UNIVERSITY OF EDUCATION, WINNEBA

IMPACT OF CLASSWIDE PEER TUTORING METHOD ON SENIOR HIGH SCHOOL STUDENTS' ACHIEVEMENT AND RETENTION IN QUADRATIC FUNCTIONS

ENOCH MAWUNYO KWASI AWUDZA

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A THESIS IN THE DEPARTMENT OF MATHEMATICS EDUCATION, FACULTY OF SCIENCE EDUCATION, SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES, UNIVERSITY OF EDUCATION, WINNEBA, IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF MASTER OF PHILOSOPHY IN MATHEMATICS EDUCATION.

DECEMBER, 2017

DECLARATION

Student's Declaration

I, **Enoch Mawunyo Kwasi Awudza**, declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

Signature……………………………………………………

Date…………………………………………………………

Supervisor's Declaration

I hereby declare that the preparation and presentation of this thesis was supervised in accordance with the guidelines on the supervision of thesis laid down by the University of Education, Winneba.

Supervisor's Name: Professor Damian Kofi Mereku

Signature……………………………………………

Date…………………………………………………

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DEDICATION

To my father Mr. Seth Awudza, Mother, Mrs. Mawuena Awudza and my brothers and sisters.

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ABSTRACT

This research work investigated the impact of Class Wide Peer Tutoring (CWPT) teaching strategy on Senior High School students' academic achievement and retention. It also looked at students' perception of using CWPT method in teaching Quadratic Functions. The design adopted was quasi-experimental design, involving pre-tests and post-tests. One Hundred and sixty-six (166) students of 2016/2017 academic year in the two public second cycle schools in the Central Tongu District of the Volta Region of Ghana, offering elective mathematics, were used as the research sample. The simple random sampling technique was used to select the experimental and control groups in each sampled school. The experimental group was exposed to Class Wide Peer Tutoring (CWPT), whilst the control group was taught with conventional method. Pre-test and post-test were used in collecting data on students' achievement. The first retention test and second retention test helped in obtaining data on students' retention of learned mathematical concept. A reliability coefficient of 0.80 for the tests was obtained. An Interview guide and observation checklist helped the researcher to obtain data on students' perception towards the CWPT procedure implemented. The data was analysed using t-tests. The hypotheses were tested at 0.05 levels of significance. The findings of the study showed that CWPT instructional strategies have significant effects on students' achievement, retention and perception in quadratic functions more than the conventional method of teaching. It is recommended that government agencies and other stake holders tasked with the responsibility of designing and revising the curriculum for Senior High Schools, incorporate the use of Class Wide Peer Tutoring technique in teaching mathematics.

CHAPTER ONE

INTRODUCTION

1.1 Overview

This opening chapter sets the study in context. It presents background of the study, statement of the problem, purpose of the study, objectives of the study, research questions and significance of the study, limitations and delimitations as well as the organization of the study.

1.2 Background of the Study

The field of education has undergone a significant shift in thinking about the nature of human learning and the conditions that best promote varied dimensions of human learning (Applefield, Huber & Moallem, 2001). A number of studies have shown that humans learn best if they repeat same behaviour several times (Kang, 2016; Horst, Parsons & Bryan, 2011; Jennifer, 2015; Horst, 2013). It means that students learn best and retain information longer when teachers create enabling learning environment for them to have meaningful practice and repetition coupled with positive feedback. It is believed that every time practice occurs, learning continues. It is clear that practice leads to improvement only when it is followed by positive feedback.

The new educational reform has undergone a paradigm shift in the mode of teaching from teacher-centred method of teaching to the learner-centred method of teaching. With the shift in the teaching and learning paradigm, maximum attention is placed on the teachers ability to take care of his/her own professional growth, create and apply innovative teaching methods needed for the development of the students competences (Marinko, et al., 2015). The student-centered instruction focuses on skills and practices that enable lifelong learning and independent problem solving (Young & Paterson, 2007 in Marinko, et al., 2015)

Several studies have shown that student-centred learning encourages deep learning that is associated with searching for meaning in task and the integration of task aspects into a whole (Beausaert & Wiltink, 2013; Hall & Sanders, 1997; Cannon & Newble, 2000; Honkimaki, Tynjala, & Valkonen, 2004).The theory and practice behind this type of learning is “The Constructivists learning theory”, which emphasizes on the learners’ critical role in constructing meaning from new information and prior experience (Marinko, et al., 2015).

In Ghana, the education fraternity advocates a move from the traditional (conventional) methods of teaching to learner- centred teaching and learning methods which focus on the students’ achievement, active knowledge construction, retention, transfer of knowledge and problem solving. The traditional method of teaching does not promote critical thinking, active learner participation and knowledge construction. In the traditional (conventional) instructional lessons/process, teachers usually explain, illustrate, demonstrate and in some cases give notes on procedures and examples. The teachers give definitions; make no use of concrete materials and practical ways to explain mathematical concepts. The learner is led deductively through small steps and closed questions to the theory being considered. The learners obtain information from the teacher without building their level of engagement in the lesson or the subject being taught (Boud & Feletti, 1999). The approach to teaching is least practical, more theoretical and memorizing (Teo & Wong, 2002). It does not promote activity based learning to arouse and sustain learners interest in the lesson and also develop the skill of handling real life problems which are based on applied knowledge. In the instructional process, the teacher focuses on transmission and sharing of knowledge when using this teaching method and much effort is made on how to maximize the delivery of information while minimizing time and effort. As a result, both the interest and the understanding of students usually get lost.

In Ghanaian Mathematics classrooms, it is an agreeable fact that most teachers embrace the traditional method of teaching in their Mathematics lessons. Teachers use traditional methods of instruction that rely solely on rote memorization as a major conduit to transfer knowledge to children and this is understood by few children (Assuah, 2013). Though the tenets of learner-centred method of teaching are known and highly acceptable to teachers, it is ironic in practice for most of these teachers. The outmost effect of these (traditional) methods of teaching that large number of our mathematics teachers practice in this country is the frequent poor performance of most students nowadays at the Senior High Schools in both internal and external exams. Explicitly, the practice has resulted into general detestation of mathematics by most students leading to their poor performance, both in the national (e.g. West African Senior Secondary Examination, WASSCE) and international (Trends in International Mathematics and Science Study, TIMSS) examinations (Anamuah-Mensah, Mereku, & Asabere-Ameyaw, 2006; Anku, 2005-2006; Djangmah & Addae-Mensah, 2012). Osafo-Affum (2001) describes the instructional strategy practiced by most Mathematics teachers (educators) in Ghanaian mathematics classrooms as ‘lecture’ instead of ‘teaching’. Anku (2014) attributes these difficulties of students to comprehend the subject to improper way of teaching the subject. He indicates that Mathematics has always been the bane of many a student at all levels of the educational ladder because there has been the perception that it is very difficult (Anku, 2014). In order to address this menace, there is the need to adopt effective teaching and learning methods which place the learners’ success at the centre of the teaching and learning process, focus on learners' achievement and promote critical thinking and active knowledge construction. Again, to ensure effective teaching to take place, the skilful mathematics teacher needs to use learner centred methods and techniques at his/her disposal. One of the many teaching methods that have numerous research support and evidence to curb this problem, yields positive effect on students' achievement and retention of information regardless of class size and diverse learners in a class is Class wide peer Tutoring (CWPT).

The CWPT is an instructional strategy which encourages students to teach and learn from one another in pairs or in small groups. It is a comprehensive instructional strategy that is developed by teachers to help them individualize instruction, actively engage an entire classroom of students in the academic tasks of learning and practicing critical developmental learning skills. The professional teacher acts as a facilitator during the lesson.

CWPT provides the opportunity for students to practice and master what they are learning and encourages positive social interaction among the students.

Retention is defined as a preservative factor of the mind (Kundu & Tutoo, 2002). The mind acquires the materials of knowledge through sensation and perception. These acquired materials in the mind need to be preserved in the form of images for knowledge to develop. Whenever a stimulating situation occurs, retained images are revived or reproduced to make memorization possible. Hence mathematics concepts need to be presented to learners in a way or method that touches their sub-consciousness which in turn can trigger a quick recall of the concept taught or learnt. Using such a teaching method as CWPT, both high ability and low ability learners would be able to collaborate in terms of understanding, explaining and retaining the concept they have learnt in a mathematics class.

The choice to focus on Quadratic Functions among other topics in mathematics was informed by students performance in both internal and external assessment of students in school setting, remedial classes and literature; several studies have shown that students have learning difficulties when it comes to teaching and learning of Functions particularly Quadratic Functions (Clement, 2001; Dubinsky & Harel, 1992; Eraslam, 2005; Looney, 2004; Maharaj, 2008; Zazkis, Liljedahl & Gadowsky, 2003). Quadratic Functions has been identified as one of the topics in which students have learning difficulty which results from poor method of teaching (Dubinsky & Harel, 1992; Eraslam, 2005). Vaiyavutjamai & Clements (2006) indicate that solving Quadratic Equations is one of the most conceptually challenging subjects in the curriculum for many secondary school students. In general, for most students, quadratic equations create challenges in various ways such as difficulties in algebraic procedures and an inability to apply meaning to the quadratics.

Various researchers (e.g., Vaiyavutjamai & Clements, 2006; Kotsopoulos, 2007; Lima, 2008;

Tall, Lima, & Healy, 2014) have illustrated that very little attention has been paid to quadratic equations in Mathematics, education literature and there is scarce research regarding the teaching and learning of Quadratic Equations. A limited number of research studies, focusing on quadratic equations, have documented the techniques students engage in when solving quadratic equations (Bosse & Nandakumar, 2005).These studies have also talked about students' understanding of and difficulties in solving quadratic equations (Kotsopoulos, 2007; Lima, 2008; Tall, Lima, & Healy, 2014; Vaiyavutjamai & Clements, 2006; Zakaria & Maat, 2010), the teaching and learning of quadratic equations in classrooms (Olteanu & Holmqvist, 2012; Vaiyavutjamai & Clements, 2006) and the application of the history of quadratic equations in teacher preparation programmes to highlight prospective teachers' knowledge (Clark, 2012).

Moreover, Quadratic Function serves as one of the fundamental topics for both Secondary Mathematics, development of Algebra and application of mathematics knowledge in real life. Movshovitzs-Hadar, Schoenfeld, & Arcavi, (1993) indicates that quadratic functions play critical role in transition from linear functions to higher-degree functions. Ellis & Grinstead, (2008) reiterates that Quadratic functions are one of the most important concepts going beyond linear functions yet students have difficulties in understanding the quadratic functions (Ellis & Grinstead, 2008). Various approaches to solving quadratic equations were used at different stages in this historical development, through representations, including arithmetic or numeracy, algebra or symbols and visuals or geometry (Katz & Barton, 2007).

In light of the above, the researcher deemed it fit to examine the impact of Class-Wide Peer Tutoring method as an instructional strategy on Senior High School Students' Achievement in Quadratic Functions. Research has supported Class Wide Peer Tutoring (CWPT) as one instructional strategy that successfully addressed this problem.

1.3 Statement of the Problem

Students express difficulty in understanding and retaining mathematical concepts being taught to them by their teachers. The utmost effects of these problems usually result to low achievement in both internal and external examinations. Meanwhile, researchers have identified that how well students retain taught mathematics concept can be traced back to the teaching approach used. The question that remains unanswered was; what teaching and learning method(s) should teachers employ in the Mathematics classroom to enhance students retention, achievement and perception in learning mathematical concept?

A critical study of the existing mathematics literatures on the impact of teaching methods on retention of learned mathematical concepts, achievement in mathematics and challenging topics to students; revealed that there were relatively few works on: " the impact of effective teaching method like CWPT method on SHS students achievement and retention of learned mathematical concept. Further study of the existing mathematics literature showed that studies or researches conducted on the impact of effective teaching method, especially the use of Class-wide Peer Tutoring method on SHS students' achievement and retention in Quadratic Functions, precisely on the students reading elective mathematics in the Central Tongu District of the Volta Region are scarce.

Again, most studies focused on the impact of teaching method(s) on students attitudes, achievements, assessment on mathematics but not much have been done on the individual topics to create room for the researchers to identify the conceptual and procedural challenges of students on the various topics or the units, their achievement and ability to retain and apply those learned concepts. Moreover a limited number of research (Bosse & Nandakumar, 2005; Allaire & Bradley, 2001; Olteanu & Holmqvist, 2012; Tall, Lima, & Healy, 2014; Vaiyavutjamai, Ellerton, & Clements, 2005; Zakaria & Maat, 2010; Vaiyavutjamai & Clements, 2006) carried out on quadratic functions, focused on the techniques of solving quadratic equations, geometric approaches used by students for solving quadratic equation, students' understanding of and difficulties with solving quadratic equations and the teaching and learning of quadratic equations in classrooms. This called for the present study on the impact of class-wide peer-tutoring strategy on senior high school students' low achievement in quadratic functions in the Central Tongu District of the Volta Region.

Meanwhile, the concept of Quadratic Functions in the SHS mathematics curriculum is the first non- linear function in which learners are introduced to in the study of mathematics. Adequate knowledge of students on Quadratic functions is a pre-requisite and very critical to students' progress, success and achievement in learning mathematics at the SHS and beyond. For example Algebra is required in studying higher degree polynomials, rational functions, binomials, trigonometry, calculus and vectors which are all very necessary at the advanced stage in the study of mathematics and its related courses. In geometry, it is required in proving and verifying theorems and solving problems. Undoubtedly, a strong understanding of quadratic functions is very essential to students' achievement in the Mathematics they study at SHS level and beyond. Quadratic Function serves as one of the fundamental topics for both Secondary Mathematics, development of Algebra and application of mathematics knowledge in real life. Movshovitzs-Hadar, Schoenfeld, & Arcavi, (1993) indicate that quadratic functions play a critical role in transition from linear functions to higher-degree functions. Ellis and Grinstead (2008) reiterate that quadratic function is one of the most important concepts going beyond linear function, yet, students have difficulties understanding it (Ellis & Grinstead, 2008). Various approaches to solving quadratic equations were used at different stages in this historical development, through representations, including arithmetic or numeracy, algebra or symbols and visuals or geometry (Katz & Barton, 2007).

In an attempt to bridge the existing gap identified from the mathematics literature, the current study examine the impact of Class-Wide Peer Tutoring method**,** as an instructional strategy, on Senior High School Students' achievement, retention and perception in learning quadratic functions.

To verify, test and confirm the need for the students in the Central Tongu District to study the concept of quadratic functions, the researcher used achievement tests prepared by him as a pre-test. This was administered before the treatment. The students' solution to, and the results of the pre-test also exposed a number of conceptual and procedural difficulties demonstrated by students. The analysis of this result and its interpretation is presented in chapter 4. The results actually confirmed the need for the treatment for the students in the district; which was just given to them by the researcher during the treatment.

1.4 Purpose of the Study

The present study explores the effects of Class-wide Peer Tutoring on Senior High School (SHS) Students' performance in Mathematics. Thus, to use the intervention (Class-wide Peer Tutoring instructional strategy) to promote academic gains, increase classroom participation and develop students' interest in the study of Mathematics at the SHS level.

Enhancing the study of Mathematics at the Senior High school has become necessary because it provides enough foundation for those who wish to continue the study of Mathematics, Sciences and other related disciplines at the university and other institutions of higher learning. It also serves as a terminal point for those who would leave school for the world of work after SHS. This will go a long way to determine the kind of work these graduates shall be doing in the future.

1.5 Objectives

The objectives of the study are;

- To explore the effects of the use of CWPT on students conceptual understanding and achievement in Quadratic Functions;
- To examine the extent to which students can keep and apply mathematical knowledge gained from lessons taught through CWPT;
- To examine students' perception on CWPT method.

1.6 Research Questions

The following research questions guided the study:

1. What is the impact of using CWPT method on SHS students' achievement in quadratic functions?
2. To what extent can students keep and apply mathematical knowledge gained from lessons taught through CWPT method?
3. How do students perceive the use of Class Wide Peer Tutoring method in teaching quadratic functions?

1.7 Hypothesis

The following null hypotheses were formulated to guide this study. They were tested at 0.05 alpha levels of significance:

H_0: "there is no significant difference in the mean scores between the Control and Experimental groups in the post test"

H_0: "there is no significant difference in the mean scores between the Control and Experimental groups in the retention test"

1.8 Significance of the Study

The study provides information regarding improving classroom instruction and student achievement (academic outcomes) for Mathematics educators. First, it demonstrates how research on classroom practice can be used to improve academic outcome (achievement) of students in Senior High Schools in Mathematics, which is important to parents, schools, and policymakers. Secondly, it helps students to develop positive attitude and interest towards the study of Mathematics. Thirdly, it offers a research-based solution to persistent concerns raised by the present Mathematics educators on contemporary classroom issues (large class size and heterogeneity or differentiation among learners) affecting learners' academic outcome. Finally, the study adds new knowledge to Mathematics education literature and also serves as a reference material at accessible areas for Mathematics educators and the general public.

1.9 Delimitations of the Study

The study was delimited to the instructional strategies (CWPT) employed to improve SHS students' Mathematical achievements and retention. It focused on student's conceptual understanding in quadratic function as enshrined in the Ghana Education Service's syllabus for Elective/Further Mathematics and other recommended curriculum materials. Explicitly, it did not consider any concept beyond the boundary of sum and product of Quadratic Equations.

It involved the two SHS in the Central Tongu District of the Volta Region. Two classes in the second year of each school were used for the study. This is due to effective management of time to complete the study within the time limit and the researcher's intension of having direct and frequent contacts with the students.

1.10 Limitations

The short duration of the study was a limitation and a longitudinal study would have been appropriate for producing a more accurate result. Interference of unplanned school activities was a drawback.

1.11 Organization of the Study

The report of the study has been discussed under five main chapters. Chapter One captures the background of the study, statement of the problem, purpose of the study, significance of the study, research questions, delimitation, limitations and organization of the study. Chapter Two covers review of related literature. It captured the theoretical framework of the study: the Constructive Theory and the concept of CWPT method, empirical evidence on CWPT method and conventional teaching method. It also discusses the concept of quadratic functions and the summary of the findings from the literature review. Chapter Three which is the methodology of the study discusses the research design, population, sample and sampling procedures, instruments and data collection procedures and data analyses.

Chapter Four presents the results and discussion of the findings of the study and Chapter Five captures the summary of the study, the key findings of the study, recommendations from the study and suggested areas for further research.

1.12 Definition of Terms

- **Class-wide Peer Tutoring(CWPT)**

  Class-wide Peer Tutoring (CWPT) method is any peer mediated teaching strategy in which the entire class is divided into smaller groups for structured learning or to execute tasks under the supervision of the teacher within or after the instructional period. Basically, CWPT method is a strategy which encourages students to teach and learn from one another

- **Achievement**

  Achievement is the learner's ability to meet the criteria or the standards to a level which demonstrates adequate understanding of the mathematical concepts tested.

- **Retention**

  Retention is the learner's ability or power to keep, recall and apply the concepts that was taught.

- **Quadratic function**

  Quadratic Function is any function having two as the highest exponent of the variable and the coefficient of the highest exponent is not zero.

- **Conventional Method**

  Conventional Method is any teacher-centred instruction where much attention is placed on the information to be delivered, and every student has to work alone during the explanation, discussion and applications of the concepts during the classroom instructions.

- **Peer:** Students of the same age/ability.
- **Tutor:** A student assigned to offer tutoring to his/her peers
- **Tutee**: A student who receives tutoring from the tutor

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

A paradigm shift in the mode of teaching from teacher centred method of teaching to learner centred method of teaching is the topmost priority for most mathematics educators at all levels of education in this country. Most educators shift their practice and principles of teaching from teaching paradigm to a learning paradigm. It is believed that a carefully-designed teaching method usually makes teaching and learning effective (Chianson, Kurumeh & Obida, 2011). The teaching paradigm helps teachers to make a means to an end.

In the learning paradigm on the other hand, the teachers create conducive teaching and learning environment and the necessary experiences that create room for students to explore new concepts. The learning paradigm exposes learners to rich and a variety of experiences which enable them learn measurably more than earlier students. For a complete change to take place in our (Ghanaian) mathematics instruction, the need to adopt and implement learner centred instructional strategies that operate on the principles of Constructivist, and that, which is well-structured, to create such conducive learning environment for learners to explore new concepts, at all costs, is paramount. One of such teaching and learning strategies is Peer Tutoring. Gaustad (1993) reiterates that "decades of research have established that well-planned peer tutoring programs can improve student achievement and self-esteem as well as overall school climate" (p. 4). Explicitly, one of the peer tutoring models that could bring such a desired end of learning is Class-Wide Peer Tutoring method. Several studies conducted on the impact of CWPT method yielded positive and promising results (Hughes & Fredrick, 2006, Greenwood, Delquadri & Carta,1997).

2.1 Theoretical Framework

The theoretical framework of a study is the structure that holds or supports the theory of the research. It sets up the philosophical basis on which a research takes place and links between the theoretical and practical components of the research under study. Obviously, it describes the theory that explains how and why that research; and connects the research to the existing knowledge. Obviously, theoretical frameworks of a research "have implications for every decision made in the research process" (Mertens,1998).The present study was based on the "Constructivist" theory of teaching. The "Constructivist Theory of teaching" is centred on the "Constructivist learning theory". The researcher deemed it fit to base the study on this principle because it is a theory on which many research works have been based, a theory recognized to be effective in helping learners and is embraced by most educators. According to Duffy and Cunningham (1996), the theories of learning that hold the greatest influence today are those based on Constructivist principles (Duffy & Cunningham, 1996). Most educational research works in mathematics literature have revealed that students learn mathematics well only when they construct their own mathematical understanding (Clements & Battista, 1990).

A call for fundamental changes in instructional practices is crucial. These instructional changes that most mathematics educators yearn for can best be understood from the *Constructivist's* perspective. In this theoretical framework, learning is viewed as building upon the existing schema or knowledge that a learner already had.

2.2 The Constructivist Theory

The Constructivist learning theory says that learners learn best when they are allowed to construct their own understanding and knowledge of the world through what they experience and how they reflect on those experiences.

Clements and Battista (1990) define constructivism as an epistemology which follows the tenets below:

- Knowledge is actively created or invented by the child, not passively received from the environment. It means that the mathematical ideas are constructed or made meaningful when learners integrate them into their existing structures of knowledge.
- Learners create new mathematical knowledge by reflecting on their physical and mental actions. It means that ideas are constructed when learners integrate them into the existing structures of knowledge. In Mathematics, the new mathematical knowledge is created through reflection on physical and mental actions.
- There is no one true reality that exists; but only individual interpretations of the world. It means that each person has his or her own reality based upon interpretations.These interpretations are shaped by experience and social interaction. Thus, learning Mathematics should be thought of as a process of adapting to and organizing one's quantitative world, not discovering pre-existing ideas imposed by others.
- Learning is a social process in which children grow into the intellectual life of those around them. It means that mathematical ideas and truths, both in meaning and in practice, are cooperatively established by the learners. Learning is a social process and is achieved by means of negotiated interaction. The constructivist mathematics learning environment is seen as a culture in which students are engaged in activities or tasks of discovery and invention in a social discourse involving explanations, negotiation, sharing, and evaluation. Learning is a social process meaning it is negotiated.

- Students learn when they are allowed to explore or encouraged to use their own methods in solving problems. The constructivist teachers view the study and nature of mathematics as "sense making", instead of helping learners learn a set of beliefs about the study and nature of mathematics. They believe that the sense-making activity of students, which is their priority, is curtailed whenever the teacher engages them in the use of a set of mathematical methods.

2.3 The Constructivist Theory and CWPT in the Present Research

The researcher used the CWPT instructional procedure to create a conducive learning environment whereby the learner was placed at the centre of the teaching learning process to achieve success in this study. Prior knowledge of students, learner-centred learning, environment and interaction, as from constructivist perspective, were achieved through the CWPT procedures employed. In each case, the teaching learning process focused on the learners' achievement and lessons were organised in such a way that the students developed their own concepts through the peer tutoring with less interference from the teacher. Afamasaga-Fuata'i (1992) indicates how the constructivism paradigm is present when students take an active and participatory role in their own learning through meaning-making activities. In order to assist students in making meaning during a learning activity, it is necessary to be deliberate in helping them connect what they are learning to their prior knowledge of the educational content.

The intention of the researcher in this regard was guided by the Constructivist Theory of learning as espoused by (Clements & Battista, 1990).

2.4 Class-Wide Peer Tutoring (CWPT) method

Class-Wide Peer Tutoring (CWPT) is a peer-mediated instructional or teaching strategy in which the entire class is put into pairs or small groups for structured learning or to execute a task. The groups include students with different or relative ability levels and mostly consist of, at most, five peers. Each group consists of, at least, a tutor and a tutee. The tutoring takes place under the supervision of the class/form teacher within the instructional time and it is led by the students offering the tutoring. Each student has the opportunity to be both the tutor and the tutee. The tutors take charge of whatever information is being tutored or reviewed in the groups. In the tutoring process, the tutors explain the work, ask questions, and provide feedback to the peer(s). The classroom or subject teacher acts as facilitator and monitors the class.

The procedures of CWPT instructional strategy usually help students grasp concepts taught, enhance understanding, retention and achieve higher. The CWPT strategy helps students' increase active learning by teaching their peers. They also learn or practise social interaction techniques. Students build confidence through increased academic and social accomplishments. It increases academic rates of response for all students, spirit of teamwork and it also helps develop empathy for peers through an increased range of interactions. The tutors gain time to focus on students with special needs during the lesson. It is mostly effective in reviewing materials or practising skills but not suitable for introducing a new content.

The teaching strategy is based on reciprocal peer tutoring and small group reinforcement. The tutoring process consists of the seven main components that make CWPT strategy successful in the classroom. These includes multi-modality format, reciprocal and distributed practice, immediate error correction and feedback, built-in reinforcement, high mastery levels, measured outcomes, game format with partner pairing and competing teams.

2.4.1 Procedure for the Peer Tutoring Stage

1. Put students in the entire class into peer dyads or small groups.
2. Let one person be the tutor and the other(s) to be the tutee(s) in each pair.
3. Provide the tutors with a list of questions and answers. Provide the tutees in each pair or group with a piece of paper and a pencil.
4. When the tutoring begins, the tutor asks the questions and the tutee provides the answers. This could take a written or oral form. In case of calculations, the tutor asks the tutee to solve the question(s) on their pieces of paper or small erasable board.
5. If the tutee gets the question correct, the tutor awards 2 points and then moves on to the next question.
6. If the tutee provides an incorrect answer, the tutor provides the correct answer and the tutee must say and write the correct answer three times before moving to the next question.
7. The tutor continues to provide questions giving 2 points to every correct answer and 1

   point for every incorrect answer that is corrected by the above technique.
8. When time is up, the roles change. The teacher resets the clock and another round begins.
9. At the end of the two rounds, the teacher confirms the points earned and updates the leader board.
10. At the end of the week, the team with the most points wins.

Research on implementing CWPT teaching and learning strategy in the classroom began around 1980. It was initially developed at Juniper Gardens Children's Project (JGCP) in Kansas City, by a group of researchers and educators (teachers) who were in search of an effective and successful teaching and learning (instructional) method for integrating children with special needs into the general educational setting.

Greenwood, Delquadri & Hall (1989) developed Class-Wide Peer Tutoring (CWPT) method, which incorporates similar principles to Reciprocal Peer Tutoring (RPT); grouping of students to prompt, monitor and evaluate each other. It was initially designed to prevent future academic failure in poor and culturally diverse schools.

Greenwood, Delquadri and Carta (1997) indicates that CWPT method uses a combination of instructional components that include partner-pairing, systematic content coverage, immediate error correction, frequent testing, team competition and point earning. Terry (2008) indicates that, with CWPT, every student in the classroom takes keen interest in the teaching and learning process. This helps them to practise basic skills in a systematic and funny way.

According to Harper and Matheady,2007), Class-Wide Peer Tutoring involves dividing the entire class into groups of two to five students with differing ability levels. The entire class participates in structured peer tutoring activities two or more times per week for approximately 30 minutes. Students then act as tutors, tutees, or both tutors and tutees during the tutoring stage of the lesson.

Maheady, Harper and Mallette (2001) indicate that CWPT instruction involves highly structured procedures, direct rehearsals, competitive teams, and posting of scores. The students pairing in CWPT instructions mostly focus on the achievement levels or student compatibility. The pairing or groups may change weekly or biweekly. It involves active engagement and repeated practice for all the students. It includes reciprocal practice in which every student has the opportunity to be a tutor and a tutee. Immediate feedback and error correction is one of the vital aspects of the tutoring stage.

According to Bowman-Perrott (2009), Class-Wide Peer Tutoring (CWPT) is a form of bi-directional peer tutoring which has several benefits for classroom instructions, the teacher, and the students. Instructions for students are individualized (one-on-one) and students learn by teaching. In the tutoring process, each student is taught and he/she also gets the opportunity to teach. There is a designed error correction system and students are engaged in social interaction.

CWPT benefits teachers by reducing teaching workload, using current curriculum, fitting lessons into current class time periods and accommodating the sharing of results with parents and administrators. The students benefit from frequent practice or repetitions of academic tasks. Again, the students experience success, improve confidence, problem solving skills, and master the materials through team work, error correction and immediate feedback from both peers and teachers.

2.4.2 CWPT and the Present Study

In this study, the researcher developed a conceptual framework that uses Class-wide Peer Tutoring (CWPT) procedure, based on the Constructivist Principle, to enhance learners' interest, conceptual understanding, retention and achievement in Mathematics (Quadratic functions).

The framework took the tutoring techniques format that aims at promoting classroom discourse, ensuring equity, and developing interest in studying the students' retention of learned concept and achievement.

In this instructional strategy, students of varying ability levels in a class are heterogeneously paired and they participate in a reciprocal tutoring format. Thus students take turns tutoring each other to ensure that both students serve the roles of a tutor and a tutee. Students learn additional skills after mastering the previous steps and procedures introduced to. The teacher monitors students' progress to ensure that established procedures, accurate review and error correction are followed; students utilize interpersonal skills and content covered.

CWPT helps students strengthen their own understanding of the subject matter. It helps develop generic skills such as communication and leadership skills among learners. Peer tutoring also develops character virtues and personal attitudes such as respect, responsibility, empathy, cooperation and persistence; and that is important in affective development.

2.4.3 The Ideal Schedule and Quantum of Instruction for CWPT method

The ideal schedule and quantum of instruction needed to use peer tutoring in the classroom depends on the subject area and model selected; one to four times in a week, which takes 30 to 45 minutes per session, can be devoted to teaching and modelling (Mastropieri & Scruggs, 2007; Spencer, 2006; Polloway, Patton, & Serna, 2008). Ideally, the development of a peer tutoring schedule is flexible but should be consistent. For example, peer tutoring can occur twice or thrice per week for 20 minutes with increasing student responsibility and minimizing feedback or supports as tutees master the selected peer tutoring process.

The principles and requisite techniques needed for implementing CWPT depends on what the researcher or teacher intends to achieve through the tutoring. Prior to the implementation of the CWPT instruction, the formation of the peer tutoring groups and selection of tutors for each group are very important. The selection of peer tutors can be carried out by means of rank and order, using scores from the pre-test, a test or an exercise before the intervention and simple random sampling. Whatever being the case, the tutee's interest should be considered.

Using the rank and order, the teachers may rank and order the students on ability level, from the highest to the lowest, to create teams of equal ability as possible. The teacher can also use partners/team chart form to plan out 3 weeks of different team configurations in advance. Once ranked, the teacher places the highest ability student in team one, the second highest ability level student in team two, third highest in team one, fourth highest in team two and so on, alternating the students on the first team and then to the other teams. With this configuration, the teacher will create 2 teams of equal ability of high, average, and low students in both teams. If all the higher ability students are in the same team, they will win every time and that would defeat the purpose of the two competing teams. The teacher must therefore create teams where both teams have an equal chance of winning on any given day. The peer partners or team formed automatically determines which student in each team takes seat and which students move to get to their partners when instructions to move are given.

CWPT involves the entire class, divided into student pairs (tutor and tutee dyads). The pairs are engaged reciprocally and simultaneously with instructional contents (Delquadri, Greenwood, Whorton, Carta, & Hall, 1986; Allsopp D. H., 1997; Heron, Welsch, & Goddard, 2003). Tutors are trained and supervised by the classroom teacher. The CWPT method utilizes an interdependent group contingency where students are held accountable for their own performance.

CWPT instructions consist of the content materials to be tutored, new partners for each week tutoring, partner-pairing strategies, teams competing for the highest team point total, contingent individual tutee point earning, tutors providing immediate error correction, score's public posting and social recognition for the winning team.

2.4.4 Implementing CWPT/Instructional Components of a CWPT Lesson

The CWPT procedure (sequence of instructions) generally follows 5 major instructional components. These are: pre-assessment, mini-lesson, CWPT activity, suggested activities and mastery post-assessment.

The pre-assessment component is used by the teacher to determine the foreknowledge/entry level (relevant previous knowledge) of every student. Explicitly, it helps the teacher to determine if any of the content to be taught is already known, and exactly who knows what in advance. It takes 7 to 10 minutes.

The mini-lessons component follows the pre-assessment stage. Mostly, teachers spend 10 to 20 minutes here. It is the stage where the teacher introduces or models the proposed tasks/skills or the new material to the learners. Explicitly, it is the stage at which the teacher introduces the lesson by stating the topic, purpose and objective(s) of the day's lesson. The teacher employs teacher-led instructions to provide a model of the lesson in a whole class discussion or close informal group format. This helps the learners to follow the lesson. It also helps learners to know what they are to do, what next and how they are to do it. The teacher finally conducts an informal assessment on learners' overall understanding of what has been achieved, what is to be done, and which students have difficulties.

The third stage captures the *CWPT activities*. It is the stage that captures the Class-Wide Peer Tutoring stage of the lesson. The entire class is put into peer dyads or small groups and every student is paired with another to learn and practise the content that was introduced or reviewed in the mini lessons. The time is divided equally between tutees and tutors. Each student performs the role of a tutor and a tutee during the peer tutoring sessions. The suggested tutoring time limit ranges from 10 to 30 minutes for each round depending on the grade level or stage.

The fourth stage captures the *suggested activities*, where individual practices take place. Each individual module or unit provides the teacher with suggestions of additional activities that they can have their students engaged in to demonstrate individual learning, mastery, and skills application on an individual basis.

The final stage includes the *post-assessments*. The post-assessments are vital to the programme because they allow the teacher to determine which students have mastered the material at or above the acceptable criterion level and how much gain the students have made since the pre-assessment. The results of the post-assessments will also guide the teacher to know how to proceed with the next instruction. The results will indicate whether or not the teacher should proceed to the next level of instruction (if all students have mastered the material) or whether there is the need to re-teach the material (if many of students fail to master the material).

Kulik and Kulik (1992) identified seven (7) important characteristics that result in successful implementation of peer mediated instruction:

1. Expectations from student learning,
2. Careful orientation to lessons,
3. Clear and focused instructions to participants,
4. Close teacher monitoring of student progress,
5. Re-teaching,
6. Use of class time for learning,
7. Positive and personal teacher and student interaction

Greenwood, Delquadri and Carta (1997) described the basic procedure to consider when planning to implement CWPT method:

a) How to introduce and review the new material to be learned;
b) Which unit content or materials will be tutored;
c) How to re-assign new partners each week;
d) How to select partner pairing strategies,
e) How to carry out reciprocal roles in each session;
f) How teams compete for the highest team point;
g) How students earn individual points;
h) How tutors provide immediate error correction;
i) How to post individual and team scores;
j) How to use social rewards for the winning team;

A CWPT procedure presented by Greenwood et al.'s (1997) revealed that a class game format should be used in CWPT lessons to enable students to measure their success through the number of points earned by themselves and their team.

Chapman (1998) recommends implementing some of the following components in any peer tutoring programme:

- Peer tutoring produces greater gains when pairs of students alternate between the roles of the tutor and the tutee.
- Peer tutors should receive training through methods such as direct instruction, modelling, and practice, using prompting and positive reinforcement, providing feedback, and systematic error correction.
- Peer tutees should receive training on following instructions, responding to questions, and seeking clarification where necessary.

- Frequent monitoring and close supervision of the peer-tutoring pairs by the teacher helps to keep students on track with their tutoring activities and goals and improve the likelihood of increase in academic achievement.

2.5 Empirical evidence on CWPT

The empirical evidence of CWPT has been placed under three thematic areas: CWPT on Achievement, CWPT on Retention of Concept; and Teaching and Learning Methods.

2.5.1 CWPT on Students Achievement

CWPT have been implemented to investigate children's academic performance, and those studies have yielded positive results (Greenwood, Delquadri & Carta, 1997, Hughes & Fredrick, 2006)

Weidright (2013) examined the effects of Class-Wide Peer Tutoring (CWPT) on the math computational fluency of a group of 41, 7th grade students in a small, rural Junior or Senior High School. Students participated in 20-minute, reciprocal tutoring sessions each day and their performance was examined under both baseline and intervention conditions a day. Points were earned for correct answers and making appropriate error corrections, and individual point totals were aggregated into daily competing team scores. Each week, the team with the most points was formally recognized for their performance. Results showed that CWPT improved all three target students' mathematics computational fluency rates.

Mkpanang (2016) examined the effects of class wide and reciprocal peer tutoring strategies on students' mathematical problem-solving achievement in electricity concepts in physics. The aim was to bring to the fore those peer tutoring strategies that enhance students' achievement in mathematical problem solving. A simple random sampling technique was used in selecting one hundred and twenty (120) SSS2 Physics students as a representative sample for the study. The results showed that Class-Wide and Reciprocal Peer tutoring strategies were both more effective than the control strategy with regard to improving students' mathematical problem solving achievement in electricity concepts in Physics. It was revealed that physics students exposed to Class-Wide Peer Tutoring strategy performed significantly better than students that were exposed to Reciprocal Peer tutoring strategy and control group strategy in mathematical problem solving in electricity concepts in physics. It was also observed that both peer tutoring strategies afforded students the opportunity to help each other to learn by addressing the students' individual differences.

Mahan (2004) examines the impact of peer tutoring on achievement in mathematics among fifth grade students (N=14) with learning disabilities. The intervention took place during a small group instructional time of 30 minutes a day over a six-week time period. The experimental class (N=7) received the peer tutoring by being paired with students of above- grade-level ability. The participants in the comparison class (N = 7) received only traditional (teacher-directed) instruction. STAR Mathematics tests, pre-tests/post-test, student questionnaires, interviews and behavioural observations were used in the data collection. The results indicated significant improvement in students' attitudes and opinions. The academic achievement in mathematics increased only slightly with the use of the peer tutoring intervention. The researcher concludes that peer tutoring was an overall useful strategy to promote active engagement in the learning process and potentially increase academic achievement among fifth grade students with learning disabilities (Mahan, 2004).

Nilson (2009) studied how students understanding and achievement were affected when peer-led reviews in a classroom setting became the main discussion method of each mathematical concept. The finding was that peer-led reviews improved student involvement and allowed students the opportunity to learn instead of only being taught.

Fraivillig, Murphy and Fuson (1999) studied three components of mathematics curriculum. These three components were: Eliciting Solution Methods, Supporting Conceptual Understanding, and Extending Mathematical Thinking. It was found out that teachers support mathematical thinking but rarely extend their thinking.

Graven (2004) studied how teachers were able to teach mathematics better and how they understood it better when they had confidence in what they had learned. The idea of confidence is hinged on understanding and explaining mathematical concept. The result showed that student achievement increased when they were exposed to more hands-on learning style where they had the chance to interact with each other and exchange ideas (Nilson, 2009).

In a study conducted by Pyle (2015) on "The Effects of Unidirectional Peer Tutoring on Mathematics Outcomes for Students with Learning Disabilities in an inclusive, Secondary Setting" on Mathematics 1 class, the selection of tutors was generally based on the ability level and the willingness of tutors to support their peers with low incidence disabilities. The students that had advanced skills in Mathematics were the tutors and students with low incidence learning disabilities or achievers in Mathematics became the tutees. The tutors were trained to deliver the unidirectional tutoring intervention to their tutees.

The result indicated increase in criterion and normative performance on teacher-developed weekly Mathematics quizzes as a result of the peer tutoring intervention.

There was improvement in quantity and quality of Mathematics problems completed as well as academic engagement during the tutoring intervention. The tutors, tutees and the general education teacher indicated that the students perceive the unidirectional tutoring intervention as effective and socially desirable (Pyle, 2015). As a teaching strategy, CWPT has proven effective for improving students' test performance and accuracy (Kamps, Barbetta, Leonard, & Delquadri, 1994).

2.5.2 CWPT on Retention

Any instructional model that elicits adequate student participation has profound effects on students' retention (Akubuilo, 2004). Retention plays a major role in the understanding, comprehensibility, reflective thinking, application of mathematical concepts and achievement in Mathematics. Retention is defined as a preservative factor of the mind (Kundu & Tutoo, 2002). The mind acquires the materials of knowledge through sensation and perception. The new materials acquired in the mind need to be retained or preserved in the form of images for knowledge to develop. Whenever a stimulating situation occurs, retained images are revived or reproduced to make memorization possible.

Kundu and Tutoo (2002) found that mathematical concepts need to be presented to the learners in a way or method that touches their sub consciousness which can trigger quick recall of the concept taught or learnt. One of the teaching methods which can help achieve this is cooperative learning. In these methods, both high ability and low ability learners have the chance to collaborate in terms of understanding, explaining and retaining the concept they have learnt in a mathematics class.

Chianson, Kurumeh and Obida (2011) investigated the effect of cooperative learning method on retention level of students in circle geometry. The study involved 358 Senior

Secondary Two (SSII) students. An independent t-test statistics was used in the analysis to determine whether a significant statistical difference existed between the cooperative learning approach and the conventional learning approach in terms of students' retention of the taught concept ($t(356)= 8.474$, $p= 0.001$). The findings of the study confirmed that students who were subjected to the cooperative learning strategy were able to retain the concepts of Circle Geometry more than those students who were taught using the Conventional Learning approach. According to Parkinson (2009), peer tutoring can produce both short-term and long-term gains for tutors and tutees in all subjects with the greatest gains seen in mathematics irrespective of its design.

Abdullahi (2016) investigates the effect of peer tutoring teaching strategy on secondary school students' academic achievement in mathematics. The findings of the study showed that student taught with peer-tutoring strategy performed better than those taught with conventional teaching method and gender has no effect on their mathematics achievement scores. The research also revealed that peer-tutoring was more effective in student retention when taught mathematics than the conventional teaching method of instruction.

Harper, Mallette, Maheady, Bentley and Moore (1995) evaluated the retention of 100 subtraction items by primary grade-age children with mild disabilities using CWPT for 10 weeks. Short and long term retention of items and rate of correct responding were assessed. Results indicated 27% improvement in accuracy. Short and longer-term retention measured on post-tests was 88.7 % and 85.0% correct, respectively. Improvement in students' rate of accurate response to subtraction items practised during CWPT was obtained.

Topping, Kearney, McGee and Pugh (2004) indicates that CWPT method is a positive tool in the enhancement of student learning and mathematics retention. According to Greenwood, Delquadri and Carta (1997), Class-Wide Peer Tutoring (CWPT) is an intervention programme that has been researched on over the past 25 years and has produced direct (academic) and indirect (socio-emotional-behavioural functioning) significant findings for children with or without special needs (Bell, Young, Blair, & Nelson, 1990; Hughes & Fredrick, 2006; Plumer & Stoner, 2005; Utley, Greenwood, Mortweet, & Bowman, 2001)

2.5.3 Students Perception on CWPT Effectiveness

Tilken and Hyle (1997) interviewed students on what they thought about peer tutoring. There were positive response from students about the effectiveness of the peer tutoring and its use in the classroom. For example, one student with disabilities reported, "*I think in any classroom someone understands it better than you. You ask a friend. You feel kind of embarrassed, but they help you. We have time in class for group work. That was pretty helpful.*" (Tilken & Hyle,1997, p. 15).

Cobb (1998) demonstrated CWPT success with low achieving sixth grade female students with ADHD. A short survey was given on CWPT after it was utilized in their inclusion classroom. Students made statements on how they felt more help was available in the classroom once CWPT strategy was in place. They stated that "*they made some smart new friends*". "*The tutors helped them with their reading skills*". Students also stated that they "*felt more confident in their reading and comprehension and wanted to read aloud in class*".

Topping, Kearney, McGee and Pugh (2004) found that there was positive feedback from tutors and tutees interviewed during the study.

Abdullahi (2016) noticed that tutees responded better to their peers than to their teachers and tend to obtain companionship from their tutors. Tutees also receive more teaching and individualized instruction than in classroom setting. It makes for better understanding of the topics; helps tackle difficult problems and topical issues as well as encourage reading habit and optimal use of time by students.

Prater and Serna (1999) discovered that students' teaching is much more effective in most cases than teachers instructing students. Students have a way of showing each other their problem solving tricks which are, in many cases, much simpler than those of the teacher (Dopp & Block, 2004)

Allsopp (1997) examined the effectiveness of using Class-Wide Peer Tutoring (CWPT) in heterogeneous middle school classrooms to teach higher order thinking skills, algebra problem-solving skills. It was found that both CWPT and independent student practice were effective strategies for helping students to learn beginning algebra problem-solving skills.

Stephenson (2001) found that the duality of student success is another powerful advantage of CWPT. The tutees gain the obvious benefits from this strategy, but there are also advantages for the tutor. CWPT allows the student tutors the opportunity to coach the tutees on how to recognize important patterns in the content, which empowers them to educate their peers during a tutoring session (McDannel, Mathot, & Thorson, 2001). This pattern recognition only reinforces the stimulus for the reinforcement of learners' understanding of the knowledge which is being shared. Stephenson (2001) states that being able to convey concepts to their peers in a familiar way greatly increase the tutors' confidence in their own learning abilities. This helps to deepen their own understanding of the topic bringing them closer to the national goal of mastery skill level in that subject.

In a study conducted by Greenwood et al. (2002) it was indicated that Class-Wide Peer Tutoring has some components that ensures its effectiveness. These includes one-on-one reciprocal peer tutoring, group contingencies of reinforcement, tutor modelling the correct response which served as an error correction strategy, tutor tasking presentation and response opportunities, tutor monitoring of tutee performance and recording the tutee's earned points, posting of performance and feedback on progress.

Gillies and Ahsman (2000) found that students used language that was more inclusive and gave more explanations to assist understanding. Students giving elaborative help to each other usually help them understand concepts and this contributes to their achievement.

2.5.4 Conventional Teaching Methods

In these methods of teaching, the mathematics teacher or educator explains, illustrates, demonstrates and, in some cases, gives notes on procedures and examples. The teaching method is simple. The learner is led deductively through small steps and closed questions to the theory being considered. In using this method in a class with lower attaining learners, the teachers usually show few examples on the chalkboard at the start of the lesson and set related exercises for the pupils to work on their own. Much time is spent on teaching than learning.

At its best, this style of teaching achieves what it is set out to do, that is, promote rote learning and prepares the pupils for examinations. At the worst, it becomes direct "telling how" by the teacher, followed by understanding of what was presented by the teacher on the part of the pupils. What is lacking in this approach, even at its best, is a sense of genuine enquiry or any stimulus to curiosity or appeal to the imagination (Ernest, 1991).

Many mathematics teachers 'lecture' instead of 'teaching' (Osafo-Affum, 2001). Teachers give definitions; make no use of concrete materials and practical ways to explain mathematical concepts. Teachers rather give notes on mathematics just as they would do in History. Students tend to verbalize their notes without any meaningful understanding. Fletcher (2003) repeated that, indeed, irrespective of the level at which mathematics is taught; the task of the Ghanaian mathematics educator has almost always been that of a lecturer and interpreter, communicating the structure of mathematics methodically. Meanwhile, the approach used by the teacher, to a larger extent, affects the building up of knowledge in learners. Coleman and Vaughn (2000) noted that teacher-directed instruction is a positive means in gaining results in student learning.

2.6 The Concept of Quadratic functions

Quadratic functions: Quadratic functions are functions which have two as the highest exponent of the variable. They are functions that take the form: $ax^2 + bx + c, a \neq 0$ where a, b and c are constant. They can be written in different forms but changing the form of the expression of the function does not change the function, the graph or the values in the table (Cooney, Beckmann, & Lloyd, 2010). Quadratic functions are the first non-linear polynomial functions. They can be expressed in three main forms. These are general form ($f(x) = ax^2 + bx + c$), vertex (standard) form ($f(x) = a(x - h)^2 + k$) and factored form ($f(x) = a((x + \alpha)(x + \beta))$)

The coefficient, a: The constant, *a,* is the leading coefficient of the polynomial in standard form. It is the constant factored out of the factored and vertex forms. In all three forms, the sign of a indicates whether the corresponding parabola opens upward (if a is positive, $a > 0$), or downward(if a is negative). The absolute value of a in the function indicates how the parabola is widened up or folded up. It widens up when the value of a is decreased and folds up when the value of a is increased. Thus the smaller the value of a, the larger it "dilates or gets widen".

Quadratics as a function means that quadratics is a single-valued mapping from one set, the domain, to another set, the range (Cooney, Beckmann, & Lloyd, 2010). It means that the relation that exists between the elements in the two sets is one-to-one and many-to–one mapping. The elements in the first set (domain) maps to one and only one element or value in the second set (range/co-domain). The graph of every quadratics usually passes the "vertical line test". It is usually not possible to draw at least one vertical line on the graph that intersects the function more than once.

The graph of a quadratic function is non-linear and takes the form of parabola. Quadratic functions are many to one function but not one-to-one. All but one value of the range, y, for any given quadratic function corresponds to two values of the domain, x. The graph of every quadratic does not pass the "horizontal line test". It is usually possible to draw, at least, one horizontal line on the graph that intersects the function more than once.

Every quadratic function has a vertex that occurs at the function's maximum or minimum. It means that the value of y is either the least or greatest in the range of the function. The vertex can be observed on the graph of the function and also in the table of values because quadratic functions have maximum or minimum values, their range is limited to all real numbers less or equal to the maximum or greater than or equal to the minimum.

The basic unit (simplest instance) of the quadratic function: $y = ax^2 + bx + c$, where $a \neq 0$ is called the parent function. It takes the form: $y = x^2$. It is formed when the *a* is one (1) while the *b* and *c* equals to zero (0). In $y = x^2$, for each value of *x* input into the equation, the opposite value of *x* gives the same result. For example, $25 = (5)^2$ and$25 = (-5)^2$. The $y = x^2$ is symmetric about the y-axis (the line x=0). This can be identified in the graph of the function and table of values representing the graph.

Expressions in standard form*:* The quadratic function is written in standard form as: $f(x) = ax^2 + bx + c$. The constant *c* indicates the value of the *y*-intercept of the graph of the parabola (The point at which the parabola and the y-axis meet).The equation of symmetry is expressed as $x = \frac{-b}{2a}$ and the minimum value is $y = f(\frac{-b}{2a})$ or $y = (\frac{4ac-b^2}{4a})$.The coordinates of the vertex is $(\frac{-b}{2a}, f(\frac{-b}{2a}))$.

The vertex or standard form of the function: The quadratic function in standard form is $f(x) = a(x-h)^2 + k)$. This form consists of perfect square binomial, multiplied by the leading coefficient (the constant, a) and summed with the constant k. The coordinates of the vertex on the parabola is (*h, k*).The line of symmetry is $x = h$. The minimum value is $y = k$ and occurs when $(x-h)^2 = 0$ or x=h.

The factored form of quadratic function: The quadratic function in factored form is $(f(x) = a((x-\alpha)(x-\beta)))$. It is the product of two binomials which mostly takes linear form and the constant, *a*. The zeros (or roots) of the quadratic function are at $x = \alpha \ and \ x = \beta$ .The students learn that transforming an expression into factored form is a method for finding the zeros. They also learn that the line of symmetry and vertex of the corresponding parabola lie halfway between the two roots: $x = \alpha \ and \ x = \beta$

Relationship between the three forms: There are relationships between the forms of the functions or the expressions. Each of the forms can be expressed from one form to the other by one of the following methods: (1) Expansion and simplification of the binomials, (2) factorization, (3) Completing the square.

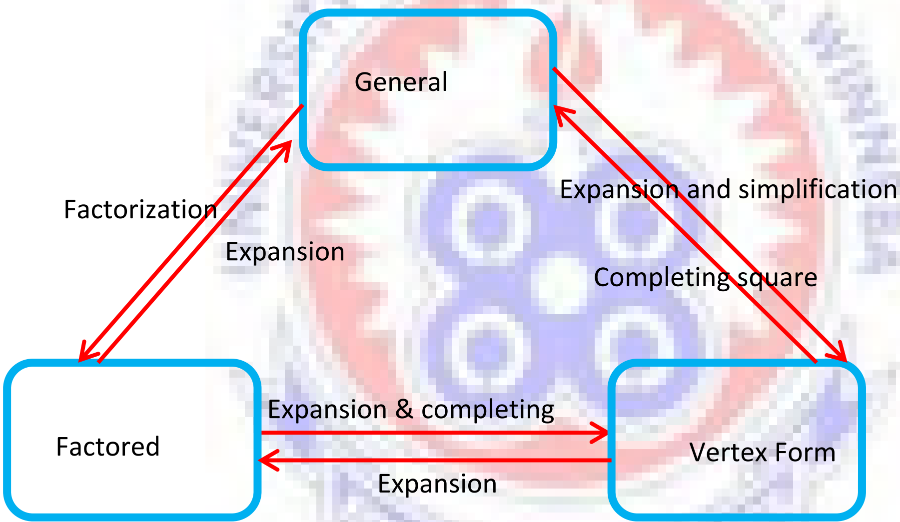

Figure2.1: Relationship between the three forms of Quadratic functions

Quadratic Equation: A quadratic equation takes the form: $ax^2 + bx + c = 0$. It is obtained if a quadratic function or the expression of a quadratic function is set to a constant or real number and solved. For example, if $f(x) = k$, the resulting equation can be solved for the independent variable, x. Solving it reveals the roots, zeros and the solution to the equation of quadratic function, $f(x)=y=k$. In other words, the solution(s) indicate at what values of x do the graph of the quadratic function and the, line y = k intersect. These solutions are the same as the solutions of the equation, which is also a quadratic equation. When the value of k is zero, solving the quadratic equation yields the zeros (or roots) of the $ax^2 + bx + c = k$;

$f(x) - d = 0.$

2.6.1 Students' Difficulties in Quadratic Functions

There are a number of studies reported in the mathematics literature on students' learning difficulties in quadratic functions. These include inadequate idea on how variables behave/many to one nature of quadratics as a function (misconception of variable), student struggle stirring between representations, and student struggle with the relationship between the diverse expressions of the algebraic forms of a quadratic function (Borgen & Manu, 2002; Ellis & Grinstead, 2008; Eraslan, Aspinwall, Knott, & Evitts, 2007; Kotsopoulos, 2007; Zaslavsky, 1997).

Eraslam (2005) identified four cognitive obstacles that students face in learning quadratic functions. These cognitive obstacles are:

1. Lack of making and investigating mathematical connections between algebraic and graphical aspects of the concepts,
2. The need to make an unfamiliar idea more familiar,
3. Disequilibrium between algebraic and graphical thinking, and
4. The image of the quadratic formula or absolute value function.

Eraslan remarked that students lack the ability to make and investigate mathematical connections between algebraic and graphical aspects of the Quadratic functions.

Kotsopoulos (2007) found that secondary students experience many difficulties when factoring quadratics. The difficulties arise as a result of the students being challenged with having to recall basic multiplication facts and quadratics being shown in a form which is not exactly like what the students are used to. For example: $x^2 + 3x + 1 = x + 4$ being not in standard form and therefore poses trouble to most students when asked to perform various tasks with it.

It has been generalized by Ellis and Grinstead (2008) that when working with quadratic functions, students' issues mainly appear with:

1. connections between algebraic, tabular, and graphical representations,
2. a view of graphs as whole objects,
3. struggles to correctly interpret the role of parameters, and
4. a tendency to incorrectly generalize from Linear Functions.

They found difficulties with connections between algebraic and graphical representations of Quadratic functions. For instance, the response from the interview conducted revealed that two-thirds of the students interviewed described the role of the parameter *a* in y = $ax^2 + bx + c$ as the "slope" of a quadratic function.

Zaslavsky (1997) researched the misconceptions that impeded students' understanding of quadratics. She identified five conceptual obstacles that impeded the students' understanding of the quadratic function:

1. interpretation of graphical information (pictorial entailments),
2. relation between a quadratic function and a quadratic equation,
3. analogy between a Quadratic functions and a linear function,
4. seeming change in form of a quadratic function whose parameter is zero, and
5. over-emphasis on only one coordinate of special points (e.g. vertex).

In a study conducted by Knuth (2000), it was found that:

1. students relied heavily on algebraic solution methods versus graphical solution methods, even if the graphical would have been quicker;

2. students seemed to have developed a ritualistic procedure for solving problems similar to those in the study; and that
3. students may have difficulties dealing with the graph-to-equation direction of solving problems.

These observations indicate that students are dependent on rote procedural understanding versus obtaining and using conceptual understanding.

Zaslavsky (1997) also identified some difficulties in student's understanding of a quadratic function. These revealed graphical interpretation, relation between a quadratic function and a quadratic equation and change in form of a quadratic. Aside these difficulties outlined, it was also evident that students preferred to work with the standard form of a quadratic function $y=ax^2+bx+c$ rather than other forms, such as vertex form.

Clement (2001) identified that definitions and images of function and the relationship between them transferred from graphical to algebraic form are perhaps most difficult for students. Different studies have shown that both in the algebraic and the graphical form, the concept and representation of images and pre-images are only partially understood (Dubinsky & Harel, 1992; Maharaj, 2008) Their studies reiterate that students find sketching graphs of Quadratic functions difficult and confusing though graphing of functions is an essential component of the study of Quadratic functions.

Zakaria & Maat (2010) found that students have difficulties in solving quadratic equations using the method of factorization, completing the square and quadratic formula. They experienced difficulties in replacing the positive and negative signs, resulting in errors in solving the equation by the various methods. The most common errors found among students in solving quadratic equations by the various methods are the transformation errors and process skill errors. Examples of process skills errors committed by students include the operation of addition, subtraction, multiplication and division. Lithner (2008) found that students' reliance on rote thinking and reasoning in mathematics are the cause of their difficulties, whether in the form of symbolical equations or word problems.

Norasia (2002) found that most students make error at the process skill level, especially in the expansion of quadratic expressions. In computation, the students make mistake in positive and negative sign when developing algebraic expressions. Errors also occur when replacing a value that has a negative sign. The findings of the study support the research of Roslina, (1997) and Parish and Ludwig (1994) that most low and average students face difficulty in factorization and simplifying algebraic expressions as well as performing algebraic operations.

A review of the literature reveals that there has been a small amount of research on difficulties students encounter with quadratics as functions, representation of functions in various forms, roots of Quadratic functions, concepts of discriminants and curve sketching. Most research on students' understanding of Quadratic functions has focused on students solving Quadratic equations.

2.6.2 The Teaching of Quadratic functions in Schools

The National Council of Teachers of Mathematics (2000) recommends that High School students should be able to "create and use tabular, symbolic, graphical, and verbal representations and to analyse and understand patterns, relations and functions" (p. 297). Eraslam (2005) therefore reported that in teaching quadratic functions, students must open up to linked concepts such as turning points, intercepts, and the effects of the parameters of the quadratic functions.

Vaiyavutjamai & Clements (2006) reiterates that students' difficulties with quadratic equations stem from their lack of both instrumental understanding and relational understanding of the specific mathematics associated with solving quadratic equations. It was suggested that while teacher-centred instruction with strong emphasis placed on the manipulation of symbols, rather than on the meaning of symbols, increases, student performance regarding solving Quadratic equations, their relational understanding would still be quite low, and they could develop misconceptions. For instance, it was found out that many students had an inadequate understanding of the "null factor law". For example, in solving $(x - 3)(x-5) = 0$, although most students gave the correct answer; $x = 3$ and $x = 5$; they considered two *x's* in the equation as representatives of different variables, and thus, they must take different values. That is, when they were asked to check their solutions, they simultaneously substituted $x = 3$ into $(x - 3)$ and $x = 5$ into $(x - 5)$ and found that $0.0 = 0$ and in doing so, decided that their solutions were correct.

Working with different representations of the quadratic is one way to promote what has been called "flexible competence"(Movshovitzs-Hadar,Schoenfeld & Arcavi, 1993). This emphasizes conceptual understanding of a concept rather than procedural mastery. Many other researchers have commented on the importance of students being able to move back and forth between the various representations of each function at hand (Ellis & Grinstead, 2008; Knuth, 2000; Leinhardt & Stein, 1990). Kotsopoulos (2007) indicates that students need to have both a strong conceptual understanding of multiplication of polynomials as well as the procedural knowledge to retrieve basic multiplication facts effectively.

2.7 Summary of Key Findings

From the empirical evidence and the conceptual framework, it's observed that little had been done by teachers in choosing interventions that is capable of shifting the trend of teaching from teacher-centred to learner-centred that is capable of meeting students' challenges with understanding, perception, retention and achievement in learning mathematical concept.

The conceptual frameworks and the empirical evidence again indicate that Class-Wide Peer Tutoring (CWPT) is one of the effective teaching strategies which are capable of meeting students' needs when used effectively. The method is capable of addressing several classroom related issues with all kinds of students if well planned and implemented.

In Ghana, only few researches have being carried out with CWPT treatment in Mathematics and content related areas. Therefore, the researcher deemed it fit to find out this evidence on CWPT treatment in Ghanaian Mathematics classroom; hence the present study looking at the impact of CWPT method on SHS students' achievement in mathematics concept (Quadratic function). Also, students' conceptual understanding, retention of learned mathematical concept and perception will also be checked in two schools in the Central Tongu District of the Volta region of Ghana.

CHAPTER THREE

METHODOLOGY

3.0 Overview

This chapter discusses the research design, population and sample. It also covers the research instruments used and procedure for carrying out the study. Finally, the method of data analysis is also discussed.

The null hypotheses that were tested in the study to help answer the research questions one (1) and two (2) are outlined below:

H_o: "There is no significant difference in the mean scores between the Control and Experimental groups in the post-test" helped the researcher to answer the research question 1.

H_o: "There is no significant difference in the mean scores between the Control and Experimental groups in the retention test" helped the researcher to answer the research question 2.

3.1 Research Design

Quasi-Experimental design involving pre-test post-test experimental and control group design were adopted as the research design for the study. Quasi-experimental designs are research designs which do not include randomization and are used when true experimental research is not feasible (Ary, Jacobs, & Razavieh, 2002). According to Gall, Gall and Borg (2003), quasi-experimental design can be used when it is not possible for the researcher to randomly sample the subject and assign them to treatment groups without disrupting the academic programmes of the schools involved in the study. Gall et al (2003) reiterated further that in a non-equivalent control group design, it is possible to have all groups receive treatments.

In this study, the non-equivalent group type of quasi-experimental design, which requires pre-test and post-test and also assigns members to a control and experimental group by a simple random technique was employed by the researcher. This type of quasi-experimental research was considered suitable for the present study at the expense of true experimental because the research was carried out in the contact periods of the schools and in a typical school situation of this kind, it was very difficult to reorganize classes to accommodate a randomized controlled trail. The researcher had limited control over the subjects (students) on scheduling of time for treatment and other difficulties posed during and after assigning subjects randomly to the control and experimental groups in the contact periods. It was, therefore, necessary to assign intact classes (non-randomized groups) to the two different groups in the study i.e. experimental and control groups. The researcher employed the technique of obtaining a simple random sample by "picking strips of paper out of a box" method; and each strips of paper was labelled "E" or "C" representing experimental (E) and control (C) to select a class for the experimental and the control groups in each school. Prior to this, the means of the two groups were statistically significantly equal in the experimental school.

The cross-sectional presentation of the research design, which is non-randomized control group, pre-test, post-test, is illustrated in the figure below:

E: → PT1 → T → PT2 → RET1 → RET2

C: → PT1 → T → PT2 → RET1 → RET2

Key: E = Experimental group; C = Control group; PT1= Pre-Test; PT2 = Post-test; T=Treatment; RET1= Retention Test One (1); RET2= Retention Test Two (2)

Figure3.1: Illustration of Research Design

In this design, both the experimental and control groups in each school, were given the pre-test and post-test. The RET1 was administered two weeks after the Post-test was carried out, and followed by RET2 in four weeks' time from the post-test in the experimental school. The experimental group was subjected to treatment using CWPT method while the Control Group was also subjected to Conventional method of teaching. According to Ary et al. (2002), the use of pre-test helps one to check on the equivalence of the groups on the dependent variable before the experiment begins. Thus if there are no significant differences in the pre-test, then the selection as a threat to internal validity is eliminated and therefore one can proceed with the study.

The independent variables in this study were the teaching methods (CWPT method and Conventional teaching method).The dependent variable was the students' test scores.

3.2 Population and Sample

The target population for this study was all second year students of the 2016/2017 academic year in the two public second cycle schools in the Central Tongu District of the Volta Region of Ghana, offering Elective Mathematics. The schools include Adidome Senior High School and Mafi-Kumase Senior High Technical School. The two second cycle schools were conveniently selected for the study because the researcher lives in the district, precisely Adidome and has become very familiar with the terrain of the district. Secondly, the two schools are the only second cycle schools in the district. In addition, the second year students were considered because the researcher believed that:

- The students were not under any immediate external examinations that could distract them from full participation in the study.
- The students have attained some level of maturity and confidence needed for participation in the study since they have received SHS Mathematics teaching for at least one year.
- The students in the two schools were taught quadratic functions and the respective teachers who taught them are graduate Mathematics teacher(s)
- The students will be writing the 2018 May/June WASCE Elective Mathematics paper.

The sample for this study was one hundred and sixty-six (166) students of the 2016/2017 academic year in the two public second cycle schools in the Central Tongu District of the Volta Region of Ghana, offering elective Mathematics. Adidome Senior High School was used as the experimental school and Mafi Kumasi as the control school, for the sake of convenience. In Adidome SHS School, the total number of students in each of the two intact classes was 54. In Mafi Kumase SHTS, the total number of students in the General Arts intact-class was 29 and the Science intact class was 29. Therefore, the total sample size was 166. In the experimental school, the 2D class (Science Class) was the experimental group/class and 2A (General Art Class) was the control group. In the control school, the General Art class was the experimental group and the Science class became the control group.

The sampling techniques employed in obtaining the population and sample through the selection of the schools, year batch, the control and the experimental schools, and classes as indicated above in this study were the purposive, convenient and simple random. According to Tashakkori and Teddlie (1998), purposive sampling and Convenience sampling techniques are used when the researcher wants to select only the cases that might best illuminate and test the hypothesis of the research. Again, they are used when, lack of time and financial constraints make it impossible to conduct a large-scale study (Creswell, 2005). In purposive sampling, the researcher employs his or her own “expert” judgment about who to include in the sample frame. In other words, it is based on deliberate choice and excludes any random process (Stout, Marden, & Travers, 2000). In this study, the convenient sampling technique was used in selecting the two secondary schools for the study. Purposive Sampling technique was employed to select all second year students offering Elective Mathematics in each of the second cycle schools as the sampling frame from which the sample, a Science intact-class and a General Art intact-classes, in each sampled schools were selected. The simple random sampling technique was used in selecting the experimental and the control groups in each of the schools.

The schools purposively selected were Adidome Senior High School and Mafi-Kumase Senior High Technical School, all in the Central Tongu District of the Volta Region of Ghana. Adidome SHS was conveniently used as the experimental school and Mafi-Kumase Secondary Technical as the control school. Two intact classes (N=166 students) were conveniently selected from each of these schools for the study. Based on the above explanations on purposive sampling, the researcher employed his own expert judgement by using the pre-test mean score of the various classes to select two different classes with “equal or equivalent class mean score” from each school (experimental and control schools) for the study.

In selecting the experimental and the control classes for the treatment, the simple random sampling technique was used to select the intact class for the experimental and control groups. In this case, the researcher used the technique of obtaining a simple random sample by "picking strips of paper out of a box" to select a class for the experimental and control group in each school as indicated above.

3.3 Research Procedure

The entire study covered a period of twelve (12) weeks, between the months of February and July, 2017, when school was in session. Every senior high school had both experimental group and a control group. The pre-test and post-test were administered to both control and experimental groups in each of the two schools sampled. The retention tests (i.e. Ret1 and Ret2) were administered to subjects in the experimental school, as indicated above under the "research design".

The experimental group received series of instructions with Class-Wide Peer Tutoring (CWPT) methods in a cooperative and collaborative setting, based on the Constructivist Principles (learner-directed method of instruction) while the control group on the other hand, received instruction through the conventional teaching method; also known as the Traditional Method (teacher directed method of instruction) in an individualized setting. The contents of the topics covered in the various lessons for both control and experimental groups were restricted to the scope of contents specified under the quadratic functions enshrined in the Ghana Education Service's syllabus for Elective/Further Mathematics and other recommended curriculum materials. The topic was divided and sequenced in a teachable unit for easy presentation and effective understanding as illustrated in the scheme of work (See Appendix A).

Prior to the first week, permission was sought from the School Management, the head of department and the subject teachers, to use the school and the concerned classes for the study. The researcher sought the consent of the subjects. The subjects were sampled from classes offering Elective Mathematics and have had lessons on quadratic functions under polynomials. This includes Science and General Arts classes. The confidentiality of participants was guaranteed because their identity in answering questions was not required.

In the first week, a pre-test, which was made up of six objective and three subjective items on Quadratic functions were administered by the researcher to both the Control and Experimental groups in each school. The mean scores obtained in the pre-test for the various classes were used to select two intact classes in both schools for the study. The two groups with equivalent mean scores were used for the study. Again, the students' responses to the pre-test also helped the researcher to: substantiate the need for the subjects to study quadratic functions, identify and describe the students' conceptual and procedural difficulties and misconceptions in learning quadratic functions. In addition, the pre-test mean scores of the groups were compared to the post-test mean scores after the intervention, to measure the potential effects of the treatment on both groups.

Training of students with regard to the different instructional treatments followed in each school. In the experimental group, the training session focused on peer tutoring procedures, tutors and tutees role: before, during and after the CWPT lesson. The training equipped students on the relevant knowledge needed for them to play the roles of tutors and tutees in the CWPT lesson. It captured the tutoring process, error correction, feedback techniques and team competition. Demonstration and role play methods of teaching were employed in taking the students through the CWPT procedures as well as the reinforcement techniques to be used by the tutors.

The second to sixth weeks were used to teach both groups the concept of quadratic functions. Each session lasted for 90minutes once a week. The teaching and learning materials used in the study include scheme of work, lesson plans on CWPT and conventional teaching methods, activity sheets and solution manual, Elective mathematics syllabus, Effective Elective Mathematics for SHS students book, WASSCE past questions, four (4) test items and their marking schemes (See Appendix F and G)

Treatment in the experimental classroom: In the experimental classroom, the researcher first reviewed learners' RPK, followed by introduction of the new materials by teacher lead instructions (introduction) and whole class discussion (mini lesson) as indicated in the lesson plan (See Appendix B). In each lesson, the topics introduced were taught in turn for at most 15 minutes, using teacher lead instruction. The researcher provided a model of the concept through demonstration while students listened and observed. The researcher used examples and non-examples to explain the concepts. This was followed by a question and answer period. The teacher wrote the questions on the board and asked students to solve them with or without the models on the board. The teacher discussed the solutions with learners. The students usually sat in rows and columns. The remaining 10 minutes involved evaluation (independent activities) on task related to quadratic functions. The outcome of this was used in forming the peer tutoring groups and selection of the tutors and tutees in some cases.

CWPT activities followed the mini-lesson (whole class discussion) where the teacher-lead instructions were employed. At the initial stage of the tutoring, the entire students in the class were paired randomly but in some cases, the result from the evaluation at the whole class discussion stage were used in forming the peer tutoring groups. Each group was made up of, at most, three students. Each group chose a student to be the peer tutor. Within each group, a tutor and tutee pair is formed, and each student had the chance to be a tutor or tutee before the lesson ended.

Activity and answer sheets were given to the various tutors after forming the peer tutoring groups (see appendix D for activity sheets and E for the answer sheets). Each activity sheet consisted of a model solution to one of the questions in the activity and sets of questions arranged in level of difficulty on quadratic functions, for the tutee and tutor interaction. The answer sheets consisted of solutions and guide to the problems outlined on the activity sheets for the tutor to make reference to. The exercises in the students' book on the topics were used as a supplementary material for groups which exhausted the items outlined on the activity sheets.

At the tutoring stage in each lesson, tutors took their tutees through the model question and their respective solution on the activity sheets. After going through the model question with the tutee and there is some level of confidence, the tutor prompt the tutee to solve the first question on the activity sheet. The tutor then asked the tutee to go over the work and pay attention to the necessary steps. This was followed by the tutor discussing the tutees work with him/her using "question and answer technique" and provides the feedback for an incorrect response. The tutee(s) repeat the same question when necessary. The tutor reinforces the tutee with terms like: "Very good", "I like that", "you're good", "yeah yeah" and the tutee moved on to the next item on the sheet. When there is a problem with tutees work, the tutor prompts the tutee to reconsider the work, compare it with the solution to the model question or asked leading questions and offer suggestions etc. until the tutee gets it. The tutor marks the tutees work using the answer sheet, award mark(s) and reinforce him/her. The tutor asked for the necessary steps used in solving the question and the tutee explained it verbally and moved on to the next tasked. The process continue until the tutee feels happy, grasp the concept and convinced the tutor by solving the remaining items without or with little support .The students switch roles and repeat the process in their new role. Thus the tutor becomes the tutee and vice versa.

The peer tutors awards marks on conceptual skills (tutee solved the problem correct) and procedural skills (provides all the necessary steps).If the steps and answers are correct, the peer tutor awards five points to the tutee (three points for correct steps and two points for solving the problem without any hints/assistance from the peer tutor). However, the peer tutor prompts the tutee to write the steps and answer again if the steps or answers given are wrong. If the tutee could not correct his/her steps or solution, no point is awarded. This scoring system was not the same as Greenwood et al's method (1989) in which peer tutor gives two points for the correct steps and answer, and giving one point for correcting the mistake. The scoring system was modified to compensate the effort made by students to set some equations themselves before solving and also quote some identities to enable them solve the questions. The objective of the tutoring game was to complete as many questions on the activity sheets as possible within a given time frame. This is to help improve learners speed in solving more mathematical tasks/problems in few times. The more items students had completed, correctly the more points they could earn for themselves and their team. The students summed up their daily points and recorded the result on the activity sheet. The class prefect and the assistant collate the scores from each dyad. The daily group’s total scores were calculated.

The Tutoring lessons took place once a week and were followed by achievement test and weekly exercise. Exercise was taken individually and students received points for each correct steps and answer. Following the weekly exercise, all points were totalled and the winning group of the week was announced the following week. The winners receive praises, handshakes and clapping of hands as reinforcement.

In the CWPT activity stage, the researcher acted as a facilitator, moved about to help dyads or triads facing problem with the given task, observed students' behaviours, checked if the tutors were following the tutoring procedures and also able to provide appropriate feedback to their tutees, but not given any extra mark.

In the tutoring process the researcher observed students with the use of the observation checklist. The purpose of this observation was to keep the tutoring dyad or triads on track and address difficulties learners faced during the tutoring. In most cases, the researcher observed the following practice or behaviour of tutors: how tutors give hints to their tutees on how tutee(s) correct their answers, informed tutees about the state of their answers (thus whether correct or wrong), notify tutees on the incorrect steps of calculation, praise tutees when their answers are correct or correct their wrong answers, and give accurate marks to correct steps or answers.

In the 40 minutes allocated for the tutoring in each lesson, the researcher tried to visit each group, spent few minutes with a tutoring pair, observed the progress of interaction in each tutoring and performance of peer tutors after tutee answered one item on the activity sheet. He also observed how the peer tutor reinforced the tutee, gave feedback and awarded points based on tutees performance.

The post-test and the semi-structured interview were carried out immediately after the instructional treatments for the 5 weeks over. Two weeks after the post-test, Ret-Test1 (Retention test one) was administered to the students. The Ret-Test1 was to test the student's retention level after the topic had been taught to them.

Four weeks after the Ret-Test1, the Ret-Test2 (Retention test two) was also administered to the students. The Ret 2 was to test the student's long term retention level after the topic had been taught. The information is presented in Fig 3.2.

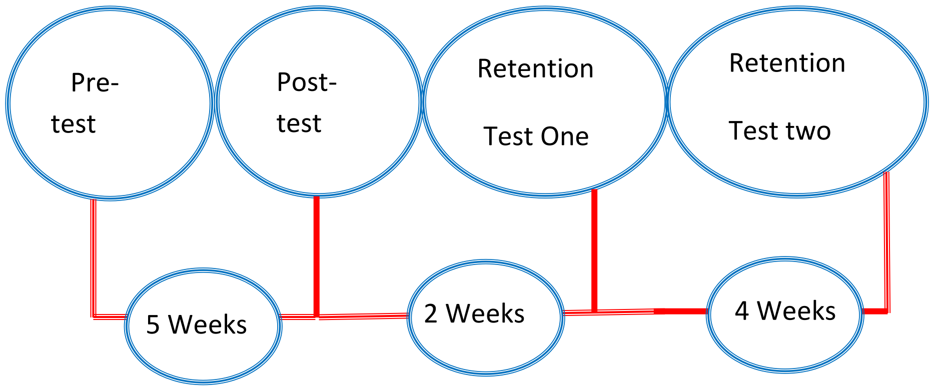

Figure 3.2: Tests and the respective intervals between them

The two weeks and four weeks gap were allotted between the tests because the researcher assumed that after the second to the fourth weeks onwards, the students might have forgotten most of the concept studied during the treatment and the post-test questions. Each set of the test items, consisted of 9 items in which 6 were objectives with 4-options and 3 subjective on quadratic functions in SHS2 elective Mathematics (See Appendices F and G). Ten (10) students from the experimental groups were interviewed. An interview guide consisting of 10 items on students' perception of Class Wide Peer Tutoring was used.

Conventional Instruction: At the beginning of the lesson, the teacher revised learners Relevant Previous Knowledge and introduced the topic(s) under study for the day to the learners. He involved the entire group of students in a group discussion on the necessary steps or procedures involved, using examples and non-examples. For instance in lesson one, the teacher defined quadratic function, provided examples of quadratic functions while students listen and copy. He moved on to discuss the various forms of writing quadratic functions and the methods used in writing quadratic functions from one form to the other. Thus general form to standard (vertex) form using completing square method. The teacher indicates the method, discuss and drill students on the necessary steps. The teacher did most of the talking and randomly asked what they had learned on that day. Popular question ask: "Do you understand", "Any question”. The teacher, at times, used illustration or charts to teach the concept to the students. This was followed by a question and answer period. The teacher writes the question on the board and asks students to solve with or without the models on the board. The teacher discussed the solution with learners. The students usually sit in rows and columns. The remaining 10 minutes involved evaluation (independent activities) thus task related to quadratic functions.

3.4 Research Instruments

Considering the research questions and the type of research design adopted, data collection was both quantitative and qualitative. Three valid and reliable instruments were used in the study for the data collection. These are Tests (Pre-Test, Post-Tests, RET1 and RET2) and interview guide.

3.4.1 Tests (Pre-test and Post–test, Retention Test 1 and Retention Test 2)

The Pre-test and Post-test were used in collecting a quantitative data that helped the researcher with more information in answering research question 1 and 3. Thus to examine the impact of CWPT method on students achievements in Quadratic Functions. The information from RET1 and RET 2 were used in obtaining information on short and long term retention ability of learners on learned mathematics concept (Quadratic function). This information enabled the researcher to answer research question two (2). Thus, how Class Wide Peer Tutoring enhances learners retention on a learned mathematics concept (Quadratic function).

Twenty-Four (24) objectives and Twelve (12) subjective questions from Elective Mathematics Textbooks and past WASSCE questions on Quadratic Functions were used for the test items. These questions were based on the Elective Mathematics syllabus, from specific objectives 1.5.1 to 1.5.4 (Ministry of Education, 2010, Pg.4).

The Twenty-four (24) objectives and the twelve (12) subjective test items were split into four different sets of test items. Each set of item has nine items. The items consisted of six (6) objective questions and three subjective questions. A simple random sampling technique was used in selecting an item for the Pre-test, Post-test, RET1 (Retention Test 1) and RET2 (Retention Test 2). Students' from both groups were given 1 hour to complete each of this tests when administered.

3.4.2 Interview guide

The researcher designed an interview guide to elicit students' perception (perspectives and experiences/views) on the use of CWPT method. The interview guide consisted of ten (10) items/Questions. The items sought information on student's perception on the effectiveness, implementation and suggestions on the CWPT intervention employed in the teaching of Quadratic functions as administered. Ten (10) students consisting of five students from each school were selected for the interview.

The interview was semi-structured type. At most 15 minutes was spent on each student. The students' response from the interview was transcribed and analysed to help the researcher in interpreting the quantitative result. The analysed data provided additional information in answering research questions 1 and 2.

3.4.3 Piloting the Study

The instruments were piloted at ST. Mary's Seminary Senior High School, Lolobi – Ashiambi (SOMASCO) in the Volta Region of Ghana. Sixty (60) students offering elective mathematics in SOMASCO were used. The pilot study enabled the researcher to restructure the test items to help elicit the right information on the test items. The school was selected because it shared similar characteristics with the Senior High Schools selected in the central Tongu District in the Volta Region for the study. In addition, the students have already received tuition on the concept of Quadratic functions.

Prior to the piloting process, the researcher sought permission from two elective Mathematics teachers to assist in carrying out the pilot study in the school. The piloting process involved first, training one of the teachers on how to use the CWPT method in teaching, a very brief orientation for students and the pre-test was administered to the 60 form two students offering elective Mathematics in SOMASCO. After One week teaching (three double lessons) on the use of CWPT method in teaching and learning some concepts of Quadratic Functions with students, the test was administered and semi-structured interview was also carried out. The ultimate aim of the piloting of the instrument was to examine the content validity and reliability. Again, to help identify and rectify ambiguity and other perceived problem likely to affect the items in the instrument.

In this study, to establish content validity, the researcher used the SHS teaching syllabus for Elective Mathematics, Effective Elective Mathematics for Senior High Schools Student's Book and some prescribed Mathematics textbooks for senior high school students to construct the instrument items. Also, the instruments (tests and interview guide) were vetted by the researcher's supervisor and were also subjected to peer-review by two (2) colleague post-graduate students, whose comments and suggestions led to some restructuring and modifications of the instruments before piloting.

The Sixty SOMASCO students that took part in the piloting exercise were tested. Split-half method was used to estimate the reliability of the test items. The scores of the test were compared using Spearman-Brown Split Half Reliability Coefficient (Spearman-Brown Coefficient). The Spearman-Brown Coefficient gave a reliability coefficient of

0.8 which was greater than 0.05, ($p>0.05$). It indicated that the two instruments were reliable (have high degrees of reliability or meets internal consistency).

To substantiate the reliability of the interview guide, the researcher prepared structured interview guide which had the same format and sequence of words and questions for each respondent (Cohen, Manion & Morrison, 2007). In most cases, the researcher avoided frequent changes in wording, context and emphasis which undermined reliability in interviews. According to Cohen, Manion & Morrison (2007) a careful piloting of interview guide and the extended use of closed questions can enhance reliability of interviews. In view of the above, the piloted interview guide contained the same structured questions as one of the reliability measures put in place in conducting the interview.

3.4.4 Data Collection Procedures

Prior to the first week, permission was sought from the School Management, head of department, the subject teachers and the concerned classes for their students to be used in the study.

In the first week, a pre-test was administered by the researcher to all students offering elective mathematics in both the control and experimental group in each school. This was followed by the selection of the control and experimental group in both schools based on the mean score of the various classes. Orientation for students' with regards to the different instructional treatments, climax the first week's activities. During the treatment period, five (90 minutes) lessons were taught, the first 35 minutes of each lesson was used to introduce new topics by the Researcher. Afterwards, CWPT was used. The entire students in the experimental groups were then paired up into peer tutoring groups (dyads and triads) and one student in each pair served as peer tutor while the other took the role of tutee for about 15 minutes. After the time limit had expired, the tutoring pairs reversed the roles for the same interval of time (Harper G. F., Maheady, Mallette, & Karnes, 1999). At the end of the sixth week, five (5) lessons were taught in each class.

In the sixth week, the post-test, checklist and interview guide were administered by the researcher in the sixth week in each school. The post test was administered to both groups, the checklist to only the Experimental groups' and the interview guide to only 10 students from the experimental groups (5 students from the experimental school and the other 5 from the control school). On the last day of the second week after the post test, Ret1 was administered likewise the last day of the fourth week after the Ret1 test; the Ret2 test was also administered to determine students' retention of the learned concept. (See Appendices F, for the post-test, checklist and interview guide respectively). The post-test was scored and analysed to answer research question 1. The Researcher also observed five different types of possible feedback required from the peer tutors to use. They were: notifying tutees the incorrect steps of calculation, informing tutees their answers are correct, praising tutees when their answers are correct or they are able to correct their wrong answers, giving hints to tutees how to correct the answers, giving accurate marks to correct steps or answers.

Finally, the responses to the interview guide and the checklist were transcribed and analysed to provide an answer to research question 3 and further information to answer research question 1 and 2.

3.5 Data Analysis

The researcher employed Quasi-experimental pre-test post-test control group designed in this study to examine the impact of Class Wide Peer Tutoring strategies on students' achievements, retention of learned concept and perception towards the study of quadratic functions. Both quantitative and qualitative data were obtained.

The quantitative data obtained from the tests' was coded and keyed into Statistical Package for Social Sciences (SPSS) version 21.0 for statistical analysis. The mean and standard deviation obtained from the output were used in the data analysis. Two inferential statistics namely Independent sample *t*-test and dependent sample *t*-test were used in the analysis to establish statistical effect of CWPT method on SHS students' achievement and retention respectively. Thus the independent sample t-test was used to detect the impact of the CWPT method on the students' achievement (information for answering research question 1) and the dependent ***t***-test for the retention (information for answering research question 2).

The independent sample t-test was used to evaluate the difference between the means of the two independent groups and also test for how the means for the two independent groups were significantly different at 0.05 levels of significance in the post-test conducted. The paired sample t-test was conducted on the experimental group to determine if significant differences existed in their performance on the post-test and retention tests given by the researcher. Thus the dependent sample t-test was used to evaluate the difference between the means of the experimental group in the post-test and the retention test also test for how the means for the two independent groups were significantly different at 0.05 levels of significance.

Prior to the use of the independent samples t-test for the analysis, the following assumptions (Independence, Normality, and Homogeneity of variance) were met:

3.5.1 Independence assumption

To meet the assumption of independence as one of the necessary conditions for using the independent sample t test, the researcher adopted Quasi-experimental pre-test- post-test control group design. Measures were put in place to ensure that the two groups (control and experimental) were independent of each other. Thus, none of the subjects belonged to both groups. The data (pre-test, post-test, Ret1 and Ret2 scores) used to run the independent samples t-test were independent of each other.

3.5.2 Normality of the dependent variable

The dependent variable (students' scores from the post-test and retention test (1&2)) were found to be approximately normally distributed within each group. The use of the Kolmogorov-Smirnov and the Shapiro-Wilk tests in SPSS helped to test for this assumption. The researcher therefore set the following null and alternative hypotheses before the Kolmogorov-Smirnov and Shapiro-Wilks Tests were used:

H_0 = "that there is no significant departure of the dependent variable from normality".

H_1 = "that there is a significant departure of the dependent variable from normality".

Table 3.1 shows the results of the normality test.

Table 3.1 Kolmogorov-Smirnov and Shapiro-Wilks Tests for Normality Assumption

Tests of Normality							
Test	Group	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	Df	Sig.	Statistic	Df	Sig.
Post Test	Experimental	.089	54	.200*	.974	54	.287
	Control	.110	54	.142	.972	54	.234
Ret1	Experimental	.099	54	.200*	.969	54	.169
	Control	.102	54	.200*	.977	54	.394
Ret2	Experimental	.108	54	.168	.962	54	.085
	Control	.101	54	.200*	.965	54	.119

From table 3.1, both normality tests, showed a Sig value greater than 0.05 in each of the groups. In the post-test for the experimental group, the Kolmogorov-Smirnov test produced a value of p = 0.200 (i.e. $p > 0.05$) and the Shapiro-Wilk also yielded p = 0.287 (i.e. $p>0.05$). Likewise in the post-test for the control group, the Kolmogorov-Smirnov test produced p = 0.142 (i.e. $p>0.05$) and the Shapiro-Wilk also generated p = 0.234 (i.e. $p>0.05$). These p values signified that there was no significant departure of the dependent variable from normality hence the null hypothesis was accepted, meaning that the normality assumption was met for the study's sample.

Also, in the retention test one (Ret1), the Kolmogorov-Smirnov test generated a value of p = 0.200 (i.e. $p>0.05$) and the Shapiro-Wilk also yielded p = 0.169 (i.e. $p>0.05$). Also in the post-test for the control group, the Kolmogorov-Smirnov test produced p = 0.200 (i.e. $p>0.05$) and the Shapiro-Wilk also generated p = 0.394 (i.e. $p>0.05$). These p values signified that there was no significant departure of the dependent variable from normality hence the null hypothesis was accepted, meaning that the normality assumption was met for the study's sample.

Similarly, in the retention test Two (Ret2), the Kolmogorov-Smirnov test generated a value of p =0.168 (i.e. $p>0.05$) and the Shapiro-Wilk also yielded $p = 0.085$ (i.e. $p>0.05$). Also in the post-test for the control group, the Kolmogorov-Smirnov test produced $p = 0.200$ (i.e. $p>0.05$) and the Shapiro-Wilk also generated $p = 0.119$ (i.e. $p>0.05$). These p values signified that there was no significant departure of the dependent variable from normality hence the null hypothesis was accepted, meaning that the normality assumption was met for the study's sample.

3.5.3 Homogeneity of variance

Homogeneity of Variance is another assumption that the researcher tested before using the independent sample t-test. The Levene's Test of Equality of Variances was used to test for this assumption in SPSS. The following null and alternative hypotheses were formulated before the use of the Levene's Test of Equality of Variances:

H_0 = "there is no significant difference between the variances of the test scores of the two groups".

H_1 = "there is a significant difference between the variances of the test scores of the two groups".

The table below presents the result of the Levene's Test of Equality of Variances in the post-test and the two retention tests.

Table 3.2 : Levene's Test of Equality of Variances for Homogeneity Assumption

Test		Levene's Test for Equality of Variances	
		F	Sig.
Post-test	Equal variances assumed	5.367	.022
	Equal variances not assumed		
Ret1	Equal variances assumed	31.094	.000
	Equal variances not assumed		
Ret2	Equal variances assumed	11.724	.001
	Equal variances not assumed		

From table 3.2, Levene's test for equality of variances was found to be violated for the present analysis. The post- test produced F (1, 15) = .71, p = .41, the retention test one (Ret1) yielded F (1, 15) = .71, p = .41 while the retention test two (Ret2) generated F (1, 15) = .71, p = .41. Because the Sig. value is less than the alpha of .05 ($p < .05$), we reject the null hypothesis (no difference) for the assumption of homogeneity of variance and conclude that there is a significant difference between the two group's variances. That is, the assumption of homogeneity of variance is not met. Owing to this violated assumption, a t statistic not assuming homogeneity of variance was computed and the data results associated with the "Equal variances not assumed," which takes into account the adjustment for the standard error of the estimate and the adjustment for the degrees of freedom (Cochran & Cox, 1957; Satterthwaite, 1946).Thus, the bottom line of information for the t test was used in each case for the present analysis. After satisfying the assumptions described above, the Independent-samples t-test statistical procedure at 0.05 significant levels was used to compare students' conception in Quadratic functions between the Control and Experimental groups, by using the sample means as the basis of comparison. The t-test results helped the researcher to answer research question 1 and 2 by rejecting the null hypothesis and accepting the alternative hypothesis. Thus, the researcher was able to ascertain the impact of the use of CWPT on SHS students' conception of Quadratic functions.

The qualitative data obtained from interview guide and observation checklist was also transcribed and analysed to provide further explanation to the quantitative result. Both descriptive and interpretive techniques based on the themes arrived at in the data collection were used. Both descriptive and inferential statistics were used in the analysis. Descriptive statistics, in the form of simple percentages, and measures of central tendency (mean and standard deviation) were all employed in the analysis. The themes were related to the research questions and interpreted on the number of issues raised by the respondents. This was based on the questions on the semi-structured interviews, checklist and classroom observation made by the researcher on the various lessons with the use of the CWPT lesson in a classroom setting. Interviews and discussions were transcribed and responses used in analysing the study. In addition, verbatim expressions of the respondents were used where applicable. Also students' conceptual and procedural skills demonstrated in their work presentation in solving questions on quadratic functions in the pre-test and post-tests were examined and used to detect the misconceptions, conceptual understanding as well as improvement in their work presentations in learning quadratic functions.

In the present study, the dependent variable was students' scores from the tests and the independent variable was the teaching approach which had two levels namely, CWPT method and Conventional (traditional) method of teaching.

To determine the effect size which is the measure of the degree (in standard deviation units) that the mean scores on the two test variables differ, the effect size statistics, "*Cohen's d*" was used. The $d = t\sqrt{\frac{n_1+n_2}{n_1 n_2}}$ where t = the obtained t value, n_1 =total sample size for group 1 and n_2= total sample size for group 2 and d=the *Cohen's d* was used for the independent sample t-test and $d = \frac{t}{\sqrt{N}}$ was also used in calculating the effect size for the paired sample t test.

The Percentage changes in either the mean or the range were obtained by the relation:

$$\frac{\boldsymbol{Post-test-Pre-test}}{\boldsymbol{Pre-test}} * \mathbf{100\%}$$

CHAPTER FOUR

RESULTS, FINDINGS AND DISCUSSION

4.0 Overview

This chapter is concerned with the analyses, interpretation and presentation of the results of the study. The qualitative and quantitative data were organised and analysed to provide information on achievement, retention of learned concepts and perception of learners. The results obtained are presented under the following research questions which guided the study:

- What is the impact of using CWPT method on SHS students' achievement in quadratic functions?
- To what extent can students keep and apply mathematical knowledge gained from lessons through CWPT method?
- How do students perceive the use of Class Wide Peer Tutoring method in teaching Quadratic Functions?

The mean scores for both tests were calculated for the Control and Experimental groups in both control and experimental school. The difference between the results of the Pre-test and Post-test was analysed to measure the actual effect of using CWPT on SHS students' performance in Quadratic Functions.

The pre-test scores of both the Control and Experimental groups provided more information on the comparability of the two groups both control and the experimental school. Table 4.1 shows an extract from the SSPS output of the analysis of the pre-test scores of students in the two groups in both Experimental and control school.

Table 4.1: Pre-test Scores showing the Comparability of both Groups in the experimental and control school

School	Group	Test	N	Mean	Std. Deviation	*t-value*	*df*	*p-value*
Experimental	Control	Pre-test	54	5.51	2.279	0.02	106	0.98
	Exp.	Pre-test	54	5.50	2.074			
Control	Control	Pre-test	29	2.50	1.414	1.10	56	0.27
	Exp.	Pre-test	29	2.10	1.319			

Table 4.1 shows the means and standard deviations of the pre-test scores of the two groups in the experimental school. The result revealed an insignificant outcome: $(t(106) = 0.01, p > 0.05)$. This implied that the mean pre-test score of the students in the experimental group is not significantly different from that of the students in the control class before the treatment.

Again, Table 4.1, above shows the means and standard deviations of the pre-test scores of the two groups in the control school. The result revealed significant outcome: $t(56) = 1.10, p < 0.05)$. This indicates that the mean pre-test score of the students in the experimental group is significantly different from that of the students in the control class at the 0.05 confidence level. An examination of the group means and standard deviations in the pre-test indicate that the experimental group (*M* =2.50, *SD* = 1.414) performed significantly higher on the post-test than the control group (*M* =2.10, *SD* = 1.319). This could be due to the criterion used in selecting students for programs and the methods used in teaching the leaners in the school.

Hence, if the mean score of the experimental group was found to be significantly different from that of the control group, in the post-test, RET1 and RET2 then the performance of the experimental group had been influenced by the CWPT method used.

4.1 Question 1: What is the impact of using CWPT method on SHS students' achievement in quadratic functions?

The first research question sought to examine the impact of using CWPT on SHS students' achievement in Quadratic Functions. To answer this question, a dependent and an independent samples *t*-test were conducted on the pre-test and post-test of both groups in the experimental and control school after the treatment. The mean scores of both the Control and Experimental groups in the pre and post tests were compared. The results of the analysis for the Control and Experimental groups in the post test, within the experimental school are summarized in Tables 4.2 and 4.3.

Table 4.2 : Pre-test and Post-test Scores for the Control Group

Test	N	Mean	Std. Deviation	Range	Percentage change	
					Mean	Range
Pre-test	54	5.51	2.279	9.50	90.63	78.95
Post-Test	54	10.58	3.696	17.00		

Table 4.3 : Pre-test and Post-test Scores for the Experimental Group

Test	N	Mean	Std. Deviation	Range	Percentage change	
					Mean	Range
Pre-test	54	5.50	2.074	9.50	190.54	89.47
Post-Test	54	15.98	4.697	18.00		

In Table 4.2, the statistics showed that within the Control group, there was a marginal increase in students' achievement in the quadratic functions in the post-test after they had been taught with conventional teaching method. In the pre-test, the range was 9.5, the minimum score was 0.0 and the maximum score was 9.50 out of the total score of 25. However, in the post-test, each of these statistics increased slightly.

The range increased slightly to 17.0, the minimum was 4 and the maximum was 21 out of the total score of 25. Moreover, the mean score also increased from 5.55 in the pre-test to 10.58 in the post-test (thus a difference of 5.03). This increase is an indication that conventional method (also known as traditional teaching method) can increase SHS students' achievement in quadratic functions.

Table 4.3, also shows the pre-test and post-test scores for the Experimental group. The results in the table show a great improvement in students' performance in Quadratic functions after they have been taught with the CWPT method. Prior to the treatment, the mean score was 5.50 which increased to 15.98 after students' had learnt with CWPT method. Again, the range increased from 9.5 to 18, the minimum score increased from 0.0 to 7 while the maximum score increased from 9.50 to 25. Thus, generally, the use of CWPT method to teach SHS students in Quadratic function has increased their performance and conception in the concept greatly. In effect, the impact of the use of CWPT method on SHS students' conception in Quadratic functions was very high.

Table 4.3, also showed the pre-test and post-test scores for the Experimental group. The results in the table show a great improvement in students' performance in Quadratic functions after they were taught with the CWPT method. Prior to the treatment, the mean score was 5.50 which increased to 15.98 after students had learnt with CWPT method. Again, the range increased from 9.5 to 18, and out of a total score of 25, the minimum score increased from 0.0 to 7 whiles the maximum score increased from 9.50 to 25. Thus, generally, the use of CWPT method to teach SHS students Quadratic function has increased their performance greatly. In effect, the impact of the use of CWPT method on SHS students' achievement in Quadratic functions was very high.

In order to test whether differences observed are statistically significant, the researcher used the paired sample t test and independent samples *t*-test to further analyse the data. Table 4.4 showed the results of the paired sample t test for differences between the post-test and pre-test scores of the experimental school.

Table 4.4 :(a) Paired Samples *t*-test Results on the Post-test and Pre-test of the experimental school

Group	Paired Differences Mean	STD. Deviation	Std. Error Mean	t	Df	Sig.
Experimental	10.48	4.446	0.605	17.34	53	0.00
Control	5.074	4.322	0.588	8.63	53	0.00

Table 4.4: (b) Paired Sample Statistics

Group	Test	Mean	N	Std. Deviation	Std. Error Mean
Experimental	Post-test	15.98	54	4.697	.639
	Pre-test	5.50	54	2.074	.282
Control	Post-test	10.58	54	3.696	.503
	Pre-test	5.51	54	2.279	.310

The paired sample t test result presented in table 4.4, showed that in the control group, there was statistically significant difference, t(53)=8.63, p<0.05, d=1.17, between the mean of students' scores in the post-test, (M=10.58, SD=3.696) and the pre-test, (M=5.51 SD=2.279).The students' scores in the Post-test, (M=10.58, SD=3.696) were higher than the scores obtained in the pre-test, (M=5.51 SD=2.279). The mean difference in the two tests was 5.074 and standard deviation of 4.32. The Cohen's *d* statistic ($d = 1.17$) indicated a large effect size .This serves as an evidence to suggest that the conventional method has large effect on student's performance in learning quadratic functions. Likewise, in the experimental group, the paired sample t test revealed a statistically significant deference between the students mean scores in the post-test, (M=15.98, SD=4.697) and the pre-test (M=5.50, SD=2.074); t(53)=7.33, p<0.05, d=2.36, The students' scores in the Post-test, (M=15.98, SD=4.697) were far more higher than the scores obtained in the pre-test, (M=5.50 SD=2.074). The mean difference in the two tests was 10.48 and standard deviation of 4.43. The Cohen's *d* statistic ($d = 2.36$) indicated a large effect size. These provide enough evidence to suggest that the CWPT method used, had a very large effect, on student's performance in learning quadratic functions. However, a comparison of the mean difference in both control, (M=5.07, SD=4.32) and the experimental group (M=10.48, SD=4.45) in both tests (post-test and pre-test) showed that the scores of the experimental group were significantly higher than that of the control group by 5.41.This provide enough evidence to suggest that the CWPT method, had a very great effect on students achievement in learning quadratic functions than the conventional method.

Again, to find out the statistically significant difference between the mean score of the control and the experimental group in the post-test, an independent sample t-test was conducted using the SPSS. A summary of the output is presented in table 4.5.

Table 4.5 Independent Sample t-test Results on the Post-test of both Groups in the experimental school

Group	No.	Mean($\overline{x}$)	*Sd*	*df*	*T*	Sig.
Control	54	10.58	3.696	100.437	-6.64	0.00
Experimental	54	15.98	4.697			

Table 4.5, showed the means and standard deviations of the post-test scores of the two groups in the experimental school. Comparison of the difference between the post-test mean scores of the two groups yielded a significant outcome, t (100.437) = -6.64, $p<0.05$, $d=1.28$.

An examination of the group means and standard deviations in the post-test indicate that the experimental group ($M = 15.98$, $SD = 4.697$) performed significantly better than the control group ($M = 10.58$, $SD = 3.696$). The Cohen's d statistic ($d = 1.28$) indicated a large effect size. It indicates that, between the two groups, the students in the experimental group (i.e. students exposed to Class Wide Peer Tutoring Method of teaching) recorded significantly better post-test achievement scores than their colleagues in the control group (exposed to conventional method of teaching) in the experimental school. Hence, the null hypothesis that "there is no significant difference in the mean scores between the Control and Experimental groups in the post test" was rejected. In effect, the alternative hypothesis that "there is significant difference in the mean scores between the Control and Experimental groups in the post test" was accepted. This tremendous achievement of the students in quadratic functions is as a result of the CWPT method employed in teaching the concept. The above findings is consistent with the findings of a study conducted by Marshak, Mastropieri, & Scruggs (2001) which indicated that students from classes which used CWPT intervention instruction statistically outperformed students exposed to only the basic teacher-led instructions. The finding also support the results in the existing literature on the impact of CPWT method on students achievement that CWPT yielded positive results when implemented (Hughes & Fredrick, 2006; Greenwood, Delquadri, & Carta,1997) or greater gains (Parkinson, 2009), CWPT helped students scored higher on content tests (academic) while in the tutoring condition and there was positive growth in their academic performance (Spencer, Scruggs, & Mastropieri, 2003) and effective in students achievement (Abdullahi, 2016).

To examine the impact of the CWPT method on students' achievement in the control school, the researcher conducted a paired t test with the students' scores of each group in both, pre-test and post- test. The results are presented in table 4.6.

Table 4.6(a) Paired Sample t test results on the post-test-pre-test within the groups in the control school

		Paired Differences						
Group	Tests	N	Mean	STD. Deviation	Std. Mean	Errort	df	Sig.
Experimental	Post-test – Pre-test	29	5.79	2.825	.525	11.04	28	.00
Control	Post-test – Pre-test	29	5.26	2.427	.451	11.67	28	.00

Table 4.6 (b) Paired Sample Statistics

Group	Test	Mean	N	Std. Deviation	Std. Error Mean
Experimental	Post-test	7.90	29	2.788	.518
	Pre-test	2.10	29	1.319	.245
Control	Post-test	7.76	29	2.900	.539
	Pre-test	2.50	29	1.476	.274

The paired sample t test result presented in table 4.6, above showed that in the control group, there was statistically significant difference, t(28)=11.67, p<0.05, d=2.17, between the means of students' scores in the post-test, (M=7.76, SD=2.900) and the pre-test, (M=2.50, SD=1.476). The students' scores in the Post-test, (M=7.76, SD=2.900) were higher than the scores obtained in the pre-test, (M=2.50, SD=1.476). The mean difference in the two tests was 5.26 and standard deviation of 2.427. The Cohen's *d* statistic (d = 2.17) indicated a large effect size. This serves as an evidence to suggest that the conventional method has some effect on student's performance in learning quadratic functions. Likewise, in the experimental group, the paired sample t test revealed a statistically significant deference between the students mean scores in the post-test, (M=7.90, SD=2.788) and the pre-test (M=1.319, SD=1.319); t (28) =11.04, $p<0.05$, d=2.05. It means that the students' scores in the Post-test, (M=7.90, SD=2.788) were far higher than the scores obtained in the pre-test, (M=1.319, SD=1.319). The mean difference in the two tests was 5.79 and standard deviation of 2.825. The Cohen's *d* statistic ($d = 2.05$) indicated a large effect size. This provides enough evidence to suggest that the CWPT method used had a very large effect on student's performance in learning quadratic functions. However, a comparison of the mean difference in both control, (M=5.79, SD=2.825) and the experimental group (M=5.26, SD=2.417) in both tests (post-test and pre-test) showed that the mean scores of the experimental group were significantly higher than that of the control group. This provides enough evidence to suggest that the CWPT method had a very great effect on students' achievement in learning quadratic functions than the conventional method.

In addition to the above analysis, the students raw scores in the post test were organised with respect to the individual test items, further organised per items and evaluated conceptually as: correct response (CR), partially correct (PR), wrong response (WR) and no response (NR). CR was awarded to student's response that demonstrated both conceptual understanding, procedural understanding of Quadratic Functions and solved correctly (used the necessary algorithms). PR was awarded if the student demonstrated either conceptual understanding and committed procedural errors or procedural errors and committed conceptual errors leading to partial result WR was given to response that demonstrated no understanding of the concept of Quadratic function (The learner committed both conceptual and procedural errors) leading to wrong result. Finally, NR was awarded if the item in the question was not attempted by the student.

Table 4.7 The Descriptive Analysis of Students' Response per Item in the Experimental and Control groups in the post-test

	Experimental Group			Control Group		
QN	CR/PR	WR/PR	NR	CR/PR	WR/PR	NR
	N(%)	N (%)	N(%)	N(%)	N (%)	N(%)
	N=54	N=54	N=54	N=54	N=54	N=54
1	40(74)	14(26)	0(0)	80	13	7
2	34(63)	19(35)	1(2)	44	52	4
3	25(46)	29(54)	0(0)	28	70	2
4	43(80)	11(20)	0(0)	50	46	4
5	42(78)	12(22)	0(0)	50	46	4
6	41(76)	13(24)	0(0)	54	44	2
7	47(87)	9(17)	0(0)	65	52	6
8	48(89)	14(26)	0(0)	89	59	6
9	37(69)	17(31)	5(9)	43	59	13

From Table 4.7, it is observed that each of the groups in the two schools had few "NR-No Response" outcomes for the items. The "WR-Wrong Response" from the control group was much higher than the experimental group in 8 out of 9 questions as in question one (1). A summary of the group's performance captured in Table 10, consisted of the wrong responses pooled from Table 4.7.

Table 4.8 The descriptive analysis of Wrong response per item in the experimental and control group in the post test

Question No.	Experimental Group %	Control group %
1	26	13
2	35	52
3	54	70
4	20	46
5	22	46
6	24	44
7	9	30
8	11	35
9	22	44
Total	223	380
Mean	24.78	42.22

From Table 4.8, it is observed that the control group gives very high wrong responses to the various questions than the experimental group. The result indicated a mean difference of 17.44 in favour of those in the control group. This revealed that the control group had the most wrong conceptual justifications than the experimental group.

In addition, the subjects' response were analysed with respect to the various mathematical concepts measured on Quadratic Functions. In the Test item, Questions 1, 4 and 9, required that students find maximum or minimum value of the quadratic function given. The use of completing square method and formula approach were necessary in finding the maximum or minimum value. Those using the completing square approach were expected to re-write the quadratic function in standard (vertex) form before writing the maximum and minimum value. Questions 2, 3, 8 also tested learners ability on how to form quadratic equation from given roots. The skill of using identities to obtain sum and products of roots was the necessary condition required in this case. Finally, Questions 5, 6, 7 examined learners' conceptual understanding on how to quote and use discriminant to find the value of a constant in a quadratic function. A summary is presented in Table 4.9.

In Table 4.9, the various concepts of Quadratic Functions tested were represented by the letters: A, B and C. The (A) represents maximum/minimum value, completing square and formula method. The (B) denote using identities on roots of quadratic functions, forming quadratic equations from given roots and the (C) for how to quote and use discriminants of a given quadratic functions.

Table 4.9 The Descriptive Analysis of response per concept of Quadratic function in the two groups in the post-test

		Experimental Group			Control Group		
QUE. No.	Concept	CR/PR S(%) S=54	WR/PR S (%) S=54	NR S(%) S=54	CR/PR S(%) S=54	WR/PR S (%) S=54	NR S(%) S=54
1,4,9	A	233(69.83)	77(23.77)	6(1.85)	46.5(14.35)	32(9.88)	13(4.01)
2,3,8	B	198(61.11)	115(35.49)	6(1.85)	43.5(13.43)	49(15.12)	6(1.85)
5,6,7	C	241(74.38)	63(19.44)	0(0)	45.5(14.04)	45.5(14.04)	6(1.85)

From the table 4.9, learners in both groups did very well. However, the learners in the experimental groups achieved higher in all the aspects of the Quadratic Functions than the respective control in both schools. It indicates that learners in the experimental groups developed conceptual understanding (grasp the concepts) of Quadratic Functions better than those in the control group. This indicates that the Class Wide Peer Tutoring treatment received by the experimental groups enhanced conceptual understanding than the conventional methods adopted in the control groups.

In addition, the result reveals that learners did better in "how to quote and use discriminant to find the value of a constant in a quadratic function" than "finding the maximum/minimum value of a quadratic function". However, they did quite well in "how to form quadratic equation from giving roots and also carry out operations involving roots of Quadratic Equations".

Also, a critical look at the learner's conceptual and procedural skills in their work presentation revealed that large number of students in the experimental group have improved upon their conceptual and procedural skills. However, only few students were affected by the procedural skills in this case. They have challenges in simplifying, expanding and factorizing, fractions involving roots of Quadratic Functions. Others were affected by the use of the equality and inequality symbols under the concept of discriminants. Large number of students in the experimental group improved on their work presentation and also solve more questions within the time frame that those in the control groups.

Its therefore means that Class Wide Peer Tutoring (CWPT) enhanced learners conceptual understanding in Mathematics concepts (Quadratic Functions), helped students to learn better and more quickly, increases the amount of class work students finish, and improved performance.

4.2 Question 2: To what extent can students keep and apply mathematical knowledge gained from lessons through CWPT?

The second research question sought to examine the statistical significance differences in the retention of the concept of learning quadratic functions between students who studied using the CWPT method (experimental group), and those who studied using the traditional method (control group). To answer this question, the researcher conducted two retention tests which were named as Ret1 and Ret2. The Ret1 was administered after 14 days from the end of the treatment and the post-test, and the Ret2, was carried out in the fourth week from the first retention test (Ret1). To analyse these results, a paired sample t test and an independent samples *t*-test were conducted on the pre-test and the respective retention tests (i.e., Ret1 and Ret2) of both control and experimental groups. The extract from the SPSS analysis showing the results of the paired and independent sample t test are presented in Table 4.10.

Table 4.10 (a) Paired Samples t-test Results on the Ret1 and Pre-test of the experimental and control group

		Paired Differences						
Group	Tests	N	Mean	STD. Deviation	Std. Mean	ErrorT	Df	Sig.
Experimental	Ret1 - Pre-test	54	8.43	5.413	.737	11.44	53	0.00
Control	Ret1 – Pre-test	54	3.63	3.313	.451	8.05	53	0.00

4.10 (b) Paired Samples Statistics

Group	Test	Mean	N	Std. Deviation	Std. Error Mean
Experimental	Ret1	13.93	54	5.895	.802
	Pre-test	5.50	54	2.074	.282

Control	Ret1	9.14	54	2.908	.398
	Pre-test	5.51	54	2.279	.310

The paired sample t test result presented in table 4.10, showed that in the control group, there was statistically significant difference, t(53)=8.05, p<0.05, $d = 1.10$, between the means of students' scores in the retention test one (Ret1), (M=9.14, SD=2.908) and the pre-test, (M=5.51 SD=2.279). The students' scores in the Retention test one (Ret1), (M=9.14, SD=2.908) were higher than the scores obtained in the pre-test, (M=5.51, SD=2.279). The mean difference in the two tests was 3.63 and standard deviation of 3.313. The Cohen's d statistic ($d = 1.10$) indicated a large effect size. This serves as an evidence to suggest that the conventional method had some effect on how students can keep and apply the knowledge gained in learning quadratic functions. Likewise, in the experimental group, the paired sample t test revealed a statistically significant deference between the students mean scores in the retention test one (Ret1), (M=13.93, SD=5.895) and the pre-test (M=5.50, SD=2.074); t(53)=11.44, p<0.05, $d = 1.56$. The students' scores in the Retention test one (Ret1), (M=13.93, SD=5.895) were far higher than the scores obtained in the pre-test, (M=5.51 SD=2.279).The mean difference in the two test was 8.43 and standard deviation of 5.413. The Cohen's d statistic ($d = 1.56$) indicated a large effect size. This provide enough evidence to suggest that the CWPT method used, had a very great effect on how students can keep and apply the knowledge gained in learning quadratic functions. However, a comparison of the mean difference in both control, (M=3.63, SD=3.313) and the experimental group (M=8.43, SD=5.413) in both tests (Ret1 and pre-test) showed that the scores of the experimental group were far higher than that of the control group by 4.80. This provide enough evidence to suggest that the

CWPT method, had a very great effect, on how students can keep and apply the knowledge gained in learning quadratic functions than the conventional method.

To determine the significant difference between the means of the two groups, an independent sample t test was conducted on the retention test one (Ret1) for both control and experimental group. The results are presented in table 4.11.

Table 4.11 Independent Samples T-test Results on the Retention test one (Ret1) of both Groups

Group	Test	N	Mean	Std. Deviation	*t*-value	*Df*	*p*-value
Control	Ret1	54	9.14	2.908	-5.352	77.354	0.00
Experimental	Ret1	54	13.93	5.885			

The result of the independent sample t test presented in table 4.11, showed that the differences in the retention test one mean scores of both the control and the experimental groups yielded a significant result, $t(77.354) = -5.352$, $p<0.05$, $d=1.03$. An examination of the group means and standard deviations in the retention test one indicated that the experimental group ($M = 13.83$, $SD = 5.895$) performed significantly better than the control group ($M = 9.14$, $SD = 2.908$). The Cohen's d statistic ($d = 1.03$) indicated a large effect size. It indicated that, between the two groups, the students in the experimental group (i.e. students exposed to Class Wide Peer Tutoring Method of teaching) recorded significantly better retention test scores than their colleagues in the control group (exposed to conventional method of teaching) by 4.69. As a result, the null hypothesis that "there is no significant difference in the mean scores between the Control and Experimental groups in the retention test one (Ret1)" was rejected. However, the alternative hypothesis that "there is significant difference in the mean scores between the Control and Experimental groups in the retention test one (Ret1)" was accepted. This tremendous improvement in how students can keep and apply the concept of quadratic functions is as a result of the CWPT method employed in teaching the concept.

Levene's test indicated unequal variances (F=31.094, p=0.00), so degrees of freedom were adjusted from 106 to 77.354. This result clearly confirms the impact of the CWPT method which was applied by the study in the retention of the concept of quadratic functions. The results showed the high level of performance in the retention of the concept of quadratic functions by the experimental group compared to the control group.

To verify how students can retain and apply knowledge gained in studying quadratic functions, beyond the two weeks tested and analysed above, retention test two was carried out in the fourth week after the Retention test one (Ret1) was administered. To analyse this result, both paired and independent sample t test were carried out and the result were presented in table 4.12 and 4.13.

Table 4.12 (a) Paired Samples *t*-test Results on the Ret2 and Pre-test of experimental and control group

		Paired Differences					
Group	Tests	Mean	STD. Deviation	Std. Mean	Errort	df	Sig.
Experimental	Ret2 – Pre-test	10.77	5.3392	.7266	14.82	53	0.00
Control	Ret2 – Pre-test	1.99	3.4374	.4678	4.26	53	0.00

4.12 (b) Paired Samples Statistics

Group	Test	Mean	N	Std. Deviation	Std. Error Mean
Experimental	Ret2	16.27	54	5.025	.6838
	Pre-test	5.50	54	2.074	.2822
Control	Ret2	7.50	54	3.107	.4228
	Pre-test	5.51	54	2.279	.3101

The paired sample t test result presented in table 4.12, showed that in the control group, there was statistically significant difference, t(53)=4.256, p<0.05, d=0.58, between the means of students' scores in the retention test two (Ret2), (M=7.50, SD=3.107) and the pre-test, (M=5.51 SD=2.279). In the analysis, the students' scores in the Retention test 2 (Ret2), (M=7.50, SD=3.107) were significantly higher than the scores obtained in the pre-test, (M=5.51 SD=2.279). The mean difference in the two tests was 1.99 and standard deviation of 3.313. The Cohen's *d* statistic ($d = 0.58$) indicated a large effect size. This serves as an evidence to suggest that the conventional method has some effect on how students can retain and apply knowledge gained in learning quadratic functions. Likewise, in the experimental group, the paired sample t test revealed a statistically significant deference between the students mean scores in the Retention test 2(Ret2), (M=16.27, SD=5.025) and the pre-test (M=5.51, SD=2.074); t(53)=14.821, p<0.05, d=2.02. The students' scores in the Retention test 2(Ret2), (M=16.27, SD=5.025) were far higher than the scores obtained in the pre-test, (M=5.50, SD=2.074). The mean difference in the two tests was 10.77 and standard deviation of 5.339. The Cohen's *d* statistic ($d = 2.02$) indicated a large effect size. This provide enough evidence to suggest that the CWPT method used, had a significant effect on how students can keep and apply knowledge gained in learning quadratic functions. However, a comparison of the mean difference in both control, (M=1.99, SD=3.437) and the experimental group (M=10.77, SD=5.339) in both tests (Ret2 and pre-test) showed that the mean score of the experimental group was significantly higher than that of the control group by 8.99.This provide enough evidence to suggest that the CWPT method, had a very great effect, on how students can keep and apply the knowledge gained in learning quadratic functions than the conventional method.

To determine the significant different between the means of the two groups (control and experimental), an independent sample t test was conducted on the Retention test 2 (Ret2) for both control and experimental group. The results are presented in table 4.13.

Table 4.13 Independent Samples T-test Results on the Retention test two (Ret2) of both Groups

Group	Test	N	Mean	Std. Deviation	*t*-value	*df*	*p*-value
Control	Ret2	54	7.50	3.107			
					-10.907	88.348	0.00
Experimental	Ret2	54	16.27	5.025			

Table 4.13 showed that the control group obtained a mean score of 7.50 and the experimental group also obtained a mean of 16.27. The result from the independent

sample t test was statistically significant, $t(88.348) = -10.91$, $p<0.05$, $d=2.10$. This indicated a significant difference between the mean scores of the Control and the Experimental groups in the retention test two. The mean of the experimental group (M =16.27, SD = 5.025) is significantly higher than the mean score of the Control group (M =7.50, SD =3.107) in the retention test 2. The Cohen's d statistic ($d = 2.10$) indicated a large effect size. Hence, the use of CWPT method in teaching Quadratic functions produces a highly significant and tremendous improvement in students' conception as against the performance of students taught by the conventional method. As a result, the null hypothesis that "there is no significant difference in the mean scores between the Control and Experimental groups in the retention test two (Ret2)" was rejected. However, the alternative hypothesis that "there is significant difference in the mean scores between the Control and Experimental groups in the retention test two (Ret2)" was accepted. This tremendous improvement in how students can keep and apply the concept of quadratic functions is as a result of the CWPT method employed in teaching the concept.

The results support the findings of the study conducted on the instructional effectiveness of CWPT by Greenwood et al., (1991), which indicates that students retain more of what they learn and make greater advances in social competence with CWPT compared to traditional teacher-led instruction also known as traditional or conventional method of teaching.

The findings also supports the studies of Chianson, Kurumeh, and Obida (2011), Abdullahi (2016), Topping et al. (2004).They all indicated the effectiveness of peer tutoring. Furthermore, analysis of the responses from the interview guide revealed that the impact of the use of CWPT method on students' interest, achievement and retention, in learning quadratic functions were very positive. Students' difficulties and interest in learning the concept of quadratic functions were addressed through the use of CWPT method employed.

4.3 Question 3: How do students perceive the use of class wide peer tutoring methods in teaching quadratic functions?

To obtain deeper understanding on how students perceive the use of Class Wide Peer Tutoring methods in teaching quadratic functions, a semi- structured interview was carried out with ten students in the experimental groups.

The thoughts and feelings of the students with regard to the Class Wide Peer Tutoring lesson's effectiveness, implementation and suggestions on the CWPT method employed in this study, the interest of the students and their overall perspective were captured. Other major things that emerged from the interview process, included learners understanding of the peer tutoring method of teaching, its effect on learning mathematical concepts, retaining and applying what is taught and learned as compared to the conventional method of teaching.

Overall, all the ten students (100%) interviewed indicated that the CWPT lesson was effective, efficient and they enjoyed everything about the lesson. Majority of them reported (a) observing significant changes in their academic achievement following CWPT treatment; (b) they enjoyed receiving immediate feedbacks from their peer tutors on the state of their work. For instance the praises and points tutors awarded for tutees whenever correct responses were given; (c) the way they practiced, helped them to understand, retain and apply what is taught (d) overall satisfaction with the lesson; (e) the manner in which the peer tutors helped them to improve in their performance or achievement was higher; (f) their willingness and readiness to have peer tutoring lessons in mathematics and other subjects could not hide (g) their recommendation on the CWPT method of teaching procedure employed for improvement most encouraging.

Most students perceived the CWPT experience as a "learning by teaching" because it creates room for them to explain things to their peers in friendly and simple ways. Four students said they *understood better* whenever they taught or explained things to their peers. They also said they learned new methods, simple ways of handling mathematical activities in a tutoring format which engage them more in the teaching process than simply listening to the teacher for the explanation of the material as its being the norm in the conventional method. It was also indicated that the CWPT method has enhanced their understanding and retention.

Specifically, when students were asked to indicate if they liked the CWPT method or not, all students said: *They liked everything about it*. They further identified the tutoring stage as a very important stage in the mathematics lessons. They explained that, the tutors used their own means (language) to explain the mathematics tasks in simple, easier to understand steps at the tutoring stage and this helped them to understand the mathematics concepts better.

In another question, students were asked to indicate if the lessons helped them as individual and how it helped them. The items demanded *Yes/No* answers with explanation. In this, all the students (100%) said *yes*; with their respective reasons on how it helped them. Many of them said it helped them understand what was taught in the various lessons, acquire new skills or ways of solving Mathematical questions, develop positive attitude towards the study of the subject. One student said it helped her *"to adopt very easy and simple ways of helping her fellow students who had the* *mind-set that Maths was difficult."* Another student also said the tutoring helped her *develop her confidence level* in studying and solving mathematical problems.

The main consensus among the students was that their peers taught them in different ways and they explained things better. They reiterated that their peers exposed them to varying ways of finding solutions or answers to questions/problems, but the teachers were the best at teaching them Mathematics.

In a nutshell, the response from the semi-structured interview showed that students had positive perception and feelings towards CWPT method. Overall, the students shared positive perception towards the quadratic functions and many showed an increase in understanding, achievement, retention, positive feeling and motivation when working with peers in CWPT lessons in Mathematics.

4.4 Discussion of findings

The findings of the study have shown that students exposed to the Class-Wide Peer Tutoring method of teaching recorded significantly better post-test achievement mean scores than their peers exposed to the conventional method of teaching in the control group in each school. The outcome of the independent sample t-test have shown and affirmed statistically significant difference in the mean scores,(t (100.437) = 6.64, $p<0.05$, d=1.28) in favour of the experimental group,(M=15.98) as against the control group(10.58) in both schools at 0.05 levels of significance with large effect size (*d=1.28*). The sufficient evidence therefore confirmed that CWPT method heightened student achievement in quadratic function concept treated than the conventional teaching method.

The above findings fell in with the findings of a study conducted by Marshak, Mastropieri, & Scruggs (2001) which indicated that students from classes which used

CWPT intervention instruction statistically outperformed students exposed to only the basic teacher-led instructions. The findings also support the results in the existing literature on the impact of CPWT method on students achievement that CWPT yielded positive results when implemented (Hughes & Fredrick, 2006; Greenwood, Delquadri, & Carta (1997) or greater gains (Parkinson ,2009), CWPT method helped students scored higher on content tests (academic) while in the tutoring condition and there was positive growth in their academic performance (Spencer, Scruggs, & Mastropieri, 2003) and effective in students achievement (Abdullahi,2016).

The research also revealed that CWPT method enhanced retention when taught the concept of Quadratic function than the conventional teaching method. This was evidenced in the analysis of students' performance in the two retention test conducted. The results showed that students taught with Class Wide Peer Tutoring method had mean score higher than those taught with conventional teaching method in each of the retention tests,(Ret1: , $t(77.354)= 5.352$, $p<0.05$, $d=1.03$; Ret2: $t(88.348)=10.91$, $p<0.05$, $d=2.10$). The students in the experimental group performed better than their colleagues in the control group.

The results support the findings of the study conducted on the instructional effectiveness of CWPT method by Greenwood et al., (1991), which indicated that students retain more of what they learn and make greater advances in social competence with CWPT compared to traditional teacher-led instruction also known as traditional or conventional method of teaching.

The findings also support the studies of Chianson, Kurumeh and Obida (2011),Abdullahi, (2016), Topping, Kearney, McGee and Pugh (2004).They all indicated the effectiveness of peer tutoring strategy in both achievement and retention of learned concept for students.

The responses from the interview conducted for the ten (10) students sampled from the experimental group were positive. It shows that CWPT method is an effective teaching method which has all it takes to accelerate all kinds of students' understanding, engagement, motivation, retention of learned concept, learning through teaching, reinforcement, high mastery level, immediate feedback and error correction (Greenwood et al, 1991).

The possible contributing factors to this positive effect of CWPT method as noticed from student's response include: sequence of instruction, active learner participation, collaborative interaction among tutors, tutees and the teacher, immediate feedback and reinforcement from tutors to their tutees, teaching and learning materials available and ample time for students to learn and practice than teaching during the treatment. Equal attention given to each learner by the tutors and the teacher to ensure equity and address diversity in the classroom as embedded in the CWPT instructional process have been the secret in all these positive gains.

This supports the findings of Arreaga-Mayer (1998) that when students are paired with a peer partner in one-to-one instruction it creates opportunities for error correction, earning points for correct responses, increased time spent on academic behaviour's, increased positive social interactions between students that would not typically occur, encouraging students to work together (an important life skill), and also help students experience more success and feel more confident.

Specifically, the findings show that CWPT was effective in helping students to develop conceptual understanding, enhanced retention, improved achievement and developed learners' interest in learning quadratic functions than the conventional teaching method only.

CHAPTER FIVE

SUMMARY, CONCLUSION, RECOMMENDATIONS

5.0 Overview of the Study

This chapter captured summary of key findings, educational implications, recommendations and conclusions.

5.1 Summary of the Study

The study explored the impact of the use of CWPT method on student's conceptual understanding and achievement in quadratic functions. It also examined the extent to which students can keep and apply mathematical knowledge gained (retention and transfer) from lessons with CWPT treatment. The students' perception on CWPT treatment was examined in addition. Mixed method (qualitative and quantitative), Quasi-Experimental design involving non-equivalent pre-test and post-test control and experimental group design were adopted as the research design for the study.

One intact class from each school was used as sample for this study. It captured one hundred and sixty-six (166) SHS 2 students of 2016/2017 academic year in two public second cycle schools in the Central Tongu District of the Volta Region, offering elective mathematics.

The schools were purposively selected and the selections of the intact classes in each school were convenient. Simple random sampling technique was employed in selecting the control and experimental groups in each school. Tests (Pre-Test, Post-Tests, RET1 and RET2) and interview guide were the main instruments used in collecting the data.

The entire study covered a period of twelve (12) weeks. The pre-test was administered before the treatment and the post-test were carried out after the treatment in both control and experimental groups in each of the two schools sampled. The retention tests (i.e.

Ret1 and Ret2) were administered to subjects in the experimental group after the post-test.

The two weeks and four weeks gap were allotted between the two retention tests after the post-test, because the researcher assumed that after the second to the fourth weeks onwards, the students might have forgotten most of the concept studied during the treatment and the post-test questions.

Both group received five lessons. The experimental group received series of instructions with class wide peer tutoring (CWPT) methods in a cooperative and collaborative setting while the control group on the other hand, received instruction through the conventional teaching method; also known as traditional method (teacher directed method of instruction) in an individualized setting. The contents of the topics covered in the various lessons for both control and experimental groups were restricted to the scope of contents specified under the quadratic functions enshrined in the Ghana Education Service's syllabus for Elective/Further Mathematics and other recommended curriculum materials. Ten lesson plans, six activity sheets and their respective answer sheets were the teaching and learning materials used.

Pre-test and post-test were used in collecting data on students' achievement. The first retention test and second retention test helped in obtaining data on students' retention of learned mathematical concept (i.e. quadratic functions). Ten (10) students, consisting of five students from each school were interviewed on the last day of the intervention. The quantitative and qualitative data were obtained. Both descriptive and inferential statistics were used in the analysis. The frequencies, percentages, means and standard deviation were the descriptive statistics used. The Inferential statistics were the dependent or paired sample t-test and independent samples t-test.

5.2 Summary of key findings

The results obtained from the entire study, analysis and interpretations with reference to the respective research questions opined that CWPT has significant effect on students' achievement, retention and perception in studying the concept of quadratic functions.

- **Effect of CWPT on Students Achievement:**

The findings from the post-test that was used in answering the research question one indicates that most students from both groups showed some level of improvement in the post test. However, those in the experimental group showed a statistically significant improvement in the post-test than their peers in the control group in both schools. In other words, students in the experimental groups in both schools achieved higher in the post-test than their peers in the respective control groups. It therefore means that the two methods employed in each case for the treatment, contribute to students performance. However, CWPT method was much effective and enhanced student's achievement better than the conventional methods in teaching Quadratic functions. It is therefore evidenced that CWPT method has positive effect on students' achievement in learning quadratic functions. It suggests that it is one of the most effective teaching and learning strategies teachers need to adopt in their mathematics classrooms to help students achieve higher in mathematics.

- **Effect of CWPT on Students Retention:**

The findings from the retention test (RET1 AND RET2) which were used in answering the research question 2 indicated that most students from both groups showed some level of improvement in the Retention tests. However, those in the experimental group showed a statistically significant improvement in the retention test than their peers. In other words, students in the experimental group achieve higher in the retention than their peers in the respective control group. It therefore means that the two methods employed in each case for the treatment, contribute to students retention of learned mathematical concept. However, CWPT was much effective and enhanced student's retention better than the conventional methods in teaching quadratic functions. It is therefore evidenced that CWPT has positive effect on students' retention in learning quadratic functions as compared to conventional methods of instruction. It suggests that one of the most effective teaching and learning strategies teachers needs to adopt in their mathematics classroom to enhance students' retention in learned mathematical concept which could help improve learners achievement in mathematics is CWPT method.

- **Effect of CWPT on Students Perception**

The finding in the study indicated that sampled students from both schools interviewed and the researchers' observation revealed that students in both groups support the view that the use of CWPT in terms of its implementation and effectiveness in teaching mathematical concept is the best. It means that in teaching mathematical concept(s), teachers should make CWPT teaching procedure as their best option.

In summary, the findings from the study showed that CWPT changes the way students perceive mathematics, enhances students' conceptual understanding, retention and finally contribute to their achievement.

In addition, it was observed that the CWPT method employed in the treatment, promotes learning by teaching and team working spirit among learners. It ensures equity or gives enough attention to: all students, improves work presentation, error correction, and quadratic problem solving skills, skills of learning concepts in diverse ways and how to share, verbalize, or communicate mathematical ideas to peers.

One reason I believe the CWPT method used in this research was beneficial to students is the structured format of the CWPT method employed. During this study, students were frequently involved in the tutoring process to enhance their understanding of quadratic function.

Although increased discourse, ensuring equity and addressing the issue of diversity in mathematics lessons was not a focus of this study, I observed both an increase in the amount of student to student discourse and positive effects of such discourse on student learning.

All students who participated in this study showed improved understanding of Quadratic function concepts according to their pre and post assessments. Some students who showed only a moderate increase on the assessments made greater gains in conceptual understanding that were not shown on the post assessment. Item analysis of students' solution to the test items typically indicated an improved conceptual understanding of quadratic functions. Students were able to write quadratic functions in the various forms, use completing square method to find the vertex, maximum or minimum point and value, solve or find the zeros of a given quadratic functions. They were able to calculate the discriminant, find the roots and the nature of roots of a given quadratic function. Again, they were able to sketch quadratic functions and also solve problems involving quadratic functions. The students who participated in the study also showed an improved ability to indicate the impact of the various constants/coefficients in a given quadratic function on a quadratic curve. Students improved in their conceptual and procedural skills.

The observation made before, during and after the treatment in the experimental class and the response from the students interviewed revealed that students perception of CWPT Method used in teaching the lesson were positive.

5.3 Conclusion

The study concludes that CWPT method is one of the effective teaching methods that will help improve students' achievement and retention in learning quadratic functions. Also the use of CWPT method in teaching quadratic functions improves students' perception and attitude.

In the course of the study, the following were identified as major contributing factors to the success of the positive findings of the study carried out:

Firstly, the CWPT method focuses on engaging all students in the tutoring process, as a tutor or tutee. That is the manner in which the process arouses and sustained learners interest in the teaching learning process through tutor-tutee interactions.

Secondly, the structured tutoring process gives more time for students to learn through practice and tutoring.

Thirdly, the CWPT method, promotes learning by teaching and team working spirit among learners. In this case, the students are able to work together as peers and gain a better understanding of the concept by learning from each other.

Last but not least, the tutoring process ensures equity or gives enough attention, improves work presentation, error correction, and problem solving skills, skills of learning concepts in diverse ways and how to share, verbalize, or communicate mathematical ideas to peers.

Finally, the feedback provided by the tutors is utilized to clarify understanding and enhance learning of the mathematical concept by the tutees.

5.4 Educational implication of the study for mathematics teaching

The findings have implications for students' learning, teachers' teaching, curriculum developers and policy makers. Those who wish to implement or promote the use of CWPT method of teaching in the classrooms or presentations; especially in most content related subject areas can make reference to the following factors:

- The series of tasks students engaged in, create room for them to put what is taught into practice, learn and assist their peers. This helped them improved on their question solving skills and also achieved higher in the various tests administered.
- Adequate preparation of tutoring materials: such as the activity sheets and their solution for the tutors, facilitated student's interaction with materials and their peers. The content of the tasks on the activity sheets should be sequenced in the level of difficulty.
- The tutors' approaches to the tutoring in the Class Wide Peer Tutoring lessons helped students develop interest in the lesson and also grasp the concept with ease. Tutors should be reminded of their respective roles in most sections. This could take place before during and after the lesson. It actually keeps the tutors on truck.
- Training of tutors and tutees in Class Wide Peer Tutoring is very crucial. This actually exposed students to the rudiments on CWPT with respect to the procedures, reinforcement and feedback techniques and tutor-tutee relationship. Its outmost effect is the established mutual understanding and cordial relationship among tutors and their respective tutees.

- Immediate feedbacks to tutees' work and correction of tutees errors arouses and sustained learners' interest in the lesson and result to massive achievement of the students in the experimental class.
- The peer tutoring groups should be heterogeneous and its formation should not focus on the practice of using the high achievers in all the time as tutors. Since the method benefits students without considering the specific roles played by the learners in the lesson.
- The method is actually effective for handling problems associated with differentiation or heterogeneous class of students, large class size, achievement, retention of learned concept and learners' interest in studying the subject.

5.5 Recommendations

Based on the findings, the following recommendations were made:

- Teachers are encouraged to use CWPT method as one of the possible ways in the teaching and learning quadratic function and other topics in mathematics.
- In-service training programmes (INSETS), seminars, workshops and other teacher professional development programmes should be organized to educate mathematics teachers on how to use CWPT method in teaching mathematical concepts.
- Recommendations for further research include studying a larger population of students. Findings in four mathematics classroom in two SHS school are not able to be generalized to larger settings.
- Research on a larger scale concerning the impact of CWPT method on students' achievement, retention and attitude towards the study of mathematical concepts as well as ensuring equity, diversity and increase classroom discourse (verbal interchange of ideas), are worthwhile.

- Further research concerning how CWPT method can be used to enhance problem-solving ability would be useful.
- This study focused on students' achievement and retention and perception towards the study of quadratic function. Further studies are needed to assess the impact of CWPT method on students' retention and achievement in other mathematics topics and other subjects at the SHS. Also how students should be assessed when CWPT method is employed.

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APPENDICES

Appendix A-Treatment Schedule /Scheme Of Work

Lessons	Procedure	Lesson taught Content covered [Experimental and control Group]	Duration Hours/min.
Lesson 1	Pre-Treatment	Pilot Testing Orientation	1.30.
Lesson 2		Pre test	1.30
Lesson 3	Treatment	Description of Quadratic Functions Completing Square Method Maximum or minimum value/points	1.30
Lesson 4		Sketching Curves of Quadratic Functions Identifying Parts of a Quadratic curve(Parabola)	1.30
Lesson 4		solve Quadratic Equation by method of completing the square; solve a quadratic equation by Formula	1.30
Lesson 5		Solving Quadratic Inequalities Roots of Quadratic Equation	1.30
Lesson 6		Relationship between roots and coefficient of a quadratic equation Forming Quadratic equation with giving roots.	1.30
Lesson 6	Post-Treatment	Post- test Questionnaire	1.30

Appendix B: Lesson Plans for the Experimental Group-CWPT Method

Lesson : One(1)

Subject: Elective Mathematics Year : 2 Duration: 90 minutes

Topic: Quadratic Functions

Sub-Topics Description of Quadratic Functions

-Completing Square Method

-Maximum or minimum value/points

Objective(s): By the end of the lesson, the student will be able to:

2.5.1(a) describe a given quadratic function

2.5.1(b) express at least a quadratic function in standard (vertex) form - using completing square method.

2.5.1(c) find the maximum or minimum point of a given quadratic function

RPK: (i)Students describe linear functions and find product of linear expressions:

(ii)Students draw the graph of a given function, using table value;

(iii) students factorize a given quadratic expressions completely;

(iv) Students identify a complete square and find the area.

1. Introduction (5min)

Use question and answer methods to revise learners RPK.

Question:

(I) What is linear function?

(II) Factorize the following expressions completely- (a)$x^2 + 5x + 6$; (b)$x^2 + x - 6$

2. Whole Class Discussion- (30 min)

i. Guide students to recognize quadratic functions, forms and describe (indicate) their respective shapes.

ii. Help students to use square (box) method in turn to model and complete the square of a quadratic expression that takes the form:$f(x) = x^2 + bx$.

iii. Assist students to rewrite the expression as perfect square trinomial and then as a perfect square.

iv. Assist students to summarize the various steps used in completing the squares from (iv)

v. Let students use the steps and the box method to express a given quadratic function in vertex form;

vi. Use varying examples to help students find the maximum or minimum value, turning point(vertex),line of symmetry and where it occurs in turn.

3. Class Wide Peer Tutoring (40 min.)

Let students practice how to express quadratic functions in standard form and find the maximum and minimum point using **Activity Sheet 1 and 2**.

4. Independent Practice(12 min.)

Discuss the salient points identified from the peer tutoring process and let students practice some questions individually for independent practice.

Assess the entire class and use the total score of each group to declare the winner for the day.

Teaching Learning Materials (TLM): Square Grid Board, White Board illustrations, drawing of quadratic graph in student's book Pg.52, Activity and Answer sheets

Evaluation: Use Questions on Activity Sheet 1 and 2 to evaluate the lesson.

5. Closure(3min.): Let students share whatever they have learnt in brief and bring the lesson to an end by reminding them to read on the next lesson (curve sketching and quadratic formula).

Remarks: The lesson was successfully taught. About 90% grasp the concept.

LESSON: TWO (2)

Subject: Elective Mathematics **Year:** 2 **Duration:** 90 minutes

Topic: Quadratic Functions

Sub-Topic(s): - Sketching Curves of Quadratic Functions

- Identifying Parts of a Quadratic curve(Parabola)

Objective(s): By the end of the lesson, the student will be able to:

2.5.2(a)sketch the curve of a quadratic function;

2.5.2(b) identify at least four parts of a quadratic curve.

RPK: **(I)** Student's identify the shape of a given Quadratic function;

(II) Student's find the intercepts of a given linear function;

(III) Students calculate the minimum or maximum point of a given quadratic function;

1. Introduction: Use question and answer methods to revise learners RPK.

Question: (I) Indicate the shape and the maximum/minimum value/points of the curve of the following quadratic functions?

a) $f(x) = x^2 + 4x + 3$ b)$f(x) = -x^2 + 4x + 3$

c) $f(x) = (x + 2)(x - 2)$ d) $f(x) = -(x + 2)^2 - 1$

(II) Find the intercepts of the following linear functions:

(e)$f(x) = 2x - 1$ (f) $f(x) = -2x + 1$

2. Whole Class Discussion

i. Using varying examples and illustrations, take the entire class through- the steps required in the process of sketching a quadratic function;
ii. Guide students to use the steps discussed in (I) above to sketch a given quadratic function.
iii. Guide students to identify the various parts of a parabola (maximum or minimum value, vertex, line of symmetry, intercepts, increasing or decreasing parts)

Class Wide Peer Tutoring(CWPT)

i. Let students practice how to sketch quadratic functions and indicates the various parts in their respective **peer tutoring groups**. *(use Activity sheet 3)*

Teaching and Learning Materials: Rule, pencil, calculator, Activity Sheet 3.

Core Points: Processes of Sketching Curves of Quadratic Functions

Identifying Parts of a Quadratic curve(Parabola)

Individual Practice: Assess the entire class and use the total score of each group to declare the winning group for the day.

Closure: Discuss the salient points identified from the peer tutoring process.

LESSON : THREE(3)

Duration: 90 minutes

Topic: Quadratic Equations

Sub-Topic(s): Solving Quadratic Equation using method of completing the square

Solving Quadratic Equation using Quadratic Formula

Objective(s): By the end of the lesson, the student will be able to:

2.5.2 (a) solve a quadratic equation by the method of completing square;

2.5.2(b) solve a quadratic equation by Formula;

RPK: (I) Student's familiar with the factorization method of solving quadratic equations.

(II) Student's expresses quadratic function as the sum of a perfect square and a constant(student's expresses quadratic functions in a vertex form by completing square method)

Introduction: Use question and answer methods to review learners RPK.

(I)Review the factorization methods, with the following examples:

Express the following functions in (I) in vertex form.Thus $a(x-h)^2 + k = 0$ (i) $x^2 - 6x + 8 = 0$ (ii) $3x^2 + x - 2 = 0$

Whole Class Discussion:

i. Present an equation(s) which does not factor easily.

ii. Let students brainstorm on the means of solving such equation since they do not factor easily as compared to the previous ones

iii. Use varying examples to introduce the concept of completing square method to students. Begin with the type they can easily solve again by factorization method. Example $x^2 - 6x + 8 = 0$.

[NB: Explain the goal of the completing square method as "a method that helps in writing a given equation in the form that can be factored and solve easily.

Thus $\boldsymbol{a(x-h)^2+k=0}$]

Class Wide Peer Tutoring:

Give out the completing square sheet to each group and help students to solve the first equation by completing square method on the white bord.(Guided practice)

Let the tutors assist the tutees in solving the remaining ones in their peer groups.

(Use Activity sheet 4)

NB: Let tutors remind tutees to pay attention to the appropriate signs(plus or minus). Go round to help students having trouble in completing any of the problems. (Independent practice)

Guide students to obtain the general solution (quadratic formula) for the quadratic equation by completing the squares method.

Help students solve the first equation by quadratic formula and allow students to solve the remaining questions solved by quadratic formula instead of the completing square method used earlier.

Independent practice/Closure:

Discuss the salient points identified from the peer tutoring process and let students practice some questions individually for independent practice.

Assess the entire class with similar task, mark and discuss the result with the whole class.

Teacher and Learning Materials: Calculator, Activity Sheet 4

Core Point: Solving Quadratic Equation using method of completing the square

-Solving Quadratic Equation using Quadratic Formula

Evaluation: Use Questions on Activity Sheet 4 to evaluate the lesson.

LESSON : FOUR(4)

Topic: Quadratic Inequalities

Sub-Topic(s): Solving Quadratic Inequalities

Roots of Quadratic Equation

Objectives: By the end of the lesson, the student will be able to:

2.5.2 solve Quadratic Inequalities using suitable method of solving Quadratic equation.

2.5.2(b) use the discriminant to describe the nature of roots of a given quadratic equation.

RPK: (I)Students solve linear in equalities.

(II) Student's used factorization, formula and calculator in solving quadratic equations.

Introduction: Use examples to review the various method of solving quadratic equations with students. Eg. (a) $x^2 + 3x + 2 = 0$; (b)$x^2 + 4x + 3 = 0$ (c) $3x^2 + 2x - 5 = 0$

Whole Class Discussion:

Activity 1: (i)Help students write a quadratic equation from the quadratic inequality. Eg. Inequality (a) $x^2 + 3x + 2 \leq 0$; (b)$x^2 + 4x + 3 < 0$; (c) $3x^2 + 2x - 5 \leq 0$

Equation (a) $x^2 + 3x + 2 = 0$; (b) $x^2 + 4x + 3 = 0$; (c) $3x^2 + 2x - 5 = 0$

(ii) Let students Sketch the parabola for the equation

(iii) Guide students to locate the region(s) that satisfies the inequality.

(iv) Help students write the solution for the quadratic inequality given.

(v) Use varying examples to help students grasp the concept.

Activity 2: Let students write the general quadratic formula and help them identify the expression called the discriminant from the formula Assist students to use the discriminant to describe the nature of the roots of

quadratic equations as equal, real and unequal and imaginary.

Class Wide Peer Tutoring:

- Give out the **Activity sheet 5** to each group and let tutors guide their tutees solve the problems in turn (Guided practice)
- Let the tutors assess the tutees work and guide then when necessary; in solving the remaining questions in their peer groups.

[**NB**: Remind students to pay attention to the inequality symbols as they write the solution. Go round to help students having trouble in solving some of the problems on the sheet]

Independent practice:

Discuss the salient points identified from the peer tutoring process and let students practice some questions individually for independent practice.

Core Points:

- Solving Quadratic Inequalities
- Required Steps:
- Write the inequality
- Write the corresponding equation
- Sketch the curve of the equation
- Locate the regions that satisfy the inequality from the curve
- Write the solution for the inequality
- Finding Roots of Quadratic Equation
- Using the discriminant to describe the nature of roots of a given quadratic

Teaching and Learning Materials: Calculator, Practice and answer sheets.

Closure: Assess the entire class with similar task and discuss the result with them.

LESSON : FIVE(5)

Duration: 90min.

Topic: Roots of quadratic Equations

Sub-Topic(s): - Relationship between roots and coefficient of a quadratic equation

-Forming Quadratic equation with giving roots.

Objective(s): By the end of the lesson, the student will be able to:

2.5.2(a) find relationship between the roots $\boldsymbol{\alpha}$ ***and*** $\boldsymbol{\beta}$ and constants: $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$

of a quadratic equations.

2.5.2(b)use these relations to write other equations with given roots.

(1) Introduction:

Use question and answer methods to review learners RPK.

Question: (I) find the solution or roots, thus α, β of the following quadratic equations by the formula method. Eg $2x^2 + 10x + 8 = 0$

ANSWER: $\alpha = x = -1$; $\beta = x = -4$

(2) Whole Class Discussions

i. let students write the general form of a quadratic equation and chose $\boldsymbol{\alpha}$and $\boldsymbol{\beta}$ to be the roots .

ii. Ask students to assign $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ to the respective roots of the equation; $ax^2 + bx + c = 0$.

iii. Guide students to find the sum and product of the roots in turn.

iv. Assist them to write a relationship between the roots and the coefficients

v. Assist students to form new equations when given the roots.

vi. Help students to note the substitute's for the following identities since they are necessary in writing other equations with given roots:

$[(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta]$ $[(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$] etc.

RPK:

(I)Students states the two general solution or roots of a quadratic equation that takes the form: $x^2 + bx + c = 0$.

(3) CWPT:

Give out the **Activity sheet 6** to each group and help students solve the first problem (Guided practice) Let the tutors assist the tutees in solving the remaining ones in their peer groups.

[NB: Remind students to pay attention to the substitutes for the identities.

Go round to help students having trouble in solving some of the problems on the sheet]

Teaching and Learning Materials: Calculator, Practice and answer sheets 6.

Core Points: Finding Relationship between roots and coefficient of a quadratic equation; Forming Quadratic equation with giving roots.

(4) Independent practice/Closure:

Let students solve similar questions on the concepts taught individually.

Evaluation: Problems on Activity Sheet 6

Closure: Let students share what they have learned from the lesson and bring the lesson to an end.

Remarks: The lesson was successfully taught and Students' responses were positive.

::

Appendix C: Lesson Plans for the Control Group-Conventional Method)

Lesson : One(1)

Subject: Elective Mathematics **Year : 2**

Duration: 90 minutes

Topic: Quadratic Functions

Sub-Topics: Description of Quadratic Functions

Completing Square Method

Maximum or minimum value/points

Objective(s): By the end of the lesson, the student will be able to:

2.5.1(a) describe a given quadratic function

2.5.1(b) express at least a quadratic function in standard (vertex) form - using completing square method.

2.5.1(c) find the maximum or minimum point of a given quadratic function

RPK: (i)Students describe linear functions and find product of linear expressions:

(ii)Students draw the graph of a given function, using table value;

(iii) students factorize a given quadratic expressions completely;

(iv) Students identify a complete square and find the area.

1. Introduction (5min)

Use question and answer methods to revise learners RPK. For example Questions (I) What is linear function? (II) Factorize the following expressions completely- (a)$x^2 + 5x + 6$; (b)$x^2 + x - 6$

<u>TEACHER AND LEARNER ACTIVITIES</u>

Actitvity1: Help students recognize quadratic functions, forms and describe (indicate) their respective shapes.

Activity 2:(i)Guide students to use square (box) method in turn to model and complete the square of a quadratic expression that takes the form: $f(x) = x^2 + bx$.

(ii)Assist students to rewrite the expression as perfect square trinomial and then as a perfect square.

Activity 3:(i)Use varying examples to help students find the maximum or minimum value, turning point(vertex),line of symmetry and where it occurs in turn.

(ii)Let students practice how to express quadratic functions in standard form.

Teaching and Learning Materials (TLM): Square Grid Board, White Board illustrations, drawing of quadratic graph in student's book Pg.52.

Core Points: Description of Quadratic Functions

The Process of Completing Square Method

Finding Maximum or minimum value/points

Evaluation: Express the following functions as a perfect square trinomial and then a perfect square: 1) f(x) = $x^2 + 10x + ?$

2)f(x) = $x^2 - 4x + ?$ etc.

Finding Maximum and Minimum Value/Point:

(1) $f(x) = x^2 + 4x + 3$ (2) $f(x) = 3x^2 + 18x + 3$

Closure: Let students share whatever they have learnt in brief and bring the lesson to an end by reminding them to read on the next lesson (curve sketching and quadratic formula).

Remarks: The lesson was successfully taught. About 60% grasped the concept. Learners' Participation was quite good

LESSON : TWO(2)

Subject: Elective Mathematics Year : 2 **Duration:** 90 minutes

Topic: Quadratic Functions

Sub-Topic(s): - Sketching Curves of Quadratic Functions

- Identifying Parts of a Quadratic curve(Parabola)

Objective(s): By the end of the lesson, the student will be able to:

2.5.2(a)sketch the curve of a quadratic function;

2.5.2(b) identify at least four parts of a quadratic curve.

RPK: **(I)** Students identify the shape of a given Quadratic function;

(II) Students find the intercepts of a given linear function;

(III) Students calculate the minimum or maximum point of a given quadratic function;

Introduction: Use question and answer methods to revise learners RPK.

Question: (I) Indicate the shape and the maximum/minimum value/points of the curve of the following quadratic functions?

a) $f(x) = x^2 + 4x + 3$; b) $f(x) = -x^2 + 4x + 3$

c) $f(x) = (x + 2)(x - 2)$ d) $f(x) = -(x + 2)^2 - 1$

(ii) Find the intercepts of the following linear functions:

(e)$f(x) = 2x - 1$ (f) $f(x) = -2x + 1$

TEACHER AND LEARNER ACTIVITIES:

i. Using varying examples and illustrations, take the entire class through- the steps required in the process of sketching a quadratic function;

ii. Guide students to use the steps discussed in (I) above to sketch a given quadratic function.(iii) Guide students to identify the various parts of a parabola (maximum or minimum value, vertex, line of symmetry, intercepts, increasing or decreasing parts). (iii) Use examples and non-examples to help students grasp the skills of sketching and dentifying parts of quadratic functions.

Teaching and Learning Materials: Rule, pencil, calculator.

Core Points: Processes of Sketching Curves of Quadratic Functions

Identifying Parts of a Quadratic curve(Parabola)

Evaluation: Refer to *Effective Maths*.Pg56, Exercise 2.5B;

Sketch the curves of the following functions: (1) $f(x) = x^2 + 4x + 3$

(2) $f(x) = x^2 + 4x + 3$ (3) $f(x) = x^2 - 6x + 8$

Closure: Discuss the salient points with students and bring the lesson to an end by stating the topics for the next lessons. **Remarks:** The Lesson was successful.

LESSON : THREE(3)

Duration: 90 minutes

Topic: Quadratic Equations

Sub-Topic(s): - Solving Quadratic Equation using method of completing the square

-Solving Quadratic Equation using Quadratic Formula

Objective(s): By the end of the lesson, the student will be able to:

2.5.2(a) solve quadratic equation by method of completing the square;

2.5.2(b) solve a quadratic equation by Formula;

RPK: **(I)** The Students solve quadratic equations by factorization method

(II) Students express quadratic function as the sum of a perfect square and a constant (student's expresses quadratic functions in a vertex form by completing square method)

Introduction: Use question and answer methods to review learners RPK. **Questions:**

(I)Review the factorization methods, with the following examples:

(II) Express the functions in (I) in vertex form. Thus $a(x - h)^2 + k = 0$

Teacher and Learner Activities:

i. Present an equation(s) which does not factor easily. For example:

ii. Let students brainstorm on the means of solving such equation since they do not factor easily as compared to the previous ones.

iii. Use varying examples to introduce the concept of completing square method to students. Begin with the type they can easily solve again by factorization method. Example: $x^2 - 6x + 8 = 0$.

[NB: Explain the goal of the completing square method as "a method that helps in writing a given equation in the form that can be factored and solve easily. Thus $\boldsymbol{a(x-h)^2+k=0}$]

iv. Guide students to obtain the general solution (quadratic formula) for the quadratic equation by completing square method.

v. Help students solve the first equation by quadratic formula and allow students to solve the remaining questions solved by quadratic formula instead of the completing square method used earlier.

Teaching and Learning Materials(TLM): Calculator

Core Point: Solving Quadratic Equation by Completing square Method
Solving Quadratic Equation by Formula Method

Evaluation: *Effective Maths.* Pg60, Exercise 2.5C

Closure: Assess the entire class with similar task, mark and discuss the result with the whole class.

Remarks: The Lesson was successful.

LESSON : FOUR(4)

Topic: Quadratic Inequalities

Sub-Topic(s): -Solving Quadratic Inequalities

Objectives: By the end of the lesson, the student will be able to:

2.5.2(a) solve Quadratic Inequalities using suitable method of solving Quadratic equation.

2.5.2(b) use the discriminant to describe the nature of roots of a given quadratic equation.

RPK: (I)Students solve linear in equalities.

(II) Student's used factorization, formula and calculator in solving quadratic equations. **Introduction:** Use examples to review the various method of solving quadratic equations with students.

Eg. (a) $x^2 + 3x + 2 = 0$; (b) $x^2 + 4x + 3 = 0$ (c) $3x^2 + 2x - 5 = 0$

TEACHER AND LEARNER ACTIVITIES:

Activity1:

i. Help students write a quadratic equation from the quadratic inequality. Eg. Inequality (a) $x^2 + 3x + 2 \leq 0$; (b)$x^2 + 4x + 3 < 0$; (c) $3x^2 + 2x - 5 \leq 0$ Equation (a) $x^2 + 3x + 2 = 0$; (b) $x^2 + 4x + 3 = 0$; (c) $3x^2 + 2x - 5 = 0$

ii. Let students Sketch the parabola for the equation

iii. Guide students to locate the region(s) that satisfies the inequality.

iv. Help students write the solution for the quadratic inequality given.

v. Use varying examples to help students grasp the concept.

Activity 2:

i. Let students write the general quadratic formula and help them identify the expression called the discriminant from the formula

ii. Assist students to use the discriminant to describe the nature of the roots of quadratic equations as equal, real and unequal and imaginary.

[NB: Remind students to pay attention to the inequality symbols as they write the solution]

Teaching and Learning Materials: Calculator,

Core Points:

- Solving Quadratic Inequalities
- Steps: Write the inequality
- Write the corresponding equation
- Sketch the curve of the equation
- Locate the regions that satisfy the inequality from the curve
- Write the solution for the inequality
- Finding Roots of Quadratic Equation
- Using the discriminant to describe the nature of roots of a given quadratic equation.

Evaluation: Effective Maths. Pg61 & 64 Ex.2.5D and 2.5E

Closure: Assess the entire class with similar task and discuss the result with them. Discuss the salient points with students and bring the lesson to an end.

Remarks: The lesson was successful.

LESSON : FIVE(5)

Duration: 90 minutes

Topic: Roots of quadratic Equations

Sub-Topic(s): - Relationship between roots and coefficient of a quadratic equation

-Forming Quadratic equation with giving roots.

Objective(s): By the end of the lesson, the student will be able to:

2.5.2(a)find relationship between the roots $\boldsymbol{\alpha}$ ***and*** $\boldsymbol{\beta}$ and constants: $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ of a quadratic equations.

2.5.2(b)use these relations to write other equations with given roots.

RPK: (I)Students states the two general solution or roots of a quadratic equation that takes

the form: $ax^2 + bx + c = 0$.

Introduction: Use question and answer methods to review learners

RPK. Question: (I) find the solution or roots, thus α, β of the following quadratic equations by the formula method. Eg $2x^2 + 10x + 8 = 0$.

Answer: $\alpha = x = -1$; $\beta = x = -4$

TEACHER AND LEARNER ACTIVITIES:

Activity 1:

i. Let students write the general form of a quadratic equation and chose $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ to be the roots .
ii. Ask students to assign $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ to the respective roots of the equation; $ax^2 + bx + c = 0$.
iii. Guide students to find the sum and product of the roots in turn.
iv. Assist them to write a relationship between the roots and the coefficients

Activity 2: Assist students to form new equations when given the roots.

Help students to note the substitute's for the following identities since they are necessary in writing other equations with given roots: $[(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta]$ $[(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$] etc.

[**NB**: Remind students to pay attention to the substitutes for the identities. Let students practice some related questions individually on the concept]

Teaching and Learning Materials: Calculator

Core Point: Finding Relationship between roots and coefficient of a quadratic equation;

Forming Quadratic equations with giving roots.

Closure: Discuss the salient points identified from the lesson and bring the lesson to an end.

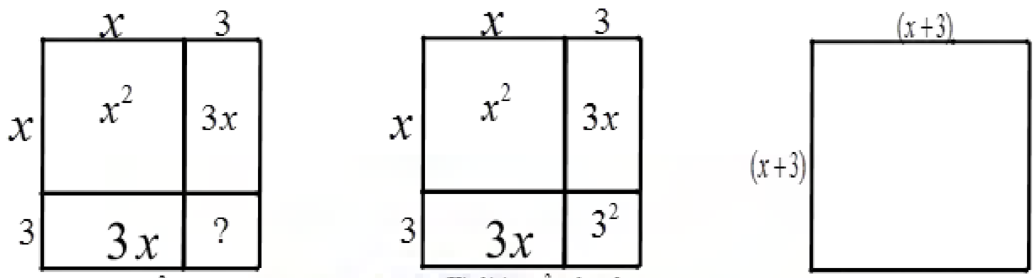
Evaluation: Effective Maths. Pg.64-65 Exx 2.5E

Remarks: Lesson was successful. Students' responses were good

Appendix D: Activity and answer sheets for the CWPT lessons

Activity Sheet One(1) :

<u>Task : Completing the Square</u>

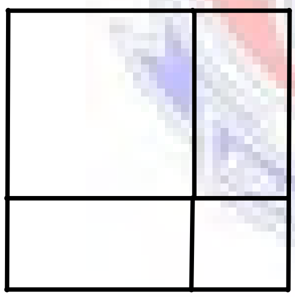


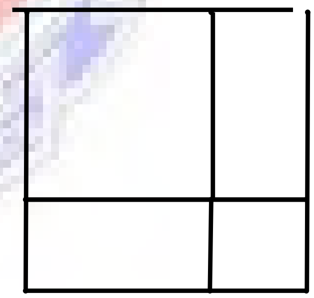
(I) $f(x)=x^2+6x+?$ (II) $f(x)=x^2+6x+9$ (III) $f(x)=(x+3)^2$

The function given : $f(x)=x^2+6x+?$

From the squares in (ii) and (iii) ,

The perfect square trinomial is $f(x)=x^2+6x+9$ and the Perfect Square is $f(x)=(x+3)^2$.





Now Try the following:

Write the following functions as a **perfect square trinomial** and a **perfect square**

1. $f(x)=x^2+10x+?$

. From the diagram,

The perfect Square Trinomial is.........

The Perfect square is ....

2. $f(x)=x^2+4x+?$

From the diagram,

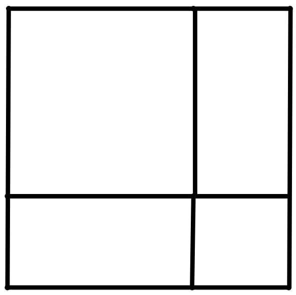
The Perfect Square Trinomial is ......

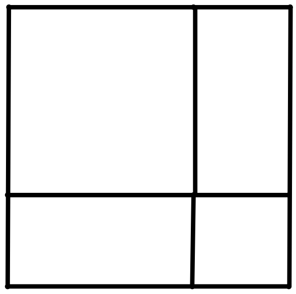
The perfect square is ....

3. $f(t)=t^2-2t+?$

4. $f(x)=x+5x+?$

From the diagram,





From the diagram,

The Perfect Square Trinomial is……… The Perfect Square Trinomial is …

The Perfect square is ………………. The perfect square is ……………..

Activity Sheet Two(2):

<u>Task: Finding Maximum and Minimum Value/Point</u>

Using the steps and the box method to express a quadratic function, $f(x) = ax^2 + bx + c$ in the form $f(x) = a(x + d)^2 + k$, where k is the maximum or minimum value.

Steps

Rewrite the following quadratic function in the form $(x) = a(x - h)^2 + k$, where k is the maximum or minimum value, Vertex, V(h,k)

Example 1: $f(x) = x^2 + 4x + 3$

$f(x) = x^2 + 4x + 3?$

$f(x) = x^2 + 4x + 3 + 4 - 4$

$f(x) = x^2 + 4x + 4 + 3 - 4$

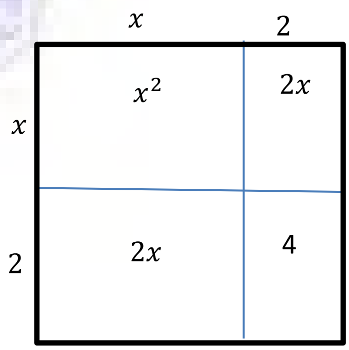
$f(x) = (x+2)^2 + 3 - 4$

$f(x) = (x+2)?^2 - 1$

$f(x) = a(x-h)^2 + k$

$Minimumvalue: y = -1$

$Vertex: (-2, -1)$



Example 2:

$$f(x) = 3x^2 + 18x + 3]$$
$$f(x) = 3[x^2 + 6x + 1]$$
$$f(x) = 3[x^2 + 6x + 1 + 9 - 9]$$
$$f(x) = 3[x^2 + 6x + 9 + 1 - 9]$$
$$f(x) = 3[(x+3)^2 + 1 - 9]$$
$$f(x) = 3[x^2 + 6x + 1]$$
$$f(x) = 3[x^2 + 6x + 1 + 9 - 9]$$
$$f(x) = 3[x^2 + 6x + 9 + 1 - 9]$$
$$f(x) = 3[(x+3)^2 + 1 - 9]$$
$$f(x) = 3[(x+3)^2 - 8]$$
$$f(x) = 3(x+3)^2 - 3(8)]$$
$$f(x) = 3(x+3)^2 - 24]$$
$$Vertex: (-3, -24)$$

Now try the following

Express the following functions in the form $f(x) = a(x-h)^2 + k$. Hence indicate the minimum or maximum value and the vertex.

1. $f(x) = 5x^2 + 10x+15$
2. $f(x) = 2x^2 - 4x + 16$
3. $f(t) = -2t^2 - 2t + 8$
4. $f(x) = ax^2 + bx + c$

<u>Activity Sheet Three(3) : CURVE SKETCHING</u>

Steps:

Function, $f(x) = ax^2 + bx + c$;

1. Let $f(x) = y$; $y = ax^2 + bx + c$
2. Determine the Nature of the turning point (thus whether maximum or minimum).
3. Find the turning point (use the method of completing the square), V(h,k)
4. Determine the Intercepts on the axes:

a. For x-intercept: put y=0 and solve for x ; (x_1-value,0) and (x_2 - value,0)

b. For y-intercept: put x=0 and solve for y; (0,y-value).

5. Plot the points on a well labeled Cartesian (X-Y) plane and sketch the curve.

Example: Sketch the curve of the function, $f(x) = x^2 + 4x + 3$.

Solution :

Let $f(x) = y$

(1)Nature of the turning Point: Minimum

(2)Turning Points

$f(x) = x^2 + 4x + 3$

$f(x) = x^2 + 4x + 3 + 4 - 4$

$f(x) = x^2 + 4x + 4 + 3 - 4$

$f(x) = (x+2)^2 + 3 - 4$

$f(x) = (x+2)^2 - 1$

$f(x) = a(x-h)^2 + k$

$Minimum value: y = -1$

$Vertex: (-2, -1)$

(3)Intercepts: Let $f(x) = y$;

$f(x) = x^2 + 4x + 3$

$y = x^2 + 4x + 3$

Let $x = 0$ and solve for y;

$y = (0)^2 + 4(0) + 3$

$y = 3$

$(0, 3)$

Let $y = 0$ and solve for x

$0 = x^2 + 4x + 3$

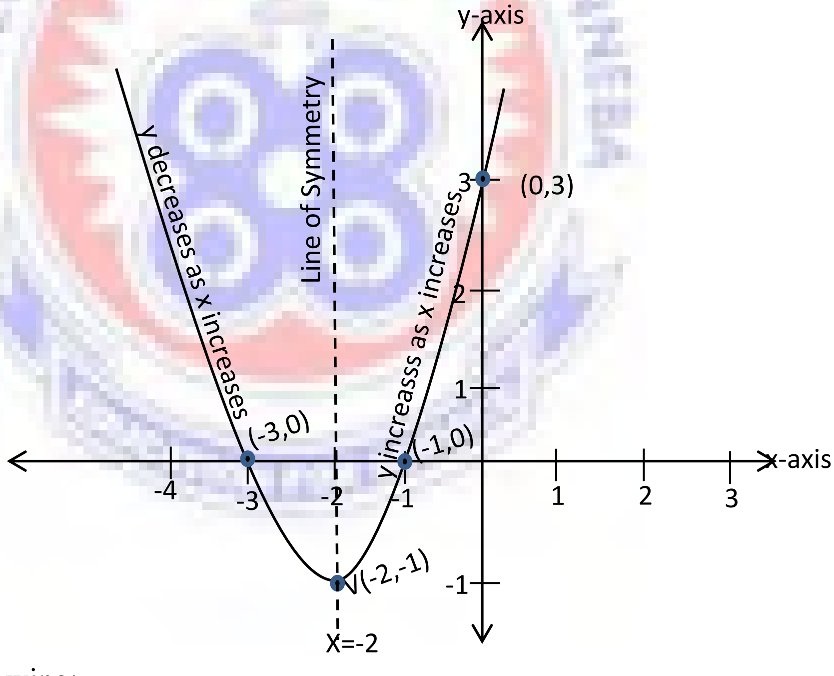
$0 = x^2 + x + 3x + 3$

$0 = x(x+1) + 3(x+1)$

$0 = (x+1)(x+3)$

$x = -1$ or $x = -3$

$(-1, 0)$ and $(-3, 0)$



Now try the following:

Sketch the following functions indicating all the turning points, intercepts and the lines of symmetry.

(1) $f(x) = x^2 - 6x + 8$

(2) $f(x) = 4x^2 + 6x - 4$

(5) $f(x) = 24 + 18x - 3x^2$

(6) $f = 3x^2 - x - 6$

(3) $f(x) = -x^2 + 6x - 2$

(4) $f(x) = 6 + 2x - x^2$

ACTIVITY SHEET FOUR(4):

A// Solve the following quadratic functions by the method of completing the square:

1. $4(x-1)^2 - 16 = 0$
2. $-3(x-2)^2 + 12 = 0$
3. $x^2 - 6x + 8 = 0$
4. $3 + 4t - 2t^2 = 0$

B// Solve the following quadratic functions by the formula method:

1. $2(x-1)^2 - 16 = 0$
2. $-3(x-2)^2 + 1 = 0$
3. $x^2 - 6x + 8 = 0$
4. $3 + 4t - 2t^2 = 0$

ACTIVITY SHEET FIVE(5)

Task: Solving Quadratic Equation

Find the set of values of the variables for each of the following:

1. $4(x-1)^2 - 16 \leq 0$
2. $-3(x-2)^2 + 12 \geq 0$
3. $x^2 - 6x + 8 < 0$
4. $3 + 4t - 2t^2 > 0$

Using discriminant, describe the nature of roots of the following quadratic equations.

1. $x^2 - 6x + 8 = 0$
2. $3 + 4t - 2t^2 = 0$
3. $2(x-1)^2 - 16 = 0$
4. $-3(x-2)^2 + 1 = 0$

Combination of discriminant and Quadratic Inequalities:

5. Find the range of values of k if the equation $kx^2 - 4(k-1)x + 9 = 0$ has real roots.

6. The roots of the quadratic equation $x^2 - 2(3k+1)x + 7(2k+3) = 0$ where k is a constant are imaginary and do not exist. Find the range of values of k.

7. If the roots of the quadratic equation $2x^2 + 3(2p+1)x - 5(p-1) = 0$ where p is constant are real, find the rang of values of p.

ACTIVITY SHEET SIX(6)

Task: Quadratic inequality and Discriminant

Solve the following quadratic Inequalities:

1. $4(x-1)^2 - 16 \leq 0$
2. $-3(x-2)^2 + 12 \geq 0$
3. $x^2 - 6x + 8 < 0$
4. $3 + 4t - 2t^2 > 0$

Using discriminant, describe the nature of roots of the following quadratic equations.

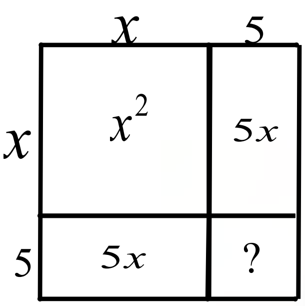
1. $x^2 - 6x + 8 = 0$
2. $3 + 4t - 2t^2 = 0$
3. $2(x-1)^2 - 16 = 0$
4. $-3(x-2)^2 + 1 = 0$

Appendix E: Answer Sheets for The Activity Sheets

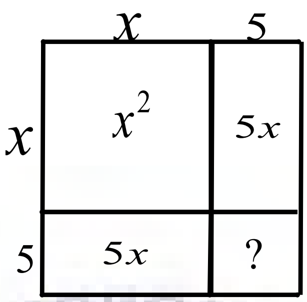
Answer sheet for Activity Sheet One

<u>Task : Completing Square Method</u>

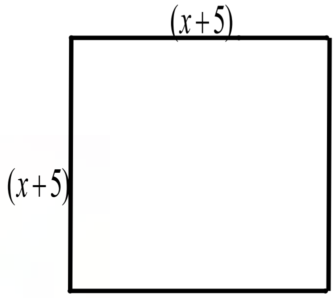
1. Example: $f(x) = x^2 + 10 + ?$



(I) $f(x) = x^2 + 10x + ?$



(II) $f(x) = x^2 + 10x + 25$



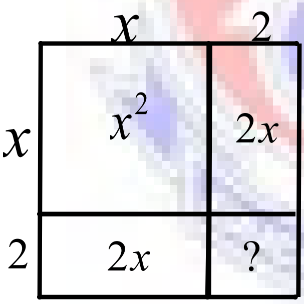
(III) $f(x) = (x+5)^2$

The function given : $f(x) = x^2 + 10x + ?$

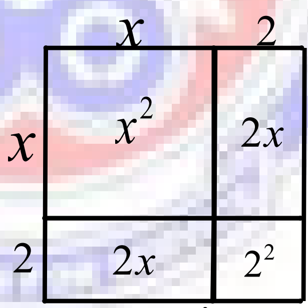
From the squares in (ii) and (iii),

The perfect square trinomial is $f(x) = x^2 + 10x + 25$ and the Perfect Square is $f(x) = (x+5)^2$

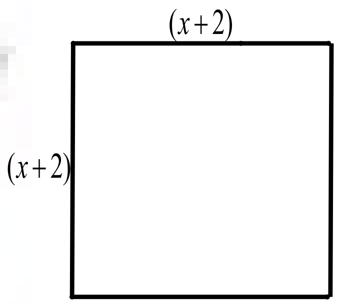
2. Example: $f(x) = x^2 + 4x + ?$



(I) $f(x) = x^2 + 4x + ?$



(II) $f(x) = x^2 + 4x + 4$



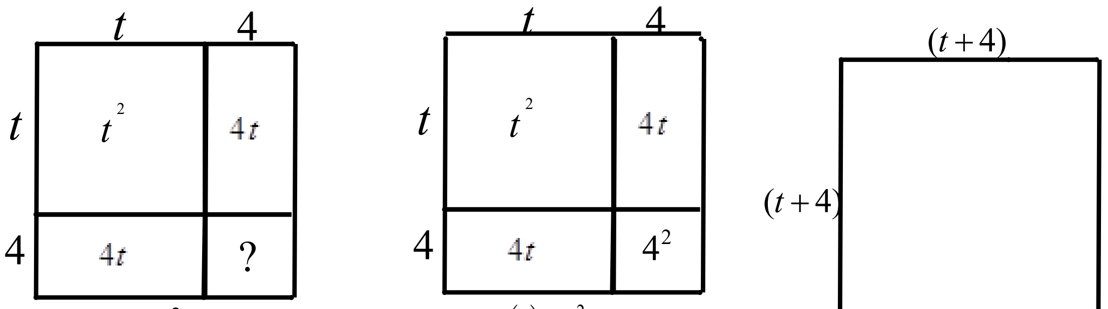
(III) $f(x) = (x+2)^2$

The function given : $f(x) = x^2 + 4x + ?$

From the squares in (ii) and (iii),

The perfect square trinomial is $f(x) = x^2 + 4x + 4$ and the Perfect Square is $f(x) = (x+2)^2$

3. $\mathbf{f(t)} = t^2 + 8t + ?$



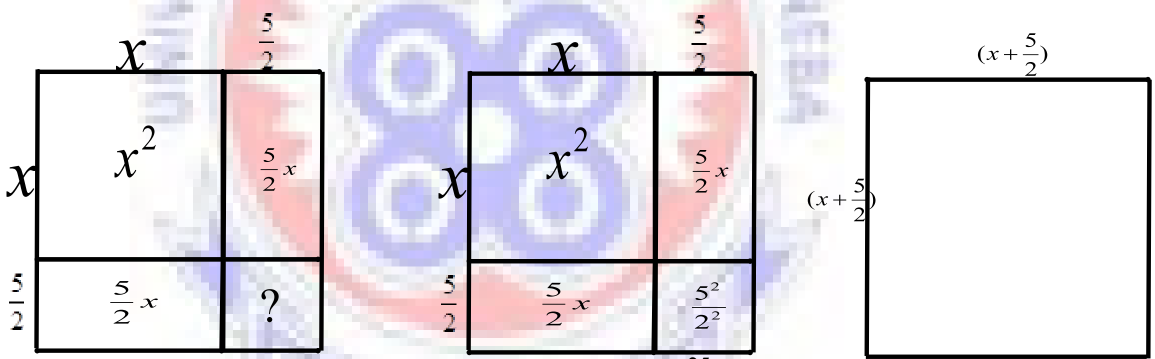
(i) $f(t) = t^2 + 8t + ?$ (II) $f(t) = t^2 + 8t + 16$ (III) $f(t) = (t+4)^2$

The function given : $f(t) = t^2 + 8t + ?$

From the squares in (ii) and (iii),

The perfect square trinomial is $f(t) = t^2 + 8t + 16$ and the Perfect Square is $f(t) = (t+4)^2$

4. $f(x) = x^2 + 5x + ?$



$f(x) = x^2 + 5x + ?$ (II) $f(x) = x^2 + 5x + \frac{25}{4}$ (III) $f(x) = (x+\frac{5}{2})^2$

The function given : $f(x) = x^2 + 5x + ?$

From the squares in (ii) and (iii),

The perfect square trinomial is $f(x) = x^2 + 5x + \frac{25}{4}$ and the Perfect Square is

$f(x) = (x+\frac{5}{2})^2$

ANSWER SHEET FOR ACTIVITY SHEET TWO(2)

Task: Finding Maximum and Minimum Value/Point

1.

$$f(x)=5x^2+10x+15$$
$$f(x)=5[x^2+2x+3]$$
$$f(x)=5[x^2+2x+3+?(1)^2-(1)^2]$$
$$f(x)=5[x^2+2x+(1)^2+3-(1)^2]$$
$$f(x)=5[(x+1)^2+3-1]$$
$$f(x)=5(x+1)^2+5(2)$$
$$f(x)=5(x+1)^2+10$$
$$f(x)=a(x-h)^2+k$$
$$Minimum value: y=10$$
$$Vertex:(-1,10)$$

$x^2+2x+?$

	x	1
x	x^2	x
1	x	$(1)^2$

2.

$$f(x)=2x^2-12x+16$$
$$f(x)=2[x^2-6x+8]$$
$$f(x)=2[x^2-6x+8+(-3)^2-(-3)^2]$$
$$f(x)=2[x^2-6x+(-3)^2+8-(-3)^2]$$
$$f(x)=2[(x-3)^2+8-9]$$
$$f(x)=2(x-3)^2+2(-1)$$
$$f(x)=2(x-3)^2-2$$
$$f(x)=a(x-h)^2+k$$
$$Minimum value: y=-2$$
$$Vertex:(3,-2)$$

$x^2-6x+?$

	x	-3
x	x^2	$-3x$
-3	$-3x$	$(-3)^2$

3.

$$f(t) = -2t^2 - 2t + 8$$
$$f(t) = -2[t^2 + t - 4]$$
$$f(t) = -2[t^2 + t - 4 + (\frac{1}{2})^2 - (\frac{1}{2})^2]$$
$$f(t) = -2[t^2 + t + (\frac{1}{2})^2 - 4 - (\frac{1}{2})^2]$$
$$f(t) = -2[(t + \frac{1}{2})^2 - 4 - (\frac{1}{2})^2]$$
$$f(t) = -2[(t + \frac{1}{2})^2 - 4 - (\frac{1}{4})]$$
$$f(t) = -2[(t + \frac{1}{2})^2 - (\frac{17}{4})]$$
$$f(t) = -2(t + \frac{1}{2})^2 - (\frac{17}{2})$$
$$Maximum\,Value = -\frac{17}{2}$$
$$Maximum\,Point = (-\frac{1}{2}, -\frac{17}{2})$$

$t^2 - t + ?$

	t	$\frac{1}{2}$
t	t^2	$\frac{t}{2}$
$\frac{1}{2}$	$\frac{t}{2}$	$(\frac{1}{2})^2$

4.

$$f(x) = ax^2 + bx + c$$
$$f(x) = a[x^2 + \frac{b}{a}x + \frac{c}{a}]$$
$$f(x) = a[x^2 + \frac{b}{a}x + \frac{c}{a} + (\frac{b}{2a})^2 - (\frac{b}{2a})^2]$$
$$f(x) = a[x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 + \frac{c}{a} - (\frac{b}{2a})^2]$$
$$f(x) = a[(x + \frac{b}{2a})^2 + \frac{c}{a} - (\frac{b}{2a})^2]$$
$$f(x) = a[(x + \frac{b}{2a})^2 + \frac{c}{a} - \frac{b^2}{4a^2}]$$
$$f(x) = a(x + \frac{b}{2a})^2 + \frac{4ac - b^2}{4a}$$
$$f(x) = a(x - h)^2 + k$$
$$Minimum\,value: y = \frac{4ac - b^2}{4a}$$
$$Vertex: (\frac{-b}{2a}, \frac{4ac - b^2}{4a})$$

$x^2 + bx + ?$

	x	$\frac{b}{2a}$
x	x^2	$\frac{b}{2a}x$
$\frac{b}{2a}$	$\frac{b}{2a}x$	$(\frac{b}{2a})^2$

ANSWER SHEET FOR ACTIVITY SHEET THREE(3)

<u>Task: CURVE SKETCHING</u>

Steps: Function, $f(x) = ax^2 + bx + c$;

1. Let $f(x) = y$; $y = ax^2 + bx + c$
2. Determine the Nature of the turning point (thus whether maximum or minimum).
3. Find the turning point (use the method of completing the square), V(h,k)
4. Determine the Intercepts on the axes:

a. For x-intercept: put y=0 and solve for x ; (x1-value,0) and (x2 value,0)

b. For y-intercept: put x=0 and solve for y; (0,y-value).

5. Plot the points on a well labeled Cartesian (X-Y) plane and sketch the curve.

Example: Sketch the curve of the function, $f(x) = x^2 + 4x + 3$.

Solution : $f(x) = x^2 + 4x + 3$

$f(x) = x^2$

Let $f(x) = y$

(1)Nature of the turning Point:Minimum

(2)*TurningPoints*

$f(x) = x^2 + 4x + 3$

$f(x) = x^2 + 4x + 3 + 4 - 4$

$f(x) = x^2 + 4x + 4 + 3 - 4$

$f(x) = (x+2)^2 + 3 - 4$

$f(x) = (x+2)^2 - 1$

$f(x) = a(x-h)^2 + k$

Minimum Value : $y = -1$

Vertex : $(-2, -1)$

(3)Intercepts:

Let $f(x) = y$;

$f(x) = x^2 + 4x + 3$

$y = x^2 + 4x + 3$

Let $x = 0$ and solve for y;

$y = (0)^2 + 4(0) + 3$

$y = 3$

(0,3)

Let $y = 0$

$0 = x^2 + 4x + 3$

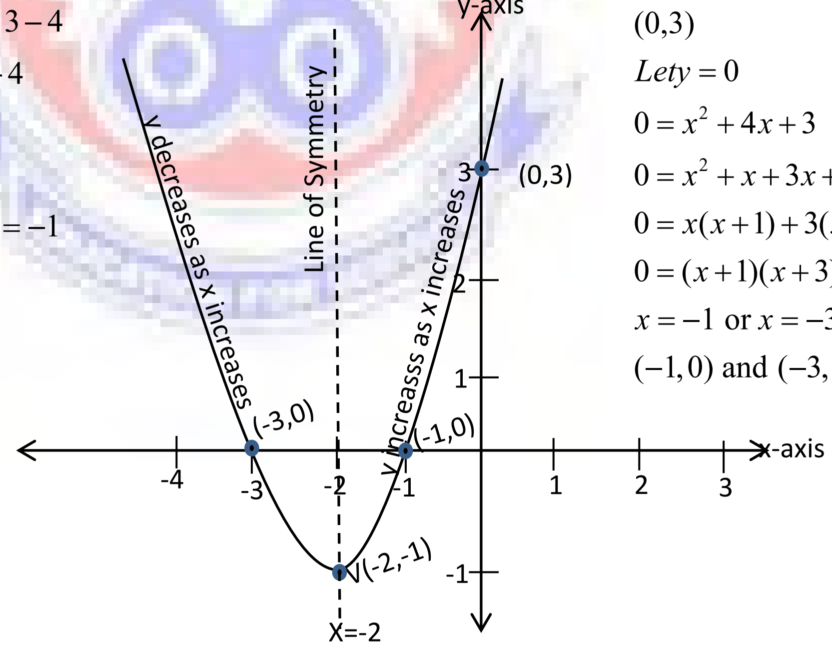
$0 = x^2 + x + 3x + 3$

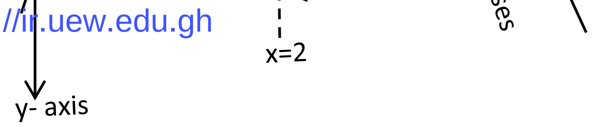
$0 = x(x+1) + 3(x+1)$

$0 = (x+1)(x+3)$

$x = -1$ or $x = -3$

$(-1, 0)$ and $(-3, 0)$





No. 3

$f(x) = -x^2 + 4x - 3$

Solution:

Let $f(x) = y$

(1)Nature of the turning Point: Minimum

(3) Intercepts:

Let f(x)=y;

$f(x) = -x^2 + 4x - 3$

$y = -x^2 + 4x - 3$

Let x=0 and solve for y;

$y = -(0)^2 + 4(0) - 3$

$y = -3$ **(0,-3)**

Let y=0

$0 = -x^2 + 4x - 3$

$0 = -x^2 + 3x + 1x - 3$

$0 = -x(x-3) + 1(x-3)$

$0 = (x-3)(-x+1)$

$x = 3 \ or \ x = 1$

(1,0) and (3,0)

(2) Turning Points

$f(x) = -x^2 + 4x - 3$

$f(x) = -[x^2 - 4x + 3]$

$f(x) = -[x^2 - 4x + (-2)^2 - (-2)^2 + 3$

$f(x) = -[(x-2)^2 - 4 + 3]$

$f(x) = -(x-2)^2 + 1$

$f(x) = a(x-h)^2 + k$

Minimum value: y= 1

No.2 $f(x) = x^2 - 6x + 8$

Solution:

Let $f(x) = y$

(1)Nature of the turning Point:Minimum

(2)*TurningPoints*

(3)Intercepts:Let $f(x) = y$;

$f(x) = x^2 - 6x + 8$

$y = x^2 - 6x + 8$

Let $x = 0$ and solve for y;

$y = (0)^2 - 6(0) + 8$

$y = 8$

$(0, 8)$

Let $y = 0$

$0 = x^2 - 6x + 8$

$0 = x^2 - 2x - 4x + 8$

$0 = x(x-2) - 4(x-2)$

$0 = (x-2)(x-4)$

$x = 2$ or $x = 4$

$(2,0)$ and $(4,0)$

ANSWER SHEET FOR ACTIVITY SHEET FOUR (4a)

Task: Solving Quadratic Equation

1. Solution:

$$4(x-1)^2 - 16 = 0$$

$$4(x-1)^2 = 16$$

$$(x-1)^2 = 4$$

$$\sqrt{(x-1)^2} = \pm\sqrt{4}$$

$$x - 1 = \pm 2$$

$$x = \pm 2 + 1$$

$x = 2 + 1$ or $x = 2 - 1$

$x = 3$ or $x = 1$

2. Solution:

$$-3(x-2)^2 + 12 = 0$$

$$-3(x-2)^2 = -12$$

$$(x-2)^2 = 4$$

$$\sqrt{(x-2)^2} = \pm\sqrt{4}$$

$$x - 2 = \pm 2$$

$$x = \pm 2 + 2$$

$x = 2 + 2$ or $x = 2 - 2$

$x = 4$ or $x = 0$

3. Solution:

$$2x^2 - 12x + 16 = 0$$

$$2[x^2 - 6x + 8] = 0$$

$$x^2 - 6x + 8 = 0$$

$$x^2 - 6x + 8 + (-3)^2 - (-3)^2 = 0$$

$$x^2 - 6x + (-3)^2 + 8 - (-3)^2 = 0$$

$$(x-3)^2 + 8 - 9 = 0$$

$$(x-3)^2 - 2) = 0$$

$$(x-3)^2 = 2$$

$$\sqrt{(x-3)^2} = \pm\sqrt{2}$$

$$x - 3 = \pm\sqrt{2}$$

$$x = \pm\sqrt{2} + 3$$

4. Solution:

$$3x^2 + 18x + 3 = 0$$

$$3[x^2 + 6x + 1] = 0$$

$$x^2 + 6x + 1 = 0$$

$$x^2 + 6x + 1 + (3)^2 - (3)^2 = 0$$

$$x^2 + 6x + (3)^2 + 1 - (3)^2 = 0$$

$$(x+3)^2 + 1 - 9 = 0$$

$$(x+3)^2 - 8 = 0$$

$$(x+3)^2 = 8$$

$$\sqrt{(x+3)^2} = \pm\sqrt{8}$$

$$x + 3 = \pm\sqrt{8}$$

$$x = \pm\sqrt{8} - 3$$

$x = \sqrt{8} - 3$ or $x = -\sqrt{8} - 3$

ANSWER SHEET FOR ACTIVITY SHEET FOUR (4b)

Task: Solving Quadratic Equation by Quadratic formula

General Quadratic Equation: $ax^2 + bx + c = 0; \quad a = o$

The quadratic formula/Solution(s)/Root(s): $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. Solution:

$x^2 - 6x + 8 = 0$

Comparing $x^2 - 6x + 8 = 0$ with

$ax^2 + bx + c = 0,$

$a = 1, \quad b = -6, \quad c = 8$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)}$

$x = \frac{6 \pm \sqrt{36 - 32}}{2}$

$x = \frac{6 \pm \sqrt{4}}{2}$

$x = \frac{6 \pm 2}{2}$

$x = \frac{6+2}{2}$ or $x = \frac{6-2}{2}$

$x = 4 \; or \; x = 2$

2. Solution:

$2x^2 - 10x + 8 = 0$

Comparing $2x^2 - 10x + 8 = 0$ with

$ax^2 + bx + c = 0,$

$a = 2, \quad b = -10, \quad c = 8$

$x = \frac{-(b) \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$

$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(8)}}{2(2)}$

$x = \frac{10 \pm \sqrt{100 - 64}}{4}$

$x = \frac{10 \pm \sqrt{36}}{4}$

$x = \frac{10 \pm 6}{4}$

$x = \frac{10+6}{4}$ or $x = \frac{10-6}{4}$

$x = 4 \; or \; x = 1$

Appendix F: Tests

3. Solution:

$6 + 4t - 2t^2 = 0$

$-2t^2 + 4t + 6 = 0$

Comparing $-2t^2 + 4t + 6 = 0$ with

$ax^2 + bx + c = 0,$

$x = t \quad a = -2, \quad b = 4, \quad c = 6$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-(4) \pm \sqrt{(4)^2 - 4(-2)(6)}}{2(-2)}$

$t = \frac{-4 \pm \sqrt{16+48}}{-4}$

$t = \frac{-4 \pm \sqrt{64}}{-4}$

$t = \frac{-4 \pm 8}{-4}$

$t = \frac{-4+8}{-4}$ or $t = \frac{-4-8}{-4}$

$t = -1$ or $t = 3$

4. Solution:

$-2x^2 - 8x + 10 = 0$

Comparing $-2x^2 - 8x + 10 = 0$ with $ax^2 + bx + c = 0,$

$a = -2, \quad b = -8, \quad c = 10$

$x = \frac{-(b) \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(-2)(10)}}{2(-2)}$

$x = \frac{8 \pm \sqrt{64+80}}{-4}$

$x = \frac{8 \pm \sqrt{144}}{4}$

$x = \frac{8 \pm 12}{-4}$

$x = \frac{8+12}{-4}$ or $x = \frac{8-12}{-4}$

$x = -5 \text{ or } x = 1$

Pre-test

Attempt all Questions Duration: 30 mins

Section A

1. Find the maximum value of $y = 4 + 3x - x^2$. (a)$\frac{3}{2}$ (b)$\frac{-25}{4}$ (c) $\frac{-3}{2}$ (d) $\frac{25}{4}$
2. Find the quadratic equation whose roots are 3 and –m

a. (a) $x^2 - 3mx - 3m = 0$ (b) $x^2 - (3 - m)x - 3m = 0$

b. (c) $x^2 + 3mx + 3m = 0$ (d) $x^2 + (3 - m)x + 3m = 0$

3. If α and β are the roots of the equation $3x^2 - 15x + 6 = 0$, find the value of $3\alpha^2 - \alpha\beta + 3\beta^2$.

   (a) -61 (b) -29 (c) 61 (d) 29
4. Find the minimum value of the function $f(x) = x^2 - 5x + 4$

   (a)$\frac{9}{4}$ (b)$\frac{5}{2}$ (c) $\frac{-9}{4}$ (d) $\frac{-5}{2}$
5. For what value of k will the equation $kx^2 - 4x + 1 = 0$ have repeated roots?

(a)1 (b)-4 (c)-1 (d)4

6. Find the values of p for which the equation $x^2 - px + 16 = 0$ has equal roots.

   (a)± 8 (b)± 4 . (c)± 6 (d) ± 9

Section B

7. Express $3x^2 - 16x + 10$ in the form: $a(x + b)^2 + c$, where a, b and c are integers. Hence state the minimum value of $3x^2 - 16x + 10$ and the value of x for which it occurs.

8. If α and β are the roots of $3x^2 - 15x - 12 = 0$, find the equation whose roots are $\left(\alpha + \frac{2}{\beta}\right)$ and $\left(\beta + \frac{2}{\alpha}\right)$

9. Given that: $p(x^2 - 10x - 2) + (2x^2 + 1) = 0$, find the values of p for which the quadratic equation has equal roots.

POST – TEST

Attempt all Questions Duration: 30 mins

Section A

1. Find the minimum value of $f(x) = x^2 - x - 6$. (a) $6\frac{1}{4}$ (b) $-6\frac{1}{12}$ (c) $-6\frac{1}{4}$ (d) $6\frac{1}{12}$

2. Find the quadratic equation whose roots are 3 and –m

   (a) $x^2 - 3mx - 3m = 0$ (b) $x^2 - (3 - m)x - 3m = 0$

   (c) $x^2 + 3mx + 3m = 0$ (d) $x^2 + (3 - m)x + 3m = 0$

3. If α and β are the roots of the equation $3x^2 - 5x + 1 = 0$, find the value of $\alpha^2 - \alpha\beta + \beta^2$.

   (a) $\frac{2}{3}$ (b) $\frac{25}{9}$ (c) $\frac{5}{3}$ (d) $\frac{16}{9}$

4. Find the maximum value of the function $f(x) = 10 + 3x - x^2$

   (a)$\frac{49}{4}$ (b)$\frac{15}{4}$ (c)16 (d) $\frac{65}{4}$

5. For what value of k will the equation $kx^2 - 8x + 4 = 0$ have repeated roots?

   (a)2 (b)4 (c)2 (d)-4

6. Find the values of m for which the equation $x^2 - mx + 9 = 0$ has equal roots.

   (a)± 3 (b)± 4 . (c)± 6 (d) ± 9

Section B

7. Express $7 - 5x - 2x^2$ in the form $a - b(x + c)^2$ state the greatest value of the expression

8. If α and β are the roots of $3x^2 + 4x + 1 = 0$, are find the equation whose roots are $\left(\alpha + \frac{1}{\beta}\right)$ and $\left(\beta + \frac{1}{\alpha}\right)$

9. Given that: $kx^2 + (k - 2)x + k = 0$, find the values of k for which the quadratic equation has equal roots.

RETENTION TEST ONE(RET1)

Section A

Attempt all Questions (30 mins)

1. Find the minimum value of $y = x^2 - 4x + 3$. (a) -4 (b) -3 (c) -2 (d) -1
2. Find the quadratic equation whose roots are k and -2.

   (a) $x^2 - 3kx - 3k = 0$ (b) $x^2 - (k - 2)x - 2k = 0$

   (c) $x^2 + 3kx + 3k = 0$ (d) $x^2 + (3 - k)x + 3k = 0$
3. Find the values of m for which the equation $3x^2 - mx + 6 = 0$ has equal roots.

   (a) $\pm 3\sqrt{2}$ (b) $\pm 4\sqrt{2}$ (c) $\pm 6\sqrt{2}$ (d) $\pm 9\sqrt{2}$
4. If α and β are the roots of the equation: $3x^2 - 5x + 1 = 0$, find the value of $\alpha^2 - \frac{2}{3}\alpha\beta + \beta^2$. (a) $\frac{19}{9}$ (b) $\frac{17}{9}$ (c) $\frac{-19}{9}$ (d) $\frac{-17}{9}$
5. Find the maximum value of the function $f(x) = 3x - 2x^2 + 10$

   (a) $\frac{-89}{8}$ (b) $\frac{89}{8}$ (c) $\frac{89}{16}$ (d) $\frac{-89}{16}$
6. For what value of m will the equation $5 - 10x + mx^2 = 0$ have repeated roots?

   (a) 10 (b) 5 (c) -5 (d) -10

Section B

7. If $2x^2 - x - 3 = A(X + B)^2 + C$, find the values of A,B, and C. Hence find the minimum value of $2x^2 - x - 3$

8. Given that α and β are the roots of the quadratic equation: $3x^2 + 4x - 3 = 0$, find the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

9. If $x^2 - 6x + 7 = k(2x - 3)$ has real equal roots, find the possible values of k.

RETENTION TEST TWO(2)(RET2)

Section A

Attempt all Questions (30 mins)

1. Find the minimum value of the function $f(x) = 2x^2 - x - 6$.

   (a) $-6\frac{1}{8}$ (b) $6\frac{1}{8}$ (c) $-6\frac{1}{4}$ (d) $6\frac{1}{4}$

2. Find the quadratic equation whose roots are (2+$\sqrt{3}$) and (2-$\sqrt{3}$) .

   (a) $x^2 - 4x - 1 = 0$ (b) $x^2 + 4x - 1 = 0$

   (c) $x^2 - 4x + 1 = 0$ (d) $x^2 + 4x + 1 = 0$

3. If α and β are the roots of the equation: $x^2 + 3x - 4 = 0$, find the value of $\alpha^2 - 3\alpha\beta + \beta^2$. (a) -11 (b) 20 (c) 21 (d) 29

4. If $y = x^2 - 2x - 3$, calculate the minimum value of y. (a)-4 (b)-1 (c) 1 (d) 4

5. If $kx^2 + 2x + k = -kx$ have equal roots, find the values of k.

   (a) 2 and $\frac{-2}{3}$ (b) 3 and 2 (c) -2 and $\frac{2}{3}$ (d) -3 and -2

6. Find the values of m for which the equation $3x^2 - 4x + m = 0$ has equal roots.

   (a) $\frac{16}{9}$ (b) $\frac{4}{3}$ (c) 9 (d) 16

Section B

7. By writing $12x^2 - 6x + 1$ in the form $p(x + m)^2 + n$, find the value of the constant. Hence find the minimum value of $y = 12x^2 - 6x + 1$

8. Let α and β be the roots of the quadratic equation: $x^2 + 2x - 5 = 0$.Without solving the equation, evaluate: (i) $\alpha^2 + \beta^2$ (ii)$(\alpha - \beta)^2$

   (iii) obtain an equation whose roots are α^2+1 and $\beta^2 + 1$

9. The roots of the quadratic equation: $x^2 - 2(3k + 1)x + 7(2k + 3) = 0$, where k is constant, are equal. Find the value of k.

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Appendix G -Marking Scheme for the tests

SECTION A

1. D
2. B
3. C
4. C
5. D
6. A

SECTION B

7. $3x^2 - 16x + 10$

$= 3[x^2 - \frac{16x}{3} + \frac{10}{3}]$

$= 3[(x^2 - \frac{16x}{3}) + \frac{10}{3}]$

$= 3[(x^2 - \frac{16x}{3} + \left(-\frac{8}{3}\right)^2) + \frac{10}{3} - \left(-\frac{8}{3}\right)^2]$

$= 3[(x - \frac{8}{3})^2 + \frac{10}{3} - \left(-\frac{8}{3}\right)^2]$

$= 3[(x - \frac{8}{3})^2 + \frac{10}{3} - \frac{64}{9}]$

$= 3[(x - \frac{8}{3})^2 - \frac{34}{9}]$

$f(x) = 3(x - \frac{8}{3})^2 - \frac{34}{3}$

Comparing $3(x - \frac{8}{3})^2 - \frac{34}{3}$ with $a(x+b)^2 + c$;

$a = 3$, $b = \frac{8}{3}$, $c = -\frac{34}{3}$

The minimum value of $3x^2 - 16x + 10$ is $\frac{-34}{3}$

and it occurred at $x = \frac{8}{3}$

8. Given: $3x^2 - 15x - 12 =$ 0

$\rightarrow$ $x^2 - 5x - 4 = 0$ ………………………………………. 1

Required Equation: $x^2 - (Sum\ of\ roots)x + Product\ of\ roots = 0$ ….2

Comparing equation 1 and 2, Sum of roots$(\alpha + \beta) = 5$

Product of Roots$(\alpha\beta) = -4$

(iii) Roots given: $(\alpha + \frac{2}{\beta})$ and $(\beta + \frac{2}{\alpha})$

Sum of roots for the required equation:

$= (\alpha + \frac{2}{\beta}) + (\beta + \frac{2}{\alpha})$

$= \frac{\alpha\beta+2}{\beta} + \frac{\alpha\beta+2}{\alpha}$

$= \frac{\alpha(\alpha\beta+2)+\beta(\alpha\beta+2)}{\alpha\beta}$

$= \frac{(\alpha\beta+2)+(\alpha+\beta)}{\alpha\beta}$

$= \frac{(-4+2)+(5)}{-4} = \frac{5}{2}$

Product of roots for the required equation $= (\alpha + \frac{2}{\beta})\ (\beta + \frac{2}{\alpha})$

$= \alpha\beta + 2 + 2 + \frac{4}{\alpha\beta}$

$= -4 + 4 + \frac{4}{-4} = -1$

Required Equation: $x^2 - \frac{5}{2}x - 1 = 0$

$\rightarrow \quad 2x^2 - 5x - 2 = 0$

9. Given: $p(x^2 - 10x - 2) + (2x^2 + 1) = 0$

$$px^2 - 10px - 2p + 2x^2 + 1 = 0$$

$$(p + 2)x^2 - 10px + (1 - 2p) = 0 \text{ ………….. } 1$$

For equal roots, the discriminant $(b^2 - 4ac)$ of the equation: $ax^2 + bx + c = 0$ is zero(0)

$$b^2 - 4ac = 0$$

$$ax^2 + bx + c = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

Comparing the two equations above, $ax^2 + bx + c = 0$ and $(p+2)x^2 - 10px + (1 - 2p) = 0$

$$a = p + 2, \quad b = -10p, \quad c = 1 - 2p$$

$$b^2 - 4ac = 0$$

$$(-10)^2 - 4(p+2)(1-2p) = 0$$

By simplification and factorization, the values of p for which the equation has equal roots are $\frac{-1}{3}$ and $\frac{2}{9}$

Sum of roots for the required equation:

$$= (\alpha + \frac{1}{\alpha}) + (\beta + \frac{1}{\beta})$$

$$= \frac{\alpha\beta + 1}{\beta} + \frac{\alpha\beta + 1}{\alpha}$$

$$= \frac{\alpha(\alpha\beta + 1) + \beta(\alpha\beta + 1)}{\alpha\beta}$$

$$= \frac{(\alpha\beta + 1)(\alpha + \beta)}{\alpha\beta}$$

$$= \frac{\left(\frac{1}{3} + 1\right)\left(-\frac{4}{3}\right)}{\frac{1}{3}} = -\frac{16}{3}$$

Product of roots for the required equation:

$$= (\alpha + \frac{2}{\beta})(\beta + \frac{2}{\alpha})$$

$$= \alpha\beta + 1 + 1 + \frac{1}{\alpha\beta}$$

$$= \frac{1}{3} + 2 + \frac{1}{\frac{1}{3}} = \frac{16}{3}$$

Required Equation: $x^2 + \frac{16}{3}x + \frac{16}{3} = 0$

$\rightarrow \quad 3x^2 + 16x + 16 = 0$

MARKING SCHEME FOR POST-TEST

SECTION A

1. C
2. B
3. D
4. A
5. B
6. C

SECTION B

7. Given: $7 - 5x - 2x^2$

$\rightarrow -2x^2 - 5x + 7$

$= -2[x^2 + \frac{5x}{2} - \frac{7}{2}]$

$= -2[(x^2 + \frac{5x}{2}) - \frac{7}{2}]$

$= -2[(x^2 + \frac{5x}{2} + \left(\frac{5}{4}\right)^2) - \frac{7}{2} - \left(\frac{5}{4}\right)^2]$

$= -2[(x + \frac{5}{4})^2 - \frac{7}{2} - \left(\frac{5}{4}\right)^2]$

$= -2[(x + \frac{5}{2})^2 - \frac{7}{2} - \frac{25}{16}]$

$= -2[(x + \frac{5}{4})^2 - \frac{81}{16}]$

$f(x) = -2(x + \frac{5}{4})^2 + \frac{81}{8}$

Comparing $\frac{81}{8} - 2(x + \frac{5}{4})^2$ with $a - b(x + c)^2$;

$a = \frac{81}{8}, \quad b = 2, \quad c = \frac{5}{4}$

The greatest value of $7 - 5x - 2x^2$ is $\frac{81}{8}$

and it occurred at $x = -\frac{5}{4}$

8. Given: $3x^2 + 4x + 1 = 0$

$\rightarrow \quad x^2 + \frac{4}{3}x + \frac{1}{3} = 0$ ………………………………………. 1

Required Equation: $x^2 - (Sum\ of\ roots)x + Product\ of\ roots = 0$ ….2

Comparing equation 1 and 2,

Sum of roots$(\alpha + \beta) = -\frac{4}{3}$

Product of Roots$(\alpha\beta) = \frac{1}{3}$

(iii) Roots given: $(\alpha + \frac{1}{\beta})$ and $(\beta + \frac{1}{\alpha})$

9. Given: $kx^2 + (k-2)x + k = 0$ ……………………………1

For equal roots, the discriminant $(b^2 - 4ac)$ of the equation: $ax^2 + bx + c = 0$ is zero(0)

$$b^2 - 4ac = 0$$

$$ax^2 + bx + c = 0 \text{ ……………………………………2}$$

Comparing the two equations above, $ax^2 + bx + c = 0$ and $kx^2 + (k-2)x + k = 0$

$$a = k, \quad b = k-2, \quad c = k$$

$$b^2 - 4ac = 0$$

$$(k-2)^2 - 4(k)(k) = 0$$

By simplification and factorization,

the values of k for which the equation has equal roots are $\frac{2}{3}$ and -2

Marking Scheme for Retention Test 1(RET1)

Section A

1.	D	4.	B
2.	B	5.	B
3.	C	6.	B

Section B

7. $f(x) = 2x^2 - x - 3$

$$f(x) = 2[x^2 - \frac{x}{2} - \frac{3}{2}]$$

$$f(x) = 2[(x^2 - \frac{x}{2}) - \frac{3}{2}]$$

$$f(x) = 2[(x^2 - \frac{x}{2} + \left(-\frac{1}{4}\right)^2) - \frac{3}{2} - \left(-\frac{1}{4}\right)^2]$$

$$f(x) = 2[(x - \frac{1}{4})^2 - \frac{3}{2} - \left(-\frac{1}{4}\right)^2]$$

$f(x) = 2[(x - \frac{1}{4})^2 - \frac{3}{2} - \frac{1}{16}]$

$f(x) = 2[(x - \frac{1}{4})^2 - \frac{25}{16}]$

$f(x) = 2(x - \frac{1}{4})^2 - \frac{\mathbf{25}}{\mathbf{8}}]$

$f(x) = A(x - B)^2 - \boldsymbol{C}]$

$A = 2\,, B = \frac{-1}{4}\,, \mathrm{C} = \frac{-25}{8}$

The minimum value of the function is $-\frac{\mathbf{25}}{\mathbf{8}}$

8. Given: $3x^2 + 4x - 3 = 0$

$$x^2 + \frac{4x}{3} - 1 = 0$$

General Equation: $x^2 - (Sum\ of\ roots)x + Product\ of\ roots$ =0

Comparing the two equations,

Sum of roots$(\alpha + \beta) = -\frac{4}{3}$

Product of Roots$(\alpha\beta) = -1$

Roots given: $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

Sum of roots for the required equation: $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta)}{\alpha\beta} = \frac{(\frac{-4}{3})^2-2(-1)}{-1}$

$= \frac{-34}{9}$

Sum of roots for the required equation: $(\frac{\alpha}{\beta})(\frac{\beta}{\alpha}) = \frac{\alpha\beta}{\alpha\beta}$ =1

Form of the required equation: $x^2 - (sum\ of\ roots)x$+product of roots=0

∴ The required equation is $x^2 - \frac{34}{9}x + 1 = 0$

9. Given: $x^2 - 6x + 7 = k(2x - 3)$

$x^2 - 6x + 7 - k(2x - 3)$ =0

$x^2 - 6x + 7 - 2kx + 3k$ =0

$x^2 - (6 - 2k)x + (7 + 3k)$ =0

Comparing the two equations, $ax^2 + bx + c = 0$ with $x^2 - (6 - 2k)x + (7 + 3k)$ =0

$$a = 1,\ \ b = -(6 - 2k),\ \ c = 7 + 3k$$

For equal roots, the discriminant $(b^2 - 4ac)$ of the equation: $ax^2 + bx + c = 0$ is zero(0)

$$b^2 - 4ac = 0$$

$$(-(6-2k))^2 - 4(1)(7+3k) = 0$$

By simplification and factorization,

the values of k for which the equation has equal roots are -1 and -2

Marking Scheme for Retention Test 2(RET2)

Section A

1. A
2. C
3. D
4. A
5. A
6. B

Section B

7. $12x^2 - 6x + 1$

$= 12[x^2 - \frac{x}{2} + \frac{x}{12}]$

$= 12[(x^2 - \frac{x}{2}) + \frac{1}{12}]$

$= 12[(x^2 - \frac{x}{2} + \left(-\frac{1}{4}\right)^2) + \frac{1}{12} - \left(-\frac{1}{4}\right)^2]$

$= 12[(x - \frac{1}{4})^2 + \frac{1}{12} - \left(-\frac{1}{4}\right)^2]$

$= 12[(x - \frac{1}{4})^2 + \frac{1}{12} - \frac{1}{16}]$

$= 12[(x - \frac{1}{4})^2 + \frac{1}{48}]$

$f(x) = 12(x - \frac{1}{4})^2 + \frac{\mathbf{1}}{\mathbf{4}}$

The minimum value of the function is $\frac{\mathbf{1}}{\mathbf{4}}$

8. Given: $kx^2 = -(k+1)x - k$

$kx^2 + (k+1)x + k = 0$

For equal roots, the discriminant $(b^2 - 4ac)$ of the equation: $ax^2 + bx + c = 0$ is zero(0)

$$b^2 - 4ac = 0$$

$$ax^2 + bx + c = 0$$

Comparing the two equations, $ax^2 + bx + c = 0$ and $kx^2 + (k+1)x + k = 0$

$$a = k, \quad b = (k+1), \quad c = k$$

$$b^2 - 4ac = 0$$

$$(k+1)^2 - 4(k)(k) = 0$$

By simplification and factorization,

the values of k for which the equation has equal roots are 1 or $\frac{-1}{3}$

9. Given: $x^2 + 2x - 5 = 0$

General Equation: $x^2 - (Sum\ of\ roots)x + Product\ of\ roots$ =0

Comparing the two equations,

Sum of roots$(\alpha + \beta) = -2$

Product of Roots$(\alpha\beta) = -5$

(i) Expression given: $\alpha^2 + \beta^2$=?

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$
$$= (-2)^2 - 2(-5)$$
$$= (-2)^2 - 2(-5) = 14$$

∴ The value of $\alpha^2 + \beta^2$ is 14

(ii) Expression given: $(\alpha - \beta)^2 = (\alpha - \beta)^2 - 4\alpha\beta$

$$= (-2)^2 - 4(-5) = 24$$

The value of the expression $(\alpha - \beta)^2$ is 24

(iii) Roots given: $(\alpha^2 + 1)$ and $(\beta^2 + 1)$

Sum of roots for the required equation: $(\alpha^2 + 1) + (\beta^2 + 1)$

$$=\alpha^2 + 1 + \beta^2 + 1$$
$$=\alpha^2 + \beta^2 + 2$$
$$= ((\alpha + \beta)^2 - 2\alpha\beta)+2$$
$$= (-2)^2 - 2(-5) + 2$$
$$= (-2)^2 - 2(-5) + 2$$
$$= 14$$

Product of roots for the required equation: $(\alpha^2 + 1)(\beta^2 + 1)$

$$=\alpha^2\beta^2 + \alpha^2 + \beta^2 + 1$$
$$=(\alpha\beta)^2 - (\alpha + \beta)^2 - 2\alpha\beta) + 1$$
$$=(-2)^2 - (-5)^2 - 2(-5) + 1 = 40$$

Form of the required equation: $x^2 - (sum\ of\ roots)x$+product of roots=0

∴ The required equation is $x^2 - 16x + 40 = 0$

::

Appendix: H Interview Guide

Student's Observation Check List/Interview Guide

1. How effective was the class wide peer tutoring? (Please underline as applicable)

   Very Good Good Quite Good Not sure

2. Did the lesson help you? (Yes or No).

   If yes, how did it help you? If no, why?

   ....

3. What did you like most about the lesson?……………………………………………
4. Was there anything you didn't like about the class wide peer tutoring?

   ....

   Did you enjoy being a peer-tutor? (yes or no).

5. How did you feel as a tutor? ....
6. How did you feel as a tutee? …………………………………………………
7. Do you think Class-wide peer tutoring should be used in other mathematics lessons?

   ……………………………………………………………………………………

8. Would you like to be a peer-tutor whenever we use CWPT in a lesson with your class?

   (Yes or No) ; Why? …………………………………………………………………

10. Any suggestion or idea that could make the class wide peer-tutoring more successful?

   ………………………………………………………………………………

Appendix I Write up on CWPT Method

Class wide Peer Tutoring (CWPT) Method

Class wide Peer Tutoring (CWPT) Method is a peer mediated teaching method in which the entire class is divided into small groups for structured learning or to execute tasks under the supervision of the teacher within or after the instructional period. It is also a type of co-operative learning where the entire students work together in pairs or in smaller groups and the more skilled student(s)-'tutor' helping the less skilled student(s)-'tutee' on a task, under the supervision of the professional teacher within or outside the contact period for structured learning. In CWPT, the paired students are the same intellectual age or level. The students take turns in assuming the expert role thus the "tutoring". CWPT methods are designed to increase practice, responding, and feedback for students, and they often result in increased student motivation, retention, application(transfer of knowledge) and achievement. Class-wide Peer Tutoring (CWPT) occurs when the teacher creates highly structured tutoring materials for use during the tutoring session. CWPT is distinguished from cooperative learning, in which students work collaboratively in groups (Scrag,2010).

The Peer Tutoring Group

The various peer tutoring groups include students with different or relatively ability levels and mostly consists of at most five peers. The student pairings are fluid and may be based on achievement levels or student compatibility. Each group consists of a tutor and at least a tutee. Each student in the group has the opportunity to play the role of both the tutor and the tutee. The tutor's takes charge of whatever information is being tutored or reviewed in the groups. In the tutoring process, the tutor's explains the work, asks questions, and provides feedback to the tutees(peers) .The classroom or subject teacher acts as facilitator and monitors the activities of the various groups.

The Tutor: is any student who offers the tutoring, guides the tutee(s) in dyads or small groups on instructional activities.

The Tutee: refers to student(s) who receive(s) the tutoring.

The instructional activities can be practicing: solving quadratic equation, finding maximum and minimum function of a quadratic function, determining the real roots of quadratic functions, sketching of the graph of a given quadratic function.

Benefits of CWPT Lesson

- It promotes academic gains, social development and maximizes students' engagement on tasks. This is usually achieved through the frequent opportunities given to students to practice academic tasks and share ideas in smaller groups (one-on-one assistance or tution). The academic gains include achievement, retention and application of learned concept, acquisition of problem solving skills and adequate means of handling the issue of diversity among students in class.
- It is effective for a wide range of students with variety of academic needs. The academic needs could be poor performance, difficulties in retaining and applying learned concepts, problem solving skills and, problems of misconceptions, poor attitudes and interest in learning a subject/concept.
- It can be used in implementing variety of curriculum areas like Mathematics, Physics, chemistry, biology, Science, English Language, Social Studies.
- It is a widely-researched practice across ages, grade levels, and subject areas

- It increases self-confidence and self-efficacy)
- Classwide Peer Tutoring method helps students to learn better and more quickly.
- Classwide Peer Tutoring method increases the amount of class work students finish.
- Classwide Peer Tutoring helps students with Attention Deficit(Hyperactivity
- Classwide Peer Tutoring is also very helpful for students with behavioural disorders when the materials used are not too difficult but matched the students' skill levels,
- CWPT enhances performance
- It intrinsically motivates students to learn.
- Knowledge gained in CWPT have been found to last. Charles Greenwood and a team of researchers.

Components of Effective CWPT Method

These includes: multi-modality format, reciprocal and distributed practice, immediate error correction and feedback, built-in reinforcement, high mastery levels, measured outcomes, game format with partner pairing and competing teams.

Instructional Components of CWPT Lesson:

The CWPT procedure (sequence of instructions) generally follows 5 major instructional components. These are: pre-assessment, mini-lesson, CWPT activity, suggested activities and mastery post-assessment.

The pre-assessment component is used by the teacher to determine the foreknowledge/entry level (relevant previous knowledge) of every student. Explicitly, it helps the teacher to determine if any of the content to be taught is already known, and exactly who knows what in advance. It takes 7 to 10 minutes.

The mini-lessons component follows the pre-assessment stage. Mostly, teachers spend 10 to 20 minutes here. It is the stage where the teacher introduces/models the proposed tasks/skills or the new material to the learners. Explicitly, it is the stage at which the teacher introduces the lesson by stating the topic, purpose and objective(s) for the day's lesson. The teacher employs teacher-led instructions to provide a model of the lesson in a whole class discussion or close informal group format. This helps the learners to follow the lesson. It also helps learners to know what they are to do, what next and how they are to do it. The teacher finally conducts an informal assessment on learners' overall understanding of what has been achieved, what is to be done, and which students have difficulties.

The third stage captures the *CWPT activities*. It is the stage that captures the Class Wide Peer Tutoring stage of the lesson. The entire class is put into peer dyads/small groups and every student is paired with another to learn and practise the content that was introduced or reviewed in the mini lessons. The time is divided equally between tutees and tutors. Each student performs the role of a tutor and a tutee during the peer tutoring sessions. The suggested tutoring time limit ranges from 10 to 30 minutes for each round depending on the grade level/stage.

The fourth stage captures the *suggested activities*, where individual practices take place. Each individual module or unit provides the teacher with suggestions of additional activities that they can have their students engaged in to demonstrate individual learning, mastery, and skills application on an individual basis.

The final stage includes the *post-assessments*. The post-assessments are vital to the programme because they allow the teacher to determine which students have mastered the material at or above the acceptable criterion level and how much gain the students have made since the pre-assessment. The results of the post-assessments will also guide the teacher to know how to proceed with the next instruction. The results will indicate whether or not the teacher should proceed to the next level of instruction (if all students have mastered the material) or whether there is the need to re-teach the material (if many of students fail to master the material).

Description of Strategy

- Train the students to use Classwide Peer Tutoring (CWPT) by explaining and modeling the procedures.
- Divide students in the entire class into pairs by pairing students of different abilities whose abilities did not differ too largely.
- Provide a lesson on the topic to the whole class.
- After the lesson, let students practice their problem-solving skills with CWPT.(For instance quadratic problem solving skills).
- Let tutees(students) use activity/worksheets as prompts. Let the tutors(student in the tutor role) use an answer sheet(answer key) to determine if the tutee's answers are correct.
- After completing half of the tasks on the answer sheets(worksheet),let the students switch roles.
- Let the tutors record the tutee's answers. Let the tutors models how to solve problems when the tutee answers them wrongly(incorrectly). [Tutors are

provided with models of problem solving methods to help them provide assistance.

The teacher monitors student performance and tutoring behavior by walking around the room during CWPT.

Tutoring Steps Stage in a CWPT Lesson

1. Put students in the entire class into peer dyads or small groups.
2. Let one person be the tutor and the other(s) to be the tutee(s) in each pair
3. Provide the tutors with a list of questions and answers. Provide the tutees in each pair or group with a piece of paper and a pencil.
4. When the tutoring begins, the tutor asks the questions and the tutee provides the answers. This could take a written or oral form. In case of calculations, the tutor asks the tutee to solve the question(s) on their pieces of paper or small erasable board.
5. If the tutee gets the question correct, the tutor awards 2 points and then moves on to the next question.
6. If the tutee provides an incorrect answer, the tutor provides the correct answer and the tutee must say and write the correct answer three times before moving to the next question.
7. The tutor continues to provide questions giving 3 points to every correct answer and 2

   point for every incorrect answer that is corrected by the above technique.
8. When time is up, the roles change. The teacher resets the clock and another round begins.
9. At the end of the two rounds, the teacher confirms the points earned and updates the leader board.

10. At the end of the week, the team with the most points wins.

Training / Orientation Guide

1. Explain and demonstrate peer tutoring and give your class time to practice tutoring before they do it for in the classroom. Show the class how to get into pairs quickly and quietly. Give clear instruction and show the learners how to begin the lesson, continue and end the lesson. Let the learners practice or role-play with their peers in the class just as the teacher(s) did in the demonstration. Give comments or feedback to learners while they practice.
2. Teach learners what good tutor and tutee behaviors are before starting Class wide Peer Tutoring to avoid many behavior problems later. Explain the need and how to talk to their peers in a respectful and cordial way when they are wrong. Give them tips and demonstrate how not to get angry when another child tells them that they made a mistake.
3. Teach learners how to keep track of their partner's right answers or their own. The tutees will then see that they are getting better over time.
4. Make sure that learners are tutoring with materials that are matched to their abilities and sequenced in level of difficulty. The tasks should not be too hard to frustrate the tutees and the tutor
5. Have learners tutor with new information as soon as they have learned the old material. This way they will not get bored.
6. Give all learners opportunities to be the tutor, even in subjects where they have problems. They will learn from tutoring other students, and they will gain more confidence in their abilities in that area.

7. Make Class wide Peer Tutoring fun – like a game. Tutors can reward their peers with points for giving answers that are right or for making progress.

METHOD ORIENTATION/TRAINING SCRIPT FOR IMPLEMENTING CWPT

A// Introduction of the Term "CWPT Method"

"In this session, I'm going to introduce you to how you can be a teacher for your fellow student (peers). We shall begin with how you can teach your peer(s), the concept of quadratic functions."

Within the time frame, I'll be dividing you into groups of at least two: In each group, one will be chosen as the tutor and the other(s), will be the student (also called "tutee"). When you're a tutor, you will be given answer sheets containing the solutions to the activity sheets which will be given to your student (tutee). When you're a tutee, you'll be given answer sheets containing problems on quadratic functions which you will be required to answer for your tutor to go through and award marks. If you are facing problems, you call on your tutor to guide you. You'll switch between being a tutor and a partner each day. We'll practice in just a few minutes."

B: Presentation of the Activity and Answer Sheet

(i). "Each tutor will be given activity sheet and an answer sheet for the tutees to work with. Each completed activity sheet will have the name of the tutee and tutor who awarded the marks, and the marks awarded.

(ii) "The tutor keeps the answer sheets away from the tutee as the tutee performed the tasks on the activity sheets.

(iii) Let's practice:

C: Model/Practical Demonstration:

Teacher models the procedures with a single student (*Teacher pretends to be a tutee*)-role play. Learners watch and receive reinforcement.

Lead each pair practices the procedures once with their tutor under the teacher's supervision—and the tutor--provides reinforcement for correct steps and presentation.

Let all pairs practice the procedures simultaneously and also exchange the expert role.

[END OF TRAINING SESSION]