**UNIVERSITY OF EDUCATION, WINNEBA**

# EFFECT OF GEOGEBRA ON THE ACADEMIC PERFORMANCE OF STUDENTS OF PRESBYTERIAN COLLEGE OF EDUCATION, AKROPONG–AKUAPEM

**ANSONG EMMANUEL KWADWO**

**MASTER OF PHILOSOPHY**

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**A thesis in the Department of Mathematics Education, Faculty of Science Education, submitted to the School of Graduate Studies in partial fulfillment**

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**Master of Philosophy**
**(Mathematics Education)**
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# DECLARATION

## STUDENT'S DECLARATION

I, Ansong Emmanuel Kwadwo, declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

SIGNATURE:…………………………….

DATE:…………………………………….

## SUPERVISOR'S DECLARATION

I hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of thesis as laid down by the University of Education, Winneba.

Supervisor: Mr. Michael E. Amppiah

SIGNATURE:…………………………

DATE:………………………… …..

# DEDICATION

To my lovely parents, Mr. and Mrs. Sampong, who believe in hard work and perseverance.

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# ABTRACT

The purpose of this study was to investigate the effect of integration of GeoGebra into the teaching and learning of circle theorem on students' performance at the Presbyterian College of Education, Akropong-Akuapem, Ghana. A sample of 390 level 100 students was selected purposively from a population of 490 level 100 students for the study. The main instruments for data collection were test and questionnaire. The four points Likert rating scale was used in administering the questionnaire and it was analysed using percentages. Data collected from the test were also analysed and presented by the independent sample t –test. Two tailed test was used in the descriptive statistics to test the null hypothesis. The reliability coefficient of the instruments was ascertained using Kuder–Richardson KR20 for the pre-test and Cronbach Alpha for the post-test. The reliability coefficient for the pre-test was 0.66 and that of the post-test was 0.65 indicating the instruments were accurate and reliable. The findings showed that there was significant difference between the mean score of students who were taught circle theorem using GeoGebra and those taught without the use of GeoGebra in favour of the GeoGebra group but no significant difference were found with respect to gender. Lack of instructional aids, students' psychological fear, poor foundation of students in basic school mathematics, poor preparations on the part of some mathematics teachers and large classes were identified as the main causes of students' difficulty in learning circle theorem. The study recommended among others that enough mathematics software especially GeoGebra should be provided in schools.

# LIST OF ACRONYMS

| | |
|---|---|
| WASSCE | West African Senior School Certificate Examination |
| WAEC | West African Examination Council |
| ICT | Information and Communication Technology |
| ANCOVA | Analysis of Co-variance |
| GLAT | Geometry Learning Assessment Test |
| BECTA | British Educational Communications and Technology |
| KR20 | Kuder-Richardson 20 |
| SPSS | Statistical Package for Social Sciences |

# CHAPTER ONE

# INTRODUCTION

## 1.0 Overview

This chapter is an introductory section of the study. It presents the background to the study, statement of the problem, purpose of the study, research objectives, research questions, hypotheses, significance of the study, delimitation, limitation and thesis organisation.

## 1.1 Background of the Study

Changing pedagogical perspectives have been influential over mathematics instruction as well as, on many other teaching fields. A great majority of the founding values of the new perspectives are reflections of those already existing in mathematics instruction. The most striking difference in the new perspectives is that, the focus of instruction has moved from teacher-centered education to student centered one. Mathematical problems have become more complex and abstract and these impede some students either from finding or spotting the solution, although they have the same knowledge levels with their peers (Okafor & Anaduaka, 2013, Yalmezer & Kiklikci, 2014). That is why topics like geometry, algebra and calculus are considered as too difficult to understand by large sections of the students from the primary school through to the university level. This perceived abstract nature of mathematics has resulted in a decline in both the skill and interest level in mathematics (Fahlberg-Stojanovska & Stajanovski, 2009). For students to understand the mathematics we teach, we need to teach effectively. “Effective mathematics teaching requires understanding what learners know and need to learn and then challenging and supporting them to learn it well (NCTM, 2000, P.16).

In Ghana, mathematics enjoys a lot of recognition and respect from policy makers. It features prominently as a core subject in the basic schools and high schools. It is a basic requirement for entering senior high school, colleges of education, nursing training colleges, polytechnics and universities.

Again, according to Haertel and Means (2004) ICT can support learning when appropriately integrated with teaching techniques, curriculum, and assessments. As a result, the mathematics curriculum in Ghana has been reviewed to conform to the New Education Reform of 2007 and ICT for Accelerated Development (ICT4AD, 2003) and this means that teachers should use ICT in their teaching and learning process.

Bos (2009) points out that when technological tools are available in mathematics classrooms, learners can pay attention on reflection, problem solving, reasoning and decision making. Similarly, the National Council of Teachers of Mathematics (NCTM, 2000) in its document, *Principles and Standards for School Mathematics*, lists technology as one of the key principles to enhance the quality of Mathematics by suggesting that, "Teachers should use technology to enhance their students' learning opportunities by selecting or creating mathematical tasks that take advantage of what technology can do efficiently and well-graphing, computing and visualizing." (NCTM, 2000, p.25). However, despite the impact educational technology and strong advocacy for the need to utilize ICT in the teaching and learning of mathematics, Ghanaian classrooms are still characterized by traditional methods of teaching.

It is on this note that Snyder and Snyder (2008) are of the view that there are instructional methods that can help students to think critically and give them better understanding (the use of ICT in mathematics teaching and learning). According to them, the traditional teaching methods focus predominantly on rote learning or memorizing of information from textbooks or lectures (McTighe & Self, 2003; Snyder & Snyder, 2008). According to Snyder & Snyder (2008), these instructional methods do not promote conceptual understanding. Also, according to Van Hiele (1986) circle geometry involves proving of theorems that require high cognitive levels of thinking (De Villiers, 2010; Mason, 1998). De Villiers (2010) posits that this is a key reason for the failure of the traditional geometry curriculum and why students struggle to understand their teachers. De Villiers (2010) contends that the teacher and students are speaking different languages (not the spoken language) and they think on different levels. Ampiah et al (2004) also reported that both pre-service and in-service programmes in mathematics predominantly reflect the traditional method of teaching and learning. This teaching method according to them is more of teacher–centred rather than student–centred. This situation produces students who are able to do all the calculations but do not know how to apply the theory to solve simple everyday life problems that involve concepts and mathematical skills. This way of teaching has resulted in general detest for mathematics by students and poor mathematics performance at the Colleges of Education and international examinations ( e.g. Trends in International Mathematics and Science study, TIMSS). Results from TIMSS in 2003 and 2007 at the junior high school level (grade 8 equivalents) are instances of poor mathematics performance in the country. In the aforementioned study, Ghana's 8 graders were ranked 43$^{rd}$ among 44$^{th}$ and 46$^{th}$ among 47 countries that participated in the study in 2003 and 2007 respectively (Mullis et al 2004, 2008). Thus, national and international reports show that Ghanaian students perform poorly in higher order thinking problems. The 2003 TIMSS report by Anamuah–Mensah, Mereku and Asabere–Ameyaw (2004) indicate that Ghanaian students scored zero in advance and higher- level thinking in the content domain tested. Unfortunately, Geometry was one of the topical areas candidates' performance was weak. Also many studies indicate that students poor performance in geometry is due to teaching methods use in teaching geometry (Lim & Hwa, 2007) The above mentioned factors indicate that, problems exist on the ground which must engage the attention of people who want to see progress in our educational set up, especially mathematics. It is against these premises that the researcher has decided to experiment to see if teaching with GeoGebra software would help students of Presbyterian college of education, Akropong-Akuapem to explore and visualize the beauty of circle theorem and hence understand circle theorem concepts. Circle Theorem is one of the topics in the SHS core mathematics syllabus which students find difficult to understand (WASSCE Chief Examiner's Report for core mathematics, 2016) and therefore by the time students get to colleges of education, because students had inappropriately early schemas on the concept of circle theorem, building on it become very difficult. Assimilation takes place when new schema can easily be absorbed into the existing cognitive structures. We demonstrate mathematical behavior when we recognize and describe patterns, construct physical and conceptual models of phenomena, create symbolic systems to help us represent, manipulate, and reflect on ideas, and invent procedures to solve problems (Battista, 2007). Circle theorem is a wonderful area of mathematics to teach. It is full of interesting problems and surprising theorems. It is open to many different approaches. It has a long history, intimately connected with the development of mathematics. It is an integral part of our cultural experience being a vital component of numerous aspects of life from architecture to design (in all its manifestations).

Geometry has been identified as one of the core mathematics concepts often posing challenge to learners and teachers. This problem came to fore when mathematics tutors of Presbyterian College of Education, Akropong-Akuapem met fresh level–100 students in the college. During one of their interactions, students were asked to write one topic that really gave them problem when they were in the senior high school. Surprisingly about 90% of the students indicated they did not understand circle theorem and as such have developed hatred for the topic. This really challenged the researcher being a practitioner in the area to find out what he can do to help improve the understanding of the concept “circle theorem” at the college. This led to the choice of the research topic: Effect of using GeoGebra to teach circle theorem on the academic performance of students’ of the Presbyterian College of Education, Akropong–Akuapem. The researcher wants to test whether integration of technology into the teaching and learning of circle theorem will help bring students’ problem in learning the concept to an end. It is the belief of the researcher that the integration of ICT into the teaching and learning of circle theorem would make the teaching of circle theorem more attractive to learners. Thus moving from the conventional way where students become passive participants in the classroom to a more active constructivist learner centeredness approach. The focus of this study would be to examine if geometry instruction through the use of GeoGebra software and through traditional methods has different effects over the learning of circle theorem at the Presbyterian college of education Akropong-Akuapem.

Again several research findings (Ezeh, 2005; Doris, Oneill & Sweetman, 2013) showed that sex differences in mathematics are varied. Forgasiz (2015) insist that gender should be a concern in mathematics education because of its importance. He argued that it is important to include gender as a variable in research studies analysis even if it is not the main focus of the study. This argument motivated the researcher to assess this variable. Hence the researcher decided to examine whether or not differences exist in the post-test score between male and female students.

## 1.2 Definition of Key Concepts

The study sought to compare two methods of instruction, instruction through the use of GeoGebra and the traditional mode of instruction ie. the chalk and chalk method of instruction.

### 1.2.1 The talk and chalk method of instruction/ Traditional Approach

This is the traditional or direct method of teaching where students focusses on what the teacher says and what he/s writes on the chalkboard. In this teaching pedagogy, the teacher is considered alpha and omega who unilaterally transmits knowledge and skills to the learner deemed to be empty and a "tabula rasa". Here the teacher is mostly an information giver instead of being a facilitator and the students are recipients of knowledge, instead of negotiators of mathematics concepts.

### 1.22 Instruction with GeoGebra

GeoGebra is one of the educational technology tools use in mathematics and any other subject. The software was developed by Markus Hohenwarter at the University of Salzburg in the year 2001 as his thesis. It is a Dynamic Mathematics Software for mathematics instruction and simple to use since it offers basic features of Computer Algebra System (CAS) to link some gaps between Statistics, Algebra, Calculus and Geometry. This software combined features of older software programs such as Maple, Derive, Cabri and Geometer's Sketchpad (Saha, Ayub, & Tarmizi, 2010). GeoGebra has several views (algebraic view; geometric view; spreadsheet view; computer algebra system view; protocol design view; and command line) all linked together. Instruction with GeoGebra enables Mathematical experiments and discoveries. Instruction with GeoGebra is largely student-centred. In addition to construct geometry dynamically, GeoGebra also provides, as a key element of learning geometry, visualization, estimation, conjecture, construction, proof and etc. GeoGebra is found to be very efficient in mathematics education and can be used effectively both in teacher training (Dogan & Karakirik, 2009) and students' learning (Doğan & Icel, 2010).

### 1.2.3 Circle Geometry

Geometry is one of the important branches of mathematics (Isil & Ubuz, 2004) and it is concerned with the study of different shapes. These shapes may be plane or solid. A plane shape is a geometrical form such that the line that joins any two points on it wholly lies on the surface. A solid shape on the other hand is bounded by surfaces which may not wholly be represented on a plane surface. Geometry according to Royal Society, 2001 does not start from formulating definitions and theorem, but it start from organizing spatial experiences that lead to the formulation of definitions and theorems. Geometry forms the building blocks of engineering and technical graphics. Also according to Aleksandrov, Kolmogorov and Lev-rent's (1963), conic section of geometry which is purely locus is of great importance to astronomy, mechanics and technology as cited by Achor, Imoko & Uloko (2009). Geometry helps us to describe and define world systematically (Canturk- Gunhan & Baser, 2007). In addition, it helps us to acquire abilities such as making new discoveries, analyzing problems and making connections between mathematics and real life situations (Bindak, 2004). Also, geometry is used to develop students' spatial awareness, intuition, visualizations and to solve practical problems and so on (Sunsuma, Masocha

& Zezekwa, 2012). Because of the above stated importance of geometry, students should be encouraged to study it.

**1.3 Statement of the Problem**

Over the year, the performance of students in circle geometry has been unimpressive. Statistics have shown that difficulty in teaching and learning of mathematics, circle geometry in particular, has resulted in mass failure in examinations (Adolphus, T. 2011). This mass failure in mathematics is real at the Presbyterian College of Education, Akropong-Akuapem and the trend of students' performance in circle geometry has been on the decline (UCC, Chief Examiner's Report for EBS 143: Geometry and Trigonometry, 2018). This may be due to the traditional approach of teaching and learning which ultimately makes students' passive learners and deficient in geometrical analysis and reasoning. Also, this method of teaching and learning Geometry lays more emphasis on how much a student can remember to the detriment of how well the student can think and reason and makes the teacher dominate the classroom and turns students to mere listeners (Mereku, 2010). For these reasons students are not encouraged to discuss, interact and explore the content collaboratively, and repeatedly fail to build the exploration and visualization skills. This mode of teaching geometry (circle theorem) leads to poor performance of students as noted by Bittista (2007) and Idris (2006).

Again, several reports of the West African Examination Council (WAEC) indicate that students who take WASSCE have been performing poorly in circle theorem questions (WAEC, 2018). In June 2018, the chief examiner for core Mathematics stated that most candidates who answered question 8 (a) demonstrated that their understanding of geometrical concepts were woefully inadequate. Candidates could not apply the cyclic quadrilateral theory and other geometrical principles to solve the problem. Mostly students avoid circle theorem questions when they have other alternatives. On rare occasion, the few students who attempt questions on the topic, most of them demonstrate lack of understanding of the topic (Fleteher & Anderson, 2012) and because the students at the colleges of education are the product of these senior high schools, they come with little or no understanding of the topic. This study therefore sought to investigate the effect of the integration of GeoGebra into the teaching and learning of circle theorem on the students' performance at the Presbyterian College of Education, Akropong-Akuapem and also find out the causes of students' difficulty in learning the concept "circle theorem. The college recognizes the pivotal role of Information and Communication Technology (ICT) to enhance the academic programmes, research initiatives and support services available for users in a safe, gender and disability friendly environment. As a result, in the 2016/2017 academic year, the college purchased Laptops and projectors for each department and also furnished the ICT laboratory with internet facility to ensure regular update of staff and students' knowledge in the use of ICT resources. Therefore the researcher believe that with the use of GeoGebra, the causes of this poor performance of students in circle theorem will be addressed. Again since students at the college will soon complete school and become teachers, there was the need to ensure that students were giving a very strong foundations in geometry so that they would be in a position to teach circle geometry when they are finally posted as teachers. This study therefore aimed at investigating whether the method of instruction (computer assisted instruction using GeoGebra) would motivate students to learn, enhances their problem-solving techniques and ultimately improve their performance in geometry (circle theorem).

Past research showed that the Van Hiele's levels of learning geometry can have implications for investigating students' difficulties and improving students' performance in geometry (Ada & Kurtulus, 2010). Again according to Ada and Kurtulus (2010), the Van Hiele's levels of learning geometry can also provide framework on which geometry instructions can be structured and taught. However, this claim has not been comprehensively investigated in the Colleges of Education in Ghana. Hence, it deserves some exploration and investigation with students at the college.

**1.4 Purpose of the Study**

The purpose of this study was to investigate the effect of integration of GeoGebra into the teaching and learning of circle theorem on students' academic performance at the Presbyterian College of Education, Akropong-Akuapem. The emphasis was to discover whether the method of instruction (computer assisted instruction using GeoGebra) would motivate students to learn, enhances their problem-solving techniques and ultimately improve their performance in geometry (circle theorem)

**1.5 Research Objectives**

The following objectives were used to guide the study:

1. Investigate the causes of students' difficulties in solving problems in circle theorem.
2. To find out whether or not there was any significant difference in performance between students taught by GeoGebra approach vis a vis students taught using the traditional approach.
3. To investigate if differences exist in the post–test scores between male and female students.

4. To find out if there exist any significant difference in the post-test average scores of the control group and the experimental group at the Van Hiele's geometric levels.

**1.6 Research Questions**

This study sought to address the following research questions:

1. What are the causes of students' difficulties in solving problems in circle theorem?
2. Is there any significant difference in performance of students taught with GeoGebra as compared to students taught without GeoGebra in circle geometry?
3. Is there any significant difference in the post-test scores of male and female students?
4. Is there any significant difference in the post–test average scores of the control and experimental group at the Van Hiele's levels?

**1.7 Research Hypotheses**

To determine the effect of using GeoGebra vis a vis the traditional method as an instructional tool in teaching Circle Theorem on the academic performance of students of Presbyterian College of Education, Akropong-Akuapem, the following hypotheses were formulated to guide the study.

1 Ho: GeoGebra as an instructional tool has no effect on the academic performance of students of Presbyterian College of Education, Akropong-Akuapem in Circle Theorem.

2 Ho: There is no significant difference in the post–test scores between male and female students.

3 Ho: There is no significant difference in the post–test mean scores at the various Van Hiele's levels

**1.8 Significance of the Study**

Institute of Education, University of Cape Coast has always been complaining about the poor performance of students in geometry. (Chief examiner's report for EBS 143: Geometry and Trigonometry, 2016, 2017, 2018, & 2019). This study therefore would serve as a source of reference material to all stakeholders of education including Colleges of Education Mathematics Tutors, Ghana Education Service, various religious education units and other organizations who have interest in education. The study would also motivate teachers at all levels of education to incorporate ICT into the teaching and learning of mathematics. This would improve the quality of education in Ghanaian schools more especially at the Presbyterian College of Education where the research was carried out. The study would also serve as a guide to mathematics educators in finding alternative and /or supplementary ways of teaching circle geometry instead of the usual talk and chalk method of teaching mathematics.

**1.9 Delimitation of the Study**

The study focused on students of Presbyterian College of Education, Akropong-Akuapem. The level 100 students were used for the study because Geometry was a course of study in level 100 and that was where the problem was predominant. Also, because geometry was an examinable course in level 100, students were motivated to participate fully in the study.

The use of GeoGebra could have been used to solve problems in other areas in Geometry but this study was limited to using the GeoGebra software to solve problems only in circle geometry (circle theorem).

### 1.10 Limitations of the Study

The use of achievement test and questionnaire in a mixed method (embedded) of data collection encountered some problems. This was because students responded to the two instruments anyhow. To ensure that students sincerely responded to the instruments, the researcher established good rapport with them. According to Dimiter and Phillip, (2003) maturation and history are major problems for internal validity in this design, whereas the interaction of pretesting and treatment is a major threat to external validity. Maturation occurs when biological and psychological characteristics of research participants change during the experiment, thus affecting their posttest scores. History occurs when participants experience an event (external to the experimental treatment) that affects their posttest scores. Interaction of pretesting and treatment comes into play when the pretest sensitizes participants so that they respond to the treatment differently than they would with no pretest. For example, participants in a job seeking skills training program take a pretest regarding job-seeking behaviors (e.g., how many applications they have completed in the past month, how many job interviews attended). Responding to questions about their job-seeking activities might prompt participants to initiate or increase those activities, irrespective of the intervention. Internal validity is the degree to which the Experimental treatment makes a difference in (or causes change in) the specific experimental settings. External validity is the degree to which the treatment effect can be generalized across populations, settings, treatment variables, and measurement instruments.

### 1.11 Thesis Organisation

This thesis was organized into five chapters. Chapter one was the introduction, which comprises the background to the study, statement of the problem and purpose of the study. It also included the significance, delimitations and limitations of the study. Chapter two took into account other literature which had a direct bearing on the research work and whose findings had been define in relation to the topic under study. Chapter three highlighted the methodology- the procedures followed in carrying out the study. Chapter four also captured the data presentation. This involved analysis of data based on the research questions and the results while chapter five dealt with summary, conclusion and recommendations.

# CHAPTER TWO

# LITERATURE REVIEW

## 2.0 Overview

The literature review focused on the role and/or impact of mathematical software, especially GeoGebra in the teaching and learning of mathematics. The researcher reviewed several research publications that had integrated computer technology, including GeoGebra, in the teaching and learning of mathematics at the various levels, ranging from primary school level to the university level. The study also reviewed literature on difficulties in learning geometry as well as the role of gender in geometrical learning. The chapter also include the theoretical and conceptual framework for the study. The aims of this review, among others, was to determine whether technology integration had a positive or negative effect on the teaching and learning of mathematics and whether gender plays any role in learning of circle geometry by students. The study also looked at identifying knowledge gaps in the integration process that need further exploration.

## 2.1 Theoretical Framework

The study adopted the Van Hiele's theory of geometrical understanding (Van Hiele, 1957) which describes the development of geometrical reasoning as its theoretical framework. It is a pedagogical theory which describes geometrical understanding levels of students by focusing on problems students face when they learn geometry (Olkun & Toluk, 2003). This theory was deliberately selected as the theoretical framework because of its relevance to the teaching and learning process of geometry; the Van Hiele theory was be used as a framework to analyse the learners' levels and/or stages that they go through when engaged in circle geometry problem-solving.

According to Van Hiele theory, students' progress through five levels of development when learning geometry, namely visualization, analysis, abstraction, deduction and rigour.

**Level 1: Visualization**

Van Hiele postulated that at these level students recognizes a figure by appearance alone, often by comparing them to a known prototype. The properties of a figure are perceived. Students recognize triangles, squares, circles, parallelogram, trapezium, kite and other shapes, but do not identify correctly the properties of these figures. At the visualization level, students make uninformed decisions because they base their arguments on perception rather than on reasoning (Ball, 1990). Also at this level, the properties of a figure cannot be understood; for example a student can say a figure is a cube, square or rectangle because he/s thinks it looks like one.

**Level 2: Analysis**

Analysis is a process of identifying and examining each element of an object or features on it in detail in order to understand it. A student operating at this level is able to identify each element of a geometric object in terms of its properties in isolation. At this level, students see figures as collections of properties. They can recognize and name the properties of a figure, but they do not see any relationships between the properties. When describing an object, a student at this level might be able to list all the properties he/s knows, but cannot make connections between figures (Battista & Clements, 1992). At this level a student sees the properties as discrete entities independent of one another. For example in circle geometry, the following two theorems can be viewed in isolation, yet they imply one another: (1) the angle subtended at the centre of a circle is twice the angle it subtends at the circumference of a circle and (11) the angle subtended by a diameter is 90°

**Level 3: Abstraction**

Abstraction is the process of formulating generalized concepts of common properties by disregarding the differences between numbers in a particular instance. Students operating at this level are able to perceive relationship between properties and figures and create meaningful definitions and give informal arguments to justify their reasoning. The key mental or cognitive activity at this stage is ordering (sequencing). The role and significance of formal deduction, however, is not understood (Oliver, 2000)

**Level 4: Deduction**

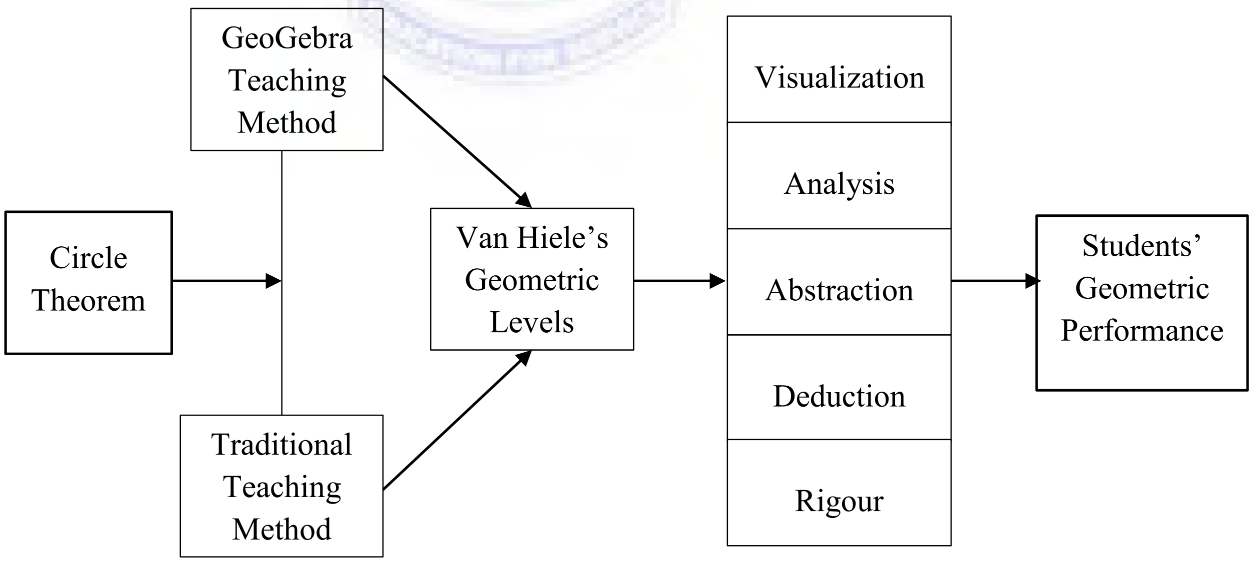
Deduction is the reasoning process by which an individual concludes something from facts or circumstances. Students at this level can construct proofs, and understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. At this level students at the Colleges of Education level should be able to construct proofs as those encountered in the secondary schools (Mason, 1998, Van Hiele, 1959 & Van Hiele–Geldof, 1984).

**Level 5: Rigour**

The last level by Van Hiele is the rigour. Students at this level understand the formal aspects of deductive reasoning, such as establishing the similarities and differences between mathematical concepts. They can also perform indirect proof and proof by contra- positive methods as well as non- Euclidean systems (Simon, 2006). Mason (1998) and Simon (2006) agree with Van Hiele theory of geometrical study and concluded that the rigour level is the level of college mathematics. Their argument was that students at this level understand the relationships between various systems of geometry, through geometrical maturity, thus they are able to describe the effect of adding and deleting an axioms on a given geometric system.

## 2.2 Conceptual Framework

The study sought to investigate the effect of integration of GeoGebra into the teaching and learning of circle theorem on students' performance at the Presbyterian College of Education, Akropong-Akuapem. The study employed two different teaching methods, i.e. the traditional teaching method and the GeoGebra teaching method. The GeoGebra teaching method was applied to the experimental group while the traditional teaching method was also applied to the control group. The Van Hiele theory of geometrical understanding was adopted as a theoretical framework to analyse the learners' levels and/stages that they go through when engaged in circle geometry problem-solving skills.




(Source: Researcher's consolidated ideas, and variables of the study)

*Figure 2.1: Conceptualized Framework of the Study.*

### *2.3 Teaching and learning mathematics using technology*

The importance of pedagogical integration of technologies into the teaching and learning of mathematics is unquestionable. A study by McClintock, Jiang, and July (2002) shown that existing technologies have increasingly become important in mathematics education because of their positive effect on students' acquisition of mathematical knowledge and skills needed for the twenty-first century. Teachers must be willing to accept change and make technology a reality in mathematics classroom. Teachers who are entrusted to educate the nation's future leaders should give serious attention to the use of technology in teaching and learning of mathematics. Educators should strive to ensure that mathematics is interesting for students to learn while focusing on important concepts in mathematics. In addition to building skills in mathematics, learning with technology can have positive long-term effect on students. By giving them the opportunity to learn and understand mathematics through technology, students are provided with knowledge to compete and function in the high-tech world. It is the responsibility of educators to provide a bright future for students in the face of the world that depends on mathematics, science and technology (Furner & Marinas, 2007).

The National Council of Teachers of Mathematics in the document "Principles and Standards for School Mathematics (NCTM, 2000) stated, Technology Principle as one of the six principles of high quality mathematics education and has guidelines and supports about the use of technology. In the Principles and Standards of School Mathematics, it is stated that "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24) and teachers should use technology to enhance their students learning opportunities by selecting or creating mathematical tasks that take advantage of what technology can do efficiently and well graphing, visualizing, and computing (p. 25). Furthermore, NCTM suggests that appropriate technology use can facilitate such applications by providing ready access to real data and information, by making the inclusion of mathematics topics useful for applications more practical (e.g., regression and recursion), and by facilitating teachers and students in handling multiple representations of mathematics topics (NCTM, 2000). The Association of Mathematics Teacher Educators (AMTE) (2006), in a recent position paper, agreed with the NCTM, 2000 and further stated that "technology has become an essential tool for doing mathematics in today's world, and thus, it is essential for the teaching and learning of mathematics" (p.1). According to the same AMTE (2006), properly implemented, technology changes how mathematics is taught by allowing teachers and students to focus on deep conceptual understanding over rote procedural skills through problem solving, reasoning, and decision making.

Confirmations from literature also show that teachers who utilize educational technology comprehensively in their Mathematics instruction environment are likely to build high confidence in pedagogical technology skills and focus their lessons on a student-centred approach which improves students' performance in Mathematics (Bos, 2009).

Again according to Boakye and Banini (2008), a mathematics teacher with pedagogical proficiency makes a difference in the learning process. They argued that if a mathematics teacher possesses limited knowledge of existing technologies, pedagogical integration will be seriously compromised. Integration of technology in education especially in teaching and learning mathematics is therefore encouraged. The Curriculum Development Division suggests that mathematics teaching and learning should be integrated with the use of technology as well as promoted with the use of dynamic software (Ministry of Education, 2013). Previous study demonstrated that the use of technology is useful as a tool to support and transform the teaching and learning process, especially for mathematics (Abdul Saha et al., 2010).

Muhtadi, Wahyudin, and Prahmana (2017) also did a study entitled: The integration of technology in teaching mathematics. Three mathematics pre-service teachers were selected from two hundred and seventeen pre-service mathematics teachers at a University in a town of Tasikmalaya for the study. That is one female and two males. Participants were introduced to the use of mathematical software such as GeoGebra, Cabri, and Geometers Sketchpad as tools and medium for learning mathematics. After the introduction of teachers into the use of those software, participants were encouraged into critical open discussion forums to consider the potential impacts of technology-based teaching methods based on TPACK framework. To investigate the TPACK transformation from the three pre-service mathematics teachers who had participated in the study, two instructional tasks were designed. The first task was that participants were to design learning materials by using mathematical software. After completion of this task, participants were assigned to complete the other two tasks by using the same mathematical software. The results from the study indicate that technology can help students develop an understanding of mathematics and therefore its usage in mathematics classroom should be encouraged. Also, the competency of the TPACK mathematics teacher can have a direct impact on students' learning in mathematics lessons.

Asiedu-Addo, Apawu and Owusu-Ansah (2016) investigated the usage of ICT in the teaching and learning of mathematics: Tracer study of Mathematics Educators. The purpose of the study was to do a follow up on in-service mathematics educators on the usage of ICTs in the teaching and learning of mathematics. The study employed the descriptive survey design. A sample of 48 participants was purposively and conveniently selected from a population of past students of UEW who had offered Mathematics Education in Ghana. The instrument used to collect data was a questionnaire. Data collected from the study were quantitatively and qualitatively analysed. Results showed that 95.8% of the participants were aware of the integration of ICTs into the teaching and learning of mathematics, however, only 41.7% were incorporating ICTs into the teaching and learning of mathematics. The results also shown that most teachers agreed or strongly agreed to the positive effect the use of ICTs have on students' learning. The researchers also recommended that schools in Ghana must be well resourced with ICT hardware, software, etc. so that in-service mathematics educators who wish to incorporate ICTs into the teaching and learning of mathematics could do so without any hindrance.

A study by Eyyam and Huseyin (2014) also examined the impact of the use of technology in mathematics lessons on students' achievement and attitudes. The population of the study was a private secondary school. The seventh grade students were placed in five heterogeneous classes; three of these classes were randomly selected into the experimental group (n= 41) with the remaining as the control group($n = 41$). The design employed in this study was a quasi-experimental one. All groups completed pre-test and post-test. For the experimental group, lessons were designed using several technological tools, whereas lessons for the control group were designed using the traditional teaching methods. Data collected from the study were analysed using one-way analysis of covariance. The results of the analysis indicated that the mathematics post-test results of students instructed using technology were significantly higher than the post-test results of those instructed without technology. The results again show that students had positive attitudes towards technology use. This means that when technological tools are used in mathematics, students are energized and therefore perform better.

Ron (2009) did a research study on the effects of the use of technology in mathematics instruction on students' achievement. The purpose of the study was to examine the effects of the use of technology on students' mathematics achievement, particularly the Florida Comprehensive Assessment Test (FCAT) mathematics results. Three schools were randomly selected from eleven schools within the Miami-Dade County Public school system who had earlier participated in a pilot program on the use of Geometers Sketchpad (GSP) for a study. Each school sent a teacher for an in-service training program on how to use GSP to teach geometry. The design for the study was an experimental one, and this consisted of GSP Class and the traditional geometry class. In each school, the GSP class and a traditional geometry class were taught by the same teacher. Students' mathematics FCAT results were examined to determine whether the GSP has had any effects on students' mathematics achievement. The findings of the study revealed a significant difference in the FCAT mathematics scores of students who were taught geometry using GSP compared to those taught with the traditional approach. Again the study revealed that no significant differences existed between FCAT scores based on gender. From the study, one is not wrong in saying; the use of technology in mathematics classroom is very useful.

Mas, Wong and Ahmad (2010) investigated the topic: Technology in mathematics teaching: the pros and cons. The study centered on the use of laptops by mathematics teachers at 28 secondary schools in Malaysia. It addresses some benefits and challenges faced by the teachers in integrating technology into mathematics teaching-learning process, as well as their daily activities related to the process such as the use of resources, and the preparation and planning of lesson. A sample of 172 mathematics teachers who were given laptops by Pengajaran dan Pembelajaran Sains danMatematik Dilam Bahasa Inggeris (PPSMIM) were purposively selected to participate in the study. This was made up of 36 males and 136 females. The main instrument employed in this study was a survey questionnaire. Data were collected using two methods namely school visits and via mail. The collected data on the benefits and challenges mathematics teachers face in integrating technology into the teaching and learning of mathematics were analysed using SPSS. The results implied that the mathematics teachers perceived laptops as a tool that benefited their classroom instruction. Teachers admitted that they could teach well with the use of laptops. They also asserted that laptop usage had indeed given them freedom to access the internet from different locations and that they could work everywhere they prefer. Teachers also remarked that laptop is a powerful tool that could enable them creates resources on their own; however, there were some teachers who were neutral in indicating their opinion.

Sung, Chang and Liu (2016) studied the effects of integrating mobile devices with teaching and learning on students' performance: A meta-analysis and research synthesis. The study performed a meta-analysis and research analysis of the effects of integrated mobile devices in teaching and learning. In all 110 experimental and quasi-experimental journal articles published during the period 1993-2013 were coded and analysed. Overall, there was a moderate mean effect size of 0.523 for the application of mobile devices to education. The effect sizes of moderate variables were analysed and the advantages and disadvantages of mobile learning in different levels of moderator variables were synthesized based on content analyses of individual studies. The results of the study show that mobile devices (technology) are useful in teaching and learning.

Hennessy, Ruthven and Brindley (2005) examined how secondary school teachers of core subjects of English, Mathematics and Science have integrated ICT into mainstream classroom practice in English schools. 18 focus-groups were interviewed by subject departments in the area examined. Teachers account emphasized both the use of ICT has enhanced classroom practice, and changed in terms of emerging forms of activity which complemented or modified practice. Teachers developed new strategies specifically for mediating ICT–supported learning. This overcame the potentially obstructive role of some form of ICT by focusing pupils' attention onto underlying learning objectives.

Mensah (2017) also did a study on Ghanaian mathematics teachers' use of ICT in instructional delivery. A stratified sampling technique was used to select 120 mathematics teachers from 24 public senior high schools with 12 schools each located in the rural and urban areas respectively. A cross- sectional survey was adopted in the study. The study also utilized questionnaire as the main instrument for data collection. The study concluded that technology is essential in teaching and learning of mathematics.

According to Ministry of Education, Youth and Sports (MOEYS) and Ghana Education Service (GES) (2002), the importance of technology in mathematics classroom cannot be underestimated. According to them, integrating technology in mathematics classroom instruction ensures greater motivation, increases self-esteem and confidence, enhances good questioning skills, promotes initiative and independent learning, improves presentation of information/outputs, develops problem solving capabilities, promote better information handling skills, increasing focus time on task, and improves social and communication skills.

### *2.2.2 Teaching and learning geometry using technology*

BECTA(2003) did a study on the integration of ICT in the teaching and learning process and came out with the following benefits; ICT promotes greater collaboration among students and encourages communication and sharing of information among learners, ICT gives rapid and accurate feedbacks to students and this contributes towards positive motivation, it also allow students to focus on strategies and interpretations of answers rather than spending time on tedious computational calculations. ICT also supports constructivist pedagogy, wherein students use technology to explore and reach an understanding of mathematical concepts. This approach promotes higher order thinking and better problem solving strategies which are in line with the recommendations forwarded by the National Council of Teachers of Mathematics (NCTM).

Ertekin (2014) investigated the effects of teaching analytical geometry using the software Cabri 3D on teacher trainees' ability to write the equation of a given special plane, identify the normal vector of a plane and draw the graph of the plane. In all 78 participants were chosen for the study. The study employed true experimental design, made up of 26 experimental group and 52 control group. All the students received written instruction made up of 14 test items. In addition the experimental group was exposed to the use of the Cabri 3D software. The result of the study indicated that students instructed with the Cabri 3D software (experimental group) were significantly more successful than those who were not instructed with the Cabri 3D software ( control group) in terms of identifying the equations of special planes and their normal vectors and drawing their graphs. Yildiz (2009) also share the same view with Ertekin (2014) and concluded that 3-D computer program and concrete manipulatives improve students' spatial visualization and mental rotation skills.

Cottrill, Marlissa and Ponesse (2012), conducted a study entitled: The views of high school geometry teachers regarding the effect of technology on student learning. The study involved 60 high school teachers who voluntarily decided to participate in the study from Columbus, Ohio. The purpose of the study was to ask these teachers which technologies they use and whether they believe technology has beneficial effects on student learning. Data were collected for the survey by asking teachers to take brief electronic surveys and conduct in-person interviews. All questions in both survey and interview were focused on the effects of technology that they see in their classrooms. Data collected from the questionnaires and interviews were descriptively analysed using graphs. From the analyses, most of the participants said that none of the technologies that they use or have used have had negative effects. However, seven teachers said that they thought students easily get over-reliant on technology and might believe they cannot solve a problem without the technology. Four of these seven participants said that the students become over-reliant on calculators specifically. Also, two teachers said that some of the technology they have incorporated has made particular concepts too easy for the students. The analyses also showed that the top three technologies that teachers found to be the most beneficial were the SMART board, Calculators, and Geometer's Sketchpad or GeoGebra

Grandgenett (2008) pointed out that the use of technology in teaching and learning of mathematics help students to develop flexibility in their mathematics thinking and enhances their imagination. Hakkarainen et al. (2000) are also of the view that ICT is a transformative tool and its full integration into the school system is necessary to prepare students for the information society they will inherit.

Karakus (2008) also determined the effects of computer-based teaching on students' achievement in transformation for geometry subjects. The experimental group were instructed using computer assisted software while the control group were instructed using the traditional approach. The study found a significant difference in favor of the experimental group. All students in the experimental group achieved high attainment level with computer-based instruction in teaching of transformation geometry. This study concluded that computer-based instruction increase students' success in learning transformation geometry.

A study conducted by Adelabu, Makgato and Ramaligela (2019) also examined the importance of dynamic geometry computer software on learners' performance in geometry. A quasi-experimental, non-equivalent control group research design was used. The main instrument used in the study was Geometry Achievement Mathematics Test (GMAT) that consisted of 15 multiple choice items. The GMAT was administered to 87 grade nine learners in two secondary schools in Tshwane South District, Guateng Province South Africa. Convenient and purposive sampling techniques were employed to select the participants for the study. Two schools were conveniently and purposively selected to serve as the control and experimental group. A pre-test (GMAT) was administered to both groups at the beginning of the experiment after which the control group were taught geometry using the traditional approach while the experimental group were instructed using the dynamic geometry computer software. Again both groups were given post-test after instruction. Data analysis employed the use of the statistical t-test independent sample. The results of the study showed that the use of the dynamic geometry computer software was important because it improved the performance of the experimental group compared to the control group. In addition, the result shown that the software affected the female learners' mathematics performance more positively than the male learners. The study recommends that the use of technology in teaching and learning of mathematics should be a priority in schools.

Perjesi–Hamori (2015) also did a study on integration of Computer Algebraic System (CAS) in teaching solutions of multivariate interpolations and found out that the use of the software enabled students with limited mathematical skills to understand more complex tasks, such as solutions of multivariate interpolations and regression, or those of partial equations. Vajda (2015) also used CAS software to introduce Classical Chebyshev polynomials. The use of computer algebra in the study was reported to have made the exploration of external polynomials easy and enjoyable for students.

Hercey and Hercey (2010) conducted a study on how to incorporate computer-based learning to reduce the working process of numerical integration. Two groups were selected to represent the control and the experimental group. The control groups were instructed using applets only whereas the experimental groups were instructed with both the applets and the GeoGebra software. The results of the study show that the experimental group gained more knowledge and skills than the control group. This study also suggested that GeoGebra use is helpful for students who face difficulty in solving mathematical problems since they have less time solving by hand.

Iranzo and Fortuny (2011) advocated that the use of GeoGebra help students in grasping mathematical understanding by enabling alternative problem resolution path and also help them to diagnose their learning difficulties.

Improvement in students' performance in mathematics is determined by how effective their teachers are in delivering instruction to them (Baumert el al, 2010). Martinovic and Manizade (2014) believe that technology should be used as partners in the geometry classroom. To support their claims, these authors presented technology-based activities to pre-service teachers, with the aim of promoting their mathematical reasoning. The technology was used as a partner in all the examples presented. At the end of the study it was realized that the pre-service teachers were able to explore each of the activities through paper folding, technology application and constructing proofs as part of reflexive pedagogy in action. The study recognized technology as an important part in developing pre-service teachers' professional integrity.

Stols (2012) also investigated the geometric cognitive growth of pre-service mathematics teachers in terms of the Van Hiele levels in a technology-enriched environment (dynamic geometry software) as compared to students in a learning environment without any technological enhancement. In order to investigate this, a quasi-experimental non-equivalent design was used. Similar course content was used for both the control and the experimental groups. In addition, dynamic geometry software was integrated into the teaching of the experimental group. The study found that the use of dynamic geometry software enhanced student teachers' geometric visualization, analysis and deduction. The study suggests that technology can help to create learning environment in which students can discover, explore, conjecture and visualize.

Karaibryamov, Tsarava and Zlanalov (2012) conducted a study on optimization of courses in geometry by using dynamic geometry software (DGS) referred to as “Sam” (mathematical software). In the study, the software was integrated into the teaching of synthetic geometry in schools and universities. At the end of the study, it came to light that the new approach increased the benefits of DGS in the teaching and learning geometry, especially optimizing the education process by saving time involved in drawing, generalizing large groups of problem and stimulating and helping investigations. Pierce and Stacey (2011) were also with the view that dynamic geometry software help bring the real world into the mathematical world.

Karakus and Peker (2015) studied the effects of dynamic geometry software and physical manipulatives on pre-service primary teachers’ Van-Hiele levels and spatial abilities. A quasi-experimental design was used in the study. A total of 61 pre-service teachers in the second year in the Department of Elementary Education at Afyon Kocatepe University were used for the study. Out of this number, 32 were put into the experimental group (computer group) whiles the control group (physical-manipulation group) consisted of 29 pre-service primary teachers. The instrument for the study was the Purdue Spatial Visualization test. The result indicated that there was no difference on the post-test of the two groups on the Van Hiele levels and spatial abilities. However, both groups had significantly higher achievement on the post-test compared to the pre-test.

Donevska-Todorova (2015) also did a study on how upper-high school students develop a conceptual understanding based on concept definitions and concept image. In this study, the author created a teaching/ learning sequence in a dynamic geometry environment (DGE), implemented and evaluated it in a high school in Berlin, following a complete cycle of design-based research (The Design-Based Research Collectives, 2003). The result of the study was that not only did the software widened students' concept image, developed multiple modes of thinking and gained deeper conceptual understanding, but also it gave insights into an eventual theoretical model of how they can further be examined.

The use of computer technology in the teaching and learning of mathematics is believed by some mathematics scholars to enhance problem-solving and creative skill in students, others are of the view that although users of technology can solve mathematical problems easily, technology in mathematics transforms the learning process from being largely mental to being largely mechanical. This traditional school of thought believes that mathematical problem-solving should minimise the use of technology in the teaching and learning process, while modern-day scholars believe mathematical problem-solving is the ability of students to perform mathematical tasks successfully, with or without computer technologies. From the above reviewed literature it is no doubt that the use of computer technology offers the learner numerous advantages and therefore its use should be encouraged.

### *2.4 Teaching and learning mathematics using GeoGebra*

Agyei and Benning (2015) researched on the topic: Pre-service teachers' use and perceptions of GeoGebra software as an instructional tool in teaching mathematics. The study made use of 85 final year pre-service mathematics teachers (74 males and 11 females) in Bachelor of Education (Mathematics) programme at the University of Cape Coast, Cape Coast, Ghana. Questionnaire, interviews and lesson artefacts developed by the teachers were the main source of data for the study. All 85 participants responded and completed a questionnaire survey which was administered to them before and after the instructional technology course. Four teams of pre-service teachers whose lesson artefacts were sampled and analysed were also interviewed after the course. Descriptive, t-test and effect size statistics were used to analyse the quantitative data whereas the interview data and lesson artefacts were analysed qualitatively. Statistical analysis confirmed that the use of the GeoGebra helped pre-service teachers expand their own understanding of mathematical concepts as well as their knowledge of instructional strategies.

Asare (2019) examined the impact of using GeoGebra software in teaching and learning rigid motion on senior high school students in Ghana. The objective of this study was to assess the applicability of GeoGebra in teaching and learning of mathematics in senior high schools in Ghana. The study population was students in the New Juaben Senior High School in the Eastern Region of Ghana. The design adopted for the study was the mixed methods design of qualitative and quantitative approaches. Here two groups of students were taught, one with the use of GeoGebra and the other with the traditional approach. The data collected from the study were analysed with the help of descriptive and inferential statistics. The result of the study indicated that GeoGebra helped improve the students' understanding of the concept 'rigid motion'. The study concluded that GeoGebra is useful in improving performance of secondary school students in rigid

Prodromou and Theodosia (2015) reported the various opportunities that teaching mathematics with technology (GeoGebra) offer. In their study, GeoGebra was integrated into the teaching and learning of introductory statistics and the results indicated that college students exposed to this software were able to perform key statistical investigative tasks, such as (1) managing data (2) understanding specific statistical concepts (3) performing data analysis and inferences. As a result of the study, the national and professional standards for teachers of Australia (AITSL, 2014) admonished teacher education graduates to demonstrate technological, pedagogical and content knowledge (TPACK) in mathematics teaching and learning.

Reis and Ozdemir (2010) used GeoGebra as a technological tool for instructional method for parabolas. The outcome from the data analysed shown that students can learn meaningfully with GeoGebra.

Bhagat and Chang (2015) also examined the impact of using GeoGebra on 9$^{th}$ grade students' achievement in learning geometry. The study utilized a quasi-experimental research design. A total of 50 middle school students were selected from a government school in the eastern part of India as the sample for the study. This sample was subdivided into control and experimental group. Each group consisted of 25 students. Before instruction students in both groups were assessed to find out their knowledge about circle theorem. The actual instruction underwent three phases. In the first phase, students in the experimental group were trained on how to use the GeoGebra software. The second phase, the experimental group was taught circle theorem using GeoGebra while the control group was taught using the traditional teaching method. In the final phase, both groups were assessed to find out whether or not the GeoGebra has had any impact on students' mathematical achievement. Data collected from both the pre-test and the post-test were analysed using the one-way analysis of co-variance (ANCOVA). All analysis was done using the Statistical Package for the Social Sciences (SPSS). At the end of the treatment, students' mathematics achievements were measured using a post-test. The results indicated that the experimental group performed much better than the control group. It was also clear from the study of Bhagat and Chang (2015) that teaching and learning Geometry with GeoGebra, helped students to improve their reasoning, visualization skills and representation of mathematical concepts in diverse ways. This notion was also supported by Dikovic' (2009) that by using GeoGebra, true exploration and visualization are possible, leading to an understandable mathematics solution to both the instructor and the learner (Fahlberg-Stojanovska & Stojanovski, 2009).

Zengin et al. (2012) conducted experimental research in order to identify the effect of using the dynamic software GeoGebra towards achievement in the subtopic of trigonometry function and graphs of trigonometry functions. The findings showed that learning using GeoGebra gives meaningful impact for students who learned in the experimental group. Overall, the research concluded that students who learned trigonometry with GeoGebra had better achievement than those who learned with a constructivist approach.

Pavethira and Leong (2017) also did a study on students' performance in geometrical reflection using GeoGebra. The research utilized an experimental research method. A total of 24 year one students were randomly selected from an international school as the sample for the study. This sample consisted of three groups namely Extension, Core and Support. The sample selected were taught and learnt by using GeoGebra software after a pre-test. Then a post- test was given. The results indicated a significant difference between pre-test and post-test results. Similarly, the results also found a statistically significant difference in scores among the three student ability groups. In conclusion, the study implies using GeoGebra enhances students' performance in geometrical studies.

Dogan and Icel, R. (2011) also conducted a quasi-experimental research study on the topic "The role of dynamic software in the process of learning: GeoGebra example about triangles". The aim of the study was to observe the possible effects of computer-based learning environment (GeoGebra software) on students' achievement. Two eighth grade classes from primary school were selected as the experimental and control group. The experimental group was made up of 9 females and 11 males while the control group consisted of 7 females and 13 males. Before the classroom activities, a pre-test was applied to both groups to determine the students' attainment levels. The control group were then taught triangles with the conventional teaching method whereas the experimental group were instructed with the GeoGebra instructional teaching approach. A post-test was then applied simultaneously on both groups to find out the effect of the instructional approaches on students' achievement. Possible comparisons between the tests and the groups were performed. The results showed that dynamic software (GeoGebra) had positive effects on students' learning and achievements. It was again observed that the GeoGebra instructional approach improved students' motivation with positive impact.

Khor and Md-Ali (2017) examined the effect of using GeoGebra dynamic geometry software on students' ability to confront geometry problem solving. A total of 102 Form Two students participated in the study. The research participants were divided into three groups, namely Experimental Group 1 ($n = 33$), Experimental Group 2 ($n = 35$) and Control Group ($n = 34$). A guideline book on the usage of GeoGebra dynamic geometry software in learning shapes and space, developed by the researchers and validated by a panel of experts, was used by the teachers and students in the experimental groups while the control group learn geometry with the traditional approach. The research instruments used in the study were the Topical Test (TT), Spatial Visualization Ability Test (SVAT), and the Teaching and Learning Observation Checklist. The quantitative data obtained via Topical Test (TT) and Spatial Visualization Ability Test (SVAT) were analysed using MANOVA while the qualitative data collected via interviews, teaching observations, video recordings and students' work were analysed thematically. The research findings indicated that the experimental groups' TT and SVAT post-test mean scores for both experimental groups were significantly higher than the control group's TT and SVAT post-test mean scores. The learning of shapes and space using GeoGebra dynamic geometry software enabled students to produce works with evidence of critical, creative and innovative elements in their solution. Students in the experimental groups agreed that the use of the GeoGebra dynamic software was something new to them and was indeed an attractive way to learn mathematics because they had had the opportunity to experience hands-on learning in mathematics using ICT.

Leong and Praveen (2013), investigated the effectiveness of using GeoGebra on students' understanding in learning circles by using quasi-experimental design. Fifty-three form three students from two intact classes selected from a population of 133 students participated in the study. Out of this number, 28 were assigned into the experimental group and 25 into the control group. The instruments for the study were the teacher made achievement test and questionnaire. Before instruction pre–test was administered to both the control as well as the experimental group. The reason for administering the pre-test was to determine the baseline knowledge or preparedness for learning the topic of circle geometry and to compare differences between experimental and control group before treatment. Afterwards the experimental group underwent instruction where they were taught circle geometry using the GeoGebra software while the control group underwent instruction without the use of the software. An independent t-test was used to determine whether there was a significant difference between the pre-test and post-test mean scores of both the control and the experimental group. The result indicated that students in the experimental group not only outperformed those in the control group in academic achievement but also in levels of learning of transformation geometry. This study shows that when technology is effectively integrated into the teaching and learning of mathematics, students turn to think critically and therefore perform better. Numrich (2010) quoted in the National forum for teacher education journal emphasize that only those who can "think" through content truly learn it". What this means is that for the learner to truly learn a concept of mathematics, he/she should be able to think critically. Another study done in Malaysia to evaluate the impact of GeoGebra in learning transformations by Baker, Ayub, Luan and Tarzimi (2010) revealed that secondary school students achieved better using the software.

Adelodun and Akanmu (2016) also did a study titled "GeoGebra; The third millennium device for mathematics instruction in Nigeria. The study adopted the non-equivalent pre-test post-test control group design. The study population comprised secondary school mathematics students in Ogbomoso North L.G.A. of Oyo State, Nigeria. SS2 mathematics students from two intact classes from each of the two purposively selected schools in the area constituted the sample. The classes were assigned into experimental and control group using simple random sampling technique. The experimental group was taught using GeoGebra, while the control group was taught using the conventional method. In the experimental group, the students interacted with different kinds of GeoGebra tools to solve problems in geometry, algebra, introductory calculus, among others. The control group was exposed to the conventional method and taught the same topics. The two main instruments used for data collection were Students Achievement Test in Mathematics (SATM) and Mathematics Attitudinal Scale (MAS). The two groups were pre- and post-tested using SATM, after which MAS was also administered to them. Data collected were analysed using mean and t-test statistics. The study concluded that the incorporation of GeoGebra and other ICT packages improved the students' learning outcomes in mathematics, while their attitude towards mathematics was also positively enhanced.

A study entitled "GeoGebra assist discovery learning model for problem solving ability and attitude toward mathematics" was a study conducted by Murni, Sariyasa and Ardana (2017). The targeted population in this study was all the 6 classes totaling 181 students of class VIII junior high school in Indonesia. The researcher used cluster random sampling to select 4 classes as a research sample with 2 classes as the experimental class and the other 2 classes as the control class. The research design for the study was a quasi-experimental and post-test only control group design. Problem solving ability data was collected through a description test and that of the attitude was collected through questionnaire. The questionnaire data was measured using the Likert scale. Data collected from the study were analysed using the one way MANOVA. The results of the data analysis showed that the utilization of GeoGebra in discovery learning led to solving problem and attitudes towards mathematics better.

Onaifoh and Ekwueme (2017) also conducted a study entitled: Innovative strategies on teaching plane geometry using GeoGebra software in secondary schools in Delta State. The design for the study was quasi-experimental with non-equivalent pre-test and post-test control group design. The population of the study consisted of all private senior secondary two (SS2) students in the 15 private schools in Oshimili-South Local

Government Area of Delta State. Two private secondary schools were selected for the study by purposive sampling technique. The total number of students who participated in the study was 59. The instrument for the study was a Performance Mathematics Ability Test (PMAT), which included pre-performance test and post-performance test. A pilot study was carried out in two secondary schools in Oshimili-South Local Government Area of Delta State which were not part of the sample. The two intact classes (sample) were then put into the control and the experimental group. Both groups were given pre-performance test after which the control groups were taught plane geometry using the conventional method while the experimental group was taught using the GeoGebra approach. The two groups were again assessed to find out the effect of the instructional approach on their performances. Data from both performance tests were then collected and analysed by SPSS Version 21. The research questions were also answered using mean and standard deviation. The results showed that GeoGebra application was more effective in improving students' understanding in mathematical plane geometry than the problem based learning approach; however, there was no significant difference in male and female performance.

Hutkemri and Sharifah (2016) also investigated the effectiveness of the GeoGebra software: The intermediary role of procedural knowledge on students' conceptual knowledge and their achievement in mathematics. The study employed a quasi-experimental approach on 345 Form Two secondary school students in Riau, Indonesia. These 345 students were randomly selected from two secondary schools in Riau, Indonesia. These students were further grouped into two, that is control group ($n = 176$) and the experimental group (n= 169). Students in both groups were given a pre-test. The students in the treatment group were then taught functions using GeoGebra while the control groups were taught functions with the conventional method. Students were then given a post-test to find the effect of the instructional approaches on their achievement. The data collected through conceptual and procedural tests, and from students' achievement on the topic: functions were then analysed using SPSS 22, AMOS 18.0 and ANATES v4 software. Findings of the study showed that students who used GeoGebra to learn Mathematics had higher mathematical conceptual and procedural knowledge compared to those who learnt mathematics through the conventional methods. Both experimental and control groups showed that procedural knowledge was a significant mediator between conceptual knowledge and students' achievement in mathematics. The study concluded that GeoGebra software was capable of enhancing students' conceptual and procedural knowledge and at the same time significantly improved students' mathematics achievement.

In Malaysia, Noorbaizura and Leong (2013) studied the effect of using GeoGebra to teach students' learning of fractions. The purpose of the study was to investigate the effect of students' achievement in fraction. A quasi-experimental research design was used. A pre-test were administered to each group after which the experimental group underwent an intervention where they learnt fraction using GeoGebra while the control group on the other hand learnt fraction by the traditional approach. The study showed that the use of GeoGebra to teach fractions is very effective. This was shown through the improved scores of the students in the experimental group. The findings highlighted that students in the experimental group performed better than those in the control group that were taught fraction using the traditional learning method. The software also enhanced visualization and understanding of the fractions concept of both the teacher and students.

Diaz-Nunja, Rordriguez-Sosa and Lingan (2018) examined teaching of geometry with GeoGebra. The purpose of the study was to evaluate the effects of the use of GeoGebra software in the teaching of geometry with high school students in the development of their capacities for reasoning and demonstration, mathematical communication and problem solving. There were 48 students who participated in the study. Of the 48 students, 40% were men and 60% were women. These 48 students were put into two groups, that is 24 each in the control and experimental group. Both groups were evaluated with test that was applied before and after treatment. The experimental group were taught geometry with the use of the GeoGebra software while the control group were instructed with the normal classroom instruction. Due to the sample size and sampling characteristics, the Wilcoxon T-test was used for the intragroup analysis, while the Mann Whitney U-test was used for the intergroup analysis. The results suggested that the use of the GeoGebra software had effects in the strengthening of the three capacities, with improvements that were significant at high levels in the experimental group as compared to those taught geometry without the GeoGebra software.

A study entitled: The effects of using GeoGebra teaching strategy in Malaysian secondary school: A case study from Sibu, Sarawak was conducted by Rohaidah, Ting, Nor'ain and Zamzamin (2016). The study examined the effects of using GeoGebra teaching strategy in learning circle 111 topics on Malaysian secondary Form Four students' performance and attitudes towards this teaching strategy. The targeted population of this study was Form Four students in national secondary school in Sibu, Sarawak. The sample selected for the study were Form Four students from a randomly selected school. In all a total of 46 students, made up of 17 students in the control group and 29 students in the experimental group. A quasi-experimental non-equivalent pre-test post-test control group design was employed in the study. The experimental group underwent learning using GeoGebra teaching strategy whereas the control group underwent learning using conventional teaching strategy. The circle 111 achievement test and the attitude questionnaire were used as instruments in the study. Data collected from the study were analysed using one way ANCOVA and one sample t-test. The analysis showed that there were no significant differences between mean performance scores of students in the experimental and control groups. However, the experimental students showed positive attitudes towards using GeoGebra software while learning circle 111 topics. This means that not only could the GeoGebra strategy be utilized in learning mathematics but also in enhancing students' performance in learning mathematics in the long run.

Syamsiah and Jasni (2018) also examined the effectiveness of using GeoGebra software in teaching angles in Balik Paulau Polytechnic in Malaysia. The research was a quasi–experimental involving 53 randomly selected students from 107 first semester students who underwent diploma in digital technology program. Before the study both the control and the experimental group underwent pre-test after which the control group was taught angles using the conventional teaching methods while the experimental group was taught the same angles but with the use of GeoGebra software. Data collected from the study were analysed using Statistical Package for Social Sciences (SPSS). The result of the study shown that there were no statistically significant difference between students taught angles using the conventional approach and those that were taught angles using the GeoGebra software. However, it was found that the experimental group shown better learning gains than the control group since the experimental group's average score was higher than control group. The study concluded that using GeoGebra in teaching can help students learning about angles in circle

Nazihatulhasanah and Nurbiha (2014) also investigated the effects of GeoGebra on students' mathematics achievement in Malaysia. The sample involved 62 students selected from Form 4 students at a secondary school in Malaysia with 32 students in the control group and 30 students in the experimental group. The sample was selected through the purposive sampling technique. The two research instruments utilized in the study were the performance test and questionnaire. Before instruction both groups were given a pre-test to find their knowledge level before instruction. During instruction, the experimental group were taught mathematics using the GeoGebra software whiles the control group were taught without the use of GeoGebra. Again students' perception about the use of the GeoGebra software was determined through questionnaire. Data collected from the study were analysed using Mann-Whitney U-test and simple percentages. The results of the Mann-Whitney U-test showed that the experimental group performed better than the control group. The results again indicated that students had positive perception towards learning and had better learning achievement using GeoGebra. This means that GeoGebra can benefit students' mathematics learning and diversify learning in mathematics classrooms.

Effect of GeoGebra on senior secondary school students' interest and achievement in statistics was the subject of investigation by Emaikwu, IJi and Abari (2015). A sample of two hundred and forty-two (242) participated in the study. This sample was selected from a population of 2,412 senior secondary one (SS1) students in 18 government co-educational secondary schools in Makurdi Local Government Area of Benue State by multistage sampling. Two research instruments were developed for the collection of data namely; Statistics Achievement Test (SAT) and Statistics Interest Inventory (SII). The test instruments were validated by 5 experts. The researchers and the research assistants administered the pre-SAT and pre-SII to both groups. That is the GeoGebra group and the non GeoGebra group. The post-SAT and the post-SII were also administered to both groups after treatment. Data collected from both pre-test and post-test were analysed using descriptive statistics of mean and standard deviation to answer the research questions asked while the hypotheses were tested at 5% significance level using the analysis of covariance (ANCOVA). Results from the study revealed that students taught statistics using the GeoGebra teaching method achieved higher and also showed greater interest in learning statistics than those taught using the conventional teaching approach. The study also revealed that both male and female students in the GeoGebra group achieved the same and also showed similar interest in statistics. The study however, recommended among others that teacher education institutions should be encouraged to include GeoGebra as a method in teaching secondary school statistics because of its numerous advantages.

Triwahyuningtyas, Rahayu and Agustin (2019) also did a study entitled "the impact of GeoGebra Classic application on learning geometry". A total of 50 fifth graders selected purposively from elementary school in Malang, Indonesia served as the sample for the study. These samples were further divided into two groups namely, experimental and control group. The main research instrument employed in this study was the teacher-made achievement test. Before treatment each group were given pen and paper test on geometry to find out their entry knowledge and to compare the groups. The experimental group then received treatment which was the use of GeoGebra Classic applications during learning sessions while the control group used Microsoft Power Point. The study used quasi-experimental non-equivalent control group as the design for the study. The groups were again tested to find out the impact of the instructional approaches on students' geometrical learning. The data collected were then analyzed using SPSS Version 21. The results of the analysis showed that there was significant differences in performance between students taught with the GeoGebra Classic application and those taught using the Microsoft Power Point in favour of the GeoGebra Classic application group. This indicated that there was a positive impact of the use of GeoGebra Classic application on geometry learning outcome of the fifth graders. Therefore it can be concluded that GeoGebra Classic application was able to help students understand geometry-related learning materials and its use must be sustained.

A study conducted by Mwingirwa and Miheso-Connor (2016) on "status of teachers' technology uptake uses of GeoGebra in teaching secondary school Mathematics in Kenya" through training thirty-three Mathematics tutors on GeoGebra use. They also tried to implement what teacher had learnt from the training. The outcomes from Mwingirwa and Miheso-O'Connor's (2016) work uncovered that the prepared educators appeared to be excited about utilizing GeoGebra in their classes. This was because GeoGebra instruction in their classes enabled students to grasp difficult and unique concepts in Geometry and also saved teachers time whereby they were able to cover the syllabus more effectively. The outcome of Mwingirwa and Miheso-O'Connor‘s study also pointed that GeoGebra was the most appropriate software or teaching learning resource for teaching Geometry due to its abstract nature. Again teachers' responses indicated that GeoGebra was perceived as useful for teaching and learning Mathematics. They also found out that teachers usually had trouble teaching geometry when they were solicited to demonstrate areas from mathematics they discovered hard to teach. The instructors accentuated these challenges they experienced because of absence of assets for teaching, the unique idea of Geometry and students' failure to envision geometrical objects.

Effect of GeoGebra-Aided REACT strategy on understanding of geometry concepts was a research conducted by Jelatu, Sariyasa and Ardana (2018). The aim of the study was to examine the effect of GeoGebra-aided REACT strategy on students understanding of geometry concepts and to investigate the interaction between learning strategy and spatial ability on the understanding of geometry concepts. The study involved 60 grade 8 students of a private Junior High School in Borong, Indonesia. The research design the study utilized was the quasi-experimental design. The instrument for the study was an achievement test. The experimental group was taught using GeoGebra while the control group was taught without GeoGebra. Data from the study was analysed using two-way ANOVA. The result of the study was that: (1) the GeoGebra-aided REACT strategy led to higher achievement of the students in the understanding of geometry concepts if compared to students in the conventional group (expository), and (2) there was no interaction effect between the learning strategy and spatial ability on students' understanding of geometry concepts. The research recommended that GeoGebra-aided REACT strategy can be used in mathematics teaching in Junior High School to improve students' conceptual understanding of geometry concepts.

Niroj, Dinesh and Binod (2019) also conducted a research study on the topic" Integration of GeoGebra in the teaching and learning of geometric transformation. The study adopted the quasi-experiment as the research design. The population of the study was all secondary schools in Karhmandu Valley, Nepal. The sample of the study was 16 students selected purposively from one of the secondary schools in

Karhmanda Valley, Nepal. These samples were subdivided into control and experimental groups. The experimental groups were taught geometric transformation with the use of GeoGebra, while the control groups were taught geometric transformation without the use of the GeoGebra software. Students in each group were assessed before and after treatment. Data collected from the study were analysed descriptively using SPSS. Results of this study showed that GeoGebra was helpful in teaching and learning abstract concepts of transformation. Findings of the study show that if GeoGebra is used in mathematics classroom, students could become an active constructor of knowledge. It was recommended by the researchers that GeoGebra should be used as an important educational tool to support the traditional lecture method of teaching mathematics.

Soheila and Kumalludeen (2018) also did a quantitative study on the intention of using GeoGebra in the teaching of mathematics among Malaysian teachers. The population for the study consisted of all mathematics teachers who had been exposed to GeoGebra and its applications through a workshop organized by government department such as Ministry of Education, Malaysia or non–governmental organizations such as GeoGebra institute of Malaysia. In all 132 teachers were selected as the sample for the study. An online survey was administered to the 132 teachers who had already participated in GeoGebra workshops in Malaysia. The study employed the cross–sectional survey as the design for the study. Out of this number, 132 teachers made up of 76 females and 56 males participated in the study. The respondents were also made of 83 users and 49 non–users of GeoGebra. Data collected via the online survey were analysed using the independent t–test. The perceived current competencies item reliability was assessed through Cronbach alpha coefficient. The results of correlation and independent t–test revealed a positive relationship between teachers' perceived competencies with the intention to use GeoGebra in mathematics teaching. The results further indicated that there was a significant differences between users and non–users of GeoGebra in their intention to use GeoGebra in their classrooms. This results shown that when GeoGebra is used in mathematics teaching and learning, not only do the students benefit but the teachers as well.

Dogän and Icel (2010) conducted an experimental design study to evaluate the success of students' learning using GeoGebra. The result show that the GeoGebra software encouraged higher order thinking among students. The software was also observed as having a positive impact in motivating students to learn and retaining their knowledge for a long time (Dogan, 2010 & Erhan and Andreasen, 2013).

Hutkemri and Zakaria (2012) also did a study on the effect of using GeoGebra on conceptual and procedural knowledge of high school mathematics students. The purpose of the study was to determine the effect of GeoGebra on conceptual and procedural knowledge of functions. The sample for the study was 124 high school students selected from Ujung Batu Rokan Hulu, Riau, Indonesia. The design for the study was a quasi-experimental made up of 60 students in the experimental group and 64 students in the control group. Data collected through the conceptual and procedural knowledge test of function were analysed using SPSS and the results indicated that students taught with the GeoGebra software performed better than students taught without the software. This study therefore show that GeoGebra is an effective tool for teaching and learning of mathematics and its usage should therefore be encouraged in mathematics classrooms.

Sudihartinih and Purniati (2019) also conducted a study on using GeoGebra to develop students' understanding on circle concept. The study utilized the quasi-experimental design. The sample for the study was selected by purposive sampling method, made up of two classes. Class A (experimental class) and class B (control class). The experimental classes were taught circle using the GeoGebra approach whiles the control class were taught circle using manipulatives. Students were assessed before and after instruction. Before instruction, all students were given written test on circle using paper and pencil. After instruction students were again tested using paper and pencil, in addition, attitude test were given. The data collected were descriptively analysed. Again the Man Whitney U-test was conducted to test the null hypothesis that the distribution of control and experimental class was the same across categories. The analyzed data showed that students' understanding of concept of circle in the experimental group (use GeoGebra) was higher than the control group (using manipulative). Students' attitude towards learning using GeoGebra was positive.

Daisy (2015) also did a study entitled: Enhancing students' achievement using GeoGebra in a technology rich environment. The study involved 112 high school students. Out of this number, 53 were males and 59 were females. The control and the treatment groups were randomly selected from four periods of geometry classes. Periods two and five were randomly selected to be the treatment group, whereas periods three and six were selected to be the control group. The study employed the mixed method design. The control group were taught geometry using the normal classroom instructions while the treatment group were instructed using GeoGebra. Students performance before and after instruction were noted. Data collected from the study were analysed descriptively and inferentially using SPSS. The findings and analyses from the qualitative and quantitative data indicated that the use of the GeoGebra software improved students' level of understanding of abstract concepts, increased students' comprehension and retention of geometric transformations, and had a positive effect on students' attitudes towards mathematics, thus enhancing their learning and achievement.

The effects of an inductive reasoning learning strategy assisted by the GeoGebra software on students' motivation for functional graph 11 topic, was the study by Abdullah, Mistom, Umar, Hamzah, Ashari, Dayana, Norazrena, Tahir and Sharifah (2020). The research design was quasi-experimental which involved 94 Form 4 students from a secondary school in Johor. The research sample was further divided into three groups: (1) study group 1 (an inductive reasoning strategy assisted by GeoGebra); (2) study group 2 (an inductive reasoning strategy without GeoGebra); (3) control group (a conventional strategy). The research instruments consisted of a motivational questionnaire set and an inductive reasoning strategy assisted by GeoGebra and without GeoGebra. The MANOVA test results shown that the overall motivation level for study group 1 was higher in terms of attention and relevance. With regard to confidence, the results indicated that control group and study group 1 shown the same motivation level. The study concluded that learning through an inductive reasoning strategy assisted by GeoGebra can increase the students' motivation in mathematics specifically for Functional Group 11 topics.

Azizul and Din (2016) also conducted a quantitative study on the use of GeoGebra in teaching and learning geometry via Massive Open Online Course (MOOC). The purpose of the study was to develop mathematics teaching and learning materials for the topic geometry based on GeoGebra for Form Four students of Sekolah Menengah Kebangsan Bandan. This study was a developmental one that used quantitative method for data collection. In all a total of 23 students were randomly selected to participate in the study. The selected students were then instructed using the learning module prepared by the researchers. The research instrument used in the study was the questionnaire. The questionnaire was in three parts. Part 1 was for demographic data, part 2 for usability of the GeoGebra module, while part 3 for elements of MOOC. The questionnaire used a five Likert scale for all the items. Data collected from the questionnaire was descriptively analysed using the SPSS. The results shown that the software was not only easy to use but also helped students understand the basic concept of geometry. Again the students agreed that GeoGebra was an effective tool for teaching geometry related topics such as straight lines, circles and trigonometry.

Kutluca (2013) also studied the effect of Geometry instruction using GeoGebra on Van Hiele Geometry Understanding Levels of students. The quasi-experimental design was employed. A total of 42 students were chosen for the study, made up of both the control and the experimental group. Kutluca (2013) found out from his study that GeoGebra instruction employed on the experimental group was better on increasing Van Hiele geometry thinking levels of students than the traditional approach of teaching geometry on the control group. He indicated that the GeoGebra helped students in creating their own geometric shapes, trying different things on the shapes, testing and constructing their own knowledge. In addition, students in the experimental group also had the opportunity to participate actively to the instructional process, share ideas comfortably, and discuss results obtained with friends and to construct their own knowledge. It is obvious from Kutluca’s (2013) study that when GeoGebra is fully utilized in classroom, it will enhance better teaching and learning, especially in Mathematics.

Also according to Ugorji and Chimuka (2017) it is virtually impossible to have passive students when computer technology, such as GeoGebra, is used in the teaching and learning process. According to the same researcher, GeoGebra changes passive students to independent explorers and the role of the teacher in the teaching and learning process is to direct and monitor students' work. Despite the benefits, this software (GeoGebra) is not widely used among mathematics teachers in Ghana (Tay and Mensah-Wonkyi, 2018) and more effects should be taken to encourage the use of GeoGebra for personalized learning.

### *2.5 Difficulties in learning circle geometry*

Idris (2006), studied the causes of students' difficulties in learning geometry and came out with the following as the main causes; geometry language, visualization abilities and ineffective instruction by teachers. She also highlighted that spatial visualization has been linked with geometric achievement because geometry is visual in nature. Geometry is the study of shapes and space; it requires visualizing abilities but many students cannot visualize three–dimensional objects in a two–dimensional perspective (Guven & Kosa, 2008). Other factors include: non–availability and obsolescence of instructional materials, gender differences, poor reasoning skills, inadequate skill, inadequate time, inadequate school curriculum and lack of proof by students (Mason, 2002; Uduosoro, 2011 and NERDC, 2012). All these contribute to students difficulties in learning circle geometry.

Johnson–Wilder and Mason (2005) have also blamed students' lack of interest and understanding of geometry on teachers' poor teaching skills and lack of resources for presenting geometrical shapes to students. According to them, the ordinary primary tutor has an anxiety of the very word 'geometry' and therefore do not handle the concept well. Again textbooks for basic schools devote little attention on geometry. Students, who enter Colleges of Education, therefore have very weak foundations in geometry in general and circle theorem in particular.

Adolphus (2011) also studied the problem of teaching and learning of geometry in secondary schools in Rivers State, Nigeria. The sample for the study was 300 students and 30 teachers drawn from a population of 10 secondary schools in River State. The research instrument for the study was questionnaire. The research also adopted the descriptive survey method as the design for the study. The data collected by the means of the questionnaires were analysed using simple means. The following were identified as some of the causes of students' difficulties in learning geometry; poor foundations of most mathematics teachers in geometry, poor foundations of students in mathematics in general, poor teaching and learning environment of most schools, poor attitude of students towards learning of geometry, lack of commitment on the part of some teachers due to lack of motivation. Based on the findings of the study, the study recommended that (a) The State government should as a matter of urgency send mathematics teachers for training and seminars for effective teaching and learning. (b) The government should endeavor to provide the necessary infrastructures and facilities that will motivate teaching and learning of mathematics.

Adegun and Adegun (2013) also carried out a study on students' and teachers' view of difficult areas in mathematics syllabus: basic requirement for science and engineering education. The population of the study consisted of all mathematics teachers and all the senior secondary III students in all the 18 secondary schools in the Local Government Area. A sample of 15 mathematics teachers and 180 senior secondary school III students were selected through simple random sampling techniques. Survey questionnaires were designed and administered by the researcher to elicit information from both students and teachers on difficult areas in mathematics. The analysed data indicated that geometry was one of the topics students perceived as difficult and the reasons assigned were: poor knowledge of the subject matter by teachers, low level of commitment by teachers and poor attitude towards and teaching and learning of geometry by students.

Surendra (2016) also conducted a qualitative research on the topic: “Problems of teaching and learning mathematics in geometry at the grade IX”. The purpose of this study was to identify the problems faced by teachers and students in teaching and learning geometry. The sample for the study was made up of two mathematics teachers, five students, a head-teacher and five parents. The sample was purposively selected from Shree RajajiTulilalJonchheJanta higher secondary school Siswa-Belhi, Saptari District. The main research tools used for data collection were observation, recorded history and interview. The collected data were descriptively analysed using SPSS. From the study, the researcher identified that the teaching-learning environment of school and home, pre-knowledge of students, learning activities which seems to be exams oriented rather than practical oriented, poor evaluation techniques, students’ weak pre-knowledge about geometry, lack of appropriate teaching methods and materials, complex and voluminous syllabus in secondary level mathematics curriculum, and no-effective management related problems faced by teachers and students in teaching and learning geometry.

Egwu, Asuque and Ofori (2018) investigated the topic: Geometry viewed as a difficult mathematics. The study focused on 450 SS 2 students, made up of 230 females and 220 males from 30 senior secondary schools in Cross River State which were randomly selected within the three senatorial District. The perceived difficult mathematics concepts were study through the research instrument of 20-item questionnaire. Data collected from the study were analysed using frequency counts and percentages. From the analysis, eight out of 20 concepts were perceived difficult by the students, these include coordinate geometry, circle theorem, construction etc. and the reasons given for viewing geometry concept as difficult is as a result of irregular class practices, unavailability of instructional materials, teachers' method of teaching, bad and inadequate timing etc. However, student gender had a great influence on the learning concepts on geometry at 0.05 level of significant in favour of female students. Based on the study, the researchers recommended appropriate teaching methods and effective instructional material if teachers want students to derive better understanding on the identified difficult geometry concepts.

Fabiyi (2017), studied geometry concepts in mathematics perceived difficult to learn by senior secondary school students in Ekiti State, Nigeria. 500 senior secondary school two (SS2) students made up of 228 males and 272 females from thirty (30) co–educational schools in Ekiti State State, Nigeria constituted the sample for the study. The proportionate and random sampling methods were used to select the sample for the study. A 23–item questionnaire on Goemetry Concept in Mathematics Perceived Difficult was used as the instrument for the study. The research questions were analysed by using frequency counts and percentages while the only hypothesis was tested using chi-square statistics The findings revealed that, out of 23 concepts, eight concepts were perceived difficult to learn by students which included: Construction, Coordinate geometry, circle theorem and so on and reasons given for perceiving geometry concepts difficult includes: unavailability of instructional materials, teachers' method of instruction and students attitudes towards the teaching and learning of geometry. It was also revealed from the analysis that students' gender had great influences on the learning of concepts in geometry at 0.05 level of significance in favour of female students.

Identification and remediation of students' learning difficulties in geometry in River State was the subject of investigation by Ejiofor-Chima and Accra (2019). The population of the study consisted of 7,719 SS2 students in Port Harcourt Local Government Area, Rivers State. A sample size of 314 was drawn from four schools out of sixteen public schools in PHALGA using purposive sampling technique. The study employed a quasi-experimental design. The instruments used for the study were Learning Difficulties Identification Test on Geometry (LDITOG) and Remediation Test on Geometry (REMTOG). The reliability coefficient of the instruments was determined using test re-test. Before treatment, the two groups were given pre-test on the identification of presence and types of learning difficulties experienced. A post-test was given after treatment to determine the effect of remediation. Data collected for the study were analysed using percentages, frequency counts, mean and standard deviations to answer the research questions while the hypotheses were tested at 0.05 significant level using Chi-Square and Analysis of Covariance (ANCOVA). The findings of the study revealed the presence of adaptive reasoning, procedural formulation, strategic competence and conceptual understanding learning difficulties among students. Again the study revealed that students' performance and mathematics ability levels improved after remediation.

Sulistiowati, Herman and Jupri (2019) conducted a research study entitled: Student difficulties in solving geometry problem based on Van Hiele thinking level. The aim of the study was to analyse students' difficulties in solving geometry problems based on Van Hiele thinking levels. A descriptive qualitative research was employed in this study. The main research instruments used in the study were the Van Hiele geometry test and problem-solving test, followed by interviews. The subjects of the study were 38 students grade VIII in one of the secondary schools in Bandung and 6 of them were interviewed afterwards. The results of the study showed that the main difficulty of students at level 1 (visualization) is interpreting problems into a mathematical model. While the main difficulty of students at level 2 (analysis) and level 3 (deduction informal) is in the solution processes. The study concluded that problem-solving ability on geometry is important to be taught to all students even though they are at different Van Hiele levels.

Ntshengedzeni (2015) did a study on the topic "Enhancement of learners' performance in geometry at secondary schools in the Vhembe District of Limpopo Province. The purpose of the study was to enhance learners' performance in Euclidian Geometry as a branch of mathematics in the Further Education and Training (FET) band of secondary schools. Two sampling techniques were used to determine the sample. Purposive sampling were used to select the participants for the study while cluster and simple random sampling were used to classify schools into three categories; low, average and high performing. Each cluster had three schools. In all nine schools were selected for the study, made up of 405 students, 6 school principals, 6 heads of departments and 6 teachers. The study adopted the mixed method design. The main instruments used in the data collection were the questionnaire and the interview. Both teachers and students were asked to response to the causes of difficulties in teaching and learning geometry as well as how these causes can be overcome. The data collected from the instruments were analysed using SPSS Version 22. The data analyzed revealed the following as the causes of students' difficulties in geometry: poor learning environment and large class size, lack of resources for teaching geometry, poor attitude towards the teaching and learning of geometry. On the other hand, the following were identified as strategies to enhance teaching and learning of geometry: provision of adequate instructional materials to teach geometry, applications of geometry in real life situations, frequent monitoring by supervisors, frequent helpful feedback to students, spending more time on the task will make the learner master geometric skills, making learners more involve in practical work than theoretical work, parents buying the necessary reading materials for students and learners' willingness to learn on their own.

Shankar (2016), carried-out a research titled "Problems faced by secondary level mathematics teachers and students in geometry. The participants of the study consisted of ten secondary level mathematics teachers, ten guardians and two hundred secondary level students. In all 220 candidates were taken as the sample for the study. These sample was selected by simple random sampling method from a population of all secondary level schools, mathematics teachers and students of grade 10 (also guardians) of Tokha. The main data collection instruments for the study were observation, interview and achievement test. The data collected from the study were analysed by survey design. Responses obtained from class observation and achievement test were analysed by three Likert scale with the help of mean weightage, content of text books. The study identified school administration, parents, teachers, publications, students and academic policy of the nation as responsible for students' difficulties in learning geometry.

Mifetu, Kpotosu, Bessah and Amegbor (2019) researched on geometry topics in mathematics perceived difficult to study by senior high school students in the Cape

Coast Metropolis. Using the descriptive survey design, the researchers collected data from 300 senior high school Form two students comprising 200 males and 100 females using simple random sampling technique. The two main research instruments employed in this study were questionnaire and teacher-made achievement test. Participants were asked some of the geometry topics they perceive as difficult to learn. In addition, 30-items multiple choice achievement test on geometry were designed for the study. Data collected from both the survey questions and the teacher-made achievement test revealed that, four of the geometry concepts perceived difficult to learn by students are: circle theorem, perpendicularity of tangent and radius of a circle, angle between tangent and chord and tangent from an external point. The main reasons given for perceiving geometry concepts difficult include: unavailability of instructional materials to make the teaching of geometry real and lack of understanding of geometrical concepts by teachers. However, students' gender had no influence on the learning of geometrical concepts.

Bosson-Amedenu (2017) also examined remedial students' perception of difficult concepts in senior high school core mathematics curriculum in Ghana. The study employed the survey design. A sample of 112 remedial students was obtained by convenience sampling for the study. This number consisted of 62 females and 50 males' remedial students graduating from various secondary schools across Ghana who had been unsuccessful (obtained grades in the range D7-F9) in the WASSCE core mathematics. The instrument used for the data collection was a 38-item questionnaire tagged Difficult Concept Identification Questionnaire in Mathematics (DCIQM). The data obtained from the study were analysed using mean with the criterion mean set at 3.05 for identifying difficult topics and 3.6 for identifying possible causes of the perceived difficulty. The findings of the study revealed that students identified some mathematics topics such as circle theorem, ratio and proportion, plane geometry, trigonometry and bearing, business mathematics and coordinate geometry as difficult. The most difficult topic identified by these students was circle theorem followed by plane geometry and the reasons assigned for these difficulties were; (1) some mathematics teachers had difficulty with some topics in mathematics themselves, (2) poor mathematics foundation by students, (3) lack of resources for teaching mathematics by instructors (4) Large class size in most senior high schools and (5) lack of motivation in learning mathematics because of its abstractness. The study recommended amongst others that workshops should be organized to train mathematics teachers on the effective and efficient strategies that should be adopted for the teaching of the identified difficult mathematics concepts.

Ahmed, Amin and Anwar (2017) also did a study on students' learning difficulties in mathematics at the secondary school level. A sample of 60 mathematics teachers and 300 students were selected through simple random sampling techniques to participate in the study. The findings of the study revealed that both teachers and students were of the view that students find it difficult in learning geometry concepts.

### *2.6 The role of gender in learning geometry*

Anas (2018) also investigated gender geometric reasoning stages and gender differences in achievements of preservice teachers of E. P. College of Education, Bimbilla, Ghana. The sample for the study was three hundred and fifty-one (351) level 200 preservice teachers of Bimbilla College of Education. The sample consisted of one hundred and thirty-three (133) female and two hundred and eighteen (218) male. These samples were selected from a population of four hundred and seventy-three (473) level 200 preservice teachers offering general programme. Convenient sampling was adopted in selecting the college for the study while simple random sampling was adopted to select the general programme preservice teachers. The survey design was used in the study while teacher made test was used as the main instrument for data collection. In order to answer the research questions and the hypotheses for the study, the general programme preservice teachers were given the Van Hiele Geometry Test (VHGT) to determine their geometric thinking levels. The questions were structured sequentially into five (5) subgroups from the very easy to the most difficult. Data collected from the study were descriptively and inferentially analysed using SPSS. The results indicated that there was no statistically significant difference in achievement between male and female, however there was mean difference of 0.03 in favour of the male preservice teachers. Again the combined results from the independent sample t-test revealed that there was no significant difference in VHGT levels between the male and female preservice teachers, even though the male performed slightly better than their female counterparts except in level 4 which favoured the female preservice teachers.

Ato and Koryoe (2015) investigated gender differences and mathematics achievement of senior high school students of Ghana National College, Cape Coast, Ghana. A sample of two intact classes were chosen by simple random sampling technique from all first-year senior high school students in the Cape Coast Metropolis for the study. In all a total of eighty-two students participated in the study. The sample comprised 42 students in the experimental group and 40 students in the control group. Again out of the 42 students in the experimental group 27 were male and 15 were female while in the control group 26 were male and 14 were female The purpose of the study was to provide insight into the differences between male and female students in mathematics (including circle geometry) learning among senior high school students in Ghana using performance-driven instruction and performance-based assessment tasks. The study also utilized quasi-experimental non-randomized control group design. Two equivalent performance-based assessment test were designed and used as the instrument for the study. The reliability estimates for pre-test and post-test were 0.67 and 0.70 respectively, using K-R formula. Data collected from the study were analysed using an independent t-test and a paired sample t-test at $\propto = 0.05$. The results of the study indicated a significant difference in the post-test score in favour of the female. The study also identified that the performance assessment–driven instruction improved students' problem-solving abilities and shown no bias between gender.

Pavethira and Leong (2017) investigated the role of gender (male and female) on students' performance in geometrical reflection using GeoGebra. The research employed the one group pre-test and post-test design. A sample of 24 students, made up of 12 boys and 12 girls were selected from students studying in year 1 in international schools. The cluster sampling method were used to select one year 1 class. Out of the sample of 24, 6 were selected from Extension (students who always achieved excellent performance in mathematics), 12 from Core group (intermediate group) and 6 from Support group (students who needed extra guidance in completing their task). All the students were taught what reflection was by using the traditional approach after which they were assessed (pre-test). The students again were introduced to the use of GeoGebra in learning reflection and assessed afterwards (post-test). The pre-test and post-test results were analysed using the SPSS software. The F test and p - value of Levene's test of equality of variance was used to determine whether or not the equal variance assumptions have been met. Again Shapiro-Wilk test was used to check normality. The results indicated that girls performed better than boys perhaps because girls have high interest in exploring the software on reflection topics. They might have also learned the concept better than they have learnt when they do it manually (without any software). However, boys were more focused on the software activity and showed less interest in doing reflection manually.

Alex and Mammen (2014) did a quantitative study on gender differences amongst South African senior secondary school learners' geometric thinking levels. The study involved a total of 359 students comprising 163 males and 196 females selected purposively from Umtata District in South African's Eastern Cape Province. The participants' geometrical levels were determined through a multiple choice test. The test results were analysed using Microsoft Excel 2007 and IBM SPSS Version 19. The results indicated that there was a slight difference in the performance in favour of the female students in terms of the mean score obtained in the test. However, the t-test showed no significant statistical difference between the genders in performance.

Achor, Imoko and Ajal (2010) investigated the effect of games and simulations on the gender related differences in mathematics achievement and interest of students in geometry. The sample for the study consisted of 287 senior secondary school (SSS 1) students comprising 158 boys and 129 girls from six out of 46 secondary schools in Gwer-West LGA of Benue State, Nigeria. Simple random sampling techniques were used to select the six schools from the 46 secondary schools with a total of 1,434 students in SSS1. Again in each of the selected schools, a simple random sampling technique was used to assign intact classes to experimental and control groups. The experimental group consisted of 74 boys and 65 girls while the control group had 84 boys and 64 girls. The study adopted a pre-test and post-test quasi-experimental design. Data collected using Geometry Achievement Test (GAT) and Geometry

Interest Inventory (GII) was analysed descriptively and analysis of covariance (ANCOVA) was used to test the hypotheses. Findings of the study indicated that male and female students taught using games and simulations did not differ significantly both in achievement and in interest. The results indicated that when the right technological tools are used in teaching and learning mathematics, especially in geometry, both genders can perform well.

Abbas and Habu (2014), conducted a study entitled "effect of gender-related differences in academic achievement and retention of senior secondary school students taught geometry using problem solving approach. A total of 70 SS3 students were selected from about 9,540 students of Jigawa state using stratified random sampling techniques. A pretest-posttest experimental control group research design was used for the study. Both male and female group were taught geometry using problem solving approach. The instruments used for data collection were Researcher Made Test (RMT), Geometry Achievement Test (GAT) and Geometry Retention-Test (GRT). Data collected from the study were analysed using t-test statistic. Finding statistically showed the existence of significant differences between male and female students' performance in Geometry Retention-Test in favour of the males

Gender differences in geometry and mathematics achievement and self–efficacy beliefs in geometry was the subject of investigation by Erdogan, Baloglu and Kesici (2011). The purpose of the study was to investigate gender differences in mathematics course achievement, geometry course achievement, and geometry self– efficacy. A total of 199 high school sophomores from an Anatolian high school in Konya were used as the sample for the study. Out of the sample 100 were men (50.3%) and 99 were women (49.7%). Students' mathematics and geometry end-of-the-year GPA were obtained from the official student records. The data collected were analysed and the results shown a significant relationship between mathematics and geometry achievement as well as significant relations between self-efficacy beliefs and geometry achievement. Again the findings of the study indicated that there were no significant differences in the achievement of males and females even though the mean score of the females were a bit higher than that of the males. This means that when boys and girls are given the needed assistance and encouragement, they can perform well in mathematics.

Udousoro (2011) studied the perceived and actual learning difficulties of students in secondary school level. The purpose of the study was to determine college students' misconceptions on geometry subjects. A total of 120 secondary school students made up of 60 males and 60 females from four co-educational secondary school in Etinam Local Government Area of Akwa Ibom State of Nigeria took part in the study. A survey design was adapted for the study. The instruments for the study were questionnaire and teacher-made achievement test. Data collected from the study were analysed using means and standard deviations, Spearman Rank Order Correlation and independent t-test The results indicated a significant difference between male and female actual learning difficulties in favour of the males.

Geometry strategic competence of junior high school students based on sex differences was the topic investigated by Zahra, Budiyono, and Isnander (2017). This study examined whether there was different geometry strategic competence between junior high school students based on sexes. This research used mix method design which consisted of two stages, that was quantitative and qualitative stage. The study involved 60 students (consisting of 34 girls and 26 boys) chosen by cluster random sampling from a population of SMPN 2 Ngawi. The instruments used for collecting data were test and interview. Students in each sex bracket were asked to perform 14 geometry task, the scores obtained were analysed quantitatively using independent t-test to compare the mean value score between the two groups. The quantitative analysed data shown that there was no different in geometry strategic competence between girls and boys. Again semi-structured interview was conducted on six students (3 girls and 3 boys) who represented each level as subjects. These six students were selected through purposive sampling approach. The chosen students were asked to do 3 chosen numbers of the task they had done before while given semi-structured interview to acquire the level of strategic competence. The students were classified based on strategic competences score into 3 groups' i.e. low, medium and high score. The qualitative data analysed shown that the girls had more positive attitude than boys towards mathematics problem in low score level and had the same strategic competence with boys in the medium score level. Then in the high score level, boys tend to use insight strategies to solve problems whereas girls used algorithm strategies.

According to Brodie (2004), gender is another background factor and anecdotal evidence gathered by the researcher over many years of teaching and his experience indicates that females tend to distance themselves from geometry as much as possible. The Australian Capital Territory Assessment Program results show a relationship between gender and success in geometry with males being more successful than females. As the majority of teachers are female (pre-school and primary 79%, senior high 56.1%, Australian Basic School, 2002) this is an important variable given the influence of a teacher on a student's success in geometry. This investigation led to the Catholic school system in Australia agitating for exemption from sex discrimination laws so that they can launch a male–only teacher recruitment program (The Daily Telegram, December, 2002).

While it is promising to see that several previous studies have demonstrated positive effects of GeoGebra instructional approach on students' performance, a reading of the literature available indicate that many of these studies are not centred on the GeoGebra instructional approach in teaching and learning Circle Theorem as a sub-topic under Geometry, particularly in the Colleges of Education in Ghana. It is therefore the belief of the researcher that learning through the use of GeoGebra may be able to fill the gap once missing in the educational process. Also, some of the findings of the literature reviewed point out the challenges teachers face in teaching Geometry to lack of resources to teach the topic, its abstract nature and inability of students to visualize geometric images. This study therefore seeks to determine whether or not using GeoGebra in teaching Circle Theorem can have any effect on the academic performance of students of the Colleges of Education, especially at the Presbyterian College of Education, Akropong-Akuapem so as to curtail the difficulties students face when learning geometry.

**2.7 Conclusion**

This chapter gave account of other studies which have relevance to the present study; discussions of past research on the use of GeoGebra in teaching and learning of mathematics were extensively discussed. The study also reviewed literature on the causes of students' difficulties in learning geometry as well as gender differences in teaching and learning of geometry. The Van Hiele's theory of geometrical understanding was also discussed. In particular, Van Hiele's level of learning geometry provided a theoretical framework which the researcher adopted in the study.

The review of related literature indicated that students had difficulties in geometry learning. From the Van Hiele's description of learning, at the basic level it emphasized visualization as an important skill in learning geometry, the researcher also agree with Van Hiele that the ability of a student to visualize geometric concept is very important in the teaching and learning of geometry. The next chapter presents the research methodology of the study.

# CHAPTER THREE

# RESEARCH METHODOLOGY

## 3.0 Overview

The chapter began by giving a justification of the research philosophy used and an outline of how the study was carried out. This includes a presentation of the research design and its justifications, population, sample and sampling procedure, data collection instruments, validity and reliability, the chapter ends with data analysis procedure.

### 3.1 Research Philosophy

This study adopted the research philosophy of post-positivism. According to Gorden and Scolt (1991), a research philosophy is a belief about the way in which data about a phenomenon should be gathered, analyzed and used. Post-Positivism subscribes to the view that theories, background knowledge and values of the researcher can influence what is observed. They believe that human knowledge is based not on unchallengeable, rock-solid foundations, rather upon human conjectures. They also assume that, "reality" and "truth" are not conditional and can be understood in different ways (Ryan, 2006). Post-positivist research principles emphasize meaning and the creation of new knowledge and are able to support committed social movements, that is, movements that aspire to change the world and contribute towards social change. Some of the characteristics of Post-positivism include:

Research is broad rather than specialized – lots of different things qualify as research;

Theory and practice cannot be kept separate – one cannot afford to ignore theory for the sake of 'just the facts'.

In this study, Post–positivism was the guiding principles because post-positivist methods require accuracy, sound reasoning and production of evidence. In this study, a field experiment (quasi-experiment) was carried out, in order to achieve greater realism and to diminish the extent to which the outcomes could be criticized and contrived (Crowther & Lancaster, 2008).

### 3.2 Research Design

The experimental research design specifically, non-equivalent quasi-experimental research design was used because intact classes of unequal number of students were used and the respondents were not randomly selected and allocated into the control and the experimental groups. The variable the study investigated were GeoGebra/teacher talk and chalk (independent variable) and Presbyterian college of Education students' performance, Van Hiele levels (dependent variable). Here the experimental group was taught circle theorem using GeoGebra, while the control group was taught on the same concept using the traditional teacher "talk and chalk approach". This design was used because 'it provides the best approach to investigating cause and effect relationship' (McMillian, 2000, p 207). This view is supported by Dinardo (2008) and Fraenkel and Wallen (2010). The design is illustrated in table 3.1 below.

***Table 3.1: Pre-Test Post-Test Control Group Design***

| **Group** | **Treatment** | | |
|---|---|---|---|
| Experimental Group (GeoGebra Instructed Group) | Pretest | GeoGebra Instructed (Computer Assisted) | Post-test |
| Control Group | Pretest | Traditional Instructed | Post-test |

### 3.3 Population

The target population for the study was all the 490 level 100 students of the Presby College of Education, Akropong-Akuapem, made up of 200 females and 290 males. The study also involved all the six (6) mathematics tutors at the college.

#### *3.3.1 Sample and sampling procedure*

A purposive sampling technique was used to select the sample for the study. A purposive sampling according to Arikunto (2010) is a non-probability sampling where participants are selected based on characteristics of a population and the objective of the study. The purposive sampling is useful because it is easier to make generalizations about the sample. Again, information collected by purposive sampling has a low margin of error. The main disadvantage is that, it provides a significant number of inferential statistical procedures that are invalid. The total number of students who participated in this study was 390 that are all students who offered geometry as a course in the semester. This number consisted of 220 males and 170 females. A simple random sampling technique was employed to select the intact classes into either the control group or the experimental group. This was done through the balloting approach by first writing "experimental" and "control" on a piece of paper, folded them, placed them in a bow, shuffled them and called one representative from each of the six intact classes, blindfolded them and then asked them to pick one paper each from the shuffled bow for their classes. The three representatives that picked experimental constituted the experimental group and those that picked the control also constituted the control group. In all, the control and the experimental group had three classes each. The total number of students in the experimental group was 220 made up of 120 males and 100 females, while 170 students made up of 90 males and 80 females in the control group. The difference in the group was as a result of the uneven distribution of students in the various programs. Again, all the 6 mathematics tutors at the college were also involve in the study.

**3.4 Instrument**

The main research instruments that were used in the study were Geometry Learning Assessment Test (GLAT) and questionnaire in an embedded mixed method. The Geometry Learning Assessment Test comprised pre-test and post-test. The test items on the pre-test were different from that of the post-test on circle geometry. The pre-test was administered to the selected groups at the same time to avoid the students discussing the test items and also to avoid leakages. The reason for the pre-test was to determine the initial entry points and to compare difference between experimental and control group before treatment. At the end of the instruction students were again assessed to find out whether or not the teaching method using the software has had any impact on their academic performance of circle geometry. Questionnaires were also administered to both the mathematics tutors and students to find out the causes of difficulties in teaching and learning circle theorem (See Appendix A1). The questionnaire was administered personally by the researcher to the students to be answered, a day after the post-test. The questionnaire was administered personally to help improve the collection and response rate of the questionnaire. The questionnaire was collected as soon as it was completed by the respondents. This enabled the researcher to obtain 100% response rate.

***3.4.1 Pre-Test***

The first research instrument the researcher used was a pre-test (see Appendix A2). The researcher began the data collection by administering a pre-test to both the control group and the experimental group. The reason for administering the pre-test was to determine the baseline knowledge or preparedness for learning the topic of circle theorem and to compare difference between experimental and control group before treatment. The pre-test consisted of 20 multiple choice questions.

### *3.4.2 Post -Test*

The second research instrument that was used was a post-test (see Appendix A3). The main reason for conducting the post-test was to determine the treatment impacts and effects on the academic performance of students of Presbyterian College of Education, Akropong-Akuapem. This test was conducted at the end of the treatment. The test was conducted under the supervision of the researcher and some colleagues mathematics tutors in the mathematics and ICT department, Akropong-Akuapem. The post-test comprised 11 multiple choice and ten essay type questions based on Colleges of Education course outline (EBC 122) and these questions were based on the principle of Van Hiele's theory of levels of geometric understanding. The questions were structured sequentially into five (5) sub-groups from the very easy to the most difficult. The post-test lasted for one hour thirty minutes after which the scripts were collected for marking. The allocation of marks for this test was dependent on the level to which the questions belong, as well as the strategy used according to Van Hiele's problem-solving skills strategies indicated in Table 3.2.

***Table 3.2 Van Hiele Levels Question Distribution***

| **Van Hiele's levels of geometric understanding** | **Question number** |
|---|---|
| Visualization | 1, 2, 3, |
| Analysis | 4, 5, 6, 7, 8, 9, 10, 11 |
| Abstraction | 12, 14, 15, 17 |
| Deduction | 13, 16, 18, 19 |
| Rigour | 20, 21 |

To mark the post-test an assessment rubrics was developed (see table 3.3)

*Table 3.3: Assessment rubrics according to Van Hiele levels*

| Level | Question number | Mark allocation | |
|---|---|---|---|
| 1. Visualization | 1, 2, 3 | 0 | Incorrect answer |
| | | 1 | Correct answer |
| 2. Analysis | 4, 5, 6, 7, 8, 9, 10, 11 | 0 | Incorrect analysis of question |
| | | 1 | Correct answer from correct analysis |
| 3. Abstraction | 12, 14, 15, 17 | 0 | Incorrect abstraction |
| | | 1 | Analysis and/or abstraction partly correct |
| | | 2 | Analysis correct but abstraction incorrect |
| | | 4 | Analysis and abstraction correct (solution correct) |
| 4. Deduction | 13, 16, 18, 19 | 0 | No understanding of axioms, theorems and definitions |
| | | 1. | Vague or partial understanding of axioms, Theorems and definitions. |
| | | 2. | Partly meaningful definitions and formal arguments |
| | | 3. | Clear logical deductions/correct answer |
| 5. Rigour | 20, 21 | 0- | No clear visualization |
| | | 2- | Clear visualization and analysis |
| | | 3 | Clear visualization, analysis and abstraction |
| | | 4 | Clear visualization, analysis, abstraction and deduction |
| | | 6 | Clear visualization, analysis, abstraction, deduction and rigor. |

### *3.4.3 Questionnaire*

The third research instrument used to collect data for this study was the questionnaire (see Appendix A1). According to Roospa and Rani (2012), a questionnaire is a series of questions asked to individuals to obtain statistically useful information about a given topic or phenomenon. According to them, questionnaires should always have a definite purpose that is related to the objectives of the research. In this study, the questionnaire was used to solicit students and tutors opinions about the causes of students' difficulties in the teaching and learning of circle theorem at the Presbyterian College of Education, Akropong-Akuapem. The questionnaire was used because it enabled the researcher to generate data specific to his research questions and also for easy comparisons. The questionnaire was again employed because it protected the privacy of the respondents as participants responded honestly to the questionnaire because their identity was hidden and their confidentiality was also maintained.

## 3.5 Treatment

### *3.5.1 The experimental group*

The experimental group was instructed using the GeoGebra teaching method. The GeoGebra approach is a constructivist learning approach which involves both the learner hands-on activities and teacher led demonstration using GeoGebra. In this approach, learners acquire a sense of ownership of the mathematics they learn since they are motivated from within themselves to learn and not to learn for any external reward; also they become active participants right from the beginning to the end of the lesson. In this study, lessons of the experimental group were organized in the computer laboratory. Each computer in the laboratory had GeoGebra software installed on it. In the computer laboratory students in the experimental group were taught circle theorem by using the GeoGebra software with worksheets design hand in hand with the lesson plan that was prepared by the researcher with students' Geometry course outline, bearing in mind curriculum aims and objectives of the topic (Circle Theorem). For example, to illustrate the concept "an angle a chord or an arc subtends at the centre of a circle is twice that it subtends at the circumference, the concept was illustrated with GeoGebra (see figure 3.1)

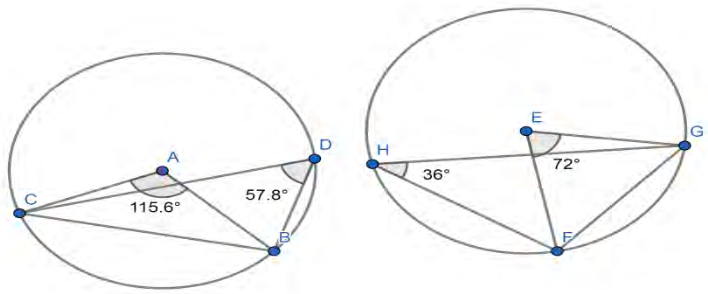



***Figure 3.1: Illustration of Circle Theorem using GeoGebra***

The teacher instructed and demonstrated all the circle theorems together with the students by connecting his laptop to an overhead projector. After a day or two of lesson introduction, content development worksheet was used during lesson delivery. The content development worksheets had the same questions for both groups (control and experimental group). Each group had a total of five lessons delivered to them; the only difference was the teaching and learning approaches used. Each lesson lasted for two-hours. Each worksheet covered one or two circle theorems depending on the length of the procedures required to prove the theorem(s). The worksheet had both open and closed ended questions and the reason was to allow students explore different solution strategies and skills in answering the circle theorem questions. The treatment lasted for 3 weeks.

### *3.5.2 The control group*

The traditional/ teacher talk and chalk approach was applied to the control group. The traditional approach of instruction refers to teaching using pen and paper for students and chalk and board for instructor. This approach is in line with the behaviorist theory that employs lecture, teacher demonstration, questions and answers as their main methods of instruction. For example, to illustrate the concept “the angle a chord or an arc subtends at the centre of a circle is twice the angle it subtends at the circumference of a circle, the teacher would have to draw several circles on the board and measure and compare angles. Here the accuracy of the results would depend on how precise the measurements and the drawings are done. In this study, five content development worksheets, similar in content to the experimental groups’ worksheets were used. All the questions and tasks were exactly the same for the two groups. The only difference was the way and manner in which students carried out their tasks. In the control group, students learn basically by listening to the teacher and focus on what the teacher says and what s/he writes on the chalkboard. Here the teacher speaks from the front of the class, explaining, guiding, controlling and deciding what students must do and occasionally writes notes, diagrams and questions on the chalkboard as students copy and follow what the teacher does.

### 3.6 Validity and Reliability

Validity of the instrument refers to the tendency of the instrument to measure exactly what it supposes to measure whilst reliability deals with consistency of the materials (the tendency of the materials to produce the same or almost the same result at any time it is used) (Phelan and Wren (2005). To ensure reliability of the instrument in the study, the researcher piloted the instruments on a small sample of 30 students drawn from the Presbyterian Women’s’ College of Education, Aburi-Akuapem. The piloting was done in this school because it has the same or similar characteristics as the sample for the study. Data from the pre-test and post-test of the pilot school was used in calculating the reliability of the instrument. Two reliability tests were calculated, the Kuder–Richardson 20 (KR20) for the pre-test and Cronbach’s Spearman–Brown formula for the post-test. The KR20 was used to measure the reliability of the pre-test because the test involved dichotomous questions (multiple choice items). The KR reliability value of the pre-test was calculated as shown in Appendices B. Again the researcher consulted some experience mathematics tutors from the Presbyterian College of Education as well as his supervisor for advice on the instruments.

### 3.7 Data Analysis Procedure

Descriptive statistics was used to analysis the data in the Statistical Package for the Social Sciences (SPSS version 21). The descriptive statistics, namely independent sample t-test was used to test if there were significant differences between the test scores of the groups in general and at the different levels of the Van Hiele's theory of geometrical understanding. The independent t-test was also used to determine whether there exists any significant difference in the post-test scores between male and female students.

# CHAPTER FOUR

# ANALYSIS AND DISCUSSIONS

## 4.0 Overview

The main purpose of this study was to investigate the effect of integration of GeoGebra into the teaching and learning of circle theorem on students' performance at the Presbyterian College of Education, Akropong-Akuapem. Four main research questions were raised namely; (1) what are the causes of students' difficulties in solving problems in circle theorem? (2) Is there any significant difference between the performances of students taught using GeoGebra as compared to students taught without GeoGebra in circle geometry? (3) Is there any significant difference in the post–test results between male and female students? (4) Is there any significant difference in the post-test average score of the control and the experimental group at the Van Hiele's levels? This chapter presents the data collected, techniques employed in the data analysis and the results that emerged from the analysis.

## 4.1 Data Analysis and Results

This part of the study deals with the analysis of the data gathered through the questionnaires and the class room test for the experimental group and the control group as well as practical observation from different sources followed by discussion of the findings. Furthermore, the main findings of the study are presented with the help of tables followed by descriptive statements for analysis to give answers to basic questions set in the study.

*Table 4.1: Demographic of Respondents*

| Respondents | Gender/Age level of Respondents | Number and % | | Total | |
|---|---|---|---|---|---|
| | | No. | % | No. | % |
| Sex | Male | 220 | 56.42 | 390 | 100 |
| | Female | 170 | 43.58 | | |
| Age | 18 - 20 | 72 | 18.46 | 390 | 100 |
| | 21-23 | 155 | 39.74 | | |
| | >24 | 163 | 41.80 | | |

Source: Field survey 2019

Table 4.1 shows the breakdown of the background of students' respondents for the study. As presented in Table 4.1, 220 or 56% of the students' respondents are male, while the remaining 170 or 44% of them are females. Concerning the age of respondents 72 (18.46%) of them are between 18-20 years of age; 155 (39.74%) of them are between 21-23 years and the rest of them who are >24 years of age are 163 (41.80%). It can be easily concluded that the majority of the respondents were above the ages of 24 years of age.

**Pre –Test Results**

A pre–test was administered to both group (control group and experimental group) one week before the interventions in order to check if the two groups were of comparable geometric abilities before the intervention. Table 4.2 shows the descriptive statistics for the pre–test results for the two groups.

***Table 4.2: Pre-test Group Statistics***

| | Groups | N | Mean | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|---|
| Pretest | Experimental Group | 220 | 10.5364 | 1.68895 | .11387 |
| | Control Group | 170 | 10.5471 | 1.76776 | .13558 |

The mean for the experimental group was 10.54 and the standard deviation was 1.69 which was lower than that of the control group with mean 10.55 and standard deviation 1.77. The mean score difference between the two groups was 0.01. To check whether the difference in performance between the experimental and the control group were statistically significant, an independent sample t–test was conducted. The following hypothesis were tested at 95% confidence interval:

$H_o$: There is no significant difference in geometric performance between the experimental and control group in the pre-test.

$H_1$: There is significance difference in geometric performance between the experimental and control group in the pre-test.

*Table 4.2.1: Independent Samples t–test for Pre-test*

| T-test for Equality of Means | | | | | | |
|---|---|---|---|---|---|---|
| t | df | Sig.(2-tailed) | Mean Difference | Std. Error Difference | 95% Confidence Interval of the Difference | |
| | | | | | Lower | Upper |
| -.061 | 388 | .952 | -.01070 | .17602 | -.35677 | .33538 |

Table 4.2.1 shows that there was no statistically significance difference between the control and the experimental group although the mean of the control group (M = 10.55, SD = 1.77) was higher than that of the experimental group (M = 10.54, SD = 1.69). According to the t-test conducted, t(388) = -.061and the p–value = 0.952. Since $0.952 > \alpha = 0.05$, it means the difference in mean of the two groups is not statistically significant. Therefore any difference in geometric performance after treatment could be attributed to the treatment.

## 4.2 Answering the Research Questions

### *4.2.1Research Question 1*

1) What are the causes of students' difficulties in solving problems in circle theorem?

Responses to the first ten items in the teachers and students questionnaire were used to answer the above research question. The four point Likert scale were coded as strongly Agree = 4, Agree = 3, Disagree = 2 and strongly Disagree = 1. The responses on agree and strongly agree were put together as agree and those on disagree and strongly disagree were put together as disagree. The responses are presented in table 4.3 and 4.4 below.

***Table 4.3: Teachers' opinion on the causes of students' difficulties in teaching and learning circle theorem at the Presbyterian College of Education, Akropong-Akuapem***

| S/N | Item | Extent of agreement | | | | |
|---|---|---|---|---|---|---|
| | | SA F(%) | A F (%) | D F (%) | SD F (%) | Total |
| 1. | Students' attitudes towards the teaching and learning of circle theorem | 4(66.6) | 1(16.7) | - | 1(16.7) | 6(100) |
| 2. | Students psychological fear for circle theorem poses a problem in teaching and learning the topic | 2(33.3) | 3(50) | 1(16.7) | - | 6(100) |
| 3. | There are inadequate mathematics teachers in terms of number and quality | - | - | 3(50) | 3(50) | 6(100) |
| 4. | Large classes makes it difficult to practicalise circle theorem. | 3(50) | 3(50) | - | - | 6(100) |
| 5. | Lack of instructional aids makes teaching and learning of circle theorem difficult. | 4(66.7) | 2(33.3) | - | - | 6(100) |
| 6. | Most mathematics teachers do not teach circle theorem well because of their poor foundation in the concept. | - | 2(33.3) | 2(33.3) | 2(33.3) | 6(100) |
| 7. | The poor foundation of students in basic school mathematics makes it difficult to teach circle theorem. | 4(66.6) | 1(16.7) | 1(16.7) | - | 6(100) |
| 8. | Parents do not buy textbooks for their wards to practice circle theorem at home. | 2(33.3) | 1(16.7) | 2(33.3) | 1(16.7) | 6(100) |
| 9. | Poor preparation on the part of some mathematics teachers in teaching circle theorem. | 2(33.3) | 2(33.3) | - | 2(33.3) | 6(100) |
| 10. | There are no incentives to motivate teachers to put up their best. | 3(50) | 2(33.3) | 1(16.7) | - | 6(100) |

From table 4.3, it could be observed that 5(83.3%) out of the 6 teachers who responded to the questionnaire indicated that the causes of students' difficulty in learning circle theorem was due to poor students' attitude towards the teaching and learning of circle theorem while 1 teacher representing 16.7% disagree with the assertion.

Similarly, 5 teachers representing 83.3% of the teachers were of the view that students' problem in circle theorem was due to students psychological fear for the topic, probably because of the way and manner they were taught circle theorem at the senior high school level, whiles one teacher representing 16.7% disagree.

Again all the six (6) teachers representing 100% disagreed with the statement that there are inadequate mathematics teachers in terms of number and quality.

It can also be inferred from table 4.3 that all the six teachers who responded to the questionnaire agreed to the statement that; large classes make it difficult to practicalise the teaching and learning of circle theorem. They added that to make the teaching and learning of mathematics in general and circle theorem in particular more effective, class size should not exceed 30.

Again, all the six (6) teachers who participated in the study agreed unanimously to the statement that lack of instructional aids makes the teaching and learning of circle theorem generally difficult.

Two teachers, representing 33.3% of the participated teachers agreed that the causes of students' difficulty in learning circle theorem can be traced to weak foundation of most mathematics teachers who handle the topic, while 4 teachers (66.6%) disagreed with the statement.

Furthermore, 50% representing 3 teachers blamed the causes of students' difficulty in learning circle theorem on parents' inability to buy textbooks and other reading materials for their wards to practice circle theorem at home whiles one-halve of the respondents representing 50% disagreed with the statement.

Poor preparation on the part of some mathematics teachers when teaching circle theorem was also one of the factors that contribute to poor performance of students in learning the concept "circle theorem" as 4(66%) out of the 6(100%) respondents alluded to that statement while 2(33.3%) of the respondents think otherwise. They further stated that there are no incentives to motivate teachers to put up there best.

***Table 4.4: Students' opinion on the causes of difficulty in learning circle theorem***

| S/N | Item | Extent of agreement | | | | |
|---|---|---|---|---|---|---|
| | | **SA F(%)** | **A F(%)** | **D F(%)** | **SD F(%)** | **Total** |
| 1. | Lack of teaching and learning aids by teachers makes it difficult to grasp the concept of circle theorem. | 200(51.3) | 120(30.8) | 50(12.8) | 20(5.1) | 390(100) |
| 2. | Lack of motivation in learning circle geometry. | 102(26.2) | 155(39.7) | 88(22.6) | 45(11.5) | 390(100) |
| 3. | There are inadequate mathematics teachers in terms of number and quality. | 112(28.7) | 165(42.3) | 66(16.9) | 47(12.1) | 390(100) |
| 4. | Students do not solve circle theorem questions in addition to what is given in school. | 118(30.3) | 153(39.2) | 67(17.2) | 52(13.3) | 390(100) |
| 5. | Students have a psychological fear for circle. | 156(40) | 141(36.2) | 53(13.6) | 40(10.2) | 390(100) |
| 6. | Parents cannot afford to buy textbooks and other learning materials for students | 72(18.5) | 100(25.6) | 140(35.9) | 78(20) | 390(100) |
| 7. | Circle theorem is not important to learn because it has no applications in real life. | 47(12.1) | 134(34.3) | 91(23.3) | 118(30.3) | 390(100) |
| 8. | The poor foundations of students in geometry at the basic level makes it difficult to learn circle theorem at the college level. | 150(38.5) | 112(28.7) | 69(17.7) | 59(15.1) | 390(100) |
| 9. | Poor teaching methods use in teaching circle geometry. | 132(33.9) | 93(23.8) | 110(28.2) | 55(14.1) | 390(100) |
| 10. | Inability to practicalise circle theorem concepts by teachers. | 95(24.4) | 120(30.8) | 93(23.8) | 82(21) | 390(100) |

Table 4.4 shows the opinion of students about the causes of difficulty in learning circle theorem at the Presbyterian College of Education, Akropong-Akuapem. From the table, it is evident that 82.1% representing 320 out of the 390 respondents agreed that lack of teaching and learning aids for teaching circle theorem makes the concept difficult to grasp whiles 70 respondents representing 17.9% think their presence does not have any effect on the teaching and learning of circle theorem.

Again, the respondents claimed generally that lack of motivation for learning circle theorem was the cause of circle theorem difficulty by students, 257(65.9%) out of the total respondents agreed to this assertion whiles 133(34.1%) thought otherwise.

Similarly, 277 respondents representing 71% were of the view that inadequate mathematics teachers in terms of number and quality is the cause of students' difficulty in circle theorem whiles 113(29%) disagreed.

Again it can be observed from the above table that 271(69.5%) out of the 390 respondents agreed to the statement that students do not solve circle theorem questions in addition to what is given in school whiles 119(30.5%) of the respondents disagreed. According to the majority of the respondents, to be good in mathematics in general and circle theorem in particular one has to solve extra questions to supplement what is given in school.

Furthermore, 297 respondents representing 72.2% attributed the causes of students' difficulty in circle theorem to students' psychological fear for circle geometry in general while 93(23.8%) disagreed.

On the issue of whether parents cannot afford to buy textbooks and other learning materials for their wards, 172 respondents representing 44% answered in the affirmative whiles 218 respondents representing 55.9% disagreed. The result indicates that though textbooks and other reading materials are important in students' geometric performance, which alone is not enough.

Again, 181 respondents representing 46.4% believed that circle theorem is not important to learn because it has no application in real life while 209 respondents which in percentage terms represent 53.6% thought otherwise.

Additionally, 262(67.2%) out of the 390 respondents attributed poor students' performance in circle theorem to students' poor foundations in geometry at the basic school level whiles 128 respondents representing 32.8% presented a contrary opinion.

Again, 296 respondents representing 57.7% believed poor teaching methods use in teaching circle theorem is the cause of students' poor performance in circle theorem while 165 respondents representing 42.3% disagreed.

Last but not least, 215(55.2%) out of the 390(100%) respondents thought students poor performance in circle theorem was due to inability of some mathematics teachers to practicalise the teaching and learning of circle theorem concept whiles 175(44.8%) respondents disagreed.

Tables 4.3 and 4.4 above reveal that the causes except there are inadequate mathematics teachers in terms of number and quality, most mathematics teachers do not teach circle theorem well because of their poor foundations in circle theorem, parents cannot afford to buy textbooks and other learning materials for students and circle theorem is not important to learn because it lacks applications in real life are responsible for the problem of teaching and learning circle theorem at the Presbyterian College of Education.

It is of importance to note that the teachers and students agree strongly to the fact that the poor foundation of students at the basic level, lack of instructional aids, students' psychological fear, large classes, poor preparation on the part of some mathematics teachers, lack of incentives to motivate teachers and students' attitudes towards circle geometry are the major problems of teaching and learning circle theorem as revealed in the tables above, the major one being lack of instructional aid in teaching the topic.

**Discussion**

This study aimed at finding the causes of students' difficulties in learning circle theorem. The major causes identified in the study were: (1) poor foundations of students at the basic school level. This finding is supported by (Adolphus, 2011; Bosson-Amedenu, 2017; Surendra, 2016), (2) lack of instructional aids for teaching circle theorem. This result is in agreement with the studies of Johnson-Wilder and Mason (2005); Egwu, Asuque and Ofori (2018); Fabiyi (2017); Ntshengedzeni (2015); Mifetu, Kpotosu, and Amegbor (2019); (3) students' psychological fear for the topic. This result confirms the findings of Ejiofor-Chima and Accra (2019); (4) lack of incentives to motivate teachers to put up their best. This finding is consistent with that of Mifetu, Kpotosu, Bessah and Amegbor (2019); and (5) students' poor attitude towards the teaching and learning of circle theorem. This result is supported by the findings of Adegun and Adegun (2013); Adolphus (2011); Fabiyi (2017) and Ntshengedzini (2015) who all concluded poor altitude of students towards learning geometry as the main problem students face as far the teaching and learning of circle theorem is concern. The study however disagreed with the finding of Johnson–Wilder and Mason (2005) who attributed the causes of students' difficulty in geometry to teachers' poor foundation in geometry.

***Table 4.5: Experimental and Control group Statistics***

| **Group** | **Male students** | **Female Students** | **Total** |
|---|---|---|---|
| Experimental Group | 120 | 100 | 220 |
| Control Group | 90 | 80 | 170 |
| **Total** | **210** | **180** | **390** |

Source: Field survey 2019

For this study, two groups of students respondents were selected - one is the control group and the other group is the experimental group. Experimental group was taught by using Geogebra and the Control group was taught by traditional teaching method (See table-4.5 above). For the teaching of Experimental group the lesson material were prepared by using Geogebra. After 21 days of teaching activities a post-test was conducted for both groups.

### *4.2.2 Research Question 2*

Is there any significant difference between the performances of students taught with GeoGebra as compared to students taught without GeoGebra in circle geometry?

To answer the research question 2, the post-test descriptive statistics were run for both the control and the experimental group. Table 4.6 gives the details of the results obtained.

***Table 4.6: Post-test results***

| | **Marks** | **N** | **Mean** | **Std. Deviation** | **Std. Error Mean** |
|---|---|---|---|---|---|
| Post-test | Experimental Group | 220 | 38.7773 | 5.20629 | .35101 |
| | Control Group | 170 | 30.0706 | 5.49187 | .42121 |

In the post-test, the average score (M = 38.77; SD = 5.21) of the experimental group, was higher than the control group average score (M = 30.07; SD = 5.49). To check if the difference between the performances of the groups were statistically significant, independent samples t-test was computed to check whether there was significant difference between the two groups' geometric performance. The following hypotheses were tested at 95% confidence interval.

$H_o$: There is no significant difference in geometric performance of the experimental group as compared to the control group after treatment.

$H_1$: There is significant difference in geometric performance of the experimental group as compared to the control group after treatment. The results are shown in Table 4.6.1.

***Table 4.6.1: Independent samples t-test for post-test***

| **t-test for Equality of Means** | | | | | | |
|---|---|---|---|---|---|---|
| **T** | **df** | **Sig.(2-tailed)** | **Mean Difference** | **Std. Error Difference** | **95% Confidence Interval of the Difference** | |
| | | | | | **Lower** | **Upper** |
| 15.989 | 388 | .000 | 8.70668 | .54454 | 7.63606 | 9.77731 |

Table 4.6.1 shows that there is a statistically significant difference in post-test scores of experimental group (M = 38.78; SD = 5.21) and control group (M =30.07; SD = 5.49); t(388) = 15.898; p = 0.00). This finding illustrated that the students in the experimental group performed better using GeoGebra than the control group that used the traditional learning method.

**Discussion**

In this study, the effect of using GeoGebra on students' mathematics performance in learning circle theorem was examined using quasi-experimental design. With the current exponential development in information and communication technology in the field of education, the present study attempted to examine the effectiveness of using GeoGebra as a tool in teaching and learning circle theorem. The results of the study indicated that there was a significant difference between the performance of the control group, which underwent the traditional method of teaching, and the experimental group, which was taught utilizing GeoGebra. This result indicated that students taught circle theorem with GeoGebra performed better than students taught without GeoGebra. The result is consistent with the study by (Saha et al., 2010; Shadaan & Eu, 2013; Soheila & Kumalludeen, 2018; Sudihartinih & Purniati., 2019 Zengin et al., 2012) which showed a positive effect of using mathematical learning softwares, thus motivating the students towards geometry learning (Dogan & Içel, 2011).

### *4.2.3 Research Question 3*

Is there any significant difference in the post–test scores between male and female students?

***Table: 4.7: Descriptive Statistics Score for the gender (Experimental Group)***

| Gender | N | Mean | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|
| Male | 120 | 39.2500 | 5.22800 | .47725 |
| Female | 100 | 38.3100 | 5.31758 | .53176 |

The descriptive table, Table 4.7, provides some very useful descriptive statistics, including the mean and standard deviation of the experimental group. The mean score for male is 39.25 that is higher than the female score of 38.31. The standard deviation for male is 5.23 while the standard deviation for female is 5.32. To check if the differences in mean is statistically significant, an independent samples *t*-test was conducted to determine if the mean of male group differs from that of the female group. Table 4.7.1 below gives further details of the result obtained.

***4.7.1: Independent Samples t-Test for gender (Experimental Group)***

| **t-test for Equality of Means** | | | | | | |
|---|---|---|---|---|---|---|
| **T** | **df** | **Sig. (2-tailed)** | **Mean Difference** | **Std. Error Difference** | **95% Confidence Interval of the Difference** | |
| | | | | | **Lower** | **Upper** |
| 1.318 | 218 | .189 | .94000 | .71341 | .46606 | 2.34606 |

Based on Table 4.7.1, we see that the significance (2-tailed) value is .189. This value is greater than .05. Thus, there was no statistically significant difference between the mean performance of male and female students on circle theorem.

***4.8: Descriptive Statistics Score for the gender (Control Group)***

| **Gender** | **N** | **Mean** | **Std. Deviation** | **Std. Error Mean** |
|---|---|---|---|---|
| Male | 90 | 29.5889 | 5.57864 | .58804 |
| Female | 80 | 30.9875 | 4.87695 | .54526 |

The descriptive statistics table 4.8 shows that the male obtained a mean score of 29.59 while the female obtained a mean score of 30.99. The mean score difference between the male and the female was 1.40. The standard deviation for the male and the female were respectively 5.58 and 4.88. To determine whether the difference between the mean score of the male and the female of the control group were statistically significant, an independent sample t-test was conducted. Table 4.8.1 gives the details of the results obtained.

**Table 4.8.1***: Independent Sample t-Test for gender (Control Group)*

| t-test for Equality of Means | | | | | | |
|---|---|---|---|---|---|---|
| T | Df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | 95% Confidence Interval of the Difference | |
| | | | | | Lower | Upper |
| -1.730 | 168 | .085 | -1.39861 | .80830 | -2.99435 | .19712 |

The results from Table 4.8.1 indicated that there was no statistically significant difference in the control group post-test scores of the male (M = 29.59, SD = 5.58) and that of the female (M = 30.99, SD = 4.88); t (168) = -1.73; p = .085). This result shows that when students are taught using GeoGebra as an instructional tool, both male and female can perform better.

**Discussion**

The study again investigated the role of gender (male and female) on students' geometric performance. From the independent sample t-test conducted, there was no statistically significant difference between the average score of males (M =39.25; SD = 5.23) and that of females (M = 38.31; SD = 5.32) in circle theorem. This result confirms the findings of Halat, (2008); Arhim and Offoe, (2015), Anas (2018), who has all found no statistical differences in their researches on gender differences in mathematics. The result however differs from that of Abbas and Habu (2014) whose study shown existence of significant differences between male and female students' performance in Geometry Retention-Test in favour of the males. This result does not provide enough evidence to show that male students perform better than female students.

*4.2.4 Research Question 4*

**Is there any significant difference in the post–test mean scores of the control and experimental group at the Van Hiele's levels?**

*Table 4.9: Group's Post-test Descriptive Statistics at Van Hiele levels*

| | Marks | N | Mean | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|---|
| Visualization (3 marks) | Experimental Group | 220 | 2.9409 | .23633 | .01593 |
| | Control Group | 170 | 2.7706 | .42170 | .03234 |
| Analysis (7 marks) | Experimental Group | 220 | 6.8909 | .35360 | .02384 |
| | Control Group | 170 | 6.2176 | .82472 | .06325 |
| Abstraction (16 marks) | Experimental Group | 220 | 12.0273 | 1.78762 | .12052 |
| | Control Group | 170 | 9.9824 | 2.07114 | .15885 |
| Deduction (12 marks) | Experimental Group | 220 | 9.1182 | 1.52143 | .10257 |
| | Control Group | 170 | 6.9059 | 1.73801 | .13330 |
| Rigour (12 marks) | Experimental Group | 220 | 7.7955 | 2.53541 | .17094 |
| | Control Group | 170 | 4.1824 | 2.09726 | .16085 |

Table 4.9 shows the descriptive statistics of both groups (control and experimental group) at the various Van Hiele levels. The results show that the average of the experimental group was higher than the average of the control group at all the Van Hiele levels of geometric understanding. The averages for the various Van Hiele levels for both groups were: visualization – (experimental group; M =2.94, SD = 0.24) which was higher than that of the control group's average of M = 2.77, SD = 0.42, analysis – (experimental group; M = 6.89, SD = 0.35) again higher than the control group's average of M = 6.22, SD = 0.82. The averages at the abstraction, deduction and rigour levels for the experimental group were respectively M =12.03, SD = 1.79, M = 9.12, SD = 1.52 and M = 7.80, SD = 2.54 which were respectively

higher than that of the control group of M = 9.98, SD =2.07, M = 6.91, SD = 1.74, M = 4.18, SD = 2.10 respectively. To check whether the difference between the averages of the groups (experimental and control group) at the various Van Hiele levels were statistically significant, independent samples t-test was conducted at 95% confidence interval to test the hypothesis:

$H_o$: There is no significant difference in the post–test mean scores of the control and experimental group at the Van Hiele's levels.

$H_1$: There is a significant difference in the post–test mean scores of the control and experimental group at the Van Hiele's levels.

*Table 4.9.1: Independent sample t-test for the groups' performance at each of the Van Hiele Levels*

| t-test for Equality of Means | | | | | | | |
|---|---|---|---|---|---|---|---|
| | t | Df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | 95 % Confidence Interval of the Difference | |
| | | | | | | Lower | Upper |
| Visualization | 5.052 | 388 | .000 | .17032 | .03371 | .10404 | .23660 |
| Analysis | 10.886 | 388 | .000 | .67326 | .06185 | .55166 | .79486 |
| Abstraction | 10.450 | 388 | .000 | 2.04492 | .19568 | 1.66019 | 2.42965 |
| Deduction | 13.379 | 388 | .000 | 2.21230 | .16536 | 1.88719 | 2.53741 |
| Rigour | 15.027 | 388 | .000 | 3.61310 | .24044 | 3.14036 | 4.08584 |

Table 4.9.1 shows that at Van Hiele visualization level, there was a statistically significant difference between average marks of the experimental group (M = 2.94, SD = 0.24) and the control group (M = 2.77, SD = 0.42); t(388) = 5.05, p = 0) in favour of the experimental group. Similarly, there is a statistically significant difference between the average post–test marks of the experimental group (M = 6.89, SD = 0.35) and control group (M = 6.22, SD =0.82); t(388) = 10.89; p = 0) at the analysis level of geometric understanding in favour of the experimental group. Similarly there are statistically significant differences in geometric performance between the experimental and control group at all levels of Van Hiele geometrical understanding in favour of the experimental group. The average post–test marks for abstraction, deduction and rigour for the experimental group are: M = 12.03, SD = 1.79; M = 9.12, SD = 1.52; M = 7.80, SD = 2.54 respectively while that of the control group are: M = 9.98, SD = 2.07; M = 6.91, SD = 1.74; M = 4.18, SD = 2.10 respectively. The analysis of the result show that, the experimental group who were taught with the use of GeoGebra performed better in circle theorem than the control group who were taught using the traditional approach.

**Discussion**

There were statistically significant differences in geometric performances between the experimental and control group at all levels of Van Hiele geometrical understanding in favour of the experimental group. The analysis of the result show that, the experimental group who were taught with the use of GeoGebra performed better in circle theorem than the control group who were taught using the traditional approach at all the levels of Van Hiele geometric understanding. The possible reasons for these findings could be attributed to the fact that GeoGebra as an instructional tool enabled students in the experimental group to check the accuracy of their work and correctness of their methods. Because GeoGebra is a dynamic teaching and learning tool and simple to use, students in the experimental group had the opportunity of re-examining their work, while those in the control group only memorized the theorems and applied them with little or no understanding as to how the theorems came by. Again in the control group, teaching was limited to few examples while the experimental group had the chance to explore the concept further using the GeoGebra software. This result is consistent with the findings of Venkataraman (2012), who found that students taught with GeoGebra made progress towards mathematical explanations which provide a foundation for further deductive reasoning in mathematics (levels 1 and 2).

# CHAPTER FIVE

# SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

## 5.0 Overview

In this chapter, a summary of findings of the study that aimed at investigating the effect of GeoGebra on the academic performance of students of Presbyterian College of Education, Akropong-Akuapem was presented in section 5.1. This was followed by concluding remarks in section 5.2, the chapter makes recommendations for practice and policy in section 5.3. The chapter ends by making suggestions for future research on the integration of mathematical software (GeoGebra) into the teaching and learning of mathematics at all levels of education in the country.

### 5.1 Summary of the Study

Students' poor performance in circle theorem at the Presbyterian College of Education, Akropong-Akuapem has been a source of worry to both students and teachers for some time now. Most teachers have done and continue to do everything within their power to ensure that students excel in circle theorem yet the problem still exists. The objectives of this study therefore was to:

- Investigate the causes of students' difficulties in solving problems in circle theorem.
- Find out whether or not there was any difference in performance between students taught with GeoGebra vis a vis students taught using the traditional approach.
- Investigate if differences exist in the post–test result between male and female students.

From the data analyses and theoretical point of view, the following findings were made.

a. If students at the Presbyterian College of Education, Akropng–Akuapem are taught Circle Theorems, using GeoGebra software as an instructional tool, their performance would be better than when they are taught using the traditional method of teaching.
b. GeoGebra makes lesson more practical, easy to understand, interesting and also enhances students' visualisation instead of memorisation of theorems.
c. The causes of students' difficulty in the teaching and learning circle geometry could be traceable to teachers' method of instruction, unavailability of instructional materials, large class size, students' psychological fear for geometry and students attitudes towards the teaching and learning of circle theorem.
d. Students' gender has no effect in the teaching and learning of circle theorem.
e. Students taught circle theorem with GeoGebra performed better than students taught without GeoGebra at all the Van Hiele levels.

**5.2 Conclusion**

The study concludes that GeoGebra as a mathematical tool can aid the improvement of the poor performance of students in questions involving circle theorem and that it enhances understanding which is key to good mathematics learning and therefore its use in mathematics classrooms should be encouraged

**5.3 Recommendations of the Study**

Based on the research findings, the following recommendations are considered appropriate:

1. Teachers should use technologically enhanced methods in teaching circle theorem.

2. Seminars/workshops should be organized for Colleges of Education Mathematics tutors on the use of appropriate technological tools such as GeoGebra in the teaching and learning of mathematical concepts by technological experts. This is because the application of GeoGebra in teaching and learning requires skills on the part of teachers.

3. College tutors should as much as possible employ gender responsive pedagogy in teaching circle theorem for the benefits of both male and female students.

4. Mathematics teachers should try as much as possible to relate their lesson to real life situation in order to reduce the perceived abstract nature of the subject.

5. Ghana Tertiary Education Commission (GTEC), Ghana Education Service (GES), Mathematics Teachers' Association of Ghana (MAG) should organized professional development workshop for all teachers to learn Van Hiele model for teaching geometry.
6. Class sizes should be reduced to a maximum of 30 students to allow teachers enough time to attend to students' needs.
7. Geometry course outline/syllabi for Colleges of Education should be revised to create rooms for Van Hiele's phases based approaches for the new 4 year bachelor degree programmes.
8. The traditional method of teaching mathematics does not promote conceptual understanding in the Colleges of Education and therefore college tutors should find alternative/or supplementary ways of teaching mathematics in general and circle theorem in particular.

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# APPENDICES

## APPENDIX A 1

### QUESTIONNAIRE FOR TEACHERS AND STUDENTS ON THE CAUSES OF DIFFICULTIES IN TEACHING AND LEARNING OF CIRCLE THEOREM.

Teachers' opinion on the causes of students' difficulties in teaching and learning circle theorem at the Presbyterian College of Education, Akropong-Akropong.

| S/N | Item | Extent of agreement | | | |
|---|---|---|---|---|---|
| | | **SA** | **A** | **D** | **SD** |
| 1. | Students' attitudes towards the teaching and learning of circle theorem | | | | |
| 2. | Students psychological fear for circle theorem poses a problem in teaching and learning the topic | | | | |
| 3. | There are inadequate mathematics teachers in terms of number and quality | | | | |
| 4. | Large classes makes it difficult to practicalise circle theorem. | | | | |
| 5. | Lack of instructional aids makes teaching and learning of circle theorem difficult. | | | | |
| 6. | Most mathematics teachers do not teach circle theorem well because of their poor foundation in the concept. | | | | |
| 7. | The poor foundation of students in basic school mathematics makes it difficult to teach circle theorem. | | | | |
| 8. | Parents do not buy textbooks for their wards to practice circle theorem at home. | | | | |
| 9. | Poor preparation on the part of some mathematics teachers in teaching circle theorem. | | | | |
| 10. | There are no incentives to motivate teachers to put up their best. | | | | |

Students' opinion on the causes of difficulty in learning circle theorem

| **S/N** | **Item** | **Extent of agreement** | | | |
|---|---|---|---|---|---|
| | | **SA** | **A** | **D** | **SD** |
| 1. | Lack of teaching and learning aids by teachers makes it difficult to grasp the concept of circle theorem. | | | | |
| 2. | Lack of motivation in learning circle geometry. | | | | |
| 3. | There are inadequate mathematics teachers in terms of number and quality. | | | | |
| 4. | Students do not solve circle theorem questions in addition to what is given in school. | | | | |
| 5. | Students have a psychological fear for circle. | | | | |
| 6. | Parents cannot afford to buy textbooks and other learning materials for students | | | | |
| 7. | Circle theorem is not important to learn because it has no applications in real life. | | | | |
| 8. | The poor foundations of students in geometry at the basic level makes it difficult to learn circle theorem at the college level. | | | | |
| 9. | Poor teaching methods use in teaching circle geometry. | | | | |
| 10. | Inability to practicalise circle theorem concepts by teachers. | | | | |

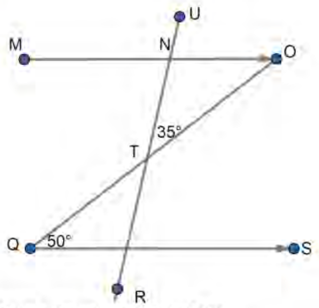
# APPENDIX A 2

## PRESBYTERIAN COLLEGE OF EDUCATION- AKROPONG AKUAPEM

## EBC 122: LEARNING, TEACHING AND APPLYING GEOMETRY AND HANDLING DATA

## LEVEL - 100

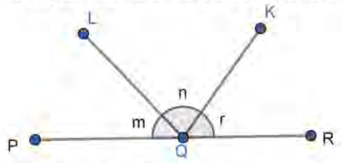
## PRE-TEST QUESTIONS: ANSWER ALL QUESTIONS (1 HOUR)




1. Calculate the value of angle ONU.
A. 35° B 50° C 85° D 95°

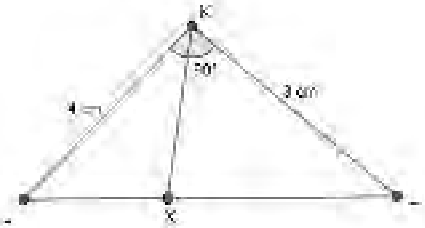
2. Find the size of angle SRT.
A.85 B. 95 C. 130 D. 145

3. In the diagram, PQR is a straight line, m+n=120° and (n+r)=100°. Find the value of m+r




A. 100° B. 120° C. 140° D. 160°

4. JKL is a triangle and KX bisects angle JKL. If |JK| =4 cm. |KL| = 3 cm and angle JKL = 90°, find |JX|




A.5cm B.6cm C.2.807cm D.2.145cm

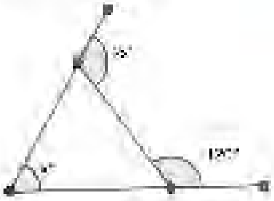
5. Which of the following is true about a rhombus?
I. The diagonals are equal
II. The diagonals bisect at right angles
III. Each diagonal bisect the area of the rhombus
A. I and II only  B. I and III only
C. II and III only  D. I, II and III

6 Two adjacent angles on a straight line are (x+50)° and (y+30)°. Find the value of (x+y)
A. 210  B. 150  C. 90  D. 30

7. Find the value of x in the figure above

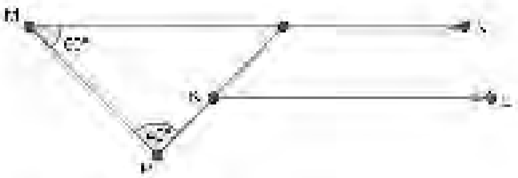

A. 50°  B. 100°  C. 20°  D. 130°

8.



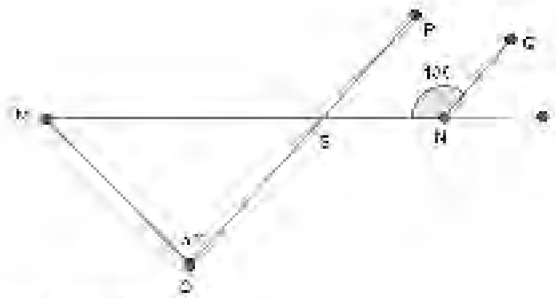

Find the value of the angle marked x in the diagram above
A. 30  B. 40  C. 50  D. 90

9. In the figure MN is parallel to KL. If angle NMP = 60° and MPK = 80°, find the angle PKL



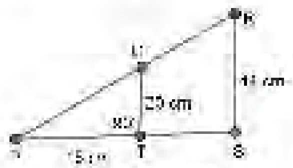

A. 40°  B. 50°  C. 80°  D. 90

10. In the diagram below ... MSN is a straight line. Angle ... = 35° and angle ... = 138. Find angle ...




A. 42° B. 48° C. 80° D. 127°

11. A sector of a circle of radius 14 cm subtends an angle of 54° at the centre. Find the length of the arc. [Take π = 22/7]
A. [illegible] cm B. [illegible] cm C. [illegible] cm D. [illegible] cm




Not drawn to scale

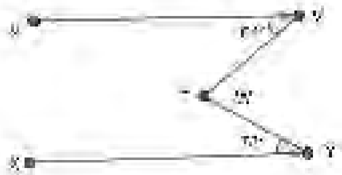
In the diagram above, angle PTU = angle TSR = 80°, |PT| = 15 cm, |UT| = 20 cm and |RS| = 14 cm.

Use the information to answer questions 12 and 13.

12. Calculate |ST|.
A. 15 cm B. [illegible] cm C. 32 cm D. 37 cm

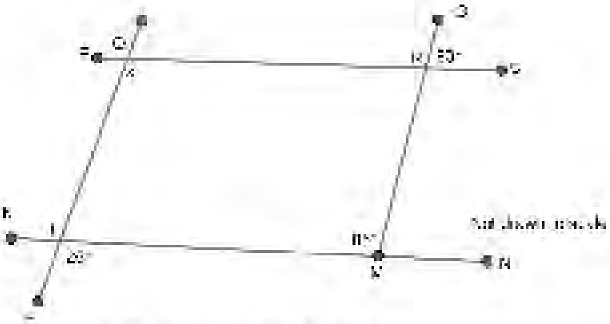
13. Find the angle PRS, correct to the nearest degree.
A. [illegible] B. [illegible] C. [illegible] D. [illegible]

14. In the figure below, line UV is parallel to line XY, angle UVW = 64° and angle XYZ = 78°. Find the value of the angle marked w.




15. The radius of a circle is 4 times that of another. What is the relationship between their areas?
A. The area of one is 64 times that of the other.
B. The area of one is 16 times that of the other.
C. The area of one is 12 times that of the other.
D. The area of one is 8 times that of the other.

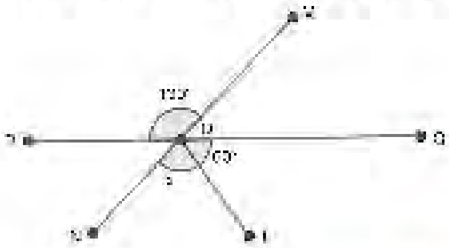
16. In the diagram below, FLS, GRM, FGRS and KLMN are straight lines. ∠FLM = 125°, ∠LMR = 35°, ∠GRS = 80° and ∠LGR = x°. Find the value of x.




A. 35° B. 105° C. 20° D. 25°

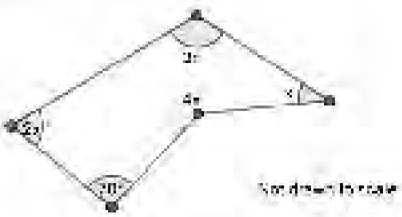
17. Triangle [illegible] is an enlargement of triangle ABC with scale factor 3. If the area of triangle [illegible] is [illegible] cm² find the area of triangle ABC.
A. 8 cm sq. B. 12 cm sq. C. 15 cm sq. D. 24 cm sq.

18. In the diagram below, POQ, MON, and OL are straight lines, ∠MOP = 130° and ∠QOL = 60°. Calculate the value of x.




A. 30° B. 50° C. 70° D. 120°

19. Find the value of x in the diagram below




A. 61° B. 51° C. 47° D. 28°

20. The length of an arc of a circle is 5.5 cm. The radius of the circle is 3.5 cm. Find the angle that the arc subtends at the centre of the circle [Take π = 22/7]
A. 75° B. 72° C. 90° D. 14°

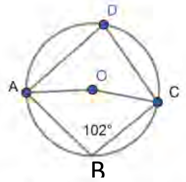
# APPENDIX A3

**PRESBYTERIAN COLLEGE OF EDUCATION- AKROPONG AKUAPEM**

**EBC 122: LEARNING, TEACHING AND APPLYING GEOMETRY AND HANDLING DATA. LEVEL - 100**

POST-TEST QUESTIONS: ANSWER ALL QUESTIONS (1 HOUR 30 MINUTES)

1.

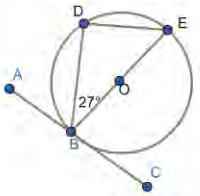



(Diagram not drawn to scale)

The points A, B, C and D lie on the circumference of the circle with centre O as shown in the diagram below. If ∠ABC = 102°, find the value of ∠AOC subtended by the minor arc AC.

A. 78° B. 152° C. 156° D. 204°

In the diagram below, B, D and E are points with centre O. ABC is a tangent to the circle, BE is a diameter and ∠DBE=27°




(Diagram not drawn to scale)

Use the information to answer questions 2 and 3

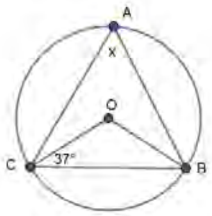
2. What is the value of angle ABD?

A. 36° B. 45° C. 63° D. 90°

3. Find the size of angle DEB

A. 45° B. 53° C. 54° D. 63°

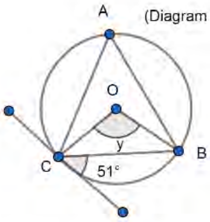
4. In the diagram below, O is the centre of the circle and ∠OCB=37°. Find the value of angle x.




(Diagram not drawn to scale)

A. 43° B. 53° C. 74° D. 106°

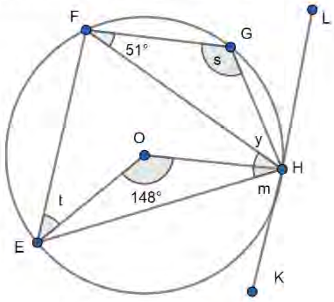
5. O is the centre of the circle and the angle between the tangent and the chord CB is 51°. Calculate the value of ∠COB marked y




(Diagram not drawn to scale)

A. 78°  B. 102°  C. 104°  D. 141°

The diagram below is a circle centre O with points E, F, G, and H on the circumference. KHL is a tangent to the circle at H. ∠EOH = 148°, ∠EHF = 40°, and ∠HFG = 51°.




(Diagram not draawn to scale)

Use the information to answer questions 6 -9

6. What is the value of ∠EHK marked m?

A. 24°  B. 50°  C. 66°  D. 74°

7. Determine the value of ∠ OEF marked t

A. 24°  B. 32°  50°  D. 66°

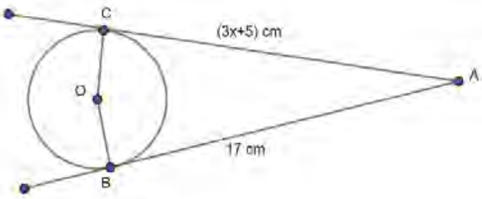
8. Find the value of ∠ FHG marked y.

A. 15°  B. 16°  C. 24°  D. 51°

9. What is the value of the angle marked s?

A. 74°  B. 78°  C. 90°  D. 114°

In the diagram below, AB, AC are tangents to the circle centre O. Tangent AB is (3x+5) cm long and tangent AC is 17 cm long.




Use the information to answer question 10 and 11.

10. Find the value of x

A. 2  B. 3  C. 4  D. 5

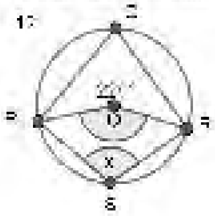
11. What is the measure of ∠OBA?

A. 60° B. 90° C. (3X+3)° D. 180°

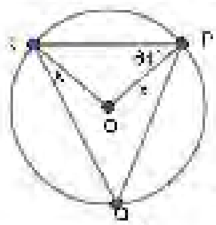
SECTION B

Find the value of the unknown angles where O is the centre of the circle, show clearly with reasons how you obtained your answers.

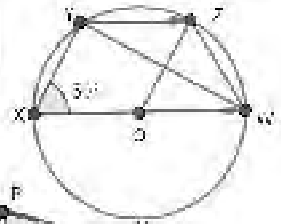
12.



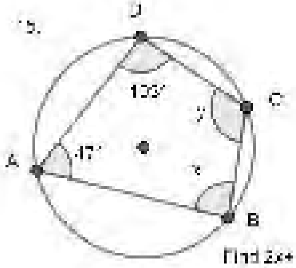
13.



14. Find the value of (i) ∠WXZ (ii) ∠YEZ

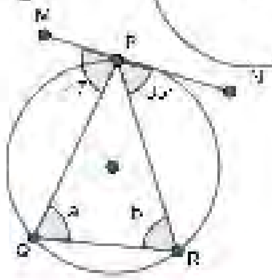


15.

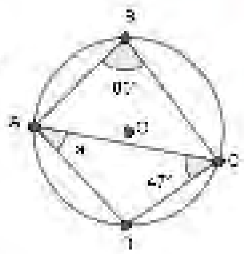


Find 2x+y

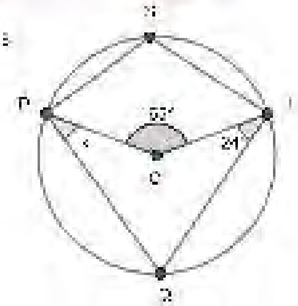
16.



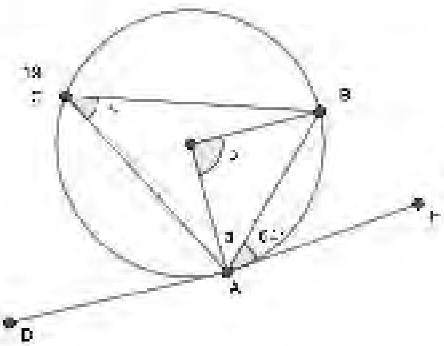
17.



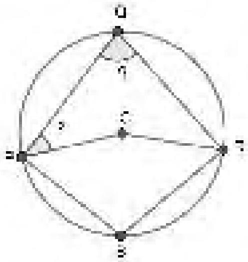
18.



19.



20. In the diagram below, O is the centre of the circle PQRS. [illegible] Find (i) the value of q (ii) the value of p [illegible] ∠ ORS




21. Prove the theorem: A line drawn from the centre of a circle perpendicular to the chord bisects the chord.

MARKING SCHEME FOR PRE-TEST QUESTIONS (See Appendix A 2)

| | | | |
|---|---|---|---|
| 1. | C | 11. | A |
| 2. | B | 12. | A |
| 3. | C | 13. | B |
| 4. | C | 14. | D |
| 5. | A | 15. | B |
| 6. | A | 16. | D |
| 7. | B | 17. | A |
| 8. | D | 18. | C |
| 9. | D | 19. | C |
| 10. | C | 20. | D |

MARKING SCHEME FOR POST TEST (See Appendix A 3)

1 C

2 C

3 D

4 B

5 B

6 D

7 C

8 A

9 D

10 C

11 B

12 $220° + \angle POR = 360°$

$\angle POR = 360° - 220° = 140°$

$140° = 2\angle PQR$ (Twice the angle subtended at the circumference equal that at the centre) $\therefore \angle PQR = 70°$

$x + 70° = 180°$ (Opposite sides of a cyclic quadrilateral sum up to 180°)

$\therefore x = 110°$

13 $\angle RPO = \angle PRO = 64°$(Base angle of an isosceles triangles are equal)

$\angle RPO + \angle PRO + \angle ROP = 180°$

$64° + 64° + \angle ROP = 180°$

$128° + \angle ROP = 180°$ $\Longrightarrow \angle ROP = 52$ $\angle ROP = \frac{1}{2}(52°)$ (angle at the circumference of a circle is half the angle at the centre of the circle

$x + x + 26° + 308° = 360°$ $2x + 334° = 360°$

$\Longrightarrow x = 13°$

14 (i) $\angle XWY = 90° - 50° = 40°$

$\angle WYZ = \angle XWY = 40°$ (Alternate angles)

$(ii) \angle WOZ = (2 \times 40°) = 80°$

($\angle$s at the centre is twice $\angle$ at the circumference)

$\angle OEW = 180° - (\angle WOZ + \angle XWY)$ (Sum of angles in a triangle)

$\angle OEW = 180° - (80° + 40°) = 60°$

$\angle YEZ = OEW = 60°$ (Vertically opposite angles)

15 $47° + y = 180°$ (Opposite angles of a cyclic quadrilaterals add up to 180°)

$y = 180° - 47° = 133°$

$x + 103° = 180° \quad \Longrightarrow x = 73°$

$2x + y = 2(73°) + 133° = 279°$

16 $a = 35°, b = 47°$ (Angle between tangent and chord).

17 $80° + \angle ADC = 180° \Longrightarrow \angle ADC = 100°$( Opposite $\angle$s of a cyclic quadrilaterals add up to 180°)

$\angle ADC + \angle DCA + a = 180°$ (Sum of angles in a triangle)

$100° + 47° + a = 180° \Longrightarrow a = 33°$

18 $165° + \widehat{O} = 360° \quad \therefore \widehat{O} = 195°$ (Angle at a point add up to 360°)

$\angle PQR = \frac{1}{2}(195°)$( Angle at the centre is twice that of the circumference)

$\angle PQR = 97.5°$

$x + \widehat{O} + 24° + 97.5° = 360°$

$x + 195° + 24° + 97.5° = 360° \quad \therefore x = 43.5°$

19 $b = 128°$(Tangent chord theorem)

$c = 64°$ (Angle at the centre of a circle is twice that of the circumference)

$a = 26°$(radius and tangent)

20 (i) $\angle PCR = 2q$ ($Angle\ a\ chord$ subtends at the centre of a circle is twice the angle it subtends at the circumference)

$\angle PSR = 180° - q$ (Opposite angles of a cyclic quadrilaterals add up to 180°)

Opposite angles of a rhombus are equal

$\therefore 2q = 180 - q \quad 3q = 180$

$\therefore q = 60°$

(ii) Triangle PCR is an isosceles triangle with base angles= $\frac{1}{2}(180 - 120) = 30°$

For triangle PQR, $60° + x + 30° + x = 180° \Longrightarrow 2x = 180° - 120°$

$\therefore x = 30°$

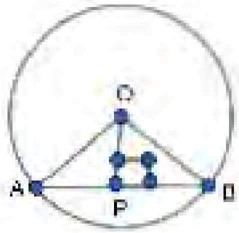
(iii) $\angle QRS = x + \angle CRS = 30° + \angle CRS \ldots \ldots . (1)$

But $\angle CRS + \angle CPS = 360° - (120° + 120°)$

$\Longrightarrow \angle CRS + CPS = 120°$ But $\angle CRS = CPS$. Thus $\angle CRS = 60°$

$\therefore$ From (1) $\angle QRS = 30° + 60° = 90°$

21




Let the chord be AB and O be the centre of the circle.

$OA^2 = OP^2 + AP^2$ (Pythagoras theorem)

$OB^2 = OP^2 + BP^2$ (Pythagoras theorem)

and OA = OB (radii)

$AP^2 = BP^2$

∴ AP = BP

**Alternative Proof**

In △ OPA and in △OPB

∠OPA = ∠OPB (given)

OA = OB (equal radii)

OP = OP (Common side)

∴ △OPA ≅ △OPB

AP = PB

Hence OP bisects AB

# APPENDIX B

## KUDER-RICHARDSON FORMULA 20

### QUESTION NUMBER

| STUDENT NO, | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | Q13 | Q14 | Q15 | Q16 | Q17 | Q18 | Q19 | Q20 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 15 |
| **2** | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 12 |
| **3** | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 7 |
| **4** | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 9 |
| **5** | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 8 |
| **6** | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 9 |
| **7** | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 8 |
| **8** | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 7 |
| **9** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 5 |
| **10** | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 5 |
| **11** | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 |
| **12** | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 |
| **13** | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 10 |
| **14** | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 9 |
| **15** | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 5 |
| **16** | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 13 |
| **17** | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 4 |
| **18** | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 11 |
| **19** | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 6 |

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **20** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 8 |
| **21** | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 9 |
| **22** | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 11 |
| **23** | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 |
| **24** | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 16 |
| **25** | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 7 |
| **26** | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 14 |
| **27** | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 8 |
| **28** | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 3 |
| **29** | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 4 |
| **30** | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 11 |
| **Total** | **14** | **14** | **16** | **12** | **6** | **14** | **12** | **12** | **14** | **14** | **12** | **13** | **13** | **11** | **14** | **14** | **15** | **12** | **8** | **6** | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P | 0.47 | 0.47 | 0.53 | 0.40 | 0.20 | 0.47 | 0.40 | 0.40 | 0.47 | 0.47 |
| Q | 0.53 | 0.53 | 0.47 | 0.60 | 0.80 | 0.53 | 0.60 | 0.60 | 0.53 | 0.53 |
| PQ | 0.25 | 0.25 | 0.25 | 0.24 | 0.16 | 0.25 | 0.24 | 0.24 | 0.25 | 0.25 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P | 0.40 | 0.43 | 0.43 | 0.37 | 0.47 | 0.47 | 0.50 | 0.40 | 0.27 | 0.20 |
| Q | 0.60 | 0.57 | 0.57 | 0.63 | 0.53 | 0.53 | 0.50 | 0.60 | 0.73 | 0.80 |
| PQ | 0.24 | 0.25 | 0.25 | 0.23 | 0.25 | 0.25 | 0.25 | 0.24 | 0.20 | 0.16 |

$$\sigma^2 = 12.51,$$

$$\sum PQ = 4.7 \qquad r_{KR20} = \left(\frac{k}{k-1}\right)\left(1 - \frac{\sum PQ}{\sigma^2}\right)$$

$$r_{KR20} = \left(\frac{20}{20-1}\right)\left(1 - \frac{4.7}{12.51}\right) = 0.66$$

**Split – Half Reliability – Spearman Brown**

**First Half Question Number**

| Student no. | Q1 | Q3 | Q5 | Q7 | Q9 | Q11 | Q13 | Q15 | Q17 | Q19 | Q21 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 2 | 4 | 4 | 3 | 6 | 24 |
| 2 | 1 | 0 | 1 | 0 | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 13 |
| 3 | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 2 | 2 | 3 | 3 | 19 |
| 4 | 1 | 1 | 1 | 1 | 0 | 1 | 2 | 2 | 1 | 3 | 3 | 16 |
| 5 | 1 | 1 | 0 | 1 | 0 | 1 | 2 | 2 | 1 | 2 | 3 | 14 |
| 6 | 1 | 1 | 0 | 1 | 1 | 0 | 2 | 1 | 2 | 2 | 0 | 11 |
| 7 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 1 | 3 | 3 | 16 |
| 8 | 1 | 1 | 1 | 1 | 0 | 1 | 3 | 2 | 4 | 3 | 4 | 21 |
| 9 | 1 | 1 | 0 | 1 | 0 | 1 | 2 | 1 | 0 | 0 | 2 | 9 |
| 10 | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 2 | 4 | 3 | 4 | 22 |
| 11 | 1 | 1 | 1 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 4 | 18 |
| 12 | 1 | 1 | 1 | 0 | 1 | 0 | 2 | 2 | 2 | 0 | 0 | 10 |
| 13 | 1 | 1 | 1 | 0 | 1 | 1 | 3 | 2 | 2 | 3 | 2 | 17 |
| 14 | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 2 | 2 | 3 | 2 | 18 |
| 15 | 1 | 1 | 1 | 0 | 1 | 1 | 2 | 4 | 4 | 3 | 3 | 21 |
| 16 | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 2 | 2 | 3 | 2 | 18 |
| 17 | 1 | 1 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 16 |
| 18 | 1 | 0 | 0 | 1 | 1 | 0 | 3 | 2 | 1 | 2 | 2 | 13 |
| 19 | 1 | 1 | 1 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 2 | 16 |
| 20 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 4 | 19 |
| 21 | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 4 | 2 | 3 | 4 | 22 |
| 22 | 1 | 1 | 1 | 0 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 11 |
| 23 | 1 | 1 | 0 | 1 | 1 | 1 | 3 | 2 | 0 | 2 | 2 | 14 |
| 24 | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 4 | 2 | 2 | 4 | 21 |
| 25 | 1 | 0 | 0 | 1 | 1 | 0 | 2 | 2 | 1 | 2 | 0 | 10 |
| 26 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 4 | 19 |
| 27 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 2 | 1 | 2 | 0 | 9 |
| 28 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 2 | 1 | 2 | 11 |
| 29 | 1 | 1 | 0 | 1 | 1 | 1 | 3 | 2 | 2 | 3 | 2 | 17 |
| 30 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 4 | 3 | 4 | 21 |

**Spearman – Brown Second Half**
**Question Number**

| Student no. | Q2 | Q4 | Q6 | Q8 | Q10 | Q12 | Q14 | Q16 | Q18 | Q20 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 4 | 4 | 2 | 3 | 6 | 24 |
| 2 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 4 |
| 3 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 0 | 15 |
| 4 | 1 | 1 | 1 | 1 | 0 | 2 | 4 | 2 | 2 | 4 | 18 |
| 5 | 1 | 1 | 0 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 14 |
| 6 | 1 | 1 | 1 | 0 | 1 | 4 | 2 | 3 | 3 | 6 | 22 |
| 7 | 1 | 1 | 1 | 1 | 0 | 2 | 2 | 3 | 3 | 0 | 14 |
| 8 | 1 | 1 | 1 | 1 | 0 | 2 | 2 | 2 | 2 | 4 | 16 |
| 9 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 0 | 0 | 9 |
| 10 | 1 | 1 | 1 | 1 | 1 | 4 | 2 | 3 | 3 | 4 | 21 |
| 11 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 3 | 2 | 4 | 18 |
| 12 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 3 | 2 | 2 | 12 |
| 13 | 1 | 1 | 1 | 1 | 1 | 4 | 2 | 3 | 2 | 4 | 20 |
| 14 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 3 | 2 | 14 |
| 15 | 1 | 1 | 1 | 0 | 1 | 4 | 4 | 3 | 3 | 6 | 24 |
| 16 | 1 | 1 | 0 | 1 | 1 | 2 | 2 | 3 | 1 | 2 | 14 |
| 17 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 3 | 2 | 4 | 18 |
| 18 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 5 |
| 19 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 3 | 1 | 2 | 14 |
| 20 | 1 | 1 | 1 | 0 | 1 | 2 | 2 | 2 | 2 | 4 | 16 |
| 21 | 1 | 1 | 1 | 1 | 1 | 4 | 4 | 2 | 3 | 3 | 21 |
| 22 | 1 | 1 | 1 | 1 | 1 | 4 | 4 | 3 | 3 | 2 | 21 |
| 23 | 1 | 1 | 1 | 1 | 1 | 0 | 2 | 3 | 2 | 2 | 14 |
| 24 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 3 | 2 | 3 | 17 |
| 25 | 1 | 0 | 1 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 12 |
| 26 | 1 | 1 | 0 | 1 | 1 | 4 | 2 | 2 | 2 | 2 | 16 |
| 27 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 2 | 0 | 0 | 8 |
| 28 | 1 | 1 | 1 | 1 | 1 | 4 | 2 | 2 | 3 | 6 | 22 |
| 29 | 1 | 1 | 1 | 1 | 1 | 4 | 2 | 3 | 3 | 3 | 20 |
| 30 | 1 | 1 | 1 | 1 | 1 | 4 | 2 | 1 | 2 | 2 | 16 |

Reliability Statistics for post-test

| Cronbach's Alpha | N of items |
|---|---|
| .653 | 2 |

Hence the reliability r of the post-test is 0.65