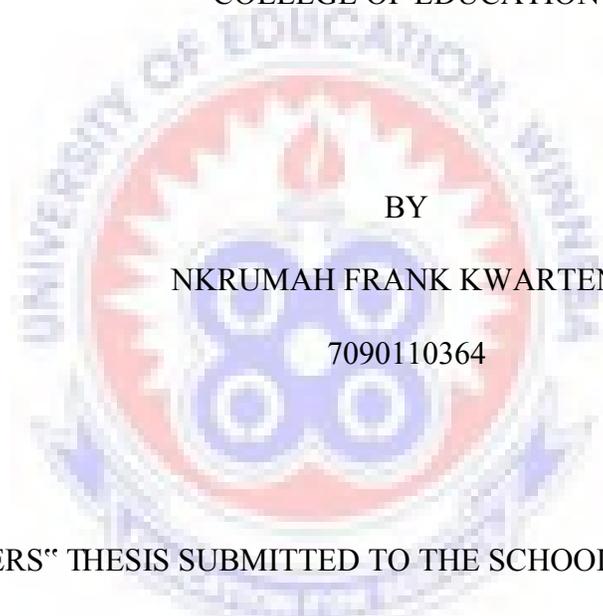


UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF SCIENCE EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION

THE USE OF DERIVE 5 IN IMPROVING PRE-SERVICE TEACHERS’
UNDERSTANDING OF MATRICES AT MAMPONG TECHNICAL
COLLEGE OF EDUCATION



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OF A MASTER OF EDUCATION DEGREE IN MATHEMATICS

AUGUST 2012.

DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this University or elsewhere.

Candidate's Name: Nkrumah Frank Kwarteng

Signature:..... Date:.....

Supervisor's Declaration

I hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on the supervision of thesis laid down by the University of Education, Winneba.

Supervisor's Name: Prof. Kofi Damain Mereku

Signature:..... Date:.....

DEDICATION

To the Nkrumah family, and my sweetheart Adwoa Akyiaa Anto for their love support and encouragement.



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ABSTRACT

This study was undertaken to help pre-service teachers to improve their understanding of matrices in the form of systems of linear equations using Drive5 at Mampong Technical College of education in the Ashanti Region of Ghana. It specifically sought to find out the impact of the Derive5 (ICT) utility application software on the performance and errors in their computational skills in handling matrices. A convenience sampling technique was used to select 30 second year pre-service teachers of 2E class due to proximity and accessibility for the study. The researcher used an observation, interview, pre-test and post-test techniques to find out the impact of the Derive5 use on the pre-service teachers' mathematical and computational skills so far as matrices are involved.

The results of the study revealed pre-service teachers' errors in computational skills such as error in sequence of multiplication, getting rid of matrix, product of matrices and concept of the use of determinant. The results indicated that there was a difference in pre-test and post-test scores with respect to the means and standard deviations with the latter better in terms of pre-test mean and SD of 3.90 and 2.05 and post-test mean and SD of 6.40 and 2.67. The results of the t-test (29df, $t=6.335$, and $p=0.00$) further indicated that the difference in means were significant at $p=0.00$. This implied that the Derive5 improved pre-service teachers' ability to deal with mathematical problems involving matrices and computational skills in mathematics

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CHAPTER 1

INTRODUCTION

1.0 Overview

This chapter gives a brief background to the study. It includes the statement of the problem, purpose and the objectives of the study. In addition, the research questions and significance of the study are discussed not forgetting the delimitation as well as the organizational plan of the study.

1.1 Background of the Study

In contemporary society, almost all domains of human knowledge have to apply mathematics and computational methods. There is therefore, no doubt that man can never be without mathematics in his day to day activities. According to Dean (1994), students have never found mathematics as an easy subject and criticisms of teaching it have been common for years. Yet the importance and the need for the subject in our ever-evolving society cannot be over emphasized. Hence, the subject has been used as a filter or a yard stick for students seeking admission into various institutions of learning and more especially in our tertiary institutions. The study of matrices as component of algebra in mathematics is one of the forces in the mathematics subject. For example matrices are taught and studied at the senior high schools, colleges of education, and other higher level of education like polytechnics and universities. This is because the active natural social interaction of man as seen in his rigorously process of manipulating the environment leads to discovery of certain objects which numbers form part. Hence the basic idea of matrices came as a result of man's daily arrangement of objects to develop a rectangular system of patterns. It is therefore not surprising that at the colleges of

education level, the insertion of matrices as content area is not only to equip the pre-service teacher with the subject matter, but also to find tune to practical ways in which matrices occur.

Hence, the theory of matrices is used in vectors to represent linear transformations, which are higher-dimensional analogs of linear functions of the form $f(x) = cx$ where c is a constant. Therefore, matrices multiplication corresponds to composition of linear transformations. Matrices are therefore the key notion of linear algebra. Matrices can also keep track of the coefficient in a system of linear equations. For example in square matrices, a matrix which has the same number of rows and column, the determinant and inverses matrix (when it exists) govern the behavior of solutions to the corresponding system of linear equations, and eigenvalues and eigenvectors

.But aside all these importance of matrices, most brilliant students who would have pursued mathematics to a higher level are scared off it because of its content of which 2×2 matrices; a system of linear equations in two variables is no exception. Thus, according to a survey conducted by the mathematics department of mathematics Mampong Technical College of Education, most of these students perceive equation involving variables as abstract and difficult to solve. For many students algebra is nothing more than rules for moving variables about an equation. Learning algebra is viewed as memorizing the rules for performing these manipulations.

Wren and Wren (1985) is of the view that large amount of difficulties may be traced to the fact that mathematics presents a radically new and difficult approach to the study of quantitative relationship characterized by new symbolism, new concepts and new language, a much higher degree of generalization and abstraction than they have encountered previously and essential dissociation of many of its parts from intuition and concrete experience. Besides, mathematics has its own language with different linguistic registers. The symbolical language aspect of system of linear equations poses difficulties to many students in many countries. According to Bennet and Nelson (1998), the concept of variables is often misunderstood by students in England. However, the problem of perceiving equation involving variables as abstract and difficult to solve is similar to students in Ghana if not worse. It was therefore not surprising to me when out of forty three second year students of Mampong Teachers College of Education students I interviewed at the Campus, on why they couldn't solve for the values of x and y in the system of equations $2x + y = 4$ and $x + 2y = 6$ using the matrix approach in their previous exams, which was compulsory, thirty two replied that solving for the variables x and y is too abstract and tedious. This was carried out during my first two weeks stay as a permanent teacher.

To me, pre-service teachers' inability to have recognized variables as quantities was a carryover effect from the junior high school through to the senior high school and to the College of Education. This is because there have been numerous reports revealed by the Diploma in Basic Education chief examiners reports, Institute of Education, UCC 2008, and 2009 and more recently the same chief

examiners report dated February 24, - March 4, 2010 (pages 2 and 3) also of Institute of Education, University of Cape Coast reporting of pre-service teachers inability to deal with questions involving matrices using the matrix approach. In my own opinion, this and many other mathematical problems have persisted without any proper attention in terms of innovation for remedy.

The mathematics courses at the Colleges of Education in Ghana require the mathematics tutors to use Information and Communication Technology (ICT) in their instructions. This has been effected by the fact that since 2005 there have been a shift from the Certificate „A“ awarded to pre-service teachers to Diploma. This has also affected the name Training Colleges now known as Colleges of Education and courses are mounted on a two-semester calendar basis instead of three-term academic year as was the case of Certificate „A“. The college of education mathematics content courses are: Number and Basic Algebra for first year first semester, Geometry and Trigonometry for first year second semester, Statistics and Probability for second year first semester and Further Algebra for second year second semester. These areas of the mathematics content syllabus require the use of ICT for effective teaching.

Furthermore, the general aims of JHS mathematics syllabus of the methods of teaching JHS Mathematics at the Colleges of Education stipulates that the tutors should teach the pre-service teachers how to explore ways in which calculators and other ICT tools could be used to enhance teaching, learning and problem solving by children. “ICT is an umbrella term that includes any communication device or application, encompassing: radio, television, cellular phones, computer and network hardware and software, satellite systems and so on, as well as the

various services and applications associated with them, such as videoconferencing and distance learning” (Rouse, 2005)

The integration of ICT into professional practice places great pressures and demands on teachers to provide students with the opportunity to develop the skills required to engage in a progressive society and become life-long learners as well as enhance the teaching of current curriculum content. McGehee and Griffith (2004) explained that to exploit the potential of ICT fully, resulting in improved student outcomes, there is need for mathematics tutors to incorporate new technologies into their teaching practice.

For instance in Kenya, government has made a huge investment in ICT in schools over the past few years and is committed to maintaining it. One aspect of this investment was the Continued Professional Development (CPD) opportunities provided for all teachers in ICT through the training groups approved by the New Opportunities Fund (NOF). The rate at which secondary school subjects, especially mathematics, have taken up the use of ICT to support teaching and learning has not been as rapid as might have been expected. This could be for a number of reasons, and I hope that this guidance document might make a contribution to improving the situation.

Back in our own country, though there is availability of computers at most of the Colleges of Education including my school and internet connectivity in some of the Colleges, according to Apaw (2009) the Mathematics tutors at these colleges in Ghana are yet to harness the potential of ICT in Mathematics. So the fundamental questions are: (a) is this limitation in using ICT due to lack of computer competency or attitudinal? (b) What is it that is preventing these tutors

from taking the advantage of the availability of computers and internet connectivity? Aside all these facilities at our disposal, students find it very difficult to discover basic concept in mathematics.

It is however disheartening to note that students find it difficult to evaluate system of linear equation using the matrix method whiles there are ICT facilities and software to arouse students interest and understanding of the matrices. Hence I was prompted to research into improving pre-service teachers' ability in solving systems of linear equation in two variables (2×2 matrices) by the use of discovery leaning techniques spear headed by Derive 5 a computer assisted instruction tool.

1.2 Statement of the Problem

Examining the extent to which matrices and its application play an important role in the social lives of people, it is therefore required by students and teachers to acknowledge matrices as ordinary systems of equations involving variables. But there is a problem in the concepts of matrices and its application with pre-service teachers in our Colleges of Education.

This is evidence of the fact that the Diploma in Basic Education chief examiners reports, Institute of Education, UCC 2008, and 2009

and more recently the same chief examiners report dated February 24, - March 4, 2010 (pages 2 and 3) also of Institute of Education, University of Cape Coast reporting of pre-service teachers inability to deal with questions involving matrices. Citing complains about poor performance of some aspects of concepts of matrices. The situation here was also similar to situation with pre-service teachers at Mampong Teachers College of Education where I happen to teach.

According to one of the reports, Students were to answer this question: if $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, then find A^2 . Some of these students gave this answer. $A^2 = \begin{bmatrix} 2^2 & 3^2 \\ 1^2 & 4^2 \end{bmatrix}$ instead of finding $A \times A$. With 60 percent of the pre-service obtaining wrong answer.

Moreover, most pre-service teachers couldn't solve for the values of x and y from systems of equations using the matrix approach. The researcher later found out that pre-service teachers were confused with the concept of rows and columns. The above attests to fact that, matrices continues to create some sort of panic in pre-service teachers and hence affecting their performance. In light of this, I decided to integrate ICT in the course of remedying this situation in my college in other to help the pre-service teacher to be more conversant with rows and columns. In recent years mathematics teaching has seen a renaissance of whole class teaching approaches. So clearly every researcher need to know what sorts of ICT resources are available to support this. In many cases, though, you will also want the students themselves to have access to ICT resources. This might be through individual access, or working in pairs, or working in groups.

Nearly all schools and colleges have special purpose ICT suites where students can have individual or paired access to computer resources. These may be available to subject departments through a booking system – but there are issues on has to consider carefully. An alternative is to make ICT resources available within mathematics teaching rooms – and here, too, learning is not restricted to timetabled lessons and need not take place solely in the school or college.

The courses at the Colleges of Education more especially Mathematics requires the tutors to teach pre-service teacher how to explore and discover ways in which ICT could be used to enhance teaching and learning of Mathematics at the basic levels, Apaw (2009).

It is perceived that the use of ICT to enhance teaching and learning is unlikely to be explored in meaningful ways in school classrooms unless there is effective modeling of technology integration during the teacher education experience (Nolan, 2004). Therefore, in my attempt to assist my pre-service teachers, I realized that some of the major among the common causes of their inability to solve systems of equations using the matrix method are: pre-service teachers' inability to recognize variables as representatives of quantities, pre-service teachers' failure to recognize matrices as system of linear equations and insufficient practice sections on square matrices.

1.3 Purpose of the Study

The purpose of the research was to help pre-service teachers to overcome their difficulties involving linear equations using matrices approach. The study also sought to find out how best pre-service teachers' could be assisted to integrate ICT in their teaching and learning. Finally, this study the study was designed to develop pre-service teachers' interest in Derive 5 as they seek to integrate ICT in their teaching.

1.4 Research Questions

In pursuance of the purposes above, the following research questions were formulated to guide the study.

1. To what extent will the use of Derive 5 help to improve upon pre-service teachers' ability to use the matrices approach to solving linear equations?
2. How would the use of Derive 5 influence pre-service teachers' interest in using technology in mathematics?
3. What is the effect of the technology on pre-service teachers' achievement in mathematics?

1.5 Significance of the Study

The research aimed to ensure that pre-service teachers acquire enough skills with regards to linear equations in the form of two dimensional matrices. This study would serve as an informative document for policy/decision makers at the Ministry of Education on how to address the concerns and needs of Mathematics tutors on matrices and ICT integration at the colleges of education in Ghana by the Ministry.

The findings of this study could also serve as a base-line data for future studies on ICT integration into the Mathematics curriculum at the Colleges of Education in Ghana. Finally, it will serve as a record for all teacher education institutions, researchers and curriculum planners to have access to alternative approach to mathematical problems involving matrices.

1.6 Delimitations of the Study

It would have been proper to cover the entire second years of the college but due to the loaded time table and in view of constraints such as time and finance, the research was restricted to only the 2E class out of the five classes.

Also, there exists a variety of mathematics software for instructing mathematics but this study focused on the use of Derive 5 software for instructing mathematics because of its friendliness and effectiveness as teaching and learning tool.

1.7 The Organization of the Study Report

The remaining chapters of the report are the relevant literature review which was presented in chapter 2. The researcher described the research design and methodology in chapter 3. Results and discussion were done in chapter 4. Chapter 5 consisted of summary of key findings, implications for practice, conclusion, recommendations, and areas for further research.

CHAPTER 2

REVIEW OF RELEVANT LITERATURE

2.0 Overview

The review of relevant literature was focused on the theoretical framework of the study, brief background on ICT in education in Ghana, national ICT for accelerated development (ICT4AD) policy, impact of ICT on learning and teaching, the role of teacher education in facilitating integration of ICT, use of ICT in Education, concept of matrices, why use a utility application software such as Derive 5 for intervention and the use of ICT in pre-service teachers' achievement in mathematics.

2.1 Potential of ICT for mathematics education

The use of ICT in the mathematics classroom has long been a topic for consideration by mathematics educators. Some examples of ICT use in mathematics include: portables, graphic calculators and computerized graphing, specialised software, programmable toys or floor robots, spreadsheets and databases. Studies have shown that a range of portable devices exists which allow pupils to collect data, and manipulate it using spreadsheets and databases for work in numeracy. Some portable equipment also enables the study of maths to move out of the classroom and to incorporate fieldwork investigations (Moseley and Higgins 1999).

The use of graphic calculators and computerized graphing in mathematics speeds up the graphing process, freeing people to analyse and reflect on the relationships between data (Hennessy 2000). Specialists software such as Computer Algebra Systems (CAS), Dynamic Geometry Systems (DGS) and Maths curriculum

software improve pupils' skills and understanding in algebra, allow pupils to manipulate and measure shapes leading to higher level of learning among them (Hennessy et al. 2001; Clements 2000). Programmable toys or floor robots controlled by instructions in programming languages (usually Logo) were one of the earliest applications of ICT to maths, and where used were the cause of significant changes in maths teaching (BECTA 2003). Logo encourages pupils to develop problem-solving skills, leads them to develop higher levels of mathematical thinking as well as learn geometric concepts (Clements 2000). According to Ittigson and Zewe (2003) ICT supports constructivist pedagogy, which allows students explore and reach an understanding of mathematical concepts.

This approach promotes higher order thinking and better problem solving strategies (Ittigson and Zewe 2003). BECTA (2003) reiterated that teachers can maximize the impact of ICT in maths teaching by using ICT as a tool in working towards learning objectives. For mathematics educators, defining the most effective uses of ICT in the teaching of mathematics can certainly be described as a "wicked problem," as represented by Mishra and Koehler (2006). A number of challenging instructional questions are associated within this wicked problem, such as: When should teachers incorporate calculators when teaching arithmetic? How should teachers incorporate the powerful new symbolic programs within basic algebra instruction? Should teachers allow student use of the many new online homework assistance web sites for mathematics? Such instructional questions illustrate that the problem of effective ICT integration into the teaching

of mathematics is a complex innovation for teachers. They do not only need to have competent knowledge of teaching mathematics but also need to be competent in the pedagogical use of ICT (Voogt, 2008).

2.2 Brief Background on ICT in Education in Ghana

In the late or mid 1990s, ICT was introduced in all the tertiary institutions as a general course popularly known as “computer literacy” in Ghana. The attempts to introduce ICT in Ghana’s educational system by the Ministry of Education (primarily through the Ghana Education Service (GES)), its development partners and other private sector agencies took over 10 years.

In addition, in the late 1990s, some of the well resourced senior secondary schools (SSS) now known as senior high schools (SHSs) in Ghana had private computer laboratories fully furnished by either PTA or donor partners of such institutions where they sometimes had lessons in computer technologies.

In one study carried out to assess and review the ICT in Education Initiatives in Ghana (2005), initiatives were selected and their impact assessed to see what could be learnt. Several positive achievements were noted. The major two are:

- The significance of the initiatives provided good examples for other schools to bring out their own ICT programmes which promote discovery learning.
- The initiatives lead to a wider number of teachers acquiring ICT skills and developing strong interests in ICT; schools involved in the initiatives were motivated to expand the project and / or acquire more ICT equipment.

This saw the collaborative efforts of private-public partners, including Parent Teachers Associations (PTAs) and civil society.

2.3 National ICT4AD Policy

Ghana has been a pace setter in Africa. In 1957 it was the first country in sub-Saharan Africa to emerge from colonialism and before an economic crisis in the late 1970s experienced the highest GNP on the continent. Moreover, Ghana stands tall in telecommunication in Africa which is driven by ICT and hence the Government of the Republic of Ghana has committed to pursuing an ICT for Accelerated Development (ICT4AD) policy (2003). This national policy outlines the plans and strategies for the development of Ghana's information society and seeks to provide a framework and plan as to how ICTs can be used to facilitate amongst other objectives the national goal of "transforming Ghana into an information and knowledge-driven ICT literate nation" (Ministry of Education and Sports, 2006, p. 11). The national policy brings out pillars, of which education is highlighted, as both a critical pillar as well as means to socio-economic development. For this, some important strategies have been underlined.

These include:

- promoting the deployment and exploitation of information, knowledge and technology within the economy and society as the priorities for socio-economic growth;
- meliorating the human resource development capacity and the Research and Development (R&D) capacity of Ghana to meet the demands and

requirements for developing the nation's information and knowledge-based economy and society.

- developing Ghana's educational system using ICTs to amend and increase access to education, and research resources and facilities, moreover to improve the quality of education and training and make the educational system responsive to the needs and requirements of the economy and society with specific reference to the development of information and knowledge-based economy and society.

Ghana's ICT policy in terms of education may be in the right direction because the researcher is of the view that, if ICT is employed in Mathematics education, it will have positive impact on academic outcomes. There will also be positive attitudes towards the study of Mathematics in schools and also make the understanding of abstract concepts in Mathematics better. In the Ghana's ICT4AD policy, it has been stated that in addition to better performance in traditional measures of academic achievements, a secondary benefit of ICTs in education is to familiarize new generations with the technologies that have become integral components of the modern world (Ministry of Education and Sports, 2006).

2.4 The Impact of ICT on Learning and Teaching

Bringing ICT into the classroom can have a considerable impact on the practice of teachers, in particular when ICT is conceptualized as a tool that supports a real change in the pedagogical approach. Not only do the teachers need to change their

roles and class organization, they also need to invest energy in themselves and their students in preparing, introducing and managing new learning arrangements. Some need to acquire basic ICT skills. Teachers also need to determine which applications have added value for learning in their subject area. While doing this they need to be aware that this is not a one-time activity, as the information environment is continuously changing. Perhaps most important and challenging for teachers is determining which basic subject, social and management skills students need to function in such environments. The change can impact on assessment tasks, with new learning environments moving away from summative methods of assessment to formative approaches and open-ended products (such as reports and research papers created by groups of students). These different aspects are time consuming, and result in an increased teacher workload.

Teachers can take time to discover that computers do not mean extra work – rather they actually make their work easier. Again, more competent students themselves can be a useful resource, this time for their peers.

There is no doubt that teachers who use ICT in classrooms have to demonstrate high levels of energy, hard work and perseverance, often in the face of considerable odds (Lankshear & Snyder, 2000, p. 110). If they are early adopters then they are required to be resourceful and overcome many barriers to make things work. Planning lessons involving computers can take considerable time and demands complex scheduling and resourcing. Therefore, teachers using computers in the classroom should not act in isolation from each other. They need access to resources which will supply ideas and material for different classroom

applications, including peers who are also developing their own pedagogies and resources (Leach et al., 2005). For while computers have great potential in education, they also present teachers with additional obstacles to overcome.

There is enough evidence from many researches that ICT can help students to learning and teachers to teach more effectively. However there is not a simple message in such evidence that ICT will make a difference simply by being used. Findings suggest that although ICT can improve learning there are a number of issues that need to be considered if such technology is going to make a difference. Some caution is therefore called for at this broad level of where and how ICT might have an impact. There are two main issues. First is the modest effect of ICT compared with other researched interventions, second is the almost negligible effect of the provision and use of ICT at a general level. There has been extensive research into computer-assisted instruction (CAI) and computer-based learning (CBL). Some major reviews of this extensive work have been undertaken. One study (Fletcher-Flynn and Gravatt, 1995) into the effectiveness of CAI limited the studies it examined to those that took place between 1987- 1992 and identified almost 400 reports of research that met these criteria.

The impact of the use of computers was then combined statistically to identify the overall impact. In this meta-analysis the mean effect size was relatively small for the five years in question but increased for more recent studies analyzed. This kind of improvement would move an „average“ class of pupils from 50th to about 40th in list of 100 classes ranked in order of attainment. This suggests two things:

first, it is possible that the impact of computers may be increasing; second, ICT only produces relatively small improvement. Other forms of educational interventions, such as peer tutoring, reciprocal teaching and homework, for example, all produce greater average impact (Hattie, 1987; Hattie, 1992). In a study of the effect of different types of study skills interventions the average effect size was .57 (Hattie, Biggs and Purdie, 1996); this would move a class from 50th to the top 30. A study of the effect of thinking skills or metacognitive approaches (Marzano, 1998) indicates the average impact would move a class from 50th into the top 20 (an effect size of .72).

A study by the British Educational Technology Association (BECTA, 2000) found no link between level of resources for ICT and either reading or mathematics grades at Key Stage 1 in 1999. At Key Stage 2 there was a significant, but very weak, association between ICT resources and pupil attainment. This indicated that ICT curriculum resourcing was at least 99.5% independent of pupil performance at Key Stage 2 (no correlation coefficient exceeded 0.07). In the USA, information about computer use from a longitudinal study was analysed (Weaver, 2000). This study also found a very small link between computer use in the curriculum in school and improvement in pupils' test scores, though again the link was very weak (no correlation coefficient was higher than 0.035 for mathematics, science and reading) which again indicates that at this general level computer use makes very little difference to pupils' achievement. Simply having more computers does not make much difference.

A similar weak link between high computer use and pupil attainment was reported in a preliminary survey for a Teacher Training Agency study in England (Moseley et al. 1999, p 82) though the authors did not interpret this as a causal link, but rather that more effective teachers (and more effective schools) tended to use more innovative approaches, or tended to use the resources that they had more effectively. If this interpretation is accepted it suggests that it is more important to think about how computers are used in schools.

In Ghana, the national curricula for the various subjects contain policy statements about the use of ICT in teaching and learning, the limitations imposed by inadequate number of computers in institutions, poorly trained educators and lack of internet connectivity pose a major challenge to the implementation of the policy to integrate ICT into teaching and learning. Pan-African research agenda on pedagogical integration of ICT (Panaf) Ghana report phase 1 (2009).

2.5 The role of teacher education in facilitating integration of ICT

According to (panaf) Ghana report phase 1 (2009) study showed that at the pre-tertiary levels, teachers are not trained to use ICT for teaching and learning, though the new curricula required that teachers integrate ICT into instruction across the curriculum. Visits to, and interviews with, teachers in some of the colleges of education revealed that the colleges of education were doing little to equip trainees with skills necessary to integrate ICT into teaching during their pre-service teacher training programmes. This study has also shown that the teaching universities are not doing much in this regard.

In many developing countries, most teachers have minimal or no ICT skills themselves and therefore cannot develop these in learners. Two of the most important supports for ICT integration into teaching and learning are effective Initial Teacher Education (ITE) and Continuing Professional Development (CPD). Both have the greatest impact on the beliefs and practice of teachers, and yet professional development time in particular is often not budgeted for (Venezky, 2004). Moreover, research into teacher learning in northern hemisphere contexts suggests that traditional, one-off external in-service workshops tend to be of limited value in developing sustained transformation of practice (e.g. Glazer & Hannafin, 2006).

The multiplicity of schemes over the last decade to introduce ICT into schools in Sub-Saharan Africa (infodev4 offers a database of such activities) have likewise often failed to live up to their aspirations because of their top-down nature and insufficient attention given to involving teachers. A growing body of research in this area shows that a more promising way forward is a sustained professional development programme that draws on teachers local professional communities, encourages ongoing peer learning by teachers of similar subjects and age groups and supports reflective classroom practice (e.g. Bowker, Hennessy, Dawes, & Deaney, 2008).

In recent years, there has been an encouraging emphasis on in-service development, supported by enlightened national ICT policy initiatives (as outlined

for the East African context by Wamakote et al, this volume). Examples of seemingly successful CPD programmes have included the Connectivity for Educator Development programme in Uganda 5 Schools Online programmes 6 in Senegal and Tanzania, World Links programmes 7 in Ghana and Uganda, 8 and the Commonwealth of Learning Southern Africa Teacher Training Programme 9. Such programmes focus on helping teachers to use technology as a tool, and to transform their classrooms into interactive learning environments.

For example the Intel Corporation Teach programme is currently supporting Kenya's transition from traditional teaching methods through educating teachers in the integration of ICT into primary and secondary school education (Panafrikan Research Agenda on the Pedagogical Integration of ICTs: (Karsenti, 2009). Using a train the trainer model, the 25 selected participants from teacher training colleges, Centre for Maths and Science and Technology Education in Africa, Kenya Institute of Education and Kenya Education Staff Institute, are working on the development of online material to then orient 250,000 teachers nationwide. Anecdotal evidence suggests, however, that teachers experiencing national and international training programmes falter after their initial learning success if they do not receive follow-up support in schools.

2.6 Use of ICT in Education

One can now think of numerous ways ICT is used by tutors and not necessary in their instructions. It has generally been observed that most Ghanaians including teachers are using ICT in communication, storage of information and preparing their documents to using it in their instructions (Yidana & Asiedu-Addo, 2001).

According to Yidana and Asiedu-Addo the preparation of personal records, managing student and timetables are some of the documents most tutors prepare.

The growing use of the internet as tools for information access and communication in educational classrooms (Oliver & Towers, 2000) are neglected by tutors and they instead use the internet for their private e-mails. According to Roblyer and Edwards (2000), as cited by Jones (2009) a number of different, yet interrelated, perspectives on the use of ICT in education by tutors have developed over the last fifty years. They went on to report that some tutors see the computer as an audiovisual system best used, instead of lectures and books, to deliver information and others build the notion of the computer as audiovisual tool and as an instructional system.

On the other hand (Romeo, 2001). views it as the notion of using computers as a mechanism to enhance teaching This view, often referred to as computers in education by Romeo (2001), very much influenced by the ideas of constructivism, is about exploiting the computer's, (and now intranets' and the internet's) power, versatility, flexibility and uniqueness to help the teacher establish powerful teaching environments (Romeo, 2001). Meanwhile, it does not mean that computer literacy and computer awareness are not important, just that both the use of the computer to enhance teaching and the development of computer literacy, are seen as part of an integrated program as opined by Anderson (1999). A range of skills are required by tutors to use technology for classroom and administrative purposes. These skills are:

- using and managing technology

- using basic computer applications
- using desktop publishing and presentation software
- using multimedia
- using communication technologies
- using Learning Technologies in the key learning areas
- using school level computer applications for administrative purposes (Department of Education, Victoria, 1998).

Reynolds (2001) is of the view that schools are using ICT to access existing information and knowledge, rather than using it as an integral part of pedagogical practice and Ward (2003) further claims that there is limited use of ICT in classroom practices. This indicates a need for teachers to gain an understanding of how ICT can be used to extend students' thinking and problem-solving skills, rather than just as a publication and research tool.

Moreover, Amarasinghe and Lambdin (2000) as cited in Rahman, Munirah & Zurida (2003), described three different varieties of technology usage; using technology as a data analysis tool, using technology as a problem-solving/mathematical modelling tool and using technology to integrate mathematics with a context. Apaw (2009) is of the view that the integration of ICT into Education will result in the creation of new possibilities for learners and teachers to engage in new ways of information acquisition and analysis. ICT may enhance access to education and improve the quality of education delivery on equitable basis (Ministry of Education and Sports, 2006).

ICT teaching and learning encompasses a variety of techniques, tools, content and resources aimed at improving the quality and efficiency of the teaching-learning process ranging from projecting media to support a lesson to multimedia self-learning module, to simulations to virtual learning environments. This exposes the teacher to a variety of options available to utilize various modes/ICT tools for effective pedagogy. Each of such devices or strategy also involves changes in the classroom environment, understanding of which has a bearing on its effectiveness. Availability of a wide range of such teaching and learning materials serves as a catalyst for the transformation of classrooms into smart classrooms. (ICT Policy in School Education, 2009)

In addition to that, BECTA (2003) explained that ICT is essential in teaching and learning mathematics. ICT improves the way mathematics should be taught and enhances student understanding of basic concepts, promotes greater teamwork among students and encourages communication and the sharing of knowledge, gives rapid and accurate feedbacks to students which gives positive motivation. The use of ICT in the mathematics classroom also allows students to focus on strategies and interpretations of answers rather than spend time on tedious computational calculations. This approach promotes higher order thinking and better problem solving strategies which are in line with the National Council of Teachers of Mathematics (NCTM) recommendations; students would then use technology to focus on problem solving processes and on calculations related to the problems.

A research conducted by Info-communications Development Authority (IDA) in Singapore in 2002 described the ways by which ICT improved the teaching-learning process and engaged the students. These included the interactive multimedia courseware which facilitated teaching and learning of abstract ideas and theories, ICT encouraged self-directed and self-paced learning on the part of the students, there was no time constraint and students had greater autonomy in learning, ICT integration served as a catalyst to boost the students' self-confidence as seen, for example, in the benefits derived by students from the use of ICT in the presentations from preparation to the final delivery of the presentations, ICT facilitated discovery learning as it encouraged students to ask and address more in-depth questions, ICT stimulated the students' sensory and cognitive curiosity. Most of the students were visual learners and the use of vibrant colours, interactive graphics and icons provided stimulation, the development of interactive courseware could involve students. When students are engaged in the design and development of multimedia, they acquire a set of life skills (<http://schools.s one.net.sg/findings1.html>).

The respondents also stated how the ICT initiative by IDA and the MOE in Singapore had enhanced the ICT integration capacity of teachers, noting the following: ICT use in training and teaching raises the teachers' ICT awareness and competency level; while ICT could never replace real-life teachers, ICT resources complement existing academic resource materials to enhance learning; ICT, including both asynchronous and synchronous communication tools,

facilitates the exchange of knowledge and resources among the teachers and ensures knowledge-based connectivity among them; as peer teaching and sharing are practiced in many schools, this fosters knowledge exchange on ICT; and ICT provides opportunities to explore beyond the teachers' academic areas and to work with ICT vendors in developing instructional design and technical skills (UNESCO, 2004).

With these benefits at the background, the Government of Ghana introduced ICT education from the primary level upwards. In the Junior High School (JHS) ICT curriculum (Ministry of Education and Sports, 2007), it is stated that teachers must integrate ICT into the Mathematics curriculum by using spreadsheet and calculators for calculations. If this must be done effectively, then teachers need to receive appropriate training in ICT. Also, schools need to have computers, technical support and maintenance, software such as Derive, Geometer's Sketchpad, Inspiration, Macromedia Author ware, Microsoft Excel, Microsoft Encarta Math Tools, Green Globe, Graphmatica and other teacher utility applications and interactive tutorials. It will be expedient for schools to also have internet connectivity.

However, the Mathematics curricular at the SHS in Ghana did not specify what type of software the Mathematics tutors should use in their instructions. The syllabus is designed to provide basic skills in Information and Communications Technology (ICT) for Senior High School (SHS) students. It was expected that the knowledge and skills gained in the course will help students to use ICT in

almost all their courses at school. The syllabus covers selected basic topics in ICT which offers hands-on activities to help students acquire the required ICT skills” (Ministry of Education, 2007). The general aims of the ICT syllabus indicates that the syllabus was designed to help the student to: acquire basic ICT literacy, develop interest and use ICT for learning in other subjects, acquire the knowledge for application of ICT in education and business, use the Internet to communicate effectively, access and share information through the Internet, follow basic ethics in the use of ICT, maintain high level speed in typing.

2.7 ICT Problems in Education

Despite all the numerous benefits of ICT in Education, there are some few problems in using ICT in education purported by some researchers. Keong, Horani and Daniel, (2005), in a study identified six major barriers. These were lack of time in the school schedule for projects involving ICT, inadequate teacher training opportunities for ICT projects, lack of adequate technical support for ICT projects, lack of knowledge about ways to integrate ICT to enhance the curriculum, integrating and using different ICT tools in a single lesson and the absence of access to the necessary technology at the homes of students.

Apaw (2009) cited that there are seven major problems that exist while integrating ICT into lessons. These problems were lack of confidence among teachers during integration, lack of access to resources, lack of time for the integration, lack of effective training, facing technical problems while the software is in use, lack of personal access during lesson preparation and the age of the teachers.

In effect, lack of resources and expertise in the school based curriculum process, combined with lack of computer literacy and unfamiliarity with the emerging of computers in education makes it difficult for teachers to use ICT in their instructions. As such if we are to exploit the true potential of ICT in education then the Mathematics teachers at the SHS in Ghana need to be empowered with the skills to use ICT in their instructions. Teachers also need to go beyond traditional approaches and become acquainted with new methods in order to get a clear understanding of the educational functionality of ICT in their educational practices. This implies that there is less use of ICT into the classroom and tutors rather depend on ICT for document preparation, keeping of students' assessment records and the like.

As it has been explained above, there exist factors that impede the adoption of ICT in education in Ghana. These included factors such as: lack of funding to support the purchase of the technology, lack of training among established teaching practitioners, and lack of motivation among teachers to adopt ICT as teaching tools (Chowdhury & Zahurul 2009). It is worth noting that the main obstacles in the growth of ICT in education in Ghana is not the high price of computers, but rather the lack of government support for equipping schools with new computers, teaching tools and sustainable hardware infrastructure (Ministry of Education, 2009).

2.8 Why the use of a Computer Algebra System such as Derive 5?

Derive 5 is a Computer Algebra System (CAS) with the ability to manipulate and solve equations, as well as analyze equations from a graphic and numeric perspective. In short, this program will perform all of the manipulations taught in College Algebra. However, algebra from a real world problem solving perspective requires more than manipulation of symbols. The problem solver must formulate the problem clearly, develop a model, apply the model, and perhaps extend the model to new situations. The focus of labs will be to develop problem solving and conceptual understanding. Derive 5 gives us the power, flexibility and time to explore problems from an algebraic perspective. Derive 5 performs the mathematical manipulations needed to apply the model, allowing us to focus on problem solving. Almost all of the Derive 5 commands that will be used in this course are introduced in the first few labs. It is a good idea to maintain a running reference sheet of useful commands as you learn them.

In order to use Derive 5 as a tool for exploring mathematics, one must first become familiar with the basic commands and syntax of the package. So the primary goal of this research is to introduce learners to Derive 5. We learn math concepts better when we actively participate in doing mathematics that is from the constructivist point of view. When students are faced with some you challenging mathematical questions, it is expected that they engage in solving them. In cases like these mathematics teachers are expected to assist when are stuck, but the student must take some time to play with the problem him/her self first. By so doing teachers become facilitators and not a teacher any more. One may ask how? Because a teacher tells, a facilitator asks; a teacher lectures from the front, a

facilitator supports from the back; a teacher gives answers according to a set curriculum, a facilitator provides guidelines and creates the environment for the learner to arrive at his or her own conclusions and at his own pace; a teacher mostly gives a monologue, a facilitator is in continuous dialogue with the learners according to Rhodes and Bellamy (1999).

A further characteristic of the role of the facilitator in the social constructivist viewpoint is that the facilitator and the learners are equally involved in learning from each other as well (Holt & Willard-Holt, 2000). All these Derive 5 does not ignore and hence, to the benefit of both the student and the teacher as well. Elsewhere in the Czech Republic, Řezanková, (2007) continue to explain that the Derive software is recommended for teaching at elementary and secondary schools by the Ministry of Education, Youth and Sports of the Czech Republic and in some universities, Derive is used in teaching Economics, Mathematics and Econometrics. I hope Ghana Education Service will in the near future recommend some software such as Derive and Microsoft Encarta Math Tools to be used to instruct Mathematics.

In this research, the researcher did not impose anything on the students but rather was a facilitator and guide them to construct their own knowledge. A facilitator is someone who should be able to adapt the learning experience by using his or her own initiative in order to steer the learning experience to where the learners want to create value.

2.9 Concept of Matrices an Integral part of Algebra

The introduction and development of the notion of a matrix and the subject of linear algebra followed the development of determinants, which arose from the study of coefficients of systems of linear equations. Leibnitz, one of the two founders of calculus, used determinants in 1693 and Cramer presented his determinant-based formula for solving systems of linear equations (today known as Cramer's Rule) in 1750. In contrast, the first implicit use of matrices occurred in Lagrange's work on bilinear forms in the late 1700s. Lagrange desired to characterize the maxima and minima of multivariate functions. His method is now known as the method of Lagrange multipliers. In order to do this he first required the first order partial derivatives to be 0 and additionally required that a condition on the matrix of second order partial derivatives hold; this condition is today called positive or negative definiteness, although Lagrange didn't use matrices explicitly.

Gauss developed Gaussian elimination around 1800 and used it to solve least squares problems in celestial computations and later in computations to measure the earth and its surface (the branch of applied mathematics concerned with measuring or determining the shape of the earth or with locating exactly points on the earth's surface is called geodesy. Even though Gauss' name is associated with this technique for successively eliminating variables from systems of linear equations Chinese manuscripts from several centuries earlier have been found that explain how to solve a system of three equations in three unknowns by "Gaussian" elimination. For years Gaussian elimination was considered part of the development of geodesy, not mathematics. The first appearance of Gauss-Jordan

elimination in print was in a handbook on geodesy written by Wilhelm Jordan. Many people incorrectly assume that the famous mathematician Camille Jordan is the Jordan in "Gauss-Jordan" elimination.

For matrix algebra to fruitfully develop one needed both proper notation and the proper definition of matrix multiplication. Both needs were met at about the same time and in the same place. In 1848 in England, J.J. Sylvester first introduced the term "matrix," which was the Latin word for womb, as a name for an array of numbers. Matrix algebra was nurtured by the work of Arthur Cayley in 1855. Cayley studied compositions of linear transformations and was led to define matrix multiplication so that the matrix of coefficients for the composite transformation ST is the product of the matrix for S times the matrix for T . He went on to study the algebra of these compositions including matrix inverses. The famous Cayley-Hamilton theorem which asserts that a square matrix is a root of its characteristic polynomial was given by Cayley in his 1858 Memoir on the Theory of Matrices. The use of a single letter A to represent a matrix was crucial to the development of matrix algebra. Early in the development the formula $\det(AB) = \det(A) \det(B)$ provided a connection between matrix algebra and determinants. Cayley wrote "There would be many things to say about this theory of matrices which should; it seems to me, precede the theory of determinants."

Mathematicians also attempted to develop of algebra of vectors but there was no natural definition of the product of two vectors that held in arbitrary dimensions. The first vector algebra that involved a noncommutative vector product (that is, \mathbf{v}

$\mathbf{v} \times \mathbf{w}$ need not equal $\mathbf{w} \times \mathbf{v}$) was proposed by Hermann Grassmann in his book *Ausdehnungslehre* (1844). Grassmann's text also introduced the product of a column matrix and a row matrix, which resulted in what is now called a simple or a rank-one matrix.

In the late 19th century the American mathematical physicist Willard Gibbs published his famous treatise on vector analysis. In that treatise Gibbs represented general matrices, which he called dyadics, as sums of simple matrices, which Gibbs called dyads. Later the physicist P. A. M. Dirac introduced the term "bra-ket" for what we now call the scalar product of a "bra" (row) vector times a "ket" (column) vector and the term "ket-bra" for the product of a ket times a bra, resulting in what we now call a simple matrix, as above. Our convention of identifying column matrices and vectors was introduced by physicists in the 20th century. Matrices continued to be closely associated with linear transformations. By 1900 they were just a finite-dimensional subcase of the emerging theory of linear transformations. The modern definition of a vector space was introduced by Peano in 1888. Abstract vector spaces whose elements were functions soon followed. But no matter how basic matrices are, they continue to pose problem hence the need for remedy through ICT.

2.10 The Use of Technology in Pre-Service Teachers' Mathematical Achievement

The impact of ICT use on students' learning has quite consistently shown that thoughtful use of ICT in mathematics classes improves student mathematics

achievement and attitudes towards mathematics. Students who learn paper-and-pencil skills in combination with technology-based instruction such as the simple four-function calculators and the most sophisticated CAS software when tested without calculators perform as well or better than students who do not use technology in instruction (Heid, 1997).

For instance, Ellington (2003) used the method of meta-analysis to combine the findings of fifty-four (54) research studies carried out between January 1983 and March 2002 and determined the effects of calculators on pupils' achievement. Each of the studies compared the outcomes of experiments in which the treatment group used calculators while a control group received equivalent method of mathematical instruction with no access to calculator. Results revealed that students' operational and problem-solving skills improved when calculators were an integral part of testing and instruction.

Loveless (2003), explained that thoughtful use of ICT improves student mathematics achievement and attitudes toward mathematics. His study was based on Hembree and Dessarts' (1986) meta-analysis of 79 non-graphing calculator studies, which was concluded that the use of hand-held calculators improved students' learning. The analysis also indicated that students' using calculators had better attitudes towards mathematics and better self-concepts in mathematics than their counterparts who did not use calculators. They also found that there was no loss in student ability to perform paper-and-pencil computational skills when calculators were used as part of mathematics instruction.

In another study, Hembree and Desserts¹¹ (1992) reported the findings of a meta-analysis of the effects of pre-college calculator use. This research analyzed results from eighty-eight studies focused on students¹² achievement and attitude. Each study involved one group of students using calculators and another group having no access to calculators. From their analysis, Hembree and Dessart concluded that the calculator did not hinder students¹³ acquisition of conceptual knowledge and that it significantly improved their attitude and self-concept concerning mathematics.

In another meta-analysis of 24 studies, Smith (1997) reports significant achievement differences in problem solving, computation and conceptual understanding favoring students who use calculators. Recent studies show students¹⁴ using non-graphing calculators perform as well or better on several measures of achievement than students who do not use calculators.

The definition, effectiveness and future of handheld technology-based Instruction for Mathematics by Prosser and Lee emphasized and explained that calculators are being used to teach a myriad of mathematical concepts beyond the four basic functions. Students are learning to factor, solve linear inequalities, perform inferential statistics, and much more. It was concluded that students score higher on summative assessments and are able to make deeper conceptual connections when they learn through the use of calculators

Asare-Inkoom, Apau-Gyamerah, and Najimudeen, (2007) cited that calculator proponents claim that calculators allow students to spend less time on tedious calculations and more time on understanding and solving problems, helps students to develop number sense, allows students to study mathematical concepts they could not attempt if they had to perform the related calculations themselves, allows students who would normally be turned of from mathematics because of frustration or boredom, to increase their mathematical understanding and makes students more confident about their mathematical abilities.

They further explained that studies conducted by some individuals also show that thoughtful use of calculators in mathematics classes improves students' mathematics achievement and attitudes towards the subject. For instance, Asare-Inkoom et al. (2007) cited that the use of hand-held calculators on students' attitudes and computational skills play a significant role in improving students' performance in mathematics.

Mereku et al (2007) in a report of the Ghana National working party on the use of calculators in the BECE mathematics paper explained that calculators are effective aids when it comes to problem solving, reinforcement of computational skills, pattern recognition, and number sense. They also help in teaching topics such as percentages and fractions, integers, perimeter, area, rates, taxes, and exponents.

It is worth emphasizing that using ICT tools such as Derive 5 in the learning of

mathematics would result in increased achievement and improved student attitudes. The Derive 5 is a valuable tool for exploration and discovery in problem-solving situations and when introducing new mathematical content. By reducing computational time and providing immediate feedback, Derive 5 helps students focus on understanding their work and justifying their methods and results. The Derive 5 is particularly useful in helping to illustrate and develop concepts and in making connections between algebraic and geometric ideas.

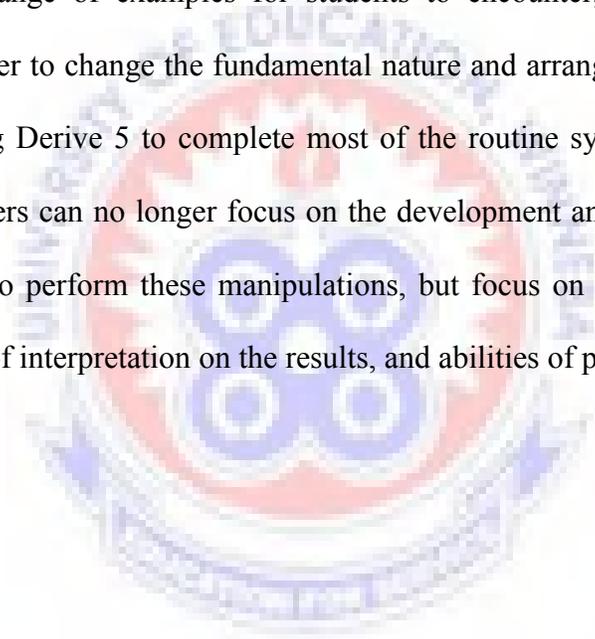
In order to accurately reflect their meaningful mathematics performance, students should be provided with more computers in practicing sessions and in achievement tests in the near future and teachers should also use the ICT facilities available during instruction. Not to do so is a major disruption in many students' usual way of doing mathematics, and an unrealistic restriction because when they are away from the school setting, they will certainly use a computer in their daily lives and in their workplaces.

2.11 Summary

Algebra is not easy for many students, several reasons were discussed, like students could easily handle numbers but not variables; students always interpret the letters of variables with some objects they related to, like a for apple, so the students may be confused when these letters are used to present other meanings in algebra; students always treat variables as symbols to be manipulated rather than

as quantities to be related and students do not have the ability to see behind the symbols to create appropriate meaning when solving algebra problems.

Literature illustrates that technology have positive influences on mathematics learning, as computer can create a useful work environment in school teaching, and Derive 5 can afford students access to high-level mathematical processes, acts as an amplifier to extend the existing curriculum, like generate a large number and a great range of examples for students to encounter, as well as serves as a reorganizer to change the fundamental nature and arrangement of the curriculum, like using Derive 5 to complete most of the routine symbolic manipulations, so the teachers can no longer focus on the development and refinement of students' abilities to perform these manipulations, but focus on development of students' abilities of interpretation on the results, and abilities of problem solving.



CHAPTER 3

RESEARCH METHODOLOGY

3.0 Overview

This chapter describes the research process that informed the study. The chapter was articulated in terms of the research design, population and sample, research instruments and piloting the instruments. These were followed by the intervention, method of data collection and summary.

3.1 Research Design

To address the different research questions, the researcher employed different methods. The research design was a combination of action research and a developmental design. A developmental research design was employed because according to Walker and Bressler (1993) developmental research is a disciplined inquiry conducted in the context of the development of a product or programme for the purpose of improving either the programme being developed or capabilities to develop better programmes of its kind, or both. In a nut shell, there are two types of developmental research. They are Type 1 and Type 2. Type 1 involves the study of a specific programme, whereas the type 2 is the evaluation, or development processes, tools aiming at generating knowledge on how to design. For this study, the researcher adopted the Type 1.

The researcher used developmental research because it helped to improve pre-service teachers' learning and ways Derive 5 can be effectively used. And on the other hand action research is a process in which participants examine their own educational practices systematically and carefully, using the techniques of research. It is based on the assumptions that teachers and students work best on problems they have identified for themselves, become more effective when

encouraged to examine and assess their own work and then consider ways of working differently, work collaboratively with colleagues who help them in their professional development.

3.2 Population and Sample

This research was conducted at the Mampong Technical College of Education one of the prominent 38 colleges of education in Ghana. Basically Mampong Technical College of Education is a male institution. The only male College of Education in the whole of West Africa designed to represent inter ethnicity and religious groups in Ghana. The population of the school is five hundred (500) excluding the third years with a teaching staff of thirty-nine. Among the thirty nine teaching staff, five are female and thirty-four are male. Also Mampong Technical College of Education can boast of 46 non-teaching staff. The courses offered are technical bias and hence, has been the nation's most renowned technical teachers college. I therefore chose the second years as the target group for which the problem of matrices is associated with. In such a study the whole population of the second year students could have been used, but due to limitations like lack of personnel to give the research assistance and the size of the computer laboratory only one class of the second years was used for the study.

In this study, non-probability sampling was used, specifically convenience sampling. Convenience sampling was employed because of logistic constraints, financial constraints and ease of accessibility. One class was selected using the convenience sampling method. The class was selected based on the fact that the

researcher teaches them and accessibility to the researcher. In qualitative studies, sampling is usually non-random. All quantitative data is based upon qualitative judgments; and all qualitative data can be described and manipulated numerically. Therefore, the researcher was of the view that the class was ideal for this study because the researcher was able to monitor the progress of their performances during their period of interaction. The 2E class was chosen conveniently because of accessibility to the researcher, which had (30) students. Hence the sample size for the study was thirty (30) students which are men of ages between 21 and 30. They were given pseudonyms for the observations, interviews, pre-test and post-test. These students were assigned to a computer which had Derive 5 application software needed for the research installed on it.

3.3 Research Instrument

Taking into consideration the nature of the research questions, instruments used were the pre-test, the post-test for the collection of data. A checklist for determining basic errors in matrices (see Appendix A) and a semi-structured interview guide (see Appendix B) were constructed for the study. The advantage of the semi-structured interview is that the interviewer is in control of the process of obtaining information from the interviewee, but is free to follow new leads as asserted by (Bernard, 1988). During interview sessions, the researcher allowed free flow of ideas from the interviewees.

3.3.1 Interview Guide

Item 1 in the interview guide had to do with whether pre-service teachers intend to use Derive 5 software again in their teaching and learning. Items 2 also solicited pre-service teachers' view on how friendly Derive 5 working environment is. Item 3 in the interview guide was meant to find out how the use of the Derive 5 could improve pre-service teachers mathematical understanding and interest at Mampong Technical College of Education. Item 4 was meant to find out pre-service teachers competence with the use of Derive 5. Item 5 was also meant to find out whether the Derive 5 could improve students computational skills in mathematics. In all, fifteen pre-service teachers were interviewed with all in favor with the use of technology.

3.3.2 Tests (Pre and Post)

The Pre-test (see Appendix C) and the post-test (see Appendix F) questions were five in number. Some M.Phil Mathematics Education students I happen to have around read through the, pre-test and post-test questions and made suggestions that were incorporated. The checklist for determining basic errors in matrices, interview, pre-test and post-test questions were further cross-checked and corrections made by the researcher's supervisor after which they were administered.

3.3.3 Rubrics for Scoring Students Pre-Test and Post-Test

The marks awarding system served the purpose of assigning the learners into various. Each correct answer to the 5-item test was assigned some marks depending on the gravity of the question. Hence, each student's score ranged from

0 – 15 marks. The percentage scores were calculated for each pre-service teacher. The percentage scores were calculated for each pre-service teacher's pre and post tests using SPSS paired t-test

3.4 Piloting the Instrument

The term *pilot study* is used in two different ways in social science research. It can refer to so-called feasibility studies which are "small scale version[s], or trial run[s], done in preparation for the major study (Polit et al., 2001: 467). However, a pilot study can also be the pre-testing or 'trying out' of a particular research instrument (Baker 1994). One of the advantages of conducting a pilot study is that it might give advance warning about where the main research project could fail, where research protocols may not be followed, or whether proposed methods or instruments are inappropriate or too complicated. Therefore the researcher not wanting to take the risk undertook the study with six student of the E class before embarking on the main research. The researchers' finding showed to be reliable hence, decided to continue the entire research.

According to (Apaw, 2009) Reliability refers to the extent to which research findings can be replicated. Golafshani (2003) cited that reliability is the extent to which results are consistent over time and an accurate representation of the total population under study is referred to as reliability and if the results of the study can be reproduced under a similar methodology, then the research instrument is considered to be reliable. In other words, it is such that if a later researcher follows the same procedure as described by an earlier researcher and then

conducts the same action research all over again, the later researcher should arrive at similar findings and conclusions.

Apaw (2009) opined that for the mere fact that a number of people have experienced the same occurrence does not make data more reliable, particularly in educational studies. Then, their view on reliability should be altered from whether findings will be found again to whether the results are consistent with the data collected. Furthermore, triangulation approach was employed to address the issue of reliability. He advocated that the use of triangulation strengthens a study by combining various methods. This means using several methods or data, including using both quantitative and qualitative approaches. Triangulation is using two or more instruments to collect data in a study. The researcher fully described how the data was collected, how categories were derived, and how decisions were made throughout the study. Data collected using interviews, observations, pre-test and post-test helped readers get „holistic understanding of the situation.

When a research measures what it is intended to measure then it is valid. Apaw (2009) stated that validity addresses the following question: did the research actually measure what it's intended to measure since one of the assumptions underlying qualitative research is that reality is holistic, multidimensional, and ever-changing; it is not a single, fixed objective phenomenon waiting to be discovered, observed, and measured as in quantitative research. To ensure internal validity, “peer review” was employed.

Reliability refers to the extent to which research findings can be replicated (Merriam, 1998). In other words, it is about if a later researcher followed the same procedures as described by an earlier researcher and conducted the same developmental research all over again, the later researcher should arrive at similar findings and conclusions (Yin, 2003).

Regarding the issue of reliability, Merriam (1998) argues that simply because a number of people have experienced the same phenomenon does not make data more reliable, particularly in educational studies. Then, the way of thinking about reliability should be changed from whether findings will be found again to whether the results are consistent with the data collected (Lincoln & Guba, 1985). In order to solve the problem concerning reliability, triangulation approach was used to answer research question 3. Triangulation is using two or more instruments to collect data in a study. Secondly, I have fully described how the data were collected, how categories were derived.

3.5 Intervention

The researcher designed an intervention in the form of lessons delivered and discussed in the class, and the ICT laboratory. This was done using Derive 5 as the major teaching and learning aid. The same software was used for practicing and exercises by the pre-service teachers with the researcher as the facilitator. The sample population for the study was 30 and they were divided into two groups with 15 students in a group. The time allocated for each group was one hour a day for five (5) days. However, pre-service teachers were given thirty (30) minutes a

day for three (3) days for each group to use the software for practice and assignments. Some pre-service teachers also practiced on their own at their own convenience. Pre-service were educated about the properties and use of Derive 5 in the context of the course. The course was taught for five weeks. The schedule of the course is shown in table 3.1

Table 3.1: the schedule of the course activities

Week	Course activities
1 st week	Introduction to matrices
2 nd week	Matrices as systems of linear equations
3 rd week	Introduction to derive 5
4 th week	Finding the general algorithm for multiplying matrices
5 th week	Getting rid of matrices

See appendix E for detailed activities.

3.6 Method of Data Collection

Diverse methods were utilised to collect data for this research. These included semi-structured interview, pre-test and post-test, and classroom observation. Before the post-test, an interview was conducted. Therefore in order to develop questions for the pre-test and post-test, the researcher made some observation and made notes on daily basis in the course of teaching matrices in the classroom. This action provided a platform for the researcher to develop an ambiance of faith and trust with the students. Moreover, class exercise, assignment and tests given

to the students during the normal school hours were collected and analysed to reveal problematic areas. After the intervention as narrated in 3.5, another interview was conducted as means of answering the third question. This is because students had had hands-on experience with the computer using the software and hence, the need to fine out from them weather they prefer the constructivist approach or not. Now with the fourth question concerning reliability, triangulation approach was used to answer it. Triangulation is using two or more instruments to collect data in a study. Secondly, I have fully described how the data were collected, how categories were derived, and how decisions were made throughout the study which is called audit trail (Merriam, 1998). In all, the results of the pre-test and the post-test were used as the data for this study.

In an attempt to find out how to improve pre-service teachers understanding of matrices using the matrix approach and the use of Derive 5 a computer assisted instruction, the researcher prepared a checklist for determining basic errors in matrices (see Appendix A), interview guide (see Appendix B), pre-test (see Appendix C) and a post-test (see Appendix F). The checklist for determining computational errors was used to observe the students scripts as they were marked. The researcher afterwards conducted interviews for the students. Students were interviewed using five semi-structured interview questions (see Appendix B). Reponses from the interview sessions were coded in the codebook and sorted using SPSS.

The pre-test was administered without the use of any computer assisted software. Pre-service teachers were given twenty-five minutes to answer the questions. The researcher and the other mathematics teachers invigilated the students after which the scripts were collected, marked and the scores recorded. The scores were grouped into two categories". The first category was for those who scored between 0 and 5 while those who scored between 5 and 15 were placed in the second category. These categories were made to fine out at the end of the intervention those individual pre-service teachers who have made improvement with the introduction of technology (Derive 5).

The intervention design (see Appendix E) was implemented some days after the pre-test was administered using a day to day activity. Moreover, (5) post-test questions were administered to the students to find out whether pre-service teachers have improve upon the understanding of matrices using the Derive 5 see (Appendix F). The researcher and other mathematics teachers invigilated the students after which the scripts were collected, marked and the scores recorded.

In this study the researcher employed mixed methods for the collection of data. Mixed methods research is the type of research in which a researcher or team of researchers combines elements of qualitative and quantitative research approaches (e.g. use of qualitative and quantitative viewpoints, data collection, analysis, inference techniques) for the broad purpose of breadth and depth of understanding and corroboration (Harrison & Reilly, 2011). Using a mixed-method design is considered to be appropriate to gain a more comprehensive picture of the

phenomena. The study used a combination of both quantitative and qualitative methodology to explain or describe the situation which gave an in-depth analysis of one or more events, settings, programs, social groups, communities, individuals, or other bounded systems. Observations, interviews, pre-test and post-test were also employed to find out the effect of the scientific calculator in improving students' computational skills as well as students' achievement in mathematics.

For the quantitative methodology, the data collected was analysed using SPSS software. SPSS gave the descriptive statistics which was used to compare the means of the post-test and the pre-test. For the qualitative methodology, interviews, semi-structured was used, also the answers provided by students formed the data and was analysed by descriptions.

3.7 Summary

The study involved thirty (30) pre-service teacher of Mampong Technical College of Education. The pre-service teachers were observed after which they responded to five interview items. The purpose of the interview was to find out how the use of mathematical software such as Derive 5 could improve their understanding/computational skills of matrices at Mampong Technical College of Education. The students were made to write a pre-test to verify the chief examiners' report 2004 institute of education University of Cape Coast. The pre-

test was also to ascertain whether the problem really existed and identify the type of errors that students committed when solving problems in matrices. A one week intervention was also organized to help students overcome their problems with the use of Derive 5 a computer assisted instruction after which students“ were made to write a post-test to find out whether students understanding as well as computational skills in mathematics had improved with respect to the usage of the ICT facility. The researcher finally discussed the post-test questions with the students.



CHAPTER 4

DATA ANALYSIS

4.0 Overview

This chapter focuses on the results of the analyses of the data and discussion of the findings. The data were organized and presented using tables, charts and descriptive statistics. The results were presented under the following themes:

1. Extent to which the use of Derive 5 help to improve pre-service teachers' ability to use the matrices approach in solving linear equations?
2. Pre-service teachers' interest in using the technology in mathematics
3. Effect of the technology on pre-service teachers' achievement in mathematics

4.1 Extent to which the use of derive 5 helps to improve pre-service teachers' ability to use the matrices approach in solving linear equations

The major research question raised for the study was to examine how the use of the utility applications such as Derive 5 could improve upon pre-serve teachers' computational skill in questions involving matrices using the matrix approach at the Mampong Technical College of Education. In order to accomplish this, the 2E students' were allowed to use the computers assigned to them in the tests described in the previous chapter. When scoring the tests data was obtained on errors the students made in during their tests. Four common errors were identified and these are sequence of multiplication, getting rid of the matrix concept (that is a concept applied in order to write one matrix in terms of other matrices), product of two matrices and the appropriate use of parenthesis. Some examples of the students' errors pre-service teachers commit without the use of any utility applications such as Derive 5 are discussed as follows.

4.1.1 Example of errors in the sequence of multiplication

Below is how the item in Box 1 was answered by most students.

Solve the system using the matrix approach

$$-x + 5y = 4$$

$$2x + 5y = -2$$

Box 1

In matrix form the above equation is written as

$$\begin{bmatrix} -1 & 5 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}, \text{ by letting } M = \begin{bmatrix} -1 & 5 \\ 2 & 5 \end{bmatrix}, N = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } P = \begin{bmatrix} 4 \\ -2 \end{bmatrix} \text{ the}$$

students then obtained the equation $MN = P$. But this time the students were to solve for N , hence the need to get rid of M by multiplying both sides of the equation by M^{-1} (*from the left*). All I realised was that the students were rather multiplying (*from the right*) giving them the equations $MM^{-1}N = PM^{-1}$ instead of the equation $M^{-1}MN = M^{-1}P$.

4.1.2 Example of errors in the use of getting rid of the matrix concept

Suppose pre-service teachers are given a matrix equation $AB = C$, how possible will the individual write B in terms of other matrices? In other words, how can you get rid of A from the left-hand side?

The answer is to multiply both sides of the equation by A^{-1} (*from the left*).

$$\therefore A^{-1}AB = A^{-1}C$$

$\therefore IB = A^{-1}C$ (where I is the multiplicative identity matrix)

$\therefore B = A^{-1}C$

Now, if pre-service teachers are given $AB = C$, how do you write A in terms of other matrices? This part, pre-service teachers must multiply both sides (*from the right*) by B^{-1} , giving

$$ABB^{-1} = CB^{-1}$$

$\therefore AI = CB^{-1}$

$\therefore A = CB^{-1}$ Done

This the researcher observed that the pre-service teachers“ ignored the approximation which affected their answers

4.1.3 Example of error in the use of product of matrices

The question item three (3) also exposed a lot of pre-service teachers weakness when it came to product of matrices. Students were to answer this question: if

$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, then find A^2 . Some of these students gave the answer as. $A^2 =$

$\begin{bmatrix} 2^2 & 3^2 \\ 1^2 & 4^2 \end{bmatrix}$. Instead of the solution below.

$$\text{That is } A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

to obtain the element of the first row and the first column

$$\begin{pmatrix} 2 & 3 \end{pmatrix} \times \begin{bmatrix} 2 \\ 1 \end{bmatrix} = (2 \times 2) + (3 \times 1) \text{ given } 4 + 3 = 7$$

to obtain the element of the first row and the second column

$$\begin{pmatrix} 2 & 3 \end{pmatrix} \times \begin{bmatrix} 3 \\ 4 \end{bmatrix} = (2 \times 3) + (3 \times 4) \text{ given } 6 + 12 = 18$$

to obtain the element of the second row and the first column

$$(1 \ 4) \times \begin{bmatrix} 2 \\ 1 \end{bmatrix} = (1 \times 2) + (4 \times 1) \text{ given } 2 + 4 = 6$$

to obtain the element of the second row and the second column

$$(1 \ 4) \times \begin{bmatrix} 3 \\ 4 \end{bmatrix} = (1 \times 3) + (4 \times 4) \text{ given } 3 + 12 = 15$$

Therefore, the solution for $A^2 = \begin{bmatrix} 7 & 18 \\ 6 & 15 \end{bmatrix}$

4.1.4 Example of errors in the use of the determinant

The number of students who used wrong computational skills for each of the five items in the Pre-test and Post-test are presented in Tables 4.1 and 4.2.

From Table 4.1, pre-service teachers' most common errors committed were on sequence of multiplication and product of matrices. There were only a few errors committed by pre-service teachers' on getting rid of matrix and concept of determinant that is 12% and 7% respectively. Majority of the students had difficulty in the pre-test but did better in the post test. In the pre-test, the total errors were ninety (90) with sequence of multiplication and product of matrices having the highest number whilst getting rid of matrix and concept of determinant were on the minimum.

Table 4.1 Number of students who used wrong computational skills for each item in the Pre-test and Post-test

Test Items	of multiplication		Sequence		Getting rid of matrix		Product of matrices		Concept of determinant		Total	
	N	%	N	%	N	%	N	%	N	%	Total	% Total
Pre-test Item 1	15	50	8	27	5	17	2	7	30	26		
Post-test Item 1	1	3	0	0	2	7	0	0	3	3		
Pre-test Item 2	8	27	4	13	7	23	0	0	19	17		
Post-test Item 2	2	7	0	0	0	0	0	0	2	2		
Pre-test Item 3	7	23	0	0	8	27	1	3	16	14		
Post-test Item 3	2	7	1	3	2	7	0	0	5	4		
Pre-test Item 4	2	7	0	0	4	13	3	10	9	8		
Post-test Item 4	2	7	0	0	3	10	2	7	7	6		
Pre-test Item 5	12	40	0	0	4	13	0	0	16	14		
Post-test Item 5	2	7	1	3	5	17	0	0	8	7		
Total	53		14		40		8		115			

Table 4.2 Number of students who used wrong computational skills for each item in the pre-test and post-test

Test Items	Pre-test		Post-test	
	N	%	N	%

Item 1	30	100	3	10
Item 2	19	63	2	6.6
Item 3	16	53	5	16.7
Item 4	9	30	7	23.3
Item 5	16	53	8	26.7
Total	90		25	

Table 4.2 indicates pre-service teachers' total errors in both the pre-test and post-test. It can be seen from the table that pre-service teachers' made more errors in the pre-test than in the post-test. Table 4.2 again indicates in the pre-test that pre-service teachers' most common errors were on items 1, 3, 4 and 5 with errors on item 2 being on the minimum.

In the post-test, the total errors were on parenthesis and operational sequences were the highest number whilst those on approximation and concept application were the least. Table 4.2 again indicates in the post-test that pre-service teachers most common errors were on items 3, 4 and 5. From those results, it is clear that pre-service teachers' difficulties reduced immensely after they were taken through the intervention.

4.2 Effect of the technology on pre-service teachers' achievement in mathematics

The third research question raised for the study was to find out the effect of the ICT on pre-service teachers' achievement in mathematics. In order to accomplish

this, the pre-service teachers were allowed computers with the utility application program installed on them in the intervention as described in the previous chapter. The descriptive statistics of pre-service teachers' overall performance on the tests are presented in Tables 4.3

Table 4.3 Descriptive statistics on pre-service teachers' pre-test and post-test scores

	N	Mean	Std. Deviation	Minimum	Maximum
Pre-service teachers' pre-test	30	3.9	2.057	0	9
Pre-service teachers' post-test	30	6.4	2.673	1	13

Table 4.3 indicates that there was a difference in pre-test and post-test scores with respect to the minimum, maximum, mean and standard deviation with the latter better than the former (i.e. with pre-test mean and SD 3.90 and 2.05 and post-test mean and SD 6.40 and 2.67).

Further analysis was conducted to find out whether the difference in means was statistically significant. A paired-sample t-test was used to test the null hypothesis that there is no significant difference between pre-service teachers' pre-test and post-test. The t-test was used because the researcher had only one group of people and collected data from them under two different conditions or occasions. The

results of the t-test (29df, $t=6.335$, and $p=0.00$) also indicated that the difference in means was significant at $p=0.00$. Hence the null hypothesis that there is no significant difference between pre-service teachers' pre-test and post-test is rejected in favor of the alternative hypothesis. It can be argued that there was statistical significant difference between the post-test and pre-test.

Table 4.4: Show results of the t-test

	N	Means	Std Dev.	Df	t	Sig.
Pre-test	30	3.9	2.057	29	6.335	0.00
Post-test	30	6.4	2.673			

Pre-service teachers' performance in the pre-test was poor as majority of the pre-service teachers had below 5 marks. The post-test results also indicated that the pre-service teachers' performance was very good since most of them had above the mark of 6. Comparatively, the performance of pre-service teachers in the post-test was far higher than the performance of pre-service teachers in the pre-test.

4.3 Pre-service teachers' interest in using the technology in mathematics

Since the researcher was committed to finding out how best pre-service teachers would respond to the technology in terms of their interest, he made a call on the pre-service teachers to find out from them an interview was conducted for fifteen (15) pre-service teachers who took part in the study. The pre-service teachers

responses from the interview revealed that most teachers“ from the basic schools and the second cycle institutions normally refer to column as row and rows as columns. For instance, a teacher goes to a class wanting to punish the whole class. He then says “*hei!! this row stand up*” mean while She/he is referring to a column thereby confusing the students. Moreover, a small percentage explained that they were not introduced to ICT when they were in SSS. In all, the fifteen (15) pre-service teachers gave a positive responds to the introduction of Derive 5 in their mathematics classroom.

As for whether the usage of ICT (utility application program such as derive5) could improve pre-service teachers“ performance/computational skills, all pre-service teachers involved responded that the ICT could improve their performance/computational and further decided to implement the usage of technology when they go out to practice teaching fully.

Moreover, most of the pre-service teachers testified that the most beneficial and appropriate tool for studying mathematics in the classroom and for practicing during leisure hours was the Derive 5. In addition to that, the pre-service teachers also stated that their computational skills could improve dramatically when they are taught how to use the Derive 5 and others well in solving mathematical problems.

For instance pre-service teacher F said “*I think if mathematics software are [made] available for [us to] use, that can help us and also if we can have one computers in the our homes that we can use for our studies before we go for the*

classes... that will also help us and if the computers in the department can constantly be repaired... it will be in the interest of using ICT in learning and teaching Mathematics as pre-service teachers”.

When student M was asked whether the use of the software made understanding of the concepts easier, he said *“Of course! Some concepts that were not clear to me were made easier by the software. For example the step-by-step elimination method of multiplying two matrices and how easier derive generates the determinant if a matrix was good to me. There are a lot also”*.

Pre-service teacher V also explained that *“if our teachers will make some time and teach us how the derive5 can be used to the maximum, then (we) can use it in our houses and not have problems in using this effective tool in teaching our students in the near future”*.

Pre-service teacher K also stated that *I think it needs constant learning to master all aspect of it. If (we) do it constantly, it will bring maximum benefit to (us) students which will improve our performance in Mathematics”*.

The major question every mathematical educator asks“ him/ herself is “how do I generate and sustain my students interest in mathematics?” The answer is that since teaching with technology is basically a Project-based learning, it is a comprehensive approach to classroom teaching and learning that is designed to engage pre-service teachers in investigation of authentic problems.

Computer-based environments challenge instructors to design new tasks for mathematics education, because it turns out to be unreasonable to continue to emphasize mathematics instruction only on teaching skills (symbol manipulation), when a powerful supportive tool such as Derive5 (a mathematics assistant) can perform better and quicker. For the purpose of enhancing pre-service teachers' mathematical knowledge and communications skills, and develop their advanced mathematical thinking and interest the researcher developed a sequence of Derive 5 based assignments for pre-service mathematics teacher students. Derive 5 serves as a convenient dynamic tool for making complicated and tedious computation, releasing the user to devote himself to the more essential aspects of mathematics: raising conjectures, investigations, reflection and reasoning. The topic matrix was chosen because of its mathematical richness, and its applicability to the current computer world. For more insight see appendix E.

CHAPTER 5

SUMMARY, DISCUSSION, AND CONCLUSION

5.0 Overview

In this chapter, the conclusion of the whole research project is provided. It includes a summary of the study and highlights the discussion of the findings. It further outlines some of the limitations, recommendations and avenues for further research studies.

5.1 Summary

This study attempted to find out the extent to which the use of Derive 5 will help to improve pre-service teachers' ability to use the matrices approach in solving systems of linear equations. According to the Mathematics Syllabus for Diploma in Basic Education (2005; 2006), the Mathematics tutors at the colleges of education are required to use ICT in their teaching and also teach the teacher trainees how to explore ways in which ICT could be used to enhance teaching and learning of Mathematics at the basic levels. Therefore, this research particularly sought to find out the sequence of multiplication errors committed by the pre-service teachers' of the 2E class of Mampong Technical College of Education, the impact of the Derive 5 on pre-service teachers computational skills and the challenging problems that require the use of the product of matrices and the concept of determinant. In all, 30 pre-service teachers from MTCE 2E were involved in the study. The researcher adapted an observation, interview, pre-test, implemented an intervention and a post-test.

5.2 Findings

The results of the study revealed pre-service teachers errors in using some basic mathematical computational skills. Some of the errors identified were in the sequence of multiplication of matrices, getting rid of matrix rule, product of matrices and application of the concept of determinant. Pre-service teachers most common errors in the pre-test were on sequence of multiplication followed product of matrices, getting rid of the matrix and application of the concept of determinant respectively. However, pre-service teachers' difficulties reduced immensely after they were taken through the intervention and their post-test scripts were marked.

The results indicated that pre-service teachers' performance in the pre-test was very poor. The post-test results indicated that there was a vast improvement in pre-service teachers' performance. Comparatively, the performance of pre-service teachers in the post-test was far higher than the performance of pre-service teachers in the pre-test which indicated a significant difference between pre-service teachers' pre-test results and their post-test results.

The results of the descriptive statistics (29df, $t=6.335$, and $p=0.00$), indicated further that there was significant difference between pre-service teachers' pre-test and their post-test in their performance in favor of the post-test ($M=6.40$, $SD=2.673$) as against the pre-test ($M=3.90$, $SD=2.057$). This implied that the derive5 improved pre-service teachers' ability to use the matrices approach in solving systems of linear equations.

5.3 Discussions on Findings

From the study, pre-service teachers' most common errors in the pre-test were on sequence of multiplication, followed product of matrices, getting rid of the matrix and application of the concept of determinant respectively. However, students' difficulties reduced enormously after they were taken through the intervention, wrote the post-test and their scripts were marked. This indicated that the proportion of students who improved their ability to use the matrix approach in the post-test was far higher than in the pre-test. The analysis on the impact of the Derive 5 on students computational skills indicated that pre-service teachers' performance improved dramatically after the post-test was marked. Pre-service teachers' general performance indicated that their performance in the pre-test was poor. The post-test results also indicated that the pre-service teachers' performance was very good. Comparatively, the performance of pre-service teachers' in the post-test was far higher than the performance of pre-service teachers' in the pre-test which brought about a massive improvement in the pre-test.

Furthermore, the results of the descriptive statistics indicated that there was significant difference between pre-service teachers' pre-test ($M=3.90$, $SD=2.057$) results and their post-test results ($M=6.40$, $SD=2.673$) in their performance in favor of the post-test results. Recent research shows that new digital technologies have the potential to revolutionize the quality of subject teaching and learning when carefully integrated into the classroom. The role of the teacher is utterly critical here. Yet a primary barrier to teachers' readiness and confidence in using

ICT, despite general enthusiasm and belief in benefits for learners, is their lack of relevant preparation, either initially or in-service. Research indicates that, until recently, training opportunities have remained limited in availability and inconsistent in quality. This has resulted in demonstrably low proficiency in using ICT, and a general lack of knowledge about technology in teaching and learning. There are some recent examples of successful practice in developing ICT use in Sub-Saharan Africa schools through its integration in teacher education. However, according to Unwin (2005), provision has often been characterized by well intentioned, but misplaced, supply-driven initiative across the continent to provide teachers and students with ICT skills. These have proved wasteful and inappropriate with limited impact. Moreover, the recent global economic downturn has amplified the shortage of public funds to devote to the already expensive business of training teachers to use ICT. Increasingly, large school classes and the designation of ICT as a discrete subject, lead to a dire lack of subject teachers trained to integrate technology into learning in their areas. These are fundamental challenges to be overcome before ICT capacity building can become a reality in African education.

5.4 Limitations of the study

A major limitation of this study is the consequence of using non-probability sample. Since this study was not based on a large population, it might have limitations in terms of generalizability of the practicality of the intervention. For instance, one may asked, can the study be applicable in similar situations?

However, the issue of difficulty in generalizing findings from a small sample study partially comes from conceiving generalizability in the same way as do investigators using experimental or correlational designs (Merriam, 1998).

Regarding this respect, Yin (2003) argues:

Small sample studies, like experiments, are generalizable to theoretical propositions and not to populations or universe. In this sense, the small sample study, like experiment, does not represent a “sample”, and in doing a small sample study, your goal will be to expand and generalize theories (analytic generalization) and not to enumerate frequencies (statistical generalization) (p. 10).

In Yin’s (2003) opinion, the above is helpful in the sense that what the small sample study seeks is not to generalize the study results from sample to a large population. In an effort to resolve the issue of generalization, the study attempts to provide a rich and thick description of events that emanated from the study.

This research could not seek to establish the effectiveness of the intervention. This was due to the limited time for the study. Effectiveness of an intervention in developmental research answers the question, what are students’ achievements after the intervention is used in the classroom? Effectiveness is usually measured in a quasi-experimental design using much larger groups of students. In terms of curriculum, effectiveness of an intervention is related to attained curriculum level.

5.5 Conclusion and Recommendations

The evidence indicates that students often adopt surface approaches to fulfil work commitments without the intention of aspiring to understand and extend prior knowledge. ICT has simply made the task easier and more efficient, and allowed the final product to be presented at a higher standard. Whilst the use and familiarization of basic technology skills can free the student to apply higher cognitive skills, the voice from students highlights an overemphasis on surface learning approaches.

The findings from this research show that:

1. The inclusion of ICT into the learning environment has created a greater focus on the collaboration and interaction between and among many students—aspects of student-centered environments. The support and guidance from the teacher remains important. The majority of students valued learning with ICT when it was relevant, gave them ownership, control and autonomy, and was conducted in environments that supported a climate of collegiality.
2. Intrinsically motivated pre-service teachers“ were more goal-oriented toward achieving and pursuing new learning with more effort and persistence. Student activity around and in a computer environment can mask the true learning approach of the individual student. This means that the „busyness“ of a student will not automatically equate to active or deep learning. Meaningful learning was often associated with project-based learning where students were encouraged to integrate content through inquiry-based tasks.

3. When the learning tasks provide little opportunity for students to explore their thinking and understanding, most students were generally passive learners. While the use of ICT has been regarded as a tool to transform thinking and learning, it can actually promote surface learning approaches. Many students often described moments where the sole intent was to reproduce or regurgitate information. The use of higher cognitive thinking was evident in tasks when ICT was purposefully integrated with subject content.

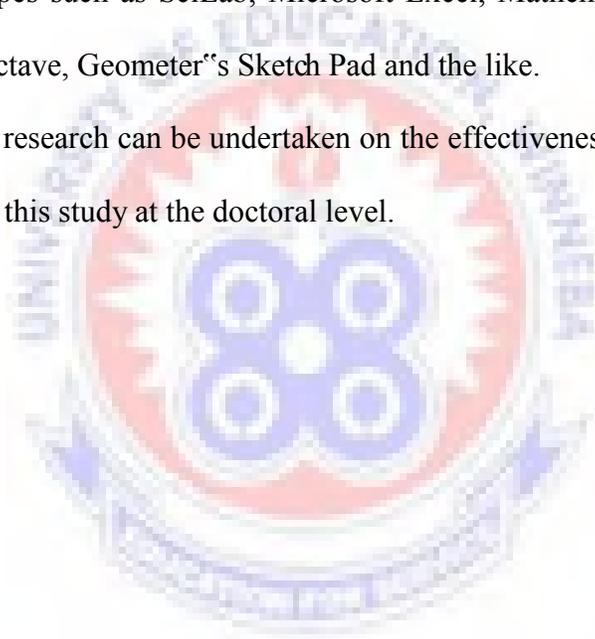
The aim of this research was to provide insights into classroom learning supported by ICT. In implementing any strategy to improve the quality of learning there is arguably considerable benefit in studying the effect on students, and looking at the experience through their eyes. This research has concentrated on two of Hargreaves (2004) gateways to personalising learning: student voice and the use of new technologies (ICT). Therefore the following recommendations are made:

1. Government budget should support ICT programmes especially ICT training programmes for Mathematics tutors at the colleges of education in Ghana so that the ICT policy on education as stipulated in the Ghana government's ICT for Accelerated Development (Republic of Ghana, 2003) policy document can be fulfilled.
2. Principals of the colleges of education in Ghana should make provisions in their budgets on technology training of Mathematics tutors.

3. To be abreast with new software that can be used in instructing Mathematics, ICT workshops should be organized periodically for the Mathematics tutors.

5.5.1 Areas for Further Research

1. It was so unfortunate that this research concentrated on the use of just one software (i.e. Derive 5 software). A study can be undertaken on other types such as SciLab, Microsoft Excel, Mathematica, Maple, MATLAB, Octave, Geometer's Sketch Pad and the like.
2. A research can be undertaken on the effectiveness of the intervention used in this study at the doctoral level.



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APPENDICES

APPENDIX A

Checklist for Determining Errors

See detailed examples in appendix E

Error indicators	Yes	No	Comments
Error in the use of sequence of multiplication ex.			
Error in getting rid of matrix			
Error in the product of matrix			
Error in the application of concept of determinant			



APPENDIX B

INTERVIEW GUIDE

Questions	Tick		Comments
Do you ever intend to use Derive 5 again in your studies?	Yes	No	
Was Derive 5 friendly in your interactions with working environment?	Yes	No	
Has the introduction of Derive 5 helped you in your understanding and interest in matrices?	Yes	No	
How do you rate your competence with the use of Derive 5?	Good	Very Good	
Will the use of Derive 5 in solving problems in mathematics improve your ability to change systems of linear equations into matrix form?	Yes	No	

APPENDIX C

PRE-TEST QUESTIONS

1. Find the product AB if

$$A = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

2. Evaluate $4V - \frac{1}{2}P$, given that $V = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$ and $P = \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix}$

3. Given that $Q = \begin{bmatrix} 3 & 9 \\ 1 & 2 \end{bmatrix}$. Find $\det Q$

4. If $M = \begin{bmatrix} 4 & 2 \\ x & 3 \end{bmatrix}$ and $N = \begin{bmatrix} 6 & 2 \\ 4 & y \end{bmatrix}$. Find the values of x and y, given that M and N are commutative under matrix multiplication.

5. Using the matrix method solve these equation simultaneously.

$$-x + 5y = 4 \text{ and } 2x + 5y = -2$$

APPENDIX D

MARKING SCHEME FOR PRE-TEST

1. $A = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ then

$$AB = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2+2 & 6-5 \\ -3-8 & -9+20 \end{bmatrix} =$$

$$\begin{bmatrix} 4 & 1 \\ -11 & 11 \end{bmatrix} \dots \mathbf{MIAI}$$

2. $4V - \frac{1}{2}P = 4 \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 12 & 8 \\ 16 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 15 & 4 \end{bmatrix} \dots \mathbf{MIAI}$$

3. $Q = \begin{bmatrix} 3 & 9 \\ 1 & 2 \end{bmatrix}$ $\det Q = ((2 \times 3) - (9 \times 1)) = 6 - 9 = -3 \dots \mathbf{MIAI}$

4. $MN = \begin{bmatrix} 4 & 2 \\ x & 3 \end{bmatrix} \begin{bmatrix} 6 & 2 \\ 4 & y \end{bmatrix} =$

$$\begin{pmatrix} 24 + 8 & 8 + 2y \\ 6x + 12 & 2x + 3y \end{pmatrix} \dots \mathbf{MI}$$

$$NM = \begin{bmatrix} 6 & 2 \\ 4 & y \end{bmatrix} \begin{bmatrix} 4 & 2 \\ x & 3 \end{bmatrix} =$$

$$\begin{pmatrix} 24 + 2x & 18 \\ 16 + xy & 8 + 3y \end{pmatrix} \dots \mathbf{MI}$$

With the given condition, $MN = NM$

$$\Rightarrow \begin{pmatrix} 32 & 8 + 2y \\ 6x + 12 & 2x + 3y \end{pmatrix} = \begin{pmatrix} 24 + 2x & 18 \\ 16 + xy & 8 + 3y \end{pmatrix} \dots \mathbf{MI}$$

$$\begin{aligned} \therefore 24 + 2x &= 32 & 8 + 2y &= 18 \\ 2x &= 32 - 24 & 2y &= 18 - 8 \\ 2x &= 8 & 2y &= 10 \\ x &= 4 \dots\dots\dots A1/2 & y &= 5 \dots\dots\dots A1/2 \end{aligned}$$

5. $-x + 5y = 4$ and $2x + 5y = -2$

The matrix equation is given by $\begin{bmatrix} -1 & 5 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix} \dots\dots\dots B1$

Inverse of $\begin{bmatrix} -1 & 5 \\ 2 & 5 \end{bmatrix} = \frac{1}{-15} \begin{bmatrix} 5 & -5 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{15} & \frac{1}{15} \end{bmatrix} \dots\dots M1$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{15} & \frac{1}{15} \end{bmatrix} \begin{bmatrix} 4 \\ -2 \end{bmatrix} \dots\dots\dots M1$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{4}{3} + -\frac{2}{3} \\ \frac{8}{15} + -\frac{2}{15} \end{bmatrix} = \begin{bmatrix} -2 \\ \frac{2}{5} \end{bmatrix} \dots\dots\dots M1$$

$$\Rightarrow x = -2 \text{ and } y = \frac{2}{5} \dots\dots\dots A1/2$$

A1/2

APPENDIX E

INTERVENTION PROCESS

The researcher designed an intervention in the form of lessons delivered and discussed in the class, and the ICT laboratory. This was done using Derive 5 as the major teaching and learning aid. The same software was used for practicing and exercises by the students with the researcher as the facilitator. The sample population for the study was 30 and they were divided into two groups with 15 students in a group. The time allocated for each group was one hour a day for five (5) days. However, students were given thirty (30) minutes a day for three (3) days for each group to use the software for practice and assignments. Some students also practiced on their own at their own convenience.

Pre-intervention

During the researcher's first two weeks stay at Mampong Technical College of Education, he observed that students had problem with solving systems of linear equations by using the matrices approach. To the researcher the problem was the fact that students were not able to recognise 2×2 matrices as a system of linear equations. Further more, when the March 2010 chief examiner's report from the institute of education, university of Cape coast came, much emphasis had been made on the fact that most students couldn't solve problem involving matrices.

The researcher quickly organised the questions (pre-test) for the thirty students from 2E to find out whether his students were part of the problem reported. See appendix C for questions.

The researcher observed that students had performed abysmally. Table 4.7 shows the performance of students see (Appendix H). And below is how question items such as five (5) was answered by most students. Solve the system using the matrix approach.

Solve the system using the matrix approach

$$-x + 5y = 4$$

$$2x + 5y = -2$$

In matrix form the above equation is written as

$$\begin{bmatrix} -1 & 5 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}, \text{ by letting } M = \begin{bmatrix} -1 & 5 \\ 2 & 5 \end{bmatrix}, N = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } P = \begin{bmatrix} 4 \\ -2 \end{bmatrix} \text{ the}$$

students then obtained the equation $MN = P$. But this time the students were to solve for N , hence the need to get rid of M by multiplying both sides of the equation by M^{-1} (*from the left*). All I realised was that the students were rather multiplying (*from the right*) giving them the equations $MM^{-1}N = PM^{-1}$ instead of the equation $M^{-1}MN = M^{-1}P$.

Therefore by looking at the seventh test item I found the above misconceptions and realised that, most of the pre-service teacher did not realise the need to write the system in matrix form. The very few that were able to write the system in a

matrix form forgot *getting rid of the matrix rule*. That is suppose you are given a matrix equation $AB = C$, how can you write B in terms of other matrices? In other words, how can you get rid of A from the left-hand side?

The answer is to multiply both sides of the equation by A^{-1} (from the left).

$$\therefore A^{-1}AB = A^{-1}C$$

$$\therefore IB = A^{-1}C \text{ (where } I \text{ is the multiplicative identity matrix)}$$

$$\therefore B = A^{-1}C$$

Now, if you are given $AB = C$, how do you write A in terms of other matrices?

This time you must multiply both sides (*from the right*) by B^{-1} , giving

$$ABB^{-1} = CB^{-1}$$

$$\therefore AI = CB^{-1}$$

$$\therefore A = CB^{-1} \text{ Done}$$

Moreover, the question item three (4) also exposed a lot of pre-service teachers weakness when it came to product of matrices. Students were to answer this question: if $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, then find A^2 . Some of these students gave this answer.

$$A^2 = \begin{bmatrix} 2^2 & 3^2 \\ 1^2 & 4^2 \end{bmatrix}. \text{ When these simple mathematical errors were recognised, the}$$

researcher then thought of something that will boost the students interest in solving more question, that will give them hands-on experience and make them learners of their own situation rather than somebody serving as a bank of knowledge moreover, something that will improve their ability to solving linear equation using the matrix approach. Hence, the need for Derive 5 as a teaching aid in the intervention to remedy the situation.

Week one

The intervention started by taking students to the ICT laboratory where definition and explanations were given on the first day on the following terms.

Matrix

A matrix is a rectangular array of numbers in bracket. Its dimensions are determined by number of rows and columns. That is the number of rows followed by the number of columns. Rows are horizontal; columns are vertical as known from sitting arrangement at the Greek theatre. Many a time, teachers make mistake by referring to columns as rows in the classroom when a task is supposed to be given to a particular column in a class. Moreover matrices are seen with square bracket such as $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$, with or the round brackets $(x \ y)$, or with roundy-square brackets such as this. $\begin{pmatrix} -2 & 3 \\ 1 & 3 \end{pmatrix}$. In this research, the researcher used square brackets for the study.

Derive 5

Derive 5 is a powerful system for doing symbolic and numeric mathematics on your personal computer. It processes algebraic variables, expressions, equations, functions, vectors, matrices and Boolean expressions like a scientific calculator processes numbers.

Problems in the fields of arithmetic, algebra, trigonometry, calculus, linear algebra, and propositional calculus can be solved with the click of the mouse.

Derive 5 make plots of mathematical expressions in two and three dimensions

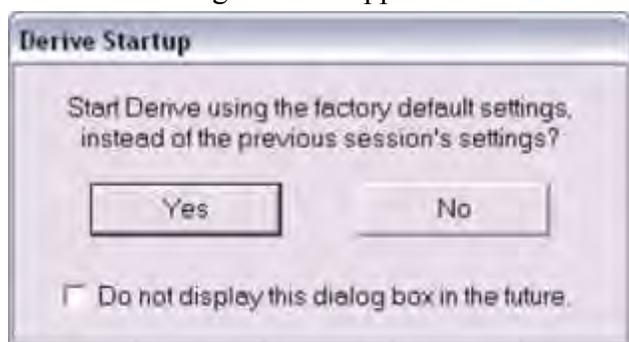
using various coordinate systems. By its seamless integration of numeric, algebraic and graphic capabilities, Derive makes an excellent tool for learning, teaching and doing mathematics.

How to launch Derive 5

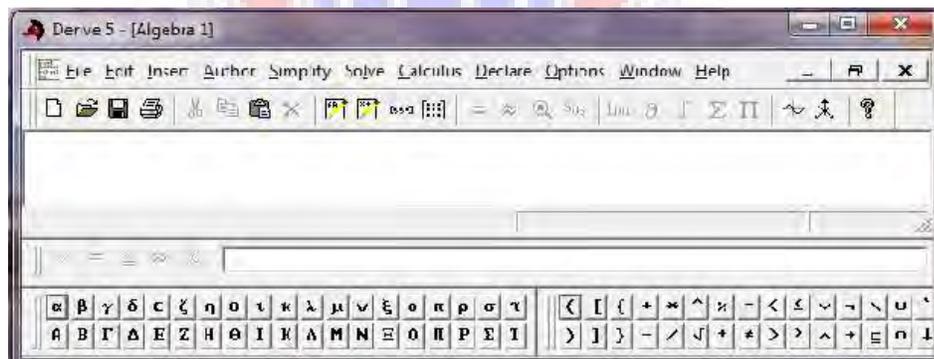
Click on the Derive 5 icon on the desktop

OR click on Start \Rightarrow All Programs \Rightarrow Derive 5 \Rightarrow Derive 5.

After that a dialog box will appear as below:



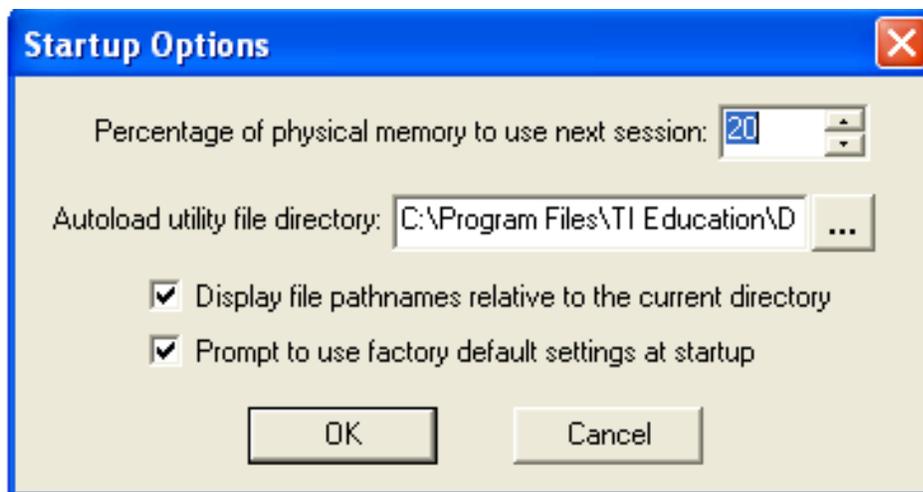
Click on Yes and the Derive 5 screen will appear after a few seconds as:



Clicking on yes means you want Derive 5 to launch with factory default settings.

If you want to change how Derive 5 will be launched without any option:

Click on Option from the menu bar \Rightarrow Startup... and a dialog box will appear as:



Unchecked Prompt to use factory default settings at startup and click on OK.-

The Derive 5 screen comprises (from top to bottom): The title bar:



The menu bar:



The command tool bar:



- A (currently empty) Algebra Window, also called the view
- The status bar: This is below the view
- The expression entry tool bar, also called the entry line: This is below the

status bar and it is on the left



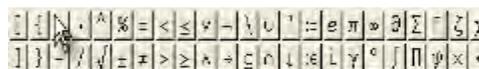
and

on the right is a white place where input is made.

- The Greek symbol tool bar:



- The Math symbol tool bar:



At a certain time in the course of the researcher's introduction of Derive 5 to the students, he cautioned them to note the following icons since each has a meaning and function in their learning process.

On the Algebra window

Or Solve \Rightarrow Expression:  Use to solve an equation

File \Rightarrow Save As: Save worksheet using a name

Edit \Rightarrow Expression or double-click left or right of expression: i.e. to edit highlighted expression

On the Algebra window, you can also left justify, right justify etc.

Expression entry tool bar

: Or enter key (\emptyset): Means enter expression



: Enter simplified expression



Or (Ctrl) + (\emptyset): Enter expression then simplify



Enter approximated expression



 Clear entry line

Try to explore the other items within Derive 5 for discussion.

To look for help on topics within mathematics, click on  from the menu bar and then click on additional Resources. The facilitator after taking the students through the Derive 5 environment then asked the students to explore on their own and prepare for the next lesson the next day.

Week two

On the second day, the researcher took the students to the ICT laboratory again with intention of assisting students to recognize matrices as systems of linear

equations. He therefore served as facilitator by helping the students analyze the matrices $\begin{bmatrix} 3 & x \\ -1 & y \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$ for possible linear equations using Derive 5. The illustrations are as followed.

1. open to Derive 5 environment
2. click on author matrix
3. matrix setup appears
4. select the appropriate dimensions
5. click OK

By going through the step by step instructions and choosing 2x2 as the dimension in the derive environment, the author 2x2 matrix setup appears. Students were then asked to enter the matrix $\begin{bmatrix} 3 & x \\ -1 & y \end{bmatrix}$ with the researcher facilitating. Again, the researcher asked the students to use the same process to enter matrix $\begin{bmatrix} 7 \\ 0 \end{bmatrix}$. The researcher then asked the students to equate the first matrix to the second matrix as seen $\begin{bmatrix} 3 & x \\ -1 & y \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$ like #1 = #2 in the Derive environment and make an equation out of the first row and another equation out of the second row. Finally, the students obtained these equations. $3 + x = 7$ from the first row and $-1 + y = 0$ from the second row. At this stage, the researcher asked the students to click on solve on the menu bar, then system and type the two linear equations they have generated from the matrix in the expression boxes and press enter. When this was done, students realized that matrices in that form are made up of systems of linear equation.

#1: $\begin{bmatrix} 3 & x \\ -1 & y \end{bmatrix}$

#2: $\begin{bmatrix} 7 \\ 0 \end{bmatrix}$

By typing #1=#2 Derive provides the equation of the two matrices

#3: $\begin{bmatrix} 3 & x \\ -1 & y \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$

BELOW IS HOW Derive has generated the two system of equations out of the matrices

#4: `SOLVE([3 + x = 7, -1 + y = 0], [x, y])`

#5:

`[x = 4 ^ y = 1]`

The Derive 5 work environment above shows how students were able to recognize matrices as systems of linear equations based on #4. The pre-service teachers came out that the system of equations below:

$$-x + 5y = 4 \text{ and } 2x + 5y = -2$$

can be written as: $\begin{bmatrix} -3 & 1 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$

I therefore explained to them that matrices are ideal for computer-driven solutions of problems because computers easily form *arrays*. We can leave out the algebraic symbols. A computer only requires the first and last matrices to solve the system. The researcher gave students time to repeat the process and work in their group for better performance and ended the second day.

Week three

The symbolical language aspect of system of linear equations poses difficulties to many students in many countries. According to Bennet and Nelson (1998), the concept of variables is often misunderstood by students in England. However, this problem is not different in Ghana if not worse. Therefore, on the third day the researcher saw the need to assist the students to recognize variables as quantities?

I therefore, took the students to the school laboratory and asked them to once again to use the same matrix $\begin{bmatrix} 3 & x \\ -1 & y \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$ to find out whether the variables y and x are quantities or not. Hence the researcher asked them to follow the following once again.

1. open to Derive 5 environment
2. click on author matrix
3. matrix setup appears
4. select the appropriate dimensions
5. click OK

when students have finished the process of obtaining the matrix on the left side, the researcher then asked them to repeat the same process for the matrix on the right side and The researcher then asked the students to equate the first matrix to the second matrix as seen $\begin{bmatrix} 3 & x \\ -1 & y \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$ like #1 = #2 in the Derive environment and solve for the values of variables x and y . The process on the Derive environment is as showed below as described during the second day.

```

#1:  $\begin{bmatrix} 3 & x \\ -1 & y \end{bmatrix}$ 
#2:  $\begin{bmatrix} 7 \\ 0 \end{bmatrix}$ 
By typing #1=#2 Derive provides the equation of the two matrices
#3:  $\begin{bmatrix} 3 & x \\ -1 & y \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$ 
Below is how Derive has generated the two system of equations out of the matrices
#4: SOLVE([3 + x = 7, -1 + y = 0], [x, y])
From #5 below it can be said that variables  $x$  and  $y$ 
are representative of quantities 4 and 1 respectively
#5:  $\begin{bmatrix} x = 4 & y = 1 \end{bmatrix}$ 
    
```

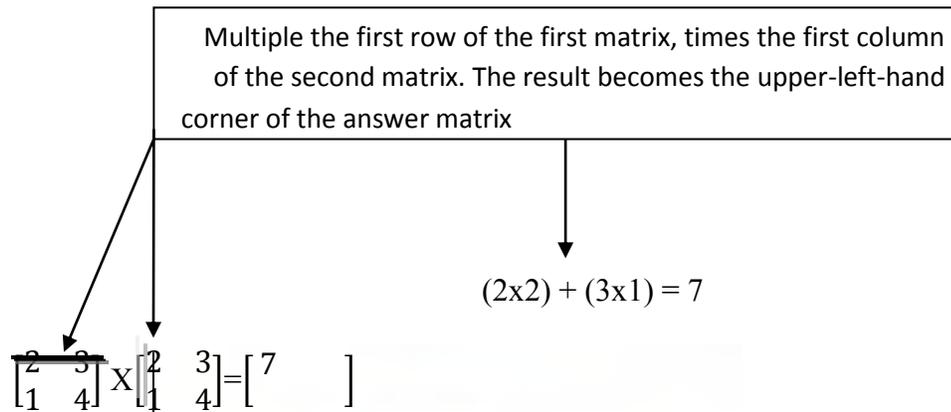
From #5 on the Derive environment, students were able to identify the fact that $x = 4$ and $y = 1$ as showed above. The researcher therefore, asked the students to continue practicing on their own and prepare for the next day's lesson.

Week four

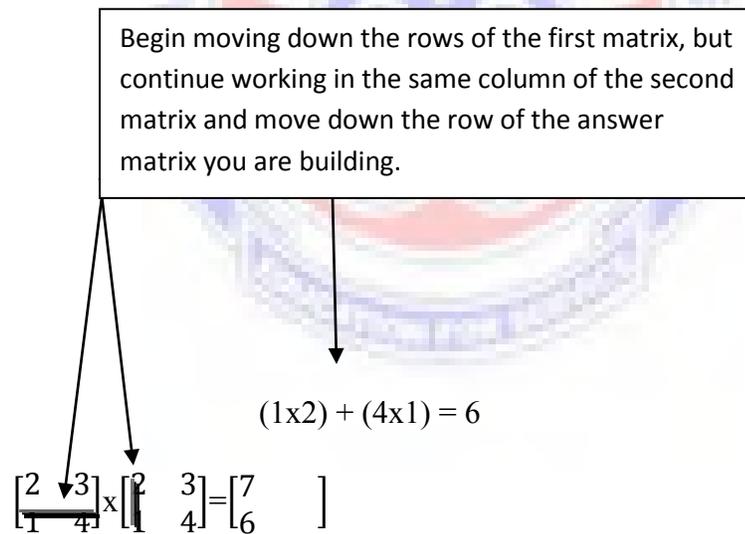
On this day, I decided to assist the students to find the general algorithm for multiplying matrices because I realized that the pre-service teachers needed the core or basic concepts first before they are able to use the matrices approach. This was done on the marker board at the computer laboratory after which I guided them to discover whether the same thing exists in the Derive 5 environment. The researcher used this approach because derive does not give the step by step procedure in the course of its solution. And moreover when the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ was given to students in their pre-test to find A^2 . Most of the students responds

were that $A^2 = \begin{bmatrix} 2^2 & 3^2 \\ 1^2 & 4^2 \end{bmatrix}$

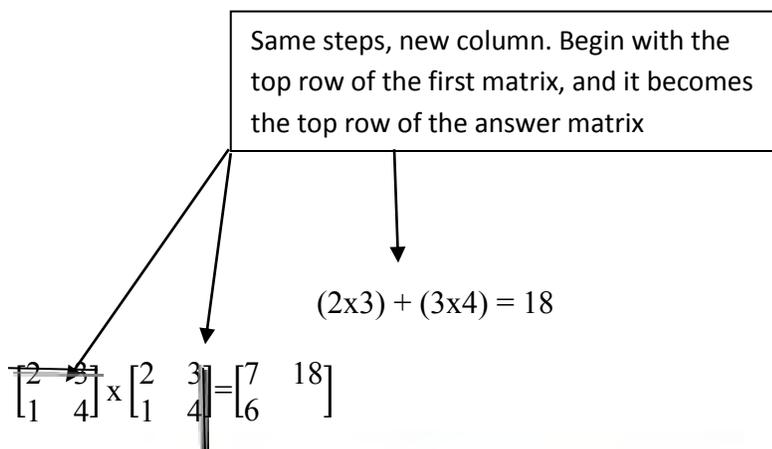
I therefore explained to the students that the key to such a problem is to think of the matrix as $A \times A$ and the first matrix as a list of rows (in this case, 2 rows), and the second matrix as a list of columns (in this case, 2 columns). I then asked the students to multiply each row in the first matrix, by each column in the second matrix. Starting from the beginning: first row, times first column.



Now, we move down to the next row. As we do so, we again move down in the answer matrix as well. The process continues using the **2nd** row of the first matrix and the **1st** column of the second matrix. The result is placed in position a_{21}

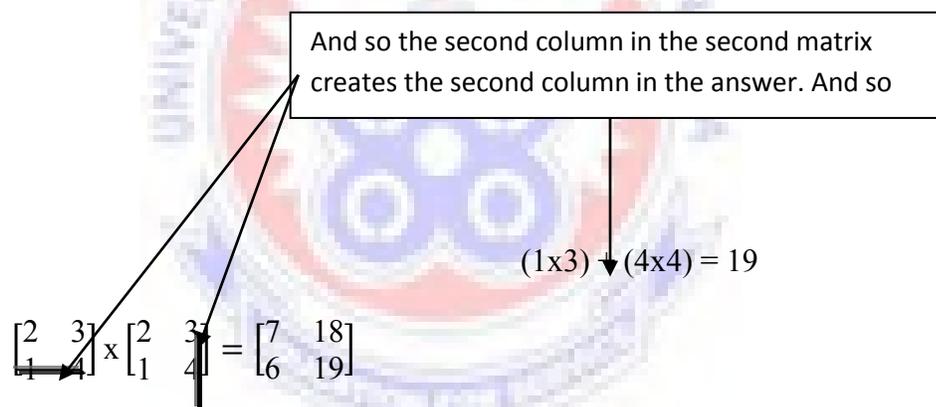


We now move on to the second column of the second matrix and repeat the entire process starting with the first row.



Finally, by working with the second row of the first matrix and the second column of the second matrix, we obtained the solution to the product of the two matrices.

Now taking a look at the whole situation, we have



We're done. We can summarize the results of this entire operation as seen above.

It's a strange and ugly process but everything we did in the rest of the fourth day builds on this, so it's vital to be comfortable with this process. The only way to become comfortable with this process is to do it. A lot of practice makes one perfect. Multiply a lot of matrices until you are confident in the steps. Note that we could add more rows to the first matrix, and that would add more rows to the answer. We could add more columns to the second matrix, and that would add more columns to the answer. In an attempt to make sure students were conversant

with the multiplication of matrices, the researcher introduced the students to Derive 5 once again to do more multiplication of matrices. Hence, the researcher asked the students to find the product of the matrices

$A = \begin{bmatrix} 1 & 5 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ using Derive 5 and guided them as followed.

1. open to Derive 5 environment
2. click on author matrix
3. matrix setup appears
4. select the appropriate dimensions that is two rows and two column
5. enter matrix A
6. enter matrix B
7. now type #1.#2 in the expression bar and press enter
8. click simplify

Immediately the simplify icon was clicked, the product of the two matrices was showed in the Derive environment as shown below #4. The researcher then asked the students to continue the same process but this time find whether #1. #2 is equal to #2. #1 that to find out whether the product of the matrices $AB = BA$.

LET THE FIRST MATRIX (A) = #1, AND THE SECOND MATRIX (B) #2

#1: $\begin{bmatrix} 1 & 5 \\ -1 & 0 \end{bmatrix}$

#2: $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

THEREFORE, AXB IS #1.#2 AS SEEN BELOW

#3: $\begin{bmatrix} 1 & 5 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

HENCE AB = #4

#4: $\begin{bmatrix} 7 & 23 \\ -2 & -3 \end{bmatrix}$

AGAIN, BXA IS #2.#1 AS SEEN BELOW

#5: $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 5 \\ -1 & 0 \end{bmatrix}$

HERE ALSO, BA = #6

#6: $\begin{bmatrix} -1 & 10 \\ -3 & 5 \end{bmatrix}$

FINALLY FROM THE ABOVE, AB IS NOT EQUAL TO BA

When this construction was completed, I assisted the students to realize the fact that multiplication of matrices is not commutative. Hence I made a summary of the days' activity by telling students that for a possible multiplication of two matrices the number of columns in the first matrix, and the number of rows in the second matrix, must be equal. Otherwise, you cannot perform the multiplication. Again matrix multiplication is not commutative which a fancy way of saying, order matters. If you reverse the order of a matrix multiplication, you may get a different answer.

Week five

Since the researcher wished to improve the pre-service teachers' ability to solving systems of simultaneous linear equations using the matrices approach, he guided the students through the instructions below and later assisted his instructions with Derive 5, in the view of generating students' interest towards solving more questions using Derive 5.

Suppose the system: $a_1x + b_1y = c_1$
 $a_2x + b_2y = c_2$

If we let $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$, $X = [y]$ and $C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

Then $AX = C$ (we have already discussed multiplication)

If we now multiply each side of $AX = C$ on the left by A^{-1} , we have

$$A^{-1}AX = A^{-1}C$$

However, we know that $A^{-1}A = I$, the identity matrix

So we obtain $IX = A^{-1}C$ but $IX = X$, so the solution is give by

$$X = A^{-1}C.$$

When the researcher was done with the above explanation, he then introduced the students again to Derive 5 and asked the students to pay attention as he assists them to solve the question (7) of the pre-test which laid much emphasis on the matrix approach.

Solution on the Derive environment

Let #1= A, #2=X and #3=C

$$\#1: \begin{bmatrix} -1 & 5 \\ 2 & 5 \end{bmatrix}$$

$$\#2: \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\#3: \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

To solve the systems, we need the inverse of A.
 Hence, we swap the leading diagonal in A and get #4

$$\#4: \begin{bmatrix} 5 & 5 \\ 2 & -1 \end{bmatrix}$$

we then change signs of the other two (2) elements and obtain #5, which knowas the ADJOINT

$$\#5: \begin{bmatrix} 5 & -5 \\ -2 & -1 \end{bmatrix}$$

We now find the determinant of A: |A|=5-10=-5

So (A) inverse is 1/determinant all times the (adjoint) as seen below

$$\#6: \frac{1}{-15} \begin{bmatrix} 5 & -5 \\ -2 & -1 \end{bmatrix}$$

Hence, (A) is \neq

#7:

$$\begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{15} & \frac{1}{15} \end{bmatrix}$$

So the solution to the systems given by

$$\#8: \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{15} & \frac{1}{15} \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

By matrix multiplication we obtained #9

#9:

$$\begin{bmatrix} -2 \\ 2 \\ 5 \end{bmatrix}$$

This means that we have the solution $x=-2$ and $v=2/5$

After the lesson with the Derive5 came to an end, the researcher observed that the students were happy and wanted to spend the rest of the day solving more question. In fact, the researcher was marveled at the eagerness and enthusiasm the students showed toward this intervention. I realized that the students were happy because of the hands on experience as they were guided to be learners of their own situation. The researcher then told the students to continue practicing and also not forgetting the tutorials he has also given them as they prepare for the post intervention.

Post intervention

In other to evaluate the outcome of the action taken, the researcher on the day of intervention decided to organize a post-test and an interview in other to ascertain whether the pre-service teachers in the 2E classes of Mampong Technical College of Education had improve upon their understanding of matrices. See appendix F for post-test. From the result of the post-test it was observed that the students have

actually embraced the use of Derive 5 in their understanding of the basic aspects of matrices. The results refer to appendix H attests the fact that, the students have improved upon their ability to solving system of linear equations using the matrix method.



APPENDIX F

POST TEST QUESTIONS

1. Find the product BA if

$$A = \begin{bmatrix} - & - \\ & \end{bmatrix} \text{ and } B = \begin{bmatrix} - & \\ & \end{bmatrix}$$

2. Evaluate $V - P$, given that $V = \begin{bmatrix} & \\ & \end{bmatrix}$ and $P = \begin{bmatrix} & \\ & \end{bmatrix}$

3. Given that $Z = \begin{bmatrix} & \\ & \end{bmatrix}$. Find $\det Z$

4. If $M = \begin{bmatrix} & \\ & x \end{bmatrix}$ and $N = \begin{bmatrix} & \\ y & \end{bmatrix}$. Find the values of x and y, given that M and N are commutative under matrix multiplication.

5. Using the matrix method solve these equation simultaneously.

$$x + y = \quad \text{and} \quad -x + y = -$$

APPENDIX G

MARKING SCHEME FOR POST-TEST

1. $A = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ then

$$BA = \begin{bmatrix} - & - \\ - & - \end{bmatrix} \begin{bmatrix} - & + \\ + & - \end{bmatrix} = \begin{bmatrix} - & - & - & + \\ - & - & + & - \end{bmatrix} = \begin{bmatrix} - & - \\ - & - \end{bmatrix} \dots MIAI$$

2. $x - \frac{1}{2}y = 4 \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$
 $= \begin{bmatrix} 8 & 8 \\ 4 & 12 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 3 & 10 \end{bmatrix} \dots MIAI$

3. $Z = \begin{bmatrix} 9 & 3 \\ 1 & 2 \end{bmatrix}$ $\det Z = ((9 \times 2) - (3 \times 1)) = 27 - 3 = 24 \dots MIAI$

4. $MN = \begin{bmatrix} 4 & 2 \\ 4 & x \end{bmatrix} \begin{bmatrix} 6 & 2 \\ y & 5 \end{bmatrix} = \begin{pmatrix} 24 + 2y & 8 + 10 \\ 24 + xy & 8 + 5x \end{pmatrix} \dots MI$

$$NM = \begin{bmatrix} y & 5 \\ 4 & x \end{bmatrix} \begin{bmatrix} 6 & 2 \\ 4 & x \end{bmatrix} = \begin{pmatrix} y + 30 & y + 5x \\ 24 + 4y & 4x + 5x \end{pmatrix} \dots MI$$

With the given condition, $MN = NM$

$$\Rightarrow \begin{pmatrix} y + 30 & y + 5x \\ 24 + 4y & 4x + 5x \end{pmatrix} = \begin{pmatrix} 24 + 2y & 8 + 10 \\ 24 + xy & 8 + 5x \end{pmatrix} \dots MI$$

$$\begin{aligned} \therefore 24 + 2y &= 32 & 12 + 2x &= 18 \\ 2y &= 32 - 24 & 2x &= 18 - 12 \\ y &= 8 & 2x &= 6 \\ \Rightarrow y &= & x &= 3 \dots \mathbf{A1/2} \quad \mathbf{A1/2} \end{aligned}$$

5. $x + 2y = 10$ and $-4x + y = -6$

The matrix equation is given by

$$\begin{bmatrix} 1 & 2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ -6 \end{bmatrix} \dots \mathbf{B1}$$

Inverse of $\begin{bmatrix} 1 & 2 \\ -4 & 1 \end{bmatrix} = - \begin{bmatrix} 1 & - \\ - & 1 \end{bmatrix} = \begin{bmatrix} 1 & - \\ - & 1 \end{bmatrix} \dots \mathbf{M1}$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & - \\ - & 1 \end{bmatrix} \begin{bmatrix} 10 \\ -6 \end{bmatrix} \dots \mathbf{M1}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & + \\ - & - \end{bmatrix} \begin{bmatrix} 10 \\ -6 \end{bmatrix} \dots \mathbf{M1}$$

$$\Rightarrow x = \text{and } y = \dots \mathbf{A1/2} \quad \mathbf{A1/2}$$

APPENDIX H

PRE-TEST RESULTS

Table 4.5 Presentation of pre-test results

MARKS	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
NO. OF STUDENTS	2	3	4	5	5	6	2	1	1	1	0	0	0	0	0	0

POST-TEST RESULTS

Table 4.6 Presentation of post-test results

MARKS	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
NO. OF STUDENTS		1	3	2	2	2	2	6	4	4	2	1	0	1	0	0