grade students for problem-based components in teaching process of probability topics and also determine the attitudes of the students. The study made use of explanatory pattern of the mixed method using qualitative and quantitative data together. The results indicated that learners showed positive attitude and achieved learning goals and also a significant change in learners, self- evaluation and tutor learning process.

Uygun and Tertemiz (2014) carried out a study on effects of problem-based learning on students' attitudes, achievements and retention of learning in mathematics course. They employed pre-test post-test control group quasi-experimental research design. Experimental group received instruction on PBL and control group instruction was based in the guide of the Ministry of Education. The study revealed that there is no significant difference between pre-test post-test attitude score means of both control and experimental groups but rather a statistical difference between experimental group pre-test and post-test achievement score means, hence PBL had positive impact on students' academic performance.

Han et al. (2016) carried out a research to explore administering problem-based learning (PBL) approach in the teaching of College-level mathematics. The purpose was to investigate the effectiveness of adopting a structured PBL approach in improving students' performance, their perceptions and attitudes toward mathematics. They employed many designs such as quasi-experimental design of pre-test post-test control groups, survey (attitudes and learning environments) and audio-recorded group interviews. The results indicated that most students agreed they would be more motivated to learning mathematical concepts that are meaningful to them and also enjoy working in groups after problem-based learning.

Demirel and Dagyar (2016) carried out a research on the effects of problem-based learning on attitude: A meta-analysis study. Their method combined different results of similar and independent studies through statistical techniques. The sample consisted of research findings of 47 studies that meet the criteria for meta-analysis of the study to determine the effects of PBL on students' attitudes as compared to traditional teaching methods. The results revealed that PBL has a low positive effect on students' attitudes. It means that PBL is effective in helping students' gain positive attitude toward courses.

Fatade, Mogari and Arigbabu (2013) carried out a research to investigate the effects of problem-based learning on Senior Secondary School students' achievement in further Mathematics. They employed a quasi-experimental design of non-equivalent group. Intact classes of 96 students participated in the study where 42 in the experimental group and 54 in the control group. The experimental group was taught using problem-based learning and the control group was taught using traditional method. The study revealed that there exists a significant difference between the experimental and control group in favour of the experimental group.

2.7 The Concept of Probability

To understand the concepts and skills associated with probability, one must develop new ways of thinking which are largely dependent on instruction (Fischbein & Schnarch, 1997). Probability is an important part of mathematics. Konold (1983) explained that College students need an "outcome orientation model" which involves making yes or no decisions about single events. He added that probability is important for College students for being careful in making objectives for instruction at the basic level. Teachers and mathematics educators have to understand probability concepts and students' conceptions of probability and develop a critical thinking as a means of informed instruction.

For that matter, the main aim of preparing teachers is the epistemological reflection which can help them understand the role of concepts with probability and other areas, its important in students' learning and students' conceptual difficulties in problem solving (Batanero, Godino, & Roa, 2004).

Lee, Doerr, Tran and Lovert (2016) defined "probability as an abstract concept that cannot be directly measured". They further stated that, there exist many objects or real-world events whose outcome cannot be completely determined unless you know all information about the actions on the object or event in the world since we cannot be able to determine the outcome of a coin toss unless we know all the physical forces acting upon the coin while it is tossed.

Gorini (2012) also defined probability as the mathematical study of random behaviour; and that it uses precise and exact methods of mathematics to find patterns in disorderly and unforeseeable situation governed by chance. She added that probability is categorised into three basic aspects which include: theoretical probability, empirical probability and subjective probability.

Theoretical probability is based on assumed understanding of situations. For example, tossing a coin or drawing a deck of cards, etc. Empirical probability is based on data collected from repeated trials or experience. Example, the probability of getting a double with loaded dice, a Biologist studying animal behaviour, Doctors evaluating a new drug and so on. Subjective probability is a personal estimation of the likelihood of an outcome or estimates that something will happen. It depends on a prior knowledge or judgement. Example, 50-50 odds of cleaning a room tomorrow, the odds of getting a ticket to the concert before all the tickets are sold out, etc.

In literature, the concept of probability is defined as the mathematical value of the realization or occurrence of anything (Kaplan, 2008). The probability of occurrence of an event is indicated by P(A). That is:

$$P(A) = \frac{Number\ of\ success\ of\ event\ A}{Total\ Number\ of\ outcomes}$$

The probability that individuals make use of when deciding on confronting uncertain situations; chance games, economics, insurance, meteorology and genetics is used in the current life and the scientific world (Kaplan, 2008). The probability of developing independent creative thinking skills has been included in the mathematics curriculum of many countries due to the fact that individuals help in the right decision-making mechanisms and their importance in the economic social field (Kaplan, 2008; Koparan & Guven, 2014). In fact, as a result of the recommendations of the National Council of Mathematics Teachers in the United States (NCTM, 2010), probability has become one of the learning areas in the curriculum including Pre-Primary, Primary, Secondary and High Schools (NCTM, 1998).

Parallel to the developments in Ghana, Ministry of Education has recently made some innovations in Primary and Junior High School Mathematics education syllabuses. In the Upper Primary School's simple probability estimates are included in the learning areas of mathematics topics while application of probability is in set theory and statistics at the Junior High Schools including non-discrete events. In the Senior High Schools mathematics education there are permutations, combinations, independent and conditional probability and statistics in the syllabuses.

Although, the PBL approach is a methodology that education experts closely witness, it is seen that this approach is not used sufficiently in mathematics education as much as science education. In the Colleges of Education mathematics curriculum, there is no planning for probability topics related to problem-based learning approach. In addition, the deficiencies concerning a teaching approach such as writing for students to reach results and using traditional teaching approaches requires that students use PBL approach in the process of learning.

Greer (2001) identified three principles of instruction to build on existing intuitions or conceptual structures and to promote the construction of new ideas.

These principles highlight the need to:

i. provide prolonged and focused experiences with situations of chance;
ii. create and use representations and generative models; and
iii. address the cultural bias toward deterministic thinking.

The last principle underscores the importance of including probability instruction in school mathematics curriculum. Children and adult students need to recognize that situations involving chance can be examined and described logically and rationally, thereby becoming the most fundamental aspect of developing an understanding of probability (Langrall & Mooney, 2005; Graham, 2005). Watson (2005) emphasised that probability in terms of chance and risk appear in many contexts especially in school curricula, traditionally suggested the teaching of probability as part of pure mathematics and the examples provided were based on finite sample spaces for which it was possible to list, count and compare outcomes explicitly, hence questions were developed based on dice, coins or deck of cards.

In contrast, early research on probabilistic understanding by Psychologists with college students tended to focus on descriptive social settings within which questions relative but not necessarily numerical, likelihood could be placed (Tversky & Kahneman, 1983; Graham, 2005). For this reason, it is imperative to acknowledge sample spaces relative to social intuition and classroom practices. The language in probability pervades almost everything we do, thereby given a great opportunity for children now and in the future which will increasingly meet chance variation and random phenomena not only in mathematics but in the Media, Meteorological and Financial forecasting and in social activities such as games, sports and gambling (Pratt, 2005; Graham, 2005). They added that, sports commutators talk about 50/50 ball, weather forecasters announce an 80% chance of rain and health is assessed based on risk factors as a result of probabilistic calculations. In order to accommodate probabilistic thinking students and tutors need the capacity to recognize uncertainty and to be able to catalogue systematically all possible combinations through the stages of formal operations (Piaget & Inhelder, 1951/1975; Pratt, 2005). Piaget's approach was to examine the theory of learning of probabilistic knowledge from genetic perspective as such he was less interested in how the setting might shape such development. In contrast, teachers deal continuously with the partial knowledge of their students and consequently they need guidance on how their actions, including the offering of certain types of resources, might shape students' knowledge development. Probability therefore provides a theoretical structure for statistical inference on the notion of considering what would happen if the method was repeated several times. Some mathematics educators used computer-based simulation approaches to improve students conceptual understanding of statistical inference and the probability distribution models that are used. This will eventually help College students' in their project work on stating hypothesis, calculating test statistics and finding P-value, hence modern research suggests better teaching methods needed to improve students conceptual understanding of sampling in relation to statistical inference (Pfannkuch, 2005; Watson, 2004).

Kaplan (2008) stated that students' failure to think critically about ideas of probability comes from their inability to handle rational number concepts and the reasoning which are used in calculations, reporting and interpreting probability concepts. Students' weaknesses in dealing basic concepts involving decimals, fractions and percentages were also identified as the main threat to the understanding of probability concepts (Tso, 2012).

Heyvaert et al. (2017) used mixed method approach in solving probability problems with students in high school. They stated that in designing mixed method study, different purposes call for different combinations of three key design dimension; that is, status (unequal Vs. equal), timing (sequential Vs. simultaneous) and independence (interactive Vs. independence). This method helped them applied pragmatism as their philosophical stance that indeed yielded positive results in solving probability problems. Many educational researchers focus on the mistakes made by children and adolescents when solving probability problem and explain these mistakes by referring to terminologies such as heuristics, biases and misconceptions (Garfield, 2003; Morsanyi, et al., 2009; Pratt , 2000).

Chng, Yew and Schmidt (2011) defined problem-based learning as learning that is driven by problems. They proposed that tutors who are more cognitive congruent would utilize subject-matter knowledge in a better way and be more socially congruent, which ultimately translates into higher student performance. Subject-matter knowledge would equip tutors with the ability to follow closely and contribute effectively to the discussions generated by students (Schmidt & Moust, 2000). A study conducted by Kassab et al. (2006) found that effective tutors were perceived by students as those who respected their opinions, were able to establish good communications, understand their feelings and advise them on how to learn. This means that possessing subject-matter alone is insufficient. Without a genuine interest in the lives and learning process of the students, tutors would lack sensitivity to the difficulties faced by students, thus hindering their ability to guide students' learning. Hypothetically, tutors exhibiting more cognitive congruent behaviours would influence knowledge construction and acquisition at each learning stage of the problem-based learning process. As learning in problem-based learning curriculum is considered to be cumulative where knowledge is built upon that which was learnt in the previous learning stage (Chng, Yew, & Schmidt, 2011). Students under tutorship of such tutors should be more extensively involved in the construction of knowledge and ultimately achieve better results at the end of the learning process.

The concept of probability is very important in making students understand their social environment and be able to do calculations and also determine various chances of specific events to occur or not to occur (Brijlall, 2014). Tutors need to understand mathematical concepts underlying questions that students ask but they also need to know how these concepts can best be explained. Tutors are not only expected to know that some concepts are as they are, tutors should also know why they are so. This requirement brings the idea of pedagogical content knowledge. The main purpose of pedagogical content knowledge was to bridge content knowledge with the practice of teaching. A tutor should know the type of context that would be interested and easy understood by their students. It was the tutor's ability to choose task that students would find motivational and the ability to identify errors common to their students. Content knowledge is necessary but is not sufficient on its own (Brijlall, 2011). A tutor to introduce a new concept in probability should know what the students already know and must therefore be able to sequence his/her lessons in a manner that is likely to enhance and facilitate learning of a new concept. Knowing the content and knowing how to teach helps the tutor to identify situations where students' suggestions can be attended to or be rejected for a later stage. Tutors therefore should have a global picture of the concept they teach. They should know what was done in lower classes that would be needed in higher classes and beyond and provide the linkage. This displays content knowledge and curriculum and is very crucial for colleges of education students. It was imperative that tutors' knowledge of content and teaching be put into correct practice. Shulman (1986) referred to this knowledge as pedagogical content knowledge when he said that this knowledge "goes beyond knowledge of subject matter knowledge for teaching" (Shulman, 1986). He added that analogies illustrations, examples and demonstrations that make the subject comprehensible to others are included in the pedagogical content knowledge. A tutor therefore has the duty to ensure that students realized the need to know that when expressing any probability, one has to know all the possible outcomes first. Tutors are expected to use the three domains of teaching: content knowledge, pedagogical content knowledge and curriculum knowledge in teaching probability problems. Pedagogical content knowledge plays a crucial role in making the teaching process more application based and meaningful. It is therefore not adequate to perceive pedagogical knowledge as the strategies used in teaching and classroom administration methods only.

2.8 Difficulties and Errors College Students' Make in Solving Probability Problems

O'Connell (2010) carried out a research investigating relationships among different types of errors occurring during probability problems solving. The subjects were 50 non-mathematically inclined college students in the Psychology and Social Sciences department enrolled in introductory probability and statistics course, who were asked to solve probability problems. The errors of these students were analysed and put under the following categories: text comprehension errors, conceptual errors, procedural errors and computational errors. The chief Examiner's report, UCC (2017, 2018), on Statistics and Probability course stated categorical that problems that College of Education students encounter in probability or probability reasoning was that of probability concepts and principles which were consistent with that of O'connell study.

Table 2: Errors Categories

CATEGORY	DESCRIPTION
Text Comprehension	Misunderstanding of information contained in a text of a problem, assigning probability value to wrong event and misinterpreting probability statements.
Conceptual	Errors that involved basic concepts/definitions of probability, wrong reporting of negative values of probability or values greater than 1.0 and misunderstanding of independence, mutually exclusive events as equally likely with good justification.
Procedural	Wrong procedures in the application of formula without preconditions checking, using incorrect formula or substituting incorrect values into the formula and inventing incorrect procedures, inappropriate methods or not completing a method during calculation.
Arithmetic	Errors that involved basic miscalculations, copy mistakes such as incorrect cancellation of terms in rational fractions and wrong transposing of digits.

Source: Ann Aileen O'connell, 1999 revised in 2010.

The difficulties experienced by students with the learning of probability are not well-documented (Pratt, 2012; Gage, 2012). The span of students' problems ranges from difficulty in understanding, reasoning, conflict between the analysis of probability in Mathematics lessons or exams, interpreting probability statements of problems and experience in real life, exposure to the abstract nature and formalized presentation in Mathematics lesson.

Due to modernity, rich society need to understand probability since people are faced more with making decision in an environment that involves a lot of uncertainty (Mutodi & Ngirande, 2014). College of education students' have difficulties in understanding fundamental ideas of probability. Most of the students lack basic rational number concepts and had difficulties with concepts involving fractions, decimals and percent which are used in calculating, reporting and interpreting probabilities (Behr, Lesh, Post,

& Silver, 1983; Carpenter, Corbitt, & Kepner, 1981). Many of the students have already developed a distaste for probability through having been exposed to its study in a highly abstract and formal way at the senior high school level.

To overcome these difficulties in probability (Garfield & Ahlgren, 1988) postulated that tutors should:

i. introduce Probability through activity-based and not in abstraction.
ii. try to arouse in students the feeling that Probability relates usefully to reality and is not just rules and formulae.
iii. point out to students' common misuses of Probability in news stories and in weather forecasting.
iv. use strategies to improve students' rational number concepts before applying Probability reasoning.
v. recognize and attack common errors in students' Probabilistic reasoning.
vi. create instances that require Probabilistic reasoning and link to students' views of the world.

The difficulties can also be attributed to Pre-service teachers not given ample opportunities for collaborative learning experiences and the application of what they have learned in daily-life and in other courses to Statistics and Probability (Olpak, Baltaci, & Arican, 2018).

CHAPTER THREE

METHODOLOGY

3.0 Overview

This chapter deals with methodology used in the study and includes the research design, the description of the population, sample and sampling procedures, data collection instruments and the procedure for data collection as well as data analysis. The study employed both quantitative and qualitative data. It also outlined briefly the research design that was used in the study because of the intervention.

3.1 Research Design

The study employed Quasi-experimental design as the research design. A research design refers to the specific methods and procedures applied in a research study to answer a research question (Privitera, 2014; Creswell, 2014; Kothari & Gaurav, 2014). Specifically, the study used a non-randomised pre-test post-test control design. Quasi-experiment is an empirical study used to estimate the causal impact of an intervention on its target population (Vanderstoep & Johnson, 2009). Quasi-experimental researchers are widely used in the evaluation of teaching interventions since it was not practical to justify assigning students to experimental and control groups by random assignment (Creswell, 2009). Quasi-experimental researchers offered the benefit of comparison between groups because of the naturally occurring treatment groups (Cohen, Manion, & Morrison, 2011). A non-equivalent control group pre-test, post-test quasi-experimental design was suitable since intact groups of students were used in the experiment rather than assigning students at random to experiment treatments. The design involved the use of methods and procedures that made observations in the study structured similar to experiments, but the conditions and experiences of students lack some control because the study lacks random assignment and includes a pre-existing factor (Wiersman & Jurs, 2005). According to Privitera (2014), a design is chosen when one wants to compare scores before and after treatment. The study compared the students' scores in probability problems achievement tests before and after treatment in both the experimental and control groups. The ultimate goal of a good research design is to provide a credible answer to the research questions (Mcmillan & Schaumacher, 2006).

A problem-based learning approach was employed to investigate College of Education students' achievement in, and attitude to probability. It involved the application of theory and practice, knowledge and skills and appropriate intervention strategies aimed at finding solutions to probability problems identified in teaching and learning situation in order to bring about change. Problem-based learning approach was preferred in this study because it dealt with a smaller scale intervention which was appropriate to a classroom situation in which the study was carried out. One aspect of the design involved interviews conducted using both simple random and purposive sampling techniques on students' level of understanding of probability based on concepts application of Pre-test and Post-test scores. The choice of the study was also informed by other studies in the same area (Espinoza & Sanchez, 2014; Uygun & Tertemiz, 2014).

Other researchers who have carried out studies on effectiveness of problem-based learning used this design (Iji, Emiakwu, & Utubaku, 2015; Han et al., 2016; Fatade, Mogari & Arigbabu, 2013; Alfred, David, & Abayomi, 2013).

They were two groups selected by simple random and convenience sampling technique, namely experimental and control groups. Both Pre-test and Post-test were conducted using Probability Achievement Test (PAT).

Table 3: Group Pre-test and Post-test Design

Group	Pre-test	Treatment	Post-test
Experimental Group	Pre-test of PAT	Problem-Based Learning	Post-test of PAT
Control Group	Pre-test of PAT	Traditional Instruction	Post-test of PAT

Table 4: Design of the Study

Research Design	Quasi-Experimental Pre-test Post-test Design
Sampling	Convenience, Purposive and Simple Random Sampling
Instruments	Probability Achievement Test (PAT)
	Probability Attitude Scale (PAS)
	Semi-Interview Guide
Data Analysis	Descriptive Statistics, Analysis of Covariance (ANCOVA), T-Test and ANOVA

Kennedy-Clark, (2013) noted that a design-based research is based on designing an instrument or a tool or an intervention that support and lead to deepened understanding of learning. Hence, a Problem-Based Learning lesson plan was design in the study.

3.2 Population

The population for the study 515 students of Colleges of Education in the Northern Region of Ghana. There were five (5) Government Colleges of Education and one (1) private College in the Northern Region, Tamale. A population in research refers to a group of individuals that have one or more characteristics in common and of an interest to the researcher (Best & Kahn, 2006). The Region was selected using convenience sampling technique. The average age of the students was twenty years and they came from different Regions of Ghana. As the students' population was definitely very large to handle, it was therefore necessary to work with a sample of the population.

3.3 Sample

The sample population was two Colleges of Education in Northern Region, Tamale. The two Colleges of Education were categorised as Control and Experimental groups. The Control group was made up of fifty-one (51) students and the Experimental group forty-nine (49) students. This group of students were chosen because they had been taught probability and Statistics at the Senior High School. The two Colleges of Education were selected by convenience sampling technique due to proximity, availability of students, accessibility and transportation convenient to the Colleges. The Colleges had a population of five hundred and fifteen (515) general programme students. These were majority of students who had not studied or done Elective Mathematics at the Senior High School level. Tuckman (1999) described a sample as a small proportion of the population selected for the study and analysis. The accessible population of the study results was generalized with respect to second year students in Tamale Metropolis. In these Colleges of Education for the study, students were already assigned to classes for instructional purposes at the beginning of the semester. The classes were assumed to be heterogeneous.

Table 5: Number of Students in the Experimental and Control Groups Based on Gender

	Female	Male	Total
Experimental Group	15	34	49

Control Group	20	31	51
Total	35	65	100

Evidently, in Table 5, 49 students were in the Experimental Group and 51 were in the Control Group. Moreover, 35 female and 65 male students took part in the study in terms of gender.

3.4 Sample Procedures

Sampling procedures for the purpose of this study was a combination of convenience sampling and simple random sampling. Simple random sampling is a process of selecting a sample from the population in a way that every different possible sample of the desired size has the same chance of being selected (Tuckman, 1999). He added that this type of sampling was also known as chance or probability sampling.

According to (Etikan, Musa, & Alkassim, 2015), convenience sampling (sometimes called accidental sampling or haphazard sampling) is a type of nonprobability or non-random sampling where members of the target population that meet certain practical procedure such as easy accessibility, geographical proximity, availability of a given time or the willingness of subjects to participate are included for the purpose of the study whilst purposive sampling technique, also called judgement sampling is the choice of a participant deliberately due to the qualities the participant possesses. In fact, purposive sampling is a non-random technique that does not need any underlying theories; the researcher decides on what needs to be known and sets out to find participants who can and are willing to provide the information by virtue of knowledge or experience. It is typically used in qualitative research to identify and select the information-rich cases for the most proper utilization of available resources (Patton, 2002). Besides knowledge and experiences, the importance of availability and willingness to participate and the ability to communicate experiences and opinions in an articulate, expressive and reflective manner (Spradley, 1979 as cited by Etikan, Musa & Alkassim, 2016). The study employed these three sampling techniques at various level. First, convenience sampling technique was used to select the region and also the colleges because of proximity, easy access and special characteristics of the college in facilitating the purpose of the research. Simple random sampling was the second sampling techniques that the researcher used to select the classes for the study and also categorizing the classes into control and experimental groups and finally purposive sampling was used to select students for interview after the pre-test. The sample units in the population was selected by a random process, using a random number table so that each person in the population would have the same probability of been selected for the study.

One of the intact classes was 82 students which were assigned numbers from 01 to 82 with their respective names. The researcher used excel software and generated list of random numbers by launching excel on the computer and typing in the cell =randbetween (bottom, top). The number of digits chosen for bottom and top would be generated with such digits. When drag would result a list of random numbers in rows and columns. The row was chosen with two digits as a start point, when the selected two-digit numbers coincide with any of the student's numbers, that student would be selected for the study. The excel random number generator technique was used to select 49 students for the experimental group. The second intact class was 51 students and was made the control group. The two Colleges of Education were labelled College A for Experimental group and College B for Control group by a tossed of the coin. This ensured that bias was eliminated while giving equal opportunities to each sample point selected.

3.5 Research Instruments

The instruments for the study were;

i. probability achievement test (Pre-test and Post-test),
ii. questionnaire and
iii. semi- structured interview guide.

Tests was designed alongside with two types of lesson plans, one for Control group (See Appendix G) and another for the Experimental group (See Appendix H). The Control group was taught using traditional/conventional method and the Experimental group was taught using Problem-Based Learning approach. A Questionnaire was also designed on students' attitude towards probability where both negative and positive questions were blended in it.

3.5.1 Probability achievement test

The achievement test was made up of pre-test and post-test. The pre-test and post-test consisted of ten (10) typed probability problems. They were administered before and after the treatment. Both tests assessed students content knowledge and understanding of probability problems. The questions were five multiple choice and five essays typed and students were expected to work and come out with their own solutions to all questions. The results of pre-test answered research question one and the post-test answered research question two while the questionnaire results answered research question three. The interviewed results answered research question one and two.

3.5.2 Questionnaire

A five-point Likert scale questionnaire results answered research question three. The questionnaire was made up of three parts for which the first part would investigate college students' background information. The second part of the questionnaire was on

College students' attitudes/misconceptions toward Probability problems and the third was to find out the effect of Problem-Based Learning on Probability which consisted of thirty statements on a five Likert-type rating scale. The College of Education students was requested to show their attitudes/misconceptions on the Likert-scale which was scored from; 1-Strongly Disagree, 2-Disagree, 3-Undecided, 4-Agree and 5-Strongly Agree. A descriptive analysis of students' percentages response was done to find out the attitude of students to probability.

3.5.3 Semi- structure interview guide

Seven students from groups of seven were selected based on simple random sampling technique and three other students were purposively selected by virtue of their knowledge and experience for interview during the intervention process. In each group six no and one yes were written on a piece of papers and put in a basket. Members within each group picked the pieces of papers. A student with yes was selected for interview to explain how they solve the probability problems and the difficulties encountered by them. In all 10 students were selected for the interview.

3.6 Validity and Reliability of Instruments

Validity of an instrument is the extent to which the items in an instrument measure what they are set to measure (Fraenkel & Wallen, 2010). Validity therefore refers to the appropriateness, meaningfulness, correctness and usefulness of the inferences a researcher makes based on the data collected. In validity of test instrument, the prescribed textbook Mathematics for Colleges of Education and course manual on Statistics and Probability were used with the assistance of a colleague tutor. The purpose was to gain insight into what learners are expected to learn under Probability to develop the instrument accordingly. After constructing both tests and questionnaire, a colleague tutor was consulted to cross-check them and also to further ensure that the content chosen was within the scope or prescribed domain of the study.

Reliability on the other hand is defined as the extent to which results are consistent over time and an accurate representation of the total population under study. Reliability refers to the consistency of scores or answers from one administration of an instrument to another and from one set of items to another. If the results of the study could be reproduced under a similar methodology, then the research instrument was considered to be reliable. The validity of the instrument was further ascertained through pilot-testing while a Test-Retest was used to estimate the degree to which the same results could be obtained in a repeated trial. One College was used to obtain responses in the same students but at different times within two weeks. It was expected that the responses in the 1st and 2nd administration would be nearly the same if the instrument was reliable (Orodho, 2010). Alternatively, the two sets of results could best be compared by calculating a correlation coefficient which was not adopted by this study. The study used test-retest reliability method to assess the test items which was given to students in a particular College of Education in the Tamale Metropolis. The test items were administered after the piloting stage, two times within two weeks. The results of the 1st and 2nd test scores were almost the same, the difference was not significant which showed that the test items were reliable and standard. The Cronbach's Alpha coefficient of the Likert scale questionnaire was 0.79.

3.7 Internal Validity

Students were from different regions of Ghana and from families with similar socio-economic status. Thus, subject characteristics was not a threat. All the students were present during the collection of data. For this study, location and history were not a threat because all measuring instruments administered in the classrooms almost at the same time. Physical conditions were not a problem because all the classes were in the same floor with equal conditions, therefore implementation was not a threat.

Biasedness during instruction or intervention was not a threat since an observer researcher's colleague in a different College, observed lessons in two of the groups and found instruction suitable and not biased. Besides, scoring Pre-test and Post-test, researcher made photocopies with the same marking scheme discussed with a colleague tutor and scripts were marked and scores were the same except few minus one and plus one, marks in some few scripts. Therefore, data collection characteristics and bias could not be a threat.

3.8 External and Population Validity

The nature of the sample, the environmental conditions and the setting within which the study takes place must be considered in thinking about generalisation. The extent to which the results of a study was generalised determined the external validity of the study. In this study, convenience and random sampling was used so generalisation of the results was limited to only second year students at the Colleges of Education in the Northern Region, Tamale. Generalisation was done to subjects who had similar characteristics to that of the subjects in the study.

3.9 Ecological Validity

Ecological validity refers to the degree which results of a study could be extended to other settings or conditions Frankel and Wallen, (2010). In this study, the interventions using PBL were conducted in classroom of the treatment group (Experimental) while the control group received regular/traditional method of teaching. The results of this study could be generalised to classroom setting similar to the present study.

3.10 Pilot Testing of the Instruments

A pilot study was done in one of the Colleges of Education in Tamale with the same characteristics as those sampled for the study. The purpose of the pilot study was to get a clearer picture of the demands of the main study with regard to time. Piloting the instrument was also to test its reliability and content validity and to identify and rectify problem areas in the questions. The pilot test consisted of five (5) multiple choice typed questions with seven (7) essay typed questions. The test was administered to fifteen (15) students selected randomly from General Programme class. The results of the Pilot test necessitated corrections such as reducing the number of questions from 12 to 10, thus five (5) multiple choice and five (5) essay typed question was finally administered to students The changing of certain questions and amount of time within which the students were to answer the questions confirmed what (Creswell, 2012) stated that, "Piloting of test is a procedure in which a researcher makes changes in an instrument based on feedback from a small number of individuals who complete and evaluate the instrument". The semi-structured interview guide was tested with three other students from a particular College within Tamale with similar characteristics before applying it to the sample of 10 students. The interview lasted 10-15minutes.

This exercise was very useful in the study as it revealed unforeseen problems that may emerge during the main investigation.

A pilot study is a small study conducted prior to a larger piece of research to determine whether the methodology, sampling, the instruments and analysis are adequate and appropriate (Bless & Higson, 2000). In fact, the mini-research of the study exposed deficiencies such as rewording of key terms in probability problems, duration for completion of probability problems and how to formulate nature of probability problems relating to real life situation. Piloting is done to determine the feasibility of using a particular research instrument in a major study.

3.11 Data Collection Procedure

The researcher sought permission from Colleges of Education where the study was carried out. The study began with pre-test of ten (10) standard questions made of five multiple-choice and five theory to be answered by both groups (Control and Experimental groups) marked out of 50% (See Appendix A). The researcher used five weeks to teach both the experimental group and the control group with their appropriate lessons, this teaching was done by the researcher in both Colleges of Education. This was because lesson presentation and delivery varied. Effective tutoring should respect the views of students, possessed good communication skills, understand how students feel and advise them on how to learn. The tutor should exhibit cognitive congruent behaviours that influence construction of knowledge and application at each learning stage of Problem-Based Learning process. There was no influence between students and tutor in the teaching and learning environment. Independence of students were assured since two Colleges of Education were used for the study. The five weeks tuition ended with post-test of ten (10) standard questions also made up of five multiple-choice and five theory different from the Pre-test given to both groups to complete in order to help the researcher to answer research question three (See Appendix B). A questionnaire was administered to students to ascertain their attitudes/misconceptions on probability problems (See Appendix C). Marking schemes for Pre-test (See Appendix D) and marking scheme for Post-test (See Appendix E).

3.12 Data Analysis Procedure

According to Tuckman (1999), data analysis is the execution of the research plan. Quantitative data was employed in the study. Basically, quantitative analysis in educational research is of two types; descriptive data analysis and inferential data analysis (Durrhein, 1999). Descriptive analysis seeks to organize and describe the data by investigating how the scores on different constructs are related to each other (Durrhein, 1999). Therefore, the study employed descriptive data analysis in an attempt to understand, interpret and describe the experiences of the research participants. An inferential statistic with independent samples t-test at 95% confidence level was used to compare students' conceptual understanding of probability problems. Analysis of covariance (ANCOVA) was used on data of post-test of both groups since there was statistically significant difference in the means of both Control and Experimental groups pre-test and descriptive analysis was also used on data from the attitudes of students toward Probability. To illustrate and compare the pre-test mean score of Probability Achievement Test (PAT) and Probability Attitude Test (PAS) and T-test was used. All statistical analysis was done by SPSS IBM 20. The Pre-test results of both Experimental and Control groups were used to answer research question one and the post-test of both groups was used to answer research question two. Research question three was answered by the Likert questionnaire results.

The descriptive and inferential statistics were used to analyse data, paying particular attention to students' ability to:

i. analyse the probability questions correctly.
ii. represent information using tables and outcome tree diagrams.
iii. choose suitable strategy leading to the correct solution.
iv. determine the solutions to probability problems relating to real life situations.

3.13 Intervention

Problem-based learning approach provides students with guided experience in learning through solving complex, real- world problems (Padmavathy & Mareesh, 2013). The following Problem-based learning steps were involved:

i. problem encounter: explain statements and concepts.
ii. problem analysis: brainstorm/try to explain the problem.
iii. research and generation of learning issues: formulate learning issues and self-directed learning.
iv. discovery ways of tackling problem and peer teaching: group meetings on report and evaluation of self-directed learning.
v. solution presentation: report of presentation and evaluation process.
vi. reflection and evaluation: reflect on learning issues and define further action.

The researcher devised series of statements that also guided the learner step by step making a series of discoveries that led to a single predetermined goal.

Besides, the researcher initiated a stimulus and the learner reacts by engaging in active inquiry thereby given the appropriate responses. The ten (10) questions outlined the various difficulties students experienced in solving Probability problems. This was to investigate their background about Probability concepts and its applications. The questions were answered within 60 minutes. A marking scheme was prepared (See Appendix D) and was used to mark and record pre-test raw scores. Students were randomly drawn from the seven groups for the interview after the pre-test. The purpose of this sampling strategy was to capture major variations (Patton, 1990).

Ten (10) students participated in the interview phase. The interviews conducted were semi-structured, implying that the topics and questions were predefined, but that of the sequence of the questions were reworded. Semi-structured interview was used because it allows the interviewee to express himself/herself in his/her own words and to a certain degree to give direction to the interview. One additional advantage was that, semi-structured interviews allow the interviewer to add questions during the interview based on the answers of the respondent, thus to deviate from a priori constructed list of relevant topics and questions. The questions asked during the interview were open-ended. The 10 students selected were interviewed in order to study the students reasoning ability when confronted with Probability problems and the use of problem-based learning approach. The quantitative test formed the basis for these interviews.

The semi-structured interviews were conducted in one of the smallest lecture room where students meet for group discussions. It was the perfect location for the interview because of its accessibility and conducive nature. The conducive nature of the room setting was assumed to allow more open interviews with more input from the students. The interviews were sound recorded. An interview guide was developed to aid the researcher in conducting the interviews. This guide helped to guard the structure and flow of the interviews without the loss of openness and was designed to encourage probing. The interview started with a brief introduction to the interview and the topic, dealing with important ethical issues, that is asking permission for sound recording, confidentiality, the anonymous analysis and reporting of data and some warming up questions that were meant to make students feel at ease.

Thereafter, students were made aware that questions consisted of ten items from the test and the effect of problem-based learning. The interviewer presented each item together with answer (initially) marked by the student and asked him/her to tell him how he/she handle this probability problem and how he/she arrived at an answer to the problem. Example of probing question that was asked was as follows: could you tell me a bit more about how you handled this probability problem? How would you explain this probability problem to your fellow student who is not familiar with probability? Could you explain a bit more detail how you arrived at this answer?

Students who changed their initial answer during the semi-interview were also asked about the rationale behind their change and how he/she arrived at this (new) answer. When some students said they doubted between several answers, the interviewer asked them about their rationales behind these different answers. When the interviewer was about to end the interview, students were asked whether there was anything else they wanted to talk about the topic or about the interview itself. The interviewer finally thanked each student and appreciated the time that they took for the interview. The interviewer was an 'active listener' who was only interested in the responses given by students through follow-up questions and body language.

The findings from the pre-test results necessitated the design and implementation of an intervention. This was as a result of learning being optimised in the old style of classroom where students sit quietly and passively and listen to knowledge being given to them by a lonely instructor standing in front of the class (Robert, 1994). Modern ways of learning emphasised the integration of specific concepts and classroom contents for enhancing students' critical thinking skills and problem-solving ability (Shahbodin & Zareema, 2013). The challenge of globalization today requires students to master problem-solving skills and positive attitudes and values besides good conceptual knowledge of Mathematics. Interventions were therefore put in place taking into consideration the outcome of the interview the researcher had with the students which revealed that most of them had problem in analysis probability problems, application of probability concepts and determining the solution to probability problems.

At the intervention stage for the Experimental group, the researcher applied Polya's heuristics and Othman et al. model of teaching that helped addressed College of Education students understanding of probability problems. The researcher also applied tasks in the form of practical activities that enhanced students understanding of probability problems. This meant relating classroom activities to learner's life experiences to enable them see the relationship between what was taught in the College of Education and what was done at home thereby facilitating transfer of teaching and learning. During the intervention stage of the Experimental group the students were given strategies, opportunities and response to what they were doing. The intervention period took five weeks of two credit hours Mathematics session (60 minutes per session) through Polya's instructional activities on Problem-based learning.

In the first two weeks, College of Education students were taken through by activity one, in order for them to discover fundamental principles and properties (Axioms) of Probability. The researcher as a facilitator based on his pedagogical content knowledge guided them through the conditions necessary for applying the concepts and properties/statements in probability.

In the third week, the students were taken through the use of outcome tree diagram in solving probability problems with replacement and without replacement. Activity two was done to reveal the concepts on how to use tree diagram to come out with sample space, for example tossing a coin once, twice etc. The main concepts were explained to students after observing their answers. In the fourth week, activity three, students were made to work in groups of seven to help each other solve probability problems. Activity three was done to let students do research and self-directed learning before meeting in their various groups to socially, collaboratively, cooperatively interact based on problem-based learning approach using Polya's heuristics that helped students discovered and comprehended difficult concepts. This also improved their communication skills.

The use of Polya's problem solving technique in solving the problem 1:

A bag contains 4 red balls and 3 green balls. A ball is taken out at random and then put back, a second ball is then taken from the bag. What is the probability that:

a) both balls are of the same colour?
b) at least one of the balls is green?

Solution

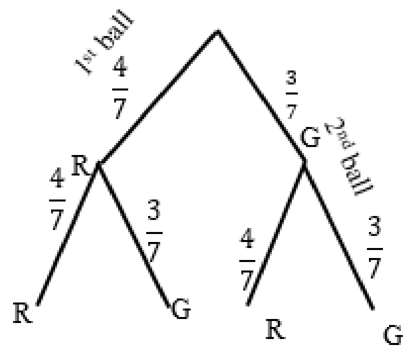
Students were made to apply Polya's problem solving techniques based on the four steps: understanding the problem, devising a strategy to solve the problem, carrying out the strategy to solve the problem and checking the results of your problem whether they are the results you want to arrive at (looking back).

Students illustrated the following in solving the question given:

1. **read** the problem. We are asked to find the probability of balls with the same colour and at least one colour being green with the concept of with replacement **(Understanding the Problem).**
2. **select** variables to represent red balls and green balls (**Devising strategies of solving the problem).**
3. **think** of a plan: translate the information on a tree diagram **(Carry out the plan).**
4. **use** your plan to solve for the solution by representing R with red balls and G with green balls **(look back at your final results).**

   n(R) = 4; n(G) = 3; total number of balls, n(U) = 7

   Tree Diagram



Outcomes [RR RG GR GG]

Probabilities: $\frac{4}{7} \times \frac{4}{7}$; $\frac{4}{7} \times \frac{3}{7}$; $\frac{3}{7} \times \frac{4}{7}$; $\frac{3}{7} \times \frac{3}{7}$

a) P (both are same colour) = P (RR) or P(GG)

$$= \frac{4}{7} \times \frac{4}{7} + \frac{3}{7} \times \frac{3}{7}$$

$$= \frac{16}{49} + \frac{9}{49}$$

$$= \frac{25}{49}$$

b) P (at least one green ball) = P(GG) or P(GR) or P(RG)

$$= \frac{3}{7} \times \frac{3}{7} + \frac{3}{7} \times \frac{4}{7} + \frac{4}{7} \times \frac{3}{7}$$

$$= \frac{9}{49} + \frac{12}{49} + \frac{12}{49}$$

$$= \frac{33}{49}$$

Verify: $\frac{4}{7} \times \frac{4}{7} + \frac{4}{7} \times \frac{3}{7} + \frac{3}{7} \times \frac{4}{7} + \frac{3}{7} \times \frac{3}{7} = \frac{16}{49} + \frac{12}{49} + \frac{12}{49} + \frac{9}{49} = \frac{49}{49} = 1$

Students check to make sure the results satisfy one of the axioms of Probability.

In the fifth week, activity four was conducted to guide students to solve probability problems based on problem-based learning approach and construction of knowledge. The researcher took time and guided the students to solve some real-life probability problems and they did presentations on their solutions. Reflection and evaluation took place to provide further action on solutions to probability problems. Students were provided with resource materials and information on the internet to guide them in solving probability problems.

Problem 2 (Problem-based learning approach)

There are four keys and four boxes, each of the keys opens one box. If one of the boxes contains GH₵50,000.00 and in each of the other boxes is a lump of coal. You can choose one of the keys and one of the boxes. What is the chance that the key will open the box? What is the chance that you will win the GH₵50,000.00?

Problem analysis/Understanding the problem

There are $4 \times 4 = 16$ Possible Combination of the Keys and Boxes. Four of them give a Key that matches the lock.

Devising a plan/ Discovery ways of tackling the problem

Students use table to illustrate the Sample Space and possible ways of opening the box and if possible, winning the GH₵50,000.00.

Boxes were denoted as B1, B2, B3 and B4 whiles Keys as K1, K2, K3 and K4

Table 6: Possible Combination of Keys and Boxes

Keys/Boxes	B1	B2	B3	B4
K1	K1B1	K1B2	K1B3	K1B4
K2	K2B1	K2B2	K2B3	K2B4
K3	K3B1	K3B2	K3B3	K3B4
K4	K4B1	K4B2	K4B3	K4B4

Carrying out the Plan/ Solution Presentation

Sample Space for both Keys and Boxes is 16 but Sample Space for boxes is 4.

Since there are four boxes the probability of opening a box $= \frac{1}{4} = 0.25 = 25\%$ but the probability of winning the money $= \frac{1}{16} = 0.0625 = 6.25\%$

Further calculations for explanation purposes.

Probability of not being able to open the box $= 1 - \frac{1}{4} = \frac{3}{4} = 0.75$ and the probability of not winning the money $= 1 - 0.0625 = 0.9375$

Looking back/ Reflection and Evaluation

The results showed that there was 75% of one not being able to open the box with the right key and 93.75% probability of not winning the GH₵50,000.00

Application

In the event of doing anything in your life, always calculate the chances of successes and failures before taking decisions.

One special thing about problem-based learning was that, students did a lot of research and generated their own learning issues and finally demonstrated their knowledge of understanding of probability and its application and argued their solutions constructively with optimism. The intervention process through the use of problem-based learning instructional activities also helped students to:

i. develop effective problem-solving skills.
ii. develop self-directed learning.
iii. become effective communicators, collaborators and
iv. become intrinsically motivated to learn (Hmelo-Silver, 2004).

Unlike the traditional method approach (Control group) which involved the tutor imparting knowledge to students and they sat down quietly and passively listened to the tutor. Lesson was purely tutor-centred and students only response when called upon to answer questions. During teaching and learning process students asked questions rarely. When they were given group assignments, most of them did not participate fully they even did their own things at the neglect of group assignment. They mostly relied on the brilliant students to do it on behalf of the group. This was realized during group presentation and questions posed to group members. Some remained silence and few who responded to questions showed that they did not take part in group discussions due to the nature of their response to most questions. The traditional method approach did not challenged students to do more research, self-directed learning, generating of learning ideas and finally demonstrating knowledge of understanding of concepts and application during solution presentation. On the sixth week, students in the control group were taught using problem-based approach to bridge the gap they were lacking in order for them to be able to answer probability problems very well in order to pass their end of semester examination. This was done after the post-test.

3.14 Post-Test

After the intervention a post-test was carried out to find out how far the intervention had helped students using problem-based learning on experimental group compared with the traditional method approach on the control group. The post-test was also made up of ten questions different from the pre-test but similar. The same time was used as the pre-test. A marking scheme was produced for the post-test (See Appendix E) and scripts marked and recorded. The Researcher carefully examined wrong answers given in order to find out possible causes. Discussions were held with students to find out why such mistakes were committed. A questionnaire in the form of Likert-scale was given to students to fill in order to ascertain the views and attitudes toward probability problems and to see the extent to which the PBL approach had helped improved their understanding of probability concepts and its application to real life situations. Responses were recorded and examined. Finally, the means and standard deviations of the pre-test and post-test were found to be used in the analysis of data.

CHAPTER FOUR

RESULTS/ FINDINGS AND DISCUSSIONS

4.0 Overview

In this chapter, the data collected from the research were analysed and discussed. The analysis of the data was in two parts. The first part dealt with descriptive analysis and the second part dealt with inferential analysis. The sequence of the presentation and discussion of the results obtained in this study were discussed in accordance with the research questions formulated for the study.

4.1 Descriptive Analysis of Pre-Test and Post-Test Scores of Control and Experimental Group

4.1.1 Pre-test scores

A Probability achievement Pre-test was administered to the sample size of 100 students of the both the Experimental and Control groups. This was to find their existing knowledge and understanding level of Probability problems. An intervention was carried out and lasted five (5) weeks with the Experimental group which served as treatment group and the normal traditional method was applied to the Control group. A Post-test was administered to both groups particularly to determine the effect of the intervention using problem-based learning approach on the probability achievement of College of Education students in the Northern Region, Tamale. The Pre-test was conducted based on the students' relevance of previous knowledge on solving probability problems. Students were asked to use any method of their choice for the pre-test. The marks obtained by students were marked out of 50. The pre-test raw scores of all 100 students of both groups were carefully analysed. The researcher took time to critically analyse students' solutions to the questions one by one to find the number of students who had the questions correct or wrong. The 10 questions given to students in both groups were meant for the following:

i. student's ability to analyse probability concepts;
ii. student's ability to use outcome tree diagrams and tables for probability problems;
iii. student's ability to apply and determine the solution for probability problems.

For the researcher to be able to answer the first research questions, all the solutions of the 100 students of both groups were carefully scrutinized. It was observed that the poor responses to the 10 questions were due to the following:

i. students find it very difficult to understand basic concepts such as probability with or without replacement, mutually inclusive or exclusive events, compound events, sample space of the coin depending on the number of times tossed with the ludo die, conditional probability and axioms of probability.
ii. students could not apply outcome tree diagrams and tables to determine solutions to probability problems.
iii. students find it difficult in determining solution to real-life situations on probability problems.

4.1.2 Post-test scores

The post-test was administered after students were introduced to the intervention with regard to the use of problem-based learning approach and those exposed to the conventional or traditional method approach. The post-test questions were similar to the pre-test questions and this assisted the researcher to assess the effectiveness of the problem-based learning approach. The researcher analysed the questions one after the other in order to obtain sufficient information about the students' responses to each of them. The post-test was marked out of 50 percent. The post-test raw scores of all 100 students of both groups were analysed. The Table 7 gave a summary of both pre-test and post-test analyses of raw scores of the control and experimental groups.

Research Question 1: What difficulties do Colleges of Education students' have in solving probability problems?

Pre-test results of the Control group and the Experimental group on Table 7, below answered research question 1.

Table 7: Students' Scores on the Key Abilities and Skills Assessed in the Pre-test and Post-test of Control and Experimental Groups

Function/ Skill	Students Ability	Control Group Number and Percentage		Experimental Group Number and Percentage	
		Pre-test	Post-test	Pre-test	Post-test
Concept on Probability	Able to analyse	23(45%)	24(47%)	19(39%)	36(73%)
	Unable to analyse	28(55%)	27(53%)	30(61%)	13(27%)
Use of tree diagram and tables	Able to solve	15(29%)	20(39%)	12(25%)	39(80%)
	Unable to solve	36(71%)	31(61%)	37(75%)	10(20%)
Application to real-life and Solution	Able to apply and determine solution	11(22%)	21(41%)	14(29%)	41(84%)
	Unable to apply and determine solution	40(78%)	30(59%)	35(71%)	8(16%)

The results on Table 7 indicated that, 55% of students in the Control group and 61% of students in the experimental group were unable to analyse Probability concepts whilst 71% of students in the Control group and 75% in the experimental group were also unable to solve probability problems using tree diagrams and tables. Finally, student's inability to apply and determine probability problems related to real-life situation, represented 78% of students in the Control group and 71% in the experimental group. In conclusion, the descriptive analyses revealed students' difficulties in solving

Probability problems and only few were able to answer questions related to the skills in Table 7.

4.2 Independent Samples T-test of Pre-test Results

The independent samples t-test was performed to see whether or not there was a statistically significant difference of pre-test scores of both groups. Before the independent t-test the following assumptions were looked at: independence of observation and level of measurement. In this study, it was assumed that the observation of the data was independent. In other words, it was considered that students' responses were not influenced from each other. Besides, the achievement and attitudes were the dependent variables and problem-based learning the independent variable, hence it was assured that the level of measurement assumption was assured. The assumptions were met, t-test analysis was conducted to check if there was significance mean difference in pre-test scores of both Control and Experimental groups.

Table 8: Independent Samples T-test of Pre-test of Probability Achievement Test (PAT)

Group	N	M	SD	F	T	df	P
Experiment Group	49	9.22	7.94				
				.03	2.82	98	0.01
Control Group	51	13.69	7.91				

The results in Table 8, of Pre-test scores of Probability achievement was statistically significant [t (98) = 2.82, P = 0.01]. It was therefore concluded that, there was a statistically significant difference between the Control and the Experimental groups regarding student's pre-test scores of Probability Achievement Test before the treatment.

Research Question 2: What is the effect of Problem-Based Learning on College of Education students' achievement in probability?

If there was no statistically significant difference between pre-test scores of both Control and Experimental groups, then independent t-test analysis should be done to determine the effect of PBL on College of Education students' achievement in probability. Since there was significant difference in the pre-test mean scores of both Control and Experimental groups and on the assumptions to reduce errors of variance and also eliminate confounding variable, an analysis of covariance (ANCOVA) had been carried out to adjust post-test mean scores in order to determine the effect of PBL on College Students achievement in probability as shown in Table 9.

Table 9: The ANCOVA Results for Adjusted Post-test Means Scores

Source	Type III Sum of Squares	Df	Mean Square	F	Partial Eta Squared	P-value
Intercept	2148.85	1	2148.85	33.93	0.26	0.00
Post-test	9.42	1	9.42	0.15	0.00	0.70
Treatment	453.05	1	453.05	7.15	0.07	0.01
Error	6144.09	97	63.34			
Total	**19876.000**	**100**				

R Squared = 0.08 (Adjusted R Squared = 0.06

To understand if there was a meaningful statistical difference in the two groups post-test scores, an ANCOVA analysis was conducted as shown in Table 9. The results indicated a statistically significant difference of the two groups while adjusting post-test scores [$F (1, 97) = 7.15$, $P = 0.01$] with Partial Eta Squared = 0.07 suggesting that Probability achievement test of the Experimental group exposed to Problem-Based Learning performed better in the post-test than the Control group exposed to traditional method. The Experimental post-test scores were adjusted according to their pre-test scores. The post-test and adjusted mean post-test scores were calculated as 27.57 to

27.69 respectively. The Control group mean post-test and adjusted mean post-test scores were also calculated as 16.88 to 16.77 respectively.

In order to further investigate the effect of problem-based learning on probability achievement test of College of Education students and the extent to which the intervention had impacted, a paired samples t-test difference of post-test and pre-test scores of the Experimental group was analysed as shown in Table 10.

Table 10: Descriptive Statistics of Students' Scores on Post-test and Pre-test of Experimental Group

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Experimental Post-test	27.57	49	11.13	1.59
	Experimental Pre-test	9.22	49	7.94	1.13

The results on Table 10 indicated that means and standard deviations of both pre-test and post-test of the Experimental group when exposed to problem-based learning of College of Education students. Table 11 showed paired sample t-test of post-test and pre-test scores of the Experimental group.

Table 11: Paired Samples T-test of Post-test and Pre-test Scores of Experimental Group

Group	Tests	N	Mean	SD	t-value	df	P-value
Experiment Group	Post-test	49	27.57	11.13	9.03	48	0.00
Experimental Group	Pre-test	49	9.22	7.94			

Table 11 showed a paired sample test carried out which compared the mean difference of post-test and pre-test scores of the Experimental group as the data met all the assumptions of paired sample t-test. The results of the paired samples t-test conducted, verify mean difference between the post-test and pre-test scores indicated that there was a significant improvement in the achievement of post-test scores (M = 27.57, SD = 11.13) over pre-test scores (M = 9.22, SD = 7.94) at $\alpha < 0.05$ level of significance, with conditions [t (48) = 9.03, P = 0.00). It was therefore concluded that, there was a statistically significant difference between the post-test and pre-test scores of students when taken through the use of problem-based learning approach. The results of the analysis also showed that, there was a significant change in the understanding and achievement of students in solving probability problems as also confirmed in Table 7.

The researcher therefore concluded that there was a significant difference between the post-test and pre-test scores and that the use of problem-based learning approach had indeed a positive effect on students' achievement in probability problems at the College level of Ghana's education.

On the other hand, a paired samples t-test of post-test and pre-test scores of the Control group was conducted to determine the effect of the Traditional method of Instruction as shown in Table 12 and 13.

Table 12: Descriptive Statistics of Students Scores on Post-test and Pre-test Scores of Control Group

		Mean	N	Std. Deviation	St. Error Mean
Pair 1	Control Post-Test Scores	16.88	51	10.19	1.43
	Control Pre-Test Scores	13.69	51	7.91	1.11

The results on Table 12 indicated the means and standard deviations of Control group when exposed to traditional method of instruction. Table 13 showed paired sample t-test of post-test and pre-test scores of the Control group.

Table 13: Paired Samples T-test of Post-test and Pre-test Scores of the Control Group

Group	**Tests**	**N**	**Mean**	**SD**	**t-value**	**df**	**P-value**

Control Group	Post-test	51	16.88	10.19			
					1.90	50	0.06
Control Group	Pre-test	51	13.69	7.91			

The results on Table 13 showed that there was no statistically significant difference of the pre-test scores (M = 13.69, S.D = 7.91) against post-test scores (M = 16.88, S.D = 10.19) at [t (50) = 1.90, P = 0.06] , the traditional method seems not to be significant difference in teaching of probability as compared to PBL. This was because the mean difference of 3.20 showed that the control group exposed to the traditional method was not effective in the understanding of probability concepts and its application to real life situations.

4.3 Probability Attitude Scale (PAS)

Research Question 3: What are the attitudes of College of Education students toward probability problems?

Probability attitude refers to liking or disliking of Probability, confidence in ability to apply Probability concepts. In this study, attitudes toward Probability were measured by a 30-item Likert questionnaire. Each of the items involved a 5-point scale (Strongly disagree = 1, Disagree = 2, Undecided = 3, Agree = 4 and Strongly agree = 5). The Probability attitude scale had four dimensions or constructs, 1^{st}, 4^{th}, 8^{th}, 14^{th}, items were about 'personal confidence', 6^{th}, 10^{th}, 11^{th}, 13^{th}, 19^{th}, were about 'usefulness of Probability', 21^{st}, 22^{nd}, 23^{rd}, 24^{th}, 25^{th}, 26^{th}, 30^{th}, items were about 'effectiveness of Problem-Based Learning on Probability', 2^{nd}, 3^{rd}, 5^{th}, 7^{th}, 9^{th}, 12^{th}, 15^{th}, 16^{th}, 17^{th}, 18^{th}, 20^{th}, 27^{th},28^{th}, 29^{th}, were about students 'views about Probability'.

Moreover, the alpha reliability coefficient of Probability Attitude Scale (PAS) with 30 items was reported as 0.79 and Standardised item Alpha as 0.78. Table 4.14 and 4.15 showed results of College of Education students' attitude towards Probability.

Table 15: Number of Student Percentage Responses to Probability Attitude Scale

Constructs	Item	Agree	Disagree	Undecided
	2	36.8%	63.2%	0.0%
	3	25.6%	61.7%	12.7%
	5	31.9%	57.5%	10.6%
	7	36.2	59.5%	4.3%
	9	31.9%	65.9%	2.2%
	12	34.0%	39.3%	27.7
Views about probability	15	34.1%	57.4%	8.5%
	16	31.9%	51.1%	17.0%
	17	67.9%	10.6%	25.5%
	18	2.1%	87.2%	10.7%
	20	85.1	8.5%	6.4%
	27	40.8%	55.1%	4.1%
	28	26.5%	65.3%	8.2%
	29	51.0%	44.9%	4.1%
	1	57.2%	42.8%	0.0%
Personal confidence	4	53.2%	34%	12.8%
	8	87.2%	2.1%	10.7%
	14	85.1%	14.9%	8.0%
Application of Polya's heuristics	21	79.6%	0.0%	20.4%
	6	44.7%	34%	21.3%
Usefulness of Probability	10	41.3%	34.0%	27.7%
	11	48.9%	21.3%	29.8%
	13	70.2%	23.4%	6.4%
	19	29.8%	53.2%	17.0%
	22	85.7%	8.2%	6.1%
Effectiveness of Problem-Based Learning	23	69.4%	18.3%	12.2%
	24	55.1%	32.7%	12.2%
	25	87.8%	6.1%	6.1%
	26	61.2%	26.5%	12.2
	30	85.7%	8.2%	6.1%

The results on Table 15 answers research question 3 using the constructs "Personal confidence" and "views about Probability". From Table 15, "Strongly Agree" and "Agree" were collapsed into "Agree" and "Strongly Disagree" and "Disagree" were also collapsed to "Disagree". Students responses to negative statements with items; 2, 3, 5, 9, 15, 18, 27 and 28 indicated students view in favour of such statements which now becomes positive in addition to positive statements of item 17 and 20. The results showed that, students had positive view or attitude toward probability problems. With item 29, the result indicated that PBL approach did not make students overcome their difficulties in communication with their peers.

With the construct of "Personal Confidence" the high percentage responses indicated student's confidence in solving probability problems resulted in positive attitude toward probability problems. The construct "Application of Polya's Heuristics" the result indicated 79.6% of students agree that, they used Polya's problem solving techniques in solving probability and this had improved their understanding of probability concepts, whilst no student disagree to the statement and 20.4% were undecided whether Polya's heuristics really helped them solve probability problems.

With the construct "Usefulness of Probability" the results indicated the importance of probability to students and how it benefitted them for example 70.2% understood most of concepts in probability as against 23.4% of students disagree to that statement whilst 6.4% were undecided to whether they understood concepts in probability. In addition to this construct of usefulness of probability 53.2% disagree that when they learn probability, they will get jobs whilst 29.8% agree to the statement and 17.0% were undecided.

Finally, with the construct "Effectiveness of Problem-Based Learning" the high percentage responses on Agree column indicated the effectiveness of PBL in those statements, for example 55.1% of students responded that, PBL helped them to think critically about different solution before making decisions whilst 32.7% disagree to the statement and 12.2% were undecided.

In conclusion, the results indicated that students hold positive views about probability and their confidence level has increased tremendously toward probability problems.

This result showed that the intervention was successful and students saw probability concepts not difficult but rather interesting and joyful.

4.4 Analysis of Interview Results

The sound interviews recorded were transcribed verbatim. See APPENDIX F for samples of student transcription of interview data. The few sketches of outcome tree diagrams made by the interviewees were added to these transcriptions. All transcriptions were read carefully at different times to become familiar with the data and to achieve emersion of data. When the researcher read the transcripts, initial reflective notes were made and coded in relevance to: Polya's techniques reasoning, analytical reasoning and reasoning related to outcome trees orientation and the representativeness views of students toward Probability. Besides, analytical and procedural memos were written and semi-structured interviews became clear to most students as they did not follow a linear path between receiving a Probability problem and giving solution to an answer. They did not just respond to the Probability problem, but rather determined a technique they assumed to be the right one and later on applied that technique to arrive at their solution in a clear manner. The decision of the students was as a result of two main processes contributed to this phenomenon.

Firstly, most of the students held different conflicting beliefs on many of the probability problems. They repeatedly evaluated the different alternative answers based on their personal experiences (Polya's technique reasoning) to a more formal Probability beliefs (analytical reasoning). This decision process between different alternative answers and different techniques to arrive at an answer on the Probability problems proved to be very hard for some students. During the semi-structured interviews students often struggled before coming out with the answer in an explicit way. This phenomenon was noticeable in the answers to items 5 and 10 in which the students were asked to evaluate the likelihood of a coin tossed twice with a die and also a die thrown twice. Not only did they doubt between different sequences because they conceptualized and understood techniques in a different way. They also doubted between different 'beliefs' on the matter as a whole, that was between the 'fact' that as such every sequence was equally likely on one hand (Analytical reasoning). Items 2 and 7 were linked to one another, however, the students gave divergent answers on the linked Probability items and followed divergent approaches to answer these linked items.

Moreover, item 9 was purely real-life probability problem, most students said they were not taught Probability at the Senior High School using outcome tree diagram hence could not apply (outcome tree orientation) in solving the problem, but they confirmed after the intervention such problems have now been easy for them to solve. Many of the students now see Probability problems as useful and would apply knowledge to teach students at basic level (Representative view).

The researcher analysed the results based on the themes: trustworthiness, dependability and confirmability and transferability.

Trustworthiness

Trustworthiness refers to the validity and reliability of the instruments. The test items were designed based on the mathematics course manual of Colleges of Education with the help of a colleague tutor in a different College hence validity was assured. The test items with the questionnaire and semi-interview guide were piloted to test reliability of the instruments. The pilot study resulted in changes in the instruments with regard to number of questions, time and understanding of key words.

Dependability

Dependability is the extent to which the study could be repeated by other researchers and the findings would be consistent. This was assured because a colleague tutor from a different College of Education observed instructions on both the control and experimental group and admitted that lessons delivery was neutral without bias and also monitored the data collection process and analysis stage, hence the results of the findings can be dependable.

For example, a respondent reported that

> "*I used to dodge Statistics and Probability lessons at the Senior High School level because it was difficult to understand the concepts and I only guess the objectives part of the test and left the theory unanswered. But with the new approach using problem-based learning, to be honest with you (Researcher) if I were not be part, I would have done myself a great loss and that Probability is very simple especially using tree diagram to solve probability questions relating to real life situations*" (**Dependability**).

Confirmability

The interview results confirmed the true reflection of the results of the pre-test and post-test scores. After the post-test the researcher made a photocopy of the marking scheme and gave it to a colleague tutor to mark in order to avoid biasness and personal influence. In fact, the difference in the results of the marking was just minus one and plus one with other scripts being the same marks. A colleague was made to also transcribe the recorded interview and both the researcher and that colleague results were the same. This confirmed what a respondent said with regard to the theme of confirmability.

Specifically, on the issue of how students answer the pre-test question, respondent E and B responded that;

> ***Responder E****: Sir, back at SHS I didn't know there was a tree diagrams to solve probability questions. So, using the probability tree diagram I was struggling, along the lines, I got somewhere and got confused, as how to proceed because I didn't have any idea about the tree diagram.*
>
> ***Respondent B****: I couldn't do it because I remembered I was taught probability back at Senior High School but I couldn't get the concept well, because probability questions are 'trickiest' so if the teacher who is to teach is not well verse with it or does not understand the concepts, then students will find it very difficult to solve questions on probability. I had no idea about tree diagram in solving probability questions, but thank God, after the intervention I now find probability questions interesting and understandable due to the application of problem-based learning and use of Polya's heuristics in solving problems and the use of tree diagram is now a thing of the past* (**Confirmability).**

The researcher probe on the pedagogical content knowledge by asking respondents

How could you explain this probability problem to your fellow student who is not familiar with probability (Polya's techniques reasoning and outcome tree orientation).

Most of them explained that based on the intervention using Problem-based learning approach and Polya's problem solving techniques the concepts and application of probability using outcome tree diagrams they could comfortably explain to the understanding of their colleagues who are not familiar with probability and that they are now in better position to impact knowledge to students at the basic level of the Ghanaian education (Analytical reasoning).

Transferability

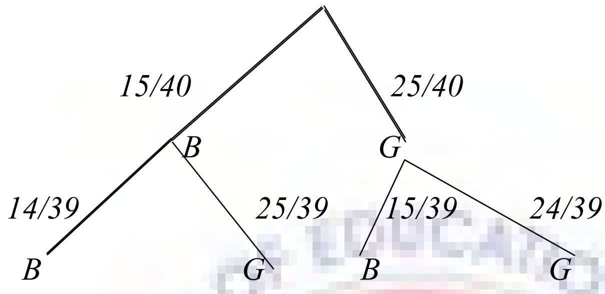
This refers to how the findings can be applicable to another College of Education. The results of the findings can be transferable to similar results if the same research is carried out to different College of Education. The difficulty with regard to use of outcome tree diagrams and application of probability concepts to real life situation had been confirmed by the Chief Examiner's report of UCC (2017, 2018) on Colleges of Education throughout the country Ghana that students had difficulties in the use of tree diagrams to answer Probability problems.

In conclusion, most of the students interviewed admitted that the new methodology of problem-based learning instructional approach had helped them overcome the difficulties in solving probability problems.

> ***Responder D**: I learnt to communicate during PBL, I also learnt to collaborate with my colleagues to find solution to probability problems. I also learnt to collaborate and cooperate with colleagues during PBL session. It helps me to talk with my colleagues now, I used to feel shy in talking but because you encourage all of me to take part and do presentation, I have to force myself to talk.*

Interviewer: I realised that you had a particular question right after the intervention meanwhile a similar one was given in the pre-test and you had it wrong. What will you say about that please? The question was on: "In a class of 40 students, 15 are boys and the remaining girls, if 2 students are selected at random without replacement, what is the probability that they are both girls?

Responder H*: During the pre-test I couldn't solve it rightly, with the intervention of the probability session I was able to solve it correctly. Before I work it out, I have to apply Polya's problem solving techniques by understanding the problem, devising a plan, carrying out the plan and finally, looking back. In this problem I first of all, find the number of girls that were in the class because the number of girls were not given in the question. So, thereby I subtract the number of boys from the whole class it will give me the number of girls, that is 25, so using the tree diagram, I draw the diagram as shown below:*



Representing B as boys and G as girls, the 15/40 represent the probability of boys and 25/40 representing the probability of girls. When you select a boy the second time, the total number of boys reduces and that affects total number of students in the class but the number of girls remain the same as shown in the tree diagram. This concept is applied to the girls. Probability of both girls, it has to be 25/40 × 24/39 = 5/13

4.5 Discussions of the Findings

The results of this study had shown that students taught probability using problem-based learning (PBL) performed better than their colleagues taught using traditional method. The analysis of covariance results showed that PBL students were found to be statistically significantly different from those of their colleagues in the Control group. The findings revealed the efficacy of the use of PBL in enhancing student's achievement in probability. This finding contradicts Abdullah, Tarmizi and Abu (2010), who found that students in the PBL group and the Control group performance was not significant in the performance of Mathematics. The stated that the overall performance of scores of both groups using ANCOVA, the PBL group appeared to obtained a higher mean score compared to the control group. However, the difference was not significant since their P = 0.23 > 0.05. The findings result however, was consistent with that of (Iji, Emiakwu & Utubaku, 2015; Uygun & Tertemiz, 2014; tan 2011; Olpak, Baltaci & Arican, 2018) who attested that PBL students performed better than the conventional/traditional method. The partial Eta squared = 0.07 indicated a moderate effect size of the independent variable (Problem-Based Learning) as an effective methodology in teaching the concepts of Probability, the use of tree diagrams in applying to real life probability problems. The large effect size also showed that the experimental variable PBL impacted positively to those students in the Experimental group as most of them enjoyed learning probability and concluded that they would be more comfortable in teaching probability at the basic level of education in Ghana. The positive effects of PBL such as becoming a good problem solver, demonstrating effective critical thinking ability (Ajmal, Jumani & Malik, 2016), the importance of being independent to do self-directed learning and being able to work collaboratively were also shown in this study. However, the findings revealed that the traditional method of instruction was not good enough for teaching and learning of probability concepts and its application to real life situation.

The findings in this study on the attitudes of students toward Probability was contended that the PBL group used Polya's problem solving technique more effectively in solving probability problems. The results from the questionnaire and interview revealed that PBL helped students overcome their difficulties of communication with their colleagues during group discussions of probability problems. The interview results revealed that most teachers at Senior High Schools in Ghana do not use tree diagrams to solve sequential experiments of probability problems.

With the usefulness of Probability, it was revealed that most of the student's belief they have understood concepts on probability and were ready to apply knowledge in teaching at the basic level of education in Ghana. PBL had also displayed good probability communication skills and on whether PBL motivates one to learn Probability problems, most of the students agree that problem-based learning motivates students to learn probability which was in consistence with the results of (Han et al., 2016) that most students agreed they would be more motivated to learn mathematical concepts that are meaningful. From the analysis of the data obtained it can be concluded that PBL had positive impact on students' approach to work, self-directed learning and high professional practice.

The overall impact in using problem-based learning instructional approach was positive on students understanding and solving of probability problems. The findings revealed that students did not know that they could learn better from their classmates until the intervention stage. The intervention made students develop more positive attitude towards probability problems because they were more excited as they could easily answer thought provoking questions and reach conclusion. The major key finding of the study revealed that PBL approach is very effective for teaching Probability since it created a number of creative thinkers, self-directed learners, critical decision makers, problem solvers which is needed most for the 21st century world.

The findings of the study include:

i. students at the Colleges of Education had difficulties in solving probability problems.

ii. students exposed to problem-based learning demonstrated effective critical thinking ability in the analysis of probability problems.

iii. the Partial Eta squared of 0.07 indicated a moderate effect size of PBL as instructional approach to the teaching of Probability problems.

iv. problem-based learning instructional approach made students independent and self-directed learners in terms of discovery strategies and ways of tackling Probability problems.

v. problem-based learning group exposed through Polya's heuristics displayed good communicative skills, collaborative and cooperative attitude to Probability problems.

vi. students exposed to traditional method showed that, it was not very effective instructional strategy to probability problems in terms of understanding of probability concepts and application of probability to real life problems and life-long memory.

vii. with the analysis of the interview results, students demonstrated that probability problems were useful and knowledge acquired would be used to teach students at the basic level of education in Ghana.

viii. students find the use of outcome tree diagrams and tables as simple, interesting and easy to understand concepts and application of Probability to real life situation.

ix. the interview phase of the results indicated that, students applied Polya's heuristics, analytical reasoning and outcome tree diagrams to solve Probability Problems.

x. The interview results revealed most teachers at the Senior High Schools in Ghana do not use tree diagrams to teach sequential experiments and real-life probability problems.

CHAPTER FIVE

SUMMARY OF FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

5.0 Overview

In this chapter, the research findings and implication for using Problem-Based Learning instruction with students were summarized. Consequently, this chapter began with a review of the purpose and the results of specific research questions for the study. Conclusion was drawn based on the findings made. Finally, recommendations were made for effective implementation of Problem-based learning.

5.1 Summary of Findings

Probability problems were familiar to most students though they still had difficulties in finding solution to such problems. As stated in Chapter one, Colleges of Education students consistently performed poorly in Statistics and Probability especially probability problems. Why the students performed poorly at the end of semester examination, class exercises and class test made the researcher identifies the following as difficulties College of Education students faced in solving probability problems:

i. students had positive attitude toward probability problems when exposed to Problem-Based learning through collaborating, communicating and cooperating with their colleagues in solving probability tasks.

I. students could not apply outcome tree diagrams and tables to determine solutions to probability problems.

II. students find it difficult in determining solution to real-life situations on probability problems.

The main purpose of this study was to investigate the effect of problem-based learning approach on College of Education students' achievement in, and attitude toward probability. Thus, it is hoped that this study provided insights into best practices for teaching probability. The researcher came out with an intervention strategy using problem-based learning lesson plan approach that helped in the understanding of probability by sampled Colleges of Education students in Tamale, in the Northern Region of Ghana.

To investigate whether the PBL approach will result in improving performance among College of Education students, the following findings were discovered through the three research questions:

Research question 1 was answered by using descriptive statistics on student's pre-test scores of both control and experimental groups. The results indicated students' difficulties in solving probability problems as a result of their inability to analyse probability concepts, solve probability problems by using outcome tree diagrams for sequential experiments, Venn diagrams for compound events and tables for theoretical probabilities.

Independent samples t-test was used on pre-test scores of both Control and Experimental groups and result showed statistically mean difference of pre-test scores. Hence analysis of Covariance (ANCOVA) was used to answer research question 2. The results indicated statistically there is significance difference between students exposed to problem-based learning instruction as compared to those exposed to traditional mode of instruction. Students in the treatment group outperformed those in the control group.

Finally, research question 3 was answered using questionnaire results on students' attitudes toward probability problems. The results showed a positive of College of

Education students’ positive attitude toward probability problems. The alpha reliability coefficient of Probability Attitude Scale (PAS) with 30 items was reported as 0.79.

In conclusion, most of the students interviewed admitted that problem-based learning approach had helped them overcome the difficulties in solving probability problems.

5.3 Conclusion

It can be argued from the results that students’ weak performance and difficulties in solving probability problems were largely due to their inability to understand the problem, devising a correct plan, carrying out their plans and inability to state the correct answer. The intervention enabled students to do team work, collaborate, cooperate and communicate effectively together; and also made them discover new strategies and apply to probability concepts and problems.

Problem-based learning approach developed in students; critical thinking, good problem solvers and self-directed learners. This would lead to life-long memory of probability concepts. It also motivated students and made them to develop interest in learning probability problems related to real life situations.

Problem-Based Learning approach made students to become good problem solvers, critical thinkers, self-directed learners and critical decision makers which is very much needed for this competitive world and in the field of mathematics education.

5.4 Recommendations

Based on the results of this study the following recommendations were made:

1. heads of Colleges of Education should organise training for mathematics tutors on the use of problem-based learning.
2. tutors in Colleges of Education should use problem-based learning in teaching and learning probability.
3. mathematics tutors at Colleges of Education should employ behaviours during teaching and learning process that would influence positive attitudes to students in learning Probability.

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APPENDIX A

PRE-TEST

Test ID.

Dear Student,

This test is not a college test or examination but a research exercise. You are kindly invited to participate in the study by answering the test questions. You are assured that, your solutions given is solely for research purposes and would be kept confidential. Please do not write your name on this paper.

From items 1 to 5, each item is followed by four options lettered A to D. Read each statement carefully and circle the letter that corresponds to the correct or best options.

1. The probability that Mabel would get first class is 0.89 and the probability that Josephine would not get first class is 0.57. Find, correct to two decimal places the probability that both will get first class.
   A. 0.25 B. 0.35 C. 0.38 D. 0.51
2. A box contains 30 similar balls of which 18 are blue and 12 are green. Calculate the probability of selecting a blue and a green ball if two balls are selected at random **with replacement** from the box.
   A. $\frac{3}{5}$ B. $\frac{12}{25}$ C. $\frac{2}{5}$ D. $\frac{6}{25}$
3. A college football team of 16 players and a volleyball team of 10 players form a squad of 22 players to represent the college in an Inter-College Competition. A student is selected at random from the squad. What is the probability that the student plays only one game?
   A. $\frac{8}{11}$ B. $\frac{9}{11}$ C. $\frac{8}{13}$ D. $\frac{11}{13}$
4. Calculate the probability of selecting a prime number at random among the set of natural numbers from 9 to 19.
   A. $\frac{2}{15}$ B. $\frac{4}{15}$ C. $\frac{6}{19}$ D. $\frac{4}{11}$

5. Two fair dice are tossed together once. What is the probability that the sum of the numbers that show up is divisible by 3?
   A. $\frac{2}{3}$ B. $\frac{1}{3}$ C. $\frac{1}{12}$ D. $\frac{1}{36}$

For questions 6 to 10, write your solution in the space provided under each question.

6. A red and a white die are tossed simultaneously. Find the probability of getting a total of
   a) Exactly 9
   b) At least 10

7. A bag contains seven discs, two of which are red and five green. Two discs are removed one after the other **without replacement**.
   i) Draw a tree diagram to illustrate the probabilities.
   ii) What is the probability that they are of different colours, correct to 2 decimal places?
8. Ibrahim writes a computer program that produce at random one of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
   What is the probability that the program produces;
   i) An even number?
   ii) A multiple of 4?
9. You choose a month at random then a day of the week at random. What is the probability that you will get a Saturday in June? Illustrate the answer using a tree diagram.
10. Two coins and a ludo die were tossed together once.
    i) Draw a table to illustrate the sample space
    ii) Use your table to find the probability of obtaining at least one Head and a Prime number.

APPENDIX B

POST-TEST

Test ID.

Dear Student,

This test is not a college test or examination but a research exercise. You are kindly invited to participate in the study by answering the test questions. You are assured that, your solutions given is solely for research purposes and would be kept confidential. Please do not write your name on this paper.

From items 1 to 5, each item is followed by four options lettered A to D. Read each statement carefully and circle the letter that corresponds to the correct or best options.

1. A card is selected at random from a pack of 52 playing cards, find the probability that the card selected will **not** be a club.

   A. $\frac{1}{52}$ B. $\frac{1}{4}$ C. $\frac{4}{13}$ D. $\frac{3}{4}$

2. Two fair dice are tossed together once. What is the probability of obtaining at least 10?

   A. $\frac{5}{18}$ B. $\frac{1}{6}$ C. $\frac{1}{9}$ D. $\frac{1}{36}$

3. In a class of 40 students 15 are boys. If two students are selected at random, one after the other without replacement, what is the probability that they are both girls?

   A. $\frac{7}{52}$ B. $\frac{2}{13}$ C. $\frac{5}{13}$ D. $\frac{8}{13}$

4. The police line-up 10 people in an identity parade; only one of the people is a criminal. A witness does not recognize the criminal and so chooses a person at random. What is the probability that the criminal is chosen?

   A. 0.1 B. 0.3 C. 0.8 D. 0.9

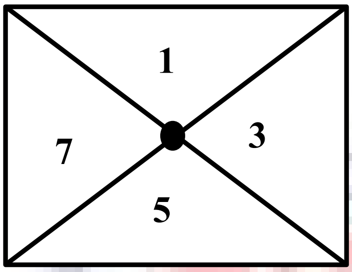
5. Five pairs of twins attended a special event for twins. Two of them are chosen at random to win the door prize. What is the probability that the two are sibling?

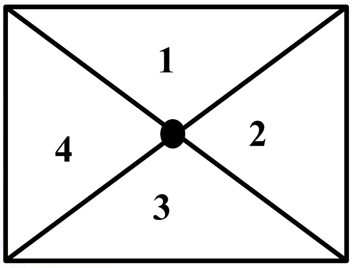
   A. 0.50 B. 0.45 C. 0.11 D. 0.04

For questions 6 to 12, write your solution in the space provided under each question.

6. The diagrams show two spinners which are both spun. What is the probability that the total score on the two spinners is:

   a) 7 b) less than 5?





7. Eleven students, consisting of six males and five females are in a class. They are called one after the other, without returning to class, for questioning on an issue by the Dean of Students' Affairs. Using a tree diagram, find the probability that, the first two students called are of different sexes.
8. Two coins and a ludo die were tossed together once. Draw a table to illustrate the sample space. Use your table to find the probability of obtaining at most one Head **and** a Prime **or** an odd number.
9. Mr. Bukari develops a computer program that produces prime numbers at random among the set of natural numbers from 9 to 19. Calculate the probability of selecting a prime number?
10. Given that A and B are events such that P(A) = 0.6, P(B) = 0.5 and $P(A \cup B) = 0.8$

    a) Find P(A/B)

    b) Are A and B independent?

APPENDIX C

QUESTIONNAIRE FOR STUDENTS

Questionnaire ID.

Dear Student,

This questionnaire is not a college test or examination but a research exercise. There are no right or wrong answers to the questions. You are kindly invited to participate in the study by completing the questionnaire. You are assured that any information given is solely for research purposes and would be kept confidential. Please do not write your name on this questionnaire.

INSTRUCTION: Tick [√] only once in the statements below by indicating the option which best suits you and write down the short response to the statements provided.

DEMOGRAPHIC PROFILE OF STUDENTS

i) Name of college of Education……………………………………

ii) District…………………………………………………………

iii) Gender: Male [ ] Female [ ]

iv) How old are you………………

Rate the statements below according to: Strongly Disagree (SD), Disagree (D), uncertain (UN), Agree (A), strongly Agree (SA)

Values of measurement : 1=SD ; 2=D ;3=UN ;4=A ;5=SA	**SD**	**D**	**UN**	**A**	**SA**
A. Students 'Attitude to Probability problems					
1. Probability is interesting					
2. I am not good at Probability					
3. I prefer other topics in Mathematics than Probability					
4. I will need the concept of Probability to teach my students at basic level					
5. The topic I hate most is Probability					
6. Probability is useful in life					

	SD	D	UN	A	SA
7. The probability I am taught is a waste of time					
8. Probability is the easier topic so far					
Values of measurement: 1=SD; 2=D ;3=UN ;4=A ;5=SA	**SD**	**D**	**UN**	**A**	**SA**
9. Probability is more difficult to understand than other topics in Mathematics					
10. I study probability because I know how useful it is					
11. I think everyone should learn Probability					
12. Probability questions terrify me					
13. I understand most of the concepts in Probability					
14. I enjoy learning Probability					
15. Probability is dull and boring					
16. When I do not understand a concept in Probability, I know that I will never really understand it					
17. I think it is important to do well in Probability in College					
18. You can never apply Probability after college to solve your everyday life situations					
19. We learn Probability in order to get jobs					
20. Exam questions from probability should be optional					
B. Engagement in Problem-Based Learning Session	**SD**	**D**	**UN**	**A**	**SA**
21. Helps me apply the problem-solving approach and identify possible solutions.					
22. Facilitates the task of team work with colleagues during group discussion.					

23. Gives me the opportunity and tools to know how to search for information.					
24. Teaches me to think critically about different solutions before making a decision.					
25. Helps me to realize the importance of being independent.					
26. Helps me realize about my strengths and weaknesses.					
27. Helps me to do my own things without paying attention to group discussion.					
28. Problem based learning approach to Probability is boring and difficult to understand					
29. Allows me overcome difficulties of communication with my colleagues					
30. Problem-based learning approach motivates me to learn Probability					

APPENDIX D

Marking Scheme for Pre-Test

Question Number	Details of Solution	Marks
1-5	Objective Questions 1. **C** 2. **C** 3. **B** 4. **D** 5. **B**	**5**
6	Illustrating using a table B_6 Exactly 9 outcomes are: {3,6}, {6,3}, {4,5}{5,4} $\boldsymbol{M_1A_1}$ P (exactly 9) = $\frac{4}{36} = \frac{1}{9}$ $\boldsymbol{M_1A_1}$	**10**
7	Illustrating using tree diagram M_1A_2 P (different colours) = $\left(\frac{2}{7} \times \frac{5}{6}\right) + \left(\frac{5}{7} \times \frac{2}{6}\right)$ $\boldsymbol{M_1A_1M_1}$ $= \frac{10}{42} + \frac{10}{42}$ $\boldsymbol{M_1}$ $= \frac{20}{42}$ M_1 $= \frac{10}{21}$ M_1 = 0.4762 = 0.48 (2dps) $\boldsymbol{A_1}$	**10**
8	Let n(E) be even numbers and n(S) be sample space Even numbers are: 0, 2, 4, 6, 8 $\boldsymbol{B_1}$ a) P (E) = $\frac{n(E)}{n(S)} = \frac{5}{10} = \frac{1}{2}$ $\boldsymbol{M_1A_1}$ b) Let n(M) be number of multiples of 4 Multiples of 4: 4, 8 $\boldsymbol{B_1}$ P (M) = $\frac{n(M)}{n(S)} = \frac{2}{10} = \frac{1}{5}$ $\boldsymbol{M_1A_1}$	**6**
9	Probability of getting June as $\frac{1}{12} = 0.083$ $\boldsymbol{M_1A_1}$ Probability of getting Saturday as $\frac{1}{7} = 0.143$ $\boldsymbol{M_1A_1}$ Illustrating using tree diagram $\boldsymbol{M_1A_2}$ P (Saturday in June) = 0.083×0.143 $\boldsymbol{M_1}$ = 0.012 $\boldsymbol{A_1}$	**9**
10	Illustrating using table B_6 Let E represent at least one Head and a Prime number E= HH2, HT2, TH2, TH3, HT3, HH3, HH5, TH5, HT5 $\boldsymbol{B_1}$ P (E) = $\frac{9}{24} = \frac{3}{8}$ $\boldsymbol{M_1A_1A_1}$	**10**

APPENDIX E

Marking Scheme for Post-Test

Question Number	Details of Solution	Marks
1-5	Objective Questions 2. **D** 2. **B** 3. **C** 4. **A** 5. **C**	**5**
6	Illustrating using a table for both spun $\boldsymbol{B_6}$ P (spinning 7) = $\frac{2}{16} = \frac{1}{8} = 0.125\ or\ 12.5\%$ $\boldsymbol{M_1 A_1}$ P (n < 5) = $\frac{4}{16} = \frac{1}{4} =\ 0.25\ or\ 25\%$ $\boldsymbol{M_1 A_1}$	**10**
7	Illustrating using tree diagram $M_1 A_2$ P (different sexes) = $\left(\frac{6}{11} \times \frac{5}{10}\right) + \left(\frac{5}{11} \times \frac{6}{10}\right)$ $\boldsymbol{M_1 A_1 M_1}$ $= \frac{30}{110} + \frac{30}{110}$ $\boldsymbol{M_1}$ $= \frac{60}{110}$ $\boldsymbol{M_1 A_1}$ $= \frac{6}{11}$ $\boldsymbol{A_1}$	**10**
8	Illustrating using table $\boldsymbol{B_6}$ Let E_1 be at most one Head and a Prime or Odd E_1 =HT1, TH1, TH2, HT2, TH3, HT3, TH5, HT5, TT1, TT2, TT3, TT5 $\boldsymbol{M_1 A_1}$ P (E_1) = $\frac{12}{24} = \frac{1}{2}$ $\boldsymbol{M_1 A_1}$	**10**
9	Natural numbers from 9 to 19 include: 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 $\boldsymbol{B_1 M_1}$ The Prime numbers include: 11, 13, 17, 19 $\boldsymbol{M_1 A_1}$ P (Prime Numbers) = $\frac{4}{11}$ $\boldsymbol{M_1 A_1}$	**6**
10	P (A∩ $B) = P(A) + P(B) - P(A \cup B)$ = 0.6 + 0.5 – 0.8 $\boldsymbol{M_1}$ = 0.11 – 0.8 $\boldsymbol{M_1}$ = 0.3 $\boldsymbol{A_1}$ i) P (A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.5} = 0.6$ $\boldsymbol{M_1 A_1}$ ii) P (A). P(B) = (0.6) (0.5) = 0.3 = P (A∩ B) $\boldsymbol{B_1 M_1 A_1}$ Which means that A and B are independent $\boldsymbol{B_1}$	**9**

APPENDIX F

TRANSCRIPTION OF INTERVIEW RECORDING VERBATIN

Course Instructor (Interviewer): What was your opinion about the Pre-test questions that was given to you?

Responder A: when you started it with us, I realized I have learned it in secondary school but I was lacking some basic skills that made me not understand it easier and simpler as when you gave us those pre-test questions since I was lacking prerequisite skills, I was not able to do anything better.

Interviewer: Could you tell me a bit more about how you handle this probability problem which was spinners questions.

Responder A: Thank you very much, with the pre-test, I was not able to answer but after you took us through a series of skills which made us to understand it clearer but with this one , I used split method table to solve the problem, now I realized that the total score of the two spinners was 7, you realized that using the split table, I have 7 appearing two times in the table, adding everything I had 2/16 and to simplify it I get 1/8.

Interviewer: How would you explain this probability problem to your fellow student who is not familiar with Probability?

Respondent A: Ok, you (Researcher) made me to understand that, there is another way of solving this probability with this one, I was not going to split anything but I go direct using the numbers by so doing just add them, with your own because is not split you can easily identify the number of 7 in the table which is 2, divided by the total number of spinners in the table which is 2/16, so to simplify everything you have 1/8 with the your (researcher) own it was easily identify the numbers on the table.

Interviewer: What did you learn during PBL on problem solving?

Responder A: You made me to do problem solving by using Polya's problem solving technique, one has to understand the problem of the question first, later on devise a strategy to solve the problem using concept of skills that will make me easily understand or how I was going to tackle the problem, then later on carry out the strategy of the plan by so doing I apply the problem given to me. In fact, it was good, I realized that when applied the problem-solving technique everything went on well, after that looking back to check whether the problem given to me was carried out successful, I realized that it was easy and I had it clear than how the problem was solved.

Interviewer: How did PBL facilitate the task of team work with peers?

Responder A: I learnt to communicate mathematically during PBL session, I also learnt to collaborate with peers to find solution to problems, lastly, I learnt to cooperate with peers no matter the situation during PBL problem solving.

Interviewer: Did PBL session make group discussion noisy?

Responder A: No

Interviewer: Is it true that PBL session is boring?

Responder A: No, is rather interesting.

Interviewer: What is your opinion on the use of PBL on probability problems?

Respondent A: Is simple to understand when related to real life situation, it also made learning on Probability concepts interesting and fully of joy. Thank you for your response God bless you.

Course Instructor (Interviewer): What is your opinion about the pre-test questions that were given to you?

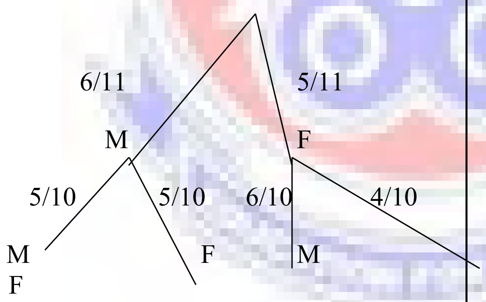
Responder B: I couldn't do it, because I remember I was taught probability back at SHS but I couldn't get the concept well, when I got to training college, I could not get the concept well as also, because probability questions are 'trickiest' so if the master who is to teach is not well verse with it or does not understand the concepts, I the student will find it very difficult to solve.

Interviewer: Can you tell me how you handle this probability problem? You were asked to solve a question on 11 students consist of 5 females and 6 males are in a class and are called one after the other without returning to the class for questioning by the Dean of Students Affairs, using a tree diagram find the probability that the first two students called are of different sex? I notice that you used decimals to represent your probabilities and you can see how you performed what made you to go by such approach?

Responder B: As I earlier said, back at SHS I didn't know that there is a tree diagrams to solve probability questions. So, using the probability tree diagram I was struggling, along the lines, I got somewhere and got confused, as how to proceed because I didn't have any idea about the tree diagram.

Interviewer: How would you explain this probability problem to a student who is not familiar with probability?

Responder B: Thank you sir, with the same question when you took us through, you made me understand that, the 11 student represent the total number of people in the class, the male are 6 and the female are 5 with the male using the tree diagram, it will be represented by 6 /11, with the female, it will be represented by 5 /11, now without replacement, this simply means that when you take a male out of the class, it will reduce the total number of male in the class, likewise it will reduce total number of people in the class which can be illustrated diagrammatically as:



Male students now become 5/10, the same applies to the females but the total number will reduce but the total number of females remain the same in the class which will be 5/10. When it comes to the other side of the tree diagram which represent females, there are 5 females in the class if one is pick, it will reduce the total number of females in the class likewise number of females in the class if one is pick, it will reduce the total number of females in the class likewise number of females in the class with the tree diagram it will be represented by 4/10 to the male at the other side of the diagram as the same concept was applied on the side, it will also apply at the same side of the diagram which is the total number of people in the class will be reduced to one but the total number of males remain the same, it will end up at 6/10.

Interviewer: Could you explain a bit detail how you arrive at this answer 6/11.

Responder B: With the 6/11 I realise that the probability with different sexes, MF or FM looking at the tree diagram male is represented by 6/11 whilst female represents 5/10 multiplying, we get 30/110 also 5/10* 6/11 = 30/110 when you add 30/110 + 30/110 = 60/110 = 6/11

Interviewer: Alright, what did you learn during PBL on problem solving?

Responder B: Thank you very much, based on the steps of problem solving, I learnt how to use Polya's problem solving technique, ie., understanding the problem, then devising a strategy to solve the problem, then you carry out the plan, then you look back to check the strategy used to solve the problem, I realise that after checking I now understood and can now solve the problem.

Interviewer: How did PBL facilitate the task of team work with your colleagues?

Responder B: I learnt to communicate mathematically during PBL session, I learnt to also collaborate with my colleagues and lastly, I learnt to cooperate with my colleagues no matter the situation.

Interviewer: Did PBL make group discussion noisy?

Responder B: No

Interviewer: Is it true that PBL session is boring?

Responder B: No

Interviewer: What is your opinion on using PBL to solve probability problems?

Responder B: Is simple to understand and make learning of probability concepts interesting and fully of joy when using Polya's problem solving technique.

Course Instructor (Interviewer): What is your opinion about the pre-test questions that you answered?

Responder D: I enjoyed it because probability is something, I learnt at SHS. I learnt new things and I'm happy about it.

Interviewer: You were asked about a probability question concerning a ludo die and a coin tossed twice and you are to draw a table to illustrate the sample space and I realised your table was correct but your final answer was 11/24, why?

Responder D: I had the answer but I realised I made a mistake, one (1) is not a prime number and I added it, that is the reason why I had the answer wrong.

Interviewer: If you say one (1) is not a prime number can you please explain what a prime number is to me?

Responder D: A prime number is a positive integer that has exactly two divisors, however, one (1) has only 1 divisor, 1 and itself.

Interviewer: Based on your explanation, you now agree that 1 is not a prime number, looking at the results on the table can you now give the outcomes for your answer.

Responder D: Yes, the outcomes are HT3, HT3, HH2, HH3, HH5, TH2, HT2, TH5, HT5

Interviewer: Can you now tell me how you arrive at your final answer.

Responder D: I now realised that my first answer was wrong and the correct answer is 9/24 = 3/8

Interviewer: How would you explain this to your fellow student who is not familiar with probability?

Responder D: By the explanation I had, I will let him use the tree diagram by tossing the coin once, H or T and when you tossed it twice you get HH, HT, TH, TT

Diagrammatically,

combining with the die you get:

	1	2	3	4	5	6
HH	HH 1	HH 2	HH 3	HH 4	HH 5	HH 6
HT	HT 1	HT 2	HT 3	HT 4	HT 5	HT 6
TH	TH 1	TH 2	TH 3	TH 4	TH 5	TH 6
TT	TT 1	TT 2	TT 3	TT 4	TT 5	TT 6

With this I can help my colleague to complete the table as illustrated.

Interviewer: What did you learn during PBL on problem-solving

Responder D: I learnt some techniques by George Polya which involve the steps;

1. Understanding the problem
2. Devising strategy to solve the problem
3. Carrying out the strategy
4. Reflect on the results

Interviewer: How did PBL facilitate the task of team work with your colleagues?

Responder D: I learnt to communicate during PBL, I also learnt to collaborate with my colleagues to find solution to probability problems. I also learnt to collaborate and cooperate with colleagues during PBL session. It helps me to talk with my colleagues now, I used to feel shy in talking but because you encourage all of us to take part and do presentation, I have to force myself to talk.

Interviewer: Some people say PBL session is noisy. What do you say about that?

Responder D: Yes, our group was too noisy.

Interviewer: Why was it noisy?

Responder D: Members argue a lot; they say anything to distract the attention of the group. Moreover, some of them used their mobile phones to play games during group discussion.

Interviewer: Can you say your group was also boring?

Respondent D: No, it wasn't.

Interviewer: What is your opinion on the use of PBL on probability problems?

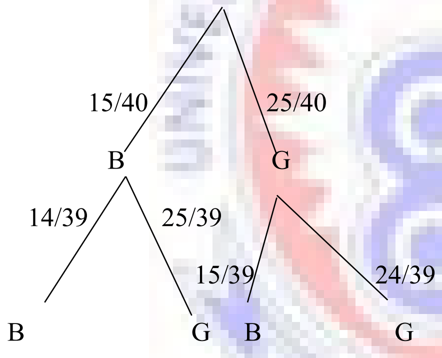
Responder D: It made learning of probability concepts interesting and I really enjoyed it.

Course Instructor (Interviewer): What was your opinion on the pre-test questions that you answered?

Responder E: The pre-test was difficult in finding the solution because I had little knowledge about them.

Interviewer: I realised that you had a particular question right after the intervention meanwhile a similar one was given in the pre-test and you had it wrong. What will you say about that please? The question was on: "In a class of 40 students, 15 are boys and the remaining girls, if 2 students are selected at random without replacement, what is the probability that they are both girls?

Responder E: During the pre-test I couldn't solve it rightly, with the intervention of the probability session I was able to solve it correctly. Before I work it out, I have to find the number of girls that were in the class because the number of girls were not given in the question. So, thereby I subtracted the number of boys from the whole class it gave me the number of girls, that is 25, so using the tree diagram:



Representing B as boys and G as girls, the 15/40 represent the boys and 25/40 representing the girls.

Interviewer: So, 15/40 what do you mean?

Responder E: 15/40 is the probability of boys in the class 25/40 is the probability of girls in the class. One (1) had been selected out, the class will be affected, i.e., 14/39. So, the concept is applied to the other side of the branch

Interviewer: How do you arrived at that answer?

Responder E: Probability of both girls, it has to be 25/40 * 24/39 = 5/13

Interviewer: How would you explain this probability problem to your fellow student who is not familiar with probability.

Responder E: Actually, is not something difficult, I will be able to use the concept of tree diagram to explain to my colleague to understand.

Interviewer: What did you learn during PBL on problem solving?

Responder E: I learnt a lot, and I have learnt that is good to learn in groups and use "Youtube" to search for information on solution to probability problems. I have also learnt to tolerant one another during group discussion. I have learnt concepts about probability and is interesting.

Interviewer: How did PBL session facilitate the task of team work?

Responder E: PBL encourages me a lot, and i learnt to collaborate with peers to find solutions to problems, I also learn to cooperate with my peers no matter the situations. I learnt to tolerant each other and to work together.

Interviewer: Is PBL session noisy?

Responder E: No

Interviewer: Is it true that PBL session is boring?

Responder E: No, is very interesting.

Interviewer: What is your opinion generally about this approach of PBL on probability problems?

Responder E: It was actually a perfect one and I think the tutors should start using right away to teach us at the college level.

Course Instructor (Interviewer): What is your opinion about the pre-test questions that you wrote?

Responder F: To be frank with you it was not easy at all, probability back at SHS I didn't get easy, for that reason I used to dodge classes, anytime is on probability lesson I feel like I don't understand, so I always dodge anytime it is Statistics and Probability. I didn't enjoy the way the master was teaching it.

Interviewer: So, the objectives you guess the answers?

Responder F: I just guess the objectives alone and couldn't attempt the theory

Interviewer: You realised that when I taught you using PBL on probability tree diagrams, how do you see the approach?

Responder F: Master, to be honest with you, if I were not be part, I would have done myself a great loss, you taken us through the new intervention I now see probability to be very simple, I never knew I can be able to use tree diagram in solving probability questions. To be honest with you I now see probability to be simple thing for me.

Interviewer: How do you see PBL when it comes to group work?

Respondent F: Is true, anytime we have presentations our group always come first, I enjoyed the group presentations.

Interviewer: What were you doing and you were getting new concepts like that?

Responder F: We were doing research using the internet, some of us had our SHS textbooks, we used them and the internet (You tube) to solve many of the problems.

Interviewer: Other group members told me during group discussions it was noisy?

Responder F: It was never true, my group they were all serious, I'm glad I got that group.

Interviewer: Was your group boring?

Responder F: No, the concept under discussion were interesting I don't see why it should be boring.

Interviewer: Generally, what is your opinion about PBL on Probability?

Responder F: I wish our end of semester exams they should bring such problems because with the concepts we had, I can assure you that if such problems are brought, we can assure you that we will pass very well.

Course Instructor (Interviewer): What is your opinion about the pre-test questions that you wrote?

Responder G: At first it was a challenge to me, but because your research it was better and understandable for me.

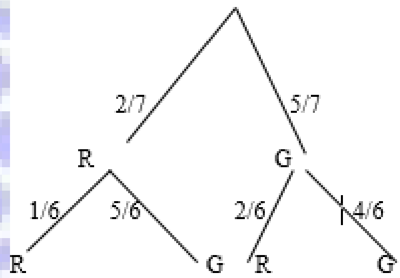
Interviewer: Looking at your solution, I realised you answered a question, could you explain a bit detail for me? The question was on: A bag contains 7 discs 2 of which were red and 5 green. 2 discs were removed after the other without replacement and you were asked to draw a tree diagram to illustrate these probabilities, from that you should find the probability that they are of different colours? I realised that during the pre-test you only attempted this question, now I want to know after the intervention do you now see the solution different?

Responder G: Yes, represent probability of red by 2/7 and probability of green by 5/7 but the question was without replacement so, picking one (1) from 2 out of 7 you get 1/6 for probability of red and for the green it would be the same except the total would be reduced. For the green 5/7 would be 4/6.

Interviewer: So, how do you arrive at the answer?

Responder G: Is simple, it would be (2/7 * 5/6) + (5/7 * 2/6) = 10/42 + 10/42 = 20/42 = 10/21

As shown in the diagram below:



P (different colours) = P (RG) or P (GR) as being calculated above.

Interviewer: How would you explain this to your fellow student who is not familiar with probability?

Responder G: You first of all help him/her to use the tree diagram to represent the probabilities and by guiding him/her to come out with the outcomes of the probabilities. You take first 2/7 * 5/6 + 5/7 * 2/6 then by multiplying you would be able to arrive at the correct answer.

Interviewer: What did you learn during PBL on problem solving?

Responder G: First understanding the problem, devising ways of solving the problem, carry out the plan and you reflect on the problem and evaluate the solution to check corrections.

Interviewer: Did PBL facilitate the task of team work and how do you see the spirit of team work?

Responder G: I may not have an idea but members from the group may have ideas so it made it easy for us.

Interviewer: Some argue that PBL is noisy, what is your view about that?

Responder G: It was interesting because you are meeting your colleagues, and you share ideas so at the end there is something you achieved.

Interviewer: Generally, what is your opinion about PBL on probability?

Responder G: Is a good initiative and should be used in teaching us at the college level to help us impart knowledge to students at the basic level.

Interviewer: Thank you for your time I really appreciate, God bless you.

Course Instructor (Interviewer): What was your opinion on the pre-test given to you to answer?

Responder H: It was nice because I had an idea back at SHS but some questions were quite challenging.

Interviewer: How do you see the new method PBL in solving probability questions, what were the steps? What will you do before solving a question?

Responder H: It was interesting and you need to analyse before solving the question.

Interviewer: How do you see the intervention in terms of team work?

Responder H: It was good because we used you tube videos on tree diagrams to help us find solution to the group work for presentation.

Interviewer: Was your group noisy during PBL sessions?

Responder H: It was not noisy

Interviewer: Was it boring?

Responder H: It was interesting

Interviewer: What is your opinion of PBL on probability?

Responder H: It was interesting and lovely.

Interviewer: Thank you God bless you.

Course instructor (Interviewer): What is your opinion on the pre-test questions you answered?

Responder I: Based on my knowledge at SHS I was able to answer some of them, but question relating to tree diagram I was not able to answer because during our SHS we were not taught probability using tree diagram.

Interviewer: I realised that you were able to answer many of the questions correctly, your challenge was the used of tree diagram. You realised also that I carried out an intervention using PBL to let you understand concepts and application of probability. After I have employed that methodology how do you now find probability questions?

Responder I: The used of tree diagram is very simple to understand probability problems if teachers are trained to teach probability using tree diagram. When you used the tree diagram to teach us, in fact, I can now use the tree diagram to explain to my colleagues to understand well in probability. The tree diagram is very important when it comes to teaching of probability.

Interviewer: Good to hear about that recommendation with the used of tree diagrams on probability. It means that when confronted with a question and someone is not familiar with probability you will be in position to explain.

Responder I: I can do that with more confidence in explaining the concepts using tree diagram.

Interviewer: I want you to tell me what you learnt during PBL on problem- solving?

Responder I: In fact, we learnt a lot during PBL, some of the things you taught us we understood that before you solve a problem you have to follow guidelines or steps. These are the steps:

1. Understanding the problem is part of finding solution to the problem.
2. Then, you devise strategies to solve the problem.
3. Carry out the strategy.
4. Checking around by looking at the strategies.

Interviewer: Once you get your solution you must look back to check to make sure everything is alright.

Responder I: Yes sir.

Interviewer: How did PBL facilitate the task of team work with your colleagues, like when you were in group discussion?

Responder I: In fact, during PBL we learn a lot during group discussion and tolerant each other. In fact, you become self-confident, you learn a lot of skills, thinking ability because you think critically to come out with solutions. In fact, reflective thinking is employed here, so we learn a lot in PBL.

Interviewer: Some people argue that during PBL session group discussion is a little noisy?

Responder I: No.

Interviewer: Some say is boring?

Responder I: Is very interesting because coming out with solution to problems with little guidance or no guidance, it becomes very interesting in fact, you become self-directed learning very interesting.

Interviewer: Generally, what is your opinion on PBL on problem-solving?

Responder I: In fact, it is simple to understand, easy to memorise. In fact, when you used PBL knowledge acquired is very useful in life situation, knowledge acquired also sustained in you for very long time. I wish that all teachers should used the PBL to teach in all institutions especially at the college level of the Ghanaian education.

Course Instructor (Interviewer): What is your opinion about the pre-test questions that you answered?

Responder J: It was nice and interesting we really enjoyed it.

APPENDIX G

Weekending: 18th February, 2019 **LESSON PLAN FOR TRADITIONAL METHOD APPROACH (CONTROL)** GROUP
College of Education: College B
Course: Statistics and Probability REFERENCE: Mathematics for Diploma Colleges, PP.425-451

Date /day/ Time/duration	Topic/ Sub- Topic	Objectives/ R.P.K	Teaching/Learning Material Teacher/ Learning Activities	Core-Points	Evaluation
Date 18-02-19 Day Monday Time 9:30am - 11:30am Duration 2hrs.	Topic: Statistics and Probability <u>Sub- Topic</u> Probability problems with replacement and without replacement	Objectives: By the end of the lesson the will be able to; 1. Solve problems on probability with replacement using formulae approach 2. Solve probability problems without replacement using formulae approach R.P.K	TLMS: Marble Keywords; With replacement, without replacement. **INTRODUTION** Tutor gives students the following problems as exercises to review their R.P.K. 1. An urn contains 10 marbles of which 8 are blue and 2 are red. Find the probability of choosing a blue marble from the urn? 2. If event A and event B are mutually exclusive, find the P(AUB), give P(A)= $\frac{1}{3}$ and P(B)= $\frac{2}{5}$.	Probability with replacement is the chance of picking an event at random from an urn or container and putting it back for the next chance of picking another event whilst probability without replacement is the chance of picking an event from a container or a bag and not putting it back before picking the next event.	

		Students have solved problem involving simple probability events.	**PRESENTATION** ACTIVITY1. Tutor assists students in class to solve probability problems with the concepts of with replacement and without replacement, using RSTVV algorithm. 1**. Read** the problems carefully and take note of the key words. 2. **Select** variables to represent events or parameters. 3. **Think** of a plan to help you represent the information into the formula. 4.**Use** your plan to solve the problem. 5. **Verify** the answer where necessary. **Teacher/Learner Activities** Example1 A bag contains 4 red marbles and 3 green marbles. A marble is taken out at random and then put back; a second marble is then taken from the bag.	RSTVV Algorithm for problem solving R=Read S=select T=Think U=Use V=Verify	

			What is the probability that: a) Both marbles are the same colour? b) At least one of the marbles is green? **Solution** 1. Read the problem: we are able to find the probability of marbles with the same colour and at least one of the marbles being green, with the concept of with replacement. 2. Select variable to represent red marbles and green marbles 3. Think of a plan or formula: Translate the information in your plan or formula 4. Use your plan or formula to solve for the solution by representing R with red marbles and G with green marbles = n(R)=4, n(G)=3, Total number of marbles, n(U)=7 a) P(Same colour)=P(1R∩2R) or P(1G∩2G) =P(1R∩2R) +P(1G∩2G) $=\frac{4}{7}\times\frac{4}{7}+\frac{3}{7}\times\frac{3}{7}$		

			$=\frac{16}{49}+\frac{9}{49}$ $=\frac{25}{49}$ b) P(at least one green marble) =P(1G∩2G) or P(1G∩ 2R) or P(1R∩2G) $=\left(\frac{3}{7}\times\frac{3}{7}\right)+\left(\frac{3}{7}\times\frac{4}{7}\right)+\left(\frac{4}{7}\times\frac{3}{7}\right)$ $=\frac{9}{49}+\frac{12}{49}+\frac{12}{49}=\frac{33}{49}$ Teacher/Learner Activities ACTIVITY 2 Tutor assist student in class to work examples in probability problems without replacement. Example 2. A bag contains 7 yellow marbles and 5 red marbles. One marble is taken from the bag at random and is not replaced. A second marble is then taken from the bag. Determine the probability that: i. The marble s are different colours		

			ii. At least one marble is yellow **Solution** Denote y to be Yellow ,n(Y)=7 R to be RED, n (R)=5 i. P (marble are different colours) =P(1Y∩2R) or P(1R∩2Y) $=\left(\frac{7}{12} \times \frac{5}{11}\right) + \left(\frac{5}{12} \times \frac{7}{11}\right)$ $=\frac{35}{132} + \frac{35}{132}$ $=\frac{70}{132}$ $=\frac{35}{66}$ Alternatively: P(both marbles are of the same colour) =P(1Y∩2Y) or P(1R∩2R) $=\left(\frac{7}{12} \times \frac{6}{11}\right) + \left(\frac{5}{12} \times \frac{4}{11}\right)$ $=\frac{42}{132} + \frac{20}{132}$ $=\frac{62}{132}$ $=\frac{31}{66}$		

			Teacher/Learner Activities Hence P(marble are different colours) $=1-P(\text{Same colour})$ $=1-\frac{31}{66}$ $=\frac{35}{66}$ ii. P(at least one Yellow) $=\left(\frac{7}{12}\times\frac{6}{11}\right)+\left(\frac{7}{12}\times\frac{5}{11}\right)\left(\frac{5}{12}\times\frac{7}{11}\right)$ $=\frac{42}{132}+\frac{35}{132}+\frac{35}{132}$ $=\frac{112}{132}$ $=\frac{28}{33}$ Alternatively: P(at least one Yellow)$=1-P(1R\cap 2R)$ $=1-\left(\frac{5}{12}\times\frac{4}{11}\right)$ $=1-\frac{20}{132}$ $=\frac{112}{132}$ $=\frac{28}{33}$		

APPENDIX H

LESSON PLAN FOR THE PROBLEM-BASED LEARNING (EXPERIMENTAL) GROUP

Weekending: 11/02/19
College: College A
Course: Statistics and Probability
Level: L200 (second year student)

References: Mathematics for Diploma Colleges by Rev. Kusi Appau, PP.425-451

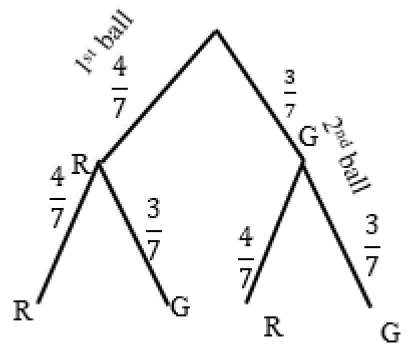
INTERVENTION

Date/Day/Time/ Duration	**Topic/Sub-Topic**	**Objectives/R.P.K**	**Teaching/learning Materials Teacher /Learner Activities**	**Core-Points**	**Evaluation**
Date 11/02/19 Day Monday	Topic Statistics and Probability Sub-Topic Probability problems with replacement and without replacement.	Objectives By the end of the lesson the student will be able to: 1. Solve probability problems with replacement using tree diagrams. 2. Solve probability problems without	TLM: Pictorial representation of tree diagram. Key words: With replacement, without replacement, tree diagram. INTRODUCTION Tutor gives students the following problems as exercises to review their R.P.K. 1. If a bag contains 3 red balls and 4 Blue balls, find the probability of picking a red ball.	Probability with replacement is the chance of picking an event at random from an urn or container and then putting it back for the next chance of picking another	

Time 7:30am – 9:30am. Duration 2hours.		replacement using tree diagram and also solve probability problems related to spuns.	2. If the event A and event B are independent events, given that P(A) = 0.42 and P(B) = 0.15, Find the P(AUB)	event whilst probability without replacement is the chance of picking an event from a container or a bag and not putting it back before picking the next event. The tree diagram is a diagram used to show the total outcomes in a probability experiment.	
		R.P.K. Students have ideas about simple probability problems.	**PRESENTATION** Activity 1 Assist students in a group to solve probability problems with replacement using outcome tree or tree diagram using RSTUV algorithm. 1. Read the problem carefully and take note of the key words. 2. Select a variable to represent the parameters given.	**RSTUV** Algorithm for problem solving R= Read S = Select T= Think U= Use	

			3. Think of a plan to help you represent the information on a tree diagram. 4. Use your plan to solve the problem. 5. Verify the solution /answer where necessary. Example 1 A bag contains 4 red balls and 3 green balls. A ball is taken out of random and then put back, a second ball is then taken from the bag. What is the probability that: a) Both balls are the same colour? b) At least one of the balls is green?	V= Verify	

			Solution 1. Read the problem: We are asked to find the probability of balls with the same colour and at least one colour being green with the concept of with replacement. 2. Select variables to represent red balls and green balls. 3. Think of a plan: Translate the information on a tree diagram. 4. Use your plan to solve for the solution by representing R with red balls and G for green balls. $\Rightarrow n(R) = 4$, $n(G) = 3$, Total number of balls, $n(U) = 7$	Representation of the events of red balls and green balls on a tree diagram with outcomes and probabilities.	

			 [RR RG GR GG] $\left[\frac{4}{7}\ X\ \frac{4}{7};\ \frac{4}{7}\ X\frac{3}{7};\ \frac{3}{7}\times\frac{4}{7}=;\frac{3}{7}\ X\ \frac{3}{7}\right]$ Probabilities $=\frac{12}{49}=\frac{12}{49}=\frac{9}{49}$ Verify your answers. $\frac{16}{49}+\frac{12}{49}+\frac{12}{49}+\frac{9}{49}=\frac{49}{49}=1$ Now that the probabilities are shown on the tree diagram, we can now answer the questions as follows: a. P(both are of the same colour)		

			$= P(RR \text{ or } GG)$ $= \frac{16}{49} + \frac{9}{49}$ $= \frac{25}{49}$ b. P(at least one green ball) $= P(GG \text{ or } GR \text{ or } RG)$ $= \frac{9}{49} + \frac{12}{49} + \frac{12}{49}$ $= \frac{33}{49}$ **ACTIVITY 2** Tutor group students in a group of five and assist them to work example on probability problem without replacement. Example 2: A bag contains 7 yellow balls and 5 red balls. One ball is taken from the bag at random and is not replaced. A second ball is then taken from the bag. Determine the probability that:		

i. The balls are different colours.
ii. At least one ball is yellow.

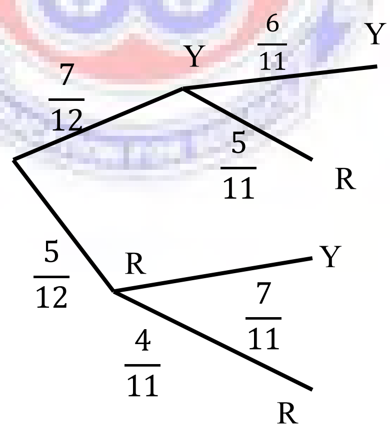
Solution

Denote y to be yellow, n (Y) =7

R to be red, n(R) = 5

Total number of balls, n (U) = 12

Tree Diagram



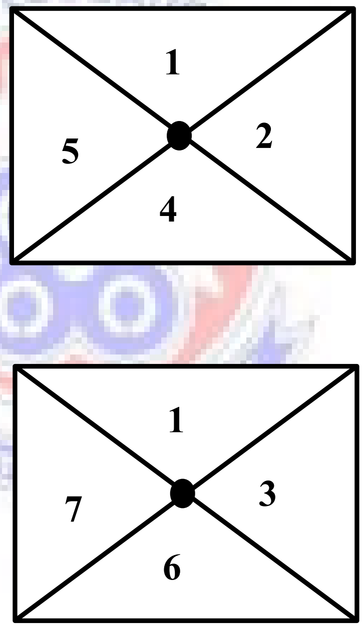
			$YY = \frac{7}{12} \times \frac{6}{11} = \frac{42}{132}$		

			$YR = \frac{7}{12} \times \frac{5}{11} = \frac{35}{132}$ $RY = \frac{5}{12} \times \frac{7}{11} = \frac{35}{132}$ $RR = \frac{5}{12} X \frac{4}{11} = \frac{20}{132}$ Check: Total probability = $\frac{42}{132} + \frac{35}{132} + \frac{35}{132} + \frac{20}{132} = \frac{132}{132} = 1$ (i) P(balls are different colours) = P(YR) or P(RY) = P(YR) + P(RY) $= \frac{35}{132} + \frac{35}{132}$		

			$= \frac{70}{132}$ $= \frac{35}{66}$ Alternatively: P(both balls are of the same colour) = P(YY) or P(RR) = P(YY) + P(RR) $= \frac{42}{132} + \frac{20}{132}$ $= \frac{62}{132}$ $= \frac{31}{66}$		

			Hence P(Different colours) = 1 – P (same colour) $= 1 - \left(\frac{31}{66}\right)$ $= \frac{35}{66}$ (ii) P(at least one yellow) = P(YY) or P(YR) or P(RY) $= \frac{42}{132} + \frac{35}{132} + \frac{35}{132}$ $= \frac{112}{132}$ $= \frac{28}{33}$ Alternatively P(at least one yellow) = 1-P(RR)		

			$= 1 - \frac{20}{132}$ $= \frac{112}{132}$ $= \frac{28}{33}$ Check: $\frac{42}{132} + \frac{35}{132} + \frac{35}{132} + \frac{20}{132} = \frac{132}{132} = 1$ **Teacher /Learner Activities** **ACTIVITY 3** Tutor group students in groups of five and assist them to work examples on probability spinners. Example 3		

			The diagrams show two spinners which are both spun. What is the probability that the total score on the two spinners is a) 7 b) less than 5  		

Solution

Form a table using the above diagram

Spun	1	3	6	7
1	2	4	7	8
2	3	5	8	9
3	4	7	10	11
4	5	8	11	12

Let n(s) be the sample space, ie. n(s) = 16.

n(E) be the event of spinning 7, n(E) =2

n(A) be the event of scores less than 5. Ie. n (A) = 4.

a) P(Spinning 7) $\frac{n(E)}{n(S)} = \frac{2}{16} = \frac{1}{8}$ or 12.5%

b) P(A<5) = $\frac{4}{16} =$ or 0.25%

			ACTIVITY 4 Tutor ask students in groups to work examples as he goes round to assist groups who might encounter difficulties after which he appoints any member of any of the groups to present their findings to the class while acting as a facilitator in scaffolding problem and other groups engage the presenter in dialogue to arrive at consensus.		**Home work** 1. A box contains 10 marbles, 7 of which are black and 3 are red. Two marbles are drawn one after the other without replacement. Find the probability of getting: a. A red, than a black marble. b. Two black marbles 2. A man has 9 identical balls in a bag. Out of these, 3 are black, 2 are blue and the

					remaining are red. a. If a ball is drawn, what is the probability that it is (i) not blue (ii) not red?

			1. A bag contains 12 blue and 8 red pens. 2 pens are chosen at random from the bag without replacement. What is the probability that: a. both are of the same colour? b. both are of different colours? 2. Three balls are drawn one after the other with replacement from the bag containing 5 red, 9 white and 4 blue identical balls. What is the probability that they are one red, one white and one blue? 3. A fair die and unbiased coin are tossed once. What is the probability of obtaining 3 a tail? **CLOSURE** Tutor concludes lesson / session by summarizing the salient points and give students homework.		b. If a balls is drawn at random, one after the other, what is the probability that both of them will be: i. black if there is no replacement? ii. blue if there is replacement?